Green Intermediary Asset Pricing

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Preliminary - Comments welcome
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Abstract Can environmentally-minded investors impact the cost of capital of green firms even when they invest through financial intermediaries? To answer this and related questions, I build an equilibrium intermediary asset pricing model with three investors, two risky assets, and a riskless bond. Specifically, two heterogeneous retail investors invest via a financial intermediary who decides on the portfolio allocation that she offers between a green and a brown equity. Both retail investors and the financial intermediary can tilt towards the green asset, beyond pure financial considerations. Perhaps surprisingly, the green retail investor can have substantial impact on the pricing of green assets, even when she invests via an intermediary who does not tilt: a sizable green premium –that is, a lower cost of capital– can emerge on the equity of the green firm. This good news comes with important qualifications, however: the green retail investor has to take large leveraged positions in the portfolio offered by the intermediary, her strategy must be inherently state-dependent, and economic conditions or the specification of preferences can overturn or limit the result. When the financial intermediary decides (or is made) to tilt instead, the impact on the green premium is substantially larger, although it is largest when preference are aligned with retail investors. I also study what happens when the green retail investor does not know the weights in the portfolio offered by the intermediary, the potential impact of greenwashing, and the effect of portfolio constraints. Taken together, these findings highlight the central role that financial intermediaries can play in channeling (or not) financing towards the green transition.

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1 Introduction

A large and growing literature has recently studied the impact of green and ESG-conscious investors on risk premia and on the financing of the green transition. Although still subject to debate, the main conclusion so far is that the presence of such investors and of environmental risks should lead the expected returns on green assets to be lower, either for pecuniary or non-pecuniary reasons, therefore fostering investment towards green firms due to this lower cost of capital.[1] Importantly, most of the literature uncovering this "green premium" does so in economies in which investors can directly trade green and brown assets, and can therefore freely expressed their preferences in their pricing.

In practice, however, the vast majority of investors do not directly trade individual assets. Instead, they invest the bulk of their wealth through financial intermediaries, such as mutual funds, pension funds, index funds, hedge funds, private equity funds, or banks. Those intermediaries in turn are responsible for deciding how to allocate the wealth of their investors into specific assets and strategies. This distinction is not innocuous: e.g., as of 2023, the four largest asset managers in the world, BlackRock, Vanguard, Fidelity, and State Street, were in charge of allocating close to 25 trillions U.S. dollars on behalf of clients.

While a number of financial intermediaries (including large ones) have committed to certain sustainable objectives in recent years, there is considerable debate about the extent to which they implement them in practice (Pástor et al., 2024), or whether such announcements are mostly used as a marketing tactic to attract fund flows and are therefore more akin to greenwashing. Further, even with the best of intentions, asset managers and banks have come under intense regulatory and political pressure in recent years, notably in the United States, to stick to a narrow financial interpretation of fiduciary duty that does not include environmental considerations.[2] As a

[1] On the theoretical front, recent papers include Pástor et al. (2021), Hsu et al. (2022), Pedersen et al. (2021), Zerbib (2022), among others. The debate has mostly focused on the empirical side, e.g., Bolton and Kacperczyk (2021, 2022), Goldstein et al. (2021), Avramov et al. (2021), Ardia et al. (2021), Pástor et al. (2022), Lontzek et al. (2023), Zhang (2021), but also include theoretical reasons, e.g., Pedersen et al. (2021), Sauzet and Zerbib (2024), Chen et al. (2024).

[2] For instance, in October 2022, 19 State Attorneys General (including for Arizona, Kentucky, Missouri, and Texas), issued subpoenas on six large U.S. banks (Bank of America, Citigroup, Goldman Sachs, JP Morgan Chase, Morgan Stanley, and Wells Fargo) requesting information on their involvement in the United Nations (UN)'s Net-Zero Banking Alliance. Similarly, a March 2023 letter

result, in the last year alone, Vanguard, and various large U.S. banks, have pulled out from, or expressed concerned about, the Glasgow Financial Alliance for Net Zero (Gfanz), a highly visible U.N.-backed international organization aiming to leverage large financial actors to finance the transition to net zero emissions. Similarly, in February 2024, State Street and JPMorgan Asset Management left, and BlackRock limited its involvement in, Climate Action 100+, an investor-led initiative aiming to ensure that the world's largest corporate greenhouse gas emitters take actions on climate change.

Against this backdrop, at least two broad sets of questions naturally arise. On the more negative side, in a world in which financial intermediaries do not account for environmental and ESG considerations, can there be any hope for green end-investors to have any effect whatsoever on expected returns and therefore on the green transition? On the other hand, if financial intermediaries started to tilt towards green assets, or were made to tilt (say, by regulators), how large could the effect be? Under what conditions? For instance, could they also harness the wealth of neutral end-investors to have more impact? Ultimately, given the amount of wealth being allocated by those intermediaries, could even small environmental tilts on their part make it possible to overcome the limited impact on the cost of capital of green firms that have been suggested in the literature (e.g., Berk and van Binsbergen, 2021, De Angelis et al., 2022)?

In this paper, I develop an intermediary asset pricing equilibrium model of sustainable investing to answer those questions.[3]

Specifically, the economy consists of three heterogeneous investors –two retail investors, and one financial intermediary–, and three assets –the equity asset of a green

from state 21 Attorneys General (including Alabama, Iowa, Kentucky, Ohio, Utah, Texas, Virginia) warned large asset managers against the adoption of ESG initiatives, including those relating to Climate Action 100+ and the UN Net Zero Asset Managers (NZAM) initiative. Both are based on the grounds that such initiatives may impede the adherence of financial institution to their fiduciary requirements, and could represent breath in antitrust laws. Various states have also pulled hundreds of millions of dollars in pension funds out of BlackRock, and others, or have threatened to do so.

[3]While a theoretical approach is not the only way to proceed, it seems particularly apt in this context given that (i) the questions at hand are fundamentally about "counterfactuals", and (ii), we do not necessarily observe enough ESG tilts in practice to be able to study all those cases empirically. The model can be more broadly understood as one of investing with an eye on specific values, be they environmental, social, governance, or other (including of the anti-ESG kind). Focusing on an economy that includes financial intermediaries has the added benefit of helping to tackle some of the difficulties of consumption-based asset pricing models.

firm, the equity asset of a brown firm, and a riskless bond. Retail investors consume a single consumption good and decide how much of their wealth to invest in the financial intermediary, so that they do not directly control their investment in each risky asset (although they can borrow and lend in the riskless bond). The financial intermediary also consumes in general, and decides how to allocate the total of her wealth and that of her investors into the three assets. As such, she has a central role in determining how much capital is allocated to the green firm.

Most importantly, investors can have a tilt towards the green asset, beyond financial considerations. Namely, one retail investor has an environmental tilt (henceforth, the green [retail] investor), one retail investor has no tilt (henceforth, the neutral [retail] investor), and I study both the situations when the financial intermediary does not have a environmental tilt, and when she does. Apart from that, preferences are recursive in the most general specification, although I discuss various simpler cases. The output of both firms are exogenous and follow standard geometric Brownian motions, but all prices, expected returns, and other variables, such as the wealth share of investors, are endogenous and determined in equilibrium.[4]

I first study the case in which the financial intermediary focuses solely on financial considerations, and does not consider environmental motives when deciding on her investment. More specifically, I start with a pure indexing economy in which the financial intermediary is a passthrough investor who does not consume or has any wealth of her own, and simply sets the portfolio that she proposes to investors equal to the market portfolio.[5] This can be seen as a reasonable approximation of the current situation in which large asset managers, such as BlackRock and others, are in charge of investing a large share of retail investors' wealth, and do so mostly in the form of index funds that do not have strong ESG tilts.

Perhaps surprisingly, and this is the "good news", the green retail investor is still able to exert a substantial impact on the pricing of green assets in the economy, even though she is not able to trade equity assets separately. The exact magnitude varies across specifications and parameterizations, but it can go as far as bringing the risk premium on the green asset close to zero or even negative when the green investor

[4]The equilibrium depends on three state variables: the wealth share of the green retail investor in the total wealth of retail investors (x_t, endogenous), the output share of the green firm (y_t, exogenous), and the wealth share of the financial intermediary in total wealth (u_t, endogenous).

[5]In this specific case, the neutral retail investor is allowed to trade both risky assets in the economy so that at least one agent can price all securities.

holds a large share of the wealth in the economy. In other words, there can exist a significant green premium, that is, a lower cost of capital for green firms. This effect emerges purely endogenously: even though her environmental tilt directly impacts the pricing of the index, the green investor's effect on the green asset itself comes from impacting the market/myopic part of its pricing. Specifically, even though she cannot directly decide of what share of her risky portfolio goes into the green asset, the green retail investor can do so indirectly (and imperfectly) by modulating how much she invests in the index itself. In practice, she invests substantially more in the index in states of the world in which the green asset dominates it. The other investor, who prices both assets, has to take the flip side of this trade by market clearing, so that he invests significantly less in the green asset. In turn, the green asset therefore provides him with ample diversification benefits, is less risky, and earns lower risk premium in equilibrium.

Turning to the case in which the financial intermediary is an investor in her own right who consumes and hold a share of the economy's wealth, but still does not tilt towards the green asset, adds a layer of interesting nuance. The main finding and mechanism remain: the green retail investor *can* substantially impact the pricing of the green equity asset, despite the fact that the intermediary leans against it by tilting the portfolio that she offers away from it. Introducing difference in risk attitudes, namely a more risk-tolerant financial intermediary, which leads the intermediary to leverage up in equilibrium, can mitigate some of the large green retail investor portfolio positions that arise in the pure indexing case, and also influences average returns.

What is the bad news, then? While it is here, this impact of the green retail investor on the cost of capital of the green firm, even when the financial intermediary does not tilt towards it, is subject to considerable qualifications.

First, to be able to influence the pricing of assets, the green retail investor has to resort to taking large positions in the index (or more generally in the risky portfolio offered by the intermediary), sometimes extremely so. In fact, she often borrows in the riskless bond so as to leverage her risky position. In addition, she must invest in an inherently state-dependent fashion, closely monitoring the composition of the index in order to increase her position mostly when the green asset dominates it. Relatedly, this assumes in the first place that the retail investor can even *observe* the composition of this index, or of the portfolio offered by the intermediary. Those

elements could be difficult to square with the financial market access, information set, and investing practices, of the average retail or end investor in practice.

The impact of green investing on green expected excess returns is also inherently state-dependent, that is, it varies a lot with economic conditions. Beyond the fact that the green investor must hold enough of total wealth, the emergence of a significant green premium can mostly occur when green firms dominate the economy. Again, this comes from the fact that, even though the green investor can tilt her portfolio towards the green asset to some extent, she can only do so in an indirect and imperfectly-targeted fashion. In short, she increases her share in the index (and therefore relatively speaking in the green asset) only when the output share of the green firm is above 50%, which broadly corresponds to the green asset representing more than half of the market portfolio. Conversely, the green premium stemming from the green investor's influence is more limited, and in fact can switch sign, if the output share of green firms is below 50%.[6] Empirically, depending on the approach and scope used, the green output share could be on the order of 30-40%, so that this can constitute a relevant caveat for the time being, even though this number is likely to rise in the future.

Further, the specification and calibration of the preferences of investors can have a significant impact. For instance, while the results above remain broadly valid for various parameters in the case of constant relative risk aversion (CRRA), although they vary in magnitude, counterproductive effects can arise when preferences are recursive. Namely, the impact on risk pricing discussed above can lead the expected returns not only on the green asset, but also on the brown asset, to decrease, so that the green premium can be negligible or in fact turn negative. This happens especially as the output share of the green firm is small. In addition, for non-zero but moderate wealth share for the green investor, the expected returns on the green asset can at first increase compared to a case without green tilt, and compared to that on the brown asset for comparable output shares, although this disappears when the green investor holds enough wealth. The CRRA case also is imperfect. In this situation, (i) the risk-free rate is often (too) large in magnitude when risk aversion is large enough

[6]The magnitude of the impact on risk premia, and on the green premium, is in fact more nuanced and non-linear. For instance, the green investor can end up investing significantly in the index even for a low green output share when her green tilt is substantial, and the green premium can flip sign for moderate but non-zero wealth share for the green investor. Those are discussed in more details below, and in Sections 4 and 5.

to deliver sizable risk premia, as is well-known, and (ii) the risk-free rate also increases with the wealth share of the green investor because she uses borrowing to lever up her risky position. As a result, even when the risk premium on the equity of the green firm decreases by more, the total *discount rate* (riskfree rate + risk premium, i.e., the cost of capital) for both firm increases substantially under CRRA preferences, and this can dwarf any helpful mild reduction in relative risk premium.[7]

Lastly, beyond the case of pure indexing, the impact of the green retail investor can decrease markedly as the wealth share of the financial intermediary increases. This could be relevant if the intermediary under consideration is, for instance, a hedge fund, private equity fund, or a bank, although it is less likely to be problematic in practice because a large share of the money managed by financial intermediaries remain that of clients.

In summary so far, when financial intermediaries do not tilt towards green asset beyond financial considerations, the influence of green retail investors on the pricing of green assets, and therefore on fostering the green transition, can be qualified as a "partial success". On the one hand, the good news is that green investors *can* have a substantial impact, which was not necessarily obvious *ex-ante*. On the other, the impact comes with significant caveats, some of which I will relax below.

The second central question is, then: what happens when financial intermediaries do tilt towards green assets? Although recent developments and estimates (Pástor et al., 2024) suggest that this might not be the case currently, financial institutions may decide to introduce tilts in the future because of their own values and preferences, in order to attract funds from clients who might put more weights on such considerations, or because they are made to by regulators.[8] I therefore turn to introducing an environmental tilt on the part of the financial intermediary in the economy. Being able to do so, and to study the equilibrium impact on asset returns and various variables, is part of the appeal of the framework, especially for those cases that are not necessarily fully observed empirically as of yet.

[7] This point has not been emphasized much in the sustainable asset pricing literature, and highlights that allowing for recursive preferences is helpful from the perspective of uncovering the full extent of those mechanisms.

[8] Although the introduction of regulation fostering investment towards environmental and other non-financial considerations is unlikely in the United States for the time being, such possibilities are very much part of the options being actively discussed in the United Kingdom, the European Union, and other parts of the world.

In short, the impact of the financial intermediary green tilt is substantial. Although the green retail investor was able to have some influence when the intermediary was focused purely on financial considerations, it is completely dwarfed by the power of having a green intermediary, and the green premium can become substantially more negative. This comes in part because the intermediary can trade both assets separately and is therefore able to directly impact their pricing, but in particular because she decides on the portfolio for all investors in the economy. As such, she can leverage the wealth of all investors, not just her own, in order to foster favorable financing –that is, a lower cost of capital– for the green firm. This arises despite the fact that the neutral investor attempt to undo some of this tilt in his investment by reducing his position in the intermediary portfolio especially when the green firm dominates in the economy, in a pattern mirroring that of green investors previously. Unless the neutral investor controls the vast majority of the wealth in the economy, the latter effect is not enough to overturn the lowering of the green premium, especially because the neutral retail investor is very much happy to invest in the intermediary portfolio that aligns well with her preferences. In fact, the effect of an environmental tilt on the part of the intermediary is so strong that it mostly persists even when retail investors have limited tilt of their own, although the effect is largest when the preferences of retail investors and intermediary are aligned.

Taken together, these findings highlight the central and unique role that financial intermediaries can play in channeling (or not) financing towards the green transition and other important questions. When intermediaries do not account for environmental or other non-financial considerations, retail investors can still express their preferences and impact the pricing of assets, although this comes with important caveats. When intermediaries do tilt however, the magnitude of their effect can be substantial, potentially orders of magnitude larger. The latter could help alleviate the concern that the cost of capital channel of sustainable investing, by which green firms benefit from lower cost of financing, is too small to have a real impact as of now (Berk and van Binsbergen, 2021, De Angelis et al., 2022).

While as of now, such tilts appear to be limited in scope in practice, the results suggest that financial institutions could be a key and powerful actor to solve the challenges of our time, if they start accounting for them. In fact, whether there is any hope to finance and reach such solutions without the financial sector playing a role, given the trillions of dollars that it manages, is perhaps the real question. This

suggests that regulatory actions could have considerable effects, and can probably explain the heated political and regulatory debate on those questions that has emerged in recent years

I conclude the paper by investigating a number of important generalizations of the framework, in order to assess how the results evolve in those broader situations. First, I consider the case in which retail investors do not know the weights in the portfolio offered by the financial intermediary, at least from the perspective of environmental tilting. Second and relatedly, I study what happens if the financial intermediary resorts to greenwashing: she does not tilt towards the green asset in the way that retail investors believe. Third, I investigate the impact of portfolio constraints on the part of both retail investors and the financial intermediary, which could change or amplify some of those results. This part is still ongoing work.

Related literature The paper contributes to several strands of literature in asset pricing, sustainable finance, and macro-finance.

First, a large and growing literature has recently investigated the impact of green investors on asset prices. On the theoretical side, this includes Bolton and Kacperczyk (2021, 2022), Pástor et al. (2021), Hsu et al., 2022, Pedersen et al. (2021), Zerbib (2022), among others. The main conclusion is that such investors should give risk a green premium, that is, to a lower risk premium and therefore cost of capital for green firms. Certain mpirical evidence supports the existence of a green premium that is higher on the stock returns of the carbon-intensive companies (Bolton and Kacperczyk, 2021, 2022), polluting companies (Hsu et al., 2022), companies most exposed to climate change risk (Bansal et al., 2016; Barnett, 2022), and least held by green funds (Zerbib, 2022) than on the stock returns of green companies. A similar effect is documented on the cost of equity (ElGhoul et al., 2011; Chava, 2014), expected returns approximated from option-implied information (Sautner et al., 2022), bond yields (Chava, 2014; Baker et al., 2018; Zerbib, 2019; Painter, 2020; Goldsmith-Pinkham et al., 2021; Huynh and Xia, 2021; Seltzer et al., 2022), venture capital funds (Barber et al., 2021), and real estate prices (Bernstein et al., 2019; Baldauf et al., 2020; Giglio et al., 2021). However, a recent debate has emerged on the theoretical side, as well as on whether the green premium is of the right sign or large enough in the data, at least in the recent period. For instance, cf. Pedersen et al. (2021), Goldstein et al.,

2021, Avramov et al., 2021), Ardia et al. (2021), Pástor et al. (2022), Berk and van Binsbergen (2021), Lontzek et al. (2023), Zhang (2021), Sauzet and Zerbib (2024), Chen et al. (2024). Another body of the literature on sustainable asset pricing studies the impact of climate risks on asset prices (e.g., Hong et al., 2019; Alok et al., 2020; De Angelis et al., 2022). Notably, a couple of recent papers analyze climate-related financial risks in general equilibrium (Barnett, 2022, Hambel et al., 2022, Barnett et al., 2023). Engle et al. (2020) and Alekseev et al. (2021) also propose portfolio construction methods that allow to efficiently hedge these climate risks.

Compared to this strand of literature, I bring to center stage the role of financial intermediaries, which had not been emphasized much but appears central in practice.

My paper is most related to a recent subset of the sustainable asset pricing literature, which does discuss more specifically on financial institutions. Pástor et al. (2024) estimate the green (and ESG) tilts of most asset managers and financial institutions and show that it is much smaller than one would expect from news headlines and marketing material. They also develop a reduced-form model of fund alpha and flows in that context to elicit predictions that they test empirically. van der Beck (2021) and Koijen et al. (2023) study related questions, and in particular the impact of fund flows, mostly empirically, by estimated a demand-based asset pricing system.

My contribution complements these contributions by building a general equilibrium model of asset prices and portfolio choice, in the presence of a financial intermediary and several retail investors who are potentially heterogeneous in their environmental tilt and preferences. It allows to study the evolution of all variables of interest in various cases, including the counterfactual ones that have not been observed empirically. As such, it constitutes a distinct, and complementary, analysis from the vastly different perspective of a macro-finance asset pricing model.

On the theoretical side, the framework that I propose builds on, and contributes to, the recent intermediary asset pricing literature. Most related are He and Krishnamurthy (2013), Chabakauri and Rytchkov (2021), Adrian and Shin (2014), and Adrian et al. (2014), among others.

Compared to He and Krishnamurthy (2013) in particular, who develop an asset pricing model with a retail investor and a financial intermediary, I bring a second heterogeneous retail agent (so that I can talk meaningful about green and neutral retail investors), two risky assets (the equity of a green firm, and that of a brown

firm, which are central), environmental tilts that go beyond financial considerations (which I model as in Sauzet and Zerbib, 2024), as well as general preferences (moving away from the log/CRRA case). I see my contribution as a natural generalization of their framework, which could be used in various other contexts.[9] Note also that those authors emphasize the importance of portfolio and balance-sheet constraints in pricing assets. This is not the central part of my analysis, but is the subject of generalization 5.3 (ongoing).

Because there are two risky assets in the economy that I study (in addition to a riskless bond), the portfolio choice of the financial intermediary and other investors become particularly central. In that sense, this is very much in the spirit of Chabakauri and Rytchkov (2021) and other paper in the indexing literature, in particular when I study the pure indexing subcase. Compared to this paper, I bring the fact that the financial intermediary consumes, owns wealth, and takes portfolio decisions to maximize her own utility and on behalf of other investors, as opposed to setting her portfolio necessarily equal to the market. This represents a third investor in practice (who could also be leveraged). From this perspective, my framework blends aspects of the indexing, and intermediary asset pricing, strands of literature. I also introduce environmental tilts beyond financial considerations, which are central for the topic at hand, and have a large effect on risk premia and portfolios. More generally, even though I focus more on the indexing/intermediary portfolio aspect, and stay closer in spirit to the intermediary asset pricing literature, my paper is also related to recent important contribution on benchmarking, such as Basak and Pavlova (2013), Buffa and Hodor (2023), and on other aspects of intermediary asset pricing, such as Kargar (2021) and Maxted (2023).

More generally, the theoretical setting is in the spirit of a long and distinguished literature on general equilibrium asset pricing models. A few examples include Dumas (1989, 1992), Duffie and Epstein (1992), Zapatero (1995), Wang (1996), Basak and Cuoco (1998), Dumas et al. (2000), Dumas and Uppal (2001), Chan and Kogan (2002), and more recently Pavlova and Rigobon (2007, 2008, 2010), Cochrane et al. (2008), Brunnermeier and Pedersen (2009), Weinbaum (2009), Bhamra and Uppal (2009, 2014), Martin (2011, 2013), Brunnermeier and Sannikov (2014), Gârleanu and Pedersen (2011), Chabakauri (2013), Gârleanu and Panageas (2015), Stathopoulos

[9]For instance, in ongoing work (Sauzet, 2021a), I construct a similar framework to study the role of financial intermediaries, e.g., global asset managers, in international equity and currency markets.

(2017), Drechsler et al. (2018), (Fang, 2019), Borovička (2020), Sauzet (2022, 2024).

On the technical side, because the model has three state variables, solving it requires higher-dimensional computational methods. In practice, I use projection approaches to solve the model globally, and while I stick to more standard Chebyshev polynomials as function approximator instead of using neural networks, I rely on the same underlying high-dimensional machinery as in Sauzet (2021b). Specifically, I build Chebyshev bases and grids using a Python implementation (Romero-Aguilar, 2021) of the standard *CompEcon* toolbox (Miranda and Fackler, 2004), but implement the actual resolution using JAX to be able to use its automatic differentiation and just-in-time compiling capabilities. This makes a big difference: I am able to solve the model on a standard laptop with CPU only, while economies with more than one or two state variables are often prohibitively costly computationally, especially for more standard, finite-difference-type methods. This opens avenues for much higher-dimensional settings, in particular by moving the resolution to high-capacity computing clusters with GPUs or TPUs. (Sauzet, 2021b shows that models with 10 state variables, for instance, can be reachable.)

Lastly, the generalizations of the framework that I consider are broadly related to He and Krishnamurthy (2013), Adrian and Shin (2014), and Adrian et al. (2014) (portfolio constraints), and Cartellier et al. (2023) (greenwashing), again among several others.

Outline The paper proceeds as follows. Section 2 describes the set-up of the economy and introduces the three state variables that drive economic mechanisms –the wealth share of the green investor in retail wealth, the output share of the green firm, and the wealth share of the financial intermediary. Section 3 characterizes asset prices, portfolio choices, and other variables. Section 4 discusses the core results about sustainable investing in this intermediary asset pricing economy. I present generalizations of the framework in Section 5, and Section 6 concludes. Proofs and additional material are provided in Appendix.

2 The Economy

In this section, I present the theoretical setup. I introduce a pure-exchange economy with three agents: one green retail investor (G), one neutral retail investor (N), as well as a financial intermediary (I). The intermediary can be seen as a general financial institution that can represent various types of asset managers, funds, banks, or the financial sector as a whole. In this economy, two firms —green and brown ($j \in \{g, b\}$)— produce the same consumption good. Their outputs is exogenous and follow geometric Brownian motions. The equity assets of each firm are traded à la Lucas (1978), and so is a riskless bond.

Retail investors cannot trade the green and brown equity assets separately. Instead, they have to invest through the portfolio offered by the intermediary. The green retail investor has an environmental tilt, that is, a preference for holding a portfolio invested (through the intermediary) in the green asset, for reasons beyond pure financial considerations (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022). The financial intermediary trades both risky assets separately, and therefore prices them. I study both the case in which she does not have an environmental tilt, and when she does. I show that the equilibrium can be characterized as a function of three state variables: the (endogenous) wealth share of the green retail investor in retail wealth, x_t, the (exogenous) output share of the green firm, y_t, and the (endogenous) wealth share of the financial intermediary in total wealth, u_t. All other variables, such as risk premia and portfolios, are determined endogenously as a function of those state variables. The setup is summarized in Figure B.1 in Appendix. Appendices A and B gather additional results and figures that are omitted in the main text.

Time is continuous and the horizon is infinite, $t \in [0, \infty)$. Uncertainty is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ supporting a two-dimensional Brownian motion $\vec{Z} \equiv (Z_g, Z_b)^T \in \mathbb{R}^2$. The filtration $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,\infty)}$ is the usual augmentation of the filtration generated by the Brownian motions, and $\mathcal{F} \equiv \mathcal{F}_\infty$.

2.1 Endowments, prices, assets

The two firms/trees produce a single consumption good, which is taken to be the numéraire and whose price is normalized to one. Their outputs follow geometric

Brownian motions

$$\frac{dY_{j,t}}{Y_{j,t}} = \mu_{Y_j} dt + \sigma_{Y_j}^{\top} d\vec{Z}_t, \quad j \in \{g, b\}$$

The green and brown trees are traded as equity assets, with returns given by

$$dR_{j,t} = \frac{dQ_{j,t}}{Q_{j,t}} + \frac{Y_{j,t}}{Q_{j,t}} dt = \frac{d\left(Y_{j,t}/F_{j,t}\right)}{Y_{j,t}/F_{j,t}} + F_{j,t} dt \equiv \mu_{j,t} dt + \sigma_{j,t}^{T} d\vec{Z}_t, \quad j \in \{g, b\} \tag{1}$$

where $Q_{j,t}$ are the equity prices, and $F_{j,t} \equiv Y_{j,t}/Q_{j,t}$ are the dividend yields. Drifts $\mu_{j,t}$ measure conditional expected returns, and diffusion terms $\sigma_{j,t}$ measure the loadings on the shocks and, hence, the conditional volatilities. Both are obtained from Itô's Lemma and given in Appendix A. We normalize the supply of each equity asset to unity.

In addition, there exists a bond, which is traded by all agents: the two retail investors, and the financial intermediary. The bond is in net zero supply, and locally riskless in units of numéraire. Its price is B_t, and its instantaneous interest rate is r_t, so that $dB_t/B_t = r_t dt$.

Finally, the portfolio offered by the financial intermediary can be see as a third risky asset, that I denote by subscript $j = I$. In practice, it is a combination of the green and brown equities and the bond. The weights of each is set by the intermediary, as discussed below. The return on this intermediary portfolio is denoted $dR_{I,t} \equiv \mu_{I,t} dt + \sigma_{I,t}^{T} d\vec{Z}_t$, this is the only risky asset in which the green and neutral retail investors can invest in practice.

2.2 Preferences

All investors have recursive preferences à la Duffie and Epstein (1992) that are defined over consumption of the unique consumption good produced by the two trees, and the weights on the assets in their portfolios, $\boldsymbol{w}^i$. Specifically, for each investors $i \in$

$\{G, N, I\}$,

$$V_t^i = \max_{\{C_u^i, w_{\pi,u}^i\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} f^i \left(C_u^i, V_u^i, \boldsymbol{w_u^i} \right) du \right] \tag{2}$$

$$f^i(C, V, \boldsymbol{w}) \equiv \left(\frac{1-\gamma^i}{1-1/\psi^i} \right) V \left[\left(\frac{C}{[(1-\gamma)V]^{1/(1-\gamma^i)}} \right)^{1-1/\psi^i} - \rho^i + \Phi^i(\boldsymbol{w}) \right]$$

where γ^i is the coefficient of relative risk aversion, ψ^i is the elasticity of intertemporal substitution (EIS), and ρ is the discount rate.

As discussed in Sauzet (2022, 2024), Sauzet and Zerbib (2024), recursive preferences are relevant for at least two reasons: (i) investors are not myopic and the hedging terms that arise from them can be sizable, and (ii) being able to separate risk aversion and the inverse of the elasticity of intertemporal substitution, $\gamma^i \neq 1/\psi^i$ can be helpful in practice, notably in the quantitative side. It turns out that in the economy of this paper, recursive preferences can make a big difference compared to CRRA in terms of the pattern of risk premia. This is discussed in Section 4.

In general, all parameters can differ across investors, although I focus on the case in which both retail investors share the same preferences, $\gamma^G = \gamma^N$, $\psi^G = \psi^N$. The financial intermediary may or may not have identical parameters, depending on the case. This can allow her to take on more risks and to use leverage, which could be relevant in practice.

A central aspect is that I also allow the green retail investor and the intermediary to have pro-environmental preferences, that is, a preference toward the green asset that go beyond pure financial considerations. I sometimes refer to this preference as an environmental *tilt* throughout the paper. In this general equilibrium context, I introduce it as a function of the exposure of investors to the green asset in their portfolio, following Sauzet and Zerbib (2024). Denote the portfolio weights of green and brown assets, $j \in \{g, b\}$, for investor i, as $\boldsymbol{w}_t^i \equiv \left(w_{g,t}^i, w_{b,t}^i \right)$. The environmental tilt for the green retail investor and the financial intermediary, $i \in \{G, N\}$ is,

$$\Phi^i(\boldsymbol{w}) \equiv \left(1 - 1/\psi^i\right) \left(w_g^i \phi_g^i + w_g^i \phi_b^i \right) = \left(1 - 1/\psi^i\right) w_g^i \phi^i \tag{3}$$

Parameter $\phi_g^i = \phi^i > 0$ drives the additional value that an investor derives from

holding the green asset, in the spirit of Pástor et al. (2021) and Zerbib (2022).[10] For the neutral retail investor, $\phi^N = 0$.

For the financial intermediary, $\boldsymbol{w}_t^I \equiv \left(w_{g,t}^I, w_{b,t}^I\right)$ is chosen optimally so as to maximize utility. The green retail investor, however, can only decide of $w_{I,t}^G$, the amount of wealth that she invests in the portfolio offered by the intermediary. As a result, the weight that the green retail investor holds on in the green and brown assets in practice are $w_{g,t}^G = w_{I,t}^G w_{g,t}^I$, and $w_{b,t}^G = w_{I,t}^G w_{b,t}^I$. In other words, she can only imperfectly target the green asset in her portfolio by choosing $w_{I,t}^G$. Note that this formulation assumes that, from the perspective of environmental tilts, retail investors observe the weight that the intermediary invests in each asset, $w_{g,t}^I, w_{b,t}^I$. This is an important aspect that I discuss in Section 4, and relax in Section 5.1 (ongoing).

From the portfolio that the intermediary investor allocates to the green and brown equity assets, $w_{g,t}^I, w_{b,t}^I$, she earns expected returns $\mu_{g,t}, \mu_{b,t}$. Similarly, the green and neutral retail investors earn expected return, $\mu_{I,t}$, from the share of wealth they invest in the portfolio of the intermediary investor. All investors, $i \in \{G, N, I\}$, allocate the remainder of their wealth to the riskless bond, which earns r_t. Investors use the proceeds of their investments to purchase their desired consumption $c_t^i \equiv C_t^i / W_t^i$.

In total, the intermediary investor chooses her consumption and portfolios to maximize (2) subject to the following budget constraint

$$\begin{aligned} \frac{dW_t^I}{W_t^I} = &\left(r_t + w_{g,t}^I \left(\mu_{g,t} - r_t\right) + w_{b,t}^I \left(\mu_{b,t} - r_t\right) - c_t^I\right) dt \\ &+ \left(w_{g,t}^I \sigma_{g,t} + w_{b,t}^I \sigma_{b,t}\right)^T d\vec{Z}_t \end{aligned} \tag{4}$$

The green and neutral retail investors $i \in \{G, N\}$ choose their consumption and portfolios to maximize (2) subject to

$$\frac{dW_t^i}{W_t^i} = \left(r_t + w_{I,t}^i \left(\mu_{I,t} - r_t\right) - c_t^i\right) dt + \left(w_{I,t}^i \sigma_{I,t}\right)^T d\vec{Z}_t \tag{5}$$

where $\mu_{I,t} - r_t = w_{g,t}^I(\mu_{g,t} - r_t) + w_{b,t}^I(\mu_{b,t} - r_t)$, and $\sigma_{I,t} = w_{g,t}^I \sigma_{g,t} + w_{b,t}^I \sigma_{b,t}$.

Finally, all investors $i \in \{G, N, I\}$ are subject to a standard transversality condi-

[10] Without loss of generality, I normalize the parameter for the brown asset to $\phi_b^i = 0$. The $\left(1 - 1/\psi^i\right)$ factor serves as a normalization so that ϕ^i drives the green premium like in the literature (cf. Section 3.4).

tion, and W_0^i is given. Note also that $W_t^i \geqslant 0$.

Pure indexing In Section 4, I also start by studying the simpler case in which the intermediary is a passthrough entity who does not consume or hold wealth of her own, and instead just sets the portfolio that she offers equal to the market portfolio. In this situation, $w_{g,t}^I = z_t$, $w_{g,t}^I = 1-z_t$, where $z_t \equiv Q_{g,t}/(Q_{g,t}+Q_{b,t}), 1-z_t = Q_{b,t}/(Q_{g,t}+Q_{b,t})$ are the market weights of the green and brown asset, respectively. (Recall that $Q_{j,t}$ are the equity prices.) This case allows to convey some of the intuition, and can be consistent with the current situation in which large asset managers, such as BlackRock and others, are in charge of investing a large share of retail investors' wealth, and do so mostly in the form of index funds that do not have strong ESG tilts. Note that in this specific situation, the neutral retail investor is allowed to trade both risky assets so that at least one agent can price all securities.

2.3 Equilibrium and state variables

The definition of the equilibrium is standard: (i) investors solve their optimization problems by taking aggregate stochastic processes as given, and (ii) goods and equity markets clear. The detailed definition is given in Appendix A. The bond market clears by Walras's law, which gives rise to the following relationship: $W_t^G + W_t^N + W_t^I = Q_{g,t} + Q_{b,t}$. In words, the total wealth in the economy is held in the form of the two equity assets in aggregate.

Stationary recursive Markovian equilibrium The equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest are expressed as a function of three state variables $X_t \equiv (x_t, y_t, u_t)'$, whose dynamics are also solely a function of X_t.[11] x_t is the (endogenous) wealth share of the green investor in the total wealth of retail investors, y_t is the (exogenous) output share of the green firm/tree, and u_t is the (endogenous) wealth share of the financial intermediary in total wealth.

[11] Formally, this is shown using a guess and verify approach like, e.g., in Gârleanu and Panageas (2015). The variables of interest are $\{c_t^G, c_t^N, c_t^I, w_{I,t}^G, w_{I,t}^N, w_{g,t}^I, w_{b,t}^I, \mu_{g,t}, \mu_{b,t}, \mu_{I,t}, r_t, F_{g,t}, F_{b,t}\}$.

Wealth share of the green investor in the wealth of retail investors The wealth share of the green retail investor in retail wealth is defined as

$$x_t \equiv \frac{W_t^G}{W_t^G + W_t^N} \tag{6}$$

In this setting with recursive preferences and tilts in investment preferences, the wealth share is neither constant nor solely a monotonic function of the current output share of the green tree, y_t. It is therefore required, and important, as an additional state variable.

Note that in the pure indexing case, the financial intermediary does not consume or hold wealth of her own, so that $W_t^I = 0$, and x_t is the share of *total* wealth (not just retail wealth) that is held by the green retail investor: $x_t = W_t^G/(W_t^G + W_t^N + W_t^I) = W_t^G/(W_t^G + W_t^N) = W_t^G/(Q_{g,t} + Q_{b,t})$.

Output share share of the green firm The relative supply of the green tree captures the effect of current fundamentals and is defined as

$$y_t \equiv \frac{Y_{g,t}}{Y_{g,t} + Y_{b,t}} \tag{7}$$

In this setting, the two firms/trees produce the same consumption good, so that the hedging motives and mechanisms related to movements in relative goods prices uncovered in Sauzet and Zerbib (2024) do not arise. However, the relative supply is still important in driving marginal values of wealth, risk premia, and other variables, such as valuation ratios. This is consistent with the existing literature on general equilibrium asset pricing with multiple trees (but no financial intermediaries), such as Cochrane et al. (2008), Martin (2013), Sauzet (2022, 2024).

Wealth share of the financial intermediary The third state variable is the share of total wealth held by the intermediary. It is defined as

$$u_t \equiv \frac{W_t^I}{W_t^G + W_t^N + W_t^I} \tag{8}$$

This variable captures the fact that financial intermediaries are likely to have an important role in pricing assets in the economy. Its role has been emphasized in the intermediary asset pricing and macro-finance literature, such as He and Krishnamurthy (2013), Adrian and Shin (2014), and Brunnermeier and Sannikov (2014). It can be understood as the size of the financial institutions under consideration –asset managers, mutual funds, private equity funds, hedge funds, or more generally as the size of the financial system as a whole. Again, in the pure indexing case, $W_t^I = 0$, so that $u_t = 0$.

Note that because $W_t^i \geqslant 0$ and $Y_{j,t} \geqslant 0$, x_t, y_t, and u_t are evolving in the bounded interval $[0, 1]$. This has the advantage that solving for unknown functions on a bounded domain is numerically more stable. Conceptually, as x_t and/or u_t gets closer to one, the economy converges (continuously) to a natural one-investor environment. As y_t gets closer to either of the boundaries, the economy converges to a one-equity asset economy.

I focus on the solution to the decentralized, that is, Radner equilibrium throughout, instead of relying on a social planner's problem. The existence and uniqueness of the equilibrium should be guaranteed, for instance, following Duffie and Epstein (1992), who use partial differential equation techniques to prove them in an infinite-horizon Markov diffusion setting with stochastic differential utility, or Chabakauri (2013) and Bhamra and Uppal (2014), who do so constructively for economies with heterogeneous agents and incomplete and complete markets, respectively. Both are also shown in situations with potentially dynamically complete marketsusing a planner solution in Anderson and Raimondo (2008), and under complete markets with a full set of Arrow-Debreu securities in Hugonnier et al. (2012). Note that given that the intermediary trades both risky assets, and that there are two sources of risk ($dZ_{g,t}, dZ_{b,t}$, the shocks to the output of each tree), markets are dynamically complete from her perspective, although not for retail investors. (Similarly, in the case of pure indexing, the neutral retail investor trades both risky assets, so that markets are dynamically complete from his perspective, but not for the green retail investor.)

2.4 Computation of the equilibrium

Section 3 characterizes all variables of interest as a function of the state variables, $X_t \equiv (x_t, y_t, u_t)'$, and a set of unknown functions $\mathcal{G} \equiv \{F_{g,t},\, F_{b,t},\, J_t^G,\, J_t^N,\, J_t^I,\, w_{I,t}^G,\, w_{I,t}^N,\, w_{g,t}^I,\, w_{g,t}^I\}$.[12] Due to the stationary recursive Markovian structure of the equilibrium, those unknown functions are themselves functions of X_t only, and are jointly determined by a set of coupled second-order partial differential and algebraic equations.

The resolution is based on projection methods and orthogonal collocation. Specifically, each of the unknown function $g : [0,1]^3 \to \mathcal{D}^g \subseteq \mathbb{R}$ in $\mathcal{G}$ is approximated using Chebyshev polynomials and the equilibrium is solved on an grid based on the zeros of the Chebyshev polynomials.

The main appeal of this approach is that this is a global solution method, which makes it possible to trace out the evolution of the variables of interest as a function of the state of the economy. Combined with continuous time, it allows to cleanly express and solve for the main variables –risk premia, portfolios, consumption, *etc.*–, as well as their subcomponents. This is crucial when it comes to explaining the underlying mechanisms, especially in terms of their dynamics with economic conditions, X_t. Projection methods are also well-suited for settings with multiple state variables.

On the technical side, because the model has three state variables, solving it is made easier by higher-dimensional computational methods. In practice, while I stick to the more standard Chebyshev polynomials as function approximator instead of using neural networks, I rely on the same underlying high-dimensional machinery as in Sauzet (2021b). Specifically, I build Chebyshev bases and grids using a Python implementation (Romero-Aguilar, 2021) of the standard *CompEcon* toolbox (Miranda and Fackler, 2004), but implement the actual resolution using JAX to be able to use its automatic differentiation and just-in-time compiling capabilities. This makes a big difference: I am able to solve the model on a standard laptop with CPU only, while economies with more than one or two state variables are often prohibitively costly computationally, especially for more standard, finite-difference-type methods. This opens avenues for much higher-dimensional settings, in particular by moving the resolution to high-capacity computing clusters with GPUs or TPUs. Details are

[12] J_t^G, J_t^N, J_t^I capture (an increasing monotonic transformation of) the marginal values of wealth of each investor, they are introduced in Section 3 In addition, for any function g, g_t denotes $g(X_t)$, not the time-derivative of g, which is zero because the model is stationary due to the infinite horizon.

provided in Sauzet (2021b, 2022). For instance, I show that models with 10 state variables can be reachable. This could be helpful both in this and other contexts.

3 Characterization of the Equilibrium

I now characterize the equilibrium theoretically. Section 3.1 discusses the marginal values of wealth of investors, as well as their consumption, which are important underlying variables. I then discuss market clearing in Section **??**, portfolios and how they depend on environmental tilts in Section 3.3, and risk premia in Section 3.4. Appendix A presents additional theoretical results, such as the law of motions of the three state variables. The theoretical characterization delivers a system of coupled algebraic and second-order partial differential equations that serves as a basis for the resolution. The resulting variables underpin the main findings in the paper that I present in Section 4.

3.1 Marginal value of wealth, consumption

In this economy, due to the homotheticity of preferences, one can show that the value functions for all investors, $i \in \{G, N, I\}$, satisfy

$$V^i(W_t^i, x_t, y_t, u_t) = \left(\frac{W_t^{i1-\gamma^i}}{1-\gamma^i}\right) J^i(x_t, y_t, u_t)^{\frac{1-\gamma^i}{1-\psi^i}} \tag{9}$$

Functions J_t^G, J_t^N, J_t^I are determined in equilibrium, and take center stage in driving many of the decisions of investors. In what follows, as is common, I refer to them as (monotonic transformation of) the investors' marginal values of wealth. Their evolution are governed by three coupled Hamilton-Jacobi-Bellman equations, one per investor, which are shown in Appendix A. Taking first-order condition with respect to consumption per unit of wealth, $c_t^i \equiv C_t^i/W_t^i$, yield, for $i \in \{G, N, I\}$

$$c_t^i = J_t^i \tag{10}$$

3.2 Market clearing

Both firms produce the same single good, which is consumed by all investors. The market clearing condition on the goods market is therefore

$$\sum_i C_t^i = C_t^G + C_t^N + C_t^I = \sum_j Y_{j,t} = Y_{g,t} + Y_{b,t}$$

This implies

$$\begin{aligned} u_t c_t^I + (1-u_t)\left(x_t c_t^G + (1-x_t)c_t^N\right) &= \frac{Y_{g,t}}{Q_{g,t}}\frac{Q_{g,t}}{\sum_i W_t^i} + \frac{Y_{b,t}}{Q_{b,t}}\frac{Q_{b,t}}{\sum_i W_t^i} \\ &= F_{g,t} z_t + F_{b,t}(1-z_t) \end{aligned} \tag{11}$$

The market clearing conditions on both equity assets are also noteworthy. In particular, they embed the fact that the financial intermediary decides on the portfolio, $w_{g,t}^I, w_{b,t}^I$, which is also invested in by retail investors. Recall that the latter invest a share $w_{I,t}^i$ of their wealth in it, and the remainder in the riskless bond. As a result, market clearing on the green and bond equity asset read

$$w_{g,t}^I\left(u_t + (1-u_t)x_t w_{I,t}^G + (1-u_t)(1-x_t)w_{I,t}^N\right) = z_t \tag{12}$$

$$w_{b,t}^I\left(u_t + (1-u_t)x_t w_{I,t}^G + (1-u_t)(1-x_t)w_{I,t}^N\right) = 1 - z_t \tag{13}$$

where $z_t \equiv Q_{g,t}/(Q_{g,t}+Q_{b,t})$ and $1-z_t = Q_{b,t}/(Q_{g,t}+Q_{b,t})$ are the weights of both assets in the market portfolio.

Note also that using the market clearing conditions for both equtiy assets yields

$$\frac{w_{g,t}^I}{w_{b,t}^I} = \frac{z_t}{1-z_t} \tag{14}$$

However, this does not necessarily mean that $(w_{g,t}^I, w_{b,t}^I)' = (z_t, 1-z_t)'$, because the intermediary can use leverage. This will happen, for instance, if the financial intermediary is less risk averse than the green and neutral retail investors. This case is common in the intermediary asset pricing and macro-finance literature, and I study it among others in Section 4. In that situation, the financial intermediary will borrow

from retail investors using the bond market so as to leverage up.[13]

The bond market clears by Walras's law, which, again, gives rise to the following useful relationship

$$\sum_i W_t^i = W_t^G + W_t^N + W_t^I = \sum_j Q_{j,t} = Q_{g,t} + Q_{b,t} \tag{15}$$

3.3 Portfolios

Define the geometric diffusion of the marginal value of wealth of each investor, $i \in \{G, N, I\}$, as

$$\sigma_{J^i,t} \equiv \frac{J^i_{x,t}}{J^i_t} x_t \sigma_{x,t} + \frac{J^i_{y,t}}{J^i_t} y_t \sigma_{y,t} + + \frac{J^i_{u,t}}{J^i_t} u_t \sigma_{u,t} \tag{16}$$

Portfolios are obtained by taking the first-order conditions with respect to portfolio weights in the Hamilton-Jacobi-Bellman equations. The financial intermediary can freely invest in both assets so that her portfolio is

$$\begin{pmatrix} w^I_{g,t} \\ w^I_{b,t} \end{pmatrix} = \frac{1}{\gamma^I} \left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi^I_g \\ \mu_{b,t} - r_t + \phi^I_b \end{pmatrix} + \left(\frac{1-\gamma^I}{1-\psi^I}\right) \Sigma_t^T \sigma_{J^I,t} \right\} \tag{17}$$
$$b^I_t = 1 - w^I_{g,t} - w^I_{b,t}$$

where $\Sigma_t \equiv [\sigma_{g,t} \ \sigma_{b,t}]$.

For the green and neutral retail investors, $i \in \{G, N\}$, the portfolio is composed of the share of their wealth that they invest in the intermediary portfolio, and in the riskless bond[14]

$$w^i_{I,t} = \frac{1}{\gamma^i} \left(\sigma^T_{I,t} \sigma_{I,t}\right)^{-1} \left\{ (\mu_{I,t} - r_t) + \left(\frac{1-\gamma^i}{1-\psi^i}\right) \sigma^T_{I,t} \sigma_{J^i,t} \right\} + \left(w^I_{g,t} \phi^i_g + w^I_{b,t} \phi^i_b\right) \tag{18}$$
$$b^i_t \equiv (1 - w^i_{I,t}) + w^i_{I,t}(1 - w^I_{g,t} - w^I_{b,t})$$

[13] We can define leverage as $l^I_t = w^I_{g,t}/z_t$, so that $(w^I_{g,t}, w^I_{b,t})' = l^I_t (z_t, 1 - z_t)'$.

[14] Note that retail investors can also get some exposure to the bond via the intermediary portfolio. Specifically, the intermediary invests $1 - w^I_{g,t} - w^I_{b,t}$ in the bond, which could positive or negative if the intermediary borrows or lends. As a result, retail investors, $i \in \{G, N\}$, allocate $w^i_{I,t}(1 - w^I_{g,t} - w^I_{b,t})$ of their wealth towards the riskless bond via the intermediary, in addition to the $1 - w^i_{I,t}$ that they allocate to it directly. In total, their total allocation to the bond is therefore $b^i_t = (1 - w^i_{I,t}) + w^i_{I,t}(1 - w^I_{g,t} - w^I_{b,t})$.

Note that for both retail investors, and the financial intermediary, portfolios are impact by potential environmental tilts. In the case of the intermediary, the impact is direct, as she is able to trade both assets separately. For retail investors, the influence comes about as they invest in the intermediary portfolio, which in turn gives them exposure to the green and brown equity assets, $w^i_{g,t} = w^i_{I,t} w^I_{g,t}$, $w^i_{b,t} = w^i_{I,t} w^I_{b,t}$. Retail investors therefore still account for their tilts while taking decision, albeit in this indirect manner. This is central in terms of explaining the main findings in Section 4. Again, this formulation assumes that retail investors observe the weight that the intermediary invests in each asset. This is an important aspect that I also discuss in Section 4, and relax in Section 5.1 (ongoing).

Lastly, in the case of pure indexing, the financial intermediary is simply a passthrough entity that sets the portfolio that she offers to $w^I_{g,t} = z_t$, $w^I_{b,t} = 1 - z_t$. In that case, the neutral retail investor is allowed to trade both risky assets so that at least one agent can price all securities. This means, in turn, that the portfolio of the neutral investor looks like in Equation (17) in this setting.

3.4 Asset returns

Lastly, I turn to asset returns. Recall that the financial intermediary is the only investor that can freely trade both equity assets separately. In practice, this means that she is the main driver of their risk premia, which are determined as follows

$$\mu_{g,t} - r_t = \gamma^I \sigma^T_{g,t} \left\{ \sigma_{I,t} - \left(\frac{1}{\gamma^I} \right) \left(\frac{1 - \gamma^I}{1 - \psi^I} \right) \sigma_{J^I,t} \right\} - \phi^I_g \tag{19}$$

$$\mu_{b,t} - r_t = \gamma^I \sigma^T_{b,t} \left\{ \sigma_{I,t} - \left(\frac{1}{\gamma^I} \right) \left(\frac{1 - \gamma^I}{1 - \psi^I} \right) \sigma_{J^I,t} \right\} - \phi^I_b \tag{20}$$

Equations (19) and (20) highlights that an environmental tilt on the part of the intermediary, $\phi^I_g > 0$, can directly (and strongly) impact expected excess returns. In other words, this can easily give rise to a negative green premium, and is the main channel that has been emphasized in the literature so far (Bolton and Kacperczyk, 2021, 2022, Pástor et al., 2021, Hsu et al., 2022, Pedersen et al., 2021, Zerbib, 2022, Sauzet and Zerbib, 2024). On the other hand, if the financial intermediary does not have any sustainable motive, one can wonder if there can be any effect at all. In

particular, any impact of the environmental tilt of the green retail investor will have to be indirect and to emerge endogenously via the market/myopic terms, $\sigma_{j,t}^T \sigma_{I,t}$. In Section 4, I show that such an endogenous effect does arise in practice in equilibrium.[15]

Turning now to the pricing of the intermediary portfolio, recall that the excess returns on it satisfy

$$\mu_{I,t} - r_t \equiv w_{g,t}^I \left(\mu_{g,t} - r_t\right) + w_{b,t}^I \left(\mu_{b,t} - r_t\right) \tag{21}$$

$$\sigma_{I,t} \equiv w_{g,t}^I \sigma_{g,t} + w_{b,t}^I \sigma_{b,t} \tag{22}$$

Contrary to the individual assets, the intermediary portfolio is traded by retail investors. In equilibrium, combining their optimal portfolios with those of the financial intermediaries and Equations (19) and (20) yield

$$\mu_{I,t} - r_t = \gamma_t \sigma_{I,t}^T \sigma_{\widetilde{W},t} - \gamma_t \sigma_{I,t}^T \sigma_{\tilde{J},t} - \gamma_t \phi_t \tag{23}$$

where

$$\begin{aligned}
\gamma_t &\equiv \left(\frac{u_t}{\gamma^I} + \frac{(1-u_t)x_t}{\gamma^G} + \frac{(1-u_t)(1-x_t)}{\gamma^N} \right)^{-1} \\
\sigma_{\tilde{J},t} &\equiv \sum_i x_t^i \left(\frac{1}{\gamma^i} \right) \left(\frac{1-\gamma^i}{1-\psi^i} \right) \sigma_{J^i,t} \\
\sigma_{\widetilde{W},t} &\equiv z_t \sigma_{g,t} + (1-z_t)\sigma_{b,t} = (l_t^I)^{-1} \sigma_{I,t} \\
\phi_t &\equiv w_{g,t}^I \left(\sum_i x_t^i \frac{\phi_g^i}{\gamma^i} \right) + w_{b,t}^I \left(\sum_i x_t^i \frac{\phi_b^i}{\gamma^i} \right)
\end{aligned}$$

In words, γ_t is the wealth-weighted risk aversion, $\sigma_{\tilde{J},t}$ is the wealth- and preference-weighted diffusion of the average marginal investor in the economy, $\sigma_{\widetilde{W},t}$ is the diffusion of total wealth in the economy, which is nothing but the market portfolio ($\widetilde{W}_t = W_t^G + W_t^N + W_t^I = Q_{g,t} + Q_{b,t}$), and ϕ_t is the wealth- and preference-weighted environmental tilt on the intermediary portfolio. Taken together, Equation 23 high-

[15] Again, in the case of pure indexing, the neutral investor is allowed to trade both equity assets separately, and therefore plays a role akin to that of the financial intermediary in the general setting. In other words, in that situation, the risk premia on the green and brown assets in Equations 19 and 20 involve the portfolio choice of the *neutral* investor. It turns out that the insights that one can glean from that case will also help understand the more general case. Cf. Section 4.1.

lights three main components: a market/myopic term, a hedging term (involving the comovement with marginals value of wealth J_t^i), and an environmental tilt term.

The pricing of the intermediary portfolio is therefore impacted by the preference, including tilts, of all investors in the economy. This is not surprising, given that they all have access to it. Equation 23 suggests that whether environmental tilts have a large impact on the market in practice is likely to depend on the wealth share of different types of investors and their tilts, as expected, but that those also interact with their varying degrees of risk aversion. This fact will influence the quantitative magnitude of the results in Section 4. Differing attitudes towards risk can also play a further role in the background, as it can give rise to additional borrowing and lending, and therefore to leveraged positions.

The central question, however, is whether this effect on the pricing of the intermediary portfolio, as well as the endogoneous market/myopic term in Equations 19 and 20, translate into an impact on the risk premia on green and brown assets in practice. In short: can a green premium still emerge from the environmental tilt of the green retail investor, even when the financial intermediary does not account for sustainable considerations? On the other hand, how large can the impact be if the intermediary does tilt herself? Answering those questions requires solving the full model, and I now turn to describe those findings.

4 Sustainable investing in an economy with financial intermediaries

Can the presence of green retail investors impact the pricing of green assets, even when they invest through financial intermediaries? This is the central question of this paper, and this section presents the main results in various cases. In Section 4.1, I discuss the situation in which financial intermediaries do not have environmental or ESG preferences. Perhaps surprisingly, even then, green retail investors can influence the relative pricing of green securities, sometimes significantly, even though this comes with significant qualifications. Section 4.2 focuses on the situation in which financial intermediaries tilt towards environmental assets, beyond what would be dictated purely by financial considerations. In that case, the impact on the risk premia

of green assets can be substantially larger, especially when financial intermediaries are leveraged. Unless otherwise specified, parameters are calibrated according to Assumption 1. Most values are standard, and the impact of the most important ones are discussed in the main text.

Taken together, these findings highlight the central role that financial intermediaries play in determining whether enough financing is channeled towards fostering the green transition.

Assumption 1 (Baseline calibration)**.** *Unless otherwise specified, results in this section are obtained under the following calibration,* $i \in \{G, N, I\}, j \in \{g, b\}$

- *Risk aversion:* $\gamma^i \in \{1, 5, 10, 20, 30\}$*,* γ^I *(intermediary)* $\leqslant \gamma^G = \gamma^N$ *(retail),*
- *Elasticity of intertemporal substitution:* $\psi^i = 1/\gamma^i$ *(CRRA), or* $\psi^i = 1.5$ *(recursive preferences),*
- *Environmental tilt, beyond financial consideration:* $\phi_g^G \in \{0, 0.5\%, 1\%, 1.5, 2\%\}$*,* $\phi_g^I \in \{0, 0.5\%, 1\%, 1.5, 2\%\}$*,* $\phi_g^N = \phi_b^G = \phi_b^N = \phi_b^I = 0$*.*
- *Discount rate:* $\rho^i = \rho = 1\%$*,*
- *Output:* $\mu_{Y_j} = \mu_Y = 2\%, \sigma_{Y_g} = (4.1\%, 0)^T, \sigma_{Y_b} = (0, 4.1\%)^T$ *(no fundamental correlation).*

4.1 Financial intermediary with no green preferences

In this section, I consider the situation in which the green retail investor has a preference towards the green asset, beyond financial considerations, but the financial intermediary does not. Specifically, for the green retail investor, the environmental tilt is $\phi_g^G = \phi^G > 0$, while for the neutral retail investor, $\phi_g^G = 0$. Both will remain true throughout most of the paper. For the financial intermediary, this section corresponds to the situation in which, $\phi_g^I = \phi^I = 0$.[16]

I focus first on a pure indexing economy in which the financial intermediary is a passthrough investor who sets the portfolio that she proposes to investors equal to the market portfolio. I then turn to the case in which the financial intermediary is an

[16] Without loss of generality, I set the tilt for the brown asset to zero in all cases, for all investors: $\phi_b^i = 0$, for $i \in \{G, N, I\}$. This could be relaxed, for instance to further study the case in which investors have different values or preferences, perhaps opposite ones, beyond financial considerations.

investor in her own right who consumes and holds a share of the economy's wealth, and sets her portfolio optimally to maximize her utility. In practice, she decides on the portfolio allocation for total wealth: both her own wealth, and that of her clients (both retail investors). In both cases, the good news is that the green retail investor is still able to exert a substantial impact on the pricing of green assets in the economy, even though she is not able to trade equity assets separately. The downside is that this comes with significant qualifications, as I describe below.

Economy with pure indexing In this situation, the financial intermediary is a passthrough investor who does not consume or has any wealth of her own.[17] Instead, she simply sets the portfolio that she proposes to investors equal to the market portfolio: $w^I_{g,t} = z_t$, $w^I_{b,t} = 1 - z_t$, where $z_t \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}), 1 - z_t = Q_{b,t}/(Q_{g,t} + Q_{b,t})$, are the market weights of the green and brown equity assets, respectively. The green retail investor can therefore only invest in the market index. This can be seen as a reasonable approximation of the current situation in which large asset managers, such as BlackRock and others, are in charge of investing a large share of retail investors' wealth, and do so mostly in the form of index funds that do not have strong ESG tilts. (Note that in this specific case, the neutral retail investor is allowed to trade both risky assets so that at least one agent can price all securities.)

As a starting point, Figure 1 shows the expected excess returns on the green and brown assets, $\mu_{g,t} - r_t$, $\mu_{b,t} - r_t$, in a world in which even the green retail investor has no green tilt, $\phi^G = 0$. This is a baseline against which all other results can be compared, and any difference with it will only come from the impact of having a positive green tilt. The figure plots those expected excess returns as a function of the output share of the green firm, $y_t \equiv Y_{g,t}/(Y_{g,t}+Y_{b,t})$, for various values of risk aversion, $\gamma^G = \gamma^N = \gamma$, and of the elasticity of intertemporal substitution, $\psi^G = \psi^N = \psi$.[18]

As expected, the average excess returns on both assets strongly increase with risk aversion, whether preferences are CRRA, $\psi = 1/\gamma$, or recursive, $\psi = 1.5$. For $\gamma = 20$ or $\gamma = 30$, risk premia can reach up to around 4% (recursive preferences) to 14%

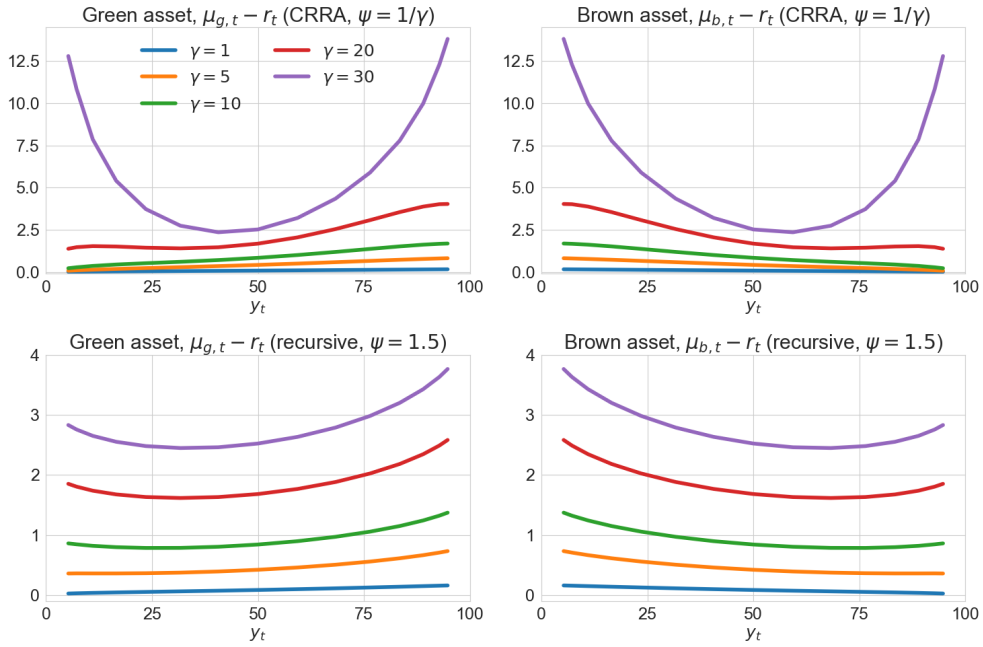
[17] Note that in this case, only two state variables are relevant: the wealth share of the green retail investor in the wealth of retail investors (x_t, endogenous), and the output share of the green firm (y_t, exogenous). The financial intermediary has no wealth so that her wealth share is $u_t = 0$, for all t, and the retail investors hold all the wealth in the economy.

[18] As long as there is no green tilt, the wealth share of the green investor, x_t, is not relevant because investors have the same preferences.

(CRRA), getting close to empirical estimates.[19]

Risk premia also vary significantly with economic conditions. For instance, as the green firm becomes dominant in the economy, that is, as her output share y_t increases, the equity of the green firm becomes dominant in the market portfolio. The green asset is therefore riskier, and commands higher risk premia in equilibrium. The pattern is symmetric for the brown asset, and their magnitude is strongly modulated by the degree of risk aversion. Those results are consistent with the existing literature such as Cochrane et al. (2008), Martin (2013), Sauzet (2022, 2024), and Sauzet and Zerbib (2024).

Figure 1: Risk premia with no green tilt



What happens when the green retail investor *does* have environmental preferences? Figure 2 shows the evolution of the risk premium on the green asset for various values of her green tilt, $\phi^G \in \{0\%, 1\%, 1.5\%\}$, and of her wealth share, $x_t \in \{5\%, 50\%, 95\%\}$.[20]

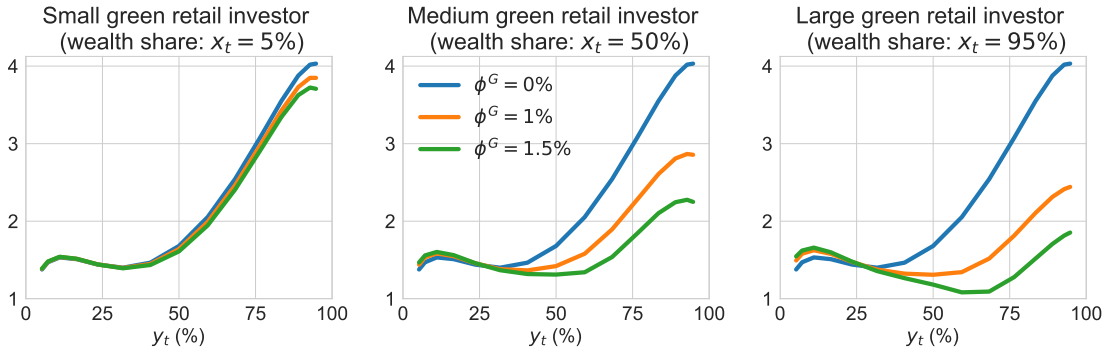
[19]The fact that somewhat large risk aversion values are required to reach sizeable risk premia is a well-known finding with consumption-based asset pricing models. The presence of a financial intermediary will help from this perspective, either when I introduce leverage below, or portfolio constraints in Section 5.3 (ongoing). In fact, this is one of the reasons why financial intermediaries were introduced in the asset pricing literature in the first place.

[20]In practice, the wealth share of the green retail investor is an endogenous variable in this

This is plotted in the case of a risk aversion of $\gamma = 20$, and CRRA preferences, $\psi = 1/\gamma$, but the pattern and relative magnitude of the changes are representative of what happens for other calibrations. Figures for other values of γ and ψ are shown in Appendix B.2.1.

As the green tilt, ϕ^G, increases, the risk premium on the green asset, $\mu_{g,t} - r_t$, decreases substantially. The effect is larger as the green investor holds more share of the economy's wealth (large x_t), as expected. For instance, when her wealth is large, $x_t = 95\%$, the risk premium on the green asset can go from 4% with no tilt, to just above 1% with $\phi^G = 1.5\%$ in certain regions of the state space. This is a decrease of about 3 percentage points, or 50%.

Figure 2: Risk premium (%) on the green asset, $\mu_{g,t} - r_t$, and green investor tilt, ϕ^G



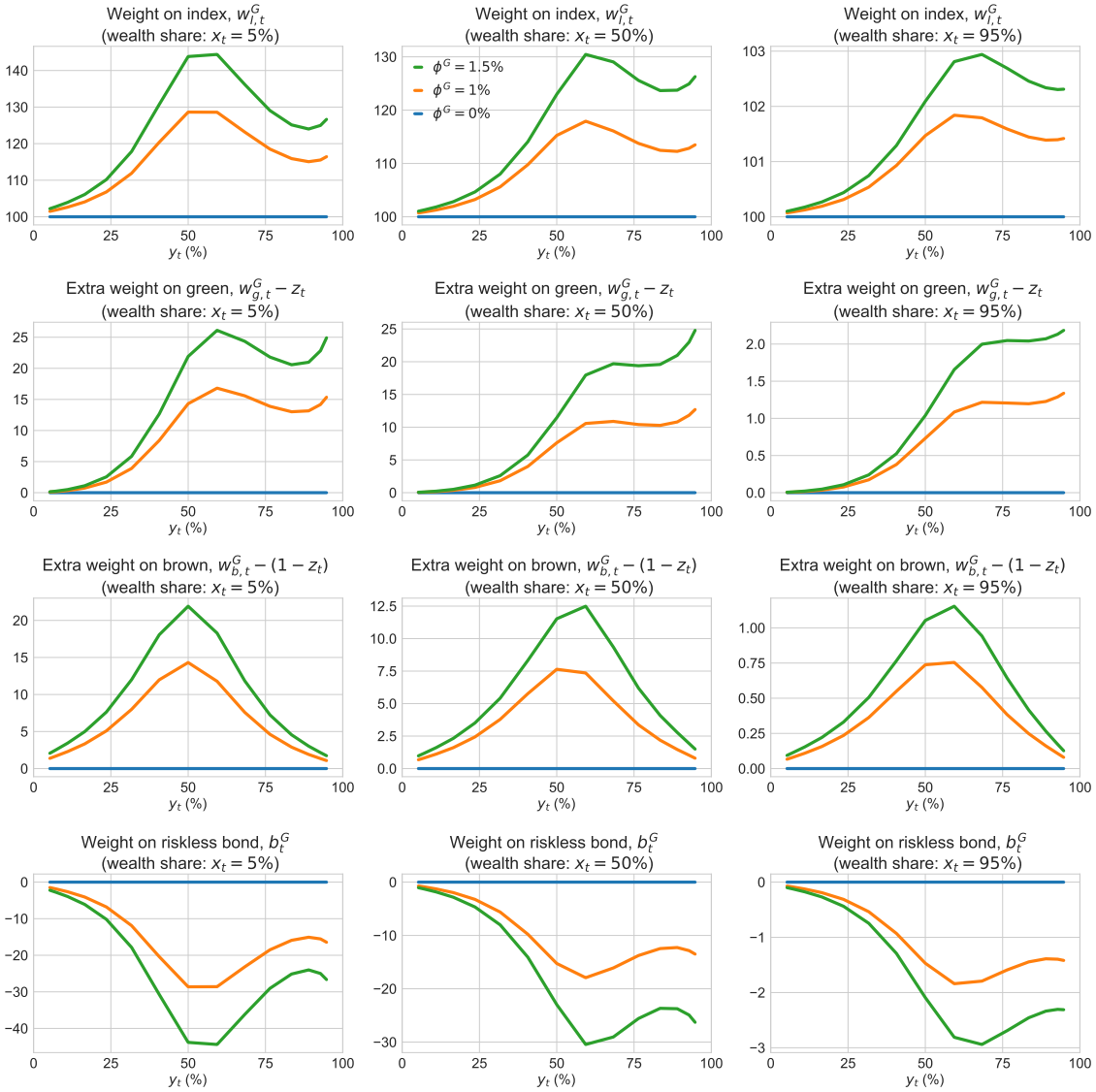
In short, this is good news: the green retail investor can have a substantial impact on reducing the cost of capital for green firms, which can in turn foster investment towards them. How is this possible, even though the green investor cannot trade assets separately, which is the usual way through which sustainable investing can influence expected returns? It turns out that not being able to trade asset separately does not mean that the green investor cannot modulate how much she invests in the green asset, albeit imperfectly.

Figure 3 presents the portfolio weights of the green retail investor, and shows that in practice, she is willing to invest up to $w^G_{I,t} = 140\%$ of her wealth in the index, in particular in situations in which the green firm dominates the economy (i.e., when the output share of the green firm, y_t is large and above 50%). To do so, she borrows

framework. Those values are used to illustrate the mechanisms.

in the riskless bond, $b_t^G < 0$, from the neutral investor. As a result, the green retail investor is able to invest in the green asset ($w_{g,t}^G = w_{I,t}^G z_t$) in excess of its market weight (z_t) by up to $w_{g,t}^G - z_t = 25\%$. Although she can target this asset only imperfectly, which results in her overweighting the brown asset as well, the latter effect is more limited: it reaches a high of $w_{b,t}^G - (1 - z_t) = 20\%$, but is much lower for most of the state space. Overall, the green retail investor is therefore able to meaningfully tilt her portfolio towards the green asset that she prefers.

Figure 3: Portfolio of the green retail investor (%)



In turn, the impact on portfolios can explain the effect on risk premia. By market clearing, the neutral investor has to take the flip side of the position of the green retail investor: he invests less in both assets, but in particular in the equity of the green firm, as shown in Figure B.2. In other words, the wealth of the neutral investor loads less on the green equity asset. Because in the case of pure indexing, the neutral investor trades both assets directly and therefore prices them, it leads the green asset to be less risky: it comoves less with the wealth of the neutral investor, offers improved diversification benefits, and therefore commands a lower risk premium in equilibrium. In other words, this effect does not come from the direct impact of the green tilt on risk premia, which cannot arise because the green retail investor does not trade assets directly. Instead, it emerges endogenously and is driven purely by the myopic or "market" term of Equation (19), which reads $\sigma_{g,t}^T \sigma_{W^N,t} = \sigma_{g,t}^T (w_{g,t}^N \sigma_{g,t} + w_{b,t}^N \sigma_{b,t})$ when the neutral retail investor does the pricing.[21]

Quantitatively, the mechanism gets larger in magnitude as the environmental tilt of the green retail investor, ϕ^G, increases, and as she controls more wealth (large x_t). In the latter case, she has to get closer back to holding the whole market, given that she is getting closer to dominating the whole economy. She therefore cannot deviate as much in her portfolio, and instead the pricing of the assets reacts more. This is also consistent with the neutral investor having more leeway to strongly tilt his portfolio away from the green asset (to take the other side of the trade), when he holds less wealth.

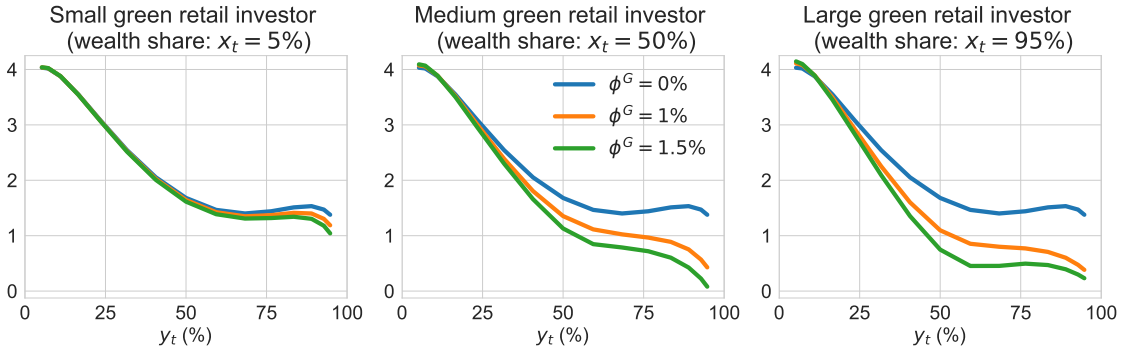
Caveats Given those results, should we declare victory, in the sense that a green retail investor can strongly influence green returns even when she can only invest through an intermediary? Not quite. I now discuss a number of important caveats and qualifications.

Risk premium on brown asset. First, the mechanism that reduces the risk premium on the green asset also is at work for the brown asset, because the neutral investor also reduces his portfolio weight for it, although in lesser amount. As a result, the risk premium on the equity of the brown firm, $\mu_{b,t} - r_t$, also decreases as the share of wealth held by the green retail investor, x_t, and her green tilt, ϕ^G, increase. This is shown in Figure 4. In other words, the fact that the green investor can only imperfectly

[21] Once I reintroduce a financial intermediary in her own right below, the pricing will revert back to being done by the intermediary herself. A lot of the intuition will remain, however.

target the green equity asset via the index can have counterproductive effects as it also decreases the expected excess returns on the brown asset.

Figure 4: Risk premium (%) on the brown asset, $\mu_{b,t} - r_t$, and green investor tilt, ϕ^G



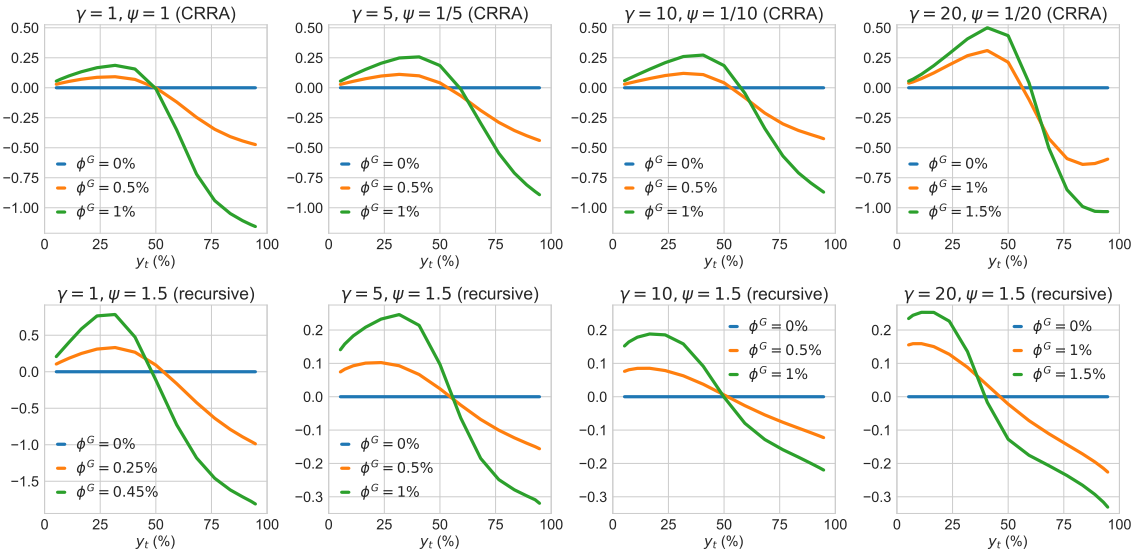
Green premium and specification. To get a sense of the relative effect, one can then compute the *green premium*, that is, the difference in risk premia between green and brown assets, $(\mu_{g,t} - r_t) - (\mu_{b,t} - r_t) = \mu_{g,t} - \mu_{b,t}$. Because the risk premia on both assets naturally vary with economic conditions even without a green tilt, as shown in Figure 1, I compute the "normalized" green premium in excess of the difference in risk premia absent green tilt, $(\mu_{g,t} - \mu_{b,t}) - (\mu_{g,t}^{\phi^G=0} - \mu_{b,t}^{\phi^G=0})$, so that any effect stems purely from the impact of the green retail investor.

Figure 5 shows the resulting normalized green premium for various risk aversion levels, under both CRRA ($\psi = 1/\gamma$) and recursive ($\psi = 1.5$) preferences. I focus on the case of a large green investor, $x_t = 95\%$, but Figure in Appendix B.2.1 shows the same variable for several values of the wealth share of the green investor, x_t. While most results so far were broadly similar across calibrations, the impact on the excess returns on the brown asset, and therefore on the green premium, varies more.

Starting with our baseline so far, $\gamma = 20, \psi = 1/\gamma$ (CRRA), the top right panel shows that the green premium can indeed be strongly negative, which is the desired result because this participates in lowering the green cost of capital. For instance, it can reach around -1 percentage point in certain regions of the state space. This finding is consistent with the picture that had emerged so far: the green investor *can* have a significant impact. As long as preferences are CRRA, this impact also persists for varying degrees of risk aversion, γ, although a larger green tilt, ϕ^G, becomes

required to reach a given green premium. The latter is consistent with investors becoming less inclined to strongly tilt their portfolios and take on undue risk, as they become more risk averse.

Figure 5: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various calibrations, in the case of a large green retail investor ($x_t = 95\%$)



Even in those cases in which the green premium can be sizable, however, the effect is inherently state-dependent, that is, it varies a lot with economic conditions. Beyond the effect of the wealth share of the green investor, x_t, that we discussed previously, the green premium can be strongly negative *only when* the green firm dominates the economy, namely, when its output share, y_t, is above 50% or even larger. This is stemming directly from the fact that the green investor can only imperfectly target the green asset, and is able to significantly increase her allocation to it via the index solely when this asset represents more than half of the market portfolio, which happens broadly when the output share of the green firm is larger than 50% (Figure 3).

As a matter of fact, when y_t falls below 50%, the green premium even turns positive in most cases. While this effect is mostly flat and moderate for low degrees of risk aversion, γ, it becomes sizable for $\gamma = 20$ or above. Once turning to recursive preferences (bottom row of Figure 5), this countervailing effect can become so large

that the green premium appear to be almost symmetric: it is negative for large green output share shares, y_t (as before), positive for low green output share shares, and of comparable magnitude in both cases. For instance, for $\gamma = 5$ and $\psi = 1.5$, the green premium is around -30 basis points for $y_t = 75\%$, and around $+30$ basis points for $y_t = 25\%$.[22]

In short, the environmental tilt of the green retail investor can sometimes backfire and lead to comparatively larger risk premia on the green asset in certain conditions, especially when y_t is low. This could constitute a relevant caveat in the current economic environment in which the output share of green firms could be on the order of $y_t = 30$-40% or less empirically, depending on the approach and scope used. This share is likely to increase, however, so that this could become less of an issue in coming years. Empirical comparison aside, those findings highlight the value of the general equilibrium setting developed in this paper, which makes it possible to investigate various cases, including those that are yet to materialize in the data but are important counterfactuals.

The specification of preferences can have further non-linear or unexpected effects, some of which are shown in Appendix.

For instance, for situations in which the green retail investor has a positive but moderate wealth share, e.g., $x_t = 30\%$, the risk premia on the green asset can be larger than that on the brown asset in a symmetric situation. This is shown in Figure in Appendix B.2.1. The green premium can also increase even for large y_t in this case, although the effect is small in magnitude.

On the other hand, even though CRRA preferences appear to be "better" from the perspective of generating a large negative green premium, they are not perfect either. Indeed, under such a specification, (i) the riskless rate is commonly too large in magnitude, often substantially so, when the degree of risk aversion is large enough to deliver sizable risk premia[23], and (ii) the riskless rate also increases with the green investor's wealth share because she uses borrowing to lever up her risky position

[22] The magnitude of the countervailing effect with recursive preferences stems from a combination of factors. E.g., the neutral investor downweights the brown asset further, compared to the CRRA case, due in part to stronger hedging motives, and the diffusion of both assets are also impacted differently by the green tilt.

[23] This is the well-known "risk-free rate puzzle" (Weil, 1989), which comes as a flip side to the "equity premium puzzle" (Mehra and Prescott, 1985), in a world with CRRA preferences. It constitutes one of the reasons why recursive preferences were introduced in practice.

(Figure 3). As a result, even when the risk premium on the equity of the green firm decreases by more (i.e., even when the green premium is negative), the *total discount rate* for both firms (i.e., risk premia + riskless rate, $\mu_{j,t} = (\mu_{j,t} - r_t) + r_t$) increases substantially. This is shown in Figure in Appendix. Because the total discount rate is the relevant cost of capital for firms, this increase can dwarf any helpful mild reduction on the relative risk premium for the green asset in that case. This point has not been emphasized much in the sustainable asset pricing literature, but highlights that allowing for recursive preferences in the framework is helpful from the perspective of uncovering the full extent of those mechanisms.

Retail investing in practice. More generally, as we discussed, the green retail investor has to resort to taking large positions in the index to be able to influence the pricing of assets, sometimes extremely so. To be able do it, she often borrows in the riskless bond so as to leverage her risky position, as shown in Figure 3. Both will remain true when I reintroduce a financial intermediary in her own right below, beyond the case of pure indexing. In addition, the green retail investor must invest in an inherently state-dependent fashion, closely monitoring the composition of the index in order to increase her position mostly when the green asset dominates it. This also assumes in the first place that she can even *observe* the composition of this index, or of the portfolio offered by the intermediary.

In practice, those elements could be difficult to square with the financial market access, information set, and investing practices, of the average retail or end investor. To alleviate some of those concerns, and assess how the results evolve as some of those elements are relaxed, I present a number of generalizations of the framework in Section 5 (ongoing). First, I consider the case in which retail investors do not know the weights in the portfolio offered by the financial intermediary, at least from the perspective of environmental tilting. Second and relatedly, I study what happens if the financial intermediary resorts to greenwashing: she does not tilt towards the green asset in the way that retail investors believe. Third, I investigate the impact of portfolio constraints on the part of both retail investors and the financial intermediary, which could change or amplify some of those results.

In summary so far, in a pure indexing economy in which the intermediary does not tilt towards the green asset, the influence of green retail investors on the pricing of green assets, and therefore on fostering the green transition, can be qualified as a

"partial success". On the one hand, the good news is that green investors *can* have a substantial impact, which was not necessarily obvious *ex-ante*. On the other, the impact comes with significant caveats, some of which are relaxed below.

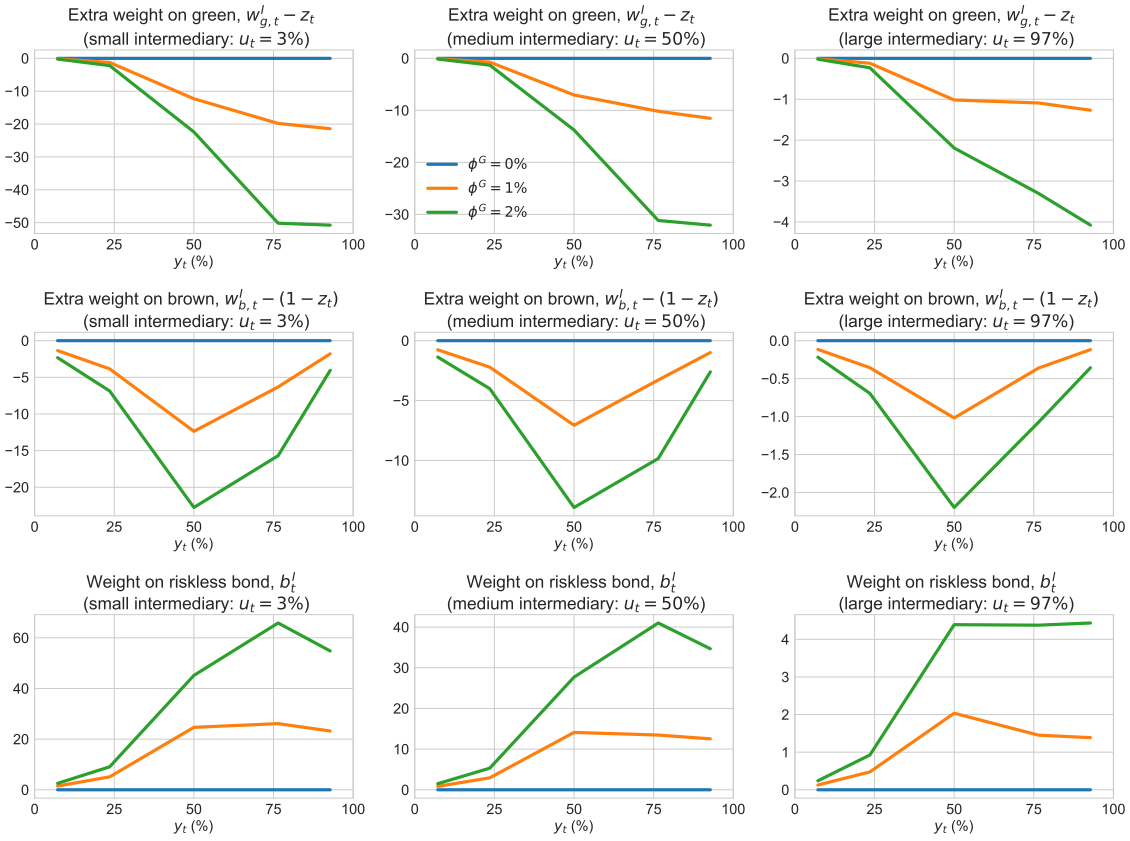
Economy with financial intermediary I now turn to the situation in which the financial intermediary is an investor in her own right, that is, she consumes, and owns a share of the economy's wealth, $u_t \equiv W_t^I/(W_t^G + W_t^N + W_t^I)$. Her wealth share ($u_t$, endogenous) therefore becomes an important state variable, in addition to the other two discussed so far: the wealth share of the green retail investor in the total retail wealth ($x_t \equiv W_t^G/(W_t^G + W_t^N)$, endogenous), and the output share of the green firm ($y_t \equiv Y_{g,t}/(Y_{g,t} + Y_{b,t})$, exogenous).

No leverage due to risk attitudes. I start with the case in which the retail investors and the financial intermediary have identical preferences, except for the green tilt. Specifically, they have identical risk aversion, $\gamma = \gamma^G = \gamma^N = \gamma^I$, and elasticity of intertemporal substitution, $\gamma = \gamma^G = \gamma^N = \gamma^I$, and no leverage emerges out of differences in risk attitudes for the time being, so that all desires to borrow and lend in the riskless asset come from the effect of the environmental tilt.

In this setting, the financial intermediary optimally sets the portfolio on total wealth, both her own and that of retail investors, in order to maximize her utility. Therefore, the portfolio that she sets and offers, $w_{g,t}^I, w_{b,t}^I$, no longer necessarily has to match the market portfolio, $z_t \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}), 1 - z_t \equiv Q_{b,t}/(Q_{g,t} + Q_{b,t})$.

Does the financial intermediary decide to deviate from the market in practice? Figure 6 (as well as Figures B.3 and B.4 in Appendix B.2), set in the same benchmark calibration as previously, shows that she does: she tilts her portfolio away from the green asset, allocating to it less than its market weight in all cases, $w_{g,t}^I < z_t$, as the environmental tilt of the green retail investor, ϕ^G, gets larger. The financial intermediary also reduces her weight on the brown asset, $w_{b,t}^I < (1 - z_t)$, but to a smaller extent. Those patterns are reminiscent of the portfolio of the neutral investor in the pure indexing case studied previously, and indeed, they occur in part for the same reason: other investors have to take the flip side of the desire of the green investor to tilt towards the green firm. This is also why these effects are stronger when the green firm dominates the economy (large y_t), like before.

Figure 6: Portfolio set and offered by the financial intermediary (%) when the green investor dominates retail wealth ($x_t = 97\%$)



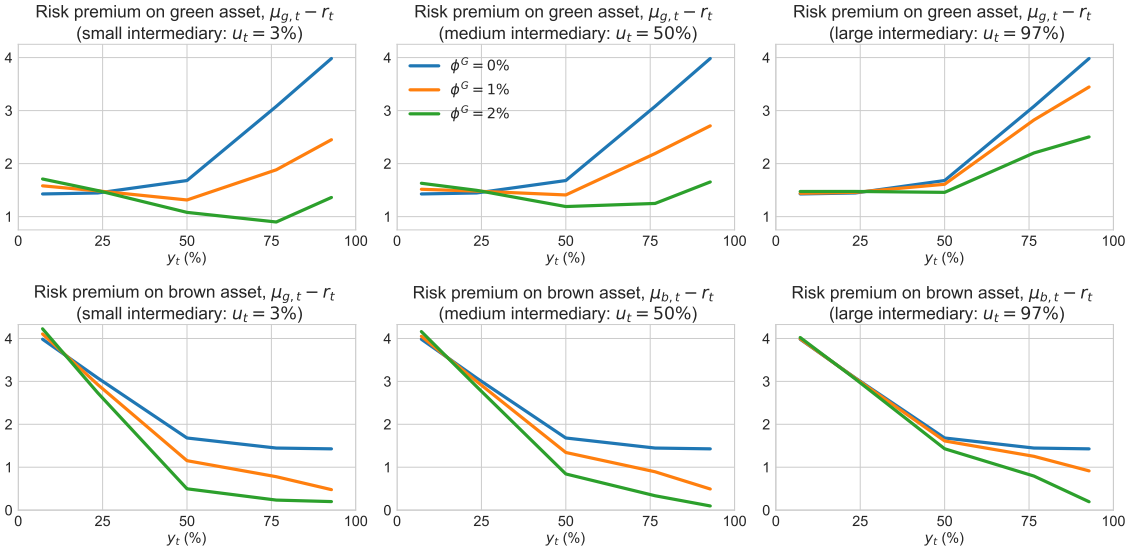
A central difference, however, is that the portfolio that the financial intermediary now sets is the only one that retail investors have access to. In other words, the intermediary replaces the pure market index from above by a portfolio that is now tilted away from the green equity, therefore working *against the green tilt*. While the neutral retail investor is entirely satisfied with this portfolio and invests all of his wealth in it, $w_{I,t} = 100\%$, the green retail investor is not, as I discuss below. In practice, the financial intermediary deviates substantially in particular when she holds a small share of wealth in the economy (low u_t): $w_{g,t} - z_t$ can reach around -50% for large y_t. In this situation, the economy is close to the case of pure indexing in the sense that other investors take a large opposite position to the green retail investor, but different in the sense that the portfolio that is offered to investors deviates significantly from the market. When the financial intermediary holds close to all the wealth in the

economy instead (large u_t), she can deviate much less given that she has to hold all assets in the market. In that case, $w_{g,t} - z_t$ only reaches around -4% at most.

Given that the portfolio set by the financial intermediary appears to be working against the green tilt, does this undo the potential impact on asset returns that arose in the pure indexing economy? In other words, can the green retail investor no longer have any ability to generate a green premium, and cheaper financing for green firms? Figure 7 shows that, once again, this does not necessarily have to be the case.

When the financial intermediary holds a relatively small share of the economy's wealth in her own right (low u_t) in particular, the environmental tilt of the green retail investor, ϕ^G, gives rise to a large decrease in the risk premium on the green asset by up to 3 percentage points, while that on the brown asset is reduced by about 1 to 2 percentage points. In other words, the (normalized) green premium can become strongly negative, as shown in Figure B.8 (as well as Figures B.9 and B.10) in Appendix B.2. This good news is consistent with our previous findings for the pure indexing economy, and also comes with similar qualifications. For instance, the green premium can be more muted, and even positive, for low green output shares, especially under recursive preferences. Most importantly, the green retail investor still very much has to resort to taking a large leveraged risky position in the portfolio offered by the intermediary, and does so by borrowing in the riskless bond. The intermediary therefore allocates part of her wealth towards lending to the green retail investor, $b_t^I = 1 - w_{g,t}^I - w_{b,t}^I > 0$ (Figure 6), as does the neutral retail investor. In fact, because the portfolio offered by the intermediary is now tilted against the green asset, the position taken by the green retail investor is even larger than in the pure indexing case. Figure B.5 in Appendix shows that $w_{I,t}^G$ can reach up to 300% of the green retail investor wealth, when the green firm dominates the economy (large y_t) and when she dominates retail wealth (large x_t), although it becomes more moderate for low x_t. *In fine*, the green retail investor invests so that the weight that she can indirectly allocate to the green asset itself in her portfolio reaches values comparable to those in the pure indexing case, but the substantial positions involved further reinforce the previous caveat: one can wonder whether retail investors would have deep enough financial market access to implement such strategies in practice.

Figure 7: Risk premia (%) on both assets, $\mu_{j,t} - r_t$, when the green investor dominates retail wealth ($x_t = 97\%$)



When the financial intermediary controls an increasingly larger amount of wealth in the economy –that is, as u_t becomes large–, the picture changes considerably. While the mechanisms described above are still at play qualitatively, they become much more muted in magnitude. For instance, Figure 7 shows that the reduction in the risk premium on the green asset is only of the order of 1.5 percentage point at most, not far from the 1-percentage-point lower risk premium on the brown asset. As a result, the (normalized) green premium is much more moderate: in most cases, it is an order of magnitude smaller compared to the situation in which the financial intermediary has a small wealth share, u_t. This comes despite the fact that the green retail investor still tilts her portfolio strongly towards the green asset, as she has more leeway to do so (Figures B.5-B.7 in Appendix). Ultimately, despite the attempts by the green retail investor, the pricing of assets has to reflect the preferences of the financial intermediary as she becomes dominant in the economy, and those do *not* include environmental tilts in this case.

In practice, the range of adequate values of the intermediary wealth share, u_t, is likely to depend on the case. On the one hand, large asset managers focusing mostly on passive and index strategies, such as BlackRock, Vanguard, Fidelity, and

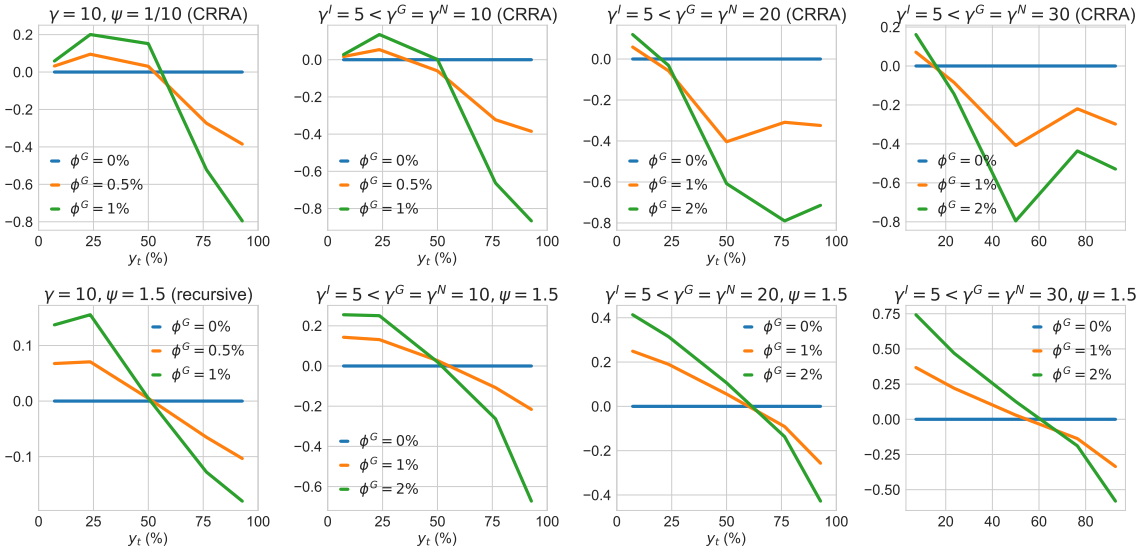
State Street, are likely to have little leverage and "wealth of their own". For those actors, a low u_t, or the pure indexing case, is probably be most realistic. On the other hand, there can be significant leverage and intermediary wealth, for instance, if the type of intermediaries under consideration are private equity, hedge funds, or other investment funds. In this instance, the wealth of general partners or of insiders can be seen as skin-in-the-game, and is consistent with a larger u_t. Even in the case of banks, or other financial institutions, the (inside) equity could be consistent with intermediary wealth, and the use of derivatives could give rise to some non-negligible amount of underlying leverage. In short, even at the aggregate level, a sizable and varying intermediary wealth share is likely to be relevant. This is consistent in spirit with the broader intermediary asset pricing literature. Overall then, I am not taking a stance on what would be the most appropriate range of values for u_t. Instead, this highlights the value of having a general and versatile framework in which intermediary wealth is determined endogenously, and in which those various cases can be investigated.

Leverage due to risk-tolerant financial intermediary. Lastly, let us consider the case in which the financial intermediary and retail investors have different attitudes towards risk. Specifically, I assume that the financial intermediary is more risk-tolerant than retail investors, $\gamma^I < \gamma^G = \gamma^N$. This assumption is consistent with the standard view that financial actors may be more willing and able to take financial risks in the economy. In this situation, provided that the difference in risk aversion is large enough, the financial intermediary is willing to borrow in the riskless bond so as to lever up her risky position. In that sense, this leverage arises from differing risk attitudes, and can allow the framework to speak more finely about specific financial players mentioned above. This flavor of leverage comes on top of any borrowing and lending due to the environmental tilt of the green retail investor that we discussed previously. Other than risk aversion, the elasticity of intertemporal substitutions are again either $\psi^I = 1/\gamma^I, \psi^G = \psi^N = 1/\gamma^G = 1/\gamma^N$ (CRRA), or $\psi^I = \psi^G = \psi^N = 1.5$ (recursive preferences).

Figure 8 (and Figures B.11-B.13 in Appendix B.2) show that, in this specification too, the environmental tilt of the green retail investor can have a significant impact on the pricing of both risky assets, and give rise to a negative green premium, in particular when the green investor dominates retail wealth (large x_t), and when

intermediary wealth is limited (low u_t). This is the good news that we have now encountered several times, and it comes with the same nuance, state-dependence, and caveats, discussed previously. The magnitude of the change in risk premia, and of the green premium, are also broadly comparable to our previous findings.

Figure 8: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various calibrations, in the case of a small green retail investor ($x_t = 3\%$)



Interestingly, although the impact on risk premia and green premium is similar, the underlying picture looks somewhat different. Figure shows that getting there only requires the green investor to allocate about $w^G_{I,t} = 30$ to 40% of her wealth in the portfolio offered by the intermediary, while she invests the remaining 60 to 70% of her wealth in the riskless bond in her own name.[24] This is a far cry from previous cases in which the green retail investor took large risky positions in the intermediary portfolio ($w^G_{I,t}$) by borrowing heavily in the riskless bond. This result emerges because the desire to borrow on the part of the green retail investor –stemming from her environmental tilt– is more than dominated by the desire to borrow from the financial intermediary –stemming from the fact that the intermediary is more risk-tolerant.

[24] In practice, the total amount of wealth that retail investors, $i \in \{G, N\}$, have invested in the bond is lower than $1 - w^i_{I,t}$, because their $w^i_{I,t}$ invested in the intermediary portfolio corresponds to $w^i_{I,t}(1 - w^I_{g,t} - w^I_{b,t})$ in the bond via the financial intermediary. The latter is negative because the intermediary borrows to leverage up in this case.

To be sure, the allocation of the green investor to the intermediary portfolio is still strongly state-dependent, and the above does not solve all, or most, caveats. However, such average values invested in risky assets via the intermediary are more consistent with the type of estimates found empirically, which is good news because they emerge from a case –risk-tolerant and levered financial intermediary– that appears reasonably plausible.

The introduction of difference in risk attitudes also has interesting implication for the pricing of the intermediary portfolio itself, $\mu_{I,t} - r_t \equiv w^I_{g,t}(\mu_{g,t} - r_t) + w^I_{b,t}(\mu_{b,t} - r_t)$. Namely, even though the average level of risk premia on each asset are determined mostly by risk aversion, γ^i (and the environmental tilt), the risk premium on the intermediary portfolio can be sizably larger, even for reasonable γ^i.[25] For instance, for $\gamma^I = 5 < \gamma^G = \gamma^N = 10$, $\mu_{g,t} - r_t$ and $\mu_{b,t} - r_t$ hover around 1% for low u_t, while $\mu_{I,t} - r_t$ is around 2% (Figure). Similarly, for $\gamma^I = 5 < \gamma^G = \gamma^N = 20$, $\mu_{g,t} - r_t$ and $\mu_{b,t} - r_t$ hover around 1% to 3% for low u_t, while $\mu_{I,t} - r_t$ is around 6-7%. This comes, again, from the fact that the financial intermediary is levered, and can help reach values of for market returns that are close to empirical estimates. As such, this can be seen as an justification or an endogenous version of the exogenous assumption that the risky asset is levered claim on consumption, as is often assumed in asset pricing. More generally, the framework also allows us to study the potential impact of environmental tilts on the market: expected market returns could very well decrease if green retail investor control enough wealth.

Summary Taken together, so far, those findings paint a nuanced picture of sustainable investing in a world with financial intermediaries, when those intermediaries do any tilt beyond financial considerations. On the one hand, green retail investors can still exert significant influence on the pricing of equity assets, even when they do not trade them separately. In turn, this can under some conditions give rise to a sizeable negative green premium, that is, a lower cost of financing for green firms. This is the good, and perhaps surprising, news: this could help finance the green transition, as they desire. On the other hand, those results come with significant qualifications.

[25] Note that the introduction of portfolio constraints as in Section 5.3 could further raise risk premia without resorting to relatively high degrees of risk aversion. This is very much in the spirit of the intermediary asset pricing literature, such as He and Krishnamurthy (2013) and Adrian et al. (2014), among others.

For instance, green retail investors must take large leverage risky positions in order to have a substantial impact (although difference in risk attitudes can partly alleviate this concern), their investing strategy is inherently state-dependent, and they must be able to observe the asset weights in the index or portfolio in which they invest in the first place. The exact specification of preferences, and economic conditions, can also have a large effect on the result, and when financial intermediaries hold a large share of the economy's wealth, the impact of any tilt on the part of retail investors become mostly negligible. I generalize the model along several dimensions to further investigate some of those issues in Section 5 (ongoing). First, however, let us turn to the case in which the financial intermediary starts considering environmental tilts of her own.

4.2 Financial intermediary with green tilt

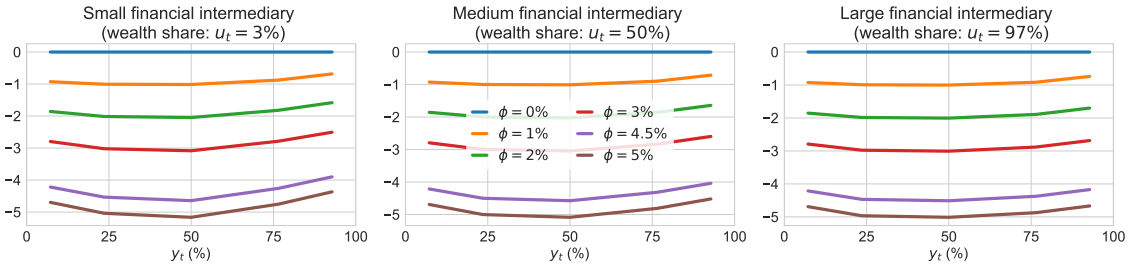
The second central question is, then: what happens when financial intermediaries do tilt towards green assets? Although recent developments and estimates (Pástor et al., 2024) suggest that this might not be the case currently, financial institutions may decide to introduce tilts in the future because of their own values and preferences, in order to attract funds from clients who might put more weights on such considerations, or because they are made to by regulators.[26] I therefore turn to introducing an environmental tilt on the part of the financial intermediary in the economy. Being able to do so, and to study the equilibrium impact on asset returns and various variables, is part of the appeal of the framework, especially for those cases that are not necessarily fully observed empirically as of yet.

The core result is that the impact of the environmental tilt of the financial intermediary is substantial. Although the green retail investor was able to have some influence when the intermediary was focused purely on financial considerations, it is completely dwarfed by the power of having a green intermediary, and the green premium can become substantially more negative. Figure 9 shows the results in the situation in which the financial intermediary tilts as much as the green retail investor, $\phi = \phi^G = \phi^I$: the green premium, which I normalize again to the case in which there

[26] Although the introduction of regulation fostering investment towards environmental and other non-financial considerations is unlikely in the United States for the time being, such possibilities are very much part of the options being actively discussed in the United Kingdom, the European Union, and other parts of the world.

is no tilt, moves broadly one-for-one with the green tilt, and can therefore easily be as large as -1 to -5%, if ϕ takes such values. In fact, the effect is so strong that for the green asset, the risk premium itself, $\mu_{g,t} - r_t$, can turn negative for most of the state space for large tilts. Interestingly, the large negative green premium also arises under recursive preferences, as shown in Figure B.14 in Appendix B.3, contrary to the situation in which the intermediary did not account for environmental considerations (Section 4.1).

Figure 9: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, when the financial intermediary tilts as much as the green investor $\phi = \phi^G = \phi^I$, for various ϕ, in the case of a large green retail investor ($x_t = 97\%$) and CRRA preferences ($\gamma = 20, \psi = 1/\gamma$)



The large influence of the intermediary green tilt comes in part because she can trade both assets separately and is therefore able to directly impact their pricing, but in particular because she decides on the portfolio for all investors in the economy. As such, she can leverage the wealth of all investors, not just her own, in order to foster favorable financing –that is, a lower cost of capital– for the green firm. This arises despite the fact that the neutral investor attempt to undo some of this tilt in his investment by reducing his position in the intermediary portfolio especially when the green firm dominates in the economy, in a pattern mirroring that of green investors previously.

Figure 10 shows that this effect can be somewhat strong when no retail investors tilt, $\phi^G = \phi^N = 0$: when the intermediary tilt , ϕ^I, is very large, retail investors dislike the portfolio that she offers so much that they deviate from it, which leads the green premium to spike back up, compared to the case in which the financial intermediary focuses purely on financial considerations.

This effect, however, arises mostly for large values of the green output share, y_t,

and when the intermediary tilt is substantial ($\phi^I > 3$ or 4%), which might not be very realistic as of now. It also dissipates as the financial intermediary holds more wealth in the economy (large u_t). In addition, as soon as the green retail investor has a tilt of her own, $\phi^G > 0$, which has been our main baseline throughout the paper, this counterproductive effect on the green premium recedes substantially provided that the green retail investor is large enough (x_t). In that baseline case, the green premium remains negative for any y_t, although with some variations (Figure 11).[27]

Figure 10: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various values of the intermediary tilt, ϕ^I, when retail investors do not tilt, $\phi^G = 0$ (therefore, x_t irrelevant), in the case of CRRA preferences ($\gamma = 20, \psi = 1/\gamma$)

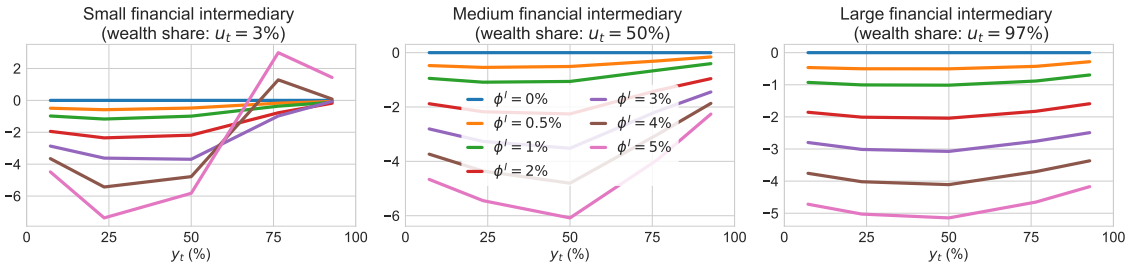
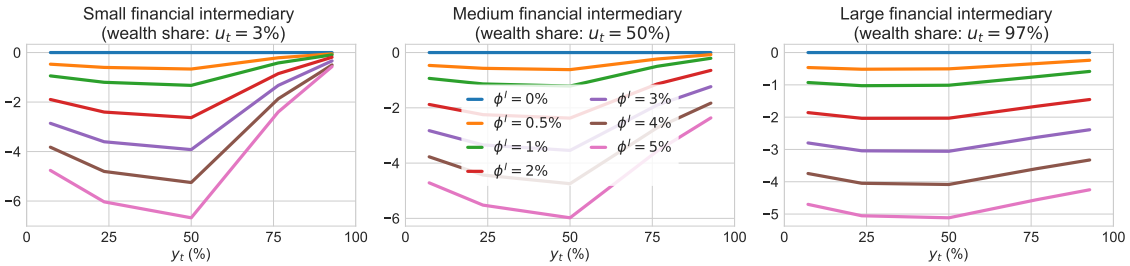


Figure 11: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various values of the intermediary tilt, ϕ^I, with green retail tilt, $\phi^G = 2\%$, in the case of a large green retail investor ($x_t = 97\%$) and CRRA preferences ($\gamma = 20, \psi = 1/\gamma$)



[27] Interestingly, for a non-zero but moderate wealth share of the green retail investor, x_t, the green premium can mildly rise zero above for low ϕ^I, even when the green firm produces a moderate share of output, y_t. This is shown in Figure, and highlights that some interesting underlying non-linearities, although the effect remains modest in magnitude, and by far dominated by the decrease in green premium coming from intermediary tilt.

Taken together, those results highlights that when the financial intermediary decides to tilt towards the green asset for reasons beyond pure financial considerations, the impact can be substantial. To be sure, there are interesting nuances, as we discussed, and some additional findings could emerge in the situation in which risk attitudes differ and leverage arises (ongoing). However, in summary, a tilt on the part of the intermediary can lead to a sizably lower cost of capital for green firms in most cases, even when retail investors have a moderate tilt. This emphasizes both the power and central role of financial intermediaries in the financing of the green transition, but also that this mechanism functions best when the preference of retail investors and intermediaries are aligned.

5 Generalizations

5.1 When retail investors do not know the index composition

5.2 Greenwashing

5.3 Portfolio constraints

6 Conclusion

In this paper, I study sustainable asset pricing in the presence of financial intermediaries. This issue in central given the vast amount of wealth managed by large asset managers, such as BlackRock, Vanguard, Fidelity, and State Street, and other financial institutions, and in light of the recent debate on whether these actors should participate more actively in the green transition or should focus narrowly on their fiduciary duty. Yet, these and related questions have received relatively little attention in the literature. To tackle them, I build an equilibrium intermediary asset pricing model with three investors, two risky assets, and a riskless bond. Specifically, two heterogeneous retail investors invest via a financial intermediary who decides on the portfolio allocation that she offers between a green and a brown equity. Both retail investors and the financial intermediary can tilt towards the green asset, beyond

pure financial considerations.[28]

Perhaps surprisingly, the green retail investor can have substantial impact on the pricing of green assets, even when she invests via an intermediary who does not tilt: a sizable negative green premium –that is, a lower cost of capital– can emerge on the equity of the green firm. This good news comes with important qualifications, however: the green retail investor has to take large leveraged positions in the portfolio offered by the intermediary, her strategy must be inherently state-dependent, and economic conditions or the specification of preferences can overturn or limit the result.

When the financial intermediary tilts instead, the impact on the green premium can be substantially larger. This comes in part because the intermediary can trade both assets separately and is therefore able to directly impact their pricing, but in particular because she decides on the portfolio for all investors in the economy. As such, she can leverage the wealth of all investors, not just her own, in order to foster favorable financing for the green firm. In fact, the effect of an environmental tilt on the part of the intermediary is so strong that it mostly persists even when retail investor have limited tilt of their own, although the effect is largest when the preferences of retail investors and intermediary are aligned. This result could help alleviate some of the concern that the cost of capital channel of sustainable investing is too small to have a real impact as of yet (Berk and van Binsbergen, 2021, De Angelis et al., 2022).

Taken together, these findings highlight the central and unique role that financial intermediaries can play in channeling (or not) financing towards the green transition and other important questions. While as of now, such tilts appear to be limited in scope in practice (Pástor et al., 2024), the results suggest that financial institutions could be a key and powerful actor to solve the challenges of our time, if they start accounting for them. In fact, whether there is any hope to reach such solutions without the financial sector playing a role, given the trillions of dollars that it manages, is perhaps the real question. This suggests that regulatory actions could have considerable effects, and can probably explain the heated political and regulatory debate on those questions that has emerged in recent years.

These and related questions represent interesting avenues for further research, and the framework could be extended and applied in several other contexts.

[28] I also study what happens when the green retail investor does not know the weights in the portfolio offered by the intermediary, the potential impact of greenwashing, and the effect of portfolio constraints. Cf. Section 5 (ongoing).

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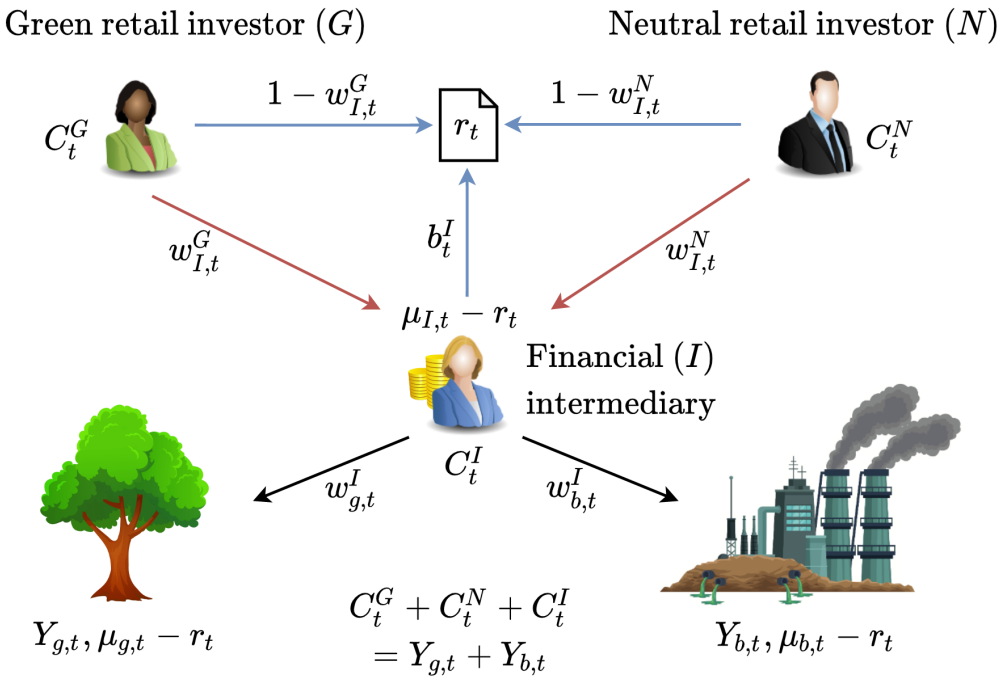
Appendix

A Additional theoretical results

B Additional figures

B.1 Economic setup

Figure B.1: Economic setup

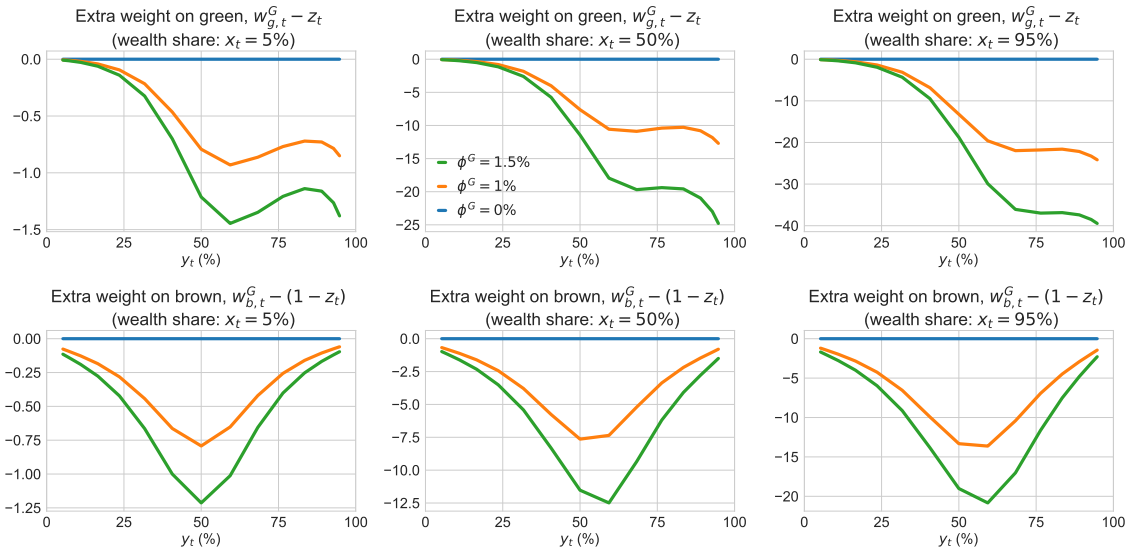


Source: Vecteezy.com. Back to main text: Section 2.

B.2 Financial intermediary with no green preferences

B.2.1 Economy with pure indexing

Figure B.2: Portfolio of the neutral (retail) investor (%)



B.2.2 Economy with financial intermediary: no leverage due to risk attitudes

Figure B.3: Portfolio set and offered by the financial intermediary (%) for small green retail investor ($x_t = 3\%$)

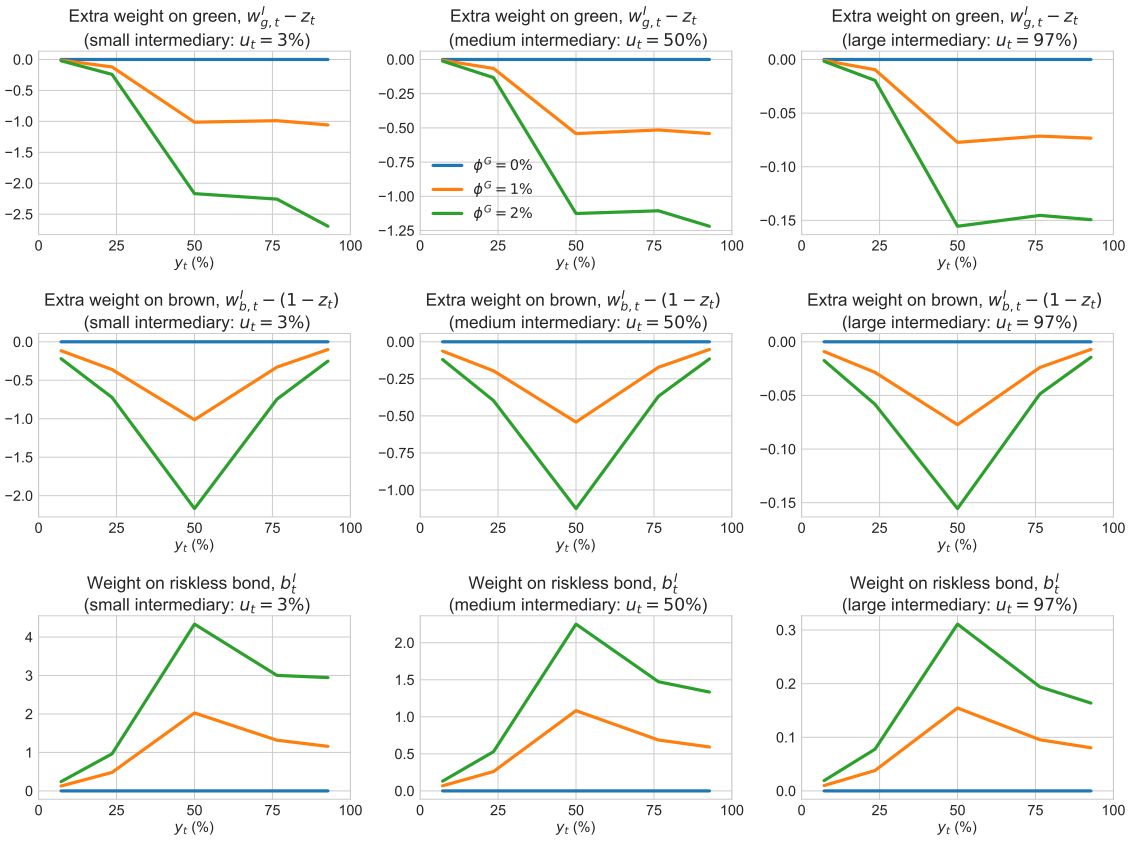


Figure B.4: Portfolio set and offered by the financial intermediary (%) for medium green retail investor ($x_t = 50\%$)

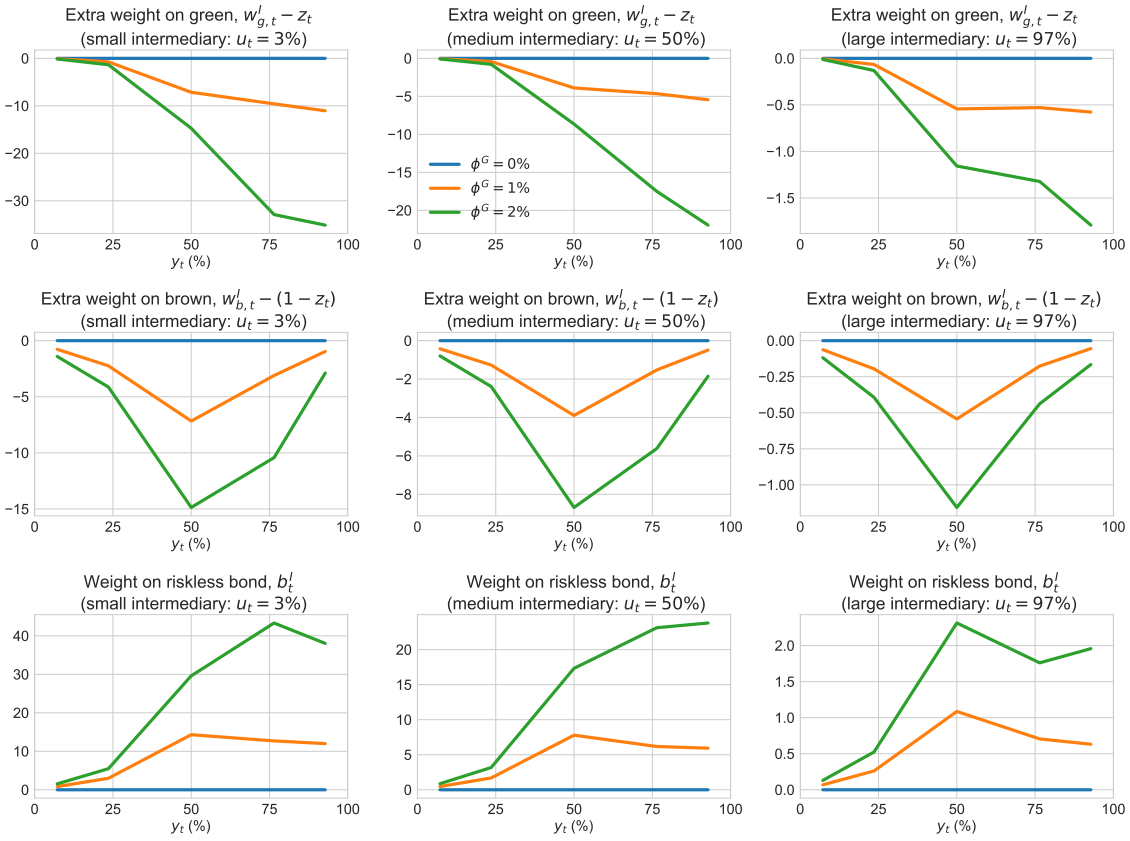


Figure B.5: Portfolio of the green retail investor (%) for small green retail investor ($x_t = 3\%$)

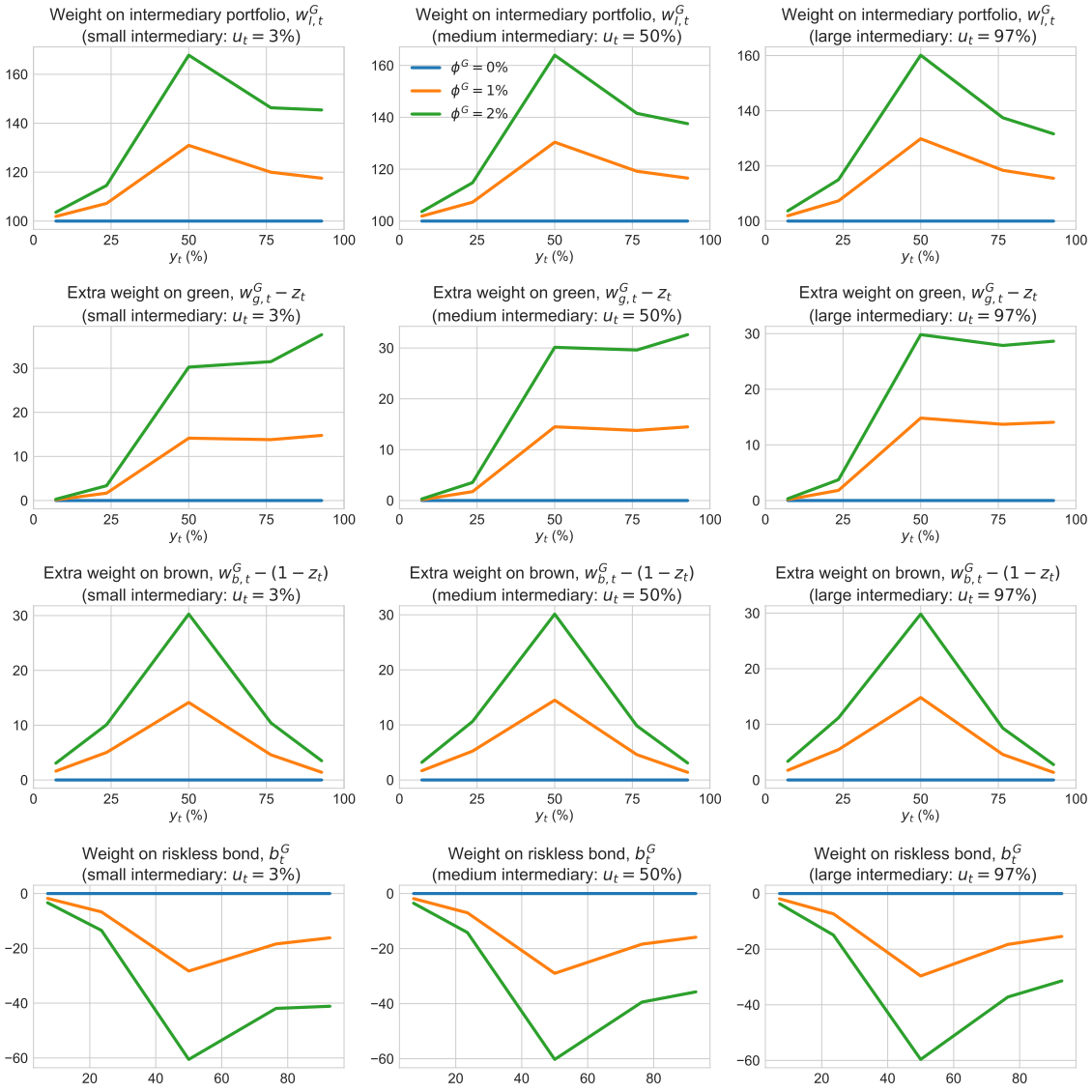


Figure B.6: Portfolio of the green retail investor (%) for medium green retail investor ($x_t = 50\%$)

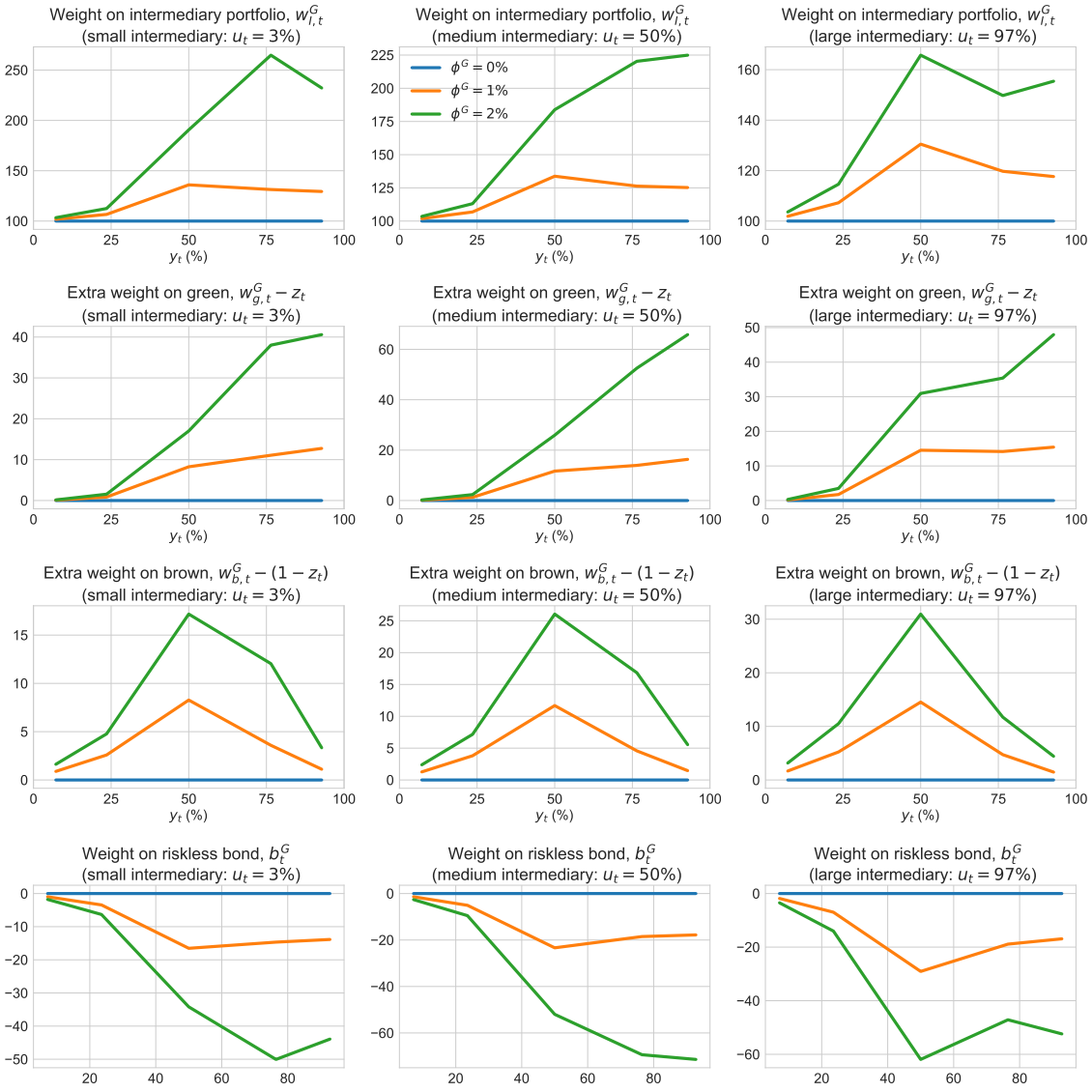


Figure B.7: Portfolio of the green retail investor (%) for large green retail investor ($x_t = 97\%$)

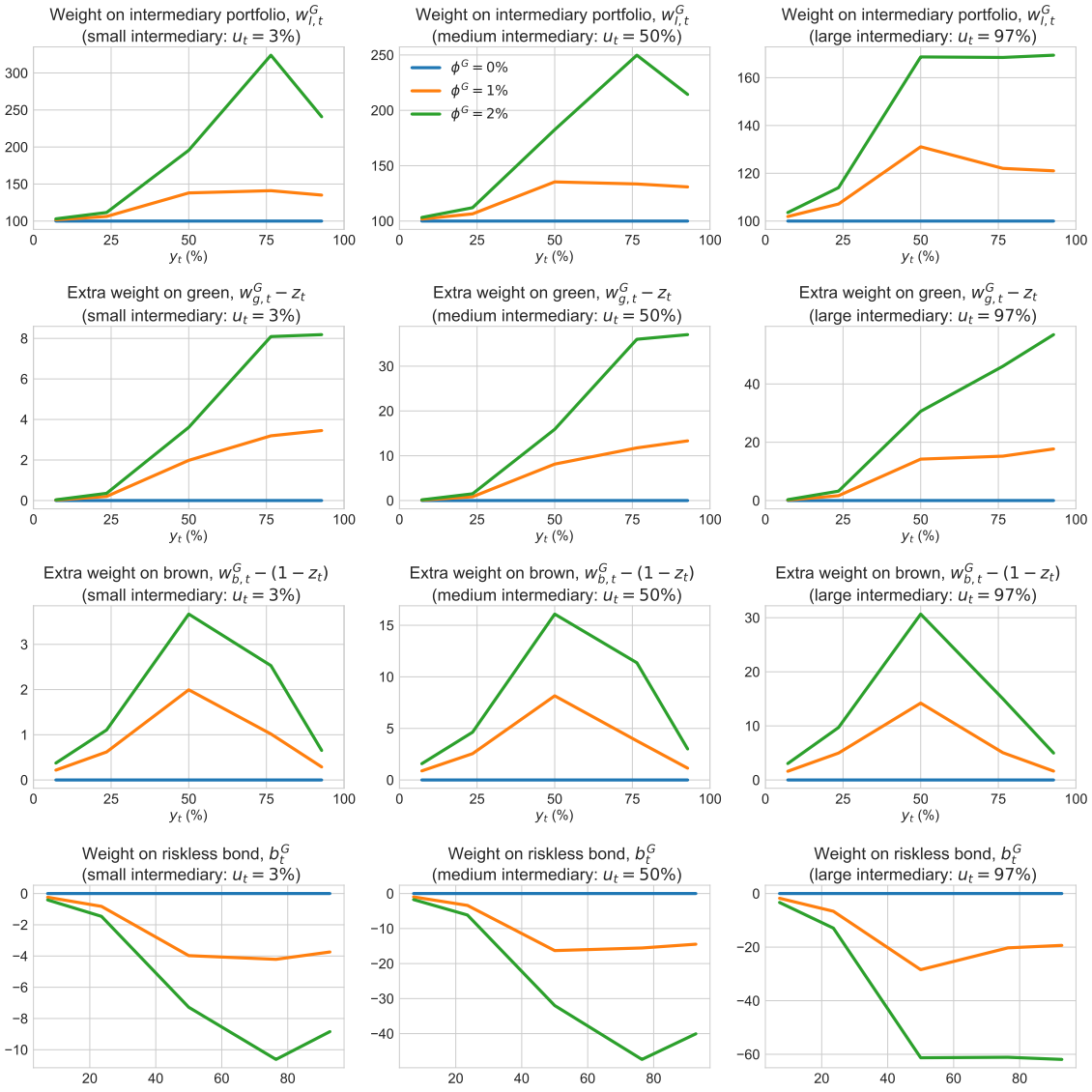


Figure B.8: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various calibrations, in the case of a small green retail investor ($x_t = 3\%$)

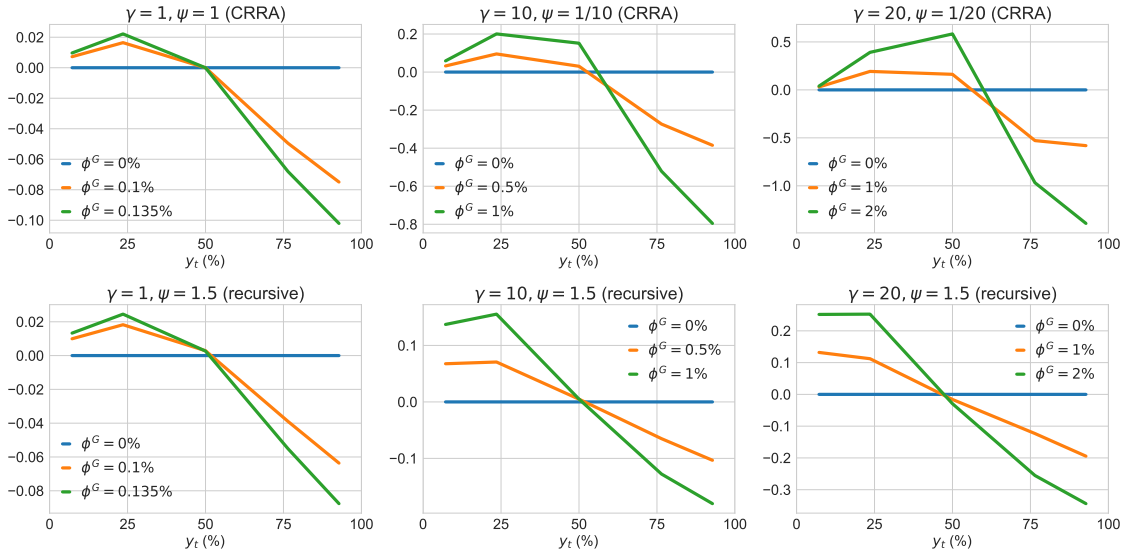


Figure B.9: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various calibrations, in the case of a medium green retail investor ($x_t = 50\%$)

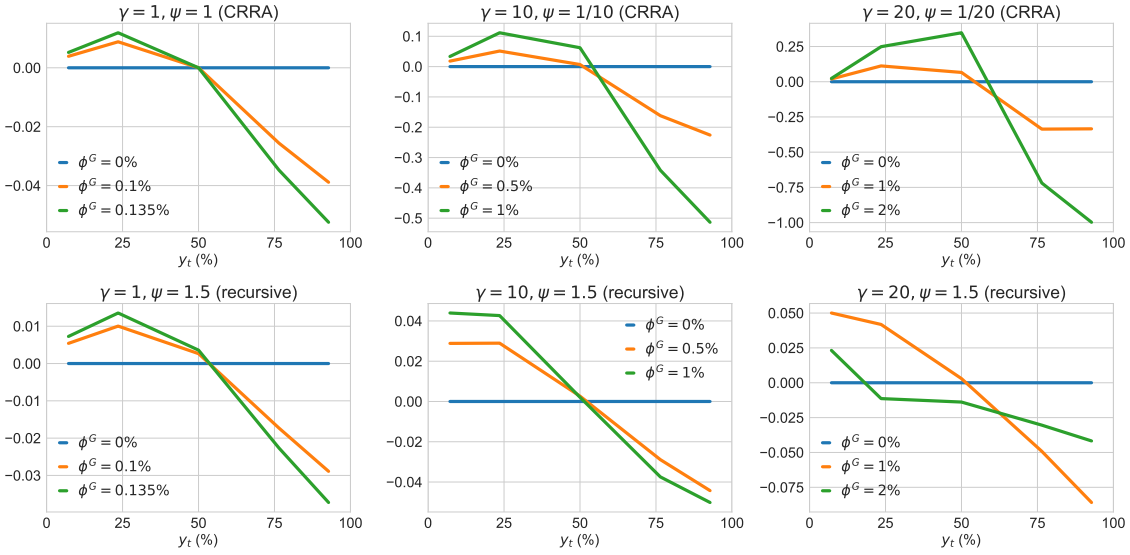
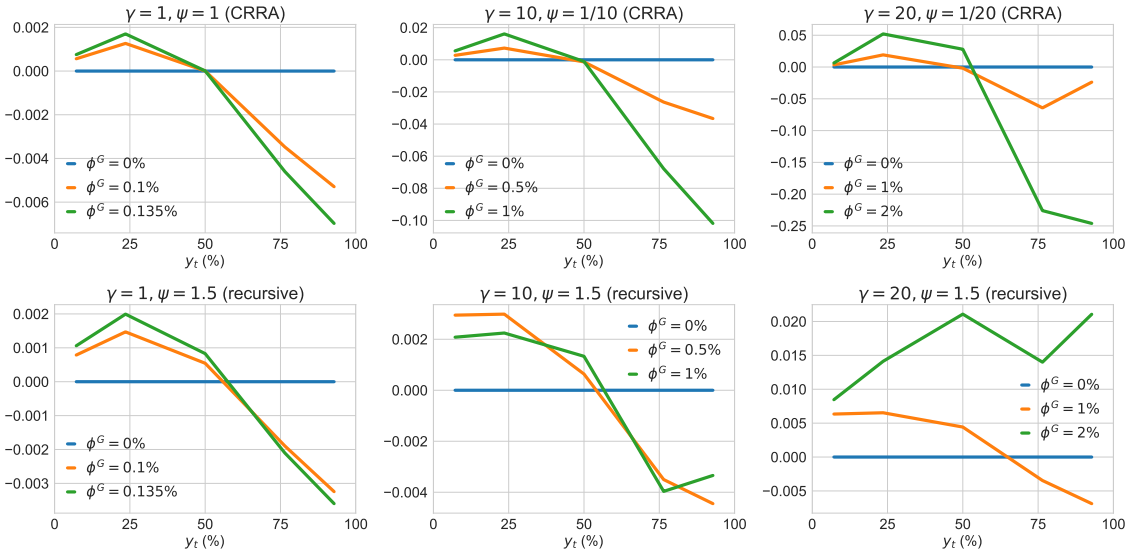


Figure B.10: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various calibrations, in the case of a large green retail investor ($x_t = 97\%$)



B.2.3 Economy with financial intermediary: leverage due to risk-tolerant intermediary

Figure B.11: Risk premia (%) on both assets with risk-tolerant financial intermediary ($\gamma^I = 5 < \gamma^G = \gamma^N = 20$), when the green investor dominates retail wealth ($x_t = 97\%$)

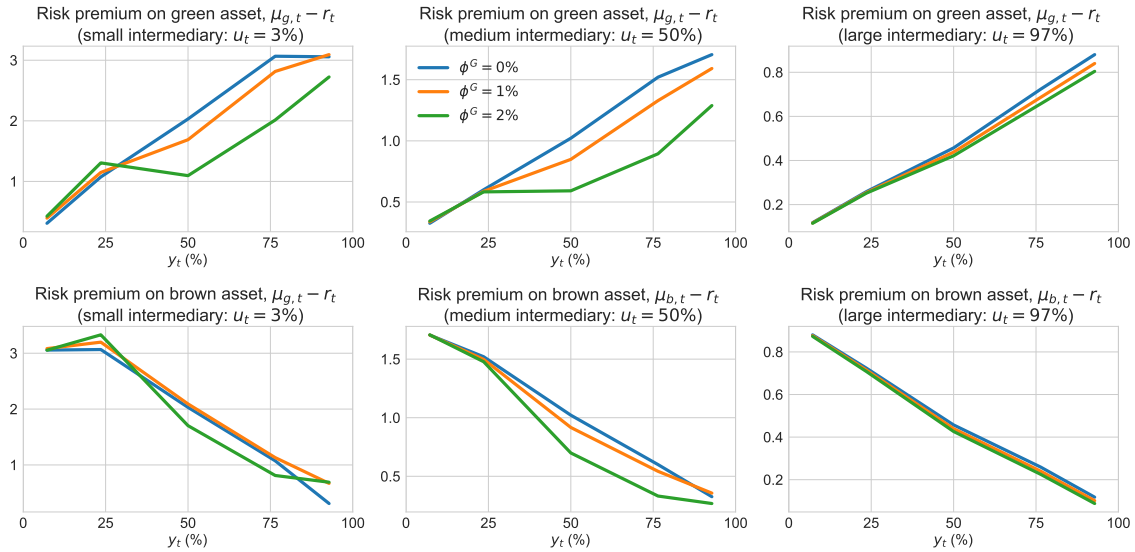


Figure B.12: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various calibrations, in the case of a medium green retail investor ($x_t = 50\%$)

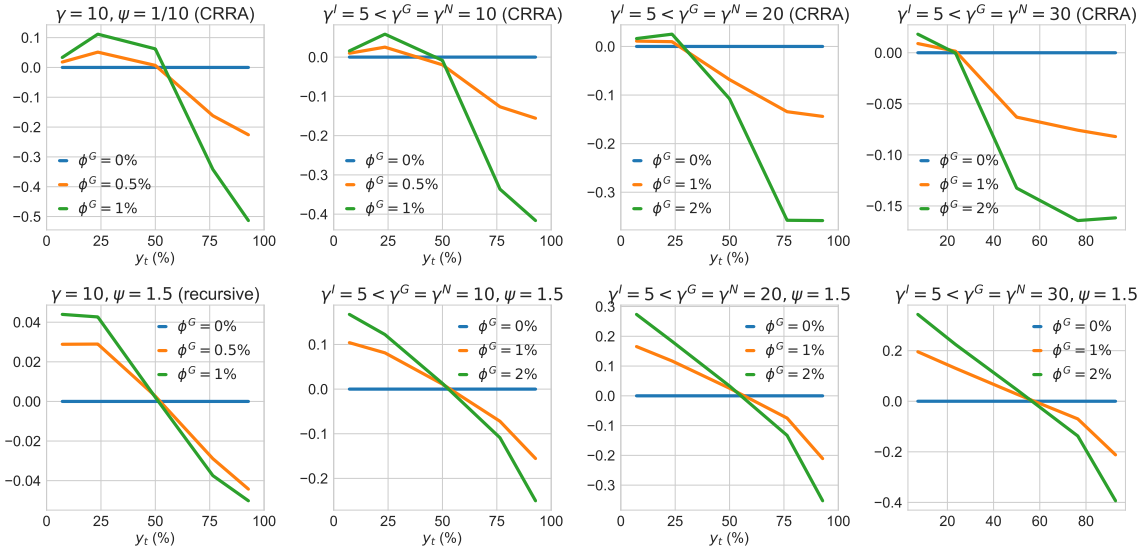
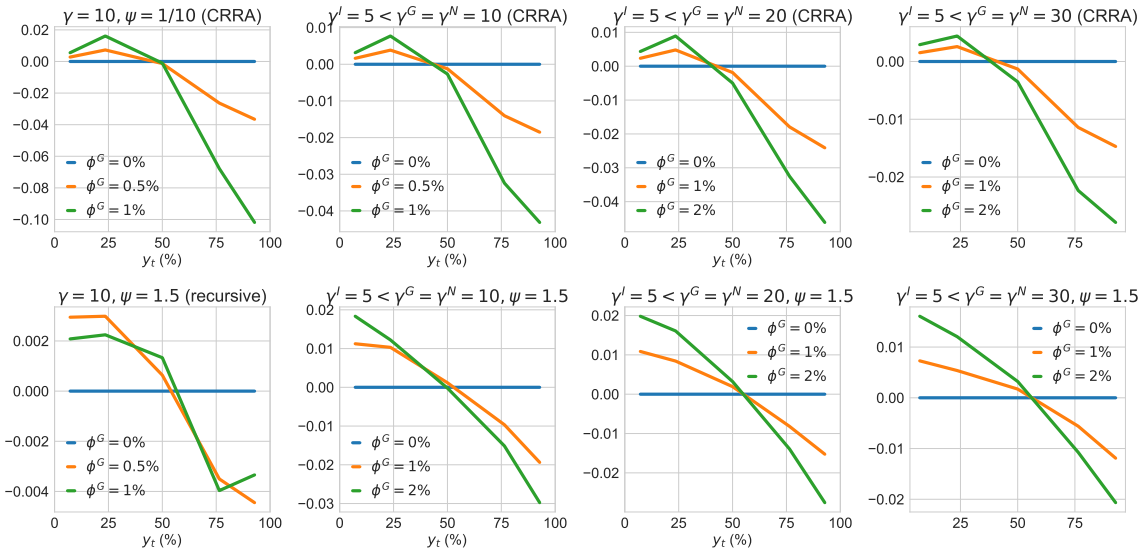


Figure B.13: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, for various calibrations, in the case of a large green retail investor ($x_t = 97\%$)



B.3 Financial intermediary with green tilt

Figure B.14: Green premium (%), $\mu_{g,t} - \mu_{b,t}$, normalized by baseline with no green tilt, when the financial intermediary tilts as much as the green investor $\phi = \phi^G = \phi^I$, for various ϕ, in the case of a large green retail investor ($x_t = 97\%$) and recursive preferences ($\gamma = 20, \psi = 1.5$)