Internet Appendix to "False Discoveries in Mutual Fund Performance: Measuring Luck in Estimated Alphas"*

A. Estimation Procedure

A.1. Determining the Value for λ^* from the Data

We use the bootstrap procedure proposed by Storey (2002) and Storey, Taylor, and Siegmund (2004) to estimate the proportion of zero-alpha funds in the population, π_0. This resampling approach chooses λ from the data such that an estimate of the Mean Squared Error (MSE) of $\widehat{\pi}_0(\lambda)$, defined as $E(\widehat{\pi}_0(\lambda) - \pi_0)^2$, is minimized. First, we compute $\widehat{\pi}_0(\lambda)$ using equation (5) of the paper across a range of λ values ($\lambda = 0.30, 0.35, ..., 0.70$). Second, for each possible value of λ, we form 1,000 bootstrap replications of $\widehat{\pi}_0(\lambda)$ by drawing with replacement from the $M \times 1$ vector of fund p-values. These are denoted by $\widehat{\pi}_0^b(\lambda)$, for $b = 1, ..., 1,000$. Third, we compute the estimated MSE for each possible value of λ:

$$\widehat{MSE}(\lambda) = \frac{1}{1,000} \sum_{b=1}^{1,000} \left[\widehat{\pi}_0^b(\lambda) - \min_{\lambda} \widehat{\pi}_0(\lambda) \right]^2 . \tag{IA.1}$$

We choose λ^* such that $\lambda^* = \arg\min_\lambda \widehat{MSE}(\lambda)$.

A.2. Determining the Value for γ^* from the Data

To estimate the proportions of unskilled and skilled funds in the population, π_A^- and π_A^+, we use a bootstrap procedure that minimizes the estimated MSE of $\widehat{\pi}_A^-(\gamma)$ and $\widehat{\pi}_A^+(\gamma)$. First, we compute $\widehat{\pi}_A^-(\gamma)$ using Equation (8) of the paper across a range of γ values ($\gamma = 0.30, 0.35, ..., 0.50$). Second, we form 1,000 bootstrap replications of $\widehat{\pi}_A^-(\gamma)$ for each possible value of γ. These are denoted by $\widehat{\pi}_A^{b-}(\gamma)$, for $b = 1, ..., 1,000$. Third, we compute the estimated MSE for each possible value of γ:

$$\widehat{MSE}^-(\gamma) = \frac{1}{1,000} \sum_{b=1}^{1,000} \left[\widehat{\pi}_A^{b-}(\gamma) - \max_{\gamma} \widehat{\pi}_A^-(\gamma) \right]^2 . \tag{IA.2}$$

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We choose γ^- such that $\gamma^- = \arg\min_\gamma \widehat{MSE}^-(\gamma)$. We use the same data-driven procedure to determine $\gamma^+ = \arg\min_\gamma \widehat{MSE}^+(\gamma)$. If $\min_\gamma \widehat{MSE}^-(\gamma) < \min_\gamma \widehat{MSE}^+(\gamma)$, we set $\widehat{\pi}_A^-(\gamma^*) = \widehat{\pi}_A^-(\gamma^-)$. To preserve the equality $1 = \pi_0 + \pi_A^+ + \pi_A^-$, we set $\widehat{\pi}_A^+(\gamma^*) = 1 - \widehat{\pi}_0 - \widehat{\pi}_A^-(\gamma^*)$. Otherwise, we set $\widehat{\pi}_A^+(\gamma^*) = \widehat{\pi}_A^+(\gamma^+)$ and $\widehat{\pi}_A^-(\gamma^*) = 1 - \widehat{\pi}_0 - \widehat{\pi}_A^+(\gamma^*)$.

A.3. Computing the Fund (alpha) p-value

We use a bootstrap procedure (instead of asymptotic theory) to compute the fund (alpha) p-value, $\widehat{p}_i$, for the two-sided test, $H_{0,i} : \alpha_i = 0$ $(i = 1, ..., M)$, with equal-tail significance level, $\gamma/2$. Since the distribution of the fund t-statistic may be asymmetric in small samples, we follow the approach proposed by Davidson and MacKinnon (2004, p. 187) and compute the p-value as

$$\widehat{p}_i = 2 \cdot \min\left(\frac{1}{Q}\sum_{q=1}^{Q} I\{\widehat{t}_i^{*q} > \widehat{t}_i\}, \frac{1}{Q}\sum_{q=1}^{Q} I\{\widehat{t}_i^{*q} < \widehat{t}_i\}\right), \tag{IA.3}$$

where Q is the number of bootstrap iterations ($Q = 1,000$), and $I\{\widehat{t}_i^{*q} > \widehat{t}_i\}$ is an indicator function that takes the value one if the bootstrap t-statistic, $\widehat{t}_i^{*q}$, is higher than the estimated t-statistic, $\widehat{t}_i$. If the fund t-statistic distribution is symmetric, equation (IA.3) is equivalent to the more familiar p-value computation: $\widehat{p}_i = \left(\frac{1}{Q}\sum_{q=1}^{Q} I\{\left|\widehat{t}_i^{*q}\right| > \left|\widehat{t}_i\right|\}\right)$.

A.4. Determining the Standard Deviation of the Estimators

We rely on the large-sample theory proposed by Genovese and Wasserman (2004) to determine the standard deviation of the estimators used in the paper.[1] The essential idea is to recognize that these estimators are all stochastic processes indexed by λ or γ that converge to a Gaussian process when the number of funds, M, goes to infinity. Proposition 3.2 of Genovese and Wasserman (2004) shows that $\widehat{\pi}_0(\lambda^*)$ is asymptotically normally distributed when $M \to \infty$, with standard deviation $\widehat{\sigma}_{\widehat{\pi}_0} = \left(\frac{\widehat{W}(\lambda^*)(M-\widehat{W}(\lambda^*))}{M^3(1-\lambda^*)^2}\right)^{\frac{1}{2}}$, where $\widehat{W}(\lambda^*)$ denotes the number of funds having p-values exceeding λ^*. Similarly, we have $\widehat{\sigma}_{\widehat{F}_\gamma^+} = (\gamma/2)\,\widehat{\sigma}_{\widehat{\pi}_0}$, $\widehat{\sigma}_{\widehat{S}_\gamma^+} = \left(\frac{\widehat{S}_\gamma^+(1-\widehat{S}_\gamma^+)}{M}\right)^{\frac{1}{2}}$ and $\widehat{\sigma}_{\widehat{T}_\gamma^+} = \left(\widehat{\sigma}^2_{\widehat{S}_\gamma^+} + (\gamma/2)^2\,\widehat{\sigma}^2_{\widehat{\pi}_0} + 2\frac{(\gamma/2)}{1-\lambda^*}\widehat{S}_\gamma^+\frac{\widehat{W}(\lambda^*)}{M^2}\right)^{\frac{1}{2}}$ (using the equality $\widehat{S}_\gamma^+ = \widehat{F}_\gamma^+ + \widehat{T}_\gamma^+$). Standard deviations for the estimators in the left tail $\left(\widehat{S}_\gamma^-,\ \widehat{F}_\gamma^-,\ \widehat{T}_\gamma^-\right)$ are obtained by simply replacing $\widehat{S}_\gamma^+$ with $\widehat{S}_\gamma^-$ in the above formulas.

Finally, if $\gamma^* = \gamma^+$, the standard deviations of $\widehat{\pi}_A^+$ and $\widehat{\pi}_A^-$ are respectively given by

$\widehat{\sigma}_{\widehat{\pi}_A^+} = \widehat{\sigma}_{\widehat{T}^+_{\gamma^*}}$ and $\widehat{\sigma}_{\widehat{\pi}_A^-} = \left(\widehat{\sigma}^2_{\widehat{\pi}_A^+} + \widehat{\sigma}^2_{\widehat{\pi}_0(\lambda^*)} - 2\left(\frac{1}{1-\lambda^*}\right)\widehat{S}^+_{\gamma^*}\frac{\widehat{W}(\lambda^*)}{M^2} - 2\left(\gamma^*/2\right)\widehat{\sigma}^2_{\widehat{\pi}_0}\right)^{\frac{1}{2}}$ (using the equality $\widehat{\pi}_A^+ = 1 - \widehat{\pi}_0^+ - \widehat{\pi}_A^-$). Otherwise, if $\gamma^* = \gamma^-$, we just reverse the superscripts $+/-$ in the two formulas above.

B. *Monte Carlo Analysis*

B.1. *Under Cross-sectional Independence*

We use Monte Carlo simulations to examine the performance of all estimators used in the paper: $\widehat{\pi}_0$, $\widehat{\pi}_A^-$, $\widehat{\pi}_A^+$, $\widehat{S}_\gamma^-$, $\widehat{F}_\gamma^-$, $\widehat{T}_\gamma^-$, and $\widehat{S}_\gamma^+$, $\widehat{F}_\gamma^+$, $\widehat{T}_\gamma^+$. We generate the $M \times 1$ vector of fund monthly excess returns, r_t, according to the four-factor model (market, size, book-to-market, and momentum factors):

$$\begin{aligned} r_t &= \alpha + \beta F_t + \varepsilon_t, \qquad t = 1, ..., T, \\ F_t &\sim N(0, \Sigma_F), \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2 I), \end{aligned} \tag{IA.4}$$

where α denotes the $M \times 1$ vector of fund alphas, and β is the $M \times 4$ matrix of factor loadings. The 4×1 vector of factor excess returns, F_t, is normally distributed with covariance matrix Σ_F. The $M \times 1$ vector of normally distributed residuals is denoted by ε_t. We initially assume that the residuals are cross-sectionally independent and have the same variance σ_ε^2, so that the covariance matrix of ε_t can simply be written as $\sigma_\varepsilon^2 I$, where I is the $M \times M$ identity matrix.

Our estimators are compared with their respective true population values defined as follows. The parameters π_0, π_A^-, and π_A^+ denote the true proportions of zero-alpha, unskilled, and skilled funds. The expected proportions of unlucky and lucky funds, $E(F_\gamma^-)$ and $E(F_\gamma^+)$, are both equal to $\pi_0 \cdot \gamma/2$. To determine the expected proportions of unskilled and skilled funds, $E(T_\gamma^-)$ and $E(T_\gamma^+)$, we use the fact that, under the alternative hypothesis $\alpha_i \neq 0$, the fund t-statistic follows a noncentral student distribution with $T-5$ degrees of freedom and a noncentrality parameter equal to $T^{\frac{1}{2}}\alpha_A/\sigma_\varepsilon$ (Davidson and MacKinnon (2004), p. 169):

$$\begin{aligned} E(T_\gamma^-) &= \pi_A^- \cdot prob\left(t < t_{T-5,\gamma/2}\,|H_A, \alpha_A < 0\right), \\ E(T_\gamma^+) &= \pi_A^+ \cdot prob\left(t > t_{T-5,1-\gamma/2}\,|H_A, \alpha_A > 0\right), \end{aligned} \tag{IA.5}$$

where $t_{T-5,\gamma/2}$ and $t_{T-5,1-\gamma/2}$ denote the quantiles of probability level $\gamma/2$ and $1-\gamma/2$, respectively (these quantiles correspond to the thresholds t_γ^- and t_γ^+ used in the text). Finally, we have $E(S_\gamma^-) = E(F_\gamma^-) + E(T_\gamma^-)$ and $E(S_\gamma^+) = E(F_\gamma^+) + E(T_\gamma^+)$.

To compute these population values, we need to set values for the (true) proportions π_0, π_A^-, π_A^+, as well as for the means of the noncentral student distributions (required to compute equation (IA.5)). In order to set realistic values, we estimate π_0, π_A^-, and π_A^+ at the end of each of the final five years of our sample (2002 to 2006) using the entire return history for each fund up to that point in time. These estimates are then averaged to produce values that reflect the recent trend observed in Figure 4 of the paper: $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$. To determine the means of the t-statistic distributions of the unskilled and skilled funds, we use a simple calibration method. First, we compute the estimated proportions of unskilled and skilled funds (at $\gamma = 0.20$), $\widehat{T}_\gamma^-$ and $\widehat{T}_\gamma^+$, at the end of each of the final five years of our sample (2002 to 2006) using the entire return history for each fund up to that point in time. These estimates are then averaged and inserted along with $\pi_A^- = 23\%$ and $\pi_A^+ = 2\%$ in equation (IA.5) in order to determine the means of the distributions that satisfy both equalities. The resulting values are -2.5 and 3, and correspond to an annual four-factor alpha of -3.2% and 3.8%, respectively (using the equality $t_A = T^{\frac{1}{2}}\alpha_A/\sigma_\varepsilon$).

The total number of funds, M, used in the simulation is equal to 1,400.[2] The input for β is equal to the empirical loadings of a random draw of 1,400 funds (among the total population of 2,076 funds). Consistent with our database, we set $T = 384$ (months) and $\sigma_\varepsilon = 0.021$ (equal to the empirical average across the 1,400 funds), and we proxy Σ_F by its empirical counterpart. To build the vector of fund alphas, α, we need to determine the identity of the unskilled and skilled funds. This is done by randomly choosing 322 funds (i.e., 23% of the entire population) to which we assign a negative alpha (-3.2% per year), and 28 funds (2% of the population) to which we assign a positive alpha (3.8% per year).

After randomly drawing F_t and ε_t ($t = 1, ..., 384$), we construct the fund return time-series according to equation (IA.4), and compute their t-statistics by regressing the fund returns on the four-factor model. To determine the alpha p-values, we use the fact that the fund t-statistic follows a Student distribution with $T - 5$ degrees of freedom under the null hypothesis $\alpha_i = 0$. We then compute $\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$ using equations (5) and (8) of the paper. The terms $\widehat{S}_\gamma^-$ and $\widehat{S}_\gamma^+$ correspond to the observed number of significant funds with negative and positive alphas, respectively. The estimated proportions of unlucky and lucky funds (at the significance level γ), $\widehat{F}_\gamma^-$ and $\widehat{F}_\gamma^+$, are computed using equation (6) of the paper, while $\widehat{T}_\gamma^-$ and $\widehat{T}_\gamma^+$, are given by equation (7) of the paper. We repeat this procedure 1,000 times.

In Table IA.I, we compare the average value of each estimator (over the 1,000 replications) with the true values. The figures in parentheses denote the lower and upper bounds of the estimator 90% confidence interval. We set γ equal to 0.05 and 0.20. In all cases, the simulation results reveal that the average values of our estimators closely match the true values, and that their 90% confidence intervals are narrow. This result is not surprising, given the large cross-section of funds available in our sample (i.e., our estimators are proportion estimators, where the Law of Large Numbers obtains).

Please insert Table IA.I here

B.2. Under Cross-sectional Dependence

The return-generating process is the same as the one shown in equation (IA.4), except that the fund residuals are cross-correlated:

$$\varepsilon_t \sim N(0, \Sigma), \tag{IA.6}$$

where Σ denotes the $M \times M$ residual covariance matrix. The main constraint imposed on Σ is that it must be positive semi-definite. To achieve this, we select all funds with 60 valid return observations over the final five years (2002 to 2006), which is the period over which we have the largest possible cross-section of funds existing simultaneously–898 funds, whose covariance matrix, Σ_1, is directly estimated from the data.[3] To assess the precision of our estimators, we also need to account for the non-overlapping returns observed in the long-term fund data due to funds that do not exist at the same time. To address this issue, we introduce 502 uncorrelated funds and write the covariance matrix for the resulting 1,400 funds as follows:[4]

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \sigma_\varepsilon^2 I \end{pmatrix}. \tag{IA.7}$$

As an input for β, we use the empirical factor loadings of the 898 funds, along with the loadings of a random draw of 502 additional funds (from the initial population of 2,076 funds). The vector of fund alphas, α, is built by randomly choosing the identity of the unskilled and skilled funds, as in the independence case. The results in Panel A of Table IA.II indicate that all estimators remain nearly unbiased ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$ exhibit small biases). Looking at the 90% confidence intervals, we logically observe that the dispersion of the estimators widens under cross-sectional dependence. However, the performance of the estimators is still very good.

Please insert Table IA.II here

Apart from this baseline dependence scenario, we also consider three other cases. First, we examine the case of block dependence. For each skill group (zero-alpha, unskilled, and skilled funds), we assume that there are blocks of funds with correlated returns (possibly because these funds follow similar bets). For each skill group s (zero-alpha, unskilled, and skilled), we take blocks of size $b_s = l \cdot M_s$, where l is the proportion of funds included in each block, and M_s is the number of funds in the skill group s. Denoting by $n = 1/l$ the total number of blocks, we compute the covariance matrix of the skill group s as $\Sigma_s = I_{n\times n} \otimes \Omega_{b_s\times b_s}$, where I is $n \times n$ identity matrix, $\Omega_{b_s\times b_s}$ is a $b_s \times b_s$ covariance matrix, and $\otimes$ denotes the Kronecker product (also known as the direct or the tensor product). The diagonal and off-diagonal elements of $\Omega_{b_s\times b_s}$ are respectively equal to σ_ε^2 and $\rho\sigma_\varepsilon^2$, where σ_ε^2 denotes the fund residual variance and ρ the cross-fund correlation coefficient. For each skill group, we set the block size l equal to 10% of the funds and the correlation ρ equal to 0.15.

In the second dependence case, we use the residual factor specification proposed by Jones and Shanken (2005) to capture the role of non-priced factors. We assume that all fund residuals depend on a common residual factor G_t. Further, the unskilled and skilled funds are affected by a specific residual factor denoted by G_t^- and G_t^+, respectively. The residual of each fund i in the population ($i = 1, ..., M$) is specified as

$$\varepsilon_{i,t} = \delta G_t + \delta G_t^- I_{\{\alpha_i=\alpha_A^-\}} + \delta G_t^+ I_{\{\alpha_i=\alpha_A^+\}} + \xi_{i,t}, \tag{IA.8}$$

where $I_{\{\alpha_i=\alpha_A^-\}}$ and $I_{\{\alpha_i=\alpha_A^-\}}$ are two indicator functions taking the value of one (zero otherwise) if fund i is unskilled ($\alpha_i = \alpha_A^-$) and skilled ($\alpha_i = \alpha_A^+$), respectively. We assume that G_t, G_t^- and G_t^+ are orthogonal to one another, and to the four-factor risk factors. The three residual factors follow a normal distribution $N(0, \sigma_G)$. Consistent with our database, we set $\sigma_G = 0.035$ (equal to the average monthly standard deviation of the size, book-to-market, and momentum factors) and $\delta = 0.11$ (equal to the average exposure of the 2,076 funds to the three factors). The remaining term $\xi_{i,t} \sim N(0, \sigma_\xi)$ is uncorrelated across funds, and its standard deviation, σ_ξ, is fixed such that the fund residual standard deviation, σ_ε, equals 0.021, as in the independence case.

Finally, we consider a scenario where the fund population only consists of the 898 correlated funds. This represents a case of extreme dependence not only because all funds are cross-correlated, but also because the number of funds (898) is much lower than the sample available in the paper (2,076).

The results under block dependence and residual factor dependence are shown in Panels B and C of Table IA.II, respectively. In both cases, we find that the estimators are nearly unbiased, and their 90% confidence intervals are narrower than those obtained under the baseline dependence scenario in Panel A. When we examine the extreme dependence case in Panel D, we find that all estimators still remain unbiased. But unsurprisingly, their confidence intervals widen slightly compared to the baseline case (on average 1.1% are added on each side of the interval).

C. Further Analysis of the Methodology

C.1. The Proportion of Zero-alpha Funds and the p-value Histogram

We examine in detail how the histogram of fund p-values is modified when the proportion of zero-alpha funds in the population, π_0, changes. We consider two different fund populations of M =2,076 funds (as in our database), whose t-statistics are drawn randomly from one of the three t-statistic distributions in Figure 1 of the paper (Panel A). While the first population contains only zero-alpha funds ($\pi_0 = 100\%$, $\pi_A^- = 0\%$, and $\pi_A^+ = 0\%$), the second population contains zero-alpha funds (75%) as well as skilled funds (25%), that is $\pi_0 = 75\%$, $\pi_A^- = 0\%$, $\pi_A^+ = 25\%$. After computing the two-sided p-values for each of the 2,076 funds for each population, we compare their respective p-value histograms in Figure IA.1.[5]

Please insert Figure A1 here

The histogram of the first population (only zero-alpha funds) is depicted by black bars. Since all funds satisfy the null hypothesis $\alpha = 0$, their p-values are drawn from the uniform distribution over the interval [0,1]. As a result, the histogram closely approximates the uniform distribution shown by the horizontal dark line at 0.10 (some black bars are slightly below or above 0.10 because of sampling variation).

The histogram of the second population (75% zero-alpha funds, 25% skilled funds) is depicted by grey bars. This histogram comprises: 1) a set of light grey bars with constant height over the interval [0,1], corresponding to the 75% of zero-alpha funds; and 2) an additional bar (dark grey) corresponding to the p-values of the 25% of skilled funds in the population. Note that the height of each grey bar is close to the horizontal grey line at 0.075 (once again, the difference comes from sampling variability). Summing the area covered by these light grey bars over the entire interval, we get the correct proportion of zero-alpha funds, π_0 (i.e., $0.075 \cdot 10 = 75\%$).

Comparing the second histogram with the first one, we observe an important increase in the proportion of extremely small p-values due to the existence of the skilled funds. But since the area covered by the histogram bars must sum to one, this increase is offset by a decline in all light grey bars over the interval [0,1] (compared to the black bars). The reason for this decline over [0,1] is straightforward: while we draw all p-values of the first population from the uniform distribution (i.e., $\pi_0 = 100\%$), the p-values of the second population comes from this uniform distribution only 75% of the time ($\pi_0 = 75\%$).

C.2. Comparison between the Bootstrap and Fixed-value Procedures

The threshold λ^* used to estimate the proportion of zero-alpha funds, π_0, determines the number of funds, $\widehat{W}(\lambda^*)$, with p-values higher than λ^*. If λ^* is too low, the estimator, $\widehat{\pi}_0(\lambda^*)$, overestimates π_0 (i.e., $\widehat{\pi}_0$ is biased upward) since $\widehat{W}(\lambda^*)$ includes the p-values of many unskilled (skilled) funds (generating Type II errors). In contrast, if λ^* is too high, we estimate π_0 using only the few p-values at the extreme right of the histogram, thus making the estimator $\widehat{\pi}_0(\lambda^*)$ extremely volatile. In the paper, we propose a simple procedure that chooses λ^* such that the estimated Mean Squared Error (MSE) of $\widehat{\pi}_0$ is minimized (see equation (IA.1)).

While the main advantage of this procedure is that it is entirely data-driven, it turns out that the estimate of π_0 is not overly sensitive to the choice of λ^*. We therefore believe that a researcher studying the performance of a large population of mutual funds can simply use a value for λ^* of 0.5 or 0.6. At these levels, we avoid including the p-values of the unskilled (skilled) funds. In addition, the estimator is not too volatile, since there are still many p-values located at the right of λ^*.[6] To illustrate, consider the baseline example used in the Monte Carlo analysis and illustrated in Figure 1 of the paper. The number of funds, M, is equal to 2,076 (as in our database). For each fund, we draw its t-statistic from one of the three t-statistic distributions shown in Panel A of Figure 1 according to the following weights: $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$. We then compute the two-sided p-value for each of the 2,076 funds and estimate π_0 using two procedures. The first one is our bootstrap procedure, while the second one (called the fixed-value procedure) sets λ^* equal to 0.5 and 0.6, respectively. The results for 10 different simulations shown in Table IA.III (Panel A) indicate that the estimated values, $\widehat{\pi}_0$, are very close to one another.

Please insert Table IA.III here

In order to estimate the proportions of unskilled and skilled funds in the population, π_A^- and π_A^+, we also need to determine the significance level γ^*. If γ^* is too low, the estimators $\widehat{\pi}_A^-(\gamma^*)$ and $\widehat{\pi}_A^+(\gamma^*)$ underestimate π_A^- and π_A^+ (i.e., $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ are biased downward), because the power of the test (i.e., the probability of detecting unskilled

(skilled) funds) is not sufficiently high (especially if the unskilled (skilled) funds are dispersed throughout the tails). On the other hand, if γ^* is too high, we inflate the variance of $\widehat{\pi}_A^-(\gamma^*)$ and $\widehat{\pi}_A^+(\gamma^*)$ by investigating a very large portion of the tails of the cross-sectional t-distribution. In the paper, we use a bootstrap procedure that minimizes the estimated MSE of $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ (see Equation (IA.2)).

Similar to λ^*, we find that the estimates of π_A^- and π_A^+ are not overly sensitive to the choice of γ^*. To illustrate, we compare our bootstrap procedure with a fixed-value approach, where γ^* is set to 0.35 and 0.4, respectively. In Panel B of Table IA.III, we compare the estimated values over 10 different simulations. The results show that these estimates are very close to one another. Thus summarizing, while we use a bootstrap approach, there is still some flexibility in the way the proportions of unskilled and skilled funds are estimated.

C.3. Comparison of the FDR Approach with Existing Methods

Figure 3 of the paper compares the different approaches using a significance level γ equal to 0.20. To assess the result sensitivity to changes in γ, we plot the different relations using a significance level γ equal to 0.10 (i.e., we measure the proportions of unlucky (lucky), unskilled (skilled) funds further into the extreme left (right) tails of the cross-sectional t-distribution). The results are displayed in Figure IA.2.

Please insert Figure IA.2 here

In Panel A, we compare the estimators of the expected proportion of unlucky funds under the no luck, full luck, and FDR approaches for different values for the proportion of zero-alpha funds in the population, π_0. This graph is similar to the one shown in Figure 3 of the paper. While the average value of our FDR estimator closely tracks $E(F_\gamma^-)$, the no luck approach (which assumes that $\pi_0 = 0$) consistently underestimates $E(F_\gamma^-)$, and the "full luck" approach (which assumes that $\pi_0 = 1$) overestimates $E(F_\gamma^-)$ when $\pi_0 < 1$. The only difference is that the bias of these two approaches is lower when γ declines from 0.20 to 0.10 (i.e., the scale of the vertical axis is lower in Figure IA.2 than in Figure 3 of the paper). To understand this result, we can write the bias of the estimators of the proportion of unlucky funds under the two approaches as

$$\begin{aligned} \text{No luck} \quad &: \quad bias = \pi_0 \cdot prob\left(t < t_\gamma^- \,|H_0\right) = \pi_0 \cdot \gamma/2, \\ \text{Full luck} \quad &: \quad bias = (1-\pi_0) \cdot prob\left(t < t_\gamma^- \,|H_0\right) = (1-\pi_0) \cdot \gamma/2. \end{aligned} \tag{IA.9}$$

In both cases, the bias depends on the probability, $\gamma/2$, of finding an unlucky fund, which declines as γ falls from 0.20 to 0.10. Stated differently, the errors in the luck measurement made by the no luck and full luck approaches increase as we investigate larger portions of the tails.[7] All these comments also apply to the expected proportion of lucky funds, $E(F_\gamma^+)$, in Panel B, since $E(F_\gamma^+) = E(F_\gamma^-)$.

Panel C displays the estimates of the expected proportion of unskilled funds, $E(T_\gamma^-)$, at $\gamma = 0.10$. Here again, the graph is very close to the one shown in Figure 3 of the paper. Our approach closely captures the negative relation between $E(T_\gamma^-)$ and π_0. In contrast, the no luck approach overestimates the expected proportion of unskilled funds (since it does not adjust for luck), while the full luck approach underestimates $E(T_\gamma^-)$ (because it overadjusts for luck).

Finally, Panel D shows that our FDR approach provides a nearly unbiased estimator of the expected proportion of skilled funds, $E(T_\gamma^+)$, at $\gamma = 0.10$, as opposed to the other approaches. The main difference with Figure 3 of the paper (Panel D) lies in the slope of the relation between π_0 and the estimators under the no luck and full luck approaches. While in Figure 3 of the paper we observe a nonsensical positive slope, it is negative in Figure IA.2 (as it should be). To understand the reason for this change, we can write the expected proportion of significant funds with positive estimated alphas, $E(S_\gamma^+)$, as

$$E(S_\gamma^+) = E(F_\gamma^+) + E(T_\gamma^+) = \gamma/2 \cdot \pi_0 + \pi_A^+ \cdot prob\left(t > t_\gamma^+ \,|H_A, \alpha_A > 0\right). \tag{IA.10}$$

Using the equalities $(1-\pi_0) = \pi_A^- + \pi_A^+$ and $\pi_A^-/\pi_A^+ = 11.5$ (0.23/0.02),[8] we can write $\pi_A^+ = (1-\pi_0)/12.5$. Replacing π_A^+ in equation (IA.10), we have

$$\begin{aligned} E(S_\gamma^+) &= \gamma/2 \cdot \pi_0 + (1-\pi_0)/12.5 \cdot prob\left(t > t_\gamma^+ \,|H_A, \alpha_A > 0\right) \\ &= \gamma/2 \cdot \pi_0 + (1-\pi_0)\, b = b + \pi_0\,(\gamma/2 - b)\,, \end{aligned} \tag{IA.11}$$

where $b = prob\left(t > t_\gamma^+ \,|H_A, \alpha_A > 0\right)/12.5$. Using this result, the average value of the estimators of skilled funds under both approaches can be written as a function of π_0:

$$\begin{aligned} \text{No luck} &: \quad E(\widehat{T}_\gamma^+) = E(S_\gamma^+) = b + \pi_0\,(\gamma/2 - b)\,, \\ \text{Full luck} &: \quad E(\widehat{T}_\gamma^+) = E(S_\gamma^+) - \gamma/2 = c + \pi_0\,(\gamma/2 - b)\,, \end{aligned} \tag{IA.12}$$

where c is a constant: $c = b - \gamma/2$. Equation (IA.12) reveals that an increase in π_0 has two contradictory effects on $E(\widehat{T}_\gamma^+)$. On the one hand, it increases the expected proportion of lucky funds that are wrongly included in $\widehat{T}_\gamma^+$ (through $\gamma/2$). On the other hand, it decreases the proportion of skilled funds in the population (through b) (i.e., a rise in π_0 leads to a decline in π_A^+).

If $\gamma/2 > b$, the no luck and full luck approaches produce a nonsensical positive relation between $E(T_\gamma^+)$ and π_0, as is the case in Figure 3 of the paper at $\gamma = 0.20$.[9] In contrast, when $\gamma/2 < b$, the relation is negative, as in Figure IA.2.[10] Therefore, the slope is positive when the proportion of skilled funds in the population is low (as we empirically find), and when the probability of finding a lucky fund, $\gamma/2$, is high (as in Figure 3 of the paper, where γ is set equal to 0.20).

Finally, note that our FDR approach is immune to this problem, as the average value of its estimator is always negatively related to π_0 :

$$\text{FDR approach: } E(\widehat{T}_\gamma^+) = E(S_\gamma^+) - \pi_0 \cdot \gamma/2 = b - \pi_0 b. \tag{IA.13}$$

The reason is that the luck adjustment, $\pi_0 \cdot \gamma/2$, depends on π_0, and correctly captures the increase in $E(S_\gamma^+)$ due to the inclusion of the additional lucky funds.

D. Additional Empirical Results

D.1. Impact of Luck on Long-term Performance: Sensitivity Analysis

Our baseline measurement of mutual fund long-term performance (Table II of the paper) uses a sample of funds which have at least 60 monthly return observations. To check the robustness of our results, we repeat our analysis after reducing the minimum fund return requirement from 60 to 36 months. This change provides us with a larger sample of 2,423 funds (as opposed to 2,076 for the original fund population). Panel A of Table IA.IV shows the estimated proportions of zero-alpha, unskilled, and skilled funds ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$) in the entire population (2,423 funds), as defined in Section I.A.1 of the paper, with standard deviations of estimates in parentheses. These point estimates are computed using the procedure described in Section I.A.3 of the paper, while standard deviations are computed using the method of Genovese and Wasserman (2004) (see Internet Appendix A.4). In the leftmost columns, we also compute the proportion of significant alpha funds in the left tail, $\widehat{S}_\gamma^-$, at four different significance levels ($\gamma = 0.05$, 0.10, 0.15, 0.20) along with its decomposition into lucky and unskilled funds ($\widehat{F}_\gamma^-$ and $\widehat{T}_\gamma^-$). The rightmost columns repeat the analysis for the significant funds in the right tail, $\widehat{S}_\gamma^+$, and decompose them into lucky and skilled funds ($\widehat{F}_\gamma^+$ and $\widehat{T}_\gamma^+$). Our results show that reducing the minimum fund return requirement to 36 has no material impact on the results. Specifically, the estimated proportions of zero-alpha, unskilled, and skilled funds are virtually unchanged (74.7%, 25.3%, 0.0% versus 75.4%, 24.5%, 0.6% in Table II of the paper).

As a second robustness test, we use the conditional four-factor model in equation (10) of the paper to measure performance (as opposed to its unconditional counterpart in equation (9)). The results shown in Panel B based on the original sample of 2,076 funds (i.e., 60 minimum return observations) are again very close to those shown in Table II of the paper.

Please insert Table IA.IV here

D.2. Performance Analysis across Investment Categories

Similar to our tests for the overall mutual fund sample in Table II of the paper and Table IA.IV, we conduct long-term performance tests for individual investment objective subgroups–Growth, Aggressive Growth, and Growth & Income categories. Panel A of Table IA.V reports the performance in the population of Growth funds (1,304 funds). We find that Growth funds show similar results to the overall universe of funds, as 76.5% of the funds are zero-alpha funds. The rest of the population (23.5%) is comprised of unskilled funds which are unable to pick stocks well enough to recover their trading costs and other expenses.

Please insert Table IA.V here

Panel B repeats these estimates for Aggressive Growth funds (388 funds). While the vast majority of these funds produce zero alphas ($\widehat{\pi}_0 = 75.5\%$), a small proportion of funds have long-term skills ($\widehat{\pi}_A^+ = 3.9\%$). Despite the high level of turnover observed for right-tail funds (between 119% and 134% per year), some of their managers are sufficiently skilled to more than compensate for these additional trading costs. We also find that 20.6% of the funds are unskilled, partly because of their high expense ratios (1.6% per year, on, average for the left-tail funds).

Finally, Panel C shows results for the Growth & Income funds (384 funds). This category produces the lowest performance: not only is the proportion of skilled funds equal to zero, but this category also contains the highest proportion of unskilled funds ($\widehat{\pi}_A^- = 30.7\%$). Despite a low level of expenses and trading costs (compared to the other categories), our results reveal that these managers do not have sufficient stockpicking skills to produce a positive performance in the long-run.

Table IA.VI repeats the short-term tests conducted for the overall mutual fund sample on the same investment objective subgroups–Growth, Aggressive Growth, and Growth & Income categories. That is, for each category, we partition our data into six non-overlapping subperiods of five years, beginning with 1977-1981 and ending with 2002-2006. For each subperiod, we include all funds having 60 monthly return observations, then compute their respective alpha p-values–in other words, we treat each fund during each five-year period as a separate "fund." We pool these five-year records together across all time periods to represent the average experience of an investor in a randomly chosen fund during a randomly chosen five-year period.

Please insert Table IA.VI here

In Panel A, the results for Growth funds are similar to the short-term performance of the overall universe of funds. First, a small fraction of funds (2.4% of the population) exhibit skill over the short run. These skilled funds are located in the extreme right tail of the cross-sectional t-distribution. For instance, at a significance level, γ, of 5%, approximately 50% of the significant Growth funds are skilled ($\widehat{T}_{\gamma}^{+}/\widehat{S}_{\gamma}^{+} = 1.7/3.5 = 48.6\%$). Proceeding toward the center of the distribution (by increasing γ to 0.05 and 0.20) produces almost no additional skilled funds and almost entirely additional zero-alpha funds that are lucky (i.e., $\widehat{T}_{\gamma}^{+}/\widehat{S}_{\gamma}^{+}$ decreases from 48.6% to 26.2%). Second, we still observe a large proportion of unskilled funds ($\widehat{\pi}_{A}^{-} = 24.6\%$), which are dispersed throughout the left tail.

The short-term performance of the Aggressive Growth funds in Panel B (i.e., $\widehat{\pi}_{A}^{-} = 24.0\%$ and $\widehat{\pi}_{A}^{+} = 4.2\%$) is similar to the long-term performance of these funds shown in Table IA.V. Similar to Growth funds, we observe that skilled Aggressive Growth funds are concentrated in the extreme right tail of the distribution. For instance, at $\gamma = 0.05$, more than 60% of the significant funds are skilled ($\widehat{T}_{\gamma}^{+}/\widehat{S}_{\gamma}^{+} = 3.1/4.9 = 63.3\%$).

Finally, Panel C shows that Growth & Income funds contain about the same proportion of unskilled funds as Growth and Aggressive-Growth funds ($\widehat{\pi}_{A}^{+} = 25.9\%$). But, contrary to these two categories, no Growth & Income funds are able to generate positive short-term alphas.

D.3. Evolution of Mutual Fund Pre-expense Performance over Time

We examine the evolution of the long-term proportions of unskilled and skilled funds on a pre-expense basis. To compute pre-expense performance, we simply add the monthly expenses of each fund (1/12 times the most recent reported annual expense ratio) to its net returns. At the end of each year from 1989 to 2006, we estimate the proportions of unskilled and skilled funds ($\widehat{\pi}_{A}^{-}$ and $\widehat{\pi}_{A}^{+}$, respectively) on a pre-expense basis using the entire return history for each fund up to that point in time. Our initial estimates, on December 31, 1989, cover the first 15 years of the sample, 1975 to 1989, while our final estimates, on December 31, 2006, are based on the entire 32 years, 1975 to 2006 (these are the estimates shown in Table VI of the paper). The results in Figure AI.3 show that the estimated proportion of pre-expense skilled funds remains above 25% until the end of 1998, and then drops to 9.6% at the end of 2006. This implies that the decline in net-expense skills noted in Figure 4 of the paper is mostly driven by a reduction in stockpicking skills over time (as opposed to an increase in expenses for pre-expense skilled funds).

Looking at the proportion of pre-expense unskilled funds, we observe that it remains equal to zero until the end of 2003. Thus, poor stockpicking skills (net of trading costs) cannot explain the large increase in the proportion of unskilled funds (net of both trading costs and expenses) from 1996 onwards. This increase is likely to be due to rising expenses charged by funds with weak stock-selection abilities, or the introduction of new funds with high expense ratios and marginal stockpicking skills.

Please insert Figure IA.3 here

D.4. Fund Selection From a Bayesian Perspective

Instead of controlling the False Discovery Rate (FDR^+) as in the paper, the Bayesian approach to fund selection consists of minimizing the investor's loss function. We denote by G_i a random variable that takes the value of -1 if fund i is unskilled, 0 if it has zero alpha, and +1 if it is skilled. The prior probabilities for the three possible values (-1, 0, +1) are given by the proportion of each skill group in the population, π_A^-, π_0, and π_A^+.

In her attempt to determine whether to include fund i ($i = 1, ..., M$) in her portfolio, the Bayesian investor is subject to two sorts of misclassification. First, she may wrongly include a zero-alpha fund in the porfolio (i.e., rejecting H_0, while it is true). Second, she may fail to include a skilled fund in the portfolio (i.e., accepting H_0, while it is wrong). Following Storey (2003), we can model the investor's loss function, BE, as a weighted average of each misclassification type for a given significance region, Γ^+:

$$BE(\Gamma^+) = (1-\psi)\, prob(T_i \in \Gamma^+) \cdot fdr_\gamma^+(\Gamma^+) + \psi \cdot prob(T_i \notin \Gamma^+) \cdot fnr_\gamma^+(\Gamma^+), \quad \text{(IA.14)}$$

where T_i is the t-statistic of fund i, $fdr^+(\Gamma^+) = prob\,(G_i = 0 |\, T_i \in \Gamma^+)$ is the False Discovery Rate (i.e., the probability of falsely including zero-alpha funds), $fnr^+(\Gamma^+) = prob\,(G_i = +1 |\, T_i \notin \Gamma^+)$ is the False Nondiscovery Rate (i.e., the probability of failing to detect skilled funds), and ψ is a cost parameter, which can be interpreted as the investor's regret after failing to detect skilled funds.

The decision problem faced by the Bayesian investor is to choose the significance threshold, $t^+(\psi)$, such that $\Gamma^+(\psi) = (t^+(\psi), +\infty)$ minimizes equation (IA.14). After updating her prior belief using the observed t-statistic of fund i, $\widehat{t}_i$, she will include fund i in her portfolio only if the posterior loss incurred when wrongly considering a skilled fund as a zero-alpha fund is larger than the posterior loss incurred when wrongly considering a zero-alpha fund as skilled,

$$\psi \cdot prob\left(G_i = +1 \middle| T_i = \widehat{t}_i\right) > (1-\psi)\, prob\left(G_i = 0 \middle| T_i = \widehat{t}_i\right), \tag{IA.15}$$

or equivalently, if $\widehat{t}_i$ belongs to the significance region, Γ^+ (see Storey (2003)),

$$\Gamma^+ = \left\{ \widehat{t}_i : \frac{\pi_0 \cdot f_0(\widehat{t}_i)}{\pi_0 \cdot f_0\left(\widehat{t}_i\right) + \pi_A^+ \cdot f_A\left(\widehat{t}_i\right)} \leq \psi \right\}, \tag{IA.16}$$

where $f_0(\widehat{t}_i) = prob\left(T_i = \widehat{t}_i \middle| G_i = 0\right)$ and $f_A(\widehat{t}_i) = prob\left(T_i = \widehat{t}_i \middle| G_i = +1\right)$. The optimal significance threshold, $t^+(\psi)$, is therefore defined as

$$t^+(\psi) = t^+ : \frac{\pi_0 \cdot f_0(t^+)}{\pi_0 \cdot f_0(t^+) + \pi_A^+ \cdot f_A(t^+)} = \psi. \tag{IA.17}$$

Equation (IA.16) reveals that a Bayesian approach requires an extensive parameterization, contrary to the frequentist approach used in the paper. This includes the exact specification of the null and alternative distributions, $f_0(\widehat{t}_i)$ and $f_A(\widehat{t}_i)$, the cost parameter, ψ, as well as the assumptions that the t-statistics are IID and homogeneous across the population (i.e., $f_0(\widehat{t}_i)$ and $f_A(\widehat{t}_i)$ must be similar across the individual test statistics).[11] In addition, a full Bayesian analysis requires to posit prior distributions for the proportions π_0, π_A^-, and π_A^+, and for the distribution parameters of $f_0(\widehat{t}_i)$ and $f_A(\widehat{t}_i)$.

Our frequentist approach to fund selection (Section III.C of the paper) consists in controling the FDR^+ of the portfolio at some specific target z^+ (z^+ =10%, 30%, 50%, 70%, and 90%). If we agree to make the additional parameterization mentioned above, we can use equation (IA.17) to determine the optimal Bayesian decision implied by each FDR^+ target. To illustrate, let us consider the hypothetical example presented in Figure 1 of the paper, where the individual fund t-statistic distributions for the three skill groups are normal, and centered at -2.5, 0, and 3.0, respectively (with a unit variance). The proportions of zero-alpha, unskilled, and skilled funds in the population (π_0, π_A^-, and π_A^+) are equal to 75%, 23%, and 2%, respectively. Since these values are directly estimated from the data (see Internet Appendix B.1), this example should provide a realistic analysis of the relation between the frequentist and Bayesian approaches.

First, we determine the significance threshold, $t^+(z^+)$, such that the fdr^+ is equal to the chosen target z^+,

$$t^+\left(z^+\right) = t^+ : fdr^+ = \frac{\pi_0(1-\Phi\left(t^+;0,1\right))}{\pi_0(1-\Phi\left(t^+;0,1\right)) + \pi_A^+(1-\Phi\left(t^+;3,1\right))} = z^+, \qquad \text{(IA.18)}$$

where $\Phi\left(x;\mu,\sigma^2\right) = prob(X < x;\mu,\sigma^2)$ is the cumulative distribution function of a normal random variable X with mean μ and variance σ^2. In Section III.C of the paper, we use the significance level, γ, (related to p-values) as opposed to the significance threshold, t^+, (related to t-statistics). Using the definition of a p-value, we can easily determine its value from $t^+\left(z^+\right)$: $\gamma\left(z^+\right) = 2\cdot\left(1-\Phi\left(t^+\left(z^+\right);0,1\right)\right)$. Second, we use Equation (IA.17) to determine the implied cost parameter, $\psi\left(z^+\right)$:

$$\psi\left(z^+\right) = \frac{\pi_0\cdot\phi(t^+\left(z^+\right);0,1)}{\pi_0\cdot\phi(t^+\left(z^+\right);0,1) + \pi_A^+\cdot\phi(t^+\left(z^+\right);3,1)}, \qquad \text{(IA.19)}$$

where $\phi\left(x;\mu,\sigma^2\right)$ is the density of the normal distribution with mean μ and variance σ^2 (at the point $X = x$). Finally, using $t^+\left(z^+\right)$ and $\psi\left(z^+\right)$, we can easily determine the implied False Nondiscovery rate, $fnr^+\left(z^+\right)$, and the Bayesian loss function, $BE(z^+)$.

In Table IA.VII we display the significance threshold $t^+\left(z^+\right)$, significance level $\gamma\left(z^+\right)$, cost parameter $\psi(z^+)$, $fnr^+\left(z^+\right)$, and loss function $BE\left(z^+\right)$ implied by the five FDR^+ targets z^+ chosen in the paper (z^+ =10%, 30%, 50%, 70%, and 90%). We observe that a high FDR^+ target (such as 90%) is consistent with the behavior of a Bayesian investor with a high cost of regret, ψ(90%) =0.997. Therefore, she chooses a very high significance level, γ(90%) = 0.477, in order to include the vast majority of the skilled funds in the portfolio (fnr^+(90%) is essentially equal to zero). In contrast, a low FDR^+ target (such as 10%) implies a low cost parameter, ψ(10%) =0.318. In this case, the Bayesian investor sets a very high significance threshold, t_γ^+(10%) = 2.96, (a low significance level, γ(10%) = 0.003) in order to avoid including a large proportion of zero-alpha funds in the portfolio.

Please insert Table IA.VII here

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Notes

[1]These are the estimated proportions of zero-alpha, unskilled, and skilled funds in the population, $\widehat{\pi}_0$, $\widehat{\pi}_A^-$, $\widehat{\pi}_A^+$; the estimated proportion of significant funds with positive (negative) estimated alpha at the significance level γ, $\widehat{S}_\gamma^+$ ($\widehat{S}_\gamma^-$); the estimated proportion of lucky (unlucky) funds (at level γ), $\widehat{F}_\gamma^+$ ($\widehat{F}_\gamma^-$); and the estimated proportion of skilled (unskilled) funds (at level γ), $\widehat{T}_\gamma^+$ ($\widehat{T}_\gamma^-$).

[2]We use this sample size to allow for comparison with the dependence case (described hereafter), which uses a sample of 1,400 correlated fund returns. Since our original sample of funds is larger than 1,400 (M=2,076), our assessment of the precision of the estimators in this section is conservative.

[3]The 25%, 50%, and 75% pairwise correlation quantiles are -0.09, 0.05, and 0.19, respectively.

[4]The total number of fund pairs, P, is given by $M(M-1)/2$, where M=1,400. If there are X uncorrelated funds in the population, the total number of uncorrelated fund pairs, I, equals $X(M-X)+X(X-1)/2$. In our data, 15% of the fund pairs do not have any return observations in common, and 55% of the observations are common to the remaining pairs (85%). Therefore, we estimate that the proportion of uncorrelated pairs is equal to 53% (15%+85%45%). With 502 uncorrelated funds, I/P amounts to 58%, and is very close to the ratio observed in the data.

[5]We have purposely separated each histogram bar, so that the two histograms can be easily compared.

[6]One may wonder why $\widehat{\pi}_0$ remains almost unchanged at λ^*=0.5 and 0.6, although there are fewer p-values at the right of λ^* when λ^*=0.6. The reason is that the area $\widehat{W}(\lambda^*)/M$ (where M is the number of funds) has to be extrapolated over the entire interval [0,1]: when λ^* rises from 0.5 to 0.6, $\widehat{W}(\lambda^*)/M$ gets smaller, but it is extrapolated more, so these effects offset each other.

[7]Note that we need to examine such large portions when estimating the proportions of unskilled and skilled funds in the population (π_A^- and π_A^+), (see Internet Appendix C.2). As a result, the no luck and full luck approaches can produce very poor estimates of π_A^- and π_A^+.

[8]The ratio π_A^-/π_A^+ is held fixed at 11.5 both in Figure 3 of the paper and in Figure IA.2 to guarantee that, as π_0 varies, the proportion of skilled funds remains low compared to the unskilled funds.

[9]Under H_A, $\alpha_A > 0$, the fund t-statistic follows a noncentral student distribution (see Internet Appendix B.1) Using t_γ^+=1.28, T=384 (the number of observations), and a t-mean equal to three (the noncentrality parameter), we find that b=0.96/12.5=0.076, implying that $b < \gamma/2$=0.10.

[10]Under H_A, $\alpha_A > 0$, the fund t-statistic follows a noncentral student distribution (see Internet Appendix B.1) Using t_γ^+=1.65, T=384 (the number of observations), and a t-mean equal to three (the noncentrality parameter), we find that b=0.89/12.5=0.071, implying that $b > \gamma/2$=0.05.

[11]See Efron et al. (2001) and Storey (2003) for further discussion in the context of genomics.

Table IA.I

Monte Carlo Analysis under Cross-sectional Independence

We examine the average value and the 90% confidence interval (in parentheses) of the different estimators based on 1,000 replications when fund residuals are independent from one another. For each replication, we generate monthly fund returns for 1,400 funds and 384 periods using the four-factor model (market, size, book-to-market, and momentum factors). The true parameter values for the proportions of zero-alpha, unskilled, and skilled funds (π_0, π_A^-, and π_A^+) are set to 75%, 23%, and 2%. We set the true four-factor annual alpha equal to -3.2% for the unskilled funds and +3.8% for the skilled ones. In each tail (left and right), we assess the precision of the different estimators at two significance levels (γ=0.05 and 0.20).

	Fund Proportion			
	True	Estimator (90% interval)		
Zero-alpha funds (π_0)	75.0	75.1 (71.7,78.6)		
Unskilled funds (π_A^-)	23.0	22.9 (19.7,25.9)		
Skilled funds (π_A^+)	2.0	2.0 (0.3,3.8)		
	Left Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^-\right)$	18.1	18.1 (16.4,19.7)	27.9	27.9 (26.1,30.0)
Unlucky funds $E\left(F_\gamma^-\right)$	1.8	1.8 (1.8,1.9)	7.5	7.5 (7.1,7.9)
Unskilled funds $E\left(T_\gamma^-\right)$	16.2	16.2 (14.6,17.9)	20.4	20.4 (18.2,22.7)
	Right Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^+\right)$	3.6	3.6 (2.8,4.4)	9.4	9.4 (8.2,10.8)
Lucky funds $E\left(F_\gamma^+\right)$	1.8	1.8 (1.8,1.9)	7.5	7.5 (7.1,7.9)
Skilled funds $E\left(T_\gamma^+\right)$	1.7	1.7 (0.9,2.5)	1.9	1.9 (0.5,3.3)

Table IA.II

Monte Carlo Analysis under Cross-sectional Dependence

We examine the average value and the 90% confidence interval (in parentheses) of the different estimators based on 1,000 replications when fund residuals are cross-sectionally correlated. For each replication, we generate monthly fund returns for 384 periods using the four-factor model (market, size, book-to-market, and momentum factors). The true parameter values for the proportions of zero-alpha, unskilled, and skilled funds (π_0, π_A^-, and π_A^+) are set to 75%, 23%, and 2%. We set the true four-factor annual alpha equal to -3.2% for the unskilled funds and +3.8% for the skilled ones. In each tail (left and right), we assess the precision of the different estimators at two significance levels (γ=0.05 and 0.20). The results shown in Panels A, B and C are based on a universe of 1,400 funds. In Panel A, we use the empirical covariance matrix of the fund residuals to determine the true covariance matrix. In Panel B, we assume that there are blocks of correlated funds within each skill group (zero-alpha, unskilled, and skilled funds). Panel C examines the case where all fund residuals depend on a common residual factor, while the residuals of the unskilled and skilled funds are affected by specific residual factors. Panel D reports the results obtained with a subgroup of 898 funds that all have cross-correlated returns.

Panel A. Baseline Case				
	Fund Proportion			
	True	Estimator (90% interval)		
Zero-alpha funds (π_0)	75.0	75.2 (68.1,81.9)		
Unskilled funds (π_A^-)	23.0	22.8 (17.1,28.3)		
Skilled funds (π_A^+)	2.0	1.9 (0.0,6.5)		
	Left Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^-\right)$	18.1	18.1 (16.4,25.6)	27.9	27.9 (24.2,32.6)
Unlucky funds $E\left(F_\gamma^-\right)$	1.8	1.8 (1.7,2.1)	7.5	7.5 (6.7,8.3)
Unskilled funds $E\left(T_\gamma^-\right)$	16.2	16.2 (13.4,19.5)	20.4	20.4 (16.3,25.6)
	Right Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^+\right)$	3.5	3.6 (2.4,5.5)	9.4	9.4 (6.6,12.9)
Lucky funds $E\left(F_\gamma^+\right)$	1.8	1.8 (1.7,2.1)	7.5	7.5 (6.7,8.3)
Skilled funds $E\left(T_\gamma^+\right)$	1.7	1.7 (0.5,3.8)	1.9	1.9 (0.0,5.8)

Table IA.II

Monte Carlo Analysis under Cross-sectional Dependence (Continued)

Panel B. Block Dependence

	Fund Proportion	
	True	Estimator (90% interval)
Zero-alpha funds (π_0)	75.0	75.6 (70.4,81.3)
Unskilled funds (π_A^-)	23.0	22.1 (17.0,27.3)
Skilled funds (π_A^+)	2.0	1.9 (0.0,6.0)

	Left Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^-\right)$	18.1	18.1 (14.6,21.2)	27.9	27.9 (24.2,31.7)
Unlucky funds $E\left(F_\gamma^-\right)$	1.8	1.8 (1.7,2.0)	7.5	7.5 (7.0,8.1)
Unskilled funds $E\left(T_\gamma^-\right)$	16.2	16.2 (12.7,19.5)	20.4	20.4 (16.2,24.4)

	Right Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^+\right)$	3.5	3.5 (2.2,4.7)	9.4	9.4 (6.6,12.3)
Lucky funds $E\left(F_\gamma^+\right)$	1.8	1.8 (1.7,2.0)	7.5	7.5 (7.0,8.1)
Skilled funds $E\left(T_\gamma^+\right)$	1.7	1.6 (0.4,2.8)	1.9	1.8 (0.0,4.9)

Panel C. Residual Factor Dependence

	Fund Proportion	
	True	Estimator (90% interval)
Zero-alpha funds (π_0)	75.0	75.5 (70.0,80.2)
Unskilled funds (π_A^-)	23.0	22.1 (15.8,27.9)
Skilled funds (π_A^+)	2.0	2.2 (0.0,7.8)

	Left Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^-\right)$	18.1	18.1 (14.4,22.2)	27.9	27.9 (23.3,33.1)
Unlucky funds $E\left(F_\gamma^-\right)$	1.8	1.8 (1.7,2.0)	7.5	7.6 (7.1,8.0)
Unskilled funds $E\left(T_\gamma^-\right)$	16.2	16.2 (12.4,20.4)	20.4	20.4 (15.6,25.8)

	Right Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^+\right)$	3.5	3.5 (2.2,5.2)	9.4	9.4 (6.2,13.3)
Lucky funds $E\left(F_\gamma^+\right)$	1.8	1.8 (1.7,2.0)	7.5	7.6 (7.1,8.0)
Skilled funds $E\left(T_\gamma^+\right)$	1.7	1.6 (0.4,3.3)	1.9	1.8 (0.0,5.7)

Table IA.II

Monte Carlo Analysis under Cross-sectional Dependence (Continued)

Panel D. Extreme Dependence				
	Fund Proportion			
	True	Estimator (90% interval)		
Zero-alpha funds (π_0)	75.0	75.2 (65.6,84.9)		
Unskilled funds (π_A^-)	23.0	22.1 (15.0,31.6)		
Skilled funds (π_A^+)	2.0	2.9 (0.0,8.9)		
	Left Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^-\right)$	18.1	18.2 (14.2,22.9)	27.9	27.9 (23.6,34.0)
Unlucky funds $E\left(F_\gamma^-\right)$	1.8	1.8 (1.5,2.2)	7.5	7.5 (6.2,8.7)
Unskilled funds $E\left(T_\gamma^-\right)$	16.2	16.3 (12.4,21.3)	20.4	20.4 (15.7,27.4)
	Right Tail			
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^+\right)$	3.5	3.5 (1.8,6.4)	9.4	9.4 (5.0,15.2)
Lucky funds $E\left(F_\gamma^+\right)$	1.8	1.8 (1.5,2.2)	7.5	7.5 (6.2,8.7)
Skilled funds $E\left(T_\gamma^+\right)$	1.7	1.6 (0.0,4.8)	1.9	1.8 (0.0,8.6)

Table IA.III

Comparison between the Bootstrap and the Fixed-value Procedures

In a population of M=2,076 funds, we draw each fund t-statistic from one of the distributions in Figure 1 of the paper (Panel A) according to the proportion of zero-alpha, unskilled, and skilled funds in the population ($\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$). We then compute the p-values of each fund, from which the different proportions are estimated. In Panel A, we compare the estimated proportion of zero-alpha funds, $\widehat{\pi}_0$, computed with the bootstrap procedure (Bootstrap) and the fixed-value procedure (Fixed-value), where λ^* is set to 0.5 and 0.6, respectively. The last column shows the difference in $\widehat{\pi}_0$ between the two approaches. In Panel B, we compare the estimated proportions of unskilled and skilled funds, $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$, computed with the bootstrap procedure (Bootstrap) and the fixed-value procedure (Fixed Value), where γ^* is set to 0.35 and 0.45, respectively. The last column shows the difference in $\widehat{\pi}_A^+$ between the two approaches (the difference in $\widehat{\pi}_A^-$ is identical (but with opposite sign), because of the equality 1-$\widehat{\pi}_0$=$\widehat{\pi}_A^-$+$\widehat{\pi}_A^+$). To assess the estimator sample variability, we run 10 simulations. All figures are expressed in percent.

Panel A. Proportion of Zero-Alpha Funds

	Bootstrap	Fixed-value ($\lambda = 0.5, 0.6$)		Difference in $\widehat{\pi}_0$	
Simulation	$\widehat{\pi}_0$	$\widehat{\pi}_0(0.5)$	$\widehat{\pi}_0(0.6)$	0.5	0.6
1	74.6	74.5	74.7	0.1	-0.1
2	75.4	76.2	76.4	-0.8	-1.0
3	74.9	74.8	74.6	0.1	0.3
4	78.3	78.9	78.5	-0.6	-0.2
5	78.1	78.1	78.4	0.0	-0.3
6	76.7	77.3	76.8	-0.6	-0.1
7	79.3	79.6	79.5	-0.3	-0.2
8	73.2	74.0	73.2	-0.8	0.0
9	74.5	75.0	74.8	-0.5	-0.3
10	78.0	78.2	78.5	-0.2	-0.5

Panel B. Proportions of Unskilled and Skilled Funds

	Bootstrap		Fixed-value ($\gamma = 0.35, 0.45$)				Difference in $\widehat{\pi}_A^+$	
Simulation	$\widehat{\pi}_A^-$	$\widehat{\pi}_A^+$	$\widehat{\pi}_A^-(0.35)$	$\widehat{\pi}_A^+(0.35)$	$\widehat{\pi}_A^-(0.45)$	$\widehat{\pi}_A^+(0.45)$	0.35	0.45
1	22.6	2.9	21.5	4.1	23.4	2.1	-1.2	0.8
2	22.0	1.9	22.2	1.6	22.8	1.0	0.3	0.8
3	21.7	1.0	21.7	1.0	21.6	1.2	0.0	-0.2
4	23.2	2.1	23.2	2.1	23.3	1.9	0.0	0.2
5	23.2	0.6	23.3	0.5	22.4	1.4	0.1	-0.8
6	21.1	0.6	21.4	0.4	21.4	0.4	0.2	0.2
7	23.5	2.9	23.5	2.9	22.7	3.6	0.0	-0.7
8	21.7	2.7	22.5	2.0	21.8	2.7	0.7	0.0
9	20.6	1.9	20.7	1.7	20.6	1.9	0.2	0.0
10	21.8	0.6	21.4	0.9	21.5	0.8	-0.3	-0.3

Table IA.IV

Long-term Performance: Sensitivity Analysis

We perform some sensitivity tests on the long-term performance measurement over the entire period 1975 to 2006. In Panel A, the fund population consists of all funds having at least 36 monthly return observations. This produces a sample of 2,423 funds (as opposed to 2,076 for the original sample). In Panel B, we use the conditional version of the four-factor model to measure the performance of the original fund sample (2,076 funds). We displays the estimated proportions of zero-alpha, unskilled, and skilled funds ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$) in the entire fund population (2,076 funds). We also count the proportions of significant funds in the left and right tails of the cross-sectional t-statistic distribution ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20). In the leftmost columns, the significant group in the left tail, $\widehat{S}_\gamma^-$, is decomposed into unlucky and unskilled funds ($\widehat{F}_\gamma^-$, $\widehat{T}_\gamma^-$). In the rightmost columns, the significant group in the right tail, $\widehat{S}_\gamma^+$, is decomposed into lucky and skilled funds ($\widehat{F}_\gamma^+$, $\widehat{T}_\gamma^+$). Finally, we present the characteristics of each significant group ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$): the average estimated alpha (% per year), expense ratio (% per year), and turnover (% per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A. Minimum of 36 Monthly Observations									
	Proportion of Unskilled and Skilled Funds								
	Zero alpha ($\widehat{\pi}_0$)		Non-zero alpha		Unskilled ($\widehat{\pi}_A^-$)		Skilled ($\widehat{\pi}_A^+$)		
Proportion	74.7 (2.3)		25.3		25.3 (2.1)		0.0 (0.8)		
Number	1,810		613		613		0		
	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-$(%)	11.8	17.6	22.4	25.6	7.5	5.6	3.8	1.9	Signif. $\widehat{S}_\gamma^+$(%)
	(0.6)	(0.8)	(0.8)	(0.9)	(0.5)	(0.5)	(0.4)	(0.3)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.9	3.8	5.6	7.5	7.5	5.6	3.8	1.9	Lucky $\widehat{F}_\gamma^+$(%)
	(0.1)	(0.1)	(0.2)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	
Unskilled $\widehat{T}_\gamma^-$(%)	9.9	13.8	16.8	18.1	0.0	0.0	0.0	0.0	Skilled $\widehat{T}_\gamma^+$(%)
	(0.7)	(0.8)	(0.9)	(1.0)	(0.6)	(0.5)	(0.4)	(0.3)	
Alpha(% year)	-5.9	-5.5	-5.0	-4.8	5.8	6.1	6.2	7.1	Alpha(% year)
	(0.2)	(0.2)	(0.2)	(0.1)	(0.5)	(0.6)	(0.6)	(1.2)	
Exp.(% year)	1.4	1.4	1.4	1.4	1.3	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	100	99	97	97	103	97	103	122	Turn.(% year)

Table IA.IV

Long-Term Performance: Sensitivity Analysis (Continued)

Panel B. Conditional Four-Factor Model									
	Proportion of Unskilled and Skilled Funds								
	Zero alpha ($\widehat{\pi}_0$)		Non-zero alpha		Unskilled ($\widehat{\pi}_A^-$)		Skilled ($\widehat{\pi}_A^+$)		
Proportion	70.5 (2.4)		29.5		28.7 (2.2)		0.8 (0.9)		
Number	1,464		612		596		16		
	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-(\%)$	11.1	17.4	20.9	25.2	7.8	6.1	4.0	2.2	Signif. $\widehat{S}_\gamma^+(\%)$
	(0.6)	(0.8)	(0.8)	(0.9)	(0.6)	(0.5)	(0.4)	(0.3)	
Unlucky $\widehat{F}_\gamma^-(\%)$	1.7	3.5	5.3	7.0	7.0	5.3	3.5	1.7	Lucky $\widehat{F}_\gamma^+(\%)$
	(0.1)	(0.1)	(0.2)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	
Unskilled $\widehat{T}_\gamma^-(\%)$	9.4	13.9	15.6	18.2	0.8	0.8	0.5	0.5	Skilled $\widehat{T}_\gamma^+(\%)$
	(0.7)	(0.9)	(1.0)	(1.1)	(0.7)	(0.6)	(0.5)	(0.3)	
Alpha(% year)	-5.2	-4.8	-4.6	-4.3	5.7	6.2	6.7	7.9	Alpha(% year)
	(0.2)	(0.2)	(0.2)	(0.1)	(0.5)	(0.6)	(0.6)	(1.1)	
Exp.(% year)	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.4	Exp.(% year)
Turn.(% year)	96	90	91	91	104	106	109	120	Turn.(% year)

Table IA.V

Long-term Performance across Investment Categories

We measure long-term performance with the unconditional four-factor model over the entire period 1975 to 2006 for three investment categories (Growth, Aggressive Growth, and Growth & Income funds) shown in Panels A, B, and C, respectively. In each panel, we display the estimated proportions of zero-alpha, unskilled, and skilled funds ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$) in the entire investment category. We aslo count the proportions of significant funds in the left and right tails of the cross-sectional t-statistic distribution ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20). In the leftmost columns, the significant group in the left tail, $\widehat{S}_\gamma^-$, is decomposed into unlucky and unskilled funds ($\widehat{F}_\gamma^-$, $\widehat{T}_\gamma^-$). In the rightmost columns, the significant group in the right tail, $\widehat{S}_\gamma^+$, is decomposed into lucky and skilled funds ($\widehat{F}_\gamma^+$, $\widehat{T}_\gamma^+$). Finally, we present the characteristics of each significant group ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$): the average estimated alpha (% per year), expense ratio (% per year), and turnover (% per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A. Growth Funds									
	Proportion of Unskilled and Skilled Funds								
	Zero alpha ($\widehat{\pi}_0$)		Non-zero alpha		Unskilled ($\widehat{\pi}_A^-$)		Skilled ($\widehat{\pi}_A^+$)		
Proportion	76.5 (3.2)		23.5		23.5 (2.9)		0.0 (1.0)		
Number	985		319		319		0		
	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-$(%)	10.8	16.1	20.5	24.2	7.6	5.7	3.8	1.9	Signif. $\widehat{S}_\gamma^+$(%)
	(0.8)	(1.0)	(1.1)	(1.2)	(0.7)	(0.6)	(0.5)	(0.4)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.9	3.8	5.7	7.6	7.6	5.7	3.8	1.9	Lucky $\widehat{F}_\gamma^+$(%)
	(0.1)	(0.2)	(0.2)	(0.3)	(0.3)	(0.2)	(0.2)	(0.1)	
Unskilled $\widehat{T}_\gamma^-$(%)	8.9	12.3	14.8	16.6	0.0	0.0	0.0	0.0	Skilled $\widehat{T}_\gamma^+$(%)
	(0.8)	(1.1)	(1.2)	(1.3)	(0.8)	(0.7)	(0.6)	(0.4)	
Alpha(% year)	-5.5	-4.9	-4.6	-4.4	5.0	5.3	5.8	7.1	Alpha(% year)
	(0.3)	(0.2)	(0.2)	(0.1)	(0.4)	(0.5)	(0.6)	(0.9)	
Exp.(% year)	1.4	1.4	1.4	1.4	1.3	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	105	100	98	98	93	90	87	99	Turn.(% year)

Table IA.V

Long-term Performance across Investment Categories (Continued)

Panel B. Aggressive Growth Funds

	Proportion of Unskilled and Skilled Funds			
	Zero alpha ($\widehat{\pi}_0$)	Non-zero alpha	Unskilled ($\widehat{\pi}_A^-$)	Skilled ($\widehat{\pi}_A^+$)
Proportion	75.5 (4.5)	24.5	20.6 (4.0)	3.9 (2.1)
Number	293	95	80	15

	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-$(%)	10.5	17.5	21.1	23.2	10.6	7.8	5.5	3.4	Signif. $\widehat{S}_\gamma^+$(%)
	(1.5)	(1.9)	(2.0)	(2.1)	(1.5)	(1.3)	(1.0)	(0.9)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.8	3.8	5.7	7.5	7.5	5.7	3.8	1.9	Lucky $\widehat{F}_\gamma^+$(%)
	(0.1)	(0.2)	(0.3)	(0.5)	(0.5)	(0.3)	(0.2)	(0.1)	
Unskilled $\widehat{T}_\gamma^-$(%)	8.7	13.7	15.4	15.7	3.1	2.1	1.7	1.5	Skilled $\widehat{T}_\gamma^+$(%)
	(1.6)	(2.0)	(2.2)	(2.4)	(1.7)	(1.4)	(1.1)	(0.9)	
Alpha(% year)	-7.8	-6.8	-6.4	-6.2	5.4	6.0	6.7	7.0	Alpha(% year)
	(0.4)	(0.3)	(0.3)	(0.2)	(0.5)	(0.6)	(1.0)	(1.4)	
Exp.(% year)	1.6	1.6	1.6	1.6	1.3	1.2	1.2	1.3	Exp.(% year)
Turn.(% year)	121	120	118	119	119	116	130	134	Turn.(% year)

Panel C. Growth & Income Funds

	Proportion of Unskilled and Skilled Funds			
	Zero alpha ($\widehat{\pi}_0$)	Non-zero alpha	Unskilled ($\widehat{\pi}_A^-$)	Skilled ($\widehat{\pi}_A^+$)
Proportion	69.3 (4.8)	30.7	30.7 (4.4)	0.0 (1.7)
Number	267	117	117	0

	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-$(%)	15.4	20.2	25.2	31.5	7.0	5.2	3.3	1.9	Signif. $\widehat{S}_\gamma^+$(%)
	(1.8)	(2.0)	(2.2)	(2.4)	(1.2)	(1.0)	(0.9)	(0.7)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.9	3.3	5.2	7.0	7.0	5.2	3.3	1.9	Lucky $\widehat{F}_\gamma^+$(%)
	(0.1)	(0.2)	(0.4)	(0.5)	(0.5)	(0.4)	(0.2)	(0.1)	
Unskilled $\widehat{T}_\gamma^-$(%)	13.5	16.9	20.0	24.5	0.0	0.0	0.0	0.0	Skilled $\widehat{T}_\gamma^+$(%)
	(0.8)	(2.1)	(2.4)	(2.6)	(1.4)	(1.2)	(1.0)	(0.7)	
Alpha(% year)	-3.8	-3.5	-3.3	-3.1	2.9	3.1	3.1	3.5	Alpha(% year)
	(0.2)	(0.2)	(0.2)	(0.1)	(0.4)	(0.5)	(0.8)	(1.1)	
Exp.(% year)	1.3	1.3	1.3	1.3	1.1	1.1	0.9	0.9	Exp.(% year)
Turn.(% year)	73	71	69	70	61	76	78	75	Turn.(% year)

Table IA.VI

Short-term Performance across Investment Categories

We measure short-term performance with the unconditional four-factor model over non-overlapping five-year periods between 1977 to 2006 for three investment categories (Growth, Aggressive Growth, and Growth & Income) in Panels A, B, and C, respectively. The different estimates shown in the table are computed from the pooled alpha p-values across all five-year periods. In each panel, we display the estimated proportions of zero-alpha, unskilled, and skilled funds ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$) in the entire investment category. We also count the proportions of significant funds in the left and right tails of the cross-sectional t-statistic distribution ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20). In the leftmost columns, the significant group in the left tail, $\widehat{S}_\gamma^-$, is decomposed into unlucky and unskilled funds ($\widehat{F}_\gamma^-$, $\widehat{T}_\gamma^-$). In the rightmost columns, the significant group in the right tail, $\widehat{S}_\gamma^+$, is decomposed into lucky and skilled funds ($\widehat{F}_\gamma^+$, $\widehat{T}_\gamma^+$). Finally, we present the characteristics of each significant group ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$): the average estimated alpha (% per year), expense ratio (% per year), and turnover (% per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A. Growth Funds

	Proportion of Unskilled and Skilled Funds			
	Zero alpha ($\widehat{\pi}_0$)	Non-zero alpha	Unskilled ($\widehat{\pi}_A^-$)	Skilled ($\widehat{\pi}_A^+$)
Proportion	73.0 (2.3)	27.0	24.4(2.1)	2.6 (0.9)
Number	1,442	534	483	51

	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-$(%)	11.3	16.6	21.4	25.2	9.9	8.1	5.9	3.5	Signif. $\widehat{S}_\gamma^+$(%)
	(0.7)	(0.8)	(0.9)	(1.0)	(0.7)	(0.6)	(0.5)	(0.4)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.8	3.6	5.5	7.3	7.3	5.5	3.6	1.8	Lucky $\widehat{F}_\gamma^+$(%)
	(0.0)	(0.1)	(0.2)	(0.2)	(0.2)	(0.2)	(0.1)	(0.0)	
Unskilled $\widehat{T}_\gamma^-$(%)	9.5	13.0	15.9	17.9	2.6	2.6	2.3	1.7	Skilled $\widehat{T}_\gamma^+$(%)
	(0.7)	(0.9)	(1.0)	(1.1)	(0.8)	(0.7)	(0.6)	(0.4)	
Alpha(% year)	-6.0	-5.6	-5.2	-5.1	6.8	6.8	6.8	7.3	Alpha(% year)
	(0.3)	(0.2)	(0.2)	(0.1)	(0.3)	(0.4)	(0.6)	(0.9)	
Exp.(% year)	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	98	96	96	97	79	79	78	79	Turn.(% year)

Table IA.VI

Short-term Performance across Investment Categories (Continued)

Panel B. Aggressive Growth Funds

	Zero alpha ($\widehat{\pi}_0$)		Non-zero alpha		Unskilled ($\widehat{\pi}_A^-$)		Skilled ($\widehat{\pi}_A^+$)		
	Proportion of Unskilled and Skilled Funds								
Proportion	71.8 (4.2)		28.2		24.0 (3.8)		4.2 (1.7)		
Number	436		171		145		26		
	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-$(%)	12.0	16.0	19.4	22.2	11.2	9.4	7.1	4.9	Signif. $\widehat{S}_\gamma^+$(%)
	(1.1)	(1.4)	(1.6)	(1.8)	(1.3)	(1.1)	(1.0)	(0.8)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.8	3.6	5.4	7.2	7.2	5.4	3.6	1.8	Lucky $\widehat{F}_\gamma^+$(%)
	(0.1)	(0.2)	(0.3)	(0.4)	(0.4)	(0.3)	(0.2)	(0.1)	
Unskilled $\widehat{T}_\gamma^-$(%)	10.2	12.4	14.0	15.0	4.0	4.0	3.5	3.1	Skilled $\widehat{T}_\gamma^+$(%)
	(1.3)	(1.6)	(1.7)	(1.9)	(1.4)	(1.2)	(1.1)	(0.9)	
Alpha(% year)	-9.3	-8.6	-8.1	-7.6	8.5	8.8	9.7	9.7	Alpha(% year)
	(0.6)	(0.4)	(0.4)	(0.3)	(0.4)	(0.7)	(1.0)	(1.1)	
Exp.(% year)	1.5	1.5	1.5	1.5	1.4	1.3	1.3	1.3	Exp.(% year)
Turn.(% year)	116	113	111	109	105	104	107	104	Turn.(% year)

Panel C. Growth & Income Funds

	Zero alpha ($\widehat{\pi}_0$)		Non-zero alpha		Unskilled ($\widehat{\pi}_A^-$)		Skilled ($\widehat{\pi}_A^+$)		
	Proportion of Unskilled and Skilled Funds								
Proportion	74.1 (3.8)		25.9		25.9 (3.5)		0.0 (1.4)		
Number	540		188		188		0		
	Impact of Luck in the Left and Right Tails								
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. $\widehat{S}_\gamma^-$(%)	11.6	17.4	22.5	26.8	7.3	5.5	3.7	1.8	Signif. $\widehat{S}_\gamma^+$(%)
	(1.1)	(1.4)	(1.5)	(1.6)	(1.0)	(0.8)	(0.7)	(0.5)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.8	3.7	5.5	7.3	7.3	5.5	3.7	1.8	Lucky $\widehat{F}_\gamma^+$(%)
	(0.1)	(0.2)	(0.3)	(0.4)	(0.4)	(0.3)	(0.2)	(0.1)	
Unskilled $\widehat{T}_\gamma^-$(%)	9.8	13.7	17.0	19.5	0.0	0.0	0.0	0.0	Skilled $\widehat{T}_\gamma^+$(%)
	(1.2)	(1.5)	(1.7)	(1.8)	(1.1)	(0.9)	(0.8)	(0.5)	
Alpha(% year)	-4.9	-4.5	-4.2	-4.0	4.9	5.3	5.1	4.9	Alpha(% year)
	(0.3)	(0.2)	(0.2)	(0.1)	(0.5)	(0.6)	(0.8)	(1.3)	
Exp.(% year)	1.3	1.2	1.2	1.2	1.1	1.1	1.0	0.9	Exp.(% year)
Turn.(% year)	69	69	67	64	59	59	54	45	Turn.(% year)

Table IA.VII

Optimal Bayesian Decision implied by the FDR Target Levels

We examine the optimal Bayesian decision implied by five False Dicovery Rate (FDR^+) targets (z^+ =10%, 30%, 50%, 70%, and 90%) in a fund population where the proportions of zero-alpha, unskilled, and skilled funds (π_0, π_A^-, and π_A^+) are equal to 75%, 23%, and 2%, respectively. For each skill group, the fund estimated t-statistic is normally distributed and centered at -2.5, 0, and 3.0, respectively (with a unit variance). The significance threshold, $t^+(z^+)$ (related to the t-statistic), and significance level, $\gamma(z^+)$ (related to the p-value), are determined such that the FDR^+ of the portfolio is equal to the target z^+. The terms $\psi(z^+)$, $fnr^+(z^+)$, and $BE(z^+)$ denote the cost parameter, False Nondiscovery Rate, and loss function implied by the target value z^+.

FDR^+ target z^+	Signif. $t^+(z^+)$	Signif. $\gamma(z^+)$	Cost $\psi(z^+)$	$fnr^+(z^+)$	Loss $BE(z^+)$
10%	2.96	0.003	0.318	0.98%	0.38
30%	2.39	0.017	0.719	0.55%	0.56
50%	2.00	0.045	0.891	0.33%	0.46
70%	1.57	0.116	0.967	0.16%	0.28
90%	0.71	0.477	0.997	0.02%	0.07

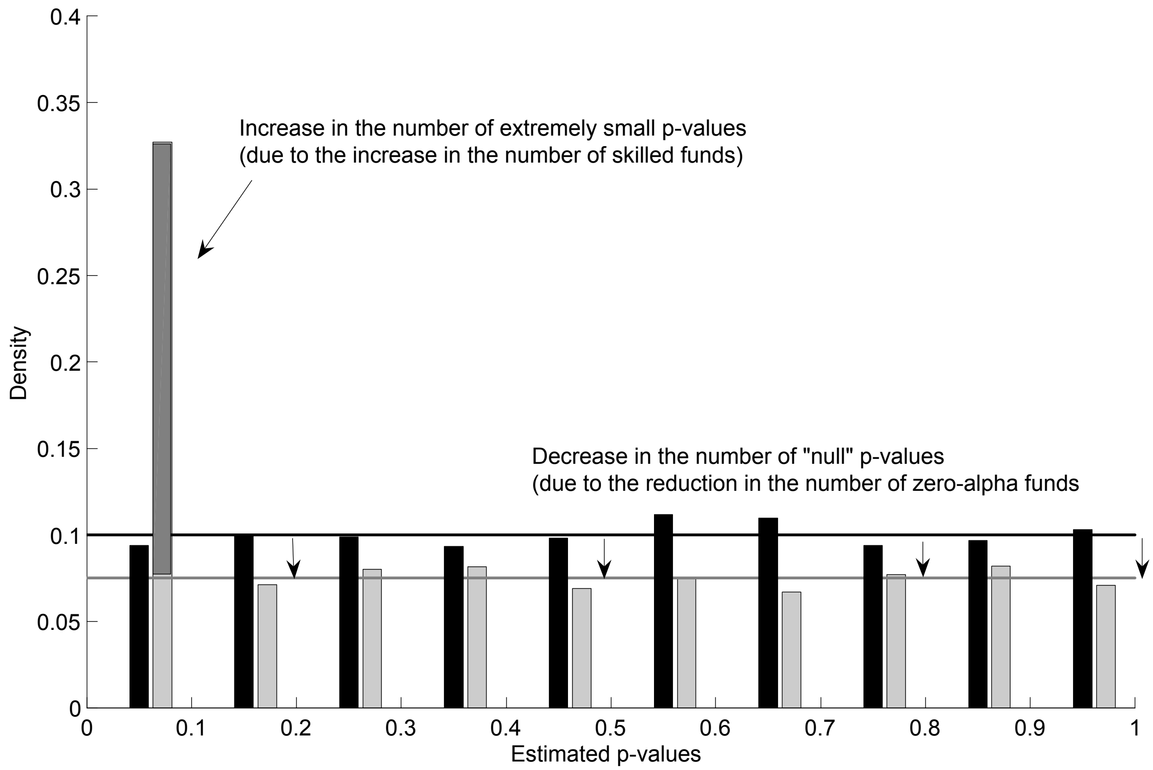

Figure IA.1. **Comparison of two p-value histograms.** This graph compares two p-value histograms of M=2,076 funds (as in our database). To plot these histograms, we draw each fund t-statistic from one of the distributions in Figure 1 of the paper (Panel A) according to the proportion of zero-alpha, unskilled, and skilled funds in the population (π_0, π_A^-, and π_A^+). We then compute the two-sided p-values of each fund from its respective t-statistic and plot them. For the first histogram (black bars), we assume that $\pi_0 = 100\%$, $\pi_A^- = 0\%$, and $\pi_A^+ = 0\%$ (i.e., there are only zero-alpha funds). For the second histogram (grey bars), we assume that $\pi_0 = 75\%$, $\pi_A^- = 0\%$, and $\pi_A^+ = 25\%$ (i.e., there are zero-alpha funds (75%), and skilled funds(25%)).

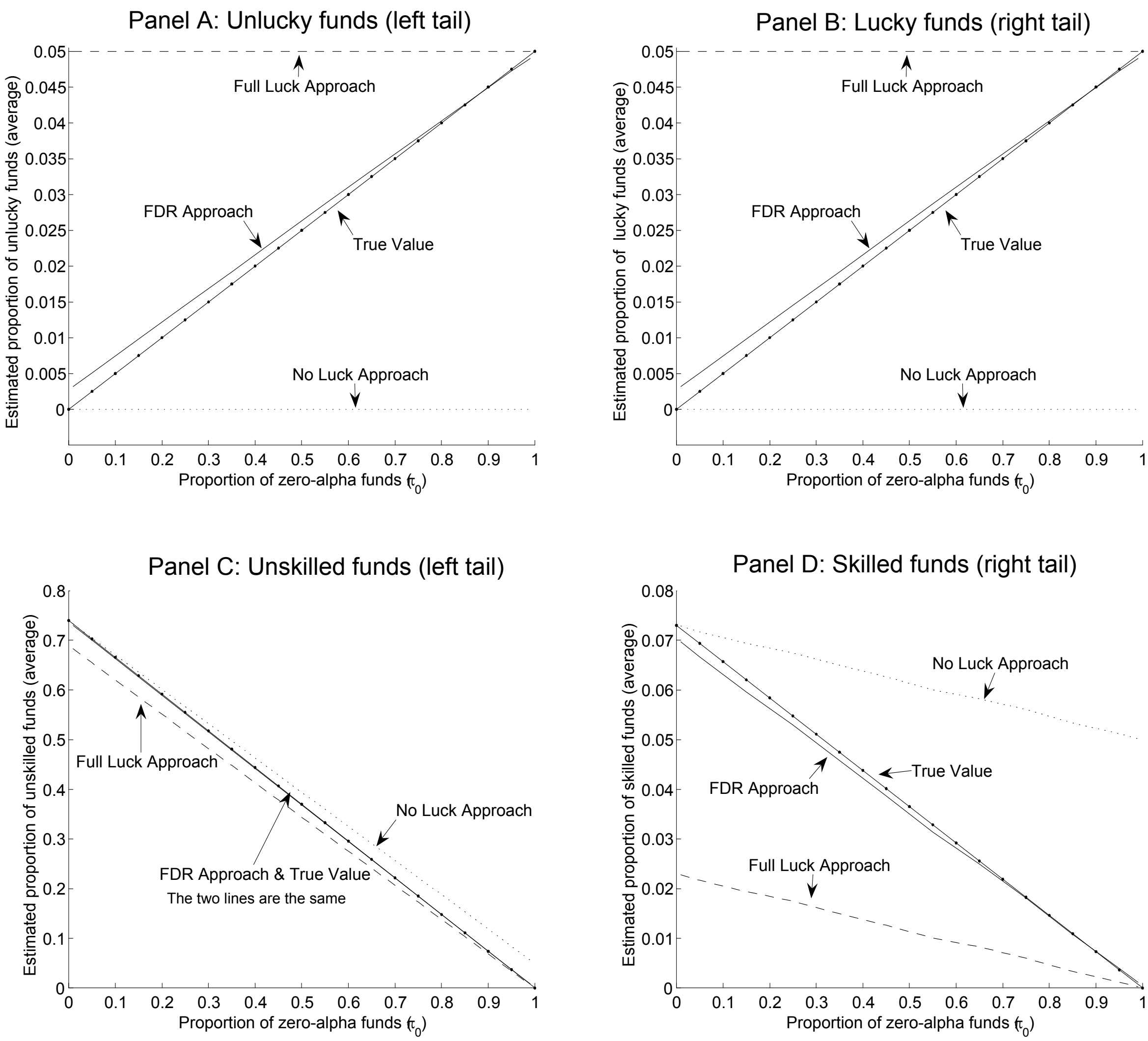

Figure IA.2. **Measuring luck: comparison with existing approaches (at $\gamma = 0.10$).** This figure examines the bias of different estimators produced by the three approaches (no luck, full luck, and FDR approach) as a function of the proportion of zero-alpha funds, π_0. We examine the estimators of the proportions of unlucky, lucky, unskilled, and skilled funds in Panel A, B, C, and D, respectively. The no luck approach assumes that π_0=0, the full luck approach assumes that π_0=1, while the FDR approach estimates π_0 directly from the data. For each approach, we compare the average estimator value (over 1,000 replications) with the true population value. For each replication, we draw the t-statistic for each fund i (i=1,...,2,076) from one of the distributions in Figure 1 of the paper (Panel A) according to the weights π_0, π_A^-, and π_A^+, and compute the different estimators at the significance level $\gamma = 0.10$. For each π_0, the ratio π_A^- over π_A^+ is held fixed at 11.5 (0.23/0.02) as in Figure 1 of the paper.

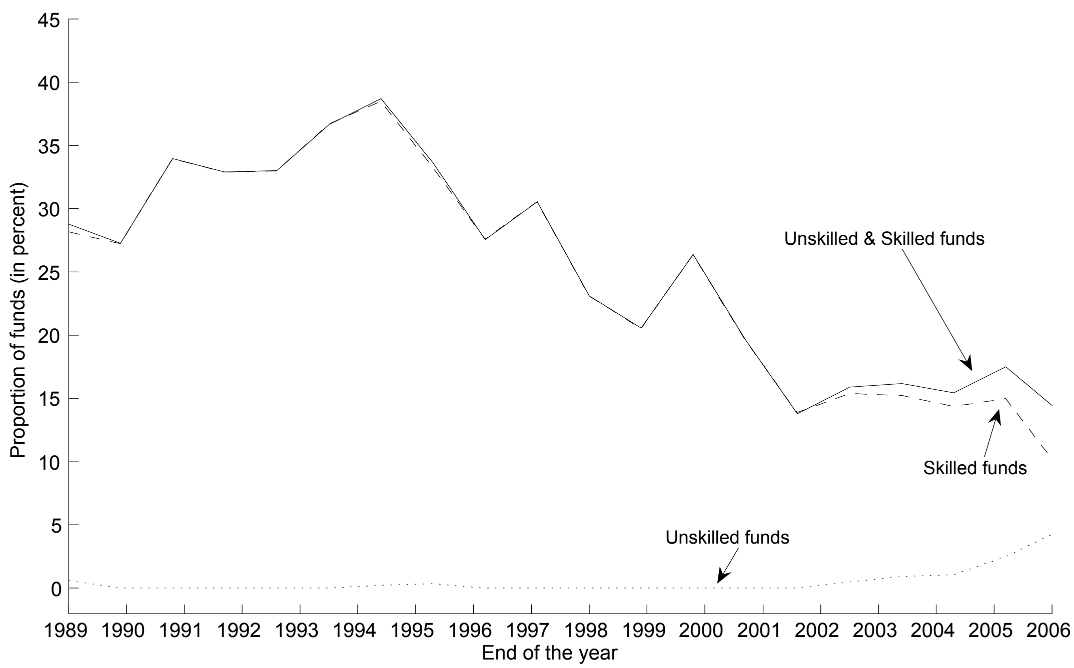

Figure IA.3. **Evolution of mutual fund pre-expense performance over time.** We plot the evolution of the estimated proportions of unskilled and skilled funds ($\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$) on a pre-expense basis between 1989 and 2006. At the end of each year, we measure $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ using the entire fund return history up to that point. The initial estimates at the end of 1989 cover the period 1975 to 1989, while the last ones in 2006 use the period 1975 to 2006. The performance of each fund is measured with the unconditional four-factor model. To compute pre-expense performance, we simply add the monthly expenses of each fund (1/12 times the most recent reported annual expense ratio) to its net returns.