

## Overview

### Motivation:

- Index funds offer diversification benefits to investors but typically lack significant stakes in their portfolio firms.
- In contrast, active funds can hold substantial shares, influencing portfolio firms and potentially providing externalities, at the cost of diversification.

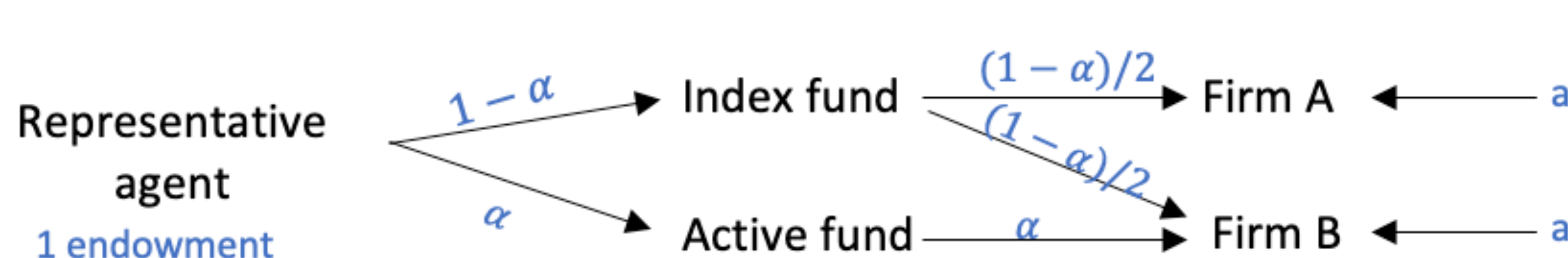
### Research Questions:

- How do investors trade-off between diversification and concentration for influence?
- How do active funds determine their degree of activism?
- Can investors collectively achieve a welfare improving outcome?

### Main Results:

- When investor is large, he holds just enough shares through active fund to influence portfolio firms.
- When investors are dispersed, active fund acts as a coordination device to offer contract for heterogeneous agents.

## Model Setup



### Firms:

- Two firms A and B, random production technology negatively correlated:
  - With prob  $p$ ,  $z_A = z + \sigma$  and  $z_B = z - \sigma$ ; with prob  $1 - p$ ,  $z_B = z + \sigma$  and  $z_A = z - \sigma$ .
- Profits  $\pi_j = z_j I_j$ ,  $j = A, B$ . Firm  $j$  receives investment  $I_j$ .
- Firm B can generate externalities:
  - at a cost of  $c$  from the profit  $\pi_B$
  - externalities happens with probability  $\theta \sim U[0,1]$ .

### Key features - Externalities:

- Public good  $H(c)$ : everyone benefits once its in place.
  - $H(0) = 0, H'(c) > 0, H''(c) < 0$
- Private good  $h(\alpha, c, \chi)$ : only active fund investor can enjoy.
  - "Worm-glow" utility, extra financial return..

### Funds:

- **Active fund:** invests all (total  $\alpha$ ) in the firm that can generate externalities (firm B).
  - has influence over firm B if enough investment received (holding  $> k$  fraction of firm B);
  - offers degree of activism  $c$  at a monitoring cost  $\Omega(c)$ .
  - Payoff  $\alpha R - \Omega(c)$ , per unit investment fee  $R$ .
- **Index fund:** invests (total  $1 - \alpha$ ) equally in both firm A and B.

### Agents:

- 1 continuum, each with 1 endowment, utility function  $U_i(c, \alpha, \eta_i) = E[\Pi_i(\alpha)] - \frac{\gamma}{2} \text{var}(\Pi_i(\alpha)) + T_i(c, \alpha, \eta_i)$ 
  - $\Pi_i(\alpha)$  is the return on portfolio,  $V_i(c, \alpha, \eta_i)$  is utility from externalities.
  - $T_i(c, \alpha, \eta_i) = \theta [H(c) + h(\alpha, c, \chi)]$
- Receives private signal  $\theta_i = \theta + \epsilon_i$ ,  $\epsilon_i \sim U[0,1]$ , of the likelihood of externalities.
- External investors with money  $a < 1$  in both A and B.

### Timing:

- T=1, active fund decides degree of activism  $c$ ;
- T=2, investors make investment decisions;
- T=3, payoffs realize.

## One Investor Equilibrium

### Lemma

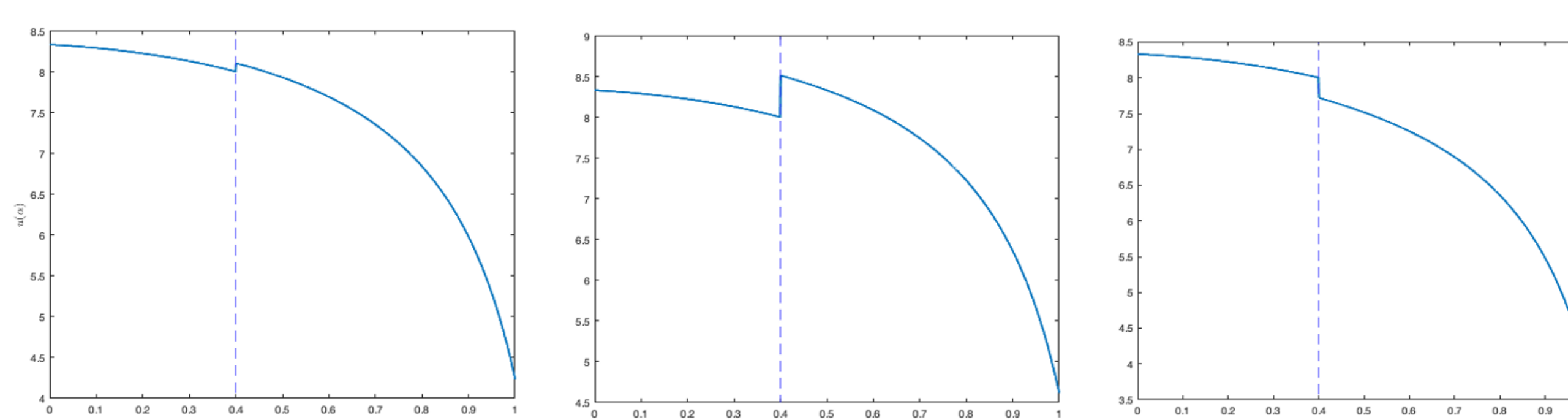
Equilibrium active fund size,  $\alpha$ , can only takes two values.  $\alpha^* = \frac{(2a+1)k}{2-k}$  if

$$\underbrace{\theta H(c) + \chi}_{\text{externality gains}} \geq \underbrace{\left[ \frac{a+1}{3a-ak+1} \right] c}_{\text{Expected profit loss}} + \underbrace{\frac{\nu \sigma^2}{2} \left( \frac{2ak+k}{2-k} \right)^2}_{\text{Volatility loss}} + \underbrace{\frac{2ak+k}{2-k} R}_{\text{Fund fees}}$$

Otherwise,  $\alpha^* = 0$ .

### T=2:

- Investor's utility decreases when deviates away from index fund, as index fund provides the best mean-variance portfolio.
- Active fund investment gives jump (up or down) to utility once externalities are in place.
- Depending on the degree of activism from active fund, investing with active fund may increase or decrease investor's utility.
- Investor either invests fully with index fund or holds just enough shares to have control of firm B.



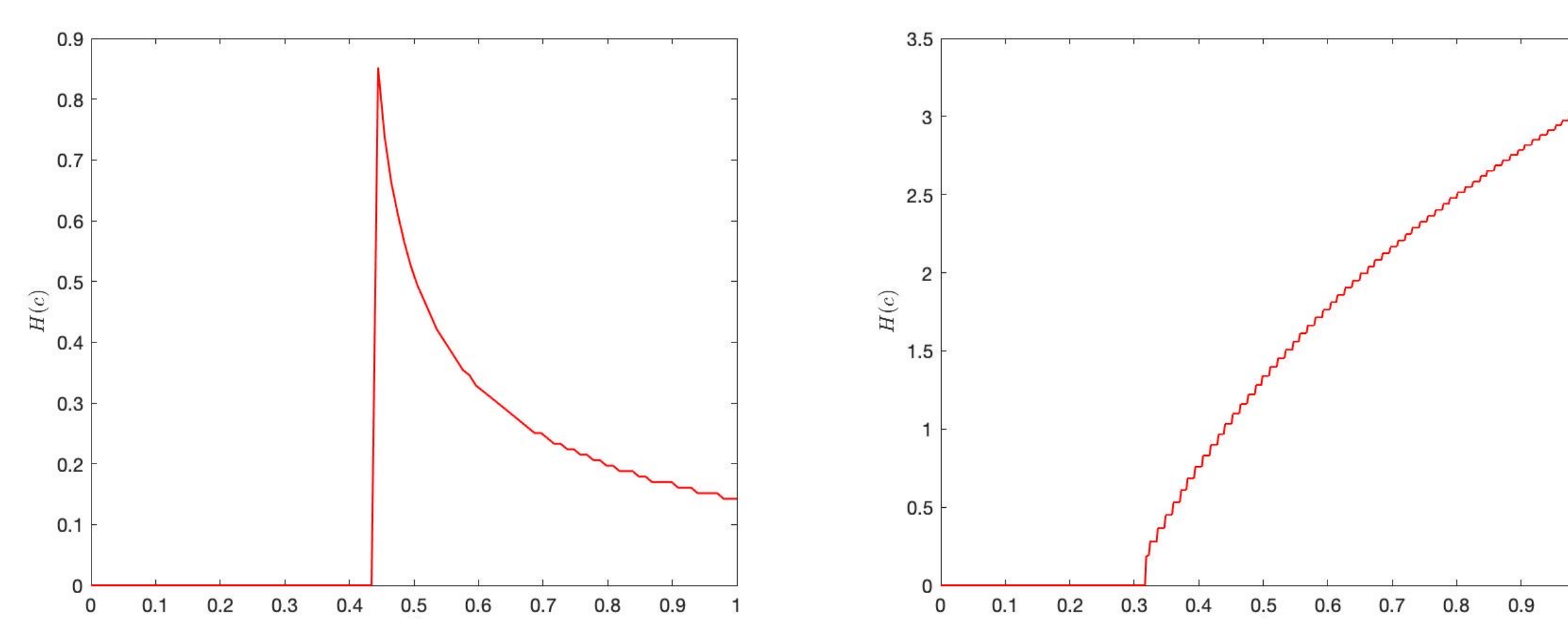
### T=1:

- Monitoring cost may reduce incentive for active fund to provide the degree of activism at the first-best level for the investor.
- Instead, active fund picks lowest possible  $c$  that makes investor indifferent between investing with active and index fund.
- Active fund size depending on how easy it is to influence firm B.
- Define a set  $F$  as all possible  $c$  that makes  $\alpha > 0$ .

### Lemma

The equilibrium are characterised as follows:

- If  $F \neq \emptyset$  and  $\min\{F\} > 0$ ,  $c^* = \min\{F\}$  and  $\alpha^* = \frac{(2a+1)k}{2-k}$
- Otherwise,  $c^* = 0$  and  $\alpha^* = 0$ .



### Proposition

When active fund size is large enough, the level of public good decreases when the likelihood of externalities increases.

### Key intuitions:

- Active fund only provides the degree of activism that makes investor indifferent between index and active fund.
- Increasing likelihood of the externalities meaning less externalities required to make investor indifferent.
- In a second-best world where social planner considers payoffs of both investor and active fund, the equilibrium level of public good increases with the likelihood.
- Always not enough externalities provided comparing to the social planner's level.

## Dispersed Investors Equilibrium

### T=2:

- Pay off function of dispersed investor follows:

	$\alpha < \frac{2ak+k}{2-k}$	$1 > \alpha \geq \frac{2ak+k}{2-k}$
Activists fund ( $\eta = 1$ )	$z - \frac{\nu}{2} \sigma^2 - R$	$z - \frac{c}{a+0.5(1+\alpha)} - \frac{\nu}{2} \sigma^2 - R + \theta_i [H(c) + h(c, \alpha, \chi)]$
Index fund ( $\eta = 0$ )	$z$	$z - \frac{c}{a+0.5(1+\alpha)} + \theta_i H(c)$

- Depending on the investment decision and the equilibrium size of active fund, the payoff for investor differs.
- Denote  $u(\eta, \alpha, x)$  is the payoff for investor choosing action  $\eta$ , give active fund size  $\alpha$  and private signal  $x$ .
- He compares his payoff between investing with active fund and index fund:

$$V(\alpha, x) = u(1, \alpha, x) - u(0, \alpha, x) = \begin{cases} -\frac{\nu \delta^2}{2} - R & \text{if } \alpha < \frac{2ak+k}{2-k} \\ -\frac{\nu \delta^2}{2} - R + xh(c, \alpha) & \text{if } \alpha \geq \frac{2ak+k}{2-k} \end{cases}$$

- He invests with active fund if  $V(\alpha, x) \geq 0$ .
- As investor's payoff from investing in active fund increases with more people doing so, the investor faces strategic complementarity.
- At the same time, the better the signal an investor receives, the more likely he would invest with active fund.
- Look for monotone Bayesian Nash equilibrium.

### Proposition

There exists a cut-off  $\theta^*$  such that for all investor  $i$  with  $\theta_i \geq \theta^*$ ,  $\eta_i = 1$  and  $\eta_i = 0$  otherwise, where

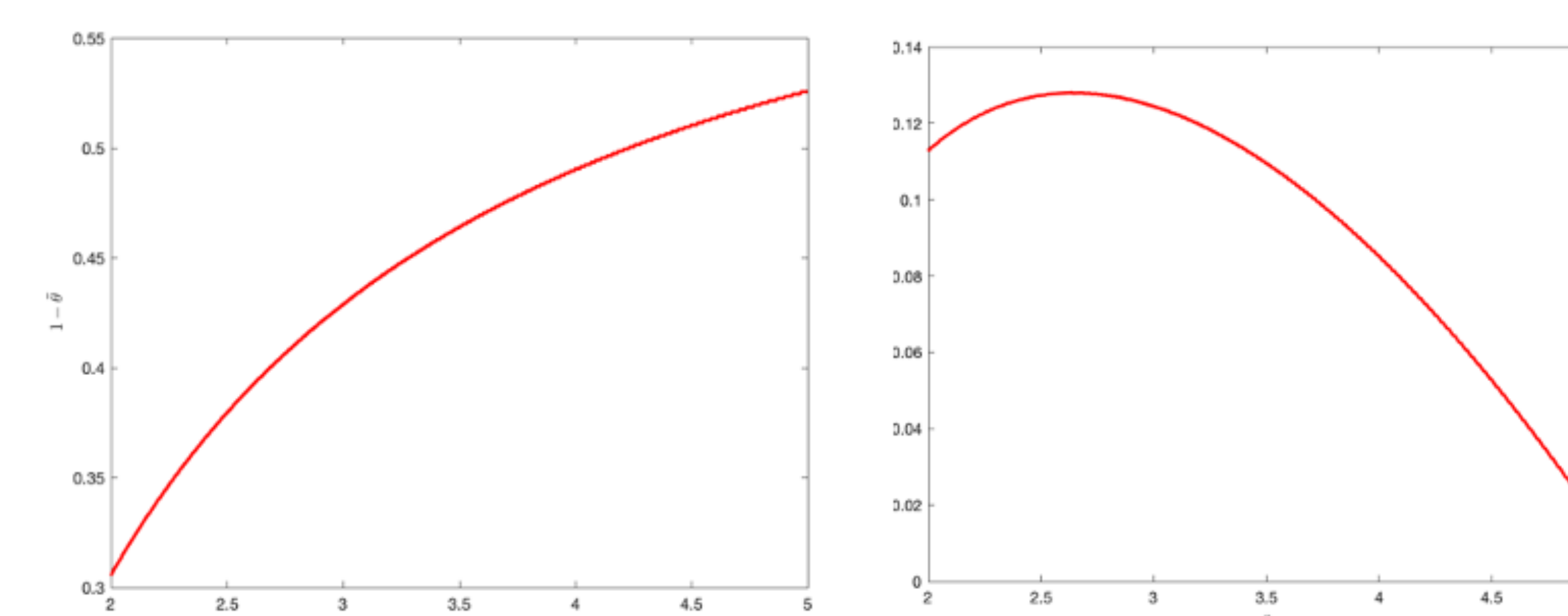
$$\theta^* = \frac{1}{h(c^*, \alpha)} \left( \frac{\nu \delta^2}{2} + R \right) \frac{2-k}{2-2k-2ak}$$

Active fund chooses:

$$c^* = \arg \max_c (1 - \theta^*) R - \Omega(c)$$

### Key Intuitions:

- As the degree of activism ( $c$ ) increases, the expected payoff from investing with active fund also increases, and the expected active fund size increases as well.
- This creates incentive for active fund to provide higher degree of activism, getting closer to the first-best level for investors.
- The active fund always stays in the market ( $\alpha > 0$ ), and the size decreases with the volatility ( $\delta$ ), fund fee ( $R$ ), control threshold ( $k$ ) and outside investors ( $a$ ).



### T=1:

- Monitoring cost gives concavity of the payoff of active fund.
- Instead of picking the lowest possible level of  $c$ , active fund now offers the level of  $c$  that maximises his payoff.
- Potential welfare increase comparing to the case with one large investor. The uncertainty in payoff creates incentive for coordination among investors.
- The degree of activism offered by active fund coordinates across heterogeneous investors with different signals.