Internet Appendix for "Stapled Finance"*

This Internet Appendix contains results for extensions of the main model.

IA.I We first analyze a model with **pure common values** (see page *IA*-2).

IA.II We then analyze an alternative auction setup, the **first-price auction** (see page *IA*-7).

IA.III Next, we show that our results extend to a setup with **more than two bidders** (see page *IA*-11).

IA.IV After that, we analyze a model with a different information structure, where **signals are affiliated with realized valuations** (see page *IA*-20).

IA.V Finally, we analyze a model in which **one of the bidders is a trade buyer**, say, a competitor, a supplier or a customer of the target firm. Trade buyers (also known as strategic buyers) plan to integrate the target into their existing operations if they win, so as to realize synergies. This makes it hard or impossible to later separate the target's cash flows and assets from those of the trade buyer's other operations, so if a trade buyer accepts the stapled finance she does not benefit from limited liability (her existing assets and cash flows support the debt too). We show that the results are unchanged: the seller always benefits from arranging stapled finance (see page *IA*-23).

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IA.I. A Model With Common Values

Suppose the valuation function is as follows:

$$V(s_i, s_j) = \frac{\varphi}{2}s_i + \frac{\varphi}{2}s_j + (1-\varphi)E[t]. \tag{IA1}$$

That is, the full-information value to a bidder can be her own signal realization, that of the other bidder, or that of an unobserved random variable that follows the same distribution. This is a pure common values model because given the signals, both bidders value the target firm equally. Further, both bidders receive signals that are somewhat, but not completely, informative.

We assume that f has full support on $\mathbb{R}_+$, that is, $f(s_i) > 0 \; \forall s_i > 0$.

Without stapled finance, it is a dominant strategy in the ascending auction to bid $V(s_i, s_i)$, that is, the bidder's valuation assuming the rival bidder realized the same signal (this maximizes the chances of winning with a positive expected net payoff, without affecting the winning price). With stapled finance, a bidder needs to compute the expected net payoff increase from having the option to accept the stapled finance.

LEMMA IA.1: *If $L > \varphi D + (1-\varphi)E\left[\min\{t, D\}\right]$, then the bidders plan to (and do) accept the stapled finance with certainty, irrespective of their signal realizations.*

Proof: Limited liability limits the expected repayment of the winner to

$$\frac{\varphi}{2}\min\{s_i, D\} + \frac{\varphi}{2}\min\{s_j, D\} + (1-\varphi)E\left[\min\{t, D\}\right], \tag{IA2}$$

which is no higher than $\varphi D + (1-\varphi)E\left[\min\{t, D\}\right]$. So, for any signal realization, the expected repayment is less than the amount borrowed. Q.E.D.

LEMMA IA.2: *There is no benefit from setting $L > \varphi D + (1-\varphi)E\left[\min\{t, D\}\right]$; the seller can equivalently set $L = \varphi D + (1-\varphi)E\left[\min\{t, D\}\right]$.*

Proof: The higher L increases all bids by the same amount and thus the net effect (higher price less larger expected loss on the loan) is nil. Q.E.D.

LEMMA IA.3: *If the seller sets $L > 0$, then it is (weakly) optimal to set $D > 0$.*

Proof: Setting $D = 0$ turns the loan into a pure subsidy, and the net benefit is zero: all bids are higher, raised exactly by the amount of the subsidy. Q.E.D.

Define the function $X(s_i, s_j)$, describing a bidder's expected net payoff from accepting the stapled finance, if her signal is s_i and her rival's is s_j:

$$X(s_i, s_j) = L - \frac{\varphi}{2}\min\{s_i, D\} - \frac{\varphi}{2}\min\{s_j, D\} - (1-\varphi)E\left[\min\{t, D\}\right], \tag{IA3}$$

$$X^+(s_i, s_j) = \max\{0, X(s_i, s_j)\}. \tag{IA4}$$

Also define a cutoff signal $\widehat{s}$ implicitly:

$$\widehat{s} = \min\{\; s \;\mid X(s, s) = 0 \;\}. \tag{IA5}$$

Note that $X(s_i, s_i)$ is decreasing in s_i if $s_i < D$ and constant if $s_i > D$. This implies that $\widehat{s} \leq D$. Also, since for any s_j bidder i's total valuation of winning $V(s_i, s_j) + X^+(s_i, s_j)$ is weakly increasing in s_i, a symmetric equilibrium bidding strategy in the ascending auction must be weakly increasing in s_i.

LEMMA IA.4: *The symmetric equilibrium bidding strategy is constant in s_i for $s_i \leq \widehat{s}$.*

Proof: Let b^* be an equilibrium symmetric bidding function. Suppose there exists an interval $[s_a, s_b] \subseteq [0, \widehat{s}]$, such that b^* is strictly increasing on $[s_a, s_b]$. If bidder i wins with a signal $s_i \in (s_a, s_b)$, then $s_j < s_i < \widehat{s}$, and bidder i will certainly accept the stapled finance. Anticipating this, a bidder with any signal $s_i \in (s_a, s_b)$ should stay in the bidding as long as the current price is below her expected valuation of winning (contingent on her current beliefs about the rival's signal realization, and including the expected net benefit from the option to accept the stapled finance). In other words, she should bid

$$B(s_i | s_i \in (s_a, s_b)) = V(s_i, s_i) + X(s_i, s_i) \tag{IA6}$$

(since the right-hand side is weakly increasing in s_i). Substituting on the right-hand side, we have

$$\begin{aligned} B(s_i | s_i \in (s_a, s_b)) =& \frac{\varphi}{2}s_i + \frac{\varphi}{2}s_i + (1-\varphi)E[t] + L - \frac{\varphi}{2}s_i - \frac{\varphi}{2}s_i - (1-\varphi)E\left[\min\{t, D\}\right] \\ =& (1-\varphi)E[t] + L - (1-\varphi)E\left[\min\{t, D\}\right] \\ =& L + (1-\varphi)E\left[\max\{0, t-D\}\right], \end{aligned} \tag{IA7}$$

which is constant in s_i. So the equilibrium bidding strategy cannot be strictly increasing for signals $s_i < \widehat{s}$. Q.E.D.

LEMMA IA.5: *A bidder with a signal $s_i < \widehat{s}$ should bid $V(\widehat{s}, \widehat{s}) = L + (1-\varphi)E\left[\max\{0, t-D\}\right]$. A bidder with a signal $s_i > \widehat{s}$ should bid $V(s_i, s_i) > V(\widehat{s}, \widehat{s})$.*

Proof: With a signal $s_i < \widehat{s}$, bidder i is certain to accept the stapled finance if she wins. Her valuation of winning is then

$$\begin{aligned} &\frac{\varphi}{2}s_i + \frac{\varphi}{2}E[s_j | s_j < \widehat{s}] + (1-\varphi)E[t] + L - \frac{\varphi}{2}s_i - \frac{\varphi}{2}E[s_j | s_j < \widehat{s}] - (1-\varphi)E\left[\min\{t, D\}\right] \\ &= (1-\varphi)E[t] + L - (1-\varphi)E\left[\min\{t, D\}\right] \\ &= L + (1-\varphi)E\left[\max\{0, t-D\}\right]. \end{aligned} \tag{IA8}$$

This bid is equal to $V(\widehat{s}, \widehat{s})$, since by definition $X(\widehat{s}, \widehat{s}) = 0$. With a signal $s_i > \widehat{s}$, a bidder should plan to stay in the auction until the current bid reaches her valuation of the target, conditional on the rival's signal being equal to s_i. If the rival drops out at a lower bid, bidder i may accept the stapled finance; but as the bid is raised and approaches $V(s_i, s_i)$, her probability of accepting the stapled finance becomes zero. Above $\widehat{s}$, the bidding strategy is thus increasing in s_i. Therefore, bidders with signals below $\widehat{s}$ cannot do better than bidding $V(\widehat{s}, \widehat{s})$. Q.E.D.

LEMMA IA.6: *Stapled finance can be effective only if its terms are set such that* $\widehat{s} > 0$.

Proof: If $\widehat{s} = 0$, then $X(s_i, s_j) \leq 0$ for all signal realizations s_i and s_j, and the winning bidder will prefer to decline the stapled finance offer (strictly if $s_i > 0$ or $s_j > 0$). Q.E.D.

PROPOSITION IA.1: *The seller benefits from arranging stapled finance if it is arranged such that* $\widehat{s} > 0$.

Proof: Distinguish three types of outcome. (*i*) If both bidders stay in the auction as the current bid goes above $V(\widehat{s}, \widehat{s})$, then the (eventual) winner is certain to decline the stapled finance and its availability will be of no further consequence. (*ii*) If both bidders drop out at $V(\widehat{s}, \widehat{s})$, then the (randomly chosen) winner's valuation of winning, including the expected value of the option to accept the stapled finance, is equal to the price that she must pay (the bid at which her rival exited, equal to her own). The bidders' expected net payoff in this outcome is thus zero: the seller extracts all rents. Since there cannot be any allocative distortions with pure common values, the seller benefits. (*iii*) If one bidder drops out at $V(\widehat{s}, \widehat{s})$, but the other bidder does not, then there are two effects. (1) The losing bid is higher than the losing bidder's expected bid in the absence of stapled finance if the expectation is taken over signals below $\widehat{s}$: the losing bid is equal to $V(\widehat{s}, \widehat{s}) > V(s_i, s_i)$. The seller benefits from this price increase. (2) The winning bidder may accept the stapled finance offer, which causes a loss to the lender that must be compensated by the seller. The winner accepts the stapled finance if

$$L - \frac{\varphi}{2} s_i - \frac{\varphi}{2} E[s_j | s_j < \widehat{s}] - (1 - \varphi) E\left[\min\{t, D\}\right] > 0. \tag{IA9}$$

The seller benefits if the expected price increase is larger than the expected loss on the stapled finance. The expected price increase is

$$L - \frac{\varphi}{2} E[s_j | s_j < \widehat{s}] - \frac{\varphi}{2} E[s_j | s_j < \widehat{s}] - (1 - \varphi) E\left[\min\{t, D\}\right]. \tag{IA10}$$

Clearly, for $s_i > \widehat{s}$, the price increase is larger than the winner's expected net payoff from accepting the stapled finance. So the winning bidder's expected net payoff is reduced if stapled finance is made available. Q.E.D.

In sum, the seller benefits from any stapled finance package whose probability of acceptance is strictly positive ex ante.

IA.II. First-Price Auction

Consider a first-price auction: the bidder who submitted the highest bid wins, and she pays her own bid to the seller; ties are resolved with a coin toss. As before, the decision regarding whether to accept or decline the stapled finance offer is made after the winner's identity and the price to be paid have been declared. At this stage, the decision depends only on s_i, L, and D. In fact, our analysis above remains valid, and we can use the same cutoff signal $\widehat{s}$.

LEMMA IA.7: *In the first-price auction with stapled finance, the following is an equilibrium bidding strategy:*

$$
\begin{aligned}
b_{FPA}(s_i) &= \varphi E\left[\max\{s,\widehat{s}\}\,|s<s_i\right] + (1-\varphi)E[t] \\
&= \begin{cases} \varphi \frac{F(\widehat{s})\widehat{s}+\int_{\widehat{s}}^{s_i} sf(s)ds}{F(s_i)} + (1-\varphi)E[t] & \textit{if} \quad s_i \geq \widehat{s} \\ \varphi\widehat{s} + (1-\varphi)E[t] & \textit{if} \quad s_i \leq \widehat{s} \end{cases}
\end{aligned} \tag{IA11}
$$

Proof: A bidder's expected net payoff is zero if $s_i \leq \widehat{s}$:

$$
\begin{aligned}
U(s_i \leq \widehat{s}) =& \frac{1}{2}F(\widehat{s})\left(\varphi s_i + (1-\varphi)E[t] - (\varphi\widehat{s} + (1-\varphi)E[t]) + x(s_i)\right) \\
=& \frac{1}{2}F(\widehat{s})\left(\varphi s_i - \varphi\widehat{s} + x(s_i) - x(\widehat{s})\right) \\
=& \frac{1}{2}F(\widehat{s})\left(\varphi s_i - \varphi\widehat{s} - \varphi s_i + \varphi\widehat{s}\right) \\
=& 0.
\end{aligned} \tag{IA12}
$$

Deviations to bids below $b_{FPA}(\widehat{s})$ are dominated because the probability of winning would zero. Further, bidder i with a signal $s_i < \widehat{s}$ will not deviate to $b_{FPA}(z)$, for some $z > \widehat{s}$, as she would nevertheless accept the stapled finance, in which case her expected net payoff with the bid $b_{FPA}(z)$ would be

$$
U(s_i \leq \widehat{s}\;,\; z > \widehat{s}) = F(z)\left(\varphi s_i + (1-\varphi)E[t] - b_{FPA}(z) + x(s_i)\right), \tag{IA13}
$$

which is negative since

$$
\begin{aligned}
&\varphi s_i + (1-\varphi)E[t] - b_{FPA}(z) + x(s_i) \\
&= \varphi s_i + (1-\varphi)E[t] - \varphi E\left[\max\{s,\widehat{s}\}\,|s<s_i\right] - (1-\varphi)E[t] + x(s_i) \\
&< \varphi s_i + (1-\varphi)E[t] - \varphi s_i - (1-\varphi)E[t] + x(s_i) \\
&= \frac{1}{F(\widehat{s})}U(s_i \leq \widehat{s}) \\
&= 0.
\end{aligned} \tag{IA14}
$$

If $s_i > \widehat{s}$, bidder i plans to decline the stapled finance offer irrespective of the price she has to pay. Her expected payoff is thus

$$
\begin{aligned}
U(s_i > \widehat{s}) =& F(s_i)\left(\varphi s_i + (1-\varphi)E[t] - b_{FPA}(s_i)\right) \\
=& F(s_i)\left(\varphi s_i + (1-\varphi)E[t] - \varphi\frac{F(\widehat{s})\widehat{s} + \int_{\widehat{s}}^{s_i} sf(s)ds}{F(s_i)} - (1-\varphi)E[t]\right) \\
=& \varphi\left(F(\widehat{s})(s_i - \widehat{s}) + \int_{\widehat{s}}^{s_i}(s_i - s)f(s)ds\right),
\end{aligned} \tag{IA15}
$$

which is strictly positive if $s_i > \widehat{s}$. Bidder i would not deviate by bidding below $b_{FPA}(\widehat{s})$, since doing so would reduce her probability of winning to zero. If she bid exactly $b_{FPA}(\widehat{s})$, her expected payoff would be

$$\begin{aligned} U(s_i > \widehat{s}\ ,\ z = \widehat{s}) =& \frac{1}{2}F(\widehat{s})\left(\varphi s_i + (1-\varphi)E[t] - b_{FPA}(\widehat{s})\right) \\ =& \frac{1}{2}F(\widehat{s})\left(\varphi s_i + (1-\varphi)E[t] - \varphi\widehat{s} - (1-\varphi)E[t]\right) \\ =& \frac{1}{2}F(\widehat{s})\varphi(s_i - \widehat{s}), \end{aligned} \tag{IA16}$$

which is strictly less than $U(s_i > \widehat{s})$. Finally, if she deviated to $b_{FPA}(z)$ for some $z > \widehat{s}$, her expected payoff would be

$$\begin{aligned} U(z > \widehat{s}) =& F(z)\left(\varphi s_i + (1-\varphi)E[t] - b_{FPA}(z)\right) \\ =& \varphi\left(F(\widehat{s})(s_i - \widehat{s}) + \int_{\widehat{s}}^{z}(s_i - s)f(s)ds\right), \end{aligned} \tag{IA17}$$

which is strictly less than $U(s_i > \widehat{s})$. Q.E.D.

PROPOSITION IA.2: *The seller benefits from arranging stapled finance if it is arranged such that $\widehat{s} > 0$.*

Proof: We need to analyze two cases: the winning bidder's signal is either below or above $\widehat{s}$. With a signal $s_i \leq \widehat{s}$, a bidder's probability of winning is $\frac{1}{2}$ if the rival's signal is also below $\widehat{s}$ (the bids are constant for all $s_i < \widehat{s}$, and a coin toss determines the winner if both signals are below $\widehat{s}$), and the probability is zero if it is the rival's signal is higher than $\widehat{s}$. If both signal realizations are below $\widehat{s}$, the bidders compete away their rents. However, stapled finance also distorts the allocation since the winning bidder is not necessarily the bidder with the highest valuation. Nevertheless, the seller benefits. Without stapled finance, the optimal bidding strategies are

$$b_{FPA}^{noD}(s_i) = \varphi E[s|s < s_i] + (1-\varphi)E[t]. \tag{IA18}$$

The expected price, contingent on both signals being below $\widehat{s}$, is thus

$$\varphi E[E[s|s < s_i]|s_i < \widehat{s}] + (1-\varphi)E[t]. \tag{IA19}$$

With stapled finance, the seller's expected net payoff, contingent on both signals being below $\widehat{s}$, is

$$\varphi\widehat{s} + (1-\varphi)E[t] - E[x(s_i)|s_i < \widehat{s}]. \tag{IA20}$$

An easier way to describe the seller's expected net payoff is to recall that the winner's expected net payoff is zero if both signals are below $\widehat{s}$. The seller's expected net payoff is therefore equal to the value created, which is (recall that a coin flip determines the winner's identity)

$$\varphi E[s_i|s_i < \widehat{s}] + (1-\varphi)E[t]. \tag{IA21}$$

This is clearly larger than $\varphi E[E[s|s < s_i]|s_i < \widehat{s}] + (1-\varphi)E[t]$. If the winning bidder's signal is $s_i > \widehat{s}$, her equilibrium bid with stapled finance is higher than in the absence of stapled finance. She will not accept the stapled finance, since $s_i > \widehat{s}$, so the seller benefits from arranging stapled finance. Q.E.D.

In sum, the seller benefits from any stapled finance package whose probability of acceptance is strictly positive ex ante.

Our results should equally apply to other auction formats. With private values, the decision to accept or decline the stapled finance depends on s_i and $\widehat{s}$, only. Put differently, the function $x(s_i)$ (which describes the winning bidder's expected gain from accepting the stapled finance) is the same for any auction type. So the accept-decline decision is the same for any auction type. Next, the slopes of $v(s_i)$ and $x(s_i)$ are such that when added, the terms with s_i cancel out — a bidder with a signal $s_i < \widehat{s}$ has a total valuation (including stapled finance) equal to that of a bidder with a realization $s_i = \widehat{s}$. These arguments are not based on the details of the auction, but are derived directly from the private values model and from the payoff schedule of debt contracts. The allocation must therefore be the same irrespective of the type of auction that a seller wants to use. Revenue equivalence then implies that any auction that maintains the same probability of a specific bidder winning and that gives a bidder with a signal realization $s_i = \underline{t}$ an expected payoff of zero (like the first-price auction does) should generate the same expected net payoff for the seller when paired with the same stapled finance offer.

IA.III. A Model With More Than Two Bidders

Assume that there are $N \geq 2$ bidders, indexed by i. Let $s_i \in [\underline{t}, \overline{t}]$. Assume $\forall s,\ f(s) > 0$. For simplicity, assume that $\varphi = 1$ (it is straightforward to generalize the proofs to $\varphi < 1$).

Suppose the seller chooses a stapled financing package such that for $s_i \leq \widehat{s}$, bidders bid $\widehat{s}$, and for $s_i > \widehat{s}$, bidders bid s_i in an ascending bid or a second-price auction (see equation (12) in the paper). Consider the expected payment from a specific bidder, say bidder 1 with signal s_1. Let $m(s_1)$ be the expected payment from this bidder. Also define Y_1 as the highest signal among the other $(N-1)$ bidders, which has a distribution of G. Thus, for all y, $G(y) = F^{N-1}(y)$ and $g(y) = (N-1)F^{N-2}(y)f(y)$.

If $s_1 > \widehat{s}$, then

$$\begin{aligned} m(s_1|s_1 > \widehat{s}) =& \Pr[\text{Win}]\ E\left[\max\left\{Y_1, \widehat{s}\right\}|Y_1 < s_1\right] \\ =& G(s_1)\left(\int_{\underline{t}}^{\widehat{s}} \widehat{s} g(y|y < s_1)dy + \int_{\widehat{s}}^{s_1} y g(y|y < s_1)dy\right) \\ =& G(s_1)\left(\int_{\underline{t}}^{\widehat{s}} \widehat{s}\frac{g(y)}{G(s_1)}dy + \int_{\widehat{s}}^{s_1} y\frac{g(y)}{G(s_1)}dy\right) \\ =& \int_{\underline{t}}^{\widehat{s}} \widehat{s} g(y)dy + \int_{\widehat{s}}^{s_1} y g(y)dy. \end{aligned} \tag{IA22}$$

If $s_1 \leq \widehat{s}$, then $\Pr[\text{Win}] = \frac{1}{N}$ and the winning bidder's payment minus the expected net benefit from the stapled finance is equal to s_1. Thus, for $s_1 \leq \widehat{s}$,

$$m(s_1|s_1 \leq \widehat{s}) = \frac{1}{N}G(\widehat{s})s_1. \tag{IA23}$$

The expected revenue received by the seller from bidder 1 is

$$\begin{aligned}
E\left[m(s_1)\right] &= \int_{\underline{t}}^{\bar{t}} m(s_1)f(s_1)ds_1 \\
&= \int_{\underline{t}}^{\widehat{s}} \frac{1}{N}G(\widehat{s})s_1 f(s_1)ds_1 + \int_{\widehat{s}}^{\bar{t}} \left(\int_{\underline{t}}^{\widehat{s}} \widehat{s}g(y)dy + \int_{\widehat{s}}^{s_1} yg(y)dy\right) f(s_1)ds_1 \\
&= \frac{1}{N}\int_{\underline{t}}^{\widehat{s}} G(\widehat{s})s_1 f(s_1)ds_1 + \int_{\widehat{s}}^{\bar{t}} \int_{\underline{t}}^{\widehat{s}} \widehat{s}g(y)dyf(s_1)ds_1 + \int_{\widehat{s}}^{\bar{t}} \int_{\widehat{s}}^{s_1} yg(y)dyf(s_1)ds_1 \\
&= \frac{1}{N}\int_{\underline{t}}^{\widehat{s}} G(\widehat{s})s_1 f(s_1)ds_1 + \widehat{s}G(\widehat{s})(1-F(\widehat{s})) + \int_{\widehat{s}}^{\bar{t}} \left(\int_{\widehat{s}}^{s_1} yg(y)dy\right) f(s_1)ds_1 \\
&= \frac{1}{N}\int_{\underline{t}}^{\widehat{s}} G(\widehat{s})s_1 f(s_1)ds_1 + \widehat{s}G(\widehat{s})(1-F(\widehat{s})) + \int_{\widehat{s}}^{\bar{t}} y\left(\int_{y}^{\bar{t}} f(s_1)ds_1\right) g(y)dy \\
&= \frac{1}{N}G(\widehat{s})\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1 + \widehat{s}G(\widehat{s})(1-F(\widehat{s})) + \int_{\widehat{s}}^{\bar{t}} y(1-F(y))g(y)dy. \qquad \text{(IA24)}
\end{aligned}$$

The seller's expected revenue is given by

$$\begin{aligned}
\Pi^{SF}(N,\widehat{s}) &= N\ E[m(s_1)] \\
&= N\frac{1}{N}G(\widehat{s})\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1 + N\widehat{s}G(\widehat{s})(1-F(\widehat{s})) + N\int_{\widehat{s}}^{\bar{t}} y(1-F(y))g(y)dy \\
&= G(\widehat{s})\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1 + N\widehat{s}G(\widehat{s})(1-F(\widehat{s})) + N\int_{\widehat{s}}^{\bar{t}} y(1-F(y))g(y)dy. \qquad \text{(IA25)}
\end{aligned}$$

Taking the derivative with respect to $\widehat{s}$, we have

$$\begin{aligned}
&\frac{\partial}{\partial \widehat{s}}\left(G(\widehat{s})\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1 + N\widehat{s}G(\widehat{s})(1-F(\widehat{s})) + N\int_{\widehat{s}}^{\bar{t}} y(1-F(y))g(y)dy\right) \\
=&g(\widehat{s})\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1 + G(\widehat{s})\widehat{s}f(\widehat{s}) \\
&+ NG(\widehat{s})(1-F(\widehat{s})) + N\widehat{s}g(\widehat{s})(1-F(\widehat{s})) - N\widehat{s}G(\widehat{s})f(\widehat{s}) \\
&- N\widehat{s}(1-F(\widehat{s}))g(\widehat{s}) \\
=&g(\widehat{s})\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1 - (N-1)\widehat{s}G(\widehat{s})f(\widehat{s}) + NG(\widehat{s})(1-F(\widehat{s}))
\end{aligned}$$

$$
\begin{aligned}
&= (N-1)F^{N-2}(\widehat{s})f(\widehat{s})\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1 - (N-1)\widehat{s}F^{N-1}(\widehat{s})f(\widehat{s}) + NF^{N-1}(\widehat{s})(1-F(\widehat{s}))\\
&= (N-1)F^{N-1}(\widehat{s})f(\widehat{s})\frac{\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1}{F(\widehat{s})} - (N-1)\widehat{s}F^{N-1}(\widehat{s})f(\widehat{s}) + NF^{N-1}(\widehat{s})(1-F(\widehat{s}))\\
&= F^{N-1}(\widehat{s})f(\widehat{s})(N-1)\frac{\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1}{F(\widehat{s})} - F^{N-1}(\widehat{s})f(\widehat{s})(N-1)\widehat{s} + F^{N-1}(\widehat{s})f(\widehat{s})N\frac{1-F(\widehat{s})}{f(\widehat{s})}\\
&= F^{N-1}(\widehat{s})f(\widehat{s})\left((N-1)\frac{\int_{\underline{t}}^{\widehat{s}} s_1 f(s_1)ds_1}{F(\widehat{s})} - (N-1)\widehat{s} + N\frac{1-F(\widehat{s})}{f(\widehat{s})}\right)\\
&= F^{N-1}(\widehat{s})f(\widehat{s})\left((N-1)\frac{[sF(s)]_{\underline{t}}^{\widehat{s}} - \int_{\underline{t}}^{\widehat{s}} F(s_1)ds_1}{F(\widehat{s})} - (N-1)\widehat{s} + N\frac{1-F(\widehat{s})}{f(\widehat{s})}\right)\\
&= F^{N-1}(\widehat{s})f(\widehat{s})\left((N-1)\widehat{s} - (N-1)\frac{\int_{\underline{t}}^{\widehat{s}} F(s_1)ds_1}{F(\widehat{s})} - (N-1)\widehat{s} + N\frac{1-F(\widehat{s})}{f(\widehat{s})}\right)\\
&= F^{N-1}(\widehat{s})f(\widehat{s})\left(N\frac{1-F(\widehat{s})}{f(\widehat{s})} - (N-1)\frac{\int_{\underline{t}}^{\widehat{s}} F(s_1)ds_1}{F(\widehat{s})}\right)\\
&= NF^{N-1}(\widehat{s})f(\widehat{s})\left(\frac{1-F(\widehat{s})}{f(\widehat{s})} - \frac{N-1}{N}\frac{\int_{\underline{t}}^{\widehat{s}} F(s_1)ds_1}{F(\widehat{s})}\right).
\end{aligned}
\tag{IA26}
$$

If $\widehat{s} = \underline{t}$, the FOC is equal to zero since $F(\widehat{s}) = 0$. If $\widehat{s} = \bar{t}$, the FOC is negative:

$$
\begin{aligned}
&\lim_{\widehat{s}\uparrow\bar{t}} NF^{N-1}(\widehat{s})f(\widehat{s})\left(\frac{1-F(\widehat{s})}{f(\widehat{s})} - \frac{N-1}{N}\frac{\int_{\underline{t}}^{\widehat{s}} F(s_1)ds_1}{F(\widehat{s})}\right)\\
&= Nf(\bar{t})\left(\frac{1-F(\bar{t})}{f(\bar{t})} - \frac{N-1}{N}\frac{\int_{\underline{t}}^{\bar{t}} F(s_1)ds_1}{1}\right)\\
&= Nf(\bar{t})\left(\frac{1-F(\bar{t})}{f(\bar{t})} - \frac{N-1}{N}\left([sF(s)]_{\underline{t}}^{\bar{t}} - \int_{\underline{t}}^{\bar{t}} s_1 f(s_1)ds_1\right)\right)\\
&= Nf(\bar{t})\left(\frac{1-F(\bar{t})}{f(\bar{t})} - \frac{N-1}{N}\left(\bar{t} - E[t]\right)\right).
\end{aligned}
\tag{IA27}
$$

(This is negative since $F(\bar{t}) = 1$ and $f(\bar{t}) > 0$.)

Finally, if $\widehat{s} \in (\underline{t}, \bar{t})$, the FOC is equal to zero if

$$
\frac{1-F(\widehat{s})}{f(\widehat{s})} - \frac{N-1}{N}\frac{\int_{\underline{t}}^{\widehat{s}} F(s_1)ds_1}{F(\widehat{s})} = 0. \tag{IA28}
$$

PROPOSITION IA.3: *If f is log-concave, then the optimal value of $\widehat{s}$ lies in the interior of $[\underline{t}, \bar{t}]$.*

Proof: The first term of (IA28) is the inverse of the hazard rate, so it is decreasing in $\widehat{s}$ since f is log-concave:

$$
\begin{aligned}
\frac{\partial}{\partial s}\frac{f(s)}{1-F(s)} &= \frac{f'(s)\left(1-F(s)\right)+\left(f(s)\right)^2}{\left(1-F(s)\right)^2} \\
&\propto \left(-f(s)\right)^2-\left(-f'(s)\right)\left(1-F(s)\right) \\
&= \left(\frac{\partial}{\partial s}\left(1-F(s)\right)\right)^2-\left(\frac{\partial^2}{\partial s^2}\left(1-F(s)\right)\right)\left(1-F(s)\right). \qquad \text{(IA29)}
\end{aligned}
$$

This is positive if $(1-F(s))$ is log-concave, which is the case if f is log-concave (see An (1998)). The second term of (IA28) is increasing in $\widehat{s}$ also because f is log-concave: a function $q(s)$ is log-concave if and only if (again, see An (1998))

$$
q''(s)q(s)-[q'(s)]^2 \leq 0. \qquad \text{(IA30)}
$$

Substitute $q(s)=\int_0^s F(t)dt$. We then have

$$
\begin{aligned}
\frac{\partial}{\partial s}\frac{\int_0^s F(t)dt}{F(s)} &= \frac{\frac{\partial}{\partial s}\left(\int_0^s F(t)dt\right)F(s)-\int_0^s F(s)dt\cdot f(s)}{F^2(s)} \\
&= \frac{F^2(s)-\int_0^s F(s)dt\cdot f(s)}{F^2(s)} \\
&\propto F^2(s)-\int_0^s F(s)dt\cdot f(s) \\
&\geq 0 \text{ if } \int_0^s F(s)dt \text{ is log-concave.} \qquad \text{(IA31)}
\end{aligned}
$$

The term $\int_0^s F(t)dt$ is log-concave if F is log-concave, which holds if f is log-concave. Now consider (IA28) evaluated at the boundaries of the support. First,

$$
\begin{aligned}
&\lim_{\widehat{s}\downarrow\underline{t}}\left(\frac{1-F(\widehat{s})}{f(\widehat{s})}-\frac{N-1}{N}\frac{\int_{\underline{t}}^{\widehat{s}}F(s_1)ds_1}{F(\widehat{s})}\right) \\
&= \lim_{\widehat{s}\downarrow\underline{t}}\frac{1}{f(\widehat{s})}-\frac{N-1}{N}\lim_{\widehat{s}\downarrow\underline{t}}\frac{F(\widehat{s})}{f(\widehat{s})} \text{ (using l'Hopital's Rule)} \\
&= \lim_{\widehat{s}\downarrow\underline{t}}\frac{1}{f(\widehat{s})}-\frac{N-1}{N}\lim_{\widehat{s}\downarrow\underline{t}}\frac{0}{f(\widehat{s})} \\
&= \lim_{\widehat{s}\downarrow\underline{t}}\frac{1}{f(\widehat{s})}, \qquad \text{(IA32)}
\end{aligned}
$$

which is positive since $f(\underline{t})>0$. This implies that while the FOC is equal to zero at $\widehat{s}=\underline{t}$, it is strictly positive for any $\widehat{s}=\underline{t}+\varepsilon$, for small $\varepsilon>0$. Hence, $\widehat{s}=\underline{t}$ is a local minimum of the seller's objective function. Next,

$$
\begin{aligned}
&\lim_{\widehat{s}\uparrow\bar{t}}\left(\frac{1-F(\widehat{s})}{f(\widehat{s})}-\frac{N-1}{N}\frac{\int_{\underline{t}}^{\widehat{s}}F(s_1)ds_1}{F(\widehat{s})}\right)\\
=&\lim_{\widehat{s}\uparrow\bar{t}}\frac{1-F(\widehat{s})}{f(\widehat{s})}-\frac{N-1}{N}\frac{\int_{\underline{t}}^{\bar{t}}F(s_1)ds_1}{1}\\
=&\lim_{\widehat{s}\uparrow\bar{t}}\frac{1-F(\widehat{s})}{f(\widehat{s})}-\frac{N-1}{N}\frac{\bar{t}-E[t]}{1}.
\end{aligned}
\tag{IA33}
$$

Since $f(\bar{t})>0$, this limit is negative. Thus, by continuity, there must exist a value $\widehat{s}^*\in(\underline{t},\bar{t})$ such that (IA28) is satisfied. Also, since the first term of (IA28) is decreasing in $\widehat{s}$ and the second term is increasing, this value must be unique. Finally, since the first-order condition is positive for all $\widehat{s}\in(\underline{t},\widehat{s}^*)$ and negative for all $\widehat{s}>\widehat{s}^*$, the value $\widehat{s}^*$ must be a global optimum. Q.E.D.

PROPOSITION IA.4: *If f is log-concave, then $\widehat{s}^*(N)$ is decreasing in N.*

Proof: Rearrange (IA28), which describes the optimal $\widehat{s}\in(\underline{t},\bar{t})$:

$$
\frac{f(\widehat{s}^*)}{1-F(\widehat{s}^*)}\frac{\int_{\underline{t}}^{\widehat{s}^*}F(s)ds}{F(\widehat{s}^*)}=\frac{N}{N-1}.
\tag{IA34}
$$

The right-hand side is decreasing in N. The left-hand side is increasing in $\widehat{s}^*$ if both fractions are increasing in $\widehat{s}^*$, which follows from the log-concavity of f. Q.E.D.

IA.III.A. An Alternative Proof Based on Reserve Prices

We now prove that it is always optimal to arrange stapled finance using an indirect argument: we first show that it is optimal to post a reserve price; we then argue that since arranging stapled finance weakly dominates posting a reserve price (for the seller), arranging stapled finance must be optimal. Note that the first result is standard (see, for example, Krishna (2002)), but we provide it here for completeness.

Again, assume that there are N bidders, that $\varphi=1$, and that $f(s_i)>0\ \forall s_i\in[\underline{t},\bar{t}]$.

Suppose the seller sets a reserve price of R in an ascending-bid or a second-price auction. Consider the expected payment of a specific bidder, say bidder 1 with signal s_1. Let $m(s_1)$ be the expected payment by this bidder. Also define Y_1 as the highest signal among the other $(N-1)$ bidders, which has a distribution of G. Thus, for all y, $G(y)=F^{N-1}(y)$. Now, if $s_1>R$, then

$$
\begin{aligned}
m(s_1)&=\Pr[\text{Win}]\ E[\max(Y_1,R)\,|Y_1<s_1]\\
&=\int_{\underline{t}}^{R}Rg(y)dy+\int_{R}^{s_1}yg(y)dy.
\end{aligned}
\tag{IA35}
$$

Given the reserve price, for $s_1 < R$, $\Pr[\text{Win}] = 0$ and $m(s_1) = 0$. The expected revenue received by the seller from bidder 1 is thus

$$\begin{aligned}
E[m(s_1)] &= \int_{\underline{t}}^{R} 0\ f(s_1)ds_1 + \int_{R}^{\bar{t}} (m(s_1))\, f(s_1)ds_1 \\
&= \int_{R}^{\bar{t}} \left(\int_{\underline{t}}^{R} Rg(y)dy \right) f(s_1)ds + \int_{R}^{\bar{t}} \left(\int_{R}^{s_1} yg(y)dy \right) f(s_1)ds \\
&= R \left(\int_{\underline{t}}^{R} g(y)dy \right) \left(\int_{R}^{\bar{t}} f(s_1)ds \right) + \int_{R}^{\bar{t}} y \left(\int_{y}^{\bar{t}} f(s_1)ds_1 \right) g(y)dy \\
&= RG(R)\,(1 - F(R)) + \int_{R}^{\bar{t}} y\,(1 - F(y))\, g(y)dy. \qquad \text{(IA36)}
\end{aligned}$$

The seller's expected revenue is given by

$$\Pi^{RP}(N, R) = NE\,[m(s_1)] + \underline{t}F^N(R) \qquad \text{(IA37)}$$

(the seller's valuation is $\underline{t}$ if the target is not sold). Substituting for $E\,[m(s_1)]$ from (IA36), we have

$$\Pi^{RP}(N, R) = N \left[RG(R)\,(1 - F(R)) + \int_{R}^{\bar{t}} y\,(1 - F(y))\, g(y)dy \right] + \underline{t}F^N(R). \qquad \text{(IA38)}$$

The first-order condition is give by

$$\begin{aligned}
\frac{\partial \Pi^{RP}(N, R)}{\partial R} &= N \left[\begin{array}{c} G(R)\,(1 - F(R)) + Rg(R)\,(1 - F(R)) - RG(R)f(R) \\ -R\,(1 - F(R))\, g(R) + \underline{t}F^{N-1}(R)f(R) \end{array} \right] \\
&= N\,[G(R)\,(1 - F(R)) - RG(R)f(R) + \underline{t}G(R)f(R)] \ \ (\text{using } G(R) = F^{N-1}(R)) \\
&= NG(R)\,[(1 - F(R)) - (R - \underline{t})f(R)]\,. \qquad \text{(IA39)}
\end{aligned}$$

Notice that the FOC is independent of N. The FOC (IA39) is satisfied at $R = \underline{t}$:

$$\begin{aligned}
\left.\frac{\partial \Pi^{RP}(N, R)}{\partial R}\right|_{R=\underline{t}} &= NG(\underline{t})\,[(1 - F(\underline{t})) - (\underline{t} - \underline{t})\, f(\underline{t})] \\
&= 0. \qquad \text{(IA40)}
\end{aligned}$$

At $R = \bar{t}$, the FOC is negative (since $f(\bar{t}) = 0$):

$$\begin{aligned}
\left.\frac{\partial \Pi^{RP}(N, R)}{\partial R}\right|_{R=\bar{t}} &= NG(\bar{t})\,\left[\left(1 - F(\bar{t})\right) - \left(\bar{t} - \underline{t}\right) f(\bar{t})\right] \\
&= -N\left[\left(\bar{t} - \underline{t}\right) f(\bar{t})\right]. \qquad \text{(IA41)}
\end{aligned}$$

For $R \in (\underline{t}, \bar{t})$, the FOC is satisfied if

$$(1 - F(R)) - (R - \underline{t})f(R) = 0$$
$$1 - (R - \underline{t})\frac{f(R)}{1 - F(R)} = 0. \tag{IA42}$$

Since $f(s) > 0\ \forall s$, we have that

$$\lim_{R \to \bar{t}} \left[1 - (R - \underline{t})\frac{f(R)}{1 - F(R)}\right] = \left[1 - \left(\bar{t} - \underline{t}\right) \frac{f(\bar{t})}{\lim_{R \to \bar{t}}\left[1 - F(R)\right]}\right] = -\infty \tag{IA43}$$
$$\lim_{R \to \underline{t}} \left[1 - (R - \underline{t})\frac{f(R)}{1 - F(R)}\right] = 1 - (0)\,\frac{f(\underline{t})}{1} = 1. \tag{IA44}$$

So by continuity, there must exist a value $R \in (\underline{t}, \bar{t})$ such that (IA42) is satisfied. Also, since f is log-concave, the hazard rate is increasing (see An (1998)). Therefore, the value $R \in (\underline{t}, \bar{t})$ that solves (IA42) is unique. Denote this value by R^*. It follows that the FOC is positive for all $R \in (\underline{t}, R^*)$ and negative for all $R > R^*$. Hence, the value R^* represents a global optimum. (The FOC is also equal to zero at $R = \underline{t}$, but since it is strictly positive for any $\widehat{s} = \underline{t} + \varepsilon$, for small $\varepsilon > 0$, the value $R = \underline{t}$ is a local minimum of the seller's objective function.)

We can now indirectly prove that the seller always benefits from providing stapled finance. We have shown that the seller always benefits from posting a reserve price. Next, arranging stapled finance with $\widehat{s} = R$ is weakly better for the seller than posting a reserve price (it generates a strictly higher expected net payoff if all valuations are below the threshold, and the same expected net payoff otherwise). Therefore, the seller always benefits from arranging stapled finance.

LEMMA IA.8: *For all N the optimal $\widehat{s} > \underline{t}$.*

Proof: Suppose not, that is, suppose that expected revenue is maximized at $\widehat{s} = \underline{t}$ (which is equivalent to $\widehat{s} < \underline{t}$), earning the seller an expected net payoff of $\Pi^{SF}(\widehat{s} = \underline{t})$. Then the seller could improve her expected net payoff by arranging stapled finance such that $\widehat{s} = R^*$, where R^* is the optimal reserve price. Her expected net payoff would then be

$$\begin{aligned}\Pi^{SF}(\widehat{s} = R^*) =& \Pi^{RP}(N, R^*) + \Pr[\max(s_1, \ldots, s_N) < R^*] E[s|s < R^*] \\ >& \Pi^{RP}(N, R^*) \\ >& \Pi^{RP}(N, R = \underline{t}) \text{ [From the optimality of } R^*] \\ =& \Pi^{SF}(\widehat{s} = \underline{t}).\end{aligned} \tag{IA45}$$

(Setting $\widehat{s} = R^*$ is likely suboptimal, so the expected net payoff can be increased further.) Q.E.D.

IA.IV. A Model in Which Valuations Are Affiliated with Signals

A target firm is for sale, and two bidders $i, j \in \{1, 2\}$ are interested in acquiring it. Bidder i privately observes a signal $s_i \in [\underline{t}, \overline{t}]$; the signals s_i and s_i are drawn from a distribution function f and they are i.i.d. After the end of the auction, a random variable θ is realized, with pdf g and support $\mathbb{R}_+$. Bidder i's final valuation (if she wins) is $V(s_i, \theta)$. Assume that $V_{s_i} > 0$, $V_\theta > 0$, and $V(\underline{t}, \cdot) = V(\cdot, 0) = 0$.

The seller arranges a stapled finance loan commitment before the auction starts: the winner has the right (but not the obligation) to borrow an amount L and repay an amount D after the valuation is realized. Assume that this is a non-recourse loan: if the realized valuation $V(s_i, \theta)$ is below D, then the winning bidder can default on the promised repayment (but she loses the realized value, $V(s_i, \theta)$). For given values of L, D, and s_i, the bidder will default if the realization of θ is sufficiently low. Denote by $\widehat{\theta}(L, D, s_i)$ the cutoff value of θ, defined implicitly by

$$V(s_i, \widehat{\theta}(L, D, s_i)) = D. \tag{IA46}$$

Bidder i will accept the loan if, given s_i, L, and D,

$$\int_0^\infty V(s_i, \theta) g(\theta) d\theta < L + \int_0^{\widehat{\theta}(L,D,s_i)} 0 \cdot g(\theta) d\theta + \int_{\widehat{\theta}(L,D,s_i)}^\infty \left(V(s_i, \theta) - D\right) g(\theta) d\theta, \tag{IA47}$$

which can be rewritten as

$$\int_0^{\widehat{\theta}(L,D,s_i)} V(s_i, \theta) g(\theta) d\theta + \int_{\widehat{\theta}(L,D,s_i)}^\infty D g(\theta) d\theta < L. \tag{IA48}$$

That is, the loan is accepted if the borrowed amount is higher than the promised repayment (contingent on observing the true signal s_i) — that is, bidder i expects the lender not to break even.

LEMMA IA.9: *Lower realizations of the signal s_i make a given loan offer (L, D) more attractive to bidder i.*

Proof: The left-hand side of (IA48) is increasing in s_i:

$$\int_0^{\widehat{\theta}(L,D,s_i)} V_{s_1}(s_i, \theta) g(\theta) d\theta + V(s_i, \widehat{\theta}(L, D, s_i))\ g(\widehat{\theta}(L, D, s_i)) \tfrac{\partial \widehat{\theta}(L,D,s_i)}{\partial s_i} - D\ g(\widehat{\theta}(L, D, s_i)) \tfrac{\partial \widehat{\theta}(L,D,s_i)}{\partial s_i}$$

$$= \int_0^{\widehat{\theta}(L,D,s_i)} V_{s_1}(s_i, \theta) g(\theta) d\theta \tag{IA49}$$

(The equality follows from $V(s_i, \widehat{\theta}(L, D, s_i)) = D$.) This integral is positive since $V_{s_i}(s_i, \theta) > 0$. Q.E.D.

For a given L and D, define $x(s_i)$, a bidder's valuation of the right to accept the stapled finance, in the presence of limited liability:

$$x(s_i) = \max \left\{ 0\ ,\ L - \int_0^{\widehat{\theta}(L,D,s_i)} V(s_i, \theta) g(\theta) d\theta - \int_{\widehat{\theta}(L,D,s_i)}^\infty D\ g(\theta) d\theta \right\} \tag{IA50}$$

We can define a cutoff value $\widehat{s} \in \mathbb{R}$ for bidder i's signal (again, given L and D) such that $x(s_i) > 0$ if $s_i < \widehat{s}$ and $x(s_i) < 0$ if $s_i > \widehat{s}$. It is defined implicitly by $x(\widehat{s}) = 0$,

$$\int_0^{\widehat{\theta}(L,D,\widehat{s})} V(\widehat{s},\theta)g(\theta)d\theta + \int_{\widehat{\theta}(L,D,\widehat{s})}^{\infty} Dg(\theta)d\theta = L. \tag{IA51}$$

The stapled finance offer weakly increases the valuation of winning, and therefore a bidder's bid is

$$b^D(s_i) = \int_0^{\infty} V(s_i,\theta)g(\theta)d\theta + x(s_i). \tag{IA52}$$

Depending on the realized signal s_i, the term $x(s_i)$ is either strictly positive (if $s_i < \widehat{s}$) or equal to zero (if $s_i \geq \widehat{s}$).

LEMMA IA.10: *The bidding strategies are strictly increasing in s_i.*

Proof: For $s_i \geq \widehat{s}$, we have $x(s_i) = 0$, and the bidding strategy is strictly increasing because $\int_0^\infty V(s_i,\theta)g(\theta)d\theta$ is increasing in s_i (since $V_{s_i}(s_i,\theta) > 0$). If $s_i < \widehat{s}$, then

$$b^D(s_i) = \int_0^{\infty} V(s_i,\theta)g(\theta)d\theta + L - \int_0^{\widehat{\theta}(L,D,s_i)} V(s_i,\theta)g(\theta)d\theta - \int_{\widehat{\theta}(L,D,s_i)}^{\infty} D\ g(\theta)d\theta. \tag{IA53}$$

Taking derivatives with respect to s_i (and recalling that $V(s_i,\widehat{\theta}(L,D,s_i)) = D$), we have

$$\frac{\partial}{\partial s_i} b^D(s_i) = \int_{\widehat{\theta}(L,D,s_i)}^{\infty} V_{s_i}(s_i,\theta)g(\theta)d\theta, \tag{IA54}$$

which is positive since $V_{s_i}(s_i,\theta) > 0$. Q.E.D.

PROPOSITION IA.5: *The seller benefits from arranging stapled finance if it is arranged such that $\widehat{s} > 0$.*

Proof: If both s_1 and s_2 are weakly larger than $\widehat{s}$, then offering stapled finance has no effect on the auction outcome. If $s_1 < \widehat{s} \leq s_2$ or $s_1 < \widehat{s} \leq s_2$, then the identity of the winner is not changed (since the bidding strategies are strictly increasing in s_i), but the winner's price is increased: the stapled finance offer increases the losing bid, and this bid is the winner's price. Finally, if both s_1 and s_2 are smaller than $\widehat{s}$, the identity of the winner is not affected (since the bidding strategies are strictly increasing in s_i), the winner's price is increased, and the winner accepts the stapled finance, expecting a strictly positive benefit (implying an expected loss to the investment bank, which the seller must compensate). Thus, we need to determine the *net* effect on the seller's expected payoff if both signal realizations are low. Denote (without loss of generality) the winning bidder by i. The winner's price increase is equal to the increase in the losing bid, $x(s_j)$. Her expected net benefit from accepting the stapled finance is $x(s_i)$. The net effect on the seller's expected payoff is thus $x(s_j) - x(s_i)$ (the winner's benefit $x(s_i)$ causes a loss to the seller, in equal amount). Since the winner had a higher signal realization, and since $x(s)$ is strictly decreasing in s for all $s < \widehat{s}$, the difference $x(s_j) - x(s_i)$ is strictly positive. This concludes the proof: for all signal realizations (s_i, s_j), offering stapled finance either benefits the seller, or her payoff is unaffected; and if $\widehat{s} > \underline{t}$, then the probability of the seller benefiting is strictly positive. Q.E.D.

In sum, the seller benefits from any stapled finance package whose probability of acceptance is strictly positive ex ante.

IA.V. Financial Buyers and Trade Buyers

In practice, some bidders are interested in a target firm because they own operations that are related, and they plan to realize synergies by combining their existing assets with the target's assets. Such trade buyers are different from financial buyers because after integrating the target into their other operations, the merged assets and cash flows are hard to disentangle. By integrating the target, trade buyers abandon the potential benefits of limited liability that financial buyers enjoy when accepting the stapled finance, since all assets and cash flows support debt that the merged entity issues. Intuitively, this makes stapled finance less attractive to a trade buyer, and thus stapled finance should become less beneficial for the seller. We therefore analyze a model with one financial buyer (bidder 1) and one trade buyer (bidder 2). As we show in this section, the presence of a trade buyer reduces the benefit from offering stapled finance, but it remains effective: the seller always benefits from arranging stapled finance.

Suppose the lender arranges stapled finance (L, D). The bidders' dominant strategy is to bid their valuation, including the value of the option to accept the stapled finance. As before, the bidders always accept the stapled finance if $L > D$, so without loss of generality we can focus on cases in which $L \leq D$. The trade buyer plans to always decline the stapled finance if $L \leq D$, since these terms are unattractive in the absence of limited liability. The financial buyer plans to accept or decline the offer depending on whether $x(s_1)$ is positive or negative. Our earlier analysis remains valid: for any L and D, we can define a cutoff signal $\widehat{s}$. The seller's problem of choosing the optimal values of L and D is equivalent to choosing the optimal cutoff signal $\widehat{s}$. Figure IA.1 shows the outcomes for different signal realizations if stapled finance is arranged

If $s_1 < \widehat{s}$, then bidder 1 bids $b^D(s_1) = v(\widehat{s})$; she wins with certainty if bidder 2's signal is below $\widehat{s}$ (Region A) and loses with certainty if bidder 2's signal is above $\widehat{s}$ (Region B). If $s_1 > \widehat{s}$, then the availability of stapled finance has no effect on the bids and therefore the seller's expected price is unchanged. The seller clearly benefits in Region B, since bidder 2's price is higher (it is $\varphi v(\widehat{s}) + (1-\varphi)E[t]$ instead of $\varphi E[s_1 | s_1 < \widehat{s}] + (1-\varphi)E[t]$). In Region A, however, the allocation is distorted since bidder 1 wins with certainty, so less value is created, and bidder 1 accepts the stapled finance, causing an expected loss for the lender that the seller must compensate ex ante. The seller must therefore design a stapled finance package such that the expected benefit in Region B outweighs the expected loss in Region A.

PROPOSITION IA.6: *With one financial buyer and one trade buyer in the pool of bidders, the seller benefits from arranging stapled finance if $\widehat{s}$ is above $\underline{t}$ but not higher than the median*

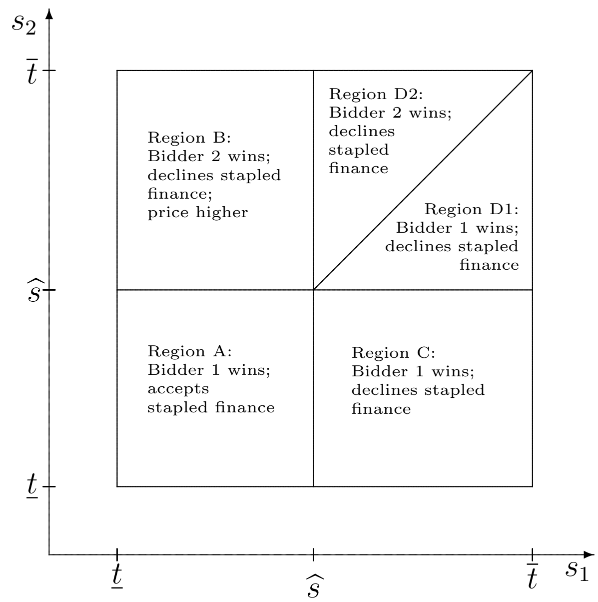

Figure IA.2. Auction outcomes with a financial buyer (bidder 1) and a trade buyer (bidder 2). The support of the bidders' signals s_1 and s_2 is $[\underline{t}, \bar{t}] \times [\underline{t}, \bar{t}]$. The trade buyer always declines the stapled finance offer. With a signal $\widehat{s}$, the financial buyer's expected net benefit $x(s_i)$ from accepting stapled finance is equal to zero. This cutoff defines the four regions A, B, C, and D. With a signal $s_1 < \widehat{s}$, the financial buyer bids above her expected valuation of the target, which affects the outcome of the auction as described in the four regions.

of the distribution F.

Proof: The seller's expected net payoff increase in Region B is

$$\begin{aligned} &F(\widehat{s})(1 - F(\widehat{s}))\left(v(\widehat{s}) - E[v(s_1)|s_1 < \widehat{s}]\right) \\ &= F(\widehat{s})(1 - F(\widehat{s}))\varphi\left(\widehat{s} - E[s_1|s_1 < \widehat{s}]\right). \end{aligned} \tag{IA55}$$

The expected value destruction (caused by the inefficient allocation) in Region A is

$$\begin{aligned} &F(\widehat{s})F(\widehat{s})\left(E[v(s_1)|s_1 < \widehat{s}] - E[\max\{v(s_1), v(s_2)\}|s_1, s_2 < \widehat{s}]\right) \\ &= F(\widehat{s})F(\widehat{s})\varphi\left(E[s_1|s_1 < \widehat{s}] - E[\max\{s_1, s_2\}|s_1, s_2 < \widehat{s}]\right), \end{aligned} \tag{IA56}$$

and the expected rent reduction for the bidders (from which the seller benefits) is

$$\begin{aligned}
&F(\widehat{s})F(\widehat{s})\left(E[\max\{v(s_1),v(s_2)\}|s_1,s_2<\widehat{s}]-E[\min\{v(s_1),v(s_2)\}|s_1,s_2<\widehat{s}]\right)\\
&\quad-F(\widehat{s})F(\widehat{s})\left(E[v(s_1)+x(s_1)|s_1<\widehat{s}]-E[v(s_2)|s_2<\widehat{s}]\right)\\
=&F(\widehat{s})F(\widehat{s})\left(E[\max\{v(s_1),v(s_2)\}|s_1,s_2<\widehat{s}]-E[\min\{v(s_1),v(s_2)\}|s_1,s_2<\widehat{s}]\right)\\
&\quad-F(\widehat{s})F(\widehat{s})\left(v(\widehat{s})-E[v(s_2)|s_2<\widehat{s}]\right)\\
=&F(\widehat{s})F(\widehat{s})\varphi\left(E[\max\{s_1,s_2\}|s_1,s_2<\widehat{s}]-E[\min\{s_1,s_2\}|s_1,s_2<\widehat{s}]+E[s_2|s_2<\widehat{s}]-\widehat{s}\right).
\end{aligned} \tag{IA57}$$

Adding the expected value destruction and the expected rent reduction for the bidders, we get the net effect on the seller's expected payoff (in Region A):

$$F(\widehat{s})F(\widehat{s})\varphi\left(E[s_1|s_1<\widehat{s}]-E[\min\{s_1,s_2\}|s_1,s_2<\widehat{s}]+E[s_2|s_2<\widehat{s}]-\widehat{s}\right). \tag{IA58}$$

The overall effect is (adding the effects for Regions A and B)

$$\begin{aligned}
\Delta\widetilde{\Pi}=&F(\widehat{s})F(\widehat{s})\varphi\left(E[s_1|s_1<\widehat{s}]-E[\min\{s_1,s_2\}|s_1,s_2<\widehat{s}]\right)\\
&+F(\widehat{s})(1-2F(\widehat{s}))\varphi\left(\widehat{s}-E[s_1|s_1<\widehat{s}]\right).
\end{aligned} \tag{IA59}$$

Since $E[s_1|s_1<\widehat{s}]\geq E[\min\{s_1,s_2\}|s_1,s_2<\widehat{s}]$ and $\widehat{s}\geq E[s_1|s_1<\widehat{s}]$, a sufficient (but not necessary) condition for stapled finance to be beneficial is that $F(\widehat{s})\in\left(0,\frac{1}{2}\right]$. ■

With one financial buyer and one trade buyer, the seller's trade-off is somewhat more complicated than in the case of two financial buyers, but it remains intuitive that arranging stapled finance is optimal. (Notice that setting $\widehat{s}$ below the median is a sufficient but not necessary condition.) The main benefit for the seller stems from her ability to extract a higher price from the trade buyer:

COROLLARY IA.1: *Assume that a trade buyer is competing with a financial buyer. Then the trade buyer's expected price (contingent on winning) is higher if the seller arranges stapled finance.*

Proof: In Region A of Figure IA.i, the trade buyer loses the chance of winning at low prices: In Region B, she wins, but at a higher price ($v(\widehat{s})$ instead of $E[v(s_1)|s_1<\widehat{s}]$). Regions C and D are not affected. Q.E.D.

Bargeron, Schlingemann, Stulz and Zutter (2008) report that trade buyers tend to pay higher prices in takeover contests when competing with financial buyers. They interpret this as evidence of agency problems or poor governance (leading to overbidding by the trade buyers), but our results show that it may merely be evidence of the sellers' sophistication in marketing targets to a wide pool of bidders, making use of stapled finance to increase competition. How stapled finance affects the financial buyers' expected prices, contingent on winning, is not clear. In Proposition 1 in the main article, we show that prices are higher if two financial buyers are competing. But the effect is ambiguous if a financial buyer is competing with a trade buyer: her probability of winning in Region A of Figure IA.1 increases to one, so her chance of winning with a low price increases; but she also pays more than her valuation of the target firm with a positive probability in that region, so the expected price may be increased.

Focusing on average realized winning prices may be misleading, however. Even if the financial buyer's expected winning price is higher when the seller arranges stapled finance, the financial buyer unambiguously benefits from it:

COROLLARY IA.2: *If a trade buyer and a financial buyer are competing, the arrangement of stapled finance is on average beneficial for the financial buyer and harmful for the trade buyer.*

Proof: The trade buyer loses her entire expected payoff in Region A of Figure IA.1, and the expected payoff is reduced in Region B because the winning price is higher. In Regions C and D, the expected payoff is unchanged. For the financial buyer, the only effects arise in Region A. Here, her expected net payoff increase is

$$\int_{\underline{t}}^{\widehat{s}} (v(s_1) + x(s_1) - v(s_2))f(s_2)ds_2 - \int_{\underline{t}}^{s_1} (v(s_1) - v(s_2))f(s_2)ds_2$$
$$= \varphi \int_{\underline{t}}^{\widehat{s}} (\widehat{s} - s_2)f(s_2)ds_2 - \varphi \int_{\underline{t}}^{s_1} (s_1 - s_2)f(s_2)ds_2. \tag{IA60}$$

This difference is positive for all $s_1 < \widehat{s}$. Q.E.D.

So it may happen that a financial buyer's expected price increases if stapled finance is offered, and yet she benefits on average because the stapled finance may be attractive and because she wins with certainty if $s_1 < \widehat{s}$. Average observed takeover prices are therefore not good indicators of whether bidders benefit or suffer from the introduction of stapled finance.

These results suggest that stapled finance is a less effective tool for rent extraction when the seller is facing a financial buyer and a trade buyer. The benefit remains obvious in Region B of Figure IA,1, where the trade buyer wins, there is no allocative distortion, the price is higher (because the financial buyer bids $v(\widehat{s}) > v(s_1)$), and the stapled finance offer is declined. In other words, there is no allocative distortion, and the bidders' rents are reduced, so the difference between the two (equal to the seller's expected net payoff) is increased. In Region A, an examination of (IA58) shows that the net effect of stapled finance is negative for the seller. Bidder 1 wins with certainty, even if her valuation is below that of the trade buyer. In part, this is beneficial for the seller, since Bidder 1 pays a high price if the trade buyer's valuation is high (if s_2 is close to $\widehat{s}$). However, the financial buyer accepts the stapled finance offer, causing a loss to the lender that requires ex ante compensation. The seller's expected net payoff is thus reduced in Region A and increased in Region B. Trading off the costs and benefits determines the optimal cutoff $\widehat{s}$. We now derive sufficient conditions for this optimal value of $\widehat{s}$ to be unique. We then compare it with the optimal value of $\widehat{s}$ when the seller is facing two financial buyers.

PROPOSITION IA.7: *Assume that f is log-concave, and that it has a nonnegative slope for all s_i below the median. Also assume that a trade buyer and a financial buyer are bidding for the target firm. Then there exists a unique optimal value of $\widehat{s} > \underline{t}$. It is the value of $\widehat{s}$ that solves*

$$(1-2F(\widehat{s}))F(\widehat{s}) = f(\widehat{s})\int_{\underline{t}}^{\widehat{s}} F(s_i)ds_i. \tag{IA61}$$

Proof: Substitute $\Pr\{A\}$ by $(F(\widehat{s}))^2$, and $\Pr\{B\}$ and $\Pr\{C\}$ by $F(\widehat{s})(1-F(\widehat{s}))$ in (IA59). After partial integration, we can rewrite (IA59) as

$$\Delta\widetilde{\Pi} = \varphi\int_{\underline{t}}^{\widehat{s}}\int_{\underline{t}}^{s_i} F(s_j)ds_j f(s_i)ds_i + (1-2\widehat{F})\varphi\int_{\underline{t}}^{\widehat{s}} F(s_i)ds_i. \tag{IA62}$$

The first-order condition is

$$\frac{\partial}{\partial\widehat{s}}\Delta\widetilde{\Pi} = \varphi(1-2F(\widehat{s}))F(\widehat{s}) - \varphi f(\widehat{s})\int_{\underline{t}}^{\widehat{s}} F(s_i)ds_i. \tag{IA63}$$

Rearranging (IA63), we have

$$\frac{1-2F(\widehat{s})}{f(\widehat{s})} - \frac{\int_{\underline{t}}^{\widehat{s}} F(s_i)ds_i}{F(\widehat{s})} = 0. \tag{IA64}$$

We now show that the first term in (IA64) is decreasing in $\widehat{s}$, and the second term is increasing in $\widehat{s}$, which implies that the FOC can be satisfied (the two terms are equal) for only one value of $\widehat{s} > \underline{t}$. The first term is decreasing over the relevant range (for $\widehat{s}$ below the median of f) since by assumption, $\frac{\partial}{\partial\widehat{s}}f(\widehat{s}) \geq 0$ for all $\widehat{s}$ below the median. The second term is weakly increasing in $\widehat{s}$ if

$$\frac{\partial}{\partial\widehat{s}}\frac{\int_{\underline{t}}^{\widehat{s}} F(s_i)ds_i}{F(\widehat{s})} = \frac{[F(\widehat{s})]^2 - f(\widehat{s})\int_{\underline{t}}^{\widehat{s}} F(s)ds}{[F(\widehat{s})]^2} \geq 0. \tag{IA65}$$

This is the case if $\int_{\underline{t}}^{\widehat{s}} F(s)ds$ is log-concave, which is implied by the log-concavity of F (see An (1998)). So the first term in (IA64) is decreasing over the relevant range of $\widehat{s}$, and the second term is increasing over this range. Hence, there can exist at most one value of $\widehat{s}$ where the two terms are equal and the first-order condition is satisfied (again, ignoring $\widehat{s} = \underline{t}$, which is a local minimum). Since the first-order condition is positive for low values of $\widehat{s}$ and negative at the median and above, an optimum must exist, and it must lie between $\underline{t}$ and the median. Q.E.D.

Two sufficient (but not necessary) conditions ensure the existence of a unique optimal value of $\widehat{s}$: the log-concavity of f (see Proposition 4 in the main article), and $\frac{\partial}{\partial s}f(s) \geq 0$ for all s below the median. The latter condition excludes distributions that are everywhere decreasing and many distributions that are positively skewed. Notice that neither of the two conditions is needed for the result that arranging stapled finance is optimal (see Propositions 3 and IA.6).

PROPOSITION IA.8: *Assume that f is log-concave, and that it has a positive slope for all s_i below the median. Then the seller arranges more aggressive stapled finance (with a higher $\widehat{s}$) if she is facing two financial buyers, compared with the stapled finance she offers when facing a trade buyer and a financial buyer.*

Proof: Compare the first-order conditions for the two models (see the proofs of Propositions 4 and IA.7), which both have unique solutions. Evaluate the left-hand side of (5, in the paper) at the value of $\widehat{s}$ that solves (IA63). The first-order condition (A5, appendix of the paper) is positive at that value, so the optimal $\widehat{s}$ must be higher if the seller faces two financial buyers instead of a trade buyer and a financial buyer. Q.E.D.

This result suggests that it is harder for the seller to extract additional value from the bidders if one is a trade buyer and the other a financial buyer. Making stapled finance available turns the financial buyer into a stronger competitor, letting the seller extract value from the trade buyer (see Corollaries IA.1 and IA.2), but the financial buyer benefits in part from having stapled finance available, so extracting value from her is harder. This is reflected in the seller's expected net payoff, depending on the mix of the pool of bidders:

COROLLARY IA.3: *The seller's expected net payoff is higher if she is facing two financial buyers than if she is facing a trade buyer and a financial buyer.*

Proof: Compare the seller's expected net payoff increases from arranging stapled finance (denoted by $\Delta\Pi$ and $\Delta\widetilde{\Pi}$) for the two setups (see A4 in the appendix of the paper) and (IA62) in the proofs of Propositions 4 and IA.7). The value of $\widehat{s}$ that is optimal when facing a trade buyer and a financial buyer is also feasible when facing two financial buyers, and clearly, $\Delta\Pi > \Delta\widetilde{\Pi}$ for all $\widehat{s}$ below the median. So when comparing $\Delta\Pi$ and $\Delta\widetilde{\Pi}$, evaluated at their respective optimal levels of $\widehat{s}$, we must have $\Delta\Pi > \Delta\widetilde{\Pi}$. Q.E.D.

It is again instructive to compare the effects of arranging stapled finance with the effects that setting a reserve price can achieve. Setting a reserve price is beneficial if it forces a high-valuation bidder to pay a higher price when defeating a low-value bidder. But, without stapled finance, setting a reserve price is costly if both bidders have low valuations and the target is not sold. Stapled finance, in contrast, guarantees that the target is sold, but not always at a high price. With one financial buyer and one trade buyer, if the trade buyer's signal s_2 is below $\widehat{s}$, then the financial buyer wins and pays a price of $v(s_2) < v(\widehat{s})$. So compared with the situation in which the seller faces two financial bidders, stapled finance is a less effective rent extraction tool.

There is no need to rely *exclusively* on one tool, however. Indeed, it is optimal to use both tools when facing a trade buyer and a financial buyer.

PROPOSITION IA.9: *When facing a trade buyer and a financial buyer, the seller can increase her expected payoff by posting a reserve price $v(\widehat{s})$. The seller's expected payoff is then the same as what she would expect when facing two financial buyers.*

Proof: Posting a reserve price $v(\widehat{s})$ affects the outcome only in Region A of Figure IA.1. The financial buyer (bidder 1) wins with certainty, but the winning price is increased to $v(\widehat{s})$, which is equal to bidder 1's overall valuation (including the benefit of accepting the stapled finance). So it remains incentive compatible for bidder 1 to bid $v(\widehat{s})$. The expected value created in Region A is the same as in the case of two financial buyers, $E[v(s_1)|s_1 < \widehat{s}]$, and both bidders expect a net payoff of zero in Region A, as they do in the model with two financial bidders. So the seller's expected net payoff in Region A (the difference between the value created and the bidders' expected payoffs) is therefore the same as in the case of two financial buyers. The outcomes in Regions B, C, and D are exactly the same as in the case of two financial buyers too. So the seller's overall expected net payoff must be the same as in the case of two financial buyers. Q.E.D.

We have shown in Corollary IA.3 that stapled finance is less effective if a seller is facing a trade buyer and a financial buyer. However, if the seller can also post a reserve price, then the expected net payoff is the same in both situations. Posting a reserve price normally requires an ability to commit not to sell the asset that is being auctioned if all bids are below the reserve price. Commitment is required, and sometimes difficult to establish, if the seller values the asset less than the bidders, even if the bidders' valuations are (relatively) low. In that case, it would be in the seller's ex post interest to sell the asset to the highest bidder, even if the price is below the reserve price. These commitment issues do not arise in our setting, because the financial bidder is always willing to bid at least $v(\widehat{s})$ and to buy the target firm at that price. Thus, posting a reserve price is beneficial in this setting and commitment is not an issue.

References

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