Internet Appendix to "The Internal Governance of Firms"*

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This online appendix derives model outcomes under the first-best case, the constrained efficient case, and the myopic CEO case. It also provides key steps in the proofs of Propositions 1 and 5.

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I. Derivation of the First-best, Constrained Efficient, and Myopic CEO Outcomes

A. First-best

The first-best outcome is investment and managerial learning pairs (k_t, s_t) $\forall\ t$ that maximize the sum of all current and future cash flows net of investment and learning effort:

$$\max_{\{k_t, s_t\}} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \left[\theta_{t+i} \left(k_{t+i-1}\right)^{\gamma} [f(s_{t+i-1}) + g(s_{t+i})] - (k_{t+i} - k_{t+i-1}) - s_{t+i} \right].$$

Let s_t^{FB} be the first-best level of learning effort whose first-order condition takes into account the effect of learning on this period's cash flow as well as next period's:

$$\theta_t (k_{t-1}^{FB})^{\gamma} g'(s_t^{FB}) + \frac{\theta_{t+1}}{1+r} \left(k_t^{FB}\right)^{\gamma} f'(s_t^{FB}) = 1.$$

This equation determines s_t^{FB} in terms of k_t^{FB}. Next, denote cash flows net of investment and learning as $V_{t+1}(k_t, k_{t+1}) = \left[\theta_{t+1} \left(k_t\right)^{\gamma} [f(s_t^{FB}(k_t)) + g(s_{t+1}^{FB}(k_{t+1}))] - (k_{t+1} - k_t) - s_{t+1}^{FB}(k_{t+1}) \right]$. The capital choice problem can then be rewritten as

$$\max_{k_t} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} V_{t+i}(k_{t+i-1}, k_{t+i}).$$

In turn, the first-order condition with respect to capital stock k_t is

$$\frac{\partial V_t(k_{t-1}, k_t)}{\partial k_t} + \frac{1}{(1+r)} \frac{\partial V_{t+1}(k_t, k_{t+1})}{\partial k_t} + \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} \left[\frac{\partial V_{t+i}(k_{t+i-1}, k_{t+i})}{\partial k_{t+i}} + \frac{1}{(1+r)} \frac{\partial V_{t+i+1}(k_{t+i}, k_{t+i+1})}{\partial k_{t+i}} \right] \frac{dk_{t+i}}{dk_t} = 0.$$

Note that these derivatives take into account the effect of s_t on k_t in cash flow V_t (but not of s_{t+1} on k_t in cash flow V_{t+1}, as that is captured in the dependence of s_{t+1} on k_{t+1}). Now, it is clear that recursively if we have $\left[\frac{\partial V_{t+i}(k_{t+i-1}, k_{t+i})}{\partial k_{t+i}} + \frac{1}{(1+r)} \frac{\partial V_{t+i+1}(k_{t+i}, k_{t+i+1})}{\partial k_{t+i}} \right] = 0\ \forall\ i > 1$, then indeed the first-order condition with respect to capital stock k_t is also $\frac{\partial V_t(k_{t-1}, k_t)}{\partial k_t} + \frac{1}{(1+r)} \frac{\partial V_{t+1}(k_t, k_{t+1})}{\partial k_t} = 0.$[1] This, in turn, is

$$-1 + \frac{1}{(1+r)} \left[\theta_{t+1}\, \gamma \left(k_t\right)^{\gamma-1} [f(s_t^{FB}) + g(s_{t+1}^{FB})] + 1 \right] +$$

$$\left[\theta_t \left(k_{t-1}\right)^{\gamma} g'(s_t^{FB}) + \frac{1}{(1+r)} \theta_{t+1} \left(k_t\right)^{\gamma} f'(s_t^{FB}) - 1 \right] \frac{ds_t^{FB}}{dk_t} = 0.$$

[1] Given the recursive structure of the problem, this essentially ensures that we can ignore the effect of this period's choice of capital and effort on next period's choice of these variables.

But given the managerial first-order condition that also maximizes the entire stream of cash flows, we have that $\left[\theta_t\left(k_{t-1}\right)^{\gamma}\ g'(s_t^{FB})+\frac{1}{(1+r)}\theta_{t+1}\left(k_t\right)^{\gamma}\ f'(s_t^{FB})-1\right]$=0. Thus, the capital stock is given by the condition

$$-1+\frac{1}{(1+r)}\left[\theta_{t+1}\,\gamma\left(k_t\right)^{\gamma-1}\,[f(s_t^{FB})+g(s_{t+1}^{FB})]+1\right]=0,$$

which can be rewritten as

$$k_t^{FB}=\left[\frac{\gamma\theta_{t+1}}{r}\left(f(s_t^{FB})+g(s_{t+1}^{FB})\right)\right]^{\frac{1}{1-\gamma}}.$$

B. Constrained Efficient Case: Long-term CEOs

Under the constrained efficient outcome, the CEO at time *t* maximizes the discounted sum of cash flows net of investment,

$$\max_{k_t}\sum_{i=0}^{\infty}\frac{1}{(1+r)^i}\left[\theta_{t+i}\left(k_{t+i-1}\right)^{\gamma}\,[f(s_{t+i-1})+g(s_{t+i})]-(k_{t+i}-k_{t+i-1})-s_{t+i}\right],$$

recognizing the moral hazard in the choice of managerial effort:

$$s_t\in\arg\max_{\hat{s}_t}\frac{1}{\left(1+r\right)}\left[\theta_{t+1}\left(k_t\right)^{\gamma}\,[f(\hat{s}_t)+g(s_{t+1})]-(k_{t+1}-k_t)\right]-\hat{s}_t.$$

In this case, note that there is moral hazard at the level of the manager when she invests in learning, so that the effects of learning on cash flows other than in the period when the manager is the CEO are ignored. The manager chooses s_t to maximize her future rents as the CEO. She maximizes

$$\frac{1}{1+r}\left[\theta_{t+1}\left(k_t\right)^{\gamma}\,[f(s_t)+g(s_{t+1})]-(k_{t+1}-k_t)\right]-s_t.$$

Differentiating and setting the result equal to zero, we get $\frac{\theta_{t+1}}{1+r}\left(k_t\right)^{\gamma}\ f'(s_t)=1$. Thus, if s_t^{CE} is the constrained efficient level of learning effort, then it satisfies

$$\frac{\theta_{t+1}}{1+r}\left(k_t\right)^{\gamma}\ f'(s_t^{CE})=1.$$

Now, since the CEO does not internalize the cost of managerial learning, the CEO must take into account the effect of his investment choice on managerial action, as that affects the firm's cash flows. Denote the managerial learning as $s_t^{CE}(k_t)$. Then, using the same recursive reasoning as in the case of the first-best, we obtain that the capital stock is given by the condition

$$-1+\frac{1}{(1+r)}\left[\theta_{t+1}\,\gamma\left(k_t\right)^{\gamma-1}\,[f(s_t^{CE})+g(s_{t+1}^{CE})]+1\right]+$$

$$\left[\theta_t\left(k_{t-1}\right)^{\gamma}\ g'(s_t^{CE})+\frac{1}{(1+r)}\theta_{t+1}\left(k_t\right)^{\gamma}\ f'(s_t^{CE})-1\right]\frac{ds_t^{CE}}{dk_t}=0.$$

Now, by the manager's first-order condition, we have that $\frac{\theta_{t+1}}{1+r}\left(k_t\right)^{\gamma}\ f'(s_t^{CE})=1$, so that the first-order condition for capital stock can be rewritten as

$$\theta_{t+1}\,\gamma\left(k_t\right)^{\gamma-1}\,[f(s_t^{CE})+g(s_{t+1}^{CE})]+(1+r)\theta_t\left(k_{t-1}\right)^{\gamma}\ g'(s_t^{CE})\frac{ds_t^{CE}}{dk_t}=r.$$

This equation gives the law of motion of capital. Note that totally differentiating the first-order condition for managerial learning and rearranging, we obtain $\frac{ds_t^{CE}}{dk_t} = \frac{-\gamma f'}{k_t f''}$, which is positive. Thus, in the constrained efficient case, the long-term CEO takes into account not just the direct effect of capital on cash flows but also its effect in terms of motivating managers who do not internalize the effect of their effort on current-period cash flows. Under specialized functions, more tractable expressions arise for the capital stock, as we will derive below.

C. Myopic CEO Case

The myopic CEO's income is extracted from the current period's cash flow net of investment:

$$\theta_t \left(k_{t-1}\right)^{\gamma} [f(s^{CEO}) + g(s_t)] - (k_t - k_{t-1}).$$

Differentiating with respect to k_t, we see that the CEO's marginal net return from investing is

$$\theta_t \left(k_{t-1}\right)^{\gamma} g' \frac{ds_t}{dk_t} - 1.$$

The manager chooses s_t to maximize her future rents as the CEO. She maximizes

$$\frac{1}{1+r}\left[\theta_{t+1}\left(k_t\right)^{\gamma} [f(s_t) + g(s_{t+1})] - (k_{t+1} - k_t)\right] - s_t.$$

Differentiating and setting the result equal to zero, we get $\frac{\theta_{t+1}}{1+r}\left(k_t\right)^{\gamma} f'(s_t) = 1$. As in the constrained efficient case, totally differentiating the first-order condition and rearranging, we obtain $\frac{ds_t}{dk_t} = \frac{-\gamma f'}{k_t f''}$, which is positive. It is clear that the managerial first-order condition is the same but the long-run CEO takes into account the more beneficial effects of capital investment on cash flows, so that the myopic CEO invests less than in the constrained efficient case.

D. Specializing Functions

Assume that the CEO and manager could each generate a cash flow *h(s)* if they were assigned all the tasks in the firm, depending on their learning *s*. The fraction of tasks assigned to the CEO is δ. The CEO's contribution to cash flows is $f(s) = \delta h(s)$, and the manager's contribution is $g(s) = (1-\delta)h(s)$. We set $h(s_t) = \frac{1}{b-1}(a+bs_t)^{\frac{b-1}{b}}$, with $a \geq 0$ and $b > 1$. To ensure convergence to steady state, we assume $1 - \gamma b > 0$. Under these assumptions, we obtain that $h'(s_t) = (a+bs_t)^{-\frac{1}{b}}$ and $h''(s_t) = \frac{-1}{b}(a+bs_t)^{\frac{-1}{b}-1}$ so that $\frac{h'(s_t)}{h''(s_t)} = -(a+bs_t)$.

We focus the following analysis on the steady state in which θ is constant over time, as are capital stock and managerial learning.

D.1. First-best Case

Under the specializing functions, the CEO's first-order condition takes the form

$$k^{FB} = \left[\frac{\gamma\theta}{r(b-1)}\left(a+bs\right)^{\frac{(b-1)}{b}}\right]^{\frac{1}{1-\gamma}},$$

and the manager's first-order condition takes the form

$$a + bs^{FB} = \left[\theta k^{\gamma}\left(\frac{\delta}{(1+r)} + (1-\delta)\right)\right]^{b}.$$

Substituting the second condition into the first, we obtain that

$$k^{FB} = \left[\frac{\gamma}{r}\frac{\theta^{b}}{(b-1)}\left(\frac{\delta}{1+r} + (1-\delta)\right)^{b-1}\right]^{\frac{1}{1-\gamma b}}.$$

D.2. Constrained Efficient Case: Long-term CEOs

Under the specializing functions, the CEO's first-order condition takes the form

$$k^{CE} = \left[\frac{\gamma\theta}{r}\left(\frac{1}{(b-1)} + (1+r)(1-\delta)\right)\left(a+bs\right)^{\frac{(b-1)}{b}}\right]^{\frac{1}{1-\gamma}},$$

and the manager's first-order condition takes the form

$$a + bs^{CE} = \left[\frac{\theta\delta}{(1+r)}k^{\gamma}\right]^{b}.$$

Substituting the second condition into the first, we obtain that

$$k^{CE} = \left[\frac{\gamma}{r}\frac{\theta^{b}\delta^{(b-1)}}{(b-1)(1+r)^{b-1}}\left(1 + (b-1)(1+r)(1-\delta)\right)\right]^{\frac{1}{1-\gamma b}}.$$

D.3. Myopic CEO Case

Under the specializing functions, the CEO's first-order condition takes the form

$$k^{SS} = \left[\gamma\theta(1-\delta)\left(a+bs\right)^{\frac{(b-1)}{b}}\right]^{\frac{1}{1-\gamma}},$$

and the manager's first-order condition takes the form

$$a + bs^{SS} = \left[\frac{\theta\delta}{(1+r)}k^{\gamma}\right]^{b}.$$

Substituting the second condition into the first, we obtain that

$$k^{SS} = \left[\gamma\left(1-\delta\right)\delta^{b-1}\frac{\theta^{b}}{(1+r)^{b-1}}\right]^{\frac{1}{1-\gamma b}}.$$

II. Decentralization of Tasks in an Internally Governed Firm

Our main result for the steady-state comparisons between the first-best, the constrained-efficient, and the myopic CEO cases can be summarized as follows.

PROPOSITION 1: *When the CEO has a long-term horizon, it is efficient for the CEO to make all cash flow-relevant contributions (*$\delta = 1$ *is optimal). When the CEO is myopic, firm value is maximized when the CEO's contribution to the firm's cash flows is neither too large nor too small relative to the manager's contribution (*$0 < \delta < 1$*).*

Proof: Using the steady-state outcomes derived in Section I of this Internet Appendix and comparing the constrained efficient case (long-horizon CEO) with the first-best, we get

$$\frac{k^{CE}}{k^{FB}} = \left[\left(\frac{\delta}{\delta+(1-\delta)(1+r)}\right)^{(b-1)}\left(1+(b-1)(1+r)(1-\delta)\right)\right]^{\frac{1}{1-\gamma b}}.$$

It is easy to see that this ratio is zero when $\delta = 0$, is increasing in δ, and reaches its maximum value of one when $\delta = 1$.

Similarly, comparing the myopic CEO case with the first-best based on steady-state outcomes in Section I, we obtain

$$\frac{k^{SS}}{k^{FB}} = \left[\frac{r\left(1-\delta\right)\delta^{b-1}}{\left(b-1\right)\left(\delta+(1-\delta)(1+r)\right)^{b-1}}\right]^{\frac{1}{1-\gamma b}}.$$

It is easy to see that this ratio is zero when $\delta = 0$ and when $\delta = 1$, is positive throughout the relevant range, and is therefore maximized in between. Indeed, it can be shown that the maximum is unique. To see this, we define $\psi(\delta) = \left[\frac{\left(1-\delta\right)\delta^{b-1}}{\left(\delta+(1-\delta)(1+r)\right)^{b-1}}\right]$.Then, taking logs and differentiating with respect to δ, it can be shown that $\psi'(\delta) = 0$ is equivalent to the quadratic equation $r\delta^2 - b(1+r)\delta + (b-1)(1+r) = 0$. This equation has two roots. After some algebra, it can be shown that one of the roots is between zero and one, and the other is greater than one. Since the relevant range is over zero and one, the maximum of $\frac{k^{SS}}{k^{FB}}$ is unique between zero and one.

Finally, note that

$$\frac{k^{SS}}{k^{FB}} = \left[\frac{r\left(1-\delta\right)(b-1)}{1+\left(b-1\right)(1-\delta)(1+r)}\right]^{\frac{1}{1-\gamma b}} < 1\forall\delta.$$

Q.E.D.

It can be similarly shown that cash flows and cash flows net of investment and managerial learning follow qualitatively the same properties as investment as stated in Proposition 1. The proofs of these additional results are available from the authors upon request.

III. Private Partnerships and Public Firms

Comparing the private (rolling) partnership, where each CEO sells the firm to the manager, to the publicly governed firm, where both have myopic CEOs, we obtain the following result.

PROPOSITION 5: *For external governance β sufficiently close to one and the CEO's contribution to cash flows δ also sufficiently close to one , we have $k^*\left(\beta\right) > k^{CE}$, $s^*\left(\beta\right) > s^{CE}$, and $CF^*\left(\beta\right) < CF^{CE}$, that is, the externally governed firm invests more, exerts employees more, and produces a smaller steady-state cash flow compared to a rolling partnership (the constrained efficient case).*

Proof: We focus the analysis on the case when $\beta \to 1$.The claim in the proposition will follow by continuity in the range for which β is sufficiently close to one. Recall that when

$\beta \to 1$, the CEO incurs no private cost of investment and hence investment is determined by the manager's participation constraint that the next period's cash flow net of rental payment on capital is only enough to cover their cost of learning in the current period. That is, $\frac{1}{1+r}\left(CF^* - rk^*\right) = s^*$, where in the steady state we have $CF^* = \theta\left(k^*\right)^\gamma \left[f\left(s^*\right)+g\left(s^*\right)\right]$. In other words, we can rewrite the steady-state capital as given by the implicit condition $k^* = \frac{1}{r}\left(CF^* - (1+r)s^*\right)$, where s^* is given by the managerial first-order condition $\frac{\theta}{1+r}\left(k^*\right)^\gamma f'\left(s^*\right) = 1$.

Now, under our specializing assumptions, the fraction of tasks assigned to the CEO is δ. The CEO's contribution to cash flows is $f(s) = \delta h(s)$, and the manager's contribution is $g(s) = (1-\delta)h(s)$. We set $h(s_t) = \frac{1}{b-1}(a+bs_t)^{\frac{b-1}{b}}$, with $a \geq 0$ and $b > 1$. To ensure convergence to steady state, we assume $1-\gamma b > 0$. Under these assumptions, we obtain that $h'(s_t) = (a+bs_t)^{-\frac{1}{b}}$ and $h''(s_t) = \frac{-1}{b}(a+bs_t)^{\frac{-1}{b}-1}$ so that $\frac{h'(s_t)}{h''(s_t)} = -(a+bs_t)$.

We obtain from the manager's first-order condition that $a + bs^* = \left[\frac{\theta\delta}{(1+r)}\left(k^*\right)^\gamma\right]^b$, which when substituted into the participation constraint yields the equation for the steady-state capital of the public firm whose external governance is perfect ($\beta \to 1$):

$$k^* = \frac{1}{r}\left(CF^* - (1+r)s^*\right) = \frac{1}{r}\left[\frac{\theta^b\delta^{(b-1)}\left(k^*\right)^\gamma}{b(b-1)(1+r)^{b-1}}\left(\delta + (1-\delta)b\right) + \frac{a(1+r)}{b}\right].$$

As we showed earlier in Section II of this Internet Appendix, under the same assumptions the steady-state capital for the constrained efficient case is given by

$$k^{CE} = \left[\frac{\gamma}{r}\frac{\theta^b\delta^{(b-1)}}{(b-1)(1+r)^{b-1}}\left(1+(b-1)(1+r)(1-\delta)\right)\right]^{\frac{1}{1-\gamma b}}.$$

While the rest of the proof can be shown more generally for $a \geq 0$ for sake of transparency of the argument we set $a = 0$. In this case, the equation for steady-state capital in the perfectly governed firm yields an explicit solution:

$$k^* = \left[\frac{1}{r}\frac{\theta^b\delta^{(b-1)}}{b(b-1)(1+r)^{b-1}}\left(\delta + (1-\delta)b\right)\right]^{\frac{1}{1-\gamma b}}.$$

The steady-state capital stock under perfectly governed firm and under the constrained efficient case are both zero when the CEO does not contribute to cash flows ($\delta = 0$) and are both increasing in the CEO's share δ. Comparing them yields the ratio:

$$\frac{k^*}{k^{CE}} = \left[\frac{1}{\gamma b}\frac{\left(\delta + (1-\delta)b\right)}{\left(1+(b-1)(1+r)(1-\delta)\right)}\right]^{\frac{1}{1-\gamma b}}.$$

Now, if the CEO's share of cash flow, δ , is one, this ratio exceeds one since $\gamma b < 1$. More generally, a limit argument can be used to show that as $\delta \to 1$, $\frac{(\delta+(1-\delta)b)}{(1+(b-1)(1+r)(1-\delta))} \to 1$:

$\forall \varepsilon > 0,\ \exists \mu > 0$ such that if $|\delta - 1| < \mu$ then $\left|\frac{(\delta+(1-\delta)b)}{(1+(b-1)(1+r)(1-\delta))} - 1\right| < \varepsilon$.[2]

Using the standard procedures, we get $\mu = \frac{\varepsilon[b+r(b-1)]}{r(b-1)}$. Note that this means that for every γ such that $\gamma b < 1$, there is a $\delta < 1$ such that the ratio $\frac{k^*}{k^{CE}}$ exceeds one. This relation between steady-state capital in the perfectly governed and constrained efficient cases carries over to managerial learning *s* and cash flows *CF*. The details are available upon request.

Q.E.D.

[2] Thanks to Maryam Farboodi for contributions to this proof.