Token-Based Platform Governance*

Joseph Abadi† Markus Brunnermeier‡

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Abstract

We develop a model to compare the governance of traditional shareholder-owned platforms to that of platforms that issue tokens. A traditional shareholder governance structure leads a platform to extract rents from its users. A platform that issues tokens for its services can mitigate this rent extraction, as rent extraction lowers the platform owners' token seigniorage revenues. However, this mitigation from issuing "service tokens" is effective only if the platform can commit itself not to dilute the "service token" subsequently. Issuing "hybrid tokens" that bundle claims on the platform's services and its profits enhances efficiency even absent ex-ante commitment power. Finally, giving users the right to vote on platform policies, by contrast, redistributes surplus but does not necessarily enhance efficiency.

Keywords: Utility Tokens, Platforms, Decentralized Finance, Corporate Governance

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†Federal Reserve Bank of Philadelphia (joseph.abadi@phil.frb.org)

‡Princeton University (markus@princeton.edu)

1 Introduction

The emergence of platforms that issue their own digital currencies or credits to users – called *tokens* – has offered an alternative to traditional platforms' model of financing and governance. Traditional platforms are owned and governed by shareholders (i.e., residual cash flow claimants). A platform's users, by contrast, do not necessarily play a role in financing or decision-making. Shareholders may not govern a traditional platform in users' best interests: they can exercise the platform's market power to extract rents from users. Proponents of cryptocurrency and decentralized finance (DeFi) argue that by giving users a stake in the system, token-financed platforms are more likely to be governed in accordance with users' preferences.

Tokens can offer users several types of claims or rights. Some platforms issue tokens that offer *transaction services* while retaining the traditional shareholder governance model. For example, the (centralized) Binance cryptocurrency exchange issues a token (BNB) that users can redeem to receive a discount on trading fees. Blockchain-based platforms often issue "utility tokens" that can be used to purchase a digital service: e.g., the Golem platform's tokens (GLM) can be used to rent out computational resources, and Chainlink's token (LINK) is used to pay network operators to retrieve data for smart contracts. Other tokens function essentially like shares of a traditional platform, granting *cash flow claims* and *voting rights*. The Kyber decentralized cryptocurrency exchange, for instance, issues a token (KNC) that can be "staked" to receive a share of the platform's revenues and to participate in governance. Still other tokens *bundle* transaction services with cash flow claims and/or voting rights. The archetypal example is a proof-of-stake cryptocurrency with "on-chain" governance, like Tezos or Algorand. On these blockchains, tokens play a dual role: users can hold them to transact with others, and "validators" can set tokens aside as collateral (called "staking") to verify blockchain transactions, earn monetary rewards, and vote on policies.

The advent of token-financed platforms raises key economic questions. Can token-financed platforms succeed in mitigating rent extraction and aligning policies with users' interests? How should tokens be designed to promote efficient platform governance? Should tokens grant transaction services, cash flows claims, voting rights, or some combination of features?

To answer these questions, we develop a unified model of a platform economy that is general enough to encompass traditional platforms as well as platforms that issue tokens with various features. The model is set in continuous time and has two groups of agents: users, who enjoy the platform's transaction services, and investors, who hold cash flow claims on the platform but do not engage in transactions.

In our benchmark model, the platform's policy consists of a transaction fee charged to users and (possibly) a rate of token seigniorage, when we consider a platform that issues tokens. These policies dictate the *split of surplus* between users and investors, since fees and seigniorage both represent transfers from users to investors, as well as *total surplus*, since the costs users face to use the platform determine their demand for its transaction services. Governance decisions (i.e., opportunities to vote on the platform's policies) occur periodically over time. Hence, there is *limited commitment* in governance: instead of committing to a full sequence of policies at $t = 0$, the platform can rewrite its policies in each governance decision.

The platform's *market power* generates scope for inefficiencies. The platform faces no competition, so it can set fees higher than the marginal cost of processing transactions without losing its user base. Importantly, fees are *distortionary* because the platform cannot fully extract surplus from users: higher fees lead to lower transaction volumes and deadweight losses.

For each of the platform designs we consider, there are two types of assets. There is a *transaction asset* that users must hold to receive the platform's transaction services: users' flow payoffs depend on their real balances of transaction assets, as in many models of platforms that issue tokens (Cong, Li, and Wang, 2021; Gryglewicz, Mayer, and Morellec, 2021). There is also a *cash flow asset* held by investors that grants claims on the platform's profits. Either type of asset can have voting rights, depending on the setting. We consider three different platform designs in this general environment.

- **Traditional platform**: Users transact with an asset that originates outside the platform, such as cash, deposits, or other liquid assets. The platform issues *shares* to investors that confer cash flow and governance rights (so shares are the cash flow asset). Investors choose the platform's policies to maximize its equity value.

- **Service tokens**: Users transact with *tokens* that are issued by the platform (so tokens are the transaction asset). Investors hold shares (the cash flow asset) that grant claims on profits. At first, we assume the platform maintains the shareholder governance model, but we later extend the model to permit token-holding users to vote as well.

- **Hybrid tokens**: The platform does not issue shares – it issues a *token* that serves as both a transaction asset and a cash flow asset. Users hold tokens in order to transact on the platform, whereas investors "stake" tokens to receive a claim on the platform's profits. Again, we begin by assuming that only the holders of staked tokens (investors) can vote, and then extend the model to permit users to vote as well.

We study the efficiency of each platform design in terms of total surplus vis-à-vis the first-best allocation.

Traditional platform: The shareholders of a traditional platform simply set fees to maximize the present value of profits without regard for user surplus. Consequently, as is typical of models with imperfect competition, the platform's fees are set higher than the marginal cost of processing transactions. Transaction volumes are therefore *inefficiently low* from a social perspective.

Service tokens: When the platform issues service tokens, investors choose both fees and the rate of seigniorage. They have a reason to internalize user surplus: the equilibrium price of tokens, and therefore the platform's seigniorage revenues from its initial token issuance, depends on the *service flow* that users expect to receive from tokens in the future (i.e., the marginal benefit of holding a token). Policies that enhance users' welfare (e.g., a promise of lower fees or less future seigniorage) increase service flows and thus token prices. If investors' ability to commit to future policies is strong enough, then the prospect of greater initial seigniorage revenues will incentivize them to choose policies that are more beneficial to users. Therefore, under commitment, investors will set lower fees than a traditional platform, leading to greater transaction demand. The equilibrium outcome is unambiguously more efficient than in the traditional case.

However, if investors' ability to commit to future policies is weak, then this logic breaks down. Investors no longer have as strong an incentive to pass policies that benefit users: they can frequently rewrite policies and do not internalize any reduction in value of tokens issued *in the past*. Instead, users bear those costs. Investors are tempted to set fees too high and over-issue tokens to boost seigniorage revenues. This temptation is so severe that in the limit of no commitment, there does not exist an equilibrium in which tokens are valued: realizing that investors will attempt to extract high rents in the future, users are unwilling to purchase tokens in the first place. Commitment is thus crucial for service tokens to enhance efficiency on their own. Of course, there are several mechanisms that could enhance platform owners' commitment to policies that benefit users, such as token retention or smart contracts that pre-program a specified sequence of policies. We show, however, that so long as investors have limited commitment power, a hybrid token that *bundles transaction services with cash flow claims* can serve as an effective substitute for commitment.

Hybrid tokens: A hybrid token is held by users for its transaction services and staked by investors for its cash flows. Equilibrium token prices reflect both their *service value* (the present value of service flows, which is users' valuation) and their *cash flow value* (the present value of dividends, which is investors' valuation). Even when investors govern the platform without strong commitment to future policies, equilibrium outcomes are unambiguously more efficient than in the traditional platform case. In fact, equilibrium governance decisions are precisely the same as in the case of a platform that issues service tokens with full commitment.

Why does a hybrid token overcome the time-inconsistency problem? The key intuition is that investors hold an asset whose value reflects users' future service flows, so investors bear part of the costs if they pass policies that harm users. When a platform issues service tokens to users without commitment, by contrast, investors may seek to pass policies that increase the value of their equity while reducing token prices.

Extensions: We then extend the model to accommodate token-issuing platforms that permit users to vote, rather than just investors. We consider both a platform that issues service tokens and one that issues hybrid tokens. When users can participate in governance, they receive a greater share of total surplus, but equilibrium outcomes are not necessarily more efficient. If users acquire a majority share of voting power, they pass policies that increase their surplus at the expense of lower profits – transaction quantities may be inefficiently *high*. Intuitively, simply *reallocating voting power* does not cause users to internalize investor welfare. By contrast, *bundling cash flow claims with transaction services* causes investors to internalize user surplus, unlike the owners of a traditional platform.

We also consider an extension in which we add investment to the model: the platform can expend resources to improve the quality of its transaction technology, increasing users' marginal utility of transacting. In this setting, a traditional platform under-invests. Since the platform cannot fully extract user surplus, investors forgo some socially efficient investments that would benefit users at the expense of lower platform profits. Token issuance alleviates the under-investment problem because investors internalize some of the benefits of investment that accrue to users. Importantly, the ability to finance investments by issuing tokens is non-neutral: token issuance is not a re-tranching of the platform's cash flows, so the Modigliani-Miller theorem does not apply. Unlike share issuance, token issuance can finance investments that benefit users while reducing the platform's future profits. Hence, our model's insights about the benefits of token issuance extend beyond our benchmark environment, in which the platform's policies mainly concern the pricing of its services.

Organization. In the remainder of this section, we give a review of the related literature. Section 2 gives a brief overview of the types of tokens issued by platforms in practice. Section 3 introduces the economic environment and other preliminary elements of the benchmark model. Section 4 studies the governance of a traditional platform as a benchmark. Section 5 introduces service tokens and outlines how token issuance affects equilibrium governance decisions. The hybrid token scheme is analyzed in Section 6. Sections 7 and 8 lay out extensions with user voting and investment, respectively. Section 9 discusses the model's main assumptions. Section 10 concludes. All proofs are in the Appendix.

Related literature. Our paper is most closely related to the emerging literature that studies the role of tokens in DeFi platforms' governance. In the context of a platform with network externalities, Sockin and Xiong (2023) study the introduction of a token that grants platform membership and permits users to vote on platform policies, preventing the platform from exploiting their data. However, users are not able to share the costs of investments in the platform and therefore cannot subsidize the admission of new users to the platform. Bakos and Halaburda (2023) study a platform that issues tokens that offer cash flow claims and voting rights. They highlight conditions under which token holdings become concentrated among non-users, leading to rent extraction. Similarly, Han, Lee, and Li (2023) develop and empirically test a model in which concentrated token holdings by a large investor can undermine efficient governance. Relatedly, Bena and Zhang (2023) and Gan, Tsoukalas and Netessine (2023) compare the inefficiencies in governance of a platform that issues service tokens to those of a traditional platform. While our analysis shares some of these themes, it is complementary: we characterize the separate roles of tokens' transaction services, cash flow claims, and voting rights, providing novel insights into the optimal *design* of tokens.

The broader literature on financing through token sales and ICOs is also related to our work. Closest to our paper, Goldstein, Gupta, and Sverchkov (2022) show that by issuing utility tokens, a platform can commit to charge lower prices to users, as in our model. Their mechanism, however, is related to the Coase (1972) conjecture and is quite distinct from ours. Gryglewicz, Mayer, and Morellec (2021) and Cong, Li, and Wang (2022) study the optimal issuance of tokens by a financially-constrained platform, demonstrating how seigniorage policies can be used to reward platform owners for investments. Li and Mann (2018), Chod and Lyandres (2021), and Lee and Parlour (2021) study other reasons why firms might finance themselves through the issuance of utility tokens. Li and Mayer (2022) and d'Avernas, Maurin, and Vandeweyer (2022) present models to study the optimal issuance of stablecoins. Similarly, You and Rogoff (2023) study how the tradability of a platform's utility tokens affects the revenue raised by a token offering. Relative to this literature, our paper differs in that it considers the role of tokens *exclusively* for governance – there are no financial frictions that motivate token issuance.

Of course, our paper connects to the corporate governance literature. There is an extensive body of work on control rights, ownership structure, and the theory of the firm stemming from the work of Coase (1937), Williamson (1979), and Grossman and Hart (1986). Our paper contributes to this literature by characterizing the specific governance consequences brought about by different token designs – we show that despite the fact that users can potentially be exploited by the platform, it is not always most efficient to give them control rights (Hansmann, 1988). Our work is complementary to the literature that studies how different control structures aggregate information in governance decisions (see Aghion and Tirole, 1997, among many others). Recent work has extended this literature to the study of

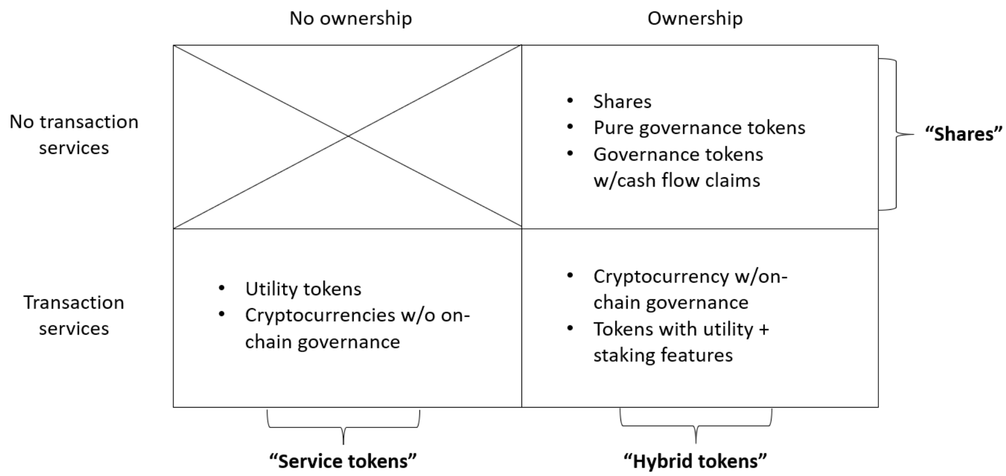

Figure 1: A taxonomy of the types of tokens issued by platforms in practice. Tokens are categorized by whether they confer ownership in the platform (cash flow claims and/or voting rights) and whether they offer transaction services. We also indicate how we refer to each type of token in the model.

DeFi platforms (Tsoukalas and Falk, 2020; Benhaim, Falk, and Tsoukalas, 2023).

2 An Overview of Tokens

Before introducing the model, we briefly outline the different claims or rights that tokens may confer and examples of tokens that are issued in practice. Tokens typically grant at least one of (1) claims on a platform's transaction services, (2) claims on cash flows, or (3) voting rights. Figure 1 provides a taxonomy of tokens based on whether they provide transaction services and whether they confer platform ownership (cash flow claims and/or voting rights). This section discusses each class of tokens in this taxonomy as well as how the different types of tokens used in practice relate to the assets in the model.

Transaction services only: Tokens that grant only claims on transaction services are typically referred to as *utility tokens*. Examples are the Binance token (BNB) or the Golem token (GLM) discussed in the Introduction, which users can redeem for specific services or perks. However, tokens can facilitate transactions on a platform even if they are not redeemable for any particular service. In the context of our model, a *pure cryptocurrency* that has no intrinsic value but is used for transactions among a platform's users could also be viewed as a token that offers an implicit claim on the platform's transaction processing services. Even outside of DeFi, some platforms have begun to issue, or have considered issuing, their own currencies (e.g., Alibaba's "Alipay" or Facebook's now-defunct Libra/Diem project).[1]

Ownership features only: Other tokens grant claims on the platform's cash flows (e.g., transaction fees or seigniorage revenues).[2] Usually, such tokens have voting rights in governance decisions as well. The (previously discussed) KNC token issued by the Kyber decentralized exchange (DEX) is a leading example. Kyber provides automated cryptocurrency market-making services and collects transaction fees from traders. Token holders can "stake" their tokens in order to receive a share of these fees. They may also participate in *on-chain governance*: the community regularly votes on referenda that determine the platform's policies, including fees and software upgrades, and voting power is allocated proportionally to token holdings. Most of the assets that users transact on the platforms are cryptocurrencies that originate elsewhere.[3] In the context of our model, a token that offers *only* cash flow claims and voting rights is equivalent to a share of a traditional platform. So, in the case of Kyber, the "investors" would be KNC token holders and "users" would be those who trade various other cryptocurrencies on the platform.

Pure governance tokens offer only voting rights. For example, the Uniswap DEX issues the UNI token, which entitles holders to vote on changes to the market-making protocol. UNI does not currently pay its holders any dividends, but in principle, token holders could vote to pay themselves a dividend at some point in the future.[4] The COMP token issued by the popular Compound lending platform carries similar voting rights. Tokens that have only voting rights are beyond the scope of our model.

Transaction services and ownership features: Finally, we turn to platforms that issue *native tokens* that bundle transaction services with ownership features. We have already given the example of proof-of-stake cryptocurrency blockchains with on-chain governance, like Algorand or Tezos. On these platforms, tokens are held by users who wish to transact with others. They are staked by "validators" (the analogue of investors in the model) who run the computational hardware needed to verify transactions and collect monetary rewards, which could take the form of transaction fees or newly minted tokens. Some blockchain platforms permit any token holder to stake and vote on proposed policies (e.g., Algorand), whereas others allow users to delegate their votes to validators who they trust to act on their behalf

[1]A key difference, however, is that tokens issued by non-DeFi platforms are typically backed at least partially by existing fiat currencies.

[2]These are distinct from *security tokens*, which usually represent a claim on another firm's profits or a claim on a financial asset that exists outside of the blockchain.

[3]Kyber users, for example, may access a "liquidity pool" that permits them to trade Ether for the Tether stablecoin.

[4]See `https://protos.com/to-fee-or-not-to-fee-that-is-the-question-does-uniswap-have-an-answer/`. Whether UNI will eventually pay dividends has been a topic of intense speculation in the community.

(e.g., Tezos). Typical policies adjust the blockchain's transaction fees or upgrade transaction verification protocols.

This setup is not restricted to proof-of-stake cryptocurrencies, however. Some DeFi platforms issue tokens that bundle voting rights, cash flow claims, and direct claims on the platform's services. The Aave lending platform issues a token (AAVE) with voting rights that (1) borrowers can post as collateral to receive discounted interest rates and (2) investors can stake to provide a liquidity backstop and receive a share of Aave's profits. Similarly, some platforms that enable interoperability across DeFi applications, such as the Cosmos and Polkadot networks, issue tokens with voting rights that users hold to pay network fees and validators stake to provide transaction security.

3 Model

Environment: We consider a continuous-time, infinite-horizon economy in which agents interact on a *platform*. There are two commodities: a numeraire good (referred to as a "dollar") and transaction services (henceforth "transactions") produced by the platform at a marginal cost $c > 0$. The economy is populated by a unit mass of two types of agents: *users* $i \in [0, 1]$ and *investors* $j \in [0, 1]$. Users enjoy the platform's transaction services: a user i who consumes a quantity x_{it} of transactions at time t receives utility $\frac{x_{it}^{1-\gamma}}{1-\gamma}$, where $\gamma \in (0, 1)$.[5] Investors, on the other hand, hold cash flow claims on the platform but do not enjoy its transaction services. All agents are risk-neutral over consumption of dollars and share a common discount rate $r > 0$.[6]

Assets: In this economy, assets can play two roles. First, there is a *transaction asset* (with endogenous price Q_t^T and supply A_t^T) that can be held by users to receive the platform's transaction services: a user i enjoys transaction services equal to her real balance of transaction assets, as is typical in the literature on tokens (as well as models with money in the utility function, e.g., Sidrauski, 1967; Feenstra, 1986).[7] So a user i who holds a quantity of transaction assets a_{it} at time t receives transaction services

$$x_{it} = Q_t^T a_{it}. \tag{1}$$

Second, there is a *cash flow asset* (with price Q_t^C and supply A_t^C) that is held by investors[8]

[5]We assume $\gamma < 1$ to ensure users' transaction demand is sufficiently elastic that a profit-maximizing policy for the platform exists.

[6]Throughout, we make the additional parametric assumption $\frac{\gamma}{1-\gamma}c > r$ to streamline the presentation of results. However, the main results remain unchanged if we drop this assumption.

[7]Biais et al. (2023) micro-found a similar utility function in an overlapping-generations model.

[8]Section 9 discusses the assumption that users do not hold cash flow assets.

and provides a pro-rata claim on the platform's profits: a cash flow asset pays a dividend $dD_t = \frac{d\Pi_t}{A_t^C}$ at time t, where $d\Pi_t$ denotes the platform's profits.

We consider three schemes for the design of assets in this economy: a traditional platform, a platform that issues service tokens, and a platform that issues hybrid tokens. The Introduction specifies what plays the role of the cash flow asset and the transaction asset under each scheme. We will mostly study the case in which only cash flow assets have voting rights (so investors govern the platform), but Section 7 extends the model to allow users to vote as well.

Platform governance: The platform's *policy* at time t consists of a *transaction fee* $f_{t+s} \geq 0$ and (possibly) a *token seigniorage* policy at all future dates $t+s$.[9] The platform's policies are determined in *governance decisions* at times $\{\tau_0 = 0, \tau_1, \tau_2, \ldots\}$ that arrive according to a Poisson process at rate λ. In a governance decision at time τ_k, any previous policy commitments are torn up, and new policies (for all future dates $\tau_k + s$) are chosen by a vote among the agents who govern the platform. Hence, the parameter λ indexes the degree of *commitment* to future policies: in the limit $\lambda \to 0$, the platform's policies are fully determined at $t = 0$ (as with an immutable smart contract), whereas in the limit $\lambda \to \infty$, a new policy is chosen at each instant. Agents are infinitesimal, so no individual agent's vote is ever pivotal. Therefore, all agents take policies as given.

Payoffs: A user i who engages in x_{it} transactions at time t pays a fee f_t per transaction, so the user receives a total flow payoff from transactions

$$U_{it}dt = \big(\frac{x_{it}^{1-\gamma}}{1-\gamma} - f_t x_{it}\big)dt.$$

The platform receives the transaction fees paid by users and incurs a marginal cost c per transaction. When we consider token-issuing platforms, the platform will also receive (endogenous) seigniorage revenues from token issuance, which we denote by dS_t for now. Thus, letting $X_t = \int_0^1 x_{it}di$ denote the aggregate quantity of transactions, the platform's flow profits at t are

$$d\Pi_t = (f_t - c)X_t dt + dS_t.$$

In the remainder of this section, we describe the elements of our model that are held constant across the different settings considered. In subsequent sections, we analyze each platform design individually.

[9] The assumption that $f_t \geq 0$ (i.e., that the platform cannot subsidize transactions) has similar implications to an assumption of limited liability: if the platform were to commit to subsidizing transactions, it would have a negative equity value. This formulation just simplifies the exposition.

Remark. *We illustrate the logic of our main results in a benchmark model that makes several specific assumptions. However, we will extend the model to more general settings to show that the main results continue to hold:*

- *Section 8 considers a platform that must make governance decisions about investment as well as fees and seigniorage;*
- *Appendix G.1 considers a platform in which users' utility exhibits network effects;*[10]
- *Appendix G.2 considers a setting with monopolistic competition across platforms, rather than a single monopolistic platform.*[11]
- *Appendix G.3 considers a setting in which the platform permits users to redeem transaction assets in exchange for a service (instead of the money-in-the-utility assumption used in the benchmark model);*

3.1 Individual optimization problems

We first lay out agents' individual portfolio optimization problems. Users choose their transaction asset holdings to maximize their expected lifetime utility subject to a standard budget constraint, taking the price of transaction assets Q_t^T and fees f_t as given. In Appendix A.1, we show that this problem reduces to a static one:

$$\max_{x_{it},a_{it}} \big(\frac{x_{it}^{1-\gamma}}{1-\gamma} - f_t x_{it}\big)dt + (\mathbb{E}_t[dQ_t^T] - rQ_t^T dt)a_{it} \quad \text{s.t.} \quad (1).$$

The first term in parentheses is the flow utility of transactions x_{it}, whereas the second term represents the expected return on transaction assets net of holding costs $rQ_t^T a_{it}$. All users i optimally choose the same transaction quantity x_{it}. Users' optimality condition can be used to show that aggregate transaction demand X_t obeys

$$X_t^{-\gamma} dt = f_t dt + \big(rdt - \mathbb{E}_t\big[\frac{dQ_t^T}{Q_t^T}\big]\big). \tag{2}$$

Transaction demand is decreasing in the net marginal cost of transacting, which consists of two components: the transaction fee f_t and the opportunity cost $r - \frac{1}{dt}\mathbb{E}_t\big[\frac{dQ_t^T}{Q_t^T}\big]$ of holding transaction assets. Of course, the aggregate demand for transaction assets is equal to

[10]We assume that a user's utility depends on their own transactions x_{it} as well as other users' aggregate transaction activity X_t via $\frac{(X_t^{\nu} x_{it}^{1-\nu})^{1-\gamma}}{1-\gamma}$ with $\nu \in (0,1)$; i.e., ν captures the strength of network effects.

[11]We assume a continuum of platforms k that produce differentiated transaction services. A user's total transaction services x_{it} are a CES aggregate of transactions x_{ikt} on each platform k, $x_{it} = (\int_0^1 x_{ikt}^{\frac{\eta-1}{\eta}} dk)^{\frac{\eta}{\eta-1}}$.

aggregate transaction demand,

$$X_t = Q_t^T A_t^T. \tag{3}$$

Similarly, investors choose their holdings of cash flow assets to maximize lifetime utility subject to a standard budget constraint. Investors' problem is formally stated in Appendix A.1. Their first-order condition implies that cash flow assets are priced according to the present value of dividends:

$$rQ_t^C dt = dD_t + \mathbb{E}_t[dQ_t^C] \quad \text{where} \quad dD_t = \frac{d\Pi_t}{A_t^C}. \tag{4}$$

Equations (3) and (4) summarize the *demand* for transaction assets and cash flow assets, respectively. The *supply* of each asset is determined in a different way for each of the platform designs we consider.

3.2 Governance decisions

The platform's *status quo policy* at time t consists of a transaction fee and (possibly) a seigniorage rate at all future dates, $\{f_{t+s}, dS_{t+s}\}_{s\geq 0}$. In a governance decision at time τ_k, the platform's status quo policy can be revised. Each agent who has the right to do so votes for a new policy. If some policy attains a majority, then the status quo is abandoned and the new policy is implemented. Otherwise, the status quo is maintained.

In principle, this voting game could be quite complicated since there are infinitely many potential policies. However, our environment offers a useful simplification: within each constituency (investors or users), agents have identical policy preferences. We assume that agents in each constituency vote unanimously for their most-preferred policy.[12] Throughout most of the analysis, we will focus on the case in which only cash flow assets confer the right to vote. In this case, investors will hold all of the voting power, so they will implement their most-preferred policy. However, we will later extend the model to allow users the opportunity to vote as well.

Each constituency's preferences over policies is determined by the lifetime utility its members expect to obtain after a new policy is passed.[13] Investors' expected lifetime utility V_t^I is equal to the market value of their cash flow assets, whereas users' expected lifetime utility V_t^U

[12]Hence, we ignore self-fulfilling equilibria in which agents do not vote for their most-preferred policy (as is typical in the corporate governance literature, see Levit, Malenko, and Maug, 2024). For instance, if all agents vote for a random policy, then no policy will ever attain a majority, so it is in fact individually rational for each agent to vote randomly.

[13]Given an expected path of policies, there may be multiple equilibrium paths of transaction quantities $\{X_t\}$ consistent with those policies. Following convention in the literature, we select the "best-case" equilibrium that generates the greatest total surplus.

is equal to the market value of their transaction assets plus the present value of *infra-marginal rents* R_t that they earn from transactions, defined below.

Proposition 1. *Following the announcement of a new policy at time* τ*, investors' expected lifetime utility is*

$$V_\tau^I = Q_\tau^C A_{\tau^-}^C, \tag{5}$$

where $A_{\tau^-}^C$ *denotes investors' cash flow asset holdings before the new policy announcement, and users' expected lifetime utility is*

$$V_\tau^U = Q_\tau^T A_{\tau^-}^T + R_\tau, \quad \textit{where} \quad R_\tau \equiv \mathbb{E}_\tau \left[\int_0^\infty e^{-rs} \frac{\gamma}{1-\gamma} X_{\tau+s}^{1-\gamma} ds \right]. \tag{6}$$

3.3 Notation

Throughout the analysis, we will focus on *Markov equilibria.* Suppose that the most recent governance decision occurred at time τ. Then, the only relevant state variable for outcomes at time t is the time $s = t - \tau$ that has elapsed since the most recent decision.

As is typical in models of policy-making with limited commitment, both current policies and anticipated future policies influence agents' behavior in equilibrium. It is therefore necessary to distinguish between *actual* policies chosen at τ and the policies that were *anticipated* before the governance decision at τ, since, at least in principle, the constituency that governs the platform can deviate from the anticipated policy. We index actual policies and outcomes simply by the subscript s: for example, f_s denotes the chosen level of fees at time $\tau + s$, X_s denotes actual aggregate transaction quantities at that time, and so on. We denote anticipated policies and outcomes with a hat as functions of s: so $\hat{f}_s$ denotes the level of fees at time $\tau + s$ that was anticipated *before* the decision at τ, $\hat{X}_s$ denotes anticipated transaction quantities, etc. Of course, in equilibrium, actual outcomes must coincide with anticipated outcomes.

We focus on equilibria in which jumps in variables may occur at the time of a governance decision, but variables evolve smoothly between governance decisions. The drift of a variable is denoted with a dot, so, for example, $dX_s = \dot{X}_s ds$.

3.4 The first-best

Before examining specific platform designs, we derive the properties of optimal allocations in this environment. This analysis will facilitate a comparison of the inefficiencies that arise under each platform design studied in subsequent sections.

There is transferable utility in this environment, so an allocation is efficient if and only if it maximizes utilitarian social welfare (i.e., the sum of agents' payoffs). Note that fees will be irrelevant for total welfare, since they are just a transfer from users to investors.

An *allocation* is therefore summarized simply by a sequence of aggregate transaction quantities X_t.[14] Total surplus at time t is just $X_t^{1-\gamma}/(1-\gamma) - cX_t$. To maintain symmetry with our model of governance, we assume a social planner with limited commitment: at the time of a governance decision, the planner chooses an allocation $\{X_s\}_{s\geq 0}$ that is maintained until the next governance decision at time τ (which arrives at rate λ).

Proposition 2. *A first-best allocation solves*

$$\hat{V}_0^P = \max_{X_s} \ \mathbb{E}_0 \left[\int_0^\infty e^{-rs} \left(\frac{X_s^{1-\gamma}}{1-\gamma} - cX_s \right) ds + e^{-r\tau} \hat{V}_0^P \right], \tag{7}$$

where $\hat{V}_s^P$ is the planner's value function. An allocation is first-best if and only if

$$X_s = X^{FB} \equiv c^{-\frac{1}{\gamma}} \quad \forall \ s. \tag{8}$$

The first-best level of transactions, X_s^{FB}, is set so that the marginal utility of an additional transaction, $X_s^{-\gamma}$, is equal to the marginal cost c of processing that transaction. Thus, the optimal level of aggregate transactions is constant over time and decreasing in transaction processing costs c.

4 Traditional platform

In this section, we study the case of a *traditional platform* as a simple benchmark. We demonstrate that as in most models with a monopolistic firm, a traditional platform charges inefficiently high fees, distorting transaction volumes downwards.

4.1 Setup

In the case of a traditional platform, there are two distinct assets: *shares* that are issued by the platform and *transaction assets* that originate outside the platform (such as cash, stablecoins, or another cryptocurrency). Users hold transaction assets for their transaction services but cannot vote in governance decisions. We assume transaction assets are supplied

[14] Users have identical concave utility functions. Without loss of generality, then, we can restrict attention to allocations in which each user transacts the same amount, $x_{it} = X_t$ for all i.

elastically at a price $Q_t^T = 1$.[15] Shares serve as the economy's cash flow asset and grant investors the right to vote on the platform's policies. The supply of shares is normalized to $A_t^C = 1$ – we assume, without loss of generality, that the platform does not issue new shares or buy them back from investors.[16]

Since the platform does not issue tokens in this environment, it receives no seigniorage revenues. Therefore, it derives profits only from processing transactions:

$$d\Pi_t = (f_t - c)X_t dt. \tag{9}$$

4.2 Equilibrium

We look for a Markov equilibrium in one state variable: the time s elapsed since the most recent governance decision. We begin by solving for users' transaction demand. The price of transaction assets is constant, so (2) implies that users' transaction demand is downward-sloping in the level of fees:

$$X_s = \big(f_s + r\big)^{-\frac{1}{\gamma}}. \tag{10}$$

Equations (4) and (5) imply that investors' expected lifetime utility is equal to the present value of profits $\mathbb{E}_0[\int_0^\infty e^{-rt} d\Pi_t]$. In a governance decision, investors unanimously vote in favor of the policy $\{f_s\}_{s\geq 0}$ that maximizes the present value of profits. Investors' governance problem can then be written as

$$\hat{V}_0^I = \max_{f_s, X_s} \ \mathbb{E}_0 \left[\int_0^\tau e^{-rs}(f_s - c)X_s ds + e^{-r\tau}\hat{V}_0^I \right] \ \text{s.t.} \ \ (10), \ f_s \geq 0. \tag{11}$$

where τ denotes the (random) time interval from $s = 0$ until the next governance decision and $\hat{V}_s^I$ denotes investors' value function at time s since the most recent governance decision.

A *Markov equilibrium* consists of a value function $\hat{V}_s^I$ for investors and outcomes $\{\hat{f}_s, \hat{X}_s\}$ that solve (11).

4.3 Welfare and efficiency under the traditional scheme

Under the traditional governance scheme, the platform is just a monopolistic firm that maximizes the present value of its profits. Investors do not internalize how changes in the

[15] What matters is not that the price of transaction assets is constant, but rather that (1) their price is exogenous to the platform's policies, and (2) their rate of return is below the discount rate r (that is, there is a *liquidity premium* on transaction assets, as with cash, Treasury bills, or deposits).

[16] There are no financial frictions, so the Modigliani-Miller theorem applies in the setting of a traditional platform. The platform's share issuance policy is irrelevant.

platform's policies affect user surplus. As a result, equilibrium policies are inefficient: they maximize investor surplus at the expense of total surplus.

Proposition 3. *Under the traditional scheme, the equilibrium sequence of policies $\{f_t\}$ maximizes expected investor surplus (but not total surplus) over all feasible sequences of policies. That is, regardless of the degree of commitment λ, equilibrium policies solve*

$$\max_{f_t, X_t} \int_0^\infty e^{-rt}(f_t - c)X_t dt \quad s.t. \quad X_t = \left(f_t + r\right)^{-\frac{1}{\gamma}}. \tag{12}$$

Under the equilibrium policy, fees are

$$f_t = \frac{1}{1-\gamma}c + \frac{\gamma}{1-\gamma}r \tag{13}$$

and transaction quantities are inefficiently low:

$$X_t = X^{trad} \equiv \left(\frac{c+r}{1-\gamma}\right)^{-\frac{1}{\gamma}} < X^{FB}. \tag{14}$$

Note, moreover, that a traditional platform's equilibrium policy is time-consistent, so limited commitment to future policies (parameterized by the frequency λ of governance decisions) is irrelevant. Intuitively, this is the case because time-t transaction demand depends only on time-t policies. When we study a token-issuing platform, this will no longer be true: future token issuance policies will affect the current return on tokens and therefore current transaction demand.

Equilibrium transaction quantities X_t are inefficiently low because users' marginal cost of transacting, fees f_t plus the opportunity cost r of holding transaction assets, is greater than the social marginal cost c of processing transactions. By (13),

$$f_t + r = \frac{c+r}{1-\gamma} > c.$$

This equation for the effective transaction cost faced by users reveals two distortions, each corresponding to a distinct source of user surplus that investors neglect. First, the discount rate r appears in the numerator on the right-hand side. This distortion arises because investors do not take into account the aggregate *service flows*

$$SF_t \equiv X_t^{1-\gamma} - f_t X_t = r X_t, \tag{15}$$

that users earn on their transaction asset holdings, defined as users' marginal utility per transaction times the aggregate quantity of transactions.[17]

Second, an additional factor $1-\gamma$ appears in the denominator on the right-hand side. This is because investors do not internalize the *inframarginal rents*

$$IR_t \equiv \big(\frac{X_t^{1-\gamma}}{1-\gamma} - f_t X_t\big) - SF_t = \frac{\gamma}{1-\gamma} X_t^{1-\gamma} \tag{16}$$

that users receive from infra-marginal transactions: since users' utility is concave in transactions, user surplus is greater than their marginal valuation of transaction assets' services.[18]

When we analyze a platform that issues tokens for transactions, we will demonstrate that, by contrast, investors take users' service flows into account. Hence, the first distortion will vanish, while the second will remain. Figure 2 illustrates how these distortions cause investors to set fees too high and destroy surplus.

There are three necessary ingredients for the inefficiency in this model. First, the platform has market power, so shareholder value maximization is not equivalent to maximization of social surplus. Put differently, the platform's market power creates a *conflict of interest* between the two constituencies. Second, the platform's owners can extract rents from users only by charging *distortionary* fees that cause deadweight losses: it is not possible for the platform to use a more complex pricing scheme (like a two-part tariff) that fully extracts user surplus. However, the Coase Theorem implies that absent restrictions on contracting, investors and users would nevertheless contract around these inefficiencies and arrive at an efficient outcome. The third necessary ingredient for inefficiency, therefore, is *limited contracting*: users cannot sign a contract in which they commit to compensate investors for choosing a more socially beneficial policy. These limits to contracting could be micro-founded, for instance, by assuming that users are unable to commit to a sequence of payments in response to the policies chosen by investors.

How can agents overcome the problem of limited contracting? In the next section, we outline conditions under which *token issuance* can partially substitute for the missing contracts between users and investors.

5 Service tokens

In this section we consider a platform that issues tokens that provide transaction services *only*. The platform continues to be governed by shareholders (investors) who hold all cash flow

[17] That is, SF_t is defined as $X_t \times \frac{\partial}{\partial X_t}(\frac{X_t^{1-\gamma}}{1-\gamma} - f_t X_t)$. Service flows are equal to rX_t in equilibrium by (10).

[18] Note that the discounted inframarginal rents that enter users' lifetime utility in (6) are just $R_t = \mathbb{E}_t[\int_0^\infty e^{-rs} IR_{t+s} ds]$.

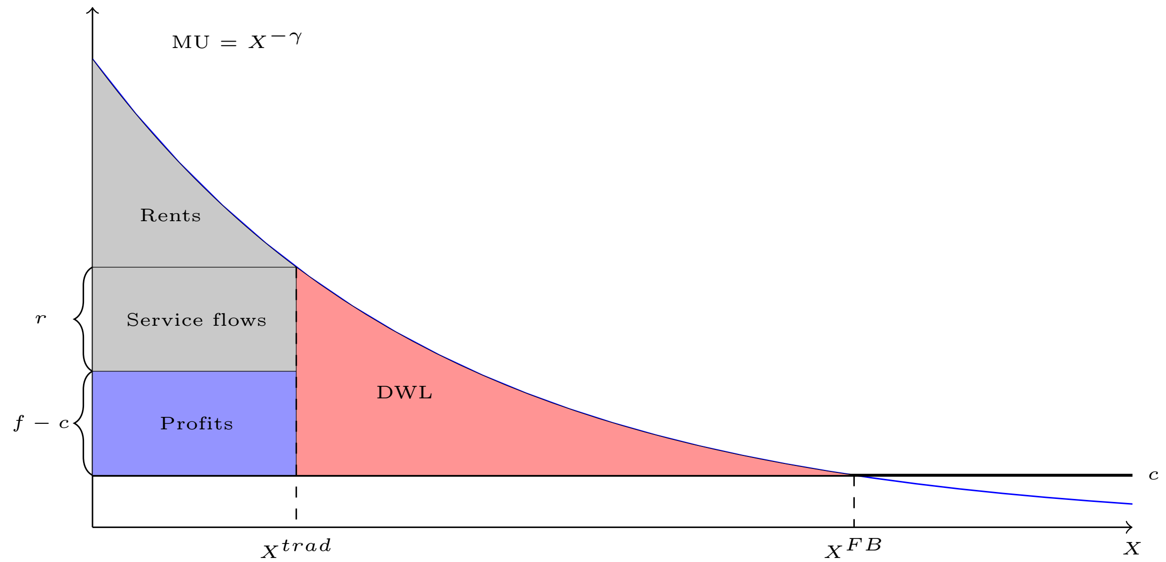

Figure 2: An illustration of the sources of neglected surplus in the case of a traditional platform. Aggregate transaction quantities X are on the horizontal axis. The downwards-sloping curve represents users' marginal utility of transacting (as a function of X). The horizontal line corresponds to the marginal cost c of processing a transaction. The profit-maximizing transaction quantity is X^{trad}, which is below the first-best X^{FB}.

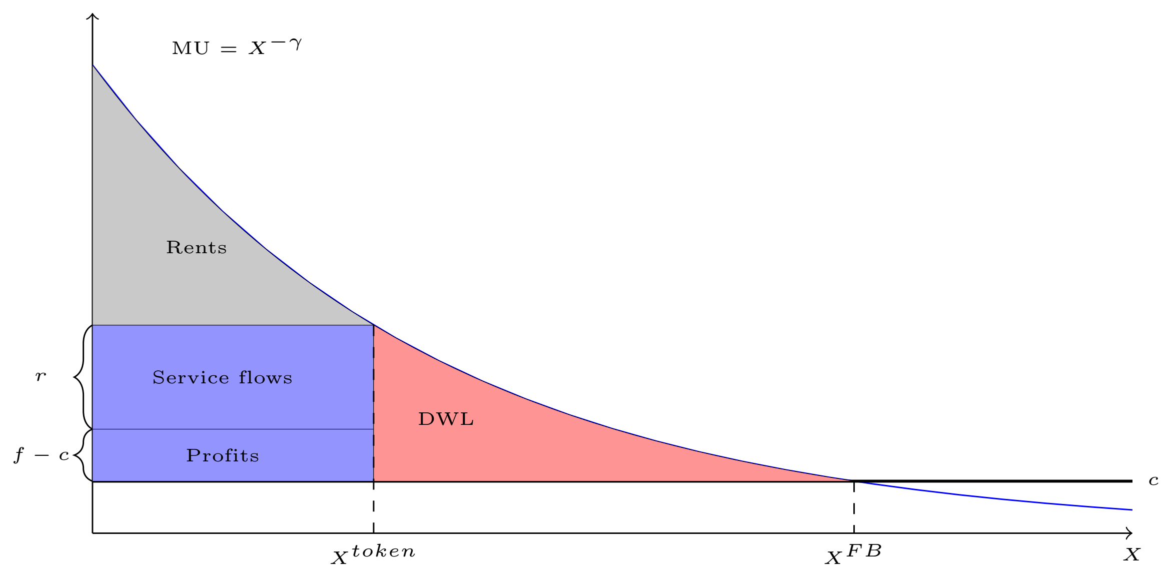

Figure 3: Illustration of the sources of surplus in the case of a platform that issues service tokens. Unlike in the case of a traditional platform (Figure 2), investors internalize users' service flows. Profits are maximized by setting lower fees than in the traditional case, raising transaction quantities to X^{token} (which remains below the first-best X^{FB}).

claims and voting rights. In contrast to the traditional setting, investors' ability to commit to future policies is key. We show that service tokens can substitute for missing contracts between users and investors (and therefore increase welfare) only if investors' commitment power is strong enough. When commitment power is weak, they are tempted to inflate away the value of tokens, depressing transaction quantities and resulting in a less efficient equilibrium.

5.1 Setup

Consider an environment in which the two assets are *shares* held by investors, which serve as the cash flow asset, and *tokens* that the platform issues to users, which serve as the transaction asset. Since the transaction asset is issued by the platform rather than supplied elastically, in this case its price Q_t^T evolves endogenously. We maintain the assumptions that (1) all voting rights are allocated to shareholders, and (2) the platform neither issues nor buys back shares, so the supply of shares is normalized to $A_t^C = 1$. The platform in this setting can be thought of as the issuer of a "utility token" or as a tech platform that issues its own currency.[19]

Unlike a traditional platform, a token-issuing platform can earn seigniorage revenues by minting new tokens. We denote the growth rate of the token stock by

$$d\mu_t = \frac{dA_t^T}{A_t^T} \geq 0.$$

To ensure that an optimal policy exists, we assume that the rate of token issuance is bounded above by some large positive constant, $\frac{A_t^T - A_{t-}^T}{A_{t-}^T} \leq \Delta$, which implies $d\mu_t \leq \frac{\Delta}{1+\Delta}$.[20] The token issuance rate $d\mu_t$ is a policy determined in governance decisions. seigniorage revenues are equal to the current token price times the quantity of tokens issued at time t, $dS_t = Q_t^T dA_t^T = X_t d\mu_t$. The platform's profits at time t are then

$$d\Pi_t = (f_t - c)X_t dt + X_t d\mu_t.$$

5.2 Equilibrium

We again consider a Markov equilibrium in which all outcomes depend only on the time s since the most recent governance decision. We look for an equilibrium in which variables

[19] In our benchmark model, tokens have no intrinsic value (agents enjoy transaction services only if the price of tokens is positive). Appendix G.3 outlines an extension in which tokens have intrinsic value: they can be redeemed for a service at a fixed exchange rate, as is common for utility tokens.

[20] All of our substantive results continue to hold in the limit of unrestricted token issuance, $\Delta \to \infty$.

may jump at the time of a governance decision ($s = 0$) but evolve smoothly thereafter. In particular, the platform may issue a discrete quantity of tokens when a new policy is implemented, $d\mu_0 > 0$, but afterwards, the supply of tokens evolves smoothly, $d\mu_s = \dot{\mu}_s ds$ for $s > 0$.

When the platform issues service tokens, users' transaction demand is no longer statically pinned down by fees as in (10). Transaction demand also depends on expected changes in token prices: all else equal, higher expected returns on tokens will increase demand. The following lemma characterizes the returns on tokens in this setting.

Lemma 1. *When the platform issues service tokens, the expected return on tokens satisfies*

$$\frac{1}{ds}\mathbb{E}_s\left[\frac{dQ^T}{Q^T}\right] = \frac{\dot{X}_s}{X_s} - \dot{\mu}_s + \lambda\left((1 - \hat{d\mu_0})\frac{\hat{X}_0}{X_s} - 1\right).$$

The first two terms represent returns in the absence of a new policy, while the third term represents returns conditional on a new policy (Recall that $\hat{d\mu_0}$ denotes anticipated token issuance at the time of the next governance decision, and $\hat{X}_0$ is anticipated transaction demand at that time.) Simply put, this lemma implies that the expected return on tokens is equal to the expected growth rate of transaction quantities minus the expected growth rate of the token stock. If the token stock grows without a commensurate increase in transaction demand, there will be inflation (a decrease in the price of tokens and a low return).

Lemma 1 allows us to characterize transaction demand in this setting – (2) implies

$$(r + \lambda)X_s = \underbrace{X_s^{1-\gamma} - (f_s + \dot{\mu}_s)X_s + \dot{X}_s}_{\text{current policy}} + \underbrace{\lambda\big(1 - \hat{d\mu_0}\big)\hat{X}_0}_{\text{exp. future policy}}. \tag{17}$$

Users' transaction demand is equal to the present value of service flows net of the costs of dilution from additional token issuance – new seigniorage reduces the price of tokens, representing an implicit transfer from token holders to investors. This transaction demand condition illustrates why lack of commitment matters. Transaction demand depends not only on current policy, but also on anticipated policy after the next governance decision, which arrives at rate λ. If users expect that investors will be tempted to issue a large quantity of tokens in the next governance decision ($\hat{d\mu_0} > 0$), then they expect a reduction in the value of their tokens, lowering transaction demand.

As before, (4) and (5) imply that investors choose fees f_s and a token issuance policy

$(d\mu_0, \{\dot{\mu}_s\}_{s>0})$ to maximize the present value of profits:

$$\hat{V}_0^I = \max_{f_s, d\mu_0, \dot{\mu}_s, X_s} \mathbb{E}_0 \left[X_0 d\mu_0 + \int_0^\tau e^{-rs}(f_s + \dot{\mu}_s - c)X_s ds + e^{-r\tau}\hat{V}_0^I \right]$$
$$\text{s.t. } (17),\ f_s, \dot{\mu}_s \geq 0,\ d\mu_0 \in [0, \frac{\Delta}{1+\Delta}], \tag{18}$$

where τ denotes the time of the next governance decision. We search for a Markov equilibrium consisting of a value function $\hat{V}_s^I$ for investors and outcomes $\{\hat{f}_s, \hat{d\mu}_0, \hat{\dot{\mu}}_s, \hat{X}_s\}$ that solve (18).

We begin our analysis of equilibrium with some simplifying observations. Note that at the time when the governance decision takes place ($s = 0$), investors can issue tokens without affecting transaction demand for $s \geq 0$. Transactions instead depend on *future* token issuance ($\dot{\mu}_s$ for $s > 0$ and $\hat{d\mu}_0$ in the *next* governance decision), since that is what determines expected returns on tokens by Lemma 1. Therefore, at the time of a governance decision, investors issue a large quantity of new tokens and inflate away the value of existing tokens.

Lemma 2. *When the platform issues service tokens, investors issue the maximum allowable quantity of tokens at the time of a governance decision ($d\mu_0 = \frac{\Delta}{1+\Delta}$).*

In our benchmark model, this is the source of time-inconsistency – ex ante, investors would like to commit not to inflate away the value of tokens, but ex post (at the time of a governance decision), it is optimal to do so.

Lemma (2) can be used to write the platform's expected profits in a simple form. The platform's expected profits are equal to users' discounted service flows SF_s (given in (15)), plus discounted future profits from transaction processing, $(f_s - c)X_s$, plus a term that depends on anticipated future policies.

Proposition 4. *When the platform issues service tokens, the expected value of platform profits on $[0, \tau)$ is*

$$\mathbb{E}_0 \left[\int_0^\tau e^{-rs} d\Pi_s \right] = \mathbb{E}_0 \left[\int_0^\tau e^{-rs} \Big(\underbrace{X_s^{1-\gamma} - f_s X_s}_{\textit{service flow } SF_s} + \underbrace{(f_s - c)X_s}_{\textit{profits}} \Big) ds + \underbrace{e^{-r\tau} \frac{1}{1+\Delta} \hat{X}_0}_{\textit{future policy}} \right]. \tag{19}$$

The intuition behind this result is straightforward. The platform makes profits in two ways: by charging transaction fees and by issuing new tokens. Users value tokens according to the present value of their service flows. Hence, when the platform issues new tokens at $s = 0$, it receives revenues equal to the present value of service flows. Thereafter, it earns the transaction processing fees that make up the remainder of its profits.

Unlike in the case of a traditional platform, then, investors take users' service flows into account when choosing policies. By passing policies that raise users' anticipated service flows, they increase token prices and therefore seigniorage revenues. Hence, investors' policy preferences will be more aligned with users' when the platform can issue tokens. Nevertheless, welfare will still fall short of the first-best: despite the fact that investors take users' service flows into account, they still fail to internalize how their policies affect users' inframarginal rents.

Investors' ability to commit will be key in determining equilibrium outcomes. Commitment power is parameterized by the rate λ at which governance decisions take place– the more often investors get to rewrite the platform's policies, the weaker their commitment power. We will distinguish between two regimes in what follows: the *strong commitment regime* (λ small enough) and the *weak commitment regime* (λ large enough). Equilibrium outcomes will differ sharply across these two regimes. We analyze each in turn.

5.3 The strong commitment regime

We begin by analyzing the strong commitment regime. We show that when investors' commitment power is strong enough, their governance problem is *time-consistent* – that is, they choose the same policies that they would have chosen if they could commit to a full sequence of policies at $t = 0$. In this regime, equilibrium policies maximize the present value of service flows plus platform profits.

Proposition 5. *There exists λ^* such that if $\lambda \leq \lambda^*$, investors' governance problem (18) is time-consistent. Equilibrium policies solve*

$$\max_{f_t, \dot{\mu}_t, X_t} \int_0^\infty e^{-rt}\left(X_t^{1-\gamma} - cX_t\right)dt \quad s.t.\ f_t, \dot{\mu}_t \geq 0,$$
$$rX_t = X_t^{1-\gamma} - (f_t + \dot{\mu}_t)X_t + \dot{X}_t. \tag{20}$$

Under the equilibrium policy, transaction quantities satisfy

$$X_t = X^{token} = \left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}}. \tag{21}$$

The quantity of transactions X_t and welfare are both higher than in the case of a traditional platform (but below their first-best levels).

In the Appendix, we take a Lagrangian approach to solve for equilibrium policies and transaction quantities, but we summarize the main results here. Transfers from users to investors (fees plus seigniorage revenues) are set statically to maximize service flows plus transaction processing profits,

$$X_t^{1-\gamma} - cX_t = \underbrace{X_t^{1-\gamma} - f_t X_t}_{\text{service flows } SF_t} + \underbrace{(f_t - c)X_t}_{\text{profits}}.$$

The equilibrium level of transactions is given in (21). Equilibrium transaction quantities are greater than in the case of a traditional platform (see (14)) but nevertheless remain below the first-best level (8).

When commitment to future policies is strong enough, then, token issuance enhances efficiency. The key idea is that since investors care about maintaining a high token price to maximize their seigniorage revenues at $s = 0$, they are reluctant to set fees too high. High fees imply low service flows for users, reducing the token price and the platform's seigniorage revenues. Hence, investors commit to lower fees than in the traditional setting. However, investors still fail to internalize how the platform's policies affect inframarginal rents, so transaction quantities remain distorted downwards (hence the additional factor of $1 - \gamma$ in the denominator of (21) relative to the first-best level (8)). Figure 3 illustrates this point.

Why is the degree of commitment (i.e., the precise value of λ) irrelevant in this setting? The answer lies in how investors choose to raise revenues from users. Conceptually, there are two ways the platform can raise revenues: by charging transaction fees or by issuing additional tokens. Recall that when a governance decision takes place, investors decide to issue a large quantity of tokens that inflates away a fraction $\frac{\Delta}{1+\Delta}$ of the value of existing tokens. These events occur at rate λ. Then, between governance decisions, users pay investors transaction fees $f_t X_t dt$ as well as seigniorage revenues $\dot{\mu}_t X_t dt$. The transaction demand condition (17) then implies that if $f_t + \dot{\mu}_t$ is constant, transaction demand satisfies

$$X_t = \left(r + f_t + \dot{\mu}_t + \lambda \frac{\Delta}{1+\Delta}\right)^{-\frac{1}{\gamma}} = \left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}},$$

where the second equality follows from (21). When λ is higher (i.e., commitment power is weaker) then investors know that in the future, they will be more tempted to raise revenues by issuing tokens. Then, they choose to charge lower fees and issue fewer tokens in the present to boost transaction demand and keep total transfers from users $(f + \dot{\mu} + \lambda \frac{\Delta}{1+\Delta})X_t$ constant. The platform's fee policy and its seigniorage policy are therefore substitutes: what matters for the platform's profits and transaction demand is the *sum* $f_t + \dot{\mu}_t + \lambda \frac{\Delta}{1+\Delta}$, not how revenues are split between fees, current seigniorage, and anticipated future seigniorage.

Proposition 6. *Under the optimal policy with commitment, the level of fees f_t and the growth* *rate of the token stock $\dot{\mu}_t$ are indeterminate. Their sum, $f_t + \dot{\mu}_t$, is uniquely determined in equilibrium:*

$$f_t + \dot{\mu}_t = \frac{c}{1-\gamma} - (r + \lambda \frac{\Delta}{1+\Delta}). \tag{22}$$

5.4 The weak commitment regime

We now turn to the case in which investors' commitment power is weak. In this regime, investors' temptation to inflate away the value of tokens is strong enough that it impairs transaction demand. In the limit of no commitment, expected inflation is so high that transaction quantities go to zero. The following proposition summarizes these results.

Proposition 7. *When $\lambda > \lambda^*$ (defined in Proposition 5), investors' policy problem is no longer time-consistent. Equilibrium transaction quantities are*

$$X_t = \big(r + \lambda \frac{\Delta}{1+\Delta}\big)^{-\frac{1}{\gamma}} < X^{token}. \tag{23}$$

In the no-commitment limit ($\lambda \to \infty$), transaction quantities $X_t \to 0$.

When investors lack commitment power, their temptation to issue tokens creates inflation and undermines transaction demand. Specifically, Lemma 2 implies that in governance decisions (which occur at rate λ), investors choose to issue a large quantity of new tokens and inflate away the value of existing tokens. Recall that *effective fees* paid by users to investors are fees f_t plus current seigniorage $\dot{\mu}_t$ plus $\lambda \frac{\Delta}{1+\Delta}$, which determines anticipated future seigniorage. Previously, we argued that the optimal transfer from users to investors satisfies (22). When anticipated future seigniorage is greater (higher λ), investors either cut fees f_t or current seigniorage $\dot{\mu}_t$. Suppose that $\lambda > \lambda^* \equiv \frac{1+\Delta}{\Delta}\big(\frac{c}{1-\gamma} - r\big)$. Then, even if investors cut fees and seigniorage to zero, users still face such a high cost $\lambda \frac{\Delta}{1+\Delta}$ from expected future inflation that the total transfer to investors is greater than its optimal level,

$$\lambda \frac{\Delta}{1+\Delta} > \frac{c}{1-\gamma} - r.$$

Then, users' transaction demand falls below investors' desired level X^{token} (given in (21)).

These results demonstrate that when the commitment problem is severe enough, transaction quantities and welfare may even fall below the levels attained by a traditional platform. Put differently, the introduction of service tokens is socially beneficial only if a platform has sufficient mechanisms to commit to future policies – otherwise, the introduction of tokens *reduces* welfare.

Proposition 7 is reminiscent of results in financing problems without commitment (Admati et al., 2018; DeMarzo and He, 2021). Typically, an issuer that cannot commit to future issuance effectively competes with its future self: it does not internalize the price impact of current issuance decisions and therefore over-issues relative to the commitment outcome. Goldstein, Gupta, and Sverchkov (2022) demonstrate that if a monopolistic platform can commit to a token's redemption value but not an issuance policy, then token issuance mitigates the platform's tendency to under-supply its services.

Our result is similar in spirit to those in the previous literature, but it is distinct. While it is true that the platform's investors are tempted to extract rents from users by over-issuing new tokens, outcomes with weak commitment would exhibit similar inefficiencies even if new seigniorage were prohibited after $t = 0$. Investors have another tool – fees – that they can use to extract rents in the absence of new token issuance. Instead of issuing new tokens at the time of a governance decision, investors would choose to charge very high transaction fees immediately following the implementation of a new policy. Anticipating this rent extraction, users would not place a high value on tokens and would be reluctant to transact on the platform.

A severe enough lack of commitment power is detrimental to the platform's profits, so investors may therefore seek mechanisms that permit them to commit. Of course, in reality, a platform's founders and investors can use smart contracts to commit to future token issuance, or they could use token retention schemes to incentivize them to pass policies that benefit users. To the extent that such mechanisms are imperfect, though, the next section argues that bundling transaction services with cash flow claims can provide an effective substitute for missing commitment mechanisms.

6 Hybrid tokens

We now consider a platform that issues a single *hybrid* token that bundles transaction services with cash flow claims. Users hold the token for transaction services, whereas investors "stake" the token for cash flows. In this setting, we assume tokens must be staked in order to participate in governance – as a result, only investors vote in governance decisions. This assumption is close to the reality for several DeFi platforms: for example, many proof-of-stake cryptocurrency blockchains (like Algorand) allow *only* validators who stake their tokens to vote on policy changes. Our main result is that by issuing a hybrid token, a platform governed by investors can achieve the full-commitment outcome of Section 5.3 even if investors lack commitment power.

6.1 Setup

In this economy, there is a single asset called a *token.* Tokens serve as both the economy's transaction asset and as its cash flow asset: users can hold tokens for their transaction services, whereas investors can hold tokens to receive dividends. Given that there is only one asset, we let Q_t denote the price of tokens (dispensing with our previous notation Q_t^C, Q_t^T). Furthermore, we denote the total supply of tokens at t by $A_t = A_t^C + A_t^T$, where A_t^C (resp. A_t^T) denotes the quantity of tokens held by investors (users) at t. Henceforth, let $M_t = Q_t A_t$ denote the total market capitalization of tokens, and let $\zeta_t = \frac{A_t^C}{A_t^C + A_t^T}$ denote the *fraction* of tokens that are held by investors ("staked").

As before, user i's transaction services are equal to her real balance of token holdings, $x_{it} = Q_t a_{it}^T$, where a_{it}^T is the quantity of tokens held by i. Aggregate transaction demand is equal to the real value of tokens held by users, so

$$X_t = Q_t A_t^T = (1 - \zeta_t) M_t.$$

The platform can earn revenues both by charging fees and by issuing tokens. Again, $d\mu_t$ will denote the rate of seigniorage,

$$d\mu_t = \frac{dA_t}{A_t} \in [0, \frac{\Delta}{1 + \Delta}],$$

so the platform's seigniorage revenues are $dS_t = M_t d\mu_t$, and total profits are

$$d\Pi_t = (f_t - c) X_t dt + M_t d\mu_t.$$

Dividends are distributed pro rata to the holders of staked tokens, so each staked token pays a dividend $dD_t = \frac{d\Pi_t}{\zeta_t A_t}$. When a greater fraction ζ_t of tokens are staked, the per-token dividend is lower, all else equal. When investors stake tokens, they compete over a fixed pool of dividends, as is common in practice.[21] The fraction of staked tokens is a key equilibrium variable: it adjusts until users and investors place an equal valuation on tokens.

For now, we assume that staking is required to vote, so only investors participate in governance decisions. In Section 7, though, we extend the model to the case in which users may vote on policies as well.

[21] For example, in proof-of-stake blockchains, the rate at which new blocks are produced (and therefore the rate at which block rewards are issued) typically does not scale linearly with the quantity of staked tokens. When an additional token is staked, then the chance that any other staked token is selected to propose a new block is reduced. Several papers in the literature, such as John, Rivera, and Saleh (2022) and Jermann (2023), make a similar assumption.

6.2 Equilibrium

We look for a Markov equilibrium in the time s since the most recent policy change in which variables can jump at the time of a governance decision and evolve smoothly thereafter, as in previous sections. The expected return on tokens is equal to the expected growth rate in the market capitalization of tokens net of the issuance rate, analogously to the return on tokens in Lemma 1 in the previous section.

Lemma 3. *When the platform issues hybrid tokens, their expected return satisfies*

$$\frac{1}{ds}\,\mathbb{E}_s\left[\frac{dQ}{Q}\right] = \frac{\dot{M}_s}{M_s} - \dot{\mu}_s + \lambda\left(\frac{\hat{M}_0}{M_s}(1-\hat{d\mu}_0) - 1\right) \quad \textit{for} \quad s > 0, \tag{24}$$

Recall that in (24), $\hat{d\mu}_0$ denotes the anticipated jump in the token stock at the time of the *next* governance decision, whereas $\hat{M}_0$ is the anticipated market capitalization of tokens at that time.

When the platform issues a hybrid token, then both constituencies must be willing to hold tokens at the same price. That is, both users and investors are marginal in the market for tokens. This is the key difference between a hybrid token platform and a platform that issues tokens that offer only transaction services. When investors pass policies, they have an additional incentive to consider how those policies affect users: a policy that is detrimental to users will decrease users' token valuation, reducing the price of the tokens held by investors. We therefore begin by studying how each constituency values tokens.

Users price tokens according to their *service value* (i.e., the present value of tokens' service flows). The transaction demand condition (2) can be combined with (24) for $s > 0$ to obtain

$$(r+\lambda)M_s = \frac{X_s^{1-\gamma} - f_s X_s}{1-\zeta_s} + \dot{M}_s - \dot{\mu}_s M_s + \lambda(1-\hat{d\mu}_0)\hat{M}_0 \tag{25}$$

The way to understand this equation is that users earn aggregate service flows $X_s^{1-\gamma} - f_s X_s$ while holding a stock of tokens worth $(1-\zeta_s)M_s$ (hence the first term). The remaining terms represent the expected return on tokens.

By contrast, investors value tokens for their dividends when staked. Their optimality condition (4) requires that the token price be equal to tokens' *cash flow value* (i.e., the present value of dividends), which can be written for $s > 0$ as

$$(r+\lambda)M_s = \frac{(f_s - c)X_s + \dot{\mu}_s M_s}{\zeta_s} + \dot{M}_s - \dot{\mu}_s M_s + \lambda(1-\hat{d\mu}_0)\hat{M}_0. \tag{26}$$

The pricing conditions (25)-(26) then imply a relationship between tokens' *service flows* and their *dividend yields* that must hold so that tokens' service value is equal to their cash flow value:

$$\underbrace{\frac{(f_s - c)X_s + \dot{\mu}_s M_s}{\zeta_s}}_{\text{dividend yield}} = \underbrace{\frac{X_s^{1-\gamma} - f_s X_s}{1 - \zeta_s}}_{\text{service flow}}. \tag{27}$$

How are users' and investors' token valuations kept in line with one another? The fraction of staked tokens ζ_s adjusts in equilibrium. Off-equilibrium, if the dividend yield were higher than the service flow, then tokens would be more attractive for their cash flows than for their transaction services. Investors would therefore purchase and stake additional tokens, increasing ζ_s and driving down the per-token dividend $dD_s = \frac{d\Pi_s}{\zeta_s A_s}$.

Under the hybrid token scheme, then, token prices reflect both their usefulness in transactions and staking dividends. In fact, the market capitalization of tokens is equal to the present value of transaction processing profits plus service flows.

Lemma 4. *Under the hybrid token system, for $s > 0$ the market capitalization of tokens satisfies*

$$M_s = \mathbb{E}_s \left[\int_s^{\tau} e^{-r(u-s)} \left(X_u^{1-\gamma} - cX_u \right) du + e^{-r(\tau - s)} \hat{M}_0 (1 - d\hat{\mu}_0) \right]. \tag{28}$$

where τ is the time of the next governance decision.

This equation conveys a key intuition: when investors pass a policy, they succeed in increasing the cash flows from their staked tokens only if that policy increases current service flows plus profits. This result may seem counter-intuitive: why is it that they cannot always increase cash flows by raising fees, as in the case of a platform that issues a service token? Indeed, raising fees may increase the platform's *profits* while decreasing the per-token *dividend yield*. If a policy increases the platform's profits at the expense of a larger *decrease* in tokens' service flows, then users will sell tokens to investors, causing investors to stake more tokens (increasing ζ_s) and decreasing the dividend yield below its initial level.

Of course, despite the fact that fees and the seigniorage rate do not appear in the valuation equation (28), they are not irrelevant in equilibrium. Fees and the seigniorage rate matter because they determine transaction demand through (25). All else equal, higher fees or seigniorage increase users' costs of transacting on the platform. Nevertheless, the pricing equation (28) does imply an important *monetary neutrality* result.

Lemma 5 (Monetary neutrality)**.** *For a given path $\{X_s\}$, the token market capitalization $\{M_s\}$ is independent of $\{\dot{\mu}_s\}$ for $s > 0$, and*

$$M_0(1 - d\mu_0) = M_{0^+}, \tag{29}$$

where M_{0+} denotes the market capitalization of tokens immediately after the governance decision at $s = 0$.

Holding transaction quantities fixed, the path of seigniorage is irrelevant for the total market capitalization of tokens. If investors immediately choose to issue a large quantity of tokens $d\mu_0$ when they make a governance decision, (29) implies that this issuance is immediately offset by a drop in the token price, so that the market capitalization of tokens before the issuance, $M_0(1 - d\mu_0)$, is equal to the market capitalization after, M_{0+}. That is, when the platform issues a hybrid token, *seigniorage imposes a capital loss on investors.* This is the main difference from the setting with service tokens: in that setting, the value of investors' assets (shares) would not drop upon the issuance of new tokens.

In governance decisions, only investors are permitted to vote on the level of fees f_s and the seigniorage rate $d\mu_s$. Per (5), investors choose policies to maximize the value of their tokens. The value of tokens outstanding at the time of the governance decision is $M_0(1 - d\mu_0)$, with investors holding some fraction of those. Therefore, investors' governance problem is equivalent to maximizing the market capitalization $M_0(1-d\mu_0)$ of tokens outstanding, which by Lemmas 4-5 satisfies

$$\hat{M}_0(1 - \hat{d\mu}_0) = \max_{\substack{f_s, d\mu_0, \dot{\mu}_s, \\ X_s, M_s, \zeta_s}} \mathbb{E}_0 \left[\int_0^\tau e^{-rs} \left(X_s^{1-\gamma} - cX_s \right) ds + e^{-r\tau} \hat{M}_0(1 - \hat{d\mu}_0) \right]$$

$$\text{s.t. } (25),\ (28),\ 1 - \zeta_s = \frac{X_s}{M_s},\ f_s, \dot{\mu}_s \geq 0, d\mu_0 \in [0, \frac{\Delta}{1 + \Delta}]. \tag{30}$$

6.3 Attaining the full-commitment outcome

We now prove our main result: equilibrium allocations are identical to those attained under the full-commitment outcome in Section 5.3. The logic underlying this result has two steps. First, as is plain from (30), the value of investors' staked tokens is proportional to future aggregate service flows plus platform profits. Second, investors' problem is *time-consistent*: the degree of commitment is irrelevant to equilibrium policies.

Proposition 8. *When the platform issues a hybrid token, investors' governance problem (30) is time-consistent: optimal policies and equilibrium allocations are invariant to the frequency λ of governance decisions.*

Time-consistency implies that no matter how strong investors' commitment power, they will choose policies to maximize the present value of service flows plus profits.

How, exactly, does a hybrid token overcome the time-inconsistency problem? After all, even in the case where the platform issues a token for transactions only, investors' profits depend on users' anticipated service flows. This logic is deceptive, though. When the platform issues a service token, share prices reflect the present value of profits and token prices reflect anticipated future service flows net of inflation. Investors can capture future service flows by issuing new tokens, diluting the value of existing tokens held by users. This is precisely the source of time-inconsistency: investors are tempted to issue a large quantity of new tokens after each governance decision. However, this token issuance is socially detrimental: it causes inflation, raising the cost of transacting on the platform and reducing transaction demand. Investors inefficiently choose to issue tokens and boost share prices at the cost of lowering the value of users' tokens.

When the platform issues a hybrid token, on the other hand, there is a single asset (tokens) held by both constituencies whose value reflects the present value of future profits plus service flows. Regardless of whether investors vote to issue new tokens, the value of their assets will depend on users' future service flows, so they have an incentive to keep service flows high. Indeed, the monetary neutrality result demonstrates that investors cannot benefit from issuing new tokens: any seigniorage revenues are offset by a decrease in the value of their tokens. The fact that investors' tokens lose value when they vote to issue new tokens ($d\mu_0 > 0$) eliminates the time-inconsistency problem. Formally, one can see this fact from (30): seigniorage at $s = 0$ does not enter investors' objective function (unlike investors' problem (18) in the case of a platform that issues service tokens). Intuitively, under the hybrid token system, users are protected from devaluation because they hold the same asset as the investors who govern the platform. With service tokens, by contrast, seigniorage reduces the price of *users' tokens* rather than *investors' shares*.

Since investors pass policies to maximize the present value of service flows plus profits, the equilibrium allocation is precisely the same as the full-commitment outcome of Section 5.3.

Proposition 9. *When governed by investors, a platform with a hybrid token achieves the full-commitment outcome of Section 5.3. Equilibrium policies solve*

$$\max_{\substack{f_t,\dot{\mu}_t,X_t,\\ M_t,\zeta_t}} \int_0^\infty e^{-rt}\left(X_t^{1-\gamma} - cX_t\right)dt \quad s.t.\ f_t, \dot{\mu}_t \geq 0, \tag{31}$$

$$rX_t = X_t^{1-\gamma} - (f_t + \dot{\mu}_t)X_t + (1-\zeta_t)\dot{M}_t,$$

$$rM_t = X_t^{1-\gamma} - cX_t + \dot{M}_t,\ 1-\zeta_t = \frac{X_t}{M_t}.$$

Equilibrium transaction quantities are $X_t = X^{token}$, defined in (21).

The equilibrium is unambiguously more efficient than the equilibrium with a traditional *platform but less efficient than the first-best.*

We should note that in this benchmark model a hybrid token can enhance efficiency by alleviating time-inconsistency in *seigniorage* policies. However, the benefits of hybrid tokens are actually more general. Section 8 will study an extension with investment in which a hybrid token can resolve time-inconsistency in *investment* policies as well as seigniorage.

7 Giving users the right to vote

Up until this point, we have assumed that the platform is governed by investors. Our main results demonstrated how various token designs can align investors' policy preferences with users' (or not). An important feature of the DeFi landscape, however, is the prevalence of platforms that give users the power to vote on policies directly. In this section, we discuss the consequences of giving token-holding users the right to vote in our model.

The model's basic ingredients are unchanged. Governance decisions arrive at a Poisson rate λ. Now, at the time τ of a governance decision, a new policy is chosen by whichever constituency (users or investors) has a voting majority at that time. If investors control the platform, as before, they choose policies that maximize the value of their assets. However, if users control the platform, then they choose a sequence of policies that maximizes the combined value of their tokens plus their expected future infra-marginal rents R_τ,

$$Q_\tau^T A_{\tau-}^T + R_\tau \quad \text{where} \quad R_\tau = \mathbb{E}_\tau \left[\int_0^\infty e^{-rs} \frac{\gamma}{1-\gamma} X_{\tau+s}^{1-\gamma} ds \right],$$

as shown by Proposition 1. We analyze both types of platforms that issue tokens in our model:

- **Service token platform**: Users are allocated voting power in proportion to their token holdings. Without loss of generality, we assume that users hold a majority of voting power[22] and choose a fee policy and a seigniorage policy $\{f_s, d\mu_0, \dot{\mu}_s\}$ at the time of each governance decision.

- **Hybrid token platform**: *All* agents, both users and investors, are allocated voting power proportional to their token holdings. In reality, this setup is akin to a DeFi

[22]Consider a platform that issues service tokens and allocates some voting power to users in proportion to their token holdings, with the remaining votes being allocated to shareholders. Each constituency always votes unanimously, so all that matters is whether token holders are allocated a minority or a majority of votes in the aggregate. When token holders have a minority of voting power, we are back in the case of an investor-controlled platform analyzed in Section 5.

platform that does not limit voting power to those who stake their tokens. At the time of a governance decision, users get to choose fee and seigniorage policies $\{f_s, d\mu_0, \dot{\mu}_s\}$ if they hold a majority of the token stock, whereas investors choose the policy otherwise, as in Section 6. (Control may pass from one constituency to the other over time as their token holdings change.)

In both cases, we look for a Markov equilibrium as before. The analysis leads to optimization problems quite similar to (18) or (30), so we present the main results here but defer the formal analysis to Appendix E.

The main difference is that users seek to pass policies that maximize the value of their transaction assets plus future infra-marginal rents, which is different from investors' objective of maximizing the value of cash flow assets. At the time τ of a governance decision, the value of users' tokens satisfies

$$Q_\tau^T A_{\tau-}^T = \begin{cases} \mathbb{E}_\tau \left[\int_0^\infty e^{-rs} \left(X_{\tau+s}^{1-\gamma} - (f_{\tau+s} + \dot{\mu}_{t+s}) X_{\tau+s} \right) ds \right] & \text{Service token} \\ \mathbb{E}_\tau \left[\int_0^\infty e^{-rs} \left(X_{\tau+s}^{1-\gamma} - c X_{\tau+s} \right) ds \right] & \text{Hybrid token} \end{cases}$$

The value of a service token is equal to the present value of service flows net of future token dilution, whereas the value of a hybrid token is equal to the present value of service flows plus profits.

Our main result is that giving users the right to vote redistributes economic rents away from investors, but it does not necessarily enhance efficiency.

Proposition 10. *Giving users the right to vote can increase user surplus, but it does not necessarily increase total surplus. Moreover, when users control the platform, transaction quantities may be above the first-best level X^{FB}. Specifically,*

1. *When the platform issues service tokens, users vote to set fees and seigniorage equal to zero ($f_t = \dot{\mu}_t = 0$), and transaction quantities satisfy*

$$X_t = X^{user} \equiv r^{-\frac{1}{\gamma}} \quad \forall\ t. \tag{32}$$

2. *When the platform issues a hybrid token, if users have a voting majority in the long run ($t \to \infty$), then there exists $\zeta^* \in (0, \frac{1}{2}]$ such that*

$$X_t \to \min \left\{ \left(\frac{1-\zeta^*}{(1-\gamma)(1-\zeta^*)+\gamma} c \right)^{-\frac{1}{\gamma}}, X^{user} \right\} \quad \text{as } t \to \infty$$

where $1-\zeta^*$ is the fraction of tokens held by users as $t \to \infty$.

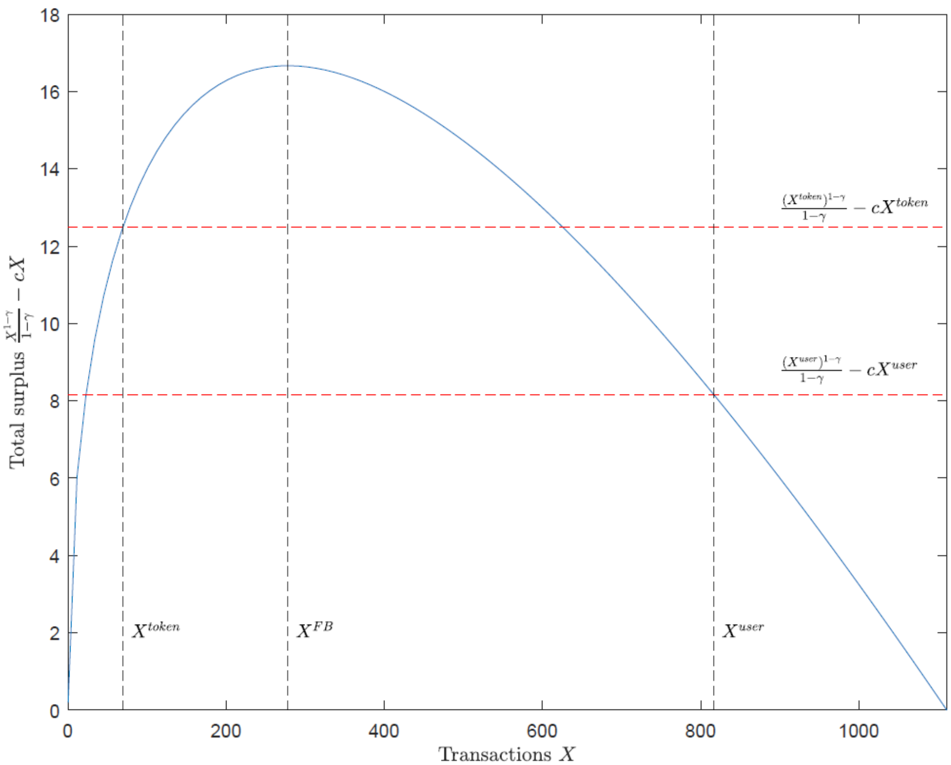

Figure 4: A comparison of total welfare (for a platform that issues service tokens) under governance by each constituency. The figure is plotted with parameters $c = 0.06, r = 0.035$. The blue curve plots total surplus as a function of transaction demand X. Transaction demand under investor governance is X^{token}, whereas transactions under user governance are X^{user}. The red lines mark the level of surplus in each case.

Figure 4 shows that a service token-issuing platform may indeed achieve lower surplus when governed by users rather than investors.

Proponents of DeFi sometimes claim that enfranchising users is key to mitigating inefficiencies. This argument contains a kernel of truth, but it is incomplete. When users are given the right to vote, the platform is more likely to be run in accordance with their interests, so there is less rent extraction. Users unambiguously benefit from the right to vote. However, users have incentives to run the platform in their own favor, to investors' detriment. In particular, users do not bear the platform's costs of operation – those are instead borne by investors. Users are therefore willing to pass policies that increase their inframarginal rents at the expense of the platform's profits. This desire to increase their own rents is why users pass policies that lead to inefficiently high transaction quantities. A redistribution of voting power can increase welfare only to the extent that it brings the *median* voter's preferences closer in line with maximizing total surplus (Hart and Moore, 1998). Just as there is no reason to presume that consumer cooperatives are more efficient than stockholder corporations, then, there is no reason to presume that a user-governed platform is more efficient than an investor-governed platform.

Note, however, that the policies preferred by users depend on whether the platform issues a service token or a hybrid token. In particular, users vote for policies that result in (weakly) lower transaction quantities under the hybrid token scheme – that is, they vote for higher fees and platform profits. Why would users vote to give some profits to investors? In this case, the value of users' tokens depends not only on their own service flows but also on the expected dividends that will be paid out to investors, as explained in Section 6. Consequently, users do not want to reduce the platform's profits too much.

The key to the hybrid token scheme is that it introduces an asset whose value is determined by the welfare of *both* constituencies. The fact that the value of investors' token holdings depends on users' welfare moderates their desire to extract rents from users. Similarly, if users govern the platform, their desired policies are moderated by the fact that they hold tokens whose value depends on the platform's profits. It is this alignment of preferences, not a redistribution of voting rights, that mitigates inefficiencies in platform policies.

8 Adding investment to the model

In our benchmark model, we showed that by issuing tokens, a platform's owners may be able to commit to future favorable *pricing* and enhance welfare by promoting greater *transaction demand*. It is natural to ask whether this conclusion is specific to pricing policies or whether token issuance can promote efficient policies in general. To answer this question, we extend the model to incorporate investment as well and show that indeed, token issuance incentivizes a platform's owners to implement better investment policies (from a welfare perspective). This conclusion is relevant to DeFi, since platforms often issue tokens to finance upgrades to platform functionality and transaction protocols.

In this extension, the platform can invest to upgrade its transaction technology whose quality is summarized by a state variable $Z_t \in \mathbb{R}_+$. A higher value of Z_t corresponds to a superior technology (e.g. lower transaction latency, a better transaction-matching algorithm, or greater transaction functionality). We assume a constant-returns-to-scale investment technology: an investment of $I_t dt$ dollars at time t increases productivity by $\Phi(\frac{I_t}{Z_t})Z_t dt$, where Φ in an increasing, concave, and differentiable function.[23] Then, denoting the *investment rate*

[23]This type of investment technology is typical in macroeconomic models in which new capital produced is a constant-returns-to-scale function of investment and the quantity of existing capital (Hayashi, 1982). To

by $\iota_t \equiv \frac{I_t}{Z_t}$, productivity evolves according to

$$dZ_t = \Phi(\iota_t) Z_t dt. \tag{33}$$

The platform's productivity determines users' utility of transacting on the platform. We assume that the payoff of a user i who consumes x_{it} transaction services at time t is

$$U_{it}dt = \big(\frac{Z_t^{\gamma} x_{it}^{1-\gamma}}{1-\gamma} - f_t x_{it}\big)dt.$$

Users' transaction payoff is Cobb-Douglas in the platform's productivity Z_t and the quantity of transactions x_{it}, so that a higher productivity increases the marginal utility of transacting on the platform.[24]Investment is financed out of the platform's earnings,[25] so the platform's profits are

$$d\Pi_t = \big((f_t - c)X_t - \iota_t Z_t\big)dt + dS_t.$$

Now, in addition to fees and seigniorage policies, the platform's governance determines the investment policy $\{\iota_s\}_{s\geq 0}$ as well. We consider the same three platform designs as in the benchmark model: a traditional platform, a platform that issues service tokens, and a platform that issues a hybrid token (assuming, for simplicity, that the platform is governed by investors). Investors still vote for policies that maximize the value of their cash flow assets as in (5). Again, we obtain optimization problems nearly identical to those in the benchmark model, so we postpone most of the formal analysis to Appendix F. The main difference from the benchmark model is that users' marginal utility of transacting depends on the platform's productivity Z_t: instead of (2), users' transaction demand condition is

$$X_t^{-\gamma}dt = Z_t^{-\gamma} \times \Bigg(f_t dt + \big(rdt - \mathbb{E}_t\big[\frac{dQ_t^T}{Q_t^T}\big]\big)\Bigg). \tag{34}$$

Just as in the benchmark model, token issuance can incentivize investors to choose investment policies that are more aligned with users' interests. The intuition is exactly the same. The value of a service token captures the present value of service flows. Hence, investors internalize users' service flows, which depend on the platform's productivity (since higher productivity increases the marginal utility of transactions). Therefore, if investors are able

ensure that optimization problems have unique solutions, we also impose the technical assumption that $\frac{\Phi'(\iota)}{r-\Phi(\iota)}$ is a decreasing function of ι (i.e., $\Phi(\cdot)$ is sufficiently concave).

[24]The Cobb-Douglas form of the utility function is analytically convenient but is not economically essential. What matter is that (1) users' utility is concave in transactions x_{it}, and (2) a higher productivity increases the marginal utility of transactions.

[25]This assumption is without loss of generality, since there are no financial frictions in this model.

to commit to future policies, the platform will invest more when it issues service tokens than when it does not.

Proposition 11. *In the model with investment, the following hold.*

1. *The first-best allocation has a constant investment rate $\iota_t = \iota^{FB}$, and transactions scale linearly with Z_t, $X_t = \tilde{X}^{FB} Z_t$, where*

$$\frac{1}{\Phi'(\iota^{FB})} = \frac{\frac{\gamma}{1-\gamma} c^{1-\frac{1}{\gamma}} - \iota^{FB}}{r - \Phi(\iota^{FB})} \quad \text{and} \quad \tilde{X}^{FB} = c^{-\frac{1}{\gamma}}.$$

2. *A traditional platform invests less than the first-best, $\iota_t = \iota^{trad} < \iota^{FB}$, and has lower transaction quantities, $X_t = \tilde{X}^{trad} Z_t$ with $\tilde{X}^{trad} < \tilde{X}^{FB}$*

3. *A platform that issues a service token with sufficient commitment power (λ small enough) invests more than a traditional platform and has higher transaction quantities. That is, $\iota_t = \iota^{token}$, $X_t = \tilde{X}^{token} Z_t$, with*

$$\iota^{trad} < \iota^{token} < \iota^{FB} \quad \text{and} \quad \tilde{X}^{trad} < \tilde{X}^{token} < \tilde{X}^{FB}.$$

Despite the fact that there are no financial frictions in the model, the mix of assets used to finance the platform is non-neutral. Specifically, the investment policies chosen by a token-financed platform are different from those chosen by a platform that is financed by shareholders only. This result stands in stark contrast to frictionless models of corporate finance in which the Modigliani-Miller theorem holds: the types of securities issued by a firm are irrelevant for investment. When token issuance is possible and investors have sufficient commitment power, they undertake investments that benefit users but decrease platform value ex post because doing so increases their initial seigniorage revenues.

Hence, the introduction of tokens may permit the financing of socially beneficial platform upgrades that would not have otherwise been in investors' interest. Conceptually, the reason for the non-neutrality of token issuance is that tokens are effectively a claim on future *user surplus* rather than *cash flows*: the precise manner in which investors tranche claims on cash flows is irrelevant in this model, but the introduction of long-lived claims on user surplus introduces new possibilities for profitable investments. The motive to issue tokens at a high price permits investors to partially internalize how policies change future user surplus as well as profits.

Hybrid tokens serve precisely the same role as in the benchmark model. When investors govern the platform, the introduction of a hybrid token overcomes the time-consistency problem, eliminating the need for commitment. Here, there is time-inconsistency not only in seigniorage policies but also in investment. Investors face a "token overhang" problem: some benefits of investment accrue to users because greater productivity increases the price of tokens issued *in the past*. A hybrid token permits investors to internalize a greater share of these benefits, resolving the token overhang.

Proposition 12. *In the model with investment, the following hold.*

1. *When commitment power is sufficiently weak (λ large enough), a platform that issues service tokens invests at a constant rate that is lower than ι^{token} in the commitment outcome of Proposition 11. The investment rate is decreasing in λ.*

2. *An investor-governed platform that issues a hybrid token achieves the commitment outcome ($\iota_t = \iota^{token}, X_t = \tilde{X}^{token} Z_t$) in Proposition (11).*

9 Discussion of assumptions

Here we comment on the model's main assumptions. The model has three key features that shape the results. First, there are distinct groups of agents who interact on the platform, generating scope for conflicts of interest. Second, the agents who run the platform lack the ability to commit to a sequence of policies. Third, we limit the types of contracts that can be written between users and investors.

Conflicts of interest: In our model, there are two distinct groups of agents: users who enjoy the platform's services and investors who hold all cash flow claims on the platform. This separation of users and investors may seem unnatural, since the users of a platform's service are typically not prevented from holding the platform's equity.

We make two comments on this issue. First, while this stark assumption helps to illustrate the logic of our model, it could be substantially relaxed without significantly altering the main results. What really matters for our results is that there must be some heterogeneity in preferences: there must be some agents who are more interested in the platform's services and others who are more natural investors (even if investors also enjoy the platform's services to some extent). For instance, investors could be interpreted as individuals who are more patient or who have deeper pockets. In the context of DeFi, those who stake tokens and earn cash flows often need some technical aptitude or computational resources – for example, blockchain "validators" typically use powerful hardware to certify transactions. As long as there is some heterogeneity in preferences, there will be conflicts of interest: natural users will prefer low fees and greater investment in the platform's technology, whereas natural investors will prefer to maximize profits at users' expense (see Hart and Moore, 1998; or Bakos and Halaburda, 2023; who emphasize a similar theme).

Second, even within the context of our model, our main results would mostly go through if we were to permit users to hold cash flow claims on the platform as well. In this case, there would be a continuum of equilibria. There would still be an equilibrium in which investors hold all of the cash flow claims on the platform. However, for each $k \in (0, 1]$, there would also be an equilibrium in which users hold a fraction k of all cash flow claims, with investors holding the remaining fraction $1 - k$. The inefficiencies that we highlight would continue to arise in this setting: investors would vote for policies that maximize profits at users' expense, whereas users would try to maximize their rents at investors' expense. The *only* efficient equilibrium would be the case in which users hold *all* cash flow claims on the platform ($k = 1$) – when investors sit out entirely, then conflicts of interest become irrelevant. By focusing on the case in which investors hold all cash flow claims and voting rights, our model addresses concerns about the possible "centralization" of DeFi platforms via a concentration of token holdings among non-users.

Lack of commitment: While a lack of commitment is of course central to our main results, we consider it natural to assume that investors cannot make a fully binding commitment to a sequence of policies. In reality, it is quite common for digital platforms to change their terms of service and fees. Even in the context of DeFi, despite the fact that some policies can be hard-coded at the time a platform is founded, it is possible for the rules of almost any platform to be amended in some way. While some blockchains require a "hard fork" (i.e., a split in the blockchain) to re-write the initially set rules, other platforms allow for rules to be updated in arbitrary ways by a simple vote among stakers. For example, the DeFi platforms MakerDAO, Curve, and Uniswap operate in this way. The possibility of amending the rules, moreover, is typically considered to be a desirable feature rather than a detriment to the platform's viability. However, our main results of course also encapsulate the case of full commitment as $\lambda \to 0$ (which could be achieved, e.g., if the platform is governed by a smart contract whose rules can never be superseded).

Limited contracting: The last key assumption in our model is limited contracting between users and investors. For one, we assume away the Coase-style solution in which users and investors collectively agree on a contract that rewards investors for passing a socially beneficial sequence of policies. We view this as a reasonable restriction for several reasons. It may be costly for users to coordinate and collectively bargain with investors. Moreover, if a contract that anticipates all possible future contingencies were feasible, then it would not even be necessary to have platform governance in the first place – all decisions could simply be encoded in the initial contract. We take the view of Grossman and Hart (1986): if the platform's ownership structure is to play an essential role, contracts must be incomplete to some extent.

10 Conclusion

We develop a general model of platform governance that is flexible enough to capture both traditional platforms as well as platforms that issue tokens with some combination of transaction services, cash flow claims, and voting rights.

A traditional platform extracts rents from its users by setting fees above its marginal costs. This discourages users from transacting and distorts volumes downwards, below the first-best level.

The issuance of *service tokens* can partially align shareholders' policy preferences with those of users, as long as shareholders are able to commit to future policies ex ante. If the platform passes policies that benefit users, they will be willing to pay a greater price to purchase tokens. Hence, if investors commit to pass such policies, they can reap large seigniorage revenues. However, if investors lack the ability to commit, this mechanism no longer aligns preferences: after selling tokens to users, investors will again be tempted to extract rents from them by inflating away tokens' value.

The platform can overcome the commitment problem by issuing a *hybrid token* that bundles transaction services with cash flow claims. The key idea is that by issuing a single asset whose value reflects *both* constituencies' welfare, investors can be disincentivized from extracting rents. If they pass policies that are detrimental to users, the token price will fall, hurting investors as well.

Giving users the right to vote, on the other hand, redistributes economic rents to users but does not necessarily enhance welfare. Just like investors, users are self-interested and pass policies that increase their rents at investors' expense.

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A Model

A.1 Agents' optimization problems

Users: Users' optimization problem is

$$U_i = \max_{x_{it}, a_{it}, c_{it}} \ \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \Big((\frac{x_{it}^{1-\gamma}}{1-\gamma} - f_t x_{it}) dt + dc_{it} \Big) \right]$$

$$\text{s.t.} \ \ Q_t^T da_{it} + dc_{it} = 0, \ x_{it} \leq Q_t^T a_{it}, \ a_{i0} \text{ given,}$$

The only individual state variable for user i is her asset holdings a_{it}.

Using integration by parts, it is possible to rearrange an agent's lifetime utility from consumption of dollars:

$$\begin{aligned} \int_0^\infty e^{-rt} dc_t = - \int_0^\infty e^{-rt} Q_t^T da_{it} & \\ &= -e^{-rt} Q_t^T a_{it} \Big|_0^\infty + \int_0^\infty e^{-rt} \big(a_{it} dQ_t^T - r a_{it} Q_t^T dt \big) \\ &= Q_0^T a_{i0} + \int_0^\infty e^{-rt} a_{it} \big(dQ_t^T - r Q_t^T dt \big). \end{aligned}$$

Using this equation, agent i's optimization problem can be reformulated as

$$U_i = \max_{x_{it}, a_{it}} Q_0^T a_{i0} + \int_0^\infty e^{-rt} \Big((\frac{x_{it}^{1-\gamma}}{1-\gamma} - f_t x_{it}) dt - a_{it} (r Q_t^T dt - \mathbb{E}_t [dQ_t^T]) \Big) \tag{35}$$

$$\text{s.t.} \ \ x_{it} \leq Q_t^T a_{it}, \ a_{i0} \text{ given.}$$

Users' problem then reduces to a sequence of static optimizations over (x_{it}, a_{it}).

The optimal x_{it} solves

$$\max_{x_{it}, a_{it}} \frac{x_{it}^{1-\gamma}}{1-\gamma} - f_t x_{it} - \big(r Q_t^T dt - \mathbb{E}_t \left[dQ_t^T \right] \big) a_{it} \ \ \text{s.t.} \ \ x_{it} \leq Q_t^T a_{it},$$

so

$$x_{it}^{-\gamma} dt = f_t dt + \left(r dt - \mathbb{E}_t \left[\frac{dQ_t^T}{Q_t^T} \right] \right), \tag{36}$$

as claimed in (2).

We also derive users' value functions, since those will be important in determining users' preferences over sequences of policies. Users' first-order condition (2) implies that the flow utility they receive at time t is

$$\left(\frac{X_t^{1-\gamma}}{1-\gamma} - f_t X_t\right)dt - X_t\left(rdt - \mathbb{E}_t\left[\frac{dQ_t^T}{Q_t^T}\right]\right) = \frac{\gamma}{1-\gamma}X_t^{1-\gamma}dt.$$

Then, plugging this expression into the right-hand side of (35),

$$U_i = Q_0^T A_{0^-}^T + R_0, \quad \text{where } R_0 \equiv \mathbb{E}_0\left[\int_0^\infty e^{-rt}\frac{\gamma}{1-\gamma}X_t^{1-\gamma}dt\right], \tag{37}$$

as claimed in Proposition 1.

Investors' lifetime utility can be derived with analogous calculations. Investor j's problem is

$$U_j = \max_{a_{jt}, c_{jt}} \ \mathbb{E}_0\left[\int_0^\infty e^{-rt}dc_{jt}\right] \ \text{ s.t. } \ Q_t^C(da_{jt} - a_{jt}dD_t) + dc_{jt} = 0, \ a_{j0} \text{ given.}$$

Integration by parts yields

$$U_j = \max_{a_{jt}} \ Q_0^C a_{j0} + \mathbb{E}_0\left[\int_0^\infty e^{-rt}a_{jt}\left(dD_t + dQ_t^C - rQ_t^C dt\right)\right] \ \text{ s.t. } \ a_{j0} \text{ given.}$$

The first-order condition implies that at an optimum (4) holds, and

$$U_j = Q_0^C A_{0^-}^C. \tag{38}$$

Proof of Proposition 1. The result is immediate from (37) and (38). □

A.2 The first-best

In this section, we formally lay out and solve the planner's problem:

$$\hat{V}_0^P = \max_{X_s} \ \mathbb{E}_0\left[\int_0^\tau e^{-rs}\left(\frac{X_s^{1-\gamma}}{1-\gamma} - cX_s\right)ds + e^{-r\tau}\hat{V}_0^P\right]. \tag{39}$$

We now prove Proposition (2), which characterizes the first-best allocation.

Proof of Proposition 2. First, note that the planner's problem is time-consistent by standard arguments (see Lucas and Stokey, 1986): it is a standard dynamic programming problem with control X_s and no state variables. Therefore, the optimal policy solves the corresponding sequence problem. The sequence problem is

$$\max_{X_t} \int_0^\infty e^{-rt}\left(\frac{X_t^{1-\gamma}}{1-\gamma} - cX_t\right)dt \tag{40}$$

with first-order condition

$$X_t^{-\gamma} = c,$$

which immediately implies (8). □

B Traditional platform

In this section, we prove our main results about the traditional scheme in Proposition 3.

Proof of Proposition 3. Standard arguments (Lucas and Stokey, 1986) imply that investors' governance problem (11) is time-consistent: it is a standard dynamic programming problem with control f_s and no state variables. Therefore, investors' problem can be written as the sequence problem (12). We substitute the transaction demand constraint (10) into investors' profits to obtain

$$(f_t - c)X_t = X_t^{1-\gamma} - (c+r)X_t$$

and use this result to write the sequence problem as.

$$\max_{X_t} \int_0^\infty e^{-rt}\left(X_t^{1-\gamma} - (c+r)X_t\right)dt.$$

The first-order condition is

$$(1-\gamma)X_t^{-\gamma} = c + r.$$

Therefore, the optimal transaction quantity is X^{trad} given by (14), which is clearly less than X^{FB} in (8). Since X^{FB} is the first-best quantity of transactions, it is immediate that welfare is lower than the first-best with a traditional platform. □

C Service tokens

In this section, we prove the results in Section 5.

Proof of Lemma 1. In equilibrium, conditional on no governance decision, transactions and the stock of tokens grow at rates $\frac{\dot{X}_s}{X_s}$ and $\dot{\mu}_s$, respectively. When a governance decision takes place, transactions jump to $\hat{X}_0$ and the stock of tokens immediately grows by $\hat{d\mu}_0$. Then, using the fact that $Q_t^T = \frac{X_t}{A_t^T}$, we have

$$\frac{dQ_t^T}{Q_t^T} = \left(\frac{\dot{X}_s}{X_s} - \dot{\mu}_s\right)ds + \left(\frac{\hat{X}_0}{X_s}(1 - \hat{d\mu}_0) - 1\right)dJ_s.$$

where dJ_s denotes a Poisson process that arrives at rate λ (representing the next governance decision). Taking expectations of both sides, we obtain the equation in Lemma 1. □

Proof of Lemma 2. This result is immediate from investors' governance problem (18). The rate of seigniorage $d\mu_0$ at $s = 0$ can be increased up to the maximum limit $d\mu_0 = 1$ without affecting profits or the constraint (17) at any future date. □

Proof of Proposition 4. Using the result of Lemma 2, we have

$$\begin{aligned}
\mathbb{E}_0\left[\int_0^\tau e^{-rs} d\Pi_s\right] &= \mathbb{E}_0\left[X_0 + \int_0^\tau e^{-rs}(f_s + \dot{\mu}_s - c)X_s ds\right] \\
&= X_0 + \mathbb{E}_0\left[\int_0^\tau e^{-rs}\left(X_s^{1-\gamma} - (c + r + \lambda)X_s + \dot{X}_s + \lambda\hat{X}_0(1 - \frac{\Delta}{1+\Delta})\right)ds\right] \\
&= X_0 + \int_0^\infty \lambda e^{-\lambda\tau} \int_0^\tau e^{-rs}\left(X_s^{1-\gamma} - (c + r + \lambda)X_s + \dot{X}_s + \lambda\frac{1}{1+\Delta}\hat{X}_0\right)ds d\tau \\
&= X_0 + \int_0^\infty e^{-(r+\lambda)s}\left(X_s^{1-\gamma} - (c + r + \lambda)X_s + \dot{X}_s + \lambda\frac{1}{1+\Delta}\hat{X}_0\right)ds \\
&= X_0 + \int_0^\infty e^{-(r+\lambda)s}\left(X_s^{1-\gamma} - (c + r + \lambda)X_s + \lambda\frac{1}{1+\Delta}\hat{X}_0\right)ds \\
&+ e^{-(r+\lambda)s}X_s\Big|_0^\infty + \int_0^\infty e^{-rs}(r + \lambda)X_s ds \\
&= \int_0^\infty e^{-(r+\lambda)s}\left(X_s^{1-\gamma} - cX_s + \lambda\frac{1}{1+\Delta}\hat{X}_0)\right)ds,
\end{aligned}$$

as desired. The second equality uses (17). The fifth equality integrates $\int_0^\infty e^{-(r+\lambda)s}\dot{X}_s ds$ by parts. □

Proof of Propositions 5 - 7. We characterize the properties of equilibrium in both the strong commitment and weak commitment regime. We begin by taking a Lagrangian approach to investors' governance problem. Following Proposition (4), investors' problem can be written as

$$\hat{V}_0^I = \max_{f_s, \dot{\mu}_s, X_s} \int_0^\infty e^{-(r+\lambda)s}\Big(X_s^{1-\gamma} - cX_s + \lambda(\frac{1}{1+\Delta}\hat{X}_0 + \hat{V}_0^I)$$

$$- \psi_s\big((r+\lambda)X_s - X_s^{1-\gamma} + (f_s+\dot{\mu}_s)X_s - \dot{X}_s\big) - \chi_s^f f_s - \chi_s^\mu \dot{\mu}_s\Big)ds$$

The Euler-Lagrange conditions are

$$(f_s) : 0 = \psi_s X_s + \chi_s^f;$$
$$(\dot{\mu}_s) : 0 = \psi_s X_s + \chi_s^\mu;$$
$$(X_s) : \dot{\psi}_s = (1+\psi_s)(1-\gamma)X_s^{-\gamma} - c - \psi_s(f_s + \dot{\mu}_s)$$

We will conjecture an equilibrium in which fees $f_s = f$, seigniorage $\dot{\mu}_s = \dot{\mu}$, transaction quantities $X_s = X$, and the Lagrange multipliers are constant as well. Define λ^* as

$$\lambda^* = \frac{1+\Delta}{\Delta}\left(\frac{c}{1-\gamma} - r\right).$$

We proceed by analyzing two distinct cases.

Case 1: Strong commitment ($\lambda \leq \lambda^*$). We conjecture that in this case, the Lagrange multiplier ψ is equal to zero. The Euler-Lagrange condition for X_s then implies that transaction quantities must satisfy

$$(1-\gamma)X_s^{-\gamma} = c \quad \Rightarrow \quad X_s = \left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}}.$$

This quantity of transactions is greater than X^{trad} but below the first-best X^{FB}.

To confirm that this is indeed an equilibrium allocation, we must show that there exist non-negative levels of fees f and seigniorage $\dot{\mu}$ that support it. The transaction demand condition (i.e., the condition corresponding to the multiplier ψ_s) yields

$$f + \dot{\mu} = \frac{c}{1-\gamma} - (r + \lambda\Delta). \tag{41}$$

Then it is possible that $f, \dot{\mu} \geq 0$ only if $\lambda \leq \lambda^*$. Our conjectured equilibrium is indeed an equilibrium, then, if and only if $\lambda \leq \lambda^*$.

Note, moreover, that this argument proves Proposition 6. Any combination $(f, \dot{\mu})$ satisfying (41) implements the same equilibrium.

Case 2: Weak commitment ($\lambda > \lambda^*$). In this case, we conjecture and verify that fees and seigniorage are equal to zero, $f = \dot{\mu} = 0$. With a constant Lagrange multiplier ψ, we have

$$(1+\psi)(1-\gamma)X^{-\gamma} = c \quad \Rightarrow \quad X = \left(\frac{c}{(1+\psi)(1-\gamma)}\right)^{-\frac{1}{\gamma}}.$$

The transaction demand condition pins down $\tilde{x}$:

$$X = \left(\frac{r + \lambda\frac{\Delta}{1+\Delta}}{1-\gamma}\right)^{-\frac{1}{\gamma}}.$$

When $\lambda > \lambda^*$, clearly, the Lagrange multiplier ψ is indeed non-zero, justifying the fact that both fees and seigniorage are at their constraints.

□

D Hybrid tokens

D.1 Equilibrium

In this section, we prove general results about the equilibrium with a hybrid token.

Proof of Lemma 4. Multiply (25) by $1 - \zeta_s$, multiply (26) by ζ_s, and add the two resulting equations together to obtain

$$(r + \lambda)M_s = X_s^{1-\gamma} - cX_s + \dot{M}_s + \lambda\hat{M}_0(1 - \hat{d\mu_0}).$$

Imposing the transversality condition $\lim_{s\to\infty} e^{-(r+\lambda)s}M_s = 0$, this differential equation can be solved forward to obtain

$$M_s = \int_0^\infty e^{-(r+\lambda)u}\left(X_{s+u}^{1-\gamma} - cX_{s+u} + \lambda\hat{M}_0(1 - \hat{d\mu_0})\right)$$

$$= \mathbb{E}_s \left[\int_0^\tau e^{-r(u-s)} \left(X_u^{1-\gamma} - cX_u \right) + e^{-r(\tau-s)} \hat{M}_0 (1 - \hat{d\mu}_0) \right],$$

as desired. □

Proof of Lemma 5. That $\{M_s\}$ is independent of $\dot{\mu}_s$ for $s > 0$ is immediate from Lemma 4. It remains to prove that $M_0(1 - d\mu_0) = M_{0+}$. First, note that if $d\mu_0 > 0$, then (27) implies $\zeta_0 = 1$. Then investors' pricing equation for tokens yields

$$\begin{aligned} 0 &= dS_0 + \mathbb{E}_0[dM_0] \\ &= M_0 d\mu_0 + M_{0+} - M_0 \\ \Rightarrow M_0(1 - d\mu_0) &= M_{0+}. \end{aligned}$$

□

D.2 Attaining the commitment outcome

Next, we prove our main results about the hybrid token system with investor governance.

Proof of Proposition 8. Problem (30) is time-consistent by standard arguments. When written in Lagrangian form, it can be viewed as a typical Bellman equation with controls $(f_s, \dot{\mu}_s, X_s, M_s, \zeta_s)$. Therefore, optimal policies are independent of the times at which governance decisions took place: the policy chosen at time τ for $s \geq \tau$ is precisely the same as the policy chosen for $s \geq \tau$ at $t = 0$. Hence, optimal policies and equilibrium allocations are independent of the frequency λ of governance decisions. □

Proof of Proposition 9. Given that Problem (30) is time-consistent, the optimal policy must solve (31). Note that this is precisely the same problem as (20), so all of the results carry over. Equilibrium transaction quantities are precisely the same as in Proposition 5, and just as in that proposition, the equilibrium allocation is more efficient than that with a traditional platform but less efficient than the first-best. □

E Giving users the right to vote

In this section, we prove the results in Proposition 10. The Proposition deals with two cases – a setting with a platform that issues service tokens and one with a platform that issues hybrid tokens. The following two subsections address each in turn.

E.1 A service token platform

As discussed in Section 7, when the platform permits service token holders to vote, we can restrict attention to the case in which users control a majority of voting power for all t. Hence, users choose their most-preferred policy in each governance decision.

We look for a recursive equilibrium in which quantities may jump at the time of a governance decision but evolve smoothly thereafter, with the same notation as in the benchmark model. Users maximize their lifetime utility, which per (6) is equal to the value of their tokens plus the present value of their inframarginal rents, which will be denoted by

$$\hat{R}_0 = \mathbb{E}_0\left[\int_0^\infty e^{-rs}\frac{\gamma}{1-\gamma}X_s^{1-\gamma}ds + e^{-r\tau}\hat{R}_0\right].$$

The value of users' tokens at the time $s=0$ of a governance decision satisfies

$$Q_0^T A_{0-}^T = X_0(1-d\mu_0),$$

since users' initial token holdings are only a fraction $1-d\mu_0$ of the initial token stock if new token issuance at $s=0$ is $d\mu_0$. Then, users' governance problem can be written recursively in Lagrangian form as

$$\begin{aligned}(1-\hat{d\mu_0})\hat{X}_0 + \hat{R}_0 = \max_{f_s,\dot{\mu}_s,d\mu_0,X_s} \int_0^\infty e^{-(r+\lambda)s}\Big((1-d\mu_0)\big(X_s^{1-\gamma}-(f_s+\dot{\mu}_s)X_s\big) + \frac{\gamma}{1-\gamma}X_s^{1-\gamma} \\ + \lambda\big((1-\hat{d\mu_0})\hat{X}_0+\hat{R}_0\big) + \chi_s^f f_s + \chi_s^\mu \dot{\mu}_s + \psi_s\big((r+\lambda)X_s \\ - X_s^{1-\gamma} + (f_s+\dot{\mu}_s)X_s - \dot{X}_s - \lambda(1-\hat{d\mu_0})\hat{X}_0\big)\Big)ds., \qquad (42)\end{aligned}$$

subject to the additional constraint $d\mu_0 \in [0, \frac{\Delta}{1+\Delta}]$.

Users will always choose to set initial seigniorage $d\mu_0$ to zero – unanticipated seigniorage is just a transfer to investors. The Euler-Lagrange conditions are

$$\begin{aligned}&(f_s): (\psi_s - 1)X_s = \chi_s^f;\\ &(\dot{\mu}_s): (\psi_s - 1)X_s = \chi_s^\mu;\\ &(X_s): (r+\lambda)\psi_s = X_s^{-\gamma} - (f_s+\dot{\mu}_s) + \psi_s((r+\lambda) - (1-\gamma)X_s^{-\gamma} + f_s + \dot{\mu}_s) + \dot{\psi}_s.\end{aligned}$$

It is simple to see from this problem that users' utility is strictly increasing in transaction quantities X_s and decreasing in fees and seigniorage. Hence, there is no advantage to users of setting fees or seigniorage above their minimum allowable levels. Users thus set fees $f_s = 0$ for all s and choose zero seigniorage, $\dot{\mu}_s = 0$ for all s and $d\mu_0 = 0$. By setting $f_s = \dot{\mu}_s = 0$ and conjecturing a constant multiplier $\psi_s = \psi$, we obtain

$$0 = (1 - (1-\gamma)\psi)X_s^{-\gamma} \quad \Rightarrow \quad \psi = \frac{1}{1-\gamma}.$$

From there, one can find suitable values for the Lagrange multipliers χ_s^f, χ_s^μ using the Euler-Lagrange conditions above. Transaction quantities satisfy (17), so

$$X_s^{1-\gamma} = rX_s \quad \Rightarrow X_s = X^{user} \equiv r^{-\frac{1}{\gamma}},$$

as desired.

E.2 A hybrid token platform

Before proving the specific results in this section, we characterize equilibrium in a slightly more general setting and then show how to derive the results from this general characterization.

We consider an equilibrium in which token quantities never jump. Lemma 5 guarantees that this is without loss of generality. The equilibrium quantities are then $\{f_s, \dot{\mu}_s, X_s, M_s, \zeta_s, R_s\}$. As in the main body of the paper, several conditions must be satisfied in equilibrium. There is the token demand condition

$$(r+\lambda)X_s = X_s^{1-\gamma} - (f_s + \dot{\mu}_s)X_s + \frac{X_s}{M_s}\big(\dot{M}_s + \lambda\hat{M}_0(\zeta_s)\big), \tag{43}$$

the token pricing condition (28), the condition determining ζ_s,

$$\zeta_s = \frac{X_s}{M_s}, \tag{44}$$

and the Bellman equation determining the present value of inframarginal rents,

$$\hat{R}_0(\zeta) = \mathbb{E}_0\left[\int_0^\tau \frac{\gamma}{1-\gamma}X_s^{1-\gamma}ds + e^{-r\tau}\hat{R}_0(\zeta_\tau)\right]. \tag{45}$$

We consider a decision-maker who, starting in state ζ, chooses policies to maximize

$$\max_{\substack{f_s,\dot{\mu}_s \\ X_s, M_s, \zeta_s}} \mathbb{E}_0\left[\int_0^\tau \Big((1+b(\zeta))X_s^{1-\gamma} - cX_s\Big)ds + e^{-r\tau}\big(\hat{M}_0(\zeta_\tau) + b(\zeta)\hat{R}_0(\zeta_\tau)\big)\right]$$

$$\text{s.t. } (28),\ (43),\ (44), \tag{46}$$

where the function b is decreasing, continuous, and differentiable (except for perhaps at $\zeta = \frac{1}{2}$).

We write the optimization problem in Lagrangian form:

$$\begin{aligned}\mathcal{L} = \max_{\substack{f_s,\dot{\mu}_s \\ X_s,M_s,\zeta_s}} \int_0^\infty e^{-(r+\lambda)s}\Big(&(1+b(\zeta))X_s^{1-\gamma} - cX_s + \lambda\hat{M}_0(\zeta_s) + \lambda b(\zeta)\hat{R}_0(\zeta_s) \\ &- \phi_s\big((r+\lambda)M_s - X_s^{1-\gamma} + cX_s - \dot{M}_s - \lambda\hat{M}_0(\zeta_s)\big) \\ &- \psi_s\big((r+\lambda)X_s - X_s^{1-\gamma} + (f_s+\dot{\mu}_s)X_s - \zeta_s(\dot{M}_s + \lambda\hat{M}_0(\zeta_s))\big) - \chi_s(\zeta_s - \frac{X_s}{M_s})\Big)ds\end{aligned}$$

The Euler-Lagrange conditions are

$$\begin{aligned}(f_s) &: \psi_s X_s = 0; \\ (\dot{\mu}_s) &: \psi_s X_s = 0; \\ (\zeta_s) &: \lambda((1+\phi_s)\hat{M}_0'(\zeta_s) + b(\zeta)\hat{R}_0'(\zeta_s)) = \chi_s; \\ (X_s) &: (1+\phi_s)(1-\gamma)(1+b(\zeta))X_s^{-\gamma} = (1+\phi_s)c - \frac{\chi_s}{M_s}; \\ (M_s) &: \dot{\phi}_s - (r+\lambda)\phi_s = -(r+\lambda)\phi_s - \chi_s\frac{X_s}{M_s^2};\end{aligned}$$

We look for an equilibrium in which $X_s = X$, $M_s = M$, and $\zeta_s = \zeta'$ are constant.

Case 1: $\zeta' \neq \frac{1}{2}$. Note that if ζ_s is constant, then $\chi_s = 0$ for all s (unless $\zeta = \frac{1}{2}$, a case to which we return later). The first-order condition for M_s then also implies $\phi_s = 0$ for all s.

The first-order condition for X_s then yields

$$X = \left(\frac{c}{(1-\gamma)(1+b(\zeta))}\right)^{-\frac{1}{\gamma}}. \tag{47}$$

Case 2: $\zeta' = \frac{1}{2}$. The optimal level of X in this case can be found by solving the constrained optimization problem

$$\max_X \frac{(1+b(\zeta))X^{1-\gamma} - cX + K(\zeta)}{r+\lambda} \quad \text{s.t.} \quad X = \frac{1}{2}\frac{(1-\gamma)X^{1-\gamma} - cX + K(\zeta)}{r+\lambda}.$$

where $K(\zeta) \equiv \lambda(\hat{M}_0(\frac{1}{2}) + b(\zeta)\hat{R}_0(\frac{1}{2}))$.

The first-order condition is

$$(X) : (1-\gamma)\big(1+\frac{b(\zeta)}{1+\frac{1}{2}\chi}\big)X^{-\gamma} = c-(r+\lambda)\frac{\chi}{1+\frac{1}{2}\chi};$$

where χ denotes the Lagrange multiplier on the constraint. This condition, along with the constraint

$$X = \frac{1}{2}\frac{X^{1-\gamma}-cX+K(\zeta)}{r+\lambda},$$

pin down the two unknowns (X, χ).

So we have derived an equilibrium in which, starting from a governance decision in state ζ, ζ_s stays constant at a level $\zeta' = G(\zeta)$ until the next governance decision. We want to demonstrate that starting from any state ζ_0, the sequence $\zeta_k = G(\zeta_{k-1})$ converges to a (possibly non-unique) steady state ζ^* with $\zeta^* = G(\zeta^*)$.

We begin by proving that G is monotonically increasing.

Lemma 6. *The function $G(\zeta)$ is weakly increasing in ζ: if $\zeta' \geq \zeta$, then $G(\zeta') \geq G(\zeta)$.*

Proof. Notice that the policy problem faced by the decision-maker can be written as

$$\max_{f_s, \dot{\mu}_s} M_0 + b(\zeta)R_0,$$

where $b(\zeta)$ is a decreasing function of ζ and

$$M_0 = \int_0^\infty e^{-(r+\lambda)s}\Big(X_s^{1-\gamma} - cX_s + \lambda\hat{M}_0(\zeta_s)\Big)ds,$$

$$R_0 = \int_0^\infty e^{-(r+\lambda)s}\Big(\frac{\gamma}{1-\gamma}X_s^{1-\gamma} + \lambda\hat{R}_0(\zeta_s)\Big)ds.$$

Clearly, then, higher ζ implies that at an optimum, M_0 will be larger and R_0 will be smaller. The first-order condition (47) also implies that X_0 will be smaller for higher ζ. Thus,

$$G(\zeta) = \frac{X_0}{M_0}$$

is decreasing in ζ. □

With this lemma, we can prove the convergence of equilibrium to a steady state.

Proposition 13. *A solution to (46) converges to an equilibrium with constant ζ.*

Proof. We have demonstrated the existence of an equilibrium in which, starting from a governance decision in state ζ, ζ_s is constant at $\zeta' = G(\zeta)$ until the next governance decision. Let ζ^0 denote the initial value of ζ and let $\zeta^k = G(\zeta^{k-1})$ denote its value in the interval after the k-th governance decision. We show that the sequence ζ^k must converge.

Convergence follows from the fact that G is a monotonically increasing function that is continuous on the intervals $[0, \frac{1}{2}]$ and $(\frac{1}{2}, 1]$. There are two cases to consider.

Case 1: $G(\frac{1}{2}) \leq \frac{1}{2}$. If $\zeta_0 \in [0, \frac{1}{2}]$, Brouwer's fixed point theorem immediately implies that the sequence ζ^k converges, since the interval $[0, \frac{1}{2}]$ is mapped to itself. If $\zeta_0 > \frac{1}{2}$, then there are again two cases: either $\lim_{\zeta \to^+ \frac{1}{2}} G(\zeta) > \frac{1}{2}$ or $\lim_{\zeta \to^+ \frac{1}{2}} G(\zeta) < \frac{1}{2}$. In the first case, G maps some interval $[\frac{1}{2} + \epsilon, \frac{1}{2}]$ to itself, so ζ^k converges somewhere in that interval. In the second case, if ζ^k does not converge to a steady state in $(\frac{1}{2}, 1]$, then it eventually enters the interval $[0, \frac{1}{2}]$, after which point it converges in that interval.

Case 2: $G(\frac{1}{2}) > \frac{1}{2}$. The proof in this case is analogous. □

Note that the setting in our paper is simply the case in which

$$b(\zeta) = \begin{cases} \frac{1}{1-\zeta} & \zeta \leq \frac{1}{2} \\ 0 & \zeta > \frac{1}{2} \end{cases}$$

This is because users attach a weight $1 - \zeta$ to the value of tokens (since that is the fraction of the token stock that they hold) and a weight of one to their inframarginal rents, and they control the platform whenever $\zeta \leq \frac{1}{2}$. Investors attach a weight ζ to the value of tokens and zero to inframarginal rents, and they control the platform whenever $\zeta > \frac{1}{2}$.

Now we characterize the outcome in a steady state as $t \to \infty$. If the fraction of staked tokens converges to $\zeta^* > \frac{1}{2}$, then investors control the platform in the steady state, and we are back in the setting of Section 6. We will then consider only the outcome when $\zeta^* \leq \frac{1}{2}$. In the steady state, users' optimal policy solves the static problem

$$\max_{f,\dot{\mu}} \left(1 - \zeta^* + \frac{\gamma}{1-\gamma}\right) X^{1-\gamma} - (1 - \zeta^*) c X$$
$$\text{s.t.} \;\; X = (r + f + \dot{\mu})^{-\frac{1}{\gamma}}, \;\; f, \dot{\mu} \geq 0.$$

That is, users maximize a linear combination of $1 - \zeta$ times the token price plus inframarginal rents. When ζ is large enough, we obtain the unconstrained solution

$$X = \left(\frac{1-\zeta}{(1-\zeta)(1-\gamma)+\gamma} c\right)^{-\frac{1}{\gamma}} > c^{-\frac{1}{\gamma}} = X^{FB}.$$

Otherwise, the optimum is to set $f = \dot{\mu} = 0$, in which case

$$X = X^{user} = r^{-\frac{1}{\gamma}},$$

as claimed in Proposition 10.

F Adding investment to the model

In this section, we characterize the equilibrium of the model with investment. We begin with the first-best before moving to the case of an investor-governed platform. Investors' objective remains the same: they choose policies to maximize the value of their assets. The only difference from the benchmark is that in addition to choosing fees and a seigniorage policy, in this setting investors choose an investment policy ι_t as well.

The first-best: The planner's problem is to choose an allocation that maximizes the discounted value of total surplus,

$$\frac{Z_t^\gamma X_t^{1-\gamma}}{1-\gamma} - \iota_t Z_t.$$

The planner's problem is time-consistent, since it is just a standard optimal control problem with controls (X_t, ι_t) and state Z_t. It can be written in Lagrangian form as

$$\mathcal{L} = \max_{X_t, \iota_t, Z_t} \int_0^\infty e^{-rt} \left(\frac{Z_t^\gamma X_t^{1-\gamma}}{1-\gamma} - cX_t - \iota_t Z_t - \xi_t \big(\dot{Z}_t - \iota_t Z_t \big) \right) dt.$$

The Euler-Lagrange conditions are

$$\begin{aligned} (X_t) &: Z_t^\gamma X_t^{-\gamma} = c; \\ (\iota_t) &: \xi_t \Phi'(\iota_t) = 1; \\ (Z_t) &: \frac{\gamma}{1-\gamma} Z_t^{\gamma-1} X_t^{1-\gamma} - \iota_t + \Phi(\iota_t)\xi_t = r\xi_t - \dot{\xi}_t. \end{aligned}$$

The optimality condition for X_t yields

$$X_t^{FB} = c^{-\frac{1}{\gamma}} Z_t.$$

Then, looking for a solution with constant ξ_t and ι_t, we combine the optimality conditions for ι_t and Z_t to obtain

$$\frac{1}{\Phi'(\iota^{FB})} = \frac{\frac{\gamma}{1-\gamma} c^{1-\frac{1}{\gamma}} - \iota^{FB}}{r - \Phi(\iota^{FB})}.$$

Traditional platform: Users' transaction demand in this case implies

$$Z_t^{\gamma} X_t^{1-\gamma} = (f_t + r)X_t \quad \Rightarrow \quad (f_t - c)X_t = Z_t^{\gamma} X_t^{1-\gamma} - (c+r)X_t. \tag{48}$$

Investors' problem is again time-consistent. In this case, using the above result, the problem of maximizing the platform's share value can be written in Lagrangian form as

$$\mathcal{L} = \max_{f_t, \iota_t, X_t, Z_t} \int_0^{\infty} e^{-rt} \Big(Z_t^{\gamma} X_t^{1-\gamma} - (c+r)X_t - \iota_t Z_t - \xi_t (\dot{Z}_t - \Phi(\iota_t) Z_t) \Big) dt. \tag{49}$$

The Euler-Lagrange conditions are

$$\begin{aligned}
(X_t) &: (1-\gamma) Z_t^{\gamma} X_t^{-\gamma} = c + r; \\
(\iota_t) &: \xi_t \Phi'(\iota_t) = 1; \\
(Z_t) &: \gamma Z_t^{\gamma-1} X_t^{1-\gamma} - \iota_t + \Phi(\iota_t)\xi_t = r\xi_t - \dot{\xi}_t.
\end{aligned}$$

Therefore, the optimal transaction quantity is given by (14). We conjecture an equilibrium with a constant rate of investment ι^{trad} and Lagrange multiplier $\xi_t = \xi$. Plugging the optimal transaction quantity into the Euler-Lagrange condition for Z_t, we have

$$\gamma Z_t^{\gamma-1} X_t^{1-\gamma} = \gamma \Big(\frac{c+r}{1-\gamma}\Big)^{1-\frac{1}{\gamma}},$$

so

$$\xi = \frac{1}{\Phi'(\iota^{trad})} = \frac{\gamma\big(\frac{c+r}{1-\gamma}\big)^{1-\frac{1}{\gamma}} - \iota^{trad}}{r - \Phi(\iota^{trad})}.$$

To see that investment is inefficiently low in the case of a traditional platform, note that for any constant K, there exists a unique solution $\iota(K)$ to

$$\frac{1}{K - \iota(K)} = \frac{\Phi'(\iota(K))}{r - \Phi(\iota(K))}. \tag{50}$$

This is because we have assumed $\frac{\Phi'(\iota)}{r-\Phi(\iota)}$ is a decreasing function of ι and $\Phi'(\cdot)$ satisfies Inada conditions. Hence, the right-hand side of (50) is decreasing and approaches infinity as $\iota \to 0$, whereas the right-hand side is increasing and approaches infinity as $\iota \to K$. Furthermore, the solution $\iota(K)$ must be increasing in K, since all else equal, higher K increases the left-hand side while keeping the right-hand side unchanged.

We have that $\iota^{FB} > \iota^{trad}$ because $\iota^{FB} = \iota(K)$ for $K = \frac{\gamma}{1-\gamma} c^{1-\frac{1}{\gamma}}$ and $\iota^{trad} = \iota(K)$ for $K = \gamma\big(\frac{c+r}{1-\gamma}\big)^{1-\frac{1}{\gamma}}$.

Service tokens: Now, investors' problem can be written in Lagrangian form as

$$\hat{V}_0^I(Z) = \max_{f_s,\iota_s,\dot{\mu}_s,X_s,Z_s} \; -\frac{1}{1+\Delta}X_0 + \int_0^\infty e^{-(r+\lambda)s}\Big(Z_s^\gamma X_s^{1-\gamma} - cX_s - \iota_s Z_s + \lambda(\hat{V}_0^I(Z_s) + \frac{1}{1+\Delta}\hat{X}_0(Z_s)$$
$$- \psi_s\big((r+\lambda)X_s - Z_s^\gamma X_s^{1-\gamma} + (f_s + \dot{\mu}_s)X_s - \dot{X}_s\big)$$
$$- \xi_s(\dot{Z}_s - \Phi(\iota_s)Z_s) - \chi_s^f f_s - \chi_s^\mu \dot{\mu}_s\Big)ds$$

The Euler-Lagrange conditions are

$$(f_s) : 0 = \psi_s X_s + \chi_s^f;$$
$$(\dot{\mu}_s) : 0 = \psi_s X_s + \chi_s^\mu;$$
$$(\iota_s) : 1 = \xi_s \Phi'(\iota_s);$$
$$(X_s) : \dot{\psi}_s = (1+\psi_s)(1-\gamma)Z_s^\gamma X_s^{-\gamma} - c - \psi_s(f_s + \dot{\mu}_s)$$
$$(Z_s) : (r+\lambda-\Phi(\iota_s))\xi_s = \gamma(1+\psi_s)Z_s^{\gamma-1}X_s^{1-\gamma} - \iota_s + \lambda(\hat{V}_0^{I'}(Z_s) + \frac{1}{1+\Delta}\hat{X}_0'(Z_s)) + \dot{\xi}_s$$

We will conjecture an equilibrium in which the value function is linear in Z, $\hat{V}_0^I(Z) = vZ$ for some constant v. Fees $f_s = f$, seigniorage $\dot{\mu}_s = \dot{\mu}$, the investment rate $\iota_s = \iota$, and the Lagrange multipliers $\psi_s = \psi, \xi_s = \xi$ are constant as well. Transactions are a constant multiple of productivity, $X_s = \tilde{x}Z_s$ for some constant $\tilde{x}$. Under this conjecture, we have $Z_s = \exp(\iota_s)Z$ and

$$vZ = -\frac{1}{1+\Delta}\tilde{x}Z + \int_0^\infty e^{-(r+\lambda-\Phi(\iota))}\Big(\tilde{x}^{1-\gamma} - c\tilde{x} - \iota + \lambda(v + \frac{1}{1+\Delta}\tilde{x})\Big)Zds$$
$$= \Big(\frac{\tilde{x}^{1-\gamma} - c\tilde{x} - \iota}{r+\lambda-\Phi(\iota)} - \frac{r-\Phi(\iota)}{r+\lambda-\Phi(\iota)}\frac{1}{1+\Delta}\tilde{x}\Big)Z + \frac{\lambda}{r+\lambda-\Phi(\iota)}vZ$$

This implies

$$v + \frac{1}{1+\Delta}\tilde{x} = \frac{\tilde{x}^{1-\gamma} - c\tilde{x} - \iota}{r-\Phi(\iota)}. \tag{51}$$

We proceed by analyzing two distinct cases.

Case 1: Strong commitment ($\lambda \leq \lambda^*$). We conjecture that in this case, the Lagrange multiplier ψ is equal to zero. The Euler-Lagrange condition for X_s then implies that transaction quantities must satisfy

$$(1-\gamma)Z_s^{\gamma}X_s^{-\gamma} = c \quad \Rightarrow \quad \tilde{x} = \left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}}.$$

Note that

$$\begin{aligned}
\tilde{x}^{1-\gamma} - c\tilde{x} &= \tilde{x} \times \tilde{x}^{-\gamma} - c\tilde{x} \\
&= \frac{c}{1-\gamma}\tilde{x} - c\tilde{x} \\
&= \frac{\gamma}{1-\gamma}c\tilde{x} \\
&= \gamma\tilde{x}^{1-\gamma}.
\end{aligned}$$

Then the optimality condition for Z_s immediately implies $\xi = v + \frac{1}{1+\Delta}$, so the optimality condition for investment yields $\iota = \iota^{token}$, where

$$\frac{1}{\Phi'(\iota^{token})} = v + \frac{1}{1+\Delta} = \frac{\gamma\left(\frac{c}{1-\gamma}\right)^{1-\frac{1}{\gamma}} - \iota^{token}}{r - \Phi(\iota^{token})}.$$

An argument similar to the one given in the case of a traditional platform implies that $\iota^{token} \in (\iota^{trad}, \iota^{FB})$.

To confirm that this is indeed an equilibrium allocation, we must show that there exist non-negative levels of fees f and seigniorage $\dot{\mu}$ that support it. The transaction demand condition (i.e., the condition corresponding to the multiplier ψ_s) yields

$$f + \dot{\mu} = \frac{c}{1-\gamma} - (r + \lambda\frac{\Delta}{1+\Delta}) + \Phi(\iota^{token}). \tag{52}$$

Define λ^* as

$$\lambda^* = \frac{1+\Delta}{\Delta}\left(\frac{c}{1-\gamma} - r + \Phi(\iota^{token})\right).$$

Then it is possible that $f, \dot{\mu} \geq 0$ only if $\lambda \leq \lambda^*$. Our conjectured equilibrium is indeed an equilibrium, then, if and only if $\lambda \leq \lambda^*$.

Note, moreover, that this argument proves Proposition 6. Any combination $(f, \dot{\mu})$ satisfying (41) implements the same equilibrium.

Case 2: Weak commitment ($\lambda > \lambda^*$). In this case, we conjecture and verify that fees and seigniorage are equal to zero, $f = \dot{\mu} = 0$. With a constant Lagrange multiplier ψ, we have

$$(1+\psi)(1-\gamma)\tilde{x}^{-\gamma} = c \quad \Rightarrow \quad \tilde{x} = \left(\frac{c}{(1+\psi)(1-\gamma)}\right)^{-\frac{1}{\gamma}}.$$

The transaction demand condition pins down $\tilde{x}$:

$$\tilde{x} = \left(r + \lambda \frac{\Delta}{1+\Delta} - \Phi(\iota^{nc})\right)^{-\frac{1}{\gamma}},$$

where ι^{nc} denotes the constant investment rate.

Using the same argument as in the previous case, we again find that $\gamma(1+\psi)\tilde{x}^{1-\gamma} = \tilde{x}^{1-\gamma} - c\tilde{x}$. Therefore, the optimality condition for Z again yields $\xi = v + \frac{1}{1+\Delta}\tilde{x}$. The optimality condition for investment then implies

$$\frac{1}{\Phi'(\iota^{nc})} = \frac{\gamma\left(r + \lambda\frac{\Delta}{1+\Delta} - \Phi(\iota)^{nc}\right)^{1-\frac{1}{\gamma}} - \iota^{nc}}{r - \Phi(\iota^{nc})},$$

as desired. Note that, as claimed, the investment rate ι^{nc} is decreasing in λ.

When $\lambda > \lambda^*$, clearly, the Lagrange multiplier ψ is indeed non-zero, justifying the fact that both fees and seigniorage are at their constraints.

Hybrid tokens: When the platform issues a hybrid token, equilibrium policies solve the analogue of (30):

$$\begin{aligned}
\hat{M}_0(Z)(1-\hat{d\mu}_0(Z)) = \max_{\substack{f_s,\iota_s,\dot{\mu}_s,\\ X_s,M_s,\zeta_s,Z_s}} \quad & \mathbb{E}_0\left[\int_0^\tau e^{-rs}\left(Z_s^\gamma X_s^{1-\gamma} - cX_s - \iota_s Z_s\right)ds + e^{-r\tau}\hat{M}_0(Z_\tau)(1-\hat{d\mu}_0(Z_\tau))\right] \\
& \text{s.t. } (r+\lambda)M_s = \frac{Z_s^\gamma X_s^{1-\gamma} - f_s X_s}{1-\zeta_s} + \dot{M}_s - \dot{\mu}_s M_s + \lambda(1-\hat{d\mu}_0(Z_s))\hat{M}_0(Z_s), \\
& M_s = \mathbb{E}_s\left[\int_s^\tau e^{-r(u-s)}\left(Z_u^\gamma X_u^{1-\gamma} - cX_u\right)du + e^{-r(\tau-s)}\hat{M}_0(Z_s)(1-\hat{d\mu}_0(Z_s))\right], \\
& (33),\ 1-\zeta_s = \frac{X_s}{M_s},\ f_s \geq c,\ \dot{\mu}_s \geq 0,\ Z_0 \text{ given.} \qquad (53)
\end{aligned}$$

This problem is time-consistent by standard arguments. When written in Lagrangian form, it can be viewed as a typical Bellman equation in Z with value function $\tilde{V}(Z) = \hat{M}_0(Z)(1-\hat{d\mu}_0(Z))$, controls $(f_s, \dot{\mu}_s, \iota_s, X_s, M_s, \zeta_s)$, and state Z_s. Therefore, optimal policies are independent of the times at which governance decisions took place: the policy chosen at time τ for $s \geq \tau$ is precisely the same as the policy chosen for $s \geq \tau$ at $t = 0$. Hence, optimal policies and equilibrium allocations are independent of the frequency λ of governance decisions.

G Extensions

Up until this point, we have used a simple and specific model of a platform to illustrate how token issuance can affect governance decisions and welfare. In this section, we describe several extensions of the model to demonstrate that our results apply in more general environments. Specifically, we consider (1) a model with network effects, (2) a model in which there is competition across platforms, and (3) a model in which the platform issues a token with intrinsic value that can be redeemed for a service.

G.1 A model with network effects

In the benchmark model, we assumed that a user's utility depended only on her own quantity of transactions. In reality, though, platforms often exhibit *network effects*: a user's enjoyment of the platform's service depends to some extent on how much others use the platform. For example, for a DeFi lending platform to function well, it must be used by both borrowers and lenders.

We capture these considerations in our model by assuming that a user i's utility depends both on that user's *individual* transactions x_{it} and *aggregate* transactions on the platform $X_t \equiv \int_0^1 x_{i't} di'$. Specifically, user i's flow payoff takes the form

$$U_{it} = \underbrace{\frac{(X_t^{\nu} x_{it}^{1-\nu})^{1-\gamma}}{1-\gamma}}_{\text{transaction payoff}} - \underbrace{f_t x_{it}}_{\text{fees}},$$

where $\nu \in (0,1)$. Agents have concave utility over a Cobb-Douglas aggregate of individual transactions (with weight $1-\nu$) and aggregate transactions (with weight ν). The parameter ν captures the strength of network effects: the larger ν, the more users care about how much others use the platform. Of course, in equilibrium, all users continue to choose the same quantity of transactions, $x_{it} = X_t$. Note, furthermore, that for a given aggregate quantity of transactions X_t, users' aggregate transaction utility is $\frac{X_t^{1-\gamma}}{1-\gamma}$, just as in the benchmark model.

The key departure from the benchmark model is in users' optimality condition: when there are network effects, aggregate transaction demand satisfies

$$(1-\nu)X_t^{-\gamma} dt = f_t dt + \left(r dt - \mathbb{E}_t\left[\frac{dQ_t^T}{Q_t^T}\right]\right) \tag{54}$$

Comparing to aggregate transaction demand (2) in the benchmark model, the only difference is that users' marginal utility of transactions is $(1-\nu)X_t^{-\gamma}$ rather than $X_t^{-\gamma}$. Since this is the only modification to the model, all of the main results continue to go through: a traditional platform features inefficiently low transaction quantities, a platform that issues service tokens can improve on a traditional one only if investors have sufficient commitment power, and a hybrid token system overcomes the commitment problem.

Using analogous proofs to those in the main body of the paper, it is possible to show that the first-best quantity of transactions X_t^{FB}, the quantity X_t^{trad} under the traditional system, and the quantity X_t^{token} under the commitment outcome satisfy

$$X_t^{FB} = c^{-\frac{1}{\gamma}},\ X_t^{trad} = \left(\frac{c+r}{(1-\gamma)(1-\nu)}\right)^{-\frac{1}{\gamma}},\ X_t^{token} = \left(\frac{c}{(1-\gamma)(1-\nu)}\right)^{-\frac{1}{\gamma}}. \tag{55}$$

If users' transaction demand takes a similar form regardless of the value of ν, then what is economically different when we introduce network effects? Strong network effects imply that users get less utility from increasing their own transaction quantity and enjoy greater externalities from others' transactions. Therefore, when network effects are stronger, users' *service flows* from transaction assets account for a smaller share of utility and *inframarginal rents* account for a larger share,

$$SF_t = (1-\nu)X_t^{1-\gamma} - f_t X_t, \text{ and } IR_t = \frac{1-(1-\gamma)(1-\nu)}{1-\gamma} X_t^{1-\gamma}.$$

A token-issuing platform governed by investors improves upon a traditional platform by accounting for users' service flows. When network effects are strong, service flows are small, so the allocation with a token-issuing platform is further away from the first-best. On the other hand, user governance becomes relatively more efficient, since users guide policy to maximize their rents. This logic is summarized by the following proposition.

Proposition 14. *There exists $\nu^* < 1$ such that whenever $\nu \geq \nu^*$, then welfare under a service token or hybrid token system in which users can vote is at least as high as welfare when only investors can vote.*

Proof. For ν close enough to 1, welfare when only investors can vote is arbitrarily close to zero (say ϵ). By contrast, when users govern the platform, they maximize a weighted average of their rents and the value of tokens. The value of tokens cannot go negative, so

$$\int_0^\infty e^{-rs}\big((1-\nu)X_s^{1-\gamma} - cX_s\big)ds > 0.$$

Users' rents are bounded away from zero for any positive value of X_s. Hence, total welfare is bounded away from zero if users control the platform in the long run. On the other hand, if investors control the platform, total welfare is of course unchanged and is equal to ϵ in the long run. □

G.2 A model of platform competition

Our benchmark model focuses on the case of a monopolistic platform for simplicity. To show that *limited market power* is what is truly essential to our results, in this section we extend the model to a a setting with monopolistically competitive platforms.

There is a continuum of platforms indexed by $k \in [0,1]$. Some are traditional platforms, some issue service tokens, and some issue hybrid tokens. Platforms share the same marginal cost c of transaction processing, but they may set different transaction fees f_{kt} and seigniorage policies dS_{kt}. For brevity, in this section we take the degree of commitment to infinity ($\lambda \to 0$), but our main results would continue to go through for finite λ. Users enjoy the differentiated transaction services provided by all platforms. A user who transacts x_{kt} on each platform k receives payoff

$$U_t = \frac{X_t^{1-\gamma}}{1-\gamma} - \int_0^1 f_{kt} x_{kt} dk \quad \text{where} \quad X_t = \left(\int_0^1 x_{kt}^{\frac{\eta-1}{\eta}} dk \right)^{\frac{\eta}{\eta-1}}.$$

That is, the total transaction services X_t consumed by a user is a CES aggregate of the transaction services x_{kt} consumed on each platform, where $\eta > 1$ is the elasticity of substitution. Utility is concave in total transaction services X_t. A higher value of η corresponds to greater substitutability and therefore more intense cross-platform competition. For simplicity, we assume that each platform is governed by its own set of investors, who choose fees f_{kt} and the seigniorage policy in governance decisions that arrive at rate λ.[26]

Users' aggregate transaction demand x_{kt} on platform k satisfies

$$X_t^{\frac{1}{\eta}-\gamma} x_{kt}^{-\frac{1}{\eta}} dt = f_{kt} dt + \left(r dt - \mathbb{E}_t \left[\frac{dQ_{kt}^T}{Q_{kt}^T} \right] \right), \tag{56}$$

where Q_{kt}^T denotes the price of platform k's transaction asset. The main difference from the benchmark model, then, is that demand for transactions on platform k depends not only on its productivity and fees, but also on aggregate consumption of transaction services X_t across all platforms. That is, there is some substitutability between consumption of different platforms' transaction services. Importantly, since each platform is infinitesimal, each platform's

[26] For simplicity, we do not consider platforms governed by users in this extension. Cross-ownership by users introduces additional challenges because users would have incentives to pass policies on all platforms in order to manipulate aggregate transaction quantities X_t.

investors take X_t as exogenous when making governance decisions, which considerably simplifies the equilibrium analysis. Notice, furthermore, that transaction demand on platform k becomes perfectly elastic in the limit of perfect competition, $\eta \to \infty$.

Platform k's investors choose policies to maximize the value of their cash flow assets. In the case of a traditional platform, this is equivalent to maximizing the present value of profits, whereas in the case of a token-issuing platform, this is equivalent to maximizing the present value of profits plus service flows (as shown in the main text). We will look for a steady-state equilibrium in which all platforms choose constant policies, and the quantity of transactions produced by each platform does not change over time. An important simplification is that since each platform is infinitesimal, platform k's investors do not internalize how their policies affect X_t.

Below, we look for a Markov equilibrium in which each platform's transaction quantities and policies are constant through time.

Traditional platform: We begin by analyzing the governance problem faced by the investors of a traditional platform k. The platform's profits can be written as

$$(f_t - c)X_{kt} = X_t^{\frac{1}{\eta}-\gamma} X_{kt}^{1-\frac{1}{\eta}} - (c+r)x_{kt}.$$

Therefore, investors' problem is equivalent to maximizing

$$\max_{f_{kt}, X_{kt}} \int_0^\infty e^{-rt}\big(X_t^{\frac{1}{\eta}-\gamma} X_{kt}^{1-\frac{1}{\eta}} - (c+r)X_{kt}\big)dt \quad \text{s.t.} \quad (56), \;\; f_{kt} \geq 0.$$

Optimal fees and transaction quantities are

$$f_k = \frac{\eta}{\eta-1}c + \frac{1}{\eta-1}r, \;\; X_k = X^{\gamma\eta-1}\left(\frac{c+r}{1-\frac{1}{\eta}}\right)^{-\eta}. \tag{57}$$

Token-issuing platforms: Just as in the main text, it is possible to show that since $\lambda \to 0$, then regardless of whether k issues service tokens or hybrid tokens, investors' problem is equivalent to solving the Lagrangian

$$\mathcal{L} = \max_{X_{kt}, f_{kt}, \dot{\mu}_{kt}} \int_0^\infty e^{-rt}\Bigg(X_t^{\frac{1}{\eta}-\gamma} X_{kt}^{1-\frac{1}{\eta}} - cX_{kt} - \chi_{kt}^f f_{kt} - \chi_{kt}^\mu \dot{\mu}_{kt}$$
$$+ \psi_{kt}\big(rX_{kt} - X_t^{\frac{1}{\eta}-\gamma} X_{kt}^{-\frac{1}{\eta}} + (f_{kt} + \dot{\mu}_{kt})X_{kt} - \dot{X}_{kt}\big)\Bigg)dt,$$

The Euler-Lagrange conditions are

$$\begin{aligned}
(f_{kt}) &: \psi_{kt} X_{kt} = \chi^{f}_{kt}; \\
(\dot{\mu}_{kt}) &: \psi_{kt} X_{kt} = \chi^{\mu}_{kt}; \\
(X_{kt}) &: X_t^{\frac{1}{\eta}-\gamma} X_{kt}^{-\frac{1}{\eta}} = \frac{\tilde{c}_k}{1-\frac{1}{\eta}};
\end{aligned}$$

From these first-order conditions, we can conclude that there are two regimes: one in which the constraints on fees and seigniorage do not bind (for small enough η) and one in which they may (for large η). We obtain

$$f_k = \max\left\{\frac{\eta}{\eta-1}c - r, 0\right\}, \; X_k = X^{\gamma\eta-1}\left(\max\left\{\frac{\eta}{\eta-1}c, r\right\}\right)^{-\frac{1}{\eta}}. \tag{58}$$

Suppose that k is a token-issuing platform and k' is a traditional platform. We can then again plug in the first-order condition for X_{kt} to obtain

$$\frac{X_{kt}}{X_{k't}} = \left(\frac{\max\left\{\frac{\eta}{\eta-1}c, r\right\}}{\frac{\eta}{\eta-1}(c+r)}\right)^{-\frac{1}{\eta}}. \tag{59}$$

Token-issuing platforms still acquire a larger market share and process more transactions than traditional platforms. Hence, what matters for our results is not that the platform is a complete monopolist – even when there are outside options, platforms still benefit from issuing tokens.

It is particularly interesting to consider the case in which $c < r$. Then, the effects of competition on platforms' relative market shares are non-monotone in η. For small η, an increase in competition *decreases* the market share of traditional platforms. Both traditional platforms and token-issuing platforms cut their fees by the same proportion, and the increase in substitutability tilts users' demand towards the lower-cost token-issuing platforms. However, for large η, this relationship is flipped. Token-issuing platforms cannot cut their fees anymore as η increases further, but competitive pressures continue to drive down the fees charged by traditional platforms, reducing the gap in market shares. That is, when competition is very intense, traditional platforms are no longer at such a significant disadvantage. Token issuance should be viewed as a substitute for the incentives for socially beneficial policies that are naturally provided by market competition.

G.3 A platform with a redeemable token

We now briefly outline a model in which the platform either operates as a traditional platform or issues a *redeemable* token that users can directly exchange for the platform's services. In this setup, the platform is more similar to the issuer of a "utility token" with intrinsic value, as discussed in Section 2.

Environment: The platform sells a service to users at a price $p_t = Q_t^T k_t \geq c$ (in dollars) and incurs a constant marginal cost c per unit produced. User i gets an opportunity to buy the service according to an (independent) Poisson process with arrival rate $\rho > 0$. If user i holds a_{it}^T transaction assets, she can purchase at most $x_{it} \leq \frac{Q_t^T a_{it}}{p_t}$ units of the service. Hence, user i's problem is

$$\max_{x_{it}, a_{it}} \rho\left(\frac{x_{it}^{1-\gamma}}{1-\gamma} - p_t x_{it}\right) - \left(rQ_t^T - \frac{1}{dt}\mathbb{E}_t\left[dQ_t^T\right]\right)a_{it}^T \text{ s.t. } p_t x_{it} \leq Q_t^T a_{it}.$$

We impose that in equilibrium, all users choose the same quantity of transactions, $X_t = x_{it}$. Clearly,

$$p_t X_t = Q_t^T A_t^T$$

at an optimum. The first-order condition is

$$\rho X_t^{1-\gamma} = p_t X_t\left(\rho + r - \frac{1}{dt}\mathbb{E}_t\left[\frac{dQ_t^T}{Q_t^T}\right]\right). \tag{60}$$

Investors' optimization problem remains unchanged, so cash flow assets are still priced according to the present value of dividends.

The first-best allocation solves

$$\max_{X_t} \int_0^\infty e^{-rt}\rho\left(\frac{X_t^{1-\gamma}}{1-\gamma} - cX_t\right)dt. \tag{61}$$

The optimality conditions is analogous to those derived earlier:

$$X_t = c^{-\frac{1}{\gamma}}, \tag{62}$$

Traditional platform: We begin with the traditional platform. The platform's profits are simply

$$d\Pi_t = \rho(p_t - c)X_t dt.$$

Users' demand for the service is

$$X_t = \left(\frac{\rho + r}{\rho} p_t\right)^{-\frac{1}{\gamma}}. \tag{63}$$

Investors' governance problem is again time-consistent. They choose the price of the service p_t to maximize the present value of profits:

$$\max_{p_t} \int_0^\infty e^{-rt}(p_t - c)X_t dt \ \text{ s.t. } \ (63), \ p_t \geq c. \tag{64}$$

From users' first-order condition (63), we can rewrite

$$p_t X_t = \frac{\rho}{\rho + r} X_t^{1-\gamma},$$

so investors' problem can be written as

$$\max_{X_t} \int_0^\infty e^{-rt}\rho\left(\frac{\rho}{\rho + r} X_t^{1-\gamma} - cX_t\right) dt \ \text{ s.t.}.$$

From our previous analysis, we can immediately conclude that X_t is constant:

$$X_t = \left(\frac{c(\rho + r)}{\rho(1-\gamma)}\right)^{-\frac{1}{\gamma}}. \tag{65}$$

Service tokens: We now solve for the equilibrium when the platform issues service tokens. The platform sets a price k_t of the service in terms of tokens, so that the real price of the service is $p_t = Q_t^T k_t$. At each instant, if users purchase a quantity $\rho X_t dt$ of the service, the platform redeems $\rho X_t k_t$ tokens and receives revenues $\rho X_t Q_t^T k_t$. Since users hold only as many tokens as they need to purchase X_t units of the service, we have

$$Q_t^T A_t^T = p_t X_t \ \text{ and } \ A_t^T = k_t X_t.$$

Hence, a fraction ρdt of the token stock A_t^T is redeemed at each instant. We continue to impose the constraint that $p_t \geq c$, which in this setting is equivalent to $M_s \geq cX_s$.

If we continue to let $d\mu_t$ denote the platform's seigniorage policy, the token stock evolves according to

$$\frac{dA_t^T}{A_t^T} = d\mu_t \in [0, \Delta].$$

The platform's profits are then

$$d\Pi_t = \rho(Q_t^T k_t - c)X_t + dS_t \quad \text{where} \quad dS_t = Q_t^T A_t^T d\mu_t.$$

We again look for a Markov equilibrium in s which quantities may jump only at the time of a governance decision. We first find a convenient representation of equilibrium quantities in this environment. Let

$$M_t \equiv Q_t^T A_t^T$$

denote the market capitalization of tokens. The return on tokens can be written as

$$\frac{1}{ds}\mathbb{E}_s\left[\frac{dQ_s}{Q_s}\right] = \frac{\dot{M}_s}{M_s} - \dot{\mu}_s + \lambda\bigg((1-\hat{d\mu}_0)\frac{\hat{X}_0}{X_s} - 1\bigg).$$

Users' transaction demand (60) can be written as

$$(r+\lambda)M_s = \rho X_s^{1-\gamma} - (\rho + \dot{\mu}_s)M_s + \dot{M}_s + \lambda(1-\hat{d\mu}_0)\hat{M}_0, \tag{66}$$

and the platform's profits are

$$d\Pi_s = M_s(d\mu_s + \rho) - cX_s.$$

Combining these two results, we have

$$\begin{aligned}
\mathbb{E}_0\left[\int_0^\tau e^{-r\tau} d\Pi_s\right] &= \mathbb{E}_0\left[\int_0^\tau e^{-rs}\bigg((\rho+\dot{\mu}_s)M_s - \rho c X_s\bigg)\right] \\
&= \int_0^\infty e^{-(r+\lambda)s}\bigg(\rho X_s^{1-\gamma} + \dot{M}_s - (r+\lambda)M_s - \rho c X_s + \lambda(1-\hat{d\mu}_0)\hat{M}_0\bigg)ds \\
&= -M_0 + \int_0^\infty e^{-(r+\lambda)s}\bigg(\rho X_s^{1-\gamma} - \rho c X_s + \lambda(1-\hat{d\mu}_0)\hat{M}_0\bigg)ds.
\end{aligned}$$

Investors' governance problem can then be written as

$$\begin{aligned}
\hat{V}_0^I = \max_{d\mu_0, \dot{\mu}_s, X_s, M_s} \; & M_0(d\mu_0 - 1) + \int_0^\infty e^{-rs}\bigg(\rho X_s^{1-\gamma} - \rho c X_s + \lambda(\hat{V}_0^I + (1-\hat{d\mu}_0)\hat{M}_0)\bigg)ds \\
& \text{s.t.} \;\; (66), \;\; M_s \geq cX_s, \;\; \dot{\mu}_s \geq 0, \;\; d\mu_0 \in [0, \frac{\Delta}{1+\Delta}].
\end{aligned} \tag{67}$$

We can apply the same Lagrangian methods as we did in previous sections to show that there are again two regimes: a strong commitment regime and a weak commitment regime, based on whether λ is above some threshold λ^*. In the strong commitment regime,

$$X_s = \left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}}, \tag{68}$$

This equilibrium is feasible as long as $M_s \geq cX_s$. The market capitalization of tokens M_s is maximized when $\dot{\mu}_s = 0$ for all s, in which case

$$M_s = \frac{\rho}{\rho + r + \lambda\frac{\Delta}{1+\Delta}} X_s^{1-\gamma},$$

so

$$\frac{\rho}{\rho + r + \lambda^*\frac{\Delta}{1+\Delta}} \left(\frac{c}{1-\gamma}\right)^{1-\frac{1}{\gamma}} = c.$$

For $\lambda > \lambda^*$, we must have $M_s = cX_s$ for all s. The quantity of transactions X_s must adjust to ensure this is the case. Then,

$$\frac{\rho}{\rho + r + \lambda\frac{\Delta}{1+\Delta}} X_s^{1-\gamma} = c, \tag{69}$$

Hybrid tokens: Finally, we turn to the case in which the platform issues hybrid tokens. We use the same notation as we did for a platform that issues service tokens. As in the main body of the paper, in this case we will not have to consider the possibility that the token stock jumps, so we denote the seigniorage policy simply by $\dot{\mu}_s$. The platform's profits are as in the previous section.

Users hold a fraction $1 - \zeta_s$ of the token stock. Given that their token holdings must equal the value of the services they purchase,

$$p_s X_s = (1 - \zeta_s) M_s.$$

Then, (2) implies

$$(r + \lambda) M_s = \frac{\rho X_s^{1-\gamma}}{1 - \zeta_s} - \dot{\mu}_s + \dot{M}_s + \lambda \hat{M}_0.$$

Investors' pricing equation for tokens is

$$(r + \lambda) M_s = \frac{\dot{\mu}_s M_s - \rho c X_s}{\zeta_s} - \dot{\mu}_s + \dot{M}_s + \lambda \hat{M}_0.$$

Combining these two conditions, we obtain

$$(r+\lambda)M_s = \rho\big(X_s^{1-\gamma} - cX_s\big) + \dot{M}_s + \lambda\hat{M}_0. \tag{70}$$

Investors' problem is again to maximize the value of the token stock. We conjecture an equilibrium in which the value of the token stock is linear in Z, $\hat{M}_0(Z) = mZ$:

$$\hat{M}_0 = \max_{\dot{\mu}_s, X_s, M_s} \int_0^\infty e^{-(r+\lambda)s}\Big(\rho\big(X_s^{1-\gamma} - cX_s\big) + \lambda\hat{M}_0\Big)$$
$$\text{s.t.}\;\; (70),\;\; M_s \geq cX_s,\;\; \dot{\mu}_s \geq 0. \tag{71}$$

The optimality conditions yield

$$X_s = \left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}}, \tag{72}$$

which is exactly the same as the full-commitment outcome in the case of a platform that issues service tokens.