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MANAGEMENT AND POLICY

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FINANCIAL MANAGEMENT AND POLICY, 2nd EDITION
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To Mimi, Drew, Stuart, and Stephen

Preface

Though significant portions of *Financial Management and Policy* have been changed in this revision, its purpose remains: first, to develop an understanding of financial theory in an organized manner so that the reader may evaluate the firm's investment, financing, and dividend decisions in keeping with an objective of maximizing shareholder wealth; second, to become familiar with the application of analytical techniques to a number of areas of financial decision-making; and third, to expose the reader to the institutional material necessary to give him a feel for the environment in which financial decisions are made.

In revising, I have attempted to reflect changes that have occurred in financial theory and practice since the first edition as well as to sharpen and update existing material so that it is better structured and more easily comprehended. There is an increased emphasis upon valuation and upon linking various financial decisions with valuation. In this regard, Chapter

2, "The Valuation of the Firm," is new. Also, there is an increased emphasis upon financial decision making.

The book has been substantially revised. Major changes were undertaken in: Chapter 4, "Cost of Capital;" Chapter 5, "Capital Budgeting for Risky Investments: The Single Proposal;" Chapter 6, "Multiple Risky Investments, Acquisitions, and Divesture;" Chapter 9, "Dividends and Valuation;" Chapter 15, "Working Capital Management;" Chapter 17, "Management of Accounts Receivable;" and Chapter 22, "Lease Financing." More moderate, but nonetheless significant, changes occur in: Chapter 3, "Methods of Capital Budgeting;" Chapter 7, "Theory of Capital Structure;" Chapter 8, "Capital Structure Decision of the Firm;" Chapter 11, "Obtaining Long-Term Funds Externally;" Chapter 14, "Convertible Securities and Warrants;" Chapter 16, "Management of Cash and Marketable Securities;" Chapter 20, "Short-Term Loans;" Chapter 23, "Mergers and Consolidations;" and Chapter 26, "Funds Flow Analysis and Financial Forecasting." Pertinent improvements are undertaken in the remaining chapters. Problems at the end of chapters have been retained, reworked, or augmented in keeping with changes in the text. Selected references have been updated. Hopefully, these changes will make *Financial Management and Policy* more relevant.

The book continues to assume that the reader has a background in elementary algebra and statistics, including some probability concepts. Some knowledge of accounting and economics also is helpful. Special topics treated in the appendixes are somewhat more complex; here, a knowledge of calculus and mathematical programming is in order. Because the appendixes deal with special topics, however, the book's continuity is maintained even if this material is not covered.

I am grateful to Professor Charles W. Haley and John Wood for their suggestions in revising specific portions of the book. In addition, the comments of a number of professors and readers who have used the book were helpful to me in changing difficult passages, correcting mistakes, and bringing to my attention new material to be covered. I am grateful also to M. Chapman Findlay, III, who revised the problems that appear at the end of each chapter. Finally, special thanks are due my wife, Mimi, who typed and read the manuscript.

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INTRODUCTION

PART

The Goals
and Functions
of Finance

The role of the financial manager in a modern company is ever chang-
ing. His responsibilities are broadening and becoming more vital to the
company's overall development. Once, these responsibilities were
mainly confined to keeping accurate financial records, preparing reports,
managing the firm's cash position, and providing the means for the pay-
ment of bills. When liquidity was insufficient for the firm's prospective
cash needs, the financial manager was responsible for procuring addi-
tional funds. However, this procurement often included only the me-
chanical aspects of raising funds externally on either a short-, an inter-
mediate-, or a long-term basis.

In recent years, the influence of the financial manager has expanded
far beyond these limited functions. Now his concern is with (1) deter-
mining the total amount of funds to employ in the firm, (2) allocating these
funds efficiently to various assets, and (3) obtaining the best mix of financ-

ing in relation to the overall valuation of the firm.[1] As we shall see in the remainder of this chapter, the financial manager needs to have a much broader outlook than ever before, for his influence reaches into almost all facets of the enterprise and into the external environment as well.

THE EVOLUTION OF FINANCE

In order to understand better the changing role of the financial manager and the evolution of his functions, it is useful to trace the changing character of finance as an academic discipline.[2] In the early part of this century, corporation finance emerged as a separate field of study, whereas before it was considered primarily as a part of economics. By and large, the field encompassed only the instruments, institutions, and procedural aspects of the capital markets. At that time, there were a large number of consolidations, the largest of which was the colossal formation of U.S. Steel Corporation in 1900. These combinations involved the issuance of huge blocks of fixed-income and equity securities. Consequently, there was considerable interest in promotion, and in consolidations and mergers. Accounting data and financial records, as we know them today, were nonexistent. Only with the advent of regulations did disclosure of financial data become prevalent.

With the era of technological innovation and new industries in the 1920s, firms needed more funds. The result was a greater emphasis on liquidity and financing of the firm.[3] Considerable attention was directed to describing methods of external financing, and little to managing a firm internally. One of the landmark texts of this period was Arthur Stone Dewing's *The Financial Policy of Corporations,* which, in a scholarly fashion, drew together existing thought, promulgated certain new ideas, and served to pattern the teaching of finance for many years to come.[4] During this period, there was widespread interest in securities, particularly in common stock. This interest became intense toward the end of the decade, and the role and function of the investment banker was particularly important in the study of corporate finance at this time.

The depression of the thirties necessarily focused the study of finance on the defensive aspects of survival. A great deal of attention was directed toward the preservation of liquidity and toward bankruptcy, liquidation, and reorganization. The principal concern in external fi-

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[1] See Ezra Solomon, *The Theory of Financial Management* (New York: Columbia University Press, 1963), Chapter 1.

[2] See Ezra Solomon, "What Should We Teach in a Course in Business Finance?" *Journal of Finance,* XXI (May, 1966), 411–15; and J. Fred Weston, *The Scope and Methodology of Finance* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1966), Chapter 2.

[3] Weston, *The Scope and Methodology of Finance,* p. 25.

[4] Arthur S. Dewing, *The Financial Policy of Corporations* (New York: The Ronald Press Company, 1920).

nancing was how a lender could protect himself. Conservatism, naturally, reigned supreme, with considerable emphasis on a company's maintaining a sound financial structure. The large number of abuses with debt — particularly those that occurred in connection with public utility holding companies — were brought into the limelight when many companies collapsed. These failures, together with the fraudulent maltreatment of numerous investors, brought cries for regulation. Regulation and increased controls on business by government were quick to follow. One result of these regulations was an increase in the amount of financial data disclosed by companies. This disclosure made financial analysis more encompassing, because the analyst was able to compare different companies as to their financial condition and performance.

Finance, during the forties through the early fifties, was dominated by a "traditional" approach. This approach, which had evolved during the twenties and thirties, was from the point of view of an outsider — such as a lender or investor — analyzing the firm and did not emphasize decision making within the firm. The study of external financing was still largely descriptive. During this period, however, a greater emphasis on analyzing the cash flows of the firm and on the planning and control of these flows from within did develop.

In the middle fifties, great interest developed in capital budgeting and allied considerations. Of all the areas of finance, probably this topic has shown the greatest advance in recent years. With the development of new methods and techniques for selecting capital investment projects came a framework for the efficient allocation of capital within the firm. New fields of responsibility and influence for the financial manager included management of the total funds committed to assets and the allocation of capital to individual assets on the basis of an appropriate and objective acceptance criterion.

As a result of these developments, the financial manager had to come squarely to grips with how investors and creditors valued the firm and how a particular decision affected their respective valuations. As a result, valuation models were developed for use in financial decision making. Security analysis and financial management are closely related, and we are seeing an integration of these two previously separate areas of study. With this concern for valuation came a critical evaluation of the capital structure and the dividend policy of the firm in relation to its valuation as a whole. As a result of the widespread interest in capital budgeting, considerable strides have been made toward an integrated theory of finance.[5] In the future, valuation will be an even more important

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[5] In the early fifties, Friederich and Vera Lutz expounded a comprehensive theory of the firm in their famous book *The Theory of Investment of the Firm* (Princeton, N.J.: Princeton University Press, 1951). Much of the work on capital budgeting owes its origin to Joel Dean's renowned book *Capital Budgeting* (New York: Columbia University Press, 1951). These works served as building blocks for subsequent theoretical and managerial development in finance.

concept in the direction of the firm. Not only will security analysis and financial management become more intertwined, but there is likely to be an integration of capital-markets analysis into these two areas.

The use of the computer as an analytical tool added much to the development of finance during the fifties and sixties. With its advent, complex information systems have been developed which provide the financial manager with the data needed to make sound decisions. In addition, great strides have been made in the application of analytical tools to financial problems. Increasingly, operations research techniques are proving their worth. As better methods and applications are developed, more disciplined and fruitful financial analysis will be possible.

Overall, then, finance has changed from a primarily descriptive study to one that encompasses rigorous analysis and normative theory; from a field that was concerned primarily with the procurement of funds to one that includes the management of assets, the allocation of capital, and the valuation of the firm as a whole; and from a field that emphasized external analysis of the firm to one that stresses decision making within the firm. Finance today is best characterized as ever changing, with new ideas and techniques. The role of the financial manager is considerably different from what it was fifteen years ago and from what it will no doubt be in another fifteen years. Academicians and financial managers must grow to accept the changing environment and master its challenge. In this regard, they must thoroughly understand the underlying objective of the firm.

THE OBJECTIVE OF THE FIRM

In this book, we assume that the objective of the firm is to maximize its value to its shareholders. Value is represented by the market price of the company's common stock, which, in turn, is a reflection of the firm's investment, financing, and dividend decisions. When a company's stock is closely held with no public market for it, value must be approximated. Here, one should try to determine the likely value of the firm if its stock were traded publicly. Although no method of approximation is completely satisfactory, perhaps the most feasible approach is to find companies of similar risk and size, with similar growth in earnings, whose stocks have a viable public market. The market values of these stocks then are used as benchmarks for the opportunity value of the firm in question.[6]

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[6] For such an approach, see L. R. Johnson, Eli Shapiro, and Joseph O'Meara, Jr., "Valuation of Closely Held Stock for Tax Purposes: Approaches to an Objective Method," *University of Pennsylvania Law Review,* 100 (November, 1951), 166–95.

PROFIT MAXIMIZATION VERSUS
WEALTH MAXIMIZATION

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CHAP. I
The Goals
and Functions
of Finance

Frequently, maximization of profits is regarded as the proper objective of the firm, but it is not as inclusive a goal as that of maximizing shareholder wealth. For one thing, total profits are not as important as earnings per share. A firm could always raise total profits by issuing stock and using the proceeds to invest in Treasury bills. Even maximization of earnings per share, however, is not a fully appropriate objective, partly because it does not specify the timing of expected returns. Is the investment project that will produce a $100,000 return 5 years from now more valuable than the project that will produce annual returns of $15,000 in each of the next 5 years? An answer to this question depends upon the time value of money to the firm and to investors at the margin. Few existing stockholders would think favorably of a project that promised its first return in 100 years, no matter how large this return. We must take into account the time pattern of returns in our analysis.

Another shortcoming of the objective of maximizing earnings per share is that it does not consider the risk or uncertainty of the prospective earnings stream. Some investment projects are far more risky than others. As a result, the prospective stream of earnings per share would be more uncertain if these projects were undertaken. In addition, a company will be more or less risky depending upon the amount of debt in relation to equity in its capital structure. This risk is known as financial risk; and it, too, contributes to the uncertainty of the prospective stream of earnings per share. Two companies may have the same expected future earnings per share, but if the earnings stream of one is subject to considerably more uncertainty than the earnings stream of the other, the market price per share of its stock may be less.

Finally, this objective does not allow for the effect of dividend policy on the market price of the stock. If the objective were only to maximize earnings per share, the firm would never pay a dividend. At the very least, it could always improve earnings per share by retaining earnings and investing them in Treasury bills. To the extent that the payment of dividends can affect the value of the stock, the maximization of earnings per share will not be a satisfactory objective by itself.

For the reasons given above, an objective of maximizing earnings per share may not be the same as maximizing market price per share. The market price of a firm's stock represents the focal judgment of all market participants as to what the value is of the particular firm. It takes into account present and prospective future earnings per share, the timing and risk of these earnings, the dividend policy of the firm, and any other factors that bear upon the market price of the stock. The market price serves as a performance index or report card of the firm's progress; it indicates how well management is doing in behalf of its stockholders.

MANAGEMENT VERSUS STOCKHOLDERS

In certain situations, the objectives of management may differ from those of the firm's stockholders. In a large corporation whose stock is widely held, stockholders exert very little control or influence over the operations of the company. When the control of a company is separate from its ownership, management may not always act in the best interests of the stockholders.[7] Managements sometimes are said to be "satisficers" rather than "maximizers";[8] they may be content to "play it safe" and seek an acceptable level of growth, being more concerned with perpetuating their own existence than with maximizing the value of the firm to its shareholders. The most important goal to a management of this sort may be its own survival. As a result, it may be unwilling to take reasonable risks for fear of making a mistake, thereby becoming conspicuous to outside suppliers of capital. In turn, these suppliers may pose a threat to management's survival. It is true that in order to survive over the long run, management may have to behave in a manner that is reasonably consistent with maximizing shareholder wealth. Nevertheless, the goals of the two parties do not necessarily have to be the same.

A NORMATIVE GOAL

Maximization of shareholder wealth, then, is an appropriate guide for how a firm *should* act. When management does not act in a manner consistent with this objective, we must recognize that this is a constraint, and we must determine the opportunity cost. This cost is measurable only if we determine what the outcome would have been had the firm attempted to maximize shareholder wealth. Because the principle of maximization of shareholder wealth provides a rational guide for running a business and for the efficient allocation of resources in society, we shall use it as our assumed objective in considering how financial decisions should be made.[9]

This is not to say that management should ignore the question of social responsibility. As related to business firms, social responsibility concerns such things as protecting the consumer, paying fair wages to employees, maintaining fair hiring practices, supporting education, and becoming actively involved in environmental issues like clean air and water. Many people feel that a firm has no choice but to act in socially responsible

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[7] For a discussion of this question, see Gordon Donaldson, "Financial Goals: Management vs. Stockholders," *Harvard Business Review,* 41 (May–June, 1963), 116–29.

[8] Herbert A. Simon, "Theories of Decision Making in Economics and Behavioral Science," *American Economic Review,* XLIX (June, 1959), 253–83. See also Weston, *The Scope and Methodology of Finance,* Chapter 2.

[9] See Solomon, *The Theory of Financial Management,* Chapter 2.

ways; they argue that shareholder wealth and, perhaps, the corporation's very existence depend upon its being socially responsible.

Social responsibility, however, creates certain problems for the firm. One is that it falls unevenly on different corporations. Another is that it sometimes conflicts with the objective of wealth maximization. Certain social actions, from a long-range point of view, unmistakably are in the best interests of stockholders, and there is little question that they should be undertaken. Other actions are less clear, and to engage in them may result in a decline in profits and in shareholder wealth in the long run. From the standpoint of society, this decline may produce a conflict. What is gained in having a socially desirable goal achieved may be offset in whole or part by an accompanying less efficient allocation of resources in society. The latter will result in a less than optimal growth of the economy and a lower total level of economic want satisfaction. In an era of unfilled wants and scarcity, the allocation process is extremely important.

Many people feel that management should not be called upon to resolve the conflict posed above. Rather, society, with its broad general perspective, should make the decisions necessary in this area. Only society, acting through Congress and other representative governmental bodies, can judge the relative tradeoff between the achievement of a social goal and the sacrifice in the efficiency of apportioning resources that may accompany realization of the goal. With these decisions made, corporations can engage in wealth maximization and thereby efficiently allocate resources, subject, of course, to certain governmental constraints. Under such a system, corporations can be viewed as producing both private and social goods, and the maximization of shareholder wealth remains a viable corporate objective.

FUNCTIONS OF FINANCE

The functions of finance can be broken down into the three major decisions the firm must make: the investment decision, the financing decision, and the dividend decision. Each must be considered in relation to the objective of the firm; an optimal combination of the three decisions will maximize the value of the firm to its shareholders. As the decisions are interrelated, we must consider their joint impact on the market price of the firm's stock. We now briefly examine each of them and their place in the subsequent chapters of this book.

INVESTMENT DECISION

The investment decision, perhaps, is the most important of the three decisions. Capital budgeting, a major aspect of this decision, is the allocation of capital to investment proposals whose benefits are to be realized

in the future. Because the future benefits are not known with certainty, investment proposals necessarily involve risk. Consequently, they should be evaluated in relation to their expected return and the incremental risk they add to the firm as a whole, for these are the factors that affect the firm's valuation in the marketplace. Included also under the investment decision is the decision to reallocate capital when an asset no longer economically justifies the capital committed to it. The investment decision, then, determines the total amount of assets held by the firm, the composition of these assets, and the business-risk complexion of the firm. The theoretical portion of this decision is taken up in Part II. Also taken up in this part is the use of the cost of capital as a criterion for accepting investment proposals.

In addition to selecting new investments, a firm must manage existing assets efficiently. The financial manager is charged with varying degrees of operating responsibility over existing assets. He is more concerned with the management of current assets than with fixed assets, and we consider the former topic in Part V. Our concern in Part V is with ways to manage current assets efficiently in order to maximize profitability relative to the amount of funds tied up in an asset. Determining a proper level of liquidity for the firm is very much a part of this management. Although the financial manager has little or no operating responsibility for fixed assets, he is instrumental in allocating capital to these assets by virtue of his involvement in capital budgeting.

In Parts II and VII, we consider mergers and acquisitions from the standpoint of an investment decision. These external investment opportunities can be evaluated in the same general manner as an investment proposal that is generated internally. Also, in Part VII, we take up failures and reorganizations, which involve a decision to liquidate a company or to rehabilitate it, often by changing its capital structure. This decision should be based upon the same economic considerations that govern the investment decision.

FINANCING DECISION

The second major decision of the firm is the financing decision. Here, the financial manager is concerned with determining the best financing mix or capital structure for his firm. If a company can change its total valuation simply by varying its capital structure, an optimal financing mix would exist in which market price per share is maximized. The financing decision should take into account the firm's present and expected future portfolio of assets, for they determine the business-risk complexion of the firm as perceived by investors. In turn, perceived business risk affects the real costs of the various methods of financing.

In Chapters 7 and 8 of Part III, we take up the financing decision in relation to the overall valuation of the firm. Our concern is with exploring

the implications of variation in capital structure on the valuation of the firm. In Part IV, we examine the various methods by which a firm goes to the market for the long-term funds that comprise its capital structure. In Part VI, following our discussion of working-capital management in the previous part, we take up short- and intermediate-term financing. The emphasis in Parts IV and VI is on the managerial aspects of financing; we analyze the features, concepts, and problems associated with alternative methods of financing. In Part III, on the other hand, the focus is primarily theoretical.

DIVIDEND DECISION

The third important decision of the firm is its dividend policy, which is examined in Chapters 9 and 10 of Part III. The dividend decision includes the percentage of earnings paid to stockholders in cash dividends, the stability of absolute dividends over time, stock dividends, and the repurchase of stock. The dividend-payout ratio determines the amount of earnings retained in the firm and must be evaluated in the light of the objective of maximizing shareholder wealth. If investors at the margin are not indifferent between current dividends and capital gains, there will be an optimal dividend-payout ratio that maximizes shareholder wealth. The value of a dividend to investors must be balanced against the opportunity cost of the retained earnings lost as a means of equity financing. Thus, we see that the dividend decision must be analyzed in relation to the financing decision.

FINANCIAL MANAGEMENT

Financial management involves the solution of the three decisions of the firm discussed above. Together, they determine the value of the firm to its shareholders. Assuming that our objective is to maximize this value, the firm should strive for an optimal combination of the three decisions. Because these decisions are interrelated, they should be solved jointly. As we shall see, their joint solution is difficult to implement. Nevertheless, with a proper conceptual framework, decisions can be reached that tend to be optimal. The important thing is that the financial manager relate each decision to its effect on the valuation of the firm.

Because of the importance of valuation concepts, they are investigated in depth in Chapter 2. Thus, Chapters 1 and 2 serve as the foundation for the subsequent development of the book.

In an endeavor to make optimal decisions, the financial manager makes use of certain analytical tools in the analysis, planning, and control activities of the firm. Financial analysis is a necessary condition, or prerequisite, for making sound financial decisions; we examine the tools of analysis in Part VIII. This material appears at the end of the book in

order to set it apart from the book's sequence of development. Depending on the reader's background, it can be taken up early or used for reference purposes throughout.

PROBLEMS

1. Examine the functions of financial managers in several large U.S. corporations. Try to ascertain how the role of the financial manager has changed in these concerns over the past fifty years.

2. Inquire among several corporations in your area to find out if these firms have determined specific objectives. Is maximizing the value of the firm to its shareholders the major objective of most of these companies?

3. "A basic rationale for the objective of maximizing the wealth position of the stockholder as a primary business goal is that such an objective may reflect the most efficient use of society's economic resources and thus lead to a maximization of society's economic wealth." Briefly evaluate this observation.

4. Think of several socially responsible actions in which a corporation might engage. Evaluate these actions in relation to the allocation of resources in society under a wealth maximization objective.

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The
Valuation
of the Firm

Given the objective discussed in Chapter 1, the firm should choose that combination of investment, financing, and dividend policy decisions that will maximize its value to its stockholders. These policies affect the firm's value through their impact on its expected return-risk character. This character, in turn, determines the view that investors hold regarding returns on their stock. Because these returns are not known with certainty, risk necessarily is involved. It can be defined as the possibility that the actual return will deviate from that which was expected. Expectations are continually revised on the basis of new information. For our purposes, information can be categorized as to whether it emanates from the investment, financing, or dividend policies of the firm.[1] In other words, on information based on these three decisions, investors formulate

[1]See D. E. Peterson, *A Quantitative Framework for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc., 1969), pp. 28–29.

expectations as to the return and risk involved in holding a common stock.

Theoretically, the most appropriate and most efficient way to combine the three categories mentioned above is through a stock valuation model, where the price of a share of stock is made a function of the three decisions. In the broadest sense, the model might be represented as

$$V = f(r, \sigma) = g(I, F, D) \tag{2-1}$$

where V is the market price per share of the company's stock, r is the return investors expect, σ is the standard deviation, a measure of dispersion, of the probability distribution of possible returns, I represents the firm's portfolio of investment projects, F is its financing mix or capital structure, and D is its dividend policy as denoted by the dividend-payout ratio and the stability of dividends. In order to maximize share price, we would vary I, F, and D jointly to maximize V through r and σ. In the equation, it is important to recognize that the firm does not influence share price directly through its investment, financing, and dividend policies. Rather, share price is determined by investors who use information with respect to these policies to form expectations as to return and risk.

Share-price models, such as the one shown in Eq. (2-1), have considerable theoretical merit. This type of model allows us to take account of the important decision variables that affect the market price of a company's stock. Moreover, we are able to consider directly the interrelationships between these variables. For example, the decision to invest in a new capital project necessitates the financing of the investment. The financing decision, in turn, influences and is influenced by the dividend decision, for retained earnings used in internal financing represent dividends foregone by stockholders. In a share-price model, these interrelationships are considered directly, allowing the firm's important decisions to be solved jointly.

From this brief explanation of how the important decisions of the firm affect its value,[2] it is clear that financial management and security analysis are closely related. With an objective of maximizing the value of the firm to its shareholders, financial decisions must be made in light of their likely impact on value. In the remainder of this chapter, we examine in more depth the valuation of common stocks. It will serve as a foundation for our subsequent analysis of the investment, financing, and dividend decisions of the firm.

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[2] Because the investment decision involves not only investment in new projects but the management of existing assets as well, it embodies a host of decisions with respect to level of output, pricing, and the combination of factor inputs in the firm's production process. As these policies involve considerations beyond the scope of this book, we do not consider them directly, but assume that they are embraced in the cash-flow information used in the evaluation of existing and new investment projects. For an excellent integration of production decisions into an overall valuation model of the firm, see Douglas Vickers, *The Theory of the Firm: Production, Capital, and Finance* (New York: McGraw-Hill Book Company, 1968).

When the individual investor purchases a common stock he gives up current consumption in the hope of attaining increased future consumption. His expectation of higher future consumption is based on the dividends he expects to receive and, hopefully, the eventual sale of the stock at a price higher than his original purchase price. The individual must allocate his wealth at a given moment in keeping with his desired lifetime consumption pattern, which includes any bequest he wishes to make. If the future were certain and the time of death known, the individual could apportion his wealth so as to obtain the maximum possible satisfaction from present and future consumption. He would know the exact returns available from investment, the timing of these returns, as well as future income from noninvestment sources. Investment would be merely a means of balancing present against future consumption.[3]

RETURN ON INVESTMENT

Not knowing what lies in the future, the investor is unable to plan his lifetime consumption pattern with certainty. Because the returns from investment and the timing of those returns are uncertain, he compensates for the lack of certainty by requiring an expected return sufficiently high to offset it. But what constitutes the return on a common stock? For a one-year holding period, most would agree that it is the sum of cash dividends received plus any capital gain or loss, all over the purchase price, minus one. Suppose that an individual were to purchase a share of DSS Corporation for $50 a share. The company was expected to pay a $2 dividend at the end of the year, and its market price after the payment of the dividend was expected to be $53 a share. The expected return would be

$$r = \frac{2.00 + 53.00}{50.00} - 1 = 0.10 \qquad (2\text{-}2)$$

where r is the expected return. Another way to solve for r is

$$50.00 = \frac{2.00}{(1 + r)} + \frac{53.00}{(1 + r)} \qquad (2\text{-}3)$$

When we solve for the rate of discount that equates the dividend and terminal value at the end of one year with the purchase price of the stock at time 0, we find it to be 10 per cent. Thus, the investor expects a 10 per cent return on his investment.

Now suppose that instead of holding the security one year, he intends to hold it two years and sell it at the end of that time. Moreover,

[3] For a rigorous analysis of lifetime consumption and investment decisions, see Eugene F. Fama, "Multiperiod Consumption – Investment Decisions," *American Economic Review*, LX (March, 1970), 163–74.

suppose he expects the company to pay a $2.20 dividend at the end of year 2 and the market price of the stock to be $56.10 after the dividend is paid. His expected return can be found by solving the following equation for r

$$50 = \frac{2.00}{(1+r)} + \frac{2.20}{(1+r)^2} + \frac{56.10}{(1+r)^2} \qquad (2\text{-}4)$$

When we solve for r, we find it to be 10 per cent also.[4] For general purposes, the formula can be expressed as

$$P_0 = \sum_{t=1}^{2} \frac{D_t}{(1+r)^t} + \frac{P_2}{(1+r)^2} \qquad (2\text{-}5)$$

where P_0 is the market price at time 0, D_t is the expected dividend at the end of period t, the capital Greek sigma denotes the sum of discounted dividends at the end of periods 1 and 2, and P_2 is the expected terminal value at the end of period 2.

If an investor's holding period were ten years, the expected rate of return would be determined by solving the following equation for r

$$P_0 = \sum_{t=1}^{10} \frac{D_t}{(1+r)^t} + \frac{P_{10}}{(1+r)^{10}} \qquad (2\text{-}6)$$

Now, suppose that the investor were a perpetual trust fund and that the trustee expected to hold the stock forever. In this case, the expected return would consist entirely of cash dividends and perhaps a liquidating dividend. Thus, the expected rate of return would be determined by solving the following equation for r

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \qquad (2\text{-}7)$$

where ∞ is the sign for infinity.

It is clear that the intended holding period of different investors will vary greatly. Some will hold a stock only a few days, while others might expect to hold it forever. Investors with holding periods shorter than infinity expect to be able to sell the stock in the future at a price higher than they paid for it. This assumes, of course, that at that time there will be investors willing to buy it. In turn, these investors will base their judgments as to what the stock is worth on expectations of future dividends and future terminal value beyond that point. That terminal value, however, will depend upon other investors at that time being willing to buy the stock. The price they are willing to pay will depend upon their expectations of dividends and terminal value. And so the process goes through successive

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[4] See Chapter 3 for an explanation of how to solve for r. It corresponds to the internal rate of return.

investors. Note that the total cash return to all successive investors in a stock is the sum of the dividends paid, including any liquidating dividend. Thus, cash dividends are all that stockholders as a whole receive from their investment; they are all the company pays out. Consequently, the foundation for the valuation of common stocks must be dividends.

This notion can be illustrated in a slightly different way. Instead of r representing the expected return for the individual investor, suppose that we replace it with k_e, which represents the market discount rate appropriate for the risk stock involved. This rate can be thought of as the required rate of return by investors. For an investor with a limited time horizon, the market price may be viewed as the discounted value of the stream of expected future dividends plus the discounted value of the expected market price per share at the end of his holding period.[5]

$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1 + k_e)^t} + \frac{P_n}{(1 + k_e)^n} \qquad \text{(2-8)}$$

However, the expected price at the end of period n will be the discounted value of expected future dividends beyond that point, or

$$P_n = \sum_{t=n+1}^{\infty} \frac{D_t}{(1 + k_e)^{t-n}} \qquad \text{(2-9)}$$

When we substitute Eq. (2-9) into Eq. (2-8), the current market price of the stock is

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + k_e)^t} \qquad \text{(2-10)}$$

Thus, the value of a common stock is based upon expected future dividends, whether they are regular or liquidating. They are the foundation for valuation.

The logical question to be raised at this time is why do the stocks of companies that pay no dividends have positive, and often quite high, values? The obvious answer is that investors expect to be able to sell the stock in the future at a price higher than they paid for it. Instead of dividend income plus terminal value, they rely only upon the terminal value. In turn, terminal value will depend upon the expectations of the marketplace at the end of the horizon period. The ultimate expectation is that the firm eventually will pay dividends, either regular or liquidating ones, and that future investors will receive a cash return on their investment. In the interim, however, investors are content with the expectation that they will be able to sell the stock at a subsequent time because there will be a market for it. In the meantime, the company is reinvesting earnings and, hopefully, enhancing its future earning power and ultimate dividends.

[5] See Eugene M. Lerner and Willard T. Carleton, *A Theory of Financial Analysis* (New York: Harcourt Brace Jovanovich, Inc., 1966), pp. 123–25.

We saw in Eq. (2-10) that the market price of a share of stock can be expressed as the discounted value of the stream of expected future dividends, where k_e is the market rate of discount appropriate for the risk company involved. If dividends of a company are expected to grow at a constant rate, g, in keeping, say, with a growth in earnings, Eq. (2-10) becomes

$$P_0 = \frac{D_0(1 + g)}{(1 + k_e)} + \frac{D_0(1 + g)^2}{(1 + k_e)^2} + \ldots + \frac{D_0(1 + g)^\infty}{(1 + k_e)^\infty} \qquad (2\text{-}11)$$

where D_0 is the dividend per share paid at time 0. Thus, the dividend expected in period n is equal to the most recent dividend times the compound growth factor, $(1 + g)^n$.

If k_e is greater than g, Eq. (2-11) can be expressed as[6]

$$P_0 = \frac{D_1}{k_e - g} \qquad (2\text{-}12)$$

The critical assumption in this valuation model is that dividends per share are expected to grow perpetually at a compound rate of g. For certain companies, this may be a fairly realistic approximation of reality. To illustrate the use of Eq. (2-12), suppose that A & G Company's dividend per share at $t = 1$ was expected to be \$3, was expected to grow at a 4 per cent rate forever, and that the market capitalization rate, k_e, was 10 per cent. The value of a share of stock at time 0 would be

$$P_0 = \frac{3.00}{0.10 - 0.04} = \$50 \qquad (2\text{-}13)$$

When the pattern of expected growth is such that a perpetual growth model is not appropriate, modifications of Eq. (2-10) can be used. A num-

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[6] If we multiply both sides of Eq. (2-11) by $(1 + k_e)/(1 + g)$ and subtract Eq. (2-11) from the product, we obtain

$$\frac{P_0(1 + k_e)}{(1 + g)} - P_0 = D_0 - \frac{D_0(1 + g)^\infty}{(1 + k_e)^\infty}$$

As k_e is greater than g, the second term on the right side will be zero. Consequently,

$$P_0\left[\frac{1 + k_e}{1 + g} - 1\right] = D_0$$

$$P_0\left[\frac{(1 + k_e) - (1 + g)}{1 + g}\right] = D_0$$

$$P_0[k_e - g] = D_0(1 + g)$$

$$P_0 = \frac{D_1}{k_e - g}$$

If k_e were less than g, it is easy to determine that the market price of the stock would be infinite. See David Durand, "Growth Stocks and the Petersburg Paradox," *Journal of Finance*, XII (September, 1957), 348–63.

ber of valuation models are based upon the premise that the growth rate will taper off eventually.[7] For example, the transition might be from a present above-normal growth rate to one that is considered normal. If dividends per share were expected to grow at an 8 per cent compound rate for ten years and then grow at a 4 per cent rate, Eq. (2-10) would become

$$P_0 = \sum_{t=1}^{10} \frac{D_0(1.08)^t}{(1 + k_e)^t} + \sum_{t=11}^{\infty} \frac{D_{10}(1.04)^{t-10}}{(1 + k_e)^t} \tag{2-14}$$

The transition from an above-normal to a normal rate of growth could be specified as more gradual than the rate above. For example, we might expect dividends to grow at an 8 per cent rate for five years, followed by a 6 per cent rate for the next five years, and a 4 per cent growth rate thereafter. The more growth segments that are added, the more closely the growth in dividends will approach a curvilinear function.

It seems clear that a company will not grow at an above-normal rate forever. Typically, companies tend to grow at a very high rate initially, after which their growth opportunities slow down to a rate that is normal for companies in general. If maturity is reached, the growth rate may stop altogether.[8] For any growth model, Eq. (2-10) can be modified so that it portrays the expected stream of future dividends. Tables have been prepared to solve for market value under various assumptions of growth in dividends, the duration of growth, and the discount rate employed.[9]

DISCOUNT RATE

In the previous section, we assumed that the discount rate, k_e, was somehow determined by the market and could be taken as given. We need now to consider the determination of k_e in depth. When an investor purchases a share of common stock, he expects to receive a stream of future dividends. If he were absolutely certain that he would receive these dividends, the appropriate rate of discount would be the risk-free rate. For many investors, the risk-free rate might be approximated by

[7] See W. Scott Bauman, "Investment Returns and Present Values," *Financial Analysts Journal,* 25 (November–December, 1969), 107–18; Burton G. Malkiel, "Equity Yields, Growth, and the Structure of Share Prices," *American Economic Review,* LII (December, 1963), 1004–31; Charles C. Holt, "The Influence of Growth Duration on Share Prices," *Journal of Finance,* XVII (September, 1962), 465–75; Eugene F. Brigham and James L. Pappas, "Duration of Growth, Changes in Growth Rates, and Corporate Share Prices," *Financial Analysts Journal,* 22 (May–June, 1966), 157–62; and Paul F. Wendt, "Current Growth Stock Valuation Methods," *Financial Analysts Journal,* 21 (March–April, 1965), 3–15.

[8] See Holt, "The Influence of Growth Duration on Share Prices," pp. 466–67.

[9] Robert M. Soldofsky and James T. Murphy, *Growth Yields on Common Stock: Theory and Tables* (Iowa City: Bureau of Business and Economic Research, University of Iowa, 1961).

the current yield on a government fixed-income security whose maturity coincides with the end of the investor's expected holding period.[10]

Risk Premium. If the stream of expected future dividends is less than certain, the rational investor will discount these dividends with a rate higher than the risk-free rate. In other words, he will require an expected return in excess of the risk-free rate in order to compensate him for the risk associated with receiving the expected dividend stream. The greater the uncertainty, the greater the expected return that he will require. Thus, the required rate of return for an investor consists of the risk-free rate, i, plus a risk premium, θ, to account for the uncertainty associated with receiving the expected return.

$$k_e = i + \theta \qquad (2\text{-}15)$$

In determining the appropriate risk premium for a common stock, an investor might be thought to formulate subjective probability distributions of dividends per share expected to be paid in various future periods. If his time horizon were limited, he would formulate probability distributions of future dividends over his intended holding period as well as a probability distribution of market prices per share to prevail at the end of this period. To illustrate, consider an investor with a one-year holding period who estimates the dividend and market price per share for SB Tool Company one year hence to be that shown in the first two columns of Table 2-1. Because the company has announced its dividend

TABLE 2-1
Probability distributions of possible dividends and market prices at end of year 1

Dividend		Market Price		Joint Dividend and Market Price	
Probability of Occurrence	Event	Probability of Occurrence	Event	Probability of Occurrence	Event
0.15	$2.20	0.05	$62	0.05	$64.20
0.70	2.00	0.10	59	0.10	61.20
0.15	1.80	0.20	56	0.20	58.00
		0.30	53	0.30	55.00
		0.20	50	0.20	52.00
		0.10	47	0.10	48.80
		0.05	44	0.05	45.80

[10] Because interest is expressed in terms of money and because the monetary standard changes over time, the real rate of return on a security can differ considerably from its money, or nominal, return. If investors buy securities on the basis of their expected real return, an increase in anticipated inflation will result in an increase in nominal interest rates. Thus, the current yield on a government security embodies in it an element attributable to anticipated inflation. As a result, the discount rate used in the valuation of common stocks takes account of anticipated inflation.

intentions, the investor is reasonably certain of the dividend he will re-
ceive at the end of the year. Consequently, the probability distribution is
relatively narrow. However, he is far less certain of the market price
per share that will prevail at the end of his holding period. As discussed
earlier, his estimates of future market prices are based upon dividends
expected to be paid beyond that point. Thus, his probability distribution
of market prices at the end of one year is based upon probability distri-
butions of expected future dividends beyond that point. Because he is
less certain of distant dividends than he is of near dividends, the prob-
ability distribution of possible market prices, shown in columns 3 and 4,
is wider than that for the dividend to be paid at the end of the year.

Suppose now that the investor believes that the amount of dividend
and the market price at the end of the year are highly correlated. That
is, a high dividend at the end of the year is closely associated with a
bright future, which in turn is closely associated with a high market
price per share. More specifically, suppose that the investor expects
the dividend to be $2.20 when the market price is $62 or $59, to be $2.00
when the market price is $56, $53, or $50, and to be $1.80 when the
market price is $47 or $44. The joint probability distribution of these
two events is shown in the last two columns of the table. If we were to
divide the values shown in the last column by the current market price,
say $50 a share, and subtract one from the result, we would obtain the
probability distribution of expected returns for the year expressed as a
per cent. These returns are shown in Table 2-2.

TABLE 2-2
Probability distribution of possible returns
for one-year holding period

Probability of Occurrence	Possible Return
0.05	0.284
0.10	0.224
0.20	0.160
0.30	0.100
0.20	0.040
0.10	−0.024
0.05	−0.084

As suggested earlier, the greater the dispersion of the probability dis-
tribution, the more risk we would say the security possesses. The con-
ventional measure of dispersion of a probability distribution is the stan-
dard deviation, which, for our one-period example, is

$$\sigma = \sqrt{\sum_{x=1}^{n} (R_x - \bar{R})^2 P_x} \tag{2-16}$$

where R_x is the return for the xth possibility, P_x is the probability of occurrence of that event, n is the total number of possibilities, and $\bar{R}$ is the expected value of the combined dividend and market price. The expected value is calculated by

$$\bar{R} = \sum_{x=1}^{n} R_x P_x \tag{2-17}$$

To illustrate these concepts, the expected value of possible returns for our example is

$$\bar{R} = 0.05(0.284) + 0.10(0.224) + 0.20(.160) + 0.30(.100) +$$
$$0.20(0.040) + 0.10(-0.024) + 0.05(-0.084) = 10 \text{ per cent}$$

The standard deviation of the probability distribution is

$$\sigma = [0.05(0.284 - 0.100)^2 + 0.10(0.224 - 0.100)^2 + 0.20(0.160 -$$
$$0.100)^2 + 0.30(0.100 - 0.100)^2 + 0.20(.040 - 0.100)^2 + 0.10$$
$$(-0.024 - 0.100)^2 + 0.05(-0.084 - 0.100)]^{1/2} = 8.9 \text{ per cent}$$

For the normal, bellshaped probability distribution, approximately two-thirds of the distribution falls within one standard deviation of the mean, 0.95 falls within two standard deviations, and 0.997 within three standard deviations.[11] By expressing differences from the mean value in terms of standard deviations, we are able to determine the probability that the actual return will be greater or less than a certain amount.

We note that the standard deviation is expressed in absolute terms. To evaluate it in relation to the expected value, we use a measure of relative dispersion called the coefficient of variation. This measure is simply the standard deviation of a probability distribution over its expected value. In our case, the coefficient of variation is

$$\sigma/\bar{R} = 8.9/10.0 = 0.89 \tag{2-18}$$

The greater the coefficient of variation for a stock, the greater its risk to the investor. The coefficient of variation simply expresses in quantitative terms his views as to the uncertainty surrounding the payment of expected future dividends. The risk premium, θ, for investor j might be thought to be some function of the coefficient of variation

$$\theta_j = f(\sigma/\bar{R}) \tag{2-19}$$

[11] The valuation process described assumes that the standard deviation of the probability distribution of security returns is finite. Eugene F. Fama, "The Behavior of Stock-Market Prices," *Journal of Business*, XXXVII (January, 1965), 34–105, building on an earlier investigation by Benoit Mandelbrot, showed that stock-market price changes conformed to a stable paretian distribution – a "fat-tailed" distribution – for which the variance and standard deviation does not exist. Fama concludes, however, that the insights on diversification gained from the mean-standard deviation model are valid when the distribution is a member of the stable family. See Fama, "Risk, Return and Equilibrium: Some Clarifying Comments," *Journal of Finance*, XXIII (March, 1968), 64.

For the risk averter, the higher the $\sigma/\bar{R}$, the greater the θ. The required rate of return for the jth investor would be

$$k_{ej} = i + f(\sigma/\bar{R}) \qquad (2\text{-}20)$$

Thus, investors can be viewed as determining the return they would require for investing in a particular stock on the basis of the risk-free rate plus some premium to compensate them for the risk associated with whether expected dividends actually will be received.

*MARKET
EQUILIBRIUM*

Once the individual investor has established his subjective required rate of return for a stock, k_{ej}, he then must decide whether to buy, sell, or hold the stock. Using Eq. (2-10) as his model, he would discount expected future dividends to their present value, using k_{ej} as the discount rate.[12] If the discounted value obtained exceeds the current market price of the stock, he would buy the stock, or continue to hold it if he already owns it. Its value to him exceeds the current market price. By the same token, if his discounted value is less than the current market price, he would want to sell the stock if he owns it. Other investors in the market can be thought to formulate judgments as to the value of the stock in a similar manner. These values are likely to differ considerably, because individual investors will have different expectations of future dividends and different risk preferences.

Assuming no short sales,[13] the demand schedule for the stock is determined by the values interested investors ascribe to it. We use the word "interested" rather than "all" investors because only a limited number of investors form judgments on a particular stock. An investor is able to follow only a portion of all the stocks available in the market. For any given stock, most investors in the market simply have no judgments; consequently, they cannot be considered potential buyers.[14] A hypothetical demand schedule of interested investors is shown in Figure 2-1. The demand schedule is established by the value an individual investor deems appropriate and the amount of stock he is willing to buy. The latter depends upon his wealth, income, preferences for other assets, and his ability and willingness to issue financial liabilities. As consideration

[12] The steps illustrated have been made sequential for ease of understanding. Actually, the market discount rate, k_e, and the market price of the stock, P_0, are determined simultaneously.

[13] A short sale occurs where an individual borrows stock from someone else and sells it. He hopes that the stock will decline in price so that he will be able to buy it back (cover) at a lower price for delivery to the person from whom the stock is borrowed.

[14] See John Lintner, "The Aggregation of Investor's Diverse Judgments and Preferences in Purely Competitive Security Markets," *Journal of Financial and Quantitative Analysis,* IV (December, 1969), 398.
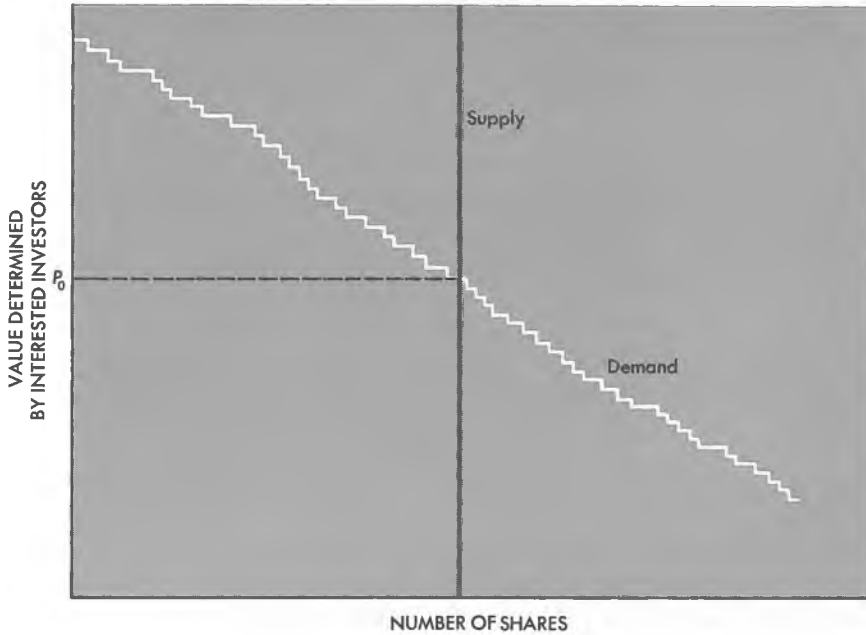
FIGURE 2-1
Demand and supply schedules for common stock

of these factors is beyond the scope of this book, we assume that each interested investor has established an amount of stock that he is willing to purchase.[15]

If we assume the total amount of stock is fixed in the sense that the company will neither issue nor retire any, the supply of stock function is depicted by the straight up-down line in the figure. The intersection of the demand and supply schedules determines the current market price of the stock, P_0. Investors who formulate subjective values above P_0 would hold the stock, whereas those who determined values below P_0 would not purchase it. Investors whose value corresponds exactly to the market price, P_0, are called *investors at the margin*. We shall use this term throughout the book to describe investors who are at the intersection of the supply and demand schedules.

The market price, P_0, is not fixed. As expectations, risk preferences, wealth, income, and other factors influencing interested investors change, so will the demand schedule change. In turn, this change will lead to a

[15] For a theoretical analysis of the amount he might demand, see James C. Van Horne, *The Function and Analysis of Capital Market Rates* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970), appendix to Chapter 3. Later in this chapter, we consider the portfolio problem.

change in market price per share. Suppose, for example, that the economic outlook turns bleak and that investors lower their estimates of future dividends and that the risk associated with receiving these dividends increases simultaneously. During such times, investor aversion to risk usually increases. All these factors will lead to a lower demand schedule and a lower market price per share, all other things remaining the same. On the other hand, interest rates generally fall in a recession so that the risk-free rate, i, will be lower. This factor will exert an upward pressure on the demand schedule, partially offsetting the downward shift caused by the first three factors. The reader can easily visualize other combinations of factors that would lead to a shift in the demand schedule and a resulting higher or lower market price. When the demand schedule shifts for any one of a number of reasons, individual investors do not necessarily occupy the same positions as before. Indeed, investors may change their relative positions in the schedule, change the amount of stock they are willing to purchase, or lose interest in the stock and drop out of the demand schedule altogether.

Purposely, our discussion of the price mechanism for securities has been elementary.[16] We treat portfolio considerations for the investor later in this chapter, but consideration of other factors influencing his behavior would involve us in a theoretical discussion beyond the scope of a basic text. Our discussion has been directed toward illustrating the major aspects of valuation on which to build our discussion of financial management.

DOWNSIDE RISK

Before proceeding, it is desirable to digress briefly and consider in more depth the risk preferences of investors. We assumed in our previous analysis that the risk premium, θ_j, for investor j was only a function of the dispersion of the probability distribution of possible returns. Now it is obvious that the risk to the investor is not dispersion per se but the possibility of downside deviations from the expected value of return. An investor would not consider upside deviations undesirable. For this reason, the shape of the probability distribution may be important to him in assessing risk. To illustrate, the two distributions in Figure 2-2 have the same expected value and standard deviation. However, distribution A is skewed to the right; while distribution B is skewed to the left. To the extent that the investor prefers one distribution to the other, the standard deviation is not a sufficient measure of risk. Many investors would prefer the distribution skewed to the right, for it has a greater degree of downside protection and upside potential.

[16] For a more detailed analysis of investor behavior, see Lintner, "The Aggregation of Investor's Diverse Judgments and Preferences in Purely Competitive Security Markets"; and Van Horne, *The Function and Analysis of Capital Market Rates,* Chapter 3.

PROBABILITY OF OCCURRENCE

R̄

EXPECTED RETURN
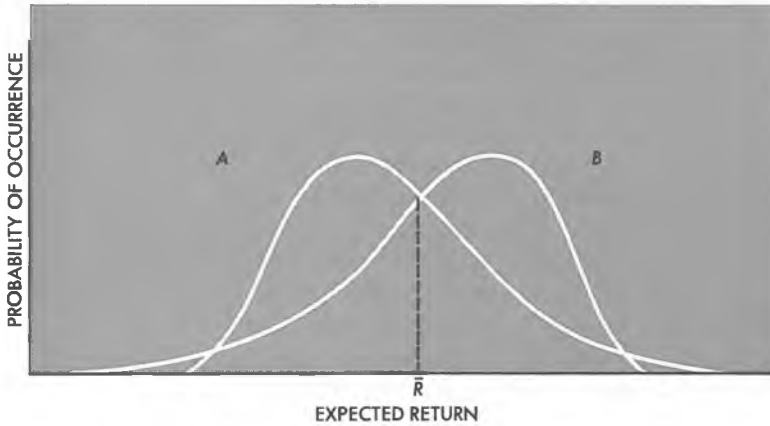
FIGURE 2-2
Illustration of skewness

If the investor is concerned solely with the possibility of actual loss, he would be interested in only that portion of the probability distribution that represents a loss. The rest of the distribution would be ignored. Such an investor simply might establish a maximum tolerance or probability for loss and treat this per cent as a constraint in his selection process.[17] Obviously, a measure of downside potential would be useful. Unfortunately, the mathematical calculation of a skewness measure is possible only for a simple problem; it is unfeasible for a security with a large number of possible returns. Because of the difficulty of dealing mathematically with moments of the probability distribution higher than the second, our analysis is confined to the first two moments—the expected value and the standard deviation. For distributions that are reasonably symmetric, this approach may approximate closely investor attitudes toward risk. That is, the distribution with the greater dispersion would consistently represent the riskier security.

Another problem is that our measure of risk does not allow for different states of nature. Recall that it implies that risk is simply the possibility that actual returns will deviate from those that are expected. However, if risk is the deviation of actual returns from those that are desired, our measure may not be entirely satisfactory. Suppose an investor had different utilities for a given return, depending upon the state of nature in which it occurred. For example, a dollar return may be far more valuable to him if there is a recession than if there is a period of prosperity. The state-

[17] Robert E. Machol and Eugene M. Lerner, "Risk, Ruin and Investment Analysis," *Journal of Financial and Quantitative Analysis,* IV (December, 1969), 473–92, formulate a decision situation of this sort as a chance-constrained problem where risk is defined as the cumulative probability of the return falling below some level of ruin. For an earlier integration of financial disaster into investment choice, see A. D. Roy, "Safety First and the Holding of Assets," *Econometrica,* XX (July, 1952), 431–49.

preference approach to security valuation suggests that returns should be estimated across various states of nature. Investment selection, then, would depend upon the utility for money in the different states as well as upon the probability of occurrence of the various states. The security that maximized the investor's utility would be preferred. The state-preference approach implies that risk is the possibility that the desired, rather than the expected, return will not be achieved.[18] Although the state-preference approach has considerable theoretical merit, it is not as yet operationally feasible due to the difficulty of formulating returns and utilities for different states of nature. Consequently, we shall use the dispersion of the probability distribution of possible returns as representing a reasonable approximation of risk for the great body of investors.

PORTFOLIO CONSIDERATIONS

For most investors, the risk premium, θ_j, for a common stock is not determined in isolation; in part, it depends upon his overall portfolio.[19] This suggests that an investor should be interested in the marginal contribution of a particular stock to the risk of his overall portfolio, and not necessarily in the risk of the stock itself. In this section, we consider the important problem of portfolio selection. This review has implications not only for security valuation but for capital budgeting for combinations of risky investments, to be considered in Chapter 6. As with the individual security, our concern is with the expected return and standard deviation of the probability distribution of possible returns.

The expected rate of return on a portfolio is simply the weighted average of the expected rates of return of the securities comprising that portfolio.

$$\overline{R}_p = \sum_{x=1}^{m} A_x R_x \qquad (2\text{-}21)$$

where A_x is the portion of funds invested in security X, $\overline{R}_x$ is the expected value of return for that security, and m is the total number of securities in the portfolio.

In contrast, the standard deviation of the probability distribution of

[18] See Alexander A. Robichek, "Risk and the Value of Securities," *Journal of Financial and Quantitative Analysis*, IV (December, 1969), 513–38; J. Hirshleifer, "Investment Decision under Uncertainty: Application of the State-Preference Approach," *Quarterly Journal of Economics* (May, 1966), 552–77; Stewart C. Myers, "A Time-State Preference Model for Security Valuation," *Journal of Financial and Quantitative Analysis*, III (March, 1968), 1–33; and William F. Sharpe, *Portfolio Theory and Capital Markets* (New York: McGraw-Hill Book Company, 1970), Chapter 10.

[19] Parts of this section are adapted from Van Horne, *The Function and Analysis of Capital Market Rates*, Chapter 3.

possible portfolio returns is not the sum of the individual standard deviations but

$$\sigma_p = \sqrt{\sum_{j=1}^{m} \sum_{k=1}^{m} A_j A_k r_{jk} \sigma_j \sigma_k} \tag{2-22}$$

where m is the total number of securities in the portfolio, A_j is the proportion of the total funds invested in security j, A_k is the proportion invested in security k, r_{jk} is the expected correlation between returns for securities j and k, σ_j is the standard deviation about the expected value of return for security j, and σ_k is the standard deviation for security k. These standard deviations are calculated with Eq. (2-16). Thus, the standard deviation of a portfolio depends upon: (1) the correlation between expected returns of the various securities comprising that portfolio; (2) the standard deviation of each security; and (3) the proportion of funds invested in each security.

CORRELATION BETWEEN SECURITIES AND DIVERSIFICATION

The correlation between returns may be positive, negative, or zero, depending upon the nature of the association. A correlation coefficient of 1.00 indicates that the returns from two securities vary positively, or directly, in exactly the same proportions; a correlation coefficient of -1.00 indicates that they vary inversely in exactly the same proportions; and a zero coefficient indicates an absence of correlation. The correlation of returns between two securities can be expressed as

$$r_{jk} = \sum_{x=1}^{n} \left(\frac{R_{xj} - \overline{R}_j}{\sigma_j} \right) \left(\frac{R_{xk} - \overline{R}_k}{\sigma_k} \right) P_{xjk} \tag{2-23}$$

where R_{xj} is the xth possible return for security j, $\overline{R}_j$ is the expected value of return for security j, R_{xk} is the xth possible return for security k, R_k is the expected value of return for security k, P_{xjk} is the joint probability that R_{xj} and R_{xk} will occur simultaneously, and n is the total number of joint possible returns. In other words, deviations from expected values of return for the two securities are normalized by dividing them by their respective standard deviations. When these normalized deviations are multiplied by each other, and their product is multiplied by the joint probability of occurrence and then summed, we obtain the correlation coefficient.

To illustrate the determination of the standard deviation for a portfolio using Eq. (2-22), consider an investor who holds a stock whose expected value of annual return is 10 per cent, with a standard deviation of 15 per cent. Suppose further that he is considering another stock with an expected value of annual return of 8 per cent, a standard deviation of

12 per cent, and that the expected correlation between the two stocks is 0.40. By investing equal portions in each of the stocks, the expected value of return for the portfolio would be:

$$R_p = (0.5)0.10 + (0.5)0.08 = 9 \text{ per cent} \qquad \textbf{(2-24)}$$

This contrasts with a 10 per cent return when the portfolio is comprised entirely of the first stock. However, the standard deviation for the probability distribution of possible returns for the new portfolio is

$$\sigma_p = [(0.5)^2(1.00)(0.15)^2 + (2)(0.5)(0.5)(0.4)(0.15)(0.12) + (0.5)^2(1.00)(0.12)^2]^{1/2} = 11.3 \text{ per cent} \qquad \textbf{(2-25)}$$

From Eq. (2-22) we know that the covariance between the two stocks must be counted twice. Therefore, we multiply the covariance by two. When $j = 1$ and $k = 1$ for stock 1, the proportion invested (0.5) must be squared, as must the standard deviation (0.15). The correlation coefficient, of course, is 1.00. The same thing applies for stock 2 when $j = 2$ and $k = 2$.

It is obvious from Eq. (2-22), however, that for even a moderate-sized portfolio we must compute a large number of correlation coefficients (one for each possible pairing of individual securities). For a large portfolio, the number of computations involved is unfeasible. Instead of estimating the correlation between security returns directly, one may estimate the correlation between a return for a security and some index. This index, for example, might be the Gross National Product. From these estimates, the total variance of a portfolio can be approximated. The index-model approach, which reduces considerably the number of correlation coefficient estimates, is described in the appendix to this chapter. For large portfolios, it is the only feasible means by which to obtain the standard deviation.

The previous example suggests that by diversifying one's holdings to include securities with less than perfect positive correlation among themselves, the risk-averse investor is able to reduce the dispersion of the probability distribution of possible returns relative to the expected value of return for that distribution. In so doing, he reduces the risk of holding securities. However, this diversification must be among the right type of securities. It is not enough for an investor simply to spread his investment among a number of securities; diversification must be among securities not possessing high degrees of positive correlation among themselves.[20] It is evident from Eq. (2-22) that the dispersion of the proba-

[20] For a more detailed analysis of diversification, see William F. Sharpe, *Portfolio Theory and Capital Markets*, Chapters 2–6; John Lintner, "Security Prices, Risk, and Maximal Gains from Diversification," *Journal of Finance*, XX (December, 1965), 587–615; and Michael C. Jensen, "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," *Journal of Business*, XXXXII (April, 1969), 167–247. All of the work along this line is an outgrowth of Harry Markowitz, *Portfolio Selection: Efficient Diversification of Investments* (New York: John Wiley & Sons, Inc., 1959).

bility distribution for a portfolio could be reduced to zero if securities with perfect negative correlation could be found. The objective of diversification, however, is not to reduce dispersion per se but to obtain the best combination of expected value of return and standard deviation.

PORTFOLIO SELECTION

The best combination will depend upon the investor's preference function with respect to expected value of return and risk from holding a portfolio of securities. If an investor is averse to risk and associates risk with divergence from expected value of return, his utility schedule may be similar to that shown in Figure 2-3. The expected value of return is plotted on the horizontal axis, while the standard deviation is along the vertical. The curves are known as indifference curves; the investor is indifferent between any combination of expected value and standard deviation on a particular curve. The greater the slope of the indifference curves, the more averse the investor is to risk. As we move to the right in Figure 2-4, each successive curve represents a higher level of expected utility.

The individual investor will want to hold that portfolio of securities that places him on the highest indifference curve, choosing it from the opportunity set of available portfolios. An example of an opportunity set, based upon the subjective probability beliefs of an individual investor, is shown in Figure 2-3. This opportunity set reflects all possible portfolios of securities as envisioned by the investor. The dark line at the bottom of the set is the line of efficient combinations, or the efficient frontier. It depicts the tradeoff between risk and expected value of re-

STANDARD DEVIATION

EXPECTED RETURN
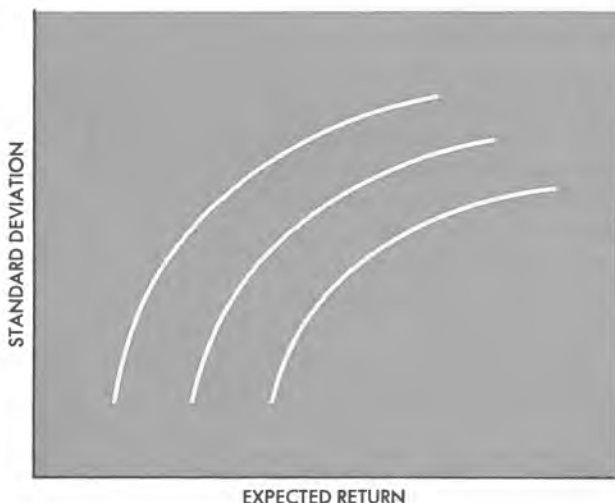
FIGURE 2-3
**Hypothetical   indifference
curves**
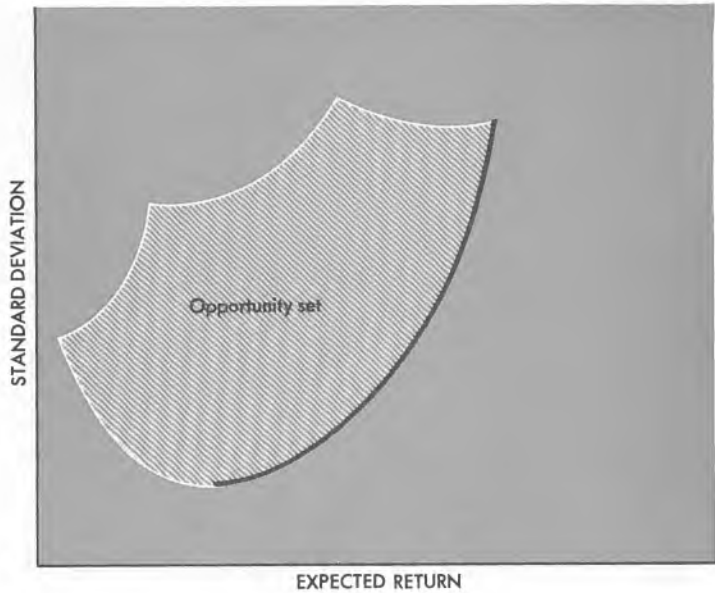
FIGURE 2-4
Hypothetical opportunity
set

STANDARD DEVIATION

Opportunity set

EXPECTED RETURN

turn. According to the Markowitz mean-variance maxim, an investor should seek a portfolio of securities that lies on the efficient frontier.[21] A portfolio is not efficient if there is another portfolio with a higher expected value of return and a lower standard deviation, a higher expected value and the same standard deviation, or the same expected value but a lower standard deviation. If an investor's portfolio is not efficient, he can increase the expected value of return without increasing the risk, decrease the risk without decreasing the expected value of return, or obtain some combination of increased expected value and decreased risk by switching to a portfolio on the efficient frontier.[22]

The objective of the investor is to choose the best portfolio from those

[21]Markowitz, *Portfolio Selection: Efficient Diversification of Investments,* Chapters 7 and 8.

[22]Baumol has proposed a modification of Markowitz's efficiency criterion for evaluating expected value and variance. This criterion involves the use of a lower confidence limit, represented by $E - K\sigma$, where E is the expected value, σ is the standard deviation from expected value, and K is a confidence coefficient. K is specified in terms of the number of standard deviations from expected value and represents the lowest plausible outcome from the standpoint of the investor. Given a lower confidence limit, an investor is able to determine how much risk he must assume in order to obtain a certain expected value. According to Baumol, only when $E - K\sigma$ decreases as portfolios with greater expected value are considered would there be a sacrifice of safety. Baumol's efficient-combination line is represented by the downward sloping portion of the $E - K\sigma$ curve and is a subset of Markowitz's line. As K increases, Baumol's efficient set approaches the Markowitz efficient set as a limit. See William J. Baumol, "An Expected Gain-Confidence Limit Criterion for Portfolio Selection," *Management Science,* 10 (October, 1963), 174–82. It would be possible to incorporate Baumol's efficiency criterion into the above analysis.
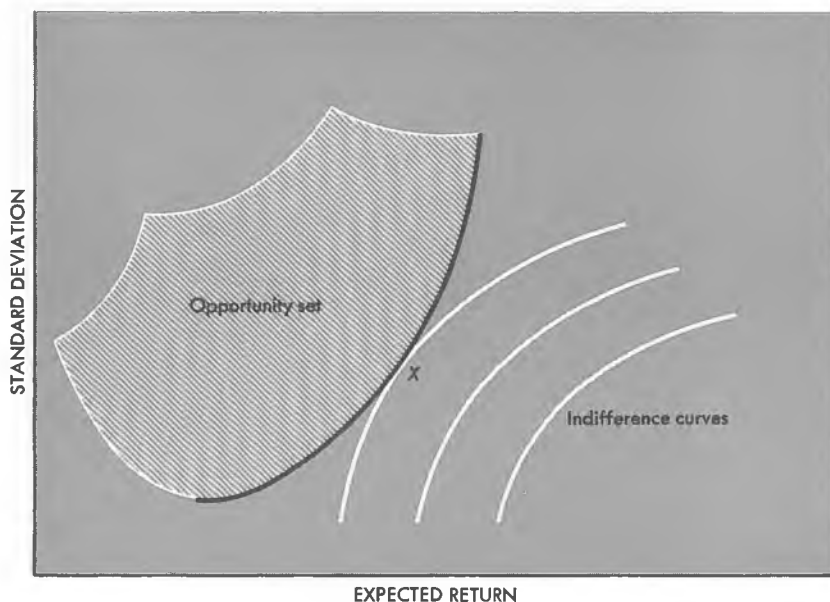
FIGURE 2-5
Selection of optimal portfolio

that lie on the efficient frontier. The portfolio with the maximum utility is the one at the point of tangency of the opportunity set with the highest indifference curve. This tangency is illustrated in Figure 2-5; and the portfolio represented by the point of tangency is the optimal one for an investor with those expectations and utility preferences.

Presence of Risk-Free Security. If a risk-free security exists that yields a certain future return, the portfolio selection process described above must be modified. Suppose the investor is able to borrow or lend at a risk-free rate, i. We would then draw a line from i through its point of tangency with the efficient frontier, as illustrated in Figure 2-6. This line then becomes the new efficient frontier. Note that only one portfolio of risky securities — namely, m — would be considered. Any point on the line tells us the proportion of the risky portfolio and the proportion of the risk-free security to be held. The optimal investment policy would be determined by the point of tangency between the straight line in Figure 2-6 and the highest indifference curve. As shown in the figure, this point is portfolio x.[23]

In a portfolio context, the risk of an individual security depends upon

[23] For a much more extensive examination, see Sharpe, *Portfolio Theory and Capital Markets*, Chapter 4.
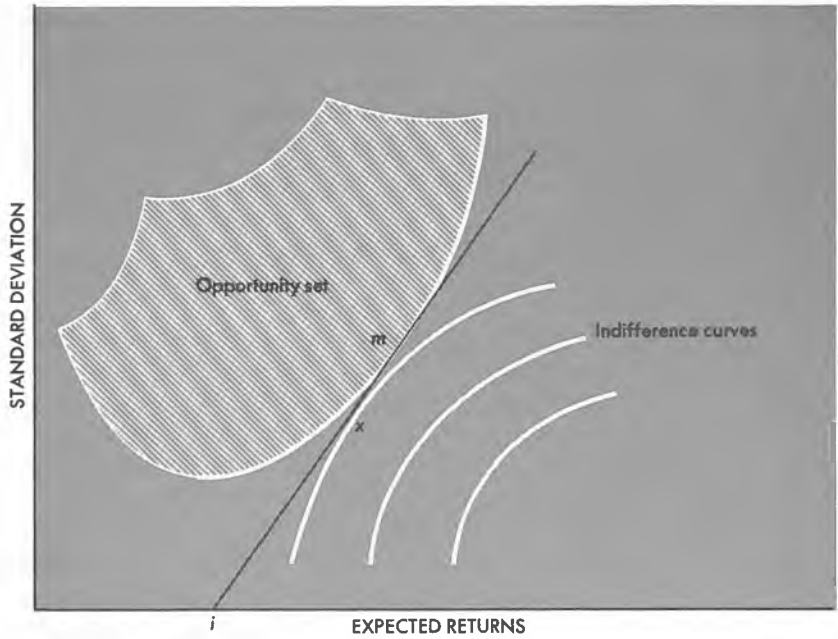
FIGURE 2-6
Selection of optimal portfolio when risk-
free asset exists

its expected correlation with other securities in the portfolio. The less the
degree of positive correlation, or the greater the degree of negative cor-
relation, the less the incremental risk of the security. According to the
Sharpe-Lintner portfolio model, the risk premium θ_j for security j would
be:[24]

$$\theta_j = \frac{\overline{R}_p - i}{\sigma_p^2}(r_{jp}\sigma_j\sigma_p) \qquad (2\text{-}26)$$

where $\overline{R}_p$ is the expected value of return for portfolio p, i is the risk-free
rate, σ_p^2 is the variance of the probability distribution of possible returns
for the portfolio, and the term $(r_{jp}\sigma_j\sigma_p)$ is the covariance of returns for
security j with those of the portfolio. Frequently, the portfolio used in
asset-pricing models is the market portfolio, which consists of all securi-
ties in the market. As can be seen in the equation, the risk premium for a
stock depends importantly upon the correlation, r_{jp}, between its returns
and those of the market. According to this notion, the greater the cor-
relation of expected returns for a security with those of other common

[24] For a discussion of this model, see Jensen, "Risk, the Pricing of Capital Assets, and
the Evaluation of Investment Portfolios," 176–77.

stocks, the greater the risk premium embodied in the discount rate for that security, all other things being the same. As this rate is used to discount expected future dividends, the greater the risk premium, the lower the share price, all other things being equal. Thus, we see how the required rate of return for a particular security might be measured in a market context.

IMPLICATIONS AND APPROACH
IN THIS BOOK

The foregoing discussion gives rise to certain implications for financial management. It suggests that the effect of investment, financing and dividend decisions on the firm's overall risk should be considered in a market portfolio context. In other words, a decision should be judged not only in terms of its impact on the dispersion of the probability distribution of possible returns for the firm itself, but also in terms of the correlation of the firm's returns with those of other firms. This agrees with the notion that the risk of a security depends not only upon its own variance, but upon its covariance with other securities as well. In other words, the risk an investor associates with realizing a stream of expected future dividends depends upon the correlation of that stream with expected dividend streams available from other securities in his portfolio.

Although the idea of taking account of the impact of an investment, financing, or dividend decision on the perceived risk of the firm in a market portfolio context is appealing theoretically,[25] it is not as yet feasible operationally. For one thing, there is the question of what portfolio is important. Different investors hold different portfolios. By and large, the firm has no knowledge of the portfolios held by investors in its stock. By using a market portfolio of all stocks, individual portfolios do not have to be considered. But a market portfolio may not be entirely appropriate for investors in the stock of a particular firm.

A more important limitation is the difficulty of estimating the effect of a decision on the correlation of the firm's returns with the returns available on other securities in the portfolio. Even though this problem is lessened if we use an index model, estimating the perceived incremental effect of a policy change on portfolio risk is extremely difficult. To date, empirical testing has not been precise enough to allow predictions that merit much confidence. About all that can be conjectured is that a de-

[25] For different theoretical approaches to the problem, see Robert S. Hamada, "Portfolio Analysis, Market Equilibrium and Corporation Finance," *Journal of Finance,* XXIV (March, 1969), 13–32; James C. D. Mao and John F. Helliwell, "Investment Decisions under Uncertainty: Theory and Practice," *Journal of Finance,* XXIV (May, 1969), 323–38; Nils H. Hakansson, "On the Dividend Capitalization Model under Uncertainty," *Journal of Financial and Quantitative Analysis,* IV (March, 1969), 65–87; and Jan Mossin, "Security Pricing and Investment Criteria in Competitive Markets," *American Economic Review,* LIX (December, 1969), 749–56.

cision is likely to make the firm somewhat more or less risky relative to the market. Even here, estimates are subject to considerable error.

Because of the operational difficulty in measuring and interpreting the effect of an investment, financing, or dividend decision on overall portfolio risk, we do not incorporate this dimension into our analysis. Instead, we estimate only the effect of a decision on the expected value of return and risk for the individual firm. Given information about this effect, we then draw implications for market price per share. While this approach may oversimplify the valuation process by ignoring important portfolio considerations, it allows us to come to grips with the effect of financial decisions on share price in an operational sense.

Although a portfolio valuation model would be more realistic, unfortunately, it also is more general. Even the realism of the portfolio model can be criticized because it ignores the utility associated with holding real assets, consumption, and the issuing of financial liabilities. All of these factors affect the valuation of common stocks.[26] The more realistic a model becomes, the less its explanatory power. To be operationally feasible, a model must have predictive value. Thus, we choose to examine financial management using a relatively simple valuation model. By tracing through the effects of certain decisions on valuation using this model, we hope to come to a better understanding of financial management.

SUMMARY

In this book, we examine the impact of investment, financing, and dividend decisions on the value of the firm's common stock. These decisions affect value through their perceived effect upon the expected return and risk associated with that return. In purchasing a common stock, the investor gives up present consumption with the expectation of increasing future consumption. We view the value of a share of stock to him as being the stream of expected future dividends, discounted by a rate appropriate for the risk associated with actually receiving the expected dividend stream. Because cash dividends are all that investors as a whole receive from their investment, these dividends are the foundation for valuation. A dividend-valuation model is consistent with the fact that many investors expect to sell their stock in the future and realize a capital gain. Given the basic valuation model, we saw how it could be simplified to allow for different expectations as to future growth.

The discount rate employed by an investor might be thought to consist of the risk-free rate plus a risk premium to compensate him for the uncertainty associated with receiving the expected return from the stock. On the basis of the dispersion of a subjective probability distribution

[26] See, for example, Van Horne, *The Function and Analysis of Capital Market Rates,* Chapter 3.

of possible returns, an investor can determine the risk premium necessary to compensate him for the risk associated with realizing the expected return. Once he has established the appropriate discount rate, the investor can determine the value of the security to him by discounting expected future dividends by this rate. The demand schedule for the stock is determined by the values individual investors feel the stock is worth and the number of shares they are willing to buy. The market price of a stock is the point at which the downward sloping demand curve crosses the supply schedule. Investors at the margin are those who are at the intersection of the two schedules.

Next, portfolio considerations were introduced. The risk of an individual security in a portfolio context depends not only upon the dispersion of its probability distribution of possible returns, but also upon the correlation of returns for that security with those for other securities in the portfolio. By diversifying into securities with less than perfect correlation with each other, an investor is able to reduce the dispersion of the probability distribution of possible returns relative to the expected value of return. The best portfolio for the investor is determined by his utility preferences as to return and risk. It is represented by the point of tangency between the opportunity set of available portfolios — the efficient frontier — and the highest indifference curve, which depicts his risk preferences. If discount rates used by investors are determined in a portfolio context, the firm should judge the impact of investment, financing, and dividend decisions on the correlation of the firm's returns with those of other stocks in the portfolio. Because of the operational difficulty in judging such decisions in a portfolio context, we used a relatively simple valuation model. Consideration was given only to the effect of decisions on the expected return and risk of the firm involved. In the chapters that follow, we shall study the effect of the firm's investment, financing, and dividend decisions on its value.

APPENDIX

Index Models

For large portfolios, it is extremely difficult to estimate the standard deviation of a portfolio with Eq. (2-22). The number of correlation coefficients to be estimated for an N-security portfolio is $(N^2 - N)/2$. For a very large portfolio, the number of estimates becomes astronomical. To reduce the number of calculations to workable proportions, Sharpe devised a "diagonal" model for approximating the total variance of a portfolio.[27] In this model, a linear relationship is postulated between

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[27] Sharpe, *Portfolio Theory and Capital Markets,* Chapter 7. Our description of the index model draws heavily upon Sharpe.

returns for an individual security and some index. This index might be the Gross National Product or some market index such as the Standard & Poor's 500 stock index. Returns for security j then could be expressed as

$$R_j = a_j + b_j I + u_j \qquad (2A\text{-}1)$$

where a_j is a constant, b_j is a constant, I is the index, and u_j is the error term. The coefficient, b_j, indicates the responsiveness of security j's return to changes in the level of the index. It is assumed that the expected value of the distribution of error terms is zero and that error terms for different securities are uncorrelated. The standard deviation of the distribution of error terms reflects the risk of security j over and above the risk due to association with the index. It is assumed also that the error term is uncorrelated with the index.

The responsiveness of the overall portfolio to changes in the level of the index can be expressed as

$$b_p = \sum_{j=1}^{n} A_j b_j \qquad (2A\text{-}2)$$

where n is the total number of securities comprising the portfolio, and A_j is the proportion of funds invested in security j. If the rates of return on various securities are correlated with each other only through the index, the standard deviation of the distribution of possible returns for the portfolio is

$$\sigma_p = \sqrt{b_p^2 \sigma_I^2 + \sum_{j=1}^{n} A_j^2 \sigma_j^2} \qquad (2A\text{-}3)$$

where σ_I^2 is the variance for the index. The first term on the right represents the risk associated with the index, while the second represents the unique risk attributable to the individual securities making up the portfolio. Instead of calculating $(N^2 - N)/2$ variances and covariances for a portfolio of securities, Eq. (2A-3) allows us to calculate only N variances plus the relation of each security to the index. For large portfolios, there is an enormous savings in the number of computations. Without such an approach, the calculation of the variance and the efficiency of a portfolio would be unfeasible. The index model can be extended to use multiple indexes if different securities are responsive to different indexes.[28]

The greater the diversification of the portfolio, the less the unique risk of the portfolio in Eq. (2A-3). Assuming securities are uncorrelated except through the index, unique dispersion can be reduced by spreading one's investment over a number of securities. For the well-diversified

[28] *Ibid.*

portfolio, the second term on the right will approach zero. For such a portfolio, a useful measure of the marginal risk of security j is the responsiveness of its return to changes in the level of the index, as denoted by b_j. On the basis of the responsiveness of a particular security relative to that for other securities, the investor can determine the risk premium, θ_j, required of security j. It will represent the approximate incremental risk of the security to the overall portfolio. The index-model method is extremely useful and will be employed again when we analyze capital budgeting for risky capital investments.

PROBLEMS

1. The stock of the Health Corporation is currently selling for $20 and is expected to pay a $1 dividend at the end of the year. If an investor bought the stock now and sold it for $23 after receiving the dividend, what rate of return would he earn?

2. Mrs. Hogg has a tax rate of 60 per cent on ordinary income and 25 per cent on capital gains. She expects the stock of the Townsend Corporation to pay a $3 dividend at the end of this year and a $5 dividend at the end of next year, at which time she would hope to sell the stock for $70. If she purchases the stock now for $50, what rate of return is she anticipating?

3. Suppose that Mrs. Hogg (see problem 2) requires a 25 per cent after-tax return on all her investments. What is the maximum price she would be willing to pay for Townsend stock now? What is the maximum price she would pay a year from now?

4. James Farnsworth Tuttle, founder and holder of the controlling interest in Tuttle, Inc., has vowed that his firm will never pay a dividend as long as he lives. His current life expectancy is twenty-five years. After that time it is estimated that Tuttle, Inc., could pay dividends of $50 per year per share forever. At present the firm could afford to pay $10 per share forever. The marginal Tuttle shareholder requires a 20 per cent pretax return.

  (a) What is the current value of Tuttle stock?
  (b) What is the cost to each shareholder of James Farnsworth Tuttle's policies?

5. The Pueblo Corporation paid a dividend of $1.50 per share last year; dividends of Pueblo are expected to grow at a rate of 10 per cent indefinitely. The Pueblo stockholders are known to demand a 20 per cent pretax return. At what price should Pueblo stock sell?

6. Suppose that Pueblo (see problem 5) dividends were expected to grow at 10 per cent for only five more years, after which they would grow at 6 per cent forever. At what price should Pueblo stock now sell?

7. The stock of the Margin Corporation is currently selling for $90 a share. Possible dividend payments and the corresponding market prices at the end of the year are given below. Compute the expected value of the probability distribution of the possible returns and its standard deviation and coefficient of variation. You may ignore taxes. Assuming the distribution of possible returns to be normal, what is the probability of earning less than 15 per cent?

| Dividend | | | Market Price | | Joint |
Amount	Probability		Amount	Probability	Probability
		→	$ 95	.10	.10
$ 0	.30				
		→	105	.20	.20
5	.40	→	115	.40	.40
		→	125	.20	.20
10	.30				
		→	135	.10	.10
					1.00

8. The securities of Companies A and B have the expected return and standard deviations given below; the expected correlation between the two stocks is 0.20.

	R	σ
Company A	20%	20%
Company B	10%	3%

(a) Compute the risk and return for the following portfolios:
   (1) 100 per cent A (5) 60 per cent A — 40 per cent B
   (2) 100 per cent B (6) 60 per cent B — 40 per cent A
   (3) 80 per cent A — 20 per cent B (7) 50 per cent A — 50 per cent B
   (4) 80 per cent B — 20 per cent A
(b) Graph your results.
(c) Which of these portfolios is optimal? Why?
(d) Suppose that the investor could borrow or lend at 6 per cent. How would this affect your graph?

1. The portfolio of James Sutten consists of 30 per cent X, 40 per cent Y, and 30 per cent Z. When the returns of these securities are compared with the Dow-Jones Average (which is assumed to have a standard deviation of 100), it is discovered that $b_x = 0.001$, $b_y = 0.0006$, and $b_z = 0.01$. Furthermore, $\sigma_x = 0.1$, $\sigma_y = 0.05$, and $\sigma_z = 0.2$. Determine the responsiveness of the overall portfolio to changes in the Dow-Jones Average and also the standard deviation of the possible returns of the portfolio.

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INVESTMENT IN ASSETS AND ACCEPTANCE CRITERION

P A R T

Methods
of Capital
Budgeting

3

Capital budgeting involves a current investment in which the benefits are expected to be received beyond one year in the future. The use of one year as a line of demarcation is somewhat arbitrary. It suggests that the investment in any asset with a life of less than a year falls into the realm of working-capital management, whereas any asset with a life of more than a year involves capital budgeting. Obviously, there is a great deal of overlap. A new distribution system may call for both a new warehouse and an additional investment in inventory. An investment proposal of this nature must be evaluated as a single package, not as an investment in a fixed asset — the warehouse — and in a current asset — inventory. Consequently, we evaluate such a project as a capital-budgeting decision.

An investment proposal should be judged in relation to whether it provides a return equal to or greater than that required by investors at the margin. Thus, the required rate of return is the link by which we re-

late the effect of an investment decision to share price; and it is examined in detail in the subsequent chapter when we take up the cost of capital. In order to simplify our investigation of the methods of capital budgeting in this chapter, we assume the required rate of return is given and is the same for all investment projects. This assumption necessitates our holding constant the financing and dividend decisions of the firm. Moreover, it implies that the selection of any investment project or combination of projects does not alter the business-risk complexion of the firm as a whole.

We use the term "business risk" to mean the risk associated with the operation of the firm. Business risk exists apart from the risk inherent in the way the firm is financed. The latter is known as financial risk, which we consider in Part III. We define business risk as the relative dispersion of the net operating income of the firm. For example, suppose that the expected annual operating incomes over the next five years for Firms A and B were subjective random variables. Suppose further that the means of the unimodal probability distributions were $500,000 and $2 million, respectively; and the standard deviations were $200,000 and $600,000, respectively. The amount of relative dispersion can be expressed as the coefficient of variation, which is the standard deviation over the mean of the probability distribution of expected net operating income. For Firm A

$$\text{Coefficient of variation} = \frac{\$200,000}{\$500,000} = 0.40$$

while for Firm B

$$\text{Coefficient of variation} = \frac{\$600,000}{\$2,000,000} = 0.30$$

As the measure of relative dispersion for Firm A is greater than that for Firm B, we would say that A had the greater degree of business risk. Thus, the coefficient of variation serves as a relative measure of the degree of business risk of a firm, and is the one we shall employ. In this chapter, we assume that the coefficient of variation of the firm as a whole remains unchanged with the acceptance of investment proposals.

In Chapters 5 and 6, we relax this assumption and allow for the fact that different investment projects have different degrees of business risk. Consequently, the selection of an investment project may affect the business-risk complexion of the firm, which, in turn, may affect the risk premium embodied in the required rate of return of investors at the margin. In addition, the risk premium required by creditors may be affected. As a result, the acceptance of a project can affect share price. In the same chapters, we consider ways to take account of risk for individual investment projects, combinations of projects, and external acquisitions. Although these methods are far from perfect, they do allow

us to take account of the important dimension of risk in capital budgeting. In addition, we take up the question of the divesture of a portion of the enterprise within a capital-budgeting context.

Capital budgeting involves the generation of investment proposals; the estimate of cash flows for the proposals; the evaluation of cash flows; the selection of projects based upon an acceptance criterion; and, finally, the continual reevaluation of investment projects after their acceptance. The first four are examined in this chapter. The last aspect, the continual reevaluation of existing projects, is taken up in Chapter 6, when we consider the question of divesture.

Depending upon the firm involved, investment proposals can emanate from a variety of sources. For purposes of analysis, projects may be classified into one of five categories:

1. New products or expansion of existing products
2. Replacement of equipment or buildings
3. Research and development
4. Exploration
5. Others

The last category comprises miscellaneous items such as the expenditure of funds to comply with certain health standards or the acquisition of a pollution control device. In the case of a new product, the proposal usually originates in the marketing department. On the other hand, a proposal to replace a piece of equipment with a more sophisticated model usually emanates from the production area of the firm. In each case, it is important to have efficient adminstrative procedures for channeling investment requests.

Most firms screen proposals at multiple levels of authority. For a proposal originating in the production area, the hierarchy of authority might run from section chiefs to (1) plant managers to (2) the vice-president for operations to (3) a capital-expenditures committee under the financial manager to (4) the president to (5) the board of directors. How high a proposal must go before it is finally approved usually depends upon its size. The greater the capital outlay, the greater the number of screens usually required. Plant managers, for example, may be able to approve moderate-sized projects on their own; but final approval for larger projects is received only at higher levels of authority. Because the administrative procedures for screening investment proposals vary greatly from firm to firm, it is not possible to generalize. The best procedure will depend upon the circumstances. Where projects are approved

at multiple levels, it is very important that the same acceptance criterion is applied objectively and consistently throughout the organization.[1] Otherwise, capital is likely to be misallocated in the sense that one division might accept a project that another would reject.

ESTIMATING CASH FLOWS

One of the most important tasks in capital budgeting is estimating future cash flows for a project. The final results we obtain are really only as good as the accuracy of our estimates. The reason we express the benefits expected to be derived from a project in terms of cash flows rather than in terms of income is that cash is what is central to all decisions of the firm. The firm invests cash now in the hope of receiving cash returns in a greater amount in the future. Only cash receipts can be reinvested in the firm or paid to stockholders in the form of dividends.[2] Thus, cash, not income, is what is important in capital budgeting.

For each investment proposal, we need to provide information on expected future cash flows on an after-tax basis. In addition, the information must be provided on an incremental basis so that we analyze only the difference between the cash flows of the firm with and without the project. For example, if a firm contemplates a new product that is likely to compete with existing products, it is not appropriate to express cash flows in terms of the estimated sales of the new product. We must take into account that probably "cannibalization" of existing products will come about and make our cash-flow estimates on the basis of incremental sales.

To illustrate the information needed for a capital-budgeting decision, consider the following situation. Suppose a firm is considering the introduction of a new product. In order to launch the product, it will need to spend $150,000 for special equipment and the initial advertising campaign. The marketing department envisions the product life to be six years and expects incremental sales revenue to be:

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$60,000	$120,000	$160,000	$180,000	$110,000	$50,000

Cash outflows include labor and maintenance costs, material costs, and various other expenses associated with the product. As with sales, these

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[1] For a discussion of administrative procedures for capital budgeting as well as a review of company practices, see Robert G. Murdick and Donald D. Deming, *The Management of Corporate Expenditures* (New York: McGraw-Hill Book Company, 1968), Chapters 7–9.

[2] See D. E. Peterson, *A Quantitative Framework for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc., 1969), p. 335.

costs must be estimated on an incremental basis. In addition to these outflows, the firm will need to pay higher taxes if the new product generates higher profits; and this incremental outlay must be included. Suppose that on the basis of these considerations the firm estimates total incremental cash outflows to be:

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$40,000	$70,000	$100,000	$100,000	$70,000	$40,000

Because depreciation is a noncash expense, it is not included in these outflows. The expected net cash flows from the project are:

	Initial Cost	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Cash inflows		$60,000	$120,000	$160,000	$180,000	$110,000	$50,000
Cash outflows	$150,000	40,000	70,000	100,000	100,000	70,000	40,000
Net cash flows	−$150,000	$20,000	$ 50,000	$ 60,000	$ 80,000	$ 40,000	$10,000

Thus, for an initial cash outflow of $150,000, the firm expects to generate net cash flows of $20,000, $50,000, $60,000, $80,000, $40,000, and $10,000 over the next six years. These cash flows represent the relevant information we need in order to judge the attractiveness of the project.

To go to a somewhat more complicated replacement-decision example, suppose that we are considering the purchase of a turret lathe to replace an old lathe and that we need to obtain cash-flow information in order to evaluate the attractiveness of this project. The purchase price of the new machine is $18,500; and it will require an additional $1,500 to install, bringing the total cost to $20,000. The old machine can be sold for its depreciated book value of $2,000. The initial net cash outflow for the investment project, therefore, is $18,000. The new machine is expected to cut labor and maintenance costs and effect other cash savings totaling $7,600 a year before taxes for the next five years, after which it is not expected to provide any savings, nor is it expected to have a salvage value. These savings represent the net savings to the firm if it replaces the old machine with the new. In other words, we are concerned with the difference between the cash flows resulting from the two alternatives — continuing with the old machine or replacing it with a new one.

Because machines of this sort have useful lives in excess of one year, their cost cannot be charged against income for tax purposes but must be depreciated over the depreciable life of the asset. Depreciation then is deducted from income in order to compute taxable income. If the firm employs straight-line depreciation, the annual depreciation charge is 20

per cent of the total depreciable cost of $20,000, or $4,000 a year. As-sume additionally that the corporate federal income tax rate is 50 per cent. Moreover, assume that the old machine has a remaining depreciable life of five years, that there is no expected salvage value at the end of this time, and that the machine also is subject to straight-line deprecia-tion. Thus, the annual depreciation charge on the old machine is 20 per cent of its depreciated book value of $2,000, or $400 a year. Because we are interested in the incremental impact of the project, we must subtract depreciation charges on the old machine from depreciation charges on the new one to obtain the incremental depreciation charges associated with the project. Given the information cited, we now are able to cal-culate the expected net cash flow (after taxes) resulting from the accept-ance of the project.

	Book Account	Cash-Flow Account
Annual cash savings	$7,600	$7,600
Depreciation on new machine	4,000	
Less depreciation on old machine	400	
Additional depreciation charge	3,600	
Additional income before taxes	4,000	
Income tax (50%)	2,000	2,000
Additional income after taxes	2,000	
Annual net cash flow		$5,600

In figuring the net cash flow, we simply deduct the additional cash out-lay for federal income taxes from the annual cash savings. The expected annual net cash inflow for this replacement proposal is $5,600 for each of the next five years; this figure compares with additional income after taxes of $2,000 a year. The cash flow and net profit figures differ by the amount of additional depreciation. As our concern is not with income, as such, but with cash flows, we are interested in the right-hand column. For an initial cash outlay of $18,000, then, we are able to replace an old lathe with a new one that is expected to result in net cash savings of $5,600 a year over the next five years. As in the previous example, the relevant cash-flow information for capital-budgeting purposes is expressed on an incremental, after-tax basis.

Efficient procedures must be set up to collect the information neces-sary for the capital-budgeting decision. This information must be stan-dardized as much as possible for all investment proposals; otherwise, proposals cannot be compared objectively.[3] We recognize the difficulty

[3] For a framework for collecting this information, see Harold Bierman, Jr., and Seymour Smidt, *The Capital Budgeting Decision,* 2d ed. (New York: The Macmillan Company, 1966), Chapter 13; and Murdick and Deming, *The Management of Corporate Expendi-tures,* Chapters 7–8.

in quantifying expected cash flows for certain investment proposals. The returns on all investments, other than the U.S. Treasury securities held to maturity, are subject to varying degrees of uncertainty. Despite the problems associated with uncertainty, management must make some estimate of the probable outcome if it is even to consider an investment proposal. These estimates can be subject to probability distributions; indeed, using probability distributions is desirable, as we shall see in the next chapter. In this chapter, however, we work with the expected values of these estimates.

EVALUATION OF PROPOSALS

Once we have collected the necessary information, we are able to evaluate the attractiveness of the various investment proposals under consideration. As our purpose in this chapter is to examine the basic concepts of capital budgeting, we assume that the risk or quality of all investment proposals under consideration does not differ from the risk of existing investment projects of the firm and that the acceptance of any proposal or group of investment proposals does not change the relative business risk of the firm. The investment decision will be either to accept or to reject the proposal. In this section, we evaluate four approaches to capital budgeting: the average-rate-of-return method; the payback method; the internal-rate-of-return method; and the net-present-value method. The first two represent approximate methods for assessing the economic worth of a project. For simplicity, we assume throughout that the expected cash flows are realized at the end of each year.

AVERAGE RATE OF RETURN

The average rate of return is an accounting method and represents the ratio of the average annual profits after taxes to the average investment in the project. In our first example, the average annual book earnings for the five-year period are \$2,000; and the average net investment in the project, assuming straight-line depreciation, is \$18,000/2, or 9,000. Therefore,

$$\text{Average rate of return} = \frac{\$2,000}{\$9,000} = 22.22 \text{ per cent} \qquad \text{(3-1)}$$

The average-rate-of-return method is sometimes based upon the original investment rather than upon the average investment. In the above example, the average rate of return would be \$2,000/\$18,000 = 11.11 per cent under this version of the average-rate-of-return method.

The principal virtue of the average rate of return is its simplicity; it makes use of readily available accounting information. Once the average

rate of return for a proposal has been calculated, it may be compared with a required, or cutoff, rate of return to determine if a particular proposal should be accepted or rejected. The principal shortcomings of the method are that it is based upon accounting income rather than upon cash flows and that it fails to take account of the timing of cash inflows and outflows. The time value of money is ignored; benefits in the last year are valued the same as benefits in the first year.

Suppose that we have three investment proposals, each costing $9,000 and each having an economic and depreciable life of three years. Assume that these proposals are expected to provide the following book profits and cash flows over the next three years:

	Project A		Project B		Project C	
Period	Book Profit	Net Cash Flow	Book Profit	Net Cash Flow	Book Profit	Net Cash Flow
1	$3,000	$6,000	$2,000	$5,000	$1,000	$4,000
2	2,000	5,000	2,000	5,000	2,000	5,000
3	1,000	4,000	2,000	5,000	3,000	6,000

If straight-line depreciation is employed, each proposal will have the same average rate of return—$2,000/4,500, or 44 per cent. However, few, if any, firms would be equally favorable to all three projects. Most would prefer Project A, which provides a larger portion of total cash benefits in the first year. For this reason, the average rate of return leaves much to be desired as a method for project selection.

PAYBACK METHOD

The payback period of an investment project tells us the number of years required to recover our initial cash investment. It is the ratio of the initial fixed investment over the annual cash inflows for the recovery period. For our example,

$$\text{Payback period} = \frac{\$18,000}{5,600} = 3.2 \text{ years} \qquad (3\text{-}2)$$

If the annual cash inflows are not equal, the job of calculation is somewhat more difficult. Suppose that annual cash inflows are $4,000 in the first year, $6,000 in the second and third years, and $4,000 in the fourth and fifth years. In the first three years, $16,000 of the original investment will be recovered, followed by $4,000 in the fourth year. With an initial cash investment of $18,000, the payback period is: 3 years + ($2,000/ $4,000), or 3½ years.

If the payback period calculated is less than some maximum acceptable payback period, the proposal is accepted; if not, it is rejected. For

example, if the required payback period is four years, the project in our example would be accepted. The major shortcoming of the payback method is that it fails to consider cash flows after the payback period; consequently, it cannot be regarded as a measure of profitability. Two proposals costing $10,000 each would have the same payback period if they both had annual net cash inflows of $5,000 in the first two years. However, one project might be expected to provide no cash flows after two years, while the other might be expected to provide cash flows of $5,000 in each of the next three years. Thus, the payback method can be very deceptive as a yardstick of profitability. In addition to this short-coming, the method does not take account of the magnitude or timing of cash flows during the payback period; it considers only the recovery period as a whole.

Nevertheless, the payback method continues in use, frequently as a supplement to other, more sophisticated methods. It does afford management limited insight into the risk and liquidity of a project. The shorter the payback period, supposedly, the less risky the project, and the greater its liquidity. The company that is cash poor may find the method to be very useful in gauging the early recovery of funds invested. There is some merit to its use in this regard, but the method does not take into account the dispersion of possible outcomes — only the magnitude and timing of the expected value of these outcomes relative to the original investment. Therefore, it cannot be considered an adequate indicator of risk. When the payback method is used, it is more appropriately treated as a constraint to be satisfied than as a profitability measure to be maximized.[4]

PRESENT VALUES

Because of the various shortcomings in the average-rate-of-return and payback methods described above, it generally is felt that discounted cash-flow methods provide a more objective basis for evaluating and selecting investment projects. These methods take account of both the magnitude and the timing of expected cash flows in each period of a project's life. In any economy in which capital has value, the time value of money is an important concept. For example, stockholders place a higher value on an investment project that promises returns over the next five years than on a project that promises identical returns for years six through ten. Consequently, the timing of expected future cash flows is extremely important in the investment decision.

[4] See H. Martin Weingartner, "Some New Views on the Payback Period and Capital Budgeting Decisions," *Management Science,* 15 (August, 1969), 594–607. For a detailed analysis of the payback method and the accounting-rate-of-return method as approximations of the internal rate of return, see Marshall Sarnat and Haim Levy, "The Relationship of Rules of Thumb to the Internal Rate of Return: A Restatement and Generalization," *Journal of Finance,* XXIV (June, 1969), 479–89.

Discounted cash-flow methods enable us to isolate differences in the timing of cash flows for various projects by discounting these cash flows to their present values. The present values can then be analyzed to determine the desirability of the projects. The two discounted cash-flow methods are the internal-rate-of-return method and the present-value method, and we consider each in turn.

Before proceeding, however, it is necessary to digress somewhat and examine briefly the construction of present-value tables. A more extensive discussion of the mathematics of compound interest and bond yields is found in Appendix A to this chapter. A present-value table is nothing more than a bond-yield table that takes account of compound interest. For example, the present value of $1 received at the end of the year n is

$$PV = \frac{1}{(1+k)^n} \qquad (3\text{-}3)$$

where PV is the present value and k the discount rate. To illustrate the construction of a present-value table, we make a few calculations using a discount rate of 10 per cent. Suppose that we wish to know the present value of $1 received one year from today. The formula is

$$PV = \frac{1}{(1+0.10)} = 0.90909 \qquad (3\text{-}4)$$

Similarly, if we wish to know the present value of $1 received two years from today, the formula is

$$PV = \frac{1}{(1+0.10)^2} = \frac{1}{1.21} = 0.82645 \qquad (3\text{-}5)$$

Fortunately, a present-value table has been prepared that relieves us of making these calculations every time we have a problem to solve; it is shown in Table A-1 in the appendix at the end of the book. We see in the table that for a 10 per cent discount rate, the discount factors for one and two years in the future are 0.90909 and 0.82645, respectively—just as we calculated by hand.

If we had an uneven series of cash flows—$1 one year hence, $3 two years hence, and $2 three years from now—the present value of this series, using a 10 per cent discount rate, would be:

PV of $1 to be received at the end of one year $1(0.90909) =   0.90909
PV of $3 to be received at the end of two years $3(0.82645) =   2.47935
PV of $2 to be received at the end of three years $2(0.75131) =   <u>1.50262</u>
                Present value of series $4.89106

Given a present-value table, we are able to calculate the present value for any series of future cash flows in the above manner.

However, the procedure can be simplified for a series if the cash flows

in each future period are the same. A series of this sort is known as an *annuity*. Suppose that in a series of future cash flows, $1 was to be received at the end of each of the next three years. The calculation of the present value of this stream, using the above procedure would be:

PV of $1 to be received in one year   =   0.90909
PV of $1 to be received in two years   =   0.82645
PV of $1 to be received in three years =   0.75131
                    Present value of series $2.48685

With an even series of future cash flows, it is unnecessary to go through these calculations. The discount factor, 2.48685, can be applied directly. We would simply multiply $1 by 2.48685 to obtain $2.48685. Present-value tables for even series of cash flows have been developed that allow us to look up the appropriate compound discount factor. An example is shown in Table A-2 in the appendix at the end of the book. We note that the discount factor for an even series of cash flows for three years, using a 10 per cent discount rate, is 2.4868 – as we calculated. Thus, for an even series of cash flows, we simply multiply the appropriate discount factor times the cash flow. If we wish to know the present value, using an 8 per cent discount rate, of a future stream of $5 cash flows to be received at the end of each year over a four-year period, the calculation would be:

$$\$5(3.3121) = \$16.56 \qquad (3\text{-}6)$$

Using the present-value tables shown in Tables A-1 and A-2 at the end of the book, we are able to calculate the present value of various future streams of cash flows.

INTERNAL-RATE-OF-RETURN METHOD

The internal rate of return for an investment proposal is the discount rate that equates the present value of the expected cash outflows with the present value of the expected inflows. Mathematically, it is represented by that rate, r, such that

$$\sum_{t=0}^{n} \left[\frac{A_t}{(1+r)^t} \right] = 0 \qquad (3\text{-}7)$$

where A_t is the cash flow for period t, whether it be a net cash outflow or inflow, and n is the last period in which a cash flow is expected. If the initial cash outlay or cost occurs at time 0, Eq. (3-7) can be expressed as

$$A_0 = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \ldots + \frac{A_n}{(1+r)^n} \qquad (3\text{-}8)$$

Thus, r is the rate that discounts the stream of future cash flows – A_1

through A_n — to equal the initial outlay at time $0 - A_0$. For our example, the problem can be expressed as

$$18,000 = \frac{5,600}{(1+r)} + \frac{5,600}{(1+r)^2} + \frac{5,600}{(1+r)^3} + \frac{5,600}{(1+r)^4} + \frac{5,600}{(1+r)^5} \quad (3\text{-}9)$$

Solving for the internal rate of return, r, sometimes involves a trial-and-error procedure using present-value tables. Fortunately, there are computer programs for solving for the internal rate of return; and these programs eliminate the arduous computations involved in the trial-and-error procedure. To illustrate the latter method, however, consider again our example. The cash-flow stream is represented by an even series of cash flows of $5,600, to be received at the end of each of the next five years. We want to determine the discount factor that, when multiplied by $5,600, equals the cash outlay of $18,000 at time 0. Suppose that we start with three discount rates — 14 per cent, 16 per cent, and 18 per cent — and calculate the present value of the cash-flow stream. Using the different discount rates, we find:

Discount Rate	Discount Factor	Cash Flow Each Year	Present Value of Stream
18%	3.1272	$5,600	$17,512.32
16	3.2743	5,600	18,336.08
14	3.4331	5,600	19,225.36

When we compare the present value of the stream with the initial outlay of $18,000, we see that the internal rate of return necessary to discount the stream to $18,000 falls between 16 and 18 per cent, being closer to 16 than to 18 per cent. To approximate the actual rate, we interpolate between 16 and 17 per cent as follows

	Discount Rate	Present Value
	16%	$18,336.08
	17	17,916.08
Difference	1%	$ 420.00

$$\frac{336.08}{420.00} = 0.80 \qquad 16\% + 0.80\% = 16.8\%$$

Thus, the internal rate of return necessary to equate the present value of the cash inflows with the present value of the outflows is approximately 16.8 per cent. It should be noted that interpolation gives only an approximation of the exact per cent; the relationship between the two discount rates is not linear with respect to present value.

When, as above, the cash-flow stream is an even series, and the initial outlay occurs at time 0, there really is no need for trial and error. We simply divide the initial outlay by the cash flow and search for the nearest discount factor. Using our example, we divide $18,000 by $5,600, obtaining 3.214. The nearest discount factor on the five-year row in Table A-2 at the end of the book is 3.2743, and this figure corresponds to a discount rate of 16 per cent. Inasmuch as 3.214 is less than 3.2743, we know that the actual rate lies between 16 and 17 per cent and we interpolate accordingly. When the cash-flow stream is an uneven series, the task is more difficult; and here we must resort to trial and error. However, given practice, a person can come surprisingly close in selecting discount rates from which to start.

When solving for *r,* it is important to recognize the possibility that there may be more than one internal rate of return that equates the present value of the cash inflows with the present value of cash outflows. Although the existence of multiple internal rates of return is unusual, we do examine the problem in Appendix 3B to this chapter.

Acceptance Criterion. The acceptance criterion generally employed with the internal-rate-of-return method is to compare the internal rate of return with a required rate of return, known also as the cutoff, or hurdle, rate. If the internal rate of return exceeds the required rate, the project is accepted; if not, it is rejected. For example, if the required rate of return were 10 per cent and this criterion is used, the investment proposal considered above would be accepted. If the required rate of return is the return investors expect the firm to earn on projects, accepting a project with an internal rate of return in excess of the required rate of return should result in an increase in the market price of the stock, because the firm accepts a project with a return greater than that required to maintain the present market price per share. The required rate of return often is taken to be the firm's cost of capital. Much more will be said in Chapter 4 about relating the investment decision to the objective of the firm. We assume for now that the required rate of return is given.

PRESENT-VALUE METHOD

Like the internal-rate-of-return method, the present-value method is a discounted cash flow approach to capital budgeting. With the present-value method, all cash flows are discounted to present value using the required rate of return. The net-present value of an investment proposal is

$$NPV = \sum_{t=0}^{n} \frac{A_t}{(1 + k)^t} \qquad (3\text{-}10)$$

where k is the required rate of return. If the sum of these discounted

cash flows is equal to, or greater than, 0, the proposal is accepted; if not, it is rejected. Another way to express the acceptance criterion is to say that the project will be accepted if the present value of cash inflows exceeds the present value of cash outflows. The rationale behind the acceptance criterion is the same as that behind the internal-rate-of-return method. If the required rate of return is the return investors expect the firm to earn on investment proposals, and the firm accepts a proposal with a net-present value greater than zero, the market price of the stock should rise. Again, the firm is taking on a project with a return greater than that necessary to leave the market price of the stock unchanged.

If we assume a required rate of return of 10 per cent after taxes, the net-present value of our example problem is

$$NPV = -18,000 + \frac{5,600}{(1.10)} + \frac{5,600}{(1.10)^2} + \frac{5,600}{(1.10)^3} + \frac{5,600}{(1.10)^4} + \frac{5,600}{(1.10)^5} =$$

$$-18,000 + 21,228.48 = \$3,228.48 \qquad (3\text{-}11)$$

An easier way to solve this problem, of course, is by direct reference to Table A-2 in the appendix at the end of the book, where we find the appropriate discount factor—3.7908—and multiply $5,600 by it to obtain $21,228.48. Subtracting the initial outlay of $18,000, we obtain $3,228.48. Inasmuch as the net-present value of this proposal is greater than 0, the proposal should be accepted, using the present-value method.

With the internal rate-of-return method, we are given the cash flows and solve for the rate of discount that equates the present value of the cash inflows with the present value of the outflows. The internal rate of return is then compared with the required rate of return to determine whether the proposal should be accepted. With the present-value method, we are given the cash flows and the required rate of return and solve for the net-present value. The acceptability of the proposal is determined by whether the net-present value is equal to, or greater than, 0.

It is obvious that different net-present values will be given for different required rates of return. With a 10 per cent required rate, the following cash-flow streams have equivalent net-present values—namely, $3,228:

| Year | Proposal (Cash Flows) | | |
	1	2	3
0	−$18,000	−$18,797	−$16,446
1	5,600	4,000	7,000
2	5,600	5,000	6,000
3	5,600	6,000	5,000
4	5,600	7,000	4,000
5	5,600	8,000	3,000

However, with different required rates of return, the net-present values of the proposals are:

Discount Rate	Proposal (Net-Present Value)		
	1	2	3
0%	$10,000	$11,203	$8,554
4	6,930	7,565	6,162
8	4,359	4,546	4,131
10	3,228	3,228	3,228
12	2,187	2,019	2,390
16	336	— 114	888
20	— 1,253	— 1,928	— 418

We see that the relative desirability of the proposals changes with changes in the discount rate. The higher the discount rate, the more valued is the proposal with early cash inflows, proposal 3. The lower the discount rate, the less important the timing of the cash flows and the more valued is the proposal with the greatest absolute amount of cash inflows, proposal 2. The example serves to illustrate the importance of the discount rate used in the calculations. Different answers will be given, depending upon the discount rate employed.

MUTUAL EXCLUSION AND DEPENDENCY

In evaluating a group of investment proposals, it is important to determine whether the proposals are independent of each other. A proposal is said to be mutually exclusive if the acceptance of it precludes the acceptance of one or more other proposals. For example, if the firm is considering investment in one of two temperature-control systems, acceptance of one system will rule out acceptance of the other. Two mutually exclusive proposals cannot both be accepted.

A *contingent* or *dependent* proposal is one whose acceptance depends upon the acceptance of one or more other proposals. An example of a contingent proposal might be an investment in a large machine, which depends upon the construction of an addition to a plant. A combination of investment proposals containing a contingent proposal must contain the proposal(s) upon which it is dependent.[5] When an investment proposal is not independent of all other proposals, this occurrence must be recognized and investment decisions made accordingly.

[5] For a survey of linear programming applications to capital-budgeting problems involving mutually exclusive and contingent proposals, see H. Martin Weingartner, "Capital Budgeting of Interrelated Projects: Survey and Synthesis," *Management Science*, 12 (March, 1966), 485–516.

PROFITABILITY INDEX

The profitability index, or benefit-cost ratio, of a project is the present value of future net cash flows over the initial cash outlay. It can be expressed as

$$PI = \frac{\sum_{t=1}^{n} \frac{A_t}{(1+k)^t}}{A_0} \qquad (3\text{-}12)$$

For our example,

$$PI = \frac{\$21,228.48}{\$18,000.00} = 1.18 \qquad (3\text{-}13)$$

As long as the profitability index is equal to or greater than 1.00, the investment proposal is acceptable. In calculating the profitability index, we compute the net rather than the aggregate index. The aggregate index is simply the present value of cash inflows over the present value of cash outflows. The reason we use the net index is that we wish to differentiate the initial cash outlay from subsequent cash outlays. The initial outlay is discretionary in the sense that the firm can choose to either commit funds to the project or to employ them elsewhere. Subsequent cash outflows are not discretionary in this sense; they are embodied in the system. The aggregate index does not differentiate between the cash outlay the firm has to put up initially and subsequent cash outlays.[6] For this reason, the net profitability index is a more rational measure of profitability than is the aggregate index.

For any given project, the net-present value method and the profitability index give the same accept-reject signals. If we must choose between mutually exclusive projects, however, the net-present value measure is preferred because it expresses in absolute terms the expected economic contribution of the project. In contrast, the profitability index expresses only the relative profitability. To illustrate, consider the following mutually exclusive projects:

	Project A	Project B
Present value of net cash flows	$20,000	$8,000
Initial cash outlay	15,000	5,000
Net present value	5,000	3,000
Profitability index	1.33	1.60

According to the net-present value method, Project *A* would be preferred,

[6] See Bernhard Schwab and Peter Lusztig, "A Comparative Analysis of the Net Present Value and the Benefit-Cost Ratio as Measures of the Economic Desirability of Investments," *Journal of Finance,* XXIV (June, 1969), 507–11.

whereas according to the profitability indexes, it would be Project *B*. Because the net-present value represents the expected economic contribution of a project, we should prefer *A* to *B*. Thus, the net-present value method is the better of the two methods when we must choose between mutually exclusive projects that involve different initial cash outlays.[7]

PRESENT-VALUE METHOD VERSUS INTERNAL-RATE-OF-RETURN METHOD

In general, the present-value and internal-rate-of-return methods lead to the same acceptance or rejection decision. In Figure 3-1, we illustrate graphically the two methods applied to a typical investment project. The figure shows the relationship between the net-present value of a project and the discount rate employed. When the discount rate is 0, net-present value is, simply, the total cash inflows less the total cash outflows of the project. Assuming that total inflows exceed total outflows and that outflows are followed by inflows, the typical project will have the highest net-present value when the discount rate is 0. As the discount rate increases, the present value of future cash inflows decreases relative to the present value of cash outflows. At the intercept, the net-present value of the project is 0. The discount rate at that point represents the internal rate of return that equates the present value of cash inflows with the present value of cash outflows. For discount rates greater than the internal rate of return, the net-present value of the project is negative.[8]

If the required rate of return is less than the internal rate of return, we would accept the project using either method. Suppose that the required rate were 10 per cent. As seen in Figure 3-1, the net-present value of the project then would be *Y*. Inasmuch as *Y* is greater than 0, we would accept the project using the present-value method. Similarly, we would accept the project using the internal-rate-of-return method because the internal rate exceeds the required rate. For required rates greater than the internal rate of return, we would reject the project under either method. Thus, we see that the internal-rate-of-return and present-value methods give us identical answers with respect to the acceptance or rejection of an investment project.

DIFFERENCES BETWEEN METHODS

However, important differences exist between the methods, and they must be recognized. When two investment proposals are mutually exclusive, so that we can select only one, the two methods may give con-

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[7] For further analysis of this point, see *ibid.,* pp. 511–16.

[8] Again, we must recognize the possibility of multiple internal rates of return. See Appendix B to this chapter.
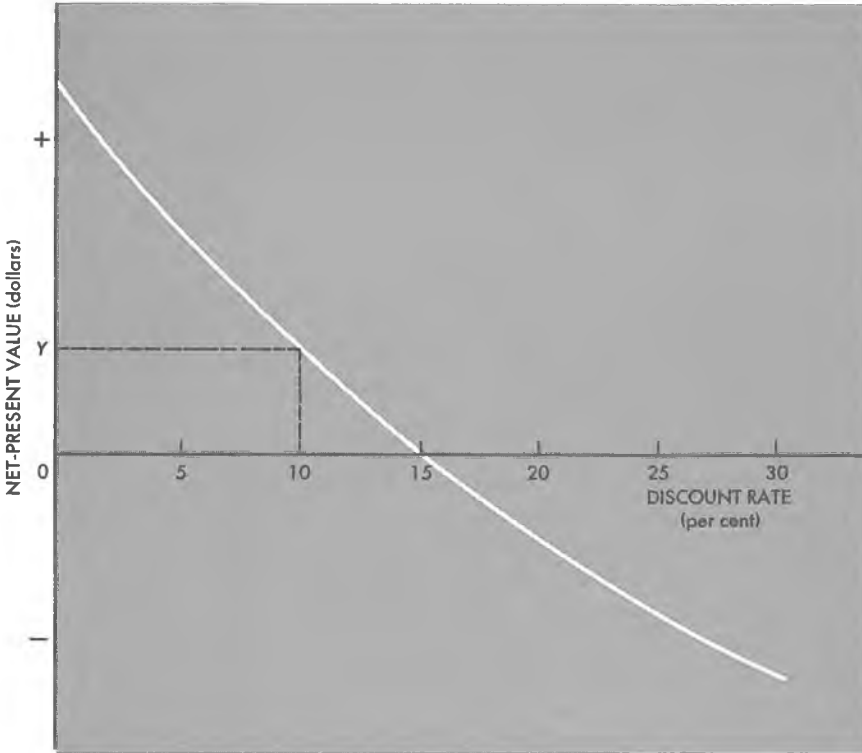
FIGURE 3-1
Relation between discount rate and net-present value

tradictory results. To illustrate the nature of the problem, suppose a firm had two mutually exclusive investment proposals that were expected to generate the following cash flows:[9]

	Cash Flows	
Year	Proposal A	Proposal B
0	−$23,616	−$23,616
1	10,000	0
2	10,000	5,000
3	10,000	10,000
4	10,000	32,675

Internal rates of return for proposals *A* and *B* are 25 per cent and 22 per cent, respectively. If the required rate of return is 10 per cent, however, and we use this figure as our discount rate, the net-present values

[9] See Solomon, *The Theory of Financial Management* (New York: Columbia University Press, 1963), pp. 134–35.

of proposals A and B are \$8,083 and \$10,347, respectively. Thus, proposal A is preferred if we use the internal-rate-of-return method, whereas proposal B is preferred if we use the present-value method. If we can choose but one of these proposals, obviously, we have a conflict.

Reinvestment Rate. The conflict between these two methods is due to different assumptions with respect to the reinvestment rate on funds released from the proposals. The internal-rate-of-return method implies that funds are reinvested at the internal rate of return over the remaining life of the proposal. For proposal A, the assumption is that cash flows of \$10,000 at the end of years 1, 2, and 3 can be reinvested to earn a return of 25 per cent, compounded annually. The present-value method implies reinvestment at a rate equivalent to the required rate of return used as the discount rate. Because of these differing assumptions, the two methods can give different rankings of investment proposals as we have seen.

To illustrate further the nature of the problem, consider two additional mutually exclusive proposals with the following cash flows

| Time | Cash Flows | |
	Proposal C	Proposal D
0	−\$155.22	−\$155.22
1	100.00	0
2	0	0
3	100.00	221.00

The net-present value of each of these proposals is \$10.82 if we assume a required rate of return of 10 per cent. However, we would be indifferent between the two proposals only if the firm had opportunities for reinvestment at a rate of 10 per cent. This concept is illustrated in Figure 3-2, where the functional relationship between net-present value and the discount rate is graphed for the two proposals. The intercepts on the 0 horizontal line represent the internal rates of return of the two proposals that equate their net-present values to 0. For proposal C, the internal rate of return is 14 per cent; for proposal D, it is 12.5 per cent. The intercepts on the vertical axis represent total cash inflows less total cash outflows for the two proposals, because the discount rate is 0. We see that proposal D ranks higher than proposal C if the required rate of return is below 10 per cent and lower if it is above 10 per cent. At the point of intersection, 10 per cent, the proposals have identical net-present values. Given a reinvestment rate of 10 per cent, then, the two proposals would have equal ranking. For reinvestment rates other than this per cent, we would prefer one proposal to the other. In a similar manner, other mutually exclusive investment proposals can be evaluated according to the intersections.[10]

[10] Under certain circumstances, it is possible to have multiple intersections. As these circumstances tend to be unusual, we will not go into an analysis of them.
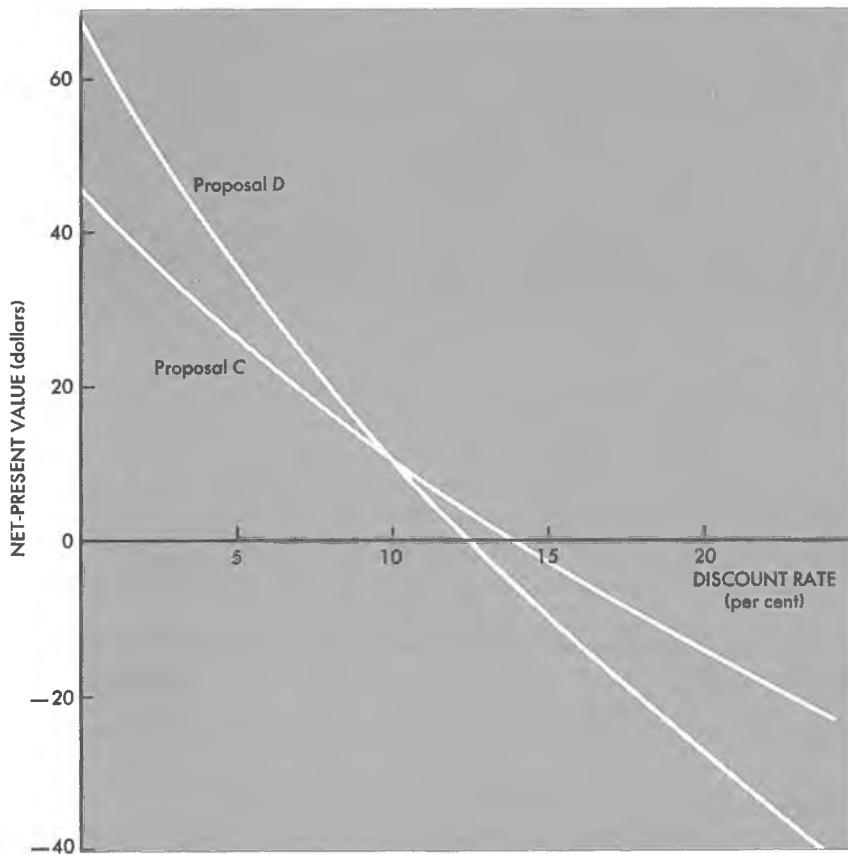
FIGURE 3-2
Relation between discount rate and net-present values, proposals C and D

WHICH METHOD PROVIDES
THE BEST RESULTS?

The question to be answered is which method — the internal-rate-of-return method or the present-value method — is best for purposes of evaluating investment proposals. Actually, the question hinges upon what is the appropriate rate of reinvestment for the intermediate cash flows. We have demonstrated that the internal-rate-of-return method implies a reinvestment rate equal to the internal rate of return, whereas the present-value method implies a reinvestment rate equal to the required rate of return used as the discount factor. Perhaps the ideal solution would be to take the expected rate of reinvestment for each period and calculate a terminal value.[11] However, this procedure involves

[11] See James T. S. Porterfield, *Investment Decisions and Capital Costs* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965), pp. 38–41.

additional computational steps that many do not feel to be worthwhile.

If a choice must be made, the present-value method generally is considered to be superior theoretically.[12] With the internal-rate-of-return method, the implied reinvestment rate will differ depending upon the cash-flow stream for each investment proposal under consideration. For proposals with a high internal rate of return, a high reinvestment rate is assumed; for proposals with a low internal rate of return, a low reinvestment rate is assumed. Only rarely will the internal rate of return calculated represent the relevant rate for reinvestment of intermediate cash flows. With the present-value method, however, the implied reinvestment rate—namely, the required rate of return—is the same for each proposal. In essence, this reinvestment rate represents the minimum return on opportunities available to the firm; for, generally, no proposal will be accepted that yields less than this rate. The reinvestment rate implied by the present-value method may be conservative, but it has the virtue of being applied consistently to all investment proposals. To the extent that we can regard the required rate of return, k, as an approximate measure of the opportunity rate for reinvestment, the present-value method is preferred over the internal-rate-of-return method.

However, the internal-rate-of-return method can be modified so that it involves an incremental type of analysis. When one is faced with two mutually exclusive investment proposals, both of whose internal rates of return exceed the required rate of return, the following additional steps could be undertaken:[13]

1. Calculate the differential cash flows between the two proposals.

2. If the internal rate of return on the differential cash flows exceeds the required rate of return, the project with the greater nondiscounted net cash flows should be selected.

To illustrate, consider again proposals A and B and the differential cash flows:

	Cash Flows		
Year	Proposal A	Proposal B	B less A
0	—$23,616	—$23,616	0
1	10,000	0	—$10,000
2	10,000	5,000	— 5,000
3	10,000	10,000	0
4	10,000	32,675	22,675

The internal rate of return that equates outflows of $10,000 and $5,000 at the end of years 1 and 2, respectively, with an inflow of $22,675 at the end of year 4 is 16.65 per cent. As this rate exceeds the required rate of return of 10 per cent, proposal B should be selected, despite the fact that its internal rate of return is 22 per cent compared with 25 per cent for proposal A.

Thus, the internal-rate-of-return method can be modified to deal with the special case of mutually exclusive investment proposals. Given such a modification, it provides a decision identical to that given by the net-present-value method. Many financial managers feel that, in practice, the internal rate of return is easier to visualize and to interpret than is the net-present value measure. In addition, one does not have to specify a required rate of return in the calculations. To the extent that the required rate of return is but a rough estimate, the use of the internal-rate-of-return method may permit a more realistic comparison of projects. The principal shortcoming of the method is the possibility of multiple internal rates of return, a subject that we take up in Appendix B to this chapter.

CASH-FLOW INFORMATION REVISITED

In our replacement example, we assumed straight-line depreciation and no salvage value. For continuity, we then investigated the various methods for evaluating investment proposals, using this example as our illustration throughout. We need now to digress for a while in order to examine the effect of accelerated depreciation and of salvage value upon the cash flows. Recall that the replacement of the old machine with a new one was expected to result in annual cash savings of $7,600 a year over the next five years. Straight-line depreciation charges on the new machine were $4,000 a year; and when we subtracted $400 annual depreciation charges on the old machine, we obtained incremental depreciation charges of $3,600 a year. Thus, the additional income before taxes was $4,000, resulting in additional taxes of $2,000 a year, assuming a 50 per cent tax rate. When this $2,000 outlay was subtracted from the $7,600 cash savings, the net cash inflow became $5,600 a year. Consider now the modification of the example occasioned by accelerated depreciation and salvage value.

ACCELERATED DEPRECIATION

Accelerated depreciation can be either the double-declining-balance method or the sum-of-the-years'-digits method. The formula for the double-declining-balance method is $2(1/n)$, or 2 divided by the depreciable life of the asset. In the case of a five-year depreciable life, the annual depreciation charge would be $2(1/5)$ or 40 per cent of the depreci-

ated book value balance at the beginning of the year. If the depreciable value were $10,000 initially, depreciation charges would be $4,000 in the first year, $2,400 in the second (40 per cent of $10,000 − 4,000), and so on. A second method of accelerated depreciation is the sum-of-the-years'-digits method. Here, we add up each year of the depreciable life of the asset to obtain the denominator. In our case, the denominator is $1 + 2 + 3 + 4 + 5 = 15$. The numerator is the depreciable life for the first year, the depreciable life minus 1 for the second year, and so on. Thus, the first-year depreciation charge on an asset having a five-year depreciable life would be 5/15, followed by 4/15, 3/15, 2/15, and 1/15 for the remaining years. Given a depreciable value of $20,000 in our example and a five-year depreciable life, the annual depreciation charges for these accelerated methods are

Year	Double-Declining Balance (40%)	Sum of the Years' Digits
1	$ 8,000.00	$ 6,666.67
2	4,800.00	5,333.33
3	2,880.00	4,000.00
4	1,728.00	2,666.67
5	2,592.00	1,333.33
	$20,000.00	$20,000.00

If the old machine were still depreciated on a straight-line basis of $400 a year, the additional depreciation charge and annual net cash flow for these two methods over the five-year period would be

	Double-Declining Balance (40%)		Sum of the Years' Digits	
Year	Additional Depreciation	Net Cash Flow	Additional Depreciation	Net Cash Flow
1	$7,600.00	$7,600.00	$6,266.67	$6,933.33
2	4,400.00	6,000.00	4,933.33	6,266.67
3	2,480.00	5,040.00	3,600.00	5,600.00
4	1,328.00	4,464.00	2,266.67	4,933.33
5	2,192.00	4,896.00	933.33	4,266.67

We see that the use of accelerated depreciation increases the depreciation charge in the early years of the project's life over what it would be if straight-line deprecation were used, resulting in lower taxes and higher cash flows in these years. The use of accelerated depreciation changes the timing of cash flows from what they would be if straight-line depreciation were used. If money does have a time value, accelerated depreciation is advantageous to the firm. For example, the internal rate

of return using the double-declining-balance method is 18.7 per cent, compared with 16.8 per cent when the straight-line method of depreciation is employed. The net-present value, assuming a required rate of return of 10 per cent, is $3,743.33 under the double-declining-balance method, compared with $3,228.48 under the straight-line method. For the sum-of-the-years'-digit method, the internal rate of return is 18.5 per cent and the net-present value, $3,708.30. Thus, the method of depreciation affects the timing of cash flows and the resulting attractiveness of the investment project. The total amount of taxes is not reduced with accelerated depreciation; taxes simply are paid at a date later than would be the case with straight-line depreciation.

SALVAGE VALUE AND TAXES

The cash-flow pattern also will differ from that shown in the example if the new machine is expected to have a salvage, or scrap, value at the end of the five-year period. When there is a salvage value it may affect depreciation charges as well as the cash flow in the last year. Assume that the salvage value of the new machine is expected to be $2,000 at the end of the fifth year. The total depreciable value becomes $18,000 instead of $20,000; and, assuming straight-line depreciation, annual depreciation charges for the new machine become $3,600 instead of $4,000. If we follow through with the previous calculations, we find that the annual net cash flow for years 1 through 4 is $5,400 instead of $5,600. The net cash flow in the fifth year is $5,400, plus the salvage value of $2,000, or $7,400.

In addition to the salvage value, there are other tax considerations. If the old machine can be sold for more than its depreciated book value, the difference is subject to income taxes if the sales price is less than the machine's original cost. For example, if the old machine could be sold for $4,000 instead of $2,000, there would be a gain of $2,000. If the tax rate were 50 per cent, the total cash proceeds realized from the sale of the old machine would be $3,000. Thus, the net cash outlay for the project would be $17,000 instead of $18,000. There are many other variations that we could illustrate, but these examples are enough to show that tax considerations are very important in calculating the cash flow.

CAPITAL RATIONING

Capital rationing occurs anytime there is a budget ceiling, or constraint, on the amount of funds that can be invested during a specific period of time, such as a year.. Such constraints are prevalent in a number of firms, particularly in those that have a policy of financing all capital expenditures internally. Another example of capital rationing is when a

division of a large company is allowed to make capital expenditures only up to a specified budget ceiling, over which the division usually has no control. With a capital rationing constraint, the firm attempts to select the combination of investment proposals that will provide the greatest profitability.

To illustrate, suppose a firm had the following investment opportunities, ranked in descending order of profitability indexes (the ratio of the present value of future net cash flows over the initial cash outlay):

Proposal	Profitability Index	Initial Outlay
4	1.25	$400,000
7	1.19	100,000
2	1.16	175,000
3	1.14	125,000
6	1.09	200,000
5	1.05	100,000
1	0.97	150,000

If the budget ceiling for initial outlays during the present period were $1 million, and the proposals were independent of each other, we would select proposals in descending order of profitability until the budget was exhausted. With capital rationing, we would accept the first five proposals, totaling $1 million in initial outlays. In other words, we do not necessarily invest in all proposals that increase the net-present value of the firm; we invest in an acceptable proposal only if the budget constraint allows such an investment. In the above example, we do not invest in proposal 5, even though the profitability index in excess of 1 would suggest its acceptance. The critical aspect of the capital rationing constraint illustrated is that capital expenditures during a period are strictly limited by the budget ceiling, regardless of the number of attractive investment opportunities.

MUTUALLY EXCLUSIVE AND CONTINGENT PROPOSALS

If any of the investment proposals are mutually exclusive or contingent, we need to take account of this factor in the allocation of capital. For example, if proposals 3 and 7 are mutually exclusive, we cannot accept both. Our choice, of course, would be proposal 7, for it is the more profitable. By the same token, if acceptance of proposal 4—the most profitable investment opportunity—depends upon the acceptance of proposal 1—the least profitable—we must evaluate both proposals together to determine their joint attractiveness. The weighted-average profitability index of proposals 1 and 4 is 1.17; this figure, rather than

the two separate profitability indexes, must be used in our analysis. Given the above example, the weighted-average profitability index is sufficiently high to justify investment in both proposals, despite the fact that proposal 1 has a profitability index of less than 1.00, indicating a negative net-present value.

SIZE OF INITIAL OUTLAY

In addition to the possibility that mutual exclusion and contingency may disrupt the order of selection of ranked investment proposals, the size of the initial outlay must be considered. Under capital rationing, the objective is to select the combination of investment proposals that provides the highest net-present value, subject to the budget constraint for the period. If this constraint is strictly enforced, it may be better to accept several smaller, less profitable proposals that allow full utilization of the budget than to accept one large proposal that results in part of the budget's being unused. Suppose that we had the following investment opportunities:

Proposal	Profitability Index	Initial Outlay
3	1.15	$200,000
1	1.13	125,000
2	1.11	175,000
4	1.08	150,000

If the budget constraint were $300,000 for the period, we should accept proposals 1 and 2 rather than proposal 3, despite the fact that the profitability indexes are lower for the first two proposals. The total net-present value of proposals 1 and 2 is

Proposal 1	$125,000(1.13 − 1) = $16,250
Proposal 2	$175,000(1.11 − 1) = $19,250
	$35,500

whereas the net-present value of proposal 3 is

Proposal 3	$200,000(1.15 − 1) = $30,000

The total net-present value of proposals 1 and 2 is higher than that of proposal 3 because we are able to use more of the available budget. Thus, the "lumpiness" of the initial outlays makes full utilization of the budget an important consideration for the firm that rations capital. Implied in the evaluation above is that uninvested capital has a net-present value of zero; that is, it corresponds to an investment yielding the required rate of return. If the net-present value of uninvested capital is less, the full utilization of the budget becomes a more important consideration.

A fixed one-period constraint is highly artificial. Seldom will a budget be set so rigidly that it does not provide for some flexibility. In addition, the cost of certain investment projects may be spread over several years. Consequently, we must consider more than just a one-period constraint. With a multiperiod analysis, the postponement of investment proposals is possible. If there is an abundance of acceptable investment proposals this year, the least profitable may be postponed until a subsequent period when the budget will permit investment. In postponing, it is important to take account of possible reductions in the expected profitability of the proposal. A project with a profitability index of 1.10 in the current period may have an expected profitability index of only 1.00 in the next period. In particular, a delay in undertaking a new product may reduce seriously the profitability of the investment project. Consequently, it may be of more value to the firm to postpone a project with a higher profitability index in the current period, say 1.18, because the project is relatively unaffected by postponement.[14]

Moreover, a one-period analysis does not take account of intermediate cash flows generated by a project. Some projects provide relatively high net cash flows in the early years; these cash flows serve to reduce the budget constraints in the early years because they may be used to finance other investment projects. A project with net cash flows expected to occur entirely in the first several years may be more valuable to the firm than a project with an identical initial outlay and profitability index, but which is expected to produce net cash flows over a longer span of time. For the reason discussed above, we should consider more than one period in the allocation of limited capital to investment projects.

Multiperiod mathematical programming models have been developed for selecting investment proposals under conditions of capital rationing. The objective is to maximize the net-present value of a combination of investment proposals subject to budget constraints in various future periods. These models are able to choose efficiently the most profitable combination of investment proposals, taking account of the problems of mutual exclusion, contingency, and the size of initial outlay. In Appendix 3C to this chapter, we examine various mathematical programming approaches to capital budgeting.

OPPORTUNITY COSTS AND
OBJECTIONS TO CAPITAL RATIONING

The cost to the firm of a budget ceiling might be regarded as the opportunity foregone on the next most profitable investment after the cutoff. In our first example, the opportunity foregone by the $1 million budget

[14]See G. David Quirin, *The Capital Expenditure Decision* (Homewood, Ill.: Richard D. Irwin, Inc., 1967), pp. 181–84.

ceiling is proposal 5, which has a profitability index of 1.05. While all cash flows are discounted at the required rate of return, we do not necessarily accept proposals that provide positive net-present values. Acceptance is determined by the budget constraint, which tells us which proposals can be accepted before the budget is exhausted. To be sure, the required rate of return sets a lower limit; we would not accept proposals yielding less than this required rate even if the budget were not exhausted.[15] However, we may reject projects that provide positive net-present values, as was shown with proposal 5. Under capital rationing, the required rate of return is not the acceptance criterion. Should it be used, then, as the discount rate in present-value calculations, or should the opportunity cost be used? The implied discount rate in any budget period is the yield foregone on the most profitable investment opportunity rejected, or the required rate of return, whichever is the higher.[16] This implied discount rate can vary significantly from period to period, depending upon variations in the total amount of investment projects from period to period and in the budget constraints.

Capital rationing usually results in an investment policy that is less than optimal. In some periods, the firm accepts projects down to its required rate of return; in others, it may reject projects that would provide returns substantially in excess of the required rate. If the required rate of return corresponds to the cost of capital, and the firm actually can raise capital at that approximate real cost, should it not invest in all projects yielding more than the required rate of return? If it rations capital and does not invest in all projects yielding more than the required rate, is it not foregoing opportunities that would enhance the market price of its stock?

From a theoretical standpoint, a firm should accept all projects yielding more than its real cost of capital.[17] By so doing, it should increase the market price per share because projects are accepted that will provide a return higher than that necessary to maintain the present market price

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[15] The exception is contingent projects, as illustrated earlier.

[16] In a mathematical programming approach, the opportunity rate for investment corresponds to the dual variable. Thus, the discount rate depends upon the optimal solution, which in turn depends upon the discount rate employed. For an analysis of this problem of mutual dependence and various proposals for its resolution, see William J. Baumol and Richard E. Quandt, "Investment and Discount Rates under Capital Rationing—a Programming Approach," *The Economic Journal*, LXXV (June, 1965), 317–29; H. Martin Weingartner, "Criteria for Programming Investment Project Selection," *The Journal of Industrial Economics*, XV (November, 1966), 65–76; Peter Lusztig and Bernhard Schwab, "A Note on the Application of Linear Programming to Capital Budgeting," *Journal of Financial and Quantitative Analysis*, III (December, 1968), 427–31; and Willard T. Carleton, "Linear Programming and Capital Budgeting Models: A New Interpretation," *Journal of Finance*, XXIV (December, 1969), 825–33.

[17] We shall examine the rationale for this criterion in Chapter 4. For theoretical support for the notion that it is the appropriate discount rate even under conditions of capital rationing, see Edwin J. Elton, "Capital Rationing and External Discount Rates," *Journal of Finance*, XXV (June, 1970), 573–84.

per share. This proposition assumes that the firm actually can raise capital, within reasonable limits, at the approximate cost-of-capital rate. Certainly, unlimited amounts of capital are not available at any one cost. However, most firms are involved in a more or less continual process of making decisions to undertake capital expenditures and to finance these expenditures.[18] Given the assumptions above, the firm should accept all proposals yielding more than its cost of capital and raise capital to finance these proposals at that approximate real cost. Certainly, there are circumstances that complicate the use of this rule. However, in general, this policy should tend to maximize the market price of the stock over the long run. If capital is rationed, and projects are rejected that would yield more than the real cost of capital of a firm, its investment policy, by definition, is less than optimal. Management could increase the value of the firm to the shareholders by accepting these projects.

SUMMARY

In this chapter, we have examined various methods of capital budgeting, concentrating in particular on the internal-rate-of-return and present-value methods. An important topic taken up initially was the collection of the cash-flow information essential for the evaluation of investment proposals. Capital-budgeting methods, including the average-rate-of-return and payback methods, were evaluated under the assumption that the acceptance of any investment proposal does not change the total business-risk complexion of the firm. It was shown that the two discounted cash-flow methods — internal rate of return and net-present value — were the only appropriate means by which to judge the economic contribution of an investment proposal.

The important distinction between the internal-rate-of-return method and the present-value method is the implied reinvestment rate. Depending upon the situation, contrary answers can be given with respect to the acceptance of mutually exclusive investment proposals. On theoretical grounds, a case can be made for the superiority of the present-value method. The problem of capital rationing was examined, and we concluded that such a policy is likely to result in investment decisions that are less than optimal.

While the capital-budgeting methods discussed in this chapter appear to be very exact, it should be pointed out that we are able only to approximate the true value of an investment proposal to the firm. The meaningfulness of the values we calculate is only as good as our cash-flow estimates and the required rate of return employed. In Chapters 5 and 6, we consider the cash-flow estimates in greater detail; in Chapter 4,

[18] If the real cost of capital does rise, this rise must be taken into consideration. See Chapter 4.

we evaluate the required rate of return. Together, these chapters provide the basis for making sound investment decisions in keeping with the objective of maximizing the market price of the company's stock. In subsequent chapters, the link between investment in capital projects and the objective of the firm will become clearer.

APPENDIX 3A

The Mathematics of Compound Interest, Bond Yields, and Perpetuities

The present-value formulations in this chapter are analogous to compound interest on a bank deposit. If \$1 is deposited in a savings account at a bank that pays r per cent a year compounded annually, the initial deposit at the end of one year increases to

$$TV_1 = \$1(1 + r) \tag{3A-1}$$

where TV_1 is the terminal value at the end of one year. At the end of two years, the terminal value is

$$TV_2 = \$1(1 + r)^2 \tag{3A-2}$$

Similarly, at the end of n years, the terminal value of the deposit is

$$TV_n = \$1(1 + r)^n \tag{3A-3}$$

If interest is paid more than once a year and compounded, Eq. (3A-3) must be modified. For example, suppose that the bank paid interest semiannually on a compound basis. The terminal value at the end of six months would be

$$TV_{1/2} = \$1\left(1 + \frac{r}{2}\right) \tag{3A-4}$$

and at the end of one year

$$TV_1 = \$1\left(1 + \frac{r}{2}\right)^2 \tag{3A-5}$$

If the interest rate paid were 4 per cent compounded semiannually, the terminal value of a \$1 deposit at the end of one year would be

$$TV_1 = \$1\left(1 + \frac{0.04}{2}\right)^2 = \$1(1.02)^2 = \$1.0404 \tag{3A-6}$$

The terminal value at the end of n years for a \$1 deposit compounded m times a year is

$$TV_n = \$1\left(1 + \frac{r}{m}\right)^{mn} \tag{3A-7}$$

Thus, if interest were compounded daily on the basis of a 365-day year, the terminal value of a \$1 deposit at the end of n years would be

$$TV_n = \$1\left(1 + \frac{r}{365}\right)^{365n} \tag{3A-8}$$

As m approaches infinity, the term $\left(1 + \frac{r}{m}\right)^{mn}$ can be expressed as e^{rn}, where e is approximately 2.71828 and is defined as:

$$e = \lim_{m \to \infty}\left(1 + \frac{1}{m}\right)^m \tag{3A-9}$$

Thus, the terminal value at the end of n years where interest is compounded continuously is

$$TV_n = \$1 e^{rn} \tag{3A-10}$$

Variations of this formula are seen frequently in the theoretical literature on growth stocks, whose earnings or dividends are expected to grow continuously at the rate g.

In the development of this chapter, we assumed that for purposes of calculating present values, interest was compounded annually. It will be recalled that the formula for the present value of a cash flow A_n received at the end of n years was

$$PV = \frac{A_n}{(1 + k)^n} \tag{3A-11}$$

If interest is compounded continuously instead of annually, the formula for the present value of a cash flow received at the end of the nth year becomes:

$$PV = \frac{A_n}{e^{kn}} = A_n e^{-kn} \tag{3A-12}$$

The present value of a continuous stream of cash flows is[19]

$$PV = \int_0^n A_t e^{-kt} dt \tag{3A-13}$$

BOND YIELDS

The yield to maturity on a bond is the same as the internal rate of return — namely, the rate of discount that equates the present value of all future interest and principal payments with the market price of the bond. If interest payments are made at the end of each year, and the face value of the bond is \$1,000, we solve the following equation for r, the yield to maturity

[19] For further discussion of compound interest, see James E. Howell and Daniel Teichroew, *Mathematical Analysis for Business Decisions* (Homewood, Ill.: Richard D. Irwin, Inc., 1963), Chapter 10.

$$P = \frac{I}{(1+r)} + \frac{I}{(1+r)^2} + \ldots + \frac{I}{(1+r)^n} + \frac{1,000}{(1+r)^n} \qquad \text{(3A-14)}$$

where P is the present market price, I is the annual interest payment as given by the coupon rate, and n is the number of years to final maturity. To illustrate, suppose that the 8% bonds of Bailey Corporation have ten years to final maturity and that the current market price is $950 a bond. Therefore

$$950 = \frac{80}{(1+r)} + \frac{80}{(1+r)^2} + \ldots + \frac{80}{(1+r)^{10}} + \frac{1,000}{(1+r)^{10}} \qquad \text{(3A-15)}$$

Solving for r in the same manner as we solve for the internal rate of return, we find the yield to maturity to be 8.77 per cent.

Suppose now that we reverse the example somewhat and that we wish to determine the market price on an 8% ten-year bond necessary for it to yield 7 per cent to maturity. The equation becomes

$$P = \frac{80}{(1.07)} + \frac{80}{(1.07)^2} + \ldots + \frac{80}{(1.07)^{10}} + \frac{1,000}{(1.07)^{10}} \qquad \text{(3A-16)}$$

Referring to Table A-2 in the Appendix at the back of the book, the present value of $80 per year for ten years, discounted at 7 per cent, is $561.89, and the present value of $1,000 received at the end of the tenth year is $508.35 (from Table A-1). The market price, P, is simply the sum of the present values of these two components, or $1,070.24.

If interest is paid more than once a year, Eq. (3A-14) needs to be modified somewhat. In keeping with the discussion in the previous section, if interest is paid semiannually, the yield to maturity can be found by solving the following equation for r

$$P = \frac{I/2}{\left(1+\frac{r}{2}\right)} + \frac{I/2}{\left(1+\frac{r}{2}\right)^2} + \frac{I/2}{\left(1+\frac{r}{2}\right)^3} + \ldots + \frac{I/2}{\left(1+\frac{r}{2}\right)^n} + \frac{1,000}{\left(1+\frac{r}{2}\right)^n}$$

$$\text{(3A-17)}$$

If the 7% bonds of IBK Corporation had ten years to final maturity, and the current market price were $1,040 per bond, Eq. (3A-17) becomes

$$1,040 = \frac{35}{\left(1+\frac{r}{2}\right)} + \frac{35}{\left(1+\frac{r}{2}\right)^2} + \frac{35}{\left(1+\frac{r}{2}\right)^3} + \ldots + \frac{35}{\left(1+\frac{r}{2}\right)^{20}} + \frac{1,000}{\left(1+\frac{r}{2}\right)^{20}}$$

$$\text{(3A-18)}$$

Solving the equation for r, we find the yield to maturity to be 6.64 per cent. Similarly, the equation for calculating bond yields where interest is compounded m times a year is:

$$P = \frac{I/m}{\left(1+\dfrac{r}{m}\right)} + \frac{I/m}{\left(1+\dfrac{r}{m}\right)^2} + \frac{I/m}{\left(1+\dfrac{r}{m}\right)^3} + \ldots + \frac{I/m}{\left(1+\dfrac{r}{m}\right)^{mn}} + \frac{1,000}{\left(1+\dfrac{r}{m}\right)^{mn}}$$

$$(3A\text{-}19)$$

Given any three of the four factors discussed above (coupon rate, final maturity, market price, and yield to maturity), we are able to solve for the fourth. Fortunately, elaborate bond value tables are available, so we need not go through the calculations. These tables are constructed in exactly the same manner as the present-value tables discussed previously. The only difference is that they take account of the coupon rate and of the fact that the face value of the bond will be paid at the final maturity date. An example of a bond-yield table for a 5 per cent coupon rate, and semiannual interest payments, is shown in Figure 3-3. In the

			YEARS AND MONTHS					5%
Mat. Yield	18-9	18-10	18-11	19-0	19-1	19-2	19-3	19-4
3.95	113.81	113.85	113.90	113.94	113.98	114.02	114.06	114.10
4.00	113.10	113.14	113.18	113.22	113.26	113.29	113.33	113.37
4.05	112.39	112.43	112.47	112.51	112.54	112.57	112.61	112.65
4.10	111.69	111.72	111.76	111.80	111.83	111.86	111.90	111.93
4.15	110.99	111.03	111.06	111.10	111.13	111.16	111.19	111.22
4.20	110.30	110.33	110.37	110.40	110.43	110.45	110.48	110.51
4.25	109.62	109.65	109.68	109.71	109.73	109.76	109.79	109.81
4.30	108.94	108.97	109.00	109.03	109.05	109.07	109.10	109.12
4.35	108.27	108.29	108.32	108.35	108.37	108.39	108.41	108.43
4.40	107.60	107.62	107.65	107.67	107.69	107.71	107.73	107.75
4.45	106.94	106.96	106.98	107.00	107.02	107.04	107.06	107.08
4.50	106.28	106.30	106.32	106.34	106.35	106.37	106.39	106.40
4.55	105.63	105.65	105.66	105.68	105.70	105.71	105.72	105.74
4.60	104.98	105.00	105.01	105.03	105.04	105.05	105.07	105.08
4.65	104.34	104.35	104.37	104.38	104.39	104.40	104.41	104.43
4.70	103.70	103.72	103.73	103.74	103.75	103.76	103.77	103.78
4.75	103.07	103.08	103.09	103.11	103.11	103.12	103.12	103.13
4.80	102.45	102.45	102.46	102.47	102.48	102.48	102.49	102.49
4.85	101.83	101.83	101.84	101.85	101.85	101.85	101.86	101.86
4.90	101.21	101.21	101.22	101.23	101.23	101.23	101.23	101.23
4.95	100.60	100.60	100.61	100.61	100.61	100.61	100.61	100.61
5.00	99.99	99.99	100.00	100.00	100.00	99.99	99.99	99.99
5.05	99.39	99.39	99.39	99.39	99.39	99.38	99.39	99.38
5.10	98.79	98.79	98.79	98.79	98.78	98.78	98.78	98.77
5.15	98.20	98.20	98.20	98.20	98.19	98.18	98.17	98.17
5.20	97.61	97.61	97.61	97.60	97.59	97.58	97.58	97.57
5.25	97.03	97.03	97.02	97.02	97.00	96.99	96.99	96.98
5.30	96.45	96.45	96.44	96.43	96.42	96.41	96.40	96.39
5.35	95.88	95.87	95.86	95.86	95.84	95.83	95.82	95.81
5.40	95.31	95.30	95.29	95.28	95.27	95.25	95.24	95.23
5.45	94.75	94.73	94.72	94.72	94.70	94.68	94.67	94.66
5.50	94.19	94.17	94.16	94.15	94.13	94.11	94.10	94.09
5.60	93.08	93.06	93.05	93.04	93.02	93.00	92.98	92.96
5.70	91.99	91.97	91.96	91.94	91.92	91.89	91.87	91.85
5.75	91.45	91.43	91.42	91.40	91.37	91.35	91.33	91.31
5.80	90.92	90.90	90.88	90.86	90.83	90.81	90.79	90.77
5.90	89.86	89.84	89.82	89.80	89.77	89.74	89.72	89.69
6.00	88.83	88.80	88.78	88.75	88.72	88.69	88.67	88.64
6.10	87.80	87.77	87.75	87.72	87.69	87.66	87.63	87.60
6.20	86.80	86.77	86.74	86.71	86.68	86.64	86.61	86.58
6.25	86.30	86.27	86.24	86.21	86.17	86.14	86.11	86.08
6.30	85.80	85.77	85.74	85.72	85.68	85.64	85.61	85.58
6.40	84.83	84.79	84.76	84.73	84.69	84.66	84.62	84.59
6.50	83.87	83.83	83.80	83.77	83.73	83.69	83.65	83.61
6.60	82.92	82.88	82.85	82.82	82.77	82.73	82.69	82.66
6.70	81.99	81.95	81.92	81.88	81.84	81.79	81.75	81.71
6.75	81.53	81.49	81.45	81.42	81.37	81.33	81.29	81.25
7.00	79.28	79.24	79.20	79.16	79.11	79.06	79.02	78.97
7.25	77.12	77.07	77.03	76.99	76.93	76.88	76.83	76.79
7.50	75.04	74.99	74.94	74.90	74.84	74.78	74.73	74.69
8.00	71.10	71.05	71.00	70.95	70.89	70.83	70.77	70.72

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FIGURE 3-3

Example of a bond-yield table. Monthly Bond Values (Boston Financial Publishing Company, 1939)

figure, the time to maturity is represented by the columns and the yield to maturity by the rows. The figures inside represent the prices, or present values, based upon a face value of $100. For example, the market price of a 5% bond with nineteen years to final maturity yielding 5.25 per cent is $97.02. By consulting a bond-yield table, we are able to determine easily the yield to maturity of a bond – given its market price, coupon rate, and maturity date – or the market price for a particular yield – given the coupon rate and maturity date.

PERPETUITIES

It is conceivable that we might be confronted with an investment opportunity that, for all practical purposes, is a perpetuity. With a perpetuity, a fixed cash inflow is expected at equal intervals forever. A case in point is the British Consul, which is a bond with no maturity date; it carries the obligation of the British government to pay a fixed coupon perpetually. If the investment required an initial cash outflow at time 0 of A_0 and were expected to pay A^* at the end of each year forever, the internal rate of return is the discount rate, r, that equates the present value of all future cash inflows with the present value of the initial cash outflow:

$$A_0 = \frac{A^*}{(1+r)} + \frac{A^*}{(1+r)^2} + \ldots + \frac{A^*}{(1+r)^n} \tag{3A-20}$$

In the case of a bond, A_0 is the market price of the bond and A^* the fixed annual interest payment. When we multiply both sides of Eq. (3A-20) by $(1 + r)$, we obtain

$$A_0(1+r) = A^* + \frac{A^*}{(1+r)} + \frac{A^*}{(1+r)^2} + \ldots + \frac{A^*}{(1+r)^{n-1}} \tag{3A-21}$$

Subtracting Eq. (3A-20) from Eq. (3A-21)

$$A_0(1+r) - A_0 = A^* - \frac{A^*}{(1+r)^n} \tag{3A-22}$$

As n approaches infinity, $A^*/(1+r)^n$ approaches 0. Thus

$$A_0 r = A^* \tag{3A-23}$$

and

$$r = \frac{A^*}{A_0} \tag{3A-24}$$

Here r is the internal rate of return or yield on a perpetual investment costing A_0 at time 0 and paying A^* at the end of each year forever.

Suppose that we had the opportunity to buy for $1,000 a security that was expected to pay $70 a year forever. The yield, or internal rate of return, of the security would be

$$r = \frac{\$70}{\$1,000} = 7 \text{ per cent} \tag{3A-25}$$

In a well-known article, Lorie and Savage pointed out that certain streams of cash flows may have more than one internal rate of return.[20] To illustrate the problem, suppose that we had the following stream of cash flows corresponding to the "pump" proposal of Lorie and Savage:

Year	Cash Flow
0	−$ 1,600
1	10,000
2	− 10,000

In this example, a new, more effective pump is substituted for an existing pump. On an incremental basis, there is an initial outlay followed by net cash inflows resulting from the increased efficiency of the new pump. If the quantity of oil, for example, is fixed, the new pump will exhaust this supply more quickly than the old pump would. Beyond this point of exhaustion, the new pump alternative would result in an incremental outflow, because the old pump would still be productive.

When we solve for the internal rate of return for the above cash-flow stream, we find that it is not one rate, but two: 25 per cent and 400 per cent. This unusual situation is illustrated in Figure 3-4, where the discount rate is plotted along the horizontal axis and net-present value along the vertical axis. At a 0 rate of discount, the net-present value of the project, −$1,600, is negative, due to the fact that the total nondiscounted cash outflows exceed total nondiscounted inflows. As the discount rate increases, the present value of the second year outflow diminishes with respect to the first year inflow; and the present value of the proposal becomes positive when the discount rate exceeds 25 per cent. As the discount rate increases beyond 100 per cent, the present value of all future cash flows (years 1 and 2) diminishes relative to the initial outflow of −$1,600. At 400 per cent, the present value of all cash flows again becomes 0.

[20] See Lorie and Savage, "Three Problems in Rationing Capital," reprinted in *Foundations for Financial Management*, ed. James Van Horne, pp. 305–7.
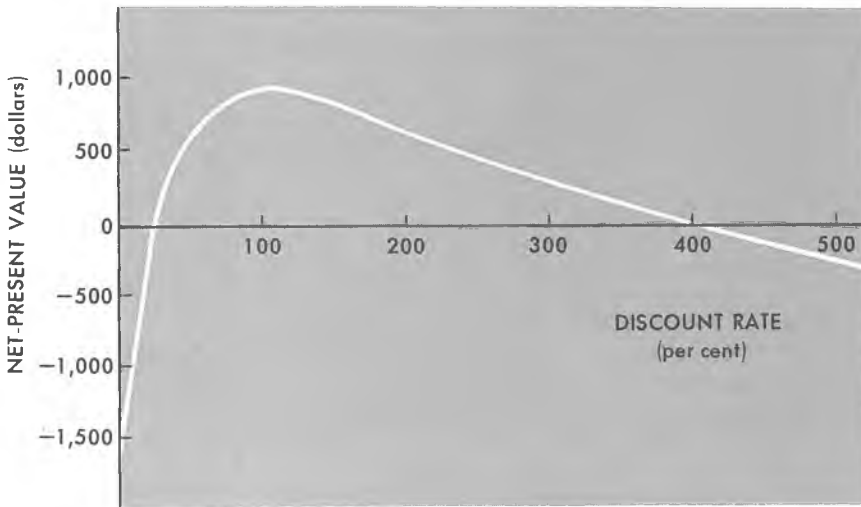
FIGURE 3-4
Dual rates of return

This type of proposal differs from the usual case, shown in Figure 3-1 in this chapter, in which net-present value is a decreasing function of the discount rate, and in which there is but one internal rate of return that equates the present value of all inflows with the present value of all outflows. An investment proposal may have any number of internal rates of return, depending upon the cash-flow pattern. Consider the following series of cash flows:

Year	Cash Flow
0	−$ 1,000
1	6,000
2	− 11,000
3	6,000

In this example, discount rates of 0, 100 per cent, and 200 per cent result in the net-present value of all cash flows equaling 0.

The number of internal rates of return is limited to the number of reversals of sign in the cash-flow stream. In the above example, we have three reversals and three internal rates of return. Although a multiple reversal in signs is a necessary condition for multiple internal rates of return, it is not sufficient for such an occurrence. The occurrence of multiple internal rates of return also depends upon the magnitude of cash flows. For the following series of cash flows, there is but one internal rate of return (32.5 per cent), despite two reversals of sign.

Year	Cash Flow
0	−$1,000
1	1,400
2	− 100

We note that the equation for solving for the internal rate of return, Eq. (3-7), is an nth degree polynomial. For most investment projects, there is but one internal rate of return, and all but one of the n roots are imaginary. For dual rates of return, n-2 roots are imaginary, and so on.

When confronted with a proposal having multiple rates of return, how does one decide which is the correct rate? In our dual-rate example, is the correct rate 25 per cent or 400 per cent? Actually, neither rate is correct, because neither is a measure of investment worth.[21] In essence, the firm has "borrowed" $10,000 from the project at the end of year 1 and will pay it back at the end of year 2. The relevant question is: What is it worth to the firm to have the use of $10,000 for one year? This question, in turn, depends upon the rate of return on investment opportunities available to the firm for that period of time. If the firm could earn 20 per cent on the use of these funds and realize these earnings at the end of the period, the value of this opportunity would be $2,000, to be received at the end of year 2. The internal rate of return that equates the present value of this amount to the present value of the initial outlay of $1,600 is 11.8 per cent. Similarly, other proposals can be evaluated to find one meaningful rate of return.[22]

Mathematical programming can be used to allocate capital under conditions of capital rationing. The most comprehensive treatment of the problem has been by Weingartner.[23] His basic model, which uses the Lorie-Savage problem as a point of departure, may be expressed as

$$\text{Maximize} \sum_{j=1}^{n} b_j x_j$$

$$\text{Subject to} \sum_{j=1}^{n} c_{tj} x_j \leq C_t \qquad \text{(3C-1)}$$

$$0 \leq x_j \leq 1$$

[21] Ezra Solomon, "The Arithmetic of Capital-Budgeting Decisions," *Journal of Business*, XXIX (April, 1956), reprinted in *Foundations for Financial Management*, ed. James Van Horne, pp. 291–94.

[22] For a rigorous analysis of the problem of multiple rates of return, see Daniel Teichroew, Alexander A. Robichek, and Michael Montalbano, "An Analysis of Criteria for Investment and Financing Decisions under Certainty," *Management Science*, 12 (November, 1965), 151–79.

[23] H. Martin Weingartner, *Mathematical Programming and the Analysis of Capital Budgeting Problems*. Copyright© H. Martin Weingartner, 1963.

where b_j is net-present value of investment proposal j, x_j is an amount between 0 and 1, c_{tj} is the net cash outlay required for proposal j in period t, and C_t is the budget constant in period t. In words, the model maximizes the net-present value from currently available investment proposals, given the budget constraint in each period.

To illustrate, consider the Lorie-Savage example, which involves budget constraints for two periods. This example was employed by Weingartner; and it is shown in Table 3C-1. The present values of the two budget constraints are $50 in period 1 and $20 in period 2. If the projects under consideration are independent, Eq. (3C-1) can be expressed as

Maximize $14x_1 + 17x_2 + 17x_3 + 15x_4 + 40x_5 + 12x_6 + 14x_7 + 10x_8 + 12x_9$

Subject to $\hspace{9cm}$ (3C-2)

$$12x_1 + 54x_2 + 6x_3 + 6x_4 + 30x_5 + 6x_6 + 48x_7 + 36x_8 + 18x_9 \leqslant 50$$
$$3x_1 + 7x_2 + 6x_3 + 2x_4 + 35x_5 + 6x_6 + 4x_7 + 3x_8 + 3x_9 \leqslant 20$$
$$0 \leqslant x_j \leqslant 1, (x_j = 1, \ldots, 9)$$

The solution of this problem is shown in Table 3C-2.[24] From the table we see that, under the direct-problem variables, we should invest to the extent of 100 per cent in projects 1, 3, 4, and 9; 97 per cent in project 6; and 4.5 per cent in project 7. Substituting these values into the objective function of Eq. (3C-2), the increase in present value from this optimal solution is found to be $70.27.

TABLE 3C-1
Investment proposals available

Investment Project	Present Value of Outlay, Period 1	Present Value of Outlay, Period 2	Present Value of Investment
1	$12	$ 3	$14
2	54	7	17
3	6	6	17
4	6	2	15
5	30	35	40
6	6	6	12
7	48	4	14
8	36	3	10
9	18	3	12

Source: Lorie and Savage, "Three Problems in Rationing Capital," p. 302.

Weingartner analyzes the dual variables as the opportunity costs, or "shadow prices," of the budget constraints for the various future periods. These variables tell us the present value that can be gained if a budget constraint is relaxed by $1.[25] For example, the dual variable of

[24] *Ibid.*, p. 18.
[25] For a more detailed analysis of dual variables, see the appendix to Chapter 21.

Optimal values for example problems

Direct	Dual*
$x_1 = 1.0$	$W_1 = 0.136$
$x_2 = 0$	$W_2 = 1.864$
$x_3 = 1.0$	
$x_4 = 1.0$	
$x_5 = 0$	
$x_6 = 0.97$	
$x_7 = 0.045$	
$x_8 = 0$	
$x_9 = 1.0$	

*W_1 and W_2 are the dual variables for the budget constraints in periods 1 and 2. The dual variables for the constraints, $0 \leq x_j \leq 1$, are not shown.

0.136 for period 1 indicates that present value can be increased by $0.136 if the budget in period 1 is increased from $50 to $51; while the dual variable of 1.864 for period 2 indicates that present value can be increased by $1.864 if the budget is increased from $20 to $21. Dual variables are valuable in deciding whether to shift funds from one period to another and in evaluating the desirability and timing of external financing.

We note in this example problem that the investment projects are divisible in the sense that investment can be anywhere between 0 and some absolute amount. In the case of investment proposal 1, the absolute amounts are $12 in period 1 and $3 in period 2.[26] In the optimal program, both projects 6 and 7 were fractional. However, most investment projects are not divisible; they are either accepted, whereupon an absolute amount is invested, or rejected, in which case investment is 0. To deal with this problem, Weingartner develops an integer programming model by requiring the x_j to be an integer of either 0 or 1. Thus, fractional investment in a proposal is precluded by virtue of the fact that the x_j is an "either/or" variable. For the example problem, the optimal solution using integer programming calls for accepting projects 1, 3, 4, 6, and 9. When compared with Table 3C-2, we see that the solutions are quite similar.

The model can also be modified to take account of mutually exclusive and contingent projects. If investment projects 1, 4, and 5 were mutually exclusive, for example, we could simply add the constraint, $x_1 + x_4 + x_5 \leq 1$. With integer programming and this constraint, we are assured that only one of these projects will appear in the final solution. If project 2 is contingent upon the acceptance of project 3, the constraint, $x_2 \leq x_3$, would assure that this contingency was recognized in the solution.

[26] As a result of the divisibility of investment projects, the budget constraints in the two periods must be binding.

In addition, the model can be extended to handle manpower and other constraints of this type.[27] For a labor constraint

$$\sum_{j=1}^{n} d_{jt}x_j \leq L_t \qquad (3\text{-C2})$$

where d_{jt} is the number of man-hours of a certain type labor input required for project j in period t; and L_t is the number of such hours available in that period. Restrictions for other scarce resources also can be formulated in this manner. Moreover, the model can be expanded to allow for liquidity requirements in various periods and for the reinvestment of intermediate cash flows from one investment project into other projects. The model also can be adapted to situations of borrowing funds in the external market, subject to certain constraints. For the deterministic case, then, the integer programming technique can be realistically adapted to many "real world" constraints.

The principal difficulty of mathematical programming approaches to capital budgeting, however, is that they are based upon the assumption that all future investment opportunities are known. In reality, the generation of investment opportunities is an unfolding process. Consequently, the budget constraints for other than the earliest years are not likely to be binding. There simply are not enough known investment opportunities. Only as new investment proposals are generated are these budgets likely to be fully utilized. Therefore, the selection process must be revised continually. As new proposals are generated, they should be evaluated on a consistent basis.

Most programming approaches for capital budgeting do not allow for uncertainty in the cash-flow estimates. Attempts have been made to introduce probabilistic concepts through chance constraint programming. With chance constraints, lower levels of cash flows must be satisfied with some minimum probability. It is possible also to specify chance constraints for the net-present value of a combination of investment projects.[28] Overall, the treatment of uncertainty in this manner has not been particularly satisfactory, primarily because of the lack of realism.[29] More encompassing results with respect to the full range of possible outcomes can be achieved with simulation. Another problem is the assumption of capital rationing and the resulting issue of investment decisions that are less than optimal. This issue was discussed earlier in the chapter. Despite these limitations, however, mathematical programming provides

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[27] See Weingartner, *Mathematical Programming*, Chapters 7–9. For an additional illustration, see Quirin, *The Capital Expenditure Decision*, pp. 185–97.

[28] See D. E. Peterson, *A Quantitative Framework for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc., 1969), pp. 459–77.

[29] For an extensive review of such issues, see Richard H. Bernhard, "Mathematical Programming Models for Capital Budgeting – A Survey, Generalization, and Critique," *Journal of Financial and Quantitative Analysis*, IV (June, 1969), 111–58.

a systematic and comprehensive means for evaluating and selecting investment projects. For this reason, it would seem that this approach will grow in importance.

PROBLEMS

1. Graph the present value of one dollar per year for 5, 10, 15, 20, and 25 years at 0, 10, 20, 30, and 40 per cent rates of discount. Explain the difference in the slopes of the curves.

2. The Symington Corporation is considering investing in one of two mutually exclusive projects. Each requires an immediate cash outlay of $1,000. Project *A* has a life of four years; project *B*, five years. Both projects will be depreciated on a straight-line basis (no salvage). The firm's tax rate is 50 per cent, and its required return is 10 per cent. Net cash flows generated by each investment have been projected as follows:

Year	A	B
1	$300	$300
2	300	300
3	400	200
4	400	200
5	-0-	200

(a) Compute the payback for each investment.
(b) Compute the average rate of return for each investment.
(c) Compute the profitability index for each investment.
(d) Which alternative would you select? Why?

3. What criticisms may be offered against the average-rate-of-return method as a capital-budgeting technique? What criticisms may be offered against the payback method?

4. A company can make either of two investments at period p_0. Assuming a required rate of return of 10 per cent, determine for each project: (a) the payback period, (b) the profitability index, (c) the internal rate of return. You may assume straight-line depreciation.

	A	B
Investment	$20,000	$28,000
Expected life		
(no salvage)	5 yrs.	5 yrs.
Projected net income		
(after interest and taxes):		
Year 1	$ 5,000	$ 8,000
Year 2	5,000	8,000
Year 3	6,000	8,000
Year 4	6,000	8,000
Year 5	6,000	8,000

5. Two mutually exclusive projects have projected cash flows as follows:

Period	A	B
0	−10,000	−10,000
1	5,000	0
2	5,000	0
3	5,000	0
4	5,000	30,000

(a) Determine the internal rate of return for each project.

(b) Assuming a required rate of return of 10 per cent, determine the present value for each project.

(c) Which project would you select? Why? What assumptions are inherent in your decision?

6. The Homes Corporation is faced with two mutually exclusive investment proposals. One would cost $100,000 and provide net cash benefits of $30,000 per year for five years. The other would cost $50,000 and provide net cash benefits of $16,000 for five years. Homes has a 10 per cent after-tax opportunity cost of funds. Compute the net-present value and profitability index of each project. Which should be accepted?

7. Rework problem 6, assuming a 17 per cent opportunity cost of funds. How would this change your answer?

8. One of the largest and most profitable companies in the country is faced with the prospect of having to replace a large stamping machine. Two machines currently being marketed will do the job satisfactorily. The Superior Stamping machine costs $50,000 and will require cash running expenses of $20,000 per year. The Peerless machine costs $75,000, but running expenses are only expected to be $15,000 per year. Both machines have a ten-year useful life with no salvage value, and would be depreciated on a straight-line basis.

(a) If the company pays a 50 per cent tax rate and has a 10 per cent after-tax required rate of return, which machine should it purchase?

(b) Would your answer be different if the required return were 8 per cent?

9. Rework problem 8, assume that the Peerless machine has a $10,000 salvage value and the Superior machine has a $5,000 salvage value.

10. The Burdick Company plans its capital budget on a two-year basis. For the next two years, a maximum $1,750,000 has been allocated; this money is currently on hand. Some proposals could be undertaken immediately, while others will not be ready until next year. The profitability index of each proposal is figured from the point of initial investment. None of the proposals will generate a cash inflow during the two-year period under consideration. The company pays taxes at a 50 per cent rate and requires an after-tax rate of return of 12 per cent. Treasury bills are yielding 6 per cent. Determine which of the following investments should be undertaken.

	Available This Year			Available Next Year	
Project	Amount	Profitability Index	Project	Amount	Profitability Index
1	$500,000	1.05	7	$500,000	1.30
2	100,000	1.31	8	300,000	1.20
3	250,000	1.17	9	400,000	1.12
4	350,000	0.97	10	100,000	1.05
5	300,000	1.22			
6	400,000	1.10			

11. The Simplex Company has $1,000,000 allocated for capital-budgeting

purposes. The following proposals and associated profitability indexes have been determined.

Project	Amount	Profitability Index
1	$500,000	1.22
2	150,000	.95
3	350,000	1.20
4	450,000	1.18
5	200,000	1.20
6	400,000	1.05

(a) Which of the above investments should be undertaken?
(b) Would the firm be maximizing the price of the common stock by turning down projects having an index larger than one?

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Cost
of Capital

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The investment decision is directly related to the financing decision because the acceptance of investment proposals depends upon how those proposals will be financed. In this chapter, we take up the question of how to measure the real cost of financing investment opportunities. In the previous chapter, we saw that the discount rate is the vehicle by which we judge the attractiveness of an investment opportunity. This rate usually is the cost of capital of the firm, and we recall that it is employed as follows.

1. Accept a project if its net-present value is positive when all cash flows are discounted at the cost-of-capital rate.
2. Accept a project if its internal rate of return is greater than the cost of capital.

If these acceptance rules are followed, the market price of the stock

supposedly will be maximized over the long run.[1] The cost of capital, then, represents a cutoff rate for the allocation of capital to investment projects; in theory, it should be the rate of return on a project that will leave unchanged the market price of the stock. In this sense, the cost of capital is the required rate of return needed to justify the use of capital.

For expository purposes, in examining the cost of capital we assume that the acceptance of any one project or combination of projects does not change the business-risk complexion of the firm as a whole. In short, we hold constant the effect of business risk upon the valuation of the firm and its cost of capital. We also assume in this chapter that the firm intends to maintain a constant dividend payout; we postpone until Chapters 9 and 10 consideration of dividend policy. Finally, we defer until Chapters 7 and 8 the analysis of the effect of changes in the capital structure of the firm on its valuation and cost of capital. In this chapter, we assume a constant financing mix. Our purpose is to measure a firm's cost of capital and to explore more fully the link between the acceptance of investment proposals and the market price of the firm's stock. Necessarily, we must hold constant the three factors above—business risk, dividend policy, and capital structure—if we are to analyze this link directly.

The cost of capital is perhaps the most difficult and controversial topic in finance. In theory, most would agree that it is the rate of return on the project that will leave unchanged the market price of the firm's stock. In practice, there are widespread differences as to how this cost should be measured. In this chapter, we examine some of these differences and, in so doing, propose a general framework for the measurement of a firm's cost of capital. Although this framework certainly is not free from possible criticism, it does give us some feeling for the important factors that go into the measurement of capital costs and the problems inherent in these measurements. The alternative of avoiding these controversies is unsatisfactory, for the cost of capital is far too important a topic. Despite the many difficulties, a firm must come to grips with the problem and attempt to approximate, as nearly as possible, its real cost of capital.

COSTS OF CAPITAL FOR SPECIFIC SOURCES OF FINANCING

Because the firm is valued as an overall entity, it is inappropriate to associate specific methods of financing with specific investment opportunities. For most firms, the sources of funds employed vary over time. A company cannot continually finance with debt without building its equity base, either through the retention of earnings or through the sale of common stock. Since our focus is on the valuation of the firm as a

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[1] For an extensive theoretical analysis of this concept, see Diran Bodenhorn, "On the Problem of Capital Budgeting," *Journal of Finance*, XIV (December, 1959), 473–92.

whole, we must use an overall cost of capital as the acceptance criterion for investment proposals, even though the firm may finance one proposal with one type of financing and another proposal with another type. It is the overall mix of financing over time that is important.

In order to measure the overall cost of capital to the firm, it is necessary first to consider the costs of specific methods of financing. In this regard, we shall be concerned with explicit costs. The explicit cost of a source of financing may be defined as the discount rate that equates the present value of the funds received by the firm, net of underwriting and other costs, with the present value of expected outflows. These outflows may be interest payments, repayment of principal, or dividends. Thus, the explicit cost of a specific method of financing can be determined by solving the following equation for k:

$$I_0 = \frac{C_1}{(1+k)} + \frac{C_2}{(1+k)^2} + \ldots + \frac{C_n}{(1+k)^n} \tag{4-1}$$

where I_0 is the net amount of funds received by the firm at time 0, and C_t is the outflow in period t.

In the remainder of this section, we shall measure the explicit costs of specific sources of funds. To the extent that the historical cost of a source of funds parallels closely the present cost, historical costs may give us some insight. However, our concern is not with the historical cost per se but with the marginal explicit cost of a specific method of financing. The use of marginal explicit costs follows from the fact that we use the cost of capital to decide whether to invest in new projects. Past costs of financing have no bearing on this decision. All costs will be expressed on an after-tax basis, so as to conform to the expression of investment project cash flows on an after-tax basis. Of the various sources of funds, the cost of equity capital is both the most difficult to measure and also the most controversial. Once we have examined the explicit costs of various sources of financing, we shall combine these costs to obtain an overall cost of capital to the firm. In the process of combining, the opportunity costs of the various sources are seen to be embodied in the overall cost. Again, it is important to point out that in this chapter we take as given the capital structure of the firm.

COST OF DEBT

The explicit cost of debt can be derived from Eq. (4-1) by solving for that discount rate, k, which equates the net proceeds of the debt issue with the present value of interest plus principal payments, and then adjusting the explicit cost obtained for the tax effect. If we denote the after-tax cost of debt by k_i, it can be approximated by

$$k_i = k(1 - t) \tag{4-2}$$

where k is the internal rate of return using Eq. (4-1), and t is the marginal tax rate. Because interest charges are tax deductible, the after-tax cost of debt is substantially less than the before-tax cost. If a company were able to sell a new issue of twenty-year bonds with an 8 per cent coupon rate and realize net proceeds (after underwriting expenses) of $1,000 for each $1,000 face value bond, k would be 8 per cent. If the federal income tax rate were 50 per cent,

$$k_i = 8.00(1 - 0.50) = 4.00 \text{ per cent}$$

We note that the 4 per cent after-tax cost in our example represents the marginal, or incremental, cost of additional debt. It does not represent the cost of debt already employed.

In general, Eqs. (4-1) and (4-2) can be used to calculate the explicit cost of debt financing.[2] The explicit cost of debt is considerably cheaper than the cost of another source of financing having the same k but where the financial charges are not deductible for tax purposes. Implied in the calculation of an after-tax cost of debt is the fact that the firm is profitable. Otherwise, it does not gain the tax benefit associated with interest payments. The explicit cost of debt for an unprofitable firm is the before-tax cost, k.

If a firm has the policy of maintaining a given proportion of debt in its capital structure, debt is never really paid. Individual debt instruments are paid, of course, but they are replaced by new debt. Thus, debt can be regarded as a permanent part of the financing mix. Under these circumstances, the appropriate formula for calculating the explicit cost of debt is not Eq. (4-1) but the formula for a perpetuity

$$k_i = \frac{C_t}{I_0}(1 - t) \qquad (4\text{-}3)$$

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[2] When the price paid for a bond differs from its face value, the premium or discount is amortized for federal income tax purposes. If the premium or discount is significant, the after-tax cost of debt should take it into account. The approximate cost of a bond sold at a discount or a premium is

$$k_i = \frac{(1 - t)\left[C_t + \frac{1}{n}(P - I_0)\right]}{\frac{1}{2}(P + I_0)}$$

where P is the face value of the bond (usually $1,000), I_0 is the price at which the bond is sold, n is the number of years to maturity, and C_t is the fixed interest cost in all periods. $1/n(P - I_0)$ represents the amortization of the discount or premium over the life of the bond, and the denominator represents the average amount outstanding. If sinking-fund payments are made, the formula must be revised. The formula above is but an approximation of the explicit cost because it does not take account of annual compounding. See G. David Quirin, *The Capital Expenditure Decision* (Homewood, Ill.: Richard D. Irwin, Inc., 1967), pp. 100–101.

Because most bond issues are sold in the capital markets at close to their face values, we do not take account specifically of the tax effect of a premium or discount in Eqs. (4-1) and (4-2).

where C_t is the fixed interest cost in all periods, and I_0 is the net proceeds of the issue.[3]

COST OF PREFERRED STOCK

The cost of preferred stock is a function of its stated dividend. As we discuss in Chapter 13, this dividend is not a contractual obligation on the part of the firm but is payable at the discretion of the board of directors. Consequently, unlike with debt, there is no risk of legal bankruptcy. However, from the standpoint of common stockholders, preferred stock represents a security senior to their interests. Because most corporations that issue preferred stock intend to pay the stated dividend, the dividend on the preferred stock represents a prior claim on income.[4] As preferred stock has no maturity date, its cost may be represented as

$$k_p = \frac{D}{I_0} \tag{4-4}$$

where D is the stated annual dividend and I_0 represents the net proceeds of the preferred stock issue. If a company were able to sell a $7\frac{1}{2}$ per cent preferred stock issue ($100 par value) and realize net proceeds of $98\frac{1}{2}$ a share, the cost of the preferred stock would be $7\frac{1}{2}/98\frac{1}{2} = 7.61$ per cent. Note that this cost is not adjusted for taxes, because the preferred stock dividend is paid after taxes. Thus, the explicit cost of preferred stock is substantially greater than that for debt.

If a preferred stock issue has a call price and the company intends to call the issue after a certain duration, Eq. (4-1) should be used to calculate the explicit cost. Usually, however, preferred stock is treated as a perpetual security and Eq. (4-4) is used.

COST OF EQUITY CAPITAL[5]

The cost of equity capital is by far the most difficult cost to measure. In theory, it may be defined as the minimum rate of return that the company must earn on the equity-financed portion of an investment project in order to leave unchanged the market price of the stock.[6]

To illustrate, suppose that the required rate of return on equity (see Chapter 2) were 10 per cent after taxes and that the cost of debt were 4 per cent after taxes. Suppose further that the company had the policy

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[3] For the mathematics of perpetuities, see Appendix A to Chapter 3.

[4] The consequences of not paying the dividend are examined in Chapter 13.

[5] The development of this section assumes that Chapter 2 has been read.

[6] It is important to emphasize the assumption that the acceptance of any investment project or combination of investment projects does not change the business-risk complexion of the company as a whole. Moreover, we assume a constant capital structure; each new investment project is financed with the same financing mix as the existing capital structure of the firm. We assume also a constant dividend payout ratio.

of financing with 50 per cent equity and 50 per cent debt, suggesting that the required rate of return on a project is

$$0.5 \times .04 = .02$$
$$0.5 \times .10 = \underline{.05}$$
$$\underline{\underline{.07}}$$

In other words, if the firm accepted an investment project costing $100 that was forever expected to return $7 per year after taxes, the project would provide a return just sufficient to leave unchanged the market price of the firm's stock. The dollar return on the equity-financed portion, $50, is

Total return (after tax)	$7
Less interest (.04 × $50)	2
Return on equity	$5

Thus, the expected rate of return is $5/$50 = 10 per cent, which just equals the required rate of return of investors. If the project were expected to return less than $7 a year, it would provide a rate of return on the equity-financed portion less than that required by investors at the margin. As a result, the market price of the stock would suffer. To measure this required rate of return, we must review the valuation of common stocks.

Dividend Valuation Model. Recall from Chapter 2 that the value of a share of stock to investors can be viewed as the present value of the expected future stream of income paid to them. Because dividends are all that stockholders as a whole receive from their investment, this stream of income is the cash dividends paid in future periods and, perhaps, a final liquidating dividend. At time 0, the value of a share of stock is

$$P_0 = \frac{D_1}{(1 + k_e)} + \frac{D_2}{(1 + k_e)^2} + \ldots + \frac{D_\infty}{(1 + k_e)^\infty}$$

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + k_e)^t}$$

(4-5)

where P_0 is the value of a share of stock at time 0, D_t is the dividend per share expected to be paid in period t, and k_e is the rate of discount appropriate for the business-risk complexion of the company.

We suggested in Chapter 2 that investors formulate subjective probability distributions of dividends per share expected to be paid in various future periods. For the individual investor, the D_t in Eq. (4-5) are the expected values, or means, of these probability distributions. For the market as a whole, the D_t represent the expected values for investors at the

margin, and k_e is the market discount factor appropriate for the risk company involved. The cost of equity capital is defined in this book as the market rate of discount, k_e, that equates the present value of all expected future dividends per share with the current market price of the stock. This cost is found by solving Eq. (4-5) for k_e. As a general rule, it is inappropriate to use the ratio of earnings per share to market price per share as the cost of equity capital. Only under certain circumstances are the two equivalent; these circumstances are examined in the appendix to this chapter.

If dividends per share are expected to grow at a constant rate, g, and k_e is greater than g, we discovered in Chapter 2 that

$$P_0 = \frac{D_1}{k_e - g} \qquad (4\text{-}6)$$

where D_1 is the dividend per share expected to be paid at the end of period 1. Thus, the cost of equity capital would be

$$k_e = \frac{D_1}{P_0} + g \qquad (4\text{-}7)$$

The critical assumption, of course, is that dividends per share are expected to grow at a compound rate of g forever. In certain situations, this assumption may be a fair approximation of investor expectations. For example, if ABC Company's expected dividend per share at the end of period 1 is \$2, the current market price is \$40, and earnings and dividends per share are expected to grow about 4 per cent per annum in keeping with growth in the economy, the company's cost of equity capital is

$$k_e = \frac{2}{40} + .04 = 9 \text{ per cent}$$

For the k_e to be realistic, expectations in the marketplace must be such that dividends per share are believed to grow in fact at a rate g. The important factor, then, is measuring the growth in dividends per share as perceived by investors at the margin.

To the extent that the past trend in earnings per share is felt to be meaningful for predicting this expected future growth, it may be used as the growth variable. For example, suppose a company had the following history of earnings per share.

	Earnings per Share		Earnings per Share
1962	\$1.50	1967	\$2.19
1963	1.68	1968	2.38
1964	1.78	1969	2.49
1965	1.92	1970	2.45
1966	2.07	1971	2.70

If one averaged the figures for the first three years and the last three years, he would find that the compound annual growth rate between these two averages is 6.4 per cent. If it is felt that this growth rate is the best approximation of expected future growth perceived by investors at the margin and that a perpetual-growth model is an appropriate valuation model, Eq. (4-7) might be used in determining the cost of equity capital.[7]

When the expected growth in dividends per share is other than perpetual, a modification of Eq. (4-5) can be used. As brought out in Chapter 2, a number of valuation models assume that the growth rate will eventually taper off. Frequently, the transition is from an above-normal growth rate to one that is considered normal. For example, if dividends were expected to grow at a 15 per cent compound rate for five years, at a 10 per cent rate for the next five years, and then grow at a 5 per cent rate, we would have

$$P_0 = \sum_{t=1}^{5} \frac{D_0(1.15)^t}{(1+k_e)^t} + \sum_{t=6}^{10} \frac{D_5(1.10)^{t-5}}{(1+k_e)^t} + \sum_{t=11}^{\infty} \frac{D_{10}(1.05)^{t-10}}{(1+k_e)^t} \qquad (4\text{-}8)$$

We see that the current dividend, D_0, is the base on which the expected

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[7] In the continuous case, if the firm engages in no external financing and investors expect a constant rate of return on investments, r, and a constant rate of earnings retention, b, the perpetual-growth valuation model becomes

$$P_0 = \frac{E_0(1-b)}{k_e - br}$$

where E_0 is current earnings per share and br represents the growth in earnings and dividends per share. This model was first developed by Gordon and Shapiro and later refined by Gordon in additional work. See Myron J. Gordon and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," *Management Science*, 3 (October, 1956), 102–10; and Gordon, *The Investment, Financing and Valuation of the Corporation* (Homewood, Ill., Richard D. Irwin, Inc., 1962), Chapter 3.

Given a constant rate of return on all future investments, br in the above model represents the additional earnings arising from retention. As retained earnings are the sole source of financing, growth is possible only if the dividend payout as a percentage of earnings is less than 100 per cent. Note, however, that if the return on investment is less than the cost of equity capital, $r < k_e$, the firm can maximize share price by adopting a zero retention rate (100 per cent payout). If $r > k_e$, the price of the stock approaches infinity as the retention rate, b, is increased. The optimal retention rate in this case would be 100 per cent if it were not for the fact that the numerator in the equation would be 0.

In order to get around this "either-or" problem, Gordon suggests that k_e increases with the distance in the future in keeping with uncertainty increasing at an increasing rate. As the retention rate, b, increases, dividends are pushed further into the future. Thus, k_e is an increasing function of the retention rate. This postulated relationship results in the possibility of an optimum dividend payout between 0 and 100 per cent.

Lerner and Carleton, on the other hand, make r a declining function of the retention rate, b, thereby allowing for the possibility of an optimal dividend payout between 0 and 1. Eugene M. Lerner and Willard T. Carleton, "The Integration of Capital Budgeting and Stock Valuation," *American Economic Review*, LIV (September, 1964), reprinted in James Van Horne, ed., *Foundations for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc.), pp. 327–46. For criticism of their approach, see Haim Ben-Shahar and Abraham Ascher, "Capital Budgeting and Stock Valuation: Comment, "*American Economic Review*, LVII (March, 1967), 209–14; Jean Crockett and Irwin Friend, "Comment," *Ibid.*, pp. 214–20; and Lerner and Carleton, "Reply," *Ibid.*, pp. 220–22.

growth in future dividends is built. By solving for k_e, we obtain the cost of equity capital as defined. For example, if the current dividend, D_0, were \$2 a share and market price per share, P_0, were \$75, k_e in Eq. (4-8) would be 9.5 per cent. For other patterns of expected future growth, the equation can be easily modified to deal with the situation. The more growth segments we specify, of course, the more the growth pattern will approximate a curvilinear relationship.

For all growth situations, the important thing is to solve for the k_e that equates the expected future dividends perceived by investors at the margin with the current market price of the stock. Since expected growth in dividends is not directly observable, it must be estimated. Herein lies the major difficulty involved in estimating the cost of equity capital. For reasonably stable patterns of past growth, one might project this trend into the future. However, we must temper the projection to take account of current market sentiment. Insight into such sentiment can come from reviewing various analyses of the industry and company by investment advisors as well as from articles about the company in financial newspapers and magazines.

On the basis of the long-range plans of the company, the financial manager can make internal estimates of expected future growth in earnings per share and dividends per share. These estimates should take account of economic and other factors that bear on the firm's future operating performance. Because the financial manager has access to a great deal of relevant information, his estimates of future earnings may be the most accurate of all. However, it is important that investors also expect these earnings. There is an obvious bias if the financial manager uses his estimate of growth to solve for k_e and his estimate differs significantly from that of the market. The important question to ask is, What growth in dividends do investors at the margin expect that leads them to pay x dollars for a share of stock? Every effort should be made to get as accurate a handle as possible on this expected growth pattern. Once it is obtained, it is an easy matter to solve for the rate of return investors expect — the cost of equity capital as defined. Thus, the financial manager must think as investors do when he estimates the cost of equity capital for his firm.

While it is reasonable to use a dividend valuation model for companies that pay dividends, it is very difficult to use such a model for measuring the cost of equity capital when a company either pays no dividend or pays a negligible one. To do so, one would have to estimate both the time and the magnitude of eventual dividends. In view of the fact that certain managements as much as state that they will never pay a dividend during their stewardship, the task is formidable indeed. It is important, however, to recognize that the rate with which we are concerned is that which investors require on the equity-financed portion of investments undertaken by the firm. One approach to the problem might be to estimate the average return that investors expect to receive from the growth in the market

price of the stock in the future. Indeed, institutional and fiduciary investors make these very estimates.

The estimates may be based upon past growth figures extrapolated into the future. For example, if the market price of the stock had increased at a compound rate of 10 per cent over the last ten years, and it was probable that this rate would continue for some time to come, 10 per cent might be used as a rough estimate of k_e. To be sure, this approach lacks precision. Nevertheless, estimates must be made. The important thing is to estimate as accurately as possible the future growth of the non-dividend-paying firm as perceived by investors at a moment in time. One danger in measurement is in extrapolation of an extremely high past growth rate into the future. No company can grow forever at a 40 per cent compound rate. It is difficult to estimate when the growth rate will taper off to a more normal rate. However, estimates can be made of what investors expect a company to earn in their behalf on investment projects. These estimates can then be employed as approximations of the cost of equity capital.

Required Rate of Return. The rate of return required by investors on the equity-financed portion of investment projects[8] might be thought to be comprised of three parts

$$k_e = i + \beta + \Phi \tag{4-9}$$

where i = the risk-free rate
β = a premium for business risk
Φ = a premium for financial risk.

We note that this equation is an extension of Eq. (2-15) in Chapter 2; we simply break the overall risk premium, θ, down into its two components, β and Φ. The premium for business risk is caused by the relative dispersion of the probability distribution of possible future operating income; whereas the premium for financial risk is caused by the dispersion of expected future earnings available to common stockholders, holding constant business risk. The distinction between these two risks is explained in Chapter 7. Together, they account for the overall risk that investors at the margin associate with receiving an expected stream of income. The greater these premiums, the lower the share price, all other factors staying the same.

Eq. (4-9) suggests that variations in the cost of equity capital are determined primarily by the types of investment to which a company allocates its capital and by the firm's degree of financial leverage. If the company has no leverage and if its investments consist entirely of Treasury securities, its cost of equity capital will approximate the risk-free rate, i. If the

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[8]Again, we must stress the assumptions of a constant capital structure and a constant dividend policy.

firm invests in assets other than Treasury securities, investors will demand a higher equity capitalization rate – the more risky the investment, the higher the rate.

Unless the firm changes the overall risk composition of its assets or its degree of financial leverage, its cost of equity capital, k_e, is likely to be relatively stable over time. The reason that the market prices of certain stocks change so dramatically from one period to the next is that investor expectations of future growth change. The greater the perceived growth for a stock, the greater the fluctuation in market price for a given percentage change in expected growth.[9] Consider two stocks with different growth rates, as perceived by investors at the margin, and suppose that the perpetual-growth valuation model is a reasonable representation of the way the market values a stock

	Company A	Company B
Dividends per share	$1.50	$1.20
Market price per share	30.00	60.00
Expected future growth rate	4%	10%

Given this information, the measured costs of equity capital for the two companies are

$$k_e \text{ Company A} = \frac{1.50}{30.00} + 0.04 = 9 \text{ per cent}$$

$$k_e \text{ Company B} = \frac{1.20}{60.00} + 0.10 = 12 \text{ per cent}$$

Assume that these measured costs are in fact the true costs of equity capital for the company. Suppose further that the companies do not change their dividends, the risk complexions of their assets, or their degrees of financial leverage, and that business and financial risks as perceived by investors at the margin do not change. Suppose, however, that the economy is now in a mild recession and that investors revise downward by 20 per cent their estimates of growth for both companies. If the perpetual-growth model still holds and if the risk-free rate remains unchanged, the new market prices of the stocks become

$$\text{Market price, Company A} = \frac{D_0}{k_e - g} = \frac{1.50}{0.09 - 0.032} = \$25.86$$

$$\text{Market price, Company B} = \frac{D_0}{k_e - g} = \frac{1.20}{0.12 - 0.08} = \$30.00$$

[9] See Burton G. Malkiel, "Equity Yields, Growth, and the Structure of Share Prices," *American Economic Review*, LIII (December, 1963), 1004–31.

Thus, given a percentage change in expected future growth, the growth stock, Company B, will decline by a far greater percentage than the stock of a company expanding at, say, a normal rate of growth. Thus, stock prices respond to changes in investor expectations, which have been brought about by new information. Investors can be thought to form expectations about the growth of a company on the basis of all available information. As fresh information is received, they revise these expectations, often resulting in a change in their valuation of the firm.

Given a change in market expectations, the cost of equity capital does not necessarily have to change; indeed, this cost may be relatively stable over time. However, we must qualify the discussion above to allow for at least some changes in the required rate of return with changes in expectations. If a recession is on the horizon, one would expect easier money and lower interest rates. Under these circumstances, the risk-free rate is likely to decline. Moreover, as expectations of operating income of the firm change, so may its business risk as perceived by investors at the margin, even though the company does not change its asset portfolio. If the premium for business risk increases during a recession, this change will cause a decrease in the price of the stock, all other things remaining the same. Similarly, in a recession, the financial risk of a company as perceived by investors at the margin may increase, causing an increase in the premium for financial risk. Again, this increase will cause the price of the stock to fall, all other things remaining the same. Increases in the premiums for business risk and financial risk during a recession may be offset partially or entirely by the decrease in the risk-free rate.

We must conclude that the cost of equity capital, k_e, can change as the risk-free rate and the premiums for business and financial risk change. This change may occur even though the firm does not alter its asset portfolio or its financing mix. However, the effect of these changes on the market price of the stock is not likely to be nearly so great as the change in future growth, as perceived by investors at the margin. We would suggest that the real cost of equity capital is relatively stable over time. The principal cause for its change is an actual change in the asset portfolio of the company and, to a lesser extent, in its capital structure. Nevertheless, the financial manager must be cognizant of changes in the perceived risk of the company by investors, apart from the investment, financing, and dividend decisions of the firm.

Flotation Costs and Underpricing. If a company is to sell a new issue of common stock to the general public, the issue will need to be priced below the current market price. Moreover, placing the issue will involve flotation costs. As a result, net proceeds from the sale of the stock will be less than the current market price of the stock times the number of shares issued. The cost of equity capital calculations should be modified to take account of this factor. The cost of new equity capital, then, is

computed by solving the following equation for k_n

$$P_f = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_n)^t} \qquad \text{(4-10)}$$

where P_f is the net proceeds to the company, after flotation costs and underpricing, from the sale of a share of common stock. The only difference between this expression and Eq. (4-5) is that P_f is substituted for P_0, the current market price, on the left-hand side of the equation. As P_f is less than P_0, the cost of new equity capital, k_n, in Eq. (4-10) will be somewhat higher than that found using Eq. (4-5).

In summary, the cost of equity capital is the rate of return required by investors at the margin on the equity-financed portion of an investment proposal, holding constant the capital structure, business risk, and dividend policy of the firm. It can be measured by solving for the rate of discount that equates the present value of all expected future dividends per share, as perceived by investors at the margin, with the current market price per share of common stock. This cost must be adjusted upward for flotation costs and underpricing if stock is to be sold externally. We turn now to the cost of retained earnings, which is related directly to our discussion of a firm's cost of equity capital.

COST OF RETAINED EARNINGS

For many firms, a large portion of their financing of investment projects comes from retained earnings. It might seem that these funds are free, but there is a very definite opportunity cost involved. This opportunity cost is simply the dividend foregone by stockholders. In the absence of taxation, the *minimum* cost of retaining these earnings is the cost of equity capital, based upon the current market price of the stock rather than upon the net proceeds of a stock sale, P_f. This cost is determined by solving Eq. (4-5) for k_e; it represents the return that investors expect to receive.

If the firm is unable to generate investment opportunities that provide a return of k_e, stockholders presumably can find stocks, in the marketplace, of other companies with the same degree of risk that can provide such a return. Rather than invest in projects providing a lower return, the firm should distribute the earnings to stockholders and let them invest on their own. In this way, stockholders are able to increase their expected wealth in keeping with their risk-return preferences. If the firm were to retain earnings and then invest in a project that was expected to provide a lower return on the equity-financed portion, the expected return to stockholders would decline. As a result, they would suffer a loss in expected wealth. Thus, k_e represents the minimum cost of retained earnings. We say minimum cost because dividends may have a value above

the stockholder investment opportunity rate. However, we defer discussion of this issue until Chapters 9 and 10.

With taxation, the stockholder will have use not of the entire distribution of earnings but only of the portion that remains after taxes. For example, if the stockholder is in the 30 per cent tax bracket, he will have the use of only 70 per cent of the dividend. In addition, to invest in another stock, he will have to pay brokerage commissions. The total return he is able to achieve by investing the dividend in the stock of a company of identical risk is

$$R = k_e(1 - T)(1 - B) \tag{4-11}$$

where T is his marginal tax rate, and B is the brokerage commission expressed as a per cent. If k_e is 10 per cent, T is 30 per cent, and B is 2 per cent, his expected return is

$$R = 0.10(1 - 0.30)(1 - 0.02) = 6.86 \text{ per cent}$$

Many authors contend that the cost of retained earnings must take into account the taxes a stockholder has to pay on dividends he receives, as well as brokerage costs. According to this view, the cost of retained earnings is represented by Eq. (4-11), where T is the weighted-average tax rate for all stockholders, B is the weighted-average brokerage commission, and k_e is the minimum cost of retained earnings computed in the absence of taxation. The difficulty in this approach is in determining the marginal tax rate for all stockholders and resolving differences in these tax rates. For example, the amount of usable funds from a dividend will differ greatly depending upon whether the recipient is an individual in a high tax bracket or an institutional investor which pays no taxes. While rough estimates suggest that the average marginal tax rate of individual stockholders is around 40 per cent,[10] the weighted average for stockholders of a specific company will depend upon the type of stockholders involved. Such estimates are extremely difficult.

A second approach to evaluating the cost of retained earnings is the "external-yield" criterion.[11] It is based upon the notion that the firm should evaluate external investment opportunities as a use for retained earnings. Accordingly, the return available on the acquisition of another company, or a portion thereof such as a division, having the same degree of risk as that of the acquiring firm determines the cost of retained earnings. Moreover, the return available externally should be approximately k_e, assuming equilibration in the marketplace between expected return

[10] See Vincent Jolivet, "The Weighted Average Marginal Tax Rate on Dividends Received by Individuals in the U.S.," *American Economic Review*, LVI (June, 1966), 473–77; and Edwin J. Elton and Martin J. Gruber, "Marginal Stockholder Tax Rates and the Clientele Effect," *Review of Economics and Statistics*, LII (February, 1970), 68–74.

[11] See Ezra Solomon, *The Theory of Financial Management* (New York: Columbia University Press, 1963), pp. 53–55.

and risk. This approach implies that the opportunity foregone by the re-tention of earnings is investment externally in another enterprise.

With the external-yield criterion, the opportunity cost of retained earnings is determined by what the firm is able to obtain on external investment of funds. In the approach presented first, the criterion is based upon what stockholders are able to obtain on other investments. Note that the external-yield criterion is not affected by personal income taxes. It is simply the return on the direct investment of funds by the firm. The external-yield criterion represents an economically justifiable opportunity cost that can be applied consistently. Moreover, there is not the problem of determining marginal tax rates for investors. For a large company whose stock is widely held, this determination is not possible. A strong case can be made for the use of k_e as the cost of retained earnings, but acceptance of this use is far from universal. Many would adjust it downward for a tax effect.

COST OF CONVERTIBLE SECURITIES

As we discuss in detail in Chapter 14, convertible securities usually are treated as a means of deferred equity financing. The price at which the securities are convertible into common stock is known as the conversion price. If a $1,000 face value bond has a conversion price of $50, this price denotes that the bond can be converted into twenty shares of common stock. Since the convertible security is delayed equity financing, its cost should be so treated. This cost can be estimated by solving for the discount rate that equates the expected after-tax interest payments, or dividend payments in the case of a convertible preferred stock, plus the expected terminal value with the offering price of the security. The expected terminal value can be represented by the expected future market price per share of common at some future date times the number of shares into which the security is convertible. If all investors had the same horizon period, the cost of a convertible bond might be found by solving the following equation for k_c:

$$B_0 = \sum_{t=1}^{n} \frac{I(1-T)}{(1+k_c)^t} + \frac{P_n CR}{(1+k_c)^n} \tag{4-12}$$

where B_0 = market value of convertible bond at time 0
  I = annual interest payments, determined by the coupon rate
  T = corporate tax rate
  n = investors' horizon period
  P_n = expected market price per share of stock at the end of period n
  CR = conversion ratio, i.e., the number of shares into which the bond is convertible.

For example, if B_0 were \$1,000, I were \$60, the corporate tax rate were 50 per cent, the conversion ratio were 20, and investors at the margin expected market price per share to be \$75 at the end of their horizon period of five years, we would have

$$\$1,000 = \sum_{t=1}^{5} \frac{60(0.5)}{(1 + k_c)^t} + \frac{75(20)}{(1 + k_c)^5}$$

When we solve for k_c, we find it to be 11.02 per cent.[12] This figure then would be taken as the cost of convertible bond financing. The critical factor in measurement, of course, is in estimating the market price per share that will prevail at the end of the horizon period. This, however, is no more difficult than estimating the expected future stream of dividends per share when we calculate the cost of equity capital. In making both estimates, the same factors are considered.

Another problem is that different investors have different horizon periods. Rather than try to specify a weighted-average horizon period, it may be more feasible for the financial manager to estimate the length of time the convertible securities are likely to remain outstanding. This estimate should be based upon considerations taken up in Chapter 14. Given a horizon period, the market price at the end of the period must be estimated. Again, the critical factor is in estimating the market price perceived by investors. As long as this is accurately done, slight errors in estimating the appropriate horizon period will not affect the results materially.

WEIGHTED-AVERAGE COST OF CAPITAL

Once the costs of the individual components of the capital structure have been computed, these costs may be weighted according to some standard and a weighted-average cost of capital calculated. As an illustration of only the mechanics of the calculations, suppose that a firm had the following capital structure at the latest statement date.

	Amount	Proportion
Debt	\$ 30 million	30%
Preferred stock	10 million	10
Common stock	20 million	20
Retained earnings	40 million	40
	\$100 million	100%

[12] For further analysis of this method, see James C. Van Horne, *The Function and Analysis of Capital Market Rates* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970), pp. 169–71.

Suppose further that the firm computed the following explicit after-tax costs for these component methods of financing:[13]

	Cost
Debt	4.0%
Preferred stock	8.0
Common stock	11.0
Retained earnings	10.0

If the above weights are used, the weighted-average cost of capital for this example problem is

(1) *Method of Financing*	(2) *Proportion*	(3) *Cost*	(4) *Weighted Cost (2 × 3)*
Debt	30%	4.0%	1.20%
Preferred stock	10	8.0	0.80
Common stock	20	11.0	2.20
Retained earnings	40	10.0	4.00
Weighted-average cost of capital			8.20%

Given the assumptions of this example, we find the measured weighted-average cost of capital to be 8.2 per cent.

With the calculation of a weighted-average cost of capital, the critical question is whether the figure represents the firm's real cost of capital to be used for evaluating investment projects. If we hold business risk constant, is 8.2 per cent the minimum rate of return that must be earned on an investment in order to leave the market price of the stock unchanged? The answer to this question depends upon how accurately we have measured the individual marginal costs, upon the weighting system, and upon certain other assumptions. Assume for now that we are able to measure accurately the marginal costs of the individual sources of financing; let us examine the importance of the weighting system.

WEIGHTING SYSTEM

Marginal Weights. The critical assumption in any weighting system is that the firm in fact will raise capital in the proportions specified. Because the firm raises capital *marginally* to make a *marginal* investment in new projects, we need to work with the marginal cost of capital to the firm as

[13] We assume that the cost of retained earnings was computed using an external-yield criterion. It is lower than the cost of common stock because of the absence of underpricing and flotation costs associated with a new stock issue.

a whole. This rate depends upon the package of funds employed to finance investment projects.[14] In order for the weighted-average cost of capital to represent a marginal cost, the weights employed must be marginal; that is, the weights must correspond to the proportions of financing inputs the firm intends to employ. If they do not, capital is raised on a marginal basis in proportions other than those used to calculate this cost. As a result, the real weighted-average cost of capital will differ from that calculated and used for capital-investment decisions. An obvious bias results. If the real cost is greater than that which is measured, certain investment projects will be accepted that will leave investors worse off than before. On the other hand, if the real cost is less than the measured cost, projects will be rejected that could increase shareholder wealth. Therefore, the 8.2 per cent weighted-average cost of capital computed in our example is realistic only if the firm intends to finance in the future in the same proportions as its existing capital structure.

It is recognized that the raising of capital is "lumpy," and strict proportions cannot be maintained. For example, a firm would have difficulty in financing each project undertaken with 35 per cent debt, 10 per cent convertible securities, and 55 per cent retained earnings. In practice, it may finance with debt in one instance and with convertible securities or retained earnings in another. Over time, most firms are able to finance in roughly a proportional manner. It is in this sense that we try to measure the marginal cost of capital for the package of financing employed. Another problem is that retained earnings, an important source of funds for most firms, are constrained by the absolute amount of earnings. If a firm's investment opportunities warrant expansion at a rate faster than the growth in earnings, financing by means of retained earnings must diminish relative to other means. Where the expansion is expected to be continuous for a number of years, the financing mix of the firm is subject to a constraint with respect to the ability of the firm to retain earnings. This constraint must be recognized. Frequently, however, expansion is concentrated in a few years so that over the long run a firm is able to finance with a roughly constant proportion of retained earnings.

Book Value versus Market Value Weights. If historical weights are used, a problem arises as to the weights. When the market value of any component method of financing differs from its book value, the weighted-average cost of capital calculated will differ according to whether book value or market value weights are employed. For example, suppose that the firm in the previous example has 2 million shares of common stock outstanding and that the current market price is $50 a share. For simplicity, assume that the market prices of the debt and the preferred stock

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[14] See Wilbur G. Lewellen, *The Cost of Capital* (Belmont, Calif.: Wadsworth Publishing Co., Inc., 1969), p. 87.

are the same as their book values. If the costs of financing are the same as before, the weighted-average cost of capital would be

(1) Method of Financing	(2) Amount	(3) Proportion	(4) Cost	(5) Weighted Cost (3 × 4)
Debt	$ 30 million	0.214	4.0%	0.86%
Preferred stock	10 million	0.072	8.0	0.58
Market value of equity*	100 million	0.714	10.0	7.14
Weighted-average cost of capital				8.58%

Two million shares times $50 a share = $100 million.

Note that the equity capital of the company is the total market value of the common stock outstanding rather than the common stock plus retained earnings. Whenever the market value of a share of stock is greater than its book value, the weighted-average cost of capital using market value weights usually will exceed the weighted-average cost using book value weights.

If the firm finances in the future according to market value weights, and these weights differ from book value weights, the book value proportions of the firm's capital structure will change. For example, suppose a firm had the following book value and market value weights:

	Book Value		Market Value	
	Amount	Proportion	Amount	Proportion
Debt	$40 million	40%	$ 40 million	25%
Equity	60 million	60	120 million	75

If the firm finances according to its market value weights and raises $20 million in additional debt funds and $60 million in equity funds, the book value proportions of the firm after this financing program is completed are

	Book Value	
	Amount	Proportion
Debt	$ 60 million	33.33%
Equity	120 million	66.66%

Thus, the capital structure of the firm has changed toward a higher proportion of equity. As additional financing occurs, the book value mix

of the capital structure will approach the market value mix. If book value weights are used and financing is undertaken in those proportions, the market value proportions of the capital structure will change. The previous example need only be reversed.

The question of book value versus market value weights is of concern primarily in a historical sense — that is, if we use historical weights based upon balance sheet figures. If the weights used are marginal in the sense that they reflect the financing proportions the firm intends to employ, we are not faced with this problem. The firm simply specifies the financing mix it intends to use over time, and these proportions are multiplied by the various marginal costs to obtain the weighted-average cost of capital. As discussed earlier, we are interested in the marginal cost of capital, not the historical cost. Therefore, we must use as weights the proportions of financing the firm intends to employ over time.

Change in Capital Stucture. A problem occurs whenever the firm wishes to change its capital structure. The costs of the component methods of financing usually are based upon the existing capital structure, and these costs may differ from those that rule once the firm has achieved its desired capital structure. As the firm cannot measure its costs directly at the desired capital structure, these costs must be estimated. During the period of transition from the present capital structure to one that is desired, the firm usually will rely upon one type of financing. For example, it might finance with debt until the desired capital structure is achieved. The question arises as to what cost of financing should be used for capital-budgeting purposes during the transition period. While there may be some discrepancy, it is best to use the estimated weighted-average cost of capital based upon the financing mix to be employed once the desired capital structure is reached. There would be no transitional problem if the firm undertook an immediate change in its capital structure by issuing debt and repurchasing stock. However, most firms are reluctant to do this; consequently, the transition period usually will be gradual and will involve some problems.

RATIONALE FOR
WEIGHTED-AVERAGE COST

The rationale behind the use of a weighted-average cost of capital is that by financing in the proportions specified and accepting projects yielding more than the weighted-average cost, the firm is able to increase the market price of its stock over the long run. This increase occurs because investment projects accepted are expected to yield more on their equity-financed portions than the cost of equity capital, k_e. Once these expectations are apparent to the marketplace, the market price of the stock should rise, all other things remaining the same. Holding business

risk constant, the firm has accepted projects that are expected to provide a return greater than that required by investors at the margin.

Using Eq. (4-5) as our valuation model and holding constant the dividend policy of the firm, we see that the acceptance of the projects raises expected future dividends per share, D_t, in the numerator of the equation. If the equity-financed portion of the new investment projects consists of a common-stock offering, market price per share will rise with an upward shift in expectations of future dividends per share. Embodied in these expectations is the dilution that necessarily will occur with the common-stock offering. In other words, the rise in expected future earnings must be sufficient to raise expected future *dividends per share,* not just expected future dividends.

If the equity-financed portion of the new investment projects consists of retained earnings, no dilution will occur. Here the rise in expected future dividends per share must be sufficient so that when the incremental increases are discounted by the market discount rate, k_e, their present value exceeds the equity capital employed in the projects. One can visualize the process as the firm's employing investors' capital at time 0 to invest in a project whose cash-flow benefits are expected to give rise to higher future dividends. In order for the project to be worthwhile, the present value of the incremental dividends must equal or exceed the equity capital employed in it. If the present value exceeds the equity capital employed in the project, the market price of the stock, P_0, will rise. If the present value is just equal to the amount of equity capital employed, P_0 will remain unchanged.[15]

We note that the capital-budgeting procedure described gives no consideration to the timing of the expected increases in future dividends, only to the present value of these increases. There are projects, of course, that provide erratic expected earnings. Various authors contend that the marketplace values stability in earnings per share about a trend. They

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[15] Over time, however, market price per share will rise if the firm invests in projects whose return on the equity-financed portion just equals the cost of equity capital. This type of situation is known as expansion and is taken up in the appendix. In essence, dividends per share would rise over time because of the increase in earnings occasioned by the reinvestment of retained earnings. As a result, the present value of expected future dividends also would rise over time. For example, if a perpetual-growth model were applicable,

$$P_0 = \frac{D_0}{k_e - g}$$

If expectations and the market rate of discount remained unchanged, the market price at time 1 would be

$$P_1 = \frac{D_1}{k_e - g}$$

As D_1 is greater than D_0, P_1 would be greater than P_0. Similarly, it can be shown that $P_t > P_{t-1}$ for all t under our assumptions.

reason that to invest in a project that is expected to result in erratic earnings per share may have a depressing effect upon share price, despite the fact that its return on the equity-financed portion exceeds the cost of equity capital.

Lerner-Rappaport and Robichek-Ogilvie-Roach propose that the firm maximize the present value of a project, subject to the constraint of maintaining some minimum trend in earnings per share.[16] Presumably, the firm would reject projects that did not satisfy this constraint, even though their expected return might exceed the firm's cost of capital. In theory, this rejection leads to a less than optimal allocation of capital. Projects are rejected that are expected to result in incremental future dividends, the present value of which exceeds the equity capital employed in the project. While the implication that investors prefer a steady growth in earnings per share cannot be ignored, it may not be a critical factor in most situations. For many projects, the expected cash-flow patterns may be relatively stable about a trend. Where the pattern in earnings per share is erratic, the firm can counterbalance this volatility by paying stable dividends—a subject taken up in Chapter 10. For these reasons, we continue to feel that as a general rule, the firm should use its cost of capital as the acceptance criterion for project selection.

Before concluding, we must return to the critical assumption that over time the firm finances in the proportions specified. If it does so, the financial risk of the company remains roughly unchanged. As we shall see in Chapter 7, the "implicit" costs of financing are embodied in the weighted-average cost of capital by virtue of the fact that a firm has to supplement nonequity financing with equity financing. It does not raise capital continually with supposedly cheaper debt funds without increasing its equity base. The firm's capital structure need not be optimal for the firm to employ the weighted-average cost of capital for capital-budgeting purposes. The important consideration is that the weights used be based upon the future financing plans of the company. If they are not, the weighted-average cost of capital calculated does not correspond to the actual cost of funds obtained; as a result, capital-budgeting decisions are likely to be suboptimal.

Even when we recognize the crudeness of estimations and the many assumptions inherent in its use, the weighted-average cost-of-capital approach can result in capital-budgeting decisions that tend to be optimal. This is particularly true if the investment proposals under consideration do not differ significantly in risk from that of the existing portfolio of assets; that is, their acceptance does not change the business-risk complexion of the firm as perceived by investors at the margin.

[16] Eugene M. Lerner and Alfred Rappaport, "Limit DCF in Capital Budgeting," *Harvard Business Review*, 46 (July–August, 1968), 133–39; and Alexander A. Robichek, Donald G. Ogilvie, and John D. C. Roach, "Capital Budgeting: A Pragmatic Approach," *Financial Executive*, XXXVII (April, 1969), 26–38.

Having taken up the concept of a weighted-average cost of capital, we can now consider the cost of depreciation as a source of funds. This consideration was deferred purposely in our evaluation of costs of capital for specific sources of financing. Like retained earnings, funds generated from depreciation have an opportunity cost. Unfortunately, these funds often are regarded as free because they do not appear on the liability side of the balance sheet. Depreciation represents the transformation of fixed assets into cash. These funds should not be reinvested into fixed assets unless the assets meet the minimum standard for acceptance. This minimum standard should be the cost of capital, because that is the opportunity cost for their use.

Suppose that a company is started with an initial capitalization of $500,000 in common stock. Suppose further that the company borrows $500,000 and invests the $1 million in fixed assets and that it has the policy of maintaining a one-to-one debt-to-equity ratio based upon book values. The company's balance sheet, then, is

		Debt	$ 500,000
Fixed assets	$1,000,000	Common stock	500,000
Total assets	$1,000,000	Total liabilities and net worth	$1,000,000

If the assets are depreciated on a straight-line basis and the depreciable life is ten years, annual depreciation charges are $100,000. For simplicity, assume that the company has zero profit in the first year and no further investment proposals on the horizon. The balance sheet after the first year of operation would be

Cash	$ 100,000		
Fixed assets	1,000,000	Debt	$ 500,000
Depreciation	−100,000	Common stock	500,000
Net fixed assets	900,000		
		Total liabilities and	
Total assets	$1,000,000	net worth	$1,000,000

If the company had the policy of distributing excess cash by reducing proportionately the right-hand side of the balance sheet, it would pay off $50,000 of debt and repurchase $50,000 of stock. The opportunity cost, then, for the use of the depreciation funds is the weighted-average cost of debt and equity, or the firm's weighted-average cost of capital. Consequently, the cost of depreciation funds does not enter into the calculation of a firm's cost of capital.

Assuming that the firm has determined an appropriate capital structure, it faces the problem of having to raise capital in roughly those proportions. The critical question is whether it can actually raise this capital at an average real cost equal to the measured weighted-average cost of capital. A number of authors postulate an upward sloping supply curve for capital – in other words, the average weighted cost of capital rises with the total amount of capital raised.[17] They suggest that, in the short run, the average cost curve rises in a manner similar, perhaps, to that shown in Figure 4-1. The exact shape of the supply curve will vary according to the size of the company and conditions in the capital markets.
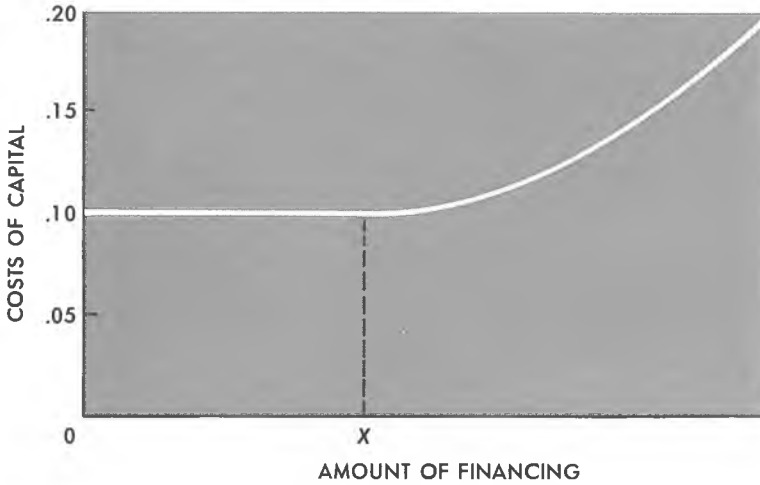
FIGURE 4-1
Supply curve of capital

An upward sloping supply curve suggests that the firm cannot raise an unlimited amount of capital at one time at the same cost. Instead, a digestion period is needed during which the company invests in projects and demonstrates to investors and creditors its ability to generate profits on those investments. Figure 4-1 implies that at a moment in time, the firm can raise capital at a real cost of 10 per cent up to an amount of x dollars. After x, the average cost of capital rises. Given a sufficient digestion period, however, the firm again will be able to raise capital at a real cost of 10 per cent.

The cost-of-capital approach described so far implies a horizontal supply curve. The assumption is that the firm can raise any amount of capital at the measured average real cost. In the short run, this occurrence is unlikely. However, most firms are investing continuously in

[17] See, for example, James S. Duesenberry, *Business Cycles and Economic Growth* (New York: McGraw-Hill Book Company, 1958), Chapter 5; and J. Robert Lindsay and Arnold W. Sametz, *Financial Management: An Analytical Approach* (Homewood, Ill.: Richard D. Irwin, Inc., 1967), Chapters 19 and 20.

projects and financing these investments. If the amount of financing in one period does not differ significantly from the amounts in previous periods, there is no reason to believe that the real cost of capital will fluctuate significantly from period to period. In each period, the firm would be raising capital in approximately the same area of the supply curve. In addition, the supply curve would probably not be sharply upward sloping in the initial stages. Nevertheless, if the firm does have to undertake substantial financing in one period, and this amount is significantly out of line with amounts in previous periods, the firm's cost of capital may be higher. In turn, this phenomenon should influence the investment decision. Certain projects providing returns close to the measured cost-of-capital cutoff rate no longer would be acceptable. The firm may choose to postpone these projects until a time when the amount of total investment, and financing, is not expected to be as large.

Thus, an upward sloping supply curve for capital adds another dimension to the investment and financing decisions. It suggests that the cost of capital cannot be determined independently of the amount of funds to
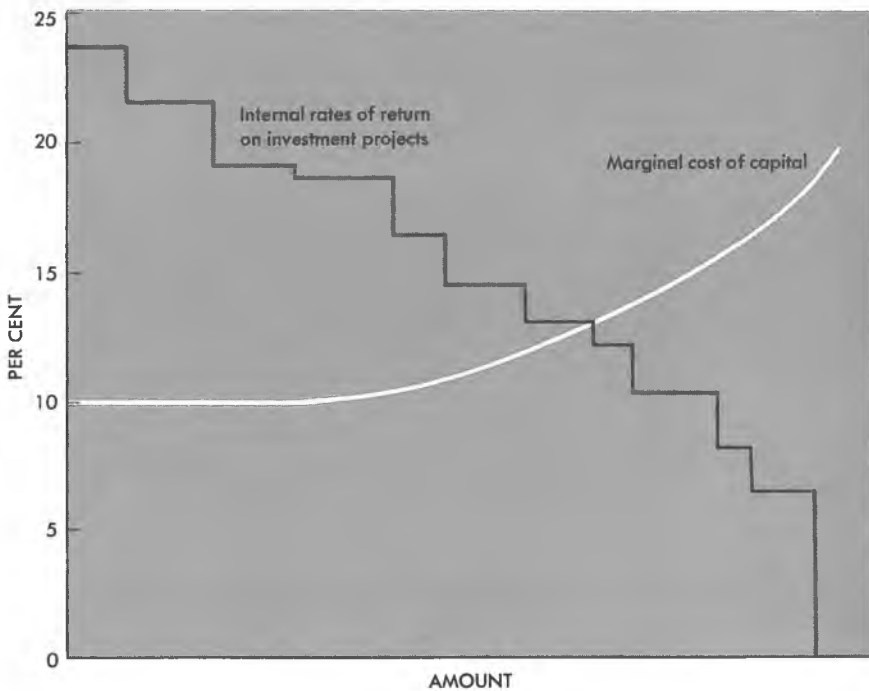
FIGURE 4-2
Internal rates of return on investment opportunities and the marginal cost of capital

be raised. However, the amount to be raised depends upon the investment opportunities available. Accordingly, the investment and financing decisions of the firm are determined simultaneously. An illustration of this occurrence is shown in Figure 4-2. The intersection of the marginal cost-of-capital line with the marginal rate of return on investment line determines the amount of funds to be invested and the amount of financing for the period. It is clear from this discussion that the financial manager must be cognizant of capital-market conditions that would cause the marginal cost of capital of the firm to rise with the amount of funds raised. These conditions must be taken into account in capital budgeting.

SUMMARY

The cost of capital may be defined as the rate of return on investment projects necessary to leave unchanged the market price of the firm's stock. A company has available a wide assortment of financing methods, each with an explicit cost. By far the most difficult cost to measure is the cost of equity capital. Using a dividend valuation model, the cost of equity is the rate of discount that equates the present value of the stream of expected future dividends per share, as perceived by investors at the margin, with the market price of the stock. The rate of return required by the marketplace on equity capital might be thought to be comprised of the risk-free rate, a premium for business risk, and a premium for financial risk.

Given the measurement of the marginal costs of the individual methods of financing, a weighted-average cost of capital can be computed. The weights employed should correspond to the proportions with which the firm intends to finance. Once computed, the weighted-average cost is used as the basis for accepting or rejecting investment projects. Holding constant business risk, capital structure, and dividend policy, projects expected to yield more are accepted, and those expected to yield less are rejected. It is important that the firm recognize the possibility that capital costs may increase if the amount of financing in one period is substantially higher than that in previous periods. The supply curve for the total capital the firm can raise at any one time may be upward sloping.

If measured properly, a weighted-average cost-of-capital approach can lead to optimal investment decisions. The cost of capital is only a means to an end—namely, that of maximizing the market price of the stock. Unfortunately, it is easy to lose sight of the objective because of the somewhat mechanical nature of the approach. By financing in specific capital structure proportions over time, the firm is able to hold financial risk constant. In the two chapters that follow, we examine how the valuation of the firm may change in keeping with changes in its business-risk complexion.

Earlier in this chapter, we suggested that, as a general rule, it is inappropriate to employ the ratio of current earnings per share to current market price per share as a measure of the cost of equity capital. In most cases, the earnings/price ratio is not a realistic estimate of the return that investors expect to receive. One need only point to the famous IBM paradox to illustrate the point. With a market price per share of around $340 in February of 1970 and earnings per share of $8.21 in 1969, IBM had an earnings/price ratio of 2.4 per cent. Certainly investors expected to receive more than a 2.4 per cent return — a yield less than that available on Treasury bills — and would be dissatisfied if IBM invested in projects that were expected to return only this amount on the equity-financed portion. Instead, investors anticipated a stream of dividend income and a terminal value that would result in an expected return on investment significantly in excess of 2.4 per cent. For growth firms with investment opportunities expected to provide returns in excess of that required by investors at the margin, the earnings/price ratio is a biased and low estimate of the cost of equity capital.

There are two cases, however, in which the cost of equity capital is equivalent to the earnings/price ratio. The first and simpler case is that of a firm whose future earnings per share are expected to be the same as current earnings per share and whose dividend-payout ratio is 100 per cent. To visualize, the dividend valuation model can be expressed as

$$P_0 = \sum_{t=1}^{\infty} \frac{E_t(1-b)}{(1+k_e)^t} \tag{4A-1}$$

where E_t is the expected earnings per share in period t as perceived by investors at the margin, and b is the percentage of earnings retained. Since all future earnings per share are the same as present earnings per share and since the retention rate, b, is zero, we have

$$P_0 = \sum_{t=1}^{\infty} \frac{E_0}{(1+k_e)^t} \tag{4A-2}$$

From Appendix 3A, we know that the market price of a perpetuity that is expected to pay a fixed return forever is

$$P_0 = \frac{E_0}{k_e} \tag{4A-3}$$

Thus, the cost of equity capital is

$$k_e = \frac{E_0}{P_0} \tag{4A-4}$$

The second case in which the cost of equity capital equals the earnings/price ratio involves an expansion situation where the firm is able to in-

vest in projects that provide an expected perpetual return of k_e on the equity-financed portion.[18] Expansion differs from a growth situation in which the firm has investment opportunities that provide an expected return in excess of k_e on the equity-financed portion. For simplicity, assume an all-equity capital structure and a retention rate of b in each period.

Suppose that at time 0, a firm momentarily will pay a dividend of $E_0(1 - b)$, where E_0 is earnings per share in that period. It will reinvest bE_0 dollars per share in assets that are expected to provide a perpetual return of k_e. Therefore, earnings per share in all future periods will be bE_0k_e higher than E_0. Expected earnings per share in period 1 will be

$$E_1 = E_0 + bE_0k_e = E_0(1 + bk_e) \tag{4A-5}$$

Expected dividends per share at the end of period 1 will be

$$D_1 = E_0(1 + bk_e)(1 - b) \tag{4A-6}$$

In period 1, the firm will retain $E_0(1 + bk_e)b$ dollars per share and will invest in assets which are expected to provide a perpetual return, beyond that point, of k_e. Therefore, expected earnings per share in period 2 would be

$$
\begin{aligned}
E_2 &= E_0(1 + bk_e) + E_0(1 + bk_e)bk_e \\
&= E_0 + 2E_0bk_e + E_0bk_e^2 \\
&= (E_0 + E_0bk_e)^2 \\
&= E_0(1 + bk_e)^2
\end{aligned}
\tag{4A-7}
$$

Thus, expected dividends per share at the end of period 2 would be

$$D_2 = E_0(1 + bk_e)^2(1 - b) \tag{4A-8}$$

In a similar manner, we are able to determine that expected dividends per share at the end of period t are

$$D_t = E_0(1 + bk_e)^t(1 - b) \tag{4A-9}$$

The dividend valuation model for this expected future stream of dividends becomes

$$P_0 = \sum_{t=0}^{\infty} \frac{E_0(1 + bk_e)^t(1 - b)}{(1 + k_e)^t} \tag{4A-10}$$

We know from our previous discussion of growth models that Eq. (4A-10) is a perpetual-growth formula, in which dividends per share are expected to grow at a compound rate of bk_e. For the case of continuous compounding, Eq. (4A-10) becomes

$$P_0 = \int_0^{\infty} (1 - b)E_0e^{bk_et}e^{-k_et}dt = \int_0^{\infty} (1 - b)E_0e^{-t(k_e-bk_e)}dt \tag{4A-11}$$

---

[18] See Solomon, *The Theory of Financial Management*, pp. 59–62.

Integrating this expression, we find

$$P_0 = \frac{(1-b)E_0}{k_e - bk_e} = \frac{(1-b)E_0}{(1-b)k_e} = \frac{E_0}{k_e} \qquad (4\text{A-}12)$$

Therefore, the cost of equity capital is

$$k_e = \frac{E_0}{P_0} \qquad (4\text{A-}13)$$

Thus, for a firm which fulfills the assumptions made earlier, the cost of equity capital is the earnings/price ratio. The most important of these is that it does not have investment opportunities that are expected to return more than k_e. For growth situations where the firm has projects yielding more than k_e on the equity-financed portion, the earnings/price ratio understates the firm's cost of equity capital. As the earnings/price ratio is appropriate only under certain circumstances, we use the more general dividend valuation model in our estimates of the cost of equity capital.

PROBLEMS

1. The Simple Corporation, which has a 50 per cent tax rate, currently has a capital structure that is one-third debt and two-thirds equity. The corporation will always raise funds in this proportion. For the current period, the costs of raising various amounts of debt and equity are given below:

Amount	Debt	Equity
$100,000	8.00	12.00%
200,000	8.50	13.00
300,000	9.00	14.00
400,000	10.00	16.00
500,000	12.00	18.00
600 000	16.00	22.00

(a) Determine the cost of capital to Simple if $300,000 or $600,000 is raised.
(b) If a project had an after-tax return of 10.5 per cent, under what conditions would it be adopted?

2. On March 10, International Copy Machines (ICOM), one of the "favor-tes" of the stock market, was priced at $300 per share. This price was based on an expected annual growth rate of at least 20 per cent for quite some time in the future. In July, economic indicators turned down, and investors revised down-ward to 15 per cent their estimate for future growth of ICOM. What should happen to the price of the stock? Assume the following:
(a) A perpetual-growth valuation model is a reasonable representation of the way the market values ICOM.

(b) The measured cost of equity capital to the firm is the true cost.

(c) The firm does not change its dividend, the risk complexion of its assets, or its degree of financial leverage.

(d) The firm pays a current dividend of $3 per share.

3. The Manx Company was recently formed to manufacture a new product. It has the following capital structure:

9% Debentures of 1982	$ 6,000,000
7% Preferred stock	2,000,000
Common stock (320,000 shs.)	8,000,000
	$16,000,000

The common stock sells for $25 and is expected to pay a $2 dividend this year which will grow at 10 per cent for the foreseeable future. The company has a marginal tax rate of 50 per cent.

(a) Compute a weighted-average cost of capital.

(b) Compute the cost of depreciation.

(c) Assume that the investment banker for the Manx Company informed the firm that it could raise an additional $4 million in debt by means of a 10 per cent subordinated debenture. This sale would result in increasing the expected dividend to $2.50 and leave the growth rate unaffected, but the added risk would cause the price of the stock to fall to $20 per share. What would be the new cost of capital?

(d) Assume that an additional $4 million in junior subordinated debentures could be sold to yield 15 per cent. This would cause the price of the common to drop to $15, yet increase the dividend to $3 and the growth rate to 15 per cent. What would be the impact upon the cost of capital?

4. Assuming the firm has a tax rate of 50 per cent, compute the after-tax cost of the following:

(a) A bond, sold at par, with a $9\frac{1}{4}$ per cent coupon.

(b) A twenty-year, $8\frac{1}{2}\%$, $1,000 par bond sold at $900 less a 5 per cent underwriting commission. (Use an approximation method.)

(c) A preferred stock sold at $100 with a 7 per cent coupon and a call price of $110 if the company plans to call the issue in five years. (Use an approximation method.)

(d) A common stock selling at $20 and paying a $2 dividend which is expected to be continued indefinitely.

(e) The same common stock if dividends are expected to grow at the rate of 5 per cent per year.

(f) A common stock, selling at $30 per share, of a company which engages in no external financing. The stock earns $5 per share, of which one-half is paid in dividends. The shareholders expect the company to earn a constant after-tax rate of 10 per cent on investments.

(g) (1) A common stock selling for $21 whose dividends are expected to be $1 per year for the next five years and $2 per year forever thereafter. (2) If expectations hold true and the discount rate is the same, what should be the price of the stock at the beginning of the year 6?

5. (a) Assume that the stock of the Mason Company pays a dividend of $5, which is expected to increase at the rate of 5 per cent per year, and the stock sells at a normalized price of $100. Further, assume that to sell this stock in the market, the Mason Company would have to offer it at a 5 per cent discount from its normal price and, in addition, pay an investment banker a 10 per cent gross spread to place the issue. Compute the cost of equity.

(b) Compute the cost of retained earnings for the Mason Company (1) ignoring taxes and (2) assuming all stockholders have a marginal tax rate of 40 per cent.

(c) Assume that the Mason Company issues a $100 par value preferred stock convertible into common at $120 and paying a dividend of $4.16⅔. If the stock is converted after three years, when the common is at $140, determine its cost.

6. The Strong Corporation has the following capital structure:

Mortgage bonds 6s of '82	$ 3,000,000
Debentures 8s of '80	4,000,000
Preferred stock 7%	3,000,000
Common stock (800,000 shares)	8,000,000
Retained earnings	2,000,000
	$20,000,000

(a) The mortgage bonds were sold in 1952 to net the company $106. The current price of these bonds is $92, though a similar issue if sold now would net the company $90.

(b) The debentures were sold in 1960 to net the company $98. The current price of these bonds is $94, though a similar issue if sold now would net the company $92.

(c) The preferred stock is $100 par and was sold to net the company $95 per share. A similar issue now would net $90.

(d) The normalized market price of the firm's stock is $60. The firm paid $4,000,000 in dividends this year, which was 80 per cent of earnings. The firm expects earnings to grow at an annual rate of 4 per cent, and it anticipates maintaining an 80 per cent payout.

(e) The firm believes it would incur a cost of $5 per share if common stock were sold. A discount of $5 per share would also be required to guarantee a successful offering.

(f) The firm's stock is widely held by individuals with varying incomes.

(g) The firm's management believes in an opportunity cost concept of retained earnings.

(h) It is anticipated that the financing of future investments will reflect the current capital structure. Further, the management believes that book values rather than market values more adequately represent the balance of the structure.

(i) The marginal tax rate for the firm is 50 per cent.

Compute the after-tax marginal weighted-average cost of capital for the Strong Corporation.

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Capital Budgeting
for Risky
Investments:
The Single Proposal

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In the previous chapter, we assumed that the acceptance of any investment proposal or group of investment proposals did not alter the total business-risk complexion of the firm as a whole. As a result, we were able to hold constant this factor in our analysis. We know, however, that different investments have different degrees of risk; so we must extend our analysis to consider this problem. If the acceptance of an investment proposal or group of proposals alters the business-risk complexion of the firm, investors and creditors are likely to view the company differently before and after the acceptance of the proposal(s). Consequently, the total valuation of the enterprise may change; the greater the increase in perceived risk, the lower the valuation—all other things being equal—and vice versa. In evaluating investment proposals, a firm should take into account the effect that their acceptance will have on its perceived risk. The project that is expected to provide a high return may increase greatly the perceived risk of the firm. The result may be an actual decrease in the market price of the stock.

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Because overall business risk is what is important when we are trying to maximize share price, our ultimate concern is with the impact of an investment decision on the risk of the firm as a whole. We know from Chapters 2 and 4 that the greater the risk that investors associate with an expected stream of dividends, the greater the return that they require and the lower the share price, all other things being the same. In turn, their perception of risk is a function of the risk associated with the gross income of the firm. However, it is extremely difficult to evaluate the overall risk of the firm at the operating level. Consequently, the evaluation of risk frequently is confined to the individual proposal; and limited attention is paid to the effect of the proposal on the risk of the firm as a whole. In this chapter, we analyze various methods for incorporating the factor of risk into the capital-budgeting decision for an individual investment proposal. In the next chapter, we extend our analysis to the evaluation of risk for the firm as a whole; projects are evaluated according to their marginal contribution of expected net-present value and risk.

Methods for taking risk into account considered in this chapter include: adjustment of the required rate of return; calculation of the certainty equivalent of cash flows; direct analysis of the probability distributions of possible outcomes under varying assumptions of dependence of cash flows over time; the use of decision trees for sequential investment decisions; and direct incorporation of the utility preferences of the decision maker into the investment decision. In this chapter and the next, we move generally toward increasingly more complex methods for taking risk into account. The ideal solution to the problem of risk would be a method whereby we could measure exactly the effect that accepting an investment proposal or group of proposals would have on share price. More specifically, we would like to determine the effect of an investment decision on the risk premiums embodied in investors' required rate of return and on expected future dividends. In turn, these factors determine the effect of the decision on share price. Unfortunately, a solution of this sort is not yet operationally feasible.

Many of the methods we examine attempt to measure quantitatively the risk of a project or group of investment projects. Given this information, together with information about expected return, management then must assess the expected impact of an investment decision on share price. As we shall see, a host of problems make the incorporation of risk into capital budgeting a difficult matter indeed. Consequently, the methods are far from perfect. Nevertheless, they do provide insight into the important dimension of risk. This dimension should not be ignored in capital budgeting simply because evaluating it is difficult. It has far too great an influence on the value of the firm to its shareholders.

The riskiness of an investment project is defined in this book as the variability of possible returns emanating from the project. Decision situations may be broken down into three types: certainty, risk, and uncertainty. The distinction between risk and uncertainty is that risk involves situations in which the probabilities of a particular event occurring are known; whereas with uncertainty, these probabilities are not known.[1] The problems we analyze in this chapter involve risk situations, although frequently the terms risk and uncertainty are used interchangeably. A wide range of factors gives rise to risk in capital investments. The economy in general, economic factors peculiar to the investment, competition, technological development, consumer preferences, labor conditions, as well as other factors make it impossible to foretell the future. Consequently, the revenues, costs, and economic life of a particular investment are less than certain.

Considering risk, a firm may not rank equally two investment proposals having the same net-present value or internal rate of return. These measures of profitability are based upon only the expected values of the possible cash flows in various future periods. Suppose that we had two investment proposals, each costing $8,000 at time 0 and having
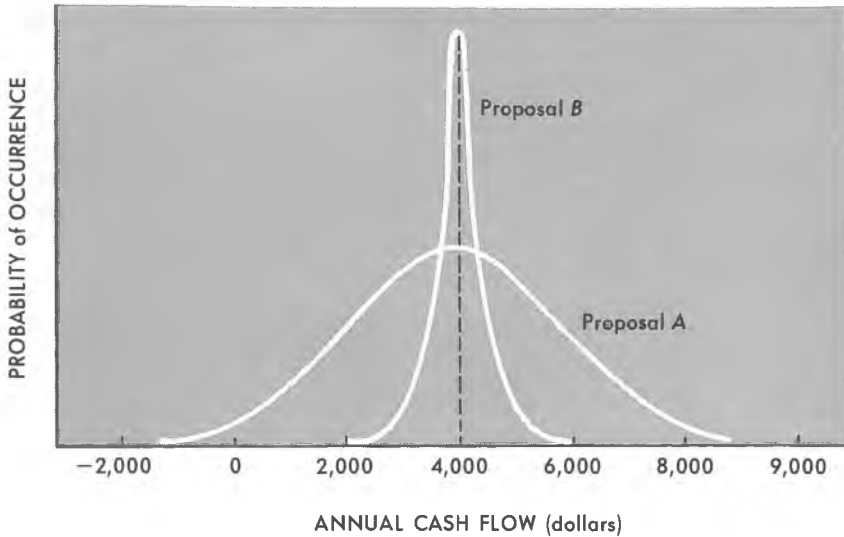
FIGURE 5-1
Comparison of probability distributions

[1] See R. Duncan Luce and Howard Raiffa, *Games and Decisions* (New York: John Wiley & Sons, Inc., 1957), p. 13.

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expected cash inflows of $4,000 in each of the next three years, after which no cash flows or salvage values were expected. According to the discussion in the previous chapter, the firm would rank these two proposals equally. However, suppose the probability distributions of the annual cash flows were those shown in Figure 5-1.

Can we conclude that the value to the firm of each of these investment opportunities is the same when we know that the dispersion of the probability distribution of possible cash flows for proposal A is greater than that for proposal B? If risk is associated with the dispersion of the probability distribution of possible cash flows, such that the greater the dispersion, the greater the risk, proposal A is the riskier investment. If management is averse to risk, it would prefer proposal B to proposal A. Naturally, we would expect most managements to be averse to risk when evaluating investment projects.[2]

STANDARD DEVIATION
AND EXPECTED VALUE

As discussed in Chapter 2, the conventional measure of dispersion of a probability distribution is the standard deviation, which, for a single period's possible outcome, is:

$$\sigma = \sqrt{\sum_{x=1}^{n} (A_{xt} - \bar{A}_t)^2 P_{xt}} \qquad (5\text{-}1)$$

where A_{xt} is a cash flow for the xth possibility in period t, P_{xt} is the probability of occurrence of that cash flow, and $\bar{A}_t$ is the expected value of cash flows in period t. The square of the standard deviation, σ^2, is the variance. The expected value is calculated by:

$$\bar{A}_t = \sum_{x=1}^{n} A_{xt} P_{xt} \qquad (5\text{-}2)$$

To illustrate these concepts, suppose two investment proposals had the following discrete probability distribution of expected cash flows in each of the next three years:

Proposal C		Proposal D	
Probability	*Cash Flow*	*Probability*	*Cash Flow*
0.10	$3,000	0.10	$2,000
0.20	3,500	0.25	3,000
0.40	4,000	0.30	4,000
0.20	4,500	0.25	5,000
0.10	5,000	0.10	6,000

[2] For the mapping of a manager's utility preferences with respect to risk, see the utility-theory section toward the end of this chapter.

The expected value of cash flows in each of the next three years for proposal C is:

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$$\bar{A} = 0.10(3,000) + 0.20(3,500) + 0.40(4,000) + 0.20(4,500)$$
$$+ 0.10(5,000) = 4,000$$

which is the same as that for proposal D:

$$\bar{A} = 0.10(2,000) + 0.25(3,000) + 0.30(4,000) + 0.25(5,000)$$
$$+ 0.10(6,000) = 4,000$$

However, the standard deviation for proposal C is

$$\sigma = [0.10(3,000 - 4,000)^2 + 0.20(3,500 - 4,000)^2 + 0.40(4,000 -$$
$$4,000)^2 + 0.20(4,500 - 4,000)^2 + 0.10(5,000 - 4,000)^2]^{1/2}$$
$$= [300,000]^{1/2} = 548$$

while that for proposal D is

$$\sigma = [0.10(2,000 - 4,000)^2 + 0.25(3,000 - 4,000)^2 + 0.30(4,000 -$$
$$4,000)^2 + 0.25(5,000 - 4,000)^2 + 0.10(6,000 - 4,000)^2]^{1/2}$$
$$= [1,300,000]^{1/2} = 1,140$$

Thus, proposal D has a significantly higher standard deviation, indicating a greater dispersion of possible outcomes.

A measure of relative dispersion is the coefficient of variation, which simply is the standard deviation of the probability distribution over its expected value. As we mentioned in Chapter 3, this coefficient serves as a relative measure of the degree of business risk. Because the coefficient of variation for proposal D, 0.29, is greater than that for proposal C, 0.14, we would say that proposal D had the greater degree of risk. Frequent reference will be made to these concepts in the remainder of this chapter.

We assume that management evaluates investment proposals on the basis of information about the expected value and dispersion of the probability distributions of possible future cash flows. As discussed in Chapter 2, management may also be concerned with the shape of a distribution as depicted by its skewness. Although it would be possible to incorporate a skewness measure into our analysis of risk, for simplicity we shall deal only with the expected value and the dispersion of the probability distribution. In the sections that follow, we examine ways in which a firm may take risk into account when analyzing investment proposals.

ADJUSTMENT OF DISCOUNT RATE

One means of adjusting for risk is simply to vary the discount rate in keeping with the degree of risk. The greater the risk, the higher the dis-

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count rate. Suppose the required rate of return, which we assume to be the cost of capital, is 10 per cent after taxes. We might be willing to use 10 per cent as our discount rate only if the proposal under consideration has the same degree of business risk as the typical existing investment project.[3] However, if the proposal were more or less risky than the typical existing investment, the discount rate would be greater or less than 10 per cent. Suppose that we have an investment proposal under consideration that costs $10,000 at time 0 and that the expected value of yearly cash flows for each of the next three years is $4,500. The net-present value of the proposal, using a 10 per cent discount rate, is

$$NPV = -10,000 + \frac{4,500}{(1.10)} + \frac{4,500}{(1.10)^2} + \frac{4,500}{(1.10)^3} = \$1,192 \qquad \text{(5-3)}$$

A positive net-present value signifies that the proposal should be accepted.

However, if management determined that this proposal were riskier than the typical existing investment project, it would want to assign a higher discount rate. Suppose that it determined that a discount rate of 18 per cent was appropriate for an investment of this nature. The net-present value then would be

$$NPV = -10,000 + \frac{4,500}{(1.18)} + \frac{4,500}{(1.18)^2} + \frac{4,500}{(1.18)^3} = -\$217 \qquad \text{(5-4)}$$

As this figure is negative, the proposal would be rejected, because its risk more than offsets the expected value of future cash flows. Thus, the risk of a project is reflected entirely in the discount rate employed and the accompanying discounting process. If the internal-rate-of-return method is used instead of the present-value method, the internal rate of return for a proposal would be compared with the risk-adjusted required rate of return. If the internal rate of return exceeded the risk-adjusted required rate, the proposal would be accepted; if not, it would be rejected.

The principal difficulty with this approach to risk adjustment is in determining the appropriate discount rate for a particular investment. This determination is likely to be somewhat arbitrary, giving rise to inconsistency. We know that the investment in a new product line requires a higher discount rate than the investment in a government bond. The real question is, how much greater a rate is appropriate? This thorny question is difficult to resolve in a consistent and objective manner. Some companies divide investments into risk classes and assign different discount rates to each class. The high risk class, consisting of such investments as

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[3] For now, we assume that all investment proposals under consideration, as well as existing investment projects, are perfectly correlated with respect to their possible outcomes.

new products, might be discounted at 20 per cent; normal risk projects at 15 per cent; and low risk projects at 10 per cent. Unfortunately, the categorization of a particular project is still largely intuitive.

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In addition to this criticism, the method does not make direct use of certain valuable information — namely, the probability distributions of expected future cash flows. To be sure, management may consider these probability distributions when determining the appropriate discount rate. However, there may be more efficient and objective ways of evaluating this information. All in all, adjustment of the discount rate for risk would have to be labeled a rather crude method of incorporating risk into the capital-budgeting decision.

CERTAINTY-EQUIVALENT APPROACH

With the risk-adjusted discount rate approach, we adjust for risk in the discount rate employed. An alternative approach is to adjust for risk by modifying the expected cash flows in the numerator of the net-present value equation. This approach may be expressed as

$$NPV = \sum_{t=0}^{n} \frac{\alpha_t A_t}{(1 + i)^t} \qquad (5\text{-}5)$$

where α_t is the certainty-equivalent coefficient for period t, and i is the risk-free rate, which we assume to be the same for all future periods.

The coefficient, α_t, is a value between 0 and 1.00.[4] It varies inversely with the degree of risk; the greater the risk associated with a future cash flow, the lower the coefficient. This coefficient is determined by management's utility preferences with respect to risk.[5] We wish to determine the value of α_t which, when multiplied by the cash flow A_t, would cause management to regard the product and a certain cash flow of A_t^* in period t as equally desirable. Thus

$$\alpha_t = \frac{A_t^*}{A_t} = \frac{\text{Certain Cash Flow}}{\text{Risky Cash Flow}} \qquad (5\text{-}6)$$

Suppose that the expected value of cash flow in period t were $10,000. Management then could be offered the choice between this amount and some certain cash flow. If management ranked an A_t of $10,000 and a certain cash flow of $6,000 as equally desirable, then α_t is 0.60. Thus, the certainty equivalent of an expected cash flow of $10,000 in period t is $6,000.

[4]The subsequent discussion of the certainty-equivalent approach is based upon Alexander A. Robichek and Stewart C. Myers, *Optimal Financing Decisions* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965), pp. 79–93.

[5]Later in this chapter, we take up the determination of certainty-equivalent coefficients in our discussion of the incorporation of utility theory into the investment decision.

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When the expected values of cash flows in all future periods have been expressed as certainty equivalents, they are discounted to present value by the risk-free rate. This rate represents the appropriate discount rate for a certain stream of cash flows, or the time value of money. Proponents of the certainty-equivalent approach contend that the discounting process should involve only the time value of money and not an adjustment for risk, as is the case with the risk-adjusted discount rate approach.

To illustrate the certainty-equivalent approach, suppose that we had an investment proposal under consideration that costs $10,000 and is expected to provide net cash flows after taxes of $5,000 in each of the next three years. Suppose further that risk is expected to increase with time, so that $\alpha_0 = 1.0$; $\alpha_1 = 0.90$; $\alpha_2 = 0.80$; $\alpha_3 = 0.70$; and i, the risk-free rate, is 4 per cent. The net-present value of the proposal is

$$NPV = 1.0(-10,000) + \frac{0.90(5,000)}{(1.04)}$$

$$+ \frac{0.80(5,000)}{(1.04)^2} + \frac{0.70(5,000)}{(1.04)^3} = \$1,137$$

(5-7)

The proposal would be accepted using this method. If the internal-rate-of-return method is employed, we would solve for the rate of discount that equates the present value of certainty-equivalent cash inflows with the present value of certainty-equivalent cash outflows. The internal rate of return then would be compared with the risk-free rate. If the internal rate of return equals or exceeds the risk-free rate, the proposal is accepted; if not, it is rejected.

**CERTAINTY-EQUIVALENT APPROACH
VERSUS RISK-ADJUSTED DISCOUNT
RATE APPROACH**

Like the risk-adjusted discount rate approach, the certainty-equivalent approach presents practical problems of implementation. For a given stream of expected future cash flows, it is difficult to specify the exact certainty-equivalent coefficients that should be applied. Management probably would look at the probability distributions of possible cash flows in specifying the coefficients, but these distributions may or may not be considered directly under the certainty-equivalent approach. Despite these shortcomings, the certainty-equivalent approach is superior theoretically to the risk-adjusted discount rate approach.

To illustrate its superiority, we compare the two approaches using the Robichek and Myers example.[6] Consider a situation in which the risk-

[6]Robichek and Myers, *Optimal Financing Decisions,* pp. 82–86. See also Houng-Yhi Chen, "Valuation under Uncertainty," *Journal of Financial and Quantitative Analysis,* II (September, 1967), 313–25.

adjusted discount rate, k, is the same for all future periods, and the risk-free rate, i, is also constant over time. If the risk-adjusted discount rate approach is a valid one, it should give the same results as the certainty-equivalent approach. Thus, for period t:

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$$\frac{\alpha_t A_t}{(1+i)^t} = \frac{A_t}{(1+k)^t} \qquad (5\text{-}8)$$

In other words, the present value of a certainty-equivalent cash flow discounted at the risk-free rate should equal the cash flow discounted at a risk-adjusted rate. Rearranging Eq. (5-8), we obtain

$$\alpha_t A_t (1+k)^t = A_t (1+i)^t$$

$$\alpha_t = \frac{A_t(1+i)^t}{A_t(1+k)^t} = \frac{(1+i)^t}{(1+k)^t} \qquad (5\text{-}9)$$

Similarly, for period $t+1$

$$\alpha_{t+1} = \frac{(1+i)^{t+1}}{(1+k)^{t+1}} \qquad (5\text{-}10)$$

Given the assumptions that i is the same for all future periods, that k is the same for all future periods, and that k exceeds i, α_{t+1} must be less than α_t. To illustrate, suppose that $k = 12$ per cent, $i = 4$ per cent and $t = 1$.

$$\alpha_1 = \frac{(1.04)}{(1.12)} = 0.9286 \qquad (5\text{-}11)$$

For $t = 2$, we have

$$\alpha_2 = \frac{(1.04)^2}{(1.12)^2} = 0.8622 \qquad (5\text{-}12)$$

Thus, a constant risk-adjusted discount rate, k, implies decreasing certainty-equivalent coefficients and increasing risk as the future becomes more distant. Furthermore, it can be shown that a constant discount rate implies that α_t decreases and risk increases at a constant rate with time. These notions are illustrated in Figure 5-2, where the relationship between certainty-equivalent coefficients and time are graphed for our example problem, $k = 12$ per cent and $i = 4$ per cent. It also is clear from Eq. (5-10) that the greater the differential between the risk-adjusted discount rate, k, and the risk-free rate, i, the lower the certainty-equivalent coefficient in all future periods and the greater the risk in those periods. In the appendix to this chapter, we consider further the resolution of uncertainty in capital budgeting.

We see then that the risk-adjusted discount rate method implies increasing risk over time when the discount rate, k, is constant. It may well be that this assumption is appropriate; however, management is
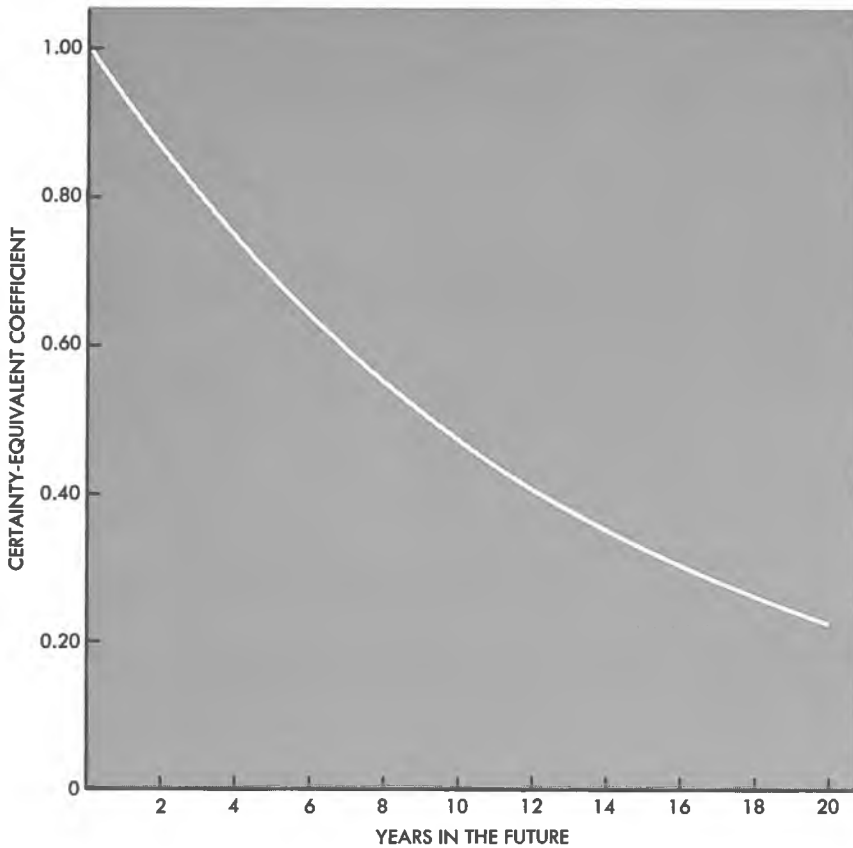
FIGURE 5-2
Relationship between certainty-equiva-
lent coefficient and time when $k = 0.12$
and $i = 0.04$

unable to consider increasing risk explicitly with this approach and may make serious errors in measuring risk over time. For many projects, risk does increase with the length of time in the future. As a result, the assumption implicit in a risk-adjusted discount rate approach may well be valid. However, not all projects conform to this pattern. With a tree farm, for example, there might be considerable risk when the trees are first planted. After a few years, however, the risk of survival is largely eliminated. Consequently, the assumption of risk increasing with the length of time in the future is not appropriate; and the project would be penalized with the risk-adjusted discount rate approach. With the certainty-equivalent approach, management is able to specify directly the degree of risk for a particular future period and then discount the cash

flow back to present value employing the time value of money. For this reason, the certainty-equivalent approach is superior to the risk-adjusted discount rate approach.

With both approaches, management determines the degree of risk for an investment project and then incorporates this specification of risk into the present-value equation. With the certainty-equivalent approach, risk is specified period by period; with the risk-adjusted discount rate approach, it is specified for the project as a whole. In either case, the really difficult problems are in specifying the appropriate degree of risk for an investment opportunity and in being consistent in these specifications from project to project and over time. The methods to be considered next take these problems into account.

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PROBABILITY DISTRIBUTION APPROACHES

In the two previous approaches, we did not consider directly the probability distributions of cash flows over time. Thus, we lacked certain valuable information. In the subsequent discussion, we analyze the probability distributions of cash flows under varying assumptions of dependence of cash flows from period to period. Of all the approaches for dealing with risky investment, the use of probability distributions is perhaps the most feasible. The idea here is to present management with pertinent information about the expected value of return and the dispersion of the probability distribution of possible returns. On the basis of this information, management will come to a decision about accepting or rejecting a proposal. For the decision to be optimal, management must consider the likely effect of the expected return and risk of the proposal on share price. As mentioned previously, we could extend our analysis to take account of the shape of the distribution by incorporating a skewness parameter; however, for simplicity, we shall look at only two parameters.

INDEPENDENCE OF CASH FLOWS OVER TIME

Suppose that we are evaluating an investment proposal in which the probability distributions of cash flows for various future periods are independent of one another. In other words, the outcome in period t does not depend upon what happened in period $t - 1$. The expected value of the probability distribution of net-present value for the proposal is

$$NPV = \sum_{t=0}^{\infty} \frac{\bar{A}_t}{(1 + i)^t} \tag{5-13}$$

where $\bar{A}_t$ is the expected value of net-cash flow in period t, and i is the risk-free rate. The risk-free rate is used as the discount rate in this analysis

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because we attempt to isolate the time value of money. To include a premium for risk in the discount rate would result in double counting with respect to our evaluation. The cost of capital embodies in it a premium for business risk. If this rate is used as the discount rate, we would be adjusting for risk in the discounting process itself. That is, we would adjust the cash-flow benefits of a proposal for the risk associated with the enterprise as a whole. We then would use the probability distribution of net-present values to judge the risk of the proposal. However, this probability distribution is obtained using a risk-adjusted discount rate. In essence, we would adjust for risk a second time in our evaluation of the relative dispersion of the probability distribution of possible net-present values. Because of the problems inherent in double counting for risk, the appropriate discount rate to use is the risk-free rate.

Given the assumption of mutual independence of cash flows for various future periods, the standard deviation of the probability distribution of net-present values is

$$\sigma = \sqrt{\sum_{t=0}^{\infty} \frac{\sigma_t^2}{(1+i)^{2t}}} \qquad (5\text{-}14)$$

where σ_t is the standard deviation of the probability distribution of possible net cash flows in period t. To illustrate the calculations involved with Eq. (5-13) and Eq. (5-14), suppose that we had an investment proposal costing \$10,000 at time 0 that was expected to generate net cash flows during the first three periods with the probabilities shown in Table 5-1. The expected values of net cash flows for periods 1, 2, and 3 are \$5,000, \$4,000, and \$3,000, respectively. The standard deviation of possible cash flows for period t, σ_t, is computed by

$$\sigma_t = \sqrt{\sum_{x=1}^{5} (A_{xt} - \bar{A}_t)^2 P_{xt}} \qquad (5\text{-}15)$$

where A_{xt} is the xth possible net cash flow, $\bar{A}_t$ is the expected value of net cash flow for period t, and P_{xt} is the probability of occurrence of A_{xt}.

TABLE 5-1
Expected cash flows for example problem

Period 1		Period 2		Period 3	
Probability	Net Cash Flow	Probability	Net Cash Flow	Probability	Net Cash Flow
0.10	\$3,000	0.10	\$2,000	0.10	\$1,000
0.25	4,000	0.25	3,000	0.25	2,000
0.30	5,000	0.30	4,000	0.30	3,000
0.25	6,000	0.25	5,000	0.25	4,000
0.10	7,000	0.10	6,000	0.10	5,000

In the above example, the standard deviation of possible net cash flows for period 1 is

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$$\sigma_1 = [0.10(3{,}000 - 5{,}000)^2 + 0.25(4{,}000 - 5{,}000)^2$$
$$+ 0.30(5{,}000 - 5{,}000)^2 + 0.25(6{,}000 - 5{,}000)^2 \quad \text{(5-16)}$$
$$+ 0.10(7{,}000 - 5{,}000)^2]^{1/2} = \$1{,}140$$

Because the probability distributions for periods 2 and 3 have the same dispersion about their expected values as that for period 1, σ_2 and σ_3 are \$1,140 also. Given this information, we are able to calculate the expected value of net-present value for the proposal as well as the standard deviation about this expected value. If we assume a risk-free rate of 4 per cent, the expected value of net-present value for the proposal is

$$NPV = -10{,}000 + \frac{5{,}000}{(1.04)} + \frac{4{,}000}{(1.04)^2} + \frac{3{,}000}{(1.04)^3} = \$1{,}173 \quad \text{(5-17)}$$

Using Eq. (5-14), under the assumption of mutual independence of cash flows over time, the standard deviation about the expected value is

$$\sigma = \sqrt{\frac{1{,}140^2}{(1.04)^2} + \frac{1{,}140^2}{(1.04)^4} + \frac{1{,}140^2}{(1.04)^6}} = \$1{,}827 \quad \text{(5-18)}$$

EVALUATION OF
PROBABILISTIC INFORMATION

The expected value and the standard deviation of the probability distribution of possible net-present values give us a considerable amount of information by which to evaluate the risk of the investment proposal. If the probability distribution is approximately normal, we are able to calculate the probability of the proposal's providing a net-present value of less or more than a specified amount. For example, suppose that we wish to determine the probability that the net-present value of the project will be zero or less. To determine this probability, we first calculate the difference between zero and the expected value of net-present value for the project. In our example, this difference is −\$1,173. We then standardize this difference by dividing it by the standard deviation of possible net-present values. We obtain −\$1,173/1,827, or −0.642 standard deviations. This figure tells us that a net-present value of 0 lies 0.642 standard deviations to the left of the expected value of the probability distribution of possible net-present values.

To determine the probability that the net-present value of the project will be zero or less, we consult a normal probability distribution table found in most statistics texts. We find that for the normal distribution, there is a 0.26 probability that an observation will be less than −0.642 standard deviations from the expected value of that distribution. Thus, there is a 0.26 probability that the net-present value of the proposal will

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be zero or less. If we assume a continuous distribution, the probability density function of our example problem can be shown in Figure 5-3.

The mean of the probability distribution of possible net-present values is $1,173. One standard deviation on either side of the mean gives us net-present values of −$654 and $3,000. With a normal distribution, 0.683 of the distribution or area under the curve falls within one standard deviation on either side of the mean or expected value. We know then that there is approximately a two-thirds probability that the net-present value of the proposal examined will be between −$654 and $3,000. We know also that
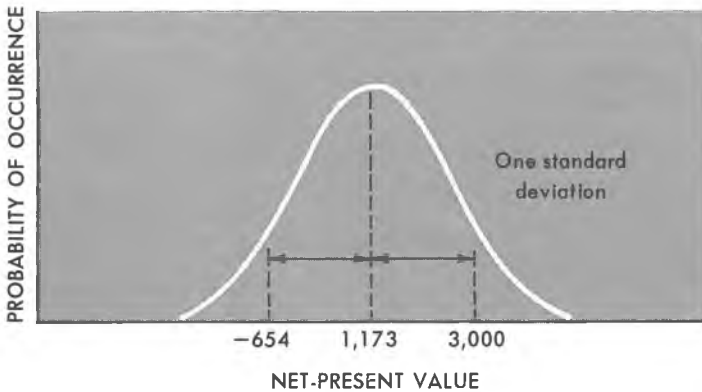
FIGURE 5-3
Probability density functions, example problem

there is a 0.26 probability that the net-present value will be less than 0 and a 0.74 probability that it will be greater than 0. By expressing differences from the expected value in terms of standard deviations, we are able to determine the probability that the net-present value for an investment proposal will be greater or less than a particular amount. Often it is useful to express the area under the curve and to the right of a particular amount as a cumulative probability distribution. For our example problem, this distribution is shown in Figure 5-4. It tells us the probability that the actual net-present value will be at least as great as the amounts shown on the horizontal axis.

Knowledge of these probabilities is fundamental for a realistic assessment of risk. For example, suppose that the firm is considering another investment project, proposal Y. The probability density function for this proposal is shown in Figure 5-5, as is that for our example problem, proposal X. We see that the expected value of net-present value for proposal Y, $2,200, is higher than that for proposal X, $1,173; but there is also greater dispersion of the probability distribution about the expected value. If risk is related positively to dispersion, proposal Y has both a higher expected profitability and a greater risk than does proposal X. Whether management prefers proposal Y to proposal X depends upon its utility preferences with respect to risk. These risk preferences are likely to vary

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according to the size of the investment project relative to the size of the firm. For example, the management of a company with a net worth of $2 million is likely to be more averse to risk with respect to a project costing $2 million than would the management of U.S. Steel. If the risk of the project is substantial, there may be a significant probability that the smaller company will be ruined; accordingly, it would probably prefer project X. Whether management's tradeoff between profitability and risk leads to investment decisions that maximize share price depends upon its perceptiveness in judging the tradeoff for investors at the margin and its willingness to adapt to this tradeoff.

Thus, the approach outlined above does not provide a decision, let alone an optimal decision. However, it does provide management with important information by which it is able to make intelligent investment decisions. This information is not provided by the conventional evaluation of expected cash flows (Chapter 3), for the dispersion of these cash flows is ignored in the capital-budgeting process. The actual assessment of risk, however, is up to management. It is unlikely that management would accept an investment proposal having an expected value of net-present value of zero unless the probability distribution had no dispersion. In this special case, the proposal, by definition, would be a riskless investment. For risky investments the net-present value would have to exceed zero. How much it would have to exceed zero before acceptance is warranted depends upon the amount of dispersion of the probability distribution and the utility preferences of management with respect to risk.

In order to facilitate project selection as well as to make it consistent over time, management may wish to formulate maximum risk profiles. To express the probability distributions in relative instead of absolute terms, we can convert the net-present value probability distribution into a distribution of possible profitability indexes. For proposal X, our example problem, the initial cash outflow is $10,000. Thus, the profitability index for the expected value of the probability distribution of possible net-present values is ($1,173 + $10,000)/$10,000 = 1.12. The profitability index for zero net-present value is (0 + $10,000)/$10,000 = 1.00. Similarly, we can convert the entire probability distribution in Figure 5-3 to a probability distribution of possible profitability indexes. The converted probability distribution for the example problem is seen in Figure 5-6.

If management has specified maximum risk profiles for various expected values of profitability indexes, one would simply compare proposal X with the maximum risk profile for an expected value of profitability index of 1.12. If the dispersion of the proposal is less than that for the risk profile, the proposal would be accepted. If not, it would be rejected. The maximum level of dispersion permitted, as depicted by the risk profile, will increase with the expected value of profitability index. For a profitability index of 1.02, the dispersion of the maximum risk profile will be
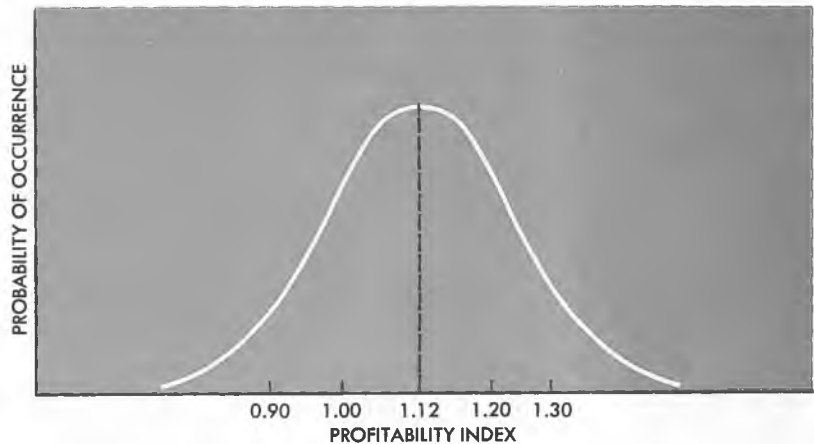
iarrower than that for a profitability index of 1.10. An illustration of some
iypothetical risk profiles is shown in Figure 5-7. We note that the greater
he expected value of profitability index, the greater the dispersion that is
olerable to management.

Once maximum risk profiles are specified, project selection becomes
iutomatic. For a given expected value of profitability index, the disper-
ion of the probability distribution for the proposal must be less than or
qual to that for the maximum risk profile in order for the proposal to be
iccepted. Management's specification of these risk profiles will depend
ipon its tolerances. If these risk tolerances are consistent with those of
nvestors at the margin, selection of proposals according to the maximum
isk profile method will tend to maximize share price.

In the above examples, we have assumed normal probability distribu-
ions. While this property is very desirable for purposes of calculation, it
s not a necessary condition for the use of the above approach. Even
vhen the distribution is not normal, we usually are able to make reason-
ibly strong probability statements by using Chebyshev's inequality.[7]
Again, we are interested in the area under the curve, or probability den-
ity function, that is to the left or right of a particular profitability index.
3y computing this area, we are able to determine the probability that the
ndex will be greater or less than a particular index and judge the risk of
he project accordingly.

[7] See John G. Kemeny *et al.*, *Finite Mathematical Structures* (Englewood Cliffs, N.J.:
'rentice-Hall, Inc., 1959), pp. 172–78.

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PROBABILITY OF OCCURRENCE

PROFITABILITY INDEXES

FIGURE 5-7
Risk profiles

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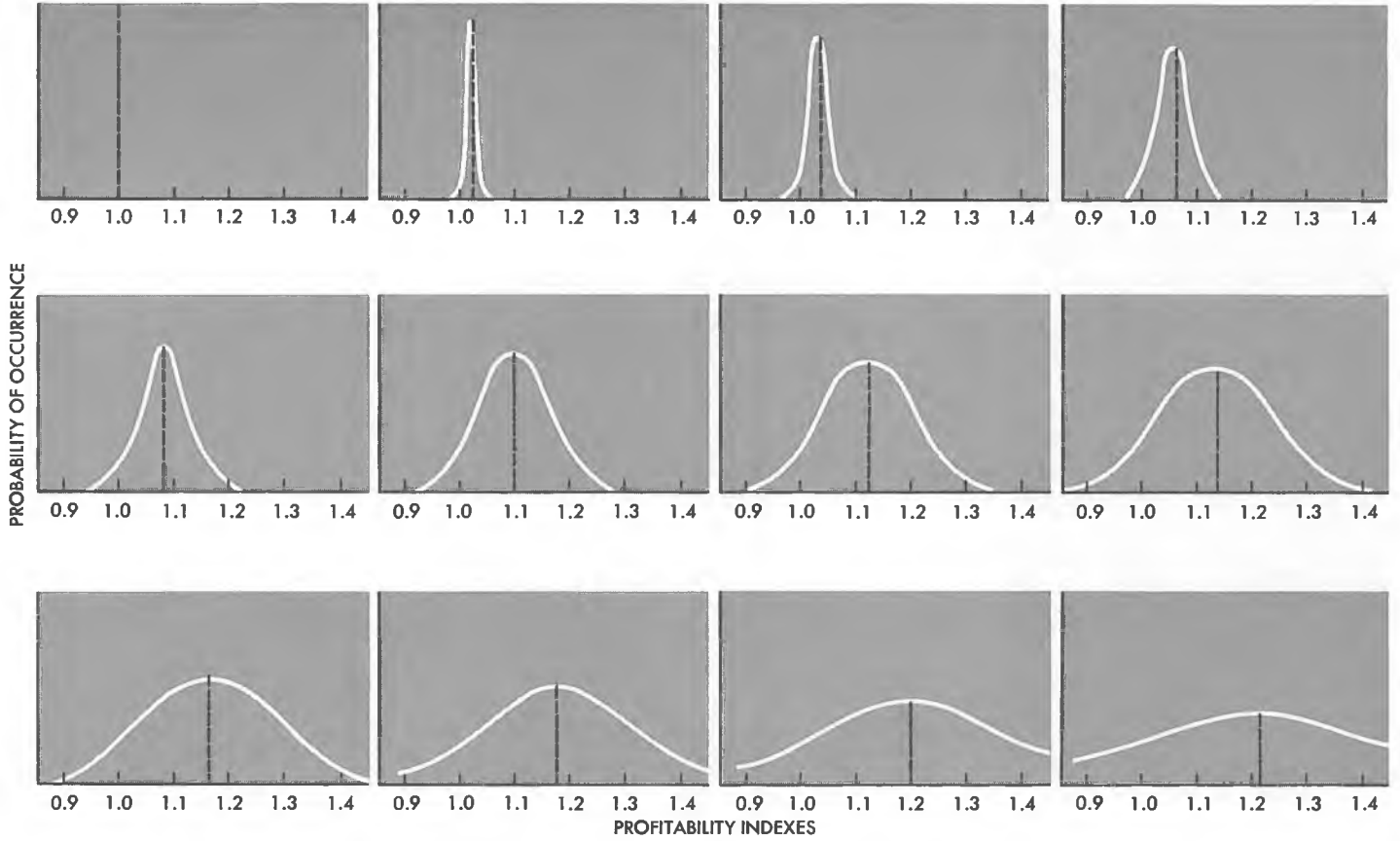
Estimating Future Cash Flows. Instead of expressing estimates of future cash flows in terms of a discrete probability distribution, we might specify them only in terms of the expected value and standard deviation of the probability distribution. For example, we might ask management for estimates of the most likely cash flow and the maximum possible deviation on either side. This type of estimating procedure corresponds to the PERT method, whereby, most likely, optimistic and pessimistic estimates are made. If we assume a normal distribution of net cash flows for various future periods, we are able to use the maximum possible deviation to determine the standard deviation.

With a normal distribution, we know that the probabilities that the actual value will fall within 1, 2, or 3 standard deviations on either side of the most likely value are 0.683, 0.954, and 0.997, respectively. If management felt that there was a 5 per cent probability that the maximum possible deviation would be positive and an equal probability that it would be negative, this probability corresponds to approximately 1.65 standard deviations on either side of the most likely value. In other words, 90 per cent of a normal probability distribution falls within 1.65 standard deviations on either side of the most likely value—the mean. By dividing the maximum deviation by 1.65, we obtain the standard deviation for the probability distribution. Thus, if the most likely value of net cash flow for period t were \$5,000 and the minimum and maximum cash flows were \$3,000 and \$7,000, respectively, the standard deviation would be \$2,000/1.65 = \$1,212.

This relatively simple approach enables us to obtain the probability information necessary to determine the expected value of net-present value for an investment proposal and the standard deviation about this expected value.[8] The approach differs somewhat in mechanics but not in concept from the discrete probability-distribution approach described earlier. To use the approach, however, it is important that the probability estimates be approximately normal.

DEPENDENCE OF CASH FLOWS OVER TIME

In the previous section, we assumed mutual independence of cash flows from one future period to another. For most investment proposals, however, the cash flow in one future period depends in part upon the cash flows in previous periods. If an investment proposal turns bad in the early years, the probability is high that cash flows in later years also will be lower than originally expected. To assume that an extremely unfavorable or favorable outcome in the early life of an investment proposal does not

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[8]See Frederick S. Hillier and David V. Heebink, "Evaluating Risky Capital Invest-ments," *California Management Review,* VIII (Winter, 1965), 72–74.

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affect the later outcome is unrealistic in most investment situations. The consequence of cash flows being correlated over time is that the standard deviation of the probability distribution of possible net-present values is larger than it would be if we assumed independence. The greater the degree of correlation, the greater the dispersion of the probability distribution. The expected value of net-present value, however, is the same, regardless of the degree of correlation over time. In this section, we explore varying degrees of dependence of cash flows over time.

Perfect Correlation. Cash flows are perfectly correlated over time if the deviation of an actual cash flow for a period from the mean of the probability distribution of expected cash flows for that period implies that cash flows in all other periods deviate in exactly the same manner. In other words, the cash flow in period t depends entirely upon what happened in previous periods. If the actual cash flow in period t is X standard deviations to the right of the expected value of the probability distribution of possible cash flows for that period, actual cash flows in all other periods will be X standard deviations to the right of the expected values of their respective probability distributions. In other words, the cash flow in any period is a linear function of the cash flows in all other periods. The formula for the standard deviation of a perfectly correlated stream of cash flows over time is

$$\sigma = \sum_{t=0}^{\infty} \frac{\sigma_t}{(1+i)^t} \tag{5-19}$$

To illustrate its use, consider the same example as before. The standard deviation about the expected value of net-present value for the proposal, using Eq. (5-19), is

$$\sigma = \frac{1,140}{(1.04)} + \frac{1,140}{(1.04)^2} + \frac{1,140}{(1.04)^3} = \$3,164$$

This compares with a standard deviation of \$1,827 when we used Eq. (5-14) under the assumption of mutual independence over time. Thus, the standard deviation, and risk, for a perfectly correlated stream of cash flows is significantly higher than the standard deviation for the same stream under the assumption of mutual independence. The standard deviation for a less than perfectly correlated stream of cash flows will be somewhere between these two values. The evaluation of a project with a perfectly correlated stream of cash flows over time is the same as that illustrated previously for a project with an uncorrelated stream. We analyze the expected value of possible net-present values, or possible profitability indexes, in relation to the standard deviation about this expected value.

Hillier's Model. Hillier combines the assumption of mutual independence and perfect correlation in developing a model to deal with mixed

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situations.[9] The model enables the analysis of investment proposals in which some of the expected cash flows over time are related closely, and others are fairly independent. To illustrate the model, suppose that a firm is considering the introduction of a new product with returns expected over the next five years. Because the product's market reception is uncertain, management feels that if initial reception exceeds expectations, reception in later years also will exceed expectations in about the same proportion. For simplicity, it is believed that the net marketing cash flows (sales minus marketing and advertising expenses) can be treated as perfectly correlated over time.

On the other hand, estimates of the initial investment in the project and of production costs are reasonably reliable, so that any deviation from expectations is assumed to be attributable to random fluctuations. Consequently, initial investment and net production cash flows are regarded as being mutually independent over time. The probability information for the introduction of the new product is shown in Table 5-2. Assume that each of the probability distributions involved can be regarded as normal. If 4 per cent is used as the risk-free rate, the expected value of net-present value for the proposal is

$$NPV = \sum_{t=0}^{5} \frac{A_t}{(1.04)^t} \tag{5-20}$$

$$= -600 + \frac{300 - 250}{(1.04)} + \frac{600 - 200}{(1.04)^2} + \frac{500 - 200}{(1.04)^3} + \frac{400 - 200}{(1.04)^4}$$

$$+ \frac{300 - 100}{(1.04)^5} = \$419$$

TABLE 5-2
Expected cash flows for new product

Year	Source	Expected Value of Net Cash Flow (in thousands)	Standard Deviation (in thousands)
0	Initial investment	−$600	$ 50
1	Production cash outflow	− 250	20
2	Production cash outflow	− 200	10
3	Production cash outflow	− 200	10
4	Production cash outflow	− 200	10
5	Production outflow — salvage value	− 100	15
1	Marketing	300	50
2	Marketing	600	100
3	Marketing	500	100
4	Marketing	400	100
5	Marketing	300	100

Source: Hillier, "The Derivation of Probabilistic Information," p. 454.

[9]Frederick S. Hillier, "The Derivation of Probabilistic Information for the Evaluation of Risky Investments," *Management Science*, 9 (April, 1963), 443–57.

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Calculating the standard deviation of the probability distribution about the expected value is considerably more complex, for we take into account both a perfectly correlated stream and an independent stream of cash flows. The formula for the standard deviation is

$$\sigma = \sqrt{\sum_{t=0}^{n} \frac{\sigma^2 y_t}{(1+i)^{2t}} + \sum_{k=1}^{m} \left(\sum_{t=0}^{n} \left[\frac{\sigma z_t^{(k)}}{(1+i)^t} \right] \right)^2}$$

where $\sigma^2 y_t$ is the variance for an independent net cash flow in period t and $\sigma z_t^{(k)}$ is the standard deviation for stream k of a perfectly correlated net cash flow in period t. For our problem, there is but one perfectly correlated stream of cash flows, so that $m = 1$ in the equation. However, Eq. (5-21) can treat any number of perfectly correlated streams. The standard deviation for the example problem is

$$\sigma = \sqrt{\sum_{t=0}^{5} \frac{\sigma^2 y_t}{(1.04)^{2t}} + \left(\sum_{t=0}^{5} \left[\frac{\sigma z_t}{(1.04)^t} \right] \right)^2} \qquad (5\text{-}22)$$

$$= \sqrt{50^2 + \frac{20^2}{(1.04)^2} + \cdots + \frac{15^2}{(1.04)^{10}} + \left(\frac{50}{(1.04)} + \cdots + \frac{100}{(1.04)^5} \right)^2}$$

$$= \$398$$

Thus, the expected value of net-present value of the proposal is \$419,000, and the standard deviation of the probability distribution is \$398,000. In the same manner as in our earlier example, we can determine the probability that the net-present value of the project will be less than zero and the probability that it will be greater or less than other specified amounts. These probabilities give management a great deal of insight into the risk of the project. Again, the tradeoff between the expected value of net-present value and dispersion will depend upon the utility preferences of management with respect to the risk borne by the firm.

Moderate Correlation. While Hillier's approach goes a long way toward coping with the correlation of cash flows over time, one obvious problem is that cash flows must be classified as either independent or perfectly correlated over time. For many investment proposals, cash flows fall into neither of these categories but show less than perfect correlation over time. Whereas it would be possible to classify a cash-flow stream into the nearest category without serious distortion if the correlation were high or low, such a classification is not appropriate if the correlation is moderate. One method for handling the problem of moderate correlation is with a series of conditional probability distributions.

To illustrate, suppose that the investment in a project costing \$10,000 at time 0 were expected to generate net cash flows in periods 1, 2, and 3 with the probabilities shown in Table 5-3. As seen in the table, there are 27 possible cash-flow series. The last column depicts the joint probability

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of occurrence of a particular cash-flow series. For series 1, the joint probability of a −$6,000 cash flow in period 1 being followed by cash flows of −$2,000 and $5,000 in periods 2 and 3, respectively, is .25 × .30 × .25 = .01875. Similarly, joint probabilities for the other cash-flow series can be determined in this manner.

The use of conditional probability distributions enables us to take account of the correlation of cash flows over time. In the above example, the cash flow in period 3 depends upon what happened in periods 1 and 2. However, the correlation of cash flows over time is not perfect. Given a cash flow in period 1, the cash flow in period 2 can vary within a range.

TABLE 5-3
Illustration of conditional probability distribution approach

Period 1		Period 2		Period 3			
Initial Probability P(1)	Net Cash Flow	Conditional Probability P(2|1)	Net Cash Flow	Conditional Probability P(3|2, 1)	Net Cash Flow	Cash Flow Series	Joint Probability P(1, 2, 3)
.25	−$6,000	.30	−$2,000	.25	$5,000	1	.01875
				.50	7,000	2	.03750
				.25	9,000	3	.01875
		.40	1,000	.25	7,000	4	.02500
				.50	9,000	5	.05000
				.25	11,000	6	.02500
		.30	4,000	.25	9,000	7	.01875
				.50	11,000	8	.03750
				.25	13,000	9	.01875
.50	−4,000	.25	3,000	.30	10,000	10	.03750
				.40	12,000	11	.05000
				.30	14,000	12	.03750
		.50	6,000	.30	12,000	13	.07500
				.40	14,000	14	.10000
				.30	16,000	15	.07500
		25	9,000	.30	14,000	16	.03750
				.40	16,000	17	.05000
				.30	18,000	18	.03750
.25	−2,000	.30	8,000	.25	15,000	19	.01875
				.50	17,000	20	.03750
				.25	19,000	21	.01875
		.40	11,000	.25	17,000	22	.02500
				.50	19,000	23	.05000
				.25	21,000	24	.02500
		.30	14,000	.25	19,000	25	.01875
				.50	21,000	26	.03750
				.25	23,000	27	.01875

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Similarly, the cash flow in period 3 can vary within a range, given the outcomes in periods 1 and 2. The calculation of the expected value of net-present value using this approach is the same as before (Eq. (5-13)). The standard deviation may be determined mathematically for the simple case by

$$\sigma = \sqrt{\sum_{x=1}^{l} (NPV_x - \overline{NPV})^2 P_x} \tag{5-23}$$

where NPV_x is the net-present value for series x of net cash flows, covering all periods, $\overline{NPV}$ is the expected value of net-present value of the proposal, and P_x is the probability of occurrence of that series. For the above example, there are 27 possible series of cash flows, so that $l = 27$. The first series is represented by a net cash flow of $-\$10,000$ at time 0, $-\$6,000$ at time 1, $-\$2,000$ at time 2, and $\$5,000$ at time 3. The probability of occurrence of that series is .01875.

Unfortunately, for complex situations, the mathematical calculation of the standard deviation is unfeasible. For these situations, we can approximate the standard deviation by means of simulation. With simulation, the net-present values for randomly selected series of cash flows are calculated. These net-present values then are ordered according to their probability of occurrence, and a probability distribution of possible net-present values is formed. The expected value and standard deviation of the probability distribution can then be calculated. Although the approach described in this section is reasonably exacting, it does not clear up the difficulty of estimating conditional probabilities over time. Simulation models, however, can be developed to generate this information.[10]

We have seen that our assumption as to the degree of correlation of cash flows over time is an important one. The risk of a project will be considerably greater if the cash flows are highly correlated over time than if they are mutually independent, all other things being the same. While independence often is assumed for ease of calculation, this assumption greatly underestimates project risk if in fact the cash flows are highly correlated over time. Thus, it is important that careful consideration be given to the likely degree of dependence of cash flows over time. Otherwise, the assessment of risk may well be distorted. Of the approaches for dealing with the problem, the use of conditional probabilities is the most accurate, although the most difficult to implement.

FULL-SCALE SIMULATION APPROACH

In an important contribution to evaluating risky investments, David B. Hertz proposed the use of a simulation model to obtain the expected return and dispersion about this expected return for an investment pro-

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[10] See Alexander A. Robichek and James C. Van Horne, "Abandonment Value and Capital Budgeting," *Journal of Finance*, XXII (December, 1967), Appendix.

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)osal.[11] Hertz considers the following factors in evaluating an investment)roposal:

Market Analysis

1. Market size
2. Selling price
3. Market growth rate
4. Share of market (which results in physical sales volume)

Investment Cost Analysis

5. Investment required
6. Residual value of investment

Operating and Fixed Costs

7. Operating costs
8. Fixed costs
9. Useful life of facilities

²robability distributions are assigned to each of these factors, based upon nanagement's assessment of the probable outcomes. Thus, the possible)utcomes are charted for each factor according to their probability of)ccurrence.

Once the probability distributions are determined, the next step is to letermine the average rate of return that will result from a random com->ination of the nine factors listed above. To illustrate the simulation pro->ess, assume that the market-size factor had the following probability listribution:

Market Size (in units)	Probability
450,000	0.05
500,000	0.10
550,000	0.20
600,000	0.30
650,000	0.20
700,000	0.10
750,000	0.05

Vow suppose that we have a roulette wheel with 100 numbers, on which numbers 1 to 5 represent a market size of 450,000 units, 6 to 15 represent ι market size of 500,000, 16 to 35 a market size of 550,000 units, and so

[11] David B. Hertz, "Risk Analysis in Capital Investment," *Harvard Business Review,* 12 (January–February, 1964), 95–106; and Hertz, "Investment Policies that Pay Off," *Harvard Business Review,* 46 (January–February, 1968), 96–108.

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on through 100. As in roulette, we spin the wheel, and the ball falls in one of the 100 slots—number 26. For this trial, then, we simulate a market size of 550,000 units. Fortunately, we do not have to have a roulette wheel to undertake a simulation; the same type of operation can be carried out on a computer in a much more efficient manner.

Simulation trials are undertaken for each of the other eight factors. The first four factors (market analysis) give us the annual sales per year, while factors 7 and 8 give us the operating costs and fixed costs per year. Together, these six factors enable us to calculate the annual earnings per year. When trial values for these six factors are combined with trial values for the required investment, the useful life, and the residual value of the project, we have sufficient information to calculate the return on investment for that trial run. Thus, the computer simulates trial values for each of the nine factors and then calculates the return on investment based upon the values simulated. The process is repeated a number of times; each time we obtain a combination of values for the nine factors and the return on investment for that combination. When the trial is repeated often enough, the rates of return obtained can be plotted in a frequency distribution.

From this frequency distribution, we are able to evaluate the expected return and the dispersion about this expected return, or risk, in the same manner as before—in other words, we can determine the probability that an investment will provide a return greater or less than a certain amount. By comparing the probability distribution of rates of return for one proposal with the probability distribution of rates of return for another, management is able to evaluate the respective merits of different risky investments.

Two points should be mentioned with respect to Hertz's simulation method. While the simulation model computes the average rate of return on investment, the method could easily be modified to calculate the internal rate of return, the net-present value, or the profitability index. In addition, although Hertz allows for dependency among the nine factors,[12] the model presented treats the factors as though they were independent. To the extent that dependency exists among factors, it must be taken into account in determining the probability distributions. For example, there is likely to be significant correlation between the market size and the selling price. These interrelationships add considerable complexity to the estimating procedure. Notwithstanding the added complexity of estimating and specifying the relationships between factors, it must be done if the model is to provide realistic results. These estimates may be based upon empirical testing when such testing is feasible. Once the relationships are incorporated in the model, those factors that are cor-

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[12] Hertz, "Risk Analysis in Capital Investment," p. 101.

related would then be simulated jointly. Rates of return for the simulated trials would be calculated and a frequency distribution of simulated trials formed in the same manner as before.

In capital budgeting, some investment opportunities involve a sequence of decisions over time. Heretofore, we have considered only a single accept-reject decision at the outset of the project. An analytical technique used in sequential decisions is the decision tree, where various decision points are studied in relation to subsequent chance events. This technique enables one to choose among alternatives in an objective and consistent manner.

To illustrate the method, let us suppose that a firm is considering the introduction of a new product. Initially, it must decide whether to distribute the product in the Midwest or nationally. Regional distribution will require an expenditure of $1 million for a new plant and for the initial marketing effort. Depending upon demand during the first two years, the firm then would decide whether or not to expand to national distribution. If it goes from regional to national distribution, it will need to spend an additional $3 million for expansion of the existing plant and to make an additional marketing effort. Of course, the firm can distribute nationally from the very outset. If it does, it will cost $3 million to construct a plant and to launch the marketing of the product. We see then that there are economies associated with distributing nationally at the outset. For one thing, building a large plant is less expensive than building a small one and having to enlarge it later. Moreover, there are economies in marketing.

The decision process is illustrated graphically by the decision tree shown in Figure 5-8. The squares represent decision points. For example, the first decision is whether to distribute regionally or nationally. The circles represent chance event nodes. If the firm decides to distribute nationally at the outset, there is 0.4 probability that demand will prove to be high, 0.4 that demand will turn out to be medium, and 0.2 that it will be low. On the other hand, if the firm distributes regionally, there is a 0.5 probability that demand will be high, a 0.3 probability that it will be medium, and a 0.2 probability that it will be low. At the end of year 2, the firm must decide whether to continue to distribute regionally, in which case demand will continue to be high, low, or medium, or whether it should distribute nationally, in which case the national demand is shown by the subsequent chance event in the figure.

In all cases, the product is expected to have a life of eight years. Moreover, we assume for simplicity's sake that at the end of the eighth

Demand   Probability   Demand   Probability

Distribute Regionally

1

Distribute Nationally

Continue Regional Distribution

High   0.5   **2**

Distribute Nationally

High   1.0

High   0.6
Medium   0.3
Low   0.1

Continue Regional Distribution

Medium   0.3   **2**

Distribute Nationally

Medium   1.0

High   0.3
Medium   0.4
Low   0.3

Continue Regional Distribution

Low   0.2   **2**

Distribute Nationally

Low   1.0

High   0.1
Medium   0.2
Low   0.7

High   0.4

Medium   0.4

Low   0.2

FIGURE 5-8
Decision tree — national versus regional distributions

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year, the plant has no more useful life and no salvage value. The expected cash flows over the eight years are shown in Table 5-4 for the various outcomes. We note in the table that if the firm switches from regional to national distribution at the end of year 2, there is a cash expenditure of $3 million at the end of the year. When this expenditure is combined with the cash flow generated from regional demand for the year, we obtain the net cash flow for year 2. In general, the lower the combination of regional and national demand, the lower the cash-flow sequence. Assuming a risk-free rate of 4 per cent after taxes, the net-present values of the various combinations are shown in the last column of the table. We need now to incorporate these net-present values into our decision tree. In Figure 5-9, they are shown at the branch tips, indicating the expected net-present values associated with the sequence of decisions and chance events comprising the branch.

TABLE 5-4
**Expected cash flows for various branches of decision tree
(000 omitted)**

	Time									
	0	1	2	3	4	5	6	7	8	NPV
Regional distribution throughout:										
High demand	−$1,000	$100	$ 300	$ 400	$ 500	$ 400	$ 300	$ 200	$100	$ 947.4
Medium demand	−1,000	0	150	250	350	250	200	100	50	136.2
Low demand	−1,000	−100	0	100	200	100	100	50	0	−637.1
Regional distr. followed by national distr.:										
High regional—high national demand	−1,000	100	−2,700	1,200	1,800	2,400	1,800	1,200	800	4,096.9
High regional—medium national demand	−1,000	100	−2,700	700	1,200	1,800	1,200	700	500	1,573.1
High regional—low national demand	−1,000	100	−2,700	200	700	1,300	700	200	200	−704.1
Medium regional—high national demand	−1,000	0	−2,850	1,100	1,700	2,300	1,700	1,100	700	3,377.4
Medium regional—medium national demand	−1,000	0	−2,850	600	1,100	1,700	1,200	600	400	932.6
Medium regional—low national demand	−1,000	0	−2,850	100	600	1,100	700	100	100	−1,426.8
Low regional—high national demand	−1,000	−100	−3,000	1,000	1,500	2,100	1,500	1,000	600	2,411.2
Low regional—medium national demand	−1,000	−100	−3,000	500	1,000	1,500	1,000	500	300	51.9
Low regional—low national demand	−1,000	−100	−3,000	0	500	900	500	0	0	−2,307.5
National distribution throughout:										
High demand	−3,000	300	1,000	1,500	2,000	1,500	1,000	700	300	4,030.5
Medium demand	−3,000	0	500	1,000	1,500	1,000	500	300	100	1,151.6
Low demand	−3,000	−300	0	500	1,000	500	0	−100	−100	−1,727.3

The optimal sequence of decisions is determined by "rolling back" the tree from the right-hand side. In other words, we appraise first the most distant decision—namely, the choice of whether or not to switch from national to regional distribution. To do so we must determine the expected value of net-present value for national distribution, given that demand for the regional distribution proves to be high, medium, or low. The expected value of net-present value is simply the net present values at the branch tips times the probability of occurrence. For high regional

Continue Regional
Distribution

Conditional
Probability

Net-Present
Value

1.0	974.4

0.5

2

| 0.6 | 4,096.9 |
| 0.3 | 1,573.1 |

Distribute
Nationally
NPV = 2,859.6

| 0.1 | −704.1 |

NPV = 1,589.8

Continue Regional
Distribution

| 1.0 | 136.2 |

0.3

2

| 0.3 | 3,377.4 |
| 0.4 | 932.6 |

Distribute
Nationally
NPV = 958.2

| 0.3 | −1,426.8 |

Distribute
Regionally

Continue Regional
Distribution

| 1.0 | −637.1 |

0.2

2

| 0.1 | 2,411.2 |
| 0.2 | 51.9 |

Distribute
Nationally
NPV = −1,363.8

| 0.7 | −2,307.5 |

1

| 0.4 | 4,030.5 |

Distribute
Nationally

NPV = 1,727.3

| 0.4 | 1,151.6 |

| 0.2 | −1,727.3 |

FIGURE 5-9
Decision tree — net-present values of branches

demand, the expected value of net-present value for subsequent national distribution is

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$$NPV = 0.6(4,096.9) + 0.3(1,573.1) + 0.1(-704.1) = \$2,859.6 \quad \textbf{(5-24)}$$

This amount appears at the chance event node for national distribution, given high regional demand.

In a similar fashion, the expected values of net-present value for national distribution, given medium and low regional demands, are computed and shown at the appropriate chance event nodes. We note that the expected value of net-present value for national distribution, given low regional demand, is $-\$1,363,800$. This figure compares with an expected net-present value of $-\$637,100$ if the firm continues with regional distribution. Thus, if regional demand is low, the firm should not distribute nationally but should continue to distribute regionally. On the other hand, if regional demand turns out to be either high or medium, the firm should go to national distribution, for the expected value of net-present value is higher than it is if the firm continues with regional distribution. By backward induction, then, we are able to determine the optimal decision at the most distant decision point.

The next step is to determine the optimal decision at the first decision point — that is, to decide whether to distribute nationally or regionally at the outset. The expected value of net-present value for regional distribution, given optimal decisions at decision point 2, is

$$NPV = 0.5(2,859.6) + 0.3(958.2) + 0.2(-637.1) = \$1,589.8 \quad \textbf{(5-25)}$$

Note that if regional demand is high or medium, we use the expected value of net-present value associated with subsequent national distribution. If regional demand is low, we use the expected net-present value associated with continuing regional distribution. The expected value of net-present value for initial national distribution is

$$NPV = 0.4(4,030.5) + 0.4(1,151.6) + 0.2(-1,727.3) = \$1,727.3$$
$$\textbf{(5-26)}$$

Thus, expected value of net-present value for initial national distribution exceeds that for initial regional distribution. Moreover, when we compute the standard deviation of the probability distribution of possible net-present values, we find it to be $\$2,123,400$ for initial national distribution compared with $\$2,145,500$ for initial regional distribution. Therefore, initial national distribution dominates initial regional distribution with respect to both expected return and dispersion. The economies associated with building the plant and initiating the marketing effort all at once as opposed to piecemeal more than offset the flexibility associated with regional distribution initially with the later possibility of national distribution. Thus, initial national distribution is the preferred alternative. Whether management will want to undertake the project, however, will

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depend upon its risk preferences. Given the expected value and standard deviation of the probability distribution of possible net-present values for initial national distribution, it must decide whether the project is worthwhile. This decision will be based upon the considerations discussed earlier.

We have seen that a decision-tree approach allows us to handle sequential investment decisions. By backward induction, we are able to determine optimal decisions at various decision points. Alternative branches are eliminated on the basis of dominance.[13] The approach will be used in the subsequent chapter when we consider the abandonment decision.

DIRECT INCORPORATION OF UTILITY THEORY

The probability distribution approaches analyzed in the last section have all been based upon monetary values. They provide management with information with which to make an investment decision, but they do not indicate the best decision. This decision depends upon the utility preferences of management with respect to the risk borne by the firm. A utility-theory approach, on the other hand, incorporates directly the utility preferences of the decision maker into the investment decision.[14] The investment project providing the greatest utility is the most desirable.

The first task with respect to the direct incorporation of an individual's utility preferences into the decision is to derive and specify his utility function numerically for risk situations. The usual procedure is to have him consider a group of lotteries in the same light as he would investment projects under consideration. Through a cardinal measure of utility, we are able to derive a utility index for money for the individual. We measure utility in units, which we call "utiles." Arbitrarily, we initially assign utile values to a pair of dollar amounts — say 0 and $1 million — of 0 and 1, respectively. It is important to stress that we measure only relative utility and not absolute utility for the individual. We have assigned utility values arbitrarily to two amounts of money; consequently, our index, or scale, has no natural origin.[15]

[13] For additional discussion of the decision-tree approach to capital investment, see John F. Magee, "Decision Trees for Decision Making," *Harvard Business Review,* 42 (July–August, 1964), 126–38; Magee, "How to Use Decision Trees in Capital Investment," *Harvard Business Review,* 42 (September–October, 1964), 79–96; and Richard F. Hespos and Paul A. Strassmann, "Stochastic Decision Trees for the Analysis of Investment Decisions," *Management Science,* 9 (August, 1966), 244–59.

[14] The utility approach taken up in this section has its origin in the classic work of John Von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior,* rev. ed. (Princeton, N.J.: Princeton University Press, 1955). For a number of very useful readings on the subject, see Alfred N. Page, ed., *Utility Theory* (New York: John Wiley & Sons, Inc., 1968).

[15] See Ralph O. Swalm, "Utility Theory — Insights into Risk Taking," *Harvard Business Review,* 44 (November–December, 1966), 124–25.

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Suppose now that we pose a risk situation to the individual by having him imagine that he owns a lottery offering him a 0.5 chance of receiving no money, and a 0.5 chance of receiving $1 million. We then ask him what he would accept in cash to sell the lottery. If his answer is $450,000, we would attach a utile value of 0.5 to the $450,000. Thus, we have determined the certainty equivalent at which the individual is indifferent between that sum and the lottery. Next, we have the individual imagine that he owns another lottery providing a 0.4 probability of receiving $450,000 and a 0.6 probability of receiving $1,000,000. Suppose now that the individual said that he would sell this lottery for $700,000. The utile value of $700,000 then is

$$U(\$700,000) = 0.4U(\$450,000) + 0.6U(\$1,000,000) \qquad \textbf{(5-27)}$$
$$= 0.4(0.5) + 0.6(1.0) = 0.80$$

We have him imagine now that he owns a lottery providing a 0.3 probability of a $400,000 loss and a 0.7 probability of a $1 million gain. Suppose that he would sell the lottery for $450,000. The utile value of −$400,000 can then be calculated as

$$0.3U(-\$400,000) + 0.7U(\$1,000,000) = U(\$450,000) \qquad \textbf{(5-28)}$$
$$0.3U(-\$400,000) + 0.7(1.0) = 0.5$$
$$U(-\$400,000) = -0.667$$

Similarly, we can pose other lotteries to the individual until we have enough observations to construct his utility function for risk situations. This utility function can be graphed by drawing a line through the points. It may look like that shown in Figure 5-10. In the figure, dollars are plotted along the horizontal axis, while utile values are plotted on the vertical. We note that the individual's utility for money increases at a
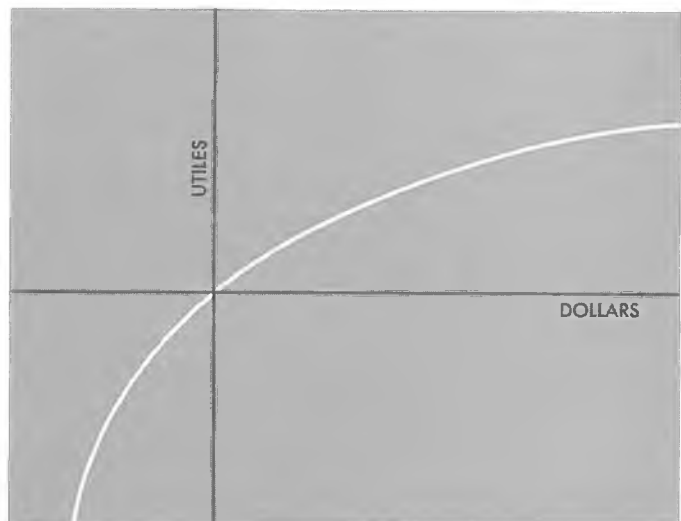
FIGURE 5-10
Marginal utility of money

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decreasing rate. This utility function is consistent with diminishing marginal utility and risk aversion throughout. Each additional dollar gives less utility or satisfaction than does the previous one. Thus, the individual is willing to take fair gambles only if the price is less than the expected value of these gambles. He is averse to risk; for, in essence, he pays an insurance premium to avoid risk.[16]

Having specified the decision maker's utility function, we are able to calculate the expected utility value of a particular investment by multiplying the utile value of a particular outcome times the probability of occurrence and adding together the products for all probabilities. To illustrate, consider the following example, in which the firm is considering two investment alternatives.

Project A		Project B	
Outcome	Probability	Outcome	Probability
−$ 400,000	0.20	0	0.20
0	0.20	$450,000	0.60
700,000	0.30	700,000	0.20
1,000,000	0.30		

The expected monetary value of project *A* is \$430,000, and that for project *B* is \$410,000.[17] On the basis of monetary value alone, project *A* would be preferred. When we calculate the expected utility values of the two alternatives, however, the results are changed.

Project A			Project B		
Utile Value of Outcome	Probability	Weighted Utility	Utile Value of Outcome	Probability	Weighted Utility
−0.667	0.20	−0.134	0	0.20	0
0	0.20	0	0.50	0.60	0.300
0.80	0.30	0.240	0.80	0.20	0.160
1.00	0.30	0.300			
Expected utility value		0.406	Expected utility value		0.460

Project *B* provides the higher expected utility value and is, therefore, the preferred alternative from the standpoint of the individual examined. The possibility of a negative outcome detracts considerably from the utility value of project *A*.

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[16] For a much more extensive discussion of utility, see Von Neumann and Morgenstern, *Theory of Games and Economic Behavior;* Milton Friedman and Leonard J. Savage, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy,* LVI (August, 1948), 279–304; Luce and Raiffa, *Games and Decisions,* Chapter 2; and Swalm, "Utility Theory," *Harvard Business Review,* 44 (November–December, 1966), 123–36.

[17] The expected monetary value is computed by multiplying the expected outcomes by their probabilities of occurrence and totaling the products.

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If this approach to capital budgeting is employed, the firm would seek to maximize the expected utility value of investment proposals under consideration, given some sort of budget constraint. From the standpoint of the individual making the decision, the expected outcomes for a proposal are reduced to their certainty equivalents. The advantage of this approach is that the utility preferences of the decision maker are incorporated directly into the project selection procedure. The approach allows the delegation of authority for the decision.[18] Knowing the utility function of a superior or the owner of a company, subordinates are able to make investment decisions on the basis of expected utility values. If the utility function is specified properly, subordinates will make decisions for the firm that are consistent with the risk preferences of the individual delegating the authority.

Despite its appeal, there is little direct use of utility theory in capital budgeting. One difficulty with the approach is in specifying a utility function that can be used consistently. Whether utility for an individual with respect to a hypothetical lottery is the same as that for an actual capital investment is open to serious question. Many executives prefer to have information presented to them and make the investment decision on the basis of this information. This preference may suggest that they are not consistent in their decisions.

When the investment decision is made by more than one individual, it is very difficult to derive a consistent utility function for the group. For example, Swalm found that in interviewing executives of different companies, many executives in the same company had significantly different utility functions with respect to risk.[19] In general, he found a bias against risk taking. Seldom will one individual exert the same degree of influence upon all decisions. When more than one owner or decision maker is dominant in making a decision, formulation of a corporate utility function is necessary. The problem of formulating a utility function for a group of executives or stockholders or both still is unresolved. For practical reasons, then, utility approaches are used very little in financial decision making.

SUMMARY

In this chapter, we recognize that investment proposals have differing degrees of business risk. In addition to the expected profitability of a proposal, we must analyze its risk contribution to the firm if we are to

[18]See C. Jackson Grayson, Jr., "The Use of Statistical Techniques in Capital Budgeting," in Alexander A. Robichek, ed., *Financial Research and Management Decisions* (New York: John Wiley & Sons, Inc., 1967), pp. 114–18; and Grayson, *Decisions under Uncertainty: Drilling Decisions by Oil and Gas Operators* (Boston: Division of Research, Harvard Business School, 1960).

[19]Swalm, "Utility Theory," *Harvard Business Review,* 44 (November–December, 1966), 123–36.

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assess properly its true economic worth. We began by examining the measurement of risk for the single investment proposal; we assumed that risk was associated positively with the dispersion of the probability distributions of expected future cash flows.

Methods considered for taking risk into account included adjusting the required rate of return, the certainty-equivalent approach, and the direct analysis of the probability distributions of possible outcomes. Varying assumptions were examined with respect to the correlation of cash flows over time. We saw that simulation techniques hold considerable promise for the evaluation of risky investments. In addition, we explored the use of decision trees to deal with projects involving sequential decisions over time. Finally, the direct incorporation of the utility preferences of the decision maker into the investment decision was examined, although this approach is seldom used in practice. Of all the methods for dealing with risk, the direct use of probability distributions is perhaps the most feasible and the most promising. In the appendix to this chapter, we incorporate the resolution of uncertainty into capital budgeting.[20]

Because of the many problems involved in taking account of risk, the methods examined are far from exact. Nevertheless, they do provide management with ways to evaluate the dispersion of possible outcomes for an investment project. Consideration of this important dimension is missing in the conventional analysis of capital budgeting. In the subsequent chapter, we extend our analysis to evaluate the risk of an investment proposal or group of proposals to the firm as a whole. Whereas this chapter was concerned only with the evaluation of an individual investment proposal, in the next chapter we consider combinations of risky investments.

APPENDIX

The Analysis
of Uncertainty
Resolution in
Capital Budgeting

In this chapter, our concern was with the expected return from a capital investment and with possible deviations from this return. Except for our brief comparison of the risk-adjusted discount rate approach with the certainty-equivalent approach, no consideration was given to when uncertainty is likely to be resolved. Yet uncertainty resolution may have important implications for the business-risk complexion of the firm and, accordingly, for the market price of its stock. In this appendix, we develop an information framework for analyzing uncertainty resolution in capital budgeting.

For many investment projects, uncertainty is not resolved at a constant rate over time. For new products in particular, the major portion

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[20]This appendix is adapted from James C. Van Horne, "The Analysis of Uncertainty Resolution in Capital Budgeting for New Products," *Management Science*, 15 (April, 1969), 376–86.

of the uncertainty is resolved in the introductory phase and in the early growth phase. Suppose that Table 5-3 in this chapter represents the conditional probabilities for an investment proposal under consideration. At time 0, any of the 27 cash-flow series is possible. As we move to period 1, however, much of the initial uncertainty with respect to future cash flows will be resolved. Suppose, for example, that the net cash flow in period 1 turned out to be $-\$6,000$. For future periods, our concern would be with only cash-flow series 1 to 9; the number of possible outcomes has been reduced considerably. The question is, how should the expected resolution of uncertainty be measured so that it is useful to management?

In measuring, we approximate relative uncertainty at a moment in time with the following statistic

$$CV_t = S_t / \overline{NPV} \tag{5A-1}$$

where S_t represents the "average" standard deviation of the various branches of the probability tree at the end of period t.

The calculation of S_t involves the following steps:

1. Discount all expected cash flows to their present value at time 0, using i as the discount rate. Thus, the entire probability tree is expressed in terms of present values of cash flows. These are labeled y_{gt}, indicating the gth discounted cash flow in period t.

2. Determine the total node value (TNV) for each node and branch tip in the probability tree by the following method:
   a. For each node, compute the expected value of all future y_{gt} in that branch.
   b. For each node and each branch tip, sum the y_{gt} involved in reaching that node or tip from time zero.
   c. Add (a) and (b) to obtain the total node value (TNV) for each node and tip.

3. Compute the weighted sum of the squares of the total node values for each period by

$$V_t = \sum_h TNV_{ht}^2 P_{ht}$$

where TNV_{ht} is the hth total node value at the end of period t, and P_{ht} is the probability of reaching that node or branch tip. When $t = 0$, there is but one total node value — the expected value of net-present value of the proposal. The probability of occurrence of this node, of course, is 1.00.

4. Determine the S_t for each period by

$$S_t = [V_n - V_t]^{1/2}$$

where n is the last period in the probability tree.

To illustrate the calculation of CV_t, our measure of relative uncertainty, consider again the probability distribution in Table 5-3. If the

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risk-free rate, i, were 5 per cent, we first would discount all cash flows by this rate to their present value at time 0. Next, we would determine the total node values and calculate their sum of squares for each of the three periods. The expected value of net-present value of the project at time 0, which is obtained in step 2 when we calculate TNV_0, is \$3,726. The V_t, S_t^2, S_t, and CV_t for the probability tree are shown in Table 5A-1.

TABLE 5A-1
Statistics for example problem

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
V_t (000 omitted)	0	57,879	68,763	70,405
S_t^2 (000 omitted)	70,405	12,526	1,642	0
S_t (000 omitted)	8,391	3,539	1,281	0
CV_t*	2.25	0.95	0.34	0

* $\overline{NPV} = 3,726$

Given the CV_t for an investment project, we can approximate the expected resolution of uncertainty for that project simply by plotting the CV_t over time. The relationship between the CV_t and length of time in the future for our example problem is shown in Figure 5A-1. The pattern of uncertainty resolution gives management considerable insight into the duration of risk for a project. In the case of Figure 5A-1, we see that uncertainty is expected to be resolved at a very fast rate through period 1, after which it is expected to be resolved more slowly.
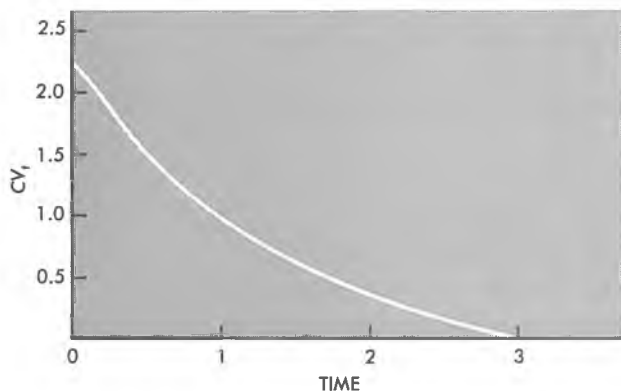
FIGURE 5A-1
Uncertainty resolution over time

Suppose that the firm were comparing this project with another project and that the two were mutually exclusive. Both projects are assumed to be perfectly correlated with the firm's existing asset portfolio. Suppose

that expectations for the second project under consideration resulted in the following:

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$$\overline{NPV} = \$3,726$$
$$S_0 = \$8,391$$
$$CV_0 = 2.25$$
$$CV_1 = 1.38$$
$$CV_2 = 0.73$$
$$CV_3 = 0.31$$
$$CV_4 = 0$$

We see that both projects have identical expected values of net-present value and dispersion about these expected values. On the basis of this information, management would be indifferent between the two. The risk, as measured by the standard deviation, is identical. However, the resolution of uncertainty is not the same. Uncertainty is expected to be resolved at a much faster rate for the first project than for the second. Given this additional information, management is unlikely to be indifferent between the two projects. Whether it prefers one to the other, however, will depend upon its preferences with respect to the pattern of uncertainty resolution for the firm as a whole.

The computations described can be extended to measure the marginal impact of an investment project on the resolution of uncertainty for the firm's overall asset portfolio.[21] The pattern of uncertainty resolution is important because it bears heavily upon the flexibility of the firm with respect to future capital budgeting decisions. If the firm wishes to maximize net-present value subject to maintaining its risk complexion, the pattern of uncertainty resolution discloses what types of projects the firm will need to generate. For example, if uncertainty is expected to be resolved very quickly in the near future, management would be able to consider relatively risky projects in its attempt to maximize net-present value subject to maintaining its risk complexion. Thus, information on uncertainty resolution is important in planning for new projects.

Another implication of knowing the pattern of uncertainty resolution has to do with the stability of the net-present value of the firm over time. If the uncertainty associated with the firm's asset portfolio is expected to be resolved very quickly and the firm takes on new projects that result in similar patterns of uncertainty resolution, the net-present value of the firm is likely to fluctuate considerably over time. With a rapid resolution of uncertainty, there is little opportunity to balance risk from period to period. As a result, the actual net-present value of an asset portfolio may differ significantly from the expected outcome. To the extent that investors at the margin value stability in the trend in earnings over time, the flexibility afforded by spreading out the uncertainty resolu-

[21] *Ibid.*, pp. 381–83.

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tion pattern of the firm's asset mix may enhance the market price of its stock, all other things being the same. Thus, it is important that management have information about the resolution of uncertainty if it is to make wise capital-budgeting decisions in keeping with the firm's risk complexion.

PROBLEMS

1. The Rettig Company can invest in one of two mutually exclusive projects. The two proposals have the following discrete probability distributions of net cash flows for period p:

A		B	
Probability	Cash Flow	Probability	Cash Flow
.20	$1,000	.10	$1,000
.30	2,000	.40	2,000
.30	3,000	.40	3,000
.20	4,000	.10	4,000

(a) Without calculating a mean and a coefficient of variation, can you select the better proposal, assuming a risk-averse management?
(b) Verify your intuitive determination.

2. The Winchell Company uses a certainty-equivalent approach in its evaluation of risky investments. Currently, the company is faced with two alternative proposals. The expected values of net cash flows for each proposal are as follows:

Year	A	B
0	−$20,000	−$15,000
1	+ 10,000	+ 8,000
2	+ 10,000	+ 9,000
3	+ 10,000	+ 10,000

(a) Risk analysis of each cash-flow distribution has provided certainty equivalents as follows:

Year	A	B
0	1.00	1.00
1	.9	.95
2	.8	.90
3	.6	.50

If the after-tax risk-free rate is 4 per cent, which of the two alternatives should be selected?
(b) If the firm were to use risk-adjusted discount rates instead of a certainty-equivalent approach, what rates would be used in order to obtain an equivalent solution?

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3. The Dewitt Corporation has determined the following discrete probability distributions for net cash flows generated by a contemplated project:

	Period 1		Period 2		Period 3
Prob.	Cash Flow	Prob.	Cash Flow	Prob.	Cash Flow
.10	$1,000	.20	$1,000	.30	$1,000
.20	2,000	.30	2,000	.40	2,000
.30	3,000	.40	3,000	.20	3,000
.40	4,000	.10	4,000	.10	4,000

(a) Assume the probability distributions of cash flows for future periods are independent. Also, assume that the after-tax risk-free rate is 4 per cent. If the proposal will require an initial outlay of $5,000, determine the expected value of the net-present value.
(b) Determine the standard deviation about the expected value.
(c) If the total distribution is approximately normal and assumed continuous, what is the probability of the net-present value being zero or less?
(d) What is the probability that the net-present value will be greater than zero?
(e) What is the probability that the profitability index will be 1.00 or less?
(f) What is the probability that the profitability index will be greater than 2?

4. The Dewitt Corporation (above) has determined that its cash-flow distributions are not independent. Further, the company has estimated that the period 1 results will affect the period 2 flows as follows:

If $P_1 = $1,000$, the distribution for P_2 is:
  .50 $1,000
  .40   2,000
  .10   3,000

If $P_1 = $2,000$, the distribution for P_2 is:
  .20 $1,000
  .50   2,000
  .30   3,000

If $P_1 = $3,000$, the distribution for P_2 is:
  .10 $1,000
  .20   2,000
  .50   3,000
  .20   4,000

If $P_1 = $4,000$, the distribution for P_2 is:
  .10 $2,000
  .30   3,000
  .50   4,000
  .10   5,000

(a) What is the most probable cash flow for P_2?
What is the probability that this flow will occur?
(b) What is the probability that the cash flow for P_2 will be $5,000?
(c) What is the probability that the cash flow for P_2 will be $6,000?
(d) If $P_1 = $3,000$, what is the probability $P_2 = $3,000$?
(e) What is the probability that $P_1 = $1,000$ and $P_2 = $1,000$?
(f) What is the probability that P_2 will be greater than $3,000?

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(g) If $P_1 = \$5,000$, what is the probability that $P_2 = \$3,000$?
(h) What is the probability that P_2 will be greater than $3,000, given the fact that P_1 is greater than \$3,000?
(i) If $P_2 = \$3,000$, what is the probability that $P_1 = \$3,000$?

5. The Hume Corporation is faced with several possible investment projects. For each, the total cash outflow required will occur in the initial period. The cash outflows, expected net-present values, and standard deviations are given below. All projects have been discounted at the risk-free rate of 4 per cent, and it is assumed that the distributions of their possible net-present values are normal.

Project	Cost	Net-present Value	σ
A	$100,000	$10,000	$20,000
B	50,000	10,000	30,000
C	200,000	25,000	10,000
D	10,000	5,000	10,000
E	500,000	75,000	75,000

(a) Construct a risk profile for each of these projects in terms of the profitability index.
(b) Ignoring size problems, are there some projects which are clearly dominated by others?
(c) May size problems be ignored?
(d) What is the probability that each of the projects will have a net-present value ≥ 0?
(e) What decision rule would you suggest for adoption of projects within this context? Which (if any) of the above projects would be adopted under your rule?

6. *Discussion questions:*
(a) Assuming that the expected net-present value of a proposal can be accurately measured, does the separate concept of risk with regard to such a proposal have meaning for General Motors?
(b) The Dupont Corporation is rumored to require in excess of a 20 per cent return on new products. May all of that 20 per cent return in excess of the risk-free interest rate be viewed as a risk premium? May any part of it be viewed as a monopoly profit?
(c) What are the theoretical justifications for considering risk in the capital-budgeting decision? Be very careful and specific in your answer.

7. Bill Rettig, owner of the Rettig Company (see problem 1 above), has determined his relative utility values for various cash flows to be as follows:

Cash Flow	Utiles
−$10,000	−1.000
− 5,000	− .300
− 1,000	− .040
0	0
+ 1,000	+ .008
+ 2,000	+ .014
+ 3,000	+ .018
+ 4,000	+ .020

Given this pattern, which project should be selected (see problem 1 above)?

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8. The Kazin Corporation is considering introducing a new product, which it can distribute initially either in the state of Georgia or in the entire Southeast. If it distributes in Georgia alone, plant and marketing will cost $5 million, and Kazin can reevaluate the project at the end of three years to decide whether to go regional. To go regional at the end of three years would cost another $10 million. To distribute regionally from the outset would cost $12 million. The risk-free after-tax cost of funds to the firm is 4 per cent. In either case, the product will have a life of six years, after which the plant will be worthless. Given the data below, what policy should Kazin follow?

*Decision Tree
Regional Versus Statewide Distribution*
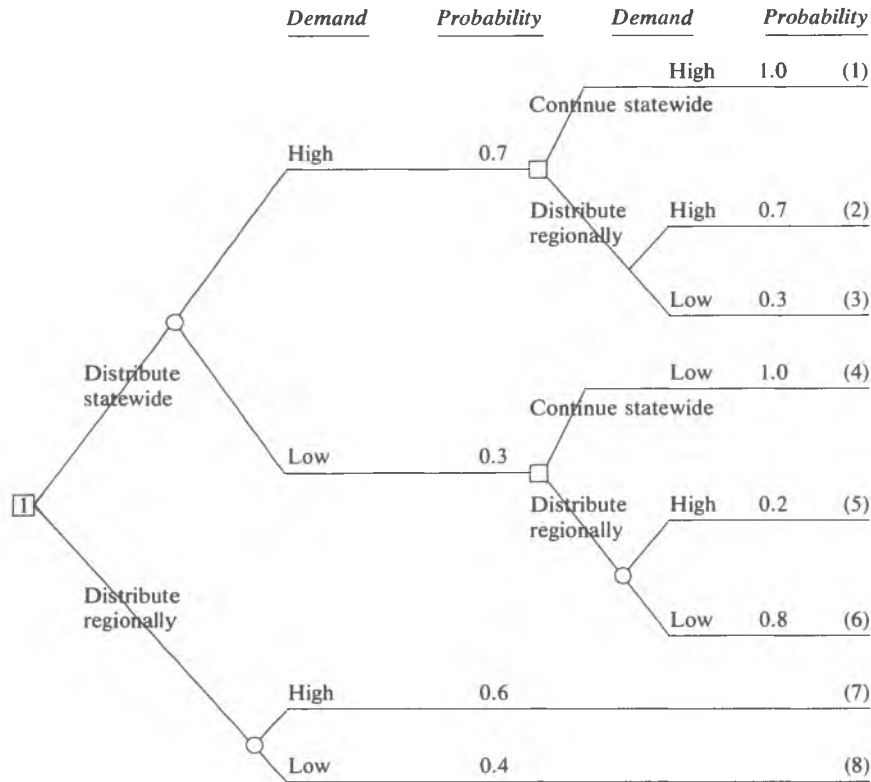
Expected cash flows (in thousands)

Branch	0	1	2	3	4	5	6
				Years			
1	−$ 5,000	$1,000	$ 3,000	$ 5,000	$ 7,000	$ 4,000	$2,000
2	− 5,000	1,000	3,000	− 7,000	10,000	20,000	8,000
3	− 5,000	1,000	3,000	− 7,000	8,000	6,000	4,000
4	− 5,000	200	400	1,000	2,000	1,000	200
5	− 5,000	200	400	− 11,000	8,000	15,000	5,000
6	− 5,000	200	400	− 11,000	3,000	4,000	4,000
7	− 12,000	3,000	10,000	15,000	20,000	12,000	5,000
8	− 12,000	1,000	2,000	3,000	4,000	3,000	1,000

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Multiple
Risky Investments,
Acquisitions,
and Divesture

6

In the previous chapter, we evaluated the expected return and risk of a single investment proposal. The approaches discussed are feasible if the firm has no existing investments or if the expected returns for all investments, existing as well as proposed, are perfectly correlated. However, if either of these conditions does not hold, the methods will not provide a true indication of the risk of an investment project to the firm. The risk of a project should not be evaluated in isolation; it must be judged in relation to its marginal additions of risk to the firm as a whole. The reason is that valuation of a firm's stock depends upon the business-risk complexion of the entire firm rather than upon the risk involved in a single investment project. As our objective is to maximize the market price of the stock, we must analyze the marginal impact of an investment on the total risk complexion of the firm.

Because most investment projects are correlated positively with each other, the firm should combine investment projects with care to obtain its

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total portfolio of assets. If it adds a project that is highly correlated with existing investments, the total risk of the firm will increase more than if it adds a project that has a low degree of correlation with others, all other things being equal. A project with a high degree of correlation may be accepted in which the incremental increase in risk to the firm as a whole more than offsets its expected profitability. As a result, the market price of the stock declines. Another project, with lower expected profitability but also with a low degree of correlation, may have a favorable enough impact on the business-risk complexion of the firm as a whole to cause an increase in the market price of the stock.

The firm should be cognizant of the potential benefits of diversifying investments to achieve the best combination of expected net-present value and risk. The firm can diversify either through investment in proposals generated internally or through external acquisitions. Capital should be allocated so as to increase shareholder wealth. An optimal combination of internal and external investments would be one that maximized the value of the firm to its shareholders. Thus, we are interested in the business risk of the firm as perceived by investors at the margin. However, a caveat is in order. Not only is risk to the firm as a whole difficult to measure but the exact link between such risk and share price is far from clear. Although empirical studies generally indicate an inverse relationship between share price and the variability of earnings — holding constant other factors — at best it is possible only to specify an approximate relationship.

Because of these difficulties, the methods proposed in the subsequent sections for evaluating risk to the firm as a whole do not, in themselves, provide the optimal set of investment decisions. Instead, they provide management with information about the risk of various combinations of investments and the expected net-present values of these combinations. Given this information, management must then select the combination of available investments that it feels will maximize the market price of the firm's stock. When possible, this selection should be made in keeping with empirical studies of the relationship between share price and expected profitability and risk as perceived by investors at the margin.

The last part of this chapter deals with the divesture of a portion of the enterprise, such as a division, when that portion no longer justifies economically the capital committed to it. In relation to the overall objectives of the firm, certain existing projects may not be earning their keep. Under such circumstances, they should be sold or abandoned outright. Divesture simply represents the reverse of capital budgeting. The economic principle is the same: Assets are acquired when they are expected to enhance shareholder wealth and are sold when they no longer fit into the overall objectives of the firm. The possibility of later abandonment should be recognized in the initial capital-budgeting analysis. Some projects have far greater abandonment value than others and, therefore, are more

valuable to the firm from the standpoint of expected value of return and of risk. We propose a method for evaluating abandonment values and incorporating these values into the investment decision.

PORTFOLIOS OF RISKY INVESTMENTS

Our purpose in this section is to propose a method by which management may evaluate investment proposals in keeping with the total business-risk complexion of the firm.[1] In this regard, we apply certain probability concepts that have been used in security portfolio analysis. From Chapter 2, we know that the total variance, or risk, of a combination of risky investments depends to a large extent upon the degree of correlation between the investments. The standard deviation of the probability distribution of possible net-present values for a portfolio of capital investments can be expressed as

$$\sigma = \sqrt{\sum_{j=1}^{m} \sum_{k=1}^{m} r_{jk} \sigma_j \sigma_k} \qquad (6\text{-}1)$$

where m is the total number of assets in the portfolio, r_{jk} is the expected correlation between the net-present values for investments j and k, σ_j is the standard deviation about the expected value of net-present value for investment j, and σ_k is the standard deviation for investment k.

Equation (6-1) indicates that the standard deviation, or risk, of a portfolio of projects depends upon (1) the degree of correlation between various projects and (2) the standard deviation of possible net-present values for each project. We note that the higher the degree of positive correlation, the greater the standard deviation of the portfolio of projects, all other things remaining constant. On the other hand, the greater the standard deviations of the individual projects, the greater the standard deviation of the portfolio, if the correlation is positive. The standard deviations of the individual investment projects, necessary for the calculation of Eq. (6-1), are obtained through the methods presented in the previous chapter.

CORRELATION BETWEEN PROJECTS

As was the case with a portfolio of securities discussed in Chapter 2, the correlation between expected net-present values of two projects may be positive, negative, or zero, depending upon the nature of the association. A correlation coefficient of 1.00 indicates that the net-present values

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[1] The development of this section assumes that the reader has covered the portfolio section in Chapter 2. It is based upon James C. Van Horne, "Capital-Budgeting Decisions Involving Combinations of Risky Investments," *Management Science*, 13 (October, 1966), 84–92.

of two investment proposals vary directly in exactly the same proportional manner; a correlation coefficient of -1.00 indicates that they vary inversely in exactly the same proportional manner; and a zero correlation coefficient usually indicates that they are independent. While independence is a sufficient condition for the absence of correlation, zero correlation does not always imply independence. Perfect positive, perfect negative, and zero correlation between the possible net-present values for two projects are illustrated in Figure 6-1. We assume three possible states of nature—recession, normal economic growth, and economic

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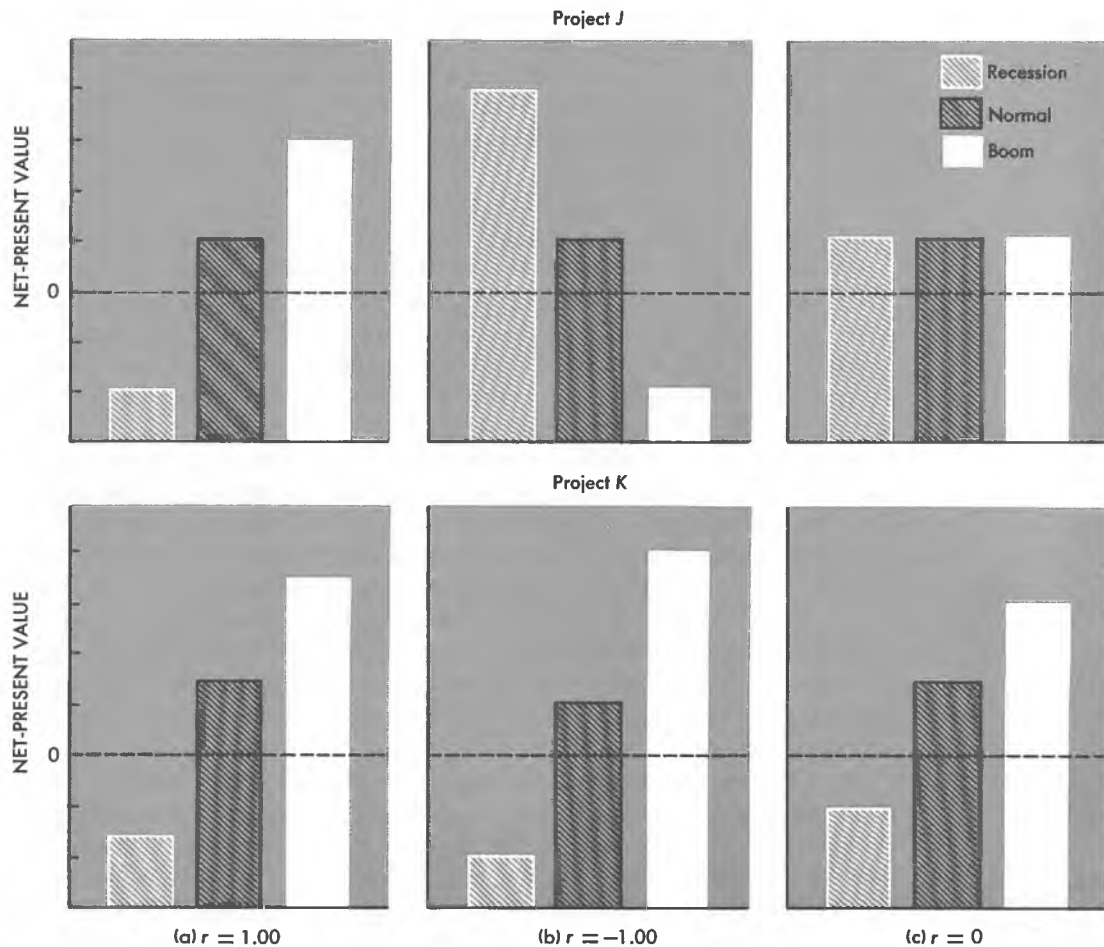
FIGURE 6-1
Relationship between net-present values and states of nature for two investment proposals

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boom — and that the probabilities of these occurrences are 0.25, 0.50, and 0.25, respectively. In panel (a), the possible net-present values for projects *J* and *K* are perfectly positively correlated; in panel (b), perfectly negatively correlated; and in panel (c), there is zero correlation.

For most pairs of investment projects, the correlation coefficient lies between 0 and 1.00. The lack of negatively correlated projects is due to most investments being correlated positively with the economy. Still it is possible to find projects having low or moderate degrees of correlation. For example, management might have reason to expect only slight correlation between an investment project involving an electronic transistor and one involving a new consumer product. It might, however, expect high positive correlation between investments in a milling machine and a turret lathe if both machines are to be used in the production of industrial lift trucks. The profit from a machine used in a production line will be highly correlated with the profit of the production line itself.[2] Projects in the same general line of business tend to be highly correlated with each other, while projects in essentially unrelated lines of business tend to have low degrees of correlation.

Estimates of correlation coefficients must be as objective as possible if the standard deviation obtained in Eq. (6-1) is to be realistic. When investment projects are like projects the firm has had experience with, it may be feasible to compute the correlation coefficients using historical data. For other investments, however, estimates of correlation coefficients must be based solely on an assessment of the future. Instead of estimating the correlation between pairs of investment projects directly, it may be better to employ an index model and estimate the correlation between an investment and some index, such as the Gross National Product or an industry production index. From these estimates, the standard deviation for a portfolio of investment projects can be approximated. The use of an index model is described in the appendix to Chapter 2. As discussed there, using such a model greatly reduces the number of correlation coefficient estimates. For a large portfolio of projects, the number of computations involved in calculating the standard deviation with Eq. (6-1) is unfeasible. Consequently, an index model must be employed.

It is not unreasonable to suppose that management is able to make fairly accurate estimates of the correlation between investment projects or the correlation between an investment project and some index. To the extent that actual correlation differs from that expected, future correlation estimates on existing projects should be revised in keeping with the learning process. The learning process also applies to future estimates of correlation between investments similar to existing investments.

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[2] J. M. English, "Economic Comparison of Projects Incorporating a Utility Criterion in the Rate of Return," *Engineering Economist,* 10 (Winter, 1965), 13.

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Instead of the approach discussed above, one may resort to methods that have been developed to simulate joint returns for a portfolio of investment projects. Cohen and Elton propose that the firm should specify the functional relationships between cash flows for a project and various factors giving rise to these cash flows.[3] The joint probability distribution of the various underlying factors must then be specified. Once these relationships are described, cash flows in each period for each investment can be simulated. For each simulation run, the net-present values of the individual investment projects are calculated. When these are summed, one obtains the net-present value of the portfolio. By simulating a number of times, a distribution of possible portfolio net-present values is formed, from which the expected value and standard deviation can be computed. A somewhat similar approach, but one which uses simulation and stochastic linear programming, is described in the appendix to this chapter.

The final output of simulation approaches—namely, the expected value and standard deviation of the probability distribution of possible portfolio net-present values—is the same as that generated through the correlation coefficient approach described earlier. Actually, the two approaches are quite similar. In the first, we specify the functional relationship between project cash flows; in the second, we specify the relationship between the net-present values for various projects. In both cases, the accuracy of the final results hinges on the accuracy of these specifications. If accurate, the two approaches will give similar results. Because using the correlation coefficients is less involved, we shall assume the use of this approach in the subsequent discussion.

EXISTING PROJECTS
AND NEW PROPOSALS

We now have a procedure for determining the total expected value of net-present value and the standard deviation of the probability distribution of possible net-present values for a combination of investments. A combination includes all existing investment projects and one or more proposals under consideration. We assume that a firm has existing investment projects generating expected future cash flows and that disinvestment with respect to these projects is not possible. Existing projects comprise a subset that is included in all combinations. Proposals under consideration are assumed to represent all future proposals on the investment horizon.

Investment proposals must be evaluated in relation to their net contribution to the total risk of the company; for this reason it is extremely

[3] Kalman J. Cohen and Edwin J. Elton, "Inter-Temporal Portfolio Analysis Based upon Simulation of Joint Returns," *Management Science,* 14 (September, 1967), 5–11.

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important that we take account of existing investment projects in computing the standard deviation. On account of covariance, the combination of investment proposals under consideration that provides the lowest total variance, or standard deviation, may not provide the lowest total variance, or standard deviation, when existing projects are included. A variance figure based solely upon investment proposals under consideration is not a measure of total variance to the firm and, consequently, is inadequate as a basis for judging total risk. Again, the incremental effect of an investment proposal on the business-risk complexion of the firm as a whole is what is important in judging a proposal's risk in relation to the objective of the firm.

EVALUATION OF COMBINATIONS

The next step involves the evaluation of all feasible combinations of existing investment projects and proposals under consideration.[4] The difficulty of calculating the expected value of net-present value and standard deviation for each feasible combination should not be underestimated, but procedures can be employed to streamline the task.[5] Again, when the combination of existing projects and proposals under consideration is large, an index model should be used. Given the expected value of net-present value and the standard deviation of the probability distribution of possible net-present values for each combination, we can plot this information on a scatter diagram. Figure 6-2 is an example of a scatter diagram; here the expected value of net-present value is along the horizontal axis, and the standard deviation is on the vertical axis. Each dot represents a feasible combination of proposals under consideration and existing investment projects for the firm.

Collectively, the dots represent the total set of feasible combinations of investment opportunities available to the firm. This set corresponds to the opportunity set of security portfolios discussed in Chapter 2, the major difference being that combinations of investment projects are not as divisible as portfolios of securities. Certain dots in Figure 6-2 dominate others in the sense that they represent a higher expected value of net-present value and the same standard deviation, a lower standard deviation and the same expected value of net-present value, or both a higher expected value and a lower standard deviation. The dots that

[4]When two or more proposals are mutually exclusive, so that acceptance of one precludes acceptance of the other(s), these proposals cannot appear in the same combination. If two or more proposals are contingent, so that acceptance of one is dependent upon acceptance of one or more other proposals, a combination containing a dependent project must also contain the proposal(s) on which it is dependent.

[5]The total expected value of net-present value and standard deviation for existing investment projects is the same for all combinations. The problem reduces to calculating the incremental net-present value and variance for the addition of one or more proposals under consideration to existing investment projects.
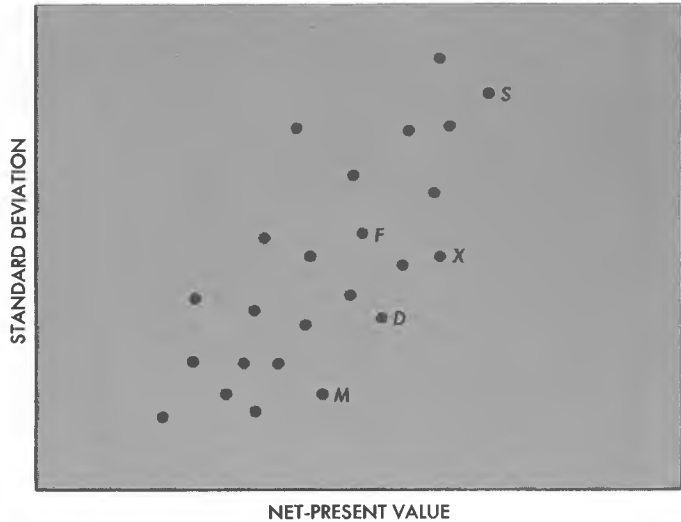
FIGURE 6-2
Opportunity set of project portfolios

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dominate others are those that are farthest to the right in Figure 6-2, and they correspond to the efficient frontier for an opportunity set of security portfolios. According to the mean-variance maxim discussed in Chapter 2, management should seek a combination of investments that lies on the right-hand edge of the opportunity set in Figure 6-2.

SELECTION OF THE
BEST COMBINATION

The selection of the most desirable combination of investments will depend upon the utility preferences of management with respect to net-present value and variance, or standard deviation. If management is averse to risk and associates risk with the variance of net-present value, its utility function may be similar to that shown in Figure 6-3.[6] As discussed in Chapter 2, the curves in the figure are indifference curves; management is indifferent to any combination of expected value of net-present value and standard deviation on a particular curve. Thus, a specific curve portrays the tradeoff between the two parameters for a particular company. The indifference curves in Figure 6-3 suggest that management's utility function is a monotonic increasing concave one, indicating decreasing marginal rates of substitution between standard deviation and net-present value. As the dispersion of possible net-present values of the firm increases, it takes increasing amounts of net-present value for management to accept additional increments of risk.

[6] See the last part of the previous chapter for a discussion of one way to map a decision maker's utility preferences.
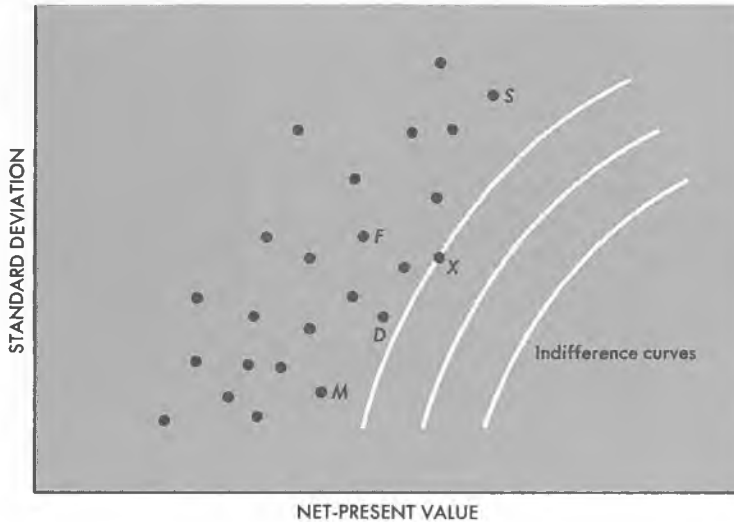
FIGURE 6-3
Selection of the best project portfolio

As we move to the right in Figure 6-3, each successive curve represents a higher level of utility. Thus, management would choose the combination of investments that lies on the highest indifference curve, the one farthest to the right, because this curve represents the greatest utility. This combination is determined by the intersection of a dot in the figure — point X — with the highest indifference curve. Point X represents the portfolio of existing projects and proposals under consideration that possesses the most desirable combination of expected value of net-present value and risk.

The framework for evaluating combinations of risky investments developed above is quite useful even if management's utility function is not defined. For example, we might present management with the information in Figure 6-2. With this, management can eliminate most combinations simply because they are dominated by other combinations. Unless management is quite averse to risk, it would probably consider only four portfolios of risky investments, M, D, X, and S. From these management would choose the one that it felt offered the best combination of expected return and risk.

This selection determines the new investment proposal or proposals that will be accepted. An exception would occur when the portfolio selected was comprised only of existing projects. In this situation, no investment proposals under consideration would be accepted. If the portfolio of existing projects were represented by portfolio F in Figure 6-2, however, the selection of any of the four portfolios would imply the acceptance of one or more new investment proposals. Those investment proposals under consideration that were not in the portfolio finally selected would be rejected, of course.

In this manner, management is able to determine which investment proposals under consideration offer the best marginal contribution of expected value of net-present value and standard deviation to the firm as a whole. In determining the standard deviation for a portfolio, consideration is given to the correlation of an investment proposal with existing investment projects, as well as with other new investment proposals. As the total risk of the firm is what is important in maximizing the market price of the stock, investment decisions should be made in light of the marginal impact on the total risk of the firm.

The framework proposed in this section enables management to appraise the various relevant factors realistically in its effort to make sound capital-budgeting decisions. Whether these decisions are optimal in the sense of maximizing share price will depend upon how accurately the tradeoff between profitability and risk is measured, the relationship between this tradeoff and share price, and whether management acts in a manner consistent with the objective of maximizing share price. As mentioned previously, the link between risk-profitability and share price is extremely difficult to estimate precisely. Consequently, investment decisions may be less than optimal if management is unable to gauge this relationship accurately. With the method proposed, however, management is in a better position to approximate the likely effects of an investment decision on expected future dividends and on the risk premium embodied in investors' required rate of return. As we know from Chapters 2 and 4, these factors determine share price.

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ACQUISITIONS

Investment proposals under consideration are not necessarily confined to proposals generated internally. A proposal can consist of the acquisition of a company, or a portion thereof. The topic of acquisitions is treated in Chapter 23; in the present chapter, we consider the capital-budgeting aspects of the problem. In principle, the prospective acquisition is much the same as any investment proposal; there is an initial outlay of cash or stock, followed by expected future benefits. The major difference is that with acquisitions, the initial cost may not be established; indeed, it is frequently subject to bargaining.

ESTIMATING CASH FLOWS

In order to consider an acquisition in a capital-budgeting framework, expected future cash flows must be expressed on a basis consistent with those for investment proposals generated internally. In evaluating the prospective acquisition, the buying company should first estimate the

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future cash income the acquisition is expected to add.[7] Because we are interested in the marginal impact of the acquisition, these estimates should embody any expected economies known as synergism, which are involved in the merger. In Chapter 23, it is established that certain mergers produce very important synergistic effects; as a result, the combination of the two companies is more valuable than the sum of the parts.

In an acquisition, there are the usual problems with respect to estimating future cash flows. However, the process may be somewhat easier than for a capital-budgeting proposal, because the company being acquired is a going concern. The acquiring company buys more than assets; it buys experience, an organization, and proven performance. The estimates of sales and costs are based upon past results; consequently, they are likely to be more accurate than the estimates for a new investment proposal.

In making the various estimates, it is important to isolate the capital structure that results from the merger. The reason is that once the merger is consummated, the buying company can modify the capital structure that results from the merger. Therefore, prospective incremental cash income from the acquisition should be estimated before interest charges. In other words, we attempt to measure the expected incremental earning power of the acquisition, apart from considerations of financing.

Expected incremental cash income should be adjusted for taxes. As was the case in Chapter 3, we deduct expected depreciation charges from cash income and compute the amount of taxes to be paid on the residual. By subtracting expected taxes from expected incremental cash income, we obtain cash income after taxes for each future period. From this expected cash income after taxes, we must in turn subtract any new investments the acquiring firm believes it will have to make in order to generate the expected stream of incremental earnings. The residual represents the expected cash flow after taxes for the period. It is extremely important that we take account of these investments; otherwise, incremental cash-flow estimates will be biased upward. An error frequently made is to treat as cash flows expected future earnings after taxes of the firm being considered for acquisition. The problem is that an earnings estimate for a future period usually is predicated upon reinvesting a portion of earnings from previous periods.[8] To use expected earnings in evaluating an acquisition results in double counting, as does the use of expected future earnings per share in a dividend valuation model.[9] (The biases associated with the latter were described in Chapter 4 and the appendix to that chapter.) In summation, the appropriate measure of incremental cash flow is

[7] See Samuel Schwartz, "Merger Analysis as a Capital Budgeting Problem," in William W. Alberts and Joel E. Segall, eds., *The Corporate Merger* (Chicago: University of Chicago Press, 1966), pp. 139–50.

[8] It is also assumed that funds generated through depreciation allowances are reinvested to maintain the company's existing level of earnings.

[9] I am grateful to H. E. Borgstrom, Jr., for pointing this out.

expected earnings after taxes plus depreciation in each future period, less any investment required in that period.

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To illustrate the information needed, suppose the incremental cash flows shown in Table 6-1 were expected from an acquisition. In the same manner, the firm should specify other possible net cash-flow series, with a probability attached to each. In other words, it needs to obtain a probability tree of possible net cash flows. The method for obtaining this probability tree was illustrated in Chapter 5; therefore, we do not discuss it here.

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TABLE 6-1

	Average for Years (in thousands)				
	1–5	6–10	11–15	16–20	21–∞
Expected cash income from acquisition before taxes	$1,500	$2,000	$2,500	$3,000	$3,200
Taxes	500	700	1,000	1,200	1,300
Expected cash income after taxes	1,000	1,300	1,500	1,800	1,900
Investment required	800	900	800	700	600
Net cash flow	200	400	700	1,100	1,300

Given the various possible net cash-flow series, each should be discounted to its present value using the risk-free rate. The result is a probability distribution of possible present values for the acquisition. One modification is necessary, however. We must subtract from each of the
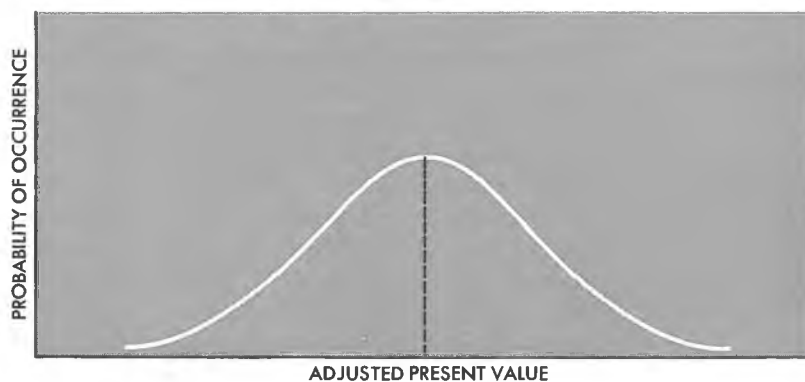
FIGURE 6-4
Probability distribution of possible adjusted present values for a prospective acquisition

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present values the amount of debt the company being considered for acquisition currently has outstanding. The reason is that the acquiring firm will assume this debt. Therefore, the value of the enterprise being considered for acquisition must be reduced by this amount. In this way, the acquisition is analyzed solely as an investment decision. In theory, the acquiring firm will finance it as it does other investment projects. When we have subtracted debt from each of the possible present values, we obtain a probability distribution of possible *adjusted* present values, as illustrated in Figure 6-4. The expected value and standard deviation of this distribution can be easily determined.

EVALUATING THE ACQUISITION

If the price to be paid for the prospective acquisition has been established, it should be subtracted from the expected value of adjusted present value to obtain the expected value of adjusted *net*-present value. The next step is to estimate the correlation coefficients for the relationship between the adjusted net-present value for the prospective acquisition and the net-present values for existing projects and investment proposals under consideration. If an index model is used, we would estimate the correlation between the prospective acquisition and the index. The acquisition then becomes one more investment proposal in a portfolio of projects to be considered. The methods of analysis and selection are the same as those employed in the previous section. Management chooses the best combination of expected value of net-present value and risk. If the portfolio of projects represented by this combination includes the prospective acquisition, the firm should acquire the company involved. In this way, an acquisition is evaluated in the same manner as any internally generated investment proposal. A decision is made in keeping with the marginal impact of the acquisition on the total risk of the firm.

Establishing a Maximum Price. If the price to be paid has not been set, the expected value–standard deviation type of analysis can be used to establish the *maximum* price that should be paid. In order to do this, we must first determine the range of possible prices. The lowest possible price, of course, is zero. If for some reason this price were to be accepted, the expected value of adjusted net-present value would simply be the expected value of adjusted present value. At the other extreme, the maximum possible price is the expected value of adjusted present value. In this case, the expected value of adjusted *net*-present value would be zero. Recall that the expected value of adjusted net-present value is the expected value of adjusted present value for the acquisition, less the price paid. Since the discount rate is the risk-free rate, the acquisition would need to be riskless in order for the maximum possible price to be justified.

For each portfolio in which the acquisition appears, we would plot the
expected value of net-present value under both of the above assumptions.
(Note that the standard deviation of the probability distribution of pos-
sible net-present values for the portfolio remains unchanged.) This dual
plotting is illustrated in Figure 6-5, where we assume that the acquisition
appears in four different project portfolios. The dots farthest to the right
represent the expected values of adjusted net-present value under the
assumption of a purchase price of zero; the dots farthest to the left repre-
sent the expected values assuming the maximum possible purchase price.
We draw horizontal lines connecting the two sets of dots.

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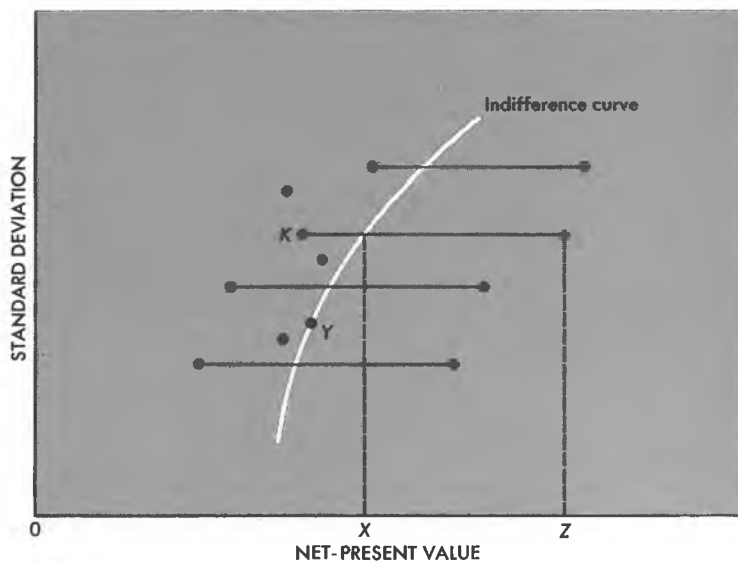
FIGURE 6-5
**Determining the maximum
price that should be paid
for an acquisition**

In order to establish the *maximum* price the firm *should* pay, we must
identify the highest indifference curve that intersects a portfolio not con-
taining the acquisition. In Figure 6-5, this portfolio is represented by
point Y. We then trace along the indifference curve to the intersections of
this curve with the horizontal lines. The maximum price the firm should
pay is determined by finding the greatest distance from the intersections
to the dots farthest to the right. Of the four portfolios in which the ac-
quisition is contained, the greatest distance is $X - Z$ for portfolio k. If
this price were paid, the expected value of net-present value of the port-
folio would be such that management would be indifferent between
portfolio K and the best portfolio which does not include the acquisition.
The distance $X - Z$ then represents the maximum purchase price the
firm should consider. Higher prices will result in a lower indifference
curve (indicating less utility) than that available from a portfolio of in-

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vestments which does not contain the acquisition. Consequently, the firm should not consider a higher price. Implied in the maximum price paid is that the firm will undertake the other investment proposals under consideration contained in the portfolio. As with our discussion in the previous section, the example can be changed to represent other utility preferences of management.

The actual price paid for the acquisition will be subject to negotiation. The maximum price we determined, however, represents an upper boundary for the acquiring company. Any price up to this amount will result in a worthwhile investment for the firm in the eyes of its management.

METHOD OF ACQUISITION

In our analysis, we assume implicitly that it makes no difference to the buying company whether the acquisition is with cash or with stock. If the acquisition is with cash, the company must raise this cash as it would to finance any investment proposal. Presumably, it could raise capital at close to its measured cost-of-capital rate. With an acquisition, we must allow for the possibility of an upward sloping supply curve for capital. It is unreasonable to assume that a firm can raise any amount of capital at the same rate. Because acquisitions often involve large amounts of capital, the real cost of capital for the acquisition may be higher than the present cost of capital.

If the acquisition is with stock, however, the firm raises funds with a particular type of financing, namely, equity. The explicit cost of equity is higher than the cost of capital, unless, of course, the firm's capital structure consists entirely of equity. With equity financing, however, the firm will build its equity base and thereby increase its borrowing capacity. This opportunity benefit lowers the real cost of equity funds. If the acquiring company is at an optimal capital structure, the marginal real cost of equity financing should equal the weighted-average cost of capital. Therefore, a case can be made for the notion that the cost of capital is the appropriate criterion for judging an acquisition and that it makes little difference whether the acquisition is with stock or with cash. In either case, however, we must take into account the possibility of an upward sloping supply curve of capital.

DIVERSIFICATION

By acquiring companies with operations having low degrees of correlation with its existing operations, a firm may be able to reduce the total dispersion of the probability distribution of possible net-present values. Diversification through acquisition usually can be accomplished more quickly and, perhaps, more efficiently than through internally gen-

erated investment proposals. Using the method of evaluation described, a company will wish to diversify in order to obtain the best combination of expected value of net-present value and risk to the firm as a whole.

Diversification and Shareholder Wealth. A question remains, however, as to whether diversification has a positive impact upon the value of the firm to its shareholders. In other words, is diversification a desirable objective of the firm? There is little question that investors have the ability to diversify in the common stocks they hold in their portfolios. As taken up in Chapter 2, investors can be viewed as choosing that portfolio of securities which provides the best combination of expected return and risk. In perfect markets, investors would be able to diversify as effectively for themselves as the firm could for them. Therefore, diversification on the part of the firm would not enhance shareholder wealth. In perfect markets, Myers argues that all investment proposals, internally generated as well as acquisition, are risk independent.[10] This argument suggests that the analysis of risky investments proposed in this chapter is not a worthwhile endeavor because investors are able to achieve risk reduction through diversification on their own. According to this view, investment proposals should be individually analyzed as to their expected value of return and risk, along the lines of the previous chapter.

When we relax the assumption of perfect markets, the assertion that risk can be analyzed on an individual project basis is far less convincing. For investment proposals generated internally, it seems unlikely that investors are able to achieve the same diversification as the firm is able to achieve for them. Therefore, when it comes to capital investments, it would seem that the firm is able to achieve better diversification for its stockholders than they are able to achieve for themselves.

The same observation would apply to the acquisition of a company whose common stock was closely held or inactively traded. The argument is far less convincing, however, when it comes to acquiring a company whose stock is publicly held and actively traded. Here, the firm invests in a company that has common stock of its own. To the extent that this stock is traded publicly, the investor is able to achieve diversification commensurate with that of the acquiring firm. In fact, he has an advantage in that he is able to diversify by buying only a few shares of stock, where the acquisition for the buying firm is much more "lumpy." If investors are able to diversify their security holdings effectively, the firm seemingly could not enhance share price by diversification through acquisitions.

This is not to say that an acquisition will not enhance the value of the

[10] Stewart C. Myers, "Procedures for Capital Budgeting Under Uncertainty," *Industrial Management Review*, 9 (Spring, 1968), 1–15. For a discussion of perfect capital markets, see Chapter 7 of this book.

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firm to its shareholders. Indeed, economies may be involved that will benefit the acquiring firm and its stockholders. In addition, the acquiring firm may be able to buy companies on favorable terms. In other words, the acquiring company simply may be a good bargainer. However, diversification itself would not be beneficial.[11] In particular, conglomerate mergers, where the sole purpose is diversification, would be suspect; they would not enhance shareholder wealth.

If investors do not diversify effectively on their own, however, corporate diversification may result in an increase in share price. In this case, the firm would be able to do something for investors that they were not able to do on their own.[12] Whether a company is in fact able to diversify more or less effectively than are individual investors, however, remains an empirical question. To date, there is little evidence casting light on the issue.

DIVESTURE

At times it may be desirable for a firm to divest itself of a portion of the enterprise. The divesture can consist of a specific asset or a division. In extreme cases, it may be desirable for the entire firm to discontinue operations. A company should be quick to recognize when an investment no longer fits into its overall objectives. Unfortunately, often there is a tendency to continue with investments long after they have ceased to be viable in an economic sense.

The economic rationale for divesture is the same as that for capital budgeting. Funds should be removed from a project, or disinvested, whenever the project does not economically justify use of these funds. For ease of understanding, we assume initially that disinvestment does not alter the business-risk complexion of the firm as a whole. Under such a circumstance, the opportunity cost of funds tied up in a project is the cost of capital. In general, an investment project should be abandoned when its abandonment value exceeds the present value of the project's subsequent expected future cash flows, discounted at the firm's cost-of-capital rate. In certain cases, this rule must be modified so that abandoning the project at a later time can be considered. The abandonment value of a project is assumed to represent its disposal value, which would be-

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[11] See William W. Alberts, "The Profitability of Growth by Merger," in Alberts and Segall, eds., *The Corporate Merger,* p. 272. See also, Haim Levy and Marshall Sarnat, "Diversification, Portfolio Analysis and the Uneasy Case for Conglomerate Mergers," *Journal of Finance,* XXV (September, 1970), 795–802.

[12] For a comparative analysis of small investors who are unable to diversify effectively and institutional investors who are, see Donald L. Tuttle and Robert H. Litzenberger, "Leverage, Diversification and Capital Market Effects on a Risk-Adjusted Capital Budgeting Framework," *Journal of Finance,* XXIII (June, 1968), 435–43.

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come available to the firm either in cash or cash savings.[13] An asset does not actually have to be sold to be abandoned; it may simply be employed in another area of the enterprise. In either case, an abandonment value can be estimated.

The abandonment rule posed above can be expressed more formally as[14]

1. Compute the present value of the sum of cash flows expected to be generated from the project and the expected abandonment value at the end of the holding period. If there were n years remaining in the life of the project, there would be n possible holding periods, and n present values to compute. Thus,

$$PV_{\tau \cdot a} = \sum_{t=\tau+1}^{a} \frac{A_t}{(1 + k)^{(t-\tau)}} + \frac{AV_a}{(1 + k)^{(a-\tau)}} \qquad (6\text{-}2)$$

where $PV_{\tau \cdot a}$ = present value at time τ of expected future net cash flows through period a, plus the present value of the expected abandonment value at the end of period a

a = period in which the project is abandoned
A_t = expected net-cash flow of the project in period t
k = cost of capital rate
AV_a = expected abandonment value at the end of period a.

Present values are computed under the assumption of abandonment in every period $\tau + 1 \leqslant a \leqslant n$, where n is the remaining life of the project.

2. Having calculated n present values, we then compare the largest such value, Max. $PV_{\tau \cdot a}$, with the current abandonment value, AV_τ. If Max. $PV_{\tau \cdot a}$ is greater than AV_τ, we continue to hold the project and evaluate it again at time $\tau + 1$, based upon our expectations at that time.

3. If Max. $PV_{\tau \cdot a}$ is equal to or less than AV_τ, we compare $PV_{\tau \cdot a}$ for $a = n - 1$ with AV_τ. If $PV_{\tau \cdot n-1} > AV_\tau$, we would hold the project and evaluate it again at time $\tau + 1$ the same as in step 2. If $PV_{\tau \cdot n-1} \leqslant AV_\tau$, we then compare $PV_{\tau \cdot n-2}$ with AV_τ. This procedure is continued either until the decision to hold is reached or $a = \tau + 1$.

4. If $PV_{\tau \cdot a} \leqslant AV_\tau$ for all $\tau + 1 \leqslant a \leqslant n$, then we would abandon the project at time τ.

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[13] For an excellent discussion of measuring cash flows and abandonment value, see Gordon Shillinglaw's two articles, "Profit Analysis for Abandonment Decision," and "Residual Values in Investment Analysis," both reprinted in Ezra Solomon, ed., *The Management of Corporate Capital* (New York: The Free Press, 1959), pp. 269–81 and 259–68, respectively.

[14] See Alexander A. Robichek and James C. Van Horne, "Abandonment Value and Capital Budgeting," *Journal of Finance*, XXII (December, 1967), 577–89; Edward A. Dyl and Hugh W. Long, "Comment," *Journal of Finance*, XXIV (March, 1969), 88–95; and Robichek and Van Horne, "Reply," *ibid.*, 96–97.

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In other words, these steps would have us abandon a project only if the present value of possible future benefits is less than the current abandonment value and if it does not appear that abandoning the project in the future would be more favorable than doing so currently. To continue with a project which the above rules suggest should be given up means that capital is being less than optimally employed. From an economic standpoint, the project simply is not earning its keep; therefore, it should be abandoned. In the discussion that follows, we assume that projects are abandoned according to the rules given above.

EFFECT UPON PROJECT SELECTION

The recognition of abandonment value may have a significant effect upon project selection. Implied in most capital-budgeting decisions is the notion that funds will be committed to an investment proposal over its entire estimated life.[15] Proposals are evaluated as though all outlays were sunk. However, many projects have significant abandonment values. Too often, this value is ignored, despite the fact that it can affect a project's expected profitability and risk. In this section, we propose a decision-tree approach for incorporating abandonment value into the analysis for capital budgeting.

To become familiar with the method, study the example that follows. Project *A*, costing $3,000 at time zero, is expected to generate net cash flows over the next two years. These cash flows and their probability of occurrence are shown as a series of conditional probabilities in Table 6-2. For simplicity of illustration, we assume that after the second year, the proposal is not expected to provide any cash flow or residual value. We also assume an expected abandonment value of $1,500 at the end of the first period. There are nine possible series of cash flows over the two-year period, the first series representing a cash flow of $1,000 in period 1, followed by a cash flow of 0 in period 2. The joint probability of each series of cash flows is shown in the last column of the table; for the first series, it is $0.25 \times 0.25 = 0.0625$.

If we assume a required rate of return of 10 per cent and use this rate as our discount factor, we are able to determine the expected value of net-present value of the proposal by (1) computing the net-present value for each cash flow series; (2) obtaining the expected net-present value for each series by multiplying the computed net-present value by the probability of occurrence of that series; and (3) adding the expected net-present values of all sequences. The standard deviation about the expected value of net-present value can be found using Eq. (5-23) in the previous chapter. When we carry out these computations, we find the expected value of

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[15]This section is adapted from Robichek and Van Horne, "Abandonment Value and Capital Budgeting."

TABLE 6-2

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Period 1		Period 2		
Cash Flow	Initial Probability P(1)	Cash Flow	Conditional Probability P(2 | 1)	Joint Probability P(1, 2)
		$ 0	0.25	0.0625
$1,000	0.25	1,000	0.50	0.1250
		2,000	0.25	0.0625
		1,000	0.25	0.1250
2,000	0.50	2,000	0.50	0.2500
		3,000	0.25	0.1250
		2,000	0.25	0.0625
3,000	0.25	3,000	0.50	0.1250
		3,500	0.25	0.0625
Abandonment value $1,500		$0		

net-present value and the standard deviation to be $444 and $1,313 respectively.

Suppose now that we have a second proposal, *B*, which also costs $3,000 at time 0 and is expected to generate net cash flows over the next two years. However, this proposal is not expected to have any abandonment value at the end of period 1; the $3,000 committed to it is sunk. Nevertheless, project *B* has an expected value of net-present value of $500 and a standard deviation of $1,200. If the two investment proposals were mutually exclusive, we would prefer project *B* to project *A* under conventional standards. It has a higher expected value of net-present value and lower risk.

When we consider the possibility of abandonment, however, the results for project *A* are changed dramatically. Following the decision rules specified earlier, we would divest ourselves of the project if its abandonment value at the end of period 1 exceeds the expected cash flows for the subsequent period, discounted at 10 per cent.[16] Because cash flows are only expected for two periods, the possibility of abandoning the project beyond period 1 does not exist. Consequently, a number of the computational steps involved in the abandonment decision rules discussed are not applicable in this case. For project *A*, we would abandon the project at the end of period 1 if the cash flow in period 1 turned out to be $1,000, because the expected value of present value of cash flows for period 2 discounted to period 1, $909, is less than the abandonment value at the end of the period, $1,500. If we allow for abandonment, the expected

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[16]For purposes of illustration, we assume that the abandonment value is known with certainty. The approach could be modified to include a probability distribution of abandonment values.

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cash flows in Table 6-2 must be revised; these revisions are shown in Table 6-3. For the first branch in Table 6-2, the cash flow for period 1 becomes $2,500, the sum of the $1,000 cash flow during the period plus the abandonment value of $1,500. Because the proposal is abandoned at the end of period 1, there is no cash flow for this branch in period 2.

When we recalculate the expected value of net-present value and the standard deviation for project *A* based upon the information in Table 6-3, we find them to be $578 and $1,110, respectively. We note the significant improvement in net-present value and the lower risk for this project when abandonment value is considered. When abandonment value is considered, project *A* would be preferred to project *B*, for the expected value of net-present value is higher and the risk lower.[17] A proportion of the downside risk for the first proposal can be eliminated if the proposal is abandoned when events turn unfavorable. For example, with abandonment,

TABLE 6-3

Period 1		Period 2		
Cash Flow	Initial Probability P(1)	Cash Flow	Conditional Probability P(2 | 1)	Joint Probability P(1, 2)
$2,500	0.25	$ 0		0.2500
		1,000	0.25	0.1250
2,000	0.50	2,000	0.50	0.2500
		3,000	0.25	0.1250
		2,000	0.25	0.0625
3,000	0.25	3,000	0.50	0.1250
		3,500	0.25	0.0625

[17]The consideration of abandonment value also affects the shape of the probability distribution as denoted by its skewness. A measure of relative skewness is $V/2SV$, where V is the variance of the distribution, and SV is the semivariance. Semivariance is the variance of the probability distribution to the left of the expected value of net-present value and may be thought to represent a measure of downside risk. Mathematically, it can be expressed as

$$SV(X) = \sum_{i=1}^{E(X)} [X_i - E(X)]^2 P(X_i)$$

where X_i is the net-present value observation, $E(X)$ is the expected value of net-present value, and the net-present value observations are ordered from low to high. For symmetrical probability distributions, $V/2SV$ equals one; for distributions skewed to the right, it is greater than one; for distributions skewed to the left, it is less than one. Using the cash flow information in Tables 6-2, and 6-3, we find $V/2SV$ to be 0.962 and 1.145 respectively.

Thus, the consideration of abandonment changes the probability distribution for the proposal from one that is skewed slightly to the left to one that is skewed to the right. To the extent that management prefers distributions skewed to the right, the change is desirable; for it indicates that some of the downside risk has been eliminated. If management has a preference with respect to the shape of probability distribution (presumably it would favor a distribution skewed to the right), allowance for abandonment will affect these preferences.

there is no probability that net-present value will be less than $228; without abandonment, there is an 18.75 per cent probability that net-present value will be less than that amount.

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IMPLICATIONS

While our example has purposely been kept simple, it does illustrate the importance of considering abandonment value when evaluating investment proposals.[18] Not to do so will result in an incomplete appraisal of the economic worth of an investment opportunity. The funds committed to certain proposals are relatively flexible, whereas those committed to others are not. For example, an investment in a multipurpose plant in a large city differs considerably from an investment in a special-purpose metal extraction complex in the wilderness of Canada. The former is reasonably marketable and may have a fairly high abandonment value; the latter represents a sunk cost that may never be recovered.[19] If the two projects had the same expected values of net-present values and standard deviations about these expected values, we would much prefer the former investment. A conventional analysis in capital budgeting, however, would not consider the important difference in abandonment values. If capital is to be allocated optimally, we must take into account possible differences in the future mobility of funds when evaluating investment proposals.

If the abandonment of a project is expected to alter the business-risk complexion of the firm as a whole, consideration must be given to this factor. One way to do so is through the portfolio approach described earlier. If a project is abandoned, it must be eliminated from the portfolio of existing projects. Its abandonment value presumably is invested in another project that is typical of the risk-return pattern available on investment projects in general. Management then must estimate the marginal impact of this project on the riskiness of the portfolio of existing projects, less the one being considered for abandonment. Once this is determined, management can compute the expected value of net-present value and standard deviation for the new portfolio in the normal way. In this regard, the risk-free rate should be used as the discount factor. If the risk-return combination of the new portfolio is more attractive than that for the portfolio of existing projects, including the one being considered for abandonment,

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[18]In Robichek and Van Horne, "Abandonment Value and Capital Budgeting," a simulation method is developed to serve as a "practical" substitute for the conditional probability approach illustrated in Tables 6-2 and 6-3. The latter approach is unfeasible when the possible number of cash-flow series is large.

[19]As mentioned before, an investment project does not have to be sold externally to have abandonment value. Abandonment value can be depicted by the economic use of the asset in fields of endeavor within the firm other than that in which it is currently being used.

the project involved should be abandoned. If not, it should be retained. In this manner, management is able to approximate the marginal impact of abandoning a project on the risk complexion of the firm as a whole.

SUMMARY

In this chapter, we proposed a framework for evaluating the marginal contribution of net-present value and risk to the firm as a whole of an investment proposal or combination of proposals. These marginal additions are what should be evaluated when deciding whether a proposal or combination of proposals should be accepted or rejected. Of crucial importance in the evaluation of risky investments is that we take account of existing investments as well as investment proposals under consideration. A portfolio approach was presented for analyzing the tradeoff between the risk and the net-present value of the firm as a whole under varying combinations of investments. The business-risk complexion of the firm is determined by the investment decisions of management. In turn, this complexion affects the market price of the firm's stock. Consequently, if management is to maximize share price over the long run, it must consider the risk of an investment project to the firm as a whole and not its risk as an isolated investment.

This approach was extended to consider acquisitions. In general, an acquisition can be evaluated in much the same manner as an investment proposal generated internally. By incorporating prospective acquisitions into a portfolio analysis framework, management is able to choose the best combination of net-present value and risk for all investment proposals, whether they be internal or external. Using this framework, a method was proposed for determining the maximum price that should be paid for an acquisition. We noted that diversification through acquisitions may not have a positive impact on share price if, in fact, investors effectively diversify their own portfolios. In this case, the substitution of corporate diversification for investor diversification would not enhance shareholder wealth. However, it would appear that stockholders would benefit from diversification of investment projects generated internally.

In order to employ capital optimally within the firm, management must be willing to give up an existing investment project if it no longer justifies the capital committed to it. A project should be abandoned whenever the incremental return on its abandonment value is less than the minimum acceptable standard and it does not appear that abandonment in the future would be more favorable than current abandonment. We considered a framework for evaluating the possibility of future abandonment of an investment proposal in a capital-budgeting context. It was seen that different decisions may be reached, depending on whether or not future abandonment is considered.

Salazar and Sen propose the use of stochastic linear programming to determine the expected return and risk of a combination of risky investments.[20] They begin with Weingartner's integer programming model for capital budgeting, where the objective function is to maximize the worth of a group of projects at some terminal date, subject to budget and certain other constraints. As Weingartner's model was taken up in Appendix 3C to Chapter 3, we shall not describe it here. Instead of assuming that future project cash flows are certain, as was the case with Weingartner's model, Salazar and Sen (SS) explore two types of uncertainty. The first involves possible changes in economic and competitive factors likely to affect cash flows. In particular, SS consider GNP, competitor's price versus the firm's price, and the introduction of a new product by a competitor. The probabilities of occurrence of various outcomes for these factors are expressed as a probability tree. The joint probability of all three occurring is then determined. Thus, it is assumed that future cash flows for investment proposals under consideration can be related directly to GNP and to competitive factors.

The second type of uncertainty involves the uncertainty of cash flows, once GNP and competitive factors are known. Given a particular branch of the probability tree of economic and competitive factors, possible cash flows are expressed as normal probability distributions. The branch of the probability tree determines the mean of the probability distribution. Thus, one can think of cash flows as being related primarily to economic and competitive factors, but having a random component.

With this probabilistic information, SS propose simulating various outcomes, using the mathematical programming model referred to earlier. The branch tree and random component are simulated for each investment proposal under consideration. This cash-flow information is then used in the programming model to select the portfolio of projects that maximizes terminal worth. Similarly, other simulation runs are undertaken, and the resulting optimal terminal worths recorded.

By categorizing simulation runs as to the specific portfolio of projects involved, one obtains the probability distribution of terminal worths for each of the portfolios. The expected values and standard deviations of the probability distributions then can be evaluated in the same manner as discussed earlier in the chapter. For complex situations involving a number of possible portfolios, SS propose a heuristic method for ranking projects according to the number of times they appear in an optimal solution. On the basis of these rankings, portfolios are generated, and risk and return are approximated for the portfolio.

The SS approach is another means for generating the information

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[20] Rudolfo C. Salazar and Subrata K. Sen, "A Simulation Model of Capital Budgeting under Uncertainty," *Management Science*, 15 (December, 1968), 161–79.

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needed for evaluating portfolios of risky investments. With this information, we can form a risk-return graph like that shown in Figure 6-2 and go on to select the best project portfolio. The SS approach is based upon the optimizing technique of mathematical programming; the problems involved with this approach to capital budgeting were discussed in Appendix 3C. Critical in the SS approach, and for that matter in any approach for evaluating risk, is the expression of the functional relationship between project outcomes and economic and competitive factors. In this regard, the SS approach is similar to that used in an index model. The end product is information about the expected return and risk for various portfolios of projects. With it, management can then make a decision. The SS approach is unique in generating this information through stochastic linear programming and simulation.

PROBLEMS

1. The Windrop Company is considering investment in two of three possible proposals, the cash flows of which are normally distributed. The expected net-present value (discounted at the risk-free rate of 4 per cent) and the standard deviation for each proposal are given as follows:

	1	2	3
Expected net-present value	$10,000	$8,000	$6,000
Standard deviation	4,000	3,000	4,000

Assuming the following correlation coefficients for each possible combination, which two proposals should be selected?

Proposals	Correlation Coefficients
1	1.00
2	1.00
3	1.00
1 and 2	.60
1 and 3	.40
2 and 3	.50

2. The Plaza Corporation is confronted with several combinations of risky investments.

Old Portfolios:	Net-present Value	σ
A	$100,000	$200,000
B	20,000	80,000
C	75,000	100,000
D	60,000	150,000
E	50,000	20,000
F	40,000	60,000

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New Portfolios:	Net-present Value	σ
G	120,000	170,000
H	90,000	70,000
I	50,000	100,000
J	75,000	30,000

(a) Plot the above portfolios.
(b) Which portfolio would you choose?

3. The Warbler Corporation is contemplating acquiring a privately held ·
company. Cash-flow estimates of the company being considered are given
below:

	Average for Years (in millions)			
	1–5	6–10	10–20	21–∞
Cash flow before taxes	$10	$15	$20	$25
Taxes	3	5	8	10
Net cash flow	7	10	12	15
Required investment	5	7	10	10

(a) The corporation to be acquired has no debt, and the after-tax risk-free
rate is 5 per cent. What is the range of possible prices for this company?
(b) The acquisition, if undertaken, would appear in the portfolios whose
current characteristics are given below:

Portfolio	N.P.V. (abscissa)	σ (ordinate)
A	$120,000	$100,000
B	100,000	75,000
C	80,000	50,000
D	60,000	25,000

One of management's indifference curves (which passes through the best portfolio
not containing the acquisition) may be approximated by the following coordinates:
$100,000, 0; $125,000, $60,000; and $150,000, $95,000. Approximately what
maximum price should be paid for the acquisition?

4. The Cenno Company has a required rate of return of 10 per cent after taxes
and pays a tax rate of 50 per cent. It currently owns a G & H Drill Press, which
it bought last year for $10,000. This drill press has a ten-year, straight-line
depreciation schedule, with no salvage value assumed. Because of a great de-
mand for drill presses, it would be possible to sell the press for a net price to the
company (after costs of removal, etc.) of the net book value of the press. The G &
H Drill Press is expected to make a contribution to profit before depreciation and
taxes of $2,000 per year for the remainder of its useful life.

At this point, the Ft. John Machinery Company offers to accept the G & H
Drill Press plus $7,000 in exchange for its new Super Drill Press. The Super
Press has an expected useful life of fifteen years, at which time it would have a
salvage value of $1,000; this machine would also be depreciated on a straight-line
basis. The Super Press would be expected to make a $3,000 yearly contribution
to profit before depreciation and taxes.

As Financial V.P. of the Cenno Company, you have adequate cash available

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for any of these alternatives and also other investment opportunities. What should you do?

5. The DeWitt Corporation has determined an abandonment value at P_1 for the project described in problem 4 of Chapter 5. If this value is $1,500 (known with certainty), and if the required after tax rate of return for the company is 10 per cent, should the project be abandoned if the cash flow for P_1 is $1,000? (Assume that at P_3, cash flow would be $0 if the flow for P_1 is $1,000.)

6. The Sniffle Corporation has determined the following distribution of net cash flows for a contemplated project:

Year 1		Year 2		Year 3	
$ 5,000	(.5)	$ 3,000	(.6)	$ 1,000	(.7)
				$ 2,000	(.3)
		$ 6,000	(.4)	$ 8,000	(.5)
				10,000	(.5)
$10,000	(.5)	$10,000	(.5)	$ 6,000	(.8)
				8,000	(.2)
		$15,000	(.5)	$20,000	(.5)
				25,000	(.5)

The firm has a required after-tax rate of return of 10 per cent. The abandonment value of the project is given below:

After Year 1		After Year 2	
Prob.	Amount	Prob.	Amount
.5	$8,000	.5	$2,000
.3	5,000	.3	1,500
.2	4,000	.2	1,000

(a) At the end of year 1, the project generated cash flows of $5,000. Should it be abandoned?
(b) At the end of year 2, the project generated a cash flow of $3,000. Should it be abandoned?

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FINANCING AND DIVIDEND POLICIES

PART III
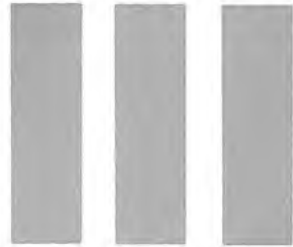
Theory
of Capital
Structure

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In Part II, we were concerned with how capital might be allocated to investment proposals, given a certain financing mix. In this chapter and the next, we are concerned with whether the way in which investment proposals are financed matters; and, if it does matter, what is the optimal capital structure? If we finance with one mix of securities rather than another, is the market price of the stock affected? If the firm can affect the market price of its stock by its financing decision, it will want to undertake a financing policy that will maximize market price. For simplicity, we examine the question of capital structure in terms of the proportion of debt to equity. However, the principles taken up in this chapter can be expanded easily to include consideration of the specific type of security being issued.

First we will explore the theory of capital structure from the standpoint of valuation. Next, we shall examine some of the implications of capital structure for the cost of capital and the financing decision. Our investiga-

tion involves a partial equilibrium analysis where we hold constant the investment and dividend decisions of the firm and try to determine the effect of a change in financing mix on share price. In the subsequent chapter, we consider how a firm in practice can determine a capital structure suitable for its particular situation. Very much a part of this consideration is an analysis of a firm's cash-flow ability to service fixed charges. Finally, we shall explore briefly the questions of timing and flexibility of a single security issue. Throughout this chapter and the next, the concept of financial risk transcends our discussion. Consequently, we define financial risk in the next section before considering the issues outlined above.

FINANCIAL RISK

Whereas the investment decision determines the basic business risk of a firm, the financing decision determines its financial risk. Broadly defined, financial risk encompasses both the risk of possible insolvency and the variability in the earnings available to common stockholders. As a firm increases the proportion of debt, lease commitments, and preferred stock in its capital structure, fixed charges increase. All other things being the same, the probability that the firm will be unable to meet these fixed charges increases also. As the firm continues to lever itself, the probability of cash insolvency, which may lead to legal bankruptcy, increases. To illustrate this notion of financial risk, suppose that two firms have different degrees of leverage but are identical in every other respect. Each has expected annual cash earnings of $80,000 before interest and taxes. However, Firm A has no debt, while Firm B has $500,000 worth of 6% perpetual bonds outstanding. Thus, the total annual financial charges for Firm B are $30,000, whereas Firm A has no financial charges. If cash earnings for both firms should be 75 per cent lower than expected — namely, $20,000 — Firm B will be unable to cover its financial charges with cash earnings. We see, then, that the probability of cash insolvency increases with the financial charges incurred by the firm.

The second aspect of financial risk involves the relative dispersion of income available to common stockholders. To illustrate, suppose that the expected future annual operating incomes over the next five years for Firms A and B were subjective random variables where the expected values of the probability distributions were each $80,000 and the standard deviations, $40,000. As before, assume that Firm A has no debt, while Firm B has $500,000 in 6% bonds. If, for simplicity, we abstract from federal income taxes, the expected value of earnings available to common stockholders would be $80,000 for Firm A and $50,000 for Firm B. Because the standard deviation about the expected values is the same

for both firms, the relative dispersion of expected earnings available to common stockholders is greater for Firm B than for Firm A. For Firm A

$$\text{Coefficient of variation} = \frac{\$40,000}{\$80,000} = 0.50$$

while for Firm B

$$\text{Coefficient of variation} = \frac{\$40,000}{\$50,000} = 0.80$$

Graphically, the relationship is shown in Figure 7-1. We see that the degree of dispersion from the expected value of earnings available to common stockholders is the same for both firms but that the expected value of these earnings is greater for Firm A than for Firm B. As a result, the relative dispersion, as measured by the coefficient of variation, is less for Firm A.
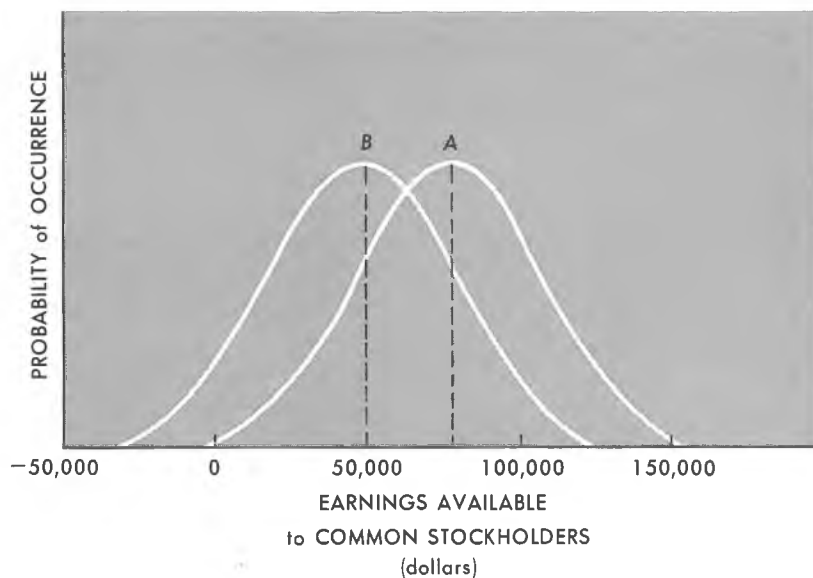

FIGURE 7-1
Probability distributions of earnings
available to common stockholders

The dispersion in earnings available to common stockholders is to be distinguished from the dispersion of operating income, known as busi-

ness risk. In our example above, both firms had the same degree of business risk, as defined, because the coefficient of variation of expected future operating income was the same

$$\text{Coefficient of variation} = \frac{\$40,000}{\$80,000} = 0.50$$

Only in the degree of financial risk did the two firms differ.[1] In summary, we regard financial risk as encompassing the volatility of earnings available to common stockholders as well as the probability of insolvency.[2] Both aspects are related directly to the dispersion of expected operating income, or the business risk, of the firm.

As a firm increases the proportion of fixed-income obligations in its capital structure, the financial risk to stockholders rises. We know from Chapter 2 that investors are concerned with the risk associated with actually receiving an expected stream of income. Recall that the required rate of return, or equity-capitalization rate, of investors at the margin can be viewed as the risk-free rate, i, plus a premium to compensate them for risk, θ,

$$k_e = i + \theta \tag{7-1}$$

In turn, θ can be represented as some function of the coefficient of variation described earlier in this section. More formally, this coefficient is expressed as

$$CV = \frac{\sigma_0}{\bar{O} - rD} \tag{7-2}$$

where σ_0 = the standard deviation of the probability distribution of possible operating income. For the sake of simplicity, this distribution is assumed to be the same for all future periods.

$\bar{O}$ = expected value of the distribution
r = average interest rate on debt outstanding
D = debt outstanding in dollars

Thus, the required rate of return for investors at the margin is

$$k_e = i + f\left(\frac{\sigma_0}{\bar{O} - rD}\right) \tag{7-3}$$

We see that the amount of debt and the average interest rate on that debt affect the required rate of return in a positive manner. The greater rD, the greater the coefficient of variation, and the greater k_e becomes if we

[1] See Alexander Barges, *The Effect of Capital Structure on the Cost of Capital* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963), Chapter 2.

[2] Again, it is important to point out as we did in Chapters 2, 5, and 6 that the dispersion of a probability distribution is but one measure of risk.

assume risk-averse investors. In turn, the greater the k_e, the lower the share price, all other things being the same.[3] Having defined financial risk and explored its relationship with valuation, we now are able to analyze the theory of capital structure.

INTRODUCTION TO THEORY

Even a casual review of the literature brings one quickly to the key question of whether or not capital structure matters. Can the firm affect its total valuation and its cost of capital by changing its financing mix? In this section, our attention is directed to the question of what happens to the total valuation of the firm and to its cost of capital when the ratio of debt to equity, or degree of leverage, is varied.

ASSUMPTIONS AND DEFINITIONS

So that the analysis that follows can be presented as simply as possible, we make the following facilitating assumptions:

1. We assume that there are no income taxes. This assumption is removed later.

2. The ratio of debt to equity for a firm is changed by issuing debt to repurchase stock or issuing stock to pay off debt. In other words, a change in capital structure is effected immediately. In this regard, we assume no transaction costs.

3. The firm has a policy of paying 100 per cent of its earnings in dividends. Thus, we abstract from the dividend decision.

4. The expected values of the subjective probability distributions of expected future operating earnings for each company are the same for all investors in the market.

5. The operating earnings of the firm are not expected to grow. The expected values of the probability distributions of expected operating earnings for all future periods are the same as present operating earnings.

6. The acceptance of an investment proposal or combination of investment proposals does not change the total business-risk complexion of the firm — in other words, business risk is held constant.

Using Solomon's symbols and some of his examples, we are concerned with the following three rates[4]

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[3] For further analysis of this point, see Douglas Vickers, *The Theory of the Firm: Production, Capital, and Finance* (New York: McGraw-Hill Book Company, 1968), pp. 58–67.

[4] Ezra Solomon, *The Theory of Financial Management* (New York: Columbia University Press, 1963), Chapters 7–9.

$$k_i = \frac{F}{B} = \frac{\text{Annual interest charges}}{\text{Market value of debt outstanding}} \qquad (7\text{-}3)$$

In this equation, k_i is the yield on the company's debt, assuming this debt to be perpetual.

$$k_e = \frac{E}{S} = \frac{\text{Earnings available to common stockholders}}{\text{Market value of stock outstanding}} \qquad (7\text{-}4)$$

From the appendix to Chapter 4, we know that the required rate of return for investors in a firm whose earnings are not expected to grow and that has a 100 per cent dividend-payout ratio, is the earnings/price ratio. Given our restrictive assumptions, then, the earnings/price ratio represents the market rate of discount that equates the present value of the stream of expected future dividends with the current market price of the stock. This is not to say that it should be used as a general rule to depict the cost of equity capital. (See Chapter 4.)

$$k_o = \frac{O}{V} = \frac{\text{Net operating earnings}}{\text{Total market value of the firm}} \qquad (7\text{-}5)$$

where $V = B + S$. Here, k_o is an overall capitalization rate for the firm. It is defined as the weighted-average cost of capital, and may also be expressed as

$$k_o = k_i\left(\frac{B}{B+S}\right) + k_e\left(\frac{S}{B+S}\right) \qquad (7\text{-}6)$$

Our concern will be with what happens to k_i, k_e, and k_o when the degree of leverage, as denoted by the ratio B/S, increases.

NET INCOME APPROACH

Durand has proposed two approaches to the valuation of the earnings of a company: the net income approach (NI) and the net operating income approach (NOI).[5] These approaches represent the extremes in valuing the firm with respect to the degree of leverage. As they give us a basis for additional discussion, we consider them in turn. To illustrate the net income approach, assume that a firm has $3,000 in debt at 5 per cent interest, that the expected value of annual net operating earnings is $1,000, and that the equity-capitalization rate, k_e, is 10 per cent. Given this information, the value of the firm may be calculated as

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[5] David Durand, "The Cost of Debt and Equity Funds for Business," in *The Management of Corporate Capital*, Ezra Solomon, ed. (New York: The Free Press, 1959), pp. 91–116.

O	Net operating earnings	$ 1,000
F	Interest	150
E	Earnings available to common stockholders	850
k_e	Equity-capitalization rate	0.10
S	Market value of stock	8,500
B	Market value of debt	3,000
V	Total value of firm	$11,500

With the net income approach, earnings available to common stockholders are capitalized at a constant rate, k_e. The implied overall capitalization rate in the above example is

$$k_o = \frac{O}{V} = \frac{\$1,000}{\$11,500} = 8.7 \text{ per cent}$$

Assume now that the firm increases its debt from $3,000 to $6,000 and uses the proceeds of the debt issue to repurchase stock. Also, suppose that the interest rate on debt remains unchanged at 5 per cent. The value of the firm then is

O	Net operating earnings	$ 1,000
F	Interest	300
E	Earnings available to common stockholders	700
k_e	Equity-capitalization rate	0.10
S	Market value of stock	7,000
B	Market value of debt	6,000
V	Total value of firm	$13,000

The implied overall capitalization rate now is

$$k_o = \frac{O}{V} = \frac{\$1,000}{\$13,000} = 7.7 \text{ per cent}$$

According to the net income approach, the firm is able to increase its total valuation, V, and lower its cost of capital, k_o, as it increases the degree of leverage. As a result, the market price per share increases. To illustrate, assume in our example that the firm with $3,000 in debt has 850 shares of common stock outstanding. Thus, the market price per share is $10 a share ($8,500/850). The firm issues $3,000 in additional debt and, at the same time, repurchases $3,000 of stock at $10 a share, or 300 shares in total. It then has 550 shares outstanding. We saw in the

example that the total market value of the firm's stock after the change in capital structure is $7,000. Therefore, the market price per share is $7,000/550 = $12.73, where before it was $10.

Graphically, the approach is illustrated in Figure 7-2. The degree of leverage, B/S, is plotted along the horizontal axis, while the percentage rate for k_i, k_e, and k_o is on the vertical axis. This graph can be constructed based upon the hypothetical examples we have shown. As can be seen, the critical assumptions of the net income approach are that k_i and, more particularly, k_e remain unchanged as the degree of leverage increases. As
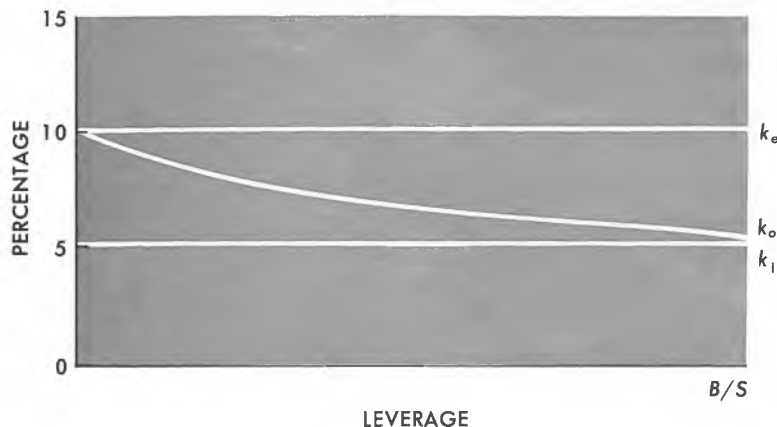
FIGURE 7-2
Capital costs: net income approach

the proportion of cheaper debt funds in the capital structure is increased, the weighted-average cost of capital, k_o, decreases and approaches the cost of debt, k_i. The optimal capital structure would be the one at which the total value of the firm is greatest and the cost of capital the lowest. At that structure, the market price per share of stock is maximized. Using the net income approach, the optimal capital structure is the one furthest to the right in Figure 7-2. The significance of this approach is that a firm can lower its cost of capital continually and increase its total valuation by the use of debt funds. Again, the critical assumption is that the firm does not become increasingly more risky in the minds of investors and creditors as the degree of leverage is increased.

NET OPERATING INCOME APPROACH

We turn now to the net operating income approach. The assumption here is that the overall capitalization rate of the firm, k_o, is constant for all degrees of leverage. Assume the same example as before but with k_o equal to 10 per cent. For $3,000 in debt, we have

O	Net operating income	$ 1,000
k_o	Overall capitalization rate	0.10
V	Total value of firm	$10,000
B	Market value of debt	3,000
S	Market value of stock	$ 7,000

The implied equity-capitalization rate in this case is[6]

$$k_e = \frac{E}{S} = \frac{850}{7,000} = 12.1 \text{ per cent}$$

With this approach, net operating income is capitalized at an overall capitalization rate to obtain the total market value of the firm. The market value of the debt then is deducted from the total market value to obtain the market value of the stock.

Suppose, as before, that the firm increases the amount of debt from $3,000 to $6,000 and uses the proceeds of the debt issue to repurchase stock. The valuation of the firm then is

O	Net operating income	$ 1,000
k_o	Overall capitalization rate	0.10
V	Total value of firm	10,000
B	Market value of debt	6,000
S	Market value of stock	$ 4,000

The implied equity-capitalization rate is

$$k_e = \frac{E}{S} = \frac{700}{4,000} = 17.5 \text{ per cent}$$

We see that the equity-capitalization rate, k_e, rises with the degree of leverage. This approach implies that the total valuation of the firm is unaffected by its capital structure. Graphically, the approach is shown in Figure 7-3.

The critical assumption with this approach is that k_o is constant regardless of the degree of leverage. The market capitalizes the value of the firm as a whole; as a result, the breakdown between debt and equity is unimportant. An increase in the use of supposedly "cheaper" debt funds is offset exactly by the increase in the equity-capitalization rate, k_e. Thus, the weighted average of k_e and k_i remains unchanged for all degrees of leverage. As the firm increases its degree of leverage, it be-

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[6] k_e also can be calculated as $k_e = k_o + (k_o - k_i)\dfrac{B}{S}$

comes increasingly more risky; and investors penalize the stock by raising the equity-capitalization rate (lowering the P/E ratio) directly in keeping with the increase in the debt-to-equity ratio. As long as k_i remains constant, k_e is a constant linear function of the debt-to-equity ratio.

According to the net operating income approach, the real cost of debt and the real cost of equity are the same — namely, k_o. The cost of debt has two parts: the explicit cost represented by the rate of interest, and
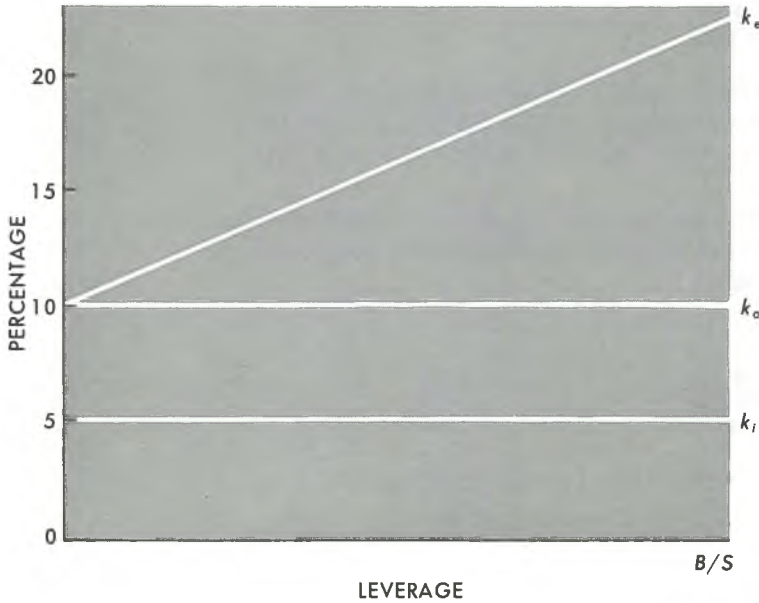
FIGURE 7-3
Capital costs: net operating income approach

the implicit cost, or "hidden" cost, which is represented by the increase in the equity-capitalization rate that accompanies an increase in the proportion of debt to equity. As the cost of capital of the firm cannot be altered through leverage, this approach implies that there is no one optimal capital structure. All capital structures are optimal, for market price per share does not change with leverage. To illustrate, assume again that our example firm with $3,000 in debt has 850 shares of common stock outstanding. The market price per share in this case is $7,000/8.50 = $8.23. With the $3,000 in additional debt, the firm repurchases $3,000 of stock at $8.23 a share, or 364 shares in total. Therefore, the market price per share after the change in capital structure is $4,000/(850–364) = $8.23, the same as before. Thus, capital structure would be a matter of indifference to the investor.

So far, our discussion of the net operating income approach has been purely definitional; it lacks behavioral significance. However, Modigliani

and Miller, in their famous 1958 article, offered behavioral support for the independence of the total valuation and the cost of capital of the firm from its capital structure.[7] Before taking up the implications of their position, however, we examine the traditional approach to valuation.

TRADITIONAL APPROACH

The traditional approach to valuation and leverage assumes that there is an optimal capital structure and that the firm can increase the total value of the firm through the judicious use of leverage. Actually, this approach encompasses all the ground between the net income approach and the net operating income approach. To illustrate one variation of the approach, assume that our hypothetical firm has $3,000 in debt at 5 per cent interest. Assume, however, that the equity-capitalization rate is 11 per cent, rather than the 10 per cent or 12.1 per cent assumed with the net income or net operating income approaches illustrated previously. The valuation of the firm then is

O	Net operating income	$ 1,000
F	Interest on debt	150
E	Earnings available to common stockholders	850
k_e	Equity-capitalization rate	0.11
S	Market value of stock	7,727
B	Market value of debt	3,000
V	Total value of firm	$10,727

The implied overall capitalization rate is

$$k_o = \frac{O}{V} = \frac{1,000}{10,727} = 9.3 \text{ per cent}$$

This example suggests that the firm can lower its cost of capital and increase the total value of the firm and share price by leverage. With no leverage, $B/S = 0$; and the overall capitalization rate, k_o, is 10 per cent. Although investors raise the equity-capitalization rate, k_e, as the firm becomes more financially risky with leverage, the increase in k_e does not offset entirely the benefit of using cheaper debt funds. As a result, total valuation and share price increase, and the cost of capital decreases. With $3,000 in debt and 850 shares outstanding, the market price per share is

[7] Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, XLVIII (June, 1958), reprinted in *Foundations for Financial Management*, James C. Van Horne, ed. (Homewood, Ill.: Richard D. Irwin, Inc., 1966), pp. 367–405.

$7,727/850 = \$9.09$. This contrasts with \$8.23 under the assumption of a net operating income approach to valuation.

The traditional approach implies that beyond some point, k_e rises at an increasing rate with leverage. Moreover, k_i also may rise beyond some point. To illustrate, suppose now that the firm increases its debt from \$3,000 to \$6,000 and uses the proceeds of the debt issue to repurchase stock. Assume also that the average rate of interest on all debt rises to 6 per cent and that the equity-capitalization rate, k_e, at that degree of leverage is 14 per cent. The valuation of the firm then is

O	Net operating income	\$ 1,000
F	Interest on debt	360
E	Earnings available to common stockholders	\$ 640
k_e	Equity-capitalization rate	0.14
S	Market value of stock	\$ 4,571
B	Market value of debt	6,000
V	Total value of firm	\$10,571

The implied overall capitalization rate is

$$k_o = \frac{O}{V} = \frac{1,000}{10,571} = 9.5 \text{ per cent}$$

Thus, the total valuation of the firm is lower and its cost of capital slightly higher than when the amount of debt was \$3,000. This result is due to the increase in k_e and, to a lesser extent, the increase in k_i. From these two observations, we know that the optimal capital structure in this example occurs before a debt-to-equity ratio of 6,000/4,571, or 1.31.

Graphically, one variation of the traditional approach is shown in Figure 7-4. As can be seen in the figure, k_e is assumed to rise at an increasing rate with leverage, whereas k_i is assumed to rise only after significant leverage has occurred. At first, the weighted-average cost of capital declines with leverage because the rise in k_e does not offset entirely the use of cheaper debt funds. As a result, the weighted-average cost of capital, k_o, declines with moderate use of leverage. After a point, however, the increase in k_e more than offsets the use of cheaper debt funds in the capital structure, and k_o begins to rise. The rise in k_o is supported further once k_i begins to rise. The optimal capital structure is the point at which k_o bottoms out; in the figure, this optimal capital structure is point X.

Thus, the traditional position implies that the cost of capital is not independent of the capital structure of the firm and that there is an optimal capital structure. At that optimal structure, the marginal real cost of debt
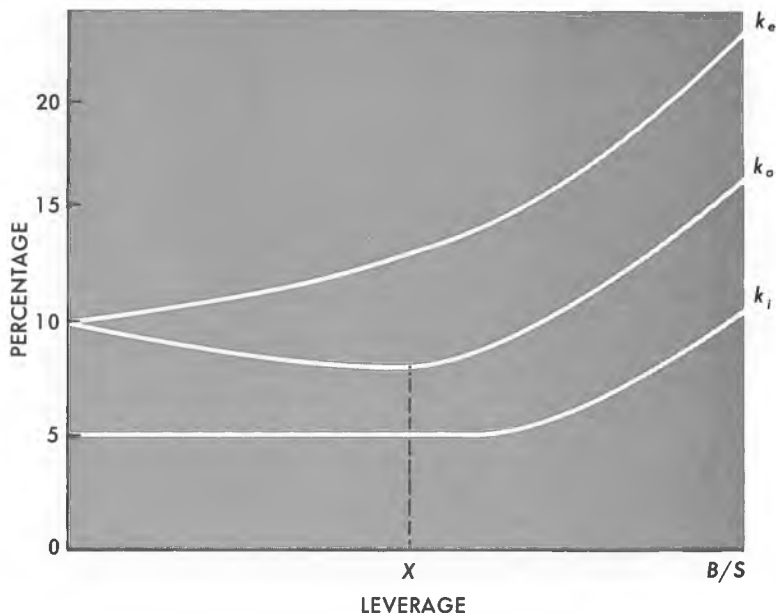
FIGURE 7-4
Illustration of the traditional approach

s the same as the marginal real cost of equity. For degrees of leverage before that point, the marginal real cost of debt is less than that of equity; beyond that point, the marginal real cost of debt exceeds that of equity.

Variations of Approach. There are wide variations in the traditional approach. As we mentioned earlier, the approach falls somewhere between the extremes, the net income and the net operating income approaches. Some members of the traditional school would contend that k_e does not actually rise until after some critical point. Only after this point is reached do investors recognize the increasing financial risk of the company and penalize the market price of the stock accordingly. This variation of the traditional position implies that a company is able to lower its cost of capital significantly with the initial use of leverage. This variation is shown in Figure 7-5.

Others view the cost-of-capital curve, k_o, as being saucer-shaped with a horizontal middle range.[8] This variation is shown in Figure 7-6. It suggests that there is a range of optimal capital structures in which the cost of capital is minimized and the total value of the firm maximized. In this range, changes in leverage have a negligible effect upon the total value of the firm. Thus, the traditional position allows for considerable variation in the optimal capital structure for different firms; no one capital structure is optimal for all companies. Having taken up the net income,

[8] Solomon, *The Theory of Financial Management,* pp. 93–98.
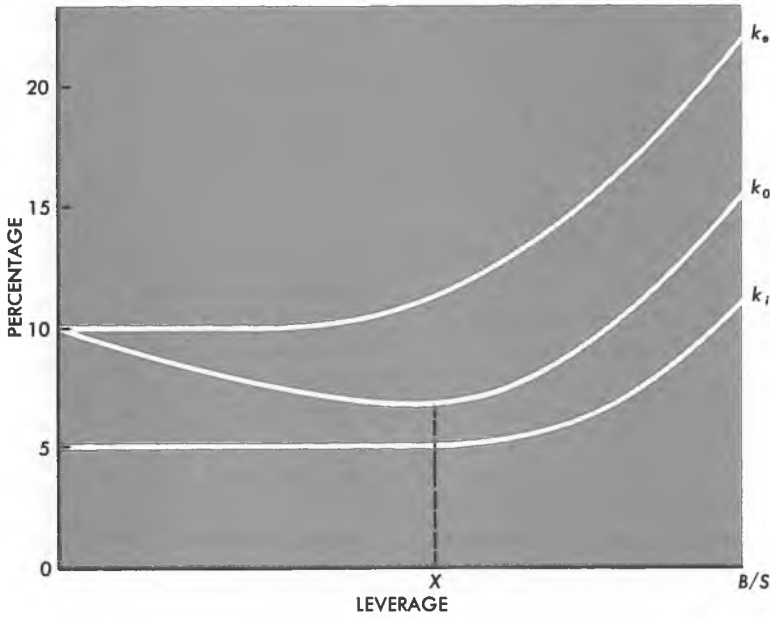
FIGURE 7-5
Illustration of the traditional approach: k_e constant at first
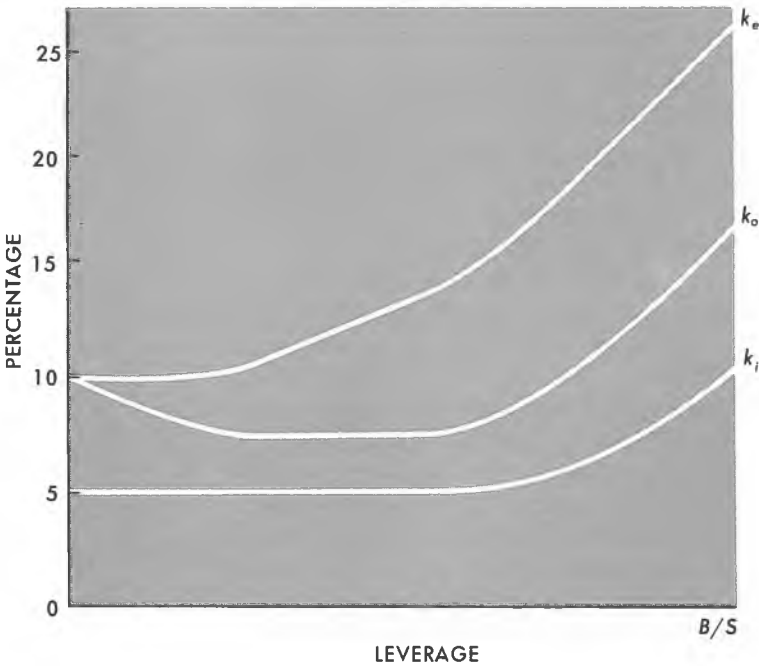
FIGURE 7-6
Traditional approach: saucer-shaped k_o curve

net operating income, and traditional approaches to valuation, we now are able to consider in more detail the question of whether capital structure matters.

MODIGLIANI-MILLER POSITION

As mentioned previously, Modigliani and Miller (MM) advocate that the relationship between leverage and the cost of capital is explained by the net operating income approach. They make a formidable attack on the traditional position by offering behavioral justification for having the cost of capital, k_0, remain constant throughout all degrees of leverage. As their assumptions are extremely important, it is necessary to spell them out.

1. Capital markets are perfect. Information is perfect, there are no transaction costs, investors are rational, and they behave accordingly.

2. The average expected future operating earnings of a firm are represented by a subjective random variable. It is assumed that the expected values of the probability distributions of all investors are the same. Implied in the MM analysis is that the expected values of the probability distributions of expected operating earnings for all future periods are the same as present operating earnings.[9]

3. Firms can be categorized into "equivalent return" classes. All firms within a class have the same degree of business risk. As we shall see later, this assumption is not essential for their proof.

4. The absence of corporate income taxes is assumed. MM remove this assumption later.

In taking the net operating income approach, MM draw essentially the same conclusions as described when we discussed that approach. Their three basic propositions are[10]

1. The total market value of the firm and its cost of capital are independent of its capital structure. The total market value of a firm is given by capitalizing the expected stream of operating earnings at a discount rate appropriate for its risk class.

2. The expected yield of a share of stock, k_e, is equal to the capitalization rate of a pure equity stream, plus a premium for financial risk equal to the difference between the pure equity capitalization rate and k_i, times

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[9] Alexander A. Robichek and Stewart C. Myers, *Optimal Financing Decisions* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965), p. 23.

[10] Modigliani and Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," in *Foundations for Financial Management,* Van Horne, ed.

the ratio B/S. In other words, k_e increases in a manner to offset exactly the use of cheaper debt funds.

3. The cutoff rate for investment purposes is completely independent of the way in which an investment is financed.

ARBITRAGE SUPPORT OF PROPOSITIONS

MM argue that the total risk for all security holders of a firm is not altered by changes in its capital structure. Therefore, the total value of the firm must be the same regardless of its financing mix. The crucial support for this hypothesis is the presence of arbitrage in the capital markets. Arbitrage precludes perfect substitutes from selling at different prices in the same market. In their case, the perfect substitutes are two or more firms in the same homogeneous risk class that differ only with respect to capital structure. MM contend that the total value of these firms has to be the same; otherwise, arbitragers will enter and drive the values of the two firms together. The essence of their argument is that arbitragers are able to substitute personal leverage for corporate leverage.[11]

Consider two firms that comprise a single risk class. These firms are identical in every respect except that Company A is not levered and Company B has $30,000 of 5% bonds outstanding. According to the traditional position, Company B may have a higher total value and lower average cost of capital than Company A. The valuation of the two firms is assumed to be the following

		Company A	Company B
O	Net operating income	$ 10,000	$ 10,000
F	Interest on debt		1,500
E	Earnings available to common stockholders	10,000	8,500
k_e	Equity-capitalization rate	0.10	0.11
S	Market value of stock	100,000	77,272
B	Market value of debt		30,000
V	Total value of firm	$100,000	$107,272
k_o	Implied overall capitalization rate	10%	9.3%
B/S	Debt-to-equity ratio	0	38.8%

MM maintain that this situation cannot continue, for arbitrage will drive the total values of the two firms together. Company B cannot command a

[11] *Ibid.,* pp. 375–76.

higher total value simply because it has a different financing mix than Company *A*. MM argue that investors in Company *B* are able to obtain the same dollar return with no increase in financial risk by investing in Company *A*. Moreover, they are able to do so with a smaller investment outlay.[12] Because investors would be better off with the investment requiring the lesser outlay, they would sell their shares in Company *B* and buy shares in Company *A*. These arbitrage transactions would continue until Company *B*'s shares declined in price and Company *A*'s shares increased in price enough so that the total value of the two firms was identical.

To illustrate, suppose that a rational investor owned 1 per cent of Company *B*, the levered firm, worth $772.72 (market value). Given this situation, he should

1. Sell his stock in Company B for $772.72.

2. Borrow $300 at 5 per cent interest. This personal debt is equal to 1 per cent of the debt of Company *B* – his previous proportional ownership of the company.

3. Buy 1 per cent of the shares of Company *A*, the unlevered firm, for $1,000.

Prior to this series of transactions, the investor's expected return on investment in Company *B* was 11 per cent on a $772.72 investment, or $85. His expected return on investment in Company *A* is 10 per cent, or $100 on an investment of $1,000. From this return, he must deduct the interest charges on his personal borrowings. Thus, his net dollar return is

Return on investment in Company A	$100
Less interest (300 × 0.05)	15
Net return	$85

We see then that his net dollar return, $85, is the same as it was for his investment in Company *B*. Moreover, his cash outlay of $700 ($1,000 less personal borrowings of $300) is less than the $772.72 investment in Company *B*, the levered firm. Because of the lower investment, the investor would prefer to invest in Company *A* under the conditions described.

The action of a number of rational investors undertaking similar arbitrage transactions will tend to drive up the price of Company *A* shares, and lower its k_e, and drive down the price of Company *B*, increasing its

[12] This arbitrage proof appears in Franco Modigliani and Merton H. Miller, "Reply to Heins and Sprenkle," *American Economic Review*, LIX (September, 1969), 592–95.

k_e. This arbitrage process will continue until there is no further opportunity for reducing one's investment outlay and achieving the same dollar return. At this equilibrium, the total value of the two firms must be the same. As a result, their average costs of capital, k_o, also must be the same.

If for some reason the value of the unlevered firm, Company A, were to exceed that of the levered firm, Company B, arbitrage would work in the opposite direction. Suppose that Company A in our example had an equity capitalization rate of 8 per cent, giving it a total market value of $125,000. Here, the investor owning 1 per cent of Company A's shares would sell his holdings for $1,250. He then would buy 1 per cent of the stock of Company B for $772.72 as well as buy 1 per cent of the debt of that company for $300. His total expected dollar return in Company B would be

Return on investment in Company B stock	$85
Return on investment in Company B debt	15
Net return	$100

This return is the same as his expected return in Company A's stock. Thus, his total return is the same as before; and he needs to invest only $1,072.72, compared with $1,250.00 previously. As a result, he would prefer investing in Company B. Again, the action of a number of investors behaving in this manner will tend to drive the total values of the two firms together.

The arbitrage argument can be extended to situations in which there are more than two firms in a risk class. Here, the equilibrium would be of a multicompany nature; but again, the average cost of capital for each company would be the same in equilibrium. The important thing is the presence of rational investors in the market who are willing to substitute personal, or "homemade," leverage for corporate leverage. The analysis can be extended further to cross risk classes and include general equilibrium in the capital markets. Here, arbitrage occurs on the basis of expected risk-return combinations for available securities.[13] Thus, the arbitrage proof of MM is not dependent upon the assumption of homogeneous risk classes, as depicted in our example. On the basis of arbitrage, MM conclude that a firm cannot change its total value or its weighted-average cost of capital by leverage. Consequently, the financing decision does not matter from the standpoint of our objective of maximizing market price per share. One capital structure is as suitable as the next.

[13] For an analysis of such an operation, see Robert S. Hamada, "Portfolio Analysis, Market Equilibrium, and Corporation Finance," *Journal of Finance*, XXIV (March, 1969), 13–31. See also Joseph E. Stiglitz, "A Reexamination of the Modigliani-Miller Theorem," *American Economic Review*, LIX (December, 1969), 784–93.

ARGUMENTS AGAINST THE MM POSITION

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CHAP. 7
*Theory
of Capital
Structure*

Given perfect capital markets, the arbitrage argument assures the validity of MM's thesis that the cost of capital and total valuation of a firm are independent of its capital structure. To dispute the MM position, one needs to look for reasons why the arbitrage process may not work perfectly. If perfect capital markets do not exist in practice, opponents of the MM position are able to contest its behavioral support and argue that the cost of capital can decline with the appropriate use of leverage. The following are the major arguments against the MM arbitrage process.

1. The perceived risks of personal leverage and corporate leverage may differ. Implied in the MM analysis is that personal and corporate leverage are perfect substitutes. In the case of corporate borrowings, the individual has only limited liability. In our first example, his loss is restricted to $727.72 if he remains invested in Company *B,* the levered firm. However, if he engages in the arbitrage transactions and invests in Company *A,* there is the possibility that he will lose his capital investment of $700 and be liable as well for borrowings of $300. Therefore his total risk exposure is greater with personal leverage and investment in the unlevered company than it is with a straight investment in the levered company.

In addition to greater risk, there are other reasons why investors may have a greater aversion to personal leverage than they do to corporate leverage. If the investor borrows personally and pledges his stock as collateral, he is subject to possible margin calls. Many investors view this possibility with considerable alarm. Moreover, personal leverage involves a certain amount of inconvenience on the part of the investor, which he does not experience with corporate leverage. For these reasons, personal leverage may not be a perfect substitute for corporate leverage in the minds of many investors.

2. The cost of borrowing may be higher for the individual than for the corporation. The corporation is larger and is likely to have a higher credit standing. Consequently, it may be entitled to a lower rate of interest than that available on a personal loan. If so, the levered company could have a somewhat greater total value than the unlevered firm for this reason alone.

3. There are institutional restrictions that may retard the arbitrage process. Many institutional investors are not allowed to engage in the "homemade" leverage that was described. Moreover, the Federal Reserve regulates the percentage of advance under a margin loan. Consequently, a significant number of investors cannot substitute personal for corporate leverage.

4. Transaction costs tend to restrict the arbitrage process. Arbitrage

will take place only up to the limits imposed by transaction costs, after which it is no longer profitable. As a result, the levered firm could have a slightly higher total value.

All of the factors listed above impede the effectiveness of the MM arbitrage process. If the arbitrage process is less than perfectly effective, a firm may be able to increase its total valuation and lower its cost of capital with an appropriate amount of leverage. As a result, the financing decision would matter, for it can affect the market value of the stock. The arbitrage argument is the behavioral foundation for the MM position.[14] Consequently, MM naturally deny the importance of these criticisms by arguing that they are too general. They suggest that as long as there are enough market participants at the margin that behave in a manner consistent with "homemade" leverage, the total value of the firm cannot be altered through leverage. According to them, the notion of "homemade" leverage as a substitute for corporate leverage cannot necessarily be rejected, even under real-world conditions.[15]

Extreme Leverage. In making their stand, MM not only deny that a judicious amount of leverage may lower the weighted-average cost of capital, but that extreme leverage will raise it. According to their hypothesis, the weighted-average cost-of-capital line, k_o, is horizontal throughout all degrees of leverage. However, we know that the cost of borrowing can rise with excessive leverage. Beyond a certain point of leverage, we would expect the firm to pay increasingly higher interest rates on borrowings. The greater the leverage, the lower the coverage of fixed charges and the more risky the loan.

Even with a rise in k_i, however, MM still maintain that the weighted-average cost-of-capital line, k_o, is horizontal. They argue that when k_i increases, k_e will increase at a decreasing rate and may even turn down eventually.[16] This notion is illustrated in Figure 7-7. MM insist that the arbitrage process will work and that as k_i increases, some investors actually become risk seekers, whereas before they avoided risk.

[14] Perfect capital markets, as defined by MM, correspond to Lintner's *fully idealized uncertainty* in which information and subjective judgments about all stocks are uniform for all market participants. This condition differs from one of *generalized uncertainty,* in which information and subjective probability distributions are not uniform. Lintner contends that only in the case of fully idealized uncertainty will arbitrage work to bring about the equality in total valuation of the levered and unlevered companies illustrated earlier. John Lintner, "Dividends, Earnings, Leverage, Stock Prices, and the Supply of Capital to Corporations," *Review of Economics and Statistics,* XLIV (August, 1962), 243–69.

[15] See Modigliani and Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment: Reply," *American Economic Review,* XLIV (September, 1959), 655–69.

[16] Modigliani and Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," in *Foundations of Financial Management,* Van Horne, ed., pp. 381–82.
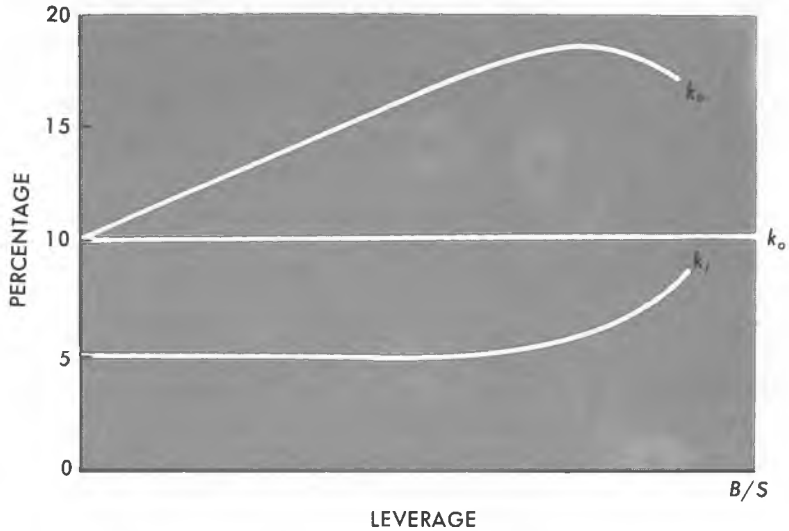
FIGURE 7-7
Capital costs: extreme lev-
erage

Given the assumption of perfect markets, k_o must remain constant.[17] Again, any denial of the cost of capital of a firm being independent of its capital structure must be based upon the belief that to assume perfect capital markets is unrealistic. In particular, a number of authors have found the contention that investors become risk seekers with extreme leverage, where before they were risk averters, to be quite objectionable.[18] In addition, the possibility of bankruptcy may cause the stock of the highly levered firm to be less attractive than that of the unlevered firm, all other things being the same. If administrative and other costs associated with bankruptcy are substantial, the risk of ruin becomes a significant consideration for the highly levered firm, resulting in a lower overall value and higher cost of capital.[19] It is clear that MM are on much weaker ground in defending their thesis for the extreme leverage case than for leverage at the other end of the scale.

In the absence of corporate income taxes, the traditional position implies that capital structure does matter and that the firm can lower its cost of capital through a judicious amount of leverage. MM, on the other

[17] Robichek and Myers (RM) stress that in perfect capital markets, k_e can never decline, for this occurrence would require the marginal interest rate to be greater than k_e. By virtue of lenders having a prior claim position, RM argue that the marginal interest rate on debt can never exceed k_e. Therefore, k_e cannot decline. Robichek and Myers, *Optimal Financing Decisions*, pp. 34–36, 48–49.

[18] See, for example, Ezra Solomon, "Leverage and the Cost of Capital," *Journal of Finance*, XVIII (May, 1963), reprinted in *Foundations for Financial Management*, Van Horne, ed., pp. 406–12; and Solomon, *The Theory of Financial Management*, pp. 409–14.

[19] See Nevins D. Baxter, "Leverage, Risk of Ruin and the Cost of Capital," *Journal of Finance*, XXII (September, 1967), 395–403.

hand, contend that the cost of capital cannot be altered with leverage. If we assume perfect capital markets, we are forced to accept the MM thesis on theoretical grounds. In practice, however, the majority of academicians and financial managers favor the traditional approach because of imperfections in the capital markets that hamper the perfect functioning of the arbitrage process.

THE INTRODUCTION OF CORPORATE INCOME TAXES

When we allow for corporate income taxes, we must reexamine the arguments presented so far. Because the payment of interest is deductible for tax purposes, leverage lowers the weighted-average after-tax cost of capital found with the MM position. To illustrate, suppose that the expected value of annual net operating income for two firms is $2,000 before taxes, the corporate income tax rate is 50 per cent, the after-tax capitalization rate is 8 per cent for both companies, and that Company A has no debt, whereas Company B has $8,000 in 5% bonds. According to the MM position, the total values of the two companies would be

		Company A	Company B
O	Net operating income	$ 2,000	$ 2,000
F	Interest on debt	0	400
	Profit before taxes	2,000	1,600
	Taxes	1,000	800
E	Profit after taxes	1,000	800
	Interest on debt	0	400
	Total income to all security holders	1,000	1,200
	After-tax capitalization rate	0.08	0.08
V	Total value of firm	$12,500	$15,000
B	Market value of debt	0	8,000
S	Market value of stock	12,500	7,000

The higher total value of Company B is due to the deductibility of interest payments.[20] Because of the tax benefit described, the firm can increase its total value with leverage under the MM position.

With taxes, the value of the firm, according to MM, is[21]

$$V = \frac{O(1-t)}{\rho_k} + tB \tag{7-7}$$

where $t =$ the corporate tax rate, ρ_k is the after-tax capitalization rate for

[20] Solomon, *The Theory of Financial Management,* pp. 114–17.

[21] Merton H. Miller and Franco Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility Industry," *American Economic Review,* LVI (June, 1966), 339–40.

a company with no debt in a given risk class, and 0 and B, as before, are expected net operating income and the market value of debt, respectively. As before, a 100 per cent dividend-payout ratio is assumed. Equation (7-7) suggests that the government subsidizes debt capital so that the greater the amount of debt, the greater the value of the firm. By the same token, the greater the leverage, the lower the cost of capital of the firm. The cost of capital on a tax-adjusted basis is expressed as[22]

$$k_o = \rho_k \left[1 - t \left(\frac{B}{B+S} \right) \right] \tag{7-8}$$

Thus, MM recognize that with the introduction of corporate income taxes the cost of capital can be lowered with leverage.[23] We note, however, that their position implies that a firm can lower its cost of capital continually with increased leverage. The greater the leverage, the higher the total value of the firm and the lower its cost of capital. In order to achieve an optimal capital structure, the firm should strive for the maximum amount of leverage. Proponents of the traditional position would argue that the cost of capital must rise with extreme leverage owing to increased financial risk. Consequently, the optimal capital structure is not the one that calls for the maximum use of debt. Again, the MM thesis is on the weakest ground when leverage is extreme. In defense, MM suggest that the firm should adopt a "target debt ratio" so as not to violate limits on leverage imposed by creditors. The implication is that debt funds simply are refused beyond some point. The introduction of debt limits, however, implies that the cost of capital rises beyond a point and that there is an optimal capital structure. Certainly, this is a position with which MM should feel uncomfortable.

EMPIRICAL
TESTING[24]

A number of empirical studies have dealt either directly or indirectly with the question whether leverage affects the cost of capital and the valuation of the firm. Most of the empirical testing has involved regression studies in which either the "measured" average cost of capital or the earnings/price ratio is used as the dependent variable and either leverage or leverage plus additional explanatory variables are used as the independent variable(s). When independent variables additional to leverage are employed, the analysis is known as a multiple-regression study. The purpose

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[22] *Ibid.,* p. 342.

[23] See Modigliani and Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review,* LIII (June, 1963), 433–42.

[24] The discussion in this section assumes a knowledge of regression analysis. It can be omitted for the student without such a background. For an excellent examination of regression analysis, see J. Johnston, *Econometric Methods* (New York: McGraw-Hill Book Company, 1963).

of a multiple-regression study is to hold constant statistically other factors that affect valuation in order to obtain an unbiased estimate of the effect of leverage on valuation. By and large, the regression studies undertaken have been based upon cross-sectional samples of firms in a particular industry.

In general, the results of empirical testing of the relationship between leverage and cost of capital have been tenuous. The major difficulty is in holding constant all other factors that affect valuation. In particular, future growth, as perceived by investors at the margin, is extremely hard to measure. Most regression studies have used the past growth in earnings per share or the past growth in total assets as a proxy for expected future growth as perceived by investors at the margin. To the extent that these past growth measures do not account fully for growth as perceived by investors at the margin, other variables, including leverage, in some measure may be a proxy for growth. For example, leverage may be associated positively with "aggressive" management. The more "aggressive" management is, the greater the leverage of a company up to a point, and the greater the prospective growth of the company. As a result, the leverage variable in a regression study may have a negative regression coefficient when the earnings/price ratio is used as the dependent variable — the opposite of what we would expect. We would expect the earnings/price ratio to increase with leverage (a positive regression coefficient), all other things being the same, as investors penalize a stock as the company becomes more financially risky. The favorable effect upon the earnings/price ratio is not the result of leverage but is due to the market's favorable valuation of "aggressive" management. As this factor is not fully taken into account in other explanatory variables, the apparent relationship between leverage and the earnings/price ratio is deceiving.

In addition to the biases described above, there are several others that plague empirical testing.[25] Frequently, the range of leverage for a sample of companies in the same industry is narrow. Without extreme values of leverage, a regression study is unlikely to denote a very strong relationship between leverage and the cost of capital or leverage and the earnings/price ratio. If all firms in a sample are at an optimal capital structure, an unbiased regression study would fail to show any relationship between leverage and valuation.[26] In summary, there are a number of

[25] For a discussion of these biases, see Irwin Friend and Marshall Puckett, "Dividends and Stock Prices," *American Economic Review,* LIV (September, 1964), reprinted in *Foundations for Financial Management,* Van Horne, ed., pp. 535–61.

[26] If the optimal financing mix could be determined by this method, and all companies in a homogeneous business-risk class adopted that mix, further regression studies would not be fruitful because all companies would have the same capital structure. If the underlying valuation parameter changed over time, the capital structure of all companies would be suboptimal; and regression analysis could not be used to determine this suboptimality.

limitations to the empirical testing of the relationship between leverage and the cost of capital. Consequently, evidence of this relationship is only suggestive.

MM, in their original paper, studied evidence relating to cross-sectional samples of electric utility and oil companies.[27] In regressing the ratio of total earnings after taxes over the market value of all securities against the ratio of the market value of senior securities over the market value of all securities, they found no significant relationship. They concluded that the evidence was consistent with the notion that capital structure does not affect the cost of capital.

Barges, in a test of the MM hypothesis, undertook cross-sectional regression studies of railroad, department store, and cement industry samples.[28] Instead of using market value weights for the leverage variable, he used book value weights, because of the bias introduced by market value weights' being associated with business risk.[29] From his samples, Barges discovered a U-shaped relationship between leverage and the cost of capital. He interpreted this finding as consistent with the traditional position and disproving the MM hypothesis. It is important to point out that the regression studies of both Barges and MM are crude because explanatory variables other than leverage are not included. Without holding other variables constant, the results are not particularly meaningful. Weston, in another test of the MM hypothesis, found that when growth was introduced as an explanatory variable, the cost of capital for the electric-utility industry sample declined with leverage.[30] He concluded that his results were consistent with the traditional position.

More recent empirical studies have introduced additional explanatory variables in the regression equation, which reduce the problem of specification bias. In one of the most interesting of them, Brigham and Gordon (BG) test for the effect of leverage on dividend yields in the electric utility industry.[31] The formulation of their valuation model is explained in detail in Chapter 9 when we examine empirical studies dealing with dividend policy. The regression model, as it relates to leverage, is

[27] Modigliani and Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," in *Foundations for Financial Management,* Van Horne, ed., pp. 388–95.

[28] Alexander Barges, *The Effect of Capital Structure on the Cost of Capital* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963).

[29] *Ibid.,* pp. 26–29.

[30] J. Fred Weston, "A Test of Cost of Capital Proposition," *Southern Economic Journal,* XXX (October, 1963), 105–12.

[31] Eugene F. Brigham and Myron J. Gordon. "Leverage, Dividend Policy and the Cost of Capital." *Journal of Finance,* XXIII (March, 1968), 85–103. For criticisms of their methodology, see comments by Michael Davenport and by Morris Mendelson as well as the reply by BG, "Leverage, Dividend Policy and the Cost of Capital: Comments and Reply," *Journal of Finance,* XXV (September, 1970), 893–908.

$$\frac{D_0}{P_0} = a_0 + a_1 g + a_2 h + a_3 u$$

where D_0 = current dividend per share

P_0 = current market price per share

g = growth measure based upon past growth in retained earnings and in assets

h = debt to equity ratio

u = an index of earnings stability.

The model was tested with a sample of sixty-nine utility stocks for each of the years 1958–1962. The authors found the regression coefficients for the leverage variable to be positive and significant. They interpret this finding as consistent with the cost of equity capital increasing with leverage. In addition, they suggest that the regression coefficients are not large enough to support the MM theorem. Instead, they contend that the evidence is consistent with the existence of an optimal capital structure, and that the cost of capital and value of the firm depend upon the firm's financing policy.

In addition to the regression studies cited, there have been a number of other empirical tests dealing with the relationship between leverage and valuation.[32] While a certain amount of the evidence seems to suggest that the cost of capital can be lowered with leverage, this finding does not necessarily refute the MM thesis. Recall that with the introduction of corporate income taxes, the MM hypothesis also implies that the cost of capital can be lowered with leverage. In view of the aforementioned limitations, the regression studies undertaken for the most part are not precise enough to constitute absolute refutation of the MM position. Additional empirical studies, particularly studies dealing with industries other than the public utility industry, are needed.

SUMMARY

A great deal of controversy has developed recently over whether the capital structure of a firm, as determined by its financing decision, affects its cost of capital. Traditionalists argue that the firm can lower its cost of

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[32]Other regression studies include Ronald F. Wippern, "Financial Structure and the Value of the Firm," *Journal of Finance*, XXI (December, 1966), 615–34; Fred D. Arditti, "Risk and the Required Return on Equity," *Journal of Finance*, XXII (March, 1967), 19–36; William Beranek, *The Effect of Leverage on the Market Value of Common Stocks* (Madison, Wisc.: University of Wisconsin, Bureau of Business Research and Service, 1964); Myron J. Gordon, *The Investment, Financing, and Valuation of the Corporation* (Homewood, Ill.: Richard D. Irwin, Inc., 1962); Burton G. Malkiel and John G. Cragg, "Expectations and the Structure of Share Prices," *American Economic Review*, LX (September, 1970), 601–17; and Richard S. Bower and Dorothy H. Bower, "Risk and the Valuation of Common Stock," *Journal of Political Economy*, LXXVII (May–June, 1969), 349–62.

capital and increase market value per share by the judicious use of leverage. However, as the company levers itself and becomes increasingly risky financially, lenders begin to charge higher interest rates on loans. Moreover, investors penalize the price/earnings ratio increasingly, all other things being the same. Beyond a point, the cost of capital begins to rise. According to the traditional position, that point denotes the optimal capital structure. Modigliani and Miller, on the other hand, argue that in the absence of corporate income taxes, the cost of capital is independent of the capital structure of the firm. They contend that the cost of capital and the total market value of the firm are the same for all degrees of leverage.

We saw that the behavioral support for their position was based upon the arbitrage process. Attacks on the MM hypothesis are centered on the validity of their assumptions. To the extent that the arbitrage process does not work perfectly, a case can be made for the view that the capital structure of the firm affects its cost of capital. With the introduction of corporate income taxes, debt has a tax advantage and serves to lower the cost of capital, even in the MM case. The traditional position implies, however, that the cost of capital will rise eventually with additional leverage, whereas the MM position implies a continually decreasing cost of capital with leverage. Unfortunately, empirical testing has been little more than suggestive with respect to the true relationship between leverage and the cost of capital. In the next chapter, we examine ways by which a firm can analyze which capital structure is appropriate.

PROBLEMS

1. The Malock Company has net operating earnings of $10 million and $20 million of debt with a 7 per cent interest charge. In all cases, assume no taxes.

   (a) Using Durand's Net Income method and an equity-capitalization rate of 12½ per cent, compute the total value of the firm and the implied overall capitalization rate.

   (b) Next, assume that the firm issues an additional $10 million in debt and uses the proceeds to retire stock; the interest rate and equity-capitalization rate remain the same. Compute the new total value of the firm and overall capitalization rate.

   (c) Using Durand's Net Operating Income concept and an overall capitalization rate of 11 per cent, compute the total market value, the stock-market value, and the implied equity-capitalization rate for the Malock Company prior to the sale of additional debt.

   (d) Determine the answers to (c) if the company were to sell the additional $10 million in debt.

2. Reconsider the Malock Company, with its $10 million in net operating income, $20 million of 7 per cent debt, and 12½ per cent equity-capitalization rate.

   (a) Compute 1(a) above if you have not already done so.

   (b) Assume that the Malock Company now issues an additional $10 million of debt at an interest rate of 8 per cent without altering the equity-capitali-

zation rate. Compute the new total value of the firm and the implied overall
capitalization rate.
(c) Recompute (b) under the assumption that the sale of additional debt would
have caused the equity capitalization to rise to 15 per cent.
(d) Recompute all of the above under the assumption that the company pays
taxes at a 50 per cent rate. Use an NI approach with a $12\frac{1}{2}$ per cent after-
tax equity-capitalization rate. In (c) above, however, continue to use the
15 per cent equity-capitalization rate.

3. The Blalock Corporation has a $1 million capital structure and will always
maintain this book value amount. Blalock currently earns $250,000 per year
before taxes of 50 per cent, has an all-equity capital structure of 100,000 shares
and pays all earnings in dividends. The company is considering issuing some debt
in order to retire some stock. The cost of the debt and the price of the stock at
various levels of debt are given below. It is assumed that the new capital structure
would be reached all at once by purchasing stock at the current price. In other
words, the following table is a schedule at a point in time.

Amount of Debt	Average Cost of Debt	Price of Stock
$ —	—	$10.00
100,000	6.0%	$10.00
200,000	6.0	10.50
300,000	6.5	10.75
400,000	7.0	11.00
500,000	8.0	10.50
600,000	10.0	9.50

(a) By observation, what do you think is the optimal capital structure?
(b) Construct a graph in terms of k_e, k_i, and k_o based upon the above data.
(c) Are your conclusions in (a) confirmed?

4. The Veblen Company and the Knight Company comprise a single risk
class. These firms are identical in every respect except that the Veblen Company
is not levered, while the Knight Company has $1,000,000 in 6% bonds out-
standing. The valuation of the two firms is assumed to be the following:

		Veblen	Knight
O	Net operating income	$ 300,000	$ 300,000
F	Interest on debt	—	60,000
E	Earnings to common	300,000	240,000
k_e	Equity-capitalization rate	.125	.140
S	Market value of stock	2,400,000	1,714,000
B	Market value of debt	—	1,000,000
V	Total value of firm	$2,400,000	$2,714,000
k_o	Implied overall capitalization rate	12.5%	11.0%
B/S	Debt/equity ratio	0	58.4%

(a) An investor owns $10,000 worth of Knight stock. Show the process and
the amount by which he could reduce his outlay through the use of
arbitrage.

(b) According to Modigliani and Miller, when will this arbitrage process cease?

(c) What arguments can be raised against this hypothesis?

5. Sam Peltz is planning to form a corporation. He has determined that $10,000,000 will be required as an initial capital investment. Several financial backers have indicated that they would be willing to buy the bonds of the new corporation or lend personally Sam the capital he needs. Sam has delineated these alternative financing plans:

(1) Form the corporation with 1,000,000 shares of $10 par stock, borrowing the entire $10,000,000 from his associates on a personal basis (paying 6 per cent interest on his note).

(2) Form the corporation with 500,000 shares of $10 par stock, borrowing the $5,000,000 from his associates on a personal basis (6 per cent note). Sell $5,000,000 worth of 6 per cent bonds to his associates.

(3) Form the corporation with 250,000 shares of $10 par stock, borrowing $2,500,000 from his associates on a personal basis (6 per cent note). Sell $7,500,000 worth of 6 per cent bonds to his associates.

(a) Assuming no corporate or personal taxes, which of the three alternatives should Sam select if the firm is expected to earn $1,000,000 per year before the payment of interest? Assume that all of net earnings are paid out in dividends.

(b) Assuming a 50 per cent corporate income tax and a personal income tax rate for Sam of 40 per cent, which of the three alternatives should Sam select if the firm were expected to earn the same amount as in (a)? Assume all earnings are paid out in dividends.

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Capital Structure
Decision
of the Firm

8

In theory, the firm should seek an optimal capital structure and finance future investment projects in those proportions. The optimal capital structure is one in which the marginal real cost of each available method of financing is the same. By real cost, we mean the sum of both the explicit and the implicit costs. The latter cost is expressed in terms of investors penalizing the price of the stock. If we confine our analysis only to debt and equity, the real cost of debt (k_i + the implicit cost) is less than the real cost of equity up to the optimal capital structure, after which it is greater than the real cost of equity. The firm should finance with debt until an optimal capital structure is reached. At that point, the marginal real cost of debt and equity are the same. If the firm then finances in those proportions, it should maximize the market value of the stock insofar as the financing decision alone will allow.

Up to now, our discussion has been theoretical. In practice, how does the financial manager determine the optimal capital structure for his par-

ticular firm? The real difficulty, of course, is in estimating the implicit costs of nonequity financing. Our concern in this chapter is with ways of coming to grips with the formidable problem of determining an appropriate capital structure. In this regard, we examine various methods of analysis that can be used in practice. None of the methods considered is completely satisfactory in itself. Taken collectively, however, they provide the financial manager with sufficient information for making a rational decision. One should hold no illusions that the financial manager will be able to identify the precise percentage of debt that will maximize share price. Rather, he should try to determine the approximate proportion of debt to employ in keeping with an objective of maximizing share price. In the last part of the chapter, we explore briefly the questions of timing and flexibility of a single security issue.

FACTORS INFLUENCING DECISION

A number of factors influence the financial manager in determining an appropriate capital structure. The greater the expected growth in sales of the firm, the greater the external financing that usually is required. As discussed in Chapter 4, the cost of common stock financing is somewhat higher than the cost of retained earnings. This difference in cost gives the firm an incentive to finance a greater proportion of its growth with debt when it builds its equity base with common stock financing than it does when it builds its equity base with retained earnings. The likely volatility and uncertainty of future sales have an important influence upon the business risk of the firm. In turn, the greater the business risk, the less debt that should be employed. Implied is a balancing of business risk with financial risk so that the total risk of the firm is kept within desirable bounds. A machine tool company with significant swings in sales, for example, should not employ a large amount of debt. On the other hand, the sales of an electric utility company are quite stable and predictable. Consequently, it can employ a large proportion of debt and preferred stock in its capital structure.

The liquidity of assets also has a bearing on the amount of debt a firm will want to employ. The greater the liquidity, the more debt that generally can be used, all other factors remaining constant. While liquidity is of primary interest in determining the maturity of the debt employed (see Chapter 15), it also affects the proportion of debt in the capital structure. The less liquid the assets of the firm, the less flexible the firm can be in meeting its fixed-charge obligations.

Other obvious factors to consider in the capital structure decision are the interest cost and availability of debt. In periods of tight money, it is extremely difficult to place long-term debt either privately or through the sale of bonds to the public. In addition, small firms and start-up compa-

nies frequently are unable to borrow on a long-term basis. The only source of financing is equity. Moreover, large firms have access to the bond markets, whereas smaller, lesser-known firms do not. All of these factors influence the capital structure decision of the firm. They should be embodied in, or considered in addition to, the methods of analysis we now examine.

EBIT-EPS ANALYSIS

One widely used means of examining the effect of leverage is to analyze the relationship between earnings before interest and taxes (EBIT) and earnings per share (EPS). Essentially, the method involves the comparison of alternative methods of financing under various assumptions as to EBIT.[1] To illustrate, suppose that a firm wished to compare the impact on earnings per share of financing a $10-million expansion program either with common stock at $50 a share or with 8 per cent bonds. The tax rate is 50 per cent, and the firm currently has an all-equity capital structure consisting of 800,000 shares of common stock. At $50 a share, the firm will need to sell 200,000 additional shares in order to raise $10 million. If we choose a hypothetical EBIT level of $8 million, earnings per share for the two alternatives would be

	Common Stock Financing	Debt Financing
EBIT	$8,000,000	$8,000,000
Interest	0	800,000
Earnings before taxes	8,000,000	7,200,000
Taxes	4,000,000	3,600,000
Earnings after taxes	4,000,000	3,600,000
Shares outstanding	1,000,000	800,000
Earnings per share	$4.00	$4.50

In order for the firm to show zero earnings per share under the debt alternative, it is clear that it will need to have EBIT of $800,000.

With this information, we are able to construct an EBIT-EPS chart, and it is shown in Figure 8-1. The intercepts on the horizontal axis represent the amount of before-tax charges. For equity, the intercept is zero; for debt, it is $800,000. We then plot earnings per share for both alternatives under the assumption of an EBIT of $8 million. When we connect the intercepts with the appropriate EPS points at an EBIT level of $8

[1] For detailed discussion of the calculations involved in this type of analysis, see Chapter 27. Our concern in the present chapter is with how the method might be used.
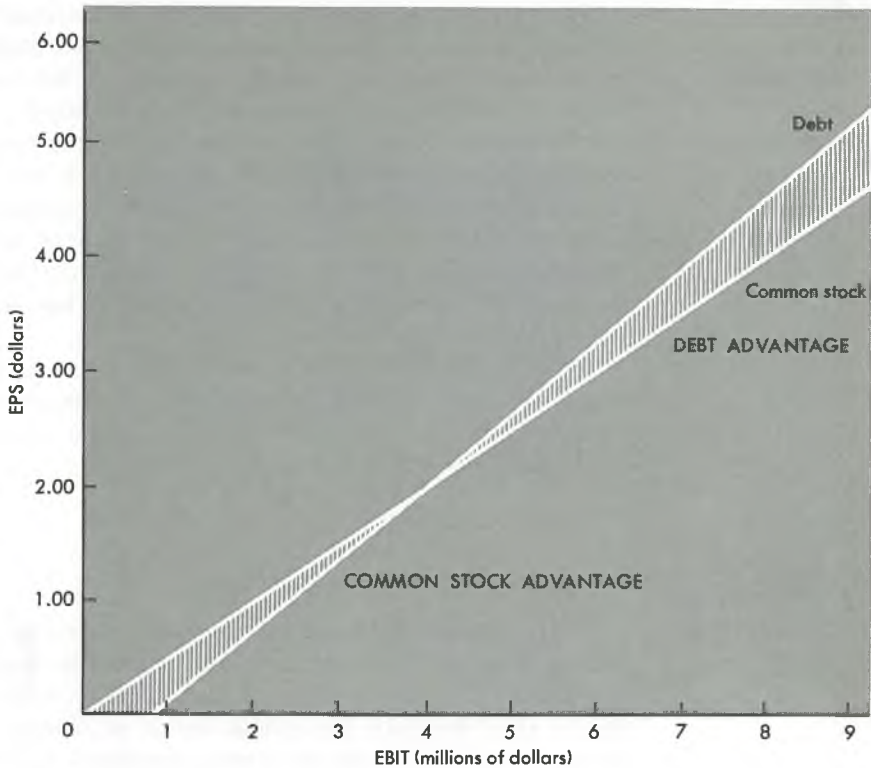
FIGURE 8-1
EBIT–EPS chart

million, we obtain the straight lines shown. They tell us the earnings per share for the two financing alternatives that will occur under varying levels of EBIT.

ANALYSIS OF INTERSECTION

Because the debt line has a steeper slope, it intersects the common stock line at an EBIT level of $4 million. At all levels of EBIT above $4 million, there is an earnings per share advantage to the use of debt. At levels of EBIT below $4 million, the advantage is in favor of common stock financing.

Constructing an EBIT-EPS chart gives the financial manager information about the differential impact on earnings per share of alternative methods of financing. We note that as long as the firm is able to earn more than 8 per cent before taxes on its investment, debt financing will show an EPS advantage. But the method does not consider directly the implicit costs associated with debt. In an analysis of financial risk, however, an EBIT-EPS chart can be useful to the financial manager. He can compare the point of intersection with the most likely level of EBIT; he can also determine the probability that EBIT will fall below that point.

For example, suppose that the EBIT in the example presently is $7 million and that the company is considering the debt alternative. Given the business risk of the company and the possible fluctuations in EBIT, the financial manager should assess the probability of EBIT's falling below $4 million. If the probability is negligible, the use of the debt alternative would be supported. On the other hand, if EBIT presently is only slightly above the indifference point and the probability of EBIT's falling below this point is high, the financial manager may conclude that the debt alternative is too risky. In summary, the greater the level of EBIT and the lower the probability of downside fluctuation, the stronger the case that can be made for the use of debt.

Although an EBIT-EPS chart does not focus on the implicit costs of senior securities, the financial manager is able to obtain insight into these implicit costs through proper analysis. While crude, the method is a useful supplement to other methods of analysis.

CASH-FLOW ANALYSIS

When considering the appropriate capital structure, it is extremely important to analyze the cash-flow ability of the firm to service fixed charges. The greater the dollar amount of senior securities the firm issues and the shorter their maturity, the greater the fixed charges of the firm. These charges include principal and interest payments on debt, lease payments, and preferred stock dividends. Before assuming additional fixed charges, the firm should analyze its expected future cash flows, for fixed charges must be met with cash. The inability to meet these charges, with the exception of preferred stock dividends, may result in financial insolvency. The greater and more stable the expected future cash flows of the firm, the greater the debt capacity of the company. Debt capacity is used in a broad sense to mean all senior securities. From an internal standpoint, the financial risk associated with leverage should be analyzed on the basis of the firm's ability to service fixed charges. This analysis should include the preparation of cash budgets to determine whether the expected cash flows are sufficient to cover the fixed obligations.[2]

PROBABILITY OF CASH INSOLVENCY

Necessarily, however, the analysis must consider the probability distribution of cash flows, for we are concerned with possible deviations in actual cash flows from those that are expected. As we discuss in Chapter 26 and illustrate in Chapter 15, cash budgets can be prepared for a range of possible outcomes, with a probability attached to each. This

[2] The preparation of cash budgets is discussed in Chapter 26.

information is extremely valuable to the financial manager in evaluating the ability of the firm to meet fixed obligations. Given the probabilities of particular cash-flow sequences, he is able to determine the amount of fixed charges and debt the company can undertake while still remaining within insolvency limits tolerable to management.

Suppose management feels that a 5 per cent probability of being out of cash is the maximum that can be tolerated, and that this probability corresponds to a cash budget prepared under pessimistic assumptions. In this case, debt might be undertaken up to the point where the cash balance under the pessimistic cash budget is just sufficient to cover the fixed charges associated with the debt. In other words, debt would be increased to the point at which the additional cash drain would cause the probability of cash insolvency to equal the risk tolerance specified by management. It is not necessary that debt be increased to this point, of course. Note that the method of analysis simply provides a means for assessing the effect of increases in debt on the risk of cash insolvency. On the basis of this information, management would arrive at the most appropriate level of debt.

Donaldson has proposed a similar type of analysis.[3] He suggests that the ultimate concern of a company is whether cash balances during some future period will be involuntarily reduced below zero. Therefore, he advocates examining the cash flows of the company under the most adverse circumstances — that is, in his definition, under recession conditions. These conditions may or may not be the most adverse; however, in keeping with the spirit of his proposal, the firm should evaluate its cash flows under adverse circumstances. Donaldson defines the net cash balance during a recession as

$$CB_r = CB_o + NCF_r \tag{8-1}$$

where CB_o = cash balance at start of recession and
NCF_r = net cash flows during recession.

Donaldson then analyzes the cash-flow behavior of a firm during a recession by calculating a probability distribution of expected net cash flows.[4] By combining the beginning cash balances, CB_o, with the probability distribution of recession cash flows, NCF_r, he prepares a probability distribution of cash balances during the recession — CB_r.

To ascertain its debt capacity, a firm first would calculate the fixed charges associated with additional increments of debt. For each addi-

[3] Gordon Donaldson, *Corporate Debt Capacity* (Boston: Division of Research, Harvard Business School, 1961). See also Donaldson, "Strategy for Financial Emergencies," *Harvard Business Review,* 47 (November–December, 1969), 67–79.

[4] The determinants of net cash flows with which he works are sales collections, other cash receipts, payroll expenditures, raw-material expenditures, and nondiscretionary cash expenditures. By analyzing each of these determinants, he determines the range and probability of recession net cash flows.

tion, the firm then would determine the probability of being out of cash, based upon the probability distribution of cash balances during the recession. As before, management could set tolerance limits on the probability of being out of cash. For example, suppose the firm were considering issuing $20 million in additional debt and that the annual fixed charges were $3 million. By subtracting $3 million from the expected cash balances shown for the probability distribution of CB_r, we obtain the probability distribution of CB_r with the addition of $20 million in debt. If the probability of being out of cash with this increment of debt is negligible, Donaldson would contend that the company has unused debt capacity. Therefore, it would be possible to increase the amount of debt until the probability of being out of cash equaled the risk tolerance of management.

Donaldson extends his analysis to calculate the probability of cash inadequacy. Our discussion before was in terms of cash insolvency, which is defined as lack of cash after all nonessential expenditures have been cut. Cash inadequacy is said to occur if the firm is out of cash after making certain desired expenditures such as dividends, R & D expendi-

TABLE 8-1
Inventory of resources to meet possible cash drains

	Available for Use Within:		
Resources	One Quarter	One Year	Three Years
I. Uncommitted reserves			
Instant reserves			
Surplus cash	$		
Unused line of credit	$		
Negotiable reserves			
Additional bank loans			
Unsecured	$		
Secured	$		
Additional long-term debt		$	
Issue of new equity		$	
II. Reduction of planned outflows			
Volume-related			
Change in production schedule	$		
Scale-related			
Marketing program		$	
R & D budget		$	
Administrative overhead		$	
Capital expenditures		$	
Value-related			
Dividend payments		$	
III. Liquidation of assets			
Shutdown		$	
Sale of unit			$
	$ ——→ $		
		$ ——→ $	
Total resources			$

Source: Donaldson, "Strategy for Financial Emergencies," 72.

tures, and capital expenditures. Thus, cash insolvency is the extreme form of cash inadequacy. In all cases, the firm should take stock of the resources it has at its disposal to meet an unexpected cash drain.[5] Typically, a number of alternatives are available, ranging from the use of surplus cash to the sale of fixed assets at distress prices. Donaldson categorizes these resources into uncommitted reserves, the reduction of outflows, and the liquidation of assets. These categories and their various subsets are shown in Table 8-1. Once an inventory of resources has been compiled, the adequacy of these resources should be judged in relation to potential cash drains. Embodied in this analysis should be the fixed charges associated with various levels of debt. Thus, the liquidity of the firm should be analyzed in relation to its debt capacity.

CASH-FLOW ANALYSIS AND DEBT-TO-EQUITY RATIOS

A probabilistic approach to analyzing the ability of a firm to service fixed charges is extremely useful. In this regard, simulation can be employed to determine the outcome under changing conditions. The analysis of the cash-flow ability of the firm to service fixed charges is perhaps the best way to analyze financial risk, but there is some question as to whether the external market analyzes a company in this manner. Sophisticated lenders and institutional investors certainly analyze the amount of fixed charges and evaluate financial risk in keeping with the ability of the firm to service these charges. However, individual investors may look more to the book value proportions of debt to equity in judging financial risk.

There may or may not be a reasonable correspondence between the ratio of debt to equity and the amount of fixed charges relative to the firm's cash-flow ability to service these charges. Some firms may have relatively high ratios of debt to equity but substantial cash-flow ability to service debt. Consequently, the analysis of debt-to-equity ratios alone can be deceiving, and an analysis of the magnitude and stability of cash flows relative to fixed charges is extremely important in determining the appropriate capital structure for the firm. To the extent that creditors and investors analyze a firm's cash-flow ability to service debt, and management's risk preferences correspond to those of investors, capital structure decisions made on this basis should tend to maximize share price.

COMPARISON OF CAPITAL STRUCTURE RATIOS

OTHER METHODS OF ANALYSIS

Another method of analyzing the appropriate capital structure for a company is to evaluate the capital structure of other companies having similar business risk. Companies used in this comparison may be those in

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[5] Donaldson, "Strategy for Financial Emergencies."

the same industry. If the firm is contemplating a capital structure signifi-cantly out of line with that of similar companies, it is conspicuous to the marketplace. This is not to say, however, that the firm is wrong; other companies in the industry may be too conservative with respect to the use of debt. The optimal capital structure for all companies in the indus-try might call for a higher proportion of debt to equity than the industry average. As a result, the firm may well be able to justify more debt than the industry average. However, if the firm is noticeably out of line in either direction, it should be able to justify its position, because invest-ment analysts and creditors tend to evaluate companies by industry.

REGRESSION STUDIES
AND SIMULATION

A number of companies are undertaking regression studies in an effort to determine the effect of capital structure on stock valuation. The results of the published studies, which we considered in the previous chapter, are mixed. In these studies, it is not possible to hold everything else sta-tistically constant so as to determine the "true" relationship between leverage and valuation. Nevertheless, regression studies based upon a sample of similar companies may give the firm some insight into the im-pact of leverage on the market price of its stock.

In addition to regression analysis, simulation is being employed increasingly as a means of providing management with valuation in-formation about the expected consequences of a particular investment, financing, or dividend decision. For example, management might be considering a large increase in the proportion of debt to equity because the firm needs to buy out certain minority interests. Before actually making the decision, management may wish to simulate the expected effect of this action on the market price of the stock. By observing the range of simulated outcomes, management has better information on which to base its decision. Most simulation problems in finance are stochastic and involve the use of probabilistic models. These models make use of the Monte Carlo technique described in Chapter 5. If the probability distributions are accurate and the interrelationship of vari-ables properly specified, simulation can be an extremely valuable aid in formulating financial decisions.[6]

However, the results achieved are no better than the probability infor-mation used and the realism of the interrelationship of variables specified. Probabilistic information usually is based upon empirical testing of valua-tion models, the results of which have been far from precise or consistent.

[6]See Alexander A. Robichek, "The Use of Computer Simulation in Financial Planning," in Robichek, ed., *Financial Research and Management Decisions* (New York: John Wiley & Sons, Inc., 1967), pp. 200–222.

An equally thorny problem is the specification of the interrelationship be-tween variables. Again, empirical studies have been less than satisfactory in sorting out the interrelationships in a consistent manner. We do not wish to undervalue the importance of regression analysis and simulation techniques in providing valuation information to the financial manager, but it is important to point out the practical limitations of the models. The state of the art is not advanced sufficiently for a financing decision to be based solely upon share-price models. Still, theoretically, the method is beyond reproach. For this reason, we can expect to see increasing efforts to lessen the difficulties that have hampered its acceptance by financial managers.

SUBJECTIVE FEELINGS
AND THE DECISION

The firm may profit also by talking with investment analysts, institu-tional investors, and investment houses to obtain their views on the ap-propriate amount of leverage. These analysts examine many companies and are in the business of recommending stocks. Therefore, they have an influence upon the market, and their judgments with respect to how the market evaluates leverage may be very worthwhile. Similarly, a firm may wish to interview lenders to see how much debt it can undertake before the cost of borrowing is likely to rise. Finally, the management of a com-pany may develop a "feel" for what has happened in the past to the mar-ket price of the stock when they have issued debt. As suggested in Chap-ter 4, the financial manager must think the way investors think if he is to evaluate the impact of a financial decision on share price.

The methods described above for analyzing the appropriate amount of leverage do not give an exact answer. Nevertheless, by undertaking a variety of analyses, the financial manager should be able to determine, within some range, the appropriate capital structure for his firm. By necessity, the final decision has to be somewhat subjective. However, it can be based upon the best information available. In this way, the firm is able to obtain the capital structure most appropriate for its situation — the one that, hopefully, will tend to maximize the market price of the stock, all other factors held constant.

TIMING AND
FLEXIBILITY

Once a firm has determined an appropriate capital structure, it still has the problem of timing security issues. When external financing is required, a company is often faced with the question of how to time an issue appro-priately and whether to use debt or common stock. Because financing is "lumpy," it is difficult for a firm to maintain strict proportions in its capital

structure. Frequently, it must decide whether to finance now with a stock issue and later with a debt issue, or vice versa. Consequently, it is forced to evaluate the alternative methods of financing in light of general market conditions and expectations for the company itself. If the market for the company's stock is depressed but is expected to improve as better earnings are realized, management may prefer to postpone the equity issue until a later date and issue debt now.

If it chooses this alternative, however, it may sacrifice a certain amount of flexibility. If the debt issue is substantial and things take a turn for the worse, the firm may be forced to issue stock on unfavorable terms in the future. In order to preserve its flexibility in tapping the capital markets, it may be better for a firm to issue stock now so as to have unused debt capacity for future needs. The preservation of unused debt capacity can be an important consideration for the company whose funds requirements are sudden and unpredictable. For a growth company, however, this approach is not altogether satisfactory. By issuing stock now, the company will probably have to issue more shares than it would if it postponed the stock issue. Consequently, there is more dilution to existing shareholders over time. The tradeoff is between preserving financial flexibility and dilution in earnings per share. If the price of the stock is high, however, and expected to fall, the firm can achieve both flexibility and minimum dilution by issuing stock now.

The cost-of-capital approach suggests some type of average cost based upon normalized market prices. However, the financial manager still can take advantage of timing in the market to assure the lowest possible cost. Although the benefits of good timing may not be particularly great for debt financing, they can be substantial for equity financing. Depending upon the stock market in general and expectations for the firm in particular, the price of the stock can vary significantly. Management has an implied responsibility to existing stockholders to try to sell a stock issue at as favorable a price as possible. In Part IV, we examine in depth specific methods of long-term financing. Included in this analysis is a discussion of the timing of a specific security issue and the flexibility afforded by the instrument.

SUMMARY

In deciding upon an appropriate capital structure, the financial manager should consider a number of factors. He can obtain considerable insight from an analysis of the cash-flow ability of the firm to service fixed charges associated with senior securities and leasing. By evaluating the probability of cash insolvency, he is able to determine the debt capacity of the firm. Another method is to analyze the relationship between earnings before interest and taxes and earnings per share for alternative meth-

ods of financing. When this analysis is expanded to consider likely fluctuations in EBIT, light is shed on the question of financial risk.

In addition, the financial manager can learn much from a comparison of capital structure ratios for similar companies, through regression studies and simulations, and through discussions with investment analysts, investment bankers, and lenders. Collectively, the methods of analysis taken up in this chapter should provide sufficient information on which to base a capital structure decision. Once an appropriate capital structure has been determined, the firm should finance investment proposals in roughly those proportions, recognizing, however, the possibility of an upward sloping supply curve for capital.

Finally, we examined the problem of timing a debt or equity issue. Where sequential financing is involved, the choice of debt or equity has an important influence on the future financial flexibility of the firm.

PROBLEMS

1. The Power Corporation currently has 2 million shares outstanding at a price of $20 each and needs to raise an additional $5 million. These funds could be raised with stock or 10 per cent debentures. Expected EBIT after the new funds are raised will be normally distributed with a mean of $4 million per year forever and a standard deviation of $2 million. Power Corporation has a 50 per cent tax rate. What is the probability that the debt alternative is superior with respect to earnings per share?

2. The Power Corporation (see problem 1) has annual noncash expenses of $3 million and will allow only a 10 per cent probability of running out of cash. On the basis of this data alone, what is the maximum amount of debt that could be sold if the initial cash balance is $1 million? What may be questioned about the EPS–EBIT approach?

3. The Great Northwest Oil Corporation has decided that it must raise $100,000,000 within the next six months to finance an expansion program. The firm believes it now has an optimal capital structure. Nevertheless, G.N.O.C. realizes that this "ideal" structure may be temporarily sacrificed unless the needed funds are secured on a proportionate basis to the current structure. It has been determined that the "normalized" price of the firm's common stock is $100 per share, though the current price is only $80 because of a general stock-market decline. Further, record high interest rates have reduced the price of the firm's AAA rated bonds to yield 8 per cent to maturity. The prime rate available to the firm for a one-year bank loan is $8\frac{1}{2}$ per cent. What alternative financing (and timing) plans are open to the firm? Outline the separate assumptions that must be made in order to justify each alternative.

G.N.O.C. Capital Structure	
Bonds	$250,000,000
Common stock	200,000,000
Retained earnings	300,000,000
	$750,000,000

4. Research Project

Compile a list of business firms operating in your area. On the basis of information available to you, group these firms into roughly equivalent risk classes.

Next, obtain a recent balance sheet for each firm. Does each firm in a given risk class have a similar capital structure? Could any divergence reflect the risk preferences of management?

Finally, compute the average capital structure of the firms in a given risk class. Place these averages on a continuum running from the average of the class of least risk to that of the greatest. Can a trend be noted? Are there significant deviations from this trend?

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Dividends and Valuation

9

The third major decision the firm makes concerns dividend policy. In this chapter, we investigate only one aspect of dividend policy – the percentage of earnings paid to stockholders in cash. In the next chapter, we take up other aspects of dividend policy. The dividend-payout ratio, of course, determines the amount of earnings retained in the firm and affects the total amount of internal financing. Consequently, it must be analyzed in relation to the overall financing decision. If the value of the firm is a function of its dividend-payout ratio, dividend policy will affect directly the firm's cost of capital.

Thus, dividend policy must be evaluated in light of the objective of the firm – namely, to choose a policy that will maximize the value of the firm to its shareholders. Shareholder wealth includes not only the market price of the stock but also current dividends.[1] In this chapter, we examine the

[1] See James T. S. Porterfield, *Investment Decisions and Capital Costs* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965), Chapter 6. The market price of the stock is the price after the payment of the dividend, and the dividend is the contemplated current dividend per share.

question of whether dividend policy can affect shareholder wealth. Again, we assume that business risk is held constant; that is, the acceptance of any investment proposal does not affect the total business-risk complexion of the firm.

MEASUREMENT OF EARNINGS

The dividend-payout ratio of a firm obviously depends upon the way earnings are measured. For ease of theoretical exposition, we use accounting net earnings, but we assume that these earnings conform to the true economic earnings of the firm. In practice, net earnings may not so conform; as a result, they may not be an appropriate measure of the ability of the firm to pay dividends. Certain writers argue that cash flow, the sum of earnings and depreciation, is a better measure of the capacity of a firm to pay dividends. John A. Brittain, for example, suggests that the liberalization of depreciation allowances in the post-World War II period renders net earnings an invalid measure of the ability of corporations to pay dividends.[2] On the basis of an empirical study of the 1920–60 period, he found that cash flow did a significantly better job of explaining corporate dividends than did net earnings. He takes this finding as support for the idea that corporations recognize the illusory nature of reported earnings and instead base dividends upon cash flow. While the argument that cash flow best approximates the "true" earnings of a firm is persuasive, we shall continue to use net earnings in the theoretical development that follows.

DIVIDEND POLICY AS A FINANCING DECISION

In order to evaluate properly the question of whether dividend policy affects shareholder wealth, it is necessary to examine first the firm's policy solely as a financing decision involving the retention of earnings. Consider the situation in which the use of funds from earnings, and the dividend policy that results, is strictly a financing decision. As long as the firm has investment projects whose returns exceed its cost of capital—that is, $r > k_0$—it will use retained earnings, and the amount of senior securities the increase in equity base will support, to finance these projects. If the firm has retained earnings left over after financing all acceptable investment opportunities, these earnings then would be distributed to stockholders in the form of cash dividends. If not, there would be no dividends. If the number of acceptable investment opportunities involved a total dollar amount that exceeded the amount of retained earnings plus the

[2]John A. Brittain, *Corporate Dividend Policy* (Washington, D.C.: The Brookings Institution, 1966), Chapter 3.

senior securities these retained earnings will support, the firm would finance the excess with a combination of a new equity issue and senior securities.

To illustrate, suppose that Windermere Corporation is expected to generate $2 million in net earnings after taxes in the coming year. Suppose further that it has an all-equity capital structure and that it was able to measure accurately the discount rate that equates the present value of future dividends expected by investors at the margin with the current market price of its stock. The discount rate is found to be 10 per cent. In keeping with our discussion in Chapter 4, this per cent is treated as the opportunity rate for retained earnings employed in external investments. Because of flotation costs and underpricing, the cost of common stock financing is higher than that of retained earnings; it is found to be 11 per cent.[3] Suppose the company has $1.5 million in projects whose expected internal rates of return exceed 10 per cent. Accordingly, it should retain $1.5 million of the $2 million in expected profits in order to finance these investments.[4] The remaining $500,000 would be distributed to stockholders as dividends. On the other hand, if the firm has $3 million in investment projects whose expected returns exceed 11 per cent, it would retain all its earnings and raise an additional $1 million through an issue of common stock. Thus, stockholders would receive no dividends.

When we treat dividend policy strictly as a financing decision, the payment of cash dividends is a passive residual.[5] The amount of dividend payout will fluctuate from period to period in keeping with fluctuations in the amount of acceptable investment opportunities available to the firm. If these opportunities abound, the percentage of dividend payout is likely to be zero. On the other hand, if the firm is unable to find profitable investment opportunities, dividend payout will be 100 per cent. For situations between these two extremes, the payout will be a fraction between 0 and 1.

WALTER'S FORMULA

To illustrate dividend policy as a financing decision determined solely by the profitability of investment opportunities available, let us examine Walter's formula.[6] His model was one of the earlier theoretical dividend

[3] For simplicity, assume that the cost of common stock financing is invariant with respect to the amount of funds raised. If the firm faces a rising supply curve of capital, of course, this occurrence must be recognized in financing with new issues of common stock.

[4] We assume that allowances for depreciation have already been reinvested in order to maintain the existing earnings potential of the firm.

[5] Ezra Solomon, *The Theory of Financial Management* (New York: Columbia University Press, 1963), pp. 139–40.

[6] James E. Walter, "Dividend Policies and Common Stock Prices," *Journal of Finance,* XI (March, 1956), 29–41.

models, and certain later models correspond to this one. His formula is

$$P = \frac{D + \frac{r}{\rho}(E - D)}{\rho} \qquad (9\text{-}1)$$

where P = market price per share of common stock
D = dividends per share
E = earnings per share
r = return on investment
ρ = market capitalization rate.

Suppose that $r = 0.12$ per cent, $\rho = 0.10$ per cent, $E = \$4$, and $D = \$2$. The market price per share would be

$$P = \frac{2 + (0.12/0.10)(4 - 2)}{0.10} = \$44$$

The optimal dividend-payout ratio is determined by varying D until you obtain the maximum market price per share. Under a strict interpretation of the Walter formula, the optimal dividend-payout ratio should be 0 if r is greater than ρ. Thus, in our example,

$$P = \frac{0 + (0.12/0.10)(4 - 0)}{0.10} = \$48$$

With a payout ratio of 0, market price per share is maximized. Similarly, if r is less than ρ, the optimal payout ratio should be 100 per cent. Suppose that $r = 0.8$ per cent, $\rho = 0.10$ per cent, $E = \$4$, and $D = \$2$. The market price per share then would be

$$P = \frac{2 + (0.08/0.10)(4 - 2)}{0.10} = \$36$$

However, with a dividend-payout ratio of 100 per cent,

$$P = \frac{4 + (0.08/0.10)(4 - 4)}{0.10} = \$40$$

Thus, market price per share can be maximized with a complete distribution of earnings. If $r = \rho$, market price per share is insensitive to the payout ratio.

The Walter formula implies that the optimal dividend payout should be determined solely by the profitability of investments. If the firm has an abundance of profitable investment opportunities, there should be no cash dividends, for the earnings are needed to finance these opportunities. On the other hand, if the firm has no profitable investment opportunities, all earnings should be distributed to stockholders in the form of dividends. In this case, the funds are not needed for financing.

The treatment of dividend policy as a passive residual determined strictly by the availability of acceptable investment proposals implies that dividends are irrelevant; the investor is indifferent between dividends and capital gains. If investment opportunities promise a return greater on the equity-financed portion than the required return, k_e, the investor would prefer to have the company retain earnings. If the return is equal to the required return, he would be indifferent between retention and dividends. Contrarily, if the return were less than the required return, he would prefer dividends. Supposedly, if the firm can earn more on projects than the required return, investors are perfectly happy to let the firm retain as much in earnings as it needs to finance the investments. With irrelevance, the required return is invariant with respect to changes in dividend payout. A question to raise is whether dividends are more than just a means of distributing unused funds. Should dividend policy in any way be an active decision variable? To answer these questions, we must examine more thoroughly the argument that dividends are irrelevant so that changes in the payout ratio (holding investment opportunities constant) do not affect shareholder wealth.

IRRELEVANCE OF DIVIDENDS

The most comprehensive argument for the irrelevance of dividends is found in Modigliani and Miller's 1961 article.[7] They assert that, given the investment decision of the firm, the dividend-payout ratio is a mere detail. It does not affect the wealth of shareholders. MM argue that the value of the firm is determined solely by the earning power on the firm's assets or its investment policy and that the manner in which the earnings stream is split between dividends and retained earnings does not affect this value. The critical assumptions of MM are[8]

1. Perfect capital markets in which all investors are rational. Information is available to all at no cost; transactions are instantaneous and without cost; and no investor is large enough to affect the market price of a security.

2. An absence of flotation costs on securities issued by the firm.

3. A world of no taxes.

[7] Merton H. Miller and Franco Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," *Journal of Business,* XXXIV (October, 1961), reprinted in James Van Horne, ed., *Foundations for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc., 1966), pp. 481–513.

[8] See also John Lintner, "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," *Review of Economics and Statistics,* 44 (August, 1962) 243–69.

4. A given investment policy for the firm, not subject to change.

5. Perfect certainty by every investor as to future investments and profits of the firm. MM drop this assumption later.

DIVIDENDS VERSUS TERMINAL VALUE

The crux of MM's position is that the effect of dividend payments on shareholder wealth is offset exactly by other means of financing. Consider first selling additional stock in lieu of retaining earnings. When the firm has made its investment decision, it must decide whether to retain earnings or to pay dividends and sell new stock in the amount of these dividends in order to finance the investments. MM suggest that the sum of the discounted value per share after financing and dividends paid is equal to the market value per share before the payment of dividends. In other words, the stocks's decline in market price because of external financing offsets exactly the payment of the dividend. Thus, the stockholder is said to be indifferent between dividends and the retention of earnings.

The market price of a share of stock at the beginning of a period is defined as equal to the present value of the dividend paid at the end of the period plus the market price at the end of the period.[9] Thus

$$P_0 = \frac{1}{1 + \rho}(D_1 + P_1) \tag{9-2}$$

where P_0 = market price per share at time 0

ρ = capitalization rate for firm in that risk class (This rate is assumed to be constant throughout time.)

D_1 = dividend per share at time 1

P_1 = market price per share at time 1.

Assume that n is the number of shares of record at time 0 and that m is the number of new shares sold at time 1 at a price of P_1. Eq. (9-2) then can be rewritten as

$$nP_0 = \frac{1}{(1 + \rho)}\left[nD_1 + (n + m)P_1 - mP_1\right] \tag{9-3}$$

In words, the total value of all shares outstanding at time 0 is the present value of total dividends paid at time 1 on those shares plus the total value of all stock outstanding at time 1, less the total value of the new stock issued. The total amount of new stock issued is

$$mP_1 = I - (X - nD_1) \tag{9-4}$$

[9] Miller and Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," in Van Horne, ed., *Foundations for Financial Management*, p. 483. See also Myron J. Gordon, "Optimal Investment and Financing Policy," *Journal of Finance*, XVIII (May, 1963), reprinted in Van Horne, ed., *Foundations for Financial Management*, pp. 526–27.

where I = total new investments during period 1
    X = total net profit of firm for the period.

The total amount of financing by the sale of new stock is determined by the amount of investments in period 1 not financed by retained earnings. By substituting Eq. (9-4) into Eq. (9-3), MM find that the nD_1 term cancels out and

$$nP_0 = \frac{1}{(1 + \rho)} [(n + m)P_1 - I + X] \qquad (9\text{-}5)$$

As D_1 does not appear directly in the expression and since X, I, $(n + m)P_1$, and ρ are assumed to be independent of D_1, MM conclude that the current value of the firm is independent of its current dividend decision.[10] What is gained by the stockholder in increased dividends is offset exactly by the decline in the terminal value of his stock. MM go on to show that nP_0 is unaffected not only by current dividend decisions but by future dividend decisions as well. Under the assumption of perfect certainty by all investors, the price of the stock at time 1, time 2, and time n is determined solely by Eq. (9-5). Thus, stockholders are indifferent between retention and the payment of dividends (and concurrent stock financing) in all future periods. As a result, shareholder wealth is unaffected by current and future dividend decisions; it depends entirely upon the expected future earnings stream of the firm.

Given MM's assumptions of perfect certainty as well as their other assumptions, the irrelevance of dividends follows. As with our example for corporate leverage, arbitrage assures that the sum of market value plus current dividends of two firms identical in all respects other than dividend-payout ratios will be the same. The individual investor can retain and invest his own earnings, and do this as well as the corporation could for him.[11]

One point needs clarification, however. In our example, we have assumed that external financing involves the sale of new stock and that the effect of this sale on the market price of the stock offsets exactly the payment of dividends. What if the external financing involved debt? MM's position then rests upon their previous indifference thesis with respect to leverage: the real cost of debt is the same as the real cost of equity financing. Therefore, according to MM, the means of external financing used to offset the payment of dividends does not affect their hypothesis that dividends are irrelevant. Thus, we see the interdependency of MM's two positions. Dividend policy does not affect their thesis

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[10] Miller and Modigliani, "Dividend Policy," in Van Horne, ed., *Foundations for Financial Management*, p. 485.

[11] For illustrations of the arbitrage process for the dividend decision, see Wilbur G. Lewellen, *The Cost of Capital* (Belmont, Calif.: Wadsworth Publishing Co., Inc., 1969), pp. 54–57; and James E. Walter, *Dividend Policy and Enterprise Valuation* (Belmont, Calif.: Wadsworth Publishing Co., Inc., 1967), pp. 106–10.

regarding leverage; in their position on dividends, the means of external financing is not a factor.

If dividends are irrelevant, a firm's cost of capital would be independent of its dividend-payout ratio. If both leverage and dividends are irrelevant, the firm would be indifferent as to whether investment opportunities were financed with debt, retained earnings, or a common stock issue. One method of financing would be as satisfactory as the next.

DIVIDENDS UNDER UNCERTAINTY

MM drop their assumption of complete certainty and consider the case of uncertainty. Despite their admission that D_1 and P_1 in Eq. (9-2) are subject to uncertainty, they conclude that dividend policy continues to be irrelevant. Their conclusion is based upon the familiar arbitrage argument. Given two firms of identical business risk and the same prospective future earnings and investment policies, the market prices of the two firms must be the same if there is "symmetric market rationality."[12] Symmetric market rationality occurs when every market participant behaves rationally in preferring more wealth to less, and believes that other market participants behave in the same manner. According to MM, differences in current and future dividend policies cannot affect the market value of these two firms, for the present value of prospective dividends plus terminal value are the same. Even under uncertainty then, MM continue to maintain that dividend policy is irrelevant, given the investment policy of the firm. Attacks upon the irrelevance doctrine have been centered on the case of uncertainty and imperfections in the markets.

ARGUMENTS FOR RELEVANCE

A number of arguments have been advanced in support of the contrary position—namely, that dividends are relevant under conditions of uncertainty.[13] In other words, the investor is not indifferent as to how the earnings stream is split between dividends and retained earnings. We shall examine these arguments under conditions of uncertainty but

[12] Miller and Modigliani, "Dividend Policy," in Van Horne, ed., *Foundations for Financial Management,* pp. 506–8.

[13] Lintner, "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," pp. 254–60, has classified uncertainty in several stages. *Fully idealized uncertainty* describes the situation in which information needed to formulate probability distributions of possible events is distributed uniformly among all market participants, and the probability distributions of possible events of all participants are identical. *Uncertainty with uniform information and diverse judgmental distributions,* as the name implies, recognizes that the subjective probability distributions formulated by market participants need not be identical. Finally, generalized uncertainty describes the situation in which both the "quality" and the "quantity" of information is not distributed uniformly. Lintner argues that only in the case of fully idealized uncertainty is dividend policy irrelevant.

will keep intact MM's other assumptions—no transaction or flotation costs, the absence of taxes, and a given fixed investment policy of the firm. Later, the first two of the assumptions will be removed when we investigate dividend policy under real-world conditions.

Resolution of Uncertainty. It has been argued that the payment of current dividends resolves uncertainty in the minds of investors, and, therefore, an investor is not indifferent between dividends and capital gains. He prefers dividends. Gordon, for example, contends that uncertainty on the part of investors increases at an increasing rate with the distance in the future of prospective cash payments.[14] If investors are averse to risk, the discount rate, ρ_t, will rise with the distance in the future, t. When a company cuts its dividend to finance investments, its near dividend is reduced, while distant dividends are increased. If the discount rate rises with the length of time in the future, the reduction in the near dividend will lead to a lower share price, all other things remaining the same.

According to Gordon, investors are not indifferent between current dividends and the retention of earnings with the prospect of future dividends, capital gains, or both. They prefer the early resolution of uncertainty and are willing to pay a higher price for the stock that offers the greater current dividend, all other things held constant. Thus, the rate of return required by investors, ρ, which represents a weighted average of the ρ_t, would rise with the percentage of earnings retained. This occurrence would make the retention of earnings an increasingly more expensive method of financing, the greater the retention rate. The relationship is illustrated in Figure 9-1, where the required rate of return is on the vertical axis and the retention rate, which represents one minus the dividend-payout ratio, is on the horizontal axis. This is not to say that superior investment opportunities cannot more than offset the rise in the required rate of return. However, the favorable effect of these opportunities upon share price is due to the investment decision and not to the dividend decision.

Informational Content of Dividends. The argument above is allied

[14]Myron J. Gordon, "Optimal Investment and Financing Policy," *Journal of Finance,* XVIII (May, 1963) and "The Savings Investment and Valuation of a Corporation," *Review of Economics and Statistics,* 44 (February, 1962), reprinted in Van Horne, ed., *Foundations for Financial Management* pp. 434–59. Unfortunately, Gordon mixes the investment and dividend decisions by restricting his analysis to internal financing. However, his basic hypothesis of an increasing market discount rate with length of time in the future can be extended easily to an external financing model. See Robert C. Higgins, "Dividend Policy and the Valuation of Corporate Shares under Uncertainty," (Ph.D. dissertation, Stanford University, 1968), pp. 33–39. For an extended analysis of the relationship between the discount rate and distance in the future in the context of the Gordon model, see Houng-Yhi Chen, "Valuation under Uncertainty," *Journal of Financial and Quantitative Analysis,* II (September, 1967), 313–25.

RETENTION RATE (1 — dividend-payout ratio)

REQUIRED RATE OF RETURN
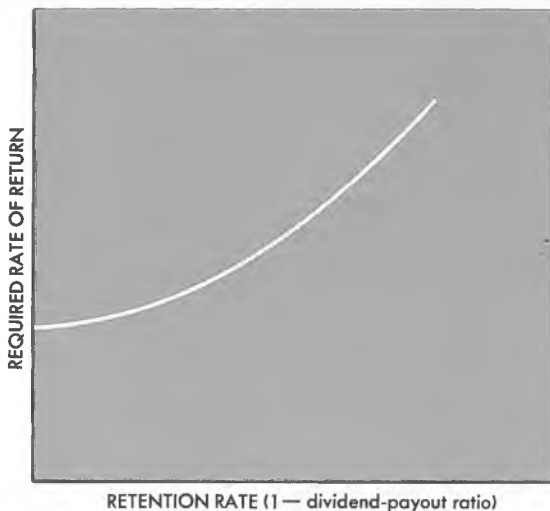
FIGURE 9-1
Relationship between required rate of return and the retention rate

closely to the "informational content of dividends" argument. The latter argument implies that dividends have an impact on share price because they communicate information to investors about the firm's profitability. When a firm has a target payout ratio that is stable over time, and it changes this ratio, investors may believe that management is announcing a change in the expected future profitability of the firm. Accordingly, the price of the stock may react to this change in dividends. Solomon contends that dividends may offer tangible evidence of the firm's ability to generate cash, and, as a result, the dividend policy of the firm affects share price. ". . . In an uncertain world in which verbal statements can be ignored or misinterpreted, dividend action does provide a clear-cut means of 'making a statement' that 'speaks louder than a thousand words.' "[15] MM do not deny the possibility of this effect but continue to maintain that present and expected future earnings are what determine value. They assert that dividends are merely a reflection of these factors and do not in themselves determine value; therefore, the irrelevance proposition holds.[16] Thus, dividends are said to be used by investors as predictors of the firm's future performance; they convey management's expectation of the future. If subsequent observed behavior is consistent with the information conveyed previously, investors will come to rely upon dividends as predictors of what is to come.[17] However, the basic factor affecting value is not dividends but expectations of future performance.

[15] Solomon, *The Theory of Financial Management*, p. 142.

[16] Miller and Modigliani, "Dividend Policy," in Van Horne, ed., *Foundations for Financial Management*, p. 509.

[17] See Walter, *Dividend Policy and Enterprise Valuation*, pp. 90–98.

Preference for Current Income. Another aspect of the uncertainty question involves investors who have a preference for current income. Under the irrelevance proposition, MM would argue that these investors can sell stock on a periodic basis to obtain income. With perfect markets, the investor always could sell part of his holdings or reinvest the dividends to satisfy his desire for consumption. Over the long run, the investor should be able to obtain the same income as he would with regular dividends. However, with uncertainty, stock prices fluctuate. Certain investors may regard as unsatisfactory the alternative of selling a portion of their stock for income at fluctuating prices. As a result, they may have a definite preference for current dividends. In addition to the uncertainty of the selling price, the inconvenience of selling a small portion of stock periodically for income may be a factor. For this reason alone, certain investors might prefer current dividends to capital gains.

Proponents of the irrelevance proposition would argue, however, that there may be no systematic preference in the market as a whole for current dividends. Whereas certain investors might prefer dividends, others prefer capital gains. At the margin, the market may behave in a manner consistent with the irrelevance proposition. MM suggest that

> If, for example, the frequency distribution of corporate-payout ratios happened to correspond exactly with the distribution of investor preferences for payout ratios, then the existence of these preferences would clearly lead ultimately to a situation whose implications were different in no fundamental respect from the perfect market case. Each corporation would tend to attract to itself a "clientele" consisting of those preferring its particular payout ratio, but one clientele would be entirely as good as another in terms of the valuation it would imply for the firm.[18]

Although the "clientele" theory is intuitively appealing, the case for the exact correspondence of the two distributions is tenuous.

Sale of Stock at a Lower Price. The irrelevance doctrine also rests upon the assumption that the sale of stock by the firm to replace the dividend will be at the current price. In order to sell the stock, however, the firm must appeal to new investors or to existing stockholders to increase their holdings. With divergent investor expectations, Lintner contends that the equilibrium price of a share of stock will decline as the firm sells additional stock to replace dividends.[19] In other words, there is a downward sloping demand curve for the stock. Those investors holding the stock believe that its value is at least as great as the prevailing price. However, investors who do not hold the stock believe that the value is less

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[18] Miller and Modigliani, "Dividend Policy," in Van Horne, ed., *Foundations for Financial Management,* p. 510.

[19] Lintner, "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," pp. 256–59.

than this price. In order to entice them into buying a new issue of stock, the issuing price must be lower than the prevailing price. Thus, "the payment of an added cash dividend . . . requires that ownership . . . be transferred to others whose expectations justify *their* holding (buying) the stock only at a lower price, so that *this* shift in finance-mix and current dividend *reduces* the (conditional) expected *aggregate* market value of the company's equity *ceteris paribus*."[20] With underpricing, the firm will need to sell more shares to replace the dividend. This dilution will cause a lower discounted value per share after financing than was true in the case of our irrelevance example. Thus, a downward sloping demand curve for new issues of stock implies a preference toward retention, as opposed to paying higher dividends.

REMOVAL OF OTHER ASSUMPTIONS

We now consider other factors that were assumed away by MM. Any attack on MM, however, must be based upon factors other than the ones we take up now. Nevertheless, the market imperfections we discuss are important in evaluating the effect of dividends upon valuation in the real world.

Tax Effect. When we remove the assumption of no personal taxes, we must take account of the fact that dividends are not taxed the same as are capital gains, at least for taxable investors. The capital gains tax is one-half that of the ordinary income tax, with a maximum of 25 per cent on capital gains. Moreover, the capital gains tax is deferred until the investor actually sells his stock. Thus, there is a strong bias in favor of capital gains as opposed to dividends, and this bias favors the retention of earnings.[21] For example, suppose a corporation pays a substantial dividend and expands by selling stock on a privileged-subscription basis to existing stockholders. These stockholders receive dividends, which are taxed at the ordinary income tax rate, and then purchase more stock. If the corporation had retained the earnings, the tax would have been avoided. The stockholder could realize value on his investment by selling some of his shares and paying only a capital gains tax. The effect of the tax differential must be qualified to take account of the growing number of tax-free institutional investors. For these investors, the tax effect would not influence their preference for dividends or capital gains.

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[20] *Ibid.*, p. 258.

[21] In testing the price behavior of stocks when they go ex-dividend in relation to the magnitude of dividend for a large sample of companies, Elton and Gruber found an inverse relationship between the dividend payout ratio and the implied tax bracket of marginal investors. They interpret this result as consistent with a clientele effect where investors in high tax brackets show a preference for capital gains over dividends and vice versa. Edwin J. Elton and Martin J. Gruber, "Marginal Stockholder Tax Rates and the Clientele Effect," *Review of Economics and Statistics,* LII (February, 1970), 68–74.

In his extensive study of dividend-payout ratios, Brittain found that for the 1920-60 period corporate payout ratios tended to vary inversely with the differential between tax rates on ordinary income and capital gains.[22] In other words, rising tax rates tended to depress dividends. This finding is consistent with stockholders' preferring lower dividends when the differential increases, and with corporations adjusting their dividend policies to the desires of their stockholders.

Flotation Costs. The irrelevance proposition is based upon the idea that, given the investment policy of the firm, funds paid out of the firm must be replaced by funds acquired through external financing. The introduction of flotation costs favors the retention of earnings in the firm. For each dollar paid out in dividends, the firm nets less than a dollar after flotation costs per dollar of external financing. Moreover, the smaller the size of the issue, the greater in general the flotation costs as a percentage of the total amount of funds raised. In addition, stock financing is "lumpy" in the sense that small issues are difficult to sell even with high flotation costs. The fact that common stock financing is less than perfectly divisible in practice favors the retention of earnings.

Transaction Costs. Transaction costs involved in the sale of securities favor current dividends relative to the retention of earnings. The stockholder who desires current income must pay a brokerage fee on the sale of a portion of his stock if the dividend paid is not sufficient to satisfy his current desire for income. This fee varies inversely, per dollar of stock sold, with the size of sale. For a small sale, the brokerage fee can be rather significant. Brokerage fees, together with the inconvenience and uncertainty associated with the sale of stock, may cause investors interested in current income to prefer dividends to capital gains.

Of the market imperfections taken up in this section, the differential tax effect on dividends and capital gains is by far the strongest. As mentioned previously, this imperfection creates a preference for the retention of earnings in the firm.

OPTIMAL DIVIDEND POLICY

The optimal dividend policy of a firm depends directly upon whether dividends are relevant. If dividends do matter, there will be an optimal dividend policy that will maximize shareholder wealth. If not, dividend policy is a mere detail. We saw in the previous section that the desire of investors to resolve uncertainty, any preference by them for current income, and the existence of transaction and inconvenience costs favor

[22] *Corporate Dividend Policy,* Chapter 4.

current dividends over capital gains. However, underpricing of stock issues, flotation costs, and a differential tax rate between dividend income and capital gains favor the retention of earnings. Whether the first set of factors more than offsets the second in their joint impact on valuation is far from clear; however, certain evidence does bear on the problem.

EMPIRICAL FINDINGS [23]

Most empirical tests suggest that dividend payout does affect the value of the firm. However, the regression studies that have been undertaken are crude and subject to a number of statistical biases.[24] Many of them are based upon cross-sectional samples of companies and employ the following regression model

$$P = a + b_1D + b_2R + e \tag{9-6}$$

where P = market price per share
D = dividends per share
R = retained earnings per share
e = error term

Usually, it is found that the regression coefficient for dividends, b_1, is greater and more statistically significant than that for retained earnings, b_2. As a result, it is concluded that dividends are more important than retained earnings in determining the market price of the stock. One difficulty with this approach is that it is not possible to determine an optimal dividend payout with a linear regression model. If information is to be given about an optimal payout, the relationship must be allowed to be curvilinear.[25] Apart from this issue, the type of analysis depicted in Eq. (9-6) involves a number of biases.

The principal bias comes from omitted variables, particularly risk and growth variables. To the extent that high risk firms have low dividend-payout ratios and low market prices, and low risk companies have high dividend-payout ratios and high market prices, there is a direct relationship between dividends and market price. The real explanatory variable, however, is the risk or quality of the firm and not its dividend payout. The omission of a growth variable is likely to work in the opposite direction. To the extent that growth companies are associated with low divi-

[23] As in Chapter 7, this section assumes a knowledge of regression analysis and may be omitted by the student not having such background.

[24] For a discussion of these biases, see Irwin Friend and Marshall Puckett, "Dividends and Stock Prices," in Van Horne, ed., *Foundations for Financial Management,* pp. 535–61.

[25] As taken up in the previous chapter, a cross-sectional regression analysis supposes that firms in the sample are not all at an optimal dividend payout. If they are, the regression study will depict a lack of significant relationship between dividend payout and share price. For an analysis of this point as well as of the biases involved in empirically testing for the effect of dividend policy, see Higgins, "Dividend Policy and the Valuation of Corporate Shares under Uncertainty," Appendix.

dend-payout ratios and high price/earnings ratios, and nongrowth companies are associated with high dividend-payout and low price/earnings ratios, the regression results are biased in favor of the retention variable.

Although the use of a multiple regression may allow the researcher to hold constant the influence of other variables, it is extremely difficult to determine the proper magnitude of these variables. As mentioned in the previous chapter, we attempt to measure the expected growth and risk of the company as perceived by investors at the margin at a moment in time. However, the specification of these expectations is likely to be only an approximation of the "true" explanatory variables. Errors in the measurement of earnings may also bias the regression results in favor of dividends. Whereas dividends are measured precisely, earnings are not. Because of differences in accounting treatment, earnings may vary from "true" earnings; and these deviations may cause a bias in favor of dividends.[26] The fact that earnings tend to vary more than dividends also will bias the results in this direction, unless earnings are normalized. When the regression results on dividend policy are analyzed critically, most of the evidence with respect to the effect of dividends on valuation turns out to be rather mixed.

In one of the most interesting recent empirical studies, however, Eugene F. Brigham and Myron J. Gordon (BG) test whether investors in electric utility stocks are indifferent between dividends and capital gains.[27] This study was described in the previous chapter in connection with its implications for leverage. BG start with a perpetual-growth dividend capitalization model

$$P_0 = \frac{D_0}{k_e - g} \qquad (9\text{-}7)$$

where P_0 is the current market price per share of stock, D_0 is the current dividend per share, k_e is the equity-capitalization rate, and g is the growth rate in dividends per share expected by investors at the margin. Rearranging,

$$\frac{D_0}{P_0} = k_e - g \qquad (9\text{-}8)$$

This equation gives rise to the following linear regression model

$$\frac{D_0}{P_0} = a_0 + a_1 g \qquad (9\text{-}9)$$

For a sample of companies, the constant term, a_0, is said to be an estimate of k_e if the regression coefficient a_1 is to equal -1. However, for a_1

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[26] Friend and Puckett, "Dividends and Stock Prices," in Van Horne, ed., *Foundations for Financial Management*, pp. 664–65.

[27] "Leverage, Dividend Policy, and the Cost of Capital," *Journal of Finance*, XXIII March, 1968), 85–103.

to equal -1, investors would have to be indifferent between current dividends and capital gains arising from growth. If $a_1 > -1$, the evidence is said to be consistent with investors preferring current dividends to capital gains.

To reduce the problem of specification bias as well as to test for the effect of leverage, BG introduce additional explanatory variables into Eq. (9-9). Their expanded regression equation is

$$\frac{D_0}{P_0} = a_0 + a_1g + a_2h + a_3u + a_4e + a_5s \qquad (9\text{-}10)$$

where g = growth measure based upon past growth in retained earnings
$\quad\ \ h$ = debt-to-equity ratio
$\quad\ \ u$ = an index of earnings stability
$\quad\ \ e$ = fraction of sales represented by electricity
$\quad\ \ s$ = an index of corporate size.

BG tested the model with a sample of sixty-nine electric utility stocks for each of the years from 1958 to 1962. They found that the a_1 regression coefficient averaged about $-.4$. BG interpret this finding as consistent with investors preferring current dividends to capital gains, all other things being the same. They suggest that the cost of equity capital increases with the corporation's retention rate. Thus, the evidence is viewed as consistent with the relevance of dividends.[28]

These results contrast with an empirical study by Modigliani and Miller dealing with stocks in the electric utility industry during the 1954–57 period.[29] They used a two-stage, instrumental variable regression analysis and concluded from the results that the effect of dividends upon valuation "is sufficiently small and uncertain to be neglected." MM conjecture that the impact of dividends is mainly informational. Unfortunately, the regression analysis is too involved to examine here. It has been the subject of several criticisms questioning the methodology and measurement of variables.[30] These attacks detract from MM's conclusion

[28] One possible bias in the model relates to the use of a perpetual-growth model. We know from Chapter 4 that no firm is likely to grow forever at a rate faster than that for the economy as a whole. Eventually, its growth tends to taper off. If the measured past growth for the companies in the sample exceeds the "true" growth expected by investors at the margin, the regression coefficient a_1, would tend to be biased upward. That is, it would be biased toward a lower negative number. However, with the stability that accompanies a regulated industry, past growth is likely to provide a reasonable estimate of future growth. For additional criticisms of the BG study, see comments by Michael Davenport and by Morris Mendelson, and the reply by BG, "Leverage, Dividend Policy and the Cost of Capital: Comments and Reply," *Journal of Finance*, XXV (September, 1970), 893–908.

[29] Merton H. Miller and Franco Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility Industry," *American Economic Review*, LVI (June, 1966), 334–91.

[30] See comments by Jean Crockett and Irwin Friend; Myron J. Gordon; and Alexander A. Robichek, John G. McDonald, and Robert C. Higgins; and the reply to these comments by Merton H. Miller and Franco Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954–1957: Comments and Reply," *American Economic Review*, LVII (December, 1967), 1258–1300.

that the evidence supports the idea that dividend policy is irrelevant.

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CHAP. 9
*Dividends
and Valuation*

Both the BG and MM empirical studies investigate the simultaneous effect of dividend policy, leverage, and growth (a proxy for investment policy) on the value of the firm. While much more empirical work needs to be done on the effect of dividend policy on share price, a simultaneous approach to the problem is likely to yield the most meaningful results.

DIVIDEND POLICY AND INVESTMENT OPPORTUNITIES

A company should endeavor to establish a dividend policy that will maximize shareholder wealth. In theory, the optimal dividend payout should be determined in keeping with the firm's investment opportunities and any preference that investors have for dividends as opposed to capital gains. Insight into such a preference can best be gained through an empirical study of the relationship between share price and dividend payout for a sample of similar companies. The key arguments supporting the idea that investors have a systematic preference for current dividends over capital gains are the resolution of uncertainty, the desire for current income, and transaction costs. A "clientele" theory reduces to some extent the validity of the current income argument; investors, in fact, can choose stocks of companies having a dividend payout that corresponds to their desire for current income. With transaction costs, however, it is both costly and inconvenient for a "clientele" of stockholders to change companies. In addition, these transaction costs and the inconvenience involved work to the disadvantage of a stockholder selling a portion of his stock for current income. Offsetting these factors is the more favorable tax treatment of capital gains relative to that of dividends. This factor, of course, creates a preference for capital gains as opposed to current dividends. Whether there is a net preference on the part of investors for dividends as opposed to capital gains or vice versa is determined to a large extent by the combined influence of uncertainty resolution, desire for current income, transaction costs, and differential tax rates.

Any net preference for current dividends must be balanced against the fact that flotation costs and underpricing make a sale of common stock a more "expensive" form of financing than the retention of earnings. If common stock financing were not more expensive and there were a net preference on the part of investors for current dividends, the firm could always sell common stock to pay dividends.[31] However, with a difference in "cost" between the two methods of financing, any net preference on the part of investors for current dividends must be judged in relation to this difference.

To illustrate, suppose a firm had a capital structure consisting entirely

[31] We assume that the firm wishes to maintain a relatively constant capital structure and not issue debt to pay dividends.

of equity and had enough acceptable investment opportunities to just exhaust its earnings.[32] Suppose further that, on the basis of an empirical test, it determined that investors had a net preference for current dividends and that share price initially could be raised by increasing the payout ratio, all other things remaining the same. This phenomenon is illustrated by the upper curve in Figure 9-2. We see that share price increases at a decreasing rate with dividend payout until eventually it turns down. This pattern is consistent with the resolution of uncertainty and preference for current dividends being quite important to investors when dividends are first declared. However, as more dividends are paid, these factors become less important in relation to the tax disadvantage of dividends as opposed to capital gains. Eventually, the tax disadvantage causes the net preference line to turn down.

Any net preference for current dividends must be balanced against the difference in "cost" between financing with common stock and with retained earnings. Recall that the difference is attributable to underpricing and flotation costs. Given our assumptions, if a dividend is paid, it must be financed with common stock. If either there is a net preference for capital gains as opposed to current dividends or if the net preference for current dividends does not offset the difference in "cost" between common stock financing and financing with retained earnings, the firm in our example should pay no dividends. Suppose that, on the basis of a study, the firm determined that the relationship between share price and the payout ratio, attributable to the difference in cost between common stock and retained earnings financing, could be depicted by the lower curve in Figure 9-2. The greater the payout ratio, of course, the greater the common stock financing that is necessary. The line would be expected to decline because of flotation costs and underpricing, both of which cause increased dilution relative to that which would occur if stock could be sold at the current price with no flotation costs. If the amount of underpricing increases with the amount of financing, the line might decline at an increasing rate, as illustrated in the figure.

The combined influence of these two factors determines the relationship between share price and the payout ratio, holding other things constant. The optimal dividend payout would be the one which maximized share price, point X in the figure. Thus, the optimal dividend-payout ratio is determined by balancing any net preferences of investors for current dividends as opposed to capital gains with the flotation cost and underpricing disadvantage of common stock financing.

If the firm does not have enough attractive investment projects to utilize its entire earnings, the optimal payout ratio is affected by this

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[32] In reality, the acceptance criterion may be affected by the dividend policy employed. However, we assume for simplicity that the last acceptable project provides a return significantly in excess of the required rate of return, while the next most profitable project provides a return significantly below the required return.
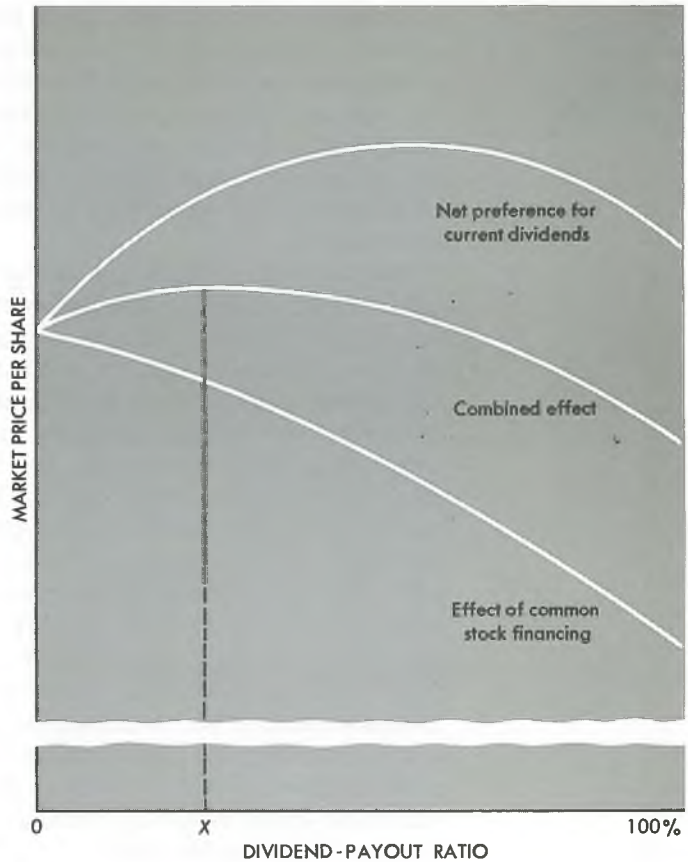
FIGURE 9-2
Hypothetical relationship between share price and dividend payout

factor as well. Under such circumstances, the firm should determine the portion of earnings that cannot be employed profitably in investment projects. These earnings should be earmarked for payment to stockholders in the form of dividends. The firm should then analyze, on the basis of the considerations discussed above, whether it should pay an even higher dividend than dictated by the amount of funds left over after investment. When this has been decided, the amount of dividend and the dividend-payout ratio can be determined.

In conclusion, if dividend policy is not relevant, a firm should choose its dividend policy solely in keeping with its long-run investment opportunities. At the point at which the return on investment is less than the cost of capital, the firm should stop investing and should pay the unused funds out as dividends. The growth company that expands faster than its growth in earnings would pay no dividends, whereas the firm in a shrinking industry might have a 100 per cent dividend payout or even a liquidating dividend. The firm need not pay out the exact unused portion of

earnings every period. Some years, the payout may be more; other years, it may be less. Indeed, the firm may want to stabilize the absolute amount of dividends paid from period to period but in such a manner that over the long run the total earnings retained, plus the senior securities the increasing equity base will support, correspond to the amount of profitable investment opportunities available. Dividend policy would still be a passive decision variable determined by the amount of investment opportunities.

For the firm to be justified in paying a dividend larger than that dictated by the amount of profitable investment opportunities, there must be a net preference of investors for current dividends as opposed to capital gains. Moreover, this net preference must more than offset the fact that owing to flotation costs and underpricing, common stock is a more expensive form of financing than is the retention of earnings. Although empirical evidence concerning the effect of dividends on the market price of a stock is far from clear in this regard, many companies appear to behave as if dividends are relevant. For example, a number of growth companies which expand at a rate faster than their growth in earnings pay small dividends. If these companies believed dividends were irrelevant, they would retain all their earnings. A number of other companies that pay significant dividends go to the capital markets for additional equity capital rather than retain a greater portion of earnings. Examples include public utilities and airlines. Whereas these actions do not support the idea that dividends are relevant, they do indicate that many companies behave as though they were. The dividend-payout ratio that these firms believe is optimal is greater than that dictated by investment opportunities alone.

SUMMARY

The critical question in dividend policy is whether dividends have an influence upon the value of the firm, given its investment decision. If dividends are irrelevant, as Modigliani and Miller believe, the firm should retain earnings only in keeping with its investment opportunities. If there are not sufficient investment opportunities to provide expected returns in excess of the cost of capital, the unused funds should be paid out as dividends.

The key issue is whether dividends are more than just a means of distributing unused funds. If they do affect the value of the common stock, dividend policy becomes more than a passive variable determined solely by the investment opportunities. The firm could affect shareholder wealth by varying its dividend-payout ratio; as a result, there would be an optimal dividend policy.

In this chapter, we have examined the various arguments for and

against the relevance of dividends. If dividends are relevant, the net preference of investors for current dividends as opposed to capital gains must be balanced against the difference in cost between the sale of stock and the retention of earnings in determining the optimal dividend-payout ratio.

Unfortunately, empirical evidence on the relevance of dividends has been little more than suggestive. Nevertheless, we know that many companies behave as if dividends do matter and can affect shareholder wealth. In the next chapter, we extend our analysis of the dividend payout and examine other aspects of dividend policy.

PROBLEMS

1. The Apex Company earns $5 per share, is capitalized at a rate of 8 per cent, and has return on investment of 10 per cent. Using the Walter dividend policy model, determine:
  (a) The optimum payout.
  (b) The price of the stock at this payout.
  (c) Indicate what would happen to the price of the stock if a payout other than the optimum were employed. Illustrate with an example.

2. The Apex Company (under the conditions outlined in problem 1) will increase earnings per share at a rate of 10 per cent per year. Utilizing the perpetual-growth valuation model, can you develop an argument to justify the observation that "capitalization rates and rates of return on investments tend to become equal in the long run"? Under what conditions might the Walter model not be a fair approximation of reality?

3. The Mann Company belongs to a risk class for which the appropriate capitalization rate is 10 per cent. It currently has outstanding 100,000 shares selling at $100 each. The firm is contemplating the declaration of a $5 dividend at the end of the current fiscal year, which just began. Answer the following questions based on the Modigliani and Miller model and the assumption of no taxes.
  (a) What will be the price of the stock at the end of the year if a dividend is not declared? What will it be if one is?
  (b) Assuming that the firm pays the dividend, has net income of $1,000,000, and makes new investments of $2,000,000 during the period, how many new shares must be issued?
  (c) Is the MM model realistic with respect to valuation? What factors might mar its validity?

4. The Combine Corporation has one million shares outstanding, earns net income of $3 million, and pays dividends of $2 million. The shareholders of Combine have a 60 per cent normal tax rate, a 25 per cent capital gains tax rate, and value Combine by the Graham and Dodd formula of $P = M(\underline{E} + D)$, with a multiple (M) of 12.   3
  (a) At what price does Combine sell?
  (b) If all of the above data remained constant every year, what after-tax return would Combine shareholders earn?
  (c) If Combine desired to raise $1 million in equity funds, show the short-run

effects upon current shareholders of (1) cutting the dividend, or (2) selling stock at 90 per cent of the current market price.

(d) If the new equity would raise earnings by $200,000, consider the long-run effects of the options presented in (c).

(e) Evaluate the validity of the model.

5. The Eureka Mining Company consists of one mine, which has a remaining useful life of five years. The owners feel that no new shafts should be sunk for at least this long; indeed, they probably will not continue in this business after five years. Since the initiation of the company, total cash flows have been paid out in dividends. All of the firm's stockholders were original investors, have a marginal tax rate of 40 per cent, and require an after-tax return of 10 per cent. The mine will generate the following income during each of the next five years.

Net operating income	$25,000
Less: Depreciation & depletion	5,000
Net Income before taxes	20,000
Less: Taxes @ 22%	4,400
Net income	$15,600

(a) Compute the total market value of Eureka if all available funds were paid in dividends at the end of the year they became available.

(b) Compute the total market value assuming that Eureka invests its available funds at the end of each year in bonds yielding 5.13 per cent which mature at the end of year 5; at the end of this time, Eureka will be liquidated.

(c) Instead of bonds, assume that Eureka chooses to invest at the end of each year in 8.27 per cent preferred stock which will be redeemed at cost at the end of year 5. Compute the total market value of Eureka (still assuming liquidation at the end of year 5).

Hint: (1) $5.13\% \times 78\% = 4\%$

(2) Dividends received by corporations are 85 per cent tax exempt.

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Dividend
Policy
of the Firm

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Given its approximate long-run investment requirements, the firm is able to adopt a long-run dividend-payout ratio. This ratio should be determined in keeping with the objective of maximizing shareholder wealth. As discussed in the previous chapter, its determination will depend upon the expected amount of acceptable investment opportunities, the perceived value to investors of dividends as opposed to capital gains, and the difference in cost between common stock financing and the retention of earnings. Although the dividend-payout ratio is a major aspect of the dividend policy of the firm, there are other aspects that may affect valuation. In this chapter, we consider the stability of dividends, certain factors that influence the dividend-payout ratio from the standpoint of the firm, stock dividends and stock splits, the repurchase of stock, and the procedural and legal elements of dividend policy.

In addition to the percentage of dividend payout of a company over the long run, investors may value stable dividends over this period. All other things being the same, the market price of the stock of a company may be higher if it pays a stable dividend over time than if it pays out a fixed percentage of earnings. To illustrate, suppose Company *A* has a long-run dividend-payout ratio of 50 per cent of earnings. Suppose further that it has the policy of paying out this percentage every year, despite the fact that its earnings are cyclical. The dividends of Company *A* are shown in Figure 10-1. Company *B,* on the other hand, has exactly the same earn-
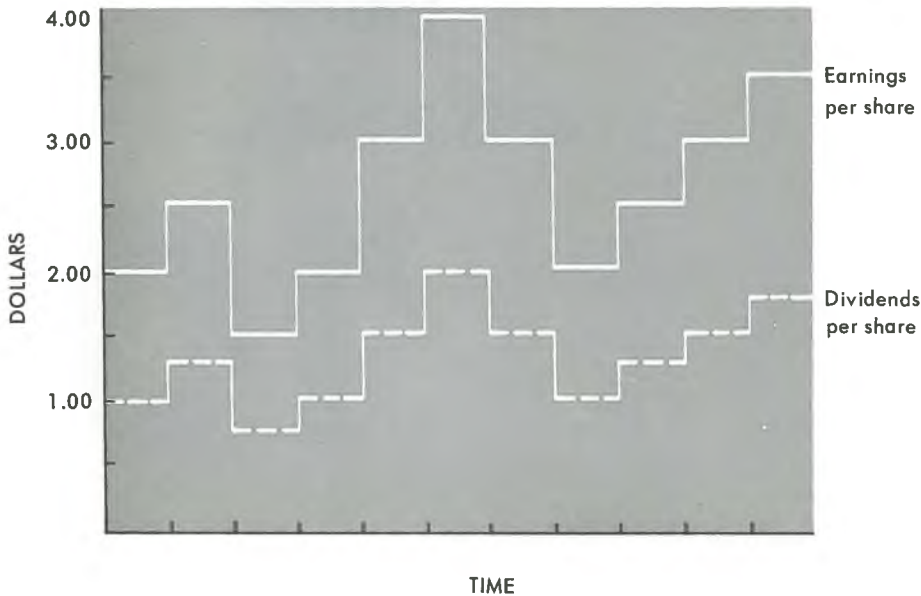
FIGURE 10-1
Hypothetical dividend policy of Company *A*

ings and a long-run dividend-payout ratio of 50 per cent, but it maintains a relatively stable dividend over time. It changes the absolute amount of dividend only in keeping with the underlying trend of earnings. The dividends of Company *B* are shown in Figure 10-2.

Over the long run, the total amount of dividends paid by these two firms is the same. However, the market price per share of Company *B* may be higher than that of Company *A,* all other things being the same. Investors may well place a positive utility on dividend stability and pay a premium for the company that offers such stability. To the extent that investors value dividend stability, the overall dividend policy of Company
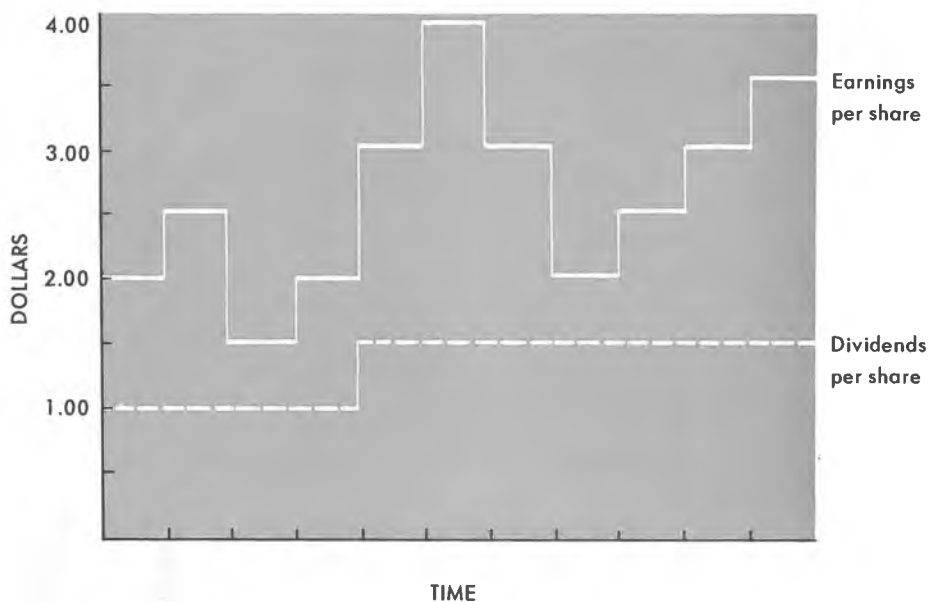
FIGURE 10-2
Hypothetical dividend policy of Company B

B would be better than that of Company A. This policy includes not only the percentage of dividend payout in relation to earnings but also the manner in which the actual dividends are paid. Rather than vary dividends directly with changes in earnings per share, Company B raises the dividend only when reasonably confident a higher dividend can be maintained.

VALUATION OF STABILITY

There are several reasons why investors may value stable dividends and pay a premium for the stock of the company providing such stability. These include the informational content of dividends, the desire of investors for current income, and certain legal considerations.

Informational Content. As we said in the previous chapter, dividends may serve to resolve uncertainty in the minds of investors. When earnings drop and a company does not cut its dividend, the market's confidence in the stock may be bolstered over what it would be if the dividend were cut. The stable dividend may convey to investors management's view that the future of the company is better than the drop in earnings suggests. Thus, management may be able to affect the expectations of investors through the informational content of dividends. Management, however, cannot "fool" the market permanently. If there is a downward trend in earnings,

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a stable dividend will not convey forever an impression of a rosy future. Moreover, if a firm is in an unstable business with wide swings in earnings, a stable dividend cannot give the illusion of underlying stability.

Current Income Desires. A second factor favoring stable dividends is that investors who desire a specific periodic income will prefer a company with stable dividends to one with unstable dividends, even though both companies may have the same pattern of earnings and long-run dividend payout. Although the investor can always sell a portion of his stock for income when the dividend is not sufficient to meet his current needs, many investors have an aversion to "dipping into principal." Moreover, when a company reduces its dividend, earnings usually are down and the market price of the stock depressed. As a result, the investor would have to sell his stock on unfavorable terms. Overall, it would seem that income-conscious investors place a positive utility on stable dividends.

Legal Considerations. Finally, a stable dividend may be advantageous from the legal standpoint of permitting certain institutional investors to invest in the stock. Various governmental bodies prepare legal lists of securities in which pension funds, savings banks, trustees, insurance companies, and others may invest. In order to qualify, a company must have an uninterrupted pattern of dividends. A cut in the dividend may result in the removal of a company from these legal lists.

The arguments presented in support of the notion that stable dividends have a positive effect upon the market price of the stock are only suggestive. There is little in the way of empirical evidence to shed light on the question. While studies of individual stocks often suggest that stable dividends buffer the market price of the stock when earnings turn down, there have been no comprehensive studies of a large sample of stocks dealing with the relationship between dividend stability and valuation. Nevertheless, most companies strive for stability in their dividend payments. This occurrence is illustrated in Figure 10-3, where total corporate dividends and net earnings after taxes are shown for the post-World War II period. Overall, corporations behave in a manner that is consistent with a belief that stable dividends have a positive effect on value. From the standpoint of public policy, dividend stability acts as a built-in stabilizer, for it tends to cushion the effect of changes in income on aggregate demand over economic cycles.[1]

TARGET PAYOUT RATIOS

It would appear that a number of companies follow the policy of a target dividend-payout ratio over the long run. Lintner contends that

[1] See John A. Brittain, *Corporate Dividend Policy* (Washington, D.C.: The Brookings Institution, 1966), p. 212.
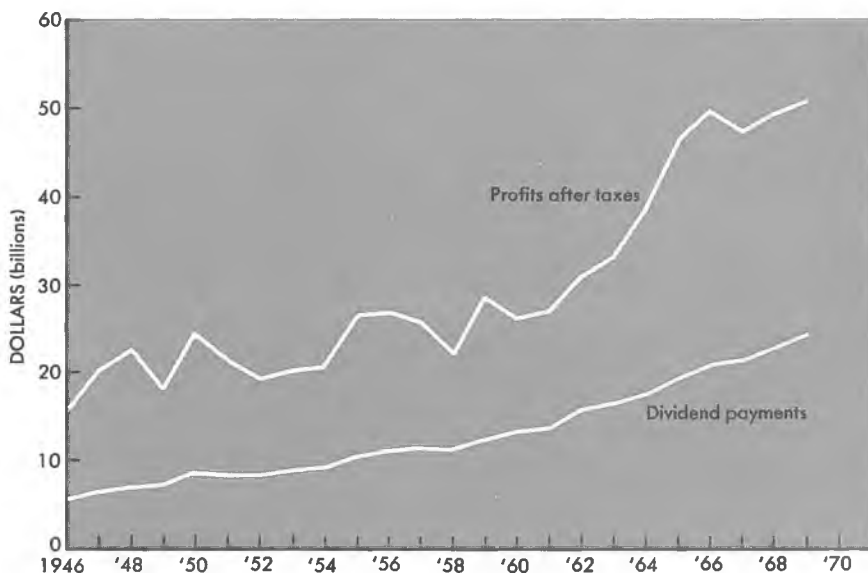
FIGURE 10-3
Total corporate earnings and dividends.
Source: *Economic Report of the President, 1970,* p. 260.

dividends are adjusted to changes in earnings, but only with a lag.[2] When earnings increase to a new level, dividends are increased only when it is felt that the increase in earnings can be maintained. In addition, there appears to be a definite reluctance on the part of companies to cut the absolute amount of their cash dividend. Both of these factors explain the lag in dividend changes behind changes in earnings. Given a lag relationship, retained earnings will increase relative to dividends in an economic upturn. In a contraction, however, retained earnings will decrease relative to dividends. Empirically, the lag of dividend changes behind changes in earnings has been verified by several investigators, using Lintner's target payout ratio model.[3]

To illustrate the use of a target payout ratio and stable dividends, consider the case of Coleman Company. This company makes outdoor recreation equipment and central heating and air-conditioning units. These lines of business are somewhat cyclical, with resulting swings in earnings. However, the company maintained stable and increasing divi-

[2] See John Lintner, "Distribution of Income of Corporations," *American Economic Review,* XLVI (May, 1956), 97–113.

[3] See Brittain, *Corporate Dividend Policy,* Chapters 2–7; and Eugene F. Fama and Harvey Babiak, "Dividend Policy: An Empirical Analysis," *Journal of the American Statistical Association,* 63 (December, 1968), 1132–61.

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dends in the sixties; it seems to raise dividends once management and the board of directors are confident that the earnings can be sustained. The dividends per share and earnings per share for the company are shown in Figure 10-4.

A target payout ratio does not necessarily connote that dividends are the active decision variable and retained earnings passive. The ratio may be set in keeping with the long-run investment needs of the company. If these needs are reasonably predictable, the firm can have an
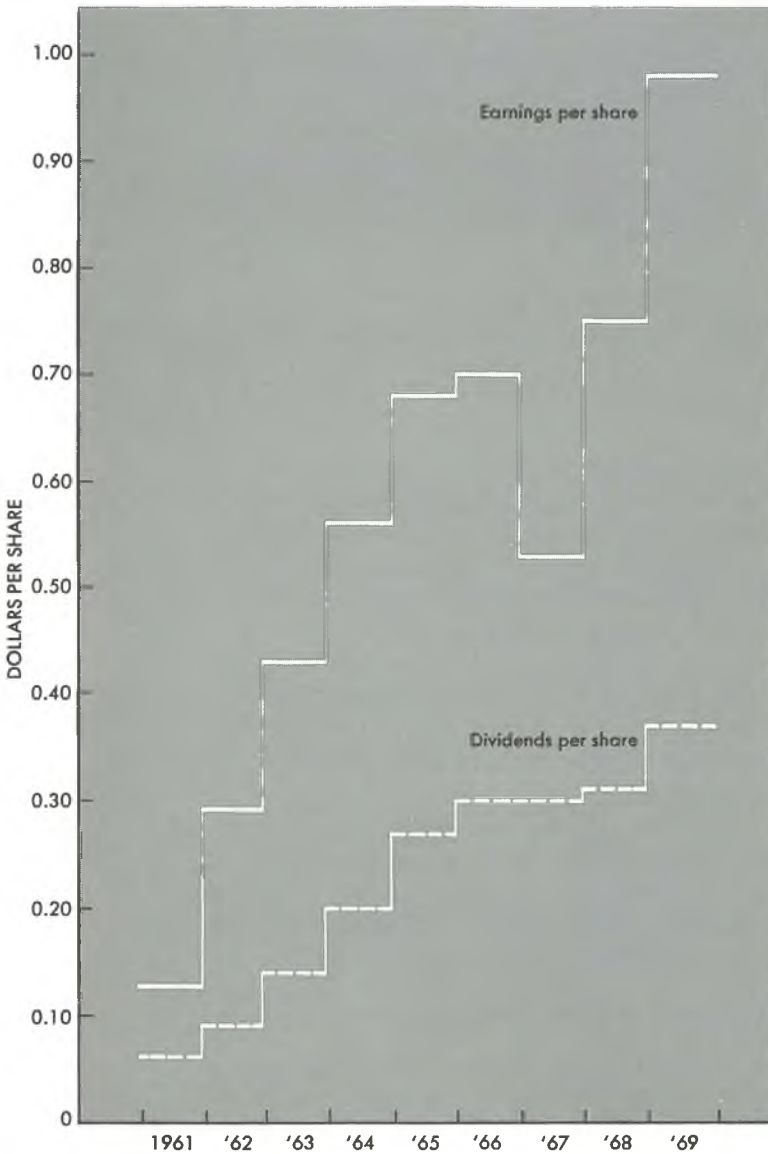

FIGURE 10-4
Coleman Company, dividends and earnings per share, 1961–1969. Source: *Value Line Investment Survey*, April 3, 1970, p. 1429.

established dividend-payout policy and still finance its growth with re-
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tained earnings.[4] In this case dividend policy would be passive.

REGULAR AND EXTRA DIVIDENDS

One way for a company to increase its cash distribution in periods of prosperity is to declare an *extra* dividend in addition to the *regular* quarterly or semiannual dividend. By declaring an extra dividend, the company attempts to prevent investors from expecting that the dividend represents an increase in the established dividend rate. The declaration of an extra dividend is suitable particularly for companies with fluctuating earnings. General Motors, for example, frequently declares extra dividends in good car years. The use of the extra dividend enables the company to maintain a stable record of regular dividends but also to distribute to stockholders some of the rewards of prosperity. By paying extra dividends only when earnings are higher than usual, the company will not lead investors to count on the increased dividends in future periods. However, a company cannot pay extra dividends continuously without conveying to the market some impression of permanency. As soon as a certain level of dividends is recurrent, investors begin to expect that level regardless of the distinction between regular and extra dividends.

OTHER CONSIDERATIONS

So far, we have related the dividend policy of a firm to the investment opportunities of that firm, to the magnitude and stability of earnings, to the possible preference of investors for dividends relative to capital gains, and to the fact that common stock is a more expensive form of financing than is the retention of earnings. However, there are a number of other considerations that influence a company in the dividend policy it undertakes. These considerations tend to be of a more practical nature than those discussed so far.

LIQUIDITY

The liquidity of a company is an important consideration in many dividend decisions. As dividends represent a cash outflow, the greater the cash position and overall liquidity of a company, the greater its ability to pay a dividend.[5] A company that is growing and profitable may not be liquid, for its funds may go into fixed assets and permanent working capital. As the management of such a company usually desires to

[4] See J. Robert Lindsey and Arnold W. Sametz, *Financial Management: An Analytical Approach* (Homewood, Ill.: Richard D. Irwin, Inc., 1967), p. 271.

[5] Brittain, *Corporate Dividend Policy,* pp. 184–87, found that for a sample of forty large firms over the 1920–60 period, dividends were related positively to corporate liquidity.

maintain some liquidity cushion to give it flexibility and a protection against uncertainty, it may be reluctant to jeopardize this position in order to pay a large dividend. The liquidity of the company, of course, is determined by the firm's investment and financing decisions. The investment decision determines the rate of asset expansion and the firm's need for funds; and the financing decision determines the way in which this need will be financed.

ABILITY TO BORROW

A liquid position is not the only way to provide for flexibility and thereby protect against uncertainty. If a firm has the ability to borrow on comparatively short notice, it may be relatively flexible. This ability to borrow can be in the form of a line of credit or a revolving credit from a bank, or simply the informal willingness on the part of a financial institution to extend credit. In addition, flexibility can come from the ability of a firm to go to the capital markets with a bond issue. The larger and more established a company, the better its access to the capital markets. The greater the ability of the firm to borrow, the greater its flexibility, and the greater its ability to pay a cash dividend. With ready access to debt funds, management should be less concerned with the effect that a cash dividend has upon its liquidity.

CONTROL

If a company pays substantial dividends, it may need to raise capital at a later time through the sale of stock in order to finance profitable investment opportunities. Under such circumstances, the controlling interest of the company may be diluted if controlling stockholders do not or cannot subscribe for additional shares. These stockholders may prefer a low dividend payout and the financing of investment needs with retained earnings. Such a dividend policy may not maximize overall shareholder wealth, but it still may be in the best interests of those in control.

Control can work two ways, however. In the case of a company being sought by another company or by individuals, a low dividend payout may work to the advantage of the "outsiders" seeking control. The outsiders may be able to convince stockholders that the company is not maximizing shareholder wealth and that they (the outsiders) can do a better job. Consequently, companies in danger of being acquired may establish a high dividend payout in order to please stockholders.

NATURE OF STOCKHOLDERS

When a firm is closely held, management usually knows the dividend desires of its stockholders and may act accordingly. If most stockholders

are in high tax brackets and prefer capital gains to current income, the firm can establish a low dividend payout. The low payout, of course, would be predicated upon having profitable investment opportunities in which to employ the retained earnings. The corporation with a large number of stockholders does not know the dividend desires of its stockholders. It can judge these desires only in terms of the market price of its stock.

TIMING OF INVESTMENT OPPORTUNITIES

A company may have profitable investment opportunities, but these opportunities may occur too sporadically for the company to be justified in retaining earnings. For example, a firm may know that it will need to build a major extension on its existing plant in about six years. If it retains earnings to finance this plant expansion, the funds will not be used for some period of time. During this period, the company will invest the funds in short-term securities yielding less than the required rate of return on retained earnings. However, shareholder wealth might be better maximized by paying out the intermediate earnings as dividends and raising the capital six years later with a stock issue. The sale of stock is a more desirable means than retained earnings by which to raise a large block of capital at one time.

RESTRICTIONS IN BOND INDENTURE OR LOAN AGREEMENT

The protective covenants in a bond indenture or loan agreement often include a restriction on the payment of dividends.[6] This restriction is employed by the lender(s) to preserve the company's ability to service debt. Usually, it is expressed as a maximum percentage of cumulative earnings. When such a restriction is in force, it naturally influences the dividend policy of the firm. There are times when the management of a company welcomes a dividend restriction imposed by lenders because, then, it does not have to justify to stockholders the retention of earnings. It need only point to the restriction.

INFLATION

Inflation also may have an influence upon dividend policy. With rising prices, funds generated from depreciation are not sufficient to replace and/or restore existing assets as they wear out or become obsolete. Consequently, a case can be made for retaining earnings simply to preserve

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[6] For a more detailed examination of these restrictions, see Chapter 21.

the earning power of the firm.[7] This decision must be based upon considerations taken up in the last chapter concerning investment policy and valuation.

STOCK DIVIDENDS AND STOCK SPLITS

In this section, we take up stock dividends and stock splits. In an economic sense, the two are very similar, although typically they are used for different purposes. Only from an accounting standpoint is there a significant difference.

STOCK DIVIDENDS

A stock dividend simply is the payment of additional stock to stockholders. It represents nothing more than a recapitalization of the company; a stockholder's proportional ownership remains unchanged. To illustrate, suppose a company had the following capital structure before issuing a stock dividend

Common stock ($5 par, 400,000 shares)	$ 2,000,000
Capital surplus	1,000,000
Retained earnings	7,000,000
Net worth	$10,000,000

Now, suppose the company pays a 5 per cent stock dividend, amounting to 20,000 additional shares of stock, and that the fair market value of the stock is $40 a share. For each twenty shares of stock owned, the stockholder receives an additional share. The balance sheet of the company after the stock dividend would be

Common stock ($5 par, 420,000 shares)	$ 2,100,000
Capital surplus	1,700,000
Retained earnings	6,200,000
Net Worth	$10,000,000

With a stock dividend, $800,000 ($40 × 20,000 shares) is transferred from retained earnings to the common stock and capital surplus ac-

[7] Economic depreciation is defined as the amount of investment needed to keep the earning power of an asset the same. This amount may differ significantly from accounting depreciation. See Eugene M. Lerner and Willard T. Carleton, *A Theory of Financial Analysis* (New York: Harcourt Brace Jovanovich, Inc., 1966), Chapter 4.

counts. As the par value stays the same, the increase in number of shares is reflected in a $100,000 increase in the common stock account ($5 par × 20,000 shares). The residual of $700,000 goes into the capital surplus account. The net worth of the company remains the same.

Because the number of shares of stock outstanding is increased by 5 per cent, earnings per share of the company are reduced proportionately. Suppose that total net profit after taxes were $1 million. Before the stock dividend, earnings per share would be $2.50 ($1 million/400,000). After the stock dividend, earnings per share are $2.38 ($1 million/420,000). Thus, the stockholder has more shares of stock but lower earnings per share. His proportion of total earnings available to common stockholders remains unchanged.

Value to Investor. If the company pays no cash dividend, what does the stockholder receive with a stock dividend? In theory, he receives nothing but an additional stock certificate. His proportionate ownership of the company is unchanged. Presumably, the market price of the stock will drop, all other things being equal, so that the total market value of his holdings stays the same. For example, if he held 100 shares of stock previously, and market price per share were $40, the total value of his holdings would be $4,000. After the stock dividend, the price of the stock should drop by $40(1 − 1.00/1.05), or by $1.90. The total value of his holdings then would be $38.10 × 105, or $4,000. Under these conditions, the stock dividend does not represent a thing of value to the stockholder. He simply has an additional stock certificate evidencing ownership.

To the extent that the investor wishes to sell a few shares of stock for income, the stock dividend may make it easier for him to do so. Without the stock dividend, of course, he could also sell a few shares of his original holdings for income. In either case, the sale of stock represents the sale of principal and is subject to the capital gains tax. However, it is probable that certain investors do not look at the sale of a stock dividend as a sale of principal. To them, the stock dividend represents a windfall gain; they can sell it and still retain their original holdings. The stock dividend may have a favorable psychological effect on these stockholders.

The stock dividend can also be a thing of value to the investor if the company maintains the same cash dividend per share after the stock dividend as before. Suppose an investor owns 100 shares of a company paying a $1 dividend and that the company declares a 10 per cent stock dividend and, at the same time, announces that the cash dividend per share will remain unchanged. The investor then will have 110 shares; and total cash dividends will be $110 rather than $100, as before. In this case, a stock dividend increases his total cash dividends. Whether this increase in cash dividend has a positive effect upon shareholder

wealth[8] will depend upon the tradeoff between current dividends and retained earnings, which we discussed in the last chapter. Clearly, the stock dividend in this case represents a decision by the firm to increase modestly the amount of cash dividends.

Use of the stock dividend per se may convey some information. Stock dividends typically are associated with growth companies. Under these circumstances, the dividend may connote to investors that management expects earnings to continue to grow and to more than offset the dilution in earnings per share arising from the increase in the total number of shares. The underlying effect on value is growth, however, and not the stock dividend itself.

Advantages to Company. Frequently, a stock dividend is employed to "conserve cash." Instead of increasing the cash dividend as earnings rise, a company may desire to retain a greater portion of its earnings and declare a stock dividend. The decision then is to lower the dividend-payout ratio, for as earnings rise and the dividend remains the same, the payout ratio will decline. Whether shareholder wealth is increased by this action will depend upon considerations taken up in the previous chapter. The decision to retain a higher proportion of earnings, of course, could be accomplished without a stock dividend. However, the stock dividend may tend to please certain investors by virtue of its informational content as well as its psychological impact. In addition, if the cash dividend per share is kept the same, total cash dividends will increase slowly in keeping with the increase in the number of shares.

In the discussion of advantages so far, the decision to issue a stock dividend has been based upon the availability of profitable investment opportunities. The percentage of cash dividend payout was reduced in order to finance a portion of these opportunities with retained earnings. Certain companies, however, have employed the stock dividend as a means of replacing the cash dividend because of financial difficulty. In these situations, the stock dividend should not connote the prospect of favorable earnings, but financial deterioration, and it should be so evaluated. It is doubtful whether many investors are fooled by the substitution.

The use of a stock dividend by a firm may also serve to keep the market price of the stock within a desired trading range. Certain companies do not like to see the market price of their stocks above a certain amount — say $60 a share — because a high price will not appeal to small investors. Consequently, they will endeavor to keep the price below a desired ceiling either by using stock dividends or, more commonly, by

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[8] In an extensive study of the effect of stock dividends on market value, Barker concluded that the stock dividend was beneficial only when the cash dividend was increased. C. Austin Barker, "Evaluation of Stock Dividends," *Harvard Business Review,* 36 (July, 1958), 99–114. Unfortunately, Barker's study suffers from his not holding other factors constant in his comparisons.

neans of stock splits. Increasing the total number of shares outstanding nay increase the total number of stockholders, resulting in greater over-ll popularity of the stock.

The principal disadvantage of stock dividends to the firm is that they ire much more costly to administer than cash dividends. Another disad-vantage is that small periodic stock dividends, perhaps 2 or 3 per cent, nay tend to distort downward the company's perceived growth in earn-ngs. Whereas investment analysts adjust earnings per share for stock plits and significant stock dividends, many do not do so for small stock lividends. If earnings per share are not adjusted, the measured growth n earnings per share will be less than the true increase in earnings for he investor who held his stock over the period measured. Consequently, t is conceivable that the price/earnings ratio might be somewhat lower hat it would be if earnings per share were adjusted.

TOCK SPLITS

With a stock split, the number of shares are increased through a pro-ortional reduction in the par value of the stock. Suppose that the apital structure of a company before a two-to-one stock split were

Common stock ($5 par, 400,000 shares)	$ 2,000,000
Capital surplus	1,000,000
Retained earnings	7,000,000
Net worth	$10,000,000

ifter the split, the capital structure is

Common stock ($2.50 par, 800,000 shares)	$ 2,000,000
Capital surplus	1,000,000
Retained earnings	7,000,000
Net worth	$10,000,000

Vith a stock dividend, the par value is not reduced, whereas with a split, is. As a result, the common stock, capital surplus, and retained earn-igs accounts remain unchanged. The net worth, of course, also stays ie same; the only change is in the par value of the stock. Except in ac-ounting treatment, the stock dividend and stock split are very similar. stock split, however, is usually reserved for occasions when a com-any wishes to achieve a substantial reduction in the market price per are. The principal purpose of a split is to place the stock in a more opular trading range. The stock of a super-growth company can rather

quickly sell in excess of several hundred dollars a share unless it is split periodically and the total number of shares increased accordingly.

As was true of the stock dividend, the stock split does not represent a thing of value to the investor. He has more shares than before, but his proportional ownership of the company remains unchanged. The market price of the stock should decline proportionately, so that the total value of his holdings stays the same. Again, however, the split may have a favorable informational content. The announcement of the split may indicate to investors that management believes that earnings will continue to grow. As a result, the market price per share may increase upon the announcement of the split, or the rumor of an announcement, and remain higher.[9] However, the underlying cause for the increase in market price, again, is growth and not the split itself. It may be possible that the total market value of the firm is slightly higher if its shares are priced in a popular trading range rather than traded at a very high price.

Very seldom will a company maintain the same cash dividends per share after a split as it did before. However, it might increase the effective dividends to stockholders. For example, suppose a company splits its stock two for one and establishes a dividend rate of $1.20 a share, whereas before the rate was $2.00 a share. A stockholder owning 100 shares before the split would receive $200 in cash dividends per annum. After the split, he would own 200 shares and would receive $240 in dividends. The market price of the stock (on an after-split basis) may react favorably to the increase in cash dividends.

Reverse Split. Rather than increasing the number of shares of stock outstanding, a company may want to reduce the number. This reduction can be accomplished with a *reverse split*. In our example above, suppose that there were a one-to-four reverse split, instead of the two-to-one straight stock split. For each four shares held, the stockholder would receive one share in exchange. The par value per share would become $20, and there would be 100,000 shares outstanding rather than 400,000. Reverse stock splits are employed to increase the market price per share when the stock is considered to be selling at too low a price. Many companies have an aversion to seeing the price of their stock fall below $10 per share. If, due to financial difficulty or other reasons, the price should fall into this range, the market price per share can be increased with a reverse split. For example, Alsco Aluminum and Studebaker Corporation have employed reverse splits for the purpose of increasing the market price per share. Overall, however, the reverse split is somewhat unusual, for it is regarded as an admission by a company that it is in financial difficulty.

[9] This occurrence is supported by the empirical work of Keith B. Johnson, "Stock Splits and Price Change," *Journal of Finance*, XXI (December, 1966), 675–86.

Increasingly, companies are turning to the repurchase of their own shares,[10] for several reasons. A number of companies repurchase stock in order to have it available for stock options. In this way, the total number of shares is not increased with the exercise of the options. Another reason for repurchase is to have shares available for the acquisition of other companies. In other situations, however, stock is repurchased with the full intention of retiring it. Under these circumstances, repurchase of stock may be treated as a part of the dividend decision of the firm.

REPURCHASING AS PART OF A DIVIDEND DECISION

If a firm has excess cash and insufficient profitable investment opportunities to justify the use of these funds, it may be in the shareholders' best interests to distribute the funds. The distribution can be accomplished either by the repurchase of stock or by paying the funds out in increased dividends. In the absence of personal income taxes and transaction costs, it should make no difference to stockholders, theoretically, which of the two alternatives is chosen.

Suppose that a company has the following earnings and market price per share:

Net profit after taxes	$2,000,000
Number of shares outstanding	500,000
Earnings per share	4.00
Market price per share, ex-dividend	60.00
Price/earnings ratio	15

Suppose further that the company is considering the distribution of $1.5 million, either in cash dividends or in the repurchase of its own stock. If investors are expecting the cash dividend, the value of a share of stock before the dividend is paid will be $63 — that is, $3 a share in expected dividends ($1.5 million/500,000) plus the $60 market price. Suppose, however, that the firm chooses to repurchase its stock and makes a tender offer to stockholders at $63 a share. It then will be able to repurchase $1.5 million/$63, or 23,810 shares. Earnings per share will be

$$EPS = \$2,000,000/476,190 = \$4.20$$

If the price/earnings ratio stays at 15, the total market price per share will be $63 ($4.20 × 15), the same total value as under the dividend alternative. If we assume that stockholders are indifferent between divi-

[10] See Douglas V. Austin, "Treasury Stock Reacquisition by American Corporations: 1961–67," *Financial Executive*, 37 (May, 1969), 41–49.

dends and capital gains, the value of the stock is not influenced by the way the funds are returned to stockholders.[11] We have seen in the previous chapter, however, that this indifference is open to question. If investors have a systematic preference either for current dividends or for capital gains, the value of the firm may be affected by the method of distribution.

The benefit arising from the repurchase of stock is that the total number of shares is reduced so that earnings per share (and dividends per share if a constant dividend-payout ratio is maintained) are increased. If the price/earnings ratio remains unchanged, the market price of the stock will increase. If the price/earnings ratio increases because of the greater growth in earnings per share, the market price per share will increase even more.

In our example, the equilibrium repurchase price to offer was $63 a share. If a lower price were offered, stockholders who sold their shares would suffer a decrease in wealth relative to those who retained their stock. A higher price would result in a redistribution of wealth in favor of those who sold their shares. The equilibrium price the firm should offer in a repurchase is:[12]

$$P^* = P_x + \frac{D}{S} \tag{10-1}$$

where P_x = market price per share on an ex-dividend basis

D = aggregate amount the firm wishes to distribute

S = number of shares outstanding prior to the distribution.

With a differential tax rate on dividends and capital gains, however, repurchase of stock offers a considerable tax advantage over payment of dividends. The market-price increase resulting from a repurchase of stock is subject to the capital gains tax, whereas dividends are taxed at the ordinary income tax rate. As a result, the investor is better off financially if the firm elects to distribute unused funds via the stock repurchase route rather than through dividends.[13]

The repurchase of stock is advantageous particularly when the firm has a large amount of unused funds to distribute. To pay the funds out

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[11] See Diran Bodenhorn, "A Cash Flow Concept of Profit," *Journal of Finance*, XIX (March, 1964), 19–20.

[12] See Edwin Elton and Martin Gruber, "The Effect of Share Repurchases on the Value of the Firm," *Journal of Finance*, XXIII (March, 1968), 136–37.

[13] If existing stockholders are equivalent with respect to tax treatment, their wealth is enhanced by an amount equal to the difference between the marginal income tax rate and marginal capital gains tax rate, times the aggregate amount of cash the firm wishes to distribute (*ibid.*, 138–39). Elton and Gruber go on to explore modifications of this amount due to the postponement and incidence of the capital gains tax, and to various types of costs associated with paying a dividend and undertaking a repurchase. With the heterogeneous tax treatment of stockholders, the preferred method of distribution will depend upon the composition of stockholders.

through an extra dividend would result in a substantial tax to stock-holders. The tax effect could be alleviated somewhat by paying the funds out as extra dividends over a period of time, but this action might result in investors' counting on the extra dividend. Since the Internal Revenue Service attempts to preclude companies from repurchasing stock in lieu of paying dividends,[14] they are unlikely not to notice a steady and definite program of repurchase. Hence, the repurchase of stock must not be used as a substitute for regular dividends or even for recurring extra dividends.

INVESTMENT OR FINANCING DECISION?

Some regard the repurchase of stock as an investment decision instead of a dividend decision. Indeed, in a strict sense, it is. However, stock held in the treasury does not provide an expected return as other invest-ments do. No company can exist by investing only in its own stock. The decision to repurchase should involve distribution of unused funds when the firm's investment opportunities are not sufficiently attractive to employ those funds, either now or in the foreseeable future. Therefore, the repurchase of stock cannot be treated as an investment decision as we define the term.

Repurchase may be regarded as a financing decision, however, pro-vided its purpose is to alter the capital structure proportions of the firm. By issuing debt and repurchasing stock, a firm can immediately change its debt-to-equity ratio toward a greater proportion of debt. In this case, the repurchase of stock is a financing decision, because the alternative is to not pay out dividends.[15] Only when there is excess cash can the re-purchase of stock be treated as a dividend decision.

METHOD OF REPURCHASE

The two most common methods of repurchase are through a tender offer and through the purchase of stock in the marketplace. With a tender offer, the company makes a formal offer to stockholders to purchase so many shares, typically at a set price. This bid price is above the current market price; stockholders can elect either to sell their stock at the speci-fied price or to continue to hold it. In open-market purchases, a company

[14] See Harold Bierman, Jr., and Richard West, "The Acquisition of Common Stock by the Corporate Issuer," *Journal of Finance,* XXI (December, 1966), 687–96.

[15] Results of an empirical study by Allan Young, "Financial, Operating and Security Market Parameters of Repurchasing," *Financial Analysts Journal,* 25 (July–August, 1969), 124, suggest that a number of companies use stock repurchase to effect major capi-alization changes. Overall, however, Young's results are consistent with the fact that repurchasing companies have less favorable operating performances than nonrepurchasing companies, which in turn is consistent with a lack of investment opportunities. Thus, the evidence is consistent with stock repurchase for most companies being a dividend decision.

buys its stock through a brokerage house in the same manner as does any other investor. If the repurchase program is gradual, its effect is to put steady upward pressure on the market price per share. This upward pressure, of course, is of benefit to stockholders. In general, the transaction costs to the firm in making a tender offer are much higher than those incurred in the purchase of stock in the open market.

Before the company repurchases stock, it is important that stockholders be informed of the company's intentions. In a tender offer, these intentions are announced by the offer itself. Even here, however, it is important that the company not withhold other information. For example, it would be unethical for a mining company to withhold information of a substantial ore discovery while making a tender offer to repurchase shares.

In open-market purchases, it is especially important to disclose the company's repurchase intentions. Otherwise, stockholders may sell their stock not knowing that a repurchase program is under way that will increase earnings per share. Given full information about the amount of repurchase and the objective of the company, the stockholder can sell his stock if he so chooses. Without proper disclosure, the selling stockholder may well be penalized.[16] When the amount of stock repurchased is substantial, a tender offer is particularly suitable, for it gives all stockholders equal treatment.[17]

DECLARATION OF DIVIDENDS

When the board of directors of a corporation declares a cash dividend, it specifies a *date of record.* Holders of record on that date are entitled to the dividend declared. After the date of record, the stock is said to trade *ex-dividend,* for investors that purchase it are not entitled to receive the declared dividend. For listed stocks, the delivery of stock sold must be made within four business days of the date of sale. As a result, the ex-dividend date for these stocks is four business days before the date of record. Investors who purchase before the fourth day are entitled to the dividend; those who purchase it after that date are not. Theoretically, the market price of the stock should decline by the amount of the dividend when the stock goes ex-dividend. Because other factors influence the market price of the stock, this effect is sometimes difficult to measure. Once a dividend is declared, stockholders become creditors of the company until the dividend is actually paid; the declared but unpaid dividend is a current liability of the company.

[16] For a discussion of the ethics surrounding repurchase, see Richard Stevenson, "Corporate Stock Reacquisitions," *Accounting Review,* XLI (April, 1966), 312–17.

[17] See Charles D. Ellis, "Repurchase Stock to Revitalize Equity," *Harvard Business Review,* 43 (July–August, 1965), reprinted in *Foundations for Financial Management,* James Van Horne, ed. (Homewood, Ill.: Richard D. Irwin, Inc., 1966), p. 352.

Although the legal restrictions we discuss below are unimportant in most dividend decisions, some companies are affected by them. We have already considered one type of legal restriction—namely, the restriction on dividends imposed in a bond indenture or loan agreement. Other legal restrictions have to do with capital impairment, insolvency, and excess accumulation of cash.

Capital Restriction. Although state laws vary considerably, most states prohibit the payment of dividends if these dividends impair capital. Capital is defined in some states as the par value of the common stock. For example, if a firm had one million shares outstanding with a $2 par value, total capital would be $2 million. If the net worth of a company were $2.1 million, the company could not pay a cash dividend totaling $200,000 without impairing capital.

Other states define capital to include not only the par value of the common stock but also the capital surplus. Under such statutes, dividends can be paid only out of retained earnings. The purpose of the capital impairment laws is to protect creditors of a corporation. For a relatively new corporation, these laws may affort creditors a degree of protection. However, for established companies that have been profitable in the past and have built up retained earnings, substantial losses will usually have been incurred before the restriction has an effect. By this time, the situation may be sufficiently hopeless that the restriction gives creditors little protection.

Insolvency. Some states prohibit the payment of cash dividends if the company is insolvent. Insolvency is defined either in a legal sense, as liabilities exceeding assets, or in a technical sense, as the firm's being unable to pay its creditors as obligations come due. As the ability of the firm to pay its obligations is dependent upon its liquidity rather than upon its capital, the technical insolvency restriction gives creditors a good deal of protection. When cash is limited, a company is restricted from favoring stockholders to the detriment of creditors.

Excess Accumulation of Cash. The Internal Revenue Code prohibits the undue retention of earnings. Although undue retention is defined vaguely, it usually is thought to be retention significantly in excess of the present and future investment needs of the company. The purpose of the law is to prevent companies from retaining earnings for the sake of avoiding taxes. For example, a company might retain all its earnings and build up a substantial cash and marketable-securities position. The entire company then could be sold, and stockholders would be subject only to a capital gains tax. If the excess earnings were distributed as dividends, the dividends would be taxed as ordinary income. If the IRS can prove

unjustified retention, it can impose penalty tax rates on the accumulation. Whenever a company does build up a substantial liquid position, it has to be sure that it can justify the retention of these funds to the IRS. Otherwise, it may be in order to pay the excess funds out to stockholders as dividends.

SUMMARY

The stability of dividends is felt by many to have a positive effect upon the market price of the stock. Stable dividends may tend to resolve uncertainty in the minds of investors, particularly when earnings per share drop. They also may have a positive utility to investors interested in current periodic income. Many companies appear to follow the policy of a target dividend-payout ratio, increasing dividends only when it is felt that an increase in earnings can be sustained. The use of an extra dividend permits a cyclical company to maintain a stable record of regular dividends while paying additional dividends whenever earnings are unusually high.

Other considerations that affect the cash-dividend policy of the firm include the liquidity of the company; its ability to borrow; the desire to maintain control; the nature of the company's stockholders; the timing of investment opportunities; dividend restrictions in a bond indenture or loan agreement; and the impact of inflation on the replacement of assets.

A stock dividend represents the payment of additional stock to stockholders. It is used frequently as a means to conserve cash and to reduce the cash dividend-payout ratio of the firm. Theoretically, the stock dividend does not represent a thing of value to the stockholder unless cash dividends per share remain unchanged or are increased. However, stock dividends may have informational content with respect to earnings and may serve to keep the market price per share in a popular trading range. A much more effective device for reducing market price per share is a stock split. With a split, the number of shares is increased by the terms of the split—for example, a three-to-one split means that the number of shares is tripled.

A company's repurchase of its own stock should be treated as a dividend decision when the firm has funds in excess of present and foreseeable future investment needs. It may distribute these funds either as dividends or by the repurchase of stock. In the absence of a tax differential between dividends and capital gains, the monetary value of the two alternatives should be about the same. With the tax differential, there is a considerable tax advantage to the repurchase of stock. Because of objections by the Internal Revenue Service, however, repur-

chase of stock cannot be used in lieu of regular dividends. Repurchases can be accomplished either with a tender offer or through purchases in the open market. In either case, the repurchase intentions of the company should be made clear to stockholders.

Finally, we discussed in this chapter the procedure by which dividends are declared and various legal aspects that upon occasion may influence the dividend policy of the firm.

PROBLEMS

1.

The Sherill Corporation
Capital Structure
Dec. 30, 19X1

Common stock ($1 par, 1,000,000 shares)	$1,000,000
Excess over par*	300,000
Retained earnings	1,700,000
Net worth	$3,000,000

* Also called capital surplus.

The firm earned $300,000 after taxes in 19X1, and paid out 50 per cent of this in cash dividends. The price of the firm's stock on Dec. 30 was $5.

(a) If the firm declared a stock dividend of 3 per cent on Dec. 31, what would the reformulated capital structure be?

(b) If the firm declared a 50 per cent stock dividend rather than the 3 per cent dividend, what would the reformulated capital structure be? [*Hint*] In the case of a large stock dividend (over 25 per cent), the reformulated capital structure should be calculated on a book value, and not a market value, basis.

(c) Assuming the firm paid no stock dividend, how much would earnings per share for 19X1 be? How much would dividends per share be?

(d) Assuming a 3 per cent stock dividend, what would EPS and DPS be for 19X1? Assuming a 50 per cent stock dividend?

(e) What would the price of the stock be after the 3 per cent dividend? After the 50 per cent dividend?

2.

Zoppo Manufacturers
Capital Structure
Dec. 30, 19X1

Common stock ($100 par, 300,000 shares)	$ 30,000,000
Excess over par	15,000,000
Retained earnings	55,000,000
Net worth	$100,000,000

On Dec. 31, Zoppo split the stock 2-for-1 and then declared a 10 per cent stock dividend. The price of the stock on Dec. 30 was $500. Reformulate the capital structure of the firm.

3. The Sampson Company is owned by several wealthy New York businessmen, all of whom are in the 70 per cent marginal income tax bracket. The firm earned $3,500,000 after taxes this year. With one million shares outstanding, earnings per share were thus $3.50. The stock has recently traded at $72 per share, among the current shareholders. Two dollars of this value is accounted for by investor anticipation of a cash dividend. As financial manager of Sampson, you have contemplated the alternative of repurchasing the company stock by means of a tender offer at $72 per share.

(a) How much stock could the firm repurchase if this alternative were selected?

(b) Ignoring taxes, which alternative should be selected?

(c) Considering taxes, which alternative should be selected?

(d) What might preclude the firm from choosing the preferred alternative?

4. The Axalt Corporation and the Baxalt Corporation have had remarkably similar earnings patterns over the last five years. In fact, both firms have had identical earnings per share. Further, both firms are in the same industry, produce the same product, and face the same business and financial risks. In short, these firms are carbon copies of each other in every respect but one: Axalt pays out a constant percentage of its earnings (50 per cent) in dividends, while Baxalt has paid a constant cash dividend. The financial manager of the Axalt Corporation has been puzzled, however, by the fact that the price of his firm's stock has been generally lower than the price of Baxalt's stock, even though in some years Axalt's dividend was substantially larger than Baxalt's.

(a) What might account for the condition which has been puzzling the financial manager of Axalt?

(b) What might be done by both companies to increase the market prices of their stock?

	Axalt			Baxalt		
Years	EPS	Div.	Mkt. Price	EPS	Div.	Mkt. Price
19X1	$ 1.00	.50	$6	$ 1.00	.23	4\frac{7}{8}$
19X2	.50	.25	4	.50	.23	4$\frac{3}{8}$
19X3	− .25	nil	2	− .25	.23	4
19X4	.30	.15	3	.30	.23	4$\frac{1}{4}$
19X5	.50	.25	3$\frac{1}{2}$	.50	.23	4$\frac{1}{2}$

5. The Xavier Cement Company has hired you as a financial consultant to advise the company with respect to its dividend policy. The cement industry has been very stable for some time, and the firm's stock has not appreciated significantly in market value for several years. However, the rapidly growing southwestern market provides an excellent opportunity for this old, traditionally midwestern cement manufacturer to undertake a vigorous expansion program into a new market area. To do so, the company has decided to sell common stock for equity capital in the near future. The company expects its entrance into the southwestern market to be extremely profitable — returning approximately 25 per cent on investment each year. Below you will find data on earnings, dividends, and common-stock prices.

	1967	1968	1969	1970	*Anticipated* 1971
Earn./Share	$ 4.32	$ 4.17	$ 4.61	$ 4.80	$ 4.75
Cash avail./Share	6.00	5.90	6.25	6.35	6.25
Dividend/Share	2.90	2.80	3.00	3.20	?
Payout ratio	67%	67%	65%	67%	?
Avg. market price	$60	$58	$60	$67	$66
P/E ratio	14/1	14/1	13/1	14/1	14/1

What dividend policy recommendations would you make to the company? Specifically, what payout would you recommend for 1971? Justify your position.

6. The Davis Company, a large manufacturing concern, is a rapidly growing corporation. It has consistently earned over 20 per cent on its investment in assets, and prospects for the future appear to be equally good. In the past, the company has retained about 30 per cent of its earnings, paying out 70 per cent in dividends. The firm's management has justified this payout policy on two grounds: (1) that a company exists for purposes of paying dividends to stockholders, and (2) that the dividends paid by the company have a favorable effect on the price of the company's stock.

(a) Evaluate the management's justification of its dividend policy in light of the fact that most of the firm's stockholders are middle-aged businessmen in rather high marginal income tax brackets.
(b) What other factors would you, as a financial manager, take into consideration in deciding the appropriate dividend policy in addition to those considered by the Davis management?

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LONG-TERM
FINANCING

PART **IV**
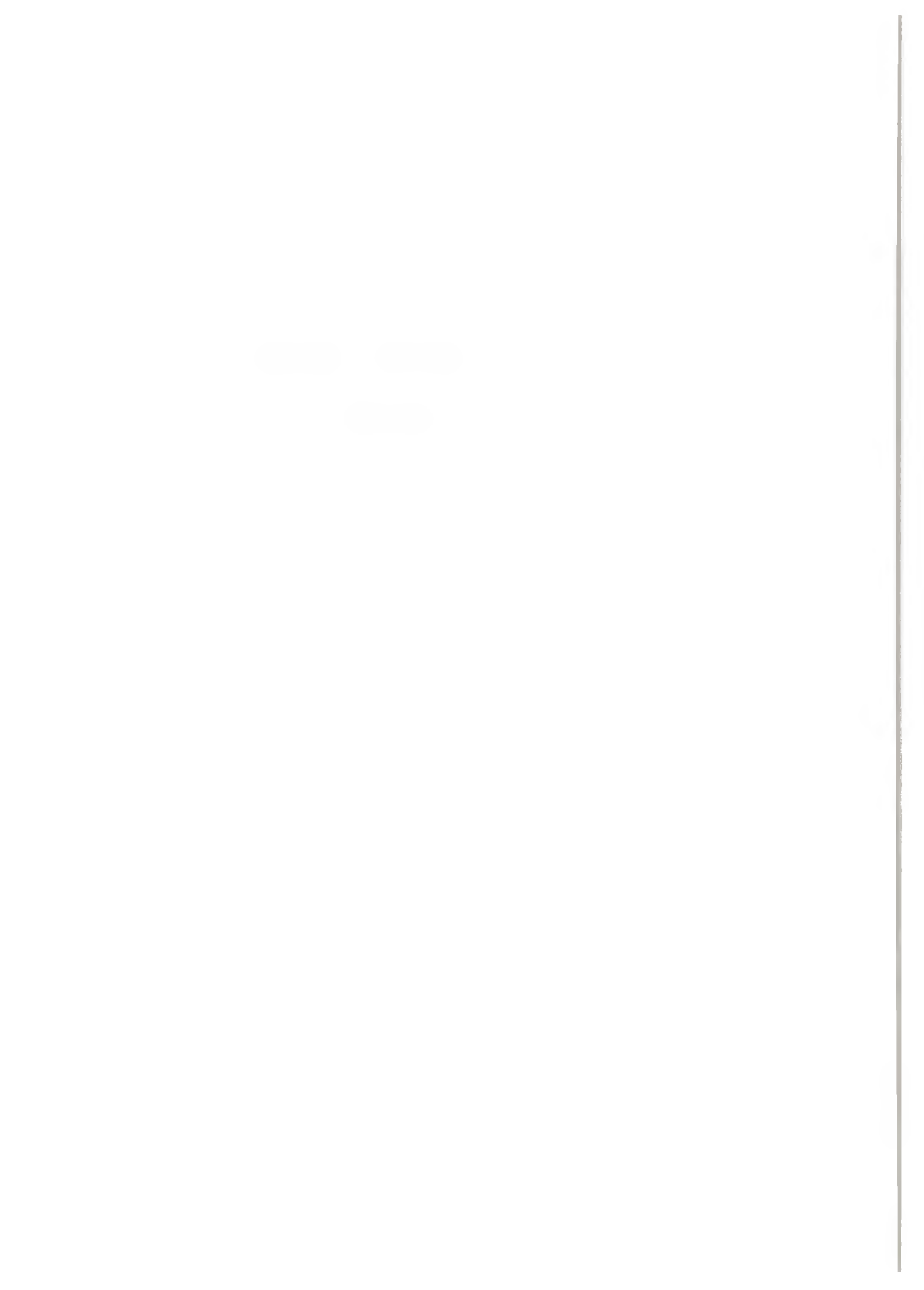
Obtaining
Long - Term
Funds Externally

11

In Part III, the focus of our analysis was primarily on the theoretical aspects of long-term financing—both external and internal. By and large, methods of financing were categorized broadly into the equity and non-equity portions of the capital structure of the firm. In this part, we explore in detail the specific methods of external long-term financing. We are concerned with the way a firm employs these various methods, their features, various valuation concepts, and, when appropriate, the integration of certain aspects of a method into the theory discussed previously. Because the financing methods taken up are long-term, this part logically follows the theoretical discussion of the capital structure of the firm in Part III. We defer discussion of the methods of short- and intermediate-term financing until we have taken up working-capital management in Part V.

The raising of funds externally automatically involves the firm in the money and capital markets. Because conditions in these markets affect the cost and availability of alternative methods of financing, obviously the financing decision is affected. By virtue of their interdependence with this decision, the investment and dividend decisions are affected as well. Consequently, it is very important that the financial manager keep abreast of financial market conditions. The more frequent the need of the firm to finance externally, of course, the closer the attention that he will need to pay. Extensive evaluation of interest rates and equity returns in financial markets is beyond the scope of this book; however, such an analysis is available in a supplementary volume.[1]

When a business firm finances its investment in real assets externally, it ultimately obtains funds from savings-surplus economic units in the economy. A savings-surplus unit can be either a business, a household, or a government whose current savings (current income less current consumption) exceeds its investment in real assets for the period under consideration. A savings-deficit unit, on the other hand, is one whose investment in real assets exceeds its current savings. If the investment in real assets always equaled the current savings for all economic units in an economy, there would be no need for any economic unit to obtain funds externally or for money and capital markets. In a modern economy, the investment in real assets for most economic units differs from their savings. Some units save more than they invest; others invest more than they save. As a result, we have money and capital markets. The more diverse these economic units are in their patterns of investment in real assets and savings, the greater the need for financial markets and the greater the amount of financial assets in the economy. While *ex post* investment must equal *ex post* savings for the economy as a whole, there can be considerable divergence between savings and investment for the individual economic unit.

One of the functions of the money and capital markets is to allocate efficiently the flow of funds from savings-surplus economic units to savings-deficit units. The efficiency of a country's money and capital markets is instrumental in the allocation of savings to the most promising investment opportunities and in the growth and development of a viable economy. The more varied the vehicles by which savings can flow from ultimate savers to ultimate users of funds, the more efficient the financial markets of an economy tend to be. In this regard, we are concerned specifically with the way business firms obtain funds – in particular long-term funds – to finance the excess of their investment in real assets over current savings for the period. The efficient functioning of financial

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[1] See James C. Van Horne, *The Function and Analysis of Capital Market Rates* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970).

markets requires a number of financial institutions and financial instruments.[2] One of these institutions is the investment banker, who acts as a middleman in the distribution of securities.

INVESTMENT BANKING

When a company issues a new securities to the public, it usually avails itself of the services of an investment banker. The principal function of the investment banker is to buy the securities from the company and then resell them to investors. For this service, the investment banker receives the difference, or spread, between the price he pays for the security and the price at which the securities are resold to the public. Because most companies make only occasional trips to the capital market, they are not specialists in the distribution of securities. To sell securities on their own would be both costly and risky. On the other hand, investment banking firms have the know-how, the contacts, and the sales organization necessary to do an efficient job of marketing securities to investors. Because they are continually in the business of buying securities from companies and selling them to investors, investment bankers can perform this service at a lower cost than can the individual firm.

COMPETITIVE BIDDING VERSUS NEGOTIATED OFFERING

A security offering through an investment banker to investors other than the firm's own stockholders (known as a public offering) can be either on a competitive bid basis or on a negotiated basis. When new securities are sold by competitive bidding, the company issuing the securities invites bids from investment bankers. Frequently, two or more investment banking firms join together for purposes of bidding on a security issue; the combination is known as a syndicate. The purpose underlying the formation of a syndicate is to spread the risk and to obtain a larger overall selling organization. The issuing company specifies the date that sealed bids will be received, and competing syndicates submit bids at the specified time and place. The syndicate with the highest bid wins the security issue. At that time, it pays the company the difference between its good-faith deposit and the bid price and then attempts to resell the issue at a higher price to investors.

With a negotiated offering, the company issuing the securities selects an investment banking firm and works directly with that firm in determining the essential features of the issue. Together, they discuss and

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[2] For a theoretical discussion of the efficiency of financial markets and savings flows, see *ibid.,* Chapters 1 and 2.

negotiate a price for the security and the timing of the issue. Depending upon the size of the issue, the investment banker may invite other firms to join it in sharing the risk and selling the issue. If a company has satisfactory experience with an investment banking firm, it usually will use the same firm for subsequent security issues.

It has been argued that the sale of securities on a negotiated basis tends to result in a lower price to the company and a wider profit spread to the investment banker than does an offering on a competitive bid basis.[3] Certain types of companies are required to issue securities on a competitive bid basis. For example, most state public utility commissions require that public utilities in their respective states issue securities by competitive bidding. In addition, railroads are required by the Interstate Commerce Commission to use competitive bidding in the sale of their securities. Offsetting the potential advantage of a higher price and narrower spread associated with competitive bidding is the fact that the company loses the benefit of the counsel of the investment banker.

FUNCTIONS OF THE INVESTMENT BANKER

One of the key functions the investment banker performs is that of bearing risk. When an investment banker or group of investment bankers buys a security issue, he *underwrites* the sale of the issue by giving the company a check for the purchase price. At that time, the company is relieved of the risk of not being able to sell the issue to investors at the established price. If the issue does not sell well, either because of an adverse turn in the market or because it is overpriced, the underwriter, and not the company, takes the loss. Thus, the investment banker or syndicate bears significant risk. For this risk-bearing function, the underwriter is compensated by an underwriting profit.

Underwriting Commission. To illustrate the underwriting commission in an offering, we turn to an example. Figure 11-1 shows the cover of the prospectus for the $75 million in $8\frac{1}{8}$ per cent sinking fund debentures and the $125 million in $8\frac{1}{4}$ per cent notes of the Ford Motor Company. The issue was a negotiated one involving some two hundred investment banking firms. Goldman, Sachs & Co., the managing agent, had the largest participation in both the debentures and the notes, with $9,725,000 in debentures and $17,325,000 in notes. Other participations ranged down to $50,000 for the debentures and to $75,000 for the notes. We see in the figure that the underwriters bought the debentures for $74,343,750, or

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[3] For a discussion of this point, see Jerome B. Cohen and Sidney M. Robbins, *The Financial Manager* (New York: Harper & Row, Publishers, 1966), pp. 536–42.

FIGURE 11-1
Prospectus of the Ford Motor Company

$991.25 a bond, from the company. In turn, the syndicate priced the debentures to the public at $1,000 a bond or $75 million in total. The spread of $8.75 per bond represents the gross commission to the syndicate for underwriting the issue, selling it, and for covering the various expenses incurred.

Of the total spread of $8.75, $5.00 represents the gross underwriting profit; the other $3.75 represents the selling concession. In other words,

for each debenture sold, the syndicate receives a profit of $5.00 regardless of whether a member of the syndicate actually sells the bond or not. A portion of the underwriting profit goes to the originating house (Goldman, Sachs) for preparing and managing the issue. Although this portion is not specified in the prospectus, we might conjecture that it was somewhat less than $1.00 on the basis of similar underwritings. The remaining $3.75 of the $8.75 total spread represents the selling concession, which we take up shortly. We see also in the figure that the notes were bought for $995.70 a note and sold to the public for $1,001.70, for a gross commission of $6.00 a note. Of the total spread, $3.00, or one-half, represents the gross underwriting profit; the other $3.00 represents the selling concession. After the bonds and notes are sold, total underwriting profits less expenses are distributed to members of the syndicate on the basis of their percentage participation.

Divided and Undivided Accounts. Underwriting syndicates can be of two types: divided and undivided. A *divided account* is one in which the liability of members is limited to their percentage participation. If the member sells all the securities allotted to him under his participation, he has no liability, regardless of whether or not other members are able to sell their allotments.[4] With an *undivided account,* each member is liable for his percentage participation in the unsold securities of the syndicate, regardless of the number of securities the individual member sells. If a member of a syndicate has a 20 per cent participation in an offering involving 40,000 bonds, and 10,000 remain unsold at the termination of the syndicate, the member would be responsible for 2,000 bonds. His liability would be the same whether he had sold 20,000 bonds or none.

Best Efforts Offering. Instead of underwriting a security issue, an investment banker may sell the issue on a *best efforts* basis. Under this arrangement, the investment banker agrees only to sell as many securities as he can at an established price. The investment banker has no responsibility for securities that are unsold. In other words, he bears no risk. Investment bankers frequently are unwilling to underwrite a security issue of a small company. For these companies, the only feasible means by which to place securities may be through a best efforts offering.

Making a Market. On occasion, the underwriter will make a market for a security after it is issued. In the case of a public offering of common stock for the first time, making a market is extremely important to investors. In making a market, the underwriter maintains a position in the stock and stands ready to buy and sell it at bid and ask prices he quotes. These quotations are based upon underlying supply and demand

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[4] A member can sell securities beyond his allotment.

conditions. With a secondary market, the stock has greater liquidity to investors; this appeal enhances the success of the original offering.

SELLING THE SECURITIES

The second major function of the investment banker is that of selling the securities to investors; for this service, he earns a selling concession. For the Ford issue, we saw that the selling concession was $3.75 a bond and $3.00 a note. The seller can be either a member of the syndicate or a qualified outside security dealer. In order to earn the full selling concession, however, he must be a member of the underwriting syndicate. An outside security dealer must purchase the bond(s) from a member, thus obtaining only a dealer concession, which is a portion of the full selling concession.

In a negotiated offering, the underwriters begin to line up prospective buyers before the actual offering date. A preliminary prospectus stating certain facts about the issue and the company is printed and given to interested investors. At this time, a price has not been established for the securities; the prospectus is known as a "red herring" because it contains, printed in red, a statement to the effect that a registration statement has been filed with the Securities and Exchange Commission but that it has not as yet been approved. Upon approval of the registration, the offering price to the public is established, and a final prospectus is printed. At that time, security salesmen seek orders from investors. If the issue is priced fairly, it will be sold within a matter of a day or two or, perhaps, even within a few hours. Sometimes "hot" issues are sold out in advance to preorder subscribers. Upon the sale of all the securities, the underwriting syndicate is dissolved.

ADVISING

In a negotiated offering, such as that of Ford, the originating house is able to advise the company on a wide variety of matters pertinent to the success of the offering. For a company that makes infrequent trips to the capital markets, this advice can be very valuable, for the matters considered include the timing of the issue, its pricing, and features that are desirable to assure a successful sale. Because of his expertise and experience in the market, the investment banker is able to recommend the best package of terms for the particular issue under consideration. When the sale of securities is by competitive bid, the issuer does not receive the benefit of this advice because the underwriter enters the picture only after the bid is accepted and the price determined. Advice from investment bankers may be of a continuing nature, with the company consulting a certain investment banker or a group of bankers regularly.

Sometimes an investment banker will serve on the board of directors of the company.

PRICING THE ISSUE

In a negotiated offering, the issuing company and the investment banker determine the price. The investment banker would like to see a price low enough to assure a successful sale, but he is aware that if the price is too low, the issuing company will be dissatisfied. An investment banker can ill afford dissatisfied customers, for news of such dissatisfaction spreads quickly in the financial community. Moreover, the issuing company must agree to the price recommended by the underwriter. Otherwise, of course, there can be no offering.

In the case of bonds, the issue will be priced in relation to the price of other new issues of the same grade. For example, if the bond being issued were rated Baa, comparisons would be made with other Baa new issues. The underwriter and the issuing company must assess the tone of the market with respect to expectations as to future interest rates. In addition to recent interest rate movements in the money and capital markets, they consider the forthcoming supply of new issues, the expected future of the economy, and expectations as to monetary and fiscal policy. Typically, a new issue will have to be sold at a lower price and higher yield to maturity than a seasoned issue of the same maturity and grade. In a competitive bidding situation, the syndicate will consider these same factors in determining the bid it will submit.[5] The syndicate wants to submit a bid high enough to win the issue but low enough to be able to sell the issue readily to investors. For the negotiated issue, the underwriter wants a price that is high enough to satisfy the issuer but low enough to make the probability of a successful sale to investors reasonably high.

For a common stock issue, the problem of pricing is perhaps more difficult because of the greater volatility of the stock market. When a company already has stock outstanding that is held by the public, the principal factor that governs the price of any new issue is the market price of the existing stock. The new issue will need to be underpriced in order to sell, however. The degree of underpricing will depend upon the volatility of the stock and the tone of the market. When it becomes known in the marketplace that a company is going to offer new stock, downward pressure usually is exerted on the market price of the outstanding stock. This pressure reflects investors' concern over dilution in earnings per share. Pressure usually develops on the day the new issue

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[5] See Ernest Bloch, "Pricing a Corporate Bond Issue: A Look Behind the Scenes," *Essays in Money and Credit,* Federal Reserve Bank of New York (December, 1964), pp. 72–76.

is announced, or before, if rumors of the new issue are out. This pressure contributes to the problem of underpricing the issue properly. With a negotiated offering where stock already is held by the public, the price usually is not established until the night before the offering.

If a company is issuing stock to the public for the first time, the pricing problem is much more difficult because there is no current market price to serve as a benchmark. For privately held companies that are going public, a comparison with similar companies usually is made to determine the appropriate price/earnings ratio. For this comparison, regression studies and other types of statistical analyses may be helpful. However, some companies may be so specialized that comparison with other companies is very difficult. The pricing of these issues is usually resolved by consideration of such essentials as present earnings, the growth rate, and the volatility of earnings. For a company going public for the first time, the underwriter and company may agree on a price well before the offering. Because there is no secondary market for existing shares, it is not necessary to delay pricing to the last minute.

STABILIZATION OF THE MARKET

During the period when the investment banker or syndicate is attempting to sell a new issue, it is important that the market price of the bond or stock be reasonably stable, to bolster investors' confidence in the issue. If the price should drop, the investment banker or syndicate stands to lose a great deal. To reduce this risk, the investment banker or managing underwriter for the syndicate often will attempt to stabilize the price during the distribution period, by placing orders to buy the security at a pegged price. For example, if the price of a bond to the public is $990, the managing underwriter may stand ready to buy any bonds offered at that price for the syndicate account. Thus, the market price cannot fall below $990 during the distribution period. In a sharply falling market, the managing underwriter may not be able to peg the price without having to buy the better part of the issue – a self-defeating process, to say the least. However, for an issue that is realistically priced in a reasonably stable market, the pegging operation does tend to reduce the risk to the underwriter. Without such stabilization, the risk to the underwriter would be greater; and he would compensate for it by bidding a lower price to the company and/or increasing the underwriter's spread.

FLOTATION COSTS

The flotation costs of a new issue of securities tend to vary with the size and the type of the issue. These costs include the underwriting spread, registration expenses, and other out-of-pocket expenses. Unfortunately, we have little in the way of recent empirical evidence on

flotation costs. The results of a survey for the 1951–55 period showed that the larger the issue, the lower the cost of flotation as a percentage of gross proceeds.[6] As certain company expenses — printing and legal fees in particular — essentially are fixed, the larger the issue, the lower their percentage cost. The underwriter also has certain "fixed" expenses. Thus, the larger the issue, the smaller the underwriting expense. Additionally and more importantly, there usually is an inverse relationship between the size of an issue and the quality of the issuing company. The study also showed that the relative cost of flotation is highest for a common stock issue and lowest for a debt issue — a fact not surprising in view of the differences in underwriting risk.

PRIVILEGED SUBSCRIPTION

Instead of selling a security issue to the general public, many firms offer the securities first to existing shareholders on a privileged-subscription basis. Frequently, the corporate charter requires that a new issue of common stock or an issue of securities convertible into common be offered first to existing shareholders because of their preemptive right.

PREEMPTIVE RIGHT

Under a preemptive right, an existing common stockholder has the right to preserve his proportionate ownership in the corporation. If the corporation issues additional common stock, he must be given the right to subscribe to the new stock so that he maintains his pro rata interest in the company. Suppose an individual owns 100 shares of a corporation and the company decides to increase by 10 per cent the number of shares outstanding through a new common stock offering. If the stockholder has a preemptive right, then he must be given the option to buy ten additional shares so that he can preserve his proportionate ownership in the company. Various states differ with respect to laws regarding preemptive rights. However, the majority of the states provide that a stockholder does have a preemptive right unless the corporate charter denies it specifically.

OFFERING THROUGH RIGHTS

When a company sells securities by privileged subscription, each stockholder is mailed one right for each share of stock he holds. With a common stock offering, the rights give him the option to purchase ad-

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[6] *Cost of Flotation of Corporate Securities,* Securities and Exchange Commission (Washington, D.C.: Government Printing Office, June, 1957), p. 37.

ditional shares according to the terms of the offering. The terms specify the number of rights required to subscribe for an additional share of stock, the subscription price per share, and the expiration date of the offering. The holder of rights has three choices: he can exercise them and subscribe for additional shares; he can sell them, as they are transferable; or he can simply do nothing and let them expire. The latter usually occurs only if the value of a right is negligible and/or if the stockholder owns but a few shares of stock. Generally, the subscription period is thirty days or less. If a stockholder wishes to buy a share of additional stock but does not have the necessary number of rights, he may purchase additional rights. For example, suppose a person presently owns eighty-five shares of stock in a company, and the number of rights required to purchase one additional share is ten. Given his eighty-five rights, he can purchase only eight full shares of stock. He can, however, buy the ninth share by purchasing an additional five rights.

In a rights offering, the board of directors establishes a date of record. Investors that buy the stock prior to that date receive the right to subscribe to the new issue. The stock is said to sell with *rights on* prior to the date of record. After the date of record, the stock is said to sell *ex-rights;* that is, the stock is traded without the rights attached. An investor who buys the stock after this date does not receive the right to subscribe to additional stock.

VALUE OF RIGHTS

The market value of a right is a function of the present market price of the stock, the subscription price, and the number of rights required to purchase an additional share of stock. The theoretical market value of one right after the offering is announced but while the stock is still selling rights-on is

$$R_o = \frac{P_0 - S}{N + 1} \tag{11-1}$$

where R_o = market value of one right when stock is selling rights-on
P_0 = market value of a share of stock selling rights-on
S = subscription price per share
N = number of rights required to purchase one share of stock.

For example, if the market price of a stock is \$100 a share and the subscription price \$90 a share, and it takes four rights to buy an additional share of stock, the theoretical value of a right when the stock is selling rights-on would be

$$R_o = \frac{100 - 90}{4 + 1} = \$2 \tag{11-2}$$

We note that the market value of the stock with rights on contains the value of one right.

When the stock goes ex-rights, the market price theoretically declines, for investors no longer receive the right to subscribe to additional shares. The theoretical value of one share of stock when it goes ex-rights is

$$P_x = \frac{(P_0 \times N) + S}{N + 1} \tag{11-3}$$

where P_x = market price of stock when it goes ex-rights. For our example

$$P_x = \frac{(100 \times 4) + 90}{4 + 1} = \$98 \tag{11-4}$$

From this example we see that, theoretically, the right does not represent a thing of value to the stockholder. His stock is worth $100 before the date of record; after the date of record, it is worth $98 a share. The decline in market price is offset exactly by the value of the right. Thus theoretically, the stockholder does not benefit from a rights offering; the right represents merely a return of capital.

The theoretical value of a right when the stock sells ex-rights is

$$R_x = \frac{P_x - S}{N} \tag{11-5}$$

where R_x = the market value of one right when the stock is selling ex-rights. If, in our example, the market price of the stock is $98 when it goes ex-rights,

$$R_x = \frac{98 - 90}{4} = \$2 \tag{11-6}$$

or the same value as before.

It is important to recognize that the actual value of a right may differ somewhat from its theoretical value on account of transaction costs, speculation, and the irregular exercise and sale of rights over the subscription period. There is an old adage on Wall Street that says a stockholder should sell his rights early in the subscription period because at that time they have the maximum value. The high value, as the reasoning goes, is the consequence of a hesitation on the part of many stockholders either to exercise or to sell their rights in the early days of the subscription period. This hesitation is said to reflect a "wait and see" attitude. As a result, there is a shortage of rights early in the subscription period; and the market price of the right rises relative to its theoretical value. The opposite occurs near the end of the subscription period; stockholders are said to unload rights. Although this behavior may seem logical enough, empirical studies have not revealed any distinct price pattern of rights

over the subscription period.[7] One reason is that arbitrage limits the deviation of actual value from theoretical value. If the price of a right is significantly higher than its theoretical value, stockholders will sell their rights and purchase the stock in the market. Such action will exert downward pressure on the market price of the right and upward pressure on its theoretical value. The latter occurs because of the upward pressure on the market price of the stock. If the price of the right is significantly lower than its theoretical value, arbitragers will buy the rights, exercise their option to buy stock, and then sell the stock in the market. This occurrence will exert upward pressure on the market price of the right and downward pressure of its theoretical value. These arbitrage actions will continue as long as they are profitable.

In the rights formulas presented, it is assumed implicitly that the relative earning power and risk complexion of the firm do not change as a result of the investment of funds raised in the offering. Implied also is that the firm's capital structure does not change — i.e., it employs the same financing mix as before. If these conditions do not hold, the market price of the common stock may well behave in a manner out of keeping with its previously computed theoretical value.

THE SUCCESS OF THE OFFERING

One of the most important aspects of a successful rights offering is the subscription price. If the market price of the stock should fall below the subscription price, stockholders obviously will not subscribe to the stock, for they can buy it in the market at a lower price. Consequently, a company will attempt to set the subscription price lower than the current market price to reduce the risk of the market price's falling below it. How much lower depends upon the volatility of the company's stock, the tone of the market, expectations of earnings, and other factors.

Amount of Discount. To avoid all risk, the issuing company can set the subscription price so far below the market price that there is virtually no possibility that the market price will fall below it. The greater the discount from the current market price, the greater the value of the right, and the greater the probability of a successful sale of stock. As long as the stockholder does not allow his rights to expire, theoretically he neither gains nor loses by the offering. Therefore, it might seem feasible to set the subscription price at a substantial discount in order to assure a successful sale. However, the greater the discount, the more shares that will have to be issued to raise a given amount of money, and the greater the dilution in earnings per share. This dilution may be an important con-

[7] See Robert M. Soldofsky and Craig R. Johnson, "Rights Timing," *Financial Analysts Journal,* 23 (July–August, 1967), 101–4.

sideration, for the investment community analyzes closely the growth trend in earnings per share. Significant underpricing of the new issue may excessively dampen the growth trend in earnings per share and result in a lower price/earnings ratio in the market. While theoretically the stockholder should be equally well off regardless of the subscription price set, in practice the market value of his stock holdings may suffer if there is unnecessary dilution.

Moreover, if the firm wishes to maintain the same dividend per share, underpricing, which will result in more shares issued, will increase the total amount of dividends the company will need to pay and lower its coverage ratio. The disadvantages of underpricing must be balanced against the risk of the market price's falling below the subscription price. The primary consideration in setting the subscription price is to reduce the probability of this occurrence to a tolerable level. If, then, the subscription price results in excessive dilution, the company should consider a public issue, wherein the amount of underpricing usually is less.

Other Factors.   There are other factors that influence the success of a rights offering. The size of the capital outlay in relation to a stockholder's existing ownership of the stock is important.[8] Stockholders are likely to be more willing to subscribe to an issue amounting to a 10 per cent addition to the stock they presently hold than to an issue amounting to a 50 per cent addition. The mix of existing stockholders may also be a factor. If a substantial number of stockholders hold only a few shares, the success of the offering may be less than if most stockholders held units of 100 shares. The breakdown between institutional and individual investors may also bear upon the success of the rights offering. The current trend and tone of the stock market are extremely important. If the trend is upward and the market is relatively stable in this upward movement, the probability of a successful sale is quite high. The more uncertain the stock market, the greater the underpricing that will be necessary in order to sell the issue. In fact, there are times when the market is so unstable that an offering will have to be postponed.

STANDBY ARRANGEMENT

A company can insure the complete success of a rights offering by having an investment banker or group of investment bankers "stand by" to underwrite the unsold portion of the issue. For this standby commitment, the underwriter charges a fee that varies with the risk involved in the offering. Often the fee consists of two parts: a flat fee, and an addi-

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[8]See Harry G. Guthmann and Herbert E. Dougall, *Corporate Financial Policy,* 4th ed. (Englewood Cliffs, N.J.: Prentice-Hall, Inc. 1962), p. 414.

tional fee for each unsold share of stock that the underwriter has to buy. From the standpoint of the company issuing the stock, the greater the risk of an unsuccessful sale, the more desirable a standby arrangement, although it also is more costly.

PRIVILEGED SUBSCRIPTION
VERSUS PUBLIC ISSUE

By offering stock first to existing stockholders, the company taps investors who are familiar with the operations of the company. As a result, a successful sale is more probable. The principal sales tool is the discount from the current market price, whereas with a public issue, the major selling tool is the investment banking organization. Because the issue is not underwritten, the flotation costs of a rights offering are lower than the costs of an offering to the general public. Moreover, many stockholders feel that they should be given the first opportunity to buy new common shares. Offsetting these advantages is the fact that a rights offering generally will have to be sold at a lower price than will an issue to the general public, with more dilution in earnings per share. As we have said, this greater dilution may work to the disadvantage of the company and its stockholders. If the company wishes to minimize dilution per share over the long run, it is better off with public issues than with rights offerings. Also, a public offering will result in a wider distribution of shares. Management can request stockholders with preemptive rights to waive them so that the company can sell stock to the general public. If the argument is persuasive enough or if management controls enough stock, the preemptive right may be waived.

GOVERNMENT REGULATIONS

Both the federal and state governments regulate the sale of new securities to the public. A company issuing securities must comply with these regulations. Of the two regulatory bodies, the federal authority is far more encompassing in its influence.

FEDERAL REGULATION

With the collapse of the stock market in 1929 and the subsequent Depression, there came a cry for the protection of investors from misinformation and fraud. Congress undertook extensive investigations and, in the end, proposed federal regulation of the securities industry. The Secur-

ities Act of 1933 dealt with the sale of new securities and required the full disclosure of information to investors. The Securities Exchange Act of 1934 dealt with the regulation of securities already outstanding. Moreover, it created the Securities and Exchange Commission to enforce the two acts.

Almost all corporations selling securities to the public must register the issue with the SEC. Certain types of corporations, such as railroads, are exempt because they are regulated by other authorities. In addition, a corporation selling $300,000 or less in new securities is required to file only a limited amount of information with the SEC. Other corporations, however, must file a detailed registration statement, which contains such information as the nature and history of the company, the use of the proceeds of the security issue, financial statements, the management and directors and their security holdings, legal opinions, and a description of the security being issued. Along with the registration statement, the corporation must file a copy of the *prospectus,* which is a summary of the essential information in the registration statement. As mentioned previously, this prospectus is known as a "red herring" because it has not yet been approved by the SEC. The prospectus must be available to prospective investors and others who request it.

The SEC reviews the registration statement and the prospectus to see that all the required information is presented and that it is not misleading. If the SEC is satisfied with the information, it approves the registration, and the company is then able to sell the securities. If not, it issues a *stop order,* which prevents the sale of the securities. Most deficiencies can be corrected by the company, and approval will usually be given eventually, except in cases of fraud or misrepresentation. For serious violations of the 1933 Securities Act, the SEC is empowered to go to court and seek an injunction. It should be pointed out that the SEC is not concerned with the investment value of the securities being issued, only with the presentation of complete and accurate information. The investor must make his own decision based upon that information. The security being issued may well be a highly speculative one subject to considerable risk. As long as the information is correct, the SEC will not prevent its sale.

The minimum period required between the time a registration statement is filed and the time it becomes effective is twenty days, sometimes known as a "cooling off" period. During this time, investors can evaluate the information in the prospectus and reach a decision. The usual time lapse, however, is longer, around thirty-five days.

The SEC regulates the sale of securities in the secondary markets in addition to the sale of new issues. In this regard, it regulates the activities of the security exchanges, the over-the-counter market, investment bankers and brokers, the National Association of Security Dealers, and investment companies. In its regulatory capacity, the SEC seeks to prevent

fraudulent practices, excessive commissions, and other abuses affecting the investment public.

STATE REGULATION

Individual states have security commissions that regulate the issuance of new securities in their states. Like the SEC, these commissions seek to prevent the fraudulent sale of securities. The laws providing for state regulation of securities are known as "blue-sky" laws, because they attempt to prevent the false promotion and sale of securities representing nothing more than "blue sky." State regulations are particularly important when the amount of the issue is $300,000 or less and not subject to the rigorous scrutiny of the SEC. Unfortunately, the laws of the individual states vary greatly in their effectiveness. Some states are strict, but others are fairly permissive, with the result that misrepresentative promotion can thrive.

PRIVATE PLACEMENT

Rather than sell securities to the public or to existing stockholders through a privileged subscription, a corporation can sell the entire issue to a single institutional investor or a small group of such investors. This type of sale is known as a private or direct placement, for the company negotiates directly with the investor over the terms of the offering, eliminating the function of the underwriter. Some issues of common stock are placed privately, but the vast majority of private placements involve debt issues. Consequently, in the discussion that follows, we shall be concerned only with the direct placement of debt issues.

Private placements increased rapidly during the early sixties and accounted for about one-half of the total funds raised externally by corporations and for over three-fifths of the total debt issues by 1964. However, with the large increase in volume of corporate bond financing since 1965, the composition of financing has shifted toward public offerings. Figure 11-2 shows the growth in private placements and public offerings and the percentage of public offerings to total offerings. The variation in private placements relative to public offerings reflects in part the limited capacity of the private placement market to handle volume. When the total volume of corporate bond financing increases sharply, the capacity of institutional investors does not increase proportionately.[9] As a result, corporate borrowers must turn to public offerings for a larger portion of their requirements. This phenomenon was evident in 1957–58 and, more dramatically, in 1965–68.

[9]See Henry Kaufman, *The Changing Dimensions of the Corporate Bond Market* (New York: Salomon Brothers and Hutzler, 1967), p. 11.
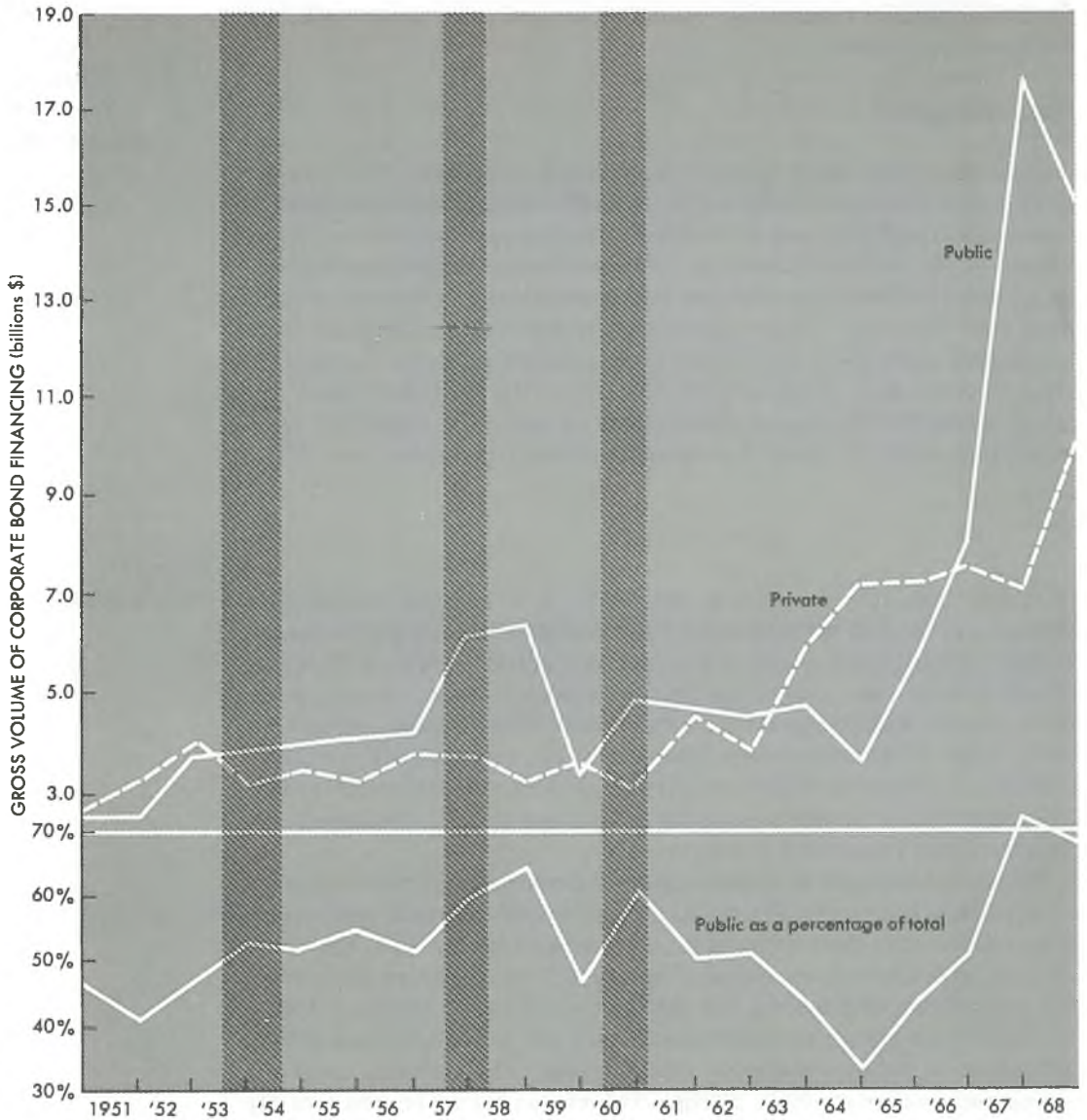
Shaded areas denote N.B.E.R. Reference Cycle Contractions

FIGURE 11-2
The varying proportion of public and private corporate bond volume. Source: Kaufman, op. cit., p. 9; and Securities and Exchange Commission.

What are the reasons for private placements? We may gain some insight by studying their advantages and disadvantages.[10]

Flexibility. One of the more frequently mentioned advantages of a private placement is the speed of the commitment. A public issue must be registered with the SEC, documents prepared and printed, and extensive negotiations undertaken; all this requires a certain lead time. In addition, the public issue always involves risks with respect to timing. With a private placement, the terms can be tailored to the needs of the borrower, and the financing can be consummated much more quickly. Because the issue is negotiated, the exact timing in the market is not a critical problem. The fact that there is but a single investor or small group of investors is attractive if it becomes necessary to change any of the terms of the issue. It is much easier to deal with a single investor than with a large group of public security holders.

Another advantage of a privately placed debt issue is that the actual borrowing does not necessarily have to take place all at once. The company can enter into an arrangement whereby it can borrow up to a fixed amount over a period of time. For this nonrevolving credit arrangement, the borrower usually will pay a commitment fee. This type of arrangement gives the company flexibility, allowing it to borrow only when it needs the funds. With a public issue, it is necessary to sell the entire issue at one time. Because the private placement does not have to be registered with the SEC, the company avoids making available to the public the detailed information required by the SEC.

Size of Issue. Private placements allow medium-sized and sometimes small companies to sell a bond issue, whereas with a public offering the flotation costs would be prohibitive. Institutional investors are willing to invest in bonds of these smaller companies, provided the company is credit-worthy. It is doubtful that institutional investors would seek an issue of less than $100,000 (and many insist upon a higher minimum), but we must remember that a $5 million bond issue is considered small as a public offering.

Cost of Issue. There are two costs to consider in comparing a private placement of debt with a public offering: the initial cost and the interest cost. As the negotiations usually are direct, private placement involves no underwriting or selling expenses. Frequently, however, a company seeks the services of an investment banker for advice in planning and negotiating

[10]See "Direct Placement of Corporate Debt," *Economic Review*. Federal Reserve Bank of Cleveland (March, 1965), reprinted in *Foundations for Financial Management,* Van Horne, ed., pp. 247–65.

the issue. Investment bankers have become increasingly involved as agents in private placements, thus offsetting to a certain degree the loss of underwriting and selling business. However, overall, the initial total cost of a private placement is significantly less than that of a public offering.[11]

The second aspect of the cost of a private placement of debt is the interest cost. Fragmentary evidence here indicates that the yield on private placements is significantly above that on public offerings. In addition to interest costs, institutional investors sometimes will request an equity "sweetener," such as warrants, to entice them to invest in the debt issue of a company. While the exact cost of this "sweetener" is difficult to measure, it certainly adds to the total cost of a private placement.[12]

In summary, it would seem that the initial cost of a private placement of debt is less than that of a public offering. However, the interest cost and any additional compensation appear to be higher. For a long-term debt issue, the total cost is likely to be somewhat higher for a private placement than for a public offering. However, the difference in cost must be compared with the advantages of the private placement.

SUMMARY

When companies finance their long-term needs externally, they may obtain funds from the capital markets or directly from a single institutional investor or a small group of them. If the financing involves a public offering, the company usually will use the services of an investment banking firm. The investment banker's principal functions are risk-bearing, or underwriting, and selling the securities. For these functions, the investment banking firm is compensated by the spread between the price it pays for the securities and the price at which it resells the securities to investors. With a negotiated offering, the investment banker provides an additional service in advising the company as to the pricing and timing of the issue and as to procedures and features involved in the issue. With an offering on a competitive bid basis, the issue is sold to the investment banker or syndicate of investment bankers that submits the highest bid.

A company may give its existing stockholders the first opportunity to purchase a new security issue on a privileged-subscription basis. This

[11]"Direct Placement of Corporate Debt," pp. 250–51; and Avery B. Cohan, *Private Placements and Public Offerings: Market Shares Since 1935* (Chapel Hill, N.C.: School of Business Administration, University of North Carolina, 1961), Chapter 11.

[12]For a discussion of the implications of equity "sweeteners" for institutional investors and borrowers, see Charles M. Williams and Howard A. Williams, "Incentive Financing—a New Opportunity," *Harvard Business Review*, 38 (March–April, 1960), 123–34; and Samuel L. Hayes, III, "New Interest in Incentive Financing," *Harvard Business Review*, 44 (July–August, 1966), 99–112.

type of issue is known as a rights offering, because existing stockholders receive one right for each share of stock they hold. A right represents an option to buy the new security at the subscription price; and it takes a specified number of rights to purchase the security. Depending upon the relationship between the current market price of the stock and the subscription price, a right will usually have a market value. Both security offerings to the general public and offerings on a privileged-subscription basis must comply with federal and state regulations. The enforcement agency for the federal government is the Securities and Exchange Commission, whose authority encompasses both the sale of new securities and the trading of existing securities in the secondary market.

Rather than offering securities to existing stockholders or the general public, a company may place them privately with an institutional investor. Private placements, almost exclusively debt issues, have grown in importance. With a private placement, the company negotiates directly with the investor; there is no underwriting and no registration of the issue with the SEC. The private placement has the virtue of flexibility and affords the medium-sized and even the small company the opportunity to sell its securities.

PROBLEMS

1. The stock of the Dunbar Company is selling for $150 per share. If the company were to issue rights to subscribe for one additional share of stock, at $125 a share, for each nine held, compute the following:
  (a) The theoretical value of a right when the stock is selling rights-on.
  (b) The theoretical value of one share of stock when it goes ex-rights.
  (c) The theoretical value of a right when the stock sells ex-rights and the actual market price goes to $143 per share.

2. The stock of the National Corporation is selling for $50 per share. The company then issues rights to subscribe to one new share at $40 for each five shares held.
  (a) What is the theoretical value of a right when the stock is selling rights-on?
  (b) What is the theoretical value of one share of stock when it goes ex-rights?
  (c) What is the theoretical value of a right when the stock sells ex-rights at $50?
  (d) Joe Speculator has $1,000 at the time National stock goes ex-rights at $50 per share. He feels that the price of the stock will rise to $60 by the time the rights expire. Compute his return on his $1,000 if he (1) buys National stock at $50 or (2) buys the rights at the price computed in (c) above if his price expectations are valid.

3. Instead of a rights offering, National Corporation (see problem 2) could undertake a public offering at $45 per share with a 6 per cent gross spread. National currently has 1 million shares outstanding and earns $4 million a year. All earnings are paid in dividends. In either case, National would sell enough shares to raise $1 million, which would be invested at an after-tax return of 10 per cent.

(a) Compute the earnings per share, dividends per share, and market price of the stock (assuming a 12.5 P/E ratio) for (1) the rights offering and (2) the public offering alternatives.

(b) Mr. Brown owns one share of National stock. On a rights offering, he will sell the right (assume for $2) and use the proceeds to reduce his investment to $48. On a public offering, he would not buy any more shares. Compute Mr. Brown's earnings and dividend return on his investment and his price gain or loss on his investment under each of the two financing alternatives facing National.

4. The Kramer Corporation wishes to raise $10 million of debt for twenty years. It can sell bonds publicly with an 8 per cent coupon and a 1.50 per cent gross spread, or it can place an $8\frac{1}{2}$ per cent note privately, with no other costs. Assuming that the Kramer Corporation would repay the principal of neither loan until maturity, make annual interest payments, and is able to earn 12 per cent before taxes on funds it employs, which plan would have the higher present value to the firm?

5. The Homex Company wishes to raise $5 million in new equity capital. Since these new shares would compete for the buyers' attention with already-outstanding shares of the Homex Company and those of comparable companies, it is felt that the securities salesmen must be offered greater compensation for the sale of the new issue than they could obtain from the sale of an equal dollar amount of an existing issue. After consideration is given to the potential difficulty of selling the shares, it is decided that the selling concession should be between 3 per cent and 4 per cent of the value of the issue. When the risks of underwriting are evaluated, it is decided that the selling concession should constitute between 50 per cent and 60 per cent of the gross spread. If the management fee (which constitutes part of the underwriting profit) is taken to be 15 per cent of the gross spread, answer the following questions:

(a) Assuming that the selling concession is set at 4 per cent of gross proceeds and 50 per cent of the gross spread, what would be the dollar value of the management fee, net underwriting profit, selling concession, and gross spread on the Homex underwriting?

(b) Rework (a), assuming that the selling concession were set at 3 per cent of gross proceeds and 60 per cent of the gross spread.

(c) Assuming that the managing underwriter underwrote 25 per cent and sold 20 per cent of the issue, what would be his total compensation under case (a) above? Under case (b)?

(d) Assuming this problem to be somewhat realistic, what are its implications for the financial manager of Homex?

6. The Tabbot Corporation, a rather new and speculative concern, wishes to sell additional stock. Its investment banker, the First Salem Corporation, offers the following two alternatives:

(1) First Salem will make a firm underwriting of 1,000,000 shares @ $4.50 per share, for a gross spread of 8 per cent.

(2) First Salem will attempt, for a $200,000 fee, a best efforts distribution, @ $4.50 per share, to the public with a gross spread of 8 per cent on those shares sold. First Salem expects sales at this price to have an approximately normal distribution, with a mean of 900,000 shares and a standard deviation of 100,000 shares.

(a) What are the expected proceeds to Tabbot under each alternative?

(b) What is the approximate probability that the best efforts distribution would yield more to Tabbot than the firm underwriting?

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Long - Term
Debt

12

In Chapters 7 and 8, the theoretical aspects of long-term debt were analyzed in conjunction with the use of equity capital. Also discussed was how in practice a firm might determine the appropriate amount of debt to employ. Our discussion was framed in terms of debt in general rather than in terms of specific types of debt. In this chapter, we evaluate the wide spectrum of long-term debt instruments available to the firm.

Our concern is with debt issues having an original maturity of more than ten years that are either privately placed or sold in a public offering. The holders of a company's long-term debt, of course, are creditors. Generally, they cannot exercise control over the company and do not have a voice in management. However, if the company violates any of the provisions of the debt contract, these holders may be able to exert some influence upon the direction of the company. In addition to not having voting power, holders of long-term debt do not participate in the residual

earnings of the company; instead, their return is fixed. Also, a debt instrument has a specific maturity, whereas a share of common or preferred stock does not. In liquidation, the claim of debt holders is before that of preferred and common stockholders. Depending upon the nature of the debt instrument, however, there may be differences in the priority of claim among the various creditors of a company.

SOME DEFINITIONS

The fixed return of a long-term debt instrument is denoted by the *coupon rate*. For example, a 7.5 per cent debenture indicates that the issuer will pay the bondholder $75 per annum for every $1,000 face value bond he holds. The yield to maturity on a bond is determined by solving for the rate of discount that equates the present value of principal and interest payments with the current market price of the bond. (See Appendix A to Chapter 3 for the mathematics of bond interest.) The yield on a bond is the same as the internal rate of return for an investment project.

With a bond issue to the public, a qualified *trustee* is designated by the company to represent the interests of the bondholders. The obligations of a trustee are specified in the Trust Indenture Act of 1939, administered by the Securities and Exchange Commission. His responsibilities are: to authenticate the bond issue as to its legality at the time of issuance; to watch over the financial condition and behavior of the borrower to make sure all contractual obligations are carried out; and to initiate appropriate actions if the borrower does not meet any of these obligations. The trustee is compensated directly by the corporation; this compensation adds to the cost of borrowing.

The legal agreement between the corporation issuing the bonds and the trustee, who represents the bondholders, is defined in the *indenture*. The indenture contains the terms of the bond issue as well as the restrictions placed upon the company. These restrictions, known as *protective covenants,* are very similar to those contained in a term-loan agreement. As we analyze protective covenants in detail in Chapter 21 and the Appendix to that chapter, it is not necessary to describe these restrictions here. The terms contained in the indenture are established jointly by the borrower and trustee. If the issue is a negotiated underwriting, the underwriter also will be involved. Naturally, the borrower does not want the terms to be unduly restrictive. Nevertheless, he is mindful of the need to appeal to investors and to conform to certain legal requirements. If the corporation defaults under any of the provisions of the indenture, the trustee, on behalf of the bondholders, can take action to correct the situation. If not satisfied, he then can call for the immediate payment of all outstanding bonds.

RETIREMENT

The retirement of bonds may be accomplished in a number of ways.[1] For example, bonds may be retired by payment at final maturity, by conversion if the bonds are convertible, by calling the bonds if there is a call feature, or by periodic repayment. Periodic repayment of the debt is possible if the bond issue is either a sinking-fund issue or a serial bond issue. Conversion is taken up in Chapter 14, and the calling of bonds is examined later in this chapter. We turn now to a discussion of sinking-fund and serial bonds.

Sinking Funds. If a bond issue has a sinking fund, the corporation makes periodic sinking-fund payments to the trustee. In turn, the trustee uses these funds to purchase or redeem bonds and retire them. This operation generally is favorable to bondholders because it tends to support the market price of the bonds and assures the steady repayment of the issue. The trustee can retire bonds in two ways. He can purchase them in the open market. To prevent the purchase of bonds at too high a price, however, most sinking-fund bond issues provide for a *call price,* which enables the trustee to call the bonds for redemption. Usually, bonds are called on a lottery basis by their serial numbers, which are published in the *Wall Street Journal* and other papers. The trustee will purchase the bonds in the open market as long as the market price is less than the call price; when the market price exceeds the call price, he will call the bonds. For example, if the market price of a bond is $99.75 and the call price is $101.25, the trustee will purchase the necessary bonds in the market.

The amount of required sinking-fund payment may be either fixed or variable, depending upon the terms in the indenture. Under the former arrangement, the corporation makes fixed, equal periodic payments to the trustee. As the bonds are retired, the interest on the outstanding bonds becomes less and less. These fixed sinking-fund payments do not necessarily have to retire all the bonds by final maturity. For example, a $20 million, twenty-year bond issue might call for annual sinking-fund payments of only $500,000. Thus, a $10 million "balloon" payment at final maturity would be required to retire the remaining bonds.

Variable periodic sinking-fund payments are those that are not equal in amount. These payments may be tied to the earnings of the corporation, so that the greater the earnings, the greater the sinking-fund payment. This type of arrangement obviously is appealing to a company and its stockholders. In periods of poor earnings, the company is not constrained by the need to make a fixed sinking-fund payment. Bondholders, of course, would prefer fixed payments, because these payments assure a

[1]See Harry Guthmann and Herbert E. Dougall, *Corporate Financial Policy,* 4th ed. (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962) pp. 218–23, on which this section is based.

steady reduction of the debt over time. If the borrower cannot meet these payments, he would be in clear default under the terms of the indenture. This default enables the trustee to take corrective actions. In the case of variable sinking-fund payments, the borrower would not be in default, and the trustee would be powerless to take corrective measures. The amount of sinking-fund payment may vary also with the number of years to final maturity. For some bond issues, the amount of sinking-fund payment may increase over the years; for others, it may decrease. Variable sinking-fund payments are employed less often than are fixed payments.

Serial Bonds. All sinking-fund bonds in an issue mature on the same date, although specific bonds are retired before that date. Serial bonds, however, mature periodically until final maturity. For example, a $20 million issue of serial bonds might have $1 million of the bonds maturing each year for twenty years. With a serial bond issue, the investor is able to choose the maturity that best suits his needs. Thus, a bond issue of this type appeals to a wider group of investors than an issue in which all the bonds have the same maturity.

TYPES OF BONDS

DEBENTURES

The term "debenture" usually applies to the unsecured bonds of a corporation; the investor looks to the earning power of the corporation as his security. Because these general credit bonds are not secured by specific property, in the event of liquidation the holder becomes a general creditor. Although the bonds are unsecured, debenture holders are protected by the restrictions imposed in the indenture. One of the more important of these restrictions is a negative pledge clause, which precludes the corporation from pledging its assets to other creditors. This provision safeguards the investor in that the borrower's assets will not be impaired in the future. Because debenture holders must look to the general credit of the borrower to meet principal and interest payments, only well-established and credit-worthy companies are able to issue debentures.

SUBORDINATED DEBENTURES

Subordinated debentures represent debt that ranks behind other unsecured debt with respect to the claim on assets. In the event of liquidation, subordinated debenture holders would receive settlement only if all unsecured and secured creditors were paid the full amount owed them. However, these holders still would rank ahead of preferred and common stockholders. In the event of liquidation, the existence of sub-

ordinated debentures works to the advantage of senior holders, because these holders are able to assume the claims of the subordinated debenture holders. To illustrate, suppose a corporation is liquidated for $600,000 and that it had $400,000 in straight debentures outstanding, $400,000 in subordinated debentures outstanding, and $400,000 in obligations owed to general creditors. One might suppose that the straight debenture holders and the general creditors would have an equal and prior claim in liquidation—that is, each would receive $300,000. However, the straight debenture holders are entitled to the subordinated debenture holders' claims, giving them $800,000 in total claims. As a result, they are entitled to two-thirds of the liquidating value, or $400,000; whereas general creditors are entitled to only one-third, or $200,000.

Because subordinated debentures are subordinate to all existing and future debt, senior creditors regard them as equity when evaluating the financial condition of the company. In fact, subordinated debt usually is employed to increase the equity base and support further borrowing.[2] Finance companies have made extensive use of this type of debt in their capital structure. The increasing popularity of the instrument stems in part from the fact that interest payments are deductible for tax purposes, whereas dividends on preferred stock, the closest substitute method of financing, are not.

Because of the nature of the claim, a straight subordinated debenture issue has to provide a yield significantly higher than a regular debenture issue in order to be attractive to investors. Frequently, however, subordinated debentures are convertible into common stock and therefore may sell at a yield that actually is less than what the company would have to pay on an ordinary debenture. From the standpoint of a creditor, the equity base of the firm is the same whether the issue remains as subordinated debentures or is converted into common stock.

MORTGAGE BONDS

A mortgage bond issue is secured by a lien on specific assets of the corporation—usually fixed assets. The specific property securing the bonds is described in detail in the mortgage, which is the legal document giving the bondholder a lien on the property. As with other secured lending arrangements, the market value of the collateral should exceed the amount of the bond issue by a reasonable margin of safety. If the corporation defaults in any of the provisions of the bond indenture, the trustee, on behalf of the bondholders, has the power to foreclose. In a foreclosure, the trustee takes over the property and sells it, using the

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[2] See Robert W. Johnson, "Subordinated Debentures: Debt that Serves as Equity," *Journal of Finance,* X (March, 1955), 1–16.

proceeds to pay the bonds. If the proceeds are less than the amount of the issue outstanding, the bondholders become general creditors for the residual amount.

A company may have more than one bond issue secured by the same property. For example, a bond issue may be secured by a *second mortgage* on property already used to secure another bond issue under a *first mortgage*. In the event of foreclosure, the first-mortgage bondholders must be paid the full amount owed them before there can be any distribution to the second-mortgage bondholders. For the obvious reason of lack of appeal to investors, second-mortgage bonds seldom are used. When they are, the connotation usually is that financing has reached a rather desperate state.

A mortgage may be either *closed-end* or *open-end*. When a mortgage is closed, additional bonds cannot be issued under that lien. In order to raise additional funds through mortgage bonds, the company must mortgage additional properties. The result is frequently a hodgepodge of mortgage bond issues outstanding. Under an open-end mortgage, however, the company can issue additional bonds under an existing lien. This arrangement allows the company to issue various series of bonds at different times under the same lien. In this respect, it gives the company considerable flexibility in its financing. In order to protect the position of the bondholders of earlier series, certain restrictions usually are imposed that limit the amount of additional debt. These restrictions include a maximum percentage on the amount of bonds in relation to the value of the property securing these bonds and a minimum earning power of the company in relation to the bonds outstanding. Public utilities and railroads have used open-end mortgages rather extensively and with notable success.

Many mortgage bond issues have an *after-acquired clause*. Under this clause, the lien covers acquisitions of property after the initial bond issue. The after-acquired clause affords investors additional protection, because any property acquired in the future is added to the lien. If the mortgage is open-end, which is almost always the case in this situation, the after-acquired clause does not restrict the company from additional mortgage financing. It merely assures existing bondholders that future bondholders will have exactly the same claim on assets as they do. It is important to recognize that *even with a mortgage bond issue, investors look to the earning power of the corporation as the primary test of creditworthiness*.

COLLATERAL TRUST BONDS

A collateral trust bond is secured by stocks or bonds pledged by the corporation to the trustee. In the case of default, the trustee can sell the

securities and pay the bondholders. Usually the securities held in colla-
teral trust are securities of some other corporation. To a large extent,
the quality of these securities determines the attractiveness of the col-
lateral trust bonds to investors. This type of bond issue, employed in the
past, now is used very frequently.

INCOME BONDS

With an income bond, a company is obligated to pay interest only
when it is earned. There may be a cumulative feature in the issue where
unpaid interest in a particular year accumulates. If the company does
generate earnings, it will have to pay the cumulative interest to the ex-
tent that earnings permit. However, the cumulative obligation usually
is limited to no more than three years. As should be evident, this type of
security offers the investor a rather weak promise of a fixed return.
Nevertheless, the income bond is still senior to preferred and common
stock as well as to any subordinated debt. Moreover, the interest pay-
ment is deductible for tax purposes, unlike preferred stock dividends.
Because income bonds are not popular with investors, they have been
used principally in reorganizations.[3]

EQUIPMENT TRUST CERTIFICATES

Although equipment trust financing is a form of lease financing, the
certificates themselves represent an intermediate- to long-term fixed-
income investment. This method of financing is used by railroads to fi-
nance the acquisition of "rolling stock." Under this method, the railroad
arranges with a trustee to purchase equipment from a railway equipment
manufacturer. The railroad signs a contract with the manufacturer for
the construction of specific equipment. When the equipment is delivered,
equipment trust certificates are sold to investors. The proceeds of this
sale, together with the down payment by the railroad, are used to pay
the manufacturer for the equipment. Title to the equipment is held by
the trustee, who in turn leases the equipment to the railroad. Lease pay-
ments are used by the trustee to pay a fixed return on the certificates
outstanding — actually a dividend — and to retire a specified portion of the
certificates at regular intervals. Upon the final lease payment by the rail-
road, the last of the certificates is retired, and title to the equipment passes
to the railroad.

The duration of the lease varies according to the equipment involved,

[3] For a discussion of income bonds, see Bowman Brown, "Why Corporations Should
Consider Income Bonds," *Financial Executive,* 35 (October, 1967), 74–78; and Frank A.
Halford, "Income Bonds," *Financial Analysts Journal,* 20 (January–February, 1964),
73–79.

but fifteen years is rather common. Because rolling stock is essential to the operation of a railroad and has a ready market value, equipment trust certificates enjoy a very high standing as fixed-income investments. As a result, railroads are able to acquire cars and locomotives on extremely favorable financing terms. In addition to railroads, airlines use a form of equipment trust certificate to finance jet aircraft. Usually these certificates are sold to institutional investors; however, some issues are sold to the public.

CONVERTIBLE BONDS

A convertible bond is one that may be converted at the option of the holder into a certain number of shares of common stock of the corporation. The number of shares into which the bond is convertible is specified in the bond; and these shares remain unissued until actual conversion. Because we consider convertible securities in depth in Chapter 14, they are not discussed at this time.

CALL FEATURE

Nearly all corporate bond issues provide for a call feature, which gives the corporation the option to buy back the bonds at a stated price before their maturity. The call price usually is above the par value of the bond and decreases over time. For example, a thirty-year bond issue might be callable at $106 ($1,060 per $1,000 face value bond) the first five years, $105 the second five years, and so on until the final five years, when it is callable at $100. Frequently, the call price in the first year is established at one year's interest above the face value of the bond. If the coupon rate is 8 per cent, the initial call price may be $108, ($1,080 per $1,000 face value).

There are two types of call provision, according to when they can be exercised. Some issues state that the call privilege can be exercised immediately after issuance; with other issues, the call privilege is deferred for a period. The most widely used deferred call periods are five years for public utility bonds and ten years for industrial bonds. During this deferment period, the investor is protected from a call by the issuer.

The call provision gives the company flexibility in its financing. If interest rates should decline significantly, it can call the bonds and refinance the issue at a lower interest cost. Thus, the company does not have to wait until the final maturity to refinance. In addition, the provision may be advantageous to the company if it finds any of the protective covenants in the bond indenture to be unduly restrictive. By calling the

bonds before maturity, the company can eliminate these restrictions. Of course, if the issue is refinanced with bonds, similar restrictions may be imposed.

VALUE OF CALL PRIVILEGE

Although the call privilege is beneficial to the issuing corporation, it works to the detriment of investors. If interest rates fall and the bond issue is called, they can invest in other bonds only at a sacrifice in yield to maturity. Consequently, the call privilege usually does not come free to the borrower. Its cost, or value, is measured at the time of issuance by the difference in yield on the callable bond and the yield that would be necessary if the security were noncallable. This value is determined by supply and demand forces in the market for callable securities. In equilibrium, the value of the call feature will be just sufficient to bring the demand for callable securities by investors into balance with the supply of callable securities by borrowers. In the equilibirating process, both borrowers and investors are influence by expectations as to the future course of interest rates.

INTEREST RATE EXPECTATIONS

When interest rates are high and expected to fall, the call feature is likely to have significant value. Investors are unwilling to invest in callable bonds unless such bonds yield more than bonds that are noncallable, all other things the same. In other words, they must be compensated for assuming the risk that the bonds might be called. On the other hand, borrowers are willing to pay a premium in yield for the call privilege in the belief that yields will fall and that it will be advantageous to refund the bonds. In equilibrium, both the marginal borrower and the marginal investor will be indifferent as to whether the bond issue is callable or non-callable.[4]

When interest rates are low and expected to rise, the call privilege may have a negligible value in that the company might pay the same yield if there were no call privilege. For the privilege to have value, interest rate expectations must be such that there is a possibility that the issue will be called. If interest rates are very low and not expected to fall further, there is little probability that the bonds will be called. The key factor is that the borrower has to be able to refund the issue at a profit. In order for him to do so, interest rates have to drop significantly; for the issuer must pay the call price, which is usually at a premium above par value,

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[4] See Gordon Pye, "The Value of the Call Option on a Bond," *Journal of Political Economy,* LXXIV (April, 1966), 200–203.

as well as the flotation costs involved in refinancing.[5] If there is no probability that the borrower can refund the issue at a profit, the call privilege is unlikely to have a value.

Empirical Studies. Because most corporate bonds have call features, empirical studies of the differential in yield on a noncallable bond and a callable bond are not possible. However, it is possible to examine the yield differential between newly issued corporate bonds having an immediate call privilege and those of the same grade having a five-year deferred call. For the immediate call privilege to have a value over the deferred call privilege, interest-rate expectations must be such that the immediately callable bond might be called during the deferment period. If there is no probability of its being called during this period, the value of the immediate over the deferred call privilege will be zero.

An examination of the yield differential between newly issued bonds of the same grade but with different call privileges reveals that the differential tends to increase in times of high interest rates and tight money, and to decline in periods of easy money and low interest rates. The differential for immediately callable and five-year deferred callable Aa public utility bonds over the period 1958–70 is shown in Figure 12-1. For the 1959, 1966, and 1968–70 periods of relatively high interest rates, the differential was fifteen to thirty basis points; whereas during the 1963–65 period, an immediately callable bond offered no premium over a deferred callable bond. Frank C. Jen and James E. Wert tested the offering yields of newly issued utility bonds over the 1960–64 period and found the yield differential to be around zero when coupon rates were low, and positive when coupon rates were high.[6] In another test, Jen and Wert computed and compared average callable yields and average call-free yields on 434 utility issues issued between 1956 and 1964.[7] The authors found that in periods of relatively high interest rates a number of issues were called. As a result, the average yield actually realized by investors on bonds issued in periods of high interest rates was only slightly higher than the average yield realized on bonds issued in moderate interest-rate periods.

Overall, the evidence is consistent with the notion that the call privilege has the most value, and the most cost to the corporation, when

[5] For an analysis of the profitability of refunding, see the subsequent section.

[6] "The Value of the Deferred Call Privilege," *National Banking Review*, 3 (March, 1966), 369–78. For an extension of this study, see Jen and Wert, "The Deferred Call Provision and Corporate Bond Yields," *Journal of Financial and Quantitative Analysis*, III (June, 1968), 157–69.

[7] "The Effect of Call Risk on Corporate Bond Yields," *Journal of Finance*, XXII (December, 1967), 637–51.
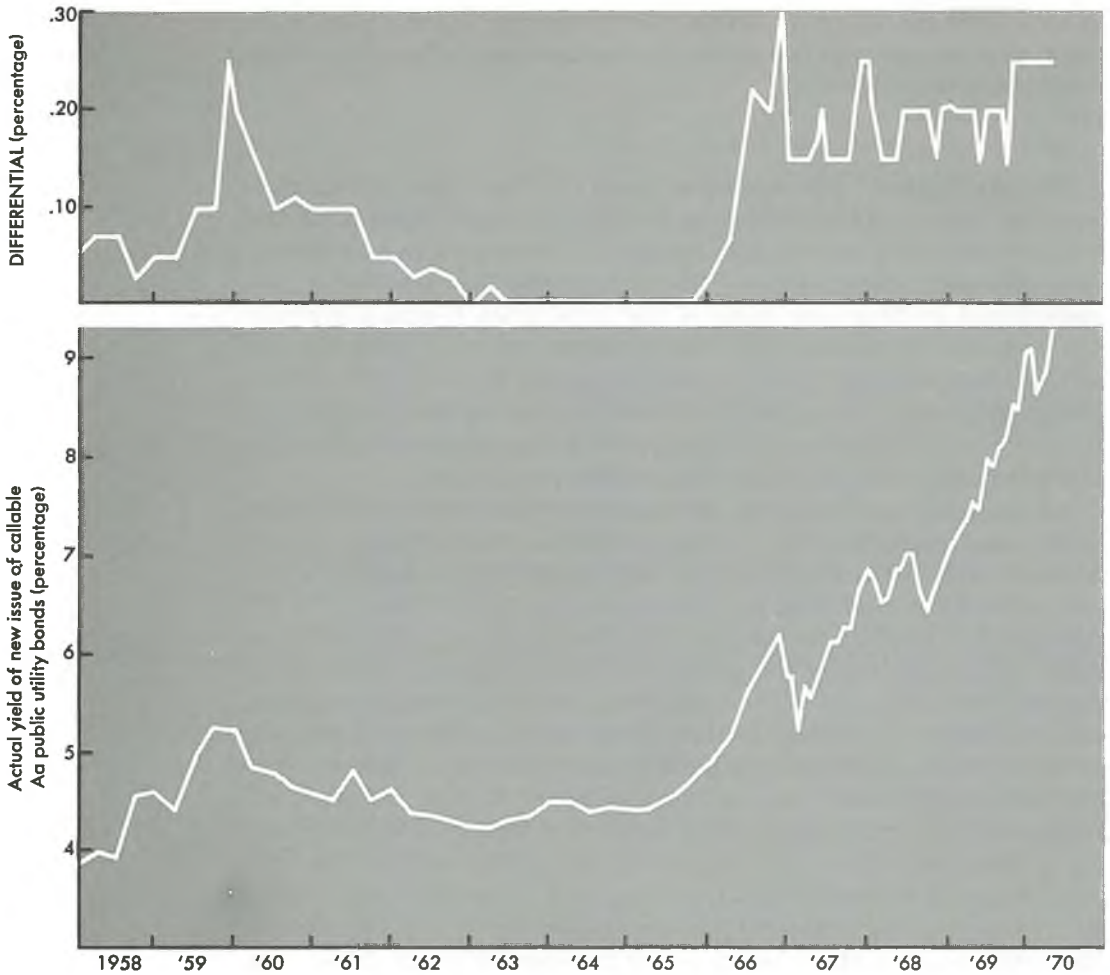
FIGURE 12-1

Yield differentials: new issues of callable and deferred callable Aa public utility bonds, 1958–April, 1970. Source: *An Analytical Record of Yields and Yield Spreads* (New York: Solomon Brothers & Hutzler, 1967).

interest rates are high and are expected to fall. By the same token, the call privilege has the greatest potential benefit to the corporation at these times. However, for this privilege, the corporation must pay a cost at the time the bonds are sold. We turn now to the question of refinancing an existing bond issue, given a call feature in the bond.

In this section, we analyze the profitability of refunding a bond issue before its maturity.[8] We assume that the decision to refund is based upon profitability alone; other considerations, such as removing restrictive protective covenants, are ignored. The refunding decision can be regarded as a form of capital budgeting; there is an initial cash outlay followed by future interest savings. These savings are represented by the difference between the annual cash outflow required under the old bonds and the net cash outflow required on the new, or refunding, bonds. Calculating the initial cash outlay is more complex. Consequently, it is best to use an example to illustrate the method of evaluation.

A HYPOTHETICAL EXAMPLE

Suppose that a company has currently a $20 million, 6 per cent debenture issue outstanding and that the issue still has twenty years to final maturity. In addition, assume that interest rates are significantly lower now than at the time of the original offering. As a result, the company can now sell a $20 million issue of twenty-year bonds at a coupon rate of 5 per cent that will net it $19,600,000 after the underwriting spread.

For federal income tax purposes, the unamortized issuing expense of the old bonds, the call premium, and the unamortized discount of the old bonds, if they were sold at a discount, are deductible as expenses in the year of the refunding. Assume that the old bonds were sold originally at a slight discount from par value and that the unamortized portion now is $200,000. Moreover, the legal fees and other issuing expenses involved with the old bonds have an unamortized balance of $100,000. Finally, let us assume a call price on the old bonds of $105, issuing expenses on the new bonds of $150,000, a federal income tax rate of 50 per cent, and a thirty-day period of overlap. The period of overlap is the lag between the time the new bonds are sold and the time the old bonds are called. This lag occurs because most companies wish to have the proceeds from the new issue on hand before they call the old issue. Otherwise, there is a certain amount of risk associated with calling the old issue and being at the "mercy" of the bond market in raising new funds. During the period of overlap, the company pays interest on both bond issues.

Framework for Analysis. With this rather involved background information in mind, we can calculate the initial cash outflow and the

[8] This section draws heavily upon Oswald D. Bowlin, "The Refunding Decision: Another Special Case in Capital Budgeting," *Journal of Finance,* XXI (March, 1966), 55–68.

future cash benefits. The net cash outflow at the time of the refunding is as follows.

Cost of calling old bonds (call price $105)		$21,000,000
Net proceeds of new bond issue		19,600,000
Difference		$ 1,400,000
Expenses:		
Issuing expense of new bonds	$ 150,000	
Interest expense on old bonds during overlap period	100,000	250,000
Gross cash outlay		$ 1,650,000
Less tax savings:		
Interest expense on old bonds during over-lap period	100,000	
Call premium	1,000,000	
Unamortized discount on old bonds	200,000	
Unamortized issuing expenses on old bonds	100,000	
Total	$1,400,000	
Tax savings (50% of amount above)		700,000
Net cash outflow		$ 950,000

The annual net cash benefits may be determined by calculating the difference between the net cash outflow required on the old bonds and the net cash outflow required on the new or refunding bonds. The annual net cash outflow on the old bonds is

Interest expense 6%		$1,200,000
Less tax savings:		
Interest expense	$1,200,000	
Amortization of bond discount ($200,000/20)	10,000	
Amortization of issuing costs ($100,000/20)	5,000	
Total	$1,235,000	
Tax savings (50% of amount above)		617,500
Annual net cash outflow—old bonds		$ 582,500

The annual net cash outflow on the new bonds is

Interest expense 5%		$1,000,000
Less tax savings:		
Interest expense	$1,000,000	
Amortization of bond discount ($400,000/20)	20,000	
Amortization of issuing costs ($150,000/20)	7,500	
Total	$1,027,500	
Tax savings (50% of amount above)		513,750
Annual net cash outflow—new bonds		$ 486,250
Difference between annual net cash outflows		$ 96,250

Discounting. Thus, for an initial net cash outflow of $950,000, the company can achieve annual net cash benefits of $96,250 over the next twenty years. Since the net cash benefits occur in the future, they must be discounted back to present value. But what discount rate should be used? Certain authors advocate the use of the cost of capital. However, a refunding operation differs from other investment proposals. Once the new bonds are sold, the net cash benefits are known with certainty. From the standpoint of the corporation, the refunding operation is essentially a riskless investment project. In keeping with our discussion in Chapter 5, the appropriate discount rate is the after-tax risk-free rate, or the time value of money. This rate will correspond to the after-tax cost of borrowing for the corporation, or about 2.5 per cent in our example. Using this rate as our discount factor, the refunding operation would be a worthwhile undertaking only if the net-present value were positive.[9] For our example, the net-present value is approximately $552,000, indicating that the refunding operation is worthwhile. The internal rate of return is 7.9 per cent, indicating again that the refunding is worthwhile, because the internal rate of return exceeds the required rate of 2.5 per cent.

TIMING OF REFUNDING

We must recognize, however, that just because a refunding operation is found to be worthwhile, it should not necessarily be undertaken right away. If interest rates are declining, and this decline is expected to continue, management may prefer to delay the refunding. At a later date, the refunding bonds can be sold at an even lower rate of interest, making the refunding operation even more worthwhile. The decision concerning timing must be based upon expectations of future interest rates. In determining whether or not to postpone refunding, the financial manager should also consider the dispersion and shape of the probability distribution of possible future interest rates. The greater the dispersion and the greater the skewness of the distribution to the right (toward higher interest rates), the less desirable it is to postpone the refunding, all other things the same.[10]

Finally, two points should be raised with respect to the calculations in our example. First, most firms refund an existing issue with a new bond issue of a longer maturity. In our example, we assumed that the new bond

[9] We recall from Chapter 3 that the net-present value is the present value of net cash benefits less the initial cash outflow.

[10] Weingartner has developed a dynamic programming model for dealing with the timing of the refunding decision. Expectations of future interest rates are based upon the term structure of interest rates at a moment in time. H. Martin Weingartner, "Optimal Timing of Bond Refunding," *Management Science,* 13 (March, 1967), 511–24. In contrast, Harold Bierman, Jr., "The Bond Refunding Decision as a Markov Process," *Management Science,* 12 (August, 1966), 545–51, used probabilities of future interest rates generated through Markov chains to decide whether to refund now or wait.

issue has the same maturity as that of the old bond issue. Our analysis needs to be modified slightly when the maturity dates are different. The usual procedure is to consider only the net cash benefits up to the maturity of the old bonds.[11] A second assumption in our example was that neither issue involved sinking-fund bonds or serial bonds. If either issue calls for periodic reduction of the debt, we must adjust our procedure for determining future net cash benefits.

SUMMARY

Our concern in this chapter has been with the various features and types of long-term debt. The decision to use long-term debt in the capital structure and the amount of debt to be employed were considered in Chapters 7 and 8. The principal features of debt include the fixed return, the priority of claim on assets, the call privilege, and the method of retirement of the debt. We saw that periodic reduction of the debt can be accomplished by issuing either sinking-fund bonds or serial bonds.

In financing with long-term debt, the company must bargain with investors over the terms of the debt instrument. If the company wishes to include terms that are not beneficial to investors, it must be prepared to pay a higher yield in order to sell the instrument. For example, if debentures are subordinated, investors will demand a higher yield than if the issue involves straight debentures. Another interesting aspect of the bargaining process between the borrower and investors relates to the call privilege. If interest rate expectations in the market are such that investors think that the issue may be called, the company will have to pay a higher yield for the privilege of being able to call it.

In the last section of the chapter, a method was proposed for analyzing the refunding of an existing bond issue before maturity. This method treats the refunding operation as a riskless capital-budgeting project.

PROBLEMS

1. The Lemand Corporation has $10 million of 8% mortgage bonds outstanding under an open-end indenture. The indenture allows additional bonds to be issued as long as all of the following conditions are met:
(a) Pretax interest coverage [(income before taxes + bond interest)/bond interest] remains greater than 4.
(b) Net depreciated value of mortgaged assets remains twice the amount of mortgage debt.

[11] Mao, however, develops a model where the expected interest rate on bonds at the time the outstanding issue matures is expressed in probabilistic terms. He then treats the probabilistic difference between the interest rate on these bonds and that on the refunding bonds as the interest savings or dissavings during the span between the two maturity dates. These savings or dissavings for the additional years are treated as future cash benefits and incorporated in the above model. James C. T. Mao, *Quantitative Analysis of Financial Decisions* (London: Macmillan & Co. Ltd., 1969), pp. 351–62.

(c) Debt/equity ratio remains below 0.5.

The Lemand Corporation has net income of $2 million and a 50 per cent tax rate, $40 million in equity, and $30 million in depreciated assets, covered by the mortgage, which are depreciated at $2 million per year. Assuming that 50 per cent of the proceeds of a new issue would be added to the base of mortgaged assets and that the company has no sinking-fund payments until next year, how much more 8 per cent debt could be sold?

2. The Hirsch Corporation is in bankruptcy. Mortgaged assets have been sold for $5 million and other assets have yielded $10 million. Hirsch has $10 million in mortgage bonds, $5 million in subordinated (to the mortgage bonds) debentures, $15 million owed to general creditors, and $10 million par value of common stock. How would distribution of the $15 million in liquidating value be made?

3. Recompute problem 4 of Chapter 11 under the assumption that the public bond issue would require a 5 per cent annual sinking fund beginning in year 10 so as to retire 50 per cent of the issue prior to maturity. Which alternative would now be more attractive? What assumptions must be made?

4. The Las Palmas Corporation wishes to borrow $10 million for ten years. The company earns 14 per cent before taxes on its funds. It can issue either a noncallable bond for 9 per cent or a bond callable at par at the end of five years for 10 per cent. For simplicity, we assume that the bond will be called only at the end of year 5 and that both bonds can be sold at par. The interest rate that the company would have to pay at the end of year 5 on a five-year bond is approximately normally distributed, with a mean of 7 per cent and a standard deviation of 1 per cent. What is the probability that the callable bond has the greater net pretax present value to the firm?

5. Recompute problem 4 above, assuming a call price of 105 at the end of year 5. How does this affect the probability that the callable bond is superior?

6. The U.S. Zither Corporation has $50 million of 10% debentures outstanding which are due in thirty years. USZ could refund these bonds in the current market with new thirty-year bonds, sold to the public at par ($1,000) with a 9 per cent coupon; total gross spread would be $2\frac{1}{2}$ per cent of the issue. The old bonds have an unamortized discount of $1 million, unamortized legal fees and other expenses of $300,000, and a call price of 107. The tax rate is 50 per cent. Treasury bills yield 6 per cent, there is a two-month overlap, and issuing expenses are $200,000. Compute the present value of the refunding, using after-tax discount rates of 3 per cent, 10 per cent, and 15 per cent.

7. *Research Project*

Obtain copies of several bond indentures. Pay particular attention to the restrictive covenants concerning such things as dividends, working capital, additional debt, and nature of the business. Try to relate the cost of debt to the firm to the relative restrictiveness of these provisions. Does management pay extra for discretion? If it does, can these covenants truly be said to be nonquantifiable? How would you go about finding a measure of degree of restriction so that trade-offs with interest could be made?

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Preferred Stock
and
Common Stock

13

In this chapter, we take up two forms of equity financing—preferred stock and common stock. Although they both fall under the same general heading, the differences between the two are far more pronounced than their similarities. From the standpoint of the ultimate owners of the corporation—namely, the common stockholders—preferred stock is a form of leverage to be evaluated in a manner similar to that of debt. As the theory behind the use of these securities was discussed in Part III, this chapter is devoted primarily to examining their features.

PREFERRED STOCK

Preferred stock is a hybrid form of financing, combining features of debt and common stock. In the event of liquidation, a preferred stockholder's claim on assets comes after that of creditors but before that of

common stockholders. Usually, this claim is restricted to the par value of the stock. For example, if the par value of a share of preferred stock is $100, the investor will be entitled to a maximum of $100 in settlement of the principal amount. Although preferred stock carries a stipulated dividend, the actual payment of a dividend is a discretionary, rather than a fixed obligation of the company. The omission of a dividend will not result in a default of the obligation or insolvency of the company. The board of directors has full power to omit a preferred stock dividend if it so chooses.

The maximum return to preferred stockholders usually is limited to the specified dividend, and these stockholders ordinarily do not share in the residual earnings of the company. Thus, if an investor owns 100 shares of 6 per cent preferred stock, $100 par value, the maximum return he can expect in any one year, usually, is $600; and this return is at the discretion of the board of directors. The corporation cannot deduct this dividend on its tax return; this fact is the principal shortcoming of preferred stock as a means of financing. In view of the fact that interest payments on debt are deductible for tax purposes, the company that treats a preferred stock dividend as a fixed obligation finds the explicit cost to be rather high.

FEATURES OF PREFERRED STOCK

CUMULATIVE FEATURE

Almost all preferred stocks have a cumulative feature, providing for unpaid dividends in any one year to be carried forward. Before the company can pay a dividend on its common stock, it must pay the dividends *in arrears* on its preferred stock. For example, suppose that the board of directors of a company omitted the preferred stock dividend on its 6 per cent cumulative preferred stock for three consecutive years. If the stock has a $100 par value, the company would be $18 per share in arrears on its preferred stock. Before it can pay a dividend to its common shareholders, it must pay preferred stockholders $18 for each share of preferred stock held. It should be emphasized that just because preferred stock dividends are in arrears, there is no guarantee that they will ever be paid. If the corporation has no intention of paying a common stock dividend, there is no need to clear up the arrearage on the preferred. The preferred stock dividend typically is omitted for lack of earnings, but the corporation does not have to pay a dividend if earnings are restored.

If the preferred stock dividends are in arrears, and the company wishes to pay a common stock dividend, it may choose not to clear up the arrearage but to make an exchange offering to preferred stockholders. For

example, suppose that the dividend arrearages on an issue of $100 par value preferred stock are $56 and that the market price of the stock is $74 a share. The company might offer preferred stockholders common stock in the company, valued at $110, for each share of preferred stock held. Although theoretically the preferred stockholder is asked to give up $156 ($100 par value plus $56 dividend arrearages), the exchange offering promises him $110 relative to a current preferred stock market value of only $74 per share. In order to eliminate the preferred stock, the company must obtain the approval of a required percentage of the stock outstanding, often two-thirds. Consequently, it probably will make its exchange offering contingent upon obtaining the required acceptance. If the incentive is attractive enough, preferred stockholders probably will accept the offer despite the fact that they are not satisfied to the full extent of the arrearages.[1]

If a preferred stock is noncumulative, dividends not paid in one year do not carry forward. As a result, a company can pay a common stock dividend without regard to any dividends it did not pay in the past on its preferred stock. From the standpoint of an investor, a noncumulative preferred stock is little more than an income bond. In fact, there is somewhat less uncertainty with income bonds, for the conditions under which interest will be paid are specified clearly, and bondholders have a prior claim on assets. Because of the obvious disadvantage to investors, noncumulative preferred stock issues are rare, although they may be used in reorganizations.

PARTICIPATING FEATURE

A participating feature allows preferred stockholders to participate in the residual earnings of the corporation according to some specified formula. For example, the preferred stockholder might be entitled to share equally with common shareholders in any common stock dividend beyond a certain amount. Suppose that a 6 per cent preferred stock ($100 par value) were participating, so that the holders were entitled to share equally in any common stock dividends in excess of $6 a share. If the common stock dividend is $7, the preferred stockholder will receive $1 in extra dividends for each share of stock owned. The formula for participation can vary greatly. The essential feature is that preferred stockholders have a prior claim on income and an opportunity for additional

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[1] In 1962, the Virginia Carolina Chemical Company offered preferred stockholders a package of prior-preferred stock, convertible preferred stock, and common stock worth about $150 for each share of $100 par value preferred stock they owned. The dividend arrearages on the preferred stock were $96 a share, giving a preferred stockholder a theoretical claim of $196 a share.

return if the dividends to common stockholders exceed a certain amount. Unfortunately for the investor, practically all preferred stock issues are nonparticipating, with the maximum return limited to the specified dividend rate.

VOTING POWER

Because of their prior claim on assets and income, preferred stockholders normally are not given a voice in management unless the company is unable to pay preferred stock dividends during a specified period of time. For example, arrearages on four quarterly dividend payments might constitute such a default. Under such circumstances, preferred stockholders as a class will be entitled to elect a specific number of directors. Usually, the number is rather small in relation to the total. Moreover, by the time the preferred stockholders are able to obtain a voice in management, the company probably is in considerable financial difficulty. Consequently, the voting power that preferred stockholders are granted may be virtually meaningless.

Depending upon the agreement between the preferred stockholders and the company, they may obtain voting power under other conditions as well. The company may default under certain restrictions in the agreement similar to those found in a loan agreement or a bond indenture. One of the more frequently imposed restrictions is that dividends on common stock are prohibited if the company does not satisfy certain financial ratios. We note, however, that default under any of the provisions of the agreement between the corporation and its preferred stockholders does not result in the obligation's becoming immediately payable, as does default under a loan agreement or bond indenture. The preferred stockholders merely are given a voice in management and assurance that common stock dividends will not be paid during the period of default. Thus, preferred stockholders do not have nearly the same legal power in default as do debt holders.

RETIREMENT OF PREFERRED STOCK

Preferred stock, like common stock, has no maturity. However, most preferred stock issues are not regarded as a means of perpetual financing, because provision for retirement of the stock invariably is made.

Call Feature. Practically all preferred stock issues have a stated call price, which is above the original issuance price and may decrease over time. Like the call feature on bonds, the call feature on preferred stock affords the company flexibility. Because the market price of a straight

preferred stock tends to fluctuate in keeping with interest-rate cycles, the value of the preferred stock call feature is determined by the same considerations as is the call feature for bonds, which we discussed in Chapter 12. However, with long-term debt, unlike with preferred stock, there is a final maturity that assures the eventual retirement of the issue. Without a call feature on preferred stock, the corporation would be able to retire the issue only by the more expensive and less efficient methods of purchasing the stock in the open market, inviting *tenders* of the stock from preferred stockholders at a price above the market price, or offering the preferred stockholders another security in its place.

Sinking Fund.   Many preferred stock issues provide for a sinking fund, which partially assures an orderly retirement of the stock. The trustee of the preferred stock issue may use the sinking-fund payments to either buy stock in the open market or call a portion of it. In either case, the stock is retired. A sinking fund is advantageous to investors because the retirement process exerts upward pressure on the market price of the shares outstanding. Also, the coverage ratio on the preferred stock dividend is improved as the number of shares outstanding is reduced. The sinking fund works to the disadvantage of common stockholders because it represents another prior charge and, therefore, contributes to the financial risk of the company from their standpoint. A preferable arrangement for them would be a sinking-fund requirement wherein payments were variable in relation to earnings. Because the sinking fund is beneficial to preferred stockholders, the company should be able to sell the issue at a lower dividend yield than if it provided for no sinking fund. Overall, sinking funds are used much less with preferred stock than with bonds.

Convertibility.   Certain preferred stock issues are convertible into common stock at the option of the holder. Upon conversion, of course, the preferred stock is retired. Since practically all convertible securities have a call feature, the company can force conversion by calling the preferred stock if the market price of the preferred is significantly above the call price. Convertible preferred stock is used frequently in the acquisition of other companies.[2] In part, its use stems from the fact that the transaction is not taxable for the company that is acquired or its stockholders at the time of the acquisition. It becomes a taxable transaction only when the preferred stock is sold.[3] We shall examine convertible securities in much more detail in Chapter 14.

[2] See Robert M. Soldofsky, "Convertible Preferred Stock: Renewed Life in an Old Form," *The Business Lawyer* (July, 1969), 1385–92.

[3] See Chapter 23 for a more detailed discussion of the tax effect.

ADVANTAGES AND DISADVANTAGES

Preferred stock is not used extensively in financing today, as it was in the past.[4] Only public utilities employ it with any degree of regularity.[5] One of the principal drawbacks to its use is the fact that the preferred dividend is not tax deductible. With a 50 per cent tax rate, the explicit cost of preferred stock is about twice that of bonds. As an investment, however, preferred stock may be more attractive to the corporate investor than bonds because 85 per cent of the dividends received is not subject to taxation. As a result, many preferred stocks sell at a lower yield than do the bonds of the same company, despite their lower priority of claim. In fact, the average yield differential between high-grade industrial bonds and high-grade preferred stocks has narrowed over the post-World War II period to where now preferred stocks yield less on the average.[6] Thus, the after-tax cost disadvantage of preferred stock financing has diminished somewhat during the last 25 years.

The advantage of preferred stock financing is that it is a flexible financing arrangement. The dividend is not a legal obligation on the part of the corporation issuing the securities; if earnings turn bad and the financial condition of the company deteriorates, the dividend can be omitted. With debt financing, interest must be paid regardless of whether earnings are good or bad. To be sure, companies that are accustomed to paying dividends on their common stock certainly regard the preferred dividend as a fixed obligation. Nevertheless, under dire circumstances, a company that omits its common stock dividend also can omit its preferred dividend.

Another advantage of a straight preferred stock issue is that it has no final maturity; in essence, it is a perpetual loan. Also, the majority of preferred stock issues do not require sinking-fund payments. Thus, a preferred stock issue gives a corporation flexibility by allowing it not to make principal payments or plan for refinancing. Moreover, from the standpoint of creditors, preferred stock adds to the equity base of the company and thereby strengthens its financial condition. The additional

[4] In 1969, only about 3 per cent of the gross proceeds from the sale of corporate securities was attributable to preferred stock financing. For an excellent review of preferred stock financing, see Donald E. Fisher and Glenn A. Wilt, Jr., "Nonconvertible Preferred Stock as a Financing Instrument," *Journal of Finance*, XXIII (September, 1968), 611–24.

[5] One reason for the use of preferred stock by utilities is that the Securities and Exchange Commission stated in 1952 that the capital structure of an electric utility should not exceed 60 per cent debt, and that common stock should not be less than 30 per cent. Thus, the 10 per cent residual could be filled by preferred stock. Another reason is that a public utility is able to pass off the higher explicit cost of preferred stock, as compared with that of debt, in the rates it charges. Public utility commissions allow utilities to base their rates on their overall measured cost of capital.

[6] For further analysis of this trend, see Fisher and Wilt, "Nonconvertible Preferred Stock as a Financing Instrument, 1950–1965," pp. 621–23.

equity base enhances the ability of the company to borrow in the future. Although the explicit cost of preferred stock is considerably higher than that of bonds, the implied benefits discussed above may offset this cost. In addition, the implicit cost of preferred stock financing, from the standpoint of investors penalizing the price/earnings ratio of the common stock, may be somewhat less than that of debt financing. To the extent that investors are apprehensive over legal bankruptcy, they would regard debt as a riskier form of leverage. Unlike creditors, preferred stockholders cannot force a company into legal bankruptcy.

AN ARGUMENT FOR PREFERRED STOCK

Donaldson has defended rigorously the use of preferred stock as a means of financing,[7] on the basis of some of the advantages mentioned above. He suggests that, from the standpoint of the common stockholders of a company, the priority list of financing might be

1. Debt
2. Preferred stock
3. Retained earnings
4. Common stock

He argues that the preferred stock capacity of a company is distinct from its debt capacity because the legal obligation of paying interest on debt creates a risk of insolvency, while the discretionary obligation of paying a preferred stock dividend does not.[8] As a result, the joint capacity of debt plus preferred stock of a company is said to be greater than its debt capacity alone. Whereas management might not be willing to add additional interest and principal charges to the corporation's existing fixed charges, it may be willing to add the obligation to pay a preferred stock dividend. Thus, preferred stock may be used *in addition* to debt, rather than as a substitute for it, when the debt capacity of the company is utilized fully.

Assuming a company can employ preferred stock in addition to debt, the decision to use preferred stock as a means of financing must still be evaluated in relation to retained earnings and common stock, according to Donaldson. Preferred stock would be favored over retained earnings as a method of financing if

$$S\left(1 - \frac{T}{100}\right) > D \times \frac{P}{E} \times (1 - 0.25) \tag{13-1}$$

[7] Gordon Donaldson, "In Defense of Preferred Stock," *Harvard Business Review*, XL (July–August, 1962), reprinted in Van Horne, ed., *Foundations for Financial Management*, pp. 194–218.

[8] *Ibid.*, p. 212.

where S = Amount of funds to be raised

T = Rate of personal income tax assumed to be representative of common shareholders

P = Established (and anticipated) market price of common stock

E = Established (and anticipated) earnings per share

D = Total preferred stock dividend

0.25 = Assumed capital gains tax

Donaldson's formula suggests that if the amount of usable funds lost to common stockholders in dividends foregone under the retained-earnings alternative exceeds the amount of usable funds lost because of market price depreciation, owing to the lower earnings caused by preferred stock dividends, preferred stock should be used for financing.[9] If not, retained earnings should be used. Donaldson argues that if retained earnings do not constitute a superior method of financing, common stock financing will not be superior, because of the issuance costs involved.[10]

In summary, Donaldson proposes that the use of preferred stock financing is beneficial to common stockholders under certain circumstances. When the company has already utilized its debt capacity, it may be able to finance further with preferred stock because the preferred stock capacity of a company is distinct from its debt capacity. The decision to use preferred will rest upon whether it is more beneficial to common stockholders than the use of retained earnings as a method of financing. Certainly, Donaldson's analysis is not free from flaws, particularly with respect to the implications for dividend policy, the second-order effects of the investment and financing decisions on the market price of the stock, and the difficulty of determining the marginal tax rate for common stockholders. However, his position stands in sharp contrast to the many arguments against preferred stock financing and, consequently, is interesting to consider.

COMMON STOCK

The common stockholders of a corporation are its residual owners; collectively, they own the company and assume the ultimate risk associated with ownership. Their liability, however, is restricted to the amount of their investment. In the event of liquidation, these stockholders have a residual claim on the assets of the company after the claims of all creditors and preferred stockholders are settled in full. Common stock, like pre-

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[9] *Ibid.*, p. 214.

[10] This argument assumes that management is able to anticipate the need for funds and retain a sufficient amount by foregoing dividends. If the company cannot wait for sufficient funds to accumulate, it will have to resort to external financing.

ferred stock, has no maturity date; and a stockholder can liquidate his
investment by selling his stock in the secondary market.

AUTHORIZED, ISSUED, AND OUTSTANDING SHARES

The corporate charter of a company specifies the number of *authorized*
shares of common stock, the maximum that the company can issue with-
out amending its charter. Although amending the charter is not a difficult
procedure, it does require the approval of existing stockholders, which
takes time. For this reason, a company usually likes to have a certain
number of shares that are authorized but unissued. These unissued shares
allow flexibility in granting stock options, pursuing mergers, and splitting
the stock. When authorized shares of common stock are sold, they be-
come *issued* stock. *Outstanding* stock is the number of shares issued that
actually are held by the public; the corporation can buy back part of its
issued stock and hold it as Treasury stock.

PAR VALUE

A share of common stock can be authorized either with or without par
value. The par value of a stock is merely a stated figure in the corporate
charter and is of little economic significance. However, a company should
not issue stock at a price less than par value, because stockholders are
liable to creditors for the difference between the price they paid and the
par value. Consequently, the par values of most stocks are set at fairly
low figures relative to their market values. Suppose a company sold
10,000 shares of new common stock at $45 a share and that the par value
of the stock was $5 per share. The equity portion of the balance sheet
would be

Common stock ($5 par value)	$ 50,000
Capital surplus	400,000
Net worth	$450,000

Stock can be authorized withour par value, in which case the stock is
carried on the books at the market price at which it is sold or at some
stated value. The difference between the issuing price and the stated
value is reflected as capital surplus.

BOOK VALUE AND LIQUIDATING VALUE

The book value of a share of stock is the net worth of a corporation
less the par value of preferred stock outstanding, divided by the number

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of shares outstanding. Suppose the equity portion of the balance sheet of a company is as follows.

Preferred stock ($100 par value)	$10,000,000
Common stock ($5 par value)	5,000,000
Capital surplus	10,000,000
Retained earnings	16,000,000
Net worth	$41,000,000

The book value of a share of common stock is $31 million/1 million shares = $31 per share. Theoretically, the book value of a share of stock should correspond to the liquidating value of the company, but this situation seldom occurs. Only if the assets of a corporation can be liquidated for the book values shown on the financial statement will book value per share correspond to the liquidating value per share. Even then, if liquidating costs are high, the liquidating value per share will be less than book value per share. For most companies, the liquidating value per share is less than book value per share because many of the assets can be liquidated only at distress prices. However, some companies carry certain assets—notably, land and mineral rights—at modest values on their books relative to the market value of the asset. For these companies, the liquidating value per share may be significantly higher than the book value. Sometimes investors calculate the net working capital per share in order to obtain a more conservative estimate of the possible liquidating value of a company.

To the extent that the liquidating value per share of a company exceeds its market value, the company may be subject to "raids." A "raider" buys a company's stock either in the open market or by a tender offer to existing stockholders until he obtains a controlling interest. Upon gaining control of the company, he liquidates it for a value in excess of the price paid for the stock. The managements of companies in which liquidating value per share exceeds market value per share watch all transactions in the company's stock very closely for signs of accumulation.

MARKET VALUE

Market value per share is the current price at which the stock is traded. For listed companies and actively traded over-the-counter stocks, market-price quotations are readily available. However, the market for the stocks of many companies is thin and inactive, so that market-price information is difficult to obtain. Even when obtainable, the information may reflect only the sale of a few shares of stock and not typify the

market value of the firm as a whole. For companies of this sort, care must be taken in interpreting market-price information.[11]

The market value of a share of common stock usually will differ considerably from its book value and its liquidating value. It is a function of the current and expected future dividends of the company and the perceived risk of the stock on the part of investors. As these factors bear only a partial relationship to the book value and liquidating value of the company, the market value per share is not tied closely to these values. Because the valuation of common stocks was studied in depth in Chapter 2, the reader is referred to that chapter for further discussion of market value.

LETTER STOCKS

A letter stock is an unregistered issue of stock sold as a block to an institutional investor. The investor is able to sell it only if the company later registers the issue or if the investor can find a qualified buyer. Because of the lack of marketability, letter stocks are sold at significant discounts from the market price of registered shares outstanding. Discounts can range from 15 per cent all the way up to 50 per cent or more. If a company's stock were trading at $60 a share and the company sold letter stock to a mutual fund at a 25 per cent discount, the fund would pay only $45 a share. One of the primary advantages of letter stock to the corporation is that it is able to sell stock in a matter of days, whereas the lead time required for a public issue often is four months or longer. In addition, small companies and start-up companies are able to sell letter stock where they are not able to sell stock to the public.

Due to the illiquid nature of the stock, the issuer usually must be a company where significant growth is expected. With growth, the investor hopes to be able to sell the stock three or so years hence, when it eventually is registered, at a substantially higher price then he paid for it. However, until registered, letter stock often can not be sold simply because no one wants to buy it. This lack of marketability occurs even though the company involved may be doing very well. Another problem is that several "go-go" mutual funds have engaged in deceptive practices in the purchase of letter stocks.[12] Because a letter stock does not have to be registered with the SEC, certain companies use it to avoid disclosure of information. In the future, the SEC is likely to increase its surveillance of letter-stock financing.

[11] For an analysis of the market price of small-bank stocks, see James Van Horne and Raymoond C. Helwig, *The Valuation of Small-Bank Stocks* (East Lansing, Mich.: Bureau of Business and Economic Research, Michigan State University, 1966).

[12] See Neil Ulman, "Bargain Securities," *Wall Street Journal* (November 18, 1969), pp. 1, 26.

LISTING

Typically, the shares of a new company are traded in the over-the-counter market. In this market, one or more security dealers maintain an inventory in the stock and buy and sell it at bid and ask prices they quote. As a company grows in financial stature, number of stockholders, and volume of transactions, it may qualify for listing on a stock exchange. In contrast to the over-the-counter market, an exchange represents an auction market where buy and sell orders are matched. The listing requirements of the New York Stock Exchange are more stringent than those of the American Stock Exchange or of the regional stock exchanges, such as the Pacific Stock Exchange and the Midwest Stock Exchange.[13]

Once a company satisfies the listing requirements of an exchange, it must decide whether or not to list. It may well want to continue in the over-the-counter market. In fact, stocks of many large companies with heavy volume are traded in the over-the-counter market. One reason often cited for listing is the increased marketability of the stock. If marketability is enhanced, stockholders will gain from the greater liquidity associated with a stock listed on an exchange. Stockholders also may gain from the greater collateral value attractiveness of a listed stock as compared with an over-the-counter one. For the company, there may be a certain amount of prestige associated with being listed on a major stock exchange. For these reasons, many feel that listing on a major exchange improves the market price of the stock. However, a recent empirical study suggests that listing of a stock from the over-the-counter market to either the New York Stock Exchange or the American Stock Exchange does not in itself create value. Moreover, stock prices do not appear to rise upon the announcement to list in any systematic manner that could be exploited profitably by market participants buying the stock upon the announcement to list and selling it upon actual listing.[14]

RIGHTS OF STOCKHOLDERS

RIGHT TO INCOME

Common stockholders are entitled to share in the earnings of the company only if cash dividends are paid. Stockholders prosper from the market value appreciation of their stock, but they are entirely dependent upon the board of directors for the declaration of dividends that give them income from the company. Thus, we see that the position of a common stockholder differs markedly from that of a creditor. If the company

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[13]For a discussion of listing requirements, see Wilford J. Eiteman, Charles A. Dice, and David K. Eiteman, *The Stock Market,* 4th ed. (New York: McGraw-Hill Book Company, 1966), Chapter 10.

[14]James C. Van Horne, "New Listings and Their Price Behavior," *Journal of Finance,* XXV (September, 1970).

fails to pay contractual interest and principal payments to creditors, the creditors are able to take legal action to assure that payment is made or the company is liquidated. Stockholders, on the other hand, have no legal recourse to a company for not distributing profits. Only if management, the board of directors, or both are engaged in fraud may they take their case to court and, possibly, force the company to pay dividends. With stock options, however, the goals of management are likely to approximate those of stockholders.

VOTING POWER

Inasmuch as the common stockholders of a company are its owners, they are entitled to elect a board of directors. In a large corporation, stockholders usually exercise only indirect control through the board of directors they elect. The board, in turn, selects the management; and management actually controls the operations of the company. With a proprietorship, partnership, or small corporation, the owners usually control the operations of the business directly. With a large corporation, however, there may be times when the goals of management differ from those of the common stockholders. The only recourse of a stockholder to management is through the board of directors. Because common stockholders often are widely dispersed geographically and, therefore, disorganized, management can often exercise effective control of a large corporation if it controls only a small percentage of the stock outstanding. By proposing a slate of directors that is favorable to its own interests, management is able to maintain control. An outside stockholder, however, does have the right to expect that the directors will administer the affairs of the corporation properly in his behalf. If the directors act in a manner that results in personal gain, a stockholder can sue to recover. These suits are known as *derivative suits.* However, derivative suits are infrequent, partly because many states require that the stockholder bear the legal expenses of the corporation if he loses the suit. These laws were instigated to prevent stockholders from undertaking derivative suits at the least provocation.

PROXIES

Each common stockholder is entitled to one vote for each share of stock he owns. Because most stockholders do not attend the annual meeting, they may vote by proxy. A proxy is simply a form by which the stockholder assigns his right to vote to another person. The SEC regulates the solicitation of proxies and also requires companies to disseminate information to its stockholders through proxy mailings. Prior to the annual meeting, management solicits proxies from stockholders to vote for the recommended slate of directors and for any other proposals requiring stockholder approval. If stockholders are satisfied with the company, they

generally sign the proxy in favor of management, giving written authorization to management to vote their shares. If a stockholder does not vote his shares, the number of shares voted at the meeting and the number needed to constitute a majority are lower. Because of the proxy system and the fact that management is able to mail information to stockholders at the company's expense, management has a distinct advantage in the voting process. As a result, it usually is able to perpetuate existing practices if it so chooses.

PROXY CONTESTS

However, outsiders can seize control of a company through a proxy contest. Obviously, outsiders would not attempt a take-over if management controlled a large percentage of shares outstanding. When an outside group undertakes a proxy raid, it is required to register its proxy statement with the Securities and Exchange Commission to prevent the presentation of misleading or false information. The outside group attempts to persuade stockholders to sign a proxy giving them the authority to vote the stockholders' shares.

In a proxy contest, the odds favor existing management to win the contest. They have both the organization and the use of the company's resources to carry on the proxy fight. Insurgents are likely to be successful only when the earnings performance of the company has been bad and management obviously ineffective. The lower the rate of return, profit margins, dividend payout, and percentage of stock owned by management, the greater the probability of success for the insurgents.[15] In recent years, proxy contests have been relatively infrequent. To a large extent, they have been replaced by tender offer take-over bids, a topic considered in Chapter 23.

VOTING PROCEDURES

Depending upon the corporate charter, the board of directors is elected either under a *majority voting system* or under a *cumulative voting system*. Under the former system, each stockholder has one vote for each share of stock he owns, and he must vote for each director position that is open. For example, if a stockholder owns 100 shares, he will be able to cast 100 votes for each director's position open. Because each person seeking a position on the board must win a majority of the total votes cast for that position, the system precludes minority interests from electing

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[15]See Richard M. Duvall and Douglas V. Austin, "Predicting the Results of Proxy Contests," *Journal of Finance,* XX (September, 1965), 464–71. They use discriminant analysis with the four variables mentioned above to predict the results.

directors. If management can obtain proxies for over 50 per cent of the shares voted, it can select the entire board.

Under a cumulative voting system, a stockholder is able to accumulate his votes and cast them for less than the total number of directors being elected. His total number of votes is the number of shares he owns times the number of directors being elected. For example, if a stockholder owns 100 shares, and twelve directors are to be elected, he will be entitled to cast 1,200 votes. He can cast these votes for whatever number of directors he chooses, the maximum being 1,200 votes for one director.

A cumulative voting system, in contrast to the majority system, permits minority interests to elect a certain number of directors. The minimum number of shares necessary to elect a specific number of directors is determined by

$$\frac{\text{Total shares outstanding times specific number of directors sought}}{\text{Total number of directors to be elected plus one}} + 1$$

(13-2)

For example, if there are three million shares outstanding, the total number of directors to be elected is fourteen, and if a minority group wishes to elect two directors, it will need at least the following number of shares.

$$\frac{3,000,000 \times 2}{14 + 1} + 1 = 400,001$$

As is evident, cumulative voting gives minority interests a better opportunity to be represented on the board of directors of a corporation. Because the system is more democratic, a number of states require that companies in the state elect directors in this way. Even with cumulative voting, however, management sometimes can preclude minority interests from obtaining a seat on the board of directors by reducing the number of directors. For example, suppose the minority group above actually owns 400,001 shares. With fourteen directors to be elected, the group can elect two directors. However, if the board is reduced to six members, the minority group can elect no directors because the minimum number of shares needed to elect a single director is.

$$\frac{3,000,000 \times 1}{6 + 1} + 1 = 428,572$$

Another method used to thwart a minority interest from obtaining representation is to stagger the terms of the directors so that only a portion is elected each year. For example, if a firm had twelve directors and the term was four years, only three would be elected each year. As a result, a minority group would need considerably more shares voted in its favor to elect a director than it would if all twelve directors came up for election each year.

PREEMPTIVE RIGHT

A preemptive right (see Chapter 11) entitles the common stockholder to maintain his proportional ownership in the corporation. He is given the first opportunity to purchase, on a pro rata basis, any new stock being offered or any new securities that are convertible into common.

RIGHT TO EXAMINE BOOKS

A stockholder legally is entitled to inspect the books and records of a corporation. However, this access is limited, for most corporations feel that the audited financial statement is sufficient to satisfy the requirement. To obtain more specific information, the stockholder may have to take his case to court in order to prove the necessity for obtaining this information. Stockholders are also entitled to a list of the stockholders of the corporation and their addresses. This list is vital to an insurgent group in a proxy contest. However, management may engage in delaying tactics by contending that the stockholder list will be misused. In these situations, the stockholder may have to go to court and demonstrate sufficient cause for obtaining the information. Upon a court order, management is required to provide the list.

CLASSIFIED COMMON STOCK

A company may have more than one class of common stock. Its common stock can be classified with respect to the claim on income and as to voting power. For example, the Class A common of a company may have no voting privilege but may be entitled to a prior claim to dividends, while the Class B common has voting rights but a lower claim to dividends. Usually, the promoters of a corporation and its management will hold the Class B common stock, whereas the Class A common is sold to the public. Actually, the Class A shares in this example are no more than a form of preferred stock. However, the Class A stock usually is given some voting power, but not as much as the Class B stock per dollar of investment. One incentive is that the New York Stock Exchange will not list a nonvoting stock, and other exchanges will do so only with reluctance.

Suppose, for example, that the Class A and Class B common stockholders of a company are entitled to one vote per share, but that the Class A stock is issued at an initial price of $20 a share. If $2 million is raised in the original offering through the issuance of 80,000 shares of Class A common for $1.6 million and 200,000 shares of Class B common for $400,000, the Class B stockholders will have over twice the number of votes as the Class A holders, despite the fact that their original investment is only one quarter as large. Thus, the Class B holders have effective control of the company. Indeed, this is the purpose of classified stock.

For this control, the Class B holders must be willing to give something up in order to make the Class A stock attractive to investors. Usually, they take a lower claim to dividends and a lower claim on assets. An appropriate balance must be struck between voting power and the claim to dividends and assets if the company is to bargain effectively for Class A equity funds. Sometimes, the Class B common simply is given to the promoters of a corporation without any cash investment on their part.

FORD MOTOR COMPANY

An example of a company with classified common stock is the Ford Motor Company. At December 31, 1969, the issued shares of capital stock for the company were

	Shares Issued
Class A stock (nonvoting)	27,971,187
Class B (voting)	12,174,972
Common stock (voting)	69,168,884

The Class A common stock is nonvoting and is held by the Ford Foundation. The Class B common is owned by members of the Ford family and constitutes 40 per cent of the total voting power of the company. The common is held by the general public and has 60 per cent of the voting power of the company. The common stock was owned originally by the Ford Foundation as Class A common, but the stock was converted into common shares and sold to the general public. A holder of common stock of Ford is entitled to one vote for each share he owns. A holder of Class B common is entitled to that number of votes per share that will make the voting power of the Class B common 40 per cent of the total voting power of the corporation. At December 31, 1969, this number was

$$\frac{69,168,884}{12,174,972} \times \frac{0.40}{0.60} = 3.79 \text{ votes}$$

Each shareholder of Class B stock was entitled to 3.79 votes per share at the end of 1969. Thus, the Ford family retains substantial voting power in the company despite the fact that they hold far fewer shares than do the general public and the Ford Foundation. All shares of common, Class A, and Class B stock share equally in dividends and equally in their claim on assets in the event of liquidation.[16] The use of classified capital stock in this case affects only the voting power.

[16]*Annual Report,* Ford Motor Company, 1969; and *Prospectus* to Ford Motor Company Stock, November 20, 1963.

SUMMARY

Preferred stock is a hybrid form of security having characteristics of both debt and common stock. The payment of dividends is not a legal but a discretionary obligation, although many companies regard the obligation as fixed. Preferred stockholders' claims on assets and income come after those of creditors but before those of common stockholders. The return on their investment is almost always limited to the specified dividend; very seldom do preferred stockholders participate in the residual earnings of the company. Although they may have some voting power, this power generally is restricted to situations where the company has evolved itself into financial difficulty.

Because preferred stock has no final maturity, almost all recent issues have had call features that give the corporation financial flexibility. Retirement of the preferred stock can also be accomplished by a sinking fund, by convertibility, or by an exchange offering. The principal disadvantages of preferred stock are that the yield generally is higher than the yield on bonds and that the dividend is not tax deductible. Offsetting in some measure the difference in explicit costs between the two methods of financing are implicit benefits associated with debt capacity and financial flexibility. Despite these implicit benefits, however, preferred stock is little used as a method of financing.

The common stockholders of a corporation are its owners. As such, they are entitled to share in the residual earnings of the company if cash dividends are paid. As owners, however, they have only a residual claim on assets in the event of liquidation. Common stockholders are also entitled to a voice in management through the board of directors they elect. These directors can be elected under a majority voting system or a cumulative voting system; the latter system allows minority interests to obtain representation on the board of directors. The use of different classes of common stock allows the promoters and management of a corporation to retain voting control without having to make a large capital contribution.

PROBLEMS

1. Eleven years ago the Delano Corporation sold 10,000 shares of 6 per cent, $100 par preferred callable at 105. After a year of paying dividends on this stock, Delano fell upon hard times, with the result that each share is now $60 in arrears. Conditions have improved, however, so that the net income after taxes has risen to a normal level of $250,000. The 50,000 common shares would ordinarily sell at a P/E multiple of 12, but the preferred arrearages have caused the common to sell at $30. The preferred has been quoted at $120, although any buying pressure would cause this price to rise significantly. Delano has a 50 per cent tax rate and is faced with the following alternatives:

(a) Exchange common for preferred on the basis of their market prices; or
(b) Call the preferred, and finance the transaction with a 9 per cent debenture.

From the standpoint of current common shareholders, which alternative is preferable? What reservations do you have?

2. D. Sent, a disgruntled stockholder of the Zebec Corporation, desires representation on the board. The Zebec Corporation, which has ten directors, has 1 million shares outstanding.

    (a) How many shares would Sent have to control to be assured of one directorship under a majority voting system?

    (b) Recompute (a), assuming a cumulative voting system.

    (c) Recompute (a) and (b), assuming the number of directors was reduced to five.

3. The Crown Corporation is contemplating the issuance of 10,000 shares of $100 par value, 6 per cent preferred stock. The Crown common stock tends to sell at a normalized price/earnings ratio of 15, and the stockholders are felt to be in a 30 per cent marginal tax bracket.

    (a) Being very careful about the above assumptions, determine whether the retention of earnings would be a preferable means of financing by the use of Donaldson's formula.

    (b) Explain why at least three of Donaldson's assumptions are highly questionable.

    (c) Recompute (a), assuming a normalized price/earnings ratio of 12.

4. Joe Miller has formed a company which can earn 12 per cent after taxes, although no investment has yet been made. Joe plans to take $100,000 in $1 par value stock for his promotion efforts. All financing for the firm will be in stock, and all earnings will be paid in dividends.

    (a) Suppose that Joe desires to keep 50 per cent control of the company after he has acquired new financing. He can do this by taking his stock in the form of $1 par value Class B, with two votes per share; while selling $1 par value Class A stock, with one vote per share, to the public. In order to buy Class A stock, however, the investors would require a dividend formula which would give them a 10 per cent return. How many Class A shares would be issued? What dividend formula would meet the investors' requirements? What dividend payment would be left for Joe's Class B shares?

    (b) Suppose that, if Joe were willing to lose control of the company, he could have just one class of common stock and sell the same amount to the public as he could Class A in (a) above. In such a case the investors would only require an 8 per cent rate of return. What would be the dividend formula and Joe's return in this case? Comparing this answer with that obtained in (a) above, what is Joe paying to retain control?

    (c) Rework (b) above under the assumption that the investors would require 9 per cent. What must Joe do?

5. The Southern Alabama Fire Insurance Company has an effective tax rate of 30 per cent. It wishes to invest a portion of its portfolio in the securities of the Southern Alabama Manufacturing Company. The SAMC preferred stock currently sells at a price to yield 7 per cent. SAFIC feels that the greater protection of a bond is worth 0.50 per cent in after-tax yield to them. At what yield to maturity would a SAMC bond have to sell to be as attractive as the preferred stock to SAFIC?

6. The stock of the Moribund Corporation is currently selling in the market for $45 per share, yet it has a liquidation value of $70 per share. The Raid Corporation has decided to make a tender offer for the shares of Moribund. Raid feels that it must obtain at least 50 per cent of the shares in order to effect the

liquidation. Assuming Raid makes its tender offer on the expected relationship below, at what price should the tender be made?

Price Per Share	Expected Percentage of Shares Tendered
$55	50%
57	60
59	70
62	80
67	90
72	100

SELECTED
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Convertible Securities
and Warrants

14

A convertible security is a bond or a share of preferred stock that can be converted at the option of the holder into the common stock of the same corporation. Once converted into common stock, the stock cannot be exchanged again for bonds or preferred stock. The ratio of exchange between the convertible security and the common stock can be stated in terms of either a *conversion price* or a *conversion ratio*. To illustrate, the Owens-Illinois, Inc., $4\frac{1}{2}$ per cent convertible subordinated debentures ($1,000 face value), issued in November, 1967, have a conversion price of $59, meaning that each debenture is convertible into 16.95 shares of common stock. We simply divide the face value of the security by the conversion price to obtain the conversion ratio, $1,000/59 = 16.95$. The conversion privilege can be stated either in terms of the conversion price or the conversion ratio.

The conversion terms are not necessarily constant over time. Many

convertible issues provide for increases in the conversion price at periodic intervals. For example, a $1,000 face value bond might have a conversion price of $100 a share for the first five years, $110 a share for the second five years, $120 for the third five, and so on. In this way, the bond converts into fewer shares of common stock as time goes by. Usually, the conversion price is adjusted for any stock splits or stock dividends that occur after the securities are sold. If the common stock were split two for one, the conversion price would be halved.

CONVERSION VALUE AND PREMIUM

The *conversion value* of a convertible security is the conversion ratio of the security times the market price per share of the common stock. If Owens-Illinois stock were selling for $80 per share, the conversion value of one convertible subordinated debenture would be $16.95 \times \$80$, or $1,356.

The convertible security provides the investor with a fixed return, in the case of a bond, or with a specified dividend, in the case of preferred stock. In addition, he receives an option to convert the security into common stock; and he thereby participates in the possibility of capital gains associated with being a residual owner of the corporation. Because of this option, the company usually is able to sell the convertible security at a lower yield than it would have to pay on a straight bond or preferred stock issue. At the time of issuance, the convertible security will be priced higher than its conversion value. The differential is known as the *conversion premium*. For example, the Owens-Illinois convertible subordinated debentures were sold to the public for $1,000 a bond. The market price of the common stock at the time of issuance (November, 1967) was approximately $52. Therefore, the conversion value of each bond was $16.95 \times \$52$, or $881; and the differential of $119 between this value and the issuing price represented the conversion premium. Frequently, this premium is expressed as a percentage; in our example, the conversion premium is $\$119/\$881 = 13.5$ per cent.

Almost without exception, convertible securities provide for a *call price*. As was true with the straight bond or preferred stock, the call feature enables the corporation to call the security for redemption. Few convertible securities, however, are ever redeemed. Instead, the purpose of the call usually is to force conversion when the conversion value of the security is significantly above its call price.

HOW SHOULD A CONVERTIBLE BE ANALYZED?

Because a convertible security is a bond or preferred stock at the time of issuance but is usually common stock later, it poses a certain amount of difficulty for the analyst examining the financial condition of the

company. Convertible subordinated debt or convertible preferred stock can be treated as a part of the equity base by a creditor when he evaluates the financial condition of the issuer. In the event of liquidation, it makes no difference to the creditor if the issue is actually converted; for in either case he has a prior claim. The situation is different, however, in the case of a convertible bond that is not subordinated. As long as the bond is not converted, its holder would be a general creditor in the event of liquidation. Consequently, creditors tend to regard the convertible bond as debt until actual conversion takes place. For this reason, there is a strong incentive for the company to make the issue subordinated.

Investors in a company's common stock tend to recognize the potential dilution in their position before actual conversion takes place. Upon the announcement of a convertible issue, or before if rumors are out, the market price for a company's stock usually declines. To illustrate the dilution effect, suppose that a company issues $20 million in 6 per cent convertible debentures and that the conversion price is $20 a share. The total number of additional shares upon conversion would be $20 million/$20 = 1 million shares. Assume further that the company has 3 million common shares outstanding and no other debt, that it expects earnings before interest and taxes two years from now to be $10 million, and that the federal income tax rate is 50 per cent. Earnings per share under the two alternatives would be

	Convertible Debentures Outstanding	Debentures Converted
Earnings before interest and taxes	$10,000,000	$10,000,000
Interest 6% debentures	1,200,000	— — — —
Profit before taxes	8,800,000	10,000,000
Taxes	4,400,000	5,000,000
Profit after taxes	4,400,000	5,000,000
Shares outstanding	3,000,000	4,000,000
Earnings per share	$1.47	$1.25

We see that upon future conversion, there is dilution in earnings per share. It is important that the investor in common stock consider the impact of this dilution upon the market price of the stock. We note also that upon conversion, the company no longer has to pay interest on the debentures; this factor has a favorable influence upon earnings per share.

CONVERTIBLES AS A MEANS OF FINANCING

Convertible securities, in most cases, are employed as deferred common stock financing. Technically these securities represent debt or preferred stock, but in essence they are delayed common stock. Compa-

nies that issue convertibles expect them to be converted in the future. By selling a convertible security instead of common stock, they create less dilution in earnings per share, both now and in the future. The reason is that the conversion price on a convertible security is higher than the issuing price on a new issue of common stock.

To illustrate, suppose that the current market price of the common stock of *ABC* Corporation is $40 per share. If the company raises capital with an issue of common stock, it will have to underprice the issue in order to sell it in the market. Suppose that the company is able to sell the stock through underwriters and to realize net proceeds of $36 per share. If the company wishes to raise $18 million, the issue would involve 500,000 shares of additional stock. On the other hand, if *ABC* Corporation sells a convertible issue, it is able to set the conversion price above the current market price per share. If the conversion premium is 15 per cent, the conversion price would be $46 per share. Assuming an $18 million issue of convertibles, the number of shares of additional stock after conversion would be:

$$\frac{\$18 \text{ million}}{\$46} = 391,305$$

We see that potential dilution with a convertible issue is less than that with a common issue because fewer shares are being added.

As a financing stragegy, management may wish to finance with convertible securities as opposed to common stock when its estimates of the firm's future are more favorable than those of the market. By so doing, it obtains lesser dilution for existing stockholders than it would if it financed with common stock. Once management's expectations are realized, the stock will presumably rise in price. Of course, the merit of such a strategy depends upon management's estimates of the future being more accurate than those of the market. When the stock is depressed in price, however, it may be wise to avoid both common stock and convertible financing. This situation will be discussed later in the chapter when we consider the timing of a convertible issue.

Another advantage to the company in using convertible securities is that the interest rate or preferred dividend rate typically is lower than the rate the company would have to pay on a straight bond or a straight preferred stock issue. The conversion feature makes the issue more attractive to investors. The greater the value of the conversion feature to investors, the lower the yield the company will have to pay in order to sell the issue. For companies with relatively low credit ratings but good prospects of growth, it may be extremely difficult to sell a straight issue of bonds or preferred stock. However, the market may regard a convertible issue of these companies in a very favorable light, not because of its quality as a bond or as preferred stock but because of its quality as com-

mon stock. Convertible securities can be sold during periods of tight money when it is very difficult for even a credit-worthy company to sell a straight bond or preferred stock. For these reasons, convertibles are attractive to many firms as a means of financing. Their use will be analyzed in the subsequent discussion.

FORCING CONVERSION

Companies usually issue convertible securities with the full expectation that these securities will be converted within a certain length of time. The investor can exercise his option voluntarily at any time and exchange the convertible security for common stock. However, he may simply prefer to hold the security, for its price will increase as the price of the common stock increases. In addition, he receives regular interest payments or preferred stock dividends. For these reasons and others discussed later in this chapter, many investors do not want to convert their security even though its conversion value is more than what they paid for it.

In order to force conversion, companies issuing convertible securities must call the issue. To do so, the market price of the security must be significantly higher than the call price, so that investors will convert rather than accept the lower call price. Many companies regard a 20 per cent premium of conversion value over call price as a sufficient cushion for possible declines in market price and for enticing investors to convert their securities. Suppose that the conversion price of a convertible debenture ($1,000 face value) were $50 and that the call price were $1,080. For the conversion value of the bond to equal the call price, the market price of the stock must be $1,080/20, or $54 a share. If the bonds are called when the market price is $54, many investors might choose to accept the call price rather than convert. The company then would have to redeem many of the bonds for cash, in part defeating the purpose of the original financing. In order to assure almost complete conversion, it might wait to call the debentures until the conversion value of the bond is 20 per cent above the call price, a value that corresponds to a common-stock market price of approximately $65 a share. At this price, the investor would suffer a significant opportunity loss if he accepted the call price.

Overhanging Issue. If a company is unable to force conversion because the market price of the stock has not risen sufficiently to entice the investor to convert, the convertible issue is said to be "overhanging." With an overhanging issue, the company is constrained in its ability to obtain new financing. It is difficult to sell another convertible security issue until the present one is converted. The overhanging issue creates apprehension in the market over the investment worthiness of

any new issue of convertibles and may even create apprehension over the worthiness of a nonconvertible security offering.

The risk of an overhanging issue and the loss of flexibility associated with such an issue may offset, at least in part, the advantage in issuing price of the convertible security over a common stock offering. With a common stock offering, the firm obtains equity capital now. With a convertible security issue, it is uncertain when, if ever, the security will convert and the company will obtain equity capital.

CONVERTIBLE FINANCING FOR THE GROWTH COMPANY

Because issuing companies usually wish to force conversion within a reasonable length of time as well as avoid an overhanging issue, the convertible security is best suited for a growth company. The faster the increase in the market price of the company's stock, of course, the quicker the company will be in a position to force conversion. To illustrate, suppose R-Tronics, Inc., is able to sell a 6 per cent convertible bond at a price of $1,000 to the public, with a conversion price of $50 and a call price of $1,060. This call price corresponds to a market price per share of $53. Suppose further that the price of the company's stock at the time of issuance is $44 a share. If R-Tronics has the policy of forcing conversion only when the conversion value of the bond is 20 per cent in excess of the call price, it will need to wait until the stock rises to approximately $64 per share ($53 × 1.2). If the company is expected to have an annual growth in earnings per share of 30 per cent in the foreseeable future, and if the price/earnings ratio is not expected to change, it will take less than 18 months for the stock to reach $64 per share. On the other hand, if the growth rate were expected to be only 5 per cent per annum, it would take almost eight years for the stock to reach a value of $64 per share.

Thus, there is considerable question whether a company that envisions only modest growth should finance with convertible securities. With a growth rate of 5 per cent in the example above, it takes an extremely long period of time before it is possible to force conversion. During this time, the company is less flexible in additional financing than it would be if it could "clear the decks" and force conversion. Moreover, investors have come to expect companies that issue convertible securities to be able to force conversion within several years after issuance. Not to be able to do so during this time is a sign that the stock has not performed as well as the company had expected originally. Thus, a company with prospects for little or no growth would be ill-advised to use convertible securities as a method of financing. They are best suited for the growth company.

**CONVERSION PREMIUMS AND
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For most issues of convertibles, the conversion premium ranges from 10 to 20 per cent. Recall that this premium is the per cent by which the issuing price of the convertible exceeds its conversion value. If a convertible bond were sold for $1,000 with a conversion price of $50, and if the market price of common at the time of issuance were $43 a share, the conversion premium would be $7/$43, or 16.3 per cent. For a growth company, the conversion premium can be in the upper part of the 10–20 per cent range, or perhaps even higher in the case of a supergrowth stock. For companies with more moderate growth, however, the conversion premium may be closer to 10 per cent. The range itself is established mainly by market tradition, in keeping, however, with the idea that the stock should be expected to rise in price so that it exceeds the conversion price within a reasonable period of time. The greater the growth in market price per share, the more quickly will the market price exceed the conversion price, all other things the same. Thus, the supergrowth company is able to command a higher conversion premium in the market than is a company with only moderate growth potential.

The higher the conversion premium, of course, the lower the dilution. If the company sets too high a conversion price, however, the issue will have to be sold as essentially a fixed-income security with a yield commensurate with what the company would pay on a straight debt or preferred stock issue. Under such circumstances, the issue cannot be considered delayed equity financing. The ability of the firm to force conversion simply is too uncertain. For this reason, there are practical limits as to how high a conversion premium can be set. For most situations, it is 20 per cent or less.

The appropriate timing of a convertible issue must be evaluated in relation to the market for the company's common stock. If it is a poor time to sell common stock because of a depressed market price, it usually is also a poor time to sell a convertible, even though the convertible issue can be sold at a conversion price higher than the price at which a common stock issue can be sold. This is due both to the depressed market price of the stock and to variations in the conversion premium with market psychology. Since the dilution associated with a convertible issue depends directly upon its conversion price, which in turn is primarily a function of the market price of the stock, it is obvious that considerable dilution occurs when the stock is depressed in price. In addition to this influence, the conversion premium at which the issue can be sold is likely to be lower when the stock is depressed than when it is buoyant. Since the conversion premium is a function of expected growth, and since investors are less likely to expect growth when the stock is depressed than when it is strong, the conversion premium nor-

mally will be lower in a depressed market. These factors will result in greater dilution, the more depressed the market price of the stock. Thus, we see that the appropriate timing of a convertible issue follows very closely the market behavior of the company's stock.

VALUE OF CONVERTIBLE SECURITIES

The value of convertible security to an investor is twofold: its value as a bond or preferred stock, and its potential value as common stock. (Because the principles of valuation of a convertible bond and a convertible preferred stock are nearly the same, our subsequent discussion will refer to convertible bonds.) The investor obtains a hedge when he purchases a convertible bond. If the market price of the stock rises, the value of the convertible is determined largely by its conversion value. However, if the market for the stock turns down, the investor still holds a bond whose value provides a floor below which the price of the convertible is unlikely to fall.[1]

BOND VALUE

The bond value of a convertible security is the price at which a straight bond of the same company would sell in the open market. It can be determined by solving the following equation for B:[2]

$$B = \sum_{t=1}^{n} \frac{I}{(1 + i)^t} + \frac{F}{(1 + i)^n} \tag{14-1}$$

where B = straight bond value of the convertible
I = annual interest payments determined by the coupon rate
F = face value of the bond
n = years to final maturity
i = market yield to maturity on a straight bond of the same company

For example, suppose that ABC Company has outstanding a 6 per cent convertible debenture issue with a final maturity twenty years hence. Suppose further that if the company is to sell a straight twenty-year debenture in the current market, the bond will have to yield 8 per cent to maturity to be attractive to investors. For a twenty-year bond with a 6 per cent coupon to yield 8 per cent to maturity, the bond has to sell at

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[1] See Ashby Bladen, *Techniques for Investing in Convertible Bonds* (New York: Salomon Brothers and Hutzler, 1966). Parts of this section draw upon Bladen's analysis.

[2] In this equation, we assume that interest payments are annual and paid at the end of the year. If payments are semiannual, the equation should be modified according to the discussion in Appendix A to Chapter 3.

a discount. More specifically, the market price has to be $804 for each $1,000 face value bond.[3] Thus, the bond-value floor of *ABC*'s convertible bond would be $804. This floor suggests that if the price of the common stock were to fall sharply so that the conversion feature had negligible value, the price of the convertible would fall only to $804. At that price, the security would sell as a straight bond in keeping with prevailing bond yields for that grade of security.

The bond value floor of a convertible is not constant over time. It varies with (1) interest rate movements in the capital markets and (2) changes in the financial risk of the company involved. If interest rates in general rise, the bond value of a convertible will decline. For example, if the yield to maturity on a straight bond in our example increases from 8 to 9 per cent, the bond value of the convertible will drop from $804 to $726. Moreover, the company's credit rating can either improve or deteriorate over time. If it improves, and the company is able to sell a straight bond at a lower yield to maturity, the bond value of the convertible security will increase, all other things held constant. However, if the company's credit standing deteriorates and the yield on a straight bond increases, the bond value floor will decline. Unfortunately for the investor, when the market price of the stock falls because of poor earnings, the company may have financial difficulty, in which case its credit standing will suffer. As a result, the straight bond value of the convertible may decline along with the decline in its conversion value, giving the investor less downside protection than he might have expected originally.[4]

PREMIUMS

Convertible securities usually sell at premiums over both their bond value and their conversion value. Recall that the conversion value of a convertible is simply the current market price per share of the company's common stock times the number of shares into which the security is convertible. The fact that the convertible bond provides the investor with

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[3] Solving for Eq. (14-1)

$$B = \sum_{t=1}^{20} \frac{\$60}{(1.08)^t} + \frac{\$1,000}{(1.08)^{20}} = \$804$$

Fortunately, we need only consult a bond table to determine the market price. Given any three of the four variables above—namely, years to maturity, coupon rate, yield to maturity, and market price—we can determine quickly the fourth variable from the table. An example of a bond table is shown in Appendix A to Chapter 3, where we take up the mathematics of bond interest.

[4] Mathematically, the straight bond value of a convertible security will rise over time, all other things held constant, if the face value of the convertible is above the straight bond value at the time of issuance. At final maturity, the straight bond value, of course, will equal the face value of the convertible. See Appendix A to Chapter 3.

a degree of downside protection, given the qualifications mentioned above, usually results in its selling at a market price somewhat higher than its conversion value. How much higher will depend upon the probability that the conversion value of the security will fall below its bond value. Suppose, in our example, that the conversion price is $50 a share and the current market price of the common stock is $60 a share. The conversion value of the $1,000 face value bond will be $1,200. If the stock falls to $25 a share, the conversion value of the bond will plummet to $500. Assuming a straight bond value of $804, however, the market price of the convertible would not be expected to fall below $804. Consequently, the investor can reduce his risk by investing in a convertible security rather than in the common stock of a corporation. In general, the more volatile the price movements of the stock, the more valuable is the downside protection afforded by the bond value floor. For this reason as well as for additional reasons discussed later, the market price of a convertible security usually is above its conversion value. The difference is known as the *premium-over-conversion value*.

Moreover, a convertible bond usually will sell at a *premium-over-bond value,* primarily because of the conversion feature. Unless the market price of the stock is very low relative to the conversion price, the conversion feature usually will have value, in that investors may eventually find it profitable to convert the securities. To the extent that the conversion feature does have value, the convertible will sell at a premium over its straight bond value. The higher the market price of the common relative to the conversion price, the greater this premium.

COMPARING THE TWO PREMIUMS

By comparing the two premiums for a sample of convertible securities of similar companies, we gain insight into the tradeoff between conversion value and straight bond value. The tradeoff has implications for what coupon rate and conversion price a company should set for a new convertible issue. If we construct a scatter diagram with the percentage premium of market price of the convertible over its straight bond value on the vertical axis and the premium of market price over conversion value on the horizontal axis, we can examine the tradeoff empirically. Taking a cross-sectional sample of convertible securities, we can plot the premiums on the scatter diagram and obtain a relationship that might be similar to that shown in Figure 14-1.

The figure suggests an inverse relationship between the two premiums.[5] At relatively high common stock price levels, the value of the

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[5] Brigham undertakes a similar analysis using the ratio of the stock's initial market price over the conversion price on the horizontal axis and the ratio of the interest rate the company would pay on a straight bond over the rate it pays on a convertible bond on the vertical axis. Eugene F. Brigham, "An Analysis of Convertible Debentures: Theory and Some Empirical Evidence," *Journal of Finance*, XXI (March, 1966), 41–48.
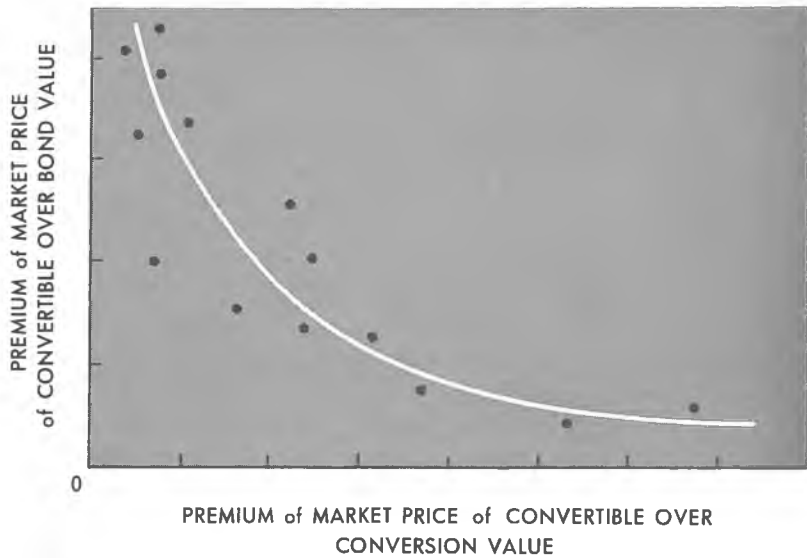
FIGURE 14-1
Convertible securities tradeoff curve

convertible as a bond is negligible. Consequently, its premium-over-bond value is high, whereas its premium-over-conversion value is slight. The security sells mainly for its stock equivalent. Investors are unwilling to pay a significant premium over conversion value for several reasons. First and foremost, the greater the premium of market price of the convertible over its bond value, the less valuable the bond-value protection to the investor. If the bond-value floor of a convertible bond is $900, for example, there is considerably more downside protection if the market price of the convertible is $1,000 than if it is $2,000. Secondly, when the conversion value is high, the convertible may be called; and if it is, the investor will want to convert rather than redeem the bond for the call price. Upon conversion, of course, the bond is worth only its conversion value. Finally, if the company increases the dividend on its common stock, the fixed return on the convertible declines relative to the return available on the stock equivalent. This occurrence contributes also to a narrowing of the premium as conversion value increases.[6]

On the other hand, when the market value of the convertible is close to its straight bond value, the conversion feature has little value. At this level, the convertible security is valued primarily as a straight bond. Under these circumstances, the market price of the convertible is likely to exceed its conversion value by a substantial premium. Otherwise, the conversion feature would have a value, and the convertible security would sell at a premium over its bond value.

[6] *Ibid.,* p. 37.

A tradeoff curve based upon a cross-sectional sample of convertible securities, such as that illustrated in Figure 14-1, is valuable to the financial manager in determining the appropriate yield and conversion premium on a new issue of convertible bonds. It gives him an idea of the additional yield that the firm would have to pay if it wanted a higher conversion premium, or, conversely, the decrease in conversion premium necessary to obtain a lower yield. In general, the higher the conversion premium, the higher must be the yield in order to entice market acceptance.

It would be more appropriate to base the scatter diagram on new issues of convertible securities of similar companies, but there simply are not enough of these new issues over a recent period on which to base a tradeoff curve. Consequently, we are forced to use outstanding issues. However, when information on similar new issues is available it is extremely valuable as a bench mark for the company issuing convertible securities. Unfortunately, there usually are only one or two recent convertible issues similar enough to be useful in this kind of analysis. The scatter diagram in Figure 14-1 also tells us which convertibles are out of line with the tradeoff curve.[7] When a convertible issue does stand out, we would want to search for reasons why. (Frequently, the cause is unusual growth.) Thus, the graph gives the analyst a standard for comparison of different convertible issues. In addition, we can compare graphs at different points in time to study shifts in the tradeoff curve over time. The curve is known to shift as the speculative appeal of convertibles increases or decreases.

OTHER REASONS FOR PREMIUMS

In addition to a convertible security's unusual appeal as both a bond and a common stock, other features contribute to its premium in market price over both its conversion value and its value as a straight bond. For one thing, a convertible security is attractive to investors who operate on margin. As the margin requirement on stock, which is set by the Federal Reserve, is higher than that on convertible securities, speculators are able to borrow more for investment in convertibles than they are for investment in stock. At the time of this writing, the requirement for convertibles was 50 per cent, which meant that the maximum loan a person could obtain from a bank or investment house was 50 per cent of the market value of the convertible securities pledged. This margin requirement, 50 per cent, contrasted with one of 65 per cent for common stock; the latter allows a person to borrow only 35 per cent of the market value of the stocks pledged. The greater collateral-value at-

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[7] For such an analysis, see Bladen, *Techniques for Investing in Convertible Bonds*, pp. 24–26.

tractiveness of convertibles relative to stock results in a greater relative demand for these securities; this demand, in turn, exerts upward pressure on the premiums.[8]

Lower transaction costs on convertible bonds relative to those on common stocks also enhance the attractiveness of these bonds. An investor who wishes to acquire common stock of a company would incur lower transaction costs by purchasing a convertible bond and converting it into common stock than he would by purchasing the stock outright. This attraction also will exert upward pressure on the premiums over conversion value and over bond value. The duration of the convertible option also should affect the premiums. In general, the longer the time to maturity, the more valuable the option and the higher the premiums should be.[9]

Another influence that may raise premiums is that certain institutional investors, such as life insurance companies, are very restricted with respect to investing in common stock.[10] By investing in convertible bonds, they gain the benefits of a common stock investment without actually investing in common stock. All these influences account for the premiums at which convertible securities sell.[11] In the Appendix to this chapter, we consider further the value of convertible securities by examining two theoretical models for valuation.

WARRANTS

A warrant is an option to purchase a specified number of shares of common stock at a stated price. When the holder exercises the option,

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[8] Prior to November, 1967, there was no margin requirement on convertible securities. Lenders were free to advance whatever portion of the market price they deemed appropriate. Consequently, speculators were able to borrow up to 90 per cent of the market value of a convertible, making convertibles extremely attractive. In late 1967, however, the Federal Reserve initiated a margin requirement on convertibles. As a result, one attraction of convertibles was reduced considerably. On the first day of trading (October 24, 1967) after the announcement, there were widespread declines in the market prices of convertible securities.

[9] See Roman L. Weil, Jr., Joel E. Segall, and David Green, Jr., "Premiums on Convertible Bonds," *Journal of Finance,* XXIII (June, 1968), 445–47.

[10] See Brigham, "An Analysis of Convertible Securities," *Journal of Finance,* p. 153.

[11] In an empirical test of the premium-over-conversion value, Weil, Segall, and Green, "Premiums on Convertible Bonds" (pp. 445–63), regressed this premium against the bond-value floor, the difference in current income streams between the convertible and the common stock, and transaction-cost differences for a sample of New York Stock Exchange convertible bonds over the 1961–63 period. Of the three independent variables, all but the bond-value floor variable were significantly different from zero. The transaction-cost variable, however, had an unexpected negative sign. The authors' major conclusion was that the bond-value floor is of negligible importance in explaining the premium-over-conversion value. While this result is interesting, no information was given about the range of observations and about certain tests undertaken with respect to the formulation of the floor variable. Without such information, it is difficult to assess the meaningfulness of the results.

he surrenders the warrant. Warrants are employed customarily as "sweeteners" to a public issue of bonds or debt that is privately placed.[12] The investor obtains not only the fixed return associated with debt but also an option to purchase common stock at a stated price. If the market price of the stock should rise, this option can be valuable. As a result, the corporation should be able to obtain a lower interest rate than it would otherwise. For companies that are marginal credit risks, the use of warrants may make the difference between being able and not being able to raise funds through a debt issue. Additionally, during periods of tight money, some financially sound companies may have to provide warrants in order to make their debt issues attractive to investors. Increasingly, institutional investors are insisting upon warrants or some other equity "kicker" before they will invest in a debt instrument. With inflation, this trend is likely to continue. In addition to a "sweetener" to debt financing, warrants are also used in the origination of a company as compensation to underwriters and venture capitalists.

FEATURES

The warrant itself contains the provisions of the option. It states the number of shares the holder can buy for each warrant he holds. For example, the Fibreboard Corporation warrants provide an option to purchase one share of common stock for each warrant held; Braniff Airways warrants provide for the purchase of three shares for each warrant held. Another important provision is the price at which the warrant is exercisable. For example, Fibreboard warrants are exercisable at $22.50 a share. The exercise price can either be fixed or "stepped-up" over time. The Indian Head Mills, Inc., warrants, for example, were exercisable at $20 a share until May 15, 1970, and are exercisable at $25 until May 15, 1975, after which the exercise price increases $5 a share every five years until final expiration of the warrants in 1990.

Finally, the warrant must specify the date on which the option expires. Certain warrants, such as those of Tri-Continental Corporation, are perpetual, having no expiration date. Most warrants, however, have a stated expiration date. Warrants may be either detachable or nondetachable. Detachable warrants may be sold separately from the bond. Consequently, the bondholder does not have to exercise his option in order to obtain the value of the warrant. He simply can sell the warrant in the marketplace. Many detachable warrants are listed on the American Stock Exchange. Recently, the New York Stock Exchange changed its rules to allow warrants to be listed for the first time on that exchange.

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[12] For an analysis of the use of warrants in financing, see Samuel L. Hayes, III, and Henry B. Reiling, "Sophisticated Financing Tool: The Warrant," *Harvard Business Review,* 47 (January–February, 1969), 137–50.

A nondetachable warrant cannot be sold separately from the bond; it can be detached only when the bondholder exercises his option and purchases stock.

Because a warrant is only an option to purchase stock, the warrant holder is not entitled to any cash dividends paid on the common stock, nor does he have voting power. If the common stock is split or a stock dividend is declared, the option price of the warrant usually is adjusted to take this change into account.

EXERCISE OF WARRANTS

Although warrants and convertible securities are similar in many respects, they differ with respect to the capitalization of the company after the option is taken. When convertible debentures are converted, new common stock is created, but the debentures are retired, and there is no infusion of new capital into the company. However, when warrants are exercised, the common stock of the company is increased, and the bonds still remain outstanding.

To illustrate the difference, let us compare the results of financing with convertible bonds and financing with a straight bond issue with warrants attached. Suppose that *ABC* Corporation is raising $20 million in debt funds with either a convertible debenture issue or a straight debenture with warrants attached. Assume that the convertible debenture issue has a coupon rate of 6 per cent and a conversion price of $50, whereas the straight debenture issue has a 7 per cent coupon rate. With the straight debenture, the investor receives one warrant entitling him to purchase three shares of common stock at $60 a share for each bond ($1,000 face value) purchased. The capitalization of the company before financing, after financing, and after complete conversion or exercise of the option, is shown in Table 14-1. We assume that the retained earnings of the company remain unchanged and that the straight debenture issue has neither matured nor been called.

TABLE 14-1
Capitalization of ABC Corporation (in millions)

	Before Financing	Convertible Debentures		Debentures With Warrants	
		Before Conversion	After Conversion	Before Exercise	After Exercise
Debentures		$20		$20	$20.0
Common stock					
($10 par value)	$10	10	$14	10	10.6
Capital surplus			16		3.0
Retained earnings	25	25	25	25	25.0
Net worth	$35	$35	$55	$35	$38.6
Total capitalization	$35	$55	$55	$55	$58.6

Upon conversion of all the debentures, the total number of shares of common stock increases by 400,000. However, total capitalization stays at $55 million, for the debentures are retired. In the case of the debentures with warrants attached, the debentures remain outstanding after all the warrants are exercised. Exercising their options, the warrant holders purchase 60,000 shares of common stock at $60 a share, or $3.6 million in total. Consequently, the total capitalization of the company is increased by $3.6 million. Dilution, of course, is greater upon conversion of the convertible debentures (400,000 new shares) than upon exercise of the warrants (60,000 new shares).

A company cannot force the exercise of the warrant option as it can force the exercise of the conversion option by calling a convertible security. Consequently, it is unable to control when the warrant will be exercised and when there will be an infusion of new equity capital into the corporation. Only the expiration date sets a limit on how long the warrants can remain outstanding and unexercised.

VALUATION OF WARRANTS

The theoretical value of a warrant can be determined by

$$NP_s - O \tag{14-2}$$

where N = the number of shares that can be purchased with one warrant

O = the option price associated with the purchase of N shares

P_s = the market price of one share of stock

On January 2, 1970, the common stock of Indian Head Mills, Inc., closed at $24\frac{1}{2}$. The option price of the Indian Head warrants was $20, and they enabled the holder to purchase one share of common stock for each warrant held. Consequently, the theoretical value of an Indian Head warrant on January 2, 1970 was

$$(1)(24\tfrac{1}{2}) - 20 = \$4.50$$

The theoretical value of a warrant is the lowest level at which the warrant will generally sell. If, for some reason, the market price of a warrant were to go lower than its theoretical value, arbitragers would eliminate the differential by buying the warrants, exercising them, and selling the stock. A warrant is unlikely to sell below its theoretical value, and many warrants sell above that value. For example, the Indian Head warrants closed at $11 on January 2, 1970.

Premium Over Theoretical Value. The primary reason that a warrant can sell at a price higher than its theoretical value is the opportunity for leverage. To illustrate the concept of leverage, consider the Textron,

Inc., warrants. For each warrant held, one share of common stock can be purchased, and the option price is $8.75. If the stock were selling at $11 a share, the theoretical value of the warrant would be $2.25. Suppose, however, that the common stock increased by 20 per cent in price to $13.20 a share. The theoretical value of the warrant would go from $2.25 to $4.45, a gain of 98 per cent.

The opportunity for increased gain is very attractive to investors when the common stock is selling near its option price. For a given investment, the investor can buy a greater number of warrants than he can shares of common stock. If the stock moves up in price, he will make more money on his investment in warrants than he would on an equal investment in common stock. Of course, leverage works both ways; the percentage change can be almost as pronounced on the downside. There is some downside protection, however, because it is unlikely that the price of the warrant will drop to zero. In order for the market price to drop to zero, there would have to be no probability that the market price of the stock would exceed the option price during the option period.

Because of the opportunity for favorable leverage as well as because of certain other factors, the market prices of most warrants are higher than their theoretical values. In particular, this event occurs when the market price of the associated common stock, NP_s in Eq. (14-2), is near the option price. When the market price of the stock increases, however, the degree of leverage decreases. For example, on January 2, 1970, the Textron Inc. common stock closed at $26\frac{1}{8}$ per share. At that price, the theoretical value of a warrant was

$$(1)(26\tfrac{1}{8}) - 8\tfrac{3}{4} = 17\tfrac{3}{8}$$

If an investor were to purchase Textron warrants at $17\frac{3}{8}$ and the market price of the stock increased 20 per cent from $26\frac{1}{8}$ to $31\frac{3}{8}$, the theoretical value of a warrant would increase 30 per cent to $22\frac{5}{8}$. Thus, there is far less opportunity for leverage when the market price of the associated common stock is high relative to the option price than when it is close to the option price. As a result, warrants tend to sell around their theoretical values when the market price of the common is relatively high. For example, the Textron warrants closed at $17\frac{1}{2}$ on January 2, 1970 — very close to their theoretical value.

The functional relationship between the market value of a warrant and the value of the associated common stock is shown in Figure 14-2. The theoretical value of the warrant is represented by the white line in the figure, and the actual market value by the dashed line. When the market value of the associated stock is less than the option price, the theoretical value of the warrant is zero. When the value of the associated common stock is greater than the option price, the theoretical value of the warrant is positive, as depicted by the white diagonal line.
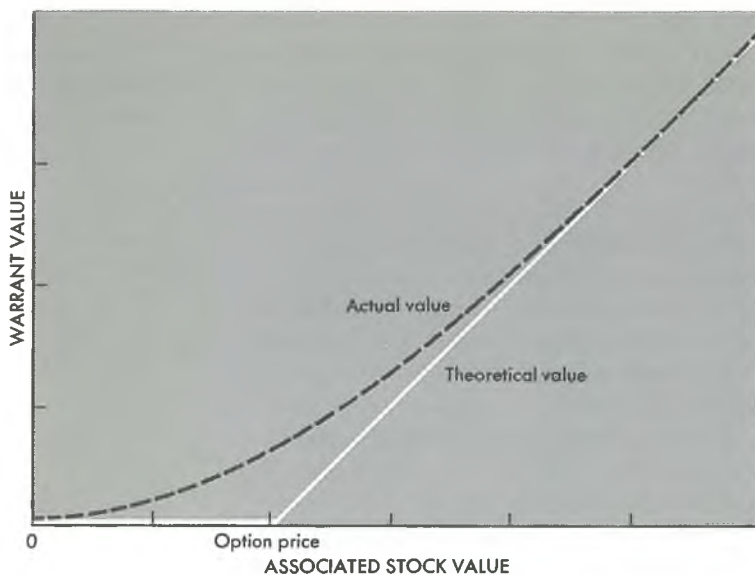
FIGURE 14-2
Relation between theoretical and actual values of a warrant

When the market value of the associated common stock is low relative to the option price, the actual market value of a warrant usually exceeds its theoretical value. However, as the market value of the associated stock rises, the market value of the warrant usually approaches its theoretical value.[13] The exact shape of the actual value line in Figure 14-2 will depend in part upon the remaining length of the option, the payment of dividends on the common stock, the volatility of the common stock, and the opportunity cost of funds to investors. Considering these factors in order, the shorter the length of time to expiration of the warrant, the less valuable the option to the investor and the more convex the actual value line — that is, the more it approaches the theoretical value line. Because the investor in a warrant does not participate in dividends paid on the common, the greater the dividend on the common stock, the less attractive the warrant in relation to its associated stock. As a result, the greater the dividend, the more the actual value line would approach the theoretical value line.

Volatility works in the opposite direction; the more volatile the stock, the more valuable the warrant. To illustrate, suppose an investor were considering a warrant that permitted him to purchase one share of com-

[13] For the development of a rigorous theory of warrant valuation, see Paul A. Samuelson, "Rational Theory of Warrant Pricing," *Industrial Management Review*, 6 (Spring, 1965), 13–39; and Paul A. Samuelson and Robert C. Merton, "A Complete Model of Warrant Pricing That Maximizes Utility," *Industrial Management Review*, 10 (Winter, 1969), 17–46.

mon stock at $20 a share. Suppose also that he expected the market price per share of common one period hence to be:[14]

Probability	Price
.20	$ 5
.30	15
.30	25
.20	35

The expected value of the probability distribution is $20. This price suggests a theoretical value of the warrant of zero one period hence. However, because a warrant cannot sell at a negative price, its expected value of theoretical value is not zero but

$$.20(0) + .30(0) + .30(25 - 20) + .20(35 - 20) = \$4.50$$

Thus, the expected value of the warrant increases with the dispersion of the probability distribution to the right of the option price. The greater the expected volatility in market price for the stock, the greater the expected value of the warrant, all other things the same, and the higher the actual value line.

The opportunity cost of funds to the investor in warrants also may affect the actual value line. This opportunity cost can be either an external lending rate for the investor or his borrowing rate. The greater the value of funds to the investor, the more likely he is to prefer investing in the warrant than in the stock, because the warrant involves a lesser investment. As a result, the warrant is likely to rise in price relative to the price of the associated common stock, exerting upward pressure on the actual value line.

All of these factors (length of the option, dividend paid on the common, volatility of the common, and the value of funds to investors) appear to affect the shape of the actual value line. They have been tested empirically, and the hypothesized relationships supported.[15] While the principal determinant of warrant prices is the value of the associated common stock, additional factors also appear to influence warrant valuation.

[14] This example comes from James C. Van Horne, "Warrant Valuation in Relation to Volatility and Opportunity Costs," *Industrial Management Review,* 10 (Spring, 1969), 20–21.

[15] See Van Horne, "Warrant Valuation in Relation to Volatility and Opportunity Costs," 9–32. See also John P. Shelton, "The Relation of the Price of a Warrant to the Price of ts Associated Stock," *Financial Analysts Journal,* 23 (May–June and July–August, 1967), 43–51 and 88–99.

In this chapter, we have examined two types of options under which the holder can obtain common stock. The conversion feature enables the investor to convert a debt instrument or preferred stock into common stock, whereas a warrant attached to a bond enables the holder to purchase a specified number of shares of common stock at a specified price. With a warrant, the exercise of the option does not result in the elimination of the bonds. Convertibles are used more than warrants in financing.

The value of the convertible in the marketplace is determined by its value as a straight bond or preferred stock and its conversion value as common stock. For the corporation, convertibles represent delayed common stock financing, and the timing of a convertible issue must be analyzed by the company in relation to the market for its common stock. There will be less dilution, for a given amount of financing, with a convertible issue than with a common stock issue, assuming, of course, that the issue eventually converts. Offsetting in some measure this advantage is the risk of an "overhanging" issue, which occurs when the company is unable to force conversion because the market price of the stock has not risen sufficiently to raise the conversion value of the security significantly above the call price. An overhanging issue results in less financing flexibility for the issuer.

Normally, warrants are employed as a "sweetener" to a public or private issue of debt. The market value of a warrant usually is higher than its theoretical value when the market value of the stock is close to the option price, because this situation gives an opportunity for favorable leverage to the investor. When the market price of the stock is high relative to the option price, warrants tend to sell at about their theoretical values. In addition to the market price of the associated common stock, the length of the option, the dividend paid on the common, the volatility of the common, and the value of funds to investors appear to affect the value of warrants.

APPENDIX

Valuation Models for Convertible Securities

The valuation of convertible securities has received considerable attention in recent studies. In order to supplement our discussion on the valuation of convertible securities, we consider two different theoretical models.

BAUMOL-MALKIEL-QUANDT MODEL

Baumol, Malkiel, and Quandt (BMQ) have developed a valuation model based upon an investor's subjective probability distribution of

future market prices of the common stock.[16] They suggest that a convertible is worth at least its conversion value plus the insurance value of the security as a bond. Thus

$$C \geqq P(to)S + V \qquad (14A\text{-}1)$$

where
- C = value of the convertible at time *to*
- S = number of shares of common stock into which the convertible can be exchanged
- $P(to)$ = the market price per share of common stock at time *to*
- V = insurance value of the convertible at time *to*

The value V depends upon the probability that the bond value of the convertible, $\bar{B}$, will exceed the conversion value of the security, $P(t)S$, sometime in the future.[17] The expected future conversion value is expressed in terms of a price relative, $i(t)$, of the market price of the stock at the time the convertible is being evaluated, *to*. The expected market price of the stock at time *t* is

$$P(t) = i(t)P(to) \qquad (14A\text{-}2)$$

and the expected conversion value at that time is $i(t)P(to)S$.

The expected value of V is the sum (integral) of all future occurrences where the bond value of the convertible security exceeds its conversion value times the probability of the occurrence, $f(i, to)di(t)$. Thus,

$$V = \int_0^{\bar{B}/P(to)S} f(i, to)[\bar{B} - i(t)P(to)S]\,di(t) \qquad (14A\text{-}3)$$

The lower limit of integration is $i(t) = 0$, because the price of the common stock cannot fall below this figure. The upper limit is $i(t) = \bar{B}/P(to)S$, because at this value $\bar{B} = i(t)P(to)S$, or the bond value equals the conversion value. At higher levels of $i(t)$, the conversion value exceeds the bond value. The higher the market price of the stock at time *to*, the lower will be this upper limit. For very high values of $P(to)$, $\bar{B}/P(to)S$ approaches 0; and V, the premium over the conversion value, will disappear. This occurrence is consistent with the fact that the market value of a convertible security depends almost entirely upon its conversion value when the conversion value substantially exceeds the straight bond value of the security.

Substituting Eq. (14A-3) into Eq. (14A-1), BMQ obtain

$$C \geqq P(t)S + \int_0^{\bar{B}/P(to)S} f(i, to)[\bar{B} - i(t)P(to)S]\,di(t) \qquad (14A\text{-}4)$$

In the same manner, BMQ state that the value of the convertible must

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[16] William J. Baumol, Burton G. Malkiel, and Richard E. Quandt, "The Valuation of Convertible Securities," *Quarterly Journal of Economics*, LXXX (February, 1966), 48–59.

[17] The bond value of the convertible security is assumed to be constant over time.

be at least as great as its straight bond value plus the value of the conversion feature. Thus

$$C \geqq \bar{B} + \int_{\bar{B}/P(to)S}^{\infty} f(i, to)[i(t)P(to)S - \bar{B}] \, di(t) \qquad \text{(14A-5)}$$

The value of the convertible, C, will be equal to the greater of the two values in Eq. (14A-4) and Eq. (14A-5).

BMQ go on to apply the model, using past distributions of stock prices as surrogates for investors' subjective probability distributions of future common stock prices. As the sample is rather limited, we shall not discuss this aspect of the article. Finally, they modify the basic model to take account of interest payments and to allow for discounting. In place of Eq. (14A-5) they present

$$C \geqq \frac{R}{\left(1 + \frac{\rho}{2}\right)} + \frac{R}{\left(1 + \frac{\rho}{2}\right)^2} + \ldots + \frac{R}{\left(1 + \frac{\rho}{2}\right)^{2n}} + \frac{C_b}{\left(1 + \frac{\rho}{2}\right)^{2n}} \qquad \text{(14A-6)}$$

where R = semiannual coupon payments[18]

$\quad\quad n$ = number of years in investors' horizon

$\quad\quad C_b$ = actuarial value of security at end of horizon period. According to BMQ, C_b is determined apparently by Eq. (14A-5) at the end of the horizon period.

$\quad\quad \rho$ = appropriate annual discount rate

BMQ contend that the appropriate discount rate is the yield on the convertible security if the market value of the convertible equals its straight bond value. On the other hand, if there is no premium over conversion value, the appropriate discount rate is said to be the equity capitalization rate. If the market value of the convertible security is above both its bond value and its conversion value, the appropriate rate would be somewhere between these two rates.

The basic model of BMQ quantifies investors' subjective probability distributions of future stock prices. We should note, however, that BMQ work with the expected values of future stock prices and do not take into account the dispersion and shape of the probability distribution.[19] Another assumption is that the straight bond value of the convertible is constant over time. As we know, this value can change in keeping with movements in interest rates and changes in financial risk. The approach of BMQ is from the standpoint of a single investor evaluating a convertible security. Assumed implicitly in their analysis is that investors at the margin behave in a manner consistent with the model. Understand-

[18] See Appendix A of Chapter 3 for the mathematics of compound interest.

[19] For an analysis of the risk of convertible securities that examines the dispersion and shape of the distribution, see Otto H. Poensgen, "The Valuation of Convertible Bonds," Parts I and II, *Industrial Management Review*, 6 and 7 (Fall, 1965 and Spring, 1966), 77–92 and 83–98.

ably, the model does not take into account institutional factors that influence the premiums; if it did, it would be considerably more complex. Finally, the horizon period is determined arbitrarily. Nevertheless, the BMQ model provides a conceptual framework for analyzing the value of a convertible security and ties in with our previous discussion.

BRIGHAM MODEL

Brigham develops a model that, in certain respects, is similar to that of BMQ. He formulates initially a graphic model in which the market value of the convertible security is a function of its conversion value and its straight bond value.[20] This model is shown in Figure 14-3. The straight
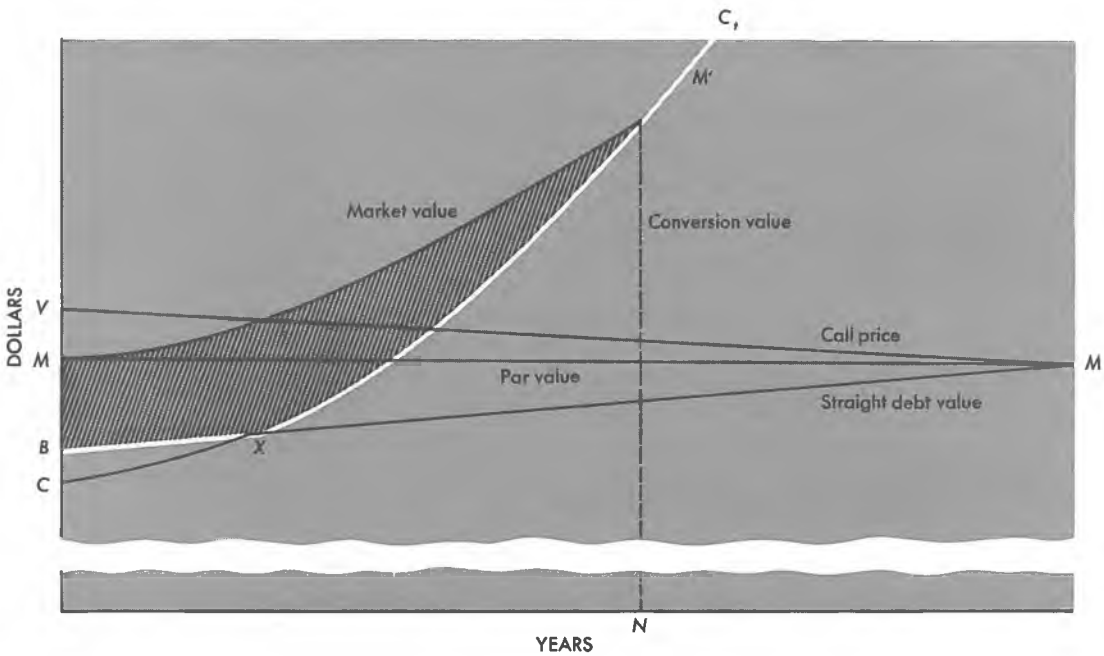
FIGURE 14-3
Hypothetical model of a convertible bond. Source: Eugene F. Brigham, "An Analysis of Convertible Debentures: Theory and Some Empirical Evidence," *Journal of Finance*, XXI (March, 1966), 37.

[20] Brigham, "An Analysis of Convertible Debentures: Theory and Some Empirical Evidence," *Journal of Finance*, XXI, 35–54.

bond value of the security at time t, B_t, is calculated as described previously in this chapter. The bond value, B_t, is shown to be a linear function of the years to final maturity. Thus, the model does not allow for changes in the bond value due to movements in interest rates and changes in financial risk.

The conversion value, C_t, of the convertible security is assumed to grow at a constant rate in keeping with the growth of the stock. Thus

$$C_t = P_0(1 + g)^t R \tag{14A-7}$$

where C_t = conversion value at time t
P_0 = initial market price of the stock
g = rate of growth of the stock's price
R = conversion ratio

The figure shows the conversion value of the security rising at a constant rate over time in keeping with g. The call price, V, is assumed to decline linearly over time and to equal the redemption price at maturity. The straight bond value line and the conversion value line form a lower boundary for the market price of the convertible; this boundary is denoted by the heavy white line. The convertible is assumed to sell at a premium above this lower boundary line; and the premium, denoted by the shaded area in the figure, is shown to narrow as the conversion value of the security rises. The reasons for the existence of this premium and for its narrowing were discussed previously in this chapter.

One of the interesting aspects of Brigham's model is the determination of the length of time, N, that the convertible will be held before conversion. This length is denoted in the graph by the intersection of the market value line with the conversion value line. This intersection can be determined mathematically if the company has a definite call policy. Using M/P_c to denote the conversion ratio, with M the issue price of the bond and P_c the conversion price, Eq. (14A-7) can be written as

$$C_t = \frac{P_0}{P_c}(1 + g)^t M \tag{14A-8}$$

If the company has a policy of calling the bonds when they reach a certain market price in relation to the call price, such as 20 per cent higher, this market price can be denoted by TV. At the time of the call, the market price of the bonds will equal the conversion value ($TV = C_t$). Therefore, Eq. (14A-8) can be written as

$$TV = \frac{P_0}{P_c}(1 + g)^N M \tag{14A-9}$$

Converting to logarithmic form and solving for N:

$$N = \frac{(\log P_c - \log P_0) + (\log TV - \log M)}{\log (1 + g)} \tag{14A-10}$$

In evaluating a convertible security, an investor will know P_c, P_0, and M, and will have expectations about TV and g. Given this information, he can use Eq. (14A-10) and solve for N, the expected holding period before conversion. Given N, the expected rate of return of the convertible can be determined by solving the following equation for k.

$$M = \sum_{t=1}^{N} \frac{I}{(1+k)^t} + \frac{TV}{(1+k)^N} \qquad \text{(14A-11)}$$

where I = dollars of interest paid per annum
k = expected rate of return
N = number of years the convertible is held
M = price paid for convertible
TV = terminal value of convertible. In the example, TV is established by the call price

Brigham suggests that if the *ex ante* expected rate of return, k, is equal to or greater than the investor's opportunity cost on investments with the same degree of risk, he would buy the bonds.[21]

Although interest payments do play a role in the determination of k, the critical factor is the conversion value of the security. The straight bond value of the security is ignored in Eq. (14A-11) because the stock is expected to grow at a constant rate. By working with the expected value of the growth rate, g, Brigham does not consider the dispersion or shape of the probability distribution. Taking these factors into account, one is forced to consider the straight bond value of the convertible security. The specification of the terminal value, TV, in the equation is very difficult, for that value is dependent largely upon an accurate prediction of the growth rate in the market price of the common stock. If the company has a definite call policy, TV can be determined, given the restrictive assumptions of the model.[22] Determining TV obviously will be a problem if the company does not have a definite call policy. We see then that the critical elements in Brigham's model are the specifications of g and TV.

The models discussed in this Appendix give us additional insight into the valuation of convertible securities. While some of the assumptions are difficult to implement, the models provide us with a better understanding of the underlying influences on valuation.

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[21] *Ibid.,* 40–41. If the marginal investor's opportunity cost just equals k, the issue price of the convertibles will be stable.

[22] If the growth rate of the common stock is such that the conversion value of the convertible does not exceed the market price at which the company will call the security, TV would equal the conversion value at final maturity. If this conversion value is less than the maturity value of the security, TV would equal the maturity value. The model assumes that the investor does not exercise his option to convert before the call.

1. Six years ago Ardordyne issued $3\frac{1}{2}\%$, 20-year convertible bonds at par. The bonds were convertible into common at $125 per share; the common was then selling at $100. The common was subsequently split two for one (the conversion price was adjusted) and currently sells at $40 with a $0.50 dividend. Nonconvertible bonds of similar quality currently yield 9 per cent. What price would you be willing to pay for one of these bonds? On what rational basis might investors in different circumstances be willing to pay a price different from yours?

2. Using Eq. (14-2), compute the theoretical value of each of the following warrants:

	N	P	O
(a)	5	$100	$400
(b)	10	10	60
(c)	2.3	4	10
(d)	3.54	$27\frac{1}{8}$	35.40

3. (a) Compute the theoretical value of each of the following five warrants as a ratio of their option price:

	N	P	O
A	1	10	20
B	1	20	20
C	1	30	20
D	1	40	20
E	1	50	20

(b) Prepare a graph with warrant value/option price on the vertical axis and common stock price/option price on the horizontal. Plot the theoretical value relationship computed in (a).

(c) Given the market values of the warrants below, fit a curve to them on the graph constructed in (b).

Warrant	Market Value
A	$3
B	7
C	15
D	23
E	30

(d) Explain the relationship between the theoretical and actual value of a warrant.

4. The Beruth Company is contemplating raising $10 million by means of a debt issue. It has the following alternatives:

(1) A 20-year, 6% convertible debenture issue with a $50 conversion price and $1,000 face value, or

(2) A 20-year, 8% straight debt issue with a detachable warrant to purchase four shares at $50 attached to each $1,000 bond.

The company has a 50 per cent tax rate, and its stock is currently selling at $40 per share. Its net income before interest and taxes is a constant 20 per cent of its total capitalization, which currently appears as follows:

Common stock (par $5)	$ 5,000,000
Capital surplus	10,000,000
Retained earnings	15,000,000
Total	$30,000,000

(a) Show the capitalizations resulting from each alternative, both before and after conversion or exercise (a total of four capitalizations).
(b) Compute earnings per share currently and under each of the four capitalizations determined in (a).
(c) If the price of Beruth stock went to $75, determine the theoretical value of each warrant issued under option 2 above.
(d) Discuss the differences in the implicit costs of (1) straight debt, (2) convertible debt, (3) debt with warrants.

5. The common stock of the Draybar Corporation earns $2.50 per share, has a dividend payout of ⅔, and sells at a P/E ratio of 16. Draybar wishes to offer $10 million of 7%, 20-year convertible debentures with an initial conversion premium of 20 per cent and a call price of 105. Draybar currently has 1 million common shares outstanding and has a 50 per cent tax rate.
(a) What is the conversion price?
(b) What is the conversion ratio per $1,000 debenture?
(c) What is the initial conversion value of each debenture?
(d) How many new shares of common must be issued if all debentures are converted?
(e) If Draybar can increase operating earnings by $1 million per year with the proceeds of the debenture issue, compute the new earnings per share and earnings retained before and after conversion.
(f) If the price of the stock is expected to increase at a compounded rate of 8 per cent per year and Draybar wishes to allow for a 20 per cent decline on the announcement of a call, how long will it be before the debentures can be called?

6. Assume that the Draybar Corporation (see problem 5) raised the $10 million through an issue of stock (total gross spread and expenses = 10 per cent of gross proceeds of issue). How many new shares would have to be issued? If operating earnings were increased by $1 million through the use of the proceeds, compute the new earnings per share and earnings retention. Compare your answers with those obtained in 5(e) above.

7. Assume that the Draybar Corporation (see problem 5) could sell $10 million in straight debt at 9 per cent as an alternative to the convertible issue.
(a) Compute the earnings per share and earnings retained after issuance of the straight debt under the assumption of a $1 million increase in operating earnings and compare your answer to those obtained in 5(e).
(b) Compute the bond value of the convertible debenture, assuming that interest is paid at the end of each year.
(c) Compute the premium over bond value at issuance of the convertible debenture.

8. *Research Project*
Examine closely some convertible preference stocks that are outstanding;

at this writing, the Litton preference stock is a good example. What are the terms of these stocks which make them unique? How should their market value be determined? Do you think their current market price can be justified? Why would a company want to issue preference stock?

APPENDIX

PROBLEMS

1. John N. Vestor has a marginal tax rate of 60 per cent and a required after-tax rate of return of 4 per cent. The Jones convertible debenture has a 5 per cent coupon paid semiannually, a call price of $105, and a conversion price of $50 per share. The Jones common is currently selling at $40, but it is known with certainty that the stock will appreciate sufficiently over the next three years to allow the Jones Company to call the debentures; the Jones Company calls debentures when the price of the stock could decline 20 per cent and still equal the call price. Using a modified BMQ model, determine the maximum price Vestor would pay for a Jones debenture.

2. Assume that another investor bought a Jones debenture (see problem 1) at par.
  (a) If the price of Jones' stock grows at 10 per cent per year and the company follows its customary call practice, use the Brigham model to determine how long it will be before the bonds are called.
  (b) Determine the approximate (to the nearest even year) rate of return.

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MANAGEMENT
OF CURRENT
ASSETS

PART **V**

Working Capital Management

15

In Parts II and III, we analyzed the investment in assets and their financing from a theoretical standpoint. By and large, this discussion was confined to long-term assets and long-term financing. In Part IV, we explored in detail the specific methods by which a firm finances externally on a long-term basis. Analysis of current assets as a whole and of short-term financing was purposely deferred until this part and the next. This important area of financial management involves special considerations that can best be treated in a separate section of inquiry. Basically, the questions we attempt to resolve are: (1) the optimal levels of investment in various current assets; (2) the optimal mix of short-term in relation to long-term financing; and (3) the appropriate means by which to finance on a short-term basis. In this chapter, we present an underlying theory of working capital before analyzing the specific management of cash and marketable securities, receivables, and inventories in the subsequent three chapters. In Chapters 19 and 20, we examine methods of short-term financing.

Current assets, by accounting definition, are assets normally converted into cash within one year. Working capital management usually is considered to involve the administration of these assets — namely, cash and marketable securities, receivables, and inventories — and the administration of current liabilities. Administration of fixed assets (assets normally not converted into cash within the year), on the other hand, is usually considered to fall within the realm of capital budgeting, which we took up in Part II. We noted there, however, that many capital-budgeting projects involve investment in current assets. In our treatment, this investment was considered in relation to the specific project under capital budgeting and not under working capital management. In this part, we are concerned with the investment in current assets as a whole and their composition. By and large, investment in current assets is more divisible than investment in fixed assets, a fact that has important implications for flexibility in financing. Differences in divisibility as well as in durability of economic life are the essential features that distinguish current from fixed assets.

Determining the appropriate levels of current assets and current liabilities, which determine the level of working capital, involves fundamental decisions with respect to the firm's liquidity and the maturity composition of its debt.[1] In turn, these decisions are influenced by a tradeoff between profitability and risk. In a broad sense, the appropriate decision variable to examine on the asset side of the balance sheet is the maturity composition, or liquidity, of the firm's assets — i.e., the turnover of these assets into cash. Decisions that affect the asset liquidity of the firm include: the management of cash and marketable securities; credit policy and procedures; inventory management and control; and the administration of fixed assets. For purposes of illustration, we hold constant the last three factors; the efficiency in managing them is taken up elsewhere in the book.[2] We assume also that the cash and marketable securities held by the firm (hereafter called liquid assets) yield a return lower than the return on investment in other assets.

For current assets, then, the lower the proportion of liquid assets to total assets, the greater the firm's return on total investment. Profitability with respect to the level of current liabilities relates to differences in costs between various methods of financing and to the use of financing during periods when it is not needed. To the extent that the explicit costs of short-term financing are less than those of intermediate- and long-term financing, the greater the proportion of short-term debt to total debt, the

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[1] Parts of this chapter are adapted from James C. Van Horne, "A Risk-Return Analysis of a Firm's Working-Capital Position," *The Engineering Economist,* 14 (Winter, 1969), 71–90.

[2] See Part II and Chapters 17 and 18.

higher the profitability of the firm. Moreover, the use of short-term debt as opposed to longer-term debt is likely to result in higher profits because debt will be paid off on a seasonal basis during periods when it is not needed.

The profitability assumptions above suggest a low proportion of current assets to total assets and a high proportion of current liabilities to total liabilities. This strategy, of course, will result in a low level of working capital, or, conceivably, even negative working capital. Offsetting the profitability of this strategy is the risk to the firm. For our purposes, risk is the probability of technical insolvency. In a legal sense, insolvency occurs whenever the assets of a firm are less than its liabilities — negative net worth. Technical insolvency, on the other hand, occurs whenever a firm is unable to meet its cash obligations.[3]

The evaluation of risk necessarily involves analysis of the liquidity of the firm. Liquidity may be defined as the ability to realize value in money, the most liquid of assets. Liquidity has two dimensions: (1) the time necessary to convert an asset into money; and (2) the certainty of the conversion ratio, or price, realized for the asset. An investment in real estate, for example, is generally a less liquid investment than an investment in marketable securities. Not only does it usually take longer to sell real estate than to sell securities, but the price realized is more uncertain. The two dimensions are not independent. If an asset must be converted into money in a short time, the price is likely to be more uncertain than if the holder has a reasonable time in which to sell the asset.[4]

In this chapter, we study the extent to which possible adverse deviations from expected net cash flows (cash inflows less cash outflows) are protected by the liquid assets of the firm. The risk involved with various levels of current assets and current liabilities must be evaluated in relation to the profitability associated with those levels. The discussion that follows concerns the financing of current assets and the level of those assets that should be maintained from a broad theoretical standpoint.

It should not be implied, however, that the investment in a particular asset is tied to a specific mix of financing. Rather, our focus is on the enterprise as a whole. The firm should not obtain financing on a piecemeal, project-by-project basis, but according to some integrated objective. The supply of the capital funds provided, as well as the terms attached to them, depend on the structure of existing financing and on the overall asset structure of the firm.[5] Consequently, we categorize the firm's overall financing into two sequential acts: (1) determining the proportion of short-

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[3] James E. Walter, "Determination of Technical Solvency," *Journal of Business,* XXX (January, 1957), 30–43.

[4] James C. Van Horne and David A. Bowers, "The Liquidity Impact of Debt Management," *The Southern Economic Journal,* XXXIV (April, 1968), 537.

[5] See Douglas Vickers, *The Theory of the Firm: Production, Capital, and Finance* (New York: McGraw-Hill Book Company, 1968), p. 81.

term versus long-term financing; and (2) determining the types and proportions of both the short-term and long-term financing to be employed. In this chapter, we intend to resolve the first of this two-part sequence, and in Chapters 19 and 20, we analyze the sources of short-term financing — the second part of the sequence. Analysis of the optimal mix of long-term financing was taken up in Chapters 7 and 8. We see, then, that the financing of working-capital requirements is part of the larger, overall financing decision of the firm.

FINANCING CURRENT ASSETS

The way in which current assets are financed involves a tradeoff between risk and profitability. For purposes of analysis, we assume that the company has an established policy with respect to payment for purchases, labor, taxes, and other expenses. Thus, the amounts of accounts payable and accruals included in current liabilities are not active decision variables.[6] These liabilities finance a portion of the current assets of the firm and tend to fluctuate with the production schedule and, in the case of taxes, with profits. As the underlying investment in current assets grows, accounts payable and accruals also tend to grow, in part financing the buildup in current assets. Our concern is with how current assets not supported by accounts payable and accruals are financed.[7]

HEDGING APPROACH

If the firm adopts a hedging approach to financing, each asset would be offset with a financing instrument of the same approximate maturity. This approach to financing is similar to hedging in the commodity futures market. For example, a miller having a contract to deliver processed grain three months from now at an established price may purchase a futures contract for delivery of the grain he needs three months hence. Thus, the miller hedges against the uncertainty of changes in the price of grain by having a contract to buy grain on approximately the same date in the future as he will need it. With a hedging approach to financing, short-term or seasonal variations in current assets — less accounts payable and accruals — would be financed with short-term debt; the permanent component of current assets would be financed with long-term debt or equity. This policy is illustrated in Figure 15-1.

[6]Delaying the payment of accounts payable can be a decision variable for financing purposes. However, there are limits to the extent to which a firm can "stretch" its payables. For simplicity, we assume in the above analysis that the firm has a definite policy with respect to paying its bills, such as taking advantage of all cash discounts and paying all other bills at the end of the credit period. See Chapter 19 for a discussion of trade credit as a means of financing.

[7]We assume the financing of fixed assets as given.
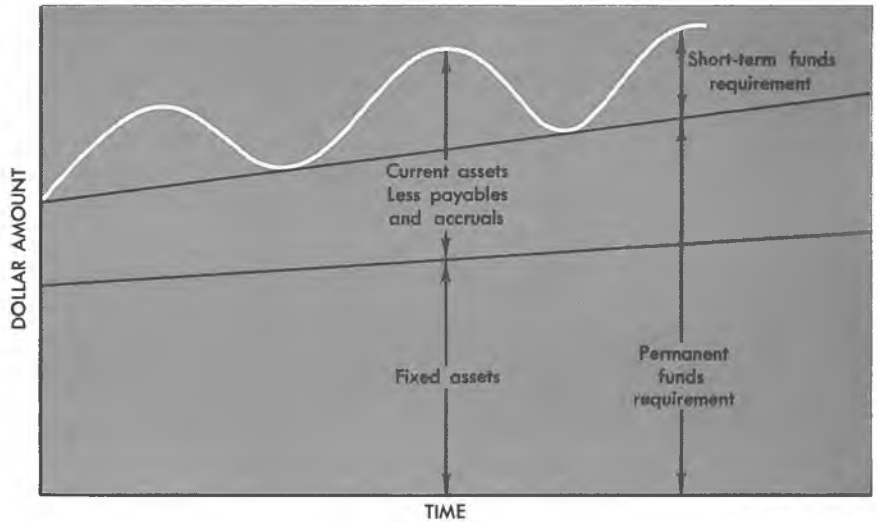
FIGURE 15-1
Funds requirement

If current assets less payables and accruals fluctuate in the manner shown in the figure, only the short-term fluctuations shown at the top of the figure would be financed with short-term debt. To finance short-term requirements with long-term debt would necessitate the payment of interest for the use of funds during times when they were not needed. This occurrence could be illustrated if we drew a straight line to represent the total amount of long-term debt and equity across the seasonal humps at the top of Figure 15-1. It is apparent that financing would be employed in periods of seasonal lull when it was not needed. With a hedging approach to financing, the borrowing and payment schedule for short-term financing would be arranged so as to correspond to the expected swings in current assets, less payables and accruals. Fixed assets and the permanent component of current assets, less payables and accruals, would be financed with long-term debt and equity.

A hedging approach to financing suggests that apart from current installments on long-term debt, a firm would show no current borrowings at the seasonal troughs in Figure 15-1. Short-term borrowings would be paid off with surplus cash. As the firm moved into a period of seasonal funds needs, it would borrow on a short-term basis, again paying the borrowings off as surplus cash was generated. In this way, financing would be employed only when it was needed. Permanent funds requirements would be financed with long-term debt and equity. In a growth situation, permanent financing would be increased in keeping with increases in permanent funds requirements.

Maturity of Debt. While an exact synchronization of the schedule of expected future net cash flows and the payment schedule of debt is appropriate under conditions of certainty, it usually is not under uncertainty. Net cash flows will deviate from expected flows in keeping with the business risk of the firm. As a result, the schedule of maturities of the debt contracts is very important in the risk-profitability tradeoff. We assume that the firm will not arrange its debt obligations so that the composite maturity schedule calls for payments of principal and interest before expected net cash flows are available. The question, however, is what margin of safety should be built into the maturity schedule in order to allow for adverse fluctuations in cash flows. The shorter the maturity schedule of the debt, the greater the risk that the firm will be unable to meet principal and interest payments. The longer the maturity schedule, the less risky the financing of the firm, all other things held constant.

The composite maturity schedule of debt for a firm will depend upon management's risk preferences. Generally, the longer the maturity schedule of debt in relation to expected net cash flows, the less the risk of inability to meet principal and interest payments. However, the longer the maturity schedule, the more costly the financing is likely to be. For one thing, the explicit cost of long-term financing usually is more than that of short-term financing.[8] In periods of high interest rates, however, the rate on short-term corporate borrowings may exceed that on long-term borrowings. Over a reasonable period of time, however, the firm typically pays more for long-term borrowings, particularly if they are negotiated privately. In addition to the generally higher costs of long-term borrowings, the firm may well pay interest on debt over periods of time when the funds are not needed. Thus, there usually is an inducement to finance funds requirements on a short-term basis.

Consequently, we have the familiar tradeoff between risk and profitability. The margin of safety, or lag between expected net cash flows and payments on debt, will depend upon the risk preferences of management. In turn, its decision as to the maturity breakdown of the firm's debt will determine the portion of current assets financed by current liabilities and the portion financed on a long-term basis.

To allow for a margin of safety, management might decide upon the proportions of short-term and long-term financing shown in Figure 15-2. Here, we see, the firm finances a portion of its expected seasonal funds requirement, less payables and accruals, on a long-term basis. If the expected net cash flows do occur, it will pay interest on debt during seasonal troughs when the funds are not needed. As we shall see in the subsequent section, however, the firm also can create a margin of safety by increasing the proportion of liquid assets. Thus, the firm can reduce the risk of cash

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[8]We ignore at this time consideration of implicit costs that might be associated with short-term financing. These costs are analyzed in Chapters 7 and 8.
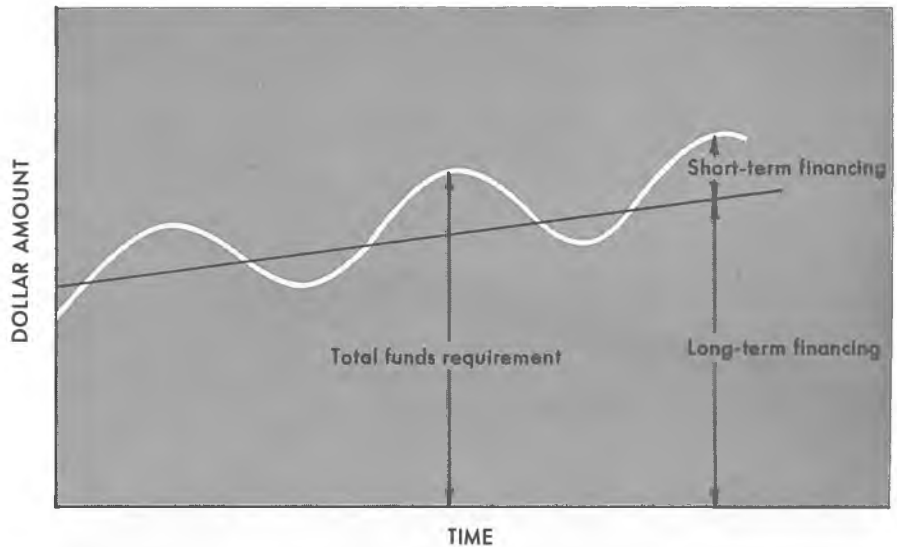
FIGURE 15-2
Funds requirement

insolvency either by increasing the maturity schedule of its debt or by decreasing the relative "maturity" of its assets. At the end of the chapter, we explore the interdependence of these two facets.

<div style="text-align: right">LEVEL OF
CURRENT AND
LIQUID ASSETS</div>

In determining the appropriate level of current assets, management must again consider the tradeoff between profitability and risk.[9] To illustrate this tradeoff, we hold constant the amount of the firm's fixed assets and vary the amount of current assets. Moreover, we assume that the management of receivables and inventories is efficient and consistent throughout the range of output under consideration. In other words, at every level of output the investment in receivables and inventories is predetermined.[10] As a result, we are concerned only with the cash and marketable securities portion of the current assets of the firm.[11]

Suppose that a firm has existing plant and equipment with which it can

[9]The development of this section draws in part upon Ernest W. Walker, "Towards a Theory of Working Capital," *Engineering Economist,* 9 (January–February, 1964), 21–35.

[10]The efficiency of management of receivables and inventory is examined in Chapters 17 and 18, respectively. The quality of these assets, as determined by the efficiency of their management, has a significant bearing upon the liquidity of the firm.

[11]The optimal allocation of funds between cash and marketable securities, near cash, is taken up in Chapter 16.

produce up to 100,000 units of output a year. Assume also that production is continuous throughout the period under consideration, given a particular level of output. For each level of output, the firm can have a number of different levels of current assets. We assume initially three current-asset alternatives. The relationship between output and current asset level for these alternatives is illustrated in Figure 15-3. We see from the figure that the greater the output, the greater the need for investment in current assets. However, the relationship is not linear; current assets increase at a decreasing rate with output. This relationship is based upon the notion that it takes a greater proportional investment in current assets when only a few units of output are produced than it does later on when the firm can use its current assets more efficiently. Fixed assets are assumed not to vary with output.
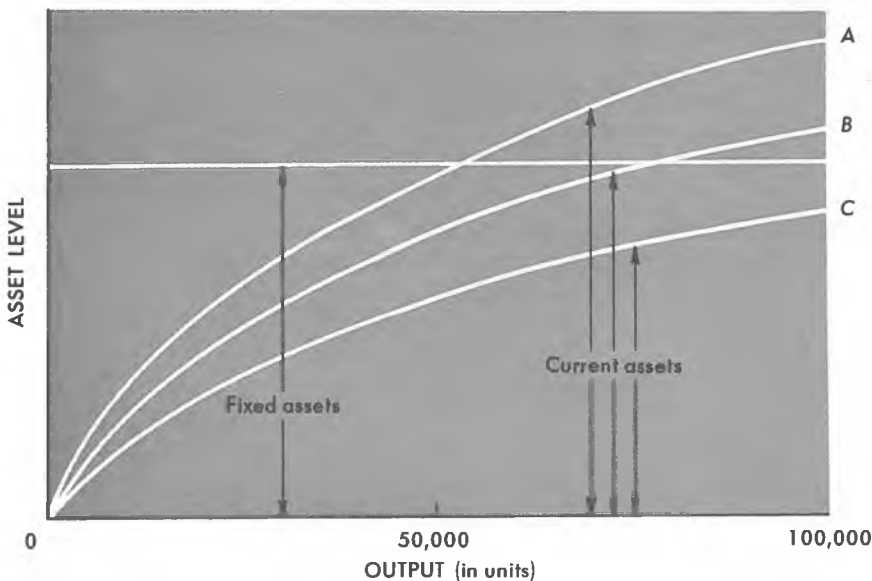
FIGURE 15-3
Current to fixed asset

Of the three alternatives, alternative *A* is the most conservative level of current assets, for the ratio of current assets to fixed assets is greatest at every level of output. The greater the proportion of current to fixed assets, the greater the liquidity of the firm and the lower the risk of technical insolvency, all other things held constant. Alternative *C* is the most aggressive policy, because the ratio of current assets to fixed assets is lowest at all levels of output. The probability of technical insolvency is greatest under alternative *C* if net cash flows are less than expected.

Suppose that for the forthcoming year a firm expects sales of $2 million on 80,000 units of output and expects to realize a profit margin before interest and taxes of 10 per cent, or $200,000 in total profits. We assume that this figure will not vary with the levels of current assets considered. Suppose also that fixed assets are $500,000 for the period under review and that management is considering current asset positions of $400,000, $500,000, or $600,000. Given this information, we are able to make the profitability calculations shown in Table 15-1. As evidenced in this table, the rate of return is lower, the greater the proportion of current assets to fixed assets. Alternative *A*, the most conservative plan, gives the firm the greatest liquidity cushion to meet unexpected needs for funds. However, it also provides the lowest rate of return of the three alternatives. Alternative *C*, on the other hand, provides the highest rate of return but has the lowest liquidity and, correspondingly, the greatest risk. This is a very simple example of the tradeoff between risk and profitability with respect to the level of current assets.

TABLE 15-1
Profitability under alternative current-asset positions

	A	B	C
Sales	$2,000,000	$2,000,000	$2,000,000
Earnings before interest and taxes	200,000	200,000	200,000
Current assets	600,000	500,000	400,000
Fixed assets	500,000	500,000	500,000
Total assets	1,100,000	1,000,000	900,000
Asset turnover (sales/total assets)	1.82:1	2:1	2.22:1
Rate of return (earnings/total assets)	18.2%	20%	22.2%

Looking at it another way, the cost of holding liquid assets is the profit foregone on the investment of these funds in other assets. A firm may reduce its risk of technical insolvency by maintaining a high level of liquidity. Without a liquidity cushion, it may be forced to convert other assets into cash if net cash flows are less than expected. Frequently, these assets can be converted only at a significant sacrifice in price; this sacrifice represents the cost of illiquidity. In most cases, we would expect this cost to increase at an increasing rate with the amount of assets to be converted.

Many assets, however, cannot be converted into cash on short notice, no matter what the sacrifice in price. The measurement of the cost of illiquidity in this case involves fewer tangible considerations than before. The firm simply is unable to pay its obligations. The cost of illiquidity here will depend upon which obligations cannot be paid; that is, whether they are payments to suppliers, payroll expenses, tax payments, bank

loans, or other obligations. Inability to pay suppliers may be less "costly" than inability to pay other obligations. However, it is very difficult to estimate the real cost of a deterioration in a firm's credit standing. Similarly, estimating the real costs of not being able to pay the other obligations mentioned above is extremely difficult, but these costs are likely to be high. In pursuing this issue, we turn to a more inclusive examination of the tradeoff between profitability and risk.

ANALYSIS OF THE TWO FACETS

In the preceding sections, we examined two broad facets of working capital management, the decision as to how current assets are to be financed, and the decision as to the proportion of liquid assets to maintain. The two facets are interdependent. All other things held constant, a firm with a high proportion of liquid assets is better able to finance its current assets, less payables and accruals, on a short-term basis than is a firm with a low proportion of liquid assets. On the other hand, a firm that finances its current assets, less payables and accruals, entirely with equity will have less need for liquidity than it would if it financed these assets entirely with short-term borrowings. Because of their interdependence, these two facets of working capital management must be considered jointly.

If the firm knows its future cash flows with certainty, it will be able to arrange its maturity schedule of debt to correspond exactly with its schedule of future net cash flows. As a result, profits will be maximized, for there will be no need to hold low-yielding liquid assets nor to have more long-term financing than is absolutely necessary. When cash flows are subject to uncertainty, however, the situation is changed. To provide a margin of safety, the firm can: (1) increase its level of liquid assets; and/or (2) lengthen the maturity schedule of its debt. To analyze the appropriate margin of safety, management must have information about the expected future cash flows of the firm and possible deviations from these expected outcomes.

CASH-FORECAST INFORMATION

To obtain this information, cash forecasts should be prepared for a range of possible outcomes, with a probability attached to each. An initial cash budget should be prepared based upon the expected value of outcomes for each future period. The procedure for this is described in Chapter 26. Instead of the cash balance, however, we wish to calculate the liquid-asset balance — the sum of cash and marketable securities. For longer-term forecasts, it is not feasible to prepare detailed cash budgets. Here, estimates of liquid-asset balances based upon major sources and uses of funds will probably be sufficient.

Given an initial cash budget, assumptions with respect to sales, average collection period, production schedule, purchasing, and expenses should be varied by management in keeping with possible deviations from expected conditions. For each change in assumptions, a new set of liquid-asset balances reflecting the change can be generated. Thus, management formulates subjective probabilities of possible future liquid-asset balances. These balances are treated as subjective random variables. In determining the effect of a change in assumptions on the liquid-asset balance, simulation techniques can be very helpful in reducing or even eliminating the detail work involved.

In summary, changes in assumptions are the bases for alternative outcomes in liquid-asset balances. For each of these outcomes, management attaches the probability of occurrence of the associated change in assumptions. For example, suppose that management felt that there were a 0.10 probability of a 20 per cent drop in sales accompanied by a slowing in the average collection period from thirty to forty days for all periods. Suppose further that production were expected to be cut back only after a month's delay. Given these changes in assumptions, a new set of liquid-asset balances for all periods would be determined, the probability of this outcome being 0.10. It is not necessary that the decline in sales or the slowing in collections be the same percentage amount for all months. If different changes over time are expected to occur, these changes should be used to determine the new set of liquid-asset balances.

By varying assumptions in this manner, management formulates subjective probability distributions of liquid-asset balances for various future periods; these distributions encompass a range of possible outcomes. To illustrate a probabilistic cash budget, consider the example in Table 15-2. Here, discrete probability distributions of ending liquid-asset balances without additional financing are shown. These balances are reported on a monthly basis for one year, followed by quarterly forecasts for the next two years. We note that the probability of occurrence of a particular liquid-asset balance in one period corresponds to specific liquid-asset balances in all other periods. For simplicity of illustration, absolute changes in liquid-asset balances are made equal over time. The realism of this example is, of course, questionable, but it will serve to illustrate the framework for analysis.

LEVEL OF LIQUID ASSETS

As discussed earlier, the level of liquid assets and the maturity composition of debt determine the margin of safety of the firm in relation to possible adverse deviations in net cash flows. The level of liquid assets is affected by: (1) the future cash flows of the firm exclusive of new financing; and (2) changes in the total financing of the firm. These factors jointly determine the expected value of the firm's liquid assets. To illustrate,

TABLE 15-2
Possible liquid-asset balances without additional financing (in thousands)

						19X1						
Probability	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
.02	−$200	−$300	−$400	−$500	−$700	−$900	−$900	−$800	−$700	−$700	−$600	−$500
.03	−100	−200	−300	−400	−600	−800	−800	−700	−600	−600	−500	−400
.05	0	−100	−200	−300	−500	−700	−700	−600	−500	−500	−400	−300
.10	100	0	−100	−200	−400	−600	−600	−500	−400	−400	−300	−200
.15	200	100	0	−100	−300	−500	−500	−400	−300	−300	−200	−100
.20	300	200	100	0	−200	−400	−400	−300	−200	−200	−100	0
.18	400	300	200	100	−100	−300	−300	−200	−100	−100	0	100
.12	500	400	300	200	0	−200	−200	−100	0	0	100	200
.07	600	500	400	300	200	−100	−100	0	100	100	200	300
.05	700	600	500	400	300	0	0	100	200	200	300	400
.03	800	700	600	500	400	100	100	200	300	300	400	500

		19X2					19X3		
Probability	Mar.	June	Sept.	Dec.		Mar.	June	Sept.	Dec.
.02	−$600	−$1000	−$700	−$500		−$700	−$1000	−$700	−$600
.03	−550	−950	−650	−450		−600	−900	−600	−500
.05	−500	−900	−600	−400		−550	−850	−550	−450
.10	−400	−800	−500	−300		−500	−800	−500	−400
.15	−300	−700	−400	−200		−400	−700	−400	−300
.20	−200	−600	−300	−100		−300	−600	−300	−200
.18	−100	−500	−200	0		−200	−500	−200	−100
.12	0	−400	−100	100		−100	−400	−100	0
.07	100	−300	0	200		0	−300	0	100
.05	200	−200	100	300		100	−200	100	200
.03	300	−100	200	400		200	−100	200	300

consider in Table 15-2 the liquid-asset balance for January, 19X1. The expected value of this balance can be found by

$$\overline{LB}_1 = \sum_{i=1}^{11} L_{i1}P_{i1} \qquad (15\text{-}1)$$

where L_{i1} is the ith possible balance, and P_{i1} is the probability of occurrence of that balance at the end of period 1. Thus, the expected value of liquid-asset balance for period 1 is

$$
\begin{aligned}
\overline{LB}_1 = &-200(.02) - 100(.03) + 0(.05) + 100(.10) + 200(.15) \\
&+ 300(.20) + 400(.18) + 500(.12) + 600(.07) \\
&+ 700(.05) + 800(.03) = \$323{,}000.
\end{aligned}
\qquad (15\text{-}2)
$$

If the firm were to increase its total financing by $150,000, the expected value of liquid-asset balance at the end of the period would be $473,000.

A decision to change the total financing of the firm will affect all probability distributions in Table 15-2. For simplicity of illustration, we assume that changes in total financing occur in exactly the same proportions of debt and equity as in the firm's existing capital structure and that any new debt financing involves perpetual debt. Changes that involve other than perpetual debt are taken into account in the last section when we consider the total information needed to evaluate alternatives. In our ex-

ample, then, a decision to increase total financing by $150,000 at time 0 will increase the liquid-asset balance in Table 15-2 by $150,000 for each probability for each of the periods.[12] As a result, there is obviously a reduced risk of cash insolvency.[13]

For each contemplated change in total financing, we can determine its effect on the probability distributions of possible liquid-asset balances shown in Table 15-2. In order to evaluate the tradeoff between risk and profitability, however, we must have information about the effect of a change in liquid assets on profitability. To determine the cost of a change in liquid assets, we multiply the cost of carrying liquid assets (expressed as a percentage) by the change. First, however, we must define a change. Although different interpretations are possible, we shall define it as

$$C_j = \sum_{t=1}^{12} (\overline{LB}_t - LB_A)/12 \qquad (15\text{-}3)$$

where C_j = change in liquid-asset balance for alternative j
$\overline{LB}_t$ = expected value of liquid-asset balance in period t for alternative j
LB_A = average liquid-asset balance during previous 12 months

In words, our measured change in the liquid-asset balance represents an average of the expected values of liquid-asset balances for the forthcoming twelve months, less the average of liquid-asset balances for the previous twelve months. If LB_A is not considered typical or appropriate, a more suitable liquid-asset balance may be substituted. The expected value of liquid-asset balance at time t is found with Eq. (15-1).

To illustrate the use of Eq. (15-3), consider the probability distribution of possible liquid-asset balances for April, 19X1, in Table 15-2. Suppose that the firm increases its total financing by $200,000. The new probability distribution of possible liquid-asset balances for April is found by adding $200,000 to each of the eleven liquid-asset balances for that month. The new expected value of liquid-asset balance is

$$
\begin{aligned}
\overline{LB}_4 = {} & -300{,}000(.02) - 200{,}000(.03) - 100{,}000(.05) + 0(.10) \qquad (15\text{-}4) \\
& + 100{,}000(.15) + 200{,}000(.20) + 300{,}000(.18) + 400{,}000(.12) \\
& + 500{,}000(.07) + 600{,}000(.05) + 700{,}000(.03) = \$214{,}000.
\end{aligned}
$$

[12] For ease of exposition, we ignore the effect of the payment of interest on new debt and dividends on new stock issued on the cash budget. These factors could be incorporated in the revised cash budget simply by deducting expected new interest and dividend payments in each future period from the $150,000 increase in liquid assets.

[13] Stephen H. Archer advocates computing the average daily transactions cash balance for a month and the standard deviation about this average. On the basis of this probability distribution, he suggests that the firm should add to its cash balance until the risk of running out of cash is reduced to an acceptable level. "A Model for the Determination of Firm Cash Balances," *Journal of Financial and Quantitative Analysis,* I (March, 1966), 1–11.

If LB_A, the average liquid-asset balance of the firm during the previous year, were $200,000, the change in liquid-asset balance for period 4 would be $214,000 − 200,000 = $14,000. Similarly, we are able to calculate the expected value of change in the liquid-asset balance for the other eleven months of the year. With Eq. (15-3), we then average the changes for the twelve months; this average represents our measure of the change in liquid assets of the firm for a specific change in total financing.

Given our measured change in liquid-asset balance, C_j, this change is multiplied by the opportunity cost of maintaining liquid assets, expressed as a percentage, in order to obtain the total cost of the change. Within a limited range, the opportunity cost of an increase in liquid assets might be approximated by the firm's cost of capital, on a before-tax basis. The product of the above multiplication represents our measure of the impact on profitability of a change in the level of the firm's liquid assets. We defer specific evaluation of the tradeoff between profitability and risk until we have considered the effect of changes in the maturity composition of the firm's debt on profitability and risk.

MATURITY COMPOSITION OF DEBT

Following a similar procedure as that for liquid-asset changes, we are able to compute the effect of changes in the maturity composition of the firm's debt on the probability distributions of liquid-asset balances shown in Table 15-3. To illustrate the impact of a change in maturity composi-

TABLE 15-3
Possible liquid-asset balances after financing

Possibility i	Liquid-Asset Balance With New Financing of $400,000	Probability of Occurrence
1	−$100,000	.02
2	0	.03
3	100,000	.05
4	200,000	.10
5	250,000	.15
6	250,000	.20
7	300,000	.18
8	400,000	.12
9	500,000	.07
10	600,000	.05
11	700,000	.03

tion, suppose that the firm had in its existing debt structure a three-year term loan that called for monthly principal payments of $25,000. These payments are assumed to be embodied in the figures in Table 15-2. If the

firm renegotiated the term loan into one of $7\frac{1}{2}$ years with equal monthly payments, the principal payment per month would be reduced from $25,000 to $10,000. We can easily recalculate the probability distributions shown in Table 15-2 by adding $15,000 to the liquid-asset balance for each probability for each monthly period.[14] Thus, the probability of cash insolvency is reduced for three years as a result of this debt lengthening. Of course, for years four through seven and one-half, the firm will be faced with a $10,000 increment in monthly principal payments. For other changes in the maturity composition of existing debt, we can also recompute the probability distributions shown in Table 15-2.

A flexible borrowing arrangement for meeting short-term funds requirements is a bank line of credit.[15] A line of credit enables a firm to borrow up to a specified maximum amount over a period of time, usually one year. With a line of credit, we must recompute the probability distributions in Table 15-2. To illustrate, suppose that the firm increases its total financing by $400,000, of which $200,000 represents a line of credit. Assume further that the firm will borrow upward to the whole line to maintain a liquid-asset balance of $250,000. For April, the probability distribution of possible liquid-asset balances after financing becomes that shown in Table 15-3. For possibilities 1 through 4, the firm would utilize the full $200,000 under the line. For possibility 5, it would borrow $150,000 under the line; for possibility 6, $50,000. For possibilities 7 through 11, it would borrow nothing under the line; but, of course, there would be $200,000 in regular financing. In a similar manner, we can recompute the other probability distributions in Table 15-2 for this financing alternative.

In our cash-flow evaluation, there is an obvious horizon problem. We have estimated cash flows for only three years hence. Given this horizon, an optimal strategy might call for all debt maturing in three years, one month. Under most circumstances, such a strategy would not be appropriate, for the firm will have funds requirements beyond three years. These requirements no doubt will preclude the paying off of all debt at that time. Consequently, the firm must arrange maturities beyond the cash-budget horizon on the basis of general estimates of future funds requirements and ability to service debt. We might point out, however, that the principles of risk and profitability are the same as those that govern debt maturities falling within the cash-budget horizon.

The opportunity cost of a change in maturity composition of debt must be estimated. If long-term borrowings command an interest rate different from that on short-term borrowings, usually higher, there exists a measurable explicit cost for the operation.[16] Suppose that, in our previous

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[14] For simplicity of exposition, we again ignore the effect of interest payments on the cash flows.

[15] For a discussion of its use, see Chapter 20.

[16] Again, we must recognize the possibility of short-term being higher than long-term rates when interest rates in general are very high.

example, the three-year term loan required an interest rate of 8 per cent, whereas the $7\frac{1}{2}$-year loan required a rate of $8\frac{1}{2}$ per cent. The difference, 0.5 per cent, represents the additional cost of lengthening the debt. If debt were assumed in perpetuity and the amount of the term loan were $900,000, the opportunity cost of debt lengthening would be $900,000 × 0.5% = $4,500 annually.

The second explicit cost involved with debt lengthening is the payment of interest on debt when it is not needed. Suppose that at January 1 a firm had a short-term loan of $600,000, of which $200,000 matured on August 31, $200,000 on October 31, and $200,000 on December 31. To reduce the risk of running out of cash, the firm might consider changing its borrowing accommodation to a one-year loan, maturing December 31. If the interest rate were 7 per cent on both loans, the firm would pay additional interest on $200,000 for four months and on $200,000 for two months. The additional interest cost would be:

$$.07 \times \$200,000 \times \frac{4}{12} = \$4,667$$
$$.07 \times \$200,000 \times \frac{2}{12} = \underline{2,333}$$
$$\text{Total} \quad \$7,000$$

The opportunity cost of a line of credit relates principally to the requirement of compensating balances. This requirement, frequently 15 per cent of the line, increases the cost of borrowing if balances must be maintained in excess of those ordinarily maintained. One way to measure the cost is to take the interest rate on borrowings times the increase in balances necessary to compensate the bank. This notion assumes that the firm will have to borrow under its line to maintain balances considered compensating.

To summarize, for each feasible change in the composition of the firm's debt, we determine the effect of the change on the probability distributions of expected future liquid-asset balances. In addition, we estimate the incremental explicit cost of the particular alternative. Again, we must point out that we have limited our attention to explicit costs. No consideration has been given to the effect of changes in the maturity composition of debt on the way investors at the margin value the firm's stock. Having taken up the effect of changes in liquid assets and debt composition individually, we now must combine the two factors.

COMBINATION OF FACTORS AND SELECTION

In reducing the risk of cash insolvency, the firm can select a combination of changes in liquid assets and in maturity composition of its debt. With a combination, we must estimate the joint effect of the two factors on the probability distributions of expected liquid-asset balances as well as the opportunity cost of the combination. Before proceeding, however,

we must digress to relax an assumption made previously. It will be recalled that we assumed that debt issued in connection with an increase in total financing was perpetual. When such is not the case, we must take account of the effect of principal payments on the schedule of expected future cash flows. For example, suppose that the firm obtained a $540,000 three-year term loan payable monthly and that this loan represented new debt. Because the liquid-asset balances previously computed to reflect the change in total financing assumed perpetual debt, we would need to reduce these balances for each probability for each monthly period over three years by $15,000 ($540,000/36), times the number of months the loan had been outstanding. In addition, we must take account of the effect of the change on explicit costs. Both of these changes should be incorporated into the information provided for evaluating alternatives.

For each feasible alternative for reducing the risk of cash insolvency, a revised schedule of probability distributions of expected future liquid-asset balances should be prepared, accompanied by the estimated opportunity cost of the alternative. Instead of providing the entire probability distribution, it may be suitable to specify only the probability of running out of cash during each future period. The total opportunity cost for each alternative should be denoted on a total annual dollar basis.[17] In this regard, it may be helpful to show not only the total opportunity cost, but also the opportunity cost of each of the changes comprising the alternative. An example of a schedule of possible alternatives is shown in Table 15-4. The probabilities of cash insolvency for these alternatives are shown in Table 15-5.

Given information similar to that found in Tables 15-4 and 15-5, management must determine the best alternative by balancing the risk of cash insolvency against the cost of providing a solution to avoid that possibility. Each solution (increasing liquidity, lengthening the maturity structure, or a combination of the two) will cost the firm something in profit-making ability. If the cost of cash insolvency were known, the best alternative could be determined easily by comparing the expected cost of a cash stockout with the opportunity cost of a particular solution to avoid that stockout.[18] The expected cost of a cash stockout is the cost associated with a particular stockout times its probability of occurrence. For example, suppose that with a particular solution there is a 10 per cent probability for a cash stockout of $50,000, and a 5 per cent probability that the stockout will be $100,000. If the costs of these stockouts are $10,000 and $25,000, respectively, the expected costs will be $0.10($10,000) = $1,000 and $0.05($25,000) = $1,250, respectively. The total expected cost of cash stockout for that solution is $2,250. The optimal solution could

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[17] If there is a change in the average cost of debt financing accompanying a change in total financing, it is necessary to multiply the change in average cost by the total amount of debt financing after the change in total financing.

[18] This assumes that the two costs are comparable.

TABLE 15-4
Schedule of alternatives for reducing risk of cash insolvency

Alternative	Description	Opportunity Cost
1	$400,000 increase in total financing	$ 5,100
2	$500,000 increase in total financing	15,100
3	$600,000 increase in total financing	25,100
4	$700,000 increase in total financing	35,100
5	$800,000 increase in total financing	45,100
6	$900,000 increase in total financing	55,100
7	$1,000,000 increase in total financing	65,100
8	Conversion of term loan maturing $200,000 quarterly through 3 years into six-year term loan with $100,000 quarterly payments	9,000
9	Conversion of term loan into ten-year term loan maturing from year 4 through 10	16,000
10	Alternatives 1 and 8	14,100
11	Alternatives 2 and 8	24,100
12	Alternatives 3 and 8	34,100
13	Alternatives 4 and 8	44,100
14	Alternatives 1 and 9	21,100
15	Alternatives 2 and 9	31,100
16	Conversion of term loan from 3 to 6 years and refunding of mortgage maturing May 19X1 into ten-year note maturing quarterly	23,800
17	Alternative 1, and refunding mortgage	19,800
18	Alternative 2, and line of credit of $250,000	17,400
19	Alternative 2, line of credit of $250,000, and extending $1 million in notes from three-year loan to 4½-year loan	26,200
.	. . .	.
.	. . .	.
32	Alternative 1, and refund intermediate-term loan into long-term loan	21,700

be determined by comparing the reduction in the expected cost of cash stockout accompanying a particular solution with the opportunity cost of implementing that solution. The optimal solution would be where the marginal opportunity cost equaled the marginal decrease in the expected cost of cash stockout.[19]

The difficulty, of course, is in estimating the cost of a cash stockout. A more practical method is for management to specify a risk tolerance for cash insolvency. For example, this risk tolerance might be 5 per cent, meaning that the firm would tolerate upward to a 5 per cent probability of not being able to pay its bills in a future period. Given an acceptable level of risk, the firm then would seek the least costly solution to reducing the probability of cash insolvency to that level. This is done simply by taking those feasible alternatives that provide a probability of cash stockout of approximately 5 per cent or less and picking the least costly. We see in Tables 15-4 and 15-5 that this alternative would be number 17. For

[19]This statement assumes that the second-order conditions for optimality are satisfied and that the solution represents a global maxima.

TABLE 15-5
Probabilities of cash insolvency for various alternatives

						19X1						
Alternative	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	.00	.00	.02	.05	.20	.55	.55	.35	.20	.20	.10	.05
2	.00	.00	.00	.02	.10	.35	.35	.20	.10	.10	.05	.02
3	.00	.00	.00	.00	.05	.20	.20	.10	.05	.05	.02	.00
4	.00	.00	.00	.00	.02	.10	.10	.05	.02	.02	.00	.00
5	.00	.00	.00	.00	.00	.05	.05	.02	.00	.00	.00	.00
5	.00	.00	.00	.00	.00	.02	.02	.00	.00	.00	.00	.00
7	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
8	.10	.20	.20	.35	.73	.85	.85	.73	.35	.35	.20	.05
9	.10	.20	.10	.20	.55	.55	.55	.35	.05	.05	.02	.00
10	.00	.00	.00	.02	.10	.20	.20	.10	.02	.02	.00	.00
11	.00	.00	.00	.00	.05	.10	.10	.05	.00	.00	.00	.00
12	.00	.00	.00	.00	.02	.05	.05	.02	.00	.00	.00	.00
13	.00	.00	.00	.00	.00	.02	.02	.00	.00	.00	.00	.00
14	.00	.00	.00	.00	.05	.05	.05	.02	.00	.00	.00	.00
15	.00	.00	.00	.00	.02	.02	.02	.00	.00	.00	.00	.00
16	.00	.00	.02	.05	.10	.15	.15	.13	.12	.08	.06	.04
17	.00	.00	.02	.05	.04	.04	.04	.04	.03	.02	.01	.01
18	.00	.04	.05	.07	.09	.10	.09	.09	.05	.04	.04	.06
19	.00	.04	.04	.05	.06	.07	.06	.06	.04	.03	.04	.03
·	·	·	·	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	·	·	·	·
32	.00	.00	.02	.05	.10	.10	.08	.05	.02	.00	.00	.00

		19X2					19X3		
Alternative	Mar.	June	Sept.	Dec.		Mar.	June	Sept.	Dec.
1	.20	.85	.35	.10		.35	.85	.35	.20
2	.10	.73	.20	.02		.20	.73	.20	.05
3	.02	.55	.10	.00		.05	.55	.05	.02
4	.00	.35	.02	.00		.02	.35	.02	.00
5	.00	.20	.00	.00		.00	.20	.00	.00
6	.00	.10	.00	.00		.00	.05	.00	.00
7	.00	.02	.00	.00		.00	.02	.00	.00
8	.10	.55	.02	.00		.00	.02	.00	.00
9	.00	.00	.00	.00		.00	.00	.00	.00
10	.00	.02	.00	.00		.00	.00	.00	.00
11	.00	.00	.00	.00		.00	.00	.00	.00
12	.00	.00	.00	.00		.00	.00	.00	.00
13	.00	.00	.00	.00		.00	.00	.00	.00
14	.00	.00	.00	.00		.00	.00	.00	.00
15	.00	.00	.00	.00		.00	.00	.00	.00
16	.07	.16	.00	.00		.00	.02	.00	.00
17	.02	.04	.02	.00		.02	.05	.04	.00
18	.06	.10	.08	.07		.06	.12	.09	.08
19	.05	.08	.05	.04		.04	.09	.06	.04
·	·	·	·	·		·	·	·	·
·	·	·	·	·		·	·	·	·
·	·	·	·	·		·	·	·	·
32	.02	.08	.05	.02		.00	.06	.00	.00

this alternative, there is a 5 per cent probability of running out of cash in April, 19X1, and a 5 per cent probability for June, 19X3. We note that the alternative has a total annual opportunity cost of $19,800.

Another way that management might select an alternative would be to formulate risk tolerances on the basis of the opportunity cost involved in reducing the risk of cash stockout to various tolerance levels. It could well be that the specification of a low risk tolerance would result in a very high cost to provide a solution. If management had information about the cost

associated with reducing risk to other levels, it might pick a higher toler-ance level that could be implemented at considerably less cost. For exam-ple, the least costly alternative in our hypothetical example that will reduce the probability of cash insolvency to 2 per cent is number 15, involving a total annual cost of $31,100. This compares with a cost of $19,800 to reduce the probability to 5 per cent. On the basis of this infor-mation, management might not feel that the additional $11,300 to reduce the probability of cash insolvency from 5 to 2 per cent was justified. Ac-cordingly, we see that it may be useful to prepare a schedule showing the least costly solution to reducing risk to various levels. In this way, man-agement can better evaluate the tradeoff between risk and profitability.

Even here, however, there is the problem of not providing enough in-formation. For example, a certain alternative may result in a probability of cash insolvency in only one future period, whereas another might result in that probability being reached in several periods. In Table 15-5, we note that for alternative 17, there is a 5 per cent probability of cash insol-vency in two periods, a 4 per cent probability in six periods, a 3 per cent probability in one period, a 2 per cent probability in five periods, and a 1 per cent probability in two periods. On the other hand, for alternative 14, there is a 5 per cent probability of cash insolvency in three periods, and a 2 per cent probability in one period. Thus, there are considerably more periods in which the firm may run out of cash with alternative 17 than with alternative 14. As the total cost of alternative 14 is only $21,000, com-pared with $19,800 for alternative 17, the firm might regard alternative 14 as more favorable.

Therefore, a strong case can be made for providing management infor-mation about the probability distributions of liquid-asset balances for all future periods for each alternative and for the opportunity cost of the alternative. In this way, management is able to evaluate the maximum probability of cash insolvency and the number of future periods in which there is a chance for a cash stockout. With this additional information, it then can assess more realistically the tradeoff between the risk of cash insolvency and the opportunity cost of reducing this risk. On the basis of this assessment, it would select and implement the most appropriate alternative. The actual implementation will determine the liquid-asset level of the firm and the maturity composition of its debt. In turn, these factors will determine the working-capital position of the firm, given the assumptions listed earlier. This position should be the one most appropri-ate with respect to considerations of risk and profitability.

SUMMARY

Working capital management involves deciding upon the amount and composition of current assets and current liabilities. These decisions involve tradeoffs between risk and profitability. The greater the relative

proportion of liquid assets, the less the risk of running out of cash, all other things being equal. However, profitability also will be less. The longer the composite maturity schedule of securities used to finance the firm, the less the risk of cash insolvency, all other things being equal. Again, however, the profits of the firm are likely to be less. Resolution of the tradeoff between risk and profitability with respect to these decisions depends upon the risk preferences of management.

A framework was proposed by which management can evaluate the level of liquid assets and the maturity composition of the firm's debt. Employing certain probability concepts, the framework allows appraisal of the risk of cash insolvency for various levels of liquid assets and different debt compositions. Given the opportunity cost of a change in liquid assets and/or maturity composition, management then is able to evaluate the tradeoff between profitability and risk. Its decision will determine the working-capital position of the firm.

In this chapter, we have been concerned with working capital management in a broad sense. We assumed, for example, the efficient management of the various components of current assets. The efficiency of credit and collection procedures and inventory control have a significant bearing upon the liquidity of the firm. Moreover, we did not differentiate between cash and marketable securites (near cash) or consider the optimal split between these two assets. In the three subsequent chapters, we analyze specifically the management of cash and marketable securities, the management of receivables, and the management of inventories. In Part VI, we consider methods of short- and intermediate-term financing.

PROBLEMS

1. The Malkiel Corporation has made the three-year projection of its asset investment given below. It has found that payables and accruals tend to equal one-third of current assets. It currently has $50 million in equity and the remainder of its capitalization in long-term debt.

Date	Fixed Assets	Current Assets
3/31/72 (now)	$ 50 (in millions)	$ 21 (in millions)
6/30/72	51	30
9/30/72	52	25
12/31/72	53	21
3/31/73	54	22
6/30/73	55	31
9/30/73	56	26
12/31/73	57	22
3/31/74	58	23
6/30/74	59	32
9/30/74	60	27
12/31/74	61	23

(a) If long-term debt will cost 8 per cent over the period and short-term debt 6 per cent, devise a financing plan for the three-year period.

(b) Suppose an officer of the company wanted to borrow enough money long-term to cover the entire three years. What would the cost difference be between your plan and his?

2. The Andersen Corporation has a sales level of $280,000 with a 10 per cent net profit margin before interest and taxes. To generate this sales volume, the firm maintains a fixed asset investment of $100,000. Currently, the firm maintains $50,000 in current assets.

(a) Determine the asset turnover for the firm. Compute the rate of return on assets.

(b) What would be the rate of return if management assumed a more conservative attitude and increased current assets by $50,000?

(c) What would be the rate of return if management assumed a less conservative attitude and decreased current assets by $25,000?

(d) Appraise the significance of increases and decreases in the level of current assets.

3. The Amos Company has determined that the distribution of expected net cash flows (available to meet current liabilities) for the next three months is approximately normal with a mean of $500,000 and a standard deviation of $350,000. The firm now earns 16 per cent on its investment in nonliquid assets; by investing in marketable securities, it could earn only 4 per cent. It has been estimated that lengthening the maturity of the firm's outstanding debt ($6,000,000) could reduce the outflow of cash in the following manner:

Added Length of Maturity	Reduced Cash Outflow
One year	$250,000
Two years	375,000
Three years	450,000
Four years	500,000
Five years	535,715
Six years	562,500
Seven years	583,333

Lengthening the maturity by five years or less would cost the company an additional 1 per cent annually, while lengthening it by six or seven years would result in a 2 per cent annual increase in interest costs.

(a) Assuming that the management of the Amos Company is willing to tolerate a 5 per cent probability of running out of cash, by how much should it increase its liquid assets (or decrease cash outflows)?

(b) There are three alternatives which will reduce the probability of running out of cash to 5 per cent:
   (1) Increase liquid assets.
   (2) Lengthen maturity of debt.
   (3) A combination of (1) and (2).
   Which alternative is optimal?

4. Assume that the firm depicted in Table 15-2 has reached December 31, 19X2, with a liquid-asset balance of $400,000 and no additional financing.

(a) What is the probability of cash insolvency by March 19X3 if no new financing is obtained? What is the probability of cash insolvency by June 19X3 if no new financing is obtained?

(b) If management were willing to tolerate insolvency only 5 per cent of the time, how much would be borrowed in March? In June?

5. Suppose that funds could be obtained on December 31, 19X2, on the following bases (see problem 4):

Amount	3-month Cost	6-month Cost
$100,000	$2,000	$3,000
200,000	4,500	8,000
300,000	7,500	15,000
400,000	12,000	26,000
500,000	17,500	40,000
600,000	24,000	60,000

(a) What is the least-cost method of avoiding cash insolvency over the six-month period? What is the cost?

(b) Construct a table depicting the probability of cash insolvency and the least-cost method of achieving it for each level of liquid balances.

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Our concern in the previous chapter was with the overall level of liquid and current assets of the firm. In this and the subsequent two chapters, we examine the firm's investment in specific current assets. Our purpose is to investigate ways in which these assets can be managed efficiently so as to contribute to the overall objective of the firm. In general, the optimum investment in a specific current asset is determined by comparing the benefits expected to be derived from a particular level of investment with the costs of maintaining that level. These costs may be both direct and opportunity costs. This chapter is devoted to examining cash management and the investment of excess funds in marketable securities. In the last twenty years, we have witnessed ever-increasing sophistication in cash management by corporations. The trend has been toward reducing cash—the firm's most liquid asset—to a minimum; the funds released are invested in earning assets. This trend can be attributed to rising interest rates on securities, which make the opportunity

cost of holding cash more expensive, to innovations in cash management, and to economies of scale in cash management as corporations grow larger.[1]

Keynes has identified three motives for holding cash: the transactions motive, the precautionary motive, and the speculative motive.[2] The transactions motive is the need for cash to meet payments arising in the ordinary course of business. These payments include such things as purchases, labor, taxes, and dividends. The precautionary motive for holding cash has to do with maintaining a cushion or buffer to meet unexpected contingencies. The more predictable the cash flows of the business, the less precautionary balances that are needed. Ready borrowing power to meet emergency cash drains also reduces the need for this type of balance. It is important to point out that not all of the firm's transactions and precautionary balances need be held in cash; indeed, a portion may be held in marketable securities — near-money assets.

The speculative motive relates to the holding of cash in order to take advantage of expected changes in security prices. When interest rates are expected to rise and security prices to fall, this motive would suggest that the firm should hold cash until the rise in interest rates ceases. When interest rates are expected to fall, cash may be invested in securities; the firm will benefit by any subsequent fall in interest rates and rise in security prices. For the most part, companies do not hold cash for the purpose of taking advantage of expected changes in interest rates. Consequently, we concentrate only upon the transactions and precautionary motives of the firm, with these balances held both in cash and in marketable securities.

AMOUNT OF TRANSACTIONS AND PRECAUTIONARY BALANCES

Influences on the amount of transactions and precautionary balances held by the firm include:

1. The expected net cash flows of the firm as determined by the cash budget. These cash forecasts should encompass both the short- and long-run cash needs of the firm.

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[1] For an excellent study documenting these causes empirically, see Robert C. Vogel and G. S. Maddala, "Cross-Section Estimates of Liquid Asset Demand by Manufacturing Corporations," *Journal of Finance*, XXII (December, 1967), 557–75.

[2] John Maynard Keynes, *The General Theory of Employment, Interest, and Money* (New York: Harcourt Brace Jovanovich, Inc., 1936), pp. 170–74.

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2. Possible deviations from expected net cash flows. As we discussed in Chapter 15, probability concepts can be applied to the cash budget to determine the variation in cash flows under different circumstances. Every effort should be made to take into account the magnitude of possible dispersions.

3. The maturity structure of the firm's debt.

4. The firm's borrowing capacity to meet emergency needs beyond transactions and precautionary balances.

5. The utility preferences of management with respect to the risk of cash insolvency.

6. The efficiency of cash management.

Factors 1 through 5 were discussed in the previous chapter in our study of working capital management. We assume that the first four of these factors have been evaluated by management in keeping with that discussion and that the appropriate transactions and precautionary balances, exclusive of the last factor, have been determined. The efficiency of cash management, however, remains to be considered. This factor, together with the previous five, will determine the appropriate level of total transactions and precautionary balances for the firm.

CASH MANAGEMENT

In this section, we analyze various collection and disbursement methods by which a firm can improve its cash management efficiency. These methods constitute two sides of the same coin; they exercise a joint impact on the overall efficiency of cash management. We consider first the acceleration of collections, or reducing the delay between the time a customer pays his bill and the time the check is collected and becomes usable funds for the firm. A number of methods have been employed in recent years to speed up this collection process and maximize available cash. These methods are designed to do one or all of the following: (1) speed the mailing time of payments from customers to the firm; (2) reduce the time during which payments received by the firm remain uncollected funds; and (3) speed the movement of funds to disbursement banks.

CONCENTRATION BANKING

Concentration banking is a means of accelerating the flow of funds of a firm by establishing strategic collection centers.[3] Instead of a single collection center located at the company headquarters, multiple collec-

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[3] See Frederick E. Horn, "Managing Cash," *Journal of Accountancy,* CXVII (April, 1964), reprinted in *Foundations for Financial Management,* ed. James Van Horne (Homewood, Ill.: Richard D. Irwin, Inc., 1966), p. 28.

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tion centers are established. The purpose is to shorten the period between the time a customer mails in his payment and the time when the company has the use of the funds. Customers in a particular geographic area are instructed to remit their payments to a collection center in that area. The selection of the collection centers usually is based upon the geographic areas served and the volume of billings in a given area. When payments are received, they are deposited in the collection center's local bank. Surplus funds are then transferred from these local bank accounts to a concentration bank or banks. A bank of concentration is one with which the company has a major account—usually a disbursement account. For example, a company headquartered in New York City might have but one concentration bank, a New York bank. Concentration banking is one way to reduce the size of the float, the difference between the amount of deposit and the amount of usable funds in a bank. A company usually cannot withdraw a deposit until the bank actually collects the checks. Until collected, the deposited checks represent float.[4]

An Illustration. To illustrate concentration banking and the transfer of funds, we examine the case of an actual large company with over twenty collection centers. At the time of the study, each collection center billed customers in its area and made daily deposits in its local bank of payments received from customers. On the average, the checks deposited in a bank were collected in one and one-fourth days. In other words, the company had use of the funds one and one-fourth days after deposit. In each of its local banks, the company maintained sufficient collected balances to compensate the bank for the costs of servicing the account.

A daily *wire transfer* arrangement was used to transfer collected balances in excess of compensating balances to one of several concentration banks. The managers of the collection centers initiated the transfer on the basis of a daily report of estimated collected balances from their local banks. Because the wire transfers were made through the Federal Reserve System, the funds transferred became available immediately at the concentration banks.

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[4]Checks deposited with a bank usually are processed for collection by that bank either through the Federal Reserve System, through a correspondent bank, or through a clearing-house system of a group of banks in a particular city. A check becomes collected funds when it is presented to the drawee bank and actually paid by that bank. In order to streamline the availability of credit, however, the Federal Reserve has established a schedule specifying the availability of credit for all checks deposited with it for collection. This schedule is based upon the average time required for a check deposited with a specific Federal Reserve bank to be collected in a particular geographic area of the country. The maximum period for which credit is deferred is two business days. This means that a check deposited with a Federal Reserve bank for collection at a distant point would become available credit for the depositing bank two days later. Correspondent banks frequently set up deferment schedules based upon that of the Federal Reserve. From the standpoint of a company, the length of the float depends upon the time it takes the bank to obtain available credit on checks processed for collection. In turn, this time will depend upon where the drawee banks are located.

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This method of transfer differs from a *depository transfer check* arrangement for the movement of funds, whereby a depository check is drawn on the local bank, payable to a concentration bank. Funds are not immediately available at the concentration bank, for the check must be collected through the usual channels. The check itself is not signed but bears the company's printed name as drawer. Given a resolution by the board of directors to the drawee bank, the printed name is sufficient authority for withdrawal. Whereas a transfer check costs only about $0.10 to process, it is not as fast as a wire transfer, which costs about $1.50. The delay must be analyzed in relation to the difference in cost. For small transfers, a wire transfer is too costly compared to a depository transfer check and should not be used. The earnings possible on investing the released funds simply do not cover the differential in cost.[5]

The advantage of a system of decentralized billings and collections over a centralized system is twofold. (Recall that we compare a system of multiple collection centers with a single collection center located at company headquarters.)

1. The time required for mailing is reduced. Because the collection center bills customers in its area, these customers usually receive their bills earlier than if the bills were mailed from the head office. In turn, when customers pay their bills, the mailing time to the nearest collection center is shorter than the time required for the typical remittance to go to the head office. The company estimated that there was a saving of approximately one day in mailing time from the customer to the company.

2. The time required to collect checks is reduced, because remittances deposited in the collection center's local bank usually are drawn on banks in that general area. The company estimated that the average collection period would be two and one-fourth days if all remittances were deposited in the company's head office bank, compared with one and one-fourth days under the present system. At the margin, then, the company was able to speed up the collection of customer checks by one day.

Thus, the company was able to accelerate overall collections by two days; one day was gained by reducing the mailing time and one day by reducing the time during which deposited checks remain uncollected. At the time of the study, average daily remittances by customers were $2.1 million. By saving two days in the collection process, approximately $4.2 million in funds were released for investment elsewhere. With the recent high levels of interest rates, it is not difficult to see the opportunity cost of tying up funds. However, profits from the investment

[5] See Frederick W. Searby, "Use Your Hidden Cash Resources," *Harvard Business Review,* 46 (March–April, 1968), 74–75.

of the released funds must be compared with any additional costs of a decentralized system over a centralized one. Also, it is important to consider any differences between the two systems in total compensating balances. The greater the number of collection centers, the greater the number of local bank accounts that must be maintained.

LOCK-BOX SYSTEM

Another means of accelerating the flow of funds is a lock-box arrangement. With concentration banking, remittances are received by a collection center and deposited in the bank after processing. The purpose of a lock-box arrangement is to eliminate the time between the receipt of remittances by the company and their deposit in the bank. A lock-box arrangement usually is on a regional basis, with the company choosing regional banks according to its billing patterns. Before determining the regions to be used, a feasibility study is made of the availability of checks that would be deposited under alternative plans. In this regard, operation research techniques have proved useful in the selection of lock-box sites.[6] If a company divided the country into five sections on the basis of a feasibility study, it might pick New York City for the Northeast, Atlanta for the Southeast, Chicago for the Midwest, Dallas for the Southwest, and San Francisco for the West Coast.

The company rents a local post office box and authorizes its bank in each of these cities to pick up remittances in the box. Customers are billed with instructions to mail their remittance to the lock box. The bank picks up the mail several times a day and deposits the checks in the company's account. The checks are microfilmed for record purposes and cleared for collection. The company receives a deposit slip and a list of payments, together with any material in the envelope. This procedure frees the company from handling and depositing the checks.

The main advantage of a lock-box system is that checks are deposited at banks sooner and become collected balances sooner than if they were processed by the company prior to deposit. In other words, the lag between the time checks are received by the company and the time they actually are deposited at the bank is eliminated. The principal disadvantage of a lock-box arrangement is the cost. The bank provides a number of services additional to the usual clearing of checks and requires compensation for them, usually preferring increased deposits. Because the cost is almost directly proportional to the number of checks deposited, lock-box arrangements usually are not profitable if the average remittance is small.

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[6] See Ferdinand K. Levy, "An Application of Heuristic Problem Solving to Accounts Receivable Management," *Management Science*, 12 (February, 1966), 236–44; and Robert F. Calman, *Linear Programming and Cash Management/CASH ALPHA* (Cambridge, Mass: The M.I.T. Press, 1968), Chapter 4.

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The appropriate rule for deciding whether or not to use a lock-box system or, for that matter, concentration banking, is simply to compare the added cost of the most efficient system with the marginal income that can be generated from the released funds. If costs are less than income, the system is profitable; if not, the system is not a profitable undertaking. The degree of profitability depends primarily upon the geographical dispersion of customers, the size of typical remittance, and the earnings rate on the released funds. Although there is disagreement as to what earnings rate to use, the most appropriate rate for our purpose is the rate on marketable securities. Because a decision as to the total amount of liquid assets to maintain was considered in Chapter 15, our present decision affects mainly the proportion of cash to marketable securities, not their sum. Thus, the opportunity cost of tying up funds in cash is the return foregone on marketable securities.

OTHER PROCEDURES

Frequently, firms give special attention to the handling of large remittances so that they may be deposited in a bank as quickly as possible. This special handling may involve personal pickup of these checks or the use of airmail or special delivery. When a small number of remittances account for a large proportion of total deposits, it may be very worthwhile to initiate controls to accelerate the deposit and collection of these large checks. The firm should exercise tight control over interbank transfers of cash and transfers between various units of the company, such as divisions or subsidiaries. Excessive funds may be tied up in various divisions of the firm.

Some companies maintain too many bank accounts, thereby creating unnecessary pockets of idle funds. A company that has an account in every city where it has either a sales office or a production facility might be able to reduce cash balances considerably if it were to eliminate some of these accounts. The banking activities of a sales office can often be handled from a larger account with little loss in service or availability of funds. Even though small accounts may create a degree of goodwill with bankers, they make little sense in the overall cash management of the firm. By closing such unnecessary accounts, a firm may be able to release funds that it then can put to profitable use.

CONTROL OF DISBURSEMENTS

In addition to accelerating collections, effective control of disbursements can result in a faster turnover of cash. Whereas the underlying objective of collections is maximum acceleration, the objective in disbursements is to slow them down as much as possible. The combination

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of fast collections and slow disbursements will result in maximum availability of funds.

For a company with multiple banks, it is important to be able to shift funds quickly to those banks from which disbursements are made, to prevent excessive balances from building up temporarily in a particular bank. Operating procedures for disbursements should be well established. If cash discounts are taken on accounts payable, procedures should aim toward eliminating or minimizing the loss of discounts due to clerical inefficiencies. The timing of payments is important. For maximum use of cash, payments should be made on the due dates, not before and not after.

In an effort to delay actual payment as long as possible, some companies have used payable-through drafts to pay bills. Unlike an ordinary check, the draft is not payable on demand. When it is presented to the issuer's bank for collection, the bank must present it to the issuer for acceptance. The funds then are deposited by the issuing firm to cover payment of the draft. The advantage of the draft arrangement is that it delays the time the firm actually has to have funds on deposit to cover the draft. Consequently, it allows the firm to maintain smaller deposits at its banks. One company making increasing use of drafts is American Telephone and Telegraph Co. "In handling its payrolls, for instance, AT&T can pay an employee by draft on Friday. The employee cashes the draft at his local bank, which sends it on to AT&T's New York bank. It may be Wednesday or Thursday before the draft arrives. The bank then sends it to the company's accounting department, which has until 3 P.M. that day to inspect and approve it. Not until then does AT&T deposit funds in its bank to pay the draft." [7] Other major users of drafts include insurance companies, auto manufacturers, and railroads. Because of the inconvenience of collecting drafts and the lower deposit balances drafts allow companies to maintain, commercial banks generally are not happy with the arrangement and have come to impose service charges for their use.

Another way of maximizing cash availability is "playing the float." In this case, float is the difference between the total dollar amount of checks drawn on a bank account and the balance shown on the bank's books. It is possible, of course, for a company to have a negative balance on its books and a positive bank balance, because checks outstanding have not been collected from the account on which they are drawn. If the size of float can be estimated accurately, bank balances can be reduced and the funds invested to earn a positive return.

As mentioned earlier, optimizing cash availability involves accelerating collections as much as possible and delaying payments as long as is

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[7] Peter Vanderwicken, "More Firms Substitute Drafts for Checks to Pay, Collect Bills," *Wall Street Journal* (August 29, 1961), pp. 1, 16.

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realistically possible. Because delaying payments may damage the firm's credit standing, the resulting cost to the firm must be taken into account. In the future, we can expect to see further improvements in check collection. As we move toward a "checkless society," the time funds remain uncollected will become shorter. In a checkless society where transfers are made through computers, payments will be immediate. There will be no uncollected funds, for one party's account will be debited the instant another's is credited. While the firm's deposits will be collected faster, so too will the checks it writes. Whether it gains or loses will depend upon the typical float on checks deposited relative to the float for checks written.

DETERMINING A MINIMUM CASH BALANCE

Establishing a minimum level of cash balances depends upon the transaction needs for cash, the efficiency of cash management, and the compensating balance requirements of banks. The efficiency of cash management determines the level of cash needed to meet transaction requirements. Although these requirements are usually not stable over time, the financial manager can determine the minimum level of cash balances needed to meet them by evaluating carefully the collection and disbursement patterns of the firm. This minimum level should be based only upon the need to have funds available to pay bills when cash outflows and inflows are less than perfectly synchronized. Because an optimal level of liquidity was considered in the previous chapter, it should not be brought into our analysis here.

The above analysis needs to be qualified for compensating balances. Seldom will the minimum amount of cash balances needed to meet transaction requirements be sufficient to compensate banks for activity in the company's accounts. For most firms, the average minimum level is determined by the compensating balance requirements of its banks. These requirements are set on the basis of the profitability of the accounts. A bank begins by calculating the average collected balances shown on the bank's books over a period of time. As brought out before, this balance often is higher than the cash balance shown on the company's books. From the average collected balance, the bank subtracts the percentage of deposits it is required to maintain at the Federal Reserve, around 17 per cent. The residual constitutes the earnings base on which income is generated. Total income is determined by multiplying the base times the earnings rate of the bank. This rate fluctuates in keeping with money market conditions.

Once the income from an account is determined, the cost of the account must be computed. Most banks have a schedule of costs on a per

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item basis for such transactions as transfers and processing checks. The account is analyzed for a typical month during which all transactions are multiplied times the per item cost and totaled. If the total cost is less than the total income from the account, the account is profitable; if more, it is unprofitable. The minimum average level of cash balances required is the point at which the account is just profitable.[8] Because banks differ in the earnings rate they use as well as in their costs and method of account analysis, the determination of compensating balances varies. The firm, therefore, may be wise to shop around and determine the bank that requires the lowest compensating balances for a given level of activity. If a firm has a lending arrangement with a bank, the firm may well be required to maintain balances in excess of those required to compensate the bank for the activity in its account. As we consider compensation for a lending arrangement in Chapter 20, no discussion of this form of compensation will be undertaken at this time.

In recent years, there has been a trend toward paying cash for services rendered by a bank instead of maintaining compensating balances. The advantage to the firm is that it may be able to earn more on funds used for compensating balances than the fee for the services. The higher the interest rate in the money markets, the greater the opportunity cost of compensating balances and the greater the advantage of service charges. It is an easy matter to determine if the firm would be better off with service charges as opposed to maintaining compensating balances. One simply compares the charges with the earnings on the funds released. Most banks resist placing normal services, such as clearing checks, on a fee basis. Nevertheless, an increasing number of bank services are being offered on such a basis.

The balances maintained at a bank and the services the bank performs should be analyzed carefully. If deposits are more than compensating, funds may be tied up unnecessarily. Linear programming has been used to structure a firm's banking relations and to determine a minimum level of cash balances. Calman develops an L.P. model for providing the optimal allocation decisions (where to disburse, where to make tax payments, where to pay fees) for a firm that has multiple banks providing different combinations of services.[9] The form of compensation (balances or fees) and the price of services will vary according to the type of service required and according to the bank providing the service. The objective function of his model is to minimize the total cost of the banking system to the firm, subject to various levels of activities requiring banking services. By analyzing the dual variables, or "shadow prices," the financial manager can determine the marginal cost of services provided

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[8] For additional discussion of account analysis, see Alfred M. King, *Increasing the Productivity of Company Cash* (Englewood Cliffs: Prentice-Hall, Inc., 1969), pp. 49–52.

[9] *Linear Programming and Cash Management/CASH ALPHA.*

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by each bank. With this information, he can attempt to renegotiate with a particular bank if the marginal cost is too high, or restructure the firm's entire banking system. The model gives indication of the value of operations research techniques in cash management.[10]

The minimum average level of cash balances for a firm is the greater of: (1) the average amount of deposits necessary to compensate banks; or (2) the average level of cash balances needed to meet transaction needs for cash. For most firms, the former rather than the latter determines the minimum average level. However, for the firm that pays for many of its banking services on a fee basis, the second factor could well determine the minimum average level of cash balances. The minimum average level of cash balances over time will differ from the minimum level at a moment in time. The minimum level at any particular moment is determined strictly by the minimum transactions needs for cash on a day-to-day basis. The concept of a minimum level of cash balances is important in determining a firm's investment in marketable securities, a subject to which we now turn.

DIVISION OF FUNDS BETWEEN CASH AND MARKETABLE SECURITIES

We assume from Chapter 15 and the earlier portion of this chapter that the firm has determined a proper level of transactions and precautionary balances. This decision was made in keeping with expected net cash flows; possible deviations of cash flows from expectation; the maturity structure of the firm's debt; the availability of borrowing; the utility preferences of management with respect to risk; and, finally, the efficiency of cash management, which encompasses the need to maintain compensating balances at commercial banks. Given the level of transactions and precautionary balances, we wish to consider the proper split between cash and marketable securities. Because these securities can be converted into cash on very short notice, they serve the precautionary need of the firm to hold cash.

As a general rule, excess cash above some minimum level should be invested in marketable securities. This rule must be qualified, however, for the fact that the expected holding period over which interest is earned must be sufficiently long to offset the transactions costs, the cost of inconvenience, and the delays involved in buying and selling securities. If

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[10] For an extensive linear programming model that deals with cash management of the firm as a whole, see Yair E. Orgler, *Cash Management* (Belmont, Calif.: Wadsworth Publishing Co., Inc., 1970), Chapters 3–6; and Orgler, "An Unequal-Period Model for Cash Management Decisions," *Management Science*, 16 (October, 1969), 77–92. The objective function encompasses payments, short-term financing, and security transactions. This function is maximized subject to managerial and institutional constraints, including minimum cash balance requirements. For a detailed analysis of the application of linear programming to a financial problem of a different sort, see the appendix to Chapter 22.

the future were known with certainty, it would be an easy matter to determine the optimal split between cash and marketable securities. Projected cash would be invested as long as the interest earnings exceeded transactions and inconvenience costs, and as long as the delays in conversion between cash and marketable securities did not hinder the firm in paying its bills. If transactions and inconvenience costs were zero and conversion between the two assets were instantaneous, the firm would hold no cash. It simply would sell securities to pay its bills. When transactions and inconvenience costs are positive, however, the firm will want to hold cash when the expected holding period for investment is not long enough to earn sufficient interest to offset them.[11] By the same token, with conversion delays, the firm may need to hold cash. Thus, even if future cash flows were known with certainty, a firm probably would hold some cash.

Under conditions of certainty, the optimal mix of cash and marketable securities can be determined with an inventory lot-size formula; and this method is illustrated in the appendix to this chapter. The more realistic decision, however, is to determine the optimal mix under conditions of uncertainty. To illustrate this decision, we shall employ a relatively simple model, using probability concepts.

ASSUMPTIONS BEHIND THE MODEL

For simplicity, we assume a one-period model. Suppose that at the beginning of each week the treasurer analyzes the firm's cash and marketable securities positions for the subsequent week relative to cash projections for that week. The frequency of analysis will depend upon the dispersion of projected cash flows, the degree to which cash flows are correlated over time, and the cost of the review. The more frequent the review, the less the risk that unfavorable cash swings will not be identified, but also the higher the total cost of review.[12] The greater the dispersion of cash flow estimates and the greater their expected correlation over time, the more desirable frequent reviews will be.

Assuming that reviews once a week are appropriate, we assume that the average yield on short-term marketable securities, such as Treasury bills, is 0.08 per cent per week. This return applies to both existing investments and any additional investments. Because the investment is for a short term and there is no default risk, we assume no risk of security price fluctuations. Also, we assume that the conversion between cash and marketable securities is instantaneous. Costs are twofold: the commission involved in buying or selling the security, and the internal fixed costs of making a transaction. The internal costs include clerical

[11] See D. E. Peterson, *A Quantitative Framework for Financial Management* (Homewood Ill.: Richard D. Irwin, Inc., 1969), pp. 212–19.

[12] Peterson, *A Quantitative Framework for Financial Management*, p. 207.

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expenses and the time required for the assistant treasurer or other in-
dividual to make the call to the security dealer. Clerical costs involve
such things as entering the transaction on the books, making the payment
for the securities, recording receipts and disbursements, and making
provisions for holding the securities. Given a number of transactions,
the procedures for placing an order can be streamlined to reduce the
average fixed cost per transaction. Nevertheless, these costs do exist
and too often are either overlooked or underestimated by management.
Included in the estimate of fixed cost should be the imputed cost of in-
convenience per transaction.

For our illustration, we assume that the commission for both buying
and selling a security is 0.03 per cent and that fixed costs per transaction
are $40. We assume also that, based upon its analysis of minimum cash
balances, the firm has established a strict policy of having at least a $3
million cash position at the end of each seven-day week and maintaining
a minimum of approximately $3 million in cash during the week. How-
ever, it is possible for the firm to fall below this minimum temporarily
during the week. We assume that the firm will not borrow to maintain the
minimum cash level as long as it has marketable securities that can be
sold either at the end of the week or during the week.

Our concern, then, is with finding the proper split between cash and
marketable securities, on the basis of cash projections. Suppose that at
the beginning of a given week, the firm has a cash balance of $3.2 million
and short-term investments totaling $1.8 million. Moreover, the probable
cash balances one week hence without the purchase or sale of securities
are estimated as follows.

Amount (in millions)	Probability of Occurrence
$2.9	0.10
3.0	0.10
3.1	0.20
3.2	0.30
3.3	0.20
3.4	0.10

These cash balance estimates are based upon the firm's cash projection
and upon its past experience and knowledge of possible variations in
cash flow.

THE MODEL ILLUSTRATED

The job now is to analyze the cash projections in relation to the other
information at hand, to determine the optimal holding of marketable
securities for the forthcoming week. The minimum level of marketable se-

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curities that might be held is $1.7 million. Under this alternative, there is no probability that the cash balance at the end of the week will be less than $3 million. Therefore, a lower level of security holdings would make no sense. This alternative would require the sale of $100,000 of existing securities, involving fixed costs of $40 and a commission of $30: ($100,000 × 0.0003). The expected net earnings under this alternative are shown in the first row of Table 16-1. From the $1,360 in gross earnings for the week ($1.7 million × 0.0008), we subtract the cost of selling $100,000 of existing securities, or $70, to obtain expected net earnings of $1,290.

The second alternative is to maintain the present $1.8 million in securities. Under this alternative, there is a 10 per cent probability that $100,000 of securities will have to be sold at the end of the period in order to restore the cash balance to $3 million at the end of the week. As shown in the second row of Table 16-1, the expected cost of selling these securities is the probability of having to sell $100,000 in securities times the fixed costs and transaction costs, or 0.10 [$40 + 0.0003(100,000)], or $7. This amount is subtracted from gross earnings to obtain expected net earnings of $1,433 for the week.

At this point in our illustration, we must consider more specifically the fact that securities might have to be sold during the week in order to maintain approximately $3 million in cash. Suppose that the firm had found from past experience that the probability of having to sell a portion of the securities during the week increases with the total amount of securities that had to be sold (shown in column 4 of Table 16-1). In other words, the need to sell securities during the week increases with the amount of the total cash drain. No longer is it sufficient simply to sell securities at the end of the week in order to maintain a cash balance of approximately $3 million; securities need to be sold during the week. If securities are sold during the week, the firm incurs on a marginal basis an additional fixed cost of $40 plus the opportunity cost of interest lost during the remainder of the week.[12] Let us assume that management is able to estimate accurately the probability that securities will have to be sold during the week.

Suppose that if the total amount having to be sold at the end of the week is $100,000, management estimates that there is no probability that any portion of this amount will have to be sold before the end of the week. However, if the total amount that must be sold is $200,000, management might estimate a 0.10 probability that half the total amount will have to be sold during the week. Assume also that on the average, the securities will have to be sold on the second day and that three-quarters of the weekly gross earnings — 0.06 per cent — will be lost on the securities sold. Thus,

[12]Note that the transaction costs already are accounted for in Table 16-1 in the sale of securities at the end of the week (column 6).

TABLE 16-1
Expected earnings from holding securities

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Gross Earnings for Week		Probability of Having to Sell		Selling	Expected Fixed Costs and Earnings Lost on Account of Sale Before	Total Expected Selling Costs	Total Expected	Expected Net
Marketable Security Level (in millions)	0.0008(1)	Buying Costs*	Amount	Probability	Costs*	End of Week	(5) × (6 + 7)	Costs (3) + (8)	Earnings (2) − (9)
$1.7	$1,360	—	$100,000	1.00	$ 70	—	$ 70.00	$ 70.00	$1,290.00
1.8	1,440	—	100,000	0.10	70	—	7.00	7.00	1,433.00
1.9	1,520	$ 70	200,000	0.10	100	10.00	11.00		
			100,000	0.10	70	—	7.00		
							18.00	88.00	1,432.00
2.0	1,600	100	300,000	0.10	130	32.50	$ 16.25		
			200,000	0.10	100	10.00	11.00		
			100,000	0.20	70	—	14.00		
							$ 41.25	141.25	1,458.75
2.1	1,680	130	400,000	0.10	160	72.00	$ 23.20		
			300,000	0.10	130	32.50	16.25		
			200,000	0.20	100	10.00	22.00		
			100,000	0.30	70	—	21.00		
							$ 82.45	212.45	1,467.55
2.2	1,760	160	500,000	0.10	190	133.00	$ 32.30		
			400,000	0.10	160	72.00	23.20		
			300,000	0.20	130	32.50	32.50		
			200,000	0.30	100	10.00	33.00		
			100,000	0.20	70	—	14.00		
							$135.00	295.00	1,465.00
2.3	1,840	190	600,000	0.10	220	220.00	$ 44.00		
			500,000	0.10	190	133.00	32.30		
			400,000	0.20	160	72.00	46.40		
			300,000	0.30	130	32.50	48.75		
			200,000	0.20	100	10.00	22.00		
			100,000	0.10	70	—	7.00		
							$200.45	390.45	1,449.55

*Includes commission of 0.0003 times the amount of the transaction plus fixed cost of $40 per transaction.

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there will be an additional fixed cost of $40, and $60 in interest will be lost ($100,000 × 0.0006). Therefore, the expected fixed costs and earnings lost on the securities sold are 0.10($100), or $10. If the total amount of securities to be sold were $300,000, there might be a 0.25 probability that one-half of this amount would need to be sold on the second day. Again, the additional fixed cost incurred is $40, but the interest lost is now ($150,000 × 0.0006), or $90. Thus, the expected additional cost is 0.25($130), or $32.50. Similarly, if we tabulate the probabilities for other amounts of securities sold during the week, we might obtain a schedule like that shown in Table 16-2. It can be seen from the table that expected costs increase with the total amount of securities that must be sold. The expected additional costs shown in the last column of Table 16-2 are incorporated in column 7 of Table 16-1.

TABLE 16-2
Expected fixed costs and earnings lost on account of sale before end of week

Total Amount of Securities Sold (Column (4) of Table 16-1)	Additional Fixed Cost	Interest Lost*	Total Additional Costs	Probability	Expected Additional Costs
$100,000	$ 0	$ 0	$ 0	0	$ 0
200,000	40	60	100	0.10	10.00
300,000	40	90	130	0.25	32.50
400,000	40	120	160	0.45	72.00
500,000	40	150	190	0.70	133.00
600,000	40	180	220	1.00	220.00

*0.0006 (one-half amount in first column)

Taking into account the additional costs associated with selling securities before the end of the week, we can consider the third alternative — a security level of $1.9 million. For this level of security holdings, the firm must acquire an additional $100,000 of securities at a cost of $70, [$40 + 0.0003($100,000)]. Moreover, there is a 0.10 probability that it will have to sell $200,000 of securities and a 0.10 probability that it will have to sell $100,000 of securities in order to restore the cash balance to $3 million at the end of the week. The costs of selling the securities at the end of the week are [$40 + 0.0003(200,000)], or $100, and [$40 + 0.0003(100,000)], or $70, respectively. If the firm has to sell $200,000 in securities at the end of the week, there is a 0.10 probability that one-half of this amount will have to be sold during the week. As calculated in the previous paragraph, the expected marginal cost of this occurrence is $10; this amount is shown in column 7 of Table 16-1. When we multiply the sum of the costs shown in columns 6 and 7 by their respective probabilities of occurrence and add the products, we obtain total expected selling costs of $18. This amount is added to the buying costs to obtain total expected

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costs of $88. Subtracting this amount from gross earnings for the week, we obtain expected net earnings of $1,432 for the week.

When we carry these calculations through for all possible levels of security holdings, we obtain the results shown in the remaining rows of Table 16-1. The table tells us the expected net earnings for each level of security holdings for the week. We see that the optimal level of security holdings for the week is $2.1 million, resulting in expected net earnings of $1,467.55. Consequently, the company should purchase $300,000 in additional securities. To increase security holdings another $100,000 to $2.2 million would involve marginal expected costs in excess of marginal gross earnings and therefore would not be profitable.

The optimal security level for the following week would be determined in a similar manner one week later, based upon the cash position and security level at that time and upon cash projections for the subsequent week. Decisions with respect to the level of cash and marketable securities would be made on a week-to-week basis.

The above model is but one of many that can be used to determine the optimal split between cash and marketable securities. In the appendix to this chapter, we examine several "inventory type" models. The model we use was meant to illustrate only some of the facets of the problem at hand. Many of the assumptions can be modified depending upon the circumstances. The purpose of this model and others is to determine the optimal investment of excess cash into marketable securities. These near-money investments earn a positive return and, at the same time, serve the liquidity needs of the company, for they may be sold on very short notice. The deterrents to investment are the transactions need for cash, including the need for compensating balances at banks, and the fixed and variable costs associated with making a transfer between the two types of assets. If cash projections are reasonably accurate, the transfer between cash and marketable securities can be highly efficient.

MARKETABLE SECURITIES

In the model, we assumed an average yield on marketable securities and no risk of fluctuations in market price. In this section, we explore the types of marketable securities available to a company as near-money investments, allowing for varying yields and for fluctuations in market price. Regardless of whether the decision to invest excess funds into marketable securities is made according to a probability model or strictly by managerial judgment, someone must decide what type of investment to make. This decision determines the composition of the firm's portfolio of marketable securities. Where formerly corporations invested primarily in short-term government securities, they now seek a more varied portfolio. By considering the full array of near-money investments available, the corporate treasurer is able to maximize portfolio income.

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To the extent that marketable securities are regarded as near moneys and serve the liquidity needs of the firm, however, companies are concerned with whether a security can be sold for approximately the amount paid for it. Available evidence suggests that for corporations as a whole, marketable securities serve as a reserve for unknown future investment outlays. Indeed, when corporations need funds, they reduce their holdings of marketable securities.[13] The ability to sell a security for approximately the amount paid depends upon its default risk, its marketability, and its maturity. These characteristics provide a useful framework for analyzing the investment of excess funds.

DEFAULT RISK

When we speak of default risk we mean the risk of default on the part of the borrower in the payment of principal or interest. Investors are said to demand a risk premium to invest in other than default-free securities.[14] The greater the possibility that the borrower will default on his obligation, the greater the financial risk and the premium demanded by the marketplace. Treasury securities are usually regarded as default-free, and other securities are judged in relation to them. For example, U.S. government agency issues might be rated next to Treasury securities in credit-worthiness. For all practical purposes, these securities are default-free. The credit-worthiness of other obligations is frequently judged on the basis of security ratings. Moody's Investors Service and Standard & Poor's, for example, grade corporate and municipal securities as to their quality. Unfortunately, these rating services do not rate money-market instruments. However, the latter securities are graded by certain large investors, dealers, and brokers. The greater the default risk of the borrower, the greater the yield of the security should be, all other things held constant. By investing in riskier securities, the firm can achieve higher returns; but it faces the familiar tradeoff between expected return and risk.

MARKETABILITY

Marketability of a security relates to the ability of the owner to convert it into cash. There are two dimensions: the price realized and the amount of time required to sell the asset. The two are interrelated in that it is often possible to sell an asset in a short period of time if enough price conces-

[13]For an excellent analysis of corporate holdings of government securities over the 1947–61 period, see Ernest Bloch, "Short Cycles in Corporate Demand for Government Securities and Cash," *American Economic Review*, LIII (December, 1963), 1958–77.

[14]For empirical investigations of default-risk premiums, see Lawrence Fisher, "Determinants of Risk Premiums on Corporate Bonds," *Journal of Political Economy*, LXVII (June, 1959), 217–37; and Ramon E. Johnson, "Term Structure of Corporate Bond Yields," *Journal of Finance*, XXII (May, 1967), 313–45. For a more extended discussion of default risk, see James C. Van Horne, *The Function and Analysis of Capital Market Rates* (Englewood Cliffs, N.J.: Prentice-Hall Inc., 1970), Chapter 5.

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sion is given. For financial instruments, marketability is judged in relation to the ability to sell a significant volume of securities in a short period of time without significant price concession. The more marketable the security, the greater the ability to execute a large transaction near the quoted price. In general, the lower the marketability of a security, the greater the yield necessary to attract investors. Thus, the yield differential between different securities of the same maturity is caused not only by differences in default risk but also by differences in marketability.

MATURITY

The maturity of a security is important from the standpoint of interest-rate risk, or risk associated with fluctuation in the value of principal on account of changes in the level of interest rates. Generally, the greater the maturity of a security, the more subject it is to fluctuations in principal value. To illustrate interest-rate risk, let us consider the period from May, 1967 to July, 1969 — a period during which there was a substantial increase in interest rates and drop in security prices. The shift in interest rates that occurred is illustrated in Figures 16-1 and 16-2, in which a yield curve for
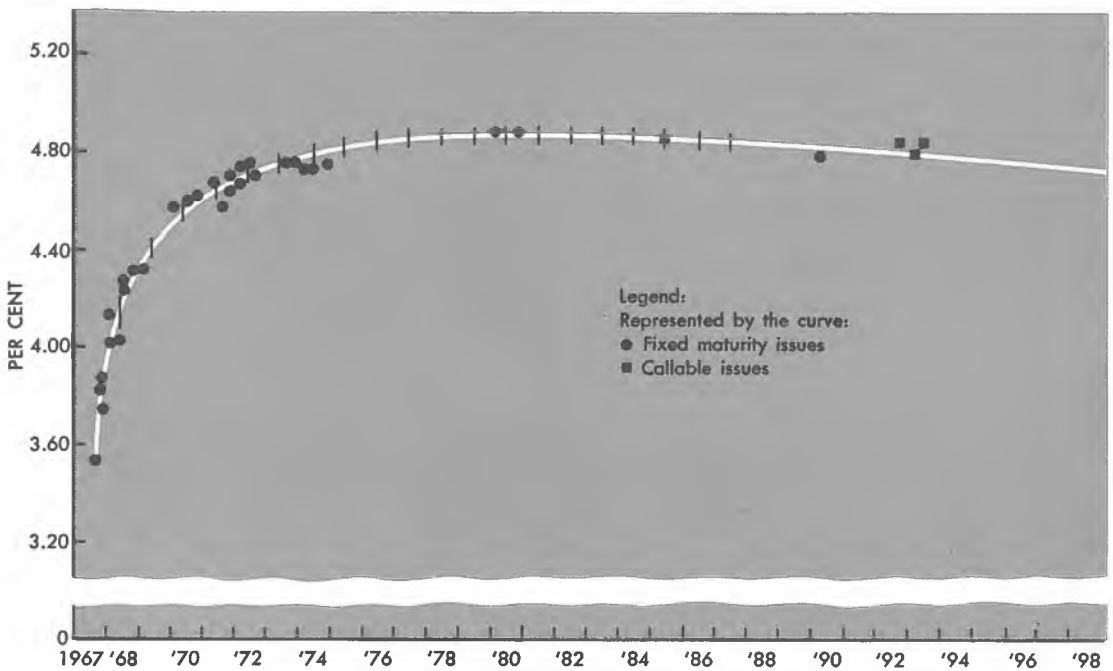
FIGURE 16-1
Yields of Treasury securities, May 31, 1967 (based on closing bid quotations). Source: *Treasury Bulletin* (June, 1967).
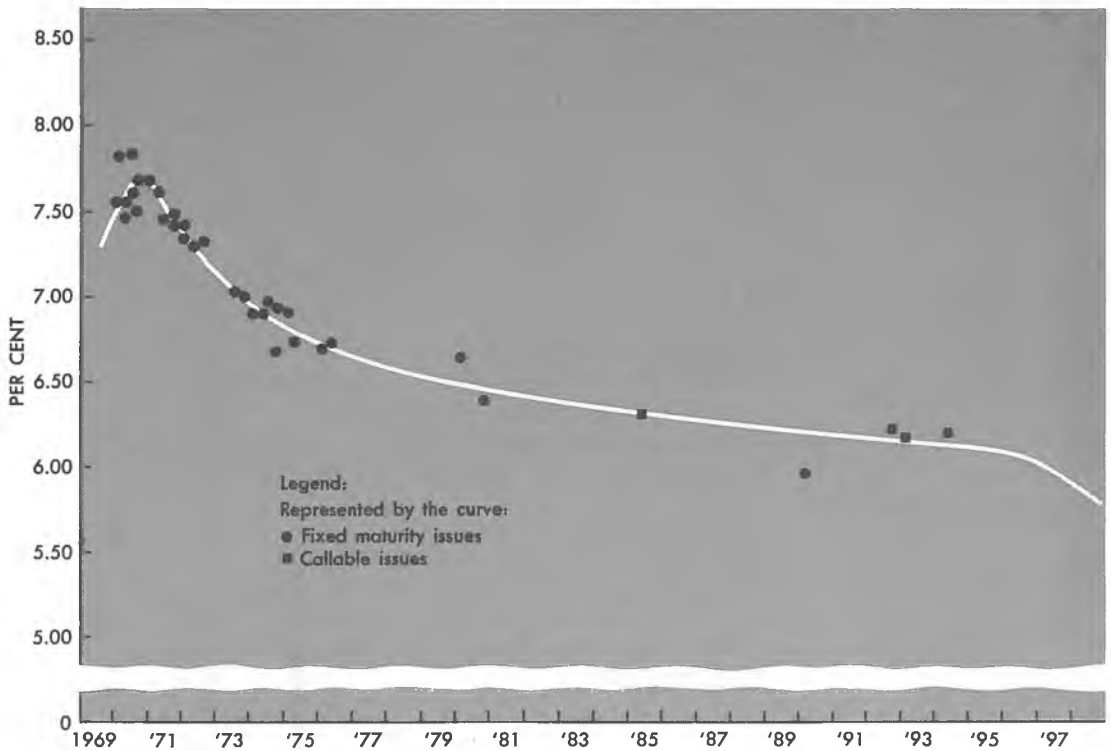
FIGURE 16-2
Yields of Treasury securities, July 31, 1969 (based on closing bid quotations). Source: *Treasury Bulletin* (August, 1969).

Treasury securities on May 31, 1967, is compared with one for July 31, 1969. A yield curve shows the relationship between yield and maturity on securities of the same default risk. In Table 16-3, we compare the market prices for specific Treasury securities of different maturity for the two dates. We see that in general the greater the maturity, the greater the loss in principal amount for the period.

Risk due to fluctuations in market price varies with the possibility that the firm will have to sell the security before maturity. When only a portion of the security position is subject to possible liquidation, the firm may be able to invest in somewhat longer-term securities and still obtain the necessary liquidity. If cash flows are relatively stable, investment policy may call for a fairly even spacing of maturities. Thus, a certain portion of securities would mature at even intervals. Under such a policy, the firm hopes to take advantage of liquidity premiums (higher yields due to lower liquid-

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ity) in the term structure of interest rates that reward the investor who is willing to invest in other than short-term securities.[15]

Some firms try to take advantage of an upward sloping yield curve by "riding the yield curve." "Riding the yield curve" involves selling securities before they mature. By investing in 180-day Treasury bills, for example, and selling them before maturity, a firm may be able to obtain a significantly higher return for the holding period than by holding the bills to maturity. If 180-day bills yield 6 per cent and 90-day bills 5 per cent, and the firm buys the 180-day bills now and sells them 90 days later at a yield of 5 per cent, its return per annum for the holding period is 7.2 per cent. This example assumes that the yield curve does not change over the holding period. Ignoring transaction costs, the investor buys the 180-day bills at a price of $97, at which price they yield 6 per cent to maturity, and sells them 90 days later for $98.75, where they yield 5 per cent to maturity. Thus, he realizes a holding-period yield of $(1.75/97)(360/90) = 7.2$ per cent.

TABLE 16-3
Bid prices of selected treasury bonds (May 30, 1967, and July 31, 1969)

			Bid Prices	
	Issue	May 31, 1967	July 31, 1969
4 %	10/1/69	$99^{16}/_{32}$	$99^{13}/_{32}$
4	8/15/70	$98^{8}/_{32}$	$96^{16}/_{32}$
4	8/15/72	$96^{26}/_{32}$	$91^{4}/_{32}$
$4^{1}/_{4}$	5/15/74	$97^{6}/_{32}$	$89^{12}/_{32}$
$4^{1}/_{4}$	5/15/75–85	$92^{28}/_{32}$	$80^{6}/_{32}$
$4^{1}/_{4}$	8/15/87–92	$91^{14}/_{32}$	76

Source: Treasury Bulletin *(June, 1967), p. 70; (August, 1969), p. 89.*

A second aspect of "riding the yield curve" involves taking advantage of expected shifts in interest rates. If interest rates are expected to fall, the firm will invest in long-term securities because these securities tend to have the greatest increase in market price when interest rates are falling. When the interest rate cycle is thought to have bottomed out, long-term securities will be sold and the firm will invest in very short-term securities, to assure a minimum of market-price decline if interest rates rise. Playing the yield curve in this manner involves risk, because expectations may prove to be wrong. Consequently, a firm that holds marketable securities as near-money investments to meet possible cash drains should be careful not to undertake such an operation on too large a scale.

[15]For an extended discussion of the term structure of interest rates, see Van Horne, *The Function and Analysis of Capital Market Rates,* Chapter 4.

TYPES OF MARKETABLE SECURITIES

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In this section, we describe very briefly certain important money-market instruments that serve the short-term investment needs of corporations.[16] Space prohibits the discussion of longer-term investments in this book, although corporations do invest excess funds in long-term securities that mature in the near future.

Treasury Securities. U.S. Treasury obligations constitute the largest segment of the money markets. The principal securities issued are bills, tax-anticipation bills, notes, and bonds. Treasury bills are auctioned weekly by the Treasury with maturities of 91 days and 182 days. In addition, one-year bills are sold periodically. Treasury bills carry no coupon but are sold on a discount basis. These securities are extremely popular with companies as short-term investments, in part because of the large amount outstanding. The market is very active, and the transaction costs involved in the sale of Treasury bills in the secondary market are small. The Treasury also offers, on occasion, tax-anticipation bills. These bills usually mature about one week after the quarterly tax-due dates. Their advantage is that they may be used to pay taxes on those dates at full face value, thereby giving the holder about one week's extra interest. These bills are offered in the same manner as Treasury bills and have the same characteristics.

The original maturity on Treasury notes is one to seven years, whereas the original maturity on Treasury bonds is over seven years. With the passage of time, of course, a number of these securities have maturities of less than one year and serve the needs of short-term investors. Notes and bonds are coupon issues, and there is an active market for them. Overall, Treasury securities are the safest and most marketable investments. Therefore, they provide the lowest yield for a given maturity of the various instruments we consider.

Federal Agency Issues. Obligations of various agencies of the federal government are guaranteed by the agency issuing the security and not usually by the U.S. government as such. Principal agencies issuing securities are the Federal Land Banks, the Federal Home Loan Banks, the Federal Intermediate Credit Banks, the Federal National Mortgage Association (Fannie Mae), and the Banks for Cooperatives. Although agency issues are being increasingly accepted by the investment community, they still provide a yield advantage over Treasury securities of the same maturity. These securities have a fairly high degree of marketability; they are sold in the secondary market through the same security dealers as are Treasury securities. With the sharp increase in agency

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[16]For a more detailed discussion of these and other instruments, see *Instruments of the Money Market* (Richmond, Va.: Federal Reserve Bank of Richmond, 1968).

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financing in recent years, marketability has been enhanced considerably. Maturities range from a month up to approximately fifteen years. However, about two-thirds of the securities outstanding mature in less than a year.

Bankers' Acceptances. Bankers' acceptances are drafts that are accepted by banks, and they are used in the financing of foreign and domestic trade. The credit worthiness of bankers' acceptances is judged relative to the bank accepting the draft, not the drawer. Acceptances generally have maturities of less than 180 days and are of very high quality. They are traded in an over-the-counter market dominated by five principal dealers. The rates on bankers' acceptances tend to be slightly higher than rates on Treasury bills of like maturity; and both are sold on a discount basis.

Commercial Paper. Commercial paper consists of short-term unsecured promissory notes issued by finance companies and certain industrial concerns. Commercial paper can be sold either directly or through dealers. Several large sales finance companies have found it profitable, because of the volume, to sell their paper directly to investors, thus bypassing dealers. Among companies selling paper on this basis are Associates Investment Company, C.I.T. Financial Corporation, Commercial Credit Company, Ford Motor Credit Company, and General Motors Acceptance Corporation (GMAC).

Paper sold through dealers is issued by industrial companies and smaller finance companies. The dealer organization for commercial paper is dominated by three firms. Overall, the total volume of paper sold through dealers is considerably less than the total volume sold directly. Dealers screen potential issuers very carefully as to their creditworthiness. In a sense, the dealer stands behind the paper he places with investors.

Rates on commercial paper are somewhat higher than rates on Treasury bills of the same maturity and about the same as the rates available on bankers' acceptances. Paper sold directly, however, generally commands a lower yield than paper sold through dealers. Usually, commercial paper is sold on a discount basis, and maturities generally range from 30 to 270 days. Most paper is held to maturity, for there is essentially no secondary market. However, direct sellers of commercial paper will often repurchase the paper on request. Arrangements may also be made through dealers for repurchase of paper sold through them. Commercial paper is sold only in large denominations, usually of at least $25,000.

Repurchase Agreements. In an effort to tap new sources of financing, government security dealers recently have offered repurchase agree-

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ments to corporations. The repurchase agreement, or "repo" as it is called, is the sale of short-term securities by the dealer to the investor whereby the dealer agrees to repurchase the securities at a specified future time. The investor receives a given yield while he holds the security. The length of the holding period itself is tailored to the needs of the investor. Thus, repurchase agreements give the investor a great deal of flexibility with respect to maturity. Rates on repurchase agreements are related to the rates on Treasury bills, federal funds, and loans to government security dealers by commercial banks.[17]

Negotiable Certificates of Deposit. Negotiable time certificates of deposit are a short-term investment that originated in 1961. The certificate (CD) is evidence of the deposit of funds at a commercial bank for a specified period of time and at a specified rate of interest. Money-market banks quote rates on CD's; these rates are changed periodically in keeping with changes in other money-market rates. The maximum rate that banks are allowed to pay, however, is regulated by the Federal Reserve System under Regulation Q. At the end of 1970, the maximum rate on CD's was 7.5 per cent.

Original maturites of CD's generally range from 30 to 360 days. A good secondary market has developed for the CD's of the large money-market banks, so CD's are reasonably marketable. When money-market rates move significantly above the maximum rate allowed, CD's offered in the primary market are no longer competitive with other money-market instruments. Rates on CD's in the secondary market, however, are not limited by Regulation Q; and, consequently, market prices can decline so that yields rise above the ceiling. Regulation Q has been raised several times in the past by the Federal Reserve, enabling rates on primary offerings to remain competitive. However, in 1969, the Federal Reserve did not raise Regulation Q in keeping with the increase in money market rates. As a result, primary offerings were not competitive.

PORTFOLIO MANAGEMENT

The decision to invest excess cash in marketable securities involves not only the amount to invest but also the type of security in which to invest. To some extent, the two decisions are interdependent. Both should be based upon an evaluation of expected net cash flows and the certainty of these cash flows. If future cash-flow patterns are known with reasonable certainty, the portfolio may be arranged so that securities will be maturing on approximately the dates when the funds will be needed. Such a cash-flow pattern gives the firm a great deal of flexibility

[17] "Repurchase Agreements," *Money Market Instruments,* Federal Reserve Bank of Cleveland, 1970, pp. 42–55.

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in maximizing the average return on the entire portfolio, for it is unlikely that significant amounts of securities will have to be sold unexpectedly.

The larger the security portfolio, the more chance there is for specialization and economies of operation. A large enough security portfolio may justify a staff whose sole responsibility is managing the portfolio. Such a staff can undertake research, plan diversification, keep abreast of market conditions, and continually analyze and improve the firm's position. When investment is made a specialized function of the firm, the number of different securities considered for investment is likely to be diverse. Moreover, continual effort can be devoted to achieving the highest yield possible in keeping with the cash needs of the firm. Trading techniques in such a firm tend to be very sophisticated. For companies with smaller security positions, however, there may be no economic justification for a staff. Indeed, a single individual may handle investments on a part-time basis. For this type of company, the diversity of securities in the portfolio will probably be limited.

While diversification of the short-term marketable security portfolio of a firm might be desirable, there is far less opportunity for such diversification than there is with a portfolio of common stocks.[18] Diversification usually is defined as the reduction of the dispersion of possible returns from a portfolio relative to the expected return from the portfolio. This reduction is achieved by investing in securities not having high degrees of covariance among themselves. Unfortunately, there is a high degree of correlation in the price movements of money-market instruments over time. Consequently, they are ill-suited for purposes of diversification. As a result, the objective of most firms is to maximize overall return subject to maintaining sufficient liquidity to meet cash drains. Because of differences in the ability to diversify, the management of a firm's marketable security portfolio differs considerably from the management of a portfolio of common stocks.

SUMMARY

The firm has three motives for holding liquid assets: the transactions motive; the precautionary motive; and the speculative motive. Our concern is with only the first two of these motives. The amount of transactions balances and precautionary balances is determined by the expected cash flows of the firm, possible deviations in these cash flows, the maturity structure of the firm's debt, its borrowing capacity, the utility preferences of management, and the efficiency of cash management.

In the management of cash, we should attempt to accelerate collections and handle disbursements so that a maximum of cash is available.

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[18] See Chapter 2 for a review of portfolio analysis and selection.

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Collections can be accelerated by means of concentration banking, a lock-box system, and certain other procedures. Disbursements should be handled so as to give maximum transfer flexibility and the optimum timing of payments. A portion of the transactions and precautionary balances of the firm may be invested in marketable securities. A model was developed for determining the optimal split between cash and market-able securities. The model embodied the properties of fixed and variable costs associated with making a transfer between cash and marketable securities, the estimated cash flows of the firm, the dispersion of the probability distribution of cash flows, and the rate of return on the securities.

There are a number of marketable securities in which the firm can invest. These securities can be evaluated in relation to their default risk, marketability, and their maturity. Depending upon the cash-flow pattern of the firm and other considerations, a portfolio can be selected in keeping with these three characteristics. Specific securities considered included Treasury securities, government agency securities, bankers' accept-ances, commercial paper, repurchase agreements, and certificates of deposit. It was shown that the management of a firm's portfolio of mar-ketable securities is considerably different from the management of a portfolio of common stocks.

Various lot-size models for the management of inventory have been applied to the management of cash and marketable securities.[19] The pur-pose of this appendix is to examine some of these models, for they pro-vide additional insight into the problem at hand.

BAUMOL MODEL

One of the first inventory models for managing cash was that of Baumol.[20] His model assumes that an individual or firm has to pay out cash in a steady transactions stream, totaling T dollars, over a period of time. Baumol assumes also that the firm or individual obtains cash either by borrowing it or by withdrawing it from investment, both of which alternatives have an opportuniy interest rate of i per dollar per period. Cash is borrowed or withdrawn instantaneously in lots of C dollars spaced evenly throughout the period. For each borrowing or withdrawal, there is a fixed cost of b dollars, which is independent of the amount

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[19] For a lengthier discussion of the models as they apply to inventory, see Chapter 18.

[20] William J. Baumol, "The Transactions Demand for Cash: an Inventory Theoretic Approach," *Quarterly Journal of Economics*, LXV (November, 1952), 545–56.

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transferred. The purpose of the model is to determine the optimal borrowing or withdrawal lot size, C.

The total cost associated with transactions demand during the period is

$$\frac{bT}{C} + \frac{iC}{2} \tag{16A-1}$$

where the first term represents total fixed costs associated with T/C borrowings or withdrawals spaced evenly over the period, and the second term is the opportunity cost of maintaining an average cash balance of $C/2$. It is assumed that each time the firm borrows or withdraws C dollars, it spends this amount in a steady stream and borrows or withdraws another C dollars when it is gone. The optimal level of C is found to be

$$C^* = \sqrt{\frac{2bT}{i}} \tag{16A-2}$$

when the derivative of Eq. (16A-1) with respect to C is set equal to zero. Thus, cash will be demanded in relation to the square root of the dollar volume of transactions. The higher the fixed cost of transfer, b, the higher the optimal borrowing or withdrawal size, C, all other things held constant. The higher the opportunity cost of funds, i, however, the lower the cash balance that is desirable and the lower the optimal borrowing or withdrawal size, C, all other things held constant.

Baumol offers his model as a crude, but useful, simplification of reality and fully recognizes certain limitations. The critical assumptions are that disbursements over time are given and steady; that there are no cash receipts other than borrowings or withdrawals during the period studied; that the opportunity cost of funds tied up in cash is constant; that the fixed cost per borrowing or withdrawal also is constant; and that only the transaction demand for cash is considered, the precautionary and speculative demands being ignored.[21] The Baumol model can be applied also when receipts are continuous and there are discontinuous large payments. The decision to be made then would be the optimal investment lot size between payments, not the optimal borrowing or withdrawal size.[22]

Tobin, in a later article, developed a model very similar to Baumol's but with certain modifications.[23] Tobin was interested in the interest elasticity of demand for cash at a given volume of transactions. He suggests that the firm or individual holds transactions balances in either cash — a nonearning asset — or in bonds — an earning asset. Tobin's model,

[21] *Ibid.*, p. 553.

[22] *Ibid.*, p. 549.

[23] James Tobin, "The Interest Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics*, XXXVII (August, 1956), 241–47.

therefore, involves determination of the optimal average cash and bond holdings. As before, this decision is based upon the yield available on bonds and the cost of a transfer between cash and bonds. However, part of the cost of a transfer is assumed to be independent of the size of the transaction, while the other part is proportional.[24]

A MODEL ILLUSTRATED

It is useful to express the Baumol model graphically for purposes of further discussion. We assume that total expenditures of T over the period are steady at a constant rate of m dollars per day; m is simply T divided by the number of days in the period. A constant transfer cost, b, is assumed; and we assume also a constant return on investment of i per dollar per day. No receipts other than those arising from the transfer of securities are assumed. The firm can vary its earnings on securities by varying the size of the transfer from securities to cash, C. The larger the C, the larger the average cash balance, $C/2$, and the smaller the average investment in securities and earnings from these securities. The smaller the C, the smaller the average cash balance and the larger the earnings from securities. However, the smaller the C, the larger the transfer costs, for more transfers, T/C, occur. These concepts can be visualized in Figure 16-3. The optimal size of transfer, C^*, is[25]

$$C^* = \sqrt{\frac{2bm}{i}} \qquad \text{(16A-3)}$$

FIGURE 16-3
Inventory model
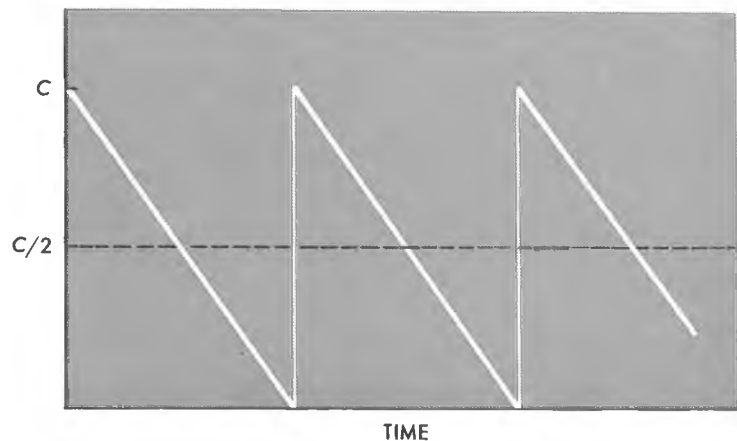
[24] For a discussion of other specific differences between the Baumol and Tobin models, see Tobin, "The Interest Elasticity," 247.

[25] This equation differs from Eq. (16A-2) in that total expenditures and the return on investment are expressed on a daily basis. Both equations give the same optimal size of transfer, C^*.

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MILLER AND ORR MODEL

Miller and Orr observe that the cash-balance patterns of business firms are much more complex than those shown in Figure 16-3.[26] They suggest that cash balances fluctuate irregularly over time as a result of fluctuations in operating receipts and disbursements. To take into account these irregular cash flows, they develop an interesting model. As in the above example, they assume that there are two assets — cash and marketable securities — and represent the marginal and average yield on the securities as i per dollar per day. They also assume a constant transfer cost, b, and no delays in transfer. Most importantly, Miller and Orr assume that net cash flows are completely stochastic and that their random behavior can be characterized as a sequence of independent Bernoulli trials. This assumption is critical to their analysis and distinguishes their model from that of Baumol.

They propose that "the cash balance [be] allowed to wander freely until it reaches either the lower bound, zero, or an upper bound, h, at which time a portfolio transfer will be undertaken to restore the balance to the level of z."[27] Graphically, the control limits are shown in Figure 16-4. When cash balances touch the upper bound, h, $h - z$ dollars of
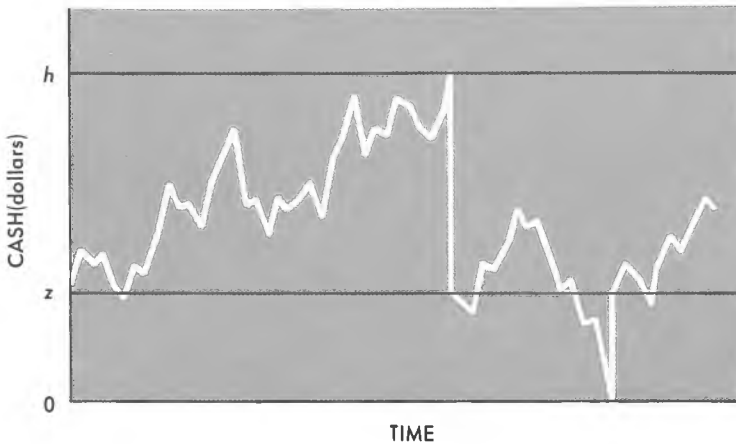
FIGURE 16-4
Source: Miller and Orr, *op. cit.*, p. 420.

cash are transferred to securities. When a cash balance of zero is touched, z dollars worth of securities are sold and transferred to cash. The objective, in their model, is to maximize earnings on the portfolio by finding optimal levels of h and z. Optimal values are found to be

[26] Merton H. Miller and Daniel Orr, "A Model of the Demand for Money by Firms," *Quarterly Journal of Economics*, LXXX (August, 1966), 413–35.

[27] *Ibid.*, p. 419.

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$$z^* = \sqrt[3]{\frac{3bm^2t}{4i}} \qquad (16A\text{-}4)$$

where t is the number of operating cash transactions per day, and

$$h^* = 3z^* \qquad (16A\text{-}5)$$

Although the model has many interesting implications,[28] the assumption of random changes in cash balances is not particularly realistic. For many firms, near-term cash flows are highly predictable. To overlook their predictability and treat all cash flows as random is likely to result in cash-management policies that are less than optimal. Although Miller and Orr do extend their model to allow for systematic drift in cash balances, this extension is cumbersome.[29]

In a second article, the authors extend the model to allow for two types of securities in which investment can be made — long-term and short-term.[30] By breaking down earning assets into two distinct types, Miller and Orr allow for differences in return and for differences in transaction costs among market instruments. On the supposition that the control of cash has little effect on the choice between long- and short-term securities, they separate the portfolio problem from the cash control problem presented in their original model. In other words, the amount of funds to be transferred between cash and marketable securities is determined independently of the mix of securities.

Miller and Orr assume that long-term securities are riskier than short-term ones and, accordingly, provide a higher return. They assume also that transactions costs are higher. Portfolio action is assumed to be of the same control limit type as that used in Figure 16-4. Transfers between cash and marketable securities occur in short-term securities unless certain control limits are breeched. If cash balances exceed the upper limit, h, and a transfer to short-term securities exceeds the upper control limit, H, for these securities, long-term securities should be purchased. More specifically, a cash transfer of $h - z$ should be used to purchase long-terms. Moreover, the firm also should sell $X - Z$ of short-term securities and purchase an equal amount of long-terms, where X is the actual level of short holdings and Z is the lower control limit for short holdings. If cash balances equal zero and actual short holdings are less than the lower cash holding control limit, z, short holdings are deemed excessively low. The firm then should sell $z + Z - X$ worth of long-term securities and buy $Z - X$ worth of shorts. The residual, z, would go into cash. All other transactions between cash and marketable securities involve short-term securities.

[28] *Ibid.*, pp. 423–27.

[29] *Ibid.*, pp. 427–29.

[30] Merton H. Miller and Daniel Orr, "The Demand for Money by Firms: Extension of Analytic Results," *Journal of Finance*, XXIII (December, 1968), 735–59.

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"The frequency of trade in longs is determined entirely by the values assigned to H and Z, the control levels on the shorts accounts, and is independent of h and z, the control levels on the cash account."[31] Thus, cash transfers occur in the same manner as with the two-asset model. Optimal values of the policy variables are found to be

$$z = \sqrt[3]{\frac{3b_sm^2t}{4i}} \qquad Z = \sqrt{\frac{3b_lm^2t}{4(i_l - i_s)}} \qquad (16A\text{-}6)$$

where b_s and b_l are transaction costs for trades in short- and long-term securities, respectively, and i_s and i_l are average yields on short- and long-term securities. As before,

$$h = 3z \qquad H = 3Z \qquad (16A\text{-}7)$$

Because the numerator for Z is larger than that for z and because the denominator is smaller, the frequency of transactions involving long-term securities is less than that for transactions in short-term securities. The policy variables are seen to be affected importantly by differences in average return and transaction costs between long- and short-term securities. According to Miller and Orr, the appropriate opportunity cost for holding cash is the greatest earnings opportunity foregone, which is assumed to be the long-term interest rate.

PROBLEMS

1. The Zindler Company currently has a centralized billing system. Payments are made by all customers to the central billing location. It requires, on the average, four days for customers' mailed payments to reach the central location. Further, an additional one and one-half days are required to process payments before a deposit can be made. The firm has a daily average collection of $500,000.

The company has recently considered the possibility of initiating a lock-box system. It has been estimated that such a system would reduce the time required for customers' mailed payments to reach the receipt location by two and one-half days. Further, the processing time could be reduced by an additional day, since each lock-box bank would pick up mailed deposits twice daily.

    (a) Determine the reduction in cash balances that can be achieved through the use of a lock-box system.

    (b) Determine the opportunity cost of the present system, assuming a 5 per cent return on short-term instruments.

    (c) If the annual cost of the lock-box system will be $75,000, should such a system be initiated?

2. The List Company, which can earn 7 per cent on money-market instruments, currently has a lock-box arrangement with a New Orleans bank for its southern customers. The bank handles $3 million a day in return for a compensating balance of $2 million.

[31] *Ibid.,* p. 750.

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(a) The List Company has discovered that it could divide the southern region into a southwestern region (with $1 million a day in collections, which could be handled by a Dallas bank for a $1 million compensating balance) and a southeastern region (with $2 million a day in collections, which could be handled by an Atlanta bank for a $2 million compensating balance). In each case, collections would be one-half day quicker than with the New Orleans arrangement. What would be the annual savings (or cost) of dividing the southern region?

(b) In an effort to retain the business, the New Orleans bank has offered to handle the collections strictly on a fee basis (no compensating balance). What would be the maximum fee the New Orleans bank could charge and still retain List's business?

3. The Excelsior Manufacturing Company follows a policy of determining its cash needs on a weekly basis and investing surplus cash in Treasury bills. The firm must maintain a minimum cash balance of $10,000,000 throughout the week. It can obtain 0.10 per cent per week on Treasury bills, though a commission of 0.02 per cent must be paid on each purchase and each sale. A fixed cost of $100 per transaction is also incurred. The firm can purchase or sell bills in minimum lots of $500,000.

As of March 7, the firm had $10,000,000 in cash and $7,000,000 in Treasury bills. The cash budget for the week ending March 14 indicates the following net receipt distribution:

Probability	Amount
.10	−$1,000,000
.20	− 500,000
.30	0
.20	+ 500,000
.20	+ 1,000,000

Assuming that the firm will not borrow to maintain the minimum level of cash balances as long as it has marketable securities on hand:

(a) Determine the optimum level of security holdings for the week.

(b) Why do you get the results you obtain? Do you suspect that the optimum level might not be attainable in this situation?

(c) How much expected net income will be earned on the security holdings?

Note: It is assumed that the sale of securities during the week will result in a loss of one-half the interest that would be received if the securities were held for the entire week.

4. Assume the initial position of the Excelsior Manufacturing Company on March 7. Further, assume that all conditions outlined in problem 3 prevail except the following:

(1) Treasury bills yield 0.08 per cent per week.

(2) Commissions are 0.03 per cent per transaction.

(3) The sale of securities during the week will result in a 75 per cent loss of the interest that would be received if the securities were held for the entire week.

(a) Determine the optimum level of security holdings for the week.

(b) Should bills be purchased or sold? In what quantity?

(c) How much expected net income will be earned on the security holdings?

(d) How much expected net income would be earned if an amount $500,000 below the optimum were held? An amount $500,000 above the optimum?

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5. The James Company has excess cash to invest for thirty days; it has narrowed the choice of money-market instruments to Treasury bills and commercial paper. Because of the greater risk associated with commercial paper, the expected return is higher but the dispersion of possible returns is also greater than in the case of Treasury bills. It is assumed that the returns on each instrument are normally distributed and perfectly positively correlated with each other (e.g., if the bill return is $\mu_{TB} + 1\sigma_{TB}$, the paper return will be $\mu_{CP} + 1\sigma_{CP}$).

(a) If $\mu_{TB} = 6$ per cent and $\sigma_{TB} = 0.25$ per cent, while $\mu_{CP} = 6.5$ per cent and $T_{CP} = 0.75$ per cent (all expressed on an annual basis), what is the probability that Treasury bills will offer a higher effective yield over the period?

(b) If the James Company has decided not to invest in commercial paper unless it will offer a superior yield to Treasury bills 90 per cent of the time, what should the company do?

(c) Rework parts (a) and (b), assuming $\sigma_{CP} = 0.5$ per cent.

6. *Research Project*

Examine quotations in the *Wall Street Journal* for each of the following money-market instruments:

(a) Treasury bills
(b) Bankers' acceptances
(c) Certificates of deposit
(d) Commercial paper, and
(e) Government agency issues.

Evaluate the yield-risk tradeoff for each instrument. Consider the appropriateness of each of these securities for the corporation's short-term investment account.

APPENDIX

PROBLEMS

1. The Schriver Company plans to have $1,000,000 in cash outlays for next year. The firm believes that it will face an opportunity interest rate of 5 per cent and will incur a cost of $100 each time it borrows (or withdraws). Cash outlays are expected to be steady over the year. Using the Baumol model:

(a) Determine the transactions demand for cash (the optimal borrowing or withdrawal lot size) for the Schriver Company.

(b) What is the total cost for the use of cash needed for transactions demand?

(c) What will be the cash cycle for the firm (velocity)?

(d) What would be the average cash balance for the firm?

2. Assume that the Schriver Company (problem 1) began the year with $1,000,000 in cash.

(a) How much would initially be invested in securities?

(b) How much would be invested in securities after 231 days?

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Accounts receivable represent the extension of open-account credit by one firm to other firms and to individuals. For many companies, accounts receivable are an extremely important investment and require careful analysis. The purpose of this chapter is to examine this vital function and the means by which receivables can be managed efficiently so that the level of investment in them is optimal. We consider first the credit and collection policies of the firm as a whole and then discuss credit and collection procedures for the individual account.

Actually, the two facets are closely related. For example, credit policy involves a tradeoff between the profits on sales that give rise to receivables on one hand and the cost of carrying these receivables plus bad-debt losses on the other. Credit analysis is instrumental in determining the amount of credit risk to be accepted. In turn, the amount of risk accepted affects the slowness of receivables, and the resulting investment in receivables, as well as the amount of bad-debt losses. Col-

lection procedures also affect these factors. Thus, the credit and collection procedures of the firm are essential to the firm's overall credit and collection policies. For expository purposes, however, it is useful to consider the two facets separately, keeping in mind their interrelationship.

CREDIT AND COLLECTION POLICIES

In this section, we examine credit and collection objectives and policies in relation to the demand for the firm's product, the investment in receivables, and the default risk on accounts. The policy variables we consider include the quality of the trade accounts accepted, the length of the credit period, the cash discount given, any special terms given, such as seasonal datings, and the collection program of the firm. Together, these elements largely determine the average collection period and the proportion of bad-debt losses. We analyze each element in turn, holding constant certain of the others as well as all exogenous variables that affect the average collection period and the percentage of bad-debt losses. In addition, we assume that the evaluation of risk is sufficiently standardized that degrees of risk for different accounts can be compared objectively.

CREDIT STANDARDS

Credit policy can have a significant influence upon sales. If competitors extend credit liberally and we do not, our policy may have a dampening effect upon the marketing effort. Trade credit is one of many factors that influence the demand for a firm's product. Consequently, the degree to which trade credit can promote demand depends upon what other factors are being employed. In theory, the firm should lower its quality standard for accounts accepted as long as the profitability of sales generated exceeds the added costs of the receivables. What are the costs of relaxing credit standards? One type of cost is the enlarged credit department and the clerical expenses involved in checking additional accounts and servicing the added volume of receivables. We assume for now that these costs are deducted from the profitability of additional sales to give a net profitability figure for computational purposes. Another cost comes from the increased probability of bad-debt losses. However, we postpone consideration of this cost to a subsequent section; we assume for now that there are no bad-debt losses.

Finally, there is the cost of the additional investment in receivables, resulting from (1) increased sales, and (2) a slower average collection period. If new customers are attracted by the relaxed credit standards, collecting from these customers is likely to be slower than collecting

from existing customers. In addition, a more liberal extension of credit may cause certain existing customers to be less conscientious in paying their bills on time. Those who decide credit policy must consider this possibility.

To determine the profitability of a more liberal extension of credit, we must know the profitability of additional sales; the added demand for products arising from the relaxed credit standards; the increased slowness of the average collection period; and the required return on investment. Suppose a firm's product sells for $10 a unit, of which $7 represents variable costs before taxes, including credit department costs. Current annual sales are $2.4 million, represented entirely by credit sales, and the average total cost per unit at that volume is $9 before taxes. The firm is considering a more liberal extension of credit, which will result in a slowing in the average collection period from one to two months. This relaxation in credit standards is expected to produce a 25 per cent increase in sales, to $3 million annually.[1] With this percentage increase, the unit sales and total costs of the firm become

Present sales × average cost	240,000 units × $9 =	$2,160,000
Additional sales × marginal cost	60,000 units × $7 =	420,000
New sales and total cost	300,000 units	$2,580,000

The average cost per unit of sale at the new level of sales is

$$\frac{\$2,580,000}{300,000} = \$8.60 \text{ per unit}$$

Assume that the firm's required return on investment is 20 per cent before taxes.[2]

Given this information, our evaluation is reduced to a tradeoff between the added profitability on the additional sales and the required return on the additional investment in receivables; and we are able to make the calculations shown in Table 17-1. Inasmuch as the profitability on additional sales, $180,000, exceeds the required return on the addi-

[1] In estimating the effect of a change in credit policy on demand, it is important to take into account the reaction of competitors to this change. Their reaction will affect demand over the long run.

[2] This required rate of return may be the cost of capital. See Chapter 4. There is some question whether all projects should require the same return. For example, should a liquid investment such as investment in receivables require the same return as an investment in fixed assets? In determining the required rate of return for investment in receivables, the firm should take account of the impact of an investment in receivables on the risk of the firm as perceived by creditors and investors. To the extent that an investment in receivables results in lesser perceived risk than an investment in other types of assets, the required rate of return should be less than the firm's cost of capital. For further discussion of this point, see the last section of this chapter, which deals with captive finance companies.

tional investment, $50,000, the firm would be well advised to relax its credit standards. An optimal credit policy would involve extending trade credit more liberally until the marginal profitability on additional sales equals the required return on the additional investment in receivables necessary to generate those sales.

TABLE 17-1
Profitability versus required return

Profitability of additional sales	=	$3 × 60,000 units = $180,000
Present average investment in receivables	=	(Annual Sales/Receivable Turnover) (Average Cost per Unit/ Selling Price per Unit) = ($2.4 million/12)($9/$10) = $180,000
Average investment in receivables after change in credit policy	=	($3.0 million/6)($8.60/$10) = $430,000
Additional investment in Receivables	=	$430,000 − $180,000 = $250,000
Required return on additional investment	=	0.20 × $250,000 = $50,000

Obviously, there are many practical problems in implementing such a policy, particularly in estimating the outcomes. In our example, we have worked with only the expected values of additional demand and of the slowing of the average collection period. It is possible, and desirable, to attach probability distributions to the increased demand and to the increased slowness in receivables and evaluate a range of possible outcomes. For simplicity of discussion, however, we shall not incorporate these dimensions into our example.

CREDIT TERMS

Credit Period. Credit terms involve both the length of the credit period and the discount given. The terms "2/10, net 30" mean that a 2 per cent discount is given if the bill is paid before the tenth day after the date of invoice; payment is due by the thirtieth day. The credit period, then, is thirty days. Although the customs of the industry frequently dictate the terms given, the credit period is another means by which a firm may be able to affect product demand — hoping to increase demand by extending the credit period. As before, the tradeoff is between the profitability of additional sales and the required return on the additional investment in receivables. Assume for the purpose of illustration that our example involves lengthening the credit period from thirty to sixty days instead of relaxing credit standards. Assume also that by lengthening the credit period, the firm expects sales to increase by 25 per cent, and the average collection period to increase from one to two months.

As the quality of account being accepted is the same, we assume that there is no change in bad-debt losses—in other words, that there are no losses. The analysis is the same as in Table 17-1. As seen, such a policy would be advantageous to the company because the profitability on additional sales—$180,000—exceeds the required return on the additional investment in receivables—$50,000.

Discount Given and The Discount Period. Varying the discount involves an attempt to speed up the payment of receivables. To be sure, the discount also may have an effect upon demand and upon bad-debt losses. However, we assume that the discount offered is not regarded as a means of cutting price and thereby affecting demand, and that the discount offered does not affect the amount of bad-debt losses. Holding constant these factors, we must determine whether a speedup in collections would more than offset the cost of an increase in the discount. If it would, the present discount policy should be changed.

Suppose, for example, that the firm has annual credit sales of $3 million and an average collection period of two months, and that the sales terms are net 45 days, with no discount given. Assume further that the annual turnover of receivables is six times. Consequently, the average receivable balance is $500,000. Now, suppose that by instigating terms of 2/10, net 45, the average collection period is reduced to one month and that 50 per cent of the customers (in dollar volume) take advantage of the 2 per cent discount. The opportunity cost of the discount to the firm is $.02 \times 0.5 \times \$3$ million, or $30,000 annually. However, the turnover of receivables has improved to twelve times a year, so that average receivables are reduced from $500,000 to $250,000. If the average cost per unit is $8.60 and the selling price $10 per unit, there is a $215,000 reduction in the investment in receivables. If the required return on investment is 20 per cent, this reduction represents an opportunity savings of $43,000.[3] In this case, the opportunity savings arising from a speedup in collections is greater than the cost of the discount. Consequently, the firm should adopt a 2 per cent discount. If the speedup in collections had not resulted in sufficient opportunity savings to offset the cost of discount, the discount policy should not be changed. It is possible, of course, that discounts other than 2 per cent may result in an even greater difference between the opportunity savings and the cost of the discount.

In addition to the size of the discount offered, the length of the discount period also may affect the average collection period. Here, the effect is not as clear as before. When a firm lengthens the discount period, two forces influence the average collection period. If the credit period is held constant, certain customers will be tempted to take the discount

[3] We do not consider the effect that the reduction of liquid assets may have on the required rate of return.

where previously they did not do so. This practice will tend to shorten the average collection period. On the other hand, customers who have been taking the discount and paying at the end of the discount period now will postpone payment until the end of the new discount period, thereby lengthening the average collection period. Whether the first force dominates the second will depend upon the mix of payment habits of the firm's customers. Given estimates of the likely effect of a change in the discount period on the average collection period, the firm can balance this effect with the increased dollar cost associated with more customers taking the discount.

Seasonal Datings. Another aspect of the credit terms given to customers involves seasonal datings. During periods of slack sales, firms will sometimes sell to customers without requiring payment for some time to come. This extension of the credit period is known as seasonal dating. Seasonal datings may be employed to stimulate demand from customers who cannot pay until later in the season. Because datings can be tailored to the cash flow of the customer, they can play an important role in selling the goods. Again, we should compare the profitability of additional sales with the required return on the additional investment in receivables to determine whether datings are an appropriate means by which to stimulate demand.

Datings also can be used to avoid inventory carrying costs. If sales are seasonal and production steady throughout the year, there will be buildups in finished goods inventory during certain times of the year. Storage involves warehousing costs; therefore, it may be profitable to give datings in order to move the goods and avoid these costs. If warehousing costs plus the required rate of return on investment in inventory exceed the required rate of return on the additional investment in receivables, it is worthwhile to give datings.

DEFAULT RISK

In the above examples, we assumed no bad-debt losses. Our concern in this section is not only with the slowness of collection but also with the portion of the receivables defaulting. Different credit policies will involve both of these factors. Suppose that we are considering the present credit policy in relation to two new ones and that these policies are expected to produce the following results.

	Present Policy	Policy A	Policy B
Additional demand (percentage)	0	25	35
Average collection period	1 month	2 months	3 months
Percentage of default losses	1	3	6

We assume that after six months an account is turned over to a collection agency and that, on the average, 1 per cent of the total receivable volume under the present credit policy is never received by the firm, 3 per cent is never received under policy *A*, and 6 per cent is never received under policy *B*.

From our previous example, we know that with a 25 per cent increase in sales to $3 million the average cost per unit is $8.60. With a 35 per cent increase in sales to $3,240,000, unit sales and total costs become

Present sales × average cost	240,000 units × $9 =	$2,160,000
Additional sales × marginal cost	84,000 units × $7 =	588,000
New sales and total cost	324,000 units	$2,748,000

The average cost per unit of sale with a sales volume of 324,000 units is

$$\frac{\$2,748,000}{324,000} = \$8.48 \text{ per unit}$$

If we go through the same type of calculations as in the previous example, we obtain the results shown in Table 17-2 for policies *A* and *B*. The profitability of the two credit policies in relation to the required return on investment can be summarized as follows.

	Policy A	Policy B
Profitability of additional sales less cost of additional bad-debt losses	$133,800	$132,720
Required return on additional investment	50,000	101,376
	$ 83,800	$ 31,344

Consequently, we would want to adopt policy *A* but would not want to go so far in relaxing our credit standards as policy *B*.[4] The reason is that the marginal benefit to the firm is greater for policy *A*. As the two policies are mutually exclusive, the best one should be chosen. It is possible, of course, that a relaxation of credit standards that fell on one side or the other of policy *A* would provide an even greater marginal benefit.

[4] The argument can be made that when we adjust for risk, the required rate of return on the additional investment in receivables should be less than the rate used for risky investments. In the example, we subtracted the cost of additional bad debts from additional profitability but continued to use the same required rate of return, which contains a premium for risk. We do not attempt to resolve this difficult problem here but do consider it in Chapter 5.

TABLE   17-2
Profitability versus required return and bad-debt losses

	Policy A	Policy B
Annual sales	$3,000,000	$3,240,000
Turnover of receivables	6	4
Average receivables	$ 500,000	$ 810,000
Average investment in receivables*	$ 430,000	$ 686,880
Additional investment in receivables above		
present investment ($180,000)	$ 250,000	$ 506,880
Required return on additional investment (20%)	$ 50,000	$ 101,376
Additional sales (units)	60,000	84,000
Profitability of additional sales	$ 180,000	$ 252,000
Bad-debt losses (per cent of annual sales)	$ 90,000	$ 194,400
Present bad-debt losses (per cent of annual sales)	$ 24,000	$ 24,000
Additional bad-debt losses	$ 66,000	$ 170,400
Cost of additional bad-debt losses†	$ 46,200	$ 119,280
Profitability of additional sales less		
cost of additional bad-debt losses	$ 133,800	$ 132,720

(Average Receivables)(Average Cost per Unit/Selling Price per Unit)
†(Bad-debt Losses)(Variable Cost per Unit/Selling Price per Unit)*

Another approach to the problem might be to combine the probability of bad-debt losses with the probability of slowness of collection for each customer. Such an approach would be practical only if the size of the account is reasonably large. For a customer with a $10,000 receivable, we might formulate the probability distribution of expected collections shown in Table 17-3.

TABLE   17-3
Possible time of receipt: single account

Time Received after Billing	Probability	Expected Collections
1 month	0.50	$5,000
2 months	0.20	2,000
3 months	0.10	1,000
4 months	0.05	500
5 months	0.05	500
6 months	0.05	500
Bad-debt loss	0.05	500
	1.00	$10,000

For this customer, there is a .05 probability that the account will not be paid after six months and that it will be charged off as a bad-debt loss.[5]

[5]Cyert, Davidson, and Thompson have made an interesting application of Markov chains to estimating the probability of a bad-debt loss for an account at various future dates. R. M. Cyert, H. J. Davidson, and G. L. Thompson, "Estimation of the Allowance for

For other customers, we could construct similar probability distributions. If the probability distributions were independent, a total probability distribution could be obtained simply by totaling the expected collections for each month for all accounts. For our hypothetical firm, the probability distribution for all accounts, totaling $500,000, might be that shown in Table 17-4. By calculating probability distributions for all accounts at different times of the year and comparing actual results with expected results, we could obtain fairly exact information as to the average investment in receivables and the average collection period for these receivables.

TABLE 17-4
Possible time of receipts: all accounts

Time Received after Billing	Probability	Expected Collections
1 month	0.60	$300,000
2 months	0.15	75,000
3 months	0.10	50,000
4 months	0.05	25,000
5 months	0.04	20,000
6 months	0.02	10,000
Bad-debt loss	0.04	20,000
	1.00	$500,000

COLLECTION POLICY

The overall collection policy of the firm is determined by the combination of collection procedures it undertakes. These procedures include such things as letters sent, phone calls, personal calls, and legal action, and they are described later in this chapter. One of the principal policy variables is the amount expended on collection procedures. Within a range, the greater the relative amount expended, the lower the proportion of bad-debt losses and the shorter the average collection period, all other things the same.

The relationships, however, are not linear. Initial collection expenditures are likely to cause little reduction in bad-debt losses. Additional expenditures begin to have a significant effect in reducing the amount of bad-debt losses. Beyond a point, however, additional expenditures tend to have little effect in further reducing these losses. The hypothesized

Doubtful Accounts by Markov Chains," *Management Science*, 8 (April, 1962), 287–303. For similar applications, see William Beranek, *Analysis for Financial Decisions* (Homewood, Ill.: Richard D. Irwin, Inc., 1963), pp. 308–20; and Harold Bierman, Jr., *Financial Policy Decisions* (New York: Macmillan Co., 1970), Chapter 3.

relationship between expenditures and bad-debt losses is shown in Figure 17-1. Likewise, the relationship between the average collection period and the level of collection expenditures is likely to be similar to that shown in the figure.
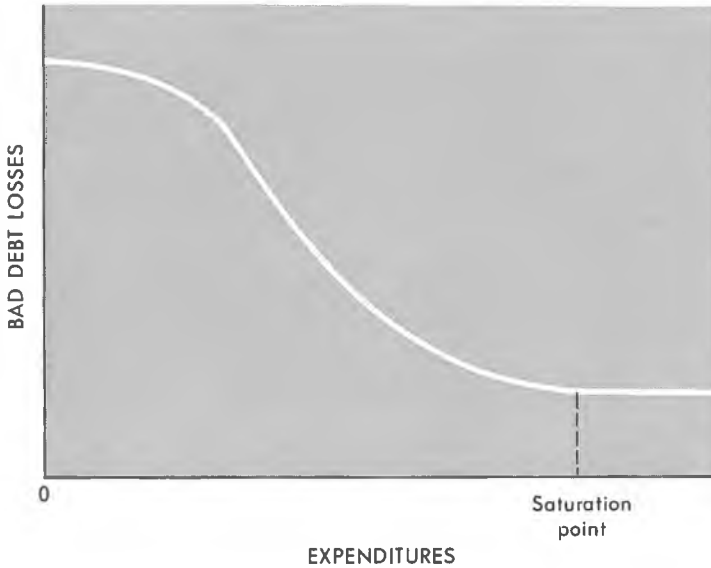
FIGURE 17-1

Relationship between amount of bad-debt losses and collection expenditures

If sales are independent of the collection effort, the appropriate level of collection expenditure again involves a tradeoff—this time between the level of expenditure on the one hand and the reduction in the cost of bad-debt losses and reduction in investment in receivables on the other. Suppose that we are considering the present collection program in relation to two new ones and that the programs were expected to produce these results.

	Present Program	Program A	Program B
Annual collection expenditures	$116,000	$148,000	$200,000
Average collection period	2 months	1½ months	1 month
Percentage of default	3	2	1

Assume that present sales are $2.4 million, price is $10 a unit, and the average cost is $9 a unit, and that these figures are not expected to change with changes in the collection effort. If we make the same type of calculations as in previous examples, we obtain the results shown in Table 17-5. In the last two rows of the table, we see that the reduction in the cost

TABLE 17-5
Evaluation of collection programs

	Present Program	Program A	Program B
Annual sales	$2,400,000	$2,400,000	$2,400,000
Turnover of receivables	6	8	12
Average receivables	$ 400,000	$ 300,000	$ 200,000
Average investment in receivables*	$ 360,000	$ 270,000	$ 180,000
Reduced investment in receivables from present investment		$ 90,000	$ 180,000
Required return on reduced investment (20%)		$ 18,000	$ 36,000
Bad-debt losses (per cent of annual sales)	$ 72,000	$ 48,000	$ 24,000
Cost of bad-debt losses†	$ 64,800	$ 43,200	$ 21,600
Reduction in cost of bad-debt losses from present cost		$ 21,600	$ 43,200
Required return on reduced investment plus reduction in cost of bad-debt losses		$ 39,600	$ 79,200
Additional collection expenditures from present expenditures		$ 32,000	$ 84,000

*(Average Receivables)(Average Cost per Unit/Selling Cost per Unit)
†(Bad-debt Losses)(Average Cost per Unit/Selling Cost per Unit)

of carrying receivables plus the reduction in the cost of bad-debt losses exceeds the additional collection expenditures for program *A* but not those for program *B*. As a result, the firm should adopt program *A* but not increase collection expenditures to the extent of program *B*.

In the example above, we have assumed that demand is independent of the collection effort. In most cases, however, sales are likely to be affected adversely if the collection efforts of the firm become too intense, as customers become increasingly irritated. If they do, we must take into account the relationship between the collection effort and demand. Reduction in demand can be incorporated into the marginal analysis of collection expenditures in the same manner as was the increase in demand accompanying a relaxation in credit standards. In addition, if the collection effort has an effect on the percentage of total sales taking a cash discount, this factor must be considered. With increased collection efforts, for example, more customers might take the cash discount.

CREDIT AND COLLECTION
POLICIES — SUMMARY

We see that the credit and collection policies of a firm involve several decisions, as to (1) the quality of account accepted, (2) the credit period, (3) the cash discount given, (4) any special terms such as seasonal dat-

ings, and (5) the level of collection expenditures. In each case, the decision should involve a comparison of what is to be gained by a change in policy with the cost of the change. Optimal credit and collection policies would be those that resulted in the marginal gains equaling the marginal costs.

To maximize profits arising from credit and collection policies, the firm should vary these policies jointly until an optimal solution is achieved. That solution will determine the best combination of credit standards, credit period, cash discount policy, special terms, and level of collection expenditures. In this regard, sensitivity analysis might be used to judge the impact of a change in policies upon profits. Once functional relationships have been specified for the relationship between a particular policy and marginal sales, average collection period, and bad-debt losses, the policy can be varied from one extreme to the other, holding constant other factors. This variation gives insight into the impact of a change in policy upon profits.
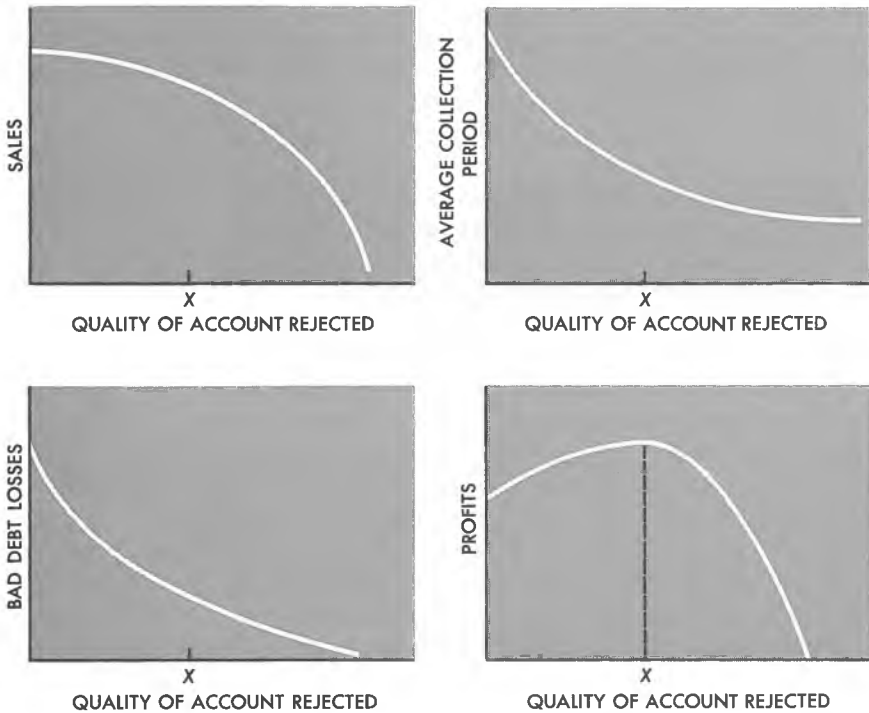

FIGURE 17-2

Relationship of sales, average collection period, bad-debt losses, and profits to the quality of account rejected

For most policy variables, profits increase at a decreasing rate up to a point and then decrease as the policy is varied from no effort to an extreme effort. This relationship is depicted in Figure 17-2 for the quality of account rejected. When there are no credit standards, that is, when all applicants are accepted, sales are maximized. However, the maximization of sales is offset by large bad-debt losses as well as by the opportunity cost of carrying a very large receivable position. As credit standards are initiated and applicants rejected, revenue from sales declines, but so do the average collection period and bad-debt losses. Because the latter two decline initially at a faster rate than do sales, profits increase. As credit standards are tightened increasingly, however, sales revenue declines at an increasing rate. At the same time, the average collection period and bad-debt losses decrease at a decreasing rate. Fewer and fewer bad credit risks are eliminated. Because of the combination of these influences, total profits of the firm increase at a diminishing rate with stricter credit standards up to a point, after which they decline. The optimal policy with respect to credit standards is represented by point X in the figure.

To the extent that decisions with respect to credit standards, the credit period, the discount given, special terms offered, and the level of collection expenditures exercise a joint influence upon profits, this interdependency must be recognized in formulating the functional relationships. With assumed relationships specified, the firm can use sensitivity analysis to experiment with different credit and collection policies in order to determine the optimal set of policies. The optimal set establishes the credit terms given, the collection procedures followed, and the cutoff point on quality of account accepted.[6]

In cases where some of the underlying parameters change, sensitivity analysis proves valuable in formulating new credit and collection policies. For example, if the marginal profit per unit of sales declines due to increased competition, the optimal set of policies is likely to change. Lower profits per unit of sale may not justify the present level of receivables carried nor the present loss rate. As a result, new credit and collection policies might be in order. Through sensitivity analysis, management could determine the new set of policies that will maximize profit.

The analysis in the last several sections has purposely been rather general, to provide insight into the important concepts of credit and collection policies. Obviously, a policy decision should be based upon a far

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[6]Greer develops a model to maximize profits based upon the number of credit applicants accepted by a creditor. Functional relationships are derived linking the number of accounts accepted to most of the factors we discussed. By differentiating the profit function with respect to the number of applicants accepted and setting the expression equal to zero, one can solve for the number of applicants that maximizes profits. Although somewhat unrealistic for trade credit, the model is useful for a retail company trying to determine a credit policy for charge accounts. Carl C. Greer, "The Optimal Credit Acceptance Policy," *Journal of Financial and Quantitative Analysis*, II (December, 1967), 399–415.

more specific evaluation than that contained in the examples above. Estimating the increased demand and increased slowness of collections that might accompany a relaxation of credit standards is extremely difficult. Nevertheless, management must make estimates of these relationships if it is to appraise realistically its existing policies.

CREDIT AND COLLECTION PROCEDURES FOR INDIVIDUAL ACCOUNTS

Having established the terms of sale to be offered, the firm must evaluate individual credit applicants and consider the possibilities of a bad debt or slow payment. The credit evaluation procedure involves three related steps: obtaining information on the applicant; analyzing this information to determine the applicant's credit worthiness; and making the credit decision. The credit decision, in turn, establishes whether credit should be extended and what the maximum amount of credit should be.

SOURCES OF INFORMATION

Though there are a number of sources of credit information, there necessarily are expenses incurred in collecting it. For some accounts, especially small ones, the cost of collecting comprehensive credit information may outweigh the potential profitability of the account. The firm extending credit may have to be satisfied with a limited amount of information on which to base a decision. Later in this chapter, we present a sequential investigation process whereby a firm may weigh the cost of collecting and analyzing additional information in relation to the expected benefits to be derived from it. In addition to cost, the firm must consider the time it takes to investigate a credit applicant. A shipment to a prospective customer cannot be delayed unnecessarily pending an elaborate credit investigation. Thus, the amount of information collected needs to be considered in relation to the time and expense required. Depending upon these considerations, the credit analyst may use one or more of the following sources of information.

Financial Statement. One of the most desirable sources of information for credit analysis is a financial statement that the seller may request from his customer at the time of the prospective sale. Some companies are perfectly willing to provide statements to suppliers, whereas others may refuse to do so. There is frequently a correlation between a company's refusal to provide a statement and weaknesses in its financial position. When possible, it is helpful to obtain interim statements, particularly for companies having seasonal patterns of sales. Needless to say, audited statements are far better than unaudited figures.

Credit Ratings and Reports. In addition to financial statements, credit ratings are available from various mercantile agencies. Dun & Bradstreet, Inc., is perhaps the best known and most comprehensive of these agencies. It provides credit ratings to subscribers for a vast number of business firms throughout the nation. A key to its individual ratings is shown in Figure 17-3. As we see in the figure, D & B ratings give the credit analyst an indication of the estimated size of net worth and a credit appraisal for companies of a particular size, ranging from "high" to "limited." D & B also indicates when the information available is insufficient to provide a rating for a given business.

In addition to its rating service, D & B provides credit reports on business firms. These reports contain a brief history of the company and its principal officers, the nature of the business, certain financial information, and a trade check of suppliers as to the length of their experience with the company and as to whether payments are discount, prompt, or past due. The quality of the D & B reports varies with the information

KEY TO RATINGS

ESTIMATED FINANCIAL STRENGTH   COMPOSITE CREDIT APPRAISAL

				HIGH	GOOD	FAIR	LIMITED
Aa	Over	$1,000,000		A1	1	1½	2
A+	Over	750,000		A1	1	1½	2
A	500,000	to	750,000	A1	1	1½	2
B+	300,000	to	500,000	1	1½	2	2½
B	200,000	to	300,000	1	1½	2	2½
C+	125,000	to	200,000	1	1½	2	2½
C	75,000	to	125,000	1½	2	2½	3
D+	50,000	to	75,000	1½	2	2½	3
D	35,000	to	50.000	1½	2	2½	3
E	20,000	to	35.000	2	2½	3	3½
F	10,000	to	20,000	2½	3	3½	4
G	5,000	to	10,000	3	3½	4	4½
H	3,000	to	5,000	3	3½	4	4½
J	2,000	to	3,000	3	3½	4	4½
K	1,000	to	2,000	3	3½	4	4½
L	Up to	1,000		3½	4	4½	5

CLASSIFICATION AS TO BOTH
ESTIMATED FINANCIAL STRENGTH AND CREDIT APPRAISAL

FINANCIAL STRENGTH BRACKET			EXPLANATION
1	$125,000 to	$1,000,000 and Over	When only the numeral (1, 2, 3, or 4) appears, it is an indication that the estimated financial strength, while not definitely classified, is presumed to be within the range of the ($) figures in the corresponding bracket and that a condition is believed to exist which warrants credit in keeping with that assumption.
2	20,000	to 125,000	
3	2,000	to 20,000	
4	Up	to 2,000	

NOT CLASSIFIED OR ABSENCE OF RATING

The absence of a rating, expressed by the dash (—), or by two hyphens (- -), is not to be construed as unfavorable but signifies circumstances difficult to classify within condensed rating symbols and should suggest to the subscriber the advisability of obtaining additional information.

FIGURE 17-3
Dun & Bradstreet key to ratings

available externally and the willingness of the company being checked to cooperate with the D & B reporter. In addition to Dun & Bradstreet, there are a number of credit agencies that specialize in a particular line of business or in geographic areas, such as New York or Chicago.

Bank Checking. Another source of information for the firm is a credit check through a bank. Many banks have large credit departments that undertake, as a service, credit checks for their customers. By calling or writing the bank(s) of account of the company being checked, a firm's bank is able to obtain information as to the average cash balance carried, loan accommodations, experience, and sometimes financial information. Because banks generally are more willing to share information with other banks than with a direct inquirer, it usually is best for the firm to initiate the credit check through its own bank rather than to inquire directly.

Exchange of Information. Credit information frequently is exchanged among companies selling to the same customer. Through various credit organizations, credit men in a particular area become a closely knit group. A company is able to check other suppliers as to their experience with an account. Useful information includes the length of time they have had the account, the maximum credit extended, the amount of the line of credit, and whether payments are prompt or slow. In addition, there are various clearing houses of credit information. The largest of these is the National Association of Credit Men; credit information is provided by this agency in the form of a report.

The Company's Own Experience. In addition to these sources, a company's own experience with an account is extremely important. A study of the promptness of past payments, including any seasonal patterns, for example, is very useful. Frequently, the credit department or a salesman will make written assessments of the quality of the management of a company to whom credit may be extended. These assessments are very important, for they pertain to the first of the famous "three C's" of credit: *character, collateral,* and *capacity.*

CREDIT ANALYSIS AND DECISION

Having collected credit information, the firm must undertake a credit analysis of the applicant. If financial statements are provided, the analyst should make a ratio analysis, as described in Chapter 25. As suggested in that chapter, empirical studies may be undertaken to determine which financial ratios have the greatest predictive power. The analyst will be particularly interested in the applicant's liquidity and ability to pay bills on time.

In addition to analyzing financial statements, the credit analyst will

consider the financial strength of the firm, the character of the company and its management, and various other matters. He then attempts to determine the ability of the applicant to service trade credit. In this regard, he assesses the probability of an applicant's not paying on time and of a bad-debt loss.

Although quantitative approaches have been developed to measure ability to service trade credit, the final decision for most companies extending trade credit rests upon the credit analyst's judgment in evaluating available information. Numerical evaluations have been used with success in consumer credit, where various characteristics of an individual are quantitatively rated and a credit decision is made on a total score.[7] Numerical rating systems also show promise for companies extending trade credit. In the appendix of this chapter, discriminant analysis is examined as a means of accepting or rejecting credit applicants. With the overall growth of trade credit and the difficulty companies have in developing and retaining competent credit analysts, it would seem that in years to come, more companies will at least explore the possibility of developing numerical credit scoring systems for the evaluation of trade credit applicants.

SEQUENTIAL INVESTIGATION PROCESS

Most credit investigations involve a sequential analysis where the collection of information and its analysis are interrelated. In general, the riskier the credit applicant, the more information the analyst will want to have. The cost of acquiring this information, however, must be balanced against the expected benefits to be derived from the order. A credit decision for an individual account may take one of three courses of action: accept, reject, or obtain additional information.[8] For the last alternative, the expected value of additional information must exceed the cost of acquiring it.

To illustrate a sequential investigation process, suppose the following stages of credit investigation were open to a company:

1. Consult past experience to see if the firm has had previous experience with the account and, if it has, what has been that experience.

2. Order a Dun & Bradstreet report on the applicant and evaluate it.

3. Undertake credit checks with banks and trade creditors.

Stages 2 and 3 could be broken down into additional steps. For ex-

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[7] See James H. Myers and Edward W. Forgy, "The Development of Numerical Credit Evaluation Systems," *Journal of the American Statistical Association,* 58 (September, 1963), 799–806.

[8] This section draws upon Dileep Mehta, "The Formulation of Credit Policy Models," *Management Science,* 15 (October, 1968), 30–50.

ample, a firm might first seek a credit rating from Dun & Bradstreet and, on the basis of that rating, decide whether or not to order a report. For stage 3 it might check banks first and then, depending upon the outcome of the bank check, check trade creditors. Moreover, the number of trade creditors checked might be a sequential function. An additional stage might be a personal interview by the credit analyst with the company placing the order. For simplicity of illustration, we consider only three stages, but recognize that the sequential investigation process could be expanded to take account of additional stages.

These three stages are broken down into various quality categories. For stage 1, an order is classified into one of the following categories, depending upon what the investigation of past experience reveals:[9]

a. Good. A good account is one in which the customer consistently pays within the credit period.

b. Fair. This rating is given to a customer who frequently pays beyond the credit period, but usually is no more than 45 days delinquent. Moreover, the customer keeps the firm informed and pays according to plan.

c. Poor. A poor account is one where the customer is often more than 45 days delinquent. In addition, the customer does not keep the firm informed and usually is inconsistent in payments.

d. New. A new order.

When a report is ordered from Dun & Bradstreet, the credit analyst evaluates it and the financial statements contained therein. On the basis of this evaluation, he assigns a letter rating of either A, B, C, or D. A is the highest grade and indicates that the prospects for the customer paying within the credit period are excellent. On the other end of the spectrum, D is the lowest rating and indicates that there is a high degree of probability the customer either will be delinquent in payment or will not pay at all. Similarly, for stage 3, a letter rating of A through D is assigned on the basis of credit checks with banks and trade creditors. In the investigation process, it is important that each stage be treated as independent of the previous stage. In other words, credit checks should be evaluated independently of the evaluation of the D & B report.

When an order is received, the credit department must make a decision as to whether or not to accept it. It would of course be desirable to undertake stages 1 through 3 for all orders, but each involves a cost. To investigate past experience, someone must check the credit file and accounts receivable ledger and categorize the applicant into one of four quality categories. For stage 2, the firm must pay for the D & B report. In addition, the credit analyst must devote time to analyzing the report and to assigning a letter rating. This analysis also involves a cost. The last stage,

[9] *Ibid.,* 33.

credit checkings, involves a considerable cost, for the credit analyst must either call or write bank(s) and trade creditors and inquire of their experience. Additionally, he must analyze the information and assign a letter rating. Suppose that the costs for the various stages of investigation are the following:

		Cost
Stage 1	Past experience	$1
Stage 2	D & B Report	8
Stage 3	Credit checkings	15

These costs will be compared shortly with the expected value of information provided by each stage.

EXPECTED BENEFITS AND COSTS

First, however, we must establish the expected benefits to be derived from an order. Assume that the firm makes but one product and that the marginal profit from each unit sold is $10 and the marginal cost to produce it $15. Moreover, suppose that the required return on investment in receivables is 15 per cent on a before-tax basis. Given this information, the firm must estimate the average collection period, average collection costs, and the probability of bad-debt loss for each of the possible quality categories. These estimates should be based upon actual experience during a sample period in which the firm undertakes all stages of investigation for each credit applicant but extends open-book credit to all. As experience unfolds for an account, the firm records the collection period, the collection cost, and whether or not it was a bad debt. On the basis of experience with a number of accounts over the sample period, it computes an average collection period, average collection costs, and the percentage of bad debt for each quality category of applicants. An example of this information for stage 1 is presented in Table 17-6. If future orders are not expected to differ significantly from the orders comprising the sample, the firm may use the information in this table to assess the profitability of an order.[10] It now is possible to compute the expected cost of accepting an order:

Acceptance cost = Probability of bad debt(Variable cost per
unit)X + (Required return)(Average collection period/360)(Variable cost per unit)X +
Average collection cost   (17-1)

where X is the number of units ordered. For our hypothetical example,

[10] It is essential that future orders correspond closely to those in the sample; otherwise, the estimates of profitability of an order will not be reliable.

TABLE 17-6
Estimates of collection period, collection costs, and bad debt for stage 1

Stage	Average Collection Period	Average Collection Costs	Probability of Bad Debt
Past experience:			
Good	29 days	0.78	0.02
Fair	53	2.78	0.16
Poor	92	9.86	0.32
New account	59	3.56	0.21

Acceptance cost = (Probability of bad debt)$(15X) + 0.15$(Average collection period/360)$(15X)$ + Average collection cost.

Recall that variable costs were assumed to be $15 per unit and the required rate of return 15 per cent. Thus, the acceptance cost embodies the costs of bad-debt losses, carrying receivables, and collection. Because collection costs are assumed to be fixed, they do not vary with the size of the order. The expected cost of rejecting an order is:

$$\text{Rejection cost} = (1 - \text{Probability of bad debt})(\text{Marginal profit per unit})X \qquad (17\text{-}2)$$

which, for our example, is

$$\text{Rejection cost} = (1 - \text{Probability of bad debt})10X$$

The rejection cost is simply the probability of payment times the marginal profit foregone on the order.

To illustrate the calculation of acceptance and rejection costs, suppose that there is an order for X units and that as a matter of policy the firm always investigates past experience. The expected costs of acceptance and rejection for a poor account are:

$$\text{Acceptance cost} = 0.32(15X) + 0.15(92/360)(15X) + \$9.86$$
$$= 5.38X + \$9.86$$
$$\text{Rejection cost} \quad = 0.68(10X) = 6.8X$$

Similarly, we can compute the expected costs of acceptance and rejection for the other quality categories in the first stage. These costs are shown in Figure 17-4. On the basis of the sample described earlier, suppose the firm has found that for the second stage of investigation 50 per cent of all orders received fall into the good category, 30 per cent into the fair category, 10 per cent into the poor category, and 10 per cent are new orders. These probabilities are shown in the figure and will become important in a short while.

From the information in Figure 17-4, we can determine easily that for
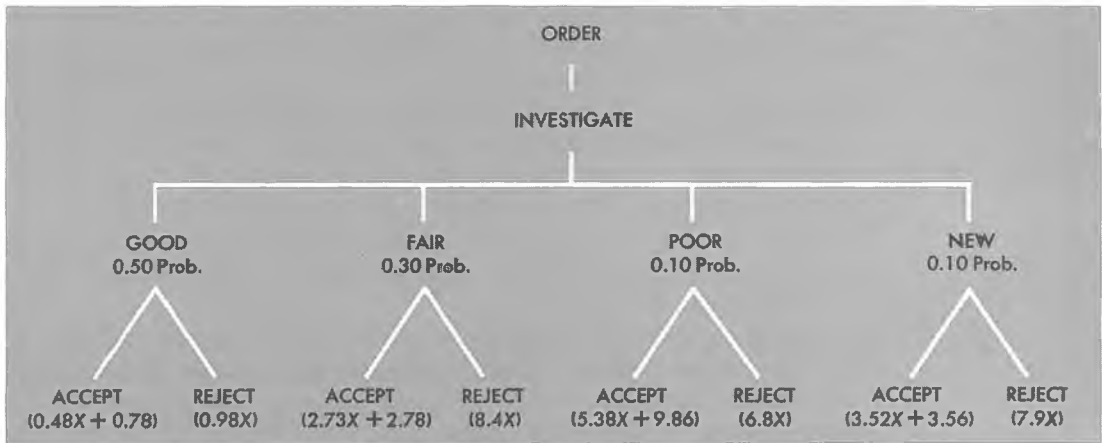
FIGURE 17-4
Investigation of past experience cost structure

the poor category, the cost of acceptance exceeds the cost of rejection for orders of six units or less. When $X \geq 7$, however, the cost of acceptance is less. If past experience (stage 1) were the only stage of investigation, an order that fell into the poor category would be rejected if it were for six units or less and accepted if it were for seven units or more. For the good, fair, and new-order categories, the cost of acceptance is less than the cost of rejection for all $X \geq 1$. Therefore, an order that fell into any of these categories would be accepted if examining past experience were the only stage of investigation.

SEQUENTIAL STAGES OF INVESTIGATION

With additional stages of investigation, however, we must decide whether to investigate further or to accept or reject the order on the basis of the information we have. A decision to go on to the next stage of investigation depends upon the benefits expected to be derived from the additional information in relation to its cost. In a nutshell, we want to establish whether the added information will improve our chances of making a correct decision.[11]

Stage 2 involves analyzing a D & B report. Assume for now that only the first two stages of investigation are open to the company. On the basis of an evaluation of the D & B report, the credit analyst assigns a

[11] The new information has value only if we change our previous accept-reject decision on the basis of it. See Harold Bierman, Jr., Charles P. Bonini, and Warren H. Hausman, *Quantitative Analysis for Business Decisions* (Homewood, Ill.: Richard D. Irwin, Inc., 1969), Chapters 4–9.

letter grade ranging from A through D. When this information is combined with that in the previous stage, the probabilities as well as the average collection periods, average collection costs, and probabilities of bad-debt losses change. New estimates of them should be based upon the sample discussed earlier. Suppose that on the basis of this sample, the firm found the conditional probabilities, average collection periods, average collection costs, and probabilities of bad-debt losses to be those shown in Table 17-7. Note that when the combination of past experience and D & B report analysis are less desirable, the factors shown in the last three columns are worse.

Similar to Figure 17-4, the expected costs of acceptance and rejection can be determined for the combinations shown in Table 17-7; and they are shown in Table 17-8. Given this information, we are able to determine whether the firm should go beyond the investigation of past experience to analyze a D & B report. The crux of the decision involves comparing the expected benefits derived from the additional information with the cost of acquiring it. In turn, the magnitude of benefits depends upon the size of the order.

TABLE 17-7
Estimates of collection period, collection costs, and bad-debt loss for D & B report analysis, given the investigation of past experience

Good Past Experience D & B Report	Conditional Probability of Occurrence	Average Collection Period	Average Collection Cost	Probability of Bad-debt Loss
A	0.70	25 days	0.50	0.00
B	0.15	30	1.00	0.02
C	0.10	40	1.50	0.07
D	0.05	50	2.50	0.15
Fair Past Experience D & B Report				
A	0.20	30 days	1.00	0.03
B	0.40	45	2.00	0.10
C	0.25	60	3.50	0.20
D	0.15	90	6.00	0.40
Poor Past Experience D & B Report				
A	0.02	40 days	1.50	0.05
B	0.18	60	3.50	0.18
C	0.40	80	8.00	0.30
D	0.40	120	15.00	0.60
New Order D & B Report				
A	0.20	32 days	1.20	0.04
B	0.30	48	2.40	0.12
C	0.30	65	4.00	0.22
D	0.20	95	7.00	0.48

TABLE 17-8
**Costs of acceptance and rejection for D & B
report analysis**

Good Past Experience D & B Report	Cost of Acceptance	Cost of Rejection
A	0.16X + 0.50	10.0X
B	0.49X + 1.00	9.8X
C	1.30X + 1.50	9.3X
D	2.56X + 2.50	8.5X
Fair Past Experience D & B Report		
A	0.64X + 1.00	9.7X
B	1.74X + 2.00	9.0X
C	3.37X + 3.50	8.0X
D	6.56X + 6.00	6.0X
Poor Past Experience D & B Report		
A	1.00X + 1.50	9.5X
B	2.17X + 3.50	8.8X
C	5.00X + 8.00	7.0X
D	9.75X +15.00	4.0X
New Order D & B Report		
A	0.86X + 1.20	9.6X
B	2.10X + 2.40	8.8X
C	3.71X + 4.00	7.8X
D	7.79X + 7.00	5.2X

AN ILLUSTRATION

To illustrate the investigation process, assume an order for ten units is received and that the firm has investigated its past experience and found it to be poor. For a cost of $8, it can go to the next stage of investigation and analyze a D & B report. The question is whether it is worthwhile for it to do so. From Figure 17-4, we know the order would be accepted if past experience were the only stage of investigation—the reason is that for orders of seven or more units, the cost of acceptance is less than the cost of rejection. We now must determine if the additional information arising from a D & B report analysis will cause us to change our decision. Only if it will does the additional information have value. The cost of acceptance and the cost of rejection for an order of ten units, given poor past experience, can be derived from Table 17-8; and they are found to be:

D & B Report Grade	Cost of Acceptance	Cost of Rejection
A	$ 11.50	$95.00
B	25.20	88.00
C	58.00	70.00
D	112.50	40.00

Thus, if ten units are ordered, the cost of acceptance exceeds the cost of rejection only if the D & B analysis grade turns out to be D. Under these circumstances, the firm would reject the order, where before the order would have been accepted. Consequently, there is value in obtaining the additional information. The expected net benefit is the difference between the cost of acceptance avoided and the cost of rejection incurred, times the conditional probability of occurrence. For our example, the expected net benefit is

$$(112.50 - 40.00)0.4 = \$29.00$$

Inasmuch as this figure exceeds the cost of investigation of $8, an analysis of a D & B report should be undertaken. When lower values of X are used, it is found that the expected benefits associated with a stage 2 investigation exceed the $8 cost for all values of X of seven or more units.

There still remains the question of whether it is worthwhile to undertake a D & B report analysis if the order is for six units or less. Here, the order would be rejected on the basis of past experience alone. We must determine if additional information will cause us to change that decision. If the order were for five units, the expected costs of acceptance and rejection would be

D & B Report Grade	Cost of Acceptance	Cost of Rejection
A	$ 6.50	$47.50
B	14.35	44.00
C	33.00	35.00
D	63.75	20.00

If a D & B report analysis were undertaken, the firm could avoid the cost of rejection for cases of grades A, B, and C. However, it would incur the lesser cost of acceptance in all three cases. Since the order would be rejected on the basis of past experience alone, there is no benefit to be derived from grade D, inasmuch as the cost of acceptance exceeds the cost of rejection. Again, the value of additional information comes only if it causes us to change a decision. The expected net benefit to the firm is determined by multiplying the differences between the costs of rejection and acceptance for the first three cases times their respective conditional probabilities of occurrence:

$$(47.50 - 6.50)0.02 + (44.00 - 14.35)0.18 + (35.00 - 33.00)0.40 = \$6.96$$

Since the expected net benefit is less than the cost of undertaking a D & B report analysis, an order for five units from a poor account should be rejected. However, when an order is for six units, the expected benefit, $9.12, exceeds the cost of investigation of $8.00. Therefore, an in-

vestigation of a D & B report is worthwhile when the order consists of six units.

As a general decision rule, then, the firm should reject any order for five units or less if an investigation of past experience reveals the account to have been poor. No further investigation should be undertaken. If the order is for six or more units, however, the firm should undertake an analysis of a D & B report because the expected benefit of the additional information exceeds its cost. Depending upon the outcome of this investigation, the order will either be accepted or rejected.[12]

Similarly, decision rules can be formulated for other categories of past experience. For good past experience, the cost of acceptance in Table 17-8 is less than the cost of rejection for all $X \geqslant 1$. Therefore, there is no benefit to be derived from further investigation. The previous acceptance decision will not be changed. As a result, the firm should accept all orders where an investigation of past experience reveals that experience to have been good.

For an account judged to be fair by past experience, the cost of acceptance exceeds the cost of rejection in Table 17-8 only if the D & B report analysis grade turns out to be D. Under these circumstances, the expected net benefit is

$$(6.56X + 6.00 - 6.0X)0.15$$

For an analysis of a D & B report to be worthwhile, the expected net benefit must exceed the cost of investigation of $8.00. To be indifferent, we must have the following equality:

$$(6.56X + 6.00 - 6.0X)0.15 = 8.00.$$

Transposing and solving for X,

$$0.56X + 6.00 = 8.00/0.15$$
$$0.56X = 53.33 - 6.00$$
$$X = 84.5$$

Thus, only if the order size is eighty-five units or more, will an analysis of a D & B report benefit the firm. If an order judged to be fair by past experience is for less than eighty-five units, it should be accepted without such analysis.

For a new account, the cost of acceptance exceeds the cost of rejection only for grade D in Table 17-8. The expected benefit is:

$$(7.79X + 7.00 - 5.2X)0.20$$

[12]This decision rule and the others presented assume a linear utility function with respect to money. As discussed in Chapter 5, there is a serious question as to whether a manager behaves in this fashion. If his utility function could be specified, the sequential investigation process could be expressed in utile values in a manner similar to the analysis of capital-budgeting projects in Chapter 5.

To be indifferent, the expected benefit must equal the cost of investigation:

$$(7.79X + 7.00 - 5.2X)0.20 = 8.00$$

Transposing and solving for X:

$$2.59X + 7.00 = 8.00/0.20$$
$$2.59X = 40.00 - 7.00$$
$$X = 12.7$$

Consequently, only if the order size is thirteen units or more should the firm analyze a D & B report for a new order. If the order is for less than this number, it should be accepted without credit investigation.

Our example could be extended to take account of the last stage of investigation—a checking of banks and trade suppliers. However, as our example is already rather long, we shall limit it to the first two stages of the investigation process. Because the desirability of additional information diminishes with each successive stage, one can visualize that the number of times where the additional information would be worthwhile would be less for credit checkings than for the D & B report analysis.

Implications. The sequential investigation process presented illustrates one means by which a firm can reach a credit decision with a minimum of investigation cost. Rather than perform all stages of investigation regardless of the size of the order and the firm's past experience, the firm should undertake investigation in stages and go to a new stage only when the expected net benefits of the additional information exceed the cost of acquiring it. When past experience has been favorable, there may be little need for further investigation. We saw that decision rules can be established whereby an order is investigated only if it meets certain size constraints and falls into a particular category in the previous stage of investigation. Because the stages of credit investigation are sequential, added sophistication is introduced only when it is beneficial to do so. In this manner, the firm can make optimal credit decisions with respect to acceptance, rejection, or further investigation.

The decision rules used in a sequential investigation process will vary according to the marginal profit and marginal cost of the product, the cost of bad debts, the average collection period, and average collection costs. To the extent that these factors change over time, the firm will need to formulate new decision rules. As experience changes and the sample on which estimates are made is no longer valid, the firm will need to revise its probability estimates and its estimates of costs and bad-debt losses in Tables 17-6 and 17-7. Perhaps the most suitable means for review is to extend credit to all applicants on a periodic basis. With

this experience recorded, new estimates of the average collection period and costs, the probability of bad-debt losses, and the probabilities of occurrence can be formulated. The more stable the experience over time, the less frequent the need for periodic review. If the underlying system is basically unstable, however, a sequential investigation process is not well suited for the problem at hand. Yet, for most situations it represents a meaningful application of decision theory to credit analysis.

LINE OF CREDIT

Another means of control for repeat orders is to establish a line of credit for an individual account. A line of credit is a maximum limit on the amount the firm will permit to be owing at any one time. In essence, it represents the maximum risk exposure that the firm will allow itself to undergo for an account. The establishment of a credit line streamlines the procedure for shipping goods, but the line must be reevaluated periodically in order to keep abreast of developments in the account. What was a satisfactory risk exposure today may be more or less than satisfactory a year from today. Despite comprehensive credit procedures, there will always be special cases that must be dealt with individually. Here, too, however, the firm can streamline the operation by defining responsibilities clearly.

COLLECTION PROCEDURES

In addition to credit procedures for the individual account, the firm must establish clear-cut collection procedures for past due or delinquent accounts. The initial question to be answered is: How past due should an account be allowed to go before collection procedures are initiated? As a receivable is only as good as the likelihood that it will be paid, a firm cannot afford to wait too long before initiating collection procedures. On the other hand, if it initiates procedures too soon, it may anger reasonably good customers, who, for some reason, fail to make payments by the due date. Procedures, whatever they are, should be firmly established. Initially, a letter is usually sent, followed, perhaps, by additional letters that become ever more serious in tone. Next may come a telephone call from the credit manager and then, perhaps, one from the company's attorney. Some companies have collection men who make personal calls on the account.

If all else fails, the account may be turned over to a collection agency. The agency's fees are quite substantial—frequently, one-half the amount of the receivable—but such a procedure may be the only feasible alternative, particularly for a small account. Direct legal action is costly, sometimes serves no real purpose, and may only force the account into bank-

ruptcy. When payment cannot be collected, compromise settlements may provide a higher percentage of collection.[13] Past experience may tell a firm what the optimal collection procedure would be relative to profitability.[14]

CREDIT INSURANCE

To protect against unusual bad-debt losses in extending trade credit, a firm may take out credit insurance. Although the firm cannot insure against losses incurred normally in the industry (known as primary losses), it can insure against above-normal losses. Insurance companies usually restrict coverage to certain acceptable risks, as determined by Dun & Bradstreet ratings. In addition, these companies usually insist upon *coinsurance,* the participation of the collecting firm in a portion of the bad-debt loss—usually 10 to 20 per cent. The insistence upon coinsurance safeguards the insurance company from the firm's becoming excessively liberal in the granting of credit. The cost of credit insurance varies directly with the risk of the accounts accepted and is calculated as a percentage of sales. The decision to use credit insurance depends upon the probability of extreme credit losses and the ability of the firm to bear these losses.

USE OF ELECTRONIC DATA
PROCESSING EQUIPMENT

Computers have been used a great deal in credit management. Their use provides certain essential up-to-date information needed for analysis. All of the information previously placed on receivable ledgers can be placed on punched cards or tapes. As a result, the credit department has very quick access to this information. At frequent intervals, it can obtain a trial balance that gives a summary of all billings, payments, discounts taken, and amounts still owed. Also, it can obtain an aging of accounts showing the total amounts owed the firm, the portion that is current, the portion that is up to thirty days past due, that which is thirty to sixty days past due, and so forth.

In addition, the computer can be programmed to provide complete reports on all delinquent accounts, and delinquency letters can be sent out mechanically at regular intervals. Frequent reports on past due accounts, which were not possible before the computer, alert the credit manager to problems as they develop. As a result, he is able to stay on

[13] For an extended discussion of these settlements, see Chapter 24.

[14] Mitchner and Peterson develop a method for determining the optimal pursuit time on defaulted loans. Morton Mitchner and Raymond P. Peterson, "An Operations-Research Study of the Collection of Defaulted Loans," *Operations Research,* 5 (August, 1957), 522–45.

top of them and take corrective action. Formerly, the situation might have deteriorated during the information lag. Management also may want to be informed when an account approaches the line of credit established for it, and computers can provide this information easily.

The computer helps the credit manager by providing timely and accurate information on the status of accounts. The payment history of a customer can be drawn from storage and printed out in seconds. Included in this history is information such as the date the account was opened, the amount owing currently, the customer's credit line, any numerical credit ratings, and the promptness of past payments. Special reports can be prepared that involve categorization or comparisons. For example, if several companies in the same industry are slow in their payments at a particular time of the year, management might want to know the firm's experience with all other companies in that particular industry. Such information enables the credit manager to analyze and deal with the problem more effectively. In another situation, management might wish to compare incoming orders from a particular customer with his payment history. This information also can be provided quickly.

Indeed, the computer can provide a vast array of detailed information, previously impractical to obtain, that may be useful not only to the credit manager but to other management as well. In addition to processing data, the computer can be programmed to make certain routine credit decisions.[15] In particular, small orders from good accounts can be approved by the computer without the order ever going to a credit analyst. All in all, electronic data processing can make a significant contribution to the credit department. As their volume of receivables grows, many firms find computer processing to be the only feasible means by which to handle receivables.

CAPTIVE FINANCE COMPANIES

Instead of carrying receivables on their own books, many companies have found it useful to establish a finance subsidiary, which holds the notes receivable generated by the sales of the parent company. These subsidiaries are known as captive finance companies; and the receivables held are principally time sales contracts. Time sales contracts are used frequently in the purchase of such things as machinery, trucks, and durable consumer goods. Companies having captive finance subsidiaries include Allis-Chalmers, Caterpillar Tractor, Clark Equipment, General Electric, International Harvester, Mack Truck, Montgomery Ward,

[15] For a discussion of credit decision making by simulation, see Roger L. Sisson and Norman L. Statland, "The Future of Computers in Credit Management," *Credit and Financial Management,* 65 (November, 1963), 40, 44.

Sears, Roebuck, and Westinghouse. Typically, the parent company makes the time sale and then sells the contract to the finance subsidiary. The contract itself carries a stipulated rate of interest and calls for periodic installment payments. The subsidiary's operating income consists of the interest received on the contract; its costs include administrative expenses and the cost of capital to carry the notes receivable.

COST OF CAPITAL

Supposedly, one of the reasons to establish a captive finance company is that the cost of capital is lower for the subsidiary than for the parent company.[16] Because it is possible to finance the subsidiary with large proportions of debt, often more than 80 per cent, the subsidiary employs a higher proportion of cheaper debt funds than does the parent. To be sure, the parent has an equity investment in the subsidiary consisting of stock and, often, subordinated debentures. However, additional financing comes from outside lenders who make substantial advances based upon the quality of the receivables, which, in turn, are judged by the delinquency experience. In the final analysis, the payment of the loan must come from the liquidation of notes receivable. The poorer the quality of the receivables, the greater the equity base lenders require the subsidiary to maintain to protect themselves.

Certain proponents of the captive finance company argue that it makes possible an increase in the total proportion of debt to equity for the corporation as a whole. They also argue that because the explicit cost of debt usually is less than that of equity, establishing a captive finance company results in a lower cost of carrying receivables. This line of reasoning does not take into account the fact that one of the most liquid of assets (receivables) is being taken away from the parent and given to the subsidiary. The parent loses a degree of credit-worthiness — a loss which, in perfect markets, will completely offset the gain achieved by the finance subsidiary. In other words, the finance subsidiary can have a high debt-to-equity ratio only at the expense of the parent, whose debt capacity is reduced. Only to the extent that lenders and others are fooled is the debt capacity of the total entity raised.[17] The parent may not have been utilizing its debt capacity before and is now doing so through its finance subsidiary, but this does not mean that total debt capacity is increased. Therefore, the argument that the cost of carrying receivables is less with a captive finance company does not hold.

[16] For an excellent analysis of this question, see Victor L. Andrews, "Captive Finance Companies," *Harvard Business Review*, 42 (July–August, 1964), 80–92.

[17] Essentially, this is the argument of Andrews, *ibid*. We have omitted purposely any discussion of the implicit costs of debt. For an analysis of that problem, see Chapters 7 and 8.

Credit and collection policies encompass the quality of accounts accepted, the credit period extended, the cash discount given, certain special terms, and the level of collection expenditures. In each case, the credit decision involves a tradeoff between the additional profitability and the cost resulting from a change in any of these elements. For example, by liberalizing the quality requirements for accounts, the firm might hope to make more on the additional sales than the cost of carrying the additional receivables plus the additional bad-debt losses. To maximize profits arising from credit and collection policies, the firm should vary these policies jointly until an optimal solution is obtained. This variation can be accomplished through simulation once the functional relationships are specified. The firm's credit and collection policies, together with its credit and collection procedures, determine the magnitude and quality of its receivable position.

In evaluating a credit applicant, the credit analyst is concerned with obtaining financial and other information about the applicant, analyzing this information, and reaching a credit decision. A sequential analysis process using decision theory was presented whereby the firm can reach a decision with respect to accepting an order, rejecting it, or obtaining additional information. Obtaining additional information is justified only when the expected benefits of the information exceed its cost. In turn, expected benefits arise only if the information allows us to correct a previously wrong decision. Because the stages of investigation are sequential, added sophistication in credit analysis is undertaken only when it is profitable to do so.

If the account is new, the firm must decide whether or not to accept the order. With repeat orders, the firm must usually decide upon the maximum credit to extend. This maximum, known as a line of credit, is based upon the credit-worthiness of the applicant. Collection procedures should be firmly established and applied consistently. The length of time an account may be delinquent before collection procedures are initiated will depend upon the billing terms and the nature of the account. To protect against unusual credit losses, a firm may take out credit insurance. We reviewed briefly some of the applications of electronic data processing to credit management. The uses of the computer are many and are likely to increase in importance.

In recent years, many companies have established wholly owned finance subsidiaries, known as captive finance companies, to carry the notes receivable generated by the parent. We examined the question of whether such a move lowered the cost of carrying the receivables. We concluded that what the subsidiary gains in borrowing capacity, the parent gives up; so that the real cost of carrying receivables is not reduced.

Discriminant analysis is a statistical tool by which we can decide which prospective accounts to accept or reject, on the basis of certain relevant variables.[18] This type of analysis is similar to regression analysis, but assumes that the observations come from two or more different universes. In our case, these universes consist of good and bad accounts. Suppose that we start with an evaluation of only two characteristics of trade credit applicants: the size of the company and its quick, or acid-test, ratio. For purposes of experiment, we extend open-book credit to all new credit applicants for a sample period. For each account, we record the size of the firm, its quick ratio, and whether or not after a length of time it defaults in payment. If the account defaults, it is classified as a bad account; if it pays on time, it is classified as a good account. With this information, we are able to undertake a linear discriminant analysis with two independent variables. We wish to determine the predictive value of these variables for the behavior of the dependent variable, whether the account is good or bad.

Suppose that we plot quick ratios and size for each account on a scatter diagram, obtaining the results shown in Figure 17-5.[19] The circles
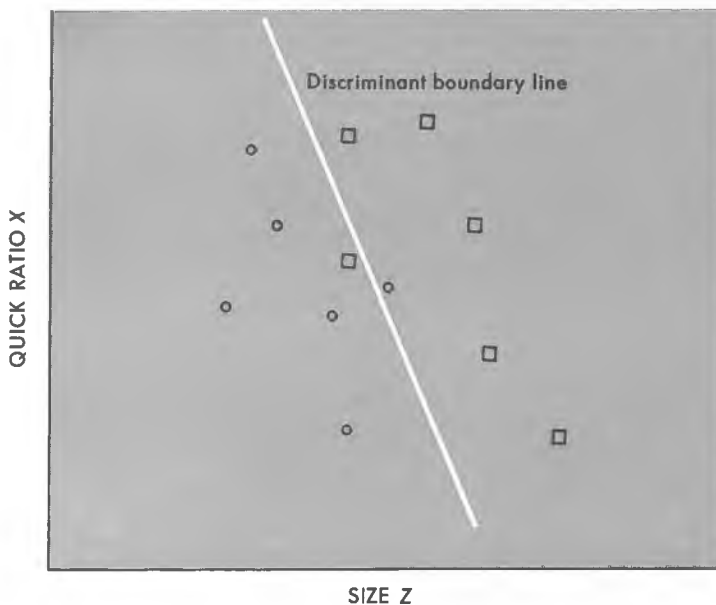
FIGURE 17-5
Discriminant analysis of account's receivable

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[18] For a discussion of discriminant analysis, see Gerhard Tintner, *Econometrics* (New York: John Wiley & Sons, Inc., 1952), pp. 96–102; and E. J. Williams, *Regression Analysis* (New York: John Wiley & Sons, Inc., 1959), Chapter 10.

[19] This example is similar to one used by William F. Massy, "Statistical Analysis of Relations between Variables," in Ronald E. Frank, Alfred A. Kuehn, and William F.

represent bad accounts; the squares represent good accounts. Using the two independent variables, our objective is to find the linear boundary line that discriminates best between good and bad accounts. We need to find the parameters, or weights, of the following discriminant function

$$f = a(X) + b(Z) \qquad \text{(17A-1)}$$

where X is the quick ratio of the firm, Z is its size, and a and b are the parameters we wish to compute. Our purpose is to obtain parameter values such that the average value of f_g in Eq. (17A-1) for good accounts will be significantly larger than the average value of f_b for bad accounts. This notion is illustrated in Figure 17-6, where the discriminant function
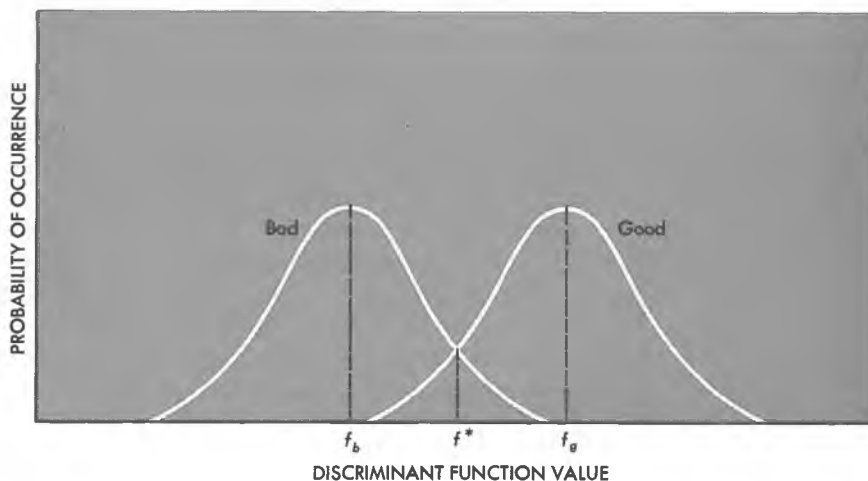
FIGURE 17-6
Universes of good and bad accounts

value is along the horizontal axis and the probability of occurrence is along the vertical. In the figure, two universes of credit applicants are shown, good to the right and bad to the left. The average value, f_b, for bad accounts is seen to be much lower than the average value, f_g, for good accounts. However, we see also that there is an area of overlap between the two universes. In general, the smaller the area of overlap, the better the ability of discriminant analysis to predict good and bad accounts.[20]

Massy, *Quantitative Techniques in Marketing Analysis* (Homewood, Ill.: Richard D. Irwin, Inc., 1962), pp. 95–100.

[20] See D. E. Peterson, *A Quantitative Framework for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc., 1969), pp. 285–89.

The coefficients a and b in Eq. (17A-1) can be computed mathematically from the sample data by

$$a = \frac{Szzdx - Sxzdz}{SzzSxx - Sxz^2} \qquad \text{(17A-2)}$$

$$b = \frac{Sxxdz - Sxzdx}{SzzSxx - Sxz^2} \qquad \text{(17A-3)}$$

where Sxx and Szz represent the variances of variables X and Z, respectively, and Sxz is the covariance of variables X and Z. The difference between the average of X's for good accounts and the average of X's for bad accounts is represented by dx. Similarly, dz represents the difference between the average of Z's for good accounts and the average of Z's for bad accounts. When we solve for a and b, we obtain the parameters of the linear discriminant function in Eq. (17A-1). The ratio a/b determines the slope of the discriminant boundary line.

We now need to determine the minimum cutoff value of the function. The idea is to refuse credit to those accounts with values of f below the cutoff value and extend credit to those with f values above the cutoff value. In theory, we wish to find the discriminant function value in Figure 17-6 denoted by f^*. Using this value for cutoff purposes will minimize the prediction of good accounts when they are bad and the prediction of bad accounts when they are good. To determine the cutoff value in practice, we start by calculating the f_i for each account, given the parameters of Eq. (17A-1). For our example, suppose that we obtained the following f_i values, arranged in ascending order of magnitude.

Account Number	Good or Bad	f_i
7	Bad	0.81
10	Bad	0.97
2	Bad	1.36
3	Bad	1.44
6	Bad	1.65
12	Good	1.77
11	Bad	1.83
4	Good	1.91
1	Good	2.12
8	Good	2.19
5	Good	2.34
9	Good	2.48

We see that there is an area of overlap for accounts 6, 12, 11, and 4. We know that the cutoff value must lie between 1.65 and 1.91. For simplicity, we may want to use the midpoint, 1.78, as our cutoff value. Given the cutoff value, we are able to draw the discriminant boundary line in

Figure 17-5 that discriminates best between good and bad accounts. We note, however, that two of the accounts, 11 and 12, are misclassified, given this cutoff value. Account 11 is classified as a good account when, in fact, it was bad; while account 12 is classified as a bad account when, in fact, it was good. Rather than assign a strict cutoff value, it may be better to allow for misclassification and designate the area between 1.65 and 1.91 as uncertain, requiring further analysis. In theory, this area would correspond to the area of overlap in Figure 17-6.

If we have reason to believe that new credit applicants will not differ significantly from the relationships found for the sample accounts, discriminant analysis can be used as a means for selecting and rejecting credit sale customers. If we use a minimum cutoff value, we will reject all sales in which the f_i value for the credit applicant is less than 1.78 and accept all sales in which the f_i value exceeds 1.78. If a range is used, we will accept all sales in which the prospective customer has a f_i value in excess of 1.91 and reject applicants with f_i values below 1.65. For applicants with f_i values lying between these two values, we might want to obtain additional credit information, along with information as to the profitability of the sale, before making a decision.

Although the example we used is simple, it illustrates the potential of discriminant analysis in selecting or rejecting credit applicants.[21] Discriminant analysis can be extended to include a number of other independent variables. In fact, additional independent variables should be added as long as the benefits of greater predictability exceed the costs of collecting and processing the additional information. For discriminant analysis to have predictive value, the credit applicants being analyzed must correspond to the sample applicants on whom the discriminant-function parameters are based. Where the parameters are no longer realistic, a new sample must be drawn. As experience provides new information, it is important to assess continually the validity of the parameters.

Where a linear discriminant function does not fit the data, it is possible to develop nonlinear functions. Discriminant analysis is sufficiently flexible to be a practical means of evaluating accounts. Because the information is processed on a high-speed computer, time spent on clerical work and credit analysis can be reduced. Credit analysts can concentrate on only those marginal accounts falling in an uncertain area. Discriminant analysis offers an efficient means by which a company can meet the mounting demands on its credit department.

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[21] Edward I. Altman, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance*, XXIII (September, 1968), 589–609, employed discriminant analysis effectively in predicting corporate bankruptcy prior to its occurrence. This study is described in Chapter 24.

1. In order to increase sales from their present annual level of $250,000, the Heap Corporation is considering a more liberal credit policy. Currently, the firm has an average collection period of 30 days; however it is believed that as the collection period is lengthened, sales will increase by the following amounts:

Credit Policy	Increase in Collection Period	Increase in Sales
A	15 days	$10,000
B	30 "	15,000
C	45 "	17,000
D	60 "	18,000

The firm has the following cost pattern at present:

Price of the only product manufactured	$1.00
Variable costs per unit (before taxes)	.60
Average costs per unit (current)	.80

If the firm requires a pretax return on investment of 20 per cent, which credit policy should be pursued? (Assume a 360-day year.)

2. If the only information available to the Heap Corporation (problem 1) was that an increase in the collection period by 60 days would increase sales by $18,000, would the firm increase the collection period?

3. The Heap Corporation (problem 1) has estimated that the following pattern of bad-debt experience will prevail if it initiates more liberal credit terms.

Increase in Collection Period	Per Cent Default
15 days	3%
30 "	6%
45 "	10%
60 "	15%

The current bad-debt loss is 1 per cent. Given the other assumptions in problem 1, which credit policy should be pursued?

4. Recalculate problem 3, assuming the following pattern of bad-debt experience:

Increase in Collection Period	Per Cent Default
15 days	1.5%
30 "	2.0%
45 "	4.0%
60 "	8.0%

5. The Chickee Corporation has a 12 per cent opportunity cost of funds and currently sells on terms of net 10, EOM. The firm has sales of $10 million a year. which are 80 per cent on credit and spread evenly over the year. Currently, the average collection period is 60 days. If Chickee offered terms of 2/10, net 30, 60 per cent of its customers would take the discount, and the collection period

would be reduced to 40 days. Should Chickee change its terms from net/10, EOM to 2/10, net 30?

6. The Pottsville Manufacturing Corporation is considering extending trade credit to the San Jose Company. Examination of the records of San Jose has produced the following financial statements.

San Jose Company
Balance Sheet
(in millions)

	19X1	19X2	19X3
Assets:			
Current assets:			
Cash	$ 1.5	$ 1.6	$ 1.6
Receivables	1.3	1.8	2.5
Inventories (at lower of cost or market)	1.3	2.6	4.0
Other	.4	.5	.4
Total current assets	$ 4.5	$ 6.5	$ 8.5
Fixed assets:			
Buildings (net)	2.0	1.9	1.8
Machinery and equipment (net)	7.0	6.5	6.0
Total fixed assets	$ 9.0	$ 8.4	$ 7.8
Other assets	1.0	.8	.6
Total assets	$14.5	$15.7	$16.9
Liabilities:			
Current liabilities:			
Notes payable (8½%)	$ 2.1	$ 3.1	$ 3.8
Trade payables	.2	.4	.9
Other payables	.2	.2	.2
Total	$ 2.5	$ 3.7	$ 4.9
Term loan (8%)	4.0	3.0	2.0
Total	$ 6.5	$ 6.7	$ 6.9
Net worth			
Common stock	$ 5.0	$ 5.0	$ 5.0
Preferred stock (6½%)	1.0	1.0	1.0
Retained earnings	2.0	3.0	4.0
Total liabilities and equities	$14.5	$15.7	$16.9

San Jose Company
Income Statement

	19X1	19X2	19X3
Net credit sales	$15.0	$15.8	$16.2
Cost of goods sold	11.3	12.1	13.0
Gross profit	3.7	3.7	3.2
Operating expenses	1.1	1.2	1.2
Net profit before taxes	2.6	2.5	2.0
Tax	1.3	1.2	1.0
Profit after taxes	1.3	1.3	1.0
Dividends	.3	.3	0
	$ 1.0	$ 1.0	$ 1.0

The San Jose Company has a Dun & Bradstreet rating of $A_A - 1\frac{1}{2}$. A bank check indicates that the firm generally carries balances in the low seven figures. A trade credit check of five suppliers to San Jose reveals that the firm takes its discounts from the three creditors offering 2/10, net 30 terms, though it is about fifteen days slow in paying the two firms offering terms net 30.

Analyze the San Jose Company's application for credit. What positive factors are present? What negative factors are present?

7. It has been determined that the San Jose Company (problem 6) will purchase about $1,000,000 worth of supplies from Pottsville (problem 6) each year. Further, it has been estimated that San Jose will meet its credit obligations to Pottsville according to the following distribution:

Payment Schedule	Probability
Payment on time (net 30)	.15
Payment one month late	.34
Payment two months late	.30
Payment three months late	.20
No payment (bad debt)	.01

If the marginal profit on each unit sold to San Jose is $5 (price = $100 per unit), should credit be extended? (Assume that no sales would be made to San Jose unless credit were extended. Further assume that Pottsville has an opportunity cost of funds of 2 per cent per month, not compounded.)

8. On the basis of their past trade credit experience, the McDougal Corporation rates customers Good, Fair, or Poor. The cost of a past experience check is $1 and of a Dun & Bradstreet report is $8. The marginal profit per item to McDougal is $5, the marginal cost is $10, and the required return is 20 per cent before taxes. Assuming McDougal runs a past experience check as a matter of policy, the following data may be estimated.

Past Experience	Proportion	Aug. Collection Period	Aug. Collection Cost	Prob. of Bad Debt
Good	40%	35 days	$ .25	.03
Fair	30%	60 "	1.50	.20
Poor	30%	100 "	4.00	.40

(a) Determine the costs of acceptance and rejection for each classification and the order size at which the two cross.

If a Dun & Bradstreet report were obtained, the analyst would classify each account as Yes or No, with the following data:

Experience	Dun & Bradstreet	Prob.	Aug. Collection Period	Aug. Collection Cost	Prob. of Bad Debt
Good	Yes	70%	30 days	$ .20	.01
	No	30%	47 "	.37	.08
Fair	Yes	50%	40 "	.80	.10
	No	50%	80 "	2.20	.30
Poor	Yes	30%	60 "	2.00	.20
	No	70%	117 "	4.86	.49

(b) Determine the costs of acceptance and rejection for each of the above. In each case at what order size would a D&B become desirable?

(c) On the basis of the above analysis, formulate the firm's policy in regard to each type of order.

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Inventory Management

18

Inventories, like receivables, represent a significant portion of most firms' assets, and, accordingly, require substantial investments. In order that this investment not become unnecessarily large, inventories must be managed efficiently. Our purpose is to investigate briefly the means by which efficient management can be achieved and to relate these methods to financial management.

Inventories provide a very important link in the production and sale of a product. For a company engaged in manufacturing, a certain amount of inventory is absolutely necessary in the actual production of the product; this inventory is known as "goods in process." While other types of inventory—namely, in-transit, raw-materials, and finished-goods inventory—are not absolutely necessary in the strictest sense, they are extremely important if the firm is to be at all flexible. For example, inventory in transit, that is, inventory between various stages of production or storage, permits efficient production scheduling and utilization of

resources. Without this type of inventory, each stage of production would be dependent upon the preceding stage's finishing its operation on a unit of production. As a result, there probably would be delays and considerable idle time in certain stages of production.

Raw-materials inventory gives the firm flexibility in its purchasing. Without it, the firm must exist on a hand-to-mouth basis, buying raw material strictly in keeping with its production schedule. Similarly, finished-goods inventory allows the firm flexibility in its production scheduling and in its marketing effort. Production does not need to be geared directly to sales. Moreover, customers generally are better served when the firm has a reasonable supply of finished goods.[1] The advantages of increased inventories, then, are several. The firm can effect economies of production and purchasing and can fill customer orders more quickly. In short, the firm is more flexible. The obvious disadvantages are the total cost of holding the inventory, including storage and handling costs, and the required return on capital tied up in the investment in inventory. Inventories, like accounts receivable, should be increased as long as the resulting savings exceed the total cost of holding the added inventory. The balance finally reached depends upon the estimates of actual savings, the cost of carrying additional inventory, and the efficiency of inventory control.

INVENTORY CONTROL

For a given level of inventory, the efficiency of inventory control affects the flexibility of the firm. Two essentially identical firms with the same amount of inventory may have significantly different degrees of flexibility in operations due to differences in inventory control. Inefficient procedures may result in an unbalanced inventory—the firm may frequently be out of certain types of inventory, and overstock other types, necessitating excessive investment. These inefficiencies ultimately have an adverse effect upon profits. Turning the situation around, differences in the efficiency of inventory control for a given level of flexibility affect the level of investment required in inventories. The less efficient the inventory control, the greater the investment required. Similarly, excessive investment in inventories affects profits adversely. Thus, the effects of inventory control on flexibility and on the level of investment required in inventories represent two sides of the same coin. Our purpose in the subsequent sections is to examine various principles of inventory control.

[1] See John F. Magee, "Guides to Inventory Policy: Functions and Lot Sizes," *Harvard Business Review,* 34 (January–February, 1956).

The economic order quantity (EOQ) is an important concept in the purchase of raw materials and in the storage of finished-goods and in-transit inventories. In our analysis, we wish to determine the optimal order quantity for a particular item of inventory, given its forecasted usage, ordering cost, and carrying cost. Ordering can mean either the purchase of the item or its production. Assume for the moment that the usage of a particular item of inventory is known with certainty. More-over, assume that ordering costs per order, O, are constant regardless of the size of the order. In the purchase of raw materials or other items, these costs represent the clerical costs involved in placing an order as well as certain costs of receiving and checking the goods once they arrive. For finished-goods inventories, ordering costs involve scheduling a production run. For in-transit inventories, ordering costs are likely to involve nothing more than record keeping. The total ordering cost for a period is simply the number of orders for that period, times the cost per order.

Carrying costs per period, C, represent the cost of inventory storage, handling, and insurance, together with the required rate of return on the investment in inventory. These costs are assumed to be constant per unit of inventory, per unit of time. Thus, the total carrying cost for a period is the average number of units of inventory for the period, times the carry-ing cost per unit. In addition, we assume for now that inventory orders are filled immediately, without delay.

If the usage of an inventory item is perfectly steady over a period of time and there is no safety stock, average inventory (in units) can be ex-pressed as:

$$\text{Average Inventory} = \frac{Q}{2} \tag{18-1}$$

where Q is the quantity (in units) ordered and is assumed to be constant for the period. The above problem is illustrated in Figure 18-1. Although the quantity demanded is a step function, we assume for analytical pur-poses that it can be approximated by a straight line. We see that zero inventory always indicates that further inventory must be ordered.

The carrying cost of inventory is the carrying cost per unit, times the average number of units of inventory, or $CQ/2$. The total number of orders for a period of time is simply the total usage (in units) of an item of inventory for that period, S, divided by Q. Consequently, total ordering costs are represented by the ordering cost per order, times the number of orders, or SO/Q. Total inventory costs, then, are the carrying costs plus ordering costs, or

$$\frac{CQ}{2} + \frac{SO}{Q} \tag{18-2}$$
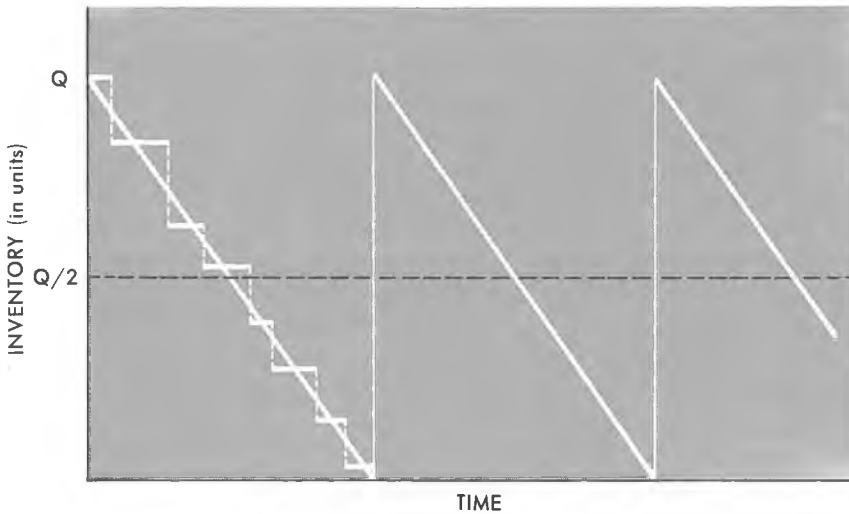
FIGURE 18-1
Order quantity example

We see from Eq. (18-2) that the higher the order quantity, Q, the higher the carrying costs but the lower the total ordering costs. The lower the order quantity, the lower the carrying costs but the higher the total ordering costs. We are concerned with the tradeoff between the economies of increased order size and the added cost of carrying additional inventory.

In order to determine the optimal order quantity, Q^*, we differentiate Eq. (18-2) with respect to Q and set the derivative equal to zero, obtaining

$$Q^* = \sqrt{\frac{2SO}{C}} \qquad \text{(18-3)}$$

This equation is known as the economic lot-size formula. To illustrate its use, suppose that usage of an inventory item is 2,000 units during a 100-day period, ordering costs are $100 an order, and the carrying costs are $10 per unit per 100 days. The most economic order quantity, then, is

$$Q^* = \sqrt{\frac{2(2,000)(100)}{10}} = 200 \text{ units}$$

With an order quantity of 200 units, the firm would order (2,000/200), or ten times, during the period under consideration or, in other words, every ten days. We see from Eq. (18-3) that Q^* varies directly with total usage, S, and order cost, O, and inversely with the carrying cost, C. However, the relationship is dampened by the square-root sign in both cases.

In our example, we have assumed that inventory can be ordered with

no delay. However, there is usually a time lapse in procurement between the time a purchase order is placed and the time the inventory is actually received, or in the time it takes to manufacture an item after an order is placed. This lead time must be considered. If it is constant and known with certainty, however, the optimal order quantity is not affected. In the above example, the firm would still order 200 units at a time and place ten orders during the specified time period, or every ten days. If the lead time for delivery were three days, the firm simply would place its order seven days after delivery of the previous order.

The EOQ function is illustrated in Figure 18-2. In the figure, we plot ordering costs, carrying costs, and total costs—the sum of the first two costs. We see that whereas carrying costs vary directly with the size of
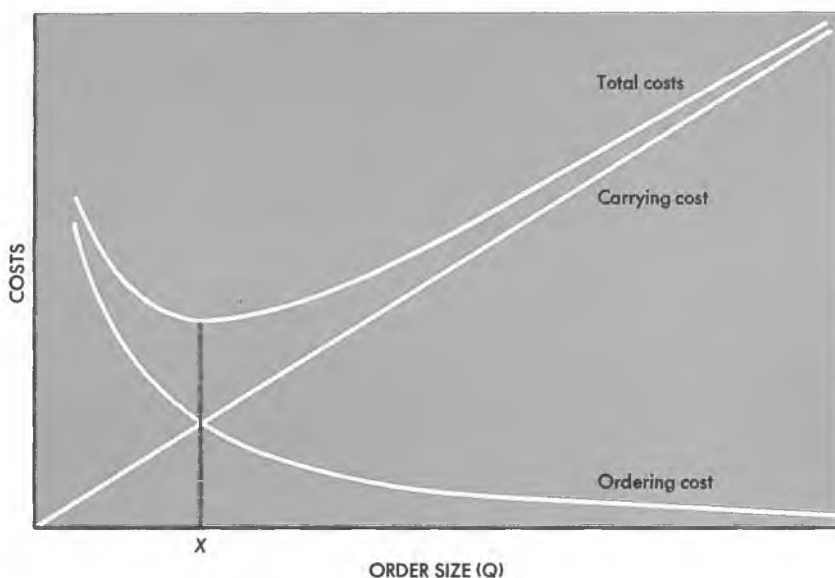
FIGURE 18-2
Economic order quantity relationship

the order, ordering costs vary inversely with the size of the order. The total cost line declines at first as the fixed costs of ordering are spread over more units. The total cost line begins to rise, however, when the decrease in average ordering cost is more than offset by the additional carrying costs. Point X, then, represents the economical order quantity, which minimizes the total cost of inventory.

When the total cost line around the EOQ point is not particularly sensitive to the number of units ordered, it may be appropriate to use an EOQ range instead of a point. To determine this sensitivity, we might

compute the percentage change in total costs for, say, a 10 per cent change in number of units ordered. If the sensitivity is not great, we may find that an EOQ range affords us greater flexibility in ordering without less economy.[2] The EOQ formula taken up in this section is a very useful tool for inventory control. In purchasing raw materials or other items of inventory, it tells us the amount to order and the best timing of our orders. For finished-goods inventory, it enables us to exercise better control over the timing and size of production runs. In general, the EOQ model gives us a rule for deciding when to replenish inventories and the amount to replenish. All inventory models, no matter how complex, address themselves to this problem of the timing and magnitude of replenishment.[3]

UNCERTAINTY AND SAFETY STOCKS

In practice, the demand or usage of inventory generally is not known with certainty; usually it fluctuates during a given period of time. Typically, the demand for finished-goods inventory is subject to the greatest fluctuation. In general, the usage of raw-materials inventory and in-transit inventory, both of which depend upon the production scheduling, is much more predictable. In addition to demand or usage, the lead time required to receive delivery of inventory once an order is placed is usually subject to some variation. Owing to these fluctuations, it is not feasible in most cases to allow expected inventory to fall to zero before a new order is expected to be received, as could be done when usage and lead time were known with certainty.

Most firms maintain some margin of safety, or safety stock; otherwise, they may at times be unable to satisfy the demand for an item of inventory. There are opportunity costs to being out of stock. In the case of finished-goods inventory, the customer is likely to become irritated and may take his business elsewhere. In the case of raw-materials and in-transit inventories, the cost of being out of stock is a delay in production. While this opportunity cost is measured more easily than that associated with finished-goods inventory, a stockout of the latter has a cost; and the firm must recognize it.

Proper Level of Safety Stock. The decision to maintain a safety stock involves balancing the cost of stockouts with the cost of carrying additional inventory. If we know the probability distribution of future demand or usage, we can assess this balance. Suppose, for example, that

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[2] See Arthur Snyder, "Principles of Inventory Management," *Financial Executive,* XXXII (April, 1964), reprinted in James Van Horne, ed., *Foundations for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc., 1966), pp. 75–83.

[3] See Harvey M. Wagner, *Principles of Operations Research—with Applications to Managerial Decisions* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1969), pp. 786–89.

the usage of an inventory item over a ten-day period is expected to be the following:

Usage (in units)	Probability
50	0.04
100	0.08
150	0.20
200	0.36
250	0.20
300	0.08
350	0.04

Moreover, assume an economic order quantity of 200 units every ten days, steady usage, 200 units of inventory on hand at the beginning of the period, and three days lead time required to procure inventory. This lead time is known with certainty, and a new order is always placed at the end of the eighth day for delivery at the end of the eleventh day. If the firm carries no safety stock, there will be no stockouts if usage is 200 units or less. However, if usage proves to be 250 units instead of 200 units, there will be a stockout of 50 units. Similarly, if usage is 300 units or 350 units, there will be stockouts of 100 units and 150 units, respectively. If we know the cost per unit of stockout, we can calculate the expected cost of stockouts and then compare this cost with the cost of carrying additional inventory.

Suppose the stockout cost is $6 a unit, and the average carrying cost for the ten-day period is $1 per unit. In Table 18-1, we show the expected stockout cost, carrying cost, and total cost for various levels of safety stock. We see that the optimal safety stock is 50 units, the level at which the total cost is lowest.

TABLE 18-1
Expected costs associated with various safety stocks

Safety Stock	Stockout	Stockout Cost ($6 per unit)	Probability	Expected Stockout Cost	Carrying Cost	Total Cost
150 units	0	0	0	$ 0	$150	$150
100 units	50	300	0.04	12	$100	$112
50 units	100	600	0.04	24		
	50	300	0.08	24		
				$ 48	$ 50	$ 98
0 units	150	900	0.04	36		
	100	600	0.08	48		
	50	300	0.20	60		
				$144	$ 0	$144

Uncertainty of Lead Time. Suppose that the lead time required for procurement, like demand or usage, is subject to a probability distribution and is expected to vary, with the following probabilities:

Lead Time	Probability
2 days	0.25
3 days	0.50
4 days	0.25

For simplicity of illustration, assume that the usage of inventory on the eleventh day is the same as the average usage during the previous ten. If the probability distributions for usage and lead time are independent, we can calculate their joint probability of occurrence. These calculations are shown in Table 18-2. The last column of the table shows the number of units of stockout if no safety stock is held and there are 200 units of inventory on hand at the beginning of the period. From the table, we see that if usage is 200 units, but lead time is four days instead of three, there

TABLE 18-2
Joint probabilities of usage and lead time

Usage		Lead Time		Joint	Stockout
Units	Probability	Days	Probability	Probability	(in units)
		2	0.25	0.01	None
50	0.04	3	0.50	0.02	None
		4	0.25	0.01	None
		2	0.25	0.02	None
100	0.08	3	0.50	0.04	None
		4	0.25	0.02	None
		2	0.25	0.05	None
150	0.20	3	0.50	0.10	None
		4	0.25	0.05	None
		2	0.25	0.09	None
200	0.36	3	0.50	0.18	None
		4	0.25	0.09	20
		2	0.25	0.05	25
250	0.20	3	0.50	0.10	50
		4	0.25	0.05	75
		2	0.25	0.02	70
300	0.08	3	0.50	0.04	100
		4	0.25	0.02	130
		2	0.25	0.01	115
350	0.04	3	0.50	0.02	150
		4	0.25	0.01	185

is a stockout of 20 units, assuming usage to be 20 units on the eleventh day. Similarly, we are able to calculate the number of units of stockout for other combinations of usage and lead time, in keeping with the above assumptions. For example, if usage is 300 units, or 30 units a day, and lead time is four days, the stockout is 130 units or the sum of the stockout occasioned by additional usage, 100 units, and by the additional lead time, 30 units.

Given the information in the last two columns of Table 18-2, together with the cost per unit of stockout and the carrying cost of inventory, we can determine the optimal level of safety stock for the period in a manner similar to that shown in Table 18-1. These calculations are shown in Table 18-3. We see from the table that the optimal safety stock is 70 units, the level at which the expected stockout cost plus carrying cost is lowest.

TABLE 18-3
Determination of Total Cost

Safety Stock	Incremental Stockout	Incremental Stockout Cost ($6 per unit)	Cumulative Probability	Incremental Expected Stockout Cost	Cumulative Expected Stockout Cost	Carrying Cost	Total Cost
185	0	$ 0	0	$ 0	$ 0	$185	$185.00
150	35	210	0.01	2.10	2.10	150	152.10
130	20	120	0.03	3.60	5.70	130	135.70
115	15	90	0.05	4.50	10.20	115	125.20
100	15	90	0.06	5.40	15.60	100	115.60
75	25	150	0.10	15.00	30.60	75	105.60
70	5	30	0.15	4.50	35.10	70	105.10
50	20	120	0.17	20.40	55.50	50	105.50
25	25	150	0.27	40.50	96.00	25	121.00
20	5	30	0.32	9.60	105.60	20	125.60
0	20	120	0.41	49.20	154.80	0	154.80

ORDER POINT FORMULA

Because the probabilistic analysis taken up in the previous section is cumbersome, many people find it unfeasible for a multiperiod problem. Instead, they propose using an order point, whereby an order is placed once inventory reaches so many units.[4] With uncertain usage, there is usually some probability of a stockout. The object is to reduce this probability to a tolerable level. Thus, the firm specifies an acceptable stockout percentage and then calculates an order point based upon this percentage, as well as upon other factors. The optimal order point is the level of inventory at which we should order the economic order quantity

[4]See Arthur Snyder, "Principles of Inventory Management," *Financial Executive,* XXXII (April, 1964), reprinted in James Van Horne, ed., *Foundations for Financial Management,* pp. 72–74, 89–90. This section is based upon his article, although the concept of an order point is found in most of the literature on inventory theory.

of additional stock. It is the point at which forecasted usage of an item of inventory, given a stockout tolerance, would just exhaust the existing inventory during the lead time required to procure additional inventory,

$$\text{Order Point*} = S(L) + F\sqrt{SR(L)} \tag{18-4}$$

where S is the usage, L is the lead time required to obtain additional inventory once an order is placed, R is the average number of units per order, and F is a stockout acceptance factor.

If 300 units were ordered during a period of time, with twenty orders, R would be fifteen. The stockout acceptance factor is based upon the probability distribution of usage. If demand or usage is distributed according to a Poisson distribution, F can be determined as in Figure 18-3.[5] For example, if the stockout acceptance percentage is specified as 10 per cent, we see from the figure that this percentage corresponds to an acceptance factor of 1.29.
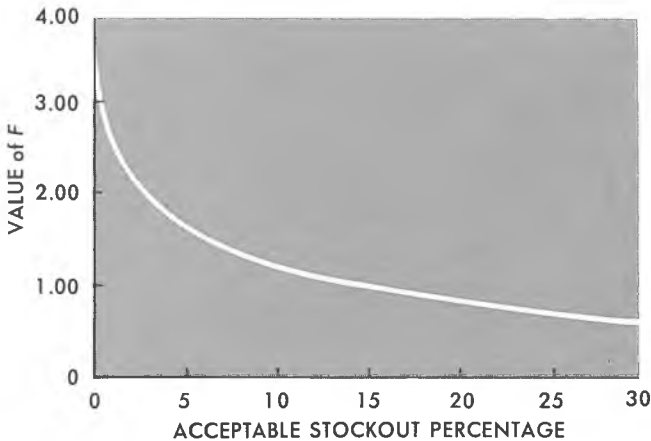
FIGURE 18-3
Stockout acceptance factor.
Source: Snyder, op. cit., p. 89.

To illustrate the use of Eq. (18-4), suppose that we consider a problem in which usage, S, is 100 units per month, lead time, L, is one-half month, the average number of units per customer order, R, is five, and the acceptable stockout percentage is 10 per cent. The optimal order point, then, would be

$$100(\tfrac{1}{2}) + 1.29\sqrt{(100)(5)(\tfrac{1}{2})} = 70 \text{ units}$$

Thus, the firm should reorder when inventory reaches 70 units. We note that the order point includes a safety stock, which is determined by the

[5]For a discussion of Poisson probability distributions, see William Feller, *Probability Theory and its Applications*, 2nd ed. (New York: John Wiley & Sons, Inc., 1957), pp. 142–49.

stockout acceptance percentage we specify. If there were no safety stock, the order point would be at 50 units, or one-half of one month's usage. The one-half month takes into account the lead time required to actually receive inventory once an order is placed. The order point formula given in Eq. (18-4) minimizes the investment in inventory relative to an acceptable level of stockout.[6]

This formula is illustrated further in Figure 18-4 in conjunction with an economic order quantity. If demand is reasonably steady over a long
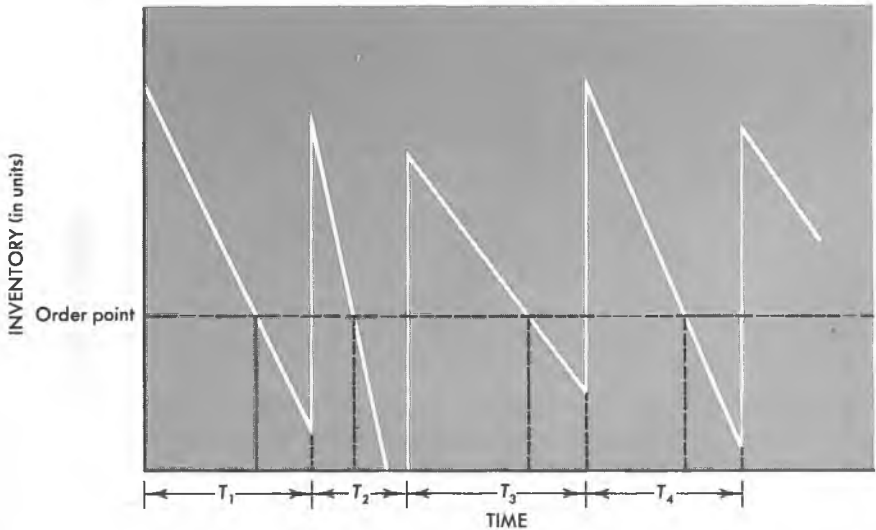
FIGURE 18-4
Order point example

period of time, the firm might employ the EOQ formula and order the same amount of inventory whenever the level of inventory touched the order point. In the figure, the order quantity is represented by the vertical distances and is the same for each order. The lead time for actual delivery of inventory is assumed to be known with certainty and to be the same for each order. However, usage is assumed to be subject to uncertainty. As a result, the time intervals T_1, T_2, T_3, and T_4 are not equal; and the low points in the level of inventory are not the same. Indeed, during period T_2, inventory reaches zero and the firm is out of stock until the new order is received. The inventory control system illustrated in Figure 18-4 is known as a "two-bin system," in which a constant re-

[6]For an analysis of the order point when usage is assumed to be normally distributed, see Harold Bierman, Jr., Charles P. Bonini, and Warren H. Hausman, *Quantitative Analysis for Business Decisions* (Homewood, Ill.: Richard D. Irwin, Inc., 1969), Chapter 12.

plenishment order is placed whenever a critical level of inventory is reached. It combines the economical order quantity formula with the order point formula to produce an optimal control system.

When the size of the order point is reduced, less inventory is held on the average and carrying costs are reduced. However, the probability of stockout is increased. Likewise, when the order quantity is reduced, carrying costs are reduced and the probability of stockout increased. Because total costs are affected by both the order point and the order quantity, an optimal control system must embody both of these factors.

INVENTORY CONTROL AND THE FINANCIAL MANAGER

The inventory control methods described in the last several sections give us a means for determining an optimal level of inventory, as well as how much should be ordered and when. These tools are necessary for managing inventory efficiently and balancing the advantages of additional inventory against the cost of carrying this inventory. With the use of computers, great improvements in inventory control have been made and are continuing to be made. Unfortunately, a review of the many applications of operations research to inventory management is beyond the scope of this book.[7]

Although inventory management usually is not the direct operating responsibility of the financial manager, the investment of funds in inventory is a very important aspect of financial management. Consequently, the financial manager must be familiar with ways to control inventories effectively so that capital may be allocated efficiently. The greater the opportunity cost of funds invested in inventory, the lower the optimal level of average inventory and the lower the optimal order quantity, all other things held constant. This statement can be verified by increasing the carrying costs, C, in Eq. (18-3).

When demand or usage of inventory is uncertain, the financial manager may try to effect policies that will reduce the average lead time required to receive inventory once an order is placed. The lower the average lead time, the lower the safety stock needed and the lower the total investment in inventory, all other things held constant. The greater the opportunity cost of funds invested in inventory, the greater the incentive to reduce this lead time. In the case of purchases, the purchasing department may try to find new vendors that promise quicker delivery or place pressure on existing vendors for faster delivery. In the case of finished goods, the production department may be able to schedule production runs for faster delivery by producing a smaller run. In either

[7] For such a review, see Arthur F. Veinott, Jr., "The Status of Mathematical Inventory Control," *Management Science,* 12 (July, 1966), 745–77.

case, there is a tradeoff between the added cost involved in reducing the lead time and the opportunity cost of funds tied up in inventory. This discussion serves to point out the importance of inventory management to the financial manager. The greater the efficiency with which the firm manages its inventory, the lower the required investment in inventory, all other things held constant.

SUMMARY

The optimal level of inventories should be judged in relation to the flexibility inventories afford. If we hold constant the efficiency of inventory management, the lower the level of inventories, the less the flexibility of the firm. The higher the amount of inventories, the greater the flexibility of the firm. In evaluating the level of inventories, management must balance the benefits of economies of production, purchasing, and increased product demand against the cost of carrying the additional inventory. Of particular concern to the financial manager is the cost of funds invested in inventory.

The efficiency of inventory control very much affects the flexibility of the firm, given a level of inventory. Conversely, given a degree of flexibility, efficiency affects the level of inventory investment. In this chapter, we have examined several tools of inventory control. One is the economic order quantity (EOQ), whereby we determine the optimal size of order to place, on the basis of the demand or usage of the inventory, the ordering costs, and the carrying costs. Under conditions of uncertainty, the firm usually must provide for a safety stock, owing to fluctuations in demand for, or usage of, inventory and in lead times. Another tool — the order point formula — tells us the optimal point at which to reorder a particular item of inventory. Together, these tools provide the means for determining an optimal average level of inventory for the firm.

PROBLEMS

1. A college book store is attempting to determine the optimal order quantity for a popular book on financial management. The store sells 5,000 copies of this book a year at a retail price of $12.50, although the publisher allows the store a 20 per cent discount from this price. The store figures that it costs $1 per year to carry a book in inventory and $100 to prepare an order for new books.
   (a) Determine the total costs associated with ordering one, two, five, ten, and twenty times a year.
   (b) Determine the economic order quantity.

2. The Hedge Corporation manufactures only one product, planks. The single raw material used in making planks is the dint. For each plank manufactured, twelve dints are required. Assume that the company manufactures 150,000

planks per year, that demand for planks is perfectly steady throughout the year, that it costs $200 each time dints are ordered, and that carrying costs are $8 per dint per year.

(a) Determine the economic order quantity of dints.

(b) What are total inventory costs for Hedge (carrying costs plus ordering costs)?

(c) How many times per year would inventory be ordered?

3. The Seguro Corporation has determined that its only raw material has an economic order quantity of 1,000 units every thirty days. Further, the firm knows with certainty that a four-day lead time is required for ordering. It has been estimated that the following inventory usage distribution will prevail each month.

Usage (in units)	Probability
900	.06
950	.14
1000	.30
1050	.16
1100	.13
1150	.10
1200	.07
1250	.04

If stockouts would cost the firm $10 per unit, and the average monthly carrying cost is $1 per unit:

(a) Determine the optimal safety stock.

(b) What is the probability of being out of stock?

4. The Seguro Corporation (problem 3) has found that the lead time required for procurement is not known with certainty but is, in fact, subject to risk. The following lead time distribution has been estimated:

Lead Time	Probability
3 days	.20
4 days	.30
5 days	.30
6 days	.20

Assuming that the probability distributions for usage and lead time are independent, that 1,000 units of inventory were on hand at the beginning of the month, and that daily inventory usage does not vary, what is the optimal safety stock?

5. The Apex Company has a policy of reordering raw materials when inventory levels reach a certain point. The firm has determined that it uses 50,000 units of inventory per month, that a lead time of ten days is required, that 5,000 orders necessitating equal raw materials usage are filled on the average each month, and that the stockout-acceptance percentage is 10 per cent (demand is Poisson distributed).

(a) What is the optimal order point?

(b) What would be the order point with no safety stock?

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SHORT- AND INTERMEDIATE-TERM FINANCING

PART

VI

Trade Credit
and
Commercial Paper

19

The three major sources of short-term financing that we examine are trade credit, commercial paper, and short-term loans. In this chapter, we consider the first two methods of financing; the last is examined in Chapter 20. From our analysis of working capital management in Chapter 15, we assume that the firm has decided on a proper proportion of short-term financing to other types of financing, i.e., the maturity composition of its debt. The decisions to be made here then, are what types of short-term financing should be employed and what their composition should be. In this chapter and the next, we analyze alternative sources of short-term financing and see how they may be used to finance seasonal and temporary fluctuations in funds requirements, as well as the more permanent needs of the firm. One source of short-term funds not considered is accrued expenses. While representing a significant current liability for most firms, this source of funds does not represent an active decision variable.

Trade credit is a form of short-term financing common to almost all businesses. In fact, it is the largest source of short-term funds for business firms collectively. In an advanced economy, most buyers are not required to pay for goods upon delivery but are allowed a short deferment period before payment is due. During this period, the seller of the goods extends credit to the buyer. Because suppliers generally are more liberal in the extension of credit than are financial institutions, trade credit is an important source of funds for small companies in particular.

There are three types of trade credit: open account, notes payable, and trade acceptances. By far the most common type is the open-account arrangement. With this arrangement, the seller ships goods to the buyer along with an invoice that specifies the goods shipped, the price, the total amount due, and the terms of the sale. Open-account credit derives its name from the fact that the buyer does not sign a formal debt instrument evidencing the amount that he owes the seller. The seller extends credit based upon his credit investigation of the buyer (see Chapter 17).

In some situations, promissory notes are employed instead of open-account credit. In this case, the buyer is asked to sign a note that evidences his debt to the seller. The note itself calls for the payment of the obligation at some specified future date. Promissory notes have been used in such lines of business as furs and jewelry. This arrangement is employed where the seller wants the buyer to recognize his debt formally. For example, a seller might request a promissory note from a buyer if the latter's open account became past due.

A trade acceptance is another arrangement by which the indebtedness of the buyer is recognized formally. Under this arrangement, the seller draws a draft on the buyer ordering him to pay the draft at some date in the future. The seller will not release the goods until the buyer accepts the time draft.[1] When the buyer accepts the draft, he designates a bank at which the draft will be paid when it comes due. At that time, the draft becomes a trade acceptance; and, depending upon the credit worthiness of the buyer, it may possess some degree of marketability. If the trade acceptance is marketable, the seller of the goods can sell it at a discount and receive immediate payment for the goods. At final maturity, the holder of the acceptance presents it to the designated bank for collection.

TERMS OF SALE

As the use of promissory notes and trade acceptances is rather limited, the subsequent discussion will be confined to open-account trade credit. With this type of credit, the terms of the sale are an important consideration. These terms, which are specified in the invoice, may be placed in

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[1] If the instrument is a sight draft, the buyer is ordered to pay the draft upon presentation. Under this arrangement, trade credit is not extended.

several broad categories according to the net period within which payment is expected and according to the terms of the cash discount.

C.O.D. and C.B.D.—No Extension of Credit. C.O.D. terms mean cash on delivery of the goods. The only risk that the seller undertakes in this type of arrangement is that the buyer may refuse the shipment. Under such circumstances, the seller will be stuck with the shipping costs. Occasionally, a seller might ask for cash before delivery (C.B.D.) to avoid all risk. Under either C.O.D. or C.B.D. terms, the seller does not extend credit. C.B.D. terms must be distinguished from progress payments, which are very common in certain industries. With progress payments, the buyer pays the manufacturer at various stages of production prior to the actual delivery of the finished product. Because large sums of money are tied up in work in progress, aircraft manufacturers request progress payments from airlines in advance of the actual delivery of aircraft.

Net Period—No Cash Discount. When credit is extended, the seller specifies the period of time allowed for payment. For example, the terms, net 30, indicate that the invoice or bill must be paid within thirty days. If the seller bills on a monthly basis, it might require such terms as net/15 EOM, which means that all goods shipped before the end of the month must be paid for by the fifteenth of the following month.

Net Period with Cash Discount. In addition to extending credit, the seller may offer a cash discount if the bill is paid during the early part of the net period. The terms, 2/10, net 30, indicate that the buyer is offered a 2 per cent discount if the bill is paid within ten days; if he does not pay within ten days, he must pay the full amount of the bill within thirty days. A cash discount differs from a trade discount and from a quantity discount. With a trade discount, one type of customer (a wholesaler, for example) is given a lower price on goods purchased than is another type of customer, say a retailer. With a quantity discount, a customer is given a discount if the shipment is above a certain amount. Under most circumstances, a cash discount is offered as an incentive to the buyer to pay early. In Chapter 17, we considered the question of the optimal cash discount to be offered by a seller.

Datings. Datings are used frequently in a seasonal business, where the seller wishes to encourage customers to place their orders before a heavy selling period. For example, a manufacturer of lawn mowers may give seasonal datings specifying that any shipment to a dealer in the winter or spring does not have to be paid for until summer. The arrangement is beneficial to the seller because, with earlier orders, he can gauge his demand more realistically and schedule production more efficiently. Also, the seller does not have to store certain finished-goods inventory.

The advantage of datings to the buyer is that he does not have to pay for the goods until he is able to sell them. Under this arrangement, credit is extended for a longer than normal period of time.

TRADE CREDIT AS A MEANS OF FINANCING

We have seen that trade credit is a source of funds, because the buyer does not have to pay for goods until after they are delivered. If the firm automatically pays its bills a certain number of days after the date of invoice, trade credit becomes a built-in source of financing that varies with the production cycle. As the firm increases its production and corresponding purchases, accounts payable increase and provide part of the funds needed to finance the increase in production. As production decreases, accounts payable tend to decrease. Although the variation of accounts payable with production may not be directly proportional, on account of shortages or gluts in inventory on hand, there is a strong degree of correspondence.

If a firm adheres strictly to the practice of paying its bills at a given time after invoice, trade credit is not a discretionary source of financing. It is entirely dependent upon the purchasing plans of the firm, which, in turn, are dependent upon its production cycle. In examining trade credit as a discretionary form of financing, we want to consider specifically situations in which (1) a firm does not take a cash discount but pays on the last day of the net period and (2) a firm pays its bills beyond the net period.

PAYMENT ON THE FINAL DUE DATE

In this section, we assume that the firm foregoes a cash discount but does pay its bill on the final due date of the net period. If no cash discount is offered, there is no cost for the use of credit during the net period. By the same token, if a firm takes the discount, there is no cost for the use of trade credit during the discount period. However, if a cash discount is offered and it is not taken, there is a definite opportunity cost. For example, if the terms of sale are 2/10, net 30, the firm has the use of funds for an additional twenty days if it does not take the cash discount but pays on the final day of the net period. In the case of a $100 invoice, it would have the use of $98 for twenty days. The annual interest cost is[2]

$$\frac{2}{98} \times \frac{360}{20} = 36.7 \text{ per cent}$$

[2] For ease of calculation, 360 rather than 365 is used as the number of days in the year.

Thus, we see that trade credit can be a very expensive form of short-term financing when a cash discount is offered.

The cost of trade credit declines the longer the net period is in relation to the discount period. For example, had the terms in the above example been 2/10, net 60, the annual interest cost would have been

$$\frac{2}{98} \times \frac{360}{50} = 14.7 \text{ per cent}$$

The relationship between the annual interest cost of trade credit and the number of days between the end of the discount period and the end of the net period is shown in Figure 19-1. In the figure, we assume 2/10 discount terms. We see that the cost of trade credit decreases at a decreasing rate as the net period increases. The point is that if a firm does
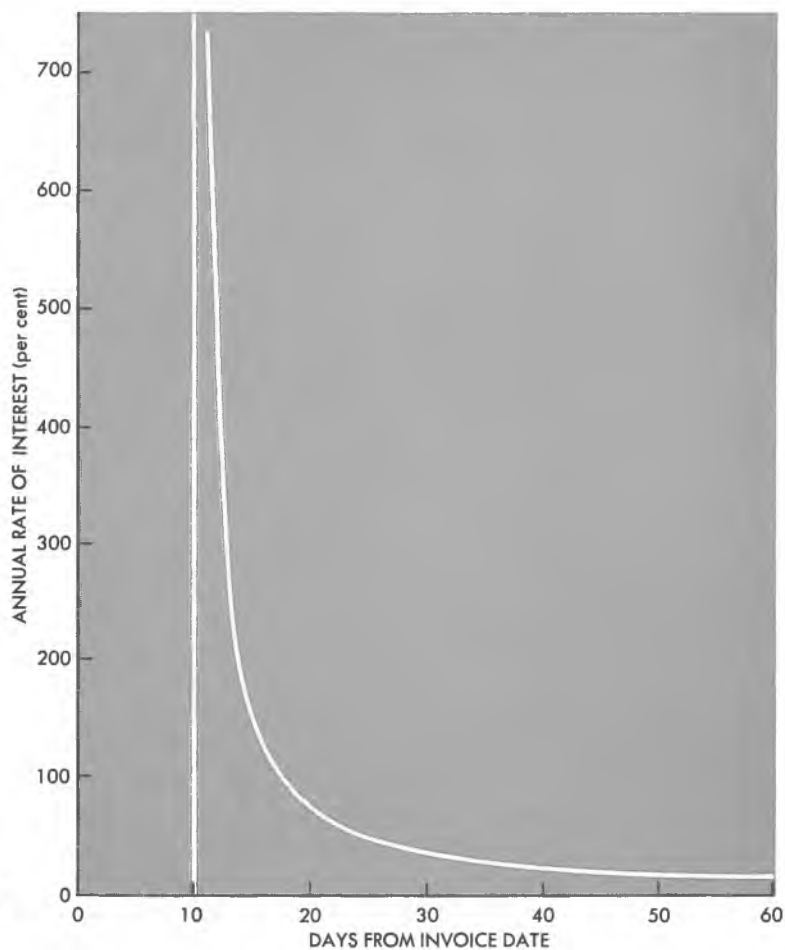
FIGURE 19-1
Annual rate of interest on accounts payable with terms of 2/10, net 30

not take a cash discount, its cost of trade credit declines the longer it is able to postpone payment.

The following terms have been used by tufters in the carpet and floor covering industry:[3]

$$5/10, \ 4/70, \ \text{net } 71.$$

These terms mean that if a firm pays within ten days after invoicing, it is entitled to a 5 per cent cash discount; while if it pays between day 10 and day 70, it is entitled to a 4 per cent discount. The final due date is seventy-one days after invoicing. If a purchaser pays on day 70, it foregoes a one per cent higher discount for the use of funds from day 10 to day 70. If the invoice is for $100, the annual interest cost is

$$\frac{1}{95} \times \frac{360}{60} = 6.3 \text{ per cent.}$$

Thus, the cost of foregoing the 5 per cent discount in favor of the 4 per cent one is relatively low in this case, and trade credit is an attractive means of financing. With these terms, the seller creates a powerful incentive to pay on the seventieth day. No one should pay on the final due date, for the cost of credit for the day is astronomical, as can be easily determined.

STRETCHING ACCOUNTS PAYABLE

In the previous section, we assumed that payment was made at the end of the due period. However, a firm may postpone payment beyond this period; we shall call this postponement "stretching" accounts payable or "leaning on the trade." The cost of stretching accounts payable is twofold: the cost of the cash discount foregone and the possible deterioration in credit rating. In Chapter 17, we discussed the rating system of such credit agencies as Dun & Bradstreet. If a firm stretches its payables excessively so that trade payables are significantly delinquent, its credit rating will suffer. Suppliers will view the firm with apprehension and may insist upon rather strict terms of sale if, indeed, they sell at all. Also, banks and other lenders do not regard excessive slowness in the trade very favorably in assessing a company. Although it is difficult to measure, there is certainly an opportunity cost to a deterioration in a firm's credit reputation.

Notwithstanding the possibility of a deteriorating credit rating, it may be possible to postpone certain payables beyond the net period without severe consequences. Suppliers are in business to sell goods, and trade credit may be a very important sales tool. A supplier may well be willing

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[3]This example is drawn from John J. Brosky, *The Implicit Cost of Trade Credit and Theory of Optimal Terms of Sale* (New York: Credit Research Foundation, 1969), p. 3.

to go along with a certain stretching of his payables, particularly if the risk of bad-debt loss is negligible. If the funds requirement of the firm is seasonal, suppliers may not view the stretching of payables in an unfavorable light during periods of peak requirements, provided that the firm is current in the trade during the rest of the year. However, there may be an indirect charge for this extension of credit in the form of higher prices. The firm should be particularly careful to consider this possibility in evaluating the cost of stretching accounts payable.

Periodic and reasonable stretching of payables is not necessarily bad per se. It should be evaluated objectively in relation to its cost and in relation to alternative sources of short-term credit. When a firm does stretch its payables, effort should be made to keep suppliers fully informed of its situation. A large number of suppliers will allow a firm to stretch payables if the firm is honest with the supplier and consistent in its payments. Sometimes a firm with seasonal funds requirements is able to obtain a dating from a supplier. When a firm obtains a dating, it does not stretch its payables; as long as it pays the bill by the final date, no deterioration in its credit ratings is likely.

ADVANTAGES OF TRADE CREDIT

The firm must balance the advantages of trade credit as a discretionary source of financing against the cost of foregoing a cash discount and the opportunity cost associated with a possible deterioration in credit reputation if it stretches its payables. There are several advantages of trade credit as a form of short-term financing. Probably the major advantage is its ready availability. The accounts payable of most firms represent a continuous form of credit. There is no need to arrange financing formally; it is already there. If the firm is now taking cash discounts, additional credit is readily available by not paying existing accounts payable until the end of the net period. There is no need to negotiate with the supplier; the decision is entirely up to the firm. In the case of stretching accounts payable, it will become necessary, after a certain degree of postponement, to negotiate with the supplier.

In other types of short-term financing, it is necessary to negotiate formally with the lender over the terms of the loan. The lender may impose restrictions on the firm and seek a secured position. Restrictions are possible with trade credit, but they are not nearly as likely. With other sources of short-term financing, there may be a lead time between the time the need for funds is recognized and the time the firm actually is able to borrow them. Trade credit is a more flexible means of financing. The firm does not have to sign a note, pledge collateral, or adhere to a strict payment schedule on the note. A supplier views an occasional delinquent payment with a far less critical eye than does a banker or other lender.

Trade credit is advantageous to small firms that have difficulty obtaining credit elsewhere, or cannot obtain it at all. In periods of tight money, it has been contended that large firms obtain credit more easily than small firms do. However, small firms still have access to trade credit as a means of financing; often this credit comes from large suppliers who, in turn, avail themselves of other sources of financing. Access to trade credit by small firms is thus said to cushion the discrimination in credit allocation during periods of tight money.[4]

The advantages of using trade credit must be weighted against the cost. As we have seen, the cost may be very high, when all factors are considered. Many firms utilize other sources of short-term financing in order to be able to take advantage of cash discounts. The savings in cost over other forms of short-term financing, however, must offset the flexibility and convenience of trade credit. For certain firms, moreover, there simply are no alternative sources of short-term credit.

WHO BEARS THE COST?

It is important to recognize that trade credit involves a cost for the use of funds over time. In the previous sections, it was implied that there is no explicit cost to trade credit if the buyer pays the invoice during the discount period or during the net period, if no cash discount is given. Although this supposition is valid from the standpoint of marginal analysis, it overlooks the fact that somebody must bear the cost of trade credit, for the use of funds over time is not free. The burden may fall on the supplier, the buyer, or both parties. The supplier may be able to pass the cost on to the buyer in the form of higher prices. In the case of a product for which demand is elastic, however, the supplier may be reluctant to increase prices and may end up absorbing most of the cost of trade credit. Under other circumstances, the supplier is able to pass the cost on to the buyer. The buyer should determine who is bearing the cost of trade credit; if he finds that he is bearing the cost, he may want to consider other suppliers to see if he can do better elsewhere.

COMMERCIAL PAPER

Large, well-established companies sometimes borrow on a short-term basis through commercial paper. Commercial paper consists of unse-

[4] See Allan H. Meltzer, "Monetary Policy and the Trade Credit Practices of Business Firms," in Commission on Money and Credit, *Stabilization Policies* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963), p. 494; Meltzer, "Mercantile Credit, Monetary Policy and the Size of Firms," *Review of Economics and Statistics*, XLII (November, 1960), 429–37, and Thomas Mayer, "Trade Credit and the Discriminatory Effects of Monetary Policy," *National Banking Review* (June, 1966), pp. 543–45.

cured short-term negotiable promissory notes sold in the money market. Because these notes are unsecured and are a money-market instrument, only the most credit-worthy companies are able to use commercial paper as a source of short-term financing. The development of the commercial paper market in this country began in the colonial period. Its explosive growth in recent years has been closely associated with the growth of the economy as a whole and the growth of installment financing of automobiles and other durable consumer goods. In addition, part of the recent growth in commercial paper financing is attributable to the fact that banks have curtailed credit in general, and credit to finance companies in particular, during periods of tight money. Increasingly, borrowers have turned to the commercial paper market as an alternative source of financing.

MARKET FOR COMMERCIAL PAPER

The commercial paper market is composed of two parts: the dealer market and the direct placement market.[5] Industrial firms, utilities, and medium-sized finance companies sell commercial paper through dealers. The dealer organization is composed of five major dealers, who purchase commercial paper from the issuer and, in turn, sell it to investors. The typical commission a dealer earns is one-eighth per cent, and maturities on dealer-placed paper generally range from one to six months. The market is a highly organized and sophisticated one; paper is sold in denominations ranging from $25,000 to several million dollars. While the dealer market has been characterized in the past by a significant number of issuers who borrow on a seasonal basis, the trend definitely is toward financing on a revolving or more permanent basis.

Table 19-1 shows the tremendous surge in commercial paper placed through dealers during the 1966 through 1969 period. This growth resulted in part from industrial firms and utilities discovering commercial paper as an appropriate alternative source of funds in periods of tight money. During these periods, commercial banks are not able to accommodate their demand for loans; therefore, the utilities and industrial firms are forced to seek other sources of short-term financing.

Since the 1920s, a number of large sales finance companies, such as General Motors Acceptance Corporation, C.I.T. Financial Corporation, and Commercial Credit Corporation, have bypassed the dealer organization in favor of selling their paper directly to investors. These issuers tailor both the maturity and the amount of the note to the needs of investors, most of which are large corporations with excess cash. Maturities on directly placed paper can range from as little as a few days up

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[5] For a discussion of commercial paper from the standpoint of a short-term investor, see Chapter 16.

TABLE 19-1
Commercial paper rates and amounts outstanding 1960–1969

	Commercial Paper Outstanding at December 31 (in millions)			Commercial Paper Average Interest Rate for Year		
	Total	Dealer-placed	Directly Placed	Dealer-Placed 4–6 Mos.	Directly Placed 3–6 Mos.	Prime Rate on Bank Loans at June 30
1960	$ 4,497	$ 1,358	$ 3,139	3.85%	3.54%	5.00%
1961	4,686	1,711	2,975	2.97	2.68	4.50
1962	6,000	2,088	3,912	3.26	3.07	4.50
1963	6,747	1,928	4,819	3.55	3.40	4.50
1964	8,361	2,223	6,138	3.97	3.83	4.50
1965	9,058	1,903	7,155	4.38	4.27	4.50
1966	13,279	3,089	10,190	5.55	5.42	5.75
1967	16,535	4,901	11,634	5.10	4.89	5.50
1968	20,497	7,201	13,296	5.90	5.69	6.50
1969	31,624	11,817	19,807	7.83	7.16	8.50

Source: Federal Reserve Bulletins.

to nine months. Unlike many industrial issuers, finance companies use the commercial paper market as a permanent source of funds. With the development of the direct-placement market, pockets of idle investment funds have been tapped for short-term financing purposes. As shown in Table 19-1, directly placed paper has recently accounted for 60 to 80 per cent of the total commercial paper outstanding. The growth in direct paper was caused by the increased demand for consumer credit along with the substitution of commercial paper for bank credit by large finance companies. Still, the percentage growth in dealer paper for the 1966–69 period far outstripped the rise in direct paper; as a result, the percentage of direct to total paper declined.

ADVANTAGES TO BORROWER

The principal advantage of commercial paper as a source of short-term financing is that it is generally cheaper than a short-term business loan from a commercial bank. Usually, the rate on prime commercial paper is 0.25 per cent to 2 per cent lower than the prime rate for bank loans to the highest-quality borrower. The differential tends to increase in periods of easy money and to decrease in periods of tight money. It is important to recognize that, unlike the prime rate on bank loans, commercial paper rates fluctuate considerably in keeping with money-market conditions. Table 19-1 shows the average rates for dealer-placed and directly placed commercial paper as well as the prime rate on business loans since 1960. In assessing commercial paper as a means of financing, the firm should weigh the relative cost and availability in comparison with alternative sources of funds. In this comparison, the cost of bank credit

should be adjusted upward for compensating-balance requirements (see the next chapter).

Many companies consider commercial paper a desirable supplement to bank credit. Ideally, a company would borrow heavily through commercial paper when the interest-rate differential was wide and borrow more from banks when the differential narrowed. This strategy would result in the lowest average interest cost and the maximum flexibility. However, commercial banks do not look favorably on credit requests only in periods of tight money. Switching from commercial paper to bank borrowings is possible, but a company must be careful not to impair relations with its bank. The commercial paper market is highly impersonal. If a firm cannot borrow from a commercial bank, it is at the mercy of the market.

In the tight money periods of 1966 and 1969, many companies had to increase their borrowings through commercial paper, partly because banks limited the amount of credit they would extend. A firm should not be too callous in its treatment of banks when the interest-rate cycle is such that commercial paper is the much cheaper form of short-term financing. In periods of easy money, banks are interested in making loans and will remember favorably the firm that makes reasonable use of bank credit during those times.[6] Still, it is evident that in a number of cases, commercial paper is supplanting rather than supplementing the use of bank credit. The proportion of commercial paper to business loans at banks has risen steadily in recent years, attesting to the popularity of commercial paper as a method of short-term financing. Availability, rather than possible cost advantage, is the principal cause for this shift. Financial managers have had to seek short-term financing from other than traditional sources.

Another advantage of commercial paper is the legal limitation on the size of a loan that a commercial bank may extend. The maximum loan a national bank can make to a single borrower is 10 per cent of its capital and surplus. The total borrowing requirements of the three largest sales finance companies exceed the legal lending limits of the fifty largest banks in this country. Consequently, these companies must turn to other sources of short-term financing—namely, direct investors. A possible disadvantage of commercial paper, however, is that it cannot be paid prior to maturity. With a bank loan, the borrower can prepay the note when he has surplus funds. This prepayment, and the flexibility it affords, is not possible with commercial paper; it can be redeemed only at maturity. Finally, because only well-established firms with high credit ratings can issue commercial paper, being in this select group may give the firm a certain amount of prestige. Obviously, this advantage alone would not be sufficient reason for issuing commercial paper.

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[6] To assure flexibility, commercial paper dealers require an issuer to maintain bank lines of credit in excess of its total peak borrowing requirements.

SUMMARY

Trade credit can be an important source of short-term financing for the firm. However, it is a discretionary source of financing only if a firm does not have a strict policy with respect to the number of days after invoice a bill is paid. When a cash discount is offered but not taken, the cost of trade credit is the cash discount foregone. However, the longer the period between the end of the discount period and the time the bill is paid, the less this opportuniy cost. "Stretching" accounts payable involves postponement of payment beyond the due period. The opportunity cost of stretching payables is the possible deterioration in the firm's credit rating. The firm must balance the costs of trade credit against its advantages and the costs of other short-term credit. The major advantage of trade credit is the flexibility it gives the firm.

Commercial paper is used only by well-established, high-quality companies. The evidence of debt is an unsecured short-term promissory note that is sold in the money market. Commercial paper is sold either through dealers or directly to investors. The latter method is used by large sales finance companies, and about two-thirds of commercial paper is placed in this manner. The principal advantage of commercial paper is that its yield is less than the rate of interest a company would have to pay on a bank loan. When used properly, it therefore is a very desirable source of short-term funds.

PROBLEMS

1. Determine the annual percentage interest cost for each of the following terms of sale, assuming the firm does not take the cash discount but pays on the final day of the net period (assume a 360-day year).
   (a) 1/20, net 30 ($500 invoice)
   (b) 2/30, net 60 ($1,000 invoice)
   (c) 2/5, net 10 ($100 invoice)
   (d) 3/10, net 30 ($250 invoice)

2. Does the dollar size of the invoice affect the annual interest cost of not taking discounts? Illustrate with an example.

3. Recompute problem 1, assuming a 10-day stretching of the payment date. What is the major advantage of stretching? What are the disadvantages?

4. The Dud Company purchases raw materials on terms of 2/10, net 30. A review of the company's records by the owner, Mr. Dud, revealed that payments are usually made fifteen days after purchases are received. When asked why the firm did not take advantage of its discounts, the bookkeeper, Mr. Grind, replied that it cost only 2 per cent for these funds, whereas a bank loan would cost the firm 6 per cent.
   (a) What mistake is Grind making?
   (b) What is the real cost of not taking advantage of the discount?
   (c) If the firm could not borrow from the bank and was forced to resort to the use of trade credit funds, what suggestion might be made to Grind which would reduce the annual interest cost?

5. The Fox Company is able to sell $1 million of commercial paper every three months at a rate of 7 per cent and a placement cost of $3,000 per issue. The dealers require Fox to maintain $200,000 in bank balances, which would otherwise not be held. Fox has a 40 per cent tax rate. What is the after-tax cost of funds from commercial paper to Fox?

6. The Sphinx Supply Company needs to increase its working capital by $100,000. It has decided that there are essentially three alternatives of financing available. They are:

(1) Forego cash discounts, granted on a basis of 3/10, net 30.

(2) Borrow from the bank at 8 per cent. This alternative would necessitate maintaining a 25 per cent compensating balance.

(3) Issue commercial paper at $7\frac{1}{2}$ per cent. The cost of placing the issue would be $500 each six months.

Assuming the firm would prefer the flexibility of bank financing, and provided the additional cost of this flexibility is no more than 1 per cent, which alternative should be selected?

SELECTED
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Short-Term Loans

20

In the previous chapter, we considered two important sources of short-term financing for the firm—namely, trade credit and commercial paper. In this chapter, we examine short-term loans, the principal sources of which are commercial banks and finance companies. For expository purposes, it is convenient to separate business loans into two categories: unsecured loans and secured loans. Almost without exception, finance companies do not offer unsecured loans, simply because a borrower who deserves unsecured credit can borrow at a lower cost from a commercial bank. Consequently, our discussion of unsecured loans will involve only commercial banks.

UNSECURED BANK CREDIT

Short-term, unsecured bank loans typically are regarded as "self-liquidating" in that the assets purchased with the proceeds generate

sufficient cash flows to pay the loan in less than a year. At one time, banks confined their lending almost exclusively to this type of loan. Fortunately, banks now provide a wide variety of business loans, tailored to the specific needs of the borrower. Still, the short-term, self-liquidating loan is an important source of business financing. It is particularly popular in financing seasonal buildups in accounts receivable and inventories. Unsecured short-term loans may be extended under a line of credit, under a revolving-credit agreement, or on a transaction basis. The debt itself is evidenced formally by a promissory note signed by the borrower, showing the time and amount of payment and the interest to be paid. From the standpoint of total funds provided, commercial banks are the largest source of financing for business firms collectively.

LINE OF CREDIT

A line of credit is an informal arrangement between a bank and its customer with respect to the maximum amount of unsecured credit the bank will permit the firm to owe at any one time. Usually, credit lines are established for a one-year period and are subject to one-year renewals. Frequently, lines of credit are set for renewal after the bank receives the audited annual report and has had a chance to review the progress of the borrower. For example, if the borrower's year-end statement date is December 31, a bank may set its line to expire sometime in March. At that time, the bank and the company would meet to discuss the credit needs of the firm for the coming year in light of its past year's performance. The amount of the line is based upon the bank's assessment of the credit-worthiness of the borrower and upon his credit needs. Depending upon changes in these conditions, a line of credit may be adjusted at the renewal date, or before, if conditions necessitate a change.

The cash budget, perhaps, gives the best insight into the borrower's short-term credit needs. For example, if maximum or peak borrowing needs over the forthcoming year are estimated at $800,000, a company might seek a line of credit of $1 million to give it a margin of safety. Whether the bank will go along with the request, of course, will depend upon its evaluation of the credit-worthiness of the firm. If the bank does agree, the firm may then borrow on a short-term basis — usually ninety days — up to the full $1 million line. As banks tend to regard borrowing under lines of credit as seasonal or temporary financing, they usually require that the borrower be out of bank debt at some time during the year. Frequently, the borrower will be required to "clean up" bank debt for at least thirty days during the year. The understanding between the bank and the borrower with respect to a "cleanup," of course, is subject to negotiation.

Despite its many advantages to the borrower, a line of credit does not constitute a formal or legal commitment on the part of the bank to extend

credit. The borrower is usually informed of the line by means of a letter indicating that the bank is willing to extend credit up to a certain amount. An example of such a letter is shown in Figure 20-1. This letter is not a legal obligation of the bank to extend credit. If the credit-worthiness of the borrower should deteriorate over the year, the bank may not want to extend credit and would not be required to do so. Under most circumstances, however, a bank feels bound to honor a line of credit.

SECOND NATIONAL BANK
Palo Alto, California

March 23, 1971

Mr. Joseph A. Ralberg
Vice President & Treasurer
Barker Manufacturing Corporation
Palo Alto, California

Dear Mr. Ralberg:

Based upon our analysis of your year-end audited statements,
we are pleased to renew your $1 million unsecured line of
credit for the forthcoming year. Borrowings under this
line will be at a rate of one-half (1/2%) over the
prime rate.

This line is subject to only the understanding that your
company will maintain its financial position and that it
will be out of bank debt for at least 45 days during the
fiscal year.

Yours very truly,

John D. Myers
Vice President

FIGURE 20-1
Sample letter extending line of credit

A revolving credit agreement represents a legal commitment on the part of the bank to extend credit up to a maximum amount. While the commitment is in force, the bank must extend credit to the borrower any time he wishes to borrow, provided total borrowings do not exceed the maximum amount specified. If the revolving credit is for $1 million, and $700,000 is already owing, the borrower can borrow an additional $300,000 at any time. For the privilege of having this formal commitment, the borrower usually is required to pay a commitment fee on the unused portion of the revolving credit. For example, if the revolving credit is for $1 million, and borrowing for the year averages $400,000, the borrower will be required to pay a commitment fee on the $600,000 unused portion. If the fee is 0.5 per cent, the cost of this privilege will be $3,000 for the year. Revolving-credit agreements frequently extend beyond one year. Because lending arrangements of more than a year must be regarded as intermediate rather than short-term credit, we shall examine revolving credits more extensively in Chapter 21. The purpose of introducing them at this time is to illustrate the formal nature of the arrangement in contrast to the informality of a line of credit.

TRANSACTION LOANS

Borrowing under a line of credit or under a revolving-credit arrangement is not appropriate when the firm needs short-term funds for only one purpose. For example, a contractor may borrow from a bank in order to complete a job. When the contractor receives payment for the job, he pays the loan. For this type of loan, a bank evaluates each request by the borrower as a separate transaction. In these evaluations, the cash-flow ability of the borrower to pay the loan usually is of paramount importance.

COMPENSATING BALANCES

In addition to charging interest on loans, commercial banks usually require the borrower to maintain demand-deposit balances at the bank in direct proportion to either the amount of funds borrowed or the amount of the commitment. These minimum balances are known as compensating balances. The amount required in the compensating balance varies according to the particular bank and the borrower, but many banks require balances equal to 15 per cent of a line of credit. If the line is $1 million, the borrower will be required to maintain average balances of at least $150,000 during the year. The effect of a compensating-balance requirement is to raise the effective cost of borrowing if the borrower is required to maintain balances above the amount the firm would maintain ordinarily.

Bankers will argue that balances compensating a lending accommo-
dation should be "free" in the sense that they are not needed to com-
pensate the bank for deposit activity in the firm's demand-deposit ac-
count. In fact, however, most banks overlook this differentiation of
compensation when it comes to evaluating whether the requirement is
being met. In other words, if a firm needed to maintain a balance of
$150,000 simply to compensate for the deposit and withdrawal activity
in its account, it also might be able to obtain a $1 million line of credit
without increasing its balances. To the extent that a compensating-
balance requirement does not require the borrower to maintain balances
above those that it would maintain ordinarily, such a requirement does
not raise the effective cost of borrowing. However, if balances above the
ordinary must be maintained, the effective cost of borrowing is raised.
For example, suppose we borrow $1 million at 8 per cent and are re-
quired to maintain $100,000 more in balances than we would ordinarily.
We would then have use of only $900,000 of the $1 million loan. The
effective annual interest cost is $80,000/$900,000 = 8.88 per cent,
rather than 8 per cent.

Compensating-balance requirements may increase the liquidity posi-
tion of the borrower from the bank's point of view. As a last resort, the
bank can exercise its legal right of offset and apply the balances on de-
posit to pay off the loan or a portion of the loan.[1]

INTEREST RATES

Unlike interest rates on such impersonal money-market instruments
as Treasury bills, bankers' acceptances, and commercial paper, those on
business loans are determined through personal negotiation between the
borrower and the lender(s). In some measure, banks try to vary the
interest rate charged according to the credit-worthiness of the borrower;
the lower the credit-worthiness, the higher the interest rate. Large,
well-established, financially sound companies are able to borrow at the
prime rate, which is the lowest rate charged on business loans. The
prime rate usually is set by large money-market banks and, typically, is
uniform throughout the country.[2] To some extent, this rate is set in
keeping with underlying market conditions; changes in the prime rate
over time are shown in Figure 20-2. We see from the figure that changes
in the prime rate tend to be "lumpy." In a very crude sense, the prime
rate can be thought to represent the equilibrium price of credit between

[1] For a comprehensive survey of compensating-balance requirements, see D. Baxter
Nevins and Harold T. Shapiro, "Compensating-Balance Requirements: The Results of a
Survey," *Journal of Finance,* XIX (September, 1964), 483–96.

[2] In recent years, there have been occasions when one bank, or a small group of banks,
has maintained a prime rate different from the prime rate maintained by the majority of
banks.
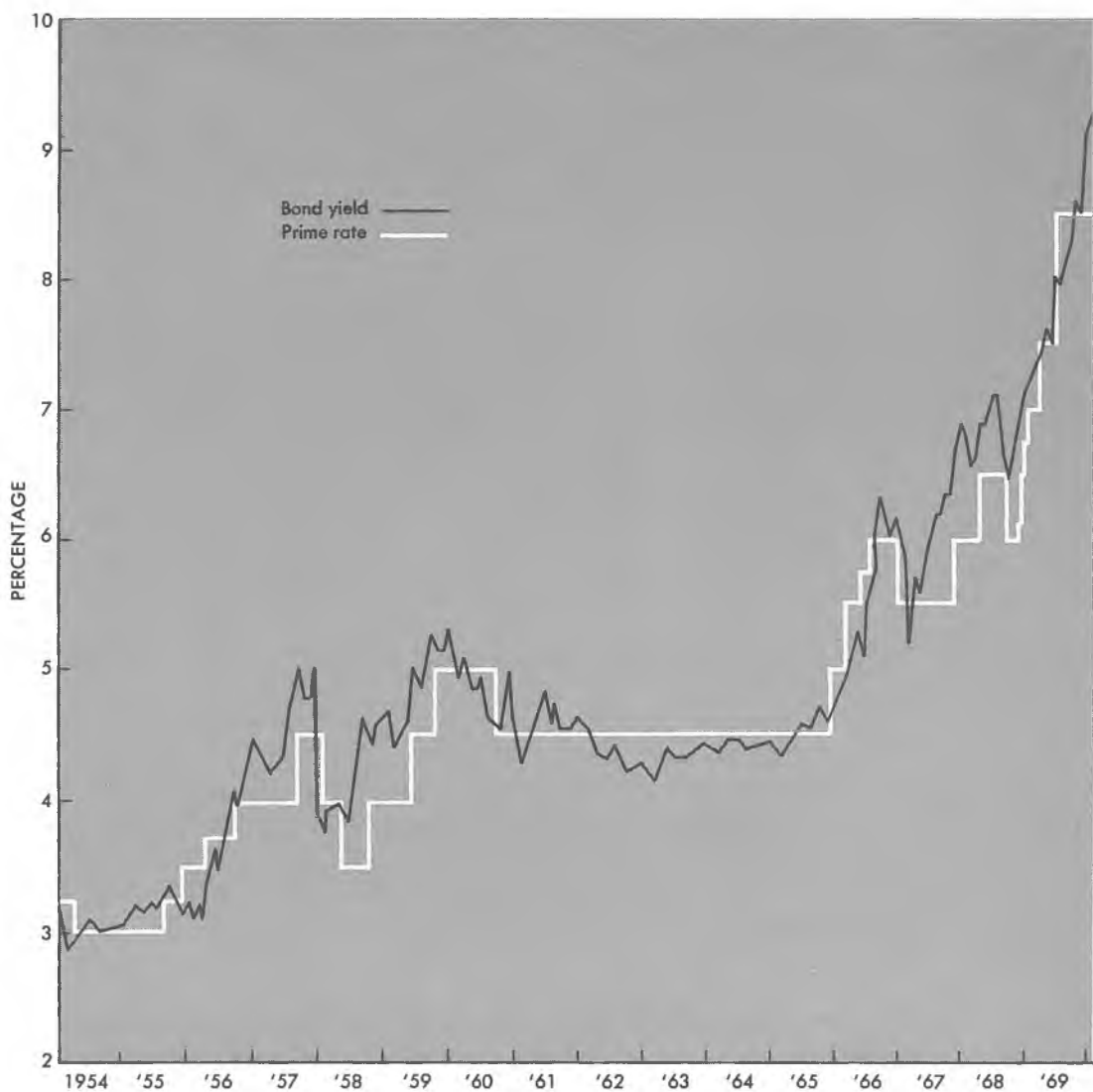
FIGURE 20-2
The relationship between callable new issue Aa utility bonds and the prime rate. Source: Salomon Brothers and Hutzler.

"prime-risk" borrowers and large commercial banks. Other borrowers are charged rates above the prime rate. For example, a bank might extend a line of credit to a company at a rate of 0.5 per cent above prime. If the prime rate is 7.5 per cent, the borrower will be charged an interest

rate of 8 per cent. If the prime rate changes to 7 per cent, the borrower will pay 7.5 per cent.

The interest-rate differential between the prime rate and the rate charged to a borrower will depend upon the relative bargaining power of the borrower and the bank. Supposedly, this differential should reflect only the borrower's credit-worthiness in relation to that of a "prime-risk" borrower. However, other factors influence the differential. The balances maintained and other business the borrower has with a bank (such as trust business) are very important considerations. A good customer who has maintained very attractive balances in the past may be able to obtain a more favorable interest rate than will a firm of equal credit-worthiness that has carried rather meager balances in the past. Although the prime rate reflects national credit conditions, many banks extend credit only in a specific geographic area. To the extent that credit conditions in that area differ from national conditions, the interest-rate differential from the prime rate will be affected. In addition, a bank that is aggressively seeking a relationship with a company may be willing to extend credit at a rate slightly lower than it might charge normally.

Thus, the interest rate charged on a short-term loan will depend upon the prevailing prime rate, the credit-worthiness of the borrower, his present and prospective relationship with the bank, and, sometimes, upon other considerations. Because of the fixed costs involved in credit investigation and in the processing of a loan, we would expect the interest rate on small loans to be higher than the rate on large loans. Table 20-1 seems to confirm this notion. (However, we must keep in mind that there is probably a correlation between credit-worthiness and the size of the loan.)

TABLE 20-1
Bank rates on short-term business loans, November 1969

Center	All Sizes	Size of Loan (in thousands of dollars)				
		1–9	10–99	100–499	500–999	1,000 and over
	Weighted Average Rates (per cent per annum)					
35 centers	8.83	9.05	9.20	9.00	8.84	8.66
New York City	8.66	9.22	9.13	8.83	8.74	8.58
7 other Northeast	9.21	9.16	9.57	9.36	9.18	8.85
8 North Central	8.83	8.77	9.16	9.11	8.81	8.70
7 Southeast	8.58	8.69	8.73	8.55	8.60	8.45
8 Southwest	8.79	9.20	9.02	8.81	8.76	8.66
4 West Coast	8.81	9.45	9.22	8.95	8.76	8.67

Source: Federal Reserve Bulletin, 53 (March, 1970), p. A32.

Methods of Computing Interest Rates. There are two ways in which interest on a loan may be paid: on a collect basis and on a discount basis. When paid on a collect basis, the interest is paid at the maturity of the note; when paid on a discount basis, interest is deducted from the initial loan. To illustrate, suppose we have a $10,000 loan at 7 per cent interest for one year. The effective rate of interest on a collect note is

$$\frac{\$700}{\$10,000} = 7.00 \text{ per cent}$$

On a discount basis, the effective rate of interest is not 7 per cent but

$$\frac{\$700}{\$9,300} = 7.53 \text{ per cent}$$

When we pay on a discount basis, we have the use of only $9,300 for the year but must pay back $10,000 at the end of that time. Thus, the effective rate of interest is higher on a discount note than on a collect note.

EURODOLLAR LOANS

Eurodollar loans are an increasingly important source of short- and intermediate-term credit. Although these loans are used primarily to finance international operations, they also may be used to finance domestic needs. A Eurodollar is defined as a dollar deposit held in a bank outside the United States. Since the late fifties, an active market has developed for these deposits. Foreign banks and foreign branches of U.S. banks, mostly in Europe, bid actively for Eurodollar deposits, paying interest rates that fluctuate in keeping with supply and demand conditions. These deposits are in large denominations, frequently $100,000 or more. The banks use the Eurodollar deposits they receive to make dollar loans to prime borrowers. These loans are made at a rate in excess of the deposit rate; the differential varies according to the relative risk of the borrower. All loans are unsecured. Essentially, the borrowing and lending of Eurodollars represent a wholesale operation, with far fewer costs than are usually associated with banking. The market itself is free from government restrictions and is truly international in scope. It grew tremendously during the sixties and by the end of the decade was estimated to be around $20 billion in size.

In the mid-sixties, the U.S. government imposed several restrictions on foreign investments by U.S. corporations which were financed by raising funds in this country. As a result, American firms have turned to the Eurodollar market to finance investments abroad. Most of this financing has taken the form of long-term bonds, called Eurobonds, but a portion of the funds obtained are short and medium term. Many American firms arrange for lines of credit and revolving credits from Eurodollar

banks. For the latter arrangement, the firm pays a commitment fee the same as it does for a domestic revolving credit. The interest rate on loans is based upon the Eurodollar deposit rate and bears only an indirect relationship to the prime rate. Typically, the rate on a Eurodollar loan exceeds the prime rate and is much more volatile. Consequently, it is more difficult to project the cost of a Eurodollar loan than that of a domestic loan. Nevertheless, no compensating balances are required, thus enhancing the attractiveness of this kind of financing. Even though most of the funds borrowed by U.S. firms in the Eurodollar market are used abroad, a portion is used domestically.[3] Several large acquisitions by U.S. companies have been financed through the Eurodollar market. For large, prime-grade corporations, Eurodollar loans are an attractive source of financing.

SECURED CREDIT

Many firms cannot obtain credit on an unsecured basis, either because they are new and unproven or because their ability to service debt is not regarded as adequate by bankers. In order to make a loan, lenders require security so as to reduce their risk of loss. With security, lenders have two sources of loan payment: the cash-flow ability of the firm to service the debt; and, if that source fails for some reason, the collateral value of the security. Most lenders will not make a loan unless the firm has sufficient expected cash flows to make proper servicing of debt probable. To reduce their risk further, however, they require security as well.

COLLATERAL VALUE

The excess of the market value of the security pledged over the amount of the loan determines the lender's margin of safety. If the borrower is unable to meet his obligation, the lender can sell the security to satisfy the claim. If the security is sold for an amount exceeding the amount of the loan and interest owed, the difference is remitted to the borrower. If the security is sold for less, the lender becomes a general, or unsecured, creditor for the amount of the difference. Because secured lenders do not wish to become general creditors, they usually seek security with a market value sufficiently above the amount of the loan to minimize the likelihood of their not being able to sell the security in full satisfaction of the loan. However, the degree of security protection a lender seeks varies with the credit-worthiness of the borrower, the security the bor-

[3] For a very capable analysis of the market, see Charles S. Ganoe, "The Eurodollar Market: A New Source of Financing," *Journal of Commercial Bank Lending*, 50 (July, 1968), 11–20.

rower has available, and the financial institution making the loan. Before taking up various short-term secured lending arrangements, we must examine briefly the means by which a lender protects himself under the Uniform Commercial Code.

SECURITY DEVICES

Because the Uniform Commercial Code has been adopted by practically every state in the nation, it is important to understand its implications for secured lending. Article 9 of the Code deals with security interests of lenders, the specific aspect with which we are concerned. Prior to the adoption of the Code, procedures by which a lender perfected a valid lien on collateral were complex and differed greatly among states. Article 9 consolidated rules governing security devices into one meaningful body of laws.[4] Because the lending arrangements taken up in subsequent sections involve security interests under the Uniform Commercial Code, we need to define certain terms in this section.

Whenever a lender requires collateral of a borrower, he obtains a *security interest* in the collateral. The collateral may be accounts receivable, inventory, equipment, or other assets of the borrower. The security interest in the collateral is created by a *security agreement,* also known as a *security device*. This agreement is signed by the borrower and lender and contains a description of the collateral. Examples of security agreements are illustrated in Figures 20-3 and 20-4. In order to "perfect" a security interest in the collateral, the lender must file a copy of the security agreement or a financing statement with a public office of the state in which the collateral is located. Frequently, this office is that of the secretary of state. The filing gives public notice to other parties that the lender has a security interest in the collateral described. Before accepting collateral as security for a loan, a lender will search the public notices to see if the collateral has been pledged previously in connection with another loan. Only the lender with a valid security interest in the collateral has a prior claim on the assets and can sell the collateral in settlement of his loan.

*RECEIVABLE
LOANS*

ASSIGNMENT OF ACCOUNTS RECEIVABLE

Accounts receivable represent one of the most liquid assets of the firm and, consequently, they make desirable security for a loan. From the standpoint of the lender, the major difficulties with this type of security

[4]See Lester E. Denonn, "The Security Agreement," *Journal of Commercial Bank Lending,* 50 (February, 1968), 32–40.

THE STANFORD BANK

COMMERCIAL LOAN DEPARTMENT

ASSIGNMENT AS SECURITY OF ACCOUNT(S) RECEIVABLE
(EXISTING ACCOUNTS)

The undersigned Assignor, as security for the payment of all indebtedness, obligations and liabilities of Assignor to The Stanford Bank (hereinafter called "Bank"), now or hereafter existing, matured or to mature, absolute or contingent, and howsoever created, evidenced or secured hereby assigns to Bank the Account(s), and all sums due and becoming due thereon, described as follows:

NAME OF ACCOUNT/DEBTOR	INVOICE DATE OR INVOICE NUMBER	AMOUNT

and to induce Bank to accept and extend financial accommodation in connection with the same, Assignor represents and warrants to, and covenants and agrees with Bank as provided on the reverse hereof.

Assignor:_____

Dated:_____   By:_____

N-441 2M 8-66 JCO

FIGURE 20-3

Example of an accounts receivable security agreement

Assignor represents, warrants, covenants and agrees as follows:

1. Assignor is the sole owner of said Account(s) and the same is/are (a) genuine, represent monies owing for goods sold or services rendered to and accepted by the purchaser(s) and enforceable against the purchaser(s) in the amount(s) set forth; (b) free of all default, dispute, counter-account, set-off and counter-claim; (c) wholly free of any prior assignment and security interest of any outstanding trust receipt or other lien, claim or encumbrance whatsoever; (d) that no part of the amounts above set forth have been received by Assignor, and Assignor will not hereafter pledge, assign, sell or encumber the same to any person other than Bank.

2. Bank, as and in such manner as it may elect from time to time, and without notice to Assignor or prejudice as to any rights of Bank against Assignor, may collect, realize upon, enforce and otherwise deal with said Account(s) and rights assigned, including any security therefor, and may accept compromises and compositions, take, release and enforce security, grant extensions, releases and discharges, and otherwise deal with debtors of Assignor and with third persons with respect to said Account(s) and rights assigned as fully and to the same extent as Assignor might do, and may apply all proceeds of said Account(s) and Security to any indebtedness of Assignor to Bank. Assignor hereby appoints Bank, by any officer thereof, as Assignor's Attorney in the name, place and stead of Assignor but for Bank's own benefit to endorse checks, drafts and other instruments for the payment of money, receive payments, and do all acts and things hereinabove provided.

3. Upon demand Assignor will reimburse Bank for all expense incurred by Bank, including reasonable attorney's fees, in exercising any of its rights, powers and privileges hereunder, and all Account(s) hereby assigned are security for the payment thereof to Bank.

4. At Bank's request, to execute and deliver to Bank, in form acceptable to it, all instruments and documents, including a General Pledge Agreement in Bank's regular form, and to do any and all acts and things which Bank may deem necessary, proper, or convenient to carry into effect the terms hereof, facilitate the collection of the Account(s) hereby assigned, or protect, effect and enforce the security of Bank hereunder.

5. Bank is under no duty or liability for failure to collect, realize upon and obtain payment of the Account(s), property and rights assigned hereby. All monies, payments, and sums, and all instruments paying or purporting to direct the payment of the same received by Assignor in connection with the assigned Account(s) received by or coming into the possession of Assignor, shall be for the account of and held in trust for Bank, and promptly delivered to Bank as received.

6. If any property sold by Assignor, the sale of which gave rise to any Account(s) herein assigned, be returned to Assignor, immediate notice of such return will be given by Assignor to Bank, and Assignor will thereafter upon demand by and at the option of Bank either (i) assign to Bank in lieu of such Account(s) another account acceptable to Bank; or (ii) pay to Bank the balance unpaid on such Account(s) as of the date of such return.

7. All rights, remedies, powers and privileges of Bank herein provided are cumulative, not alternative, and in addition to all others provided by law, and this Agreement shall inure to the benefit of Bank's successors and assigns and be binding upon the heirs, executors, administrators, successors and assigns of Assignor.

FIGURE 20-3   (continued)

SECURITY AGREEMENT: FLOORING

Pursuant to the California Uniform Commercial Code, the undersigned Borrower hereby grants to

_____ (Bank)

a security interest in the following described inventory, together with all replacements and substitutions thereof, all additions and accessions thereto, and all proceeds thereof.

YEAR	MAKE	ARTICLE	MODEL OR MOTOR NO.	SERIAL NO.	INVOICE NO.	COST	RELEASE PRICE	DATE RELEASED
								1.
								2.
								3.
								4.
								5.
								6.
								7.
								8.

1. BORROWER'S OBLIGATIONS: The security interest created hereby is given as security for the payment of $_____ , together with interest thereon payable _____ from date hereof at the rate of _____ percent per annum, provided that said rate of interest may be changed upon not less than _____ days notice to Borrower. Borrower hereby agrees to pay said sum to Secured Party at its office as follows:

_____ % of the cost of each unit of collateral on or before _____ , 19 _____ ;
_____ % of the cost of each unit of collateral on or before _____ , 19 _____ ;
and the balance of principal and interest on or before _____ , 19 _____ ,

unless the maturity is extended by Secured Party. This Agreement also secures all other Indebtedness of Borrower to Bank, including all debts, obligations, or liabilities now or hereafter existing, absolute or contingent, and future advances.

2. LOCATION OF COLLATERAL: _____

3. USE OF COLLATERAL: The inventory Collateral of this agreement is to be held for ☐ Sale   ☐ Lease

4. INCORPORATION OF PROVISIONS ON REVERSE: All provisions on the reverse side are incorporated herein as if set forth fully at this point.

Dated _____ , 19 _____

IF CORPORATION AFFIX SEAL

By _____
   SIGNATURE OF BORROWER(S) - TITLE

_____
BORROWER(S) - PRINT

_____
CHIEF PLACE OF BUSINESS OR RESIDENCE (INDIVIDUALS)

CBA-8A-3 (4-66)

ORIGINAL — TO BANK

FIGURE 20-4

Example of a trust receipt security agreement

are the cost of processing the collateral and the risk of fraud. To illustrate the nature of the arrangement, we trace through a typical assignment of accounts-receivable loan. A company may seek a receivable loan from either a commercial bank or a finance company. As the interest rate charged by a bank usually is less than that charged by a finance company, the firm will generally try to borrow first from a bank.

Quality and Size of Receivables. In evaluating the loan request, the lender will analyze the quality of the firm's receivables in order to determine the amount he is willing to lend against these receivables. The greater the quality of the accounts the firm maintains, the greater the percentage the lender is willing to advance against the face value of the receivables pledged. A lender does not have to accept all the borrower's accounts receivable; usually, he will reject accounts that have low credit ratings or that are unrated. Depending upon the quality of the receivables accepted, a lender typically advances between 50 per cent and 85 per cent of their face value.

The lender is concerned not only with the quality of receivables but also with their size. The lender must keep records on each account receivable that is pledged; the smaller the average size of the accounts, the more it costs per dollar of loan to process them. Consequently, a firm that sells low-priced items on open account will generally be unable to obtain a receivable loan regardless of the quality of the accounts. The cost of processing the loan is simply too high. Occasionally a "bulk" assignment of receivables will be used to circumvent the problem. With a "bulk" assignment, the lender does not keep track of the individual accounts but records only the total amounts in the accounts assigned and the payments received. Because preventing fraud is difficult with a "bulk" assignment, the percentage advance against the face value of receivables is likely to be low—perhaps 25 per cent.

Procedure. Suppose that a lender has decided to extend a loan to a firm on the basis of a 75 per cent advance against the face value of accounts receivable assigned. The firm then sends in a schedule of accounts showing the name of the account, the date of billing(s), and the amounts owed. An example of an assignment schedule is shown in Figure 20-3. The lender will sometimes require evidence of shipment, such as an invoice. Having received the schedule of accounts, the lender has the borrower sign a promissory note and a security agreement. The firm then receives 75 per cent of the face value of the receivables shown on the schedule of accounts.

A receivable loan can be on either a non-notification or a notification basis. Under the former arrangement, the customer of the firm is not notified that his account has been pledged to the lender. When the firm receives payment on the account, it forwards this payment, together with

other payments, to the lender. The lender checks the payments against its record of accounts outstanding, and reduces the amount the borrower owes. With a nonnotification arrangement, the lender must take precautions to make sure the borrower does not withhold a payment check, using the funds himself. With a notification arrangement, the account is notified of the assignment, and remittances are made directly to the lender. Under this arrangement, the borrower cannot withhold payments. Most firms naturally prefer to borrow on a nonnotification basis; however, the lender reserves the right to place the arrangement on a notification basis.

Means of Financing. An accounts-receivable loan is a more or less continuous financing arrangement. As the firm generates new receivables that are acceptable to the lender, they are assigned, adding to the security base against which the firm is able to borrow. New receivables replace the old, and the security base and the amount of loan fluctuate accordingly. A receivable loan is a very flexible means of secured financing. As receivables build up, the firm is able to borrow additional funds to finance this buildup. Thus, it has access to "built in" financing.

At a commercial bank, the interest cost of borrowing against accounts receivable frequently is 2 to 4 per cent higher than the prime rate. In addition, many banks have a service charge of an additional 1 to 2 per cent for processing this type of loan. Costs at commercial finance companies are higher; the total interest cost of a receivable loan may range from 12 to 24 per cent.

FACTORING RECEIVABLES

In the assignment of accounts receivable, the firm retains title to the receivables. When a firm *factors* its receivables, however, it actually sells them to a factor. The sale may be either with or without recourse, depending upon the type of arrangement negotiated. The factor maintains a credit department and makes credit checks on accounts. Based upon its credit investigation, the factor may refuse to buy certain accounts that it deems too risky. By factoring, a firm frequently relieves itself of the expense of maintaining a credit department and making collections. Any account that the factor is unwilling to buy is an unacceptable credit risk unless, of course, the firm wants to assume this risk on its own and ship the goods. If the factoring arrangement involves full recourse, the firm will want to maintain some sort of credit department in order to limit its risk exposure. On the other hand, if the receivables are sold without recourse, the factor bears both the risk of bad-debt losses and of the expenses associated with the collection of accounts. Although it is customary in a factoring arrangement to notify the customer that his account has been sold and that payments on the account should be sent directly to the factor, in many instances notification is not made. The

customer continues to remit payments to the firm, which, in turn, endorses them to the factor. These endorsements are frequently camouflaged to prevent the customer from learning that his account has been sold.

Factoring Costs. For bearing risk and servicing the receivables, the factor receives a fee of around 1 to 3 per cent of the face value of the receivables sold. This fee will vary according to the typical size of individual accounts, the volume of receivables sold, and the quality of the accounts. We must recognize, however, that the receivables sold to the factor will not be collected from the various accounts for a period of time. If the firm wishes to receive payment for the sale of its receivables before they are actually collected, it must pay interest on the advance. Advancing payment is a lending function of the factor in addition to his functions of risk bearing and of servicing the receivables. For this additional function, the factor requires compensation. For example, if the receivables sold total $10,000, and the factoring fee is 2 per cent, the factor will credit the firm's account with $9,800. If the firm wants to draw on this account before the receivables are collected, however, it will have to pay an interest charge — say 1 per cent a month — for the use of the funds. If it wishes a cash advance of the full $9,800, and the receivables are collected on the average, in one month, the interest cost will be approximately $0.01 \times 9,800$, or $98.[5] Thus, the total cost of factoring is composed of a factoring fee plus an interest charge if the firm draws upon its account before the receivables are collected. If the firm does not draw on its account until the receivables are collected, there is no interest charge. A third alternative is for the firm to leave its funds with the factor beyond the time when the receivables are collected and to receive interest on the account from the factor.

Flexibility. The typical factoring arrangement is continuous. As new receivables are acquired, they are sold to the factor, and the firm's account is credited. The firm then draws upon this account as it needs funds. Sometimes the factor will allow the firm to overdraw its account during periods of peak needs and thereby borrow on an unsecured basis. Under other arrangements, the factor may withhold a reserve from the firm's account as a protection against losses. There are about twenty old-line factors in the country, most of which are located in New York City. In recent years, commercial banks have entered the factoring business and, accordingly, are a source of such financing. As with the old-line factors, most banks that factor are located on the eastern seaboard.[6]

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[5] The actual cash advance would be $9,800 less the interest cost, or $9,702.

[6] For an analysis of the entry of commercial banks into factoring, see Robert P. Shay and Carl C. Greer, "Banks Move into High-Risk Commercial Financing," *Harvard Business Review,* 46 (November–December, 1968), 149–53, 156–61.

Factoring, like the assignment of accounts receivable, affords the firm flexibility in its financing. As sales increase and the firm needs funds, financing becomes available automatically. This eliminates the uncertainty associated with the collection cycle. Consequently, the cash flows of the firm are more predictable. Factoring is used widely in the textile industry and has found acceptance as well in the shoe and furniture industries. In many cases, factoring is a perfectly acceptable method of financing. Its principal shortcoming is that it can be expensive. We must bear in mind, however, that the factor often relieves the firm of credit checkings, the cost of processing receivables, and collection expenses. For a small firm, the savings may be quite significant.

INVENTORY LOANS

Inventories also represent a reasonably liquid asset and are therefore suitable as security for a short-term loan. As with a receivable loan, the lender determines a percentage advance against the market value of the collateral. This percentage varies according to the quality of the inventory. Certain inventories, such as grains, are very marketable and resist physical deterioration over time. The margin of safety required by the lender on a loan of this sort is fairly small, and the advance may be as high as 90 per cent. On the other hand, the market for a highly specialized piece of equipment may be so narrow that a lender is unwilling to make any advance against its reported market value. Thus, not every kind of inventory can be pledged as security for a loan. The best collateral is inventory that is relatively standard and for which a ready market exists apart from the marketing organization of the borrower.

Lenders determine the percentage that they are willing to advance by considering marketability, perishability, market-price stability, and the difficulty and expense of selling the inventory to satisfy the loan. The cost of selling some inventory may be very high indeed. The lender does not want to be in the business of liquidating collateral, but he does want to assure himself that the collateral has adequate value in case the borrower defaults in the payment of principal or interest. As is true with most secured loans, however, the actual decision to make the loan will depend upon the cash-flow ability of the borrower to service debt. There are a number of different ways a lender can obtain a secured interest in inventories, and we consider each in turn. In all cases, the inventory must be identifiable—that is, a lender must be able to verify its physical presence.

FLOATING LIEN

Under the Uniform Commercial Code, the borrower may pledge his inventories "in general" without specifying the specific inventory in-

volved. Under this arrangement, the lender obtains a floating lien on all inventory of the borrower. This lien is very general and difficult to police on the part of the lender. Frequently, a floating lien is requested only as additional protection and does not play a major role in determining whether or not the loan will be made. Even if the lender does regard the collateral as important, he usually is willing to make only a moderate advance because he cannot exercise tight control over the collateral. The floating lien can be made to cover both receivables and inventories, as well as the collection of receivables. This modification gives the lender a lien on a major portion of a firm's current assets. In addition, the lien can be made to encompass almost any length of time so that it includes future as well as present inventory as security.

CHATTEL MORTGAGE

With a chattel mortgage, inventories are identified specifically either by serial number or by some other means. While the borrower holds title to the goods, the lender has a lien on inventory. This inventory cannot be sold unless the lender gives his consent. Because of the rigorous identification requirements, chattel mortgages are ill-suited for inventory with rapid turnover and/or inventory that is not easily identified because of size or other reasons. They are well suited, however, for certain capital assets such as machine tools.

TRUST RECEIPT LOANS

Under a trust receipt financing arrangement, the borrower holds the inventory and proceeds from the sale of inventory in trust for the lender. This type of lending arrangement, known also as floor planning, has been used extensively by automobile dealers, equipment dealers, and consumer durable goods dealers. To illustrate trust receipt financing, suppose an automobile manufacturer ships cars to a dealer who, in turn, finances the payment for these cars through a finance company. The finance company pays the manufacturer for the cars shipped. The dealer signs a trust receipt security agreement, which specifies what can be done with the inventory. A copy of the security device used under a trust receipt arrangement is shown in Figure 20-4. The car dealer is allowed to sell the cars but must turn the proceeds of the sale over to the lender in payment of the loan. Inventory in trust, unlike inventory under a floating lien, is specifically identified by serial number or by other means. In our example, the finance company, periodically audits the cars the dealer has on hand. The serial numbers of these cars are checked against those shown in the security agreement. The purpose of the audit is to see if the dealer has sold cars without remitting the proceeds of the sale to the finance company.

As the dealer buys new cars from the automobile manufacturer, a new trust receipt security agreement is signed that takes account of the new inventory. The dealer then borrows against this new collateral, which he holds in trust. Although there is tighter control over collateral with a trust receipt agreement than with a floating lien, there is still the risk of inventory being sold without the proceeds being turned over to the lender. Consequently, the lender must exercise judgment in deciding to lend under this arrangement. A dishonest dealer can devise numerous ways to fool the lender.

Many durable goods manufacturers finance the inventories of their distributors or dealers. Their purpose is to encourage dealers or distributors to carry reasonable stocks of goods. It is reasoned that the greater the stock, the more likely the dealer or distributor is to make a sale. Because the manufacturer is interested in selling his product, financing terms often are more attractive than they are with an "outside" lender.

TERMINAL WAREHOUSE
RECEIPT LOANS

A borrower secures a terminal warehouse receipt loan by storing inventory with a public, or terminal, warehousing company.[7] The warehouse company issues a warehouse receipt, which evidences title to specified goods that are located in the warehouse. An example of a warehouse receipt is shown in Figure 20-5. The warehouse receipt gives the lender a security interest in the goods, against which he makes a loan to the borrower. Under such an arrangement, the warehouseman can release the collateral to the borrower only when authorized to do so by the lender. Consequently, the lender is able to maintain strict control over the collateral and will release collateral only when the borrower pays a portion of the loan. For his own protection, the lender usually requires the borrower to take out an insurance policy with a loss-payable clause in favor of the lender.

Warehouse receipts may be either nonnegotiable or negotiable. A nonnegotiable warehouse receipt is issued in favor of a specific party — in this case, the lender — who is given title to the goods and has sole authority to release them. A negotiable warehouse receipt can be transferred by endorsement. Before goods can be released, however, the negotiable receipt must be presented to the warehouseman. A negotiable receipt is useful when title to the goods is transferred from one party

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[7] For an excellent discussion of warehouse receipts, see Robert W. Rogers, "Warehouse Receipts and their Use in Financing," *Bulletin of the Robert Morris Associates,* Vol. XLVI (April, 1964), reprinted in *Foundations for Financial Management,* ed. James Van Horne. (Homewood, Ill.: Richard D. Irwin, Inc., 1966), pp. 103–14.

LAWRENCE WAREHOUSE COMPANY

LAWRENCE SYSTEM

NOT INSURED

ORIGINAL WAREHOUSE RECEIPT **No.** 31 120

RECEIVED FROM
FOR STORAGE IN
LOCATED AT _____ STREET, WAREHOUSE NO.
SUBJECT TO ALL THE TERMS AND CONDITIONS CONTAINED HEREIN AND ON THE REVERSE HEREOF. FOR THE ACCOUNT OF AND TO BE DELIVERED,
WITHOUT SURRENDER OF THIS WAREHOUSE RECEIPT UPON WRITTEN INSTRUCTION OF:

SAMPLE

WAREHOUSE RECEIPT HOLDER

IBM DATA				31 120	Depositor's Declared Values for which the Warehouseman Assumes No Responsibility
	WAREHOUSE NO.	HOLDER NO.	DATE	RECEIPT NO.	

ITEM	CODE NO.	NO. UNITS	SAID TO BE OR CONTAIN	UNIT VALUES	EXTENSION OF UNIT VALUES

(DO NOT ACCEPT THIS WAREHOUSE RECEIPT IF ANY CORRECTIONS OR ERASURES APPEAR HEREON.)

Stored under field warehousing arrangement. Subject to lien for storage, handling and other charges as per contract & lease with the industry served. A general lien is claimed for charges & expenses in regard to other goods deposited by the same depositor. Transfers of merchandise not complete unless made on the books of the warehouse company.

LAWRENCE WAREHOUSE COMPANY

PER _____

BONDED WAREHOUSE MANAGER

The Uniform Warehouse Receipts Act has been superseded by the Uniform Commercial Code. (Over)

FIGURE 20-5
Example of a warehouse receipt

to another while the goods are in storage. With a nonnegotiable receipt, the release of goods can be authorized only in writing. Most lending arrangements are based upon nonnegotiable receipts.

FIELD WAREHOUSE RECEIPT LOANS

In a terminal warehouse receipt loan, the goods are located in a public warehouse. Another arrangement, known as field warehousing, permits loans to be made against inventory that is located on the borrower's

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premises. Under this arrangement, a field warehousing company sets off a designated storage area on the borrower's premises for the inventory pledged as collateral. The field warehousing company has sole access to this area and is supposed to maintain strict control over it. (The goods that serve as collateral are segregated from the borrower's other inventory.) The field warehousing company issues a warehouse receipt as described in the previous section, and the lender extends a loan based upon the collateral value of the inventory. The field warehouse arrangement is a useful means of financing when it is not desirable, either because of the expense or because of the inconvenience, to place the inventory in a public warehouse. Field warehouse receipt lending is particularly appropriate when a borrower must make frequent use of inventory. Because of the need to pay the field warehousing company's expenses, the cost of this method of financing can be relatively high.

It is important to recognize that the warehouse receipt, as evidence of collateral, is only as good as the issuing warehousing company. When administered properly, a warehouse receipt loan affords the lender a high degree of control over the collateral. However, there have been sufficient examples of fraud to show that the warehouse receipt does not always evidence actual value. The warehouseman must exercise strict control. A grain elevator that is alleged to be full may, in fact, be empty. Upon close examination, we may find that barrels reported to contain chemical concentrate actually contain water.

Salad Oil Scandal.   Perhaps the most notorious example of fraud with field warehousing involved the Allied Crude Vegetable Oil Refining Corporation. This company had a field warehousing arrangement whereby a group of lenders extended credit based upon warehouse receipts issued, supposedly, by the American Express Field Warehousing Corporation. The principal commodity involved was soybean oil, which was readily marketable. Many warehouse receipts, some of which were forged, were issued based upon nonexistent tanks of oil or tanks that contained water instead of oil. By manipulation and deceit of the field warehousing company, the company and its president, Anthony de Angelis, were able to borrow millions. At one time, the companies involved had loans outstanding of over three times the value of the oil.[8] When the empire crumbled in 1963, lenders discovered that over $100 million worth of oil was nonexistent. A number of the nation's largest banks suffered losses, and two brokerage houses went bankrupt, as did the field warehousing company and a number of other companies. De Angelis now is serving a twenty-year prison term. The great salad oil scandal has made lenders increasingly wary of the pitfalls of warehouse receipt loans.

[8]"Salad-Oil Settlements Could be Delayed by American Express Unit's Court Action," *Wall Street Journal*, April 3, 1967, p. 4.

Collateral other than the kinds we have discussed may be used in securing short-term loans. The owners of a corporation may have outside assets that they are willing to pledge to secure a loan. For example, stocks or bonds might be assigned to a lender as security for a loan. If the company defaults on the loan, the lender can sell the securities in settlement of its loan. For bonds and listed stocks, lenders usually are willing to advance a fairly high percentage of the market value. For bonds, the percentage may be as high as 90 per cent. In addition to pledging securities, the owners of a company may pledge the cash surrender value of life insurance policies, a savings account passbook, or a building or house owned separately from the corporation itself.

Although not collateral in a strict sense, an outside party—either an individual or another company—may guarantee the loan of the borrower. A lender may not care to extend credit based upon the strength of the company but may be willing to do so if the loan is guaranteed by another party. If the borrower defaults in payment under this arrangement, the guarantor is liable for the payment of the loan. Before making the loan, the lender will analyze carefully the liquidity and net worth of the prospective guarantor. Unless the guarantor is a party of financial substance, the guarantee is meaningless as protection for a loan. An arrangement whereby the owners of a corporation guarantee a loan to the company insures that the owners will take a real interest in the fortunes of the firm. Other possible guarantors of a loan might be relatives of the owners, or a principal supplier.

In this and the preceding chapter, we considered various sources of short-term financing. Because the total amount of short-term financing was assumed to have been determined according to the framework presented in Chapter 15, only determination of the best combination need be considered in this chapter. The appropriate mix, or the weighting, of alternative sources will depend upon considerations of cost, availability, timing, flexibility, and the degree to which the assets of the firm are encumbered. Central to any meaningful analysis of alternative sources of funds is a comparison of their costs, and inextricably related to the question of cost is the problem of timing. Differentials in cost between various alternatives are not necessarily constant over time. Indeed, they fluctuate in keeping with changing financial market conditions. Whereas the differential between the prime rate and commercial paper rate was around 1.5 per cent in the early sixties, it was slightly negative in 1969. Thus, timing bears heavily on the question of the most appropriate mix of short-term financing.

Naturally, the availability of financing is important. If a firm cannot borrow through commercial paper or through a bank because of its low credit standing, it must turn to alternative sources. The lower the credit standing of the firm, of course, the fewer the sources of short-term financing available to it. Flexibility with respect to short-term financing pertains to the ability of the firm to pay off a loan as well as to its ability to renew it or increase it. With factoring and also with a bank loan, the firm can pay off the loan when it has surplus funds. As a result, interest costs are reduced relative to the case, such as commercial paper, where the firm must wait until final maturity before paying off the loan. Flexibility relates also to how easily the firm can increase its loan on short notice. With a line of credit or revolving credit at a commercial bank, it is an easy matter to increase borrowings, assuming the maximum has not been reached. With other forms of short-term financing, the firm is less flexible. Finally, the degree to which assets are encumbered bears on the decision. With secured loans, lenders obtain a lien on the assets of the firm. This secured position constrains the firm in future financing. Whereas receivables are sold under a factoring arrangement, the principle is the same. In this case, the firm sells one of its most liquid assets, thus reducing its credit-worthiness in the minds of creditors.

All of these factors influence the firm in deciding upon the most appropriate mix of short-term financing. Because cost is perhaps the key factor, differences in other factors should be compared with differences in cost. What is the cheapest source of financing from the standpoint of explicit costs may not be the cheapest source when flexibility, timing, and the degree to which assets are encumbered are considered. While it would be desirable to express sources of short-term financing in terms of both explicit and implicit costs, the latter are hard to quantify. A more practical approach is to list available sources according to their explicit costs and then consider the other factors to see if they change the ranking as it relates to total desirability. If one is willing to formulate implicit costs as constraints, another means by which the optimal mix of short-term financing can be determined is with linear programming. This method is illustrated in the appendix to this chapter. Because the financing needs of the firm change over time, multiple sources of short-term financing should be explored on a continuous basis.

SUMMARY

Short-term loans can be divided into two types, unsecured loans and secured loans. Unsecured credit is usually confined to bank loans under a line of credit, under a revolving-credit agreement, or on a transaction basis. Typically, banks require balances to compensate for a lending arrangement. If the borrower is required to maintain balances above those

that it would maintain ordinarily, the effective cost of borrowing is increased. Interest rates on business loans are a function of the existing prime rate, the credit-worthiness of the borrower, and the profitability of the relationship for the bank. A fairly recent and growing source of short- and intermediate-term financing is the Eurodollar loan, and this method was evaluated.

Many firms are unable to obtain unsecured credit and are required by the lender to pledge security. In giving a secured loan, the lender looks first to the cash-flow ability of the company to service debt and, if this source of loan repayment might fail, to the collateral value of the security. To provide a margin of safety, a lender usually will advance somewhat less than the market value of the collateral. The percentage advance varies according to the quality of the collateral pledged and the control the lender has over this collateral. Accounts receivable and inventory are the principal assets used to secure short-term business loans. Receivables may either be pledged to secure a loan or sold to a factor. Inventory loans can be under a general lien, under a trust receipt, or under terminal warehouse or field warehouse receipt arrangements. Certain collateral owned outside a corporation may be used to secure a loan for a corporation. The most appropriate mix of short-term financing will depend upon considerations of relative cost, availability, flexibility, timing, and the degree to which the assets of the firm are encumbered.

APPENDIX

Linear Programming Approach to Short-Term Financing

Robichek, Teichroew, and Jones have developed an extensive linear programming model for making short-term financing decisions.[9] As the type of approach they propose has considerable merit, we describe it briefly in this appendix. The first step is the preparation of a cash budget in which total receipts less total disbursements are tabulated for each future period. Disbursements include payments for purchases and other disbursements. Given beginning cash, the minimum amount of cash the firm desires to hold, and total receipts less total disbursements, a cumulative cash deficit or surplus can be determined for each future period in the manner described in Chapter 26.

The financial manager has available to him a number of alternatives by which the cumulative cash deficit can be financed or excess cash invested. These alternatives carry certain costs per period and are subject to constraints. The alternatives assumed to be available are:

1. Unsecured borrowings under a line of credit. This line sets the upper

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[9]See A. A. Robichek, D. Teichroew, and J. M. Jones, "Optimal Short-Term Financing Decision," *Management Science,* 12 (September, 1965), 1-36; and Alexander A. Robichek and Stewart C. Myers, *Optimal Financing Decisions,* Chapter 7.

limit on borrowings. An additional constraint pertains to the need to maintain compensating balances.

2. Accounts-receivable loan. This type of loan has an upper constraint and is limited to a percentage of the face value of accounts receivable pledged.

3. Stretching accounts payable. The cost of this alternative is the discount foregone. Stretching of accounts is limited to two periods and is further constrained by a limitation on the percentage of payables that can be stretched in any one period. In addition to the explicit cost of cash discount foregone, the authors assume an implicit cost of ill will to creditors if payables are stretched more than one period.

4. Term loan from a bank. This type of loan is constrained, having a minimum and a maximum amount, and is subject to a fixed installment payment schedule. There are additional constraints on the maximum amount of total borrowings. Here, too, the authors assume an implicit cost.

5. Investment of excess cash. The financial manager may invest excess cash in any given period at a specified rate of return.

The objective function of the linear programming problem is to show how to provide the funds needed, as shown by the cash budget, at the minimum total cost. This objective function is subject to the constraints listed under the financial alternatives. When the multiperiod linear programming problem is solved,[10] we obtain the optimal financing strategy for each period of the planning horizon under consideration. The optimal amounts of unsecured borrowings, accounts-receivable financing, stretching of payables, term loan, and investment of excess cash are specified for each period, together with the cost of this optimal financing procedure. By evaluating the dual variables, management obtains insight into the opportunity cost of the various constraints.[11] This approach provides the financial manager with a decision-making tool for solving rather complex short-term financing problems.

In a similar manner, Mao, Peterson, and Orgler propose linear programming models to deal with financing decisions.[12] Of these, Peterson's is the most comprehensive because he incorporates into the short-run operating plan of the firm the following sources of short-term financing:

[10] See Robichek, Teichroew, and Jones, "Optimal Short-Term Financing Decision," 21–25.

[11] For a detailed evaluation of dual variables in connection with a different linear programming problem, see the appendix to Chapter 21.

[12] James C. T. Mao, *Quantitative Analysis of Financial Decisions* (London: Macmillan & Company, Ltd., 1969), pp. 527–47; D. E. Peterson, *A Quantitative Framework for Financial Management* (Homewood, Ill.: Richard D. Irwin, Inc., 1969), Chapter 7; and Yair E. Orgler, *Cash Management* (Belmont, Calif.: Wadsworth Publishing Co., Inc., 1970), Chapters 3–6.

accounts payable, a bank line of credit, secured financing based upon accounts receivable and/or inventories, and the sale of commercial paper.

The most serious shortcoming of a linear programming approach to short-term financing is the need to project future cash flows as though they were known with certainty. This treatment eliminates from consideration possible deviations from expected outcomes as well as the flexibility of financing instruments to deal with unexpected cash demands or surpluses. The approach suffers also from the need to formulate certain implicit costs as constraints. For example, borrowings may not be limited in any absolute sense to a specific amount. Rather, the explicit and implicit costs simply may increase at an increasing rate, so that beyond a point borrowing is no longer feasible. By necessity, a linear programming approach tries to "force" these costs into constraints. Where the alternative sources of financing are few, a more appropriate method may be simply a comparison of explicit costs, with the financial manager assessing implicit costs on the basis of probabilistic cash-budget information. When short-term financing problems are complex, however, a linear-programming approach provides a rigorous tool for obtaining a solution. As long as its limitations are recognized, the tool can be useful.

PROBLEMS

1. The Bierman Supply Company is a recently formed firm which produces original equipment auto parts. A slowdown in the auto industry has caused Bierman's receivables collections to slacken, which, in turn, has caused it to stretch some of its payables. Bierman currently has an asset turnover of 2 and a net margin of 3 per cent.

Bierman Supply (in millions)			
Cash and securities	$ 1	Notes payable (12%)	$ 5
Accounts receivable	7	Accounts payable	10
Inventories	12	Long-term debt (9%)	10
Fixed plant	30	Common and surplus	25
Total assets	$50	Total liabilities & net worth	$50

The prospect of an upturn in the auto business would boost sales 30 per cent with no increase in fixed plant.

(a) How much financing would be required if this upturn did materialize?

(b) Is there any other financing which should be undertaken?

(c) What sources and proportions of financing appear to be available?

2. The Barnes Corporation has just acquired a large account. As a result, it needs an additional $75,000 in working capital immediately. It has been determined that there are three feasible sources of funds:

(1) Trade credit: the company buys about $50,000 of materials per month on terms of 2/30, net 90. Discounts are taken.

(2) Bank loan: the firm's bank will loan $100,000 at 9 per cent. A 20 per cent compensating balance will be required.

(3) A factor will buy the company's receivables ($100,000 per month) which have a collection period of sixty days. The factor will advance up to 75 per cent of the face value of the receivables for an annual charge of 8 per cent. The factor will also charge a 2 per cent fee on all receivables purchased. It has been estimated that the factor's services will save the company a credit department expense and bad debts expense of $1,500 per month.

Which alternative should be selected?

3. The Sphartz Company has estimated the following net cash flow pattern for the first three months of next year as follows:

January	+$150,000
February	− 200,000
March	− 300,000

These flows exclude consideration of gains secured from investing and costs associated with obtaining short-term funds. Further, it is assumed that all flows are effected at the *end* of the month.

The firm may obtain funds when needed from the following sources:

Short-Term Revolving Credit Agreement: The firm may borrow up to $500,000. The monthly fee for the privilege is $^1/_{10}$ per cent on the unused portion. It is believed that a borrowing rate of 8 per cent will prevail for the next three months. As of December 31, $200,000 of this amount was unused.

Trade Credit: The firm purchases approximately $100,000 of materials each month on terms 2/30, net 60. Currently, the firm is taking its discounts.

Factoring: The firm can borrow up to 90 per cent of its average factor balance ($200,000 per month) at a rate of $^3/_4$ per cent per month. The firm factors all of its receivables each month at a 2 per cent fee in order to keep this line of financing open. The firm saves $3,000 per month by not maintaining a credit department.

The firm may invest surplus funds in bankers' acceptances yielding 7 per cent. The commission cost of these purchases (and sales) is $^2/_{10}$ per cent.

Assuming the firm now has (Dec. 31) and must maintain a cash balance of $100,000, determine the optimum borrowing-investing pattern for the firm for the next three months. (Be sure to adjust cash flows for gains secured from investing and costs associated with obtaining short-term funds.)

4. The Bone Company has been factoring its accounts receivable for the past five years. The factor charges a fee of 2 per cent and will lend up to 80 per cent of the volume of receivables purchases for an additional $^3/_4$ per cent per month. The firm typically has sales of $500,000 per month, 70 per cent of which are on credit. By using the factor, two savings are effected:

(1) $2,000 per month that would be required to support a credit department, and

(2) A bad-debt expense of 1 per cent on credit sales.

The firm's bank has recently offered to lend the firm up to 80 per cent of the face value of the receivables shown on the schedule of accounts. The bank would charge 8 per cent per annum interest plus a 2 per cent processing charge per dollar of receivables lending. The firm extends terms of net 30, and all customers who pay their bills do so by the thirtieth of the month. Should the firm discontinue its factoring arrangement in favor of the bank's offer if the firm borrows, on the average, $100,000 per month on its receivables?

5. The Sharpless Corporation is in financial difficulty. In order to continue operations, the firm must raise $100,000 in working capital. The firm is unable to secure bank credit, though a commercial sales company has agreed to lend the

company up to $100,000 secured by a warehouse receipt. The loan will carry an annual interest charge of 20 per cent. The additional cost of maintaining a field warehouse arrangement to issue negotiable receipts is $2,000 per year.

A second alternative open to the firm is stretching its trade credit. The firm purchases on terms of 2/10, net 30. Stretching beyond the due date will result in a 3 per cent per month penalty charge, though it is believed that stretching beyond the due date by over 60 days may impair the firm's ability to get trade credit.

    (a) If the firm purchases $80,000 of raw materials every 30 days, how much will discount losses and stretching penalties cost the firm each month after the required $100,000 is raised?

    (b) Which form of financing is cheaper?

6. The Laurel Corporation needs $5 million now for one year, although there is a 0.4 probability that it can repay $2 million of this at the end of six months. The prime rate is currently 7.5 per cent, although there is a 0.5 probability that in six months it will rise to 8 per cent. Bankers' acceptances, it is assumed, can always be bought to yield 1 per cent less than the prime rate.

    (a) Laurel can get a one-year revolving-credit arrangement at prime plus $\frac{1}{2}$ per cent, with $\frac{1}{2}$ per cent on the unused balance and a 10 per cent compensating balance.

    (b) Laurel can sell one-year commercial paper at 8 per cent plus $25,000 dealers' commission. Laurel would also be required to maintain $1 million extra in the bank. Which of these plans is better? What other factors might be considered?

7. *Research Project*

Examine the annual reports of several large consumer finance companies. What sources of short-term financing do these firms employ? Why do they use these particular sources? Attempt to find out how often these firms issue commercial paper, in what volume, and at what cost. How many lines of credit (number of banks), and for what amounts, do these firms maintain? Do you find it incongruous that these firms maintain substantial lines of credit and issue commercial paper in large volumes? Why or why not?

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Intermediate-
Term Debt
Financing

The principal characteristic of short-term loans is that they are self-liquidating over a period of time less than a year. Frequently, they are employed to finance seasonal and temporary funds requirements. Intermediate-term financing, on the other hand, is employed to finance more permanent funds requirements, such as fixed assets and underlying build-ups in receivables and inventories. The means for payment of the loan usually come from the generation of cash flows over a period of years. As a result, most of these loans are paid in regular, periodic installments. We regard intermediate-term financing as involving final maturities of one to five years. These boundaries are arbitrary, although the one-year boundary is rather commonly accepted. The firm, of course, should choose those financial instruments that best serve its needs from the entire spectrum of maturities available. We assume that the choice of maturity composition has been made in keeping with the framework pre-

sented in Chapter 15. In this chapter, we examine various types of intermediate-term loans. In the chapter following, we consider lease financing, another means of intermediate-term financing.

BANK TERM LOANS

Commercial banks have become increasingly involved in providing intermediate-term financing to industry. Whereas once banks were thought to invest only in government securities and short-term loans, today the banker is called upon to provide financing for a wide variety of business activities. Currently, a large portion of business loans made by banks consist of term loans. There are two features of a bank term loan that distinguish it from other types of business loans. First, it has a final maturity of more than one year, and second, it most often represents credit extended under a formal loan agreement.[1] Both ordinary term loans and revolving credits are classified under the broad heading of bank term loans.

ORDINARY TERM LOANS

An ordinary term loan is a business loan with an original, or final, maturity of more than one year, repayable according to a specified schedule. For the most part, these loans are repayable in periodic installments, for example, quarterly, semiannually, or yearly. The payment schedule of the loan usually is geared to the borrower's cash flow ability to service the debt. Typically, this schedule calls for equal periodic installments, but it may be irregular with respect to amounts or may simply call for repayment in a lump sum at final maturity. Sometimes the loan is amortized in equal periodic installments except for the final payment, known as a "balloon" payment, which is larger than any of the others.

Maturity. Most bank term loans are written with original maturities in the one- to six-year range. Some banks are willing to make longer term loans, but only rarely will a bank make a term loan with a final maturity of more than ten years. In recent years, however, banks have been making longer term loans. Whereas a four- to six-year loan once was considered dangerous to a bank's liquidity, term loans in this maturity range now are common.

Interest Costs. Generally, the interest rate on a term loan is higher than the rate on a short-term loan to the same borrower. For example, if

[1] See Douglas A. Hayes, *Bank Lending Policies: Issues and Practices* (Ann Arbor, Mich.: Bureau of Business Research, University of Michigan, 1964), p. 95.

a firm could borrow at the prime rate on a short-term basis, it might pay 0.25 per cent to 0.50 per cent more on a term loan. The interest rate on a term loan can be set in one of two ways: (1) a fixed rate which is effective over the life of the loan may be established at the outset or (2) a variable rate may be set that is adjusted in keeping with changes in the prime rate. Under the second alternative, it is common to include a "floor" and a "ceiling." For example, if the interest rate on a term loan is specified as 0.50 per cent above the prime rate, a further condition might be that the firm would not pay more than 9 per cent or less than 5 per cent.[2] In addition to interest costs, the borrower is required to pay the legal expenses that the bank incurs in drawing up the loan agreement. Also, a commitment fee may be charged for the time during the commitment period when the loan is not taken down. For an ordinary term loan, these additional costs usually are rather small in relation to the amount of the loan. An indirect cost to the borrower is the need to maintain compensating balances, which we discussed in Chapter 20.

Advantages. The principal advantage of an ordinary bank term loan is flexibility. The borrower deals directly with the lender, and the loan can be tailored to the borrower's needs through direct negotiation. The bank usually has had previous experience with the borrower, so it is familiar with the company's situation. Should the firm's requirements change, the terms and conditions of the loan may be revised. It is considerably more convenient to negotiate with a single lender or a reasonably small group of lenders than with a large number of public security holders, as there are with a bond issue. In addition, the borrower can deal confidentially with a bank, or, for that matter, with any private lending institution, and does not have to reveal certain financial information to the public.

In many instances, bank term loans are made to small businesses that do not have access to the capital markets and cannot readily float a public issue. Large companies also may find it quicker and more convenient to seek a bank term loan than to float a public issue. A term loan can be arranged in several weeks, whereas a public issue takes a good deal longer.

Limitations. One of the limitations on the use of a bank term loan is the maturity. Banks seldom will make a term loan for more than ten years and often will want a shorter maturity. Another limitation is the restrictive provisions imposed in the loan agreement, which we will discuss in detail in this chapter. Although the borrower is restricted by these provisions, he probably would encounter them with an insurance company term-loan agreement or a bond indenture. A possible disad-

[2] *Ibid.*, pp. 107–8.

vantage of a bank term loan is the legal restriction on the maximum amount a bank can lend to a single borrower. However, many term loans are extended by a group of banks rather than a single bank. Except for the very largest of loans, the legal lending limit on banks is not a barrier.

When making loans, some banks ask for equity kickers in order to get a "piece of the action." These kickers usually take the form of stock purchase warrants, which enable the bank to purchase a number of shares of stock at a specified price. A percentage of net profit is another form of kicker, as is a percentage of the gross sales of a retail organization. With inflation, we can expect banks to demand equity kickers increasingly when they grant other than short-term loans. These kickers, of course, work to the disadvantage of the borrower; they raise the effective cost of borrowing. If the firm is doing well, the effective cost can be very high indeed. A firm regarded as risky, however, may have little choice other than to provide a kicker. This may spell the difference between obtaining a loan or being refused term credit. Moreover, the kicker usually operates in only one direction. If the firm is doing poorly, the kicker it provides has negligible value and, accordingly, costs the firm very little. If the firm is doing well, the kicker will have value. However, the firm will also be in a better position to incur the added cost.

REVOLVING CREDITS

As we said in Chapter 20, a revolving credit is a formal commitment by a bank to lend up to a certain amount of money to a company over a specified period of time. The actual notes evidencing debt are short term, usually ninety days; but the company may renew them or borrow additionally, up to the specified maximum, throughout the duration of the commitment. Many revolving-credit commitments are for three years, although it is possible for a firm to obtain a shorter commitment. As with an ordinary term loan, the interest rate is usually 0.25 to 0.50 per cent higher than the rate at which the firm could borrow on a short-term basis under a line of credit. When a bank makes a revolving-credit commitment, it is legally bound under the loan agreement to have funds available whenever the company wants to borrow. The borrower usually must pay for this availability in the form of a commitment fee, perhaps 0.50 per cent per annum, on the difference between the amount borrowed and the specified maximum.

Because most revolving-credit agreements are for more than one year, they are regarded as intermediate-term financing. This borrowing arrangement is particularly useful at times when the firm is uncertain about its funds requirements. A revolving-credit agreement has the features of both a short-term borrowing arrangement and a term loan, for the firm can borrow a fixed amount for the entire duration of the commitment. Thus, the borrower has flexible access to funds over a period of uncer-

tainty and can make more definite credit arrangements when the uncertainty is resolved.[3] Revolving-credit agreements can be set up so that at the maturity of the commitment, borrowings then owing can be converted into a term loan at the option of the borrower. To illustrate, suppose that a company introduces a new product and is faced with a period of uncertainty over the next several years. To provide maximum financial flexibility, the company might arrange a three-year revolving credit that is convertible into a five-year term loan at the expiration of the revolving-credit commitment. At the end of three years, the company, hopefully, would know its funds requirements better. If these requirements are permanent, or nearly so, the firm might wish to exercise its option and take down the term loan.

LOAN AGREEMENTS

When a bank makes a term loan or revolving-credit commitment, it provides the borrower with available funds for an extended period of time. Much can happen to the financial condition of the borrower during that period. In order to safeguard itself, the lender requires the borrower to maintain its financial condition and, in particular, its current position at a level at least as favorable as when the commitment was made. The provisions for protection contained in a loan agreement are known as protective covenants. In this section, we examine these provisions from the standpoint of the lender. However, later in the section and in the appendix to this chapter, we examine how a firm can negotiate to lessen the restrictiveness of the provisions.

The loan agreement itself simply gives the bank legal authority to step in should the borrower default under any of the provisions. Otherwise, the bank would be locked into a commitment and would have to wait until maturity before being able to effect corrective measures. If the borrower should suffer losses or other adverse developments, he will default under a well-written loan agreement; the bank will then be able to act. The action usually takes the form of working with the company to straighten out its problems. Seldom will a bank demand immediate payment, although it has the legal right to do so in cases of default.

Formulation of Provisions. The formulation of the different restrictive provisions should be tailored to the specific loan situation. These provisions are the tools by which the banker fashions the overall protection of his loan. No one provision is able by itself to provide the necessary safeguards, but together with the other provisions, it is designed to assure overall liquidity and ability to pay a loan. The important protective covenants of a loan agreement may be classified as follows: (1)

[3] Hayes, *Bank Lending Policies*, p. 96.

general provisions used in most loan agreements, which are variable to fit the situation; (2) routine provisions used in most agreements, which are not usually variable; and (3) specific provisions that are used according to the situation. Although we focus on a bank loan agreement, the protective covenants used and the philosophy underlying their use are the same for an insurance company loan agreement or the indenture for a bond issue.

General Provisions. The *working-capital requirement* probably is the most commonly used and most comprehensive provision in a loan agreement. Its purpose is to preserve the company's current position and ability to pay the loan. Frequently, a straight dollar amount, such as $2 million, is set as the minimum working capital the company must maintain during the duration of the commitment. When the bank feels that it is desirable for a specific company to build working capital, it may increase the minimum working-capital requirement throughout the duration of the loan. The establishment of a working-capital minimum normally is based upon the amounts of present working capital and projected working capital, allowing for seasonal fluctuations. The requirement should not restrict the company unduly in the ordinary generation of profit. However, should the borrower incur sharp losses or spend too much for fixed assets, purchase of stock, dividends, redemption of long-term debt, and so forth, it would probably breach the working-capital requirement.

The *cash dividend and repurchase-of-stock restriction* is another important restriction in this category. Its purpose is to limit cash going outside the business, thus preserving the liquidity of the company. Most often, cash dividends and repurchase of stock are limited to a percentage of net profits on a cumulative basis after a certain base date, frequently the last fiscal year-end prior to the date of the term-loan agreement. A less flexible method is to restrict dividends and repurchase of stock to an absolute dollar amount each year. In most cases, the prospective borrower must be willing to undergo a cash dividend and repurchase-of-stock restriction. If tied to earnings, this restriction still will allow adequate dividends as long as the company is able to generate satisfactory profits.

The *capital-expenditures limitation* is third in the category of general provisions. Capital expenditures may be limited to a fixed dollar amount each year. However, it probably is more common to limit annual capital expenditures either to depreciation or to a percentage thereof. The capital-expenditures limitation is another tool used by the banker to assure the maintenance of the borrower's current position. By limiting capital expenditures directly, the bank can be more sure that it will not have to look to liquidation of fixed assets for payment of its loan. Again, however, the provision should not be so restrictive as to prevent the adequate maintenance and improvement of facilities.

A *limitation on other indebtedness* is the last general provision. This limitation may take a number of forms, depending upon the circumstances. Frequently, a loan agreement will prohibit a company from incurring any other long-term debt. This provision protects the bank, inasmuch as it prevents future lenders from obtaining a prior claim on the borrower's assets. Usually a company is permitted to borrow within reasonable limits for seasonal and other short-term purposes arising in the ordinary course of business.

Routine Provisions. The second category of restrictions includes routine, usually invariable, provisions found in most loan agreements. Ordinarily, the loan agreement requires the borrower to furnish the bank with financial statements and to maintain adequate insurance. Additionally, the borrower normally is required not to sell a substantial portion of its assets and is required to pay, when due, all taxes and other liabilities, except those contested in good faith. A provision forbidding the pledging or mortgaging of any of the borrower's assets is almost always included in a loan agreement; this important provision is known as a negative pledge clause.

Ordinarily, the company is required not to discount or sell its receivables. Moreover, the borrower generally is prohibited from entering into any leasing arrangement of property, except up to a certain dollar amount of annual rental. The purpose of this provision is to prevent the borrower from taking on a substantial lease liability, which might endanger its ability to pay the loan. A lease restriction also prevents the firm from leasing property instead of purchasing it and thereby getting around the limitations on capital expenditures and debt. Usually, too, there is a restriction on other contingent liabilities. The provisions in this category appear as a matter of routine in most bank loan agreements. Although somewhat mechanical, they are important because they close many loopholes and provide a tight, comprehensive loan agreement.

Special Provisions. Special provisions are used in specific loan agreements by the banker in order to achieve a desired total protection o his loan. For instance, a loan agreement may contain a definite understanding regarding the use of the loan proceeds, so that there will be no diversion of funds to purposes other than those contemplated when the loan was negotiated. A provision for limiting loans and advances often is found in a bank term-loan agreement. Closely allied to this restriction is a limitation on investments, which is used to safeguard liquidity by preventing certain nonliquid investments.

If one or more executives are essential to a firm's effective operation, a bank may insist that the company carry life insurance on their lives. Proceeds of the insurance may be payable to the company or directly to the bank, to be applied to the loan. An agreement may also contain a

management clause, under which certain key individuals must remain actively employed in the company during the time the loan is owing. Aggregate executive salaries and bonuses sometimes are limited in the loan agreement, to prevent excessive compensation of executives, which might reduce profits. This provision closes another loophole; it prevents large stockholders who are officers of the company from increasing their own salaries in lieu of paying higher dividends, which are limited under the agreement.

Negotiation of Restrictions. The provisions described above represent the most frequently used protective covenants in a loan agreement. From the standpoint of the lender, the aggregate impact of these provisions should be to safeguard the financial position of the borrower and its ability to pay the loan. Under a well-written agreement, the borrower cannot get into serious financial difficulty without defaulting under the agreement, thereby giving the bank legal authority to take action. Although the lender is instrumental in establishing the restrictions, the restrictiveness of the protective covenants is subject to negotiation between the borrower and the lender. The final result will depend upon the relative bargaining power of each of the parties involved. In the appendix to this chapter, a linear programming approach for evaluating the opportunity costs of the protective covenant restrictions is presented. If it knows these opportunity costs, management is able to bargain more effectively over the restrictiveness of the covenants.

INSURANCE COMPANY TERM LOANS

Insurance companies and certain other institutional lenders, as well as banks, extend term loans to companies. The former establish protective covenants much as a bank does. However, there are important differences in the maturity of the loan extended and in the interest rate charged. In general, life insurance companies are interested in term loans with final maturities in excess of ten years. Because these companies do not have the benefit of compensating balances or other business from the borrower, and because their loans usually have a longer maturity than bank term loans, typically the rate of interest is higher. To the insurance company, the term loan represents an investment and must yield a return commensurate with the costs involved in making the loan, the risk, the maturity, and prevailing yields on alternative investments. Because an insurance company is interested in keeping its funds employed without interruption, it normally has a prepayment penalty, whereas ordinarily the bank does not. One of the simpler prepayment formulas calls for a premium of 0.25 per cent for each year remaining to maturity.

Insurance company term loans generally are not competitive with

bank term loans. Indeed, they are complementary; for they serve different maturity ranges. Sometimes a bank and an insurance company will participate in the same loan. The bank may take the early maturities, perhaps the first five years, with the insurance company taking the remaining maturities. Including an insurance company in the credit permits a longer maturity range than the bank can provide, and the bank can offer a lower interest rate on the early maturities. Usually, there will be only one loan agreement, drawn up jointly by the bank and the insurance company. A term loan of this sort may serve both the intermediate- and long-term funds requirements of the firm.

EQUIPMENT FINANCING

Equipment represents another asset of the firm that may be pledged to secure a loan. If the firm either has equipment that is marketable or is purchasing such equipment, it is usually able to obtain some sort of secured financing. Because such loans usually are for more than a year, we consider them in this chapter rather than under short-term secured loans. As with other secured loans, the lender is concerned with the marketability of the collateral. Depending upon the quality of the equipment, he will make a percentage advance against the equipment's market value. Frequently, the repayment schedule for the loan is set in keeping with the depreciation schedule of the equipment. For example, a trucking company will usually depreciate its tractors over four years and its trailers over six years. A lender might set a four-year installment payment schedule for a loan secured by tractors and a six-year schedule for a loan secured by trailers. In setting the repayment schedule, the lender wants to be sure that the market value of the tractor or trailer always exceeds the balance of the loan.

The excess of the expected market value of the equipment over the amount of the loan represents the margin of safety, which will vary according to the specific situation. In the case of the rolling stock of a trucking company, the collateral is movable and reasonably marketable. As a result, the advance may be as high as 80 per cent. Less marketable equipment, such as that with a limited use, will not command as high an advance. A certain type of lathe, for example, may have a reasonably thin market, and a lender might not be willing to advance more than 50 per cent of its reported market value. Some equipment is of such a special-purpose nature that it has no value for collateral purposes. Frequently, the lender either will have its own appraiser or will hire an appraiser to estimate the approximate value of a piece of equipment if it should have to be sold. As with other collateral, the lender is interested not only in the estimated market price of the equipment but also in the cost of selling it.

SOURCES OF EQUIPMENT FINANCING

Sources of equipment financing include commercial banks, finance companies, and the sellers of equipment. Because the interest charged by a finance company on an equipment loan usually is higher than that charged by a commercial bank, a firm will turn to a finance company only if he is unable to obtain the loan from a bank. The seller of the equipment may finance the purchase either by holding the secured note itself or by selling the note to its captive finance subsidiary. The interest charge will depend upon the extent to which the seller uses financing as a sales tool. If he uses it extensively, he may charge only a moderate interest rate, and may make up for part of the cost of carrying the notes by charging higher prices for the equipment. The borrower must consider this possibility in judging the true cost of financing. Equipment loans may be secured either by a chattel mortgage or by a conditional sales contract arrangement.

CHATTEL MORTGAGE

A chattel mortgage is a lien on property other than real property. The borrower signs a security agreement which gives the lender a lien on the equipment specified in the agreement. In order to perfect the lien, the lender files a copy of the security agreement or a financing statement with a public office of the state in which the equipment is located. Given a valid lien, the lender can sell the equipment if the borrower defaults in the payment of principal or interest on the loan.

CONDITIONAL SALES CONTRACT

With a conditional sales contract arrangement, the seller of the equipment retains title to it until the purchaser has satisfied all the terms of the contract. The buyer signs a conditional sales contract security agreement under which he agrees to make periodic installment payments to the seller over a specified period of time. These payments usually are monthly or quarterly. Until the terms of the contract are satisfied completely, the seller retains title to the equipment. Thus, the seller receives a down payment and a promissory note for the balance of the purchase price upon the sale of the equipment. The note is secured by the contract, which gives the seller the authority to repossess the equipment if the buyer does not meet all the terms of the contract.

The seller may either hold the contract himself or sell it, simply by endorsing it, to a commercial bank or finance company. The bank or finance company then becomes the lender and assumes the security interest in the equipment. If the buyer should default under the terms of

the contract, the bank or finance company could repossess the equipment and sell it in satisfaction of its loan. Often, the vendor will sell the contract to a bank or finance company with recourse. Under this arrangement, the lender has the additional protection of recourse to the seller in case the buyer defaults.

The Small Business Act of 1953 gave the Small Business Administration, an agency of the federal government, the authority to make loans to small businesses that could not get loans from other sources on reasonable terms.[4] The definition of a small business depends upon its sales in relation to those of the industry, and upon the number of employees. Any manufacturing firm with less than 250 employees is eligible for a SBA loan. The SBA seeks to make sound business loans to creditworthy borrowers, as private lenders do. A company wishing to borrow files an application with the SBA; the application receives thorough analysis before the decision to make the loan is made. SBA credit is designed primarily for firms that are unable to obtain a loan from a commercial bank.

When possible, the SBA prefers to participate with a private lending institution in extending credit. The participation by the SBA may be up to 90 per cent of the loan, with the balance being provided by the private lender. This participation does not necessarily have to consist of a loan from the SBA; the SBA may instead guarantee payment of up to 90 per cent of a loan by a private lender. When unable to participate with a private lender, the SBA can make the entire loan itself; this type of loan is called a direct loan. Some SBA loans are short term, but the vast majority are term loans with an average maturity of about five years. Increasingly, longer-term loans are being made and ten-year loans, the maximum maturity, are common. Over 85 per cent of the loans approved in 1968 were under $25,000.

Intermediate-term financing generally is thought to include maturities of one to ten years. There are a number of sources of intermediate-term financing. Commercial banks, insurance companies, and other institutional investors make term loans to business firms. Banks also provide

[4]See *Organization and Operation of the Small Business Administration,* Select Committee on Small Business, House of Representatives, 88th Congress, 2d Session (Washington, D.C.: Government Printing Office, 1964).

financing under a revolving-credit arrangement, which represents a formal commitment on the part of the bank to lend up to a certain amount of money over a specified period of time.

Lenders who offer unsecured credit usually impose restrictions on the borrower. These restrictions are called protective covenants and are contained in a loan agreement. If the borrower defaults under any of the provisions of the loan agreement, the lender may initiate immediate corrective measures. On a secured basis, firms can obtain intermediate-term financing by pledging equipment that they own or are purchasing. Banks, finance companies, and sellers of the equipment are active in providing this type of secured financing. Finally, the Small Business Administration (SBA) extends intermediate-term credit.

Typically, intermediate-term financing is self-liquidating. For this reason, it resembles short-term financing. However, intermediate-term financing can also satisfy more permanent funds requirements and, in addition, can serve as an interim substitute for long-term financing. If a firm wishes to float long-term debt or issue common stock but conditions are unfavorable in the market, the firm may seek intermediate-term debt to bridge the gap until long-term financing can be undertaken on favorable terms. Thus, intermediate-term debt may give a firm flexibility in the timing of long-term financing. It can also provide flexibility when the firm is uncertain as to the size and nature of its future funds requirements. As uncertainty is resolved, intermediate-term financing can be replaced by a more appropriate means of financing. (A bank revolving credit is well suited for providing this type of flexibility.) The most important use of intermediate-term financing, however, is to provide credit when the expected cash flows of the firm are such that the debt can be retired steadily over a period of several years. Even though it is sometimes linked to a particular asset, such as a piece of equipment, intermediate-term financing must be considered in relation to the firm's total funds requirements. It can play a major role in the overall financing decision of the firm.

APPENDIX

**A Method for
Evaluating
Restrictions
Under a
Loan Agreement[5]**

When a company enters into a bond indenture or loan agreement, certain restrictions usually are placed on it. These restrictions, known as protective covenants, may have a significant influence on the firm's profits, making the bargaining strategy of the company very important. But to bargain effectively over the restrictiveness of the protective covenants, management must know the impact that the covenants have

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[5] Adapted from James Van Horne, "A Linear-Programming Approach to Evaluating Restrictions Under a Bond Indenture or Loan Agreement," *Journal of Financial and Quantitative Analysis,* 1 (June, 1966), 68–83.

on profits. In this appendix, a method is proposed for determining the opportunity costs of the restrictions imposed under the indenture or agreement. These opportunity costs represent the additional profit a company could make if a restriction were relaxed. The framework for analysis is a linear programming model for capital-budgeting and financing decisions, using a hypothetical company as an example. Sensitivity analysis is employed to determine the opportunity costs of the restrictions, and to give management the information it needs to formulate its bargaining strategy.

STATEMENT OF THE PROBLEM

We assume that the company's objective is to maximize net-present value arising from various investment proposals. There are, of course, many practical difficulties in forecasting the cash flows expected from an investment proposal as well as in determining the cost of capital to be used as the discount factor. We shall assume here that cash flows are known with certainty and the company can measure its cost of capital accurately.

Assuming that all outlays are made at the time of the investment decision, net-present value per dollar of investment in a proposal can be determined by the profitability index. For our purposes, the profitability index is interpreted as the net-present value of all outlays and inflows, discounted at the company's existing cost of capital, divided by the amount of initial cash outlay. Table 21A-1 shows seven investment proposals available to our hypothetical company, all having positive profitability indexes, and involving varying proportions of working capital and fixed assets. We assume that these proposals are independent of each other and of the company's existing investment projects and that the company may invest in each proposal at any level between zero and the stated maximum.[6]

We assume that the size of the capital budget is limited to $800,000, financed by $400,000 in retained earnings represented by excess cash

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[6] In the model, we have assumed the divisibility of investment proposals. If a proposal is nonfractional, investment must be either zero or some absolute amount; and a dichotomy exists. Problems of this sort having "either-or" conditions may be solved by integer programming. See George B. Dantzig, "On the Significance of Solving Linear Programming Problems with Some Integer Variables," *Econometrica*, XXVIII (January, 1960), 30–44; and Ralph E. Gomory, "Outline of an Algorithm for Integer Solutions to Linear Programs," *Bulletin of the American Mathematical Society*, LXIV (September, 1958), 275–78.

Although the direct problem is straightforward when integer-valued variables are involved, there may be complications in interpreting values for the dual variables. For exposition of the dual-variable problem in integer programming, see Ralph E. Gomory and William J. Baumol, "Integer Programming and Pricing," *Econometrica*, XXVIII (July, 1960), 521–50; and H. Martin Weingartner, *Mathematical Programming and the Analysis of Capital Budgeting Problems*, copyright H. Martin Weingartner, Chapter 5.

TABLE 21A-1
Investment proposals available

Proposal	Proportion of Working Capital to Fixed Assets	Maximum Amount of Investment	Profitability Index
1	1 : 4	$350,000	0.18
2	2 : 1	225,000	0.06
3	1 : 1	170,000	0.13
4	3 : 1	200,000	0.09
5	1 : 2	150,000	0.15
6	4 : 1	250,000	0.07
7	1 : 3	300,000	0.08

and $400,000 in debt. The debt portion may be divided between short-term and long-term borrowings. In addition, when the firm finances investment proposals with equal amounts of retained earnings and borrowings, the relative proportions of borrowings and equity remain unchanged. Thus, we assume that the financing of investment proposals leaves the cost of capital used as a discount factor virtually unchanged.[7] The problem, as stated so far, involves weighing the alternative investment proposals in Table 21A-1 so as to maximize the value of the firm. If there were no protective-covenant constraints imposed on the company, we would expect it to invest, in descending order of profitability, $350,000 in investment proposal 1, $150,000 in proposal 5, $170,000 in proposal 3, and $130,000 in proposal 4. Investment in these proposals, totaling $800,000, would result in the maximum possible increase in net-present value — that is, $119,300.

We now consider the constraints imposed by the terms of the bond indenture or loan agreement; these are the constraints that we shall evaluate later. The problem will involve a single-stage decision at a point in time to invest and to borrow, subject to liquidity and other financial constraints. In order to simplify the problem, we assume that the terms of the indenture or agreement already are in force and that the impact of the restrictions occurs at the time of the decision.

The first protective covenant we consider is *a minimum working-capital constraint.* If minimum working capital is $2,250,000 and existing working capital $2,500,000, investment in fixed assets (a use of working capital) could exceed long-term borrowings (a source of working capital) by only $250,000. A *capital-expenditures constraint,* the second covenant, involves a limitation on the amount of funds that may be invested in fixed assets. We assume in this example that capital expenditures are limited to $500,000.

Protective covenants dealing with long-term debt may take a number of forms. We shall use only one — *a percentage limitation of long-term*

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[7] We are assuming that the acceptance of an investment proposal or group of proposals does not alter the business-risk complexion of the firm.

debt to working capital—but additional constraints may be fashioned to fit the situation. We assume this percentage to be 80 per cent.

The last protective-covenant constraint we consider is a *cash dividend restriction*. This restriction is most often expressed as a limitation on the proportion of dividends to annual earnings on a cumulative basis. In our example, the percentage limitation is assumed to be 50 per cent, and it becomes effective at the time of the investment decision, there being no accumulated earnings available for dividend payments. Therefore, dividends must not exceed 50 per cent of book earnings on existing projects plus book earnings generated from investment in the projects under consideration.

These protective covenants are by no means the only possible restrictions that may be imposed in a bond indenture or loan agreement. However, the covenants considered do represent some of the more widely used restrictions. In molding the above restrictions into linear programming constraints, we use the limits imposed in the indenture or agreement. Where a constraint is binding in the final program, the company would be at the verge of violation under the specific covenant involved. It would be desirable, therefore, to allow for a margin of safety. If, for example, the minimum working-capital requirement in the indenture or loan agreement is $1,250,000, we might want to use $1,350,-000 in formulating the linear programming constraint. Other restrictions likewise can be reformulated to permit a margin of safety.

FORMULATION OF PROBLEM

Given the information in Table 21A-1, the objective function for our hypothetical company would be

$$\text{Max. } Z = .18X_{11} + .06X_{21} + .13X_{31} + .09X_{41} + .15X_{51} + .07X_{61} +$$
$$.08X_{71} + .18X_{12} + .06X_{22} + .13X_{32} + .09X_{42} + .15X_{52} +$$
$$.07X_{62} + .08X_{72} \quad \text{(A-1a)}$$

where X_{11} through X_{71} represent investment in working capital for the seven proposals and X_{12} through X_{72} represent investment in fixed assets. Structural constraints are introduced by virtue of the ceilings on investment, the relative proportions of working capital and fixed assets, and the need to finance the proposals with retained earnings and debt. For constraints relating to ceilings on investment, we have

$$X_{11} + X_{12} \leqq 350,000 \quad \text{(A-1b)}$$
$$X_{21} + X_{22} \leqq 225,000 \quad \text{(A-1c)}$$
$$X_{31} + X_{32} \leqq 170,000 \quad \text{(A-1d)}$$
$$X_{41} + X_{42} \leqq 200,000 \quad \text{(A-1e)}$$
$$X_{51} + X_{52} \leqq 150,000 \quad \text{(A-1f)}$$
$$X_{61} + X_{62} \leqq 250,000 \quad \text{(A-1g)}$$
$$X_{71} + X_{72} \leqq 300,000 \quad \text{(A-1h)}$$

The structural constraints relating to proportions of working capital and fixed assets are

$$4X_{11} - X_{12} = 0 \qquad \text{(A-1i)}$$
$$X_{21} - 2X_{22} = 0 \qquad \text{(A-1j)}$$
$$X_{31} - X_{32} = 0 \qquad \text{(A-1k)}$$
$$X_{41} - 3X_{42} = 0 \qquad \text{(A-1l)}$$
$$2X_{51} - X_{52} = 0 \qquad \text{(A-1m)}$$
$$X_{61} - 4X_{62} = 0 \qquad \text{(A-1n)}$$
$$3X_{71} - X_{72} = 0 \qquad \text{(A-1o)}$$

The last set of structural constraints is

$$\sum_{i=1}^{7} \sum_{j=1}^{2} X_{ij} - \sum_{l=1}^{3} Y_l = 0 \qquad \text{(A-1p)}$$

$$\sum_{l=1}^{3} Y_l \leqq 800{,}000 \qquad \text{(A-1q)}$$

$$\sum_{l=1}^{2} Y_l - Y_3 = 0 \qquad \text{(A-1r)}$$

where Y_1 is the amount of short-term borrowings, Y_2 the amount of long-term borrowings, and Y_3 the amount of retained earnings used to finance the investment proposals.

As taken up in the previous section, there are four protective-covenant constraints for our hypothetical example. The first, the *minimum working-capital constraint*, may be expressed as

$$\sum_{i=1}^{7} X_{i2} - Y_2 \leqq 250{,}000 \qquad \text{(A-1s)}$$

where $250,000 represents existing working capital less minimum or required working capital. The *capital-expenditures constraint* may be expressed as

$$\sum_{i=1}^{7} X_{i2} \leqq 500{,}000 \qquad \text{(A-1t)}$$

If long-term debt cannot exceed 80 per cent of working capital under the *long-term debt to working-capital constraint,* we have

$$W.C. - \sum_{i=1}^{7} X_{i2} + Y_2 \geqq 1\tfrac{1}{4}\, L.T.D. + 1\tfrac{1}{4}\, Y_2$$

where $W.C.$ is existing working capital, and $L.T.D.$ is existing long-term debt. If existing working capital is $2,500,000, as before, and existing long-term debt is $1,720,000, by transposing and multiplying through by -1, we obtain

$$\sum_{i=1}^{7} X_{i2} + \frac{1}{4} Y_2 \leqq W.C. - 1\frac{1}{4} L.T.D. \leqq 350,000 \qquad \text{(A-1u)}$$

With the *cash dividend restriction,* dividends are limited to 50 per cent of book earnings on existing projects and proposals under consideration. The first-year profit (or loss) per dollar of investment for the seven investment proposals under consideration is shown in the columns below.

Proposals	First-Year Profit (Loss) Per Dollar of Investment
1	(0.08)
2	0.10
3	(0.04)
4	(0.03)
5	(0.09)
6	(0.02)
7	0.05

We assume that the company desires to pay, at the end of the first year, cash dividends totaling $200,000 and that book first-year earnings on existing projects, exclusive of proposals under consideration, will be $425,000. We have as a constraint

$$0.08X_{11} - 0.10X_{21} + 0.04X_{31} + 0.03X_{41} + 0.09X_{51} + 0.02X_{61}$$
$$- 0.05X_{71} + 0.08X_{12} - 0.10X_{22} + 0.04X_{32} + 0.03X_{42} + 0.09X_{52}$$
$$+ 0.02X_{62} - 0.05X_{72} \leqq Y - 2C \qquad \text{(A-1v)}$$

where Y equals first-year earnings on existing projects, and C equals the cash dividend the company desires to pay. Thus, $Y - 2C = \$425,000 - 2(\$200,000) = \$25,000$. It would be possible to reformulate this constraint for additional years. For simplicity, we assume that book profits for all investment proposals under consideration are positive beyond the first year and that the lowest total combination of book profits in any one year is more than twice the cash dividend the company desires to pay. Consequently, the dividend restriction is of concern only in the first year.

The complete problem may be expressed as

$$\text{Max. } Z = \sum_{i=1}^{7} \sum_{j=1}^{2} C_i X_{ij} \qquad \text{(A-2)}$$

subject to

$$\sum_{i=1}^{7} \sum_{j=1}^{2} A_{kij} X_{ij} + \sum_{l=1}^{3} A_{kl} Y_l \leqq b_k$$

and the nonnegative requirement:

$$X_{ij} \text{ and } Y_l \geqq 0 \text{ for all } ijl.$$

SOLUTION OF PROBLEM

A computer program was used to solve this problem, and optimal values of the direct-problem variables and the dual variables are shown in Table 21A-2.[8] The values for the direct-problem variables tell us that we should invest $169,500 in investment proposal 1, $54,750 in proposal 2, $170,000 in proposal 3, $200,000 in proposal 4, and $205,750 in proposal 6. Moreover, the company should borrow $320,000 on a short-term basis and $80,000 on a long-term basis. Substituting into Eq. (A-1a), the increase in net-present value resulting from this optimal solution is $88,297.50.

TABLE 21A-2
Optimal values for example problem

Direct				Dual			
$X_{11} =$	33,900	$X_{32} =$	85,000	$W_1 =$	0	$W_{12} = 0.04671$	
$X_{21} =$	36,500	$X_{42} =$	50,000	$W_2 =$	0	$W_{13} = 0.03150$	
$X_{31} =$	85,000	$X_{52} =$	0	$W_3 = 0.00758$		$W_{14} = 0.09792$	
$X_{41} =$	150,000	$X_{62} =$	41,150	$W_4 = 0.00954$		$W_{15} = 0.03333$	
$X_{51} =$	0	$X_{72} =$	0	$W_5 =$	0	$W_{16} = 0.03333$	
$X_{61} =$	164,600	$Y_1 =$	320,000	$W_6 =$	0	$W_{17} =$	0
$X_{71} =$	0	$Y_2 =$	80,000	$W_7 =$	0	$W_{18} = 0.03150$	
$X_{12} =$	135,600	$Y_3 =$	400,000	$W_8 = 0.03150$		$W_{19} =$	0
$X_{22} =$	18,250			$W_9 = 0.05250$		$W_{20} = 0.12600$	
				$W_{10} = 0.07875$		$W_{21} = 0.25833$	
				$W_{11} = 0.03937$			

From Table 21A-2 it can be seen that the optimal investment policy under the protective-covenant constraints differs considerably from what would be optimal without the constraints. As mentioned previously, the optimal policy without constraints would call for investment of $350,000 in investment proposal 1, $170,000 in proposal 3, $130,000 in proposal 4, and $150,000 in proposal 5; and this policy would result in a $119,300 increase in net-present value. Thus, the restrictions have a marked effect on the optimal investment policy of the company. For example, under the protective-covenant restrictions, the company would invest $54,750 in the least profitable proposal, 2, and invest nothing in the second most profitable, 5. The $119,300 increase in net-present value represents the limit of increase in net-present value attainable with relaxation of the

[8] For each direct problem there is a dual problem that makes use of the same data as are employed in the direct problem. The dual theorem of linear programming states that the optimal solutions to the direct and the dual problem are the same. For explanation of the dual method, see A. Charnes and W. W. Cooper, *Management Models and Industrial Applications of Linear Programming,* Vol. I (New York: John Wiley & Sons, Inc., 1961), or other texts dealing with linear programming.

protective-covenant constraints. In the analysis that follows, it is useful to relate the increase in net-present value arising from relaxation of an individual restriction to this limit.

EVALUATING THE RESTRICTIONS

The dual-variable values in Table 21A-2 enable us to determine the opportunity costs of certain protective-covenant constraints imposed under the bond indenture or loan agreement. Knowledge of these costs is extremely important to management in formulating negotiation strategy. For example, what will be the effect if the minimum working-capital requirement is relaxed? If this constraint is binding or critical in the final program, the dual variable for the restriction will be more than zero; if not, it will be zero. Thus, for the working-capital constraint, $W_{18}{}^* = 0.0315$ signifies that if required working capital is decreased by \$1, the company will be able to increase net-present value \$0.0315 if the dollar is optimally employed.[9] We see in Table 21A-2 that the capital-expenditures limitation is not binding in the final program, for $W_{19}{}^* = 0$. The constraint, as formulated in inequality (A-1t), would be critical only if investments in fixed assets increased by \$170,000, bringing total investment in fixed assets to \$500,000.

For the limitation of long-term debt to 80 per cent of working capital, (inequality (A-1u)), the value $W_{20}{}^* = 0.126$ denotes that a \$1 increase in $(W.C. - 1\frac{1}{4}L.T.D.)$, if used optimally, would result in a \$0.126 increase in net-present value.[10] More important than the effect of changes in $(W.C. - 1\frac{1}{4} L.T.D.)$ is the effect that changes in the maximum percentage of long-term debt to working capital have on the optimal solution. For example, what would be the effect if the percentage limitation is relaxed from 80 per cent to 100 per cent? Inequality (A-1u) would become

$$\sum_{i=1}^{7} X_{i2} \leqq W.C. - L.T.D. \qquad \text{(A-3)}$$

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[9] Optimal use would involve investing an additional \$0.30 in proposal 1 and \$0.15 in proposal 2, decreasing by \$0.45 the investment in proposal 6, increasing short-term borrowings by \$0.80, and decreasing long-term borrowings by \$0.80. The dual variable, $W_{18} = 0.0315$ is valid only within certain limits. We can increase short-term debt and decrease long-term debt by only \$80,000 before inequality (A-1q) and the nonnegative requirement become binding. On the other hand, investment in proposal 6 can increase only \$44,250 before inequality (A-1g) becomes binding. Consequently, $W_{18}{}^* = 0.0315$ remains valid for \$151,666.67 $\leqq (W.C. - \text{min. } W.C.) \leqq$ \$350,000.

[10] Optimal employment of a \$1 increase in the right-hand side of inequality (A-1u) would involve a \$1.20 increase in proposal 1, a \$0.60 increase in proposal 2, a \$1.80 decrease in proposal 6, an \$0.80 decrease in short-term borrowings, and an \$0.80 increase in long-term borrowings. The value $W_{20}{}^* = 0.126$ remains valid for \$325,416.67 $\leqq (W.C. - 1\frac{1}{4} L.T.D.) \leqq$ \$464,305.55. At \$325,416.67, inequality (A-1g) becomes binding and precludes further investment in proposal 6; at \$464,305.55, further reductions in investment proposal 6 would not be possible.

where $W.C. - L.T.D.$ is $2,500,000 - $1,720,000 = $780,000. If optimal adjustment is made to this relaxation of the percentage limitation, net-present value would increase by $22,223.55, to $110,521.05.[11]

For the dividend constraint (inequality (A-1v)), $W_{21}{}^* = 0.25833$. If $Y - 2C$, where Y is first-year earnings on existing projects and C is the total cash dividend the company desires to pay at the end of the first year, were to increase by $1, net-present value would increase $0.25833 if the $1 were employed optimally.[12] What effect does the payment of dividends have on the firm's net-present value? Given the amount of the first-year earnings on existing projects and the limitation of dividends to 50 per cent of earnings, the company may increase net-present value by $0.25833 for each $0.50 decrease in total cash dividends it pays at the end of the first year. Thus, given the dividend restriction, management is able to determine the effect that the payment of dividends has on net-present value.[13]

Perhaps the more important consideration is the effect that the percentage limitation has on profitability, given the dividend the company desires to pay. If the limitation is increased from 50 per cent to 66.67 per cent, what would be the effect on net-present value? The right-hand side of the constraint, inequality (A-1v), would become

$$425,000 - 1.5(200,000) = 125,000$$

If optimal adjustment is made to this relaxation in the dividend restriction, net-present value will increase by $18,858.33.[14]

IMPLICATIONS

Through sensitivity analysis, we may evaluate the effect on the optimal solution of given changes in the protective-covenant constraints. The sensitivity of the optimal solution to changes in these parameters was determined without our having to solve one or a series of new problems. Equipped with knowledge of the opportunity costs of the various re-

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[11] Optimal adjustment would involve increasing investment in proposal 1 by $180,500, to $350,000, increasing investment in proposal 2 by $129,460.53 to $184,210.53, eliminating the investment of $200,000 in proposal 4, investing $95,789.47 in proposal 5, eliminating the investment of $205,750 in proposal 6, decreasing short-term borrowings by $160,263.16, and increasing long-term borrowings by the same amount.

[12] Optimal employment would involve a $1.67 increase in proposal 1, a $7.50 decrease in proposal 2, and a $5.83 increase in proposal 6. As can be determined, $W_{21}{}^* = 0.25833$ remains valid for $2,300 \leq Y - 2C \leq $32,300. Below $2,300, inequality (A-1c) becomes binding on further increases in proposal 2. Above $32,300, further decreases in proposal 2 are not possible.

[13] We ignore the problem of deviations from desired dividends having an effect on the cost of capital by assuming that the moderate changes in dividends under consideration will not affect the cost of capital.

[14] Optimal adjustment would involve increasing investment in proposal 1 by $12,166.67 to $181,666.67, eliminating the $54,750 investment in proposal 2, and increasing investment in proposal 6 by $42,583.33, to $248,333.33.

strictions, management can bargain more rationally and effectively, giving ground when the restrictions involved have small opportunity costs or none and driving a hard bargain on those restrictions having high opportunity costs.

This knowledge is important to management if there is a possibility that the covenants under an existing indenture or agreement might be relaxed through negotiation with the lender(s). Changing protective covenants is much more likely under a loan agreement in which there is only one or a relatively small number of lenders than it is under a bond indenture. Knowledge of the opportunity costs involved in various restrictions is perhaps even more valuable in the initial negotiation of protective covenants to be imposed under an indenture or agreement.

PROBLEMS

1. The Bell Corporation has indentures that require mortgage debt to be no more than 50 per cent of all other junior debt and net worth, senior debentures to be no more than 75 per cent of all other junior debt and net worth, subordinated debentures to be no more than 25 per cent of all securities junior to it, and preferred stock to be no more than 30 per cent of common. Bell is currently at these limits and will always return to them through the immediate sale of securities.

If $5 million of subordinated debentures were converted into common, what net gain in total liabilities and net worth would take place?

2. The McDonald Company wishes to buy a $1.2 million piece of equipment over a two-year period. The bank has offered to loan the required money on the basis of a two-year note with a $300,000 amortization payment every six months. The loan would require a compensating balance equal to 15 per cent of the outstanding balance and would bear 10 per cent interest on the unpaid balance. The seller of the equipment has offered McDonald a conditional sales contract with four equal semiannual payments. How large could the payments be before McDonald would find the bank loan more attractive?

3. The Buda Company is contemplating investing in a project which will generate the following net cash flows (after taxes):

Year	Net Flows (end of year)
0	−9,000,000
1	+5,000,000
2	+5,000,000
3	+5,000,000

The firm has decided to finance the project through using intermediate-term debt; for this there appear to be two alternatives:
  (a) Utilize an additional $9,000,000 of the firm's three-year revolving bank credit. Currently $3,000,000 of the $15,000,000 commitment is used. The bank charges 1 per cent over prime on the balance and ½ per cent on the unused portion. Prime is expected to average 7 per cent in year 1, 8 per cent in year 2, and 7 per cent in year 3. Treasury bills can always be bought to yield 1.5 per cent less than the prevailing prime rate.
  (b) Privately place a $9 million, unsecured three-year note with Atonement

Mutual. The note would bear $7\frac{1}{2}$ per cent interest and involve a 0.5 per cent placement fee. The note agreement would also require the firm to maintain $1 million more in working capital than would otherwise be held.

(1) Which alternative is to be preferred? What other factors might be considered?

(2) Suppose that the Atonement note provided for an equal amortization of principal, and Buda had no other investment opportunities (except Treasury bills) over the three-year period. Would this affect your answer?

4. The Chain Corporation needs $10 million for the next ten years. The prime rate is now 6 per cent; it is expected to ascend gradually to 8 per cent at the end of year 5 and then descend gradually back to 7 per cent at the end of year 10. Chain has the following alternative sources of financing:

(a) Ten one-year bank loans at prime with a 20 per cent compensating balance. The interest cost would be computed on the basis of the average prime rate for the year.

(b) Two five-year notes placed with an insurance company. The interest cost would be set at 1 per cent above the prime rate at the time the note was negotiated.

(c) One ten-year bond sold publicly (total costs of issue = 3 per cent of gross proceeds) now at an interest rate of 7.25 per cent.

If the Chain Corporation has a 50 per cent tax rate, which alternative should be selected?

5. Research Project
Examine several bond indentures and/or loan agreements (if available). What major restrictions are imposed? Do you note any differences in the restrictions outlined in bond indentures as opposed to those in loan agreements? Why might these differences exist? Do you find any correlation between the maturity length of the instrument and the number (and degree) of the restrictions? What might account for your findings?

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Lease

Financing

22

Lease financing has developed rapidly in the past two decades; now it is an important source of financing for a wide variety of capital assets.[1] In this chapter, our concern is with financial leases rather than operating leases. A financial lease is a noncancellable contractual commitment on the part of a lessee to make a series of payments to a lessor for the use of an asset. The lessee acquires most of the economic values associated with outright ownership of the asset, even though the lessor retains title to it. With a financial lease, the lease period generally corresponds to the economic life of the asset. In addition, the total payments the lessee agrees to make must exceed the purchase price of the asset.[2] The dis-

[1] The development of most of this chapter assumes that the reader has covered Parts II and III.

[2] See Richard F. Vancil, "Lease or Borrow: New Method of Analysis," *Harvard Business Review,* 39 (September–October, 1961), reprinted in James Van Horne, ed., *Foundations for Financial Management,* 137–38.

tinguishing feature between a financial and an operating lease is cancellability; an operating lease can be cancelled by giving proper notice whereas a financial lease cannot. An example of an operating lease is one for telephone service.

Because of the contractual nature of a financial lease obligation, it must be regarded as a form of financing. It is used in place of other methods of financing to acquire the use of an asset. For example, an alternative method of financing might be to purchase the asset and finance its acquisition with debt. Both the lease payment and the payment of principal and interest on debt are fixed obligations that must be met. Inability to meet these obligations will result in financial embarrassment. Thus, lease financing and debt financing are very similar from the standpoint of analyzing the ability of the firm to service fixed obligations.

In lease financing, the nature of the obligations of the lessor and the lessee is specified in the lease contract. This contract contains

1. The basic lease period during which the lease is noncancellable.

2. The timing and amounts of periodic rental payments during the basic lease period.

3. Any option to renew the lease or to purchase the asset at the end of the basic lease period.

4. Provision for the payment of the costs of maintenance and repairs, taxes, insurance, and other expenses. With a "net lease," the lessee pays all of these costs. Under a "maintenance lease," the lessor maintains the asset and pays the insurance.

TYPES OF LEASING ARRANGEMENTS

Lease financing generally falls into one of two basic categories: a sale and leaseback arrangement, or the direct acquisition of assets under a lease. Under a sale and leaseback arrangement, a firm sells an asset it owns to another party, and this party leases the asset back to the firm. Usually, the asset is sold at approximately its market value. The firm receives the sales price in cash, which can then be employed in other parts of the business. In addition, it receives the economic use of the asset during the basic lease period. In turn, the firm contracts to make periodic lease payments and, of course, gives up title to the asset. As we shall see, the latter consideration may be very important if the asset is likely to have significant residual value. Any profit on the sale of the asset to the lessor is treated as a capital gain for tax purposes. Since World War II, the sale and leaseback arrangement has grown to be a very popular method of financing. Lessors engaged in this arrangement

include insurance companies, other institutional investors, finance companies, and independent leasing companies.

Under direct leasing, a company acquires the use of an asset it did not own previously. For example, a firm simply may lease an asset from the manufacturer: IBM leases computers; Kearney & Trecker Corporation leases machine tools. Indeed, a number of capital goods are available today on a lease-financed basis. There are a wide variety of direct leasing arrangements available to meet various needs of the firm. The major types of lessors are manufacturers, finance companies, banks, independent leasing companies, and special-purpose leasing companies. For leasing arrangements involving all but the first, the vendor sells the asset to the lessor and he, in turn, leases it to the lessee. As in any lease arrangement, the lessee has use of the asset, along with a contractual obligation to make lease payments to the lessor. Since 1963, commercial banks have been allowed to engage in direct leasing; their entry represents an important development in the leasing industry. Independent leasing companies, such as Boothe Leasing and Nationwide Leasing, finance the purchase of a wide variety of equipment. In doing so, they frequently borrow from banks, securing the loan with the assignment of the lease payments. Special-purpose leasing companies confine their operations to certain types of assets; computer leasing companies, for example, mainly lease computer hardware and peripheral equipment.

ADVANTAGES OF LEASING

Lease financing is said to have a number of advantages as well as disadvantages compared with debt financing. The advantages include flexibility, lack of restrictions, 100 per cent financing, a lower obligation in bankruptcy and reorganization, tax considerations, and the accounting treatment. We shall examine each of these arguments in turn.

FLEXIBILITY

It sometimes is contended that leasing gives the firm more flexibility than does ownership, because the firm avoids the risk of obsolescence. However, in most financial leases, the risk of obsolescence is passed on to the lessee. If the original cost of the asset is amortized completely during the basic lease period, the lessee bears all the risk. He has a fixed obligation to make lease payments whether or not the asset becomes obsolete. Most lessors are well aware of the risk of obsolescence and insist upon recovering their original investment plus interest during the basic lease period. However, leasing does afford the firm flexibility with respect to the financing of relatively small asset acquisitions that occur sporadically over time. Piecemeal financing through debt may be both

expensive and difficult to arrange. The larger the acquisition, of course, the less valid this argument for leasing.

LACK OF RESTRICTIONS

Related to flexibility is the lack of restrictions that leasing offers. The protective-covenant restrictions imposed under a loan agreement or bond indenture (see Chapter 21) usually are not found in a lease agreement. Moreover, even under existing loan agreements and indentures, leasing sometimes is not restricted. As leasing grows in importance, however, few lenders will allow such a loophole to exist in loan agreements and bond indentures.

100 PER CENT FINANCING

Lease financing permits the firm to acquire the use of an asset without having to make a down payment or initial equity investment. If the firm purchased the asset and then sought to borrow against the collateral value of the asset, it would not be able to borrow 100 per cent of its cost. To the extent that debt and lease financing are regarded as equivalent, however, the firm would use up less of its capacity to raise nonequity funds with debt than it would with leasing. Presumably, it could borrow elsewhere to make up the difference. Thus, the important consideration is not the percentage of advance but the effect that the method of financing has on the firm's total capacity to raise funds. We take up this topic shortly. .

TREATMENT IN BANKRUPTCY AND REORGANIZATION

In bankruptcy, the maximum claim of the lessor is one year's lease payments; in reorganization, the maximum claim is three years' lease payments.[3] In either case, the trustee in bankruptcy or reorganization must reject the lease. Although the argument that a firm has a lower obligation in bankruptcy under lease financing than under debt financing has some merit, it means very little if the firm is analyzed as a going concern.

TAX CONSIDERATIONS

A lease payment is deductible as an expense for federal income tax purposes. If the asset is purchased, it must be capitalized; and the annual depreciation charge then is deducted as an expense. Prior to the 1954 tax code, lease financing permitted a faster tax writeoff over the basic lease period than was possible generally with a purchased asset

[3]See Donald R. Gant, "Illusion in Lease Financing," *Harvard Business Review*, 37 (March–April, 1959), 126.

subject to depreciation. The faster writeoff was beneficial to the lessee because the payment of taxes was delayed, and the lessee had the use of funds for a longer period of time. With the passage of the 1954 tax code, which permitted accelerated depreciation, the tax advantage of leasing was largely eliminated. Companies now are able to use the double-declining-balance and the sum-of-the-years'-digits methods of depreciation for assets owned by them.

However, one tax advantage still remains. With leasing, the cost of any land is amortized in the lease payments. By deducting the lease payments as an expense for federal income tax purposes, in essence, the lessee is able to write off the original cost of the land. If the land is purchased, the firm cannot depreciate it for tax purposes. When the value of land represents a significant portion of the asset acquired, lease financing can offer a tax advantage to the firm. Offsetting this tax advantage, however, is the likely residual value of land at the end of the basic lease period. The firm also may gain certain tax advantages in a sale and lease-back arrangement when the assets are sold for less than their depreciated value.

An important tax consideration prior to 1970 was the investment tax credit. This credit, which went up to 7 per cent for assets with depreciable lives in excess of eight years, was available either to the lessor or to the lessee, depending upon the leasing arrangement. When the lessor retained the tax credit instead of passing it through to the lessee, lease payments generally were lowered. How much depended upon negotiations between the lessor and lessee. When the lessee was unable to utilize the full tax credit because of an insufficient tax liability, but the lessor was able to use it, both parties tended to gain. The lessee was able to realize part of the tax credit through lease payments that were lower than they would have otherwise been. In turn, the lessor was able to use the full tax credit. In 1969, Congress repealed the investment tax credit; thus, an important stimulant to leasing was removed.

ACCOUNTING TREATMENT

For certain companies, one of the principal attractions of leasing is the fact that they can acquire the use of an asset without having the lease obligation appear on their balance sheets as a liability. If the asset were purchased and financed by debt, both the asset and the debt incurred would be shown. At present, the lease obligation generally is disclosed in a footnote to the audited financial statement.[4]

[4]The Accounting Principles Board of the American Institute of Certified Public Accountants—in *Reporting of Leases in Financial Statements of Lessee,* No. 5 (September, 1964)—issued an opinion recommending that sufficient information about the lease should be disclosed either in the financial statements or in footnotes to the statements. Where the terms of the lease result in the creation of a significant equity interest in the property on the part of the lessee, the board recommended that the asset as well as the liability appear in the balance sheet. In general, accountants must conform to these recommendations if they are to give unqualified opinions on the financial statements of the firms they audit.

The problem of the disclosure of the lease obligation is under continuous study by the accounting profession. Some accountants argue that the annual lease payment should be capitalized at an appropriate discount rate and the capitalized liability be shown on the balance sheet, together with the amortized value of the asset. Others would argue that long-term leases and property rights should not be reflected on the balance sheet.[5] Obviously, a company that feels that the present accounting treatment is one of the attractions of leasing is not going to be favorably disposed toward a movement to show the lease on the balance sheet.

Effect upon Financial Ratios. The omission of the lease obligation on the balance sheet can have a favorable, and deceptive, effect upon the financial condition of a firm, as depicted by financial ratios, over what would be the case if the asset were purchased and financed with debt.[6] Consider a company with the balance sheet shown in the first column of Table 22-1. In this column, we assume that the company has acquired an asset costing $2 million and has financed its acquisition with $2 million in long-term debt. In the second column of the table, we assume that the asset has been leased instead of purchased and that the lease obligation does not appear on the balance sheet. Suppose that the asset acquired has a twenty-year economic life, which is also its depreciable life, and that annual depreciation charges are $100,000. We assume also that annual lease payments are $210,000 for twenty years and that the interest on the long-term debt is 7 per cent.

The effect of leasing on various financial ratios is shown in Table 22-2. As can be seen, the decision to lease instead of purchase the asset and to borrow results in a significant improvement in certain financial ratios. For one thing, the company shows a faster turnover of its assets and better earning power. Moreover, it appears to have less financial risk under the leasing alternative. The debt-to-equity ratio is less, and the coverage ratio for times interest earned is higher. Close analysis of the financial statement, however, shows that the improvement in financial ratios is an illusion. The lease payments represent just as much a contractual obligation

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[5] For arguments for and against the capitalization of leases, see John L. Hennessy, "Recording of Lease Obligations and Related Property Rights," *Journal of Accountancy,* III (March, 1961), 40–46; and Alvin Zises, "Disclosure of Long-term Leases," *Journal of Accountancy,* III (February, 1961), 37–47. For a discussion of various approaches to capitalizing a lease, see John H. Myers, *Reporting of Leases in Financial Statements,* Accounting Research Study No. 4 (New York: American Institute of Certified Public Accountants, 1962); W. J. Vatter, "Accounting for Leases," *Journal of Accounting Research,* 4 (Autumn, 1966), 133–48; and William L. Ferrara and Joseph F. Wojdak, "Valuation of Long-Term Leases," *Financial Analysts Journal,* 25 (November–December, 1969), 29–32. At present, only a small number of companies report the lease obligation in their balance sheets.

[6] See A. Thomas Nelson, "Capitalizing Leases – the Effect on Financial Ratios," *Journal of Accountancy,* 116 (July, 1963), 49–58.

TABLE 22-1
Balance sheet and income statement (in thousands)

	Financing Alternatives	
	Borrowing	Leasing
Balance sheet:		
Current assets	$ 4,000	$ 4,000
Fixed assets net	6,000	4,000
Total assets	$10,000	$ 8,000
Current liabilities	$ 2,000	$ 2,000
Long-term debt	3,000	1,000
Total debt	5,000	3,000
Net worth	5,000	5,000
Total liabilities and net worth	$10,000	$ 8,000
Income statement:		
Sales	$12,000	$12,000
Operating income before depreciation,		
interest, and lease payments	2,000	2,000
Depreciation	500	400
Interest on long-term debt (7%)	210	70
Lease-payment expense		210
Net income before taxes	1,290	1,320
Taxes (50%)	645	660
Net income after taxes	645	660

TABLE 22-2
Effect of leasing upon certain financial ratios

	Financing Alternatives	
Ratio	Borrowing	Leasing
Turnover of assets (times)	1.2	1.5
Return on assets (earning power)	6.5%	8.3%
Debt to net worth	1.0	0.6
Times interest earned (before taxes and interest)	7.1	19.9

on the part of the company as does the payment of principal and interest on debt.

If the debt contract called for equal annual payments over the twenty years, these payments would have to be $188,786 in order to retire the $2 million by maturity, and yield 7 per cent.[7] This contractual obligation compares with $210,000 under the lease alternative, the difference being

[7]See Eq. (22-1) for the determination of annual payments.

attributable to the higher interest rate embodied in the lease contract. Thus, the financial risk to the company of not being able to meet its obligations is slightly higher under the lease alternative. We see, then, that in order to analyze a company properly, the credit analyst and the investor must take account of the contractual nature of lease payments.

IMPLICATIONS FOR RAISING FUNDS

Creditors and more sophisticated investors are becoming increasingly aware of the implications of the lease obligation for the financial condition of a company. Most creditors and institutional investors at least note this contractual obligation in their analyses. Some go so far as to capitalize the obligation and prepare an adjusted balance sheet. Even when creditors and investors recognize leasing as a contractual commitment, the important question is whether they regard it as they do debt. The answer to this question is far from clear. Moreover, we know that many individual investors do not take the lease obligation into account at all.

To the extent that creditors do not recognize the full implications of the lease obligation to the firm as a whole, or do not recognize the lease obligation at all, the ability of the firm to raise funds may be enhanced by leasing. The firm may be able to raise more funds by leasing or by a combination of leasing and borrowing than it could by borrowing alone. Because the perceived risk of the firm is less, the cost of additional debt financing may be lower than if the firm had acquired the assets by debt financing. We note that only the firm's capacity to raise funds externally, not its ability to service total fixed charges internally, is improved.

Similarly, investors may not associate the same degree of financial risk with lease financing and debt financing. Consequently, they may not penalize the company as much in the equity-capitalization rate, or price/ earnings ratio, for a given amount of lease financing as they would for an equivalent amount of debt financing. As a result, lease financing would have a lower implicit cost than would debt financing. If creditors and investors do not take lease obligations fully into account, and the implicit cost of leasing is less than it would be otherwise, should the firm take advantage of this market imperfection? In theory, it should seek the optimal mix of financing wherein the total real cost of capital is lowest; therefore, it should take advantage of the situation. This criterion conflicts with the fact that the ability of the firm to service fixed charges is no better under lease financing than it is under debt financing.

In other words, if investors and creditors in fact are fooled by the alternative of lease financing, should the firm actually fool them and lease finance beyond what would be the optimal level of debt financing? Intuitively, we would probably say no. If the firm assumes fixed charges beyond its ability to service them and evolves itself into financial difficulty,

creditors and investors will recognize the financial risk of the firm and penalize it accordingly. However, most situations are not as clear-cut, for the firm may be able to service its fixed charges even though the probability of not being able to do so is increased. If a market imperfection does exist, whether management takes advantage of the imperfection will depend upon its risk preferences.

RESIDUAL VALUE

Because the asset being leased is owned by the lessor, any residual or terminal value at the end of the basic lease period goes to the lessor. The economic value of an asset lies in its use during the basic lease period; nevertheless, the fact that the lessee has neither the use nor ownership of the asset at the termination of the lease may in certain situations be a significant disadvantage. Land, for example, may show great appreciation in market value over the basic lease period. Many trucking companies lease terminal facilities in large cities. Over a period of years, the land on which many of these terminals are located has increased in value many times. At the termination of the lease, the trucking company may be able to find similar property only at a substantially higher market price than it would have had to pay to purchase the property in the first place. Although a building is unlikely to appreciate in value, the lessee still may find, at the termination of the lease, that the replacement value of the building is very substantial. Whenever there is a possibility that an asset will have a significant residual value, this value must be considered in weighing the desirability of leasing.

Depending upon the lessor, it may be possible for the lessee to obtain an option to renew the lease or to purchase the asset at the end of the basic lease period. The option terms can either be established at the time of the initial contract or be based upon the market value of the asset at the termination of the lease. By option terms, we mean the amount of the periodic lease payments and the duration of the new lease if the lease is renewed, or the purchase price of the asset if the option calls for purchase. If the option terms are established in the initial contract, the lessee can obtain a hedge against appreciation in asset values. However, if the option terms are based upon the market value at the termination of the lease, they provide no such hedge. The lessor usually is aware of the possiblity of residual value and is not willing to give it away even though the original cost of the asset is completely amortized over the basic lease period. Certain equipment leases, however, do allow the lessee to obtain the proceeds of the sale of the asset at the termination of the lease.

INTEREST COST OF LEASING

A second disadvantage of leasing involves the interest cost, which usually is higher than the interest cost of debt. Assuming that the cost of the asset is completely amortized over the basic lease period, the interest cost of lease financing is the rate of discount that equates the present value of the lease payments with the cost of the asset. The differential between the interest cost a firm would pay in leasing and what it would pay if it purchased the asset and financed the purchase with debt usually is 1 per cent or more. Thus, on the basis of interest cost alone, borrowing is cheaper than leasing in most situations.

LEASING VERSUS BORROWING

Whether lease financing or borrowing is favored will depend upon the patterns of cash outflows for each financing method and upon the opportunity cost of funds. A number of different methods may be used to compare the two alternatives. In this section, we illustrate some of the more important, using a hypothetical example. Before proceeding, we must point out that no consideration is given to differences in the effect that the two alternatives have upon the ability of the firm to raise additional funds. This topic was considered earlier in the chapter.

EXAMPLE

For comparative purposes, each method of analysis we consider will be illustrated with the same example. Suppose that a firm has decided to acquire an asset costing $200,000 and having an expected economic life of ten years, after which the asset is not expected to have any residual value. Once the investment decision is made, the question becomes, Is it better to finance the asset by leasing or by borrowing? If leasing is used, the lessor requires that cost of the asset be completely amortized over the ten-year period and that it yield a 9 per cent return. As is customary, lease payments are to be made in advance—that is, at the end of the year prior to each of the ten years. The amount of annual lease payment may be calculated by solving the following equation for x:

$$\$200,000 = \sum_{t=0}^{9} \frac{x}{(1.09)^t} \tag{22-1}$$

$$\$200,000 = x + 5.9852x$$

$$x = \frac{\$200,000}{6.9852}$$

$$x = \$28,600.$$

Because lease payments are made in advance, we solve for the annual lease payment which equates the cost of the asset, $200,000, with the present value of one lease payment at time 0, plus the present value of nine lease payments at the end of each of the next nine years. Since the discount rate is 9 per cent, we find in Table A-2 at the end of the book that the present-value discount factor for an even stream of cash flows for nine years, discounted at 9 per cent, is 5.9852. Therefore, the annual lease payment necessary to amortize the cost of the asset completely and to return the lessor 9 per cent is $28,600.

If the asset is purchased, it will be subject to depreciation. Assuming straight-line depreciation, annual depreciation charges are $20,000. As we shall see shortly, depreciation is an important consideration in the comparison between financing alternatives because it is deductible for tax purposes.

PRESENT-VALUE ANALYSIS
OF ALTERNATIVES

The first method of analysis we consider is a comparison of the present values of cash outflows for each of the alternatives. According to this method, whichever alternative has the lowest present value is the most desirable. While the method has some conceptual shortcomings, it is widely used.

Recall that the firm will make annual lease payments of $28,600 if the asset is leased. Because these payments are an expense, they are deductible for tax purposes. However, they are deductible only in the year for which the payment applies. For example, the $28,600 payment at the end of year 0 represents a prepaid expense and is not deductible for tax pruposes until year 1. Similarly, the other nine payments are not deductible until the following year. Suppose that the appropriate opportunity cost of funds to the firm as a whole is 10 per cent after taxes and that the Federal income tax rate is 50 per cent. Given this information, we are able to derive a schedule of cash outflows after taxes and compute their present value. These computations are shown in Table 22-3. The present value of the total cash outflows under the leasing alternative is seen to be $105,441. This figure, then, must be compared with the present value of cash outflows under the borrowing alternative.

If the asset is purchased, the firm is assumed to be able to finance the acquisition entirely with an 8 per cent, ten-year unsecured term loan. The loan is payable over this period in annual payments of $29,806 at the end of each year.[8] These payments include both principal and interest. The proportion of interest in each payment depends upon the unpaid

[8]This amount is computed in the same manner as in Eq. (22-1), with the exception that time goes from $t = 1$ through 10 instead of $t = 0$ through 9.

TABLE 22-3
Schedule of cash outflows: leasing alternative

End of Year	(1) Lease Payment	(2) Tax Shield	(3) Cash Outflow After Taxes (1)–(2)	(4) Present Value of Cash Outflows (10%)
0	$28,600	—	$28,600	$ 28,600
1–9	28,600	$14,300	14,300	82,354
10	—	14,300	−14,300	−5,513
				$105,441

balance of the principal amount owing during the year. For example, the principal amount owing during year 1 is $200,000; therefore, the annual interest for that year is $16,000. Table 22-4 shows the schedule of debt payments.

TABLE 22-4
Schedule of debt payments

End of Year	Interest Plus Principal Payments	Principal Amount Owing at End of Year	Annual Interest
0	$ —	$200,000	$ —
1	29,806	186,194	16,000
2	29,806	171,284	14,896
3	29,806	155,181	13,703
4	29,806	137,789	12,414
5	29,806	119,006	11,023
6	29,806	98,720	9,520
7	29,806	76,812	7,898
8	29,806	53,151	6,145
9	29,806	27,598	4,252
10	29,806	0	2,208

Given annual interest and depreciation, we are able to compute the cash outflows after taxes for the borrowing alternative; these outflows are shown in Table 22-5. Because both depreciation and interest are deductible for tax purposes, they provide a tax shield equal to their sum times the tax rate. When this shield is deducted from the total payment of $29,806, we obtain the cash outflow after taxes at the end of each year. Finally, we compute the present value of these outflows; and they are found to total $88,041. According to this analysis, the firm should acquire the asset through debt financing, because the present value of cash outflows with borrowing is less than that with leasing.

In our example, we assumed straight-line depreciation. If the company uses accelerated depreciation, it obtains a greater tax shield in the early

TABLE 22-5

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Schedule of cash outflows: borrowing alternative

End of Year	(1) Loan Payment	(2) Interest	(3) Depreciation	(4) Tax Shield [(2) + (3)]0.5	(5) Cash Outflow After Taxes (1) − (4)	(6) Present Value of Cash Outflows
1	$29,806	$16,000	$20,000	$18,000	$11,806	$11,000
2	29,806	14,896	20,000	17,448	12,358	10,000
3	29,806	13,703	20,000	16,851	12,955	9,733
4	29,806	12,414	20,000	16,207	13,599	9,288
5	29,806	11,023	20,000	15,512	14,294	8,875
6	29,806	9,520	20,000	14,760	15,046	8,493
7	29,806	7,898	20,000	13,949	15,857	8,137
8	29,806	6,145	20,000	13,072	16,734	7,807
9	29,806	4,252	20,000	12,126	17,680	7,498
10	29,806	2,208	20,000	11,104	18,702	7,210
						$88,041

years and a lower one in later years. As a result, cash outflows after taxes in the early years are reduced relative to cash outflows in later years, and the present value under the borrowing alternative is increased. For example, if sum-of-the-years'-digits depreciation is employed, the present value of cash outflows under the borrowing alternative becomes $79,335, in contrast with $88,041 when straight-line depreciation is used. Thus, accelerated depreciation makes borrowing more attractive than it is with straight-line depreciation. It should be pointed out, however, that a lessor also is able to use accelerated depreciation. In turn, the lessor may pass off some of the benefits inherent in its use to the lessee in the form of lower lease payments.

While the present-value analysis described above provides an approximate means for choosing between financing an asset with debt or with leasing, several factors detract from its usefulness. For one thing, no consideration is given to differences in the configuration of the two payments streams. The two financing alternatives simply are taken as given, and compared. In particular, this approach introduces error if the two alternatives involve different amounts of financing. For example, if less than the total cost of the asset is financed under the borrowing alternative, while 100 per cent of it is financed under the lease, the firm uses up less of its capacity to raise funds if it purchases the asset and borrows than if it leases. We must consider the implicit cost of the difference; otherwise, there is a bias in favor of leasing. This bias arises because the "down payment" under the borrowing alternative usually is treated as a cash outflow at time 0.

Even if the total financing is the same under both alternatives, the

shape of the payments streams may differ. In our example, the lease contract called for equal payments before the beginning of each year, while the debt contract called for equal payments at the end of each year. In theory, the earlier payment under the lease alternative should release capacity for raising funds by other means of financing. Therefore, the timing advantage associated with debt financing is largely an illusion.

Another problem with the present-value approach is that it mixes the investment and financing decisions if the cost of capital is used as the discount rate. The cost of capital used as the cutoff rate to evaluate investment opportunities is the result of financing decisions, among which is the use of debt or lease financing. Conceptually, it is not proper to employ the cost of capital in deciding between methods of financing. It is the result, not the cause, of these decisions.[9] If a discount rate is to be used, a strong case can be made that it be the after-tax rate on debt. Similar to the refunding of a bond issue taken up in Chapter 12, the difference in cash flows between lease financing and debt financing involves little or no risk. Therefore, it is not appropriate to use the cost of capital, which embodies a risk premium for the firm as a whole, as the discount rate. The problems described in the last few paragraphs have led to other methods for evaluating the decision to lease or borrow.

EFFECTIVE YIELD ANALYSIS OF LOAN EQUIVALENTS

Instead of computing the present value of cash outflows for the two financing alternatives, we could compute the effective interest cost. This approach avoids the problem of having to choose a rate of discount. We begin by assuming that the loan payment schedule is of the same configuration as the lease payment schedule. This assumption places the loan on an equivalent basis with the lease.[10] As a result, the bias associated with differences in the shape of the two payments streams is avoided. The firm's capacity to finance with additional fixed-charge instruments is released at approximately the same rate under either alternative. So as to conform to the lease payment stream, we assume in our example that loan payments are made in advance — namely, at the end of the year prior to each of the ten years.

For the lease alternative, the effective interest cost is determined by solving for the rate of discount that equates the cash outflows after taxes with the adjusted purchase price of the asset. The adjusted purchase price we employ is the original purchase price multiplied by one minus the tax rate. This procedure places both the purchase price and the cash

[9] See Thomas H. Beechy, "Quasi-Debt Analysis of Financial Leases," *Accounting Review*, XLIV (April, 1969), 376.

[10] See Richard S. Bower, Frank C. Herringer, and J. Peter Williamson, "Lease Evaluation," *Accounting Review*, XLI (April, 1966), 260.

outflows on an equivalent tax basis. For our example, the adjusted purchase price is $200,000 (1 − .50) = $100,000. When we solve for the rate of discount which equates the cash outflow after tax stream in Table 22-3 with $100,000, we find it to be 11 per cent.

Solving for the effective interest cost for the debt alternative is somewhat more involved. First, we must determine the cash outflows after taxes. This is done in the same manner as before, except that the debt payments are assumed to occur at the end of the prior year; the schedule is shown in Table 22-6. Note that while the loan is for $200,000, a payment of $27,598 is made at the inception so that the principal amount initially owing is $172,402.[11]

TABLE 22-6
Schedule of debt payments — yield analysis

End of Year	(1) Interest Plus Principal Payments	(2) Principal Amount Owing at End of Year	(3) Annual Interest
0	$27,598	$172,402	$ —
1	27,598	158,596	13,792
2	27,598	143,686	12,688
3	27,598	127,583	11,495
4	27,598	110,191	10,207
5	27,598	91,408	8,815
6	27,598	71,122	7,313
7	27,598	49,214	5,690
8	27,598	25,554	3,937
9	27,598	0	2,044

Given the annual interest expense, we can prepare the schedule of cash outflows shown in Table 22-7. We see that payment in advance causes a cash outflow after taxes of $27,598 at the end of year 0, while depreciation charges in the tenth year give rise to a cash inflow of $10,000 in that year. Because the cash outflows in the last column are after taxes, it is not appropriate to solve for the rate of discount which equates these outflows with the original amount of the loan. Instead, the original amount of the loan is multiplied by one minus the tax rate to give $200,000(1 − .50) = $100,000. When we solve for the rate of discount which equates the stream of cash outflows in Table 22-7 with $100,000, we find it to be 9.37 per cent. According to this method of analysis, borrowing is the preferred alternative because its effective interest cost is less than that for the leasing alternative.

If accelerated depreciation is used, cash outflows in the early years

[11] The amount of annual payment required to amortize the loan completely and to yield 8 per cent can be determined with Eq. (22-1).

TABLE 22-7
Schedule of cash outflows—borrowing alternative, yield analysis, straight line depreciation

End of Year	(1) Loan Payment	(2) Interest	(3) Depreciation	(4) Tax Shield $[(2)+(3)]\ 0.5$	(5) Cash Outflow After Taxes $(1)-(4)$
0	$27,598	—	—	—	$27,598
1	27,598	$13,792	$20,000	$16,896	10,702
2	27,598	12,688	20,000	16,344	11,254
3	27,598	11,495	20,000	15,748	11,850
4	27,598	10,207	20,000	15,104	12,494
5	27,598	8,815	20,000	14,408	13,190
6	27,598	7,313	20,000	13,657	13,941
7	27,598	5,690	20,000	12,845	14,753
8	27,598	3,937	20,000	11,969	15,629
9	27,598	2,044	20,000	11,022	16,576
10	—	—	20,000	10,000	−10,000

are reduced relative to those in later years for the borrowing alternative. As a result, the discount rate that equates the present value of cash outflows with the adjusted loan amount is less than it is with straight-line depreciation. If sum-of-the-years'-digits depreciation is used, the schedule of cash outflows becomes that shown in Table 22-8. When we solve for the discount rate that equates this stream with $100,000, we find it to be 7.70 per cent. Thus, the effective interest cost for debt is less when accelerated depreciation is used in our example problem.

TABLE 22-8
Schedule of cash outflows—borrowing alternative, yield analysis, sum-of-years'-digits depreciation

End of Year	(1) Loan Payment	(2) Interest	(3) Depreciation	(4) Tax Shield $[(2)+(3)]0.5$	(5) Cash Outflow After Taxes $(1)-(4)$
0	$27,598	—	—	—	$27,598
1	27,598	$13,792	$36,364	$25,078	2,520
2	27,598	12,688	32,727	22,708	4,890
3	27,598	11,495	29,091	20,293	7,305
4	27,598	10,207	25,455	17,831	9,767
5	27,598	8,815	21,818	15,317	12,281
6	27,598	7,313	18,182	12,748	14,850
7	27,598	5,690	14,545	10,118	17,480
8	27,598	3,937	10,909	7,423	20,175
9	27,598	2,044	7,273	4,659	22,939
10	—	—	3,636	1,818	−1,818

BOWER-HERRINGER-WILLIAMSON
METHOD

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CHAP. 22
Lease
Financing

In another approach to evaluating lease versus borrowing, Bower, Herringer, and Williamson (BHW) divide the payments streams into two parts: the cash flows associated with financing and the cash flows associated with tax savings.[12] BHW first measure the incremental financial impact of a loan by subtracting the present value of the lease payments from the present value of the loan payments, where both are discounted by the debt rate. As was done in the previous section, they place the two payments streams on an equivalent basis with respect to configuration. For our example problem, the present value of the loan payments, column (1) of Table 22-7, is obviously $200,000 if an 8 per cent discount rate is used. The present value of the lease payments is determined by discounting the lease payments in column (1) of Table 22-3 by 8 per cent; and it is found to be $207,261. Given these two present values, the *financial disadvantage* of leasing can be determined as follows:

Present value of loan payments	$200,000
Present value of lease payments	207,261
Financial advantage (disadvantage)	$ (7,261)

As long as the interest rate embodied in the lease payments, 9 per cent in our example, exceeds the rate on debt, there always will be a financial disadvantage to leasing.

The next step is to determine the incremental present value of the tax savings associated with leasing. If we use the straight-line depreciation example, we begin by subtracting the tax shield for the borrowing alternative in Table 22-7 from that for the leasing alternative in Table 22-3. The difference represents the decrease in the firm's tax payments, which is associated with leasing as opposed to borrowing. The calculations are shown in Table 22-9. When these decreases in tax payments are discounted by the cost of capital rate, we obtain what BHW call the *operating advantage* of the lease. If a cost of capital of 10 per cent is assumed, the present value of the decreased tax payments is found to be −$508 for our example problem.

The decision to lease or borrow is made on the basis of whether the operating advantage of the lease exceeds its financial disadvantage. If so, lease financing should be used; if not, debt financing should be employed. In our case, the operating advantage of the lease, −$508, is smaller than

[12] "Lease Evaluation," pp. 257–65.

TABLE 22-9

Calculation of decrease in tax payments with leasing

End of Year	(1) Leasing* Tax Shield	(2) Borrowing† Tax Shield	(3) Decreases in Tax Payments with Leasing (1) − (2)	(4) Present Value of Decreases (10%)
1	$14,300	$16,896	−$2,596	−$2,360
2	14,300	16,344	− 2,044	− 1,689
3	14,300	15,748	− 1,448	− 1,088
4	14,300	15,104	− 804	− 549
5	14,300	14,408	− 108	− 67
6	14,300	13,657	643	363
7	14,300	12,845	1,455	747
8	14,300	11,969	2,331	1,087
9	14,300	11,022	3,278	1,390
10	14,300	10,000	4,300	1,658
				−$ 508

*From Table 22-3.
†From Table 22-7.

its financial disadvantage of $7,261. Therefore, according to this analysis, debt financing should be employed.

The BHW method of analysis is similar to the *basic interest rate* approach of Richard F. Vancil.[13] Like BHW, Vancil separates the financing effect of leasing from its tax-savings effect. Holding constant the amount of financing, he discounts the tax savings associated with the non-interest portions of the two payments streams by the cost of capital. Whichever financing alternative has the lower present value is preferred.

A critical factor in both the BHW and the Vancil approaches is the discount rate employed. As discussed earlier, a strong case can be made for using the after-tax cost of debt. BHW justify the use of the cost of capital as the discount rate on the basis that investors and creditors, in their valuation of the firm, recognize the difference in tax savings between the two methods. Because investors and creditors determine the overall value of the firm, BHW view the average cost of capital as the appropriate discount rate.[14] Whether investors and creditors evaluate differences in tax savings in this manner is open to serious doubt. In addition, there is the conceptual problem, discussed earlier, of the cost of capital depending in part upon the decision to lease or borrow. An inconsistency arises in using the cost of capital as the discount rate to evaluate this decision. While the problem is not important when the project being financed is relatively small, the problem is important if the project is large. In essence,

[13]"Lease or Borrow: New Method of Analysis," *Harvard Business Review*, 39 (September–October, 1961), 138–59.

[14]"Lease Evaluation," pp. 262–63.

the tax savings that result from a financing decision are evaluated as though they were an investment.

Because of these problems a case can be made for the use of the effective yield method proposed in the previous section. By comparing effective yields for the two financing alternatives, one does not have to choose a discount rate. This approach avoids intermingling investment and financing decisions by treating the problem as one of financing alone. Nonetheless, if differences in taxes are valued in the market at an implied interest rate higher than the effective interest yield computed, the method will give biased results. To the extent that these differences are valued at a rate that approximates the firm's cost of capital, the BHW method would be preferred. Under most circumstances, the two methods will provide identical decisions. Both have the virtue of placing the two payments streams on an equivalent basis with respect to configuration.

ANALYSIS WHEN PURCHASE PRICE DIFFERS FROM CASH-EQUIVALENT PRICE

In the methods of analysis considered so far, we have assumed that the purchase price of the asset is the same to the lessee as it is to the lessor. However, there are situations where the lessor and lessee work from different cost bases; this occurrence requires a special analysis. For example, the manufacturer of a capital asset such as a computer may be willing to sell it outright or to lease it under a noncancellable lease. The selling price for outright purchase may differ from the cash-equivalent price the manufacturer uses to determine lease payments. By varying the selling price relative to the cash-equivalent price or vice versa, the manufacturer can encourage or discourage leasing. Similarly, in a third-party lease, the selling price by the manufacturer to the lessor may be different—usually lower—than the selling price to the potential lessee. To evaluate situations of this sort, the firm should determine the cash-equivalent price used by the lessor. Given this price, the firm then can evaluate leasing versus borrowing with one of the methods described earlier.

To illustrate, suppose that Carter Electronics Corporation is considering acquiring a quality-control testing machine from DSS Manufacturing Company. The purchase price of the machine is $20,000, and it is expected to have an economic life of five years, with no residual value at the end of that time. When Carter Electronics discounts the expected future cash benefits after taxes by its cost of capital rate, it finds that the present value of these benefits exceeds $20,000. Therefore, acquisition of the machine with an outright purchase is considered worthwhile. However, DSS also offers the company the opportunity to lease the machine for five years, with annual lease payments of $5,189, payable at the end of the year prior to each of the five years. If Carter Electronics believes the

interest rate being used by DSS in calculating the lease payments is 9 per cent, Carter could compute the cash-equivalent price by solving the following equation for X:

$$X = \sum_{t=0}^{4} \frac{\$5,189}{(1.09)^t} \tag{22-2}$$

$$X = 5,189(1 + 3.2397) = \$22,000.$$

In this equation, 3.2397 is the present value of $1 received at the end of each year for four years. As one payment is made at the end of year 0, we must add one to it. Thus, the cash-equivalent price of $22,000 exceeds the purchase price of $20,000. DSS Manufacturing appears to be discouraging leasing relative to outright purchase.

The difference between the two figures – $2,000 – must be incorporated into our analysis of leasing versus borrowing. If the effective yield method of analysis is used, the schedule of cash outflows for the borrowing alternative should be based upon a loan equal to the purchase price, $20,000; while the schedule of cash outflows for the lease alternatives should be based upon the lease payments required – namely, $5,189 per year. As the lease payments will be higher than they would be if the cash-equivalent price equaled the purchase price of $20,000, the lease alternative will have a higher effective yield than it would have if the two prices were the same. Thus, the difference between the two prices is incorporated in the comparison of effective yields. As before, the preferred method is the one with the lower effective yield. If the BHW method of analysis is used, the difference between the purchase price and the cash-equivalent price is automatically embodied in the financial disadvantage. Because lease payments are higher than they would be if the two prices were the same, the present value of these payments also will be higher. As a result, the financial disadvantage will be greater.

If the cash-equivalent price were less than the purchase price, the two methods of analysis described also would take account of the difference in this direction. Thus, the methods proposed previously allow us to analyze situations where the purchase price of an asset differs from the cash-equivalent price being used by the lessor.

ANALYSIS OF AN ASSET THAT CAN ONLY BE LEASED

Occasionally, the firm must evaluate an asset which can only be acquired by leasing. For example, use of certain dock facilities by ocean freighters can be obtained only through a long-term lease. An alternative purchase price is not available. In situations of this sort, the firm does not choose between leasing or borrowing; the only decision is whether or not to lease. As a result, the investment and financing decisions are inextricably intertwined.

While no method of analysis is entirely satisfactory, perhaps the best approach is to determine the merit of the project as an investment. The first step is to compute the cash-equivalent price of the lease alternative. You begin by establishing an interest rate which is consistent with other current leasing arrangements. The cash-equivalent price is the present value of all required lease payments discounted by this rate. The next step is to compute the present value of expected future cash benefits associated with the project, discounted at the cost of capital rate. Obviously, these benefits should be estimated only for the duration of the lease period. If the present value of the expected future cash benefits exceeds the cash-equivalent price, the project is worthwhile, and the firm should enter into the lease.[15] If the present value of the cash benefits is less than the cash-equivalent price, the project should be rejected.

If the project is accepted, obviously it can be financed only by leasing. Implied then in our method of analysis is that lease financing does not alter the firm's cost of capital. In other words, the firm must be able to balance this method of financing with others so that it maintains a desired cost of capital. If, for some reason, this balancing is not possible, perhaps because the project is large, then the effect of financing the project on the firm's cost of capital must be taken into account.

CALCULATION IN WEIGHTED-AVERAGE COST OF CAPITAL

When leasing is used as a method of financing, it should be included in the calculation of a weighted-average cost of capital. At times, the appropriate weight and cost may be difficult to calculate, but, nevertheless, approximations can be made. If the purchase price of the asset being leased is amortized completely over the basic lease period, the cost of leasing is easily determined. It is the rate of discount that equates the present value of lease payments with the purchase price of the asset. This cost represents the interest cost of leasing.

Determining the weight to employ is more controversial. For simplicity, assume that the weights employed for other fixed-charge obligations are book value weights and that the firm intends to finance in the future with its present financing mix. In other words, lease financing and other methods of financing will be employed on approximately the same relative basis as currently. Under these circumstances, the most appropriate weight for lease financing is the capitalized value of the lease payments. At the outset, the capitalized value is the purchase price of the asset. Subsequent capitalized values can be calculated by deducting the amount of cumulative principal payments from the original capitalized value.

[15]We assume that the acceptance of the project does not change the business-risk complexion of the firm as a whole. (See Part II.) Under no circumstances should the lease payments be deducted from the expected cash benefits and the project judged on the basis of whether or not the present value of the residuals is positive.

These computations are the same as those for a debt obligation; they are illustrated in Tables 22-4 and 22-6.

Instead of using the capitalized value of lease payments as the weight, some would contend that we should use the sum of all future lease payments required up to the termination of the lease. For example, if annual lease payments are $100,000 and six years remain in the lease, the weight would be $600,000, according to this method. Because future lease payments include both principal and interest payments, the sum of these payments would be higher than their capitalized value. This weighting scheme is based on the idea that the firm has a contractual obligation to meet all lease payments, not just the principal amount of these payments. While, from a legal standpoint, this argument has merit, it is not convincing if the firm is treated as a going concern. In keeping with our analysis in Chapter 4, the principal value of a fixed-income obligation is the appropriate weight to use, for this amount represents the actual funds employed by the firm in earning assets. To employ the sum of all future debt or lease payments overstates the funds actually available to the firm. A lease is no different from a bond that is noncallable. As we would use the principal amount owing on the bond in our weighting, so should we use the capitalized value of the lease.

Table 22-10 shows the calculation of a weighted-average cost of capital for a hypothetical firm. We assume that the capitalized value of a lease is $10 million, the interest cost is 10 per cent, and the tax rate is 50 per cent. When lease financing is employed, it must be considered directly in the determination of a weighted average cost of capital. Too often, it is ignored. We discussed the assumptions underlying the use of a weighted-average cost of capital in Chapter 4. Here, it is important to emphasize that the firm should consider the possibility of lease financing in determining its optimal capital structure, because it can be an important source of financing.

TABLE 22-10
Weighted-average cost of capital including lease financing

	Amount (in millions)	Weight	After-tax Cost	Weighted Cost
Long-term debt	$15	0.30	4%	1.2%
Lease financing	10	0.20	5%	1.0
Common equity	25	0.50	10%	5.0
	$50			
Weighted-average cost of capital				7.2%

SUMMARY

Lease financing involves the acquisition of the economic use of an asset through a contractual commitment to make periodic lease payments to a

lessor who owns the asset. Because of this contractual obligation, leasing is regarded as a method of financing similar to borrowing. Leasing can involve either the direct acquisition of an asset under a lease or a sale and leaseback arrangement, whereby the firm sells an asset it owns and leases it back from the buyer.

Because the lease obligation generally is not disclosed in the balance sheet as a liability but rather treated as a footnote to the financial statement, certain creditors and investors may not recognize the full implications of this contractual commitment. To the extent that it is not recognized, the ability of the firm to raise additional funds may seem better than it would if the assets were purchased and financed by debt. For this reason, the implicit cost of lease financing may be somewhat less than that of debt financing. One of the principal disadvantages of lease financing is that the lessee does not own the asset; any residual value after the basic lease period goes to the lessor. The second major disadvantage is that the interest cost of lease financing usually is higher than the interest cost of borrowing.

Several methods were analyzed for evaluating lease financing in relation to debt financing. It is important that the payments stream for the debt alternative be of the same configuration as that for the lease alternative. The decision to lease or borrow can be made on the basis of which alternative has the lowest effective yield, adjusted for differences in taxes. Another method of analysis calls for the separation of differences in explicit financing costs from differences in taxes, with the two methods of financing compared according to their present values. Consideration was given to situations in which the purchase price of the asset differs from its cash-equivalent price under leasing, and also to the case where leasing is the only means by which an asset can be acquired. Finally, the importance of taking into account lease financing in determining the optimal capital structure and in calculating a weighted-average cost of capital was pointed out.

PROBLEMS

1. The Alpine Company owns the building and land which comprise its home office. The building has a remaining life of twenty years, is being depreciated on a straight-line basis, and has a book value of $1 million. The land has a book value of $500,000. The company has a 50 per cent tax rate. At the end of the twenty years, the land is expected to be worth $1 million, assuming $100,000 is spent to demolish the building. The company has an 8 per cent after-tax opportunity cost of funds.

The Lease-All company has offered to pay the Alpine Company $1.5 million for the land and building and grant them a twenty-year lease at $150,000 per year, payable in advance. Alpine would pay for normal maintenance and would have no rights or interests in the property at the expiration of the lease.

(a) Compute the differential net-present value of the sale-leaseback arrangement.

(b) Are there any problems involved in this method of analysis? What are they?

2. The Simone Corporation, which has a 40 per cent tax rate, wishes to acquire a $100,000 stamping machine, which would be depreciated on a straight-line basis with an eight-year life and no salvage. It would be possible to lease the machine for $20,000 per year, payable in advance; it would also be possible to borrow at 10 per cent. Being careful to place the flows on a comparable basis, use the equivalent-yield method to determine the better alternative.

3. (a) Rework problem 2, assuming sum-of-the-years' digits depreciation.
   (b) Rework problem 2, assuming a 60 per cent tax rate.
   (c) Are there any problems involved in the equivalent-yield method of analysis? What are they?

4. Reconsider the Simone Corporation (problem 2).

(a) What is the financial advantage or disadvantage of leasing, in terms of the BHW model?

(b) If it is assumed that Simone has a 10 per cent cost of capital, what is the operating advantage of leasing?

(c) According to the BHW model, should the stamping machine be leased?

(d) What are the limitations of the BHW model?

5. Discuss the implications of the methodology of problems 1 through 4 for (a) the cost of capital to the firm, and (b) the optimal capital structure of the firm.

6. The Blough Corporation desires to produce a new product, called the monopoly. This can only be made with a Clayton machine which, in turn, can only be leased from the Sherman Corporation. Sales of the monopoly are estimated to be $5 million per year for ten years; labor, new materials, and other expenses are estimated at $2 million per year. Blough has a 14 per cent after-tax cost of capital, a 50 per cent tax rate, and figures the pretax cost of equivalent leases to be 10 per cent. Sherman is willing to lease the Clayton machine to Blough for $1 million a year, payable at the beginning of each year, on a ten-year, noncancellable lease.

(a) Should the lease be undertaken?

(b) If Blough's capital structure consists of $5 million of 8 per cent debt and $5 million of 24 per cent equity, what adjustments, if any, to the firm's cost of capital would you suggest?

(c) What would be the implications of any such adjustment for the firm's optimal capital structure?

(d) If you were in charge of leasing for Sherman, what pricing implications would this problem have?

7. The Signo Company is an independent computer-leasing company, specializing in purchasing ABC computers from the manufacturer and leasing them to commercial users. The ABC company also leases its own computers, including its Series 400, which has a list price of $1 million.

(a) If the Signo Company has a pretax opportunity cost for funds of 10 per cent and depreciates the Series 400 computer on a straight-line basis over ten years with 15 per cent salvage value, compute its monthly rental charge for a ten-year lease.

(b) If the ABC Company has a pretax cost of capital of 15 per cent and depreciates the Series 400 computer over six years with no salvage value, compute its monthly rental charge for a six-year lease.

(c) The ABC Company has the policy of renewing leases for years 7–10 at

50 per cent of the prime term rent. If a corporation with a 50 per cent tax rate, a 10 per cent after-tax opportunity cost of funds, and a need for computer usage extending over ten years were to sign a six-year lease (with option to renew) with ABC instead of a ten-year lease with Signo, what present value would the corporation be relinquishing for the option of making other arrangements in years 7–10?

(d) Suppose that there is a 50 per cent probability that the Signo Company would have to pay ABC $500,000 to retrofit each Series 400 at the end of three years for them to be of use in years 4–10. What should Signo's monthly rate be if it is neutral to risk?

8.

The Hardt Corporation
Balance Sheet
(in millions)

Current assets	$10
Fixed assets (30 years, straight-line depreciation)	30
Total assets	$40
Current liabilities	$ 5
Long-term debt (8%)	15
Total debt	20
Net worth	20
Total liabilities	$40

Income Statement

Sales	$100,000,000
Operating income	8,200,000
Depreciation	1,000,000
Interest	1,200,000
Net income before taxes	6,000,000
Taxes (50%)	3,000,000
Net income	$ 3,000,000

The Hardt Corporation is contemplating the sale and leaseback of $10 million of fixed assets. The proceeds would be used to retire debt. The total cost of the lease, including amortization, would be 10 per cent.

(a) Reformulate the financial statements under the assumption that this transaction took place.

(b) Show the effects of this sale upon the following ratios:
   (1) Asset turnover
   (2) Return on assets
   (3) Debt to net worth
   (4) Times interest earned

(c) What is the real impact of this transaction? Has the corporation's fundamental position improved? What is the implicit assumption most companies make when they undertake such transactions? Is this assumption correct?

9. The Hardt Corporation (see problem 8) has a $2/3$ dividend payout and a total market value of $30 million for its stock; its earnings have increased at a compounded rate of 4 per cent per year. Compute a weighted-average cost of capital for the firm before and after leasing.

10. The Signo Company (see problem 7(a)) has discovered that it can charge 90 per cent of ABC's rental per year on three-year leases. The company estimates that it could lease the computers at the end of three years for 40 per cent of ABC's prime rental per month for the remaining seven years; this subjective probability distribution is assumed to be normal, and the standard deviation of the expected rental for years 4–7 is $1,000 per month. If Signo's salvage expectations and cost of capital remain unchanged, what is the probability that the company could make more money with short-term leases?

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EXPANSION
AND
CONTRACTION

VII

Mergers
and
Consolidations

23

In Chapter 6, we considered the acquisition of another company as a capital-budgeting decision. Like an internally generated investment proposal, the acquisition requires an initial outlay, which is expected to be followed by future benefits. Our purpose in Chapter 6 was to develop a conceptual framework for analyzing the likely effect of an acquistion on the net-present value and risk of the firm as a whole. In this chapter, we deal with some of the more practical aspects of external growth. Nonetheless, the framework developed in Chapter 6 will underlie our discussion here. We turn now to some basic definitions.

A *statutory merger* is a combination of two corporations wherein one loses its corporate existence. The surviving company acquires both the assets and liabilities of the merged company. A merger must be distinguished from a *statutory consolidation,* which is a combination of two companies whereby an entirely new corporation is formed. Both the old companies cease to exist, and shares of their common stock are

exchanged for shares in the new company. When two companies of about the same size combine, they usually consolidate. When the two companies differ significantly in size, usually a merger is involved. While it is important to understand the distinction, the terms *merger* and *consolidation* tend to be used interchangeably to describe the combination of two companies.

PROCEDURE

A merger or consolidation often begins with negotiations between the managements of the two companies.[1] Usually, the boards of directors of the companies are kept up to date on the negotiations. When initial agreement is reached as to terms, the respective boards must ratify these terms. Upon ratification, the agreement is submitted to the common stockholders of both companies for approval. Depending upon the corporate charter, an established majority—usually two-thirds—of the total shares is required. After approval by the common stockholders, the merger or consolidation can take place once the necessary papers are filed with the states in which the companies are incorporated.

One hurdle remains, however—that neither the Antitrust Division of the Department of Justice nor the Federal Trade Commission brings suit to block the combination. In order to actually block a merger or consolidation, the government, under Section 7 of the Clayton Act, must prove that a "substantial lessening of competition" might occur on account of it. Because the costs in executive time, legal expenses, and other expenses of waging an antitrust battle are so great, most companies want to be reasonably sure that they will not be challenged before going ahead with the combination.

From an accounting standpoint, a combination of two companies is treated either as a *purchase* or as a *pooling of interests*. With a purchase, the acquired company is treated as an investment by the buyer. The excess of the price paid for the company above its net worth must be reflected as goodwill. Moreover, this goodwill usually is written off against future income, the logic being that it will be reflected in such income. Like any asset, an estimate must be made of its life; and goodwill is amortized over this period.[2] Thus, earnings are reduced by the amount of the charge. It is important to recognize that goodwill charges are not deductible for tax purposes. Therefore, the reduction of reported future earnings associated with this accounting treatment is viewed as a disadvantage by the acquiring firm.

[1] We defer consideration of a tender offer until later in this chapter.

[2] If it can be demonstrated that the life of the goodwill is indefinite, no write-off is required.

In a pooling of interests, the balance sheets of the two companies are combined, with assets and liabilities simply being added together. As a result, goodwill is not reflected in the combination, and there is no charge against future income. For this reason, the pooling of interests is quite popular. Unfortunately, there are a number of cases in which, over time, earnings per share have been distorted through a pooling-of-interests accounting treatment. Such distortion has caused the Securities and Exchange Commission and the accounting profession to take a hard look at the matter. How much the pooling of interests will be restricted remains to be seen.

PURCHASE OF ASSETS OR PURCHASE OF STOCK

The acquistion of another company can take place either by the purchase of assets or by the purchase of the common stock of the company being acquired. Under the former arrangement, the buying company may purchase all or a portion of the assets of another company and pay for this purchase either with cash or with its own stock. Frequently, the buyer acquires only the assets of the other company and does not assume its liabilities. If all the assets are purchased, the selling company is but a corporate shell. After the sale, its assets are composed entirely of cash or the stock of the buying company. The selling company can either hold the cash or stock or it can distribute the cash or stock to its stockholders as a liquidating dividend, after which the company is dissolved.

Thus, when its assets are purchased, the selling company can continue to exist if it holds the cash or stock arising from the sale. If it has cash, it may invest in other assets, such as a division of another company. Obviously, if only a portion of its assets are sold, the selling company definitely will continue as a corporate entity. If paid in cash, the transaction is taxable to the selling company or its stockholders, that is, they must recognize the capital gain or loss on the sale of the assets at the time of the sale.[3] If payment is made in preferred or common stock, however, the transaction is not taxable at the time of sale. The capital gain or loss is recognized only when the stock is sold.

A purchase of assets is easier to effect than a purchase of stock, for all that is needed on the part of the buying company is approval by the board of directors. The selling company, however, needs the approval of its stockholders. Prior to the 1950 amendment to the Clayton Act, the purchase of assets of another company was not subject to antitrust action. Consequently, the purchase of assets was a popular means by

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[3] Likewise, payment with a debt instrument of the acquiring company is also taxable at the time of sale.

which to acquire another company. The 1950 amendment, however, closed this loophole.

When an acquiring company purchases the stock of another company, the latter is merged into the acquiring company. The company that is acquired ceases to exist, and the surviving company assumes all its assets and liabilities. As with a purchase of assets, the means of payment to the stockholders of the company being acquired can be either cash or stock. If cash, the transaction is taxable to the stockholders of the acquired company at the time of the acquisition. If stock, the transaction is tax free to the stockholders until the stock is sold.

DISSENTING STOCKHOLDERS

While a combination generally depends only upon the approval of a required majority of the total number of shares outstanding, minority stockholders can contest the price paid for their stock. If a dissenting stockholder and the company fail to agree as to a just settlement on a voluntary basis, the stockholder can take his case to court and demand an appraisal of his shares and a settlement in cash. After a "fair market price" has been established by the court, the dissenting stockholder receives payment in cash for his shares. If the number of dissenting stockholders is large, they can cause considerable trouble. If the transaction is in stock, the demands for cash payments on the part of these stockholders may put a severe financial strain on the combination. Thus, most combinations depend not only upon obtaining approval of a required majority of stockholders but also upon minimizing the number of dissenting stockholders by making the offer attractive to all. Dissenting stockholders may be able to block the combination if they suspect that fraud is involved, even though the required majority of stockholders has approved it.

HOLDING COMPANIES

Instead of actually acquiring another company, a firm may purchase a portion of its stock and act as a holding company. By definition, a *holding company* owns sufficient voting stock to have a controlling interest in one or more other corporations. A holding company does not necessarily have to own 51 per cent of the stock of another company in order to have control. For a widely held corporation, ownership of 20 per cent or as little as 10 per cent of the stock outstanding may constitute effective working control. The advantage of a holding company is that it allows a company to acquire control of another with a much smaller investment than would be necessary with a merger. Moreover, by acquiring only a portion of the stock, the holding company usually does not have to pay as high a price per share as it would if it sought to purchase all the stock.

It may purchase the stock gradually without undue upward pressure on the market price of the stock. Another advantage of a holding company is the possibility that operating economies can be effected owing to centralized management. The principal disadvantage of the holding company is that 15 per cent of the dividends paid to it by the subsidiary is subject to taxation.[4] Thus, the holding company must pay a partial tax on dividends, and stockholders of the holding company also must pay a tax on dividends they receive. The partial tax could be avoided, of course, if the stockholders owned the operating companies directly.

It is possible to pyramid a series of holding companies to obtain maximum leverage. For example, Holding Company A may own 20 per cent of holding companies B, C, and D, which, in turn, own 20 per cent controlling interst in nine operating companies. Thus, for every dollar of capital in each of the operating companies—$9 in all—Company A is able to control them with an investment of $0.36, (0.20 × 0.20 × $9), or 4 per cent of the total capital of the operating companies. As long as the operating companies are profitable and able to pay dividends to the holding companies, all may go well. However, in the 1920s, there tended to be excessive pyramiding of holding companies, particularly with respect to public utilities. In the 1930s the leverage of these companies magnified the losses, and a number of the pyramids crumbled. Because of the many abuses of holding companies, the Public Utility Holding Company Act of 1935 was passed to restrict the operation of holding companies in the public utility field.

REASONS FOR COMBINATION

The reasons for a combination are many and complex. Moreover, they are not mutually exclusive; more than one usually is involved in a combination. In this section, we consider various reasons for combinations, but recognize that they must be taken collectively.

OPERATING ECONOMIES

Often, operating economies can be achieved through a combination of companies. Duplicate facilities can be eliminated, and marketing, purchasing, and other operations can be consolidated. For example, certain salesmen can be eliminated to avoid duplication of effort in a particular territory. The principal objective in a railroad merger is to realize economies of operation through elimination of duplicate facilities

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[4] If the holding company owns 80 per cent or more of the voting stock of the subsidiary, the dividend is not subject to taxation. For an extensive discussion of holding companies, see Harry G. Guthmann and Herbert E. Dougall, *Corporate Financial Policy*, 4th ed. (Englewood Cliffs, N.J., Prentice-Hall, Inc., 1962), Chapter 28.

and runs. With an industrial company merger, a firm with a product that complements an existing product line can fill out that line and, hopefully, increase the total overall demand for the products of the acquiring company.

Operating economies can best be realized with a *horizontal merger,* in which two companies in the same line of business are combined. The economies achieved by this means result primarily from eliminating duplicate facilities and offering a broader product line in the hope of increasing total demand. A *vertical merger,* whereby a company either expands forward toward the ultimate consumer or backward toward the source of raw material, may also bring about economies. This type of merger gives a company more control over its distribution and purchasing. For example, West Virginia Pulp and Paper Company acquired U.S. Envelope Company in order to provide a sales outlet for its paper. In the formation of U.S. Steel Corporation in 1900, one of the purposes was a complete vertical integration of steel from extraction of ore to the final sale of the product. There are few operating economies in a *conglomerate merger,* where two companies in unrelated lines of business are combined.

Some authors maintain that the prospect of operating economies is the only justification for a merger when the objective of a firm is to maximize shareholder wealth.[5] Their reasons essentially are those discussed in Chapter 6 — namely, that investors are able to achieve the other benefits associated with mergers on their own. In particular, conglomerate mergers are felt to lack economic justification unless the acquiring company can manage more productively the assets of the companies being acquired.

MANAGEMENT ACQUISITION

Closely related to operating economies is the acquisition of management. If a firm finds that it is unable to hire top-quality management and that it has no one coming up through the ranks, it may seek a combination with another company having aggressive and competent management. The choice may be between gradual stagnation with an existing organization or combination with another company in order to obtain aggressive management and prospects for growth. To foster the long-run wealth of stockholders, the latter may be the only feasible alternative.

DIVERSIFICATION

Diversification is the motive in some mergers. By acquiring a firm in a different line of business, a company may be able to reduce cyclical

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[5] See, for example, Dennis C. Mueller, "A Theory of Conglomerate Mergers," *Quarterly Journal of Economics,* LXXXIII (November, 1969), 652–53.

instability in earnings. To the extent that investors in the company's stock are averse to risk in the sense that they prefer less to more variation in earnings, this reduction may have a favorable effect upon the price of the stock. The Martin Company, a manufacturer of missiles and related defense products, combined with American Marietta Company, a manufacturer of cement and other building materials, to form Martin-Marietta Company for the purpose of diversification to reduce earnings instability. While it is virtually impossible to find two companies with negative correlation in earnings, it is possible to find situations in which there is only moderate correlation. Many conglomerate mergers involve companies whose earnings are only moderately correlated.

Related to the argument for diversification is the notion of spreading risk. For a small company the risk exposure of undertaking a new product line may be very significant indeed. In fact, the potential loss may be so great in relation to the capital base of the company that management is unwilling to go ahead with the product development despite its considerable promise. By combining with a larger company, however, the firm may be able to undertake the project, because the potential loss is not nearly so significant relative to the capital base of the surviving company. As taken up in Chapter 6, diversification through mergers will enhance shareholder wealth only if investors cannot diversify effectively for themselves. If they can, however, efforts by a company to diversify through acquiring other companies will go unrewarded.

GROWTH

A company may not be able to grow at a fast or balanced enough rate by internal expansion and may find that its only way of achieving a desired growth rate is by acquiring other companies. The cost of growth by acquisition may well be cheaper than the real cost of internal growth; the numerous costs and risks involved in developing and embarking upon a new product line may be avoided through acquisition of a going concern. In addition, it usually is quicker to acquire new products and facilities through mergers than through internal development. An important aspect of external growth may be the acquisition of the research capabilities of another firm. As research tends to be individually oriented, the acquiring company may be unable to develop such capabilities on its own. Closely related to research is the possession of basic patents. A company having certain patent rights may be extremely valuable for this reason alone.

For certain companies, growth in sales, assets, and profits has supplanted maximization of shareholder wealth as the primary goal of the firm. Robin Maris contends that because management's salaries, stock options, and prestige are more closely related to size than to profits,

managers have considerable incentive to maximize growth.[6] If this is true, it is not difficult to understand the attractiveness of mergers in implementing such an objective, since in most cases growth can be achieved more easily through external acquisitions than through internal development.

FINANCING

Rapidly growing companies can run into difficulty in financing their growth. Rather than curtail their expansion, they may seek to combine with a company having the liquidity and stability necessary for financing the contemplated growth. The "cash-rich" company can benefit by being able to utilize its liquidity in a growth situation. The growing company benefits in that it does not have to give up exciting opportunities in order to provide for a period of "digestion."

TAXATION

The avoidance of corporate income taxes is a factor in some mergers. A company with a tax loss carry-forward may want to acquire one or more profitable companies in order to be able to utilize its carry-forward. Otherwise, the carry-forward may expire at the end of five years for the lack of sufficient profits to utilize it completely.[7] For this reason, a company may be willing to pay a fairly substantial price to acquire a profitable company. Studebaker Corporation acquired a number of companies in the early to middle 1960s in order both to diversify and to utilize the substantial tax carry-forward that resulted from losses in its automobile division.

PERSONAL REASONS

In a tightly held company, the individuals who have controlling interest may want their company acquired by another company that has an established market for its stock. For estate tax purposes, it may be desirable for these individuals to hold shares of stock that are readily marketable and for which market-price quotations are available. The owners of a tightly held company may have too much of their wealth tied up in the

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[6] *The Economic Theory of Managerial Capitalism* (New York: The Free Press, 1964). See also Mueller, "A Theory of Conglomerate Mergers," 644–48. In contrast, Wilbur G. Lewellen and Blaine Huntsman, "Managerial Pay and Corporate Performance," *American Economic Review*, LX (September, 1970), 710–20, find in an empirical study that executive compensation is influenced more strongly by profit and stock performance than by sales.

[7] For a discussion of the accounting treatment of the carry-forward, see James C. Van Horne, "A Look at the Loss Carry-Forward," *Accounting Review*, XXXVIII (January, 1963), 56–60.

company. By merging with a publicly held company, they obtain a marked improvement in their liquidity, enabling them to sell some of their stock and diversify their investments.

FINANCIAL CONSIDERATIONS

When two companies are combined, a ratio of exchange occurs that denotes the relative weighting of the firms. In this section, we consider the ratio of exchange with respect to the earnings, the market prices, and the book values of the stocks of the two companies involved. We assume that the combination is consummated in stock rather than in cash or debt. The objective in any merger should be to maximize the long-run wealth of existing stockholders. A successful merger, then, would be one that increases the market price of the firm's stock over what it would have been if the combination had not taken place.[8]

In Chapter 6, we developed a framework for analyzing a prospective acquisition in keeping with its likely effect upon the risk-return complexion of the firm as a whole. With this information, management can assess the probable impact of the acquisition on share price. In this section, we consider the effect of a prospective acquisition on certain financial relationships. While these relationships are embodied in the framework presented in Chapter 6, most companies prefer to analyze them separately. The analysis to be described, together with that taken up in Chapter 6, serves as the foundation for negotiations.

EARNINGS

In evaluating the possibility of an acquisition, it is important to consider the effect the merger has on the earnings per share of the surviving corporation. Suppose that Company *A* is considering the acquistion, by stock, of Company *B*. The financial data on the acquisition at the time it is being considered follows.

	Company A	Company B
Present earnings (in millions)	$20.00	$ 5.00
Shares (in millions)	5	2
Earnings per share	$ 4.00	$ 2.50
Price of stock	$64.00	$30.00
Price/earnings ratio	16	12

[8] William W. Alberts, "The Profitability of Growth by Merger," William W. Alberts and Joel E. Segall, eds., *The Corporate Merger* (Chicago: University of Chicago Press, 1966), p. 236.

Assume that Company B has agreed to an offer of $35 a share to be paid in Company A stock. The exchange ratio, then, is $35/$64, or about 0.547 shares of Company A's stock for each share of Company B's stock. In total, 1,093,125 shares of Company A will need to be issued in order to acquire Company B. Assuming that the earnings of the component companies stay the same after the acquisition, earnings per share of the surviving company would be

	Surviving Company A
Earnings	$25,000,000
Shares	6,093,125
Earnings per share	$4.10

Thus, there is an immediate improvement in earnings per share for Company A as a result of the merger. Company B's former stockholders experience a reduction in earnings per share, however. For each share of B's stock they had held, they now hold 0.547 shares of A. Thus, the earnings per share on each share of Company B's stock they had held is (0.547) (4.10), or $2.24, compared with $2.50 before.

Assume, however, that the price agreed upon for Company B's stock is $45 a share. The ratio of exchange, then, would be $45/$64, or about 0.703 shares of A for each share of B. In total, 1,406,250 shares would have to be issued, and earnings per share after the merger would be

	Surviving Company A
Earnings	$25,000,000
Shares	6,406,250
Earnings per share	$3.90

In this case, there is initial dilution in Company A's earnings per share on account of the acquisition of Company B.[9] Dilution in earnings per share will occur anytime the price/earnings ratio paid for a company exceeds the price/earnings ratio of the company doing the acquiring. In our example, the price/earnings ratio in the first case was $35.00/$2.50, or 14, and in the second case, it was $45.00/$2.50, or 18. As the price/earnings ratio of Company A was 16, there was an increase in earnings per share in the first case and a decrease in the second.

Future Earnings.   If the decision to acquire another company is based solely upon the initial impact on earnings per share, we would say that

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[9]Company B's former stockholders obtain an improvement in earnings per share. Earnings per share on each share of stock they had held is $2.74.

the first case was a worthwhile acquisition and the second was not. Using this criterion, an initial dilution in earnings per share suggests that the exchange ratio is too high; Company A should not pay a price that will result in a price/earnings ratio higher than 16. However, this type of analysis does not take into account the possibility of a future growth in earnings owing to the merger. If the earnings of Company B are expected to grow at a faster rate than those of Company A, a high ratio of exchange for the stock may be justified, despite the fact that there is initial dilution in earnings per share for stockholders of Company A. The superior growth in earnings of the acquired company may result eventually in higher earnings per share for these stockholders relative to earnings without the merger.

It is useful to graph likely future earnings per share with and without the acquisition. Figure 23-1 shows this for a hypothetical merger. The graph tells us how long it will take for the dilution in earnings per share to be eliminated, and for an accretion to take place. In this example, it is three years; earnings per share drop $0.30 initially, but this relative dilution is eliminated by the start of the fourth year. The greater the duration of dilution, the less desirable the acquisition is from the standpoint
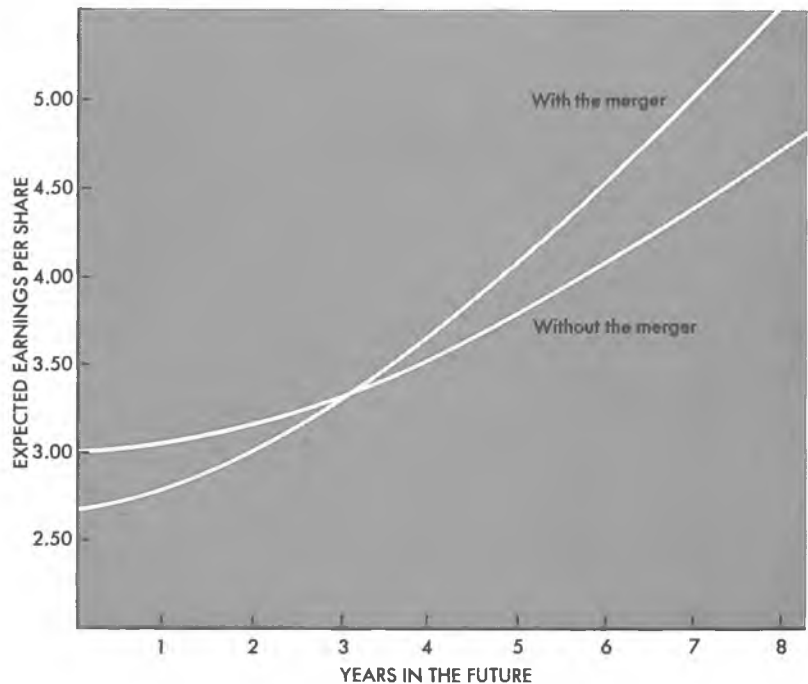
FIGURE 23-1
**Expected earnings per share with and
without the merger**

of the acquiring company. Some companies set a ceiling on the number of years dilution will be tolerated, and this ceiling serves as a constraint in establishing the exchange ratio to be paid in the acquisition.

Another drawback to using the initial impact on earnings per share as the sole criterion for judging the value of a merger is that the earnings of the surviving company are not necessarily an additive affair, such that $2 + 2 = 4$. In many cases, there are *synergistic* effects, such that $2 + 2 = 5$. Because of operating economies, increases in demand, and so forth, earnings of the surviving company may be greater than the sum of the earnings of the two companies without the merger. In our example, suppose that total earnings three years after the merger are expected to be $36 million for the surviving company, whereas total earnings of Company A three years hence without the merger are expected to be $26 million. Assuming the price paid for Company B's stock is $45 a share, the expected earnings per share three years hence with and without the merger are

	With Merger	Without Merger
Expected earnings	$36,000,000	$26,000 000
Shares	6,406,250	5,000,000
Expected earnings per share	$5.62	$5.20

We see then that despite initial dilution, the acquisition of Company B produces a favorable effect upon future earnings per share over and above the expected growth in earnings per share for Company A without the merger.[10] We can graph expected earnings per share, as in Figure 23-1, with and without the acquistion under the assumption of synergism. In fact, when an acquisition is being considered graphs should be prepared under differing assumptions as to the exchange ratio. They should also be made under differing earnings assumptions for the combination, for preparing multiple graphs gives management greater information on which to base negotiations.

Dividends per Share. The ratio of exchange of dividends per share sometimes is considered in the evaluation of a merger. However, the dividend decision really is separate from the merger decision. The important variable is prospective earnings, for dividends are a function of these earnings. The acquiring company can alter its dividend-payout ratio if the prospect of a higher total dividend is enticing to the stockholders of the company being acquired. Justification for this action, however, must be based upon an analysis of its effect upon shareholder wealth (see Chapters 9 and 10).

[10] See J. Fred Weston, "Determination of Share Exchange Ratios in Mergers," in Alberts and Segall, eds., *The Corporate Merger,* pp. 117–38.

MARKET VALUE

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CHAP. 23
*Mergers
and
Consolidations*

In addition to earnings, the major emphasis in the bargaining process is on the ratio of exchange of market prices per share. The market price of a publicly held stock is the focal judgment of investors as to the "intrinsic" value of that company. Accordingly, it reflects the earnings potential of the company, dividends, business risk, capital structure, asset values, and other factors that bear upon valuation. The ratio of exchange of market prices is simply

$$\frac{\text{Market Price per Share of Acquiring Company} \times \text{Number of Shares Offered}}{\text{Market Price per Share of Acquired Company}}$$

For example, if the market price of Company A is $60 per share and that of Company B is $30, and Company A offers a half share of its stock for each share of Company B, the ratio of exchange would be

$$\frac{\$60 \times 0.5}{\$30} = 1.00$$

In other words, the stocks of the two companies would be exchanged on a one-to-one market-price basis. If the market price of the surviving company is relatively stable at $60 a share, each set of stockholders is about as well off as before with respect to market value. However, there is little enticement to the company being acquired to accept a one-to-one market-value ratio of exchange. Consequently, the acquiring company usually must offer a price in excess of the current market price per share of the company it wishes to acquire. Instead of a half share of stock, Company A might have to offer 0.667 shares, or $40 a share in current market value.

Even when the acquiring company offers a price in excess of the current market price of the company being acquired, its own stockholders still may be better off with respect to market price per share. The reason is that there may be a difference in the price/earnings ratios of the two companies. Suppose that Company B is a moderate-sized company whose stock is traded in the over-the-counter market. Because, among other reasons, its stock is not particularly marketable, its price/earnings ratio is 10, relatively low. Company A, on the other hand, has a price/earnings ratio of 18. Assume the following financial information:

	Company A	Company B
Present earnings (in millions)	$20.00	$ 6.00
Shares (millions)	6	2
Earnings per share	$ 3.33	$ 3.00
Market price per share	$60.00	$30.00
Price/earnings ratio	18	10

With an offer of 0.667 shares of Company *A* for each share of Company *B*, or $40 a share in value, the market-price exchange ratio for Company *B* is

$$\frac{\$60 \times 0.667}{\$30} = 1.33$$

Stockholders of Company *B* are being offered a stock with a market value of $40 for each share of stock they own. Obviously, they benefit from the acquisition with respect to market price, because their stock was formerly worth $30 a share. However, the stockholders of Company *A* also stand to benefit, if the price/earnings ratio of the surviving company stays at 18. The market price per share of the surviving company after the acquisition, all other things held constant, would be

	Surviving Company
Total earnings	$26,000,000
Number of shares	7,333,333
Earnings per share	$3.55
Price/earnings ratio	18
Market price per share	$63.90

The reason for this apparent bit of magic whereby the stockholders of both companies benefit is the difference in price/earnings ratios.

Thus, companies with high price/earnings ratios are able to acquire companies with lower price/earnings ratios and obtain an immediate increase in earnings per share, despite the fact that they pay a premium with respect to the market value exchange ratio. Provided that the price/earnings ratio of the surviving company is not lowered, the market price of the stock will improve also. By acquiring a sufficient number of companies over time in this manner, a company can increase earnings per share steadily. To the extent that the marketplace values this illusory growth, the price/earnings ratio of the stock actually may increase. As a result, a company would be able to increase shareholder wealth appreciably through acquisitions alone. However, it seems unlikely that the market continually will raise the price/earnings ratio of a company that cannot demonstrate growth potential in ways other than acquiring companies with lower price/earnings ratios. The acquiring company must be able to manage the companies it acquires if the benefit of acquisitions is to be lasting.

The effect of the acquisition on the price/earnings ratio of the surviving company obviously is important. If the company being acquired is small relative to the acquiring company, it is unlikely that there will be much effect upon the price/earnings ratio. The price paid for a company may be high in terms of the relative market price of the two stocks, but the

difference may be more than offset by an improvement in present and future earnings per share. When the company being acquired is relatively large, the acquiring company must consider the possibility that its price/earnings ratio will change. If the market is relatively free from imperfections and if synergism is not anticipated, we would expect the price/earnings ratio of the surviving firm to approach a weighted average of the two previous price/earnings ratios. Under these circumstances, the acquisition of companies with lower price/earnings ratios would not enhance shareholder wealth. If synergism were expected, however, shareholder wealth could be increased through the acquisition.

Market values are unquestionably a major factor in most mergers; however, these values fluctuate greatly over time and in differing degrees for different companies. As a result, there may be considerable doubt as to just what the appropriate market value of a company is. Because of the fluctuation in market value, some companies vary their pursuit of acquisitions in keeping with the price of their stock. When the price is high, they may be aggressive in their pursuit of acquisitions; when it is relatively low, merger activity may dry up completely. Although certain mergers are based upon normalized market prices over a length of time, most are predicated upon the current market price. Consequently, fluctuations in this price are extremely important to the acquisition-minded company.

BOOK VALUE

Book value per share is rather meaningless as a basis for valuation in most mergers. Whereas once it was the dominant factor, book value per share is important now only when it is significantly above market value. When the purpose of an acquisition is to obtain the liquidity of another company, book value per share and working capital per share become important in the terms of the exchange. For example, Textron acquired American Woolen primarily for the latter's liquidity. American Woolen's book value per share was approximately $60, its working capital per share was $24, and its market price per share was $16. Textron paid $25 a share in cash. The ratios of exchange of book value per share of two companies is calculated in the same manner as is the ratio for market values. The importance of this ratio in bargaining is usually restricted to situations in which a company is acquired for its liquidity and asset values rather than for its earning power.

AN ILLUSTRATION

To illustrate the ratios of exchange, consider the acquisition of American Optical Company by Warner-Lambert Pharmaceutical Com-

pany in April, 1967, wherein 2.1 shares of Warner-Lambert were exchanged for each share of American Optical. The following figures are significant.

	Ratio of Exchange Warner-Lambert/American Optical
Book value per share	0.47
Working capital per share	0.58
Average earnings per share during previous 5 years	1.13
Earnings per share during previous year	0.87
Average dividends per share during previous 5 years	1.66
Dividends per share during previous year	1.63
Average closing market price for previous 30 weeks	1.16
Closing market price per share, 3/31/67	1.05

The exchange ratios indicate that American Optical made a substantial contribution in book value and working capital to the surviving company. In earnings per share over the past five years, Warner-Lambert contributed more to the surviving company. However, for the most recent earnings figures, American Optical made the larger contribution. The mixed results with respect to earnings were due to American Optical's faster relative growth in earnings per share.

Market price clearly was an important factor in making the merger attractive to the stockholders of American Optical. Warner-Lambert's higher price/earnings ratio enabled it to acquire American Optical on favorable terms with respect to earnings, book value, and current assets, with only a small premium over American Optical's market value. When evaluating this merger, it is certainly important to consider the synergistic effects. Both companies expected to profit from increased diversification and from the improved promotion of American Optical's product line, owing to Warner-Lambert's more extensive international sales organization.

NEGOTIATIONS

BOUNDARIES FOR NEGOTIATION

Once the financial relationships taken up in the previous section have been analyzed, the acquiring firm is ready to begin negotiations. The financial relationships establish the boundaries within which negotiation can take place. To illustrate, suppose Company *A* were considering

acquiring Company B.[11] The market price per share of Company A before the merger is

$$P_a = \frac{(P/E_a)Y_a}{N_a} \qquad (23\text{-}1)$$

where P/E_a is the price/earnings ratio for Company A, Y_a is the total current earnings of that company, and N_a is the number of shares outstanding. For Company B, the market price per share is

$$P_b = \frac{(P/E_b)Y_b}{N_b} \qquad (23\text{-}2)$$

Expected market price per share after the combination is

$$P_{ab} = \frac{(P/E_{ab})(Y_a + Y_b)}{N_a + (ER)N_b} \qquad (23\text{-}3)$$

where P/E_{ab} is the expected price/earnings ratio after the merger, and ER is the exchange ratio, that is, the number of shares of Company A offered in exchange for one share of Company B. The estimate of P/E_{ab} should take account of any expected synergistic effects.

It is clear that stockholders in Company A will be dissatisfied if market price per share after the merger is less than that which prevailed before. Thus, the maximum exchange ratio that should be offered from their standpoint is the one that results in P_{ab} equaling P_a. Setting Eq. (23-1) equal to Equation (23-3) and rearranging, Larson and Gonedes demonstrate that the maximum exchange ratio acceptable to Company A stockholders is[12]

$$ER_a = \frac{(P/E_{ab})(Y_a + Y_b) - (P/E_a)(Y_a)}{(P/E_a)(Y_a)(1/N_a)(N_b)} \qquad (23\text{-}4)$$

With this ratio of exchange, the expected market price per share after the merger would be the same as the market price per share before. Usually, the price/earnings ratio of the combined company, P/E_{ab}, is not known with any degree of certainty. Consequently, it is desirable to graph the maximum exchange ratio acceptable to Company A stockholders for a range of P/E_{ab}. An example is shown in Figure 23-2, and it is seen that ER_a is a linear function of P/E_{ab}. The greater P/E_{ab}, the greater the maximum exchange ratio acceptable to Company A stockholders. Thus, any exchange ratio on or below the straight line is acceptable.

In a similar manner, it is possible to graph the minimum exchange

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[11] This example is based upon Kermit D. Larson and Nicholas J. Gonedes, "Business Combinations: An Exchange-Ratio Determination Model," *Accounting Review*, XLIV (October, 1969), 720–28.
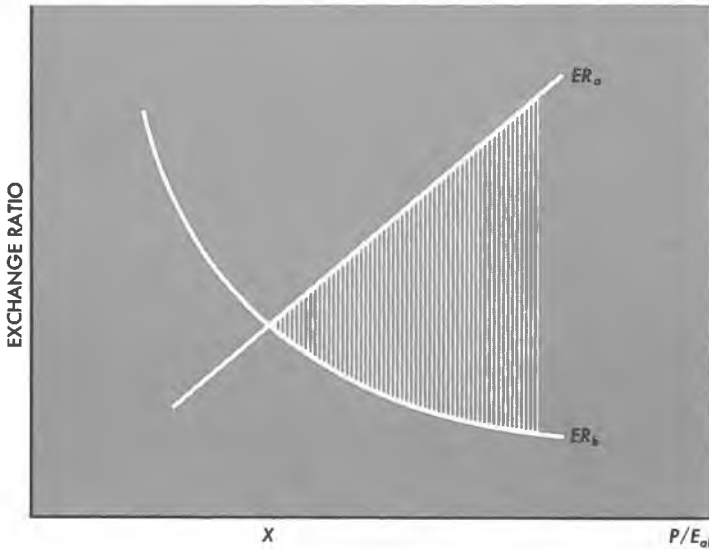
[12] *Ibid.*, p. 724.

FIGURE 23-2
Exchange ratio boundaries

ratio acceptable to Company B stockholders. With this ratio, the market price of their holdings after the merger would equal the market price of their holdings before the merger, or $P_{ab} = P_b/ER$. Rearranging Eq. (23-3) and Eq. (23-2), one obtains the minimum exchange ratio acceptable to the stockholders of Company B[13]

$$ER_b = \frac{(P/E_b)(Y_b/N_b)(N_a)}{(P/E_{ab})(Y_a + Y_b) - (P/E_b)(Y_b)} \qquad (23\text{-}5)$$

It is found that ER_b declines at a decreasing rate with P/E_{ab}. An example of this relationship also is shown in Figure 23-2. Stockholders of Company B would be satisfied only with an exchange ratio on or above the line.

Once these relationships are graphed, the resulting shaded area in the figure represents the boundaries for negotiation over the exchange ratio. With information of this sort, the management of Company A should try to bargain for an exchange ratio close to the ER_b line, while Company B's management should strive for an exchange ratio close to the ER_a line. Boundary conditions shown in the figure establish the area in which fruitful negotiations are possible.

SETTLING UPON FINAL TERMS

The exchange ratio finally settled upon depends upon the relative bargaining power of the two managements, as well as upon their expecta-

[13]*Ibid.*, p. 725.

tions of the price/earnings ratio likely to prevail once the merger is consummated. Note that if the expected price/earnings ratio is to the left of point X in Figure 23-2, a merger of the two companies should not be undertaken. Neither set of shareholders is likely to gain, and one or both will suffer a decline in the market value of their holdings. At the intersection of the two lines, the price/earnings ratio for the surviving firm represents a weighted average of the price/earnings ratios of Companies A and B prior to the merger.[14] All price/earnings ratios to the right of point X are greater than the weighted average of the premerger price/earnings ratios. In a market free from imperfections, these ratios would be due solely to the expectation of synergism. Under such circumstances, it is possible for both sets of stockholders to gain from the merger.

The financial information developed in the previous section assists management only in negotiating more effectively; it does not establish the final terms. Management of the buying company must convince its counterpart in the selling company that a merger is in the latter's best interests. One must be mindful of the fact that if the prospective acquisition is attractive to the buying company, it is probably attractive to others as well.[15] Consequently, the selling company may have its pick of offers. Naturally, it will want to select the best.

But what constitutes the best offer? Obviously, the exchange ratio is important, because it establishes the market price per share offered for the company. Apart from the exchange ratio, management of the potential acquisition often has to be convinced that a marriage of the two companies is in their best interest. Very much part of the total picture is the role this management will play in the surviving company. To come to grips with this and related issues prior to negotiation, management of the acquiring company must thoroughly understand the operations of the potential acquisition. Then it must lay out a strategy with respect to the nonfinancial aspects of the prospective merger. These aspects include not only the role and compensation of management in the surviving company but also such things as the continuation and promotion of existing products, the opportunity to go into new markets, and the provision of financial resources to assure future growth.[16] Nonfinancial considerations can loom quite large in the minds of the selling company's management, often spelling the difference between going along with a merger or turning it down.

[14]*Ibid.*, p. 726.

[15]For an excellent exposition on negotiating strategy, see Gary E. MacDougal and Fred V. Malek, "Master Plan for Merger Negotiations," *Harvard Business Review,* 48 (January–February, 1970), 77–82. See also John S. R. Shad, "The Financial Realities of Mergers," *Harvard Business Review,* 47 (November–December, 1969), 138–41; Richard M. Hexter, "How to Sell Your Company," *Harvard Business Review*, 46 (May–June, 1968), 71–77; and Willard F. Rockwell, Jr., "How to Acquire a Company," *Harvard Business Review,* 46 (May–June, 1968), 121–32.

[16]See MacDougal and Malek, "Master Plan for Merger Negotiations," 78–80.

To the extent that synergism is possible, the buyer can be relatively generous in the exchange ratio offered, as well as in nonfinancial terms. As suggested earlier, final terms depend upon the bargaining strengths of the two parties, the financial relationships described, and expectations regarding the future earnings performance of the surviving company. In the case of the buyer, negotiations should be framed in terms of what the seller will gain from the merger. Unless there is a reasonable gain, few companies will wish to sell. In contrast, the seller should think in terms of how much the merger is worth to the buyer. Simply because a merger will benefit the seller does not mean that the terms offered are the best that can be obtained by the seller. The final terms will fall somewhere within the boundaries depicted in Figure 23-2.

STOCK VERSUS CASH OFFER

In our analysis so far, we have assumed an exchange of stock. However, the acquisition of another company can be for cash or for bonds. In theory, we suggested in Chapter 6 that it should make no difference whether the method of acquisition is with cash or with stock. The valuation principles are the same. However, there may be practical problems. For one thing, the market price of the stock may be subject to considerable variation, making the valuations under the two mediums of exchange somewhat different. The company being acquired with stock may demand a higher price, all other things held constant, in order to obtain a cushion for possible declines in market price. However, with a stock acquisition, the acquiring company does not actually have to float a common stock issue in the marketplace. If it raised equity capital in the market, the issue would have to be underpriced, and there would be flotation costs. Thus, the cost of equity financing per dollar of capital raised will be somewhat lower than if a new issue of common stock is floated to finance the acquisition.

With respect to taxation, the selling company usually prefers payment in stock because the transaction is tax free at the time of payment. If the acquisition is with cash or debt, it is treated as a taxable transaction, and the capital gain or loss on it must be recognized at the time of the acquisition. For this reason, a selling company may be willing to settle for a lower price if the payment is in stock rather than in cash. On the other hand, because of certain other tax considerations, the buyer may prefer a cash purchase to a stock purchase. With a stock purchase, the buying company assumes as its tax base the book values of the assets it acquires. Subsequent depreciation is based on these values. With a cash purchase, the buying company is able to write up the value of the assets to reflect the purchase price. If the purchase price is substantially above the book value of the assets, the write-up can be significant. In turn, the purchase price, less any portion considered as goodwill, is used as the tax base by the buying company. As a result, the depreciation

charges and resulting tax shield may be greater with a cash purchase than with a stock purchase. When this is the case, the buying company would prefer to make payment in cash rather than in stock, all other things held constant. Whether the advantage of a stock sale to the seller more than offsets its disadvantage to the buyer will depend upon the individual situation.[17]

TENDER OFFERS

In our hypothetical examples, it was assumed that the two companies negotiated over the terms of the exchange. When terms are agreed upon by both companies, the plan is submitted to the respective stockholders of each company for approval. Upon approval by a required majority, the merger can be consummated. All negotiations in the examples were assumed to be confined to the managements and boards of directors of the company involved.

However, the acquiring company can make its appeal directly to the stockholders of the company it wishes to acquire, through a tender offer. A tender offer is an offer to purchase shares of stock of another company at a fixed price per share from any stockholder who "tenders" his shares. The tender price is usually set significantly above the present market price in order to provide an incentive to stockholders to tender their shares. Use of the tender offer allows the acquiring company to bypass the management of the company it wishes to acquire and, therefore, serves as a threat in any negotiations with that management. If management holds out for too high a price or otherwise balks at the offer of the acquiring company, that company can always make a tender offer.

The tender offer can also be used when there are no negotiations but when one company simply wants to acquire another. In a "surprise" tender offer, the acquiring company is very careful not to reveal its intentions prior to the actual offer. The primary selling tool is the premium that is offered over the existing market price of the stock. As a rule of thumb, many suggest a premium of 20 per cent, which is adjusted up or down depending upon the circumstances.[18] In addition, brokers are often given very attractive commissions for shares tendered through them. The tender offer itself is usually communicated through financial newspapers. Direct mailings are made to the stockholders of the company being bid for if the bidder is able to obtain a stockholders' list. While a company is legally obligated to provide such a list, it usually is able to delay delivery long enough to frustrate the bidder.

[17] See Samuel Schwartz, "Merger Analysis as a Capital Budgeting Problem," in William W. Alberts and Joel E. Segall, eds., *The Corporate Merger* (Chicago: University of Chicago Press, 1966), p. 147.

[18] See Samuel L. Hayes, III, and Russell A. Taussig, "Tactics of Cash Takeover Bids," *Harvard Business Review*, 45 (March–April, 1967), 139–40.

A bidder takes pains to keep his intentions secret until the last minute.
"Claude; Courtesy of *Fortune Magazine.*"

Management often seeks to foil a bidder by
quickly arranging a merger with another company.
"Claude; Courtesy of *Fortune Magazine.*"

Determined management opposition
can usually fend off an unwanted take-over bid.
"Claude; Courtesy of *Fortune Magazine.*"

FIGURE 23-3
Tender offers

From the standpoint of the company being bid for, a number of defensive tactics are available. First, management may try to persuade its stockholders that the offer is not in their best interests. Usually, the argument is that the bid is too low in relation to the true, long-run value of the firm. However, in the face of an attractive premium, the long run may be too distant. Some companies raise the cash dividend or declare

a stock split in hopes of gaining stockholder support. Legal actions are often undertaken, more to delay and frustrate the bidder than with the expectation of winning. To the extent that the two firms are competitors, an antitrust suit may prove a powerful deterrent to the bidder. As a last resort, management of the company being bid for may seek a merger with a "friendly" company.[19] Some of the strategies involved in tender offers are depicted in Figure 23-3 (opposite page).

The use of the tender offer has increased in importance (and success) during the last ten years. There are many who would contend that tender offers contribute to corporate "democracy" and, thereby, serve a socially useful function. If management does not behave so as to maximize the value of the firm to its shareholders, there is always the danger of a tender offer from another company. Whereas stockholders may not have enough control to effect a change otherwise, a tender offer may bring about a change and increase shareholder wealth. In recent years, the tender offer has largely displaced the proxy contest as a means of obtaining control of a company.

SUMMARY

A company may grow internally, or it may grow externally through acquisitions. The objective of the firm in either case is to maximize existing shareholder wealth. Another company can be acquired through the purchase of either its assets or its stock. In turn, the means of payment can be cash, or it can be stock. When two companies combine and one loses its corporate existence, the combination is known as a merger; if two companies combine and form a new corporation, the combination is known as a consolidation. Other than merging or consolidating, one company can purchase effective working control of another company and act as a holding company.

There are a number of reasons for acquiring another company, all of which relate to expected return and risk. Among the more important are operating economies, acquisition of management, diversification, growth potential, financing, taxation, and personal motives of the owners. The relative valuation of two companies may be based upon earnings, market values, book values, or a combination of all three. As the market price reflects the judgment of investors as to everything that affects value, it is the foundation upon which most exchange ratios are based. Once financial relationships are analyzed, rational negotiations can take place. In this regard, it is useful to establish the exchange ratio boundaries for negotiations. Within these boundaries, the wealth of both sets of stockholders is enhanced. In recent years, we have seen an increasing use of

[19] For a more extensive discussion of these points, see *Ibid.*, 135–48.

tender offers, whereby one company offers to purchase shares from the stockholders of another company.

1.

	Net Income	Number of Shares	Market Price Per Share	Tax Rate
Nimbus Company	$5,000,000	1,000,000	$100	50%
Noor Company	1,000,000	500,000	$ 20	50%

The Nimbus Company wishes to acquire the Noor Company. If the merger were effected through an exchange of stock, Nimbus would be willing to pay a 25 per cent premium for the Noor shares. If done for cash, the terms would have to be as favorable to the Noor shareholders; to obtain the cash, Nimbus would have to sell its own stock in the market.

(a) Compute the exchange ratio and the combined earnings per share if an exchange of stock were accomplished.

(b) If we assume that all Noor shareholders have held their stock for more than six months, have a 40 per cent marginal tax rate, and paid an average of $14 for their shares, what cash price would have to be offered to be as attractive as the terms in (a) above?

(c) Why might the computation in (b) overstate the premium that would have to be paid to make the cash price comparable to the exchange of stock offer? Upon what factor would the size of premium depend?

(d) If the cash (see (b) above) were obtained by means of a stock issue at the current price (with total expenses of 10 per cent), what would the earnings per share of new Nimbus be?

2. Suppose that the current balance sheets of Nimbus and Noor (see problem 1) are as follows:

	Nimbus	Noor
Cash	$ 10,000,000	$ 5,000,000
Accounts receivable	17,000,000	4,000,000
Inventories	20,000,000	5,000,000
Prepaid expenses	3,000,000	1,000,000
Total current assets	$ 50,000,000	$15,000,000
Fixed assets, net	100,000,000	10,000,000
Total assets	$150,000,000	$25,000,000
Notes payable	$ 15,000,000	$ —
Accounts payable	25,000,000	8,000,000
Accrued wages and taxes	10,000,000	2,000,000
Total current liabilities	$ 50,000,000	$10,000,000
Long-term debt	50,000,000	5,000,000
Common stock	10,000,000	2,500,000
Capital surplus	20,000,000	2,500,000
Retained earnings	20,000,000	5,000,000
Total liabilities and Net worth	$150,000,000	$25,000,000

(a) Derive the balance sheet of new Nimbus if the exchange of stock (see 1 (a)) were effected. Compute the old and new book value per share.

(b) Recompute (a), assuming instead that the Noor holders are paid in cash, as outlined in 1 (b) and (d). Assume that the Noor assets cannot be written up. Also compute an old and new net tangible assets per share.

3. Assume the exchange of Nimbus shares for Noor shares as outlined in problems 1 and 2 above.

(a) What is the ratio of exchange?

(b) Compare the earnings per Noor share before and after the merger. Compare the earnings per Nimbus share. On this basis alone, which group fared better? Why?

(c) What would you expect to happen to the share price of Nimbus after the merger? What would you expect to happen to the P/E ratio? Must they both move together?

(d) Why do you imagine that old Nimbus commanded a higher P/E than Noor? What should be the change in P/E ratio resulting from the merger? Does this conflict with your answer to (c)? Why?

(e) If the Nimbus Company is in a high-technology growth industry and Noor makes cement, would you revise your answers?

(f) In determining the appropriate P/E ratio for Nimbus, should the increase in earnings resulting from this merger be added as a growth factor?

(g) In light of the above discussion, do you feel that the Noor shareholders would have approved the merger if Noor stock paid a $1 dividend and Nimbus paid $3? Why?

4. Instead of an exchange of stock (see problem 1 (b)), suppose that Nimbus offered Noor shareholders a 5 per cent preferred stock, convertible 20 per cent above the market, at the same exchange ratio proposed for the common stock. Assume the common-stock dividend payments of problem 3 (g).

(a) If this transaction were viewed as an exchange of common stock, what is the effective exchange ratio?

(b) Compute the change in dividend income per Noor share after the exchange for Nimbus convertible preferred. Compute the change in dividend income after conversion into Nimbus common.

(c) Compute earnings per share for new Nimbus before and after conversion.

(d) Compute earnings retention before and after conversion, assuming the dividend remains the same.

5. The shares of the Navigation Company and the Matson Company have the following characteristics:

	Navigation	Matson
EPS	$4	$2
Growth in EPS expected forever	5%	10%
Number of shares (in millions)	10	3
Price per share	$40	$30

(a) If Navigation acquires Matson with an exchange of stock on the basis of their market values, what will be the new EPS?

(b) Graph Navigation EPS into the future with and without the acquisition (assume no synergism).

(c) How long would it take to eliminate the dilution?

6. Let it be assumed that a holding company can always be set up with 50 per cent debt at 8 per cent and 20 per cent preferred stock at 6 per cent. Further

assume that all companies pay a tax rate of 50 per cent, the 85 per cent inter-corporate dividend exclusion applies in all cases, and that ownership of 40 per cent of the stock of another company constitutes control. The shares of the Target Company can be obtained at their book value.

		Target Company	
Total assets	$30,000,000	Debt (8%)	$15,000,000
		Preferred (6%)	5,000,000
		Common	10,000,000
			$30,000,000

(a) A group of investors has set up Holding Company *A* to acquire control of the Target Company. If the group holds all the equity of Holding Company *A*, how much money must they put up? If Target has operating earnings equal to 20 per cent of total assets and pays all earnings in dividends, what return on investment will the group earn?

(b) Suppose the group sets up Holding Company *B* to acquire control of Holding Company *A*. If the group holds all the equity of *B*, how much money must they put up? If *A* pays all earnings in dividends, what return on investment will the group earn? How many dollars of operating assets does the group control per dollar of their own investment?

(c) How would your answers change if Target had operating earnings equal to 8 per cent of total assets?

7. Reconsider Matson and Navigation (problem 5).

(a) Develop and graph the boundary conditions for the merger negotiations.

(b) If the P/E of the combined firm were estimated to be twelve times, what would be the possible range of exchange ratios?

(c) Rework part (b) with a P/E of 10.2.

8. Collect data on situations where one company made a tender offer for the shares of another, and management hurriedly conducted negotiations with a third company. Compare the terms of the two offers, especially with regard to the price of the offer. Did the offer endorsed by management always bear the highest price? Should this not be the case under the theory? Examine the other terms of the offers, including employment contracts, options, bonuses, and retirement provisions. Correlate management's endorsement with:

(a) Those offers having the highest price, and then

(b) Those having the most favorable employment terms.

Which correlation is higher? Why? Finally, determine which offer was accepted by the stockholders. Correlate this with:

(a) Those having the highest price, and

(b) Those endorsed by management.

Which correlation is higher? Why?

SELECTED
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Business Failure and Reorganization

24

Our analysis throughout most of this book has assumed that the firm is a going concern; nevertheless, we must not lose sight of the fact that some firms do fail. Recognition of failure is important both from the standpoint of internal management, and from the standpoint of a creditor with amounts owing from a company in distress. The word "failure" is vague, partly because there are varying degrees of failure. For example, a company is regarded as technically insolvent if it is unable to meet its current obligations. However, such insolvency may be only temporary and subject to remedy.[1] Technical insolvency, then, denotes only a lack of liquidity. Insolvency in bankruptcy, on the other hand, means that the liabilities of a company exceed its assets; in other words, the net worth of the company is negative. Financial failure includes the entire range of possibilities between these extremes.

[1] See James E. Walter, "Determination of Technical Insolvency," *Journal of Business*, XXX (January, 1957), 30–43.

The remedies available to save a failing company vary in harshness according to the degree of financial difficulty. If the outlook is sufficiently hopeless, liquidation may be the only feasible alternative. However, many failing firms can be rehabilitated to the gain of creditors, stockholders, and society. Although the major purpose of a liquidation or rehabilitation is to protect creditors, the interests of the owners also are considered. (In the thirties, they were all but neglected.) Still, legal procedures favor creditors. Otherwise, they would hesitate to extend credit, and the allocation of funds in the economy would be less than efficient.

SIGNS OF FAILURE

Although the causes of financial difficulty are numerous, many failures are attributable either directly or indirectly to management. Usually, nonfinancial problems lead to losses which, in turn, lead to financial strain and eventual failure. Very seldom is one bad decision the cause of the difficulty; usually the cause is a series of errors, and the difficulty evolves gradually. Because with most companies the signs of potential distress are evident prior to actual failure, a creditor may be able to take corrective actions before failure finally occurs.

In an extensive research study, William H. Beaver used financial ratios to predict failure.[2] The study encompassed a sample of seventy-nine relatively large firms which failed during the 1954–64 period.[3] For each of these companies, another firm was selected which did not fail, but which was in the same industry and was of approximately the same size as the firm that failed. The data collected for the nonfailed companies were for the same years as those for the failed firms. These samples were used to test the predictive ability of thirty financial ratios. The mean values of the ratios for the two samples were compared over the five-year period prior to failure. An example of such a comparison, using the cash-flow/total-debt ratio, is shown in Figure 24-1. We see that the mean ratio for the failed firms differs significantly from that for the nonfailed firms. Not only is it lower, but it deteriorates markedly as failure approaches.

In addition to a comparison of mean values, Beaver tested the samples using a form of discriminant analysis and then went on to analyze the evidence using likelihood ratios. While not all of the financial ratios examined predicted failure equally well, many showed excellent predictive power. In a companion article, Beaver investigated the ability to predict failure from the changes in market prices of stocks.[4] He found that the

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[2] "Financial Ratios as Predictors of Failure," *Empirical Research in Accounting: Selected Studies,* supplement to *Journal of Accounting Research* (1966), 71–111.

[3] Failure was defined as the inability of a firm to meet its financial obligations.

[4] William H. Beaver, "Market Prices, Financial Ratios, and the Prediction of Failure," *Journal of Accounting Research,* 6 (Autumn, 1968), 179–92.
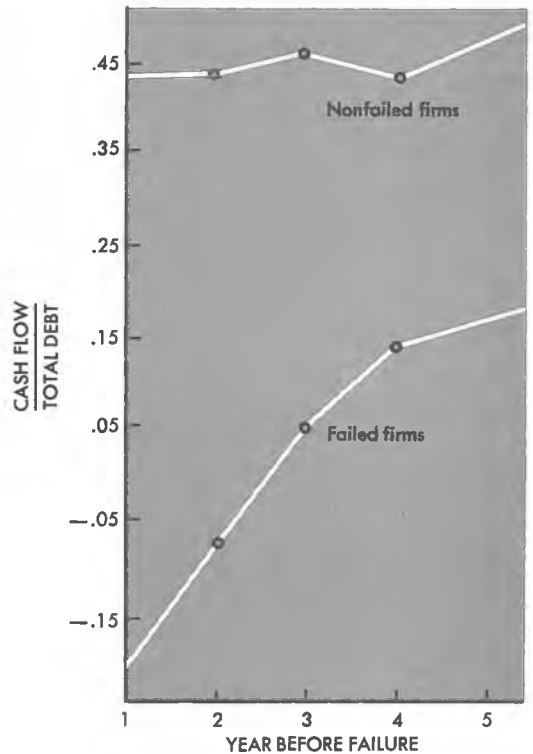
FIGURE 24-1
Comparison of mean values for failed and nonfailed firms

median market price of the failed companies declined at an increasing rate as failure approached, relative to that for the nonfailed companies. The largest price decline occurred in the final year. Beaver concluded that investors adjust stock prices to the deteriorating condition of failing companies. Moreover, he found the evidence to be consistent with investors assessing the likelihood for failure on the basis of financial ratios.

In a similar type of study, Edward I. Altman employed multiple discriminant analysis to predict bankruptcy, using various financial ratios.[5] This statistical technique is described in the appendix to Chapter 17. Altman worked with a sample of thirty-three corporations that filed for bankruptcy during the period 1946–65. Like Beaver, he collected a paired sample of thirty-three nonbankrupt firms on a stratified random basis. Starting with twenty-two financial ratios, he selected the five that did the best combined job of predicting bankruptcy. These ratios were used to discriminate between bankrupt and nonbankrupt firms, using data from one to five years prior to bankruptcy. As expected, the predictive accuracy of the multiple discriminant model declined with the number of years prior to bankruptcy. However, the model was able to forecast failure quite well up to two years before bankruptcy. Altman also tested

[5]"Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance,* XXIII (September, 1968), 589–609.

the model with secondary samples of bankrupt and nonbankrupt firms. Using the parameter estimates obtained in the original sample, he found the model to have considerable predictive accuracy when used in conjunction with the secondary samples.

In his investigation, Altman, like Beaver, found that the financial ratios of bankrupt firms deteriorated as bankruptcy approached, the greatest deterioration occurring between the third and the second year. Altman concluded that through discriminant analysis, a creditor can predict potential bankruptcy successfully. On the basis of these studies, it would appear that signs of potential failure are evident well before actual failure occurs. For the creditor, the lag allows time to take corrective actions.

Despite difficulties caused by past mistakes, many companies can be preserved as going concerns and can make an economic contribution to society. Sometimes the rehabilitation is severe, in keeping with the degree of financial difficulty. Nevertheless, these measures may be necessary if the firm is to obtain a new lease on life. In this chapter, we consider the remedies available to a company in financial distress, beginning with remedies that are voluntary on the part of creditors and the company and then examining legal actions that can be taken in connection with a failing company. Our focus is different from that in Chapter 6, where we developed a decision rule for determining whether or not a firm should liquidate in whole or in part. Recall that the rule called for liquidating the firm when the expected return on its liquidating value was less than the required rate of return. Under such circumstances, the firm would not be expected to earn its economic keep; and liquidation would be in the best interest of all concerned. In this chapter, we take up the full spectrum of remedies available to a firm in financial distress.

VOLUNTARY SETTLEMENTS

EXTENSIONS

An extension involves nothing more than creditors extending the maturity of their obligations. In cases of temporary insolvency of a basically sound company, creditors may prefer to work the problem out with the company. By not forcing the issue with legal proceedings, creditors avoid considerable legal expense and the possible shrinkage of value in liquidation. Obviously, no one creditor is going to extend his obligation unless others do likewise. Consequently, the major creditors usually form a committee whose function is to negotiate with the company and to formulate a plan mutually satisfactory to all concerned.

We must point out, however, that no one creditor is obligated to go along with the plan. If there are dissenting creditors and they have small amounts owing, they may be paid off in order to avoid legal proceedings. The number of dissenters cannot be too large, for the remaining creditors must, in essence, assume their obligations. Obviously, the remaining

creditors do not want to be left "holding the bag." If an extension is worked out, the creditors can institute controls over the company to assure proper management and to increase the probability of speedy recovery. In addition, they may elect to take security if marketable assets are available. The ultimate threat on the part of creditors is to initiate bankruptcy proceedings against the company and to force it into liquidation. By making an extension, however, they show an inclination to cooperate with the company.

COMPOSITION

A composition involves a pro rata settlement of creditors' claims in cash or in cash and promissory notes. The creditors must agree to accept a partial settlement in discharge of their entire claim. For example, a debtor may propose a settlement of sixty cents on the dollar. If creditors feel that the settlement is more than they could obtain in liquidation after legal expenses, they will probably accept. Even if it is somewhat less, they may still accept, because no company likes to be responsible for forcing another into bankruptcy. The settlement is a "friendly" one in the sense that legal proceedings are avoided.

As in an extension, however, the settlement must be agreed to by all creditors. Dissenting creditors must be paid in full, or they can force the company into bankruptcy. These creditors can be a considerable nuisance and may all but preclude a voluntary settlement. Overall, voluntary settlements can be advantageous to creditors as well as to the debtors, for they avoid legal expenses and complications.

OTHER ASPECTS OF VOLUNTARY SETTLEMENT

Creditors may agree to a voluntary settlement only if the present management is relieved of its responsibility. A creditors' committee may be appointed by creditors to control the operations of the company until the claims can be settled. The company enters into an agreement with creditors, giving them control of the company. One problem with this arrangement is the possibility of stockholder suits against the creditors for mismanagement of the company. Consequently, creditors are reluctant to become too active in the management of a failing company.

In certain circumstances, creditors may feel that the company should not be preserved, because further financial deterioration seems inevitable. When liquidation is the only realistic solution, it can be accomplished either through a private settlement or through bankruptcy proceedings. An orderly private liquidation is likely to be more efficient and result in a significantly higher settlement. A private settlement can also be through a formal assignment of assets to an appointed trustee. The trustee liqui-

dates the assets and distributes the proceeds to creditors on a pro rata basis. Because the voluntary settlement must be agreed to by all creditors, it usually is restricted to companies with a limited number of creditors and securities outstanding that are not publicly held.

LEGAL PROCEDURES

Most legal procedures undertaken in connection with failing companies fall under the Bankruptcy Act of 1898, as amended by the Chandler Act of 1938. This act provides for both the liquidation of a company and for its reorganization.[6] In most cases, the courts take over the operation of the company and preserve the *status quo* until a decision is reached whether to liquidate the company or keep it alive through reorganization.

LIQUIDATION

If there is no hope for the successful operation of a company, liquidation is the only feasible alternative. The federal district court, then, declares the firm bankrupt and proceeds with a plan for orderly liquidation. Bankruptcy proceedings may be either voluntary or involuntary. With a voluntary bankruptcy, the company files a petition of bankruptcy with a federal district court. In an involuntary bankruptcy, three or more creditors with claims in excess of $500 initiate the action by filing a petition with the court. If the total number of creditors of a firm is less than twelve, any one creditor can file a petition. The federal court will declare the company an involuntary bankrupt if it violates one of the six acts of bankruptcy.[7]

Upon the declaration of bankruptcy, the court usually appoints a *referee* to take over the operation of the company temporarily and call a meeting of the creditors. At the meeting, claims of the creditors are proven, and the creditors are given the opportunity to appoint the *trustee in bankruptcy*. The trustee has the responsibility of liquidating the assets of the company and distributing liquidating dividends to the creditors. The conduct of the trustee in carrying out these responsibilities is under the supervision of the court.

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[6] Before the thirties, companies were reorganized under equity receiverships. This process is no longer in use.

[7] The first act involves the concealment or removal of the bankrupt's property with the intent of defrauding creditors. The second act is the transfer of cash or other assets to one creditor in preference to others. The third act is the insolvent debtor's giving any creditor a lien on his property. The fourth act involves a general assignment by the debtor for the benefit of creditors. The fifth act occurs if the debtor, while insolvent, appoints a receiver or trustee to take charge of his property. The sixth act is an admission in writing by the debtor that he is unable to pay his debt and that he is willing to be adjudged a bankrupt.

In the distribution of the proceeds of a liquidation, the priority of claims must be observed. The administrative costs involved in the bankruptcy, taxes, and certain other claims must be paid before creditors are entitled to receive settlement. Secured creditors are entitled to the proceeds realized from the liquidation of specific assets on which they have a lien. If any balance of the claim is not realized from the sale of the collateral, these creditors become general creditors. General creditors are paid liquidating dividends on a pro rata basis from the total liquidation of unencumbered assets. If all of these claims are paid in full, liquidating dividends then can be paid to subordinated debt holders, to preferred stockholders, and, finally, to common stockholders. It is unlikely, however, that common stockholders will receive any distribution from a liquidation.

When a trustee cannot be appointed quickly, the court appoints a *receiver* to manage the operation of the company and conserve its assets until a trustee can be selected. After that, the procedure is the same as before. Upon the payment of all liquidating dividends, the bankrupt is discharged, thereby being relieved of any further claim. The principal objective of bankruptcy proceedings is an orderly liquidation of assets and an equitable distribution to creditors on a formal basis. The disadvantage of these proceedings is that they are slower and usually more expensive than a private liquidation. Some court-appointed officials are inefficient, being more concerned with their remuneration than with the proceeds available to creditors. As a result, a liquidation in bankruptcy may be less efficient than a private liquidation, providing creditors with a lower settlement. However, when creditors cannot come together in a voluntary manner, bankruptcy proceedings are the only recourse.

REORGANIZATION

It may be in the best interests of all concerned to reorganize a company rather than liquidate it. A reorganization is an effort to keep a company alive by changing its capital structure. The rehabilitation involves the reduction of fixed charges by substituting equity and limited-income securities for fixed-income securities.

Procedure.   Most reorganizations of industrial and public utility companies occur under Chapter X of the Bankruptcy Act. Reorganization procedures are initiated in the same general manner as a liquidation in bankruptcy. The federal district court appoints a *trustee* to operate the debtor's business until a reorganization plan is put into effect.[8] If the debts of the company are in excess of $250,000, the court must appoint a "disinterested" trustee — that is, a party independent of the debtor. In addition

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[8]The company is declared a "debtor" in a reorganization as opposed to a "bankrupt" in a liquidation.

to managing the operations of the debtor temporarily, the trustee must compile all the essential information required by the court, creditors, and — if the securities are publicly held — the SEC. Included is information pertaining to the value of assets, the nature of the liabilities, and the operating potential of the debtor from the standpoint of profitability.

Most important, the trustee is charged with the responsibility of drawing up a plan of reorganization. This plan is proposed after a thorough review of the situation and discussions with creditors and stockholders. Committees may be formed by the various classes of creditors and stockholders to represent and protect the interests of each class. The plan then is submitted to the court for hearings and approval. If liabilities exceed $3 million, and the securities are publicly held, the plan must also be submitted to the SEC. The SEC acts only in an advisory capacity to the court; it prepares a report on the proposed plan and submits it to the court. The final decision is that of the court.

If the court feels the reorganization plan is "fair, equitable, and feasible," it will approve the plan. All parties must be treated fairly and equitably; moreover, the plan must be workable with respect to the earning power and financial structure of the reorganized company. The reorganized company cannot have too great an amount of fixed financial charges in relation to its expected earning power. Upon approval by the court, the plan is submitted to the creditor and stockholder groups for approval. In order to become effective, it must be accepted by a two-thirds majority of each class of debt holders and by a simple majority of each class of stockholders. Upon approval by the majority of a particular class of security holders, the plan is binding on dissenters in that class.

The reorganization procedure for railroads is similar, except that the Interstate Commerce Commission plays an active role in the reorganization. Railroad reorganizations occur under Section 77 of the Bankruptcy Act and the Mahaffe Act of 1948. The reorganization plan must be submitted to the ICC, which approves the trustee. The ICC holds hearings and then either approves the proposed reorganization plan or submits its own plan to the court. The concern of the ICC is that the reorganization plan be compatible with the public's interest. The court, however, must approve the plan on the basis of whether it is fair, equitable, and feasible.

Reorganization Plan. The difficult aspect of a reorganization is the recasting of the company's capital structure to reduce the amount of fixed charges. In formulating a reorganization plan, the trustee must carry out three steps. First, he must determine the total valuation of the reorganized company. This step, perhaps, is the most difficult and the most important. The technique favored by trustees is a capitalization of prospective earnings. For example, if future annual earnings of the reorganized company are expected to be $2 million, and the overall capitalization rate of similar

companies averages 10 per cent, a total valuation of $20 million would be set for the company. The valuation figure is subject to considerable variation owing to the difficulty of estimating prospective earnings and determining an appropriate capitalization rate. Thus, the valuation figure represents nothing more than a best estimate of potential value. Although the capitalization of prospective earnings is the generally accepted approach to valuing a company in reorganization, the valuation may be adjusted upward if the assets have substantial liquidating value. The common stockholders of the company, of course, would like to see as high a valuation figure as possible. If the valuation figure the trustee proposes is below the liquidating value of the company, common stockholders will argue for liquidation rather than reorganization.

Once a valuation figure has been determined, the next step is to formulate a new capital structure for the company to reduce fixed charges so that there will be an adequate coverage margin. To reduce these charges, the total debt of the firm is scaled down by being partly shifted to income bonds, preferred stock, and common stock. In addition to being scaled down, the terms of the debt may be changed. The maturity of the debt can be extended to reduce the amount of annual sinking-fund obligation. The trustee may feel that a more conservative ratio of debt to equity is in equity in relation to the prospective earnings of the company. If it appears that the reorganized company will need new financing in the future, the trustee may feel that a more conservative ratio of debt to equity is in order to provide for future financial flexibility.

Once a new capital structure is established, the last step involves the valuation of the old securities and their exchange for new securities. Under an *absolute priority rule,* which is required in reorganization under Chapter X, all senior claims on assets must be settled in full before a junior claim can be settled. For example, in the exchange process, a bondholder must receive the par value of his bond in another security before there can be any distribution to preferred stockholders. The total valuation figure arrived at in step one sets an upper limit on the amount of securities that can be issued. Suppose that the existing capital structure of a company undergoing reorganization is as follows.

Debentures	$ 9 million
Subordinated debentures	3 million
Preferred stock	6 million
Common stock equity (at book value)	10 million
	$28 million

If the total valuation of the reorganized company is to be $20 million, the trustee might establish the following capital structure in step two.

Debentures	$ 3 million
Income bonds	6 million
Preferred stock	3 million
Common stock	8 million
	$20 million

Having established the "appropriate" capital structure for the reorganized company, the trustee then must allocate the new securities. In this regard, he may propose that the debenture holders exchange their $9 million in debentures for $3 million in new debentures and $6 million in income bonds; that the subordinated debenture holders exchange their $3 million in securities for preferred stock; and that preferred stockholders exchange their securities for $6 million of common stock in the reorganized company. The common stockholders then would be entitled to $2 million in stock in the reorganized company, or 25 per cent of the total common stock of the reorganized company. Before, these stockholders held 100 per cent of the stock. It is easy to see why common stockholders would like to see as high a valuation figure as possible. To encourage high valuation, they may attempt to discount the troubles of the company as temporary and argue that the earning potential of the company is favorable.

Thus, each claim is settled in full before a junior claim is settled. The example above represents a relatively "mild" reorganization. In a "harsh" reorganization, debt instruments may be exchanged entirely for common stock in the reorganized company and the old common stock eliminated completely. Had the total valuation figure in the example been $12 million, the trustee might have proposed a new capital structure consisting of $3 million in preferred stock and $9 million in common stock. Only the straight and subordinated debenture holders would receive a settlement in this case. The preferred and the common stockholders of the old company would receive nothing.

These examples serve to show that the common stockholders of a company undergoing reorganization suffer under an absolute priority rule, whereby claims must be settled in the order of their legal priority. From their standpoint, they would much prefer to see claims settled on a *relative priority basis*. Under this rule, new securities are allocated on the basis of the relative market prices of the securities. The common stockholder could never obtain senior securities in a reorganization, but they would be entitled to some common stock if their present stock had value. Since the company is not actually being liquidated, common stockholders argue that a rule of relative priority is really the fairest. Unfortunately for them, the absolute priority rule has been upheld by the Supreme Court (*Los Angeles Lumber Products Company* case, 1939). Their only recourse is to question whether the reorganization plan is fair and equitable to all security holders and not whether the absolute priority of claims is valid.

CHAPTER XI PROCEDURES

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CHAP. 24
*Business
Failure and
Reorganization*

Chapter XI of the Bankruptcy Act permits a failing company to seek an *arrangement*. In essence, an arrangement is a "legal" extension or composition. Only the company itself can initiate an arrangement, by filing a voluntary petition with a federal district court, attesting to the fact that it is unable to pay unsecured creditors and proposing a plan of action. Once the petition is filed, creditors cannot push for collection while an arrangement is being worked out. The court appoints a referee to call a meeting of creditors and discuss the plan proposed by the debtor. In addition, the court may appoint a receiver or trustee if the situation so warrants. The plan proposed by the debtor is subject to amendments by the creditors. Once a plan is approved by the majority of creditors and is judged by the court to be fair, equitable, and feasible, however, it becomes binding on all. In this respect, the arrangement has an advantage over a voluntary extension or composition, wherein creditors do not necessarily have to accept the plan. A large creditor can easily prevent a voluntary settlement from working.

The arrangement applies only to unsecured creditors, however. The claims of secured creditors are left intact; the debtor must pay secured creditors according to the terms of the obligations. An arrangement usually is a cheaper and quicker form of settlement than other types of legal settlements. The method is well suited for the company whose creditors are mostly trade creditors and that has no publicly held fixed-income securities. If a company has publicly held securities and substantial changes in its capital structure are required, these changes usually will be effected under Chapter X, rather than under Chapter XI, of the Bankruptcy Act.

SUMMARY

Business failure encompasses a wide range of financial difficulty; it occurs whenever a company is unable to meet its current obligations. The remedies applied to a failing company vary in severity with the degree of financial difficulty. Voluntary settlements are informal and must be agreed to by all creditors and the company itself. The difficulty with a voluntary settlement is in obtaining agreement of all parties concerned. Included in voluntary settlements are extensions, compositions, a creditors' committee controlling the operations of the company, and a private liquidation.

Legal settlements are effected, for the most part, under Chapters X and XI of the Bankruptcy Act, as amended by the Chandler Act of 1938. Railroads are reorganized under Section 77 of the Bankruptcy Act. The least "harsh" of the legal procedures is an arrangement under Chapter XI. An arrangement is simply a formal extension or composition. Reorganizations and liquidations occur mostly under Chapter X. In a reorganiza-

tion, the capital structure of the company is changed so as to reduce the total amount of fixed charges. The reorganized plan has to be fair, equitable, and feasible, as determined by the court and approved by a two-thirds majority of each class of debt holders and a majority of each class of stockholders. If the company cannot be rehabilitated, it will be declared bankrupt by the court and liquidated by a trustee in bankruptcy. Creditors receive liquidating dividends according to the priority of their claims.

An extended discussion of bankruptcy and reorganization would concern mostly legal matters. If a company should become involved in financial distress, counsel that is experienced in bankruptcy proceedings should be engaged.

PROBLEMS

1.

Fall Corporation
Balance Sheet

Cash	$ 1,000,000	Note payable	$ 1,000,000
Accounts receivable	2,000,000	Accounts payable	4,000,000
Inventories	5,000,000	Accrued wages	3,000,000
Prepaid expenses	1,000,000	Accrued taxes	1,000,000
Total current assets	$ 9,000,000	Total current liabilities	$ 9,000,000
Fixed assets, net	8,000,000	Long-term debt	12,000,000
Goodwill	5,000,000	Equity	1,000,000
Total assets	$22,000,000		$22,000,000

(a) Do you feel it likely that the Fall Corporation either is now or will soon be technically insolvent? Why? What steps could management take to correct this situation?
(b) Answer (a) with respect to the bankruptcy concept of insolvency.
(c) Compare and contrast these two concepts of insolvency.
(d) Is it possible that attempts to alleviate one form of insolvency could aggravate the other? How?
(e) Is the balance sheet the best tool for determining technical or fundamental insolvency? Can you suggest better ones?

2. *Research Project*
Compile data on recent bankruptcy cases involving fairly large companies. Learn as much as you can about "stockholder protective committees"; discover how many of these committees there are for each bankruptcy case you study. Finally, compare the amount of money going for legal fees, court costs, etc., as opposed to the amount the creditors finally receive. Do you begin to feel that there might be "self-interest" as well as "friendly" motives behind the creditors' acceptance of a composition? Whom do stockholder protective committees really protect? Can you find any parallels between bankruptcy law and probate law?

3. The Greenwood Corporation is in bankruptcy. The trustee has estimated that the company can earn $1.5 million before interest and taxes (50 per cent) in the future. In the new capitalization, he feels that debentures should bear a cou-

pon of 6 per cent and have coverage of five times, income bonds (6 per cent) should have overall coverage of two times, preferred stock (6.25 per cent) should have after-tax coverage of three times, and common stock should be issued on a P/E basis of twelve times. Determine the capital structure which conforms to the trustee's criteria.

4. Assume that the Greenwood Corporation (see problem 3) originally had the following capital structure:

	Book Value	Market Value
Senior debentures	$10,000,000	$ 9,000,000
Subordinated debentures	15,000,000	12,000,000
Junior subordinated debentures	5,000,000	2,000,000
Preferred stock (par $100)	5,000,000	1,000,000
Common stock (1,000,000 shares, par value $10)	—10,000,000	1,000,000
	$25,000,000	$25,000,000

Determine which of the new securities each class of old securities holders would get under:

(a) The absolute priority rule
(b) The relative priority rule

5. The Vent Corporation has been liquidated under bankruptcy proceedings. The book and liquidation values are as follows:

	Book	Liquidation
Cash	700,000	700,000
Accounts receivable	2,000,000	1,600,000
Inventory	3,500,000	2,000,000
Office building	5,000,000	3,000,000
Plant	8,000,000	5,000,000
Equipment	7,000,000	3,000,000
Total	$26,200,000	$15,300,000

The liability and equity accounts at the time of liquidation were as follows:

Accounts payable	$ 2,000,000
Accrued federal taxes	500,000
Accrued local taxes	200,000
Notes payable	1,000,000
Accrued wages	500,000
Total current liabilities	4,200,000
Mortgage on office building	3,000,000
First mortgage on plant	3,000,000
Second mortgage on plant	2,000,000
Subordinated debentures	5,000,000
Total long-term debt	13,000,000
Preferred stock	5,000,000
Common stock	7,000,000
Retained earnings	(3,000,000)
Total	9,000,000
Total:	$26,200,000

Expenses of liquidation (lawyers' fees, court costs, etc.) came to 20 per cent of the proceeds. The debentures are subordinated only to the two first mortgage bonds. All of the accrued wages are less than three months old and less than $600 per employee. Determine the appropriate distribution of the proceeds of liquidation.

6. Research Project

Read the sections of the Internal Revenue Code which apply to the reorganization of insolvent corporations. (At the time of this writing, the appropriate portions would be Sections 371 and 372 of the 1954 Code, as amended.) What are the tax implications of a transfer of property from an insolvent corporation to another corporation? What conditions must be met? What are the tax implications of the exchange of stocks or securities of an insolvent corporation for those of the corporation to which the property is transferred? What restrictions apply? If you were the financial manager of a solvent corporation negotiating the purchase of assets of an insolvent corporation, how would the information required above be of use to you?

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THE TOOLS OF
FINANCIAL ANALYSIS
AND CONTROL

PART **VIII**

Financial
Ratio
Analysis

In order to make rational decisions in keeping with the objectives of the firm, the financial manager must have at his disposal certain analytical tools. The purpose of this chapter and of the next two is to examine the more important tools of financial analysis. Financial analysis is undertaken by outside suppliers of capital—creditors and investors—and also by the firm itself. The type of analysis varies according to the specific interests of the party involved. A trade creditor is interested primarily in the liquidity of a firm. His claim is short term, and the ability of a firm to pay this claim is best judged by means of a thorough analysis of its liquidity. The claim of a bondholder, on the other hand, is long term. Accordingly, he would be more interested in the cash-flow ability of the firm to service debt over the long run. The bondholder may evaluate this ability by analyzing the capital structure of the firm, the major sources and uses of funds, its profitability over time, and projections of future profitability.

An investor in a company's common stock is concerned principally with present and expected future earnings and the stability of these earnings about a trend. As a result, the investor might concentrate his analysis on the profitability of the firm. He would be concerned with its financial condition insofar as this condition affects the stability of future earnings. Finally, in order to bargain more effectively for outside funds, the management of a firm should be interested in all aspects of financial analysis that outside suppliers of capital use in evaluating the firm. In addition, management employs financial analysis for purposes of internal control. In particular, it is concerned with profitability on investment in the various assets of the company and in the efficiency of asset management. Thus, the type of financial analysis undertaken varies according to the particular interests of the analyst.

USE OF FINANCIAL RATIOS

To evaluate the financial condition and performance of a firm, the financial analyst needs certain yardsticks. The yardstick frequently used is a ratio, or index, relating two pieces of financial data to each other. Analysis and interpretation of various ratios should give an experienced and skilled analyst a better understanding of the financial condition and performance of the firm than he would obtain from analysis of the financial data alone.[1]

The analysis of financial ratios involves two types of comparison. First, the analyst can compare a present ratio with past and expected future ratios for the same company. For example, the current ratio (the ratio of current assets to current liabilities) for the present year-end could be compared with the current ratio for the previous year-end. When financial ratios are arrayed on a spread sheet over a period of years, the analyst can study the composition of change and determine whether there has been an improvement or deterioration in the financial condition and performance of the firm over time. Financial ratios also can be computed for projected, or pro forma, statements and compared with present and past ratios. In the comparisons over time, it is best to compare not only financial ratios but also the raw figures.

The second method of comparison involves comparing the ratios of one firm with those of similar firms or with industry averages at the same point in time. Such a comparison gives insight into the relative financial condition and performance of the firm. Financial ratios for various industries are published by Robert Morris Associates, Dun & Bradstreet, and various other credit agencies and trade associations.[2] The analyst

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[1] For an excellent discussion of the history of ratio analysis, see James O. Horrigan, "A Short History of Financial Ratio Analysis," *Accounting Review*, XLIII (April, 1968), 284–94.

[2] Robert Morris Associates, an association of bank credit and loan officers, publishes industry averages based upon financial statements supplied to banks by borrowers. Eleven

should avoid using "rules of thumb" indiscriminately for all industries. For example, the criterion that all companies should have at least a 2-to-1 current ratio is inappropriate. The analysis must be in relation to the type of business in which the firm is engaged and to the firm itself. Many sound companies have current ratios of less than 2 to 1. Only by comparing the financial ratios of one firm with those of similar firms can one make a realistic judgement.

Because reported figures and the ratios computed from these figures are numerical, there is a tendency to regard them as precise portrayals of a firm's true financial status. For some firms, the accounting data may closely approximate economic reality. On many occasions, however, it is necessary to go beyond the reported figures in order to analyze properly the financial condition and performance of the firm. Such accounting data as depreciation, reserve for bad debts, and other reserves at best are estimates and may not reflect economic depreciation, bad debts, and other losses.

Moreover, accounting data from different companies should be standardized as much as possible. It is important to compare apples with apples and oranges with oranges. Even with standardized figures, however, the analyst should use caution in interpreting the comparisons.

TYPES OF RATIOS

For our purposes, financial ratios can be divided into four types: liquidity, debt, profitability, and coverage ratios. The first two types are ratios computed from the balance sheet; the last two are ratios computed from the income statement and, sometimes, from both the income statement and the balance sheet. It is important to recognize from the outset that no one ratio gives us sufficient information by which to judge the financial condition and performance of the firm. Only when we analyze a group of ratios are we able to make reasonable judgments. In addition, it is very important to take into account any seasonal character in a business. Underlying trends may be assessed only through a comparison of raw figures and ratios at the same time of year. For example, we would not compare a December 31 balance sheet with a May 31 balance sheet but would compare December 31 with December 31.

Only the more important ratios are considered in this chapter. In order to illustrate these ratios, we use the balance sheet and income statement of Sunbeam Corporation at the end of the 1970 fiscal year. The balance sheet and income statement of Sunbeam, a manufacturer of household appliances, are shown in Tables 25-1 and 25-2.

ratios are computed annually for 156 lines of business. In addition, each line of business is broken down according to four size categories. Dun & Bradstreet calculates annually 14 important ratios for 125 lines of business.

TABLE 25-1
SUNBEAM CORPORATION
Consolidated balance sheet

Assets

Current Assets:	March 28, 1970	March 29, 1969
Cash and marketable securities	$ 17,768,864	$ 17,504,225
Accounts receivable (less allowance for doubtful accounts and discounts of $1,982,893)	67,827,911	74,070,530
Inventories, at lower of cost (first-in, first-out) or market—		
Finished products and parts	101,667,379	94,509,279
Work in process	14,597,411	15,596,921
Raw materials and supplies	16,631,568	13,366,260
Prepaid expenses	2,075,552	1,719,665
Accumulated income tax prepayments	3,520,331	2,916,527
Total current assets	224,089,016	219,683,407
Investments, at cost (Note 5)	6,537,558	—
Plant and Equipment, at cost (Note 3):		
Land	3,416,136	3,127,401
Buildings and improvements...................................	50,840,076	48,546,915
Machinery and equipment	105,432,399	102,175,151
	159,688,611	153,849,467
Less — Accumulated depreciation	85,682,926	79,120,509
Net plant and equipment	74,005,685	74,728,958
Other Assets:		
Goodwill and other intangible assets, at cost	19,885,371	19,885,371
Debenture discount and expense.............................	630,411	677,109
	$325,148,041	$314,974,845

Liabilities and Stockholders' Equity

Current Liabilities:	March 28, 1970	March 29, 1969
Bank loans and notes payable.................................	$ 44,850,767	$ 35,651,163
Accounts payable...	14,842,735	13,679,315
Payrolls, taxes and other accrued liabilities	19,093,845	16,428,428
Taxes on income	3,620,252	12,745,536
Total current liabilities	82,407,599	78,504,442
Long Term Debt (Note 6)	58,028,999	58,032,497
Obligations Under Foreign Pension Plan (Note 7)	1,688,090	1,529,099
Minority Interest in Foreign Companies.........................	3,361,225	3,084,374
Stockholders' Equity (Note 4):		
Common stock..	11,751,481	42,082,397
Capital in addition to par value of stock.....................	30,608,662	—
Retained earnings	139,992,296	131,977,379
	182,352,439	174,059,776
Less — Treasury stock, at cost	2,690,311	235,343
Total stockholders' equity..................................	179,662,128	173,824,433
	$325,148,041	$314,974,845

TABLE 25-2
**Consolidated statements of earnings
and retained earnings**

Earnings	52 Weeks Ended March 28, 1970	52 Weeks Ended March 29, 1969
Net sales	$399,275,812	$372,124,149
Cost of goods sold...	268,029,757	249,996,477
Selling, general and administrative expenses...............	79,571,055	72,084,615
Depreciation (substantially by the straight-line method)...	11,150,864	11,398,925
Interest expense..	8,527,448	6,976,412
Minority interest in earnings of foreign companies.........	568,486	611,282
	367,847,610	341,067,711
Earnings before income taxes and extraordinary charge...	31,428,202	31,056,438
Income taxes, less prepayments of $603,804 in 1969–70 and $694,805 in 1968–69	16,370,777	17,244,569
Earnings before extraordinary charge........................	15,057,425	13,811,869
Extraordinary charge from sale of electronics business (less income tax credit of $916,912)	894,491	—
Earnings for the period	$ 14,162,934	$ 13,811,869
Per share (based on average shares outstanding):		
Earnings before extraordinary charge.....................	$1.29	$1.20
Extraordinary charge...	.08	—
Earnings for the period	$1.21	$1.20

Retained Earnings		
Retained earnings at beginning of period...................	$131,977,379	$126,323,701
Retained earnings of companies acquired in poolings of interests, at beginning of period (Note 2)................	3,152,200	705,214
Earnings for the period	14,162,934	13,811,869
	149,292,513	140,840,784
Less — Cash dividends paid (per share — $.79 in 1969–70 and $.77 in 1968–69)	9,229,940	8,863,405
Excess of cost of treasury shares reissued in pooling of interests over paid-in capital applicable thereto	70,277	—
	9,300,217	8,863,405
Retained earnings at end of period...........................	$139,992,296	$131,977,379

All above per share figures are adjusted to reflect the three-for-two stock split on August 12, 1969.

CURRENT RATIO

Liquidity ratios are used to judge a firm's ability to meet short-term obligations. From them, much insight can be obtained into the present cash solvency of the firm and its ability to remain solvent in the event of adversities. One of the most general and most frequently used of these ratios is the *current ratio*.

$$\frac{\text{Current Assets}}{\text{Current Liabilities}}$$

NOTE 1—PRINCIPLES OF CONSOLIDATION AND FOREIGN OPERATIONS:

The consolidated financial statements include the accounts of Sunbeam Corporation and all majority-owned subsidiaries.

The Corporation's equity in the consolidated foreign subsidiaries at March 28, 1970, was $57,811,876. Sales of these companies for the year were $101,460,419 and earnings from foreign operations, after deducting earnings accruing to minority interests, were $4,635,079.

NOTE 2—ACQUISITIONS:

During the year, the Corporation issued 63,000 common shares for Hanson Scale Company, 126,300 common shares and 3,713 treasury common shares for Mile High Equipment Company and Ice-O-Matic, Inc., and 55,730 common shares for the Downey Steel Treating group of companies (all adjusted for the three-for-two stock split on August 12, 1969). A total of 52,112 additional common shares may be issued in connection with the acquisitions under certain conditions until January, 1972. These transactions were poolings of interests and the results of operations of the acquired companies are included in the consolidated financial statements for the entire fiscal year ended March 28, 1970. The financial statements for the preceding year have not been adjusted to include the operations of the acquired companies since such adjustment would not have a significant effect on the comparability of the statements.

NOTE 3—LONG TERM LEASES:

At March 28, 1970, the Corporation and its subsidiaries occupied manufacturing facilities and service stations under long term leases expiring in three to twenty-four years. Aggregate annual net basic rentals under these agreements exclusive, in some instances, of property taxes, maintenance, insurance, etc., and after deducting rentals under subleases, are $1,580,000, and total net basic rentals payable over the remaining full primary terms of the leases are approximately $19,430,000.

NOTE 4—STOCKHOLDERS' EQUITY:

In July, 1969, the state of incorporation of the Company was changed from Illinois to Delaware, and the authorized capital was increased from 20,000,000 common shares of no par value to 35,000,000 shares, consisting of 30,000,000 common shares of $1 par value and 5,000,000 preferred shares of $1 par value. At March 28, 1970, none of the preferred shares had been issued. Changes in common stock outstanding, capital in addition to par value of stock and treasury stock during the two years ended March 28, 1970, were as follows (see table below):

NOTE 5—INVESTMENTS:

Investments include approximately 20% of the outstanding common stock of Hurst Performance, Inc., acquired in May, 1969, as a long term investment at a cost of $6,208,125. Although the market value at May 11, 1970, was $2,590,000, it is believed that there has not been any permanent impairment of this investment.

NOTE 6—LONG TERM DEBT:

Long term debt at the end of the year consisted of the following (less current maturities of $756,719 and $502,658, respectively):

	March 28, 1970	March 29, 1969
5½% sinking fund debentures due 1992 with annual prepayment of $2,000,000 starting in 1973	$50,000,000	$50,000,000
Foreign bank note due in semi-annual installments to 1974	4,575,708	4,214,937
5⅛% sinking fund notes of domestic subsidiary due $275,000 annually	1,025,000	1,300,000
7½% mortgage note of foreign subsidiary due in installments to 1982	1,371,450	1,471,800
Other long term debt	1,056,841	1,045,760
Total long term debt	$58,028,999	$58,032,497

Interest is payable on the foreign bank note at 3% above the rediscount rate of the Deutsche Bundesbank. The interest rate on this loan became 10½% on March 9, 1970.

NOTE 7—RETIREMENT PLANS

Several retirement plans, some of which are contributory and others non-contributory, are maintained by the Corporation and certain of its subsidiaries for the benefit of employees who meet eligibility requirements. The total cost of these retirement plans for the year was $3,444,276, which includes as to certain plans, amortization of prior service cost over a period of 30 years. Pension costs are funded under all but one of the retirement plans. The liability for unfunded pension costs is accrued. The amounts funded and accrued exceed the actuarially computed value of benefits vested under all plans as of March 28, 1970.

	Common Stock Outstanding		Capital in Addition to Par Value of Stock	Treasury Stock	
	Shares	Amount		Shares	Amount
Balances at March 30, 1968.................	7,570,551	$41,406,188			
Stock options exercised	23,200	481,400			
Stock issued in pooling of interests	77,216	194,809			
Purchase of treasury stock				6,700	$ 235,343
Balances at March 29, 1969	7,670,967	42,082,397		6,700	235,343
Change from no par value to $1 par value, and three-for-two split of shares	3,835,484	(30,575,946)	$30,575,946	3,350	—
Stock issued in poolings of interests*	245,030	245,030	32,716	(3,713)	(86,295)
Purchase of treasury stock*				97,407	2,541,263
Balances at March 28, 1970...	11,751,481	$11,751,481	$30,608,662	103,744	$2,690,311

At March 29, 1969, 188,850 shares of common stock were available for granting of options under the Corporation's stock option plan, 186,150 shares were under option at an average price of $25.84 per share, and options on 68,100 shares were exercisable. During the year, options for 25,100 shares were granted and options for 9,150 shares were cancelled. At the end of the year, 172,900 shares were available for grant, 202,100 shares were under option at an average price of $25.86 per share, and options on 177,000 shares were exercisable.

Shares issued or purchased prior to August 12, 1969, have been adjusted for the three-for-two stock split on that date.

For Sunbeam, the ratio for the 1970 year-end is

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Analysis*

$$\frac{\$224,089,016}{\$82,407,599} = 2.72$$

The higher the ratio, supposedly, the greater the ability of the firm to pay its bills. However, the ratio must be regarded as somewhat crude because it does not take into account the liquidity of the individual components of the current assets. A firm having current assets composed principally of cash and current receivables is generally regarded as more liquid than a firm whose current assets consist primarily of inventories.[3] Consequently, we must turn to "finer" tools of analysis if we are to evaluate critically the liquidity of the firm.

ACID-TEST RATIO

A somewhat more accurate guide to liquidity is the *quick*, or *acid-test, ratio*.

$$\frac{\text{Current Assets Less Inventories}}{\text{Current Liabilities}}$$

For Sunbeam, this ratio is

$$\frac{\$224,089,016 - \$132,896,358}{\$82,407,599} = 1.11$$

This ratio is the same as the current ratio, except that it excludes inventories — presumably the least liquid portion of current assets — from the numerator. The ratio concentrates on cash, marketable securities, and receivables in relation to current obligations and, thus, provides a more penetrating measure of liquidity than does the current ratio.

LIQUIDITY OF RECEIVABLES

To the extent that there are suspected imbalances or problems in various components of the current assets, the financial analyst will want to examine these components separately in his assessment of liquidity. Receivables, for example, may be far from current. To regard all receivables as liquid, when in fact a sizable portion may be past due, overstates the liquidity of the firm being analyzed. Receivables are liquid assets only

[3] We have defined liquidity as the ability to realize value in money — the most liquid of assets. Liquidity has two dimensions: (1) the time required to convert the asset into money, and (2) the certainty of the realized price. To the extent that the price realized on receivables is as predictable as that realized on inventories, receivables would be a more liquid asset than inventories, owing to the shorter time required to convert the asset into money. If the price realized on receivables is more certain than that on inventories, receivables would be regarded as being even more liquid.

insofar as they can be collected in a reasonable amount of time. For our analysis of receivables, we have two basic ratios, the first of which is the *average collection period ratio*.

$$\frac{\text{Receivables} \times \text{Days in Year}}{\text{Annual Credit Sales}}$$

For Sunbeam, this ratio is

$$\frac{\$67,827,911 \times 365}{\$399,275,812} = 62 \text{ days}$$

The average collection period tells us the average number of days receivables are outstanding.

The second ratio is the *receivable turnover ratio*.

$$\frac{\text{Annual Credit Sales}}{\text{Receivables}}$$

For Sunbeam, this ratio is

$$\frac{\$399,275,812}{\$67,827,911} = 5.89$$

Actually, these two ratios are inverses of each other. The number of days in the year, 365, divided by the average collection period, 62 days, gives the receivable turnover ratio, 5.89. The number of days in the year divided by the turnover ratio gives the average collection period. Thus, either of these two ratios can be employed.

When credit sales figures for a period are not available, we must resort to the total sales figures. The receivable figure used in the calculation ordinarily represents year-end receivables. However, when sales are seasonal or have grown considerably over the year, using the year-end receivable balance may not be appropriate. With seasonality, an average of the monthly closing balances may be the most appropriate figure to use. With growth, the receivable balance at the end of the year will be deceptively high in relation to sales. In this case, an average of receivables at the beginning and at the end of the year might be appropriate, if the growth in sales was steady throughout the year.

The average collection period ratio or the receivable turnover ratio indicates the slowness of receivables. Either ratio must be analyzed in relation to the billing terms given on the sales. For example, if the average collection period is 45 days and the terms given are 2/10, net 30,[4] the comparison would indicate that a sizable proportion of the receivables is past due beyond the final due date of thirty days. On the other hand,

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[4]The notation means that the supplier gives a 2 per cent discount if the receivable invoice is paid within ten days and that payment is due within thirty days if the discount is not taken.

if the terms are 2/10, net 60, the typical receivable is being collected before the final due date. A comparison of the average collection period and terms given by a specific company with those of other companies in the industry gives us additional insight into the investment in receivables. Too low an average collection period may suggest an excessively restrictive credit policy. The receivables on the books may be of prime quality and yet sales may be curtailed unduly — and profits less than they might be — because of this policy. In this situation, credit standards for an acceptable account should be relaxed somewhat. On the other hand, too high an average collection period may indicate too liberal a credit policy. As a result, a large number of receivables may be past due, with some uncollectable. Here, too, profits may be less than those possible on account of bad-debt losses and the need to finance a large investment in receivables. In this case, credit standards should be raised.

Another means by which we can obtain insight into the liquidity of receivables is through an *aging of accounts*. With this method, we categorize the receivables at a moment in time according to the proportions billed in previous months. For example, we might have the following hypothetical aging of accounts receivable at December 31:

		Proportion of Receivables Billed			
December	November	October	September	August and Before	Total
67%	19%	7%	2%	5%	100%

If the billing terms are 2/10, net 30, this aging tells us that 67 per cent of the receivables at December 31 are current, 19 per cent are up to one month past due, 7 per cent are one to two months past due, and so on. Depending upon the conclusions drawn from our analysis of the aging, we may want to examine more closely the credit and collection policies of the company. In the example above, we might be prompted to investigate the individual receivables that were billed in August and before, in order to determine if any should be charged off. The receivables shown on the books are only as good as the likelihood that they will be collected.

An aging of accounts receivable gives us considerably more information than the calculation of the average collection period, because it pinpoints the trouble spots more specifically. Of particular value is a comparison of different agings over time. With this comparison, we obtain an accurate picture of the investment of a firm in receivables and changes in the basic composition of this investment over time. Comparison of agings for different firms is difficult because most published reports do not include such information.

From a creditor's point of view, it is sometimes desirable to obtain an

aging of accounts payable. This measure, combined with the less exact turnover of payables (annual purchases divided by payables), allows us to analyze payables in much the same manner as we do receivables. Also, one can compute the average age of a firm's accounts payable. The average age of payables is

$$\frac{\text{Accounts Payable} \times 365}{\text{Purchase of Raw Materials}}$$

where accounts payable is the average balance outstanding for the year; and the denominator is the purchase of raw material during the year. This information is valuable in evaluating the probability that a credit applicant will pay on time. If the average age of payables is 48 days, and the terms in the industry are net 30, we know that a portion of the applicant's payables are not being paid on time. A credit check of other suppliers will give insight into the severity of the problem.

LIQUIDITY OF INVENTORIES

We may compute the *inventory turnover ratio* as an indicator of the liquidity of inventory.

$$\frac{\text{Cost of Goods Sold}}{\text{Average Inventory}}$$

For Sunbeam, the ratio is

$$\frac{\$268,029,757}{\$128,184,409} = 2.09$$

The figure for cost of goods sold used in the numerator is for the period being studied — usually one year; the average inventory figure used in the denominator typically is an average of beginning and ending inventories for the period. As was true with receivables, however, it may be necessary to compute a more sophisticated average when there is a strong seasonal element. The inventory turnover ratio tells us the rapidity with which the inventory is turned over into receivables through sales. This ratio, like other ratios, must be judged in relation to past and expected future ratios of the firm and in relation to ratios of similar firms, the industry average, or both.

Generally, the higher the inventory turnover, the more efficient the inventory management of a firm. However, a relatively high inventory turnover ratio may be the result of too low a level of inventory and frequent stockouts. It might also be the result of too many small orders for inventory replacement. Either of these situations may be more costly to the firm than carrying a larger investment in inventory and having a lower turnover ratio. Again, caution is necessary in interpreting the ratio. When the inventory turnover ratio is relatively low, it indicates

slow-moving inventory or obsolescence of some of the stock. Obsolescence may necessitate substantial write-downs, which, in turn, would negate the treatment of inventory as a liquid asset. Because the turnover ratio is a somewhat crude measure, we would want to investigate any perceived inefficiency in inventory management. In this regard, it is helpful to compute the turnover of the major categories of inventory to see if there are imbalances, which may indicate excessive investment in specific components of the inventory. Once we have a hint of a problem, we must investigate it more specifically to determine its cause.

DEFENSIVE POSITION

Another measure of liquidity has been proposed to indicate the defensive position of the firm.[5] The measure is the interval of time the firm can operate on existing liquid assets without having to resort to cash flows from sales or from other sources. The *basic defensive interval* (*B.D.I.*) is

$$\frac{\text{Total Defensive Assets}}{\text{Projected Daily Operating Expenditures}}$$

For Sunbeam, this ratio is

$$\frac{\$85,596,775}{\$975,693} = 87.7 \text{ days}[6]$$

Defensive assets include cash, marketable securities, and receivables; the denominator of the equation consists of projected daily operating expenditures of the firm. It has been argued that this measure and other related measures give a more meaningful picture of liquidity than do the liquidity ratios considered so far. Essentially, the underlying information is the same as that provided by the other ratios described in this section. Only the ease of interpretation is in question.

DEBT RATIOS

There are several debt ratios that may be used in financial analysis. The *debt-to-net-worth* ratio is computed by simply dividing the total debt of the firm (including current liabilities) by its net worth.

[5] See George H. Sorter and George Benston, "Appraising the Defensive Position of a Firm: The Internal Measure," *Accounting Review,* XXXV (October, 1960), 633–40; and Sidney Davidson, George H. Sorter, and Hemu Kalle, "Measuring the Defensive Position of a Firm," *Financial Analysts Journal,* 20 (January–February, 1964), 23–29.

[6] Projected daily operating expenditures are found by dividing cost of goods sold plus selling, general, and administrative expenses by the number of days in the year—365.

$$\frac{\text{Total Debt}}{\text{Net Worth}}$$

For Sunbeam, the ratio is

$$\frac{\$145,485,863}{\$179,662,128} = 0.81$$

When intangible assets are significant, they frequently are deducted from net worth to obtain the tangible net worth of the firm. Depending upon the purpose for which the ratio is used, preferred stock sometimes is included as debt rather than as net worth. Preferred stock represents a prior claim from the standpoint of the investor in common stock; consequently, he might include preferred stock as debt when analyzing a firm. The ratio of debt to equity will vary according to the nature of the business and the volatility of cash flows. An electric utility, with very stable cash flows, usually will have a higher debt ratio than will a machine tool company, whose cash flows are far less stable. A comparison of the debt ratio for a given company with those of similar firms gives us a general indication of the credit-worthiness and financial risk of the firm. Much more is said about the analysis of financial risk in Chapters 7 and 8.

In addition to the ratio of total debt to equity, we may wish to compute the following ratio, which deals with only the long-term capitalization of the firm.

$$\frac{\text{Long-term Debt}}{\text{Total Capitalization}}$$

where total capitalization represents all long-term debt and net worth. For Sunbeam, the ratio is

$$\frac{\$58,028,999}{\$237,691,127} = 0.24$$

This measure tells us the relative importance of long-term debt in the capital structure. The ratios computed above have been based upon book value figures; it is sometimes useful to calculate these ratios using market values. The use of debt ratios is considered in Chapter 7, where we take up the problem of capital structure. Again, it is important to compare ratios for the same company over time and also to compare the ratios of one firm with those of similar companies.

PROFITABILITY RATIOS

Profitability ratios are of two types: those showing profitability in relation to sales, and those showing profitability in relation to investment. Together these ratios give us indication of the firm's efficiency of operation. We do not take up the calculation of earnings per share or the