Internet Appendix for "Ambiguous Information, Portfolio Inertia, and Excess Volatility"*

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This Internet Appendix serves as a companion to my paper "Ambiguous Information, Portfolio Inertia, and Excess Volatility." It provides some additional results and proofs not reported in the main text due to space constraints. I present the results and proofs in the order in which they appear in the main paper.

(i) I assume in the main text of the paper that there is no ambiguity about the marginal distribution of the asset. In Section I of this Internet Appendix I show with an example that there is still excess volatility if investors are ambiguous about the prior dividend variance and the signal noise variance.

(ii) Theorem 2 of the main text is true even if investors are risk neutral. I provide a formal proof of this case in Section II. of this Internet Appendix.

(iii) I provide a formal proof of Lemma 2 stated in the main text of the paper in Section III of this Internet Appendix.

(iv) In Section IV of the paper I discuss properties of the equilibrium stock price

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when the economy is populated with heterogenous investors. In Section III of this Internet Appendix I show formally that there is a representative investor when investors are heterogenous with respect to initial wealth, risk aversion, and labor income and I provide conditions under which there is still a discontinuity in the equilibrium signal-to-price map when investors also differ with respect to their aversion to ambiguity.

(v) In Section V of the paper I show with a simple example that there is no portfolio inertia and excess volatility when investors are standard expected utility maximizers with a unique prior over a range of signal precisions. I provide a formal proof of these results in Section V of this Internet Appendix.

I. Ambiguity about Marginals and Conditionals

The portfolio inertia and excess volatility results are robust to adding ambiguity about the prior dividend distribution. In Footnote 5 of Section I of the main text, I mention that the results can be generalized by allowing for ambiguous prior information without changing the main results of the paper. I now provide more details.

Suppose there is no labor income and investors are ambiguous about the marginal distribution of the dividend:

$$\tilde{d} \sim \mathrm{N}\left(\bar{d}, \sigma_d^2\right), \tag{IA.1}$$

where $\sigma_d^2 \in [\sigma_{da}^2, \sigma_{db}^2] \subset [0, \infty]$.

Investors are ambiguous about the conditional distribution of $\tilde{s}$ given $\tilde{d}$:

$$\tilde{s} = \tilde{d} + \tilde{\varepsilon}, \qquad \tilde{\varepsilon} \sim \mathrm{N}\left(0, \sigma^2\right), \tag{IA.2}$$

where $\sigma^2 \in [\sigma_a^2, \sigma_b^2] \subset [0, \infty]$.

Standard normal-normal updating for each $(\sigma_d^2, \sigma^2) \in [\sigma_{da}^2, \sigma_{db}^2] \times [\sigma_a^2, \sigma_b^2]$ leads to

$$\tilde{d} \mid \tilde{s} = s \sim \mathrm{N}_{(\sigma_d^2, \sigma^2)}\left(\bar{d} + \phi(\sigma_d^2, \sigma^2)\left(s - \bar{d}\right), \sigma_d^2(1 - \phi(\sigma_d^2, \sigma^2))\right), \qquad \phi = \frac{\sigma_d^2}{\sigma_d^2 + \sigma^2}. \tag{IA.3}$$

Suppose there is a representative investor with CARA utility who is averse to ambiguity as in the main text. The equilibrium signal-to-price map is plotted in Figure IA.1. In this case there are two discontinuities: one for bad news and one for good news. Intuitively, investors are worried about a low posterior mean for large (good or bad) news surprises and about high residual variance for signals that roughly confirm the unconditional mean of the dividend. The residual variance is maximized when both variances are high, whereas the posterior mean is minimized when the prior variance is large and the signal noise variance is low for very bad news and vice versa if the news is very good.

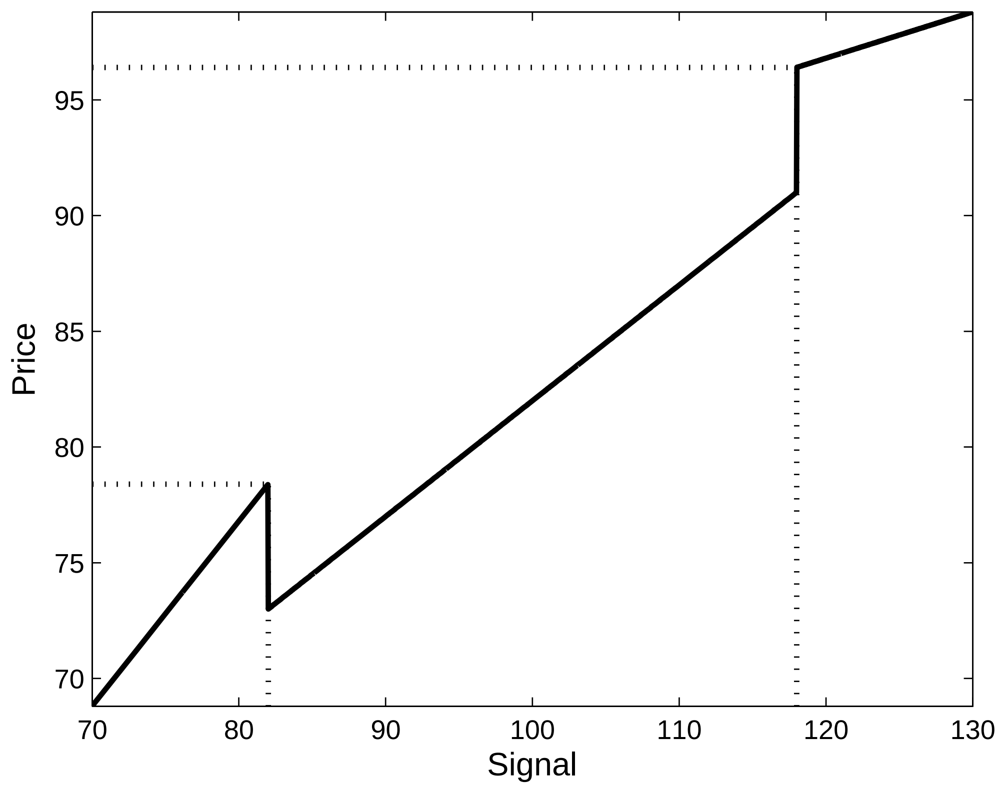

Figure IA.1. Ambiguity about marginals and conditionals. This graph shows the equilibrium signal-to-price map of the stock when investors are ambiguous about the prior dividend variance and the precision of the signal. The worst case scenario is a high (low) prior variance and a low (high) signal noise variance for very bad (good) news surprises and a high prior and high signal noise variance for moderate news surprises. The parameters are $\bar{d} = 100$, $[\sigma_{da}, \sigma_{db}] = [\sigma_a, \sigma_b] = [3, 6]$, and $\gamma = 1$.

II. Risk-Neutral and Ambiguity-Averse Investors

When investors are risk neutral, then the equilibrium signal-to-price map is given in Theorem 2 of the main text. I report utility of an investor and the equilibrium price in the next corollary before I provide a formal proof.

COROLLARY IA. 1: *If $\gamma = 0$, then the utility of an ambiguity-averse investor is*

$$U(\theta) = \min_{\phi \in [\phi_a, \phi_b]} \mathrm{E}_\phi \left[\tilde{w} \mid \tilde{s} = s\right] = \begin{cases} \mathrm{E}_{\phi_a} \left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta + z \leq 0 \quad \text{and} \quad s \leq \bar{d} \\ & \text{or} \quad \theta + z \geq 0 \quad \text{and} \quad s > \bar{d} \\ \mathrm{E}_{\phi_b} \left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta + z > 0 \quad \text{and} \quad s \leq \bar{d} \\ & \text{or} \quad \theta + z < 0 \quad \text{and} \quad s > \bar{d}, \end{cases} \tag{IA.4}$$

where

$$\begin{aligned} \mathrm{E}_\phi \left[\tilde{w} \mid \tilde{s} = s\right] &= w_0 + \left(\mu_\phi(s) - p\right)\theta + \bar{L} + z\left(\mu_\phi(s) - \bar{d}\right) \\ &= w_0 + \bar{L} + (p - \bar{d})z + \left(\mu_\phi(s) - p\right)(\theta + z). \end{aligned} \tag{IA.5}$$

There is a unique equilibrium stock price correspondence:

$$p(s) \in \begin{cases} \{\bar{d} + \phi_a(s - \bar{d})\} & \text{if} \quad s \geq \bar{d} \quad \text{and} \quad 1 + z > 0 \\ & \text{or} \quad s \leq \bar{d} \quad \text{and} \quad 1 + z < 0 \\ \{\bar{d} + \phi_b(s - \bar{d})\} & \text{if} \quad s < \bar{d} \quad \text{and} \quad 1 + z > 0 \\ & \text{or} \quad s > \bar{d} \quad \text{and} \quad 1 + z < 0 \\ \mathcal{P}_0(s) & \text{if} \qquad\qquad\qquad 1 + z = 0. \end{cases} \tag{IA.6}$$

Specifically, $p \in \mathcal{P}_0(s)$ if $\exists\, \phi \in [\phi_a, \phi_b]$ such that $p = \bar{d} + \phi(s - \bar{d})$.

Proof. Consider the three cases (i) $1 + z > 0$, (ii) $1 + z < 0$, and (iii) $1 + z = 0$.

(i) Consider the three subcases (a) $s > \bar{d}$, (b) $s < \bar{d}$, and (c) $s = \bar{d}$.

(i)(a) If $\theta + z > 0$, then the posterior mean of final wealth $\tilde{w}$ given in equation (IA.5) is minimized when $\phi = \phi_a$. If $\theta + z < 0$, then the posterior mean of final wealth is minimized when $\phi = \phi_b$. Finally, if $\theta + z = 0$, then the posterior mean of final wealth does not depend on ϕ. It follows that

$$U(\theta) = \min_{\phi \in [\phi_a, \phi_b]} \mathrm{E}_\phi \left[\tilde{w} \mid \tilde{s} = s\right] = \begin{cases} \mathrm{E}_{\phi_a} \left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta + z \geq 0 \\ \mathrm{E}_{\phi_b} \left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta + z < 0. \end{cases} \tag{IA.7}$$

This verifies equation (IA.4). We also need to check that $p = \mu_a(s) = \bar{d} + \phi_a(s - \bar{d})$ is an equilibrium. Plugging this price into utility given in equation (IA.7) leads to

$$U(\theta) = \begin{cases} w_0 + \bar{L} + (\mu_a(s) - \bar{d})z + (\mu_a(s) - \mu_a(s))\,(\theta + z) & \text{if} \quad \theta + z \geq 0 \\ w_0 + \bar{L} + (\mu_a(s) - \bar{d})z + (\mu_b(s) - \mu_a(s))\,(\theta + z) & \text{if} \quad \theta + z < 0, \end{cases} \tag{IA.8}$$

where $\mu_b(s) = \bar{d} + \phi_b(s - \bar{d})$. To verify that $p = \mu_a(s)$ is an equilibrium we need to show that $\theta = 1$ maximizes utility given in equation (IA.8). It follows from equations (IA.5) and (IA.8) that

$$U(\theta) - U(1) = (\mu_b(s) - \mu_a(s)) \min(\theta + z, 0) \leq 0 \qquad \forall\, \theta \in \mathcal{R}. \tag{IA.9}$$

Hence, $\theta = 1$ is an optimum.

(i)(b) If $\theta + z > 0$, then the posterior mean of final wealth $\tilde{w}$ given in equation (IA.5) is minimized when $\phi = \phi_b$. If $\theta + z < 0$, then the posterior mean of final wealth is minimized when $\phi = \phi_a$. Finally, if $\theta + z = 0$, then the posterior mean of final wealth does not depend on ϕ. It follows that

$$U(\theta) = \min_{\phi \in [\phi_a, \phi_b]} \mathrm{E}_\phi \left[\tilde{w} \mid \tilde{s} = s\right] = \begin{cases} \mathrm{E}_{\phi_b} \left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta + z \geq 0 \\ \mathrm{E}_{\phi_a} \left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta + z < 0. \end{cases} \tag{IA.10}$$

This verifies equation (IA.4). We also need to check that $p = \mu_b(s)$ is an equilibrium. Plugging this price into utility given in equation (IA.10) leads to

$$U(\theta) = \begin{cases} w_0 + \bar{L} + (\mu_b(s) - \bar{d})z + (\mu_b(s) - \mu_b(s))\,(\theta + z) & \text{if} \quad \theta + z \geq 0 \\ w_0 + \bar{L} + (\mu_b(s) - \bar{d})z + (\mu_a(s) - \mu_b(s))\,(\theta + z) & \text{if} \quad \theta + z < 0. \end{cases} \tag{IA.11}$$

To verify that $p = \mu_b(s)$ is an equilibrium we need to show that $\theta = 1$ maximizes utility given in equation (IA.11). It follows from equations (IA.5) and (IA.11) that

$$U(\theta) - U(1) = (\mu_a(s) - \mu_b(s)) \min(\theta + z, 0) \leq 0 \qquad \forall\, \theta \in \mathcal{R}. \tag{IA.12}$$

Hence, $\theta = 1$ is an optimum.

(i)(c) If $s = \bar{d}$, then there is no ambiguity. Hence,

$$U(\theta) = w_0 + \bar{L} + (p - \bar{d})z + \left(\bar{d} - p\right)(\theta + z) \tag{IA.13}$$

and $p(s) = \bar{d}$.

(ii) The proof of this case is similar to the previous one and thus is omitted.

(iii) Consider the three subcases (a) $s > \bar{d}$, (b) $s < \bar{d}$, and (c) $s = \bar{d}$.

(iii)(a) We have shown in case (i)(a) that utility is given in equation (IA.7). It remains to show that all

$$p(s) \in [\mu_a(s), \mu_b(s)] \tag{IA.14}$$

are equilibrium prices. Let $\theta > 1$. Then

$$U(\theta) = w_0 + \bar{L} + (p - \bar{d})z + (\mu_a(s) - p)(\theta + z) \tag{IA.15}$$

$$U(1) = w_0 + \bar{L} + (p - \bar{d})z + (\mu_a(s) - p)\underbrace{(1 + z)}_{=0} \tag{IA.16}$$

and we have $\forall\, p \in [\mu_a(s), \mu_b(s)]$ that

$$U(\theta) - U(1) = (\mu_a(s) - p)(\theta - 1) \leq 0. \tag{IA.17}$$

Similarly, let $\theta < 1$. Then

$$U(\theta) = w_0 + \bar{L} + (p - \bar{d})z + (\mu_b(s) - p)(\theta + z) \tag{IA.18}$$

$$U(1) = w_0 + \bar{L} + (p - \bar{d})z + (\mu_b(s) - p)\underbrace{(1 + z)}_{=0} \tag{IA.19}$$

and we have $\forall\, p \in [\mu_a(s), \mu_b(s)]$ that

$$U(\theta) - U(1) = (\mu_b(s) - p)(\theta - 1) \leq 0. \tag{IA.20}$$

Hence, $\theta = 1$ is an optimum.

(iii)(b) The proof of this case is similar to case (iii)(a) and thus is omitted.

(iii)(c) The proof of this case is similar to the proof of case (i)(c) and thus is omitted.

□

III. Risk Premium-to-Price Mapping

I now provide a formal proof of Lemma 2 stated in the main text of the paper.

Proof. Let $\widehat{\beta\lambda} = 2(\bar{d} - s) > 0$ and consider the three cases (i) $s < \bar{d} \Leftrightarrow \widehat{\beta\lambda} > 0$, (ii) $s > \bar{d} \Leftrightarrow \widehat{\beta\lambda} < 0$, and (iii) $s = \bar{d} \Leftrightarrow \widehat{\beta\lambda} = 0$.

(i) Consider the five subcases (a) $\beta\lambda > \widehat{\beta\lambda}$, (b) $\beta\lambda = \widehat{\beta\lambda}$, (c) $0 < \widehat{\beta\lambda} < \beta\lambda$, (d) $\beta\lambda = 0$, and (e) $\beta\lambda < 0$.

(i)(a) $\beta\lambda > \widehat{\beta\lambda}$ and thus

$$\hat{s}(\beta\lambda) = \bar{d} - \frac{1}{2}\beta\lambda = \bar{d} - \frac{1}{2}\widehat{\beta\lambda} + \frac{1}{2}\left(\widehat{\beta\lambda} - \beta\lambda\right) = s + \frac{1}{2}\left(\underbrace{\widehat{\beta\lambda} - \beta\lambda}_{<0}\right). \tag{IA.21}$$

Hence, $s > \hat{s}$ and $\beta\lambda > 0$, and it follows from Theorem 2 of the main text that

$$p(\beta\lambda) = p_a(s) = p_a(\beta\lambda) \qquad \forall\, \beta\lambda > \widehat{\beta\lambda}. \tag{IA.22}$$

(i)(b) $\beta\lambda = \widehat{\beta\lambda}$ and thus

$$\hat{s}(\beta\lambda) = \bar{d} - \frac{1}{2}\beta\lambda = \bar{d} - \frac{1}{2}\widehat{\beta\lambda} = s. \tag{IA.23}$$

Hence, $s = \hat{s}$ and $\beta\lambda > 0$, and it follows from Theorem 2 of the main text that

$$p\left(\widehat{\beta\lambda}\right) \in \mathcal{P}_{\hat{s}} = P_{\widehat{\beta\lambda}}. \tag{IA.24}$$

(i)(c) $0 < \beta\lambda < \widehat{\beta\lambda}$ and thus

$$\hat{s}(\beta\lambda) = \bar{d} - \frac{1}{2}\beta\lambda = \bar{d} - \frac{1}{2}\widehat{\beta\lambda} + \frac{1}{2}\left(\widehat{\beta\lambda} - \beta\lambda\right) = s + \frac{1}{2}\left(\underbrace{\widehat{\beta\lambda} - \beta\lambda}_{>0}\right). \tag{IA.25}$$

Hence, $s < \hat{s}$ and $\beta\lambda > 0$, and it follows from Theorem 2 of the main text that

$$p(\beta\lambda) = p_b(s) = p_b(\beta\lambda) \qquad \forall\, 0 < \beta\lambda < \widehat{\beta\lambda}. \tag{IA.26}$$

(i)(d) $\beta\lambda = 0 < \widehat{\beta\lambda}$ and thus $s < \hat{s}$ and $\beta\lambda = 0$. Hence, it follows from Theorem 2 of the main text that

$$p\,(0) \in \mathcal{P}_0(s) = P_0. \tag{IA.27}$$

(i)(e) $\beta\lambda < 0 < \widehat{\beta\lambda}$ and thus $s < \hat{s}$ and $\beta\lambda < 0$. Hence, it follows from Theorem 2 of the main text that

$$p(\beta\lambda) = p_a(s) = p_a(\beta\lambda) \qquad \forall\, \beta\lambda < 0. \tag{IA.28}$$

(ii) The proof of this case is similar to case (i) and thus is omitted.

(iii) In this case $p\left(\bar{d}\right) = p_a\left(\bar{d}\right) = p_a\left(\beta\lambda\right)\ \forall \beta\lambda \in \mathcal{R}$.

□

IV. Aggregation

Consider the one-period economy described in Section I of the main text. Suppose there is a continuum of investors with unit mass who all receive the same ambiguous signal but may differ with respect to their initial wealth, their labor income, and their aversion to risk and ambiguity. Let w_{0h} denote investor h's initial wealth, $\gamma_h > 0$ her risk aversion coefficient, $[\phi_{ah}, \phi_{bh}]$ the interval that describes her aversion to ambiguous information, and $\tilde{L}_h$ her labor income. Individual labor income consists of the systematic component $\zeta_h \tilde{L}$ and an idiosyncratic component $\tilde{\varepsilon}_h^L$. Specifically,

$$\tilde{L}_h = \zeta_h \tilde{L} + \tilde{\varepsilon}_h^L \qquad \text{where} \qquad \int_0^1 \zeta_h \, dh = 1, \tag{IA.29}$$

and where the $\tilde{\varepsilon}_h^L$ are i.i.d. zero mean normals that are independent of aggregate labor income $\tilde{L}$, the dividend $\tilde{d}$, and the ambiguous signal $\tilde{s}$.

Assume that the strong law of large numbers holds in the sense that

$$\int_0^1 \tilde{\varepsilon}_h^L \, dh = 0 \qquad \text{and thus} \qquad \int_0^1 \tilde{L}_h \, dh = \tilde{L}. \tag{IA.30}$$

An equilibrium in this economy is defined as follows:

DEFINITION IA. 1 (Equilibrium): *The signal to-price-map $p(s)$ is an equilibrium $\forall s \in \mathcal{R}$ if and only if (i) each investor chooses a portfolio θ_h to maximize*

$$\min_{\phi_h \in [\phi_{ah}, \phi_{bh}]} \mathrm{E}_{\phi_h} \left[u_h \left(w_{0h} + \left(\tilde{d} - p(s) \right) \theta_h + \tilde{L}_h \right) \mid \tilde{s} = s \right], \qquad \forall s \in \mathcal{R} \tag{IA.31}$$

and (ii) markets clear, that is $\int_0^1 \theta_h \, dh = 1$ and investors consume $\tilde{d} + \tilde{L}$ at date 1.

A. *Homogenous Ambiguity*

There exists a representative investor if all investors are standard expected utility maximizers.[1] In the next proposition I show that this is still true when all investors have the same aversion to ambiguity.[2]

PROPOSITION IA. 1 (Representative Investor): *Assume that all investors have the same aversion to ambiguous information described by $[\phi_a, \phi_b]$. Then there exists a*

[1] See Chapter 7 in Back (2010).

[2] Wakai (2007) shows that there exists a representative investor when investors have homogenous ambiguity but differ with respect to their CARA coefficient.

representative investor with aversion to ambiguous information described by $[\phi_a, \phi_b]$*, initial wealth* $w_0 = \int_0^1 w_{0h}\, dh$*, risk tolerance* $1/\gamma = \int_0^1 \frac{1}{\gamma_h}\, dh$*, and aggregate labor income* $\tilde{L} = \int_0^1 \tilde{L}_h\, dh$*.*

Proof. We can rewrite investor h's individual labor income:

$$\begin{aligned} \tilde{L}_h &= \zeta_h \left(\bar{L} + z(\tilde{d} - \bar{d}) + \tilde{\varepsilon}_L \right) + \tilde{\varepsilon}_h^L \\ &= \zeta_h \bar{L} + \zeta_h z(\tilde{d} - \bar{d}) + \zeta_h \tilde{\varepsilon}_L + \tilde{\varepsilon}_h^L. \end{aligned} \tag{IA.32}$$

Hence, it follows from Theorem 1 of the main text that the optimal demand of investor h is

$$\theta_h(p) = \begin{cases} \dfrac{\mu_a(s) - p}{\gamma_h v_a} - \zeta_h z & p \leq p_1 \\ \frac{2}{\gamma_h \sigma_d^2}\, \max\left(\bar{d} - s, 0\right) - \zeta_h z & p_1 < p \leq p_2 \\ \dfrac{\mu_b(s) - p}{\gamma_h v_b} - \zeta_h z & p_2 < p \leq p_3 \\ \frac{2}{\gamma_h \sigma_d^2}\, \min\left(\bar{d} - s, 0\right) - \zeta_h z & p_3 < p \leq p_4 \\ \dfrac{\mu_a(s) - p}{\gamma_h v_a} - \zeta_h z & p > p_4, \end{cases} \tag{IA.33}$$

where

$$p_1 = \mu_a(s) - \frac{2}{\sigma_d^2} v_a \max\left(\bar{d} - s, 0\right) \tag{IA.34}$$

$$p_2 = \mu_b(s) - \frac{2}{\sigma_d^2} v_b \max\left(\bar{d} - s, 0\right) \tag{IA.35}$$

$$p_3 = \mu_a(s) - \frac{2}{\sigma_d^2} v_a \min\left(\bar{d} - s, 0\right) \tag{IA.36}$$

$$p_4 = \mu_b(s) - \frac{2}{\sigma_d^2} v_b \min\left(\bar{d} - s, 0\right). \tag{IA.37}$$

The critical prices p_1, …, p_4 are the same for all investors. Hence, integrating over individual demands leads to aggregate demand:

$$\theta(p) = \int_0^1 \theta_h(p) dh = \begin{cases} \dfrac{\mu_a(s) - p}{v_a} \int_0^1 \frac{1}{\gamma_h}\, dh - z \int_0^1 \zeta_h \, dh & p \leq p_1 \\ \frac{2}{\sigma_d^2} \max\left(\bar{d} - s, 0\right) \int_0^1 \frac{1}{\gamma_h}\, dh - z \int_0^1 \zeta_h \, dh & p_1 < p \leq p_2 \\ \dfrac{\mu_b(s) - p}{v_b} \int_0^1 \frac{1}{\gamma_h}\, dh - z \int_0^1 \zeta_h \, dh & p_2 < p \leq p_3 \\ \frac{2}{\sigma_d^2} \min\left(\bar{d} - s, 0\right) \int_0^1 \frac{1}{\gamma_h}\, dh - z \int_0^1 \zeta_h \, dh & p_3 < p \leq p_4 \\ \dfrac{\mu_a(s) - p}{v_a} \int_0^1 \frac{1}{\gamma_h}\, dh - z \int_0^1 \zeta_h \, dh & p > p_4. \end{cases} \tag{IA.38}$$

We have that $\int_0^1 \zeta_h \, dh = 1$ and $1/\gamma = \int_0^1 \frac{1}{\gamma_h}\, dh$ and therefore $\theta(p)$ is the demand function of a representative investor with initial wealth w_0, risk aversion γ, ambiguity $[\phi_a, \phi_b]$, and labor income $\tilde{L}$. $\square$

Intuitively, the range of prices over which the demand function given in equation (IA.33) changes its slope does not depend on investor h's risk aversion γ_h or hedging demand for labor income risk $\zeta_h z$ and hence individual demands can be added up as in the standard expected utility case.

B. Heterogenous Ambiguity

Suppose there is also heterogeneity in aversion to ambiguity and define $[\phi_a, \phi_b] \equiv \bigcap_{h=0}^1 [\phi_{ah}, \phi_{bh}]$. In the next proposition I show that there is still a discontinuity in the equilibrium signal-to-price map when investors are heterogeneous in their aversion to ambiguity and $\phi_a < \phi_b$.

Proposition IA. 2 (Aggregation): *There is a discontinuity in the equilibrium signal-to-price map if $\phi_a < \phi_b$ and $z \neq 1$. The interval of equilibrium prices is* *given by $\mathcal{P}_{\hat{s}}$. Specifically, $p \in \mathcal{P}_{\hat{s}}$, if $\exists\, \phi \in [\phi_a, \phi_b]$ such that*

$$p = \mathrm{E}_\phi \left[\tilde{d} \mid \tilde{s} = \hat{s}\right] - \gamma(1+z)\mathrm{Var}_\phi \left[\tilde{d} \mid \tilde{s} = \hat{s}\right], \tag{IA.39}$$

where $\hat{s} = \bar{d} - \gamma(1+z)\sigma_d^2/2$ and $1/\gamma = \int_0^1 (1/\gamma_h)\, dh$.

Proof. The individual demands given in equation (IA.33) are continuous and non-increasing in p with $\lim\limits_{p\to-\infty} \theta_h(p) = \infty$ and $\lim\limits_{p\to\infty} \theta_h(p) = -\infty$. Hence, aggregate demand $\theta(p) = \int_0^1 \theta_h(p)\, dh$ is continuous and non-increasing in p with $\lim\limits_{p\to-\infty} \theta(p) = \infty$ and $\lim\limits_{p\to\infty} \theta(p) = -\infty$. Thus, there exists an equilibrium because the market clearing condition $\theta(p) - 1 = 0$ always has a solution.

Let $[\phi_a, \phi_b] = \bigcap_{h=0}^1 [\phi_{ah}, \phi_{bh}]$, $1/\gamma = \int_0^1 (1/\gamma_h)\; dh$, and $\hat{s} = \bar{d} - \gamma(1+z)\sigma_d^2/2$. Consider the three cases (i) $1+z > 0$, (ii) $1+z < 0$, and (iii) $1+z = 0$.

(i) Let

$$p_1 \equiv \mu_a(\hat{s}) - \max\left(2\frac{\bar{d}-\hat{s}}{\gamma\sigma_d^2}, 0\right)\gamma v_a = \mu_a(\hat{s}) + \frac{2}{\sigma_d^2} v_a \left(\hat{s} - \bar{d}\right) \tag{IA.40}$$

$$p_2 \equiv \mu_b(\hat{s}) - \max\left(2\frac{\bar{d}-\hat{s}}{\gamma\sigma_d^2}, 0\right)\gamma v_b = \mu_b(\hat{s}) + \frac{2}{\sigma_d^2} v_b \left(\hat{s} - \bar{d}\right). \tag{IA.41}$$

If $\phi_b > \phi_a$ and $z > -1$, then $p_2 > p_1$. It follows from equation (IA.33) that investor h's optimal demand for the price range $p_1 \leq p \leq p_2$ is constant:

$$\theta_h(p) = \max\left(2\frac{\bar{d}-\hat{s}}{\gamma_h\sigma_d^2}, 0\right) - \zeta_h z = \frac{2}{\gamma_h\sigma_d^2}\left(\bar{d} - \hat{s}\right) - \zeta_h z \qquad \forall p_1 \leq p \leq p_2 \tag{IA.42}$$

because $[\phi_a, \phi_b] \subseteq [\phi_{ah}, \phi_{bh}]\; \forall\, h$.

Integrating over all investors leads to

$$\begin{aligned}\theta(p) &= \int_0^1 \theta_h(p)\, dh = \frac{2}{\sigma_d^2}\,\left(\bar{d}-\hat{s}\right)\int_0^1 \frac{1}{\gamma_h}\, dh - z\int_0^1 \zeta_h\, dh \\ &= \frac{2}{\sigma_d^2}\,\left(\bar{d}-\hat{s}\right)\frac{1}{\gamma} - z \qquad \forall\, p_1 \leq p \leq p_2.\end{aligned}$$

Plugging in for $\hat{s}$ leads to $\theta(p)$=1 and hence every $p \in [p_1, p_2]$ is an equilibrium. Moreover, it is straightforward to show that $p \in [p_1, p_2]$, if $\exists\ \phi \in [\phi_a, \phi_b]$ such that

$$p = \mathrm{E}_\phi\left[\tilde{d} \mid \tilde{s} = \hat{s}\right] - \gamma(1+z)\mathrm{Var}_\phi\left[\tilde{d} \mid \tilde{s} = \hat{s}\right].$$

(ii) The proof for this case is similar to case (i) and thus is omitted.

(iii) We know from Proposition 5 of the main text that there is no ambiguity if $1 + z = 0$.

□

V. Bayesian Model Uncertainty

I show analytically that optimal demand is a strictly decreasing and smooth function of the stock price and hence there is no portfolio inertia if investors are standard expected utility maximizers with a unique prior over the range of signal noise precisions $[\phi_a, \phi_b]$. I also determine the equilibrium stock price when there is a representative investor who maximizes standard expected utility and who has a unique prior over a range of signal precisions. I show analytically that the price is a smooth function of the signal and hence there is no excess volatility.

A. *Model*

The marginal distribution of the dividend $\tilde{d}$ and labor income $\tilde{L}$ is normal:[3]

$$\begin{pmatrix} \tilde{d} \\ \tilde{L} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & \rho\sigma_d\sigma_L \\ \rho\sigma_d\sigma_L & \sigma_L^2 \end{pmatrix} \right). \tag{IA.43}$$

The investor receives a signal about the liquidating dividend:

$$\tilde{s} = \tilde{d} + \tilde{\varepsilon}, \qquad \tilde{\varepsilon} \mid \tilde{\sigma}^2 = \sigma^2 \sim \mathrm{N}\left(0, \sigma^2\right). \tag{IA.44}$$

The signal noise variance has distribution function $G_{\tilde{\sigma}}(\cdot)$ and continuous support $[\sigma_a^2, \sigma_b^2] \subset [0, \infty]$. The signal is conditionally independent of labor income.

Standard Bayesian updating leads to

$$\begin{pmatrix} \tilde{d} \\ \tilde{L} \end{pmatrix} \mid \tilde{s} = s, \tilde{\sigma}^2 = \sigma^2 \sim \mathrm{N}_\phi \left(\begin{pmatrix} \mu_\phi(s) \\ z\mu_\phi(s) \end{pmatrix}, \begin{pmatrix} v_\phi & zv_\phi \\ zv_\phi & z^2 v_\phi + \sigma_L^2(1-\rho^2) \end{pmatrix} \right), \tag{IA.45}$$

where $z = \rho\frac{\sigma_L}{\sigma_d}$, $\phi = \sigma_d^2/(\sigma_d^2 + \sigma^2)$, and

$$\mu_\phi(s) \equiv \mathrm{E}\left[\tilde{d} \mid \tilde{s} = s, \tilde{\phi} = \phi\right] = \phi s \tag{IA.46}$$

$$v_\phi \equiv \mathrm{Var}\left[\tilde{d} \mid \tilde{s} = s, \tilde{\phi} = \phi\right] = \sigma_d^2(1-\phi). \tag{IA.47}$$

It is more convenient to describe the informativeness of the signal with $\tilde{\phi}$ and hence let $G_{\tilde{\phi}}(\cdot)$ denote the distribution function of $\tilde{\phi}$, which has continuous support $[\phi_a, \phi_b] \subset [0, 1]$ with $\phi_a = \sigma_d^2/(\sigma_d^2 + \sigma_b^2)$ and $\phi_b = \sigma_d^2/(\sigma_d^2 + \sigma_a^2)$.

After observing the signal investors can make inferences about the random signal noise precisions. Let $F(\phi; s)$ denote the distribution function of $\tilde{\phi}$ conditional on

[3]For ease of notation and w.l.o.g. I set $\bar{d} = 0$ and $\bar{L} = 0$.

$\tilde{s} = s$. The utility of a Savage investor who holds θ shares of the risky asset is therefore

$$\mathrm{E}\left[u\left(w_0 + \left(\tilde{d} - p\right)\theta + \tilde{L}\right) \mid \tilde{s} = s\right] = \mathrm{E}_F\left[u\left(\mathrm{CE}\left(\theta, \tilde{\phi}\right)\right) \mid \tilde{s} = s\right], \tag{IA.48}$$

where E_F denotes the expectation taken with respect to $F(\phi; s)$ and

$$\mathrm{CE}(\theta, \phi) = w_0 + pz + \left(\mu_\phi(s) - p\right)(\theta + z) - \frac{1}{2}\gamma\,\left(v_\phi(\theta + z)^2 + \sigma_L^2(1 - \rho^2)\right). \tag{IA.49}$$

Let $\theta(p)$ denote the demand function that maximizes expected utility given in equation (IA.48). The properties of $\theta(p)$ are summarized in the next proposition.

Proposition IA. 3: *For every distribution $F(\cdot; s)$ with support $[\phi_a, \phi_b] \subset [0, 1]$ such that expected utility given in equation (IA.48) exists, we have that optimal demand $\theta(p)$ is unique, continuously differentiable, and strictly decreasing in the price p. Moreover, $\theta(p)$ is implicitly given by*

$$\theta(p) = \frac{\mathrm{E}_{Q(\phi;\theta(p))}\left[\mu_{\tilde{\phi}}(s) \mid \tilde{s} = s\right] - p}{\gamma\,\mathrm{E}_{Q(\phi;\theta(p))}\left[v_{\tilde{\phi}} \mid \tilde{s} = s\right]} - z, \tag{IA.50}$$

where $Q(\phi; \theta)$ denotes the risk-neutral distribution of $\tilde{\phi}$ conditional on $\tilde{s} = s$:

$$dQ(\phi; \theta) = \frac{u'\left(CE^S(\theta, \phi)\right)\, dF(\phi; s)}{\mathrm{E}_F\left[u'\left(CE^S(\theta, \tilde{\phi})\right) \mid \tilde{s} = s\right]}. \tag{IA.51}$$

Proof. The utility of a Savage investor with belief $F(\tilde{\phi}, s)$ who holds θ shares of the risky asset is

$$
\begin{aligned}
U(\theta) &= \mathrm{E}\left[u\left(w_0 + \left(\tilde{d} - p\right)\theta + \tilde{L}\right) \mid \tilde{s} = s\right] \\
&= \int_{\phi_a}^{\phi_b} \mathrm{E}\left[u\left(w_0 + \left(\tilde{d} - p\right)\theta\right) + \tilde{L} \mid \tilde{s} = s, \tilde{\phi} = \phi\right] dF(\phi; s) \\
&= \int_{\phi_a}^{\phi_b} u\left(\mathrm{CE}(\theta, \phi)\right) dF(\phi; s) \\
&= \mathrm{E}_F\left[u\left(\mathrm{CE}(\theta, \tilde{\phi})\right) \mid \tilde{s} = s\right],
\end{aligned}
\tag{IA.52}
$$

with $\mathrm{CE}^S(\theta, \phi)$ given in equation (IA.49).

Let $x = \theta + z$. Then maximizing $U(x)$ is equivalent to maximizing $U(\theta)$. Hence, taking the first derivative of $U(x)$ w.r.t. x and setting it equal to zero leads to the FOC

$$
\int_{\phi_a}^{\phi_b} u'\left(\mathrm{CE}(x, \phi)\right)\left(\mu_\phi(s) - p - \gamma\, x\, v_\phi\right) dF(\phi; s) = 0. \tag{IA.53}
$$

Taking the second derivative of $U(x)$ w.r.t. x and using the fact that $u''(\cdot)/u'(\cdot) = -\gamma$ leads to

$$
\frac{\partial^2 U(x)}{\partial x^2} = -\gamma \int_{\phi_a}^{\phi_b} u'\left(\mathrm{CE}(x, \phi)\right)\left(\left(\mu_\phi(s) - p - \gamma\, v_\phi x\right)^2 + v_\phi\right)\, dF(\phi, s) < 0. \tag{IA.54}
$$

Hence, $U(x)$ and thus $U(\theta)$ are concave and the solution of the FOC (IA.53) is the unique maximum of $U(\theta)$.

It remains to show that $\theta(p) = x(p) - z$ is strictly decreasing. Let $\theta(p)' \equiv \partial\theta(p)/\partial p = x(p)'$ and

$$
H(x(p), p) \equiv \int_{\phi_a}^{\phi_b} u'\left(\mathrm{CE}(x(p), \phi, p)\right)\left(\mu_\phi(s) - p - \gamma\, v_\phi \theta(p)\right) dF(\phi, s). \tag{IA.55}
$$

We have that

$$\frac{\partial \mathrm{CE}(x(p),p)}{\partial p} = z - x(p) + (\mu_\phi(s) - p)x(p)' - \gamma\, v_\phi x(p)x(p)' \tag{IA.56}$$

and thus differentiating the FOC $H(x(p),p) = 0$ with respect to p leads to

$$\begin{aligned}
&\int_{\phi_a}^{\phi_b} u''(\cdot)\ \left(z - x(p) + (\mu_\phi(s) - p)x(p)' - \gamma\, v_\phi\, x(p)x(p)'\right)(\mu_\phi(s) - p - \gamma\, v_\phi\, x(p))\ dF(\phi,s)\\
&+ \int_{\phi_a}^{\phi_b} u'(\cdot)\ \left(-1 - \gamma\, v_\phi\, x(p)'\right)\ dF(\phi,s) = 0.
\end{aligned} \tag{IA.57}$$

Solving for $x(p)'$ using $u''(\cdot)/u'(\cdot) = -\gamma$ and $H(x(p),p) = 0$ leads to

$$x(p)' = \frac{-1/\gamma}{\mathrm{E}_F\left[\xi\left(x(p),\tilde{\phi}\right)\ \left(\left(\mu_{\tilde{\phi}}(s) - p - \gamma\, v_{\tilde{\phi}} x(p)\right)^2 +\ v_{\tilde{\phi}}\right) \mid \tilde{s} = s\right]} < 0, \tag{IA.58}$$

where

$$\xi\left(x,\phi\right) := \frac{u'\left(\mathrm{CE}(x,\phi)\right)}{\mathrm{E}_F\left[u'\left(\mathrm{CE}(x,\tilde{\phi})\right) \mid \tilde{s} = s\right]}. \tag{IA.59}$$

Hence, $x(p)$ and thus $\theta(p)$ are continuously differentiable and strictly decreasing.

The function $\xi(x,\phi)$ is positive and $\mathrm{E}_F\left[\xi(\theta,\tilde{\phi})\right] = 1$ and thus $Q(\theta,\phi)$ defined in equation (IA.51) is a conditional probability distribution.

Moreover, solving for $\theta(p)$ using the FOC (IA.53) leads to equation (IA.50). □

B. Equilibrium Price

Suppose there exists a representative Savage investor who puts the prior $G_{\tilde{\phi}}$ on the signal precision $\tilde{\phi}$. In equilibrium the representative investor holds the asset $(\theta = 1)$ and consumes the dividend $\tilde{d}$ and labor income $\tilde{L}$. The properties of the equilibrium price are summarized in the next proposition.

PROPOSITION IA. 4: *Let $Q_1(\phi) = Q(\phi; 1)$ denote the risk-neutral probability measure for $\tilde{\phi}$ conditional on $\tilde{s} = s$ defined in equation (IA.51) and evaluated at $\theta = 1$. The equilibrium price is a continuously differentiable function of the signal:*

$$p(s) = \mathrm{E}_{Q_1(\phi)}\left[p_{\tilde{\phi}}(s) \mid \tilde{s} = s\right], \tag{IA.60}$$

where

$$p_\phi(s) = \mathrm{E}\left[\tilde{d} \mid \tilde{s} = s, \tilde{\phi} = \phi\right] - \gamma(1+z)\,\mathrm{Var}\left[\tilde{d} \mid \tilde{s} = s, \tilde{\phi} = \phi\right]. \tag{IA.61}$$

Proof. Plugging $\theta = 1$ in to the FOC (IA.53) and solving for $p(s)$ leads to equation (IA.60). $p_\phi(s)$ is a continuously differentiable function of the signal and hence $p(s)$ is a continuously differentiable function of the signal. □

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