Market Fragmentation and Price Informativeness

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Abstract

I study the effect of market fragmentation on the informativeness of prices. On the one hand, a higher degree of fragmentation may harm price informativeness because it lowers expected gains from trade and disincentivizes information production. On the other hand, it can benefit the aggregate informativeness since prices become less correlated. I develop a tractable trading model with two markets and two competing speculators who produce information about the fundamental value of a firm. The principal-agent framework of Holmstrom and Tirole (1993) allows me to stress the link between the informativeness of prices and the optimal managerial compensation.

1 Introduction

Modern financial markets feature a large number of stock exchanges. For example, in the US, there are 13 exchanges where stocks can be traded. Unlisted Trading Privileges regulation enables all stocks to trade on any exchange independently of where they are technically listed. All market participants, traders, market makers and platforms, respond strategically to the availability of multiple trading venues, which can impact the market quality. However, the literature in this field has not come to a unilateral agreement on the effects of fragmentation on the market quality, and in particular the informativeness of prices.

Through international brokers, one can gain access to foreign stock markets. Different countries can be more or less open to international investors which gives rise to cross-country

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segmentation. In international finance, the effect of the stock market liberalization on the price informativeness is one of the central questions (Sjöö and Zhang, 2000; Li et al., 2004; Lei and Lu, 2024). It determines the optimal government policy for introducing foreign investors to a domestic stock market. To that end, this paper compares the choice between dual-listing in different countries and direct foreign investment in one domestic stock market.

To study the effect of market segmentation on the quality of prices, this paper uses the framework of Holmstrom and Tirole (1993). The owner (principal) can choose between selling some of her shares in *one* market or *two* fragmented markets (labelled as "domestic" and "foreign"). Stock prices reveal the fundamental value of the firm which depends on the unobserved and non-contractible effort of the manager. Hence, from the principal's perspective, stock prices provide a signal about the manager's effort. The more informative the prices are, the better the signal is. However, it does not come for free: the principal ends up making losses trading against the informed traders. This can be viewed as the cost of the market monitoring.

There are sophisticated informed traders located in each market. They pay extra costs for sending cross-market orders in addition to the cost of information acquisition. This cross-market trading cost is an exogenous parameter in the model which allows us to vary the degree of market fragmentation. Higher costs mean more fragmented markets.

The first main result of the paper is that dual-listing option dominates direct investing. In other words, when the principal sells her stock to two markets, the resulting informativeness is higher compared to the single-market case. Moreover, the expected loss after trading against informed traders is smaller in the dual-listing case. Other costs of cross-listing are explicitly set to zero since the analysis is focused around informativeness.

The second result is that in the two-market case, the effect of trading barriers is non-monotone: higher cross-market trading costs are *detrimental* to price informativeness up to a certain threshold, after which they make prices *more effective* (Figure **??**). So, this paper can contribute to the market microstructure literature by characterising different channels that can potentially affect price informativeness in both directions. On the one hand, when cross-market trading is costlier, informed traders expect lower profits and cut investments in information production. On the other hand, equilibrium prices become less correlated and collectively are more informative about the fundamentals.

Fragmentation is generally understood as the presence of multiple exchanges which traders can freely choose from (Budish, Lee and Shim 2019; Degryse, De Jong, and van Kervel, 2015). While in this model, a two-market economy is counterposed to a single-market economy, there is also variation in the degree of fragmentation within the two-market case. When *cross-market trading costs are higher*, markets are considered to be *more fragmented*.

Kyle (1985) provides one way to think about the informativeness of prices. According to this seminal paper, prices are *more informative* if traders' beliefs about the fundamental variance conditional on the vector p of observed stock prices, $Var\left(\tilde{v}|p\right)$, decreases. However, in the principal-agent framework, it is also necessary to talk about the monitoring role of the prices. Market prices can be a better means for monitoring the manager if they induce a higher equilibrium level effort given the optimal contract. This paper shows that these two roles are equivalent for linear signals (a signal is a linear sum of the manager's effort, luck and noise). In other words, whenever prices are more informative in Kyle's understanding, they also motivate the manager's effort better. It is possible though that for an arbitrary signal structure this result might not hold.

As in Holmstrom and Tirole (1993), the effort is unobserved, and the principal incentivizes the manager using all available performance measures. Stock prices are noisy measures of the firm's fundamental value, which is a sum of the manager's effort, and the random component, or luck. The noise in prices comes from two sources. First, liquidity traders trade randomly. Second, the signals generated by two competing informed traders are imprecise. The informed traders choose the precision of their signals based on potential trading profits. The manager is risk-averse, and thus the optimal compensation scheme assigns such weights to each performance metrics that the overall risk in the compensation is minimized. Thus, the precision of the traders' signals and the report becomes key to understanding the impact of market fragmentation.

The endogenous information production decision leads to a violation of the Holmstrom's informativeness principle. It states that any measure that adds new information to the existing metrics of performance should be included in the optimal compensation contract (Holmstrom, 1979). More fragmented markets produce less correlated stock prices. Then, according to the informativeness principle, the optimal compensation structure would include all available stock and bond prices [1]. However, the information content of each stock price changes when markets become more fragmented. High entry barriers for foreign investors lower their potential profits and, therefore, their incentives to produce more information. Depending on which of these two effects dominates, the fragmentation may result in a more informative or less informative price system.

In Section 2, I describe the model. Section 3 solves for the equilibrium in financial markets taking the managerial effort as given and then the optimal compensation scheme. Section 4 finds the closed-form solution in the symmetric case. Section 5 is a single-market case. This section shows that fragmented markets are always preferred. Section 6 discusses the results and, finally, Section 7 concludes.

1.1 Related Literature

There is an extensive literature that studies price informativeness in the context of fragmented markets.

Some papers argue that fragmentation might reduce market efficiency. Foucault, Pagano, and Roell (2013) discuss potential drawbacks of fragmentation. First, in fragmented markets, it is easier for informed investors to exploit their information and more challenging for other traders to detect and infer their signals. Second, searching for the best price is more difficult since quotes are not centralized. Third, investors may lose a chance to take full advantage of 'liquidity externalities': the more investors enter a market, the more liquid it is (Pagano, 1989 a). Fragmented markets give the green light to high-frequency traders exploiting millisecond price deviations, which results in the 'high-frequency trading arms race' (Budish, Cramton and Shim, 2015). Some empirical research has also shown that fragmentation affects market quality, especially for less liquid securities (Bennett and Wei, 2006)

On the other hand, a significant number of empirical papers demonstrated that more fragmented markets are, on average, more efficient. O'Hara and Ye (2011) indicate that executions speeds are faster and transaction costs are lower for those stocks that can be split across multiple disintegrated stock exchanges in the US. Degryse et al.(2015) analyse a

[1] Edmans and Liu (2011) argue in favor of using bond prices in CEO compensation

more detailed dataset and distinguish between the impact stemming from fragmentation on visible and dark venues. They also find a positive association between fragmentation visible fragmentation and the depth aggregated across all visible venues. Foucault and Menkveld (2008) analyze the effect of EuroSETS entry in the Dutch stock market. They find that the aggregated depth of increases for a sample of stocks traded in both exchanges.

Literature on the theoretical side has emphasized potential benefits as well. For example, Chen and Duffie (2020) demonstrate that although more fragmented markets allow informed traders to hide their information by splitting their orders, aggregate trading becomes more aggressive and market prices reveal more information collectively. My paper adds to this literature by analysing when market fragmentation improves informativeness of stock prices and when it hurts.

In international finance, one of the integral question is the effect of market liberalization on the quality of stock markets. Mei et al (2005) study the segmented Chinese stock markets and: class A stocks traded domestically and class B stocks for foreign investors. They demonstrate that the A-B premium on stocks with identical cash-flow rights is correlated with the speculative motives, and thus suggest that the domestic Chinese market is prone to speculation. Lei and Lu (2024) find that an increase in market openness after introducing the Shanghai- Hong Kong Stock Connect in China lead to a decrease in stock price synchronicity, which presumably means that stock prices reflect their relatively uncorrelated fundamentals better [2]. However, they find a strong effect only for direct foreign investment (non-segmented foreign investment), whereas the segmented investment through class-H shares does not improve informativeness.

[2]Morck et al. (2000) studies the cross-country synchronicity and find that countries with greater impediments to informed trading have the highest synchronicity. Jin and Myers(2006) show that synchronicity decreases with a country's accounting transparency. Fernandes and Ferreira (2008,2009) and Kim and Shi (2009) also find synchronicity to be higher in emerging markets than in developed markets

2 Model

2.1 Players and Information

The owner of a firm (henceforth, the "principal") sells a proportion of firm shares at each of the two trading venues for the stock. The fraction sold in venue i is denoted as α_i. At time 0, these α_i shares are sold to liquidity traders in market i. At time 1, the liquidity traders receive a (positive or negative) liquidity shock and must trade with informed traders. Also, the principal chooses the optimal contract to incentivise the manager, whose effort is not observable and not contractible.

Each market i has an informed trader who receives a noisy signal s_i about the fundamental value, $e+\theta_i$, where e is the managerial effort and θ_i is the 'luck' component that corresponds to market i. The markets are fragmented. Specifically, if the informed trader in market i wants to trade x_{ji}, they bear additional costs of cross-market trading $\delta_i(x_{ji})^2$. Assume that this fee is proportional to the market illiquidity or price impact, λ_j:

$$\delta_i = \hat{\delta}_i \lambda_j$$

It is a Kyle's lambda that demonstrates the price reaction to the order flow of \$1 (Kyle, 1985). This assumption is mainly required for computational simplicity. However, Cohen et al.(2017) argue that cross-market trading barriers and market liquidity are positively associated.

As in Holmstrom and Tirole (1993), the firm reports earnings π at $t = 2$

$$\pi = e + \theta_1 + \theta_2 + \varepsilon,$$

where the components of the fundamental value are *iid*: $\theta_i \sim N\left(0, \sigma_\theta^2\right)$, and $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$ is the noise, or the error in the financial report. π is a verifiable and contractible earnings report, whereas the fundamental value

$$v = e + \theta_1 + \theta_2$$

is not verifiable and not contractible. Each of θ_i components corresponds to a specific market i. We can think of it as the separate performance of the two branches of an international firm in two different markets. Each informed trader gets the signal that corresponds to their domestic market and therefore their pieces of information are not correlated. At $t = 1$, they privately observe signals:

$$s_1 = e + \theta_1 + \eta_1,$$
$$s_2 = e + \theta_2 + \eta_2,$$

where $\eta_i \sim N\left(0, \sigma_{\eta_i}^2\right)$ is the noise in the signal. Define the precision of the signal as

$$\tau_i = \frac{\sigma_\theta^2}{\sigma_{\eta_i}^2 + \sigma_\theta^2}$$

When $\tau_i = 0$, there is too much noise in the signal which makes it useless. If $\tau_i = 1$, then the signal fully reveals $e + \theta_i$. I assume that errors in each signal are independent, and thus $Cov\left(\eta_1, \eta_2\right) = 0$. Then, traders cannot utilize information they have on hand to infer anything about the signal of their competitor.

Prior to trading, informed traders secretly choose the precision level τ_i at cost $g\left(\tau_i\right)$, where $g\left(\cdot\right)$ is increasing and convex with $g\left(0\right) = 0$,

All agents are risk-neutral, except for the manager. If investor i holds x_{ii} of class i shares and x_{ji} of class j shares, their utility is

$$(x_{ii} + x_{ji})\pi - p_i x_{ii} - p_j x_{ji} - \delta_j x_{ji}^2$$

Here, p_i is the market price in market i. It is set by competitive market makers and corresponds to Kyle (1985) auctions:

$$p_i(\text{order flow}_i) = E[v|\text{order flow}_i]$$

Notice that market makers in the domestic market do not observe the order flow in the foreign market.

2.2 Contracting

For tractability, we will consider a linear compensation $I()$. It is contingent on the announced earnings, π, and stock prices p_1 and p_2:

$$I = A_1 p_1 + A_2 p_2 + B\pi + F,$$

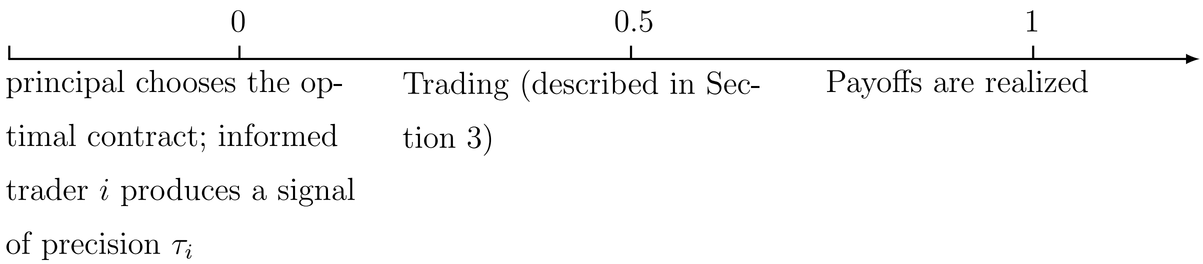


Figure 1: Timing of the model

where (A_1, A_2, B, F) are weights of each component and F is constant. Assume that the manager is compensated from the principal's deep pockets.

The manager is risk-averse. They have CARA utility over wealth w of the form:

$$u(w, e) = \exp\{-r\,(w - c(e))\}$$

with the risk-aversion coefficient r. Given that all random variables are normally distributed, maximizing $u()$ is equivalent to maximizing:

$$U\,(I, e) = E\,(I) - \frac{r}{2} var\,(I) - c\,(e)\,.$$

2.3 Timing

The timing of events is illustrated in Figure 1. At $t = 0$, the principal chooses the optimal contract (A_1, A_2, B, F). Then, liquidity traders buy α_1 and α_2 shares in each market. In the original paper by Holmstrom and Tirole (1993), the principal chooses α to maximize their objective function. However, in my model, I shy away from the choice of α_1 and α_2, so they are fixed. What I consider in this paper is the principal's choice between selling share α in a single domestic market and selling $\frac{1}{2}\alpha$ in two different markets.

At $t = 0.5$, the manager chooses effort e. The informed trader i privately chooses τ_i, and draws a signal s_i. Then, they send market orders to each market. Market makers observe order flow in their domestic markets and set prices p_i.

At $t = 1$, payoffs realize and the game ends.

3 Generalized Case with Asymmetric Markets

In this section, I do not require that both markets should be equivalent in terms of cross-market trading barriers and the amount of noise trading. In the next section, I make the assumption of symmetric market to find a closed-form solution for the infomativeness and perform comparative statics analysis.

3.1 Equilibrium in Stock Market

This section is devoted to finding the Bayesian Nash equilibrium in financial markets for a fixed level of manager's effort, e.

Markets are Kyle's auctions. Like in Kyle (1985), traders send market orders without knowing the exact price in advance. However, they can conjecture the pricing rule (Equation 1). Sophisticated informed traders can send their orders to domestic markets as well as across markets for additional costs described above. At the same time, liquidity traders' choice is unmodelled. They trade a random quantity in their respective domestic markets. However, to account for cross-market spillovers, liquidity shocks are assumed to be correlated (Equation 2). After, observing an order flow q_i in domestic market i, market makers set the price. Market maker from market i do not observe the flow in market j.

In each market, market makers are atomistic and competitive. Thus, they are playing the Bertrand game. In equilibrium, prices are set at the break-even level and market-makers earn zero ex-ante profits.

$$p_j(q_j) = E\left[\pi \middle| x_{j1}(s_1) + x_{j2}(s_2) + y_j = q_j\right]$$

Here, q_j is the observed order flow. It consists of the informative part, $x_{1j}(s_1) + x_{2j}(s_2)$, and the noise (liquidity traders), $y_j \sim N(0, \sigma_{yj}^2)$.

The pricing rule is linear. To be more precise, this is a conjecture that will be proved based on the projection theorem (see the proof of Proposition 1). Since π and q_j are normally distributed, it is not problematic to show that this conjecture holds.

$$p_j = \mu_j + \lambda_j q_j, \tag{1}$$

$\lambda_i > 0$ is Kyle's lambda and denotes the price impact of the order flow. Since the higher order flow reveals more positive information, market makers update their expectations about the fundamental value of the asset and charge a higher price.

Correlation between noise traders in each market is equal to

$$Corr\left(y_1, y_2\right) = \rho\sqrt{\frac{1}{1+\hat{\delta}_1}\frac{1}{1+\hat{\delta}_2}} \tag{2}$$

The correlation is inversely related to the intensity of cross-market barriers, $\hat{\delta}_1$ and $\hat{\delta}_2$. Notice that if the correlation coefficient is one, $\rho = 1$, and cross-market trading is frictionless, $\frac{1}{1+\hat{\delta}_1} = \frac{1}{1+\hat{\delta}_2} = 1$, then y_1 and y_2 are perfectly correlated. Then, two markets merge into a single aggregated one. The US stock exchanges featuring a high degree of synchronization fit this description.

Conditional on the chosen level of precision $\sigma^2_{\eta_i}$, informed trader i observes signal s_i. Then, his optimization problem looks as follows:

$$\max_{x_{ii}, x_{ji}} \{x_{ii}\left(E[\pi|s_i] - E[p_i|x_{ii}]\right) + x_{ji}\left(E[\pi|s_i] - E[p_j|x_{ji}]\right) - \delta_i (x_{ji})^2\}$$

The first-order condition implies:

$$\begin{aligned} x_{ii} &= \frac{E[\pi|s_i] - \mu_i - \lambda_i E[x_{ij}|s_i]}{2\lambda_i}, \\ x_{ji} &= \frac{E[\pi|s_i] - \mu_j - \lambda_j E[x_{jj}|s_i]}{2(\lambda_j + \delta_i)} \end{aligned}$$

The second order condition holds since the profit functions are concave in (x_{ii}, x_{ji}). The following proposition describes the Bayesian Nash equilibrium of the trading game:

Proposition 1 *In equilibrium, conditional on the observed signal s_i and the beliefs about the manager's effort $\bar{e}$, informed trader from market i sends the following optimal orders to the domestic and foreign markets:*

$$\begin{aligned} x_{ii}(s_i) &= \frac{\tau_i\left(s_i - \bar{e}\right)}{2\lambda_i} \\ x_{ji}(s_i) &= \frac{\tau_i\left(s_i - \bar{e}\right)}{2\lambda_j\left(1+\hat{\delta}_i\right)} \end{aligned}$$

Here, λ is Kyle's lamdba denoting the price impact of the informed order flow. Trading across markets is costly, and $\hat{\delta}$ denotes the size of the barriers.

The price impact optimally set by market makers in equilibrium is:

$$\lambda_i = \frac{1}{2}\sqrt{\left(\tau_i + \tau_j - \tau_j\left(1 - \frac{1}{1+\hat{\delta}_j}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_{y_i}^2}}$$

And constants in Equation 1 are $\mu_i = \bar{e}$, that is market beliefs about the manager's effort

We can make several observations about the optimal strategy of informed traders. First, when the cost of cross-market trading increases, they trade less in foreign markets. If cross-market trading is costless, $\hat{\delta} = 0$, then trader i sends identical orders to each market. This is the corner case of aggregated markets. If cross-market trading is prohibitively costly, that is, $\hat{\delta} = +\infty$, then markets are completely fragmented in a sense that each trader is confined to a domestic market. Second, informed trader i buys after positive information ($s_i > \bar{e}$) and sells (goes short) after negative information ($s_i < \bar{e}$).Third, when their signals get more precise, informed traders choose a more aggressive strategy. Finally, if their trading causes a higher price impact, then they trade more conservatively.

As in Kyle (1985), asymmetric information leads to a higher price impact: λ_i increases with τ_i. In particular, for given precision levels τ_i and τ_j, if market barriers are lower, the price impact increases, because foreign investors trade more actively. At the same time, the more noise there is, the more liquid the markets are: λ_i decreases with $\sigma_{y_i}^2$. However, higher uncertainty about the fundamental value σ_θ^2 makes markets less liquid.

Notice that as in Pagano (1989, b) if there is no trading costs, $\rho(y_1, y_2) = 1$ and $x_{ii} = x_{ji}$, and thus two markets are identical. This basically means a single aggregated market.

3.2 The Compensation of the Manager

The manager maximizes the objective function:

$$U(I,e) = E(I) - \frac{r}{2}var(I) - c(e).$$

Here, $c(e)$ is the costs of exerting efforts of level e. Assume a quadratic form for these costs: $c(e) = \frac{k}{2}e^2$. I parametrise the marginal cost of effort with $k > 0$.

In the first-best scenario, the effort level is observable and verifiable. Then, the principal and the manager would write a contract on the optimal level of effort, and the manager would adhere to it. In that case, the optimal effort level would be:

$$e^{FB} = \frac{1}{k} \tag{3}$$

However, in our setup, the manager's effort is not observable and not contractible. Thus, the principal needs a reliable, observable and contractible signal to incentivise the manager.

In Proposition 1, we have established that market prices are linear in signals s_1 and s_2, which, in turn, are positively related to the manager's effort. In other words, if the manager exerts more effort, informed traders are more likely to get a positive signal and drive up the market price thereby increasing the manager's compensation. This feedback effect reflects the monitoring role of the stock markets (Holmstrom and Tirole, 1993).

For simplicity, I will assume that the compensation scheme is linear and I will solve for the optimal compensation within this class of payment schemes.

$$I = A_1 p_1 + A_2 p_2 + B\pi + F,$$

In the optimum, the principal finds such (A_1, A_2, B, F), that maximize her expected profit. The principal gets $E[\pi]$ as the expected future profit of the company. The compensation of the manager, $E[I]$, is subtracted from the value of the principal's holdings. Also, the information loss ER is subtracted from her profit. Indeed, at $t = 0$, the principal sells α_1 and α_2 to noise traders. They anticipate future loss to sophisticated informed traders of the size ER (which is equivalent to what the informed traders earn), and price it in. Thus, at $t = 0$, the principal can earn $p_i = \alpha_i E[\pi] - ER_i$ from each market. Combining it with the value of the remaining shares, we can get

$$E[\pi] - E[I] - ER$$

This is the objective function that the principal is maximizing subject to the manager's participation constraint $E[I] - \frac{r}{2}Var(I) - \frac{k}{2}e^2 \geq 0$. We do not impose liability constraints on the value of F. Thus, the participation constraint is binding.

For the complete contract, which is a tuple $(e,\ I)$, the compatibility constraint should hold:

$$e = argmax_{e'_\pi} \left\{ E[I] - \frac{r}{2}Var(I) - \frac{k}{2}\left(e'_\pi\right)^2 \right\},$$

Since the participation constraint is binding, the principal's optimization problem can be rewritten as:

$$\max_{A_1,A_2,B}\left\{E[\pi]-ER_1-ER_2-\frac{r}{2}Var(I)-\frac{k}{2}\bar{e}^2\right\},$$

From Proposition 1, it follows that the stock market equilibrium is independent of the compensation scheme (A_1,A_2,B). Then, the optimization problem above is equivalent to

$$\min_{A_1,A_2,B}\left\{Var(I)\right\}, \tag{4}$$

subject to

$$e=\arg\max_{e'}\left\{E[I]-\frac{k}{2}(e')^2\right\},$$

Lemma 1 describes the optimal compensation scheme (A_1,A_2,B)

Lemma 1 *1. In equilibrium, the optimal compensation scheme includes both prices. The weight of each price is inversely related to its variance*

$$\frac{A_1}{A_2}=\frac{\frac{Cov(p_1,p_2)}{Var(p_1)}-1}{\frac{Cov(p_1,p_2)}{Var(p_2)}-1}$$

2. The weight of the earnings report, B, is inversely related to the amount of noise in it, σ_ε

3. To achieve a higher level of effort, each component of the pay should be increased proportionately to e

4. If effort is more costly, k is increasing, then to induce the same level of effort, the compensation should increase proportionately.

For the sake of sparing the reader the cumbersome closed-form expressions, I do not include them in the lemma. The main intuition from Lemma 1 is that the principal chooses the weights to balance the noisiness of each metrics (stock price 1, stock price 2, earnings report). Next section focuses on the simplifying case of symmetric markets to get a closed-formed solution.

4 Simplified Case With Symmetric Markets

4.1 Stock Market Equilibrium

Here, we will solve a simplified version of the model. From here on, assume that the cross-market costs are similar for each trader, $\hat{\delta}_1 = \hat{\delta}_2 = \hat{\delta}$, and noise traders' shocks are distributed similarly in each market, $\sigma^2_{y_1} = \sigma^2_{y_2} = \sigma^2_y$. Note that since at $t = 0$, the principal sells shares α_1 and α_2 of her stock to each market, symmetric markets require her to sell equal shares $\alpha_1 = \alpha_2 = \alpha$.

Lemma 2 *Each informed trader chooses the same level of precision*

$$\tau = \frac{\sigma_y \sigma_\theta}{2c} \sqrt{\frac{\left(1 + \frac{1}{1+\hat{\delta}}\right)^2}{2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2}} \tag{5}$$

This optimal precision τ

1. *decreases with the costs of cross-market trading,* $\tau'_{\hat{\delta}} < 0$
2. *increases with the liquidity of the markets,* $\tau'_{\sigma^2_y} > 0$
3. *increases with the fundamental uncertainty,* $\tau'_{\sigma^2_\theta} > 0$

Intuitively, each informed trader solving the same optimization problem. Thus, the solution is naturally similar. It takes more effort to show that this is the only solution, but the proof in Appendix demonstrates the it is indeed the case. Higher costs of trading reduce the informed trader's expected profits and therefore make information production less lucrative. At the same time, the more noise traders send their orders (σ^2_y goes up), the easier it is for the informed trader to hide an informed order, which incentivises the information production. Similarly, higher fundamental uncertainty σ^2_θ necessitates a more precise signal.

In this symmetric case, the price impact is similar in each market:

$$\lambda_1 = \lambda_2 = \frac{1}{2}\sqrt{\tau\left(2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\frac{\sigma^2_\theta}{\sigma^2_y}}$$

Note that even though pricing rules are similar, the prices themselves are different, because market makers receive distinct market orders q_1 and q_2.

Notice that since the price impact of the order flow is driven by the asymmetric information, higher cross-market trading barriers result in less information production and a lower price impact.

Next, consider covariance between the two stock market prices:

$$Cov(p_1, p_2) = \frac{1}{1+\hat{\delta}} \left(\frac{1}{2} \tau \sigma_\theta^2 + \rho \lambda^2 \sigma_y^2 \right) \tag{6}$$

Stock prices become less correlated as fragmentation gets more severe ($\hat{\delta}$ goes up). There are two reasons for this. First, as information production declines, prices do not reflect as much fundamental variation as with more precise signals. Second, noise trading becomes less correlated. Equation 6 may also be rewritten as:

$$Cov(p_1, p_2) = \frac{1}{1+\hat{\delta}} \frac{1}{2} \tau \sigma_\theta^2 \left(1 + \frac{1}{2} \rho \left(2 - \left(1 - \frac{1}{1+\hat{\delta}} \right)^2 \right) \right)$$

The informativeness principle would tell us that it is optimal to include both of the two imperfectly correlated signals in the manager's contract (Holmstrom 1979). It is even more true as the marginal effect on correlation gets larger when ρ increases. However, the information production, or the precision τ, decreases with $\hat{\delta}$. So, the overall effect on the second-best effort level and the price informativeness is not obvious.

4.2 The Contract of the Manager

Now, we will find the closed-form optimal contract and see how the effort induced by it depends on the cross-market barriers, $\hat{\delta}$.

From Lemma 1, it follows that the optimal compensation scheme in the symmetric case puts equal weights on each stock price: $A_1 = A_2 = A$. The exact compensation is described in Lemma 3 below. Due to the manager's risk aversion, the optimal contract should provide enough incentives without exposing him to excessive risks. The following lemma solves for the principal's problem of maximum insurance, in Equation 4, for a fixed level of induced effort e.

Lemma 3 *When the stock markets are symmetric, that is, the cross-market trading barriers are the same, $\hat{\delta}_1 = \hat{\delta}_2 = \hat{\delta}$ and the liquidity in each market is the same: $\sigma^2_{y_1} = \sigma^2_{y_2} = \sigma^2_y$, the optimal compensation puts equal weights on each market price $A_1 = A_2 = A$, with the following close-form expression:*

$$A = \frac{1}{\tau_\pi + \left(1 + \frac{1}{1+\hat{\delta}}\right)\tau(1-\tau_\pi) + \tau_\pi \frac{1}{2+\hat{\delta}}\left(1 + \frac{1}{2}\rho\left(2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\right)} ke$$

where τ_π is the precision of the earnings report:

$$\tau_\pi = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon}$$

and the weight of the earnings report in this scheme is

$$B = \left(1 - \left(1 + \frac{1}{1+\hat{\delta}}\right)\tau \frac{A}{ke}\right) ke$$

We can notice that if the earnings report is completely uninformative, $\tau_\pi = 0$, then it is not included in the optimal compensation scheme, $B = 0$. And the higher the informativeness of the report, the higher its weight in the compensation.

Note that all the broader conclusions established by Lemma 1 still hold. However, we have not yet found the effort level e that is the final part of the manager's contract. Now that we have a closed-form solution for the optimal compensation, (A, B, F), we can complete the optimal contract.

Denote the contract weights normalized by the marginal cost of effort ke as $(\hat{A}, \hat{B}, \hat{F})$. Then, we can express the variance of the manager's compensation, $Var(I)$, as:

$$Var(I) = \left(\left(2\sigma^2_\theta + \sigma^2_\varepsilon\right) - \left(\sigma^2_\theta + \sigma^2_\varepsilon\right)\left(1 + \frac{1}{1+\hat{\delta}}\right)\tau \times \hat{A}\right)(k\bar{e})^2 \equiv \widehat{Var}(I)k^2\bar{e}^2 \qquad (7)$$

Here, $\widehat{Var}(I)$ denotes a constant:

$$\widehat{Var}(I) = \left(2\sigma^2_\theta + \sigma^2_\varepsilon\right) - \left(\sigma^2_\theta + \sigma^2_\varepsilon\right)\left(1 + \frac{1}{1+\hat{\delta}}\right)\tau\hat{A}$$

Next, the principal's optimization problem can be rewritten in terms of the effort level e that needs to be enforced:

$$\max_e \left\{e - ER_1 - ER_2 - \frac{r}{2}\widehat{Var}(I)k^2e^2 - \frac{k}{2}e^2\right\}, \qquad (8)$$

Here, ER_i is the trading profit of informed trader i. The informed traders extract rents from noise traders who anticipate these losses and discount the initial offerings by the principal. However, as Preposition 1 demonstrates, these gains are not influenced by the effort level e. The effort only impacts the realization s_i, but not the ex-ante expected profit ER_i.

Then, the optimal effort level that solves 8 is

$$e = \frac{1}{rk^2\widehat{Var}(I) + k} \tag{9}$$

First, notice that this effort level is lower that the first best (Equation 3). The effort decreases by the degree of uncertainty in the compensation structure. $\widehat{Var}(I)$, and the extent of the manager's risk-aversion, r. If the variance $\widehat{Var}(I)$ is high and the risk-aversion r is high, then the manager exerts less effort. Thus, the degree of market fragmentation $\hat{\delta}$ is welfare improving if and only if it can lead to a lower variance in the manager's compensation. Intuitively, this means that the price system (p_1, p_2) should be more informative. That is, it should reveal the fundamental value without much noise. In the discussion section (Section 6), I will show that whenever the prices are more informative in Kyle's understanding [3], the optimal contract induces a higher level effort (that is, $\widehat{Var}(I)$ is falling).

The variance is influence by the degree of market fragmentation $\hat{\delta}$ through two channels. First, as prices become less correlated, more information can be extracted collectively from two markets, and $\widehat{Var}(I)$ goes down. Second, as signals acquired by the informed traders become less precise (τ falls), $\widehat{Var}(I)$ goes up. The equilibrium effect of $\hat{\delta}$ would depend on the relative importance of each channel. The following proposition describes how these two forces interact and what is the resulting effect on the price informativeness.

Proposition 2 *1. In equilibrium, as cross-market trading costs, $\hat{\delta}$, increase:*

(a) At first, the precision effect dominates and the variance $\widehat{Var}(I)$ goes up till some point $\hat{\delta}^*$. More fragmented markets make for a less informative price system

(b) Then, as $\hat{\delta}$ grows larger, $\hat{\delta} > \hat{\delta}^*$, the correlation falls significantly and the prices collectively reveal more information, which drives the variance $\widehat{Var}(I)$ down. More fragmented markets make for a more informative price system

[3] $Var(\theta|p_1, p_2)$ is lower, Kyle (1985)

(c) The threshold $\hat{\delta}^*$ only depends on the correlation between the noise in each market, ρ. As ρ goes up the marginal effect on correlation increases and δ^* decreases.

2. *As markets become more liquid, that is $\sigma_y^2 \uparrow$, prices become more informative and the marginal effect of market fragmentation falls.*

This is the main result of the paper. Proposition 2, illustrated by Figure 2, shows that at first, the negative marginal effect on the information production dominates the correlation effect and the price informativeness goes down with the degree of fragmentation $\hat{\delta}$. Then, as the cross-market trading barriers keep increasing, prices become less correlated and the positive marginal effect of the correlation channel dominates the fallen information production.

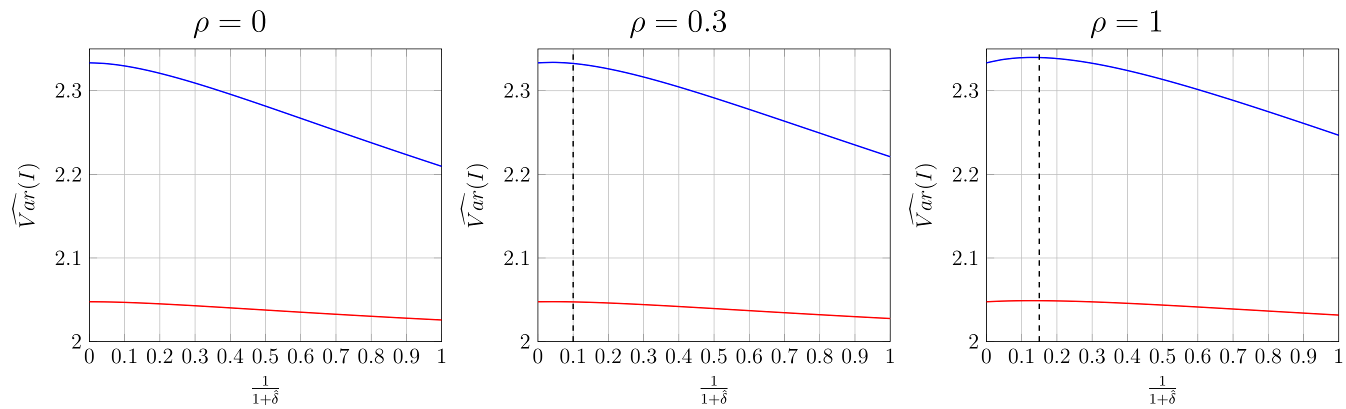
The threshold $\hat{\delta}^*$ where the positive correlation effect comes into play *does not* depend on the amount of noise in the earnings report σ_ε^2, the cost of information production c or the ratio of market liquidity to fundamental uncertainty $\frac{\sigma_y}{\sigma_\theta}$. It only depends on the strength of the correlation between the two prices. Intuitively, since all the market characteristics affect both the covariance between the prices and the information production, the trade-off between these two channels does not change. If the noise becomes more correlated, then the correlation effect is stronger.

As the liquidity σ_y^2 goes up, the precision increases, and the overall informativeness improves: $\widehat{Var}(I)$ goes down. Therefore, the marginal impact of the fragmentation parameter, $\hat{\delta}$, decreases and both channels become less pronounced.

5 Single-Market Case

5.1 Price Informativeness

Now, suppose the owner decides to issue just one type of stock. That is, either $\alpha_1 = 0$ or $\alpha_2 = 0$, whereas $\hat{\delta} > 0$. Then, there will be only one market where two informed investors, a domestic and a foreign one, and domestic noise investors trade. Therefore, there will be a



This figure plots the total variance of the manager's contract against $\frac{1}{1+\hat{\delta}}$, the degree of market integration, which lies within the interval $[0,1]$. To plot these graphs, I assigned $\frac{\sigma_\varepsilon^2}{2c} = 1$,$\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$, $\mathcal{L} = 1$ for the blue graphs, and $\mathcal{L} = 10$ for the red ones.

Figure 2: Variance of Compensation For Different Liquidity Parameters

single price that reveals the manager's effort. The equilibrium effort level in this case will still be given by Equation 9, but the market outcome and the optimal contract change. In particular, the variance $\widehat{Var}(I)$ will now take the following form:

$$\widehat{Var}(I)' = 2\sigma_\theta^2 + \sigma_\varepsilon^2 - \left(\sigma_\theta^2 + \sigma_\varepsilon^2\right) \frac{T'}{\tau_\pi + T'(1-\tau_\pi)}$$

$\widehat{Var}(I)'$ denotes the new variance of the managers compensation conditional on the optimal structure in the situation of one market being shut down. T' is the weighted sum of the two precision measures: $T' = \frac{1}{2}\left(\tau_1 + \frac{1}{1+\hat{\delta}}\tau_2\right)$. Again, our analysis of the informativeness of such a price system boils down to the analysis of function $\widehat{Var}(I)'$. In particular, we are interested in understanding whether the principal can improve the price informativeness by shutting down one of the markets, $\alpha_1 = 0$ or $\alpha_2 = 0$.

Lemma 4 *With just one stock market of the size α, equilibrium prices are less informative than in the case of two markets of the size $\frac{1}{2}\alpha$:*

$$\widehat{Var}(I)' > \widehat{Var}(I)$$

and thus

$$e' < e$$

In the single market case, there is no Holmstrom informativeness effect on the variance of the optimal contract $\widehat{Var}(I)'$

This lemma draws an interesting conclusion that the two-market case always dominates one-market case in terms of informativeness. At the same time, it abstracts away from some real-world considerations, namely the costs of dual-listing. Those may include the costly regulatory compliance among other things. For example, in China, IPOs must be approved by the government and dual-listed companies face even stricter scrutiny. Lemma 4 shies away from these concerns and zeroes in on the implications for price informativeness. However, informativeness of the prices per se is not the final goal of the principal even in my model. The principal's *expected payoff* also depends on the loss to informed traders, ER, which I dub the cost of market monitoring following Holmstrom and Tirole (1993).

In the limit case when there is almost no barriers to cross-market trading, $\hat{\delta} \to 0$, two markets merge into one. Then, upon normalizing the size of each market to $\frac{1}{2}\alpha$, it becomes evident the two-market structure is essentially the same as a single market of size α from the perspective of optimal contracting and information production.

Figure 3 illustrates that higher cross-market trading costs can only undermine information production by the informed traders. Therefore, higher barriers are always bad for price informativeness.

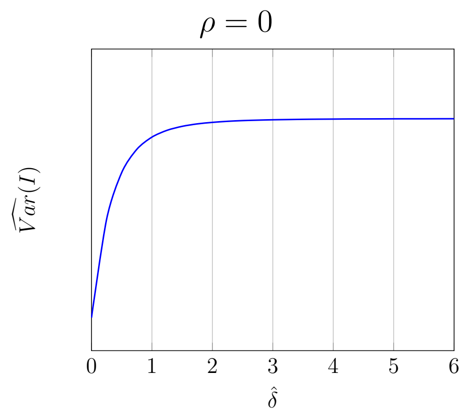
5.2 Costly Monitoring

In the single-market case, the informed traders earn collectively:

$$ER_1 + ER_2 = \frac{(\sigma_y \sigma_\theta)^{\frac{3}{2}}}{6\sqrt{2}\sqrt{c}} \frac{1 + \left(\frac{1}{1+\hat{\delta}}\right)^3}{\left(1 + \left(\frac{1}{1+\hat{\delta}}\right)^2 \left(1 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\right)^{\frac{3}{4}}}$$

In the two-market case, their expected profit is:

$$2ER = \frac{(\sigma_y \sigma_\theta)^{\frac{3}{2}}}{3\sqrt{2}\sqrt{c}} \frac{\left(1 + \frac{1}{1+\hat{\delta}}\right)^{\frac{3}{2}}}{\left(2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)^{\frac{3}{4}}}$$



This figure plots the total variance of the manager's compensation. To plot the graph, I assigned $\sigma_\theta^2 = \sigma_\varepsilon^2 = \sigma_y^2 = 1$, $c = \frac{1}{2}$.

Figure 3: Variance of Compensation For Different Liquidity Parameters

Lemma 5 *The cost of market monitoring ER for the principal is lower in the two-market case compared to the single-market case:*

$$ER_1 + ER_2 > 2ER$$

Then, from Lemma 4 and Lemma 5, it follows that two markets dominate one both in terms of the price informativeness, leading to a higher managerial effort, and in terms of the cost of market monitoring. Hence,

Proposition 3 *The owner is better-off splitting shares between two markets in equal proportion. It increases price-informativeness and reduces the cost of monitoring.*

Between having two markets with costly cross-market trading and having a single market for local and foreign investors, who still have to bear these additional trading costs, the first option always dominates. Even though there is a single market, both informed traders acquire more precise signals and earn more in expectation compared to the two-market case. It seems that having an additional market can help informed traders earn more, but since price impact adjusts for that and since each market is twice as small as the aggregated one (by assumption), it hurts their expected profits, and they produce less information as a consequence

6 Discussion

6.1 Price Informativeness

Kyle (1985) defines price informativeness in terms of the traders' beliefs about the fundamental uncertainty conditional on the observed market price.

$$Var\left(v|p\right)$$

Prices are more informative, then there is less risk or uncertainty about the fundamental value left after observing them.

In the single-market case, it is less challenging to compute this measure:

$$Var\left(\theta_1+\theta_2|p_1\right) = Var\left(\theta_1+\theta_2\right) - \frac{\left(Cov\left(\theta_1+\theta_2,p_1\right)\right)^2}{Var\left(p_1\right)} =$$
$$= 2\sigma_\theta^2 - \frac{1}{2}\sigma_\theta^2\left(\tau_1+\left(\frac{1}{1+\hat{\delta}}\right)\tau_2\right)$$

We can see that as the traders acquire more precise signals, the variance above goes down. We can also notice that based on the results from Section 5, the optimal contract also improves with better information production. So, there is a one-to-one mapping between the price informativeness in Kyle's understanding and the optimality of the contract and the manager's effort.

With two markets,

$$Var(\theta_1+\theta_2|p_1,p_2)$$

can be found as $\frac{det(\Sigma)}{det(\Sigma')}$, where $\Sigma = Cov\left(v,\begin{bmatrix}p_1\\p_2\end{bmatrix}\right)$ and $\Sigma' = Var\left(\begin{bmatrix}p_1\\p_2\end{bmatrix}\right)$. Then, it can be shown that

$$Var(\theta_1+\theta_2|p_1,p_2) = \sigma_\theta^2\left(2-\tau\frac{1+\frac{1}{1+\hat{\delta}}}{1+\frac{1}{2+\hat{\delta}}\left(1+\frac{1}{2}\rho\left(2-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)\right)}\right)$$

This second term in the brackets is a function of $\hat{\delta}$. It can be shown that it has exactly the same extrema as the function $\widehat{Var}(I)$. In other words, these two functions behave in the same way, and thus if the prices are more informative, then the manager's compensation induces a higher effort level.

6.2 Empirical predictions

First, it follows from Proposition 2 that when cross-market trading barriers are relaxed [4], the price informativeness may increase at first before it starts improving. It can decrease if $\hat{\delta}_1, \hat{\delta}_2 \in \left(\hat{\delta}^*, +\infty\right)$ or increase if $\hat{\delta}_1, \hat{\delta}_2 \in \left(0, \hat{\delta}^*\right)$ or $\hat{\delta}_2 \in \left(0, \hat{\delta}^*\right)$ and $\hat{\delta}_1 \in \left(\hat{\delta}^*, +\infty\right)$.

Second, the magnitude of the effect above decreases with the liquidity of the market σ_y^2. Intuitively, when the markets are already very liquid, the prices are efficient (Kyle, 1985). Hence, the marginal impact of market fragmentation is lower.

Third, the link between the informativeness and the optimal compensation offers an alternative explanation to the observed price jumps around earnings announcement (Watts, 1978). Stock prices may react to an announcement both because prices fail to reflect the fundamental value $\theta_1 + \theta_2$ and because the manager's incentives are not linked to the market value of the company. Since managerial compensation is not updated continuously, filed reports may reveal how ineffective the compensation scheme is, which in turn make market participants update their expectations downwards.

7 Conclusion

This paper explores the impact of market fragmentation on price informativeness and the manager's effort in the principal-agent model a la Holmstrom and Tirole (1993).

The principal always prefers to split her sale order across two markets rather than sell all shares at one venue. Cross-market barriers harm price informativeness at first, because sophisticated traders anticipate lower profits and do not invest in precise information. Then, as the barriers keep increasing, prices become less correlated and collectively reveal more information.

The principal-agent framework allows us to demonstrate how the price informativeness is connected to the optimal compensation of the manager. For the linear structure of the fundamental value, I have demonstrated that more informative prices make for the better

[4] One example of such events could be the launching of Shanghai-Hong Kong Stock Connect in 2014. Prior to the introduction of Shanghai-Hong Kong Stock Connect, mainland China investors could only trade class A shares while overseas investors were only eligible to trade H class shares. In terms of the model, that would mean $\hat{\delta} = +\infty$

compensation scheme and the higher managerial effort in equilibrium. This highlights the importance of the stock prices as the monitoring devise in managerial contracts (Holmstrom and Tirole, 1993)

Among potential policy implications, a benevolent planner may be interested in increasing entry barriers for foreign traders to redistribute the proceeds among other agents without damaging the efficiency of the stock market. Alternatively, if regulators aim to lower entry barriers, the new level should be chosen carefully taking into consideration both the correlation effect and the effect on the information production by sophisticated agents.

Appendix: Proofs

Proof of Proposition 1

Conditional on buying signal with precision $\sigma^2_{\eta_1}$, informed trader 1 observes signal s_1. Then, his optimization problem looks as follows:

$$\max_{x_{11},x_{21}} \{x_{11}\left(E[\pi|x_{11},s_1]-E[p_1|x_{11}]\right)+x_{21}\left(E[\pi|x_{21},s_1]-E[p_2|x_{21}]\right)-\delta_1 x_{21}^2\}$$

$$\begin{aligned}\max_{x_{11},x_{21}} \{&x_{11}\left(E[\pi|s_1]-\mu_1-\lambda_1(x_{11}+E[x_{12}|x_{11},s_1])\right)+\\ &+x_{21}\left(E[\pi|s_1]-\mu_2-\lambda_2(x_{21}+E[x_{22}|x_{21},s_1])\right)-\\ &-\delta_1 x_{21}^2\}\end{aligned}$$

Then,

$$\begin{aligned} x_{11} &= \frac{E[\pi|s_1]-\mu_1-\lambda_1 E[x_{12}|s_1]}{2\lambda_1}, \\ x_{21} &= \frac{E[\pi|s_1]-\mu_2-\lambda_2 E[x_{22}|s_1]}{2(\lambda_2+\delta_1)}, \end{aligned}$$

where $E[\pi|s_1]=(1-\tau_1)\bar{e}+\tau_1 s_1$, and $\tau_1=\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_{\eta_1}}$

Similarly, informed trader 2 sends:

$$\begin{aligned} x_{12} &= \frac{E[\pi|s_2]-\mu_1-\lambda_1 E[x_{11}|s_2]}{2(\lambda_1+\delta_2)}, \\ x_{22} &= \frac{E[\pi|s_2]-\mu_2-\lambda_2 E[x_{21}|s_2]}{2\lambda_2}, \end{aligned}$$

Then,

$$\begin{aligned} E[x_{12}|s_1] &= \frac{E\Big[E[\pi|s_2]\Big|s_1\Big]-\mu_1-\lambda_1 E[E[x_{11}|s_2]|s_1]}{2(\lambda_1+\delta_2)}, \\ E[x_{22}|s_1] &= \frac{E\Big[E[\pi|s_2]\Big|s_1\Big]-\mu_2-\lambda_2 E[E[x_{21}|s_2]|s_1]}{2\lambda_2}, \end{aligned}$$

Here, $E\Big[E[\pi|s_2]\Big|s_1\Big]$ is what informed trader 1 thinks about informed trader 2's expectations about future profit.

And similarly,

$$\begin{aligned} E[x_{11}|s_2] &= \frac{E\Big[E[\pi|s_1\Big|s_2\Big] - \mu_1 - \lambda_1 E[E[x_{12}|s_1]|s_2]}{2\lambda_1}, \\ E[x_{21}|s_2] &= \frac{E\Big[E[\pi|s_1\Big|s_2\Big] - \mu_2 - \lambda_2 E[E[x_{22}|s_1]|s_2]}{2(\lambda_2+\delta_1)}, \end{aligned}$$

Note that by design, signals of each trader are not correlated. That is, knowing signal s_i does not help to infer anything from signal s_j.

$$E[E[\pi|s_2]|s_1] = (1-\tau_2)\,\bar{e} + \tau_2 E[s_2|s_1] = (1-\tau_2)\,\bar{e} + \tau_2\bar{e} = \bar{e}$$

Symmetrically,

$$E[E[\pi|s_1]|s_2] = \bar{e}$$

$$\begin{aligned} E[x_{12}|s_1] = \frac{\bar{e} - \mu_1 - \lambda_1 E[E[x_{11}|s_2]|s_1]}{2(\lambda_1+\delta_2)} &= \frac{\bar{e}-\mu_1}{2(\lambda_1+\delta_2)} - \frac{\lambda_1}{2\,(\lambda_1+\delta_2)} E[E[x_{11}|s_2]] = \\ &= \frac{\bar{e}-\mu_1}{2(\lambda_1+\delta_2)} - \frac{\lambda_1}{2\,(\lambda_1+\delta_2)} E[x_{11}] \end{aligned}$$

Similarly,

$$E[x_{22}|s_1] = \frac{\bar{e}-\mu_2}{2\lambda_2} - \frac{\lambda_2}{2\lambda_2} E[x_{21}]$$

$$\begin{aligned} E[x_{11}|s_2] &= \frac{\bar{e}-\mu_1}{2\lambda_1} - \frac{\lambda_1}{2\lambda_1} E[x_{12}] \\ E[x_{21}|s_2] &= \frac{\bar{e}-\mu_2}{2\,(\lambda_2+\delta_1)} - \frac{\lambda_2}{2\,(\lambda_2+\delta_1)} E[x_{22}] \end{aligned}$$

Then, it also holds that :

$$\begin{aligned} E[x_{22}] &= \frac{\bar{e}-\mu_2}{2\lambda_2} - \frac{\lambda_2}{2\lambda_2} E[x_{21}] \\ E[x_{21}] &= \frac{\bar{e}-\mu_2}{2\,(\lambda_2+\delta_1)} - \frac{\lambda_2}{2\,(\lambda_2+\delta_1)} E[x_{22}] \end{aligned}$$

Next,

$$\begin{aligned} E[x_{22}] &= \frac{\bar{e}-\mu_2}{2\lambda_2} - \frac{1}{2}\left(\frac{\bar{e}-\mu_2}{2\,(\lambda_2+\delta_1)} - \frac{\lambda_2}{2\,(\lambda_2+\delta_1)} E[x_{22}]\right) \\ E[x_{22}]\left(1 - \frac{1}{2}\frac{\lambda_2}{2\,(\lambda_2+\delta_1)}\right) &= \frac{\bar{e}-\mu_2}{2\lambda_2} - \frac{1}{2}\frac{\bar{e}-\mu_2}{2\,(\lambda_2+\delta_1)} \\ E[x_{22}] &= \frac{\bar{e}-\mu_2}{\lambda_2}\frac{\frac{3}{2}\lambda_2 + 2\delta_1 - \lambda_2 - \delta_1}{\frac{3}{2}\lambda_2 + 2\delta_1} = \frac{\bar{e}-\mu_2}{\lambda_2}\frac{\frac{1}{2}\lambda_2 + \delta_1}{\frac{3}{2}\lambda_2 + 2\delta_1} \end{aligned}$$

Similarly,

$$E[x_{11}] = \frac{\bar{e} - \mu_1}{\lambda_1} \frac{\frac{1}{2}\lambda_1 + \delta_2}{\frac{3}{2}\lambda_1 + 2\delta_2}$$

Then,

$$E[x_{21}] = \frac{\bar{e} - \mu_2}{2(\lambda_2 + \delta_1)} \left(1 - \frac{\frac{1}{2}\lambda_2 + \delta_1}{\frac{3}{2}\lambda_2 + 2\delta_1}\right) = \frac{\bar{e} - \mu_2}{2\left(\frac{3}{2}\lambda_2 + 2\delta_1\right)} = \frac{\bar{e} - \mu_2}{3\lambda_2 + 4\delta_1}$$

Similarly,

$$E[x_{12}] = \frac{\bar{e} - \mu_1}{3\lambda_1 + 4\delta_2}$$

Then, the actual order sizes are as follows. informed trader 1 sends to market 1:

$$x_{11} = \frac{E[\pi|s_1] - \mu_1 - \lambda_1 E[x_{12}]}{2\lambda_1} = \frac{E[\pi|s_1] - \mu_1}{2\lambda_1} - \frac{1}{2}\frac{\bar{e} - \mu_1}{3\lambda_1 + 4\delta_2}$$

informed trader 1 sends to market 2:

$$x_{21} = \frac{E[\pi|s_1] - \mu_2}{2(\lambda_2 + \delta_1)} - \frac{\lambda_2}{2\left(\lambda_2 + \delta_1\right)} \frac{\bar{e} - \mu_2}{\lambda_2} \frac{\frac{1}{2}\lambda_2 + \delta_1}{\frac{3}{2}\lambda_2 + 2\delta_1} =$$
$$= \frac{E[\pi|s_1] - \mu_2}{2(\lambda_2 + \delta_1)} - \frac{\bar{e} - \mu_2}{2\left(\lambda_2 + \delta_1\right)} \frac{\frac{1}{2}\lambda_2 + \delta_1}{\frac{3}{2}\lambda_2 + 2\delta_1}$$

informed trader 2 sends to market 1:

$$x_{12} = \frac{E[\pi|s_2] - \mu_1}{2(\lambda_1 + \delta_2)} - \frac{\bar{e} - \mu_1}{2\left(\lambda_1 + \delta_2\right)} \frac{\frac{1}{2}\lambda_1 + \delta_2}{\frac{3}{2}\lambda_1 + 2\delta_2}$$

informed trader 2 sends to market 2:

$$x_{22} = \frac{E[\pi|s_2] - \mu_2}{2\lambda_2} - \frac{1}{2}\frac{\bar{e} - \mu_2}{3\lambda_2 + 4\delta_1}$$

Then market makers observe:

$$q_1 = x_{11} + x_{12} + y_1$$
$$q_2 = x_{21} + x_{22} + y_2$$

First, we will focus on market 1 and find the equilibrium stock price here. (π, q_1) has a multivariate normal distribution. Then, we can apply the projection theorem to find the optimal pricing rule which is conjectured to be linear:

$$p_1 = E[\pi|q_1] = E[\pi] + \frac{Cov(\pi, q_1)}{Var(q_1)}(q_1 - E[q_1]) = \mu_1 + \lambda_1 q_1$$

We can find each component of the price impact parameter λ_1:

$$Cov(\pi, q_1) = \frac{\tau_1}{2\lambda_1}\sigma_\theta^2 + \frac{\tau_2}{2\left(\lambda_1 + \delta_2\right)}\sigma_\theta^2$$

$$Var(q_1) = \frac{\tau_1}{\left(2\lambda_1\right)^2}\sigma_\theta^2 + \frac{\tau_2}{\left(2\left(\lambda_1 + \delta_2\right)\right)^2}\sigma_\theta^2 + {\sigma_{y1}}^2 + \frac{1}{1+\hat{\delta}_2}\sigma_{y2}^2$$

$$\lambda_1 = \frac{\frac{\tau_1}{2\lambda_1}\sigma_\theta^2 + \frac{\tau_2}{2(\lambda_1+\delta_2)}\sigma_\theta^2}{\frac{\tau_1}{(2\lambda_1)^2}\sigma_\theta^2 + \frac{\tau_2}{(2(\lambda_1+\delta_2))^2}\sigma_\theta^2 + {\sigma_{y1}}^2}$$

Or, under the assumption $\delta_i = \hat{\delta}_i\lambda_j$,

$$\lambda_1 = \frac{\frac{\tau_1}{2\lambda_1}\sigma_\theta^2 + \frac{\tau_2}{2\lambda_1\left(1+\hat{\delta}_2\right)}\sigma_\theta^2}{\frac{\tau_1}{(2\lambda_1)^2}\sigma_\theta^2 + \frac{\tau_2}{\left(2\lambda_1\left(1+\hat{\delta}_2\right)\right)^2}\sigma_\theta^2 + \sigma_{y_1}^2} = \frac{2\lambda_1\left(\tau_1\sigma_\theta^2 + \frac{\tau_2}{\left(1+\hat{\delta}_2\right)}\sigma_\theta^2\right)}{\tau_1\sigma_\theta^2 + \frac{\tau_2}{\left(1+\hat{\delta}_2\right)^2}\sigma_\theta^2 + \left(2\lambda_1\right)^2\sigma_{y_1}^2}$$

$$\tau_1\sigma_\theta^2 + \frac{\tau_2}{\left(1+\hat{\delta}_2\right)^2}\sigma_\theta^2 + \left(2\lambda_1\right)^2\sigma_{y_1}^2 = 2\left(\tau_1\sigma_\theta^2 + \frac{\tau_2}{\left(1+\hat{\delta}_2\right)}\sigma_\theta^2\right)$$

$$\left(2\lambda_1\right)^2\sigma_{y_1}^2 = \tau_1\sigma_\theta^2 + 2\frac{\tau_2}{\left(1+\hat{\delta}_2\right)}\sigma_\theta^2 - \frac{\tau_2}{\left(1+\hat{\delta}_2\right)^2}\sigma_\theta^2$$

$$\lambda_1 = \frac{1}{2}\sqrt{\left(\tau_1 + \tau_2 - \tau_2\left(1 - \frac{1}{1+\hat{\delta}_2}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_{y_1}^2}}$$

Similarly, the price impact on market 2 is

$$\lambda_2 = \frac{1}{2}\sqrt{\left(\tau_2 + \tau_1 - \tau_1\left(1 - \frac{1}{1+\hat{\delta}_1}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_{y_2}^2}}$$

Then, the constant term in the pricing rule can be found from the following condition:

$$\mu_1 = E[\pi] - \lambda_1 E[q_1] = \bar{e} - \lambda_1\left(E[x_{11}] + E[x_{12}]\right)$$

$$\lambda_1\left(\frac{\bar{e}-\mu_1}{\lambda_1}\frac{\frac{1}{2}\lambda_1 + \delta_2}{\frac{3}{2}\lambda_1 + 2\delta_2} + \frac{\bar{e}-\mu_1}{3\lambda_1 + 4\delta_2}\right) = \bar{e} - \mu_1$$

Then, $\mu_1 = \bar{e}$. Similarly, $\mu_2 = \bar{e}$.

Thus, the equilibrium orders sent by informed trader i:

$$\begin{aligned} x_{ii} &= \frac{\tau_i (s_i - \bar{e})}{2\lambda_i} \\ x_{ji} &= \frac{\tau_i (s_i - \bar{e})}{2\lambda_j \left(1 + \hat{\delta}_i\right)} \end{aligned}$$

The stock market price in market i is :

$$p_i = \bar{e} + \frac{E[\pi|s_i] - \bar{e}}{2} + \frac{1}{1 + \hat{\delta}_j} \frac{E[\pi|s_j] - \bar{e}}{2} + \lambda_i y_i$$

informed trader 1's expected profit after observing signal s_1:

$$E[\pi_1|s_1] = \lambda_1 x_{11}^2 + (\lambda_2 + \delta_1) x_{21}^2 = \frac{(E[\pi|s_1] - \bar{e})^2}{4\lambda_1} + \frac{(E[\pi|s_1] - \bar{e})^2}{4\left(1 + \hat{\delta}_1\right)\lambda_2} =$$

$$= \frac{\tau_1^2 (s_1 - \bar{e})^2}{4\lambda_1} + \frac{\tau_1^2 (s_1 - \bar{e})^2}{4\left(1 + \hat{\delta}_1\right)\lambda_2}$$

Before the signal after their signals:

$$E[\pi_1] = \tau_1 \sigma_\theta^2 \left(\frac{1}{4\lambda_1} + \frac{1}{4\left(1 + \hat{\delta}_1\right)\lambda_2} \right) - g(\tau_1)$$

Symmetrically,

$$E[\pi_2] = \tau_2 \sigma_\theta^2 \left(\frac{1}{4\lambda_2} + \frac{1}{4\left(1 + \hat{\delta}_2\right)\lambda_1} \right) - g(\tau_2)$$

The optimal precision chosen by informed trader 1:

$$\sigma_\theta^2 \left(\frac{1}{4\lambda_1} + \frac{1}{4\left(1 + \hat{\delta}_1\right)\lambda_2} \right) = g'(\tau_1) \tag{10}$$

Similarly, for informed trader 2:

$$\sigma_\theta^2 \left(\frac{1}{4\lambda_2} + \frac{1}{4\left(1 + \hat{\delta}_2\right)\lambda_1} \right) = g'(\tau_2) \tag{11}$$

Note that

$$(1 + \hat{\delta}_1)\lambda_2 = (1 + \hat{\delta}_1)\frac{1}{2}\sqrt{\left(\tau_2 + \tau_1 - \tau_1 \left(1 - \frac{1}{1 + \hat{\delta}_1}\right)^2\right) \frac{\sigma_\theta^2}{\sigma_{y_2}^2}}$$

is increasing in $\hat{\delta}_1$. Thus, the left-hand side of Equation 11 above is decreasing in $\hat{\delta}_1$. Then, since both LHS and RHF are decreasing in τ_1, it should fall for the equation to restore.

The cost of cross-market trading for informed trader 2 also affects the precision of informed trader 1 throught the effect on the price sensitivity to market order flow, λ_1.

Proof of Lemma 1

Find the optimal contract in a general case

$$\min_{A_1,A_2,B}\Big\{Var(I)\Big\},$$

subject to

$$e = argmax_{e'}\Big\{E[I]-c(e)\Big\},$$

where

$$\begin{aligned}&Var(I) = A_1^2Var(p_1) + A_2^2Var(p_2) + B^2Var(\pi) + \\ &+2A_1A_2Cov(p_1,p_2) + 2A_1BCov\left(p_1,\pi\right) + 2A_2BCov\left(p_2,\pi\right)\end{aligned}$$

and e is found from:

$$A_1\left(\frac{1}{2}\tau_1+\frac{1}{2}\frac{1}{1+\hat{\delta}_2}\tau_2\right)+A_2\left(\frac{1}{2}\tau_2+\frac{1}{2}\frac{1}{1+\hat{\delta}_1}\tau_1\right)+B=k_1e$$

Denote k_1e as $\bar{c}$, $\left(\frac{1}{2}\tau_1+\frac{1}{2}\frac{1}{1+\hat{\delta}_2}\tau_2\right)$ as T_1 and $\left(\frac{1}{2}\tau_2+\frac{1}{2}\frac{1}{1+\hat{\delta}_1}\tau_1\right)$ as T_2.

Prices are

$$\begin{aligned}p_1 &= \bar{e}+\frac{E[\pi|s_1]-\bar{e}}{2}+\frac{1}{1+\hat{\delta}_2}\frac{E[\pi|s_2]-\bar{e}}{2}+\lambda_1y_1\\ p_2 &= \bar{e}+\frac{E[\pi|s_2]-\bar{e}}{2}+\frac{1}{1+\hat{\delta}_1}\frac{E[\pi|s_1]-\bar{e}}{2}+\lambda_2y_2\end{aligned}$$

$$\begin{aligned}Var\left(p_1\right) &= \frac{1}{4}\tau_1\sigma_\theta^2+\frac{1}{4}\left(\frac{1}{1+\hat{\delta}_2}\right)^2\tau_2\sigma_\theta^2+\frac{1}{4}\left(\tau_1\sigma_\theta^2+\tau_2\sigma_\theta^2-\tau_2\sigma_\theta^2\left(1-\frac{1}{1+\hat{\delta}_2}\right)^2\right)\\ Var\left(p_2\right) &= \frac{1}{4}\tau_2\sigma_\theta^2+\frac{1}{4}\left(\frac{1}{1+\hat{\delta}_1}\right)^2\tau_1\sigma_\theta^2+\frac{1}{4}\left(\tau_2\sigma_\theta^2+\tau_1\sigma_\theta^2-\tau_1\sigma_\theta^2\left(1-\frac{1}{1+\hat{\delta}_1}\right)^2\right)\end{aligned}$$

$$Var\left(p_1\right)=\left(\frac{1}{2}\tau_1+\frac{1}{2}\frac{1}{1+\hat{\delta}_2}\tau_2\right)\sigma_\theta^2=T_1\sigma_\theta^2$$
$$Var\left(p_2\right)=\left(\frac{1}{2}\tau_2+\frac{1}{2}\frac{1}{1+\hat{\delta}_1}\tau_1\right)\sigma_\theta^2=T_2\sigma_\theta^2$$
$$Cov(p_1,p_2)=\frac{1}{4}\frac{1}{1+\hat{\delta}_2}\tau_2\sigma_\theta^2+\frac{1}{4}\frac{1}{1+\hat{\delta}_1}\tau_1\sigma_\theta^2+\rho\lambda_1\lambda_2\sqrt{\frac{1}{1+\hat{\delta}_2}\frac{1}{1+\hat{\delta}_1}}\sigma_{y_1}\sigma_{y_2}$$

$$Cov(p_1,\pi)=\left(\frac{1}{2}\tau_1+\frac{1}{2}\frac{1}{1+\hat{\delta}_2}\tau_2\right)\sigma_\theta^2=Var(p_1)$$
$$Cov(p_2,\pi)=\left(\frac{1}{2}\tau_2+\frac{1}{2}\frac{1}{1+\hat{\delta}_1}\tau_1\right)\sigma_\theta^2=Var(p_2)$$

Then,

$$Var\left(I\right)=\left(A_1^2+2A_1B\right)Var(p_1)+\left(A_2^2+2A_2B\right)Var(p_2)+B^2Var(\pi)+2A_1A_2Cov(p_1,p_2)=$$
$$=\left(A_1^2+2A_1\bar{c}-2T_1A_1^2-2T_2A_1A_2\right)Var(p_1)+\left(A_2^2+2A_2\bar{c}-2T_1A_1A_2-2T_2A_2^2\right)Var(p_2)+$$
$$+\left(\bar{c}-T_1A_1-T_2A_2\right)^2\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)+2A_1A_2Cov\left(p_1,p_2\right)$$

FOC:

$$Var(I)_{A_1}=0$$
$$Var(I)_{A_2}=0$$

$$Var(I)_{A_1}=\left(2\left(1-2T_1\right)A_1+2\bar{c}-2T_2A_2\right)Var(p_1)+\left(-2T_1A_2\right)Var(p_2)-$$
$$-2T_1\left(\bar{c}-T_1A_1-T_2A_2\right)\times\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)+2A_2Cov(p_1,p_2)=0$$

Rewrite it as

$$\left(2\left(1-2T_1\right)A_1+2\bar{c}-2T_2A_2\right)T_1\sigma_\theta^2+\left(-2T_1A_2\right)T_2\sigma_\theta^2-4\left(\bar{c}-T_1A_1-T_2A_2\right)T_1\sigma_\theta^2+2A_2Cov(p_1,p_2)$$
$$-2T_1\left(\bar{c}-T_1A_1-T_2A_2\right)\times\sigma_\varepsilon^2=$$
$$=\left(2A_1-4T_1A_1+2\bar{c}-2T_2A_2-2T_2A_1-4\bar{c}+4T_1A_1+4T_2A_2\right)T_1\sigma_\theta^2+2A_2Cov(p_1,p_2)$$
$$-2T_1\left(\bar{c}-T_1A_1-T_2A_2\right)\times\sigma_\varepsilon^2=$$
$$=\left(2A_1-2\bar{c}\right)T_1\sigma_\theta^2+2A_2Cov(p_1,p_2)-2T_1\left(\bar{c}-T_1A_1-T_2A_2\right)\times\sigma_\varepsilon^2=0$$
$$A_1\left(\sigma_\theta^2+T_1\sigma_\varepsilon^2\right)+A_2\left(\frac{Cov(p_1,p_2)}{T_1}+T_2\sigma_\varepsilon^2\right)=\bar{c}\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)$$

Then, we can guess the form of the second equation since it should be symmetric:

$$A_2\left(\sigma_\theta^2+T_2\sigma_\varepsilon^2\right)+A_1\left(\frac{Cov(p_1,p_2)}{T_2}+T_1\sigma_\varepsilon^2\right)=\bar{c}\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)$$

Next, we observe that

$$A_2\left(\sigma_\theta^2+T_2\sigma_\varepsilon^2\right)+A_1\left(\frac{Cov(p_1,p_2)}{T_2}+T_1\sigma_\varepsilon^2\right)=A_1\left(\sigma_\theta^2+T_1\sigma_\varepsilon^2\right)+A_2\left(\frac{Cov(p_1,p_2)}{T_1}+T_2\sigma_\varepsilon^2\right)$$
$$A_1\left(\frac{Cov(p_1,p_2)}{T_2}-\sigma_\theta^2\right)=A_2\left(\frac{Cov(p_1,p_2)}{T_1}-\sigma_\theta^2\right)$$
$$\frac{A_1}{A_2}=\frac{\frac{Cov(p_1,p_2)}{T_1}-\sigma_\theta^2}{\frac{Cov(p_1,p_2)}{T_2}-\sigma_\theta^2}$$

$$A_2\left(\sigma_\theta^2+T_2\sigma_\varepsilon^2+\frac{\frac{Cov(p_1,p_2)}{T_1}-\sigma_\theta^2}{\frac{Cov(p_1,p_2)}{T_2}-\sigma_\theta^2}\left(\frac{Cov(p_1,p_2)}{T_2}+T_1\sigma_\varepsilon^2\right)\right)=\bar{c}\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)$$
$$A_2=\bar{c}\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\frac{Cov(p_1,p_2)}{T_2}-\sigma_\theta^2\right)}{\left(\sigma_\theta^2+T_2\sigma_\varepsilon^2\right)\left(\frac{Cov(p_1,p_2)}{T_2}-\sigma_\theta^2\right)+\left(\frac{Cov(p_1,p_2)}{T_1}-\sigma_\theta^2\right)\left(\frac{Cov(p_1,p_2)}{T_2}+T_1\sigma_\varepsilon^2\right)}$$

Then,

$$A_1=\bar{c}\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\frac{Cov(p_1,p_2)}{T_1}-\sigma_\theta^2\right)}{\left(\sigma_\theta^2+T_2\sigma_\varepsilon^2\right)\left(\frac{Cov(p_1,p_2)}{T_2}-\sigma_\theta^2\right)+\left(\frac{Cov(p_1,p_2)}{T_1}-\sigma_\theta^2\right)\left(\frac{Cov(p_1,p_2)}{T_2}+T_1\sigma_\varepsilon^2\right)}$$

The denominator:

$$\left(\sigma_\theta^2+T_2\sigma_\varepsilon^2\right)\left(\frac{Cov(p_1,p_2)}{T_2}-\sigma_\theta^2\right)+\left(\frac{Cov(p_1,p_2)}{T_1}-\sigma_\theta^2\right)\left(\frac{Cov(p_1,p_2)}{T_2}+T_1\sigma_\varepsilon^2\right)=$$
$$-\sigma_\theta^4+\sigma_\varepsilon^2Cov(p_1,p_2)-\sigma_\theta^2T_2\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)^2}{T_1T_2}+\sigma_\varepsilon^2Cov(p_1,p_2)-\sigma_\theta^2T_1\sigma_\varepsilon^2=$$
$$=-\sigma_\theta^4+2\sigma_\varepsilon^2Cov(p_1,p_2)-\sigma_\theta^2\sigma_\varepsilon^2\left(T_2+T_1\right)+\frac{Cov(p_1,p_2)^2}{T_1T_2}$$

Proof of Lemma 2

With symmetric markets: cross-market costs are the same $\hat{\delta}_1=\hat{\delta}_2=\hat{\delta}$, liquidity is the same in both markets $\sigma_{y_1}^2=\sigma_{y_2}^2=\sigma_y^2$

$$\lambda_1 = \frac{1}{2}\sqrt{\left(\tau_1 + \tau_2 - \tau_2\left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_y^2}}$$

$$\lambda_2 = \frac{1}{2}\sqrt{\left(\tau_2 + \tau_1 - \tau_1\left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_y^2}}$$

And to find the optimal precision, I assume the simplest cost function $g(\tau) = c\frac{2}{3}\tau^{\frac{3}{2}}$

$$\sigma_\theta^2\left(\frac{1}{4\lambda_1} + \frac{1}{4\left(1+\hat{\delta}\right)\lambda_2}\right) = c\sqrt{\tau_1}$$

$$\sigma_\theta^2\left(\frac{1}{4\lambda_2} + \frac{1}{4\left(1+\hat{\delta}\right)\lambda_1}\right) = c\sqrt{\tau_2}$$

We can notice that the system is symmetric: if (τ_1^*, τ_2^*) solves the system, then (τ_2^*, τ_1^*) solves it. Next, we can notice that for any fixed τ_2, there is only one function $\tau_1 = f(\tau_2)$ that solves the first equation, since for any fixed τ_2, the left hand side decreases in τ_1 while the right-hand side increases in τ_1. From the symmetry argument it follows that if $(f(\tau_2), \tau_2)$ solves the system, then $(f^{-1}(\tau_2), \tau_2)$ solves it too. Thus, $f(\tau_2) = f^{-1}(\tau_2) = \tau_2$. So, the only solution is

$$\tau_1 = \tau_2 = \tau$$

τ that solves the system can be found from any of the equations above:

$$\frac{1}{4}\sigma_\theta^2\left(1 + \frac{1}{1+\hat{\delta}}\right)\frac{1}{\lambda} = cg(\tau)' = c\sqrt{\tau}$$

The price impact is the same in each market:

$$\lambda = \frac{1}{2}\sqrt{\tau\left(2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_y^2}}$$

Plug this in the equation above:

$$\frac{1}{2}\left(1 + \frac{1}{1+\hat{\delta}}\right)\sqrt{\frac{\sigma_y^2\sigma_\theta^2}{\left(2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\tau}} = c\sqrt{\tau}$$

$$\tau = \frac{\sigma_y \sigma_\theta}{2c} \sqrt{\frac{\left(1 + \frac{1}{1+\hat{\delta}}\right)^2}{2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2}} \tag{12}$$

It can be shown that $\tau\left(\frac{1}{1+\hat{\delta}}\right)$ is an increasing function of $\frac{1}{1+\hat{\delta}}$, and therefore is a decreasing function of $\hat{\delta}$.

And then, the price impact is

$$\lambda = \sqrt{\frac{1}{4}\tau\left(2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_y^2}}$$

λ obviously increases with $\frac{1}{1+\delta}$.

$$Cov(p_1, p_2) = \frac{1}{2}\frac{1}{1+\hat{\delta}}\tau\sigma_\theta^2 + \rho\frac{1}{1+\hat{\delta}}\lambda^2\sigma_y^2 = \frac{1}{2}\frac{1}{1+\hat{\delta}}\tau\sigma_\theta^2 + \frac{1}{4}\rho\frac{1}{1+\hat{\delta}}\tau\left(2 - \left(1 - \frac{1}{1+\hat{\delta}}\right)^2\right)\sigma_\theta^2$$

$$Cov(p_1, p_2) = \frac{1}{2}\tau\sigma_\theta^2\frac{1}{1+\hat{\delta}}\left(1 + \frac{1}{2}\rho\left(2 - \left(1 - \frac{1}{1+\delta}\right)^2\right)\right)$$

So, prices become less correlated as fragmentation becomes more severe ($\hat{\delta}$ goes up). So, the informativeness principle would tell us that it is optimal to include both of the two imperfectly correlated signals in the manager's contract (Holmstrom 1979). However, the information production is also affected by cross-market costs. So, the overall effect on the optimal contract and the moral hazzard issue is not that obvious.

The profit of each informed trader equals in this case:

$$ER = \tau\frac{1}{4}\sigma_\theta^2\left(1 + \frac{1}{1+\hat{\delta}}\right)\frac{1}{\lambda} - \frac{2}{3}c\tau^{\frac{3}{2}} = \tau\left(\frac{1}{4}\sigma_\theta^2\left(1 + \frac{1}{1+\hat{\delta}}\right)\frac{1}{\lambda} - \frac{2}{3}c\sqrt{\tau}\right) = c\frac{1}{3}\tau^{\frac{3}{2}}$$

Proof of Lemma 3 In Lemma 1, it has been shown what the optimal contract looks like: In the symmetric case, $T_1 = T_2 = T$. The solution is $A_1 = A_2 = A$:

$$A = \frac{\sigma_\theta^2 + \sigma_\varepsilon^2}{\sigma_\theta^2 + T\sigma_\varepsilon^2 + \frac{Cov(p_1,p_2)}{T} + T\sigma_\varepsilon^2}\bar{c} = \frac{\sigma_\theta^2 + \sigma_\varepsilon^2}{\sigma_\theta^2 + 2T\sigma_\varepsilon^2 + \frac{Cov(p_1,p_2)}{T}}\bar{c}$$

The optimal weight of the earnings report in the contract:

$$B = \bar{c} - 2TA = \frac{\sigma_\theta^2(1 - 2T) + \frac{Cov(p_1,p_2)}{T}}{\sigma_\theta^2 + 2T\sigma_\varepsilon^2 + \frac{Cov(p_1,p_2)}{T}}\bar{c}$$

Proof of Proposition 2 For the optimal contract from Lemma 3, the variance of the compensation is

$$\begin{aligned}
&Var\left(I\right) = 2A\left(A+2B\right)T\sigma_\theta^2 + B^2\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right) + 2A^2Cov(p_1,p_2) = \\
&= 2A\left((1-4T)A+2\bar{c}\right)T\sigma_\theta^2 + 2(\bar{c}^2-4TA\bar{c}+4T^2A^2)\sigma_\theta^2 + 2A^2Cov(p_1,p_2) + \\
&+ \sigma_\varepsilon^2\left(\bar{c}^2-4TA\bar{c}+4T^2A^2\right) = \\
&= 2T\sigma_\theta^2A^2 - 8T^2\sigma_\theta^2A^2 + 4T\sigma_\theta^2\bar{c}A + (2\sigma_\theta^2\bar{c}^2 - 8T\sigma_\theta^2\bar{c}A + 8T^2\sigma_\theta^2A^2) + 2A^2Cov(p_1,p_2) + \\
&+ \sigma_\varepsilon^2\left(\bar{c}^2-4TA\bar{c}+4T^2A^2\right) = \\
&= \left(2T\sigma_\theta^2+4T^2\sigma_\varepsilon^2\right)A^2 - 4T\bar{c}\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)A + \left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)\bar{c}^2 + 2Cov(p_1,p_2)A^2 = \\
&= 2T\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+2T\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)}{T}}\bar{c}^2 - 4T\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+2T\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)}{T}}\bar{c}^2 + \left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)\bar{c}^2 = \\
&= \left(\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right) - 2T\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+2T\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)}{T}}\right)\bar{c}^2
\end{aligned}$$

Analyse this expression in more detail. First, remember

$$T = \frac{1}{2}\tau\left(1+\frac{1}{1+\hat{\delta}}\right) \tag{13}$$

where τ is given by Equation 11. So,

$$T = \frac{1}{2}\frac{\sigma_y\sigma_\theta}{2c}\frac{\left(1+\frac{1}{1+\hat{\delta}}\right)^2}{\sqrt{2-\left(1-\frac{1}{1+\hat{\delta}}\right)^2}}$$

$$\text{sign}\{Var(I)'_x\} = \text{sign}\left\{\left(-2T\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+2T\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)}{T}}\right)'_x\right\}$$

$$2T\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+2T\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)}{T}} = \left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2\frac{2T}{\sigma_\theta^2+2T\sigma_\varepsilon^2+\sigma_\theta^2\frac{\frac{1}{1+\hat{\delta}}}{1+\frac{1}{1+\hat{\delta}}}\left(1+\frac{1}{2}\rho\left(2-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)\right)} =$$

$$= \left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2\frac{\frac{1}{2}\frac{\sigma_y\sigma_\theta}{2c}\frac{\left(1+\frac{1}{1+\hat{\delta}}\right)^2}{\sqrt{2-\left(1-\frac{1}{1+\hat{\delta}}\right)^2}}}{\frac{1}{2}\sigma_\theta^2+\frac{1}{2}\frac{\sigma_y\sigma_\theta}{2c}\frac{\left(1+\frac{1}{1+\hat{\delta}}\right)^2}{\sqrt{2-\left(1-\frac{1}{1+\hat{\delta}}\right)^2}}\sigma_\varepsilon^2+\frac{1}{2}\sigma_\theta^2\frac{\frac{1}{1+\hat{\delta}}}{1+\frac{1}{1+\hat{\delta}}}\left(1+\frac{1}{2}\rho\left(2-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)\right)} =$$

$$= \frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2\frac{1}{4c}\mathcal{L}(1+x)^3}{\frac{1}{2}\sqrt{2-(1-x)^2}\left(1+x\right)+\frac{1}{4c}\mathcal{L}\left(1+x\right)^3\sigma_\varepsilon^2+\sqrt{2-(1-x)^2}x\left(1+\frac{1}{2}\rho\left(2-(1-x)^2\right)\right)}$$

where $\mathcal{L} = \frac{\sigma_y}{\sigma_\theta}$, $x = \frac{1}{1+\hat{\delta}}$.

On the interval $x \in [0,1]$, this function decreases and then starts to increase. This function reaches its minimum at some point $x^* \in (0,1)$. x^* does not change with $\frac{\sigma_\varepsilon^2}{4c}\mathcal{L}$. The mathematical argument is as follows. If we rewrite $\widehat{Var}(I)$ as follows:

$$\frac{\frac{1}{4c}\mathcal{L}}{\frac{1}{4c}\mathcal{L}\sigma_\varepsilon^2 + \frac{1}{2}\frac{\sqrt{2-(1-x)^2}}{(1+x)^2} + \frac{\sqrt{2-(1-x)^2}x}{(1+x)^3}\left(1+\frac{1}{2}\rho\left(2-(1-x)^2\right)\right)} = \frac{\frac{1}{4c}\mathcal{L}}{\frac{1}{4c}\mathcal{L}\sigma_\varepsilon^2 + \tilde{f}(x,\rho)} \tag{14}$$

where extremum points of $\widehat{Var}(I)$ and $\tilde{f}(x,\rho)$ are the same and depend only on ρ

Proof of Lemma 4

Consider the case when the principal sells only to one market. Without loss of generality, consider market 1.

Then, the stock market equilibrium:

$$p_1 = \bar{e} + \frac{E[\pi|s_1]-\bar{e}}{2} + \frac{1}{1+\hat{\delta}}\frac{E[\pi|s_2]-\bar{e}}{2} + \lambda_1 y_1$$

The equilibrium orders sent by informed traders:

$$x_{11} = \frac{\tau_1(s_1-\bar{e})}{2\lambda_1}$$
$$x_{12} = \frac{\tau_2(s_2-\bar{e})}{2\lambda_1\left(1+\hat{\delta}\right)}$$

And the price impact is:

$$\lambda_1 = \frac{1}{2}\sqrt{\left(\tau_1+\tau_2-\tau_2\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)\frac{\sigma_\theta^2}{\sigma_y^2}}$$

The profit of each informed trader after the signals are observed:

$$E[\pi_1|s_1] = \lambda_1 x_{11}^2 = \frac{(E[\pi|s_1]-\bar{e})^2}{4\lambda_1} = \frac{\tau_1^2(s_1-\bar{e})^2}{4\lambda_1}$$
$$E[\pi_2|s_2] = (\lambda_1+\delta_2)x_{12}^2 = \frac{\tau_2^2(s_2-\bar{e})^2}{4\lambda_1\left(1+\hat{\delta}\right)}$$

The expected payoff of each informed trader prior to observing the signals:

$$\begin{cases} E[R_1] = \dfrac{\tau_1\sigma_\theta^2}{4\lambda_1} - \dfrac{2}{3}c\tau_1^{\frac{3}{2}} \\ E[R_2] = \dfrac{\tau_2\sigma_\theta^2}{4\lambda_1(1+\hat{\delta})} - \dfrac{2}{3}c\tau_2^{\frac{3}{2}} \end{cases}$$

FOC:

$$\begin{cases} \dfrac{\sigma_\theta^2}{4\lambda_1} = c\sqrt{\tau_1} \\ \dfrac{\sigma_\theta^2}{4\lambda_1(1+\hat{\delta})} = c\sqrt{\tau_2} \end{cases}$$

Then, $\tau_1 = \left(1+\hat{\delta}\right)^2 \tau_2$.

$$\lambda_1 = \frac{1}{2}\sqrt{\left(\tau_1 + \tau_1\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)\right)\frac{\sigma_\theta^2}{\sigma_y^2}} =$$

$$= \frac{1}{2}\sqrt{\tau_1}\frac{\sigma_\theta}{\sigma_y}\sqrt{1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}$$

Then,

$$\tau_1 = \frac{1}{c}\frac{\sigma_\theta\sigma_y}{2}\frac{1}{\sqrt{1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}} \tag{15}$$

$$\tau_2 = \frac{1}{c}\frac{\sigma_\theta\sigma_y}{2}\frac{\left(\frac{1}{1+\hat{\delta}}\right)^2}{\sqrt{1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}} =$$

And thus the expected payoff:

$$\begin{cases} E[R_1] = c\dfrac{1}{3}\tau_1^{\frac{3}{2}} \\ E[R_2] = c\dfrac{1}{3}\tau_2^{\frac{3}{2}} = \left(\dfrac{1}{1+\hat{\delta}}\right)^3 E[R_1] \end{cases}$$

$$Var(p_1) = \frac{1}{4}\tau_1\sigma_\theta^2 + \frac{1}{4}\left(\frac{1}{1+\hat{\delta}}\right)^2\tau_2\sigma_\theta^2 + \frac{1}{4}\tau_1\sigma_\theta^2\left(1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)\right) =$$

$$= \frac{1}{4}\tau_1\sigma_\theta^2\left(1+\left(\frac{1}{1+\hat{\delta}}\right)^4 + \left(1+\left(\frac{1}{1+\hat{\delta}}\right)^2 \times \frac{1}{1+\hat{\delta}}\left(2-\frac{1}{1+\hat{\delta}}\right)\right)\right) = \frac{1}{4}\tau_1\sigma_\theta^2\left(1+1+2\left(\frac{1}{1+\hat{\delta}}\right)^3\right) =$$

$$= \frac{1}{2}\tau_1\sigma_\theta^2\left(1+\left(\frac{1}{1+\hat{\delta}}\right)^3\right) = \frac{1}{2}\sigma_\theta^2\left(\tau_1 + \left(\frac{1}{1+\hat{\delta}}\right)\tau_2\right)$$

Then, the optimal contract solves:

$$\min_{A,B}\Big\{Var(I)\Big\},$$

subject to

$$e = argmax_{e'_\pi}\Big\{E[I]-c(e)\Big\},$$

where

$$Var(I) = A^2Var(p_1)+B^2Var(\pi)+2AB\times Cov(p_1,\pi) =$$
$$= A^2\frac{1}{2}\sigma_\theta^2\left(\tau_1+\left(\frac{1}{1+\hat{\delta}}\right)\tau_2\right)+B^2\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)+2AB\times\left(\frac{1}{2}\sigma_\theta^2\tau_1+\frac{1}{1+\hat{\delta}}\frac{1}{2}\sigma_\theta^2\tau_2\right)$$

subject to

$$A\left(\frac{1}{2}\tau_1+\frac{1}{1+\hat{\delta}}\frac{1}{2}\tau_2\right)+B=\bar{c}$$

Denote

$$T' = \frac{1}{2}\left(\tau_1+\frac{1}{1+\hat{\delta}}\tau_2\right)=\frac{1}{2}\tau_1\left(1+\left(\frac{1}{1+\hat{\delta}}\right)^3\right)$$

Recall that τ_1 is given by equation (7):

$$T' = \frac{1}{2c}\frac{\sigma_\theta\sigma_y}{2}\frac{1}{\sqrt{1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}}\left(1+\left(\frac{1}{1+\hat{\delta}}\right)^3\right)$$

$$Var(I) = A^2\sigma_\theta^2T'+\left(\bar{c}-T'A\right)^2\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)+2A\left(\bar{c}-T'A\right)T'\sigma_\theta^2 =$$
$$= T'\sigma_\theta^2A^2+\bar{c}^2\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)-2T'\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)\bar{c}A+T'^2\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)A^2+2T'\sigma_\theta^2\bar{c}A-2T'^2\sigma_\theta^2A^2 =$$
$$= T'\left(\sigma_\theta^2+T'\sigma_\varepsilon^2\right)A^2-2T'\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\bar{c}A+\bar{c}^2\left(2\sigma_\theta^2+\sigma_\varepsilon^2\right)$$

Then, the minimum is

$$A=\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\theta^2+T'\sigma_\varepsilon^2}\bar{c}$$

Then, the variance:

$$Var(I)=\left(2\sigma_\theta^2+\sigma_\varepsilon^2-T'\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+T'\sigma_\varepsilon^2}\right)\bar{c}^2=\widehat{Var}(I)\bar{c}^2$$

Similarly to the two-market case, the optimal effort level in equilibrium is

$$e=\frac{1}{rk^2\widehat{Var}(I)+k}$$

Compare the variance of the compensation under the optimal contract with one and two markets. For the comparison to be fair, assume that two markets are twice as small as a single market: $\sigma_y = \frac{1}{2}\sigma'_y$

$$\left(2\sigma_\theta^2+\sigma_\varepsilon^2 - T'\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+T'\sigma_\varepsilon^2}\right) > \left(2\sigma_\theta^2+\sigma_\varepsilon^2-2T\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\sigma_\theta^2+2T\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)}{T}}\right)$$

$$\frac{T'}{\sigma_\theta^2+T'\sigma_\varepsilon^2} < \frac{T}{\frac{1}{2}\sigma_\theta^2+T\sigma_\varepsilon^2+\frac{Cov(p_1,p_2)}{2T}}$$

$$\frac{\frac{1}{2c}\sigma_\theta\sigma_y\frac{1+\left(\frac{1}{1+\hat\delta}\right)^3}{\sqrt{1+\left(\frac{1}{1+\hat\delta}\right)^2\left(1-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)}}}{\sigma_\theta^2+\frac{1}{2c}\sigma_\theta\sigma_y\frac{1+\left(\frac{1}{1+\hat\delta}\right)^3}{\sqrt{1+\left(\frac{1}{1+\hat\delta}\right)^2\left(1-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)}}\sigma_\varepsilon^2} <$$

$$\frac{\frac{1}{2}\frac{\sigma_y\sigma_\theta}{2c}\frac{\left(1+\frac{1}{1+\hat\delta}\right)^2}{\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}}}{\frac{1}{2}\sigma_\theta^2+\frac{1}{2}\frac{\sigma_y\sigma_\theta}{2c}\frac{\left(1+\frac{1}{1+\hat\delta}\right)^2}{\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}}\sigma_\varepsilon^2+\frac{1}{2}\sigma_\theta^2\frac{\frac{1}{1+\hat\delta}}{1+\frac{1}{1+\hat\delta}}\left(1+\frac{1}{2}\rho\left(2-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)\right)}$$

$$\frac{\mathcal{L}\left(1+\left(\frac{1}{1+\hat\delta}\right)^3\right)}{\sqrt{1+\left(\frac{1}{1+\hat\delta}\right)^2\left(1-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)}+\frac{1}{2c}\mathcal{L}\left(1+\left(\frac{1}{1+\hat\delta}\right)^3\right)\sigma_\varepsilon^2} <$$

$$\frac{\frac{1}{2}\mathcal{L}\left(1+\frac{1}{1+\hat\delta}\right)^3}{\frac{1}{2}\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}\left(1+\frac{1}{1+\hat\delta}\right)+\frac{1}{4c}\mathcal{L}\left(1+\frac{1}{1+\hat\delta}\right)^3\sigma_\varepsilon^2+\frac{1}{2}\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}\frac{1}{1+\hat\delta}\left(1+\frac{1}{2}\rho\left(2-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)\right)}$$

$$\left(1+\left(\frac{1}{1+\hat\delta}\right)^3\right)\left(\frac{1}{2}\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}\left(1+\frac{1}{1+\hat\delta}\right)+\frac{1}{4c}\mathcal{L}\left(1+\frac{1}{1+\hat\delta}\right)^3\sigma_\varepsilon^2+\frac{1}{2}\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}\frac{1}{1+\hat\delta}\left(1+\frac{1}{2}\rho\left(2-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)\right)\right)$$

$$\frac{1}{2}\left(1+\frac{1}{1+\hat\delta}\right)^3\left(\sqrt{1+\left(\frac{1}{1+\hat\delta}\right)^2\left(1-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)}+\frac{1}{2c}\mathcal{L}\left(1+\left(\frac{1}{1+\hat\delta}\right)^3\right)\sigma_\varepsilon^2\right)$$

$$\left(1+\left(\frac{1}{1+\hat\delta}\right)^3\right)\left(\frac{1}{2}\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}\left(1+\frac{1}{1+\hat\delta}\right)+\frac{1}{2}\sqrt{2-\left(1-\frac{1}{1+\hat\delta}\right)^2}\frac{1}{1+\hat\delta}\left(1+\frac{1}{2}\rho\left(2-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)\right)\right) <$$

$$\frac{1}{2}\left(1+\frac{1}{1+\hat\delta}\right)^3\sqrt{1+\left(\frac{1}{1+\hat\delta}\right)^2\left(1-\left(1-\frac{1}{1+\hat\delta}\right)^2\right)}$$

Denote $x = \frac{1}{1+\hat{\delta}}$

$$(1+x) + x\left(1 + \frac{1}{2}\rho\left(2-(1-x)^2\right)\right) < \frac{1+2x+x^2}{1-x+x^2}\sqrt{\frac{1+x^2\left(1-(1-x)^2\right)}{2-(1-x)^2}}$$

$$\frac{1}{2}\rho x\left(2-(1-x)^2\right) < \frac{1+2x+x^2}{1-x+x^2}\sqrt{\frac{1+x^2\left(1-(1-x)^2\right)}{2-(1-x)^2}} - (1+2x)$$

$$\rho < \frac{2}{x(1+2x-x^2)}\frac{1+2x+x^2}{1-x+x^2}\sqrt{\frac{1+x^2\left(1-(1-x)^2\right)}{2-(1-x)^2}} - 2\frac{1+2x}{x(1+2x-x^2)}$$

which always holds for $\rho \in [0,1)$ and holds as an equality for $\rho = 1$ and $x = 1$

Proof of Lemma 5

$$\frac{(\sigma_y\sigma_\theta)^{\frac{3}{2}}}{3\sqrt{c}}\frac{1+\left(\frac{1}{1+\hat{\delta}}\right)^3}{\left(1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)\right)^{\frac{3}{4}}} > \frac{(\sigma_y\sigma_\theta)^{\frac{3}{2}}}{3\sqrt{2}\sqrt{c}}\frac{\left(1+\frac{1}{1+\hat{\delta}}\right)^{\frac{3}{2}}}{\left(2-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)^{\frac{3}{4}}}$$

Can be transformed to

$$\sqrt{2} > \left(\frac{1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}{1+\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}\right)^{\frac{3}{4}}\frac{\sqrt{1+\frac{1}{1+\hat{\delta}}}}{1-\frac{1}{1+\hat{\delta}}+\left(\frac{1}{1+\hat{\delta}}\right)^2}$$

which holds because

$$\sqrt[3]{2}\left(1+\frac{1}{1+\hat{\delta}}\right) > 1 > \left(\frac{1+\left(\frac{1}{1+\hat{\delta}}\right)^2\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}{1+\left(1-\left(1-\frac{1}{1+\hat{\delta}}\right)^2\right)}\right)^{\frac{1}{2}}$$

Since $\frac{1}{1+\hat{\delta}} \in (0,1)$.

References

[1] Budish, E., P.Cramton, J. Shim 2015. The High-Frequency Trading Arms Race: Frequent Batch Auctions as a Market Design Response. *The Quarterly Journal of Economics*, Volume 130, Issue 4

[2] Budish, E., R. Lee, J. Shim 2019. A Theory of Stock Exchange Competition and Innovation Will the Market Fix the Market. *National Bureau of Economic Research*

[3] Bennett, P. and L. Wei 2006. Market structure, fragmentation, and market quality. *Journal of Financial Markets*

[4] Cohen, B., D. Domanski, I. Fender, and H.S. Shin 2017. Global Liquidity: A Selective Review. *Annual Review of Economics*

[5] Chen, D. and D. Duffie 2020. Market Fragmentation. *National Bureau of Economic Research*

[6] Chowdhry, B. and V. Nanda, 1991. Multimarket Trading and Market Liquidity. *The Review of Financial Studies*

[7] Degryse H., F.De Jong, and V. van Kervel 2015. The Impact of Dark Trading and Visible Fragmentation on Market Quality. *Review of Finance*, Volume 19, Issue 4

[8] Domowitz, I., J. Glen, and A. Madhavan 1997. Market Segmentation and Stock Prices: Evidence from an Emerging Market. *The Journal of Finance*

[9] Foucault, T., M. Pagano, and A. Roell 2013. Market Liquidity *Oxford University Press*

[10] Foucault, T., A. Menkveld 2008. Competition for Order Flow and Smart Order Routing Systems. *The Journal of Finance*

[11] Holmstrom, B. 1979. Moral Hazard and Observability. *Bell Journal of Economics*

[12] Holmstrom, B. and J. Tirole 1993. Market Liquidity and Performance Monitoring. *Journal of Political Economy*, vol. 101, no. 4

[13] Jia, C., Y. Wang, and W. Xiong 2017. Market Segmentation and Differential Reactions of Local and Foreign Investors to Analyst Recommendations. *The Review of Financial Studies*, Volume 30, Issue 9

[14] Pagano, M. 1989 a. Endogenous Market Thinness and Stock Price Volatility. *The Review of Economic Studies*

[15] Pagano, M. 1989 b. Trading Volume and Asset Liquidity. *The Quarterly Journal of Economics*

[16] Ross, S. 1973. The Economic Theory of Agency: The Principal's Problem. *American Economic Association*

[17] R. Watts 1978. Systematic 'abnormal' returns after quarterly earnings announcements. *Journal of Financial Economics*