Production Technology, Information Acquisition and Disclosure, and Asset Prices⋆

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Abstract

We develop a model in which investors trade a long-lived asset whose dividend is contingent on a firm's production and show that a more efficient real economy can lead to a less efficient financial market. With a higher real production efficiency, measured by a larger output elasticity of capital, the sensitivity of the firm's income to its capital input decision increases. Investors who are uncertain about the firm's future decision thus perceive a higher risk of the asset's resale price and trade less aggressively. Consequently, the asset's price informativeness, trading volume, liquidity, and the investors' information extraction (the asset's risk premium, return volatility, and the investors' information production) decrease (increase). We also identify a new channel through which firms' information disclosure lowers financial price informativeness. Suggestive evidence of the negative relationship between production efficiency and market efficiency is provided.

Keywords: Production technology, Financial market efficiency, Information acquisition, Information disclosure, Output elasticity of capital

JEL Classification: D24, D82, G11, G12, G14

1. Introduction

Financial market efficiency and real efficiency are closely related. For example, a more efficient financial market can provide more precise information to firm managers, facilitating better real investment decisions (see, e.g., Benhabib et al. (2019)). However, these two kinds of efficiency are not always aligned. Goldstein and Yang (2014) find that because of the existence of externality,

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a more informative asset price renders real decision makers to overuse the information contained in the price, resulting in a lower real investment efficiency. From the perspective of competition, Peress (2010) shows that a higher real efficiency, indicated by a more competitive product market, decreases financial price informativeness because it mitigates firms' ability to transfer adverse shocks to customers, increasing the risks of the firms' securities and depressing informed trading.

In this paper, we investigate the misalignment between real efficiency and financial market efficiency from the perspective of production. We examine whether a more efficient production technology, indicated by a higher output elasticity of capital, increases or decreases financial price informativeness. The output elasticity of capital can be considered as a measure for production efficiency in the following sense. First, by definition, a higher output elasticity of capital improves the percentage increase in output caused by one percentage increase in capital input. Second, with a given amount of capital input, a higher output elasticity of capital increases the amount of output (i.e., expands the production possibility frontier). Third, as will be shown in our results, a higher output elasticity of capital corresponds to a higher profit for a firm.

We extend the overlapping-generation financial market framework in Farboodi and Veldkamp (2020) to a setting in which a firm produces and sells a real product. Capital is used as the factor of production. The firm faces both productivity shocks and demand shocks in the process of producing and selling. In each period, the firm discloses a noisy signal about its productivity to a financial market. In the financial market, investors trade a long-lived risky asset that pays a dividend contingent on the firm's operation in each period. Each investor is endowed with a limited data processing technology that can be allocated to acquire private information about the firm's product demand and/or noise traders' order flow.

Our analysis starts from a benchmark case in which investors' information acquisition decisions are exogenous, and the output elasticity of capital and public signal precision can vary over time. We analytically show that the investors' perceived asset payoff risk in a period is not affected by the current period's output elasticity of capital, but is increasing in the next period's output elasticity of capital. The mechanism is that the firm makes its capital input decision based on the public signal about its productivity and on the asset price, and the investors can also observe these two pieces of information. Therefore, the investors perfectly foresee the firm's decision in the current period. Consequently, although a higher output elasticity of capital in the current period increases the sensitivity of output to capital input, it does not increase the uncertainty faced by the investors.

In contrast, investors in the current period are less certain about the next period's capital input decision, because they cannot observe the information used by the firm in the next period. Since a higher output elasticity of capital in the next period increases the sensitivity of the next period's output to capital input, the next period's asset price will also be more sensitive to the next period's capital input which cannot be precisely predicted by the investors in the current period. Therefore, the current period's investors' perceived variance of the asset's resale price is increasing in the output elasticity of capital in the next period. In other words, a future advance in production efficiency increases the risk faced by the current investors. Moreover, the higher risk caused by a larger output elasticity of capital in the next period lowers the current period's investors' confidence in their prediction about the asset payoff. Therefore, they trade less aggressively, and less private information about the firm's product demand is injected into the price. This result indicates a negative relationship between real efficiency and financial market efficiency.

We also investigate the impacts of public information disclosure on financial price informativeness in the analytical benchmark model. We find that an increase in the current period's public disclosure increases price informativeness, while an increase in the next period's public disclosure decreases price informativeness. In the current period, if the firm discloses more information about its productivity, investors can better forecast the firm's value and the dividend, lowering their perceived dividend risk. Therefore, the investors will be more confident in their predictions about the asset payoff and trade more actively, injecting more information into the asset price. However, if the next period's public information disclosure increases, the investors in the next period will rely more on the public information, and the asset price in the next period will also be more sensitive to the public information. Since the investors in the current period do not know the realization of public information in the next period, the increase in the sensitivity of the next period's asset price to public information can increase the investors' perceived resale price risk and make the investors trade less actively. Therefore, an increase in future public disclosure may decrease current price informativeness.

Our results of the benchmark model suggest that an overall increase in the output elasticity of capital (i.e., the output elasticity of capital increases in every period) decreases financial price informativeness, and an overall increase in public disclosure can also decrease financial price informativeness. We investigate the effects of overall changes in output elasticity of capital and public disclosure using the full model in which investors' information acquisition decisions are endogenous.

Specifically, an investor optimally allocates her data processing capacity to information production (i.e., learning about real product demand) and information extraction (i.e., learning about noise traders' order flow). Moreover, we focus on stationary solutions in which the output elasticity of capital, public signal precision, and endogenous price coefficients are constant over time.

The solutions to our full model show that the financial price informativeness (investors' perceived asset payoff risk) is decreasing (increasing) in the output elasticity of capital, which is consistent with the analytical results in the benchmark model. Correspondingly, we find that information production (i.e., learning about real product demand) is increasing in output elasticity, and information extraction (i.e., learning about noise traders' order flow) is decreasing in output elasticity. We call investors' learning about noise traders' order flow "information extraction" because, with more information on the order flow, the investors can better filter out the noise contained in the price signal, and extract more information about the product demand from the asset price. When the asset price informativeness decreases because of an increase in the output elasticity of capital, the asset payoff risk perceived by the informed investors tends to increase because they learn less information about the real product demand from the asset price. Therefore, the investors optimally increase their private learning about the real product demand and decrease their learning about the noise traders' order flow since the data capacity is limited. Our full model also shows that the financial price informativeness is decreasing in public information disclosure, which is consistent with the analytical results in the benchmark model. Correspondingly, information production (information extraction) is increasing (decreasing) in public disclosure.

We also investigate the implications of the output elasticity of capital for cross-sectional asset pricing. Since a higher output elasticity of capital increases the sensitivity of the firm's income to the capital investment decision, it also increases the risky asset's return volatility. The risk premium is also increasing in the output elasticity of capital because a higher output elasticity of capital brings more risk to the risk-averse investors. Notably, the return volatility increases faster with the output elasticity of capital than the risk premium does, so the Sharpe ratio decreases with the output elasticity of capital. Moreover, we find that the asset's liquidity is decreasing in the output elasticity of capital. The reason is that when the output elasticity increases, informed investors' perceived payoff risk also increases, and their trading becomes less active. Therefore, less liquidity demand from the noise traders can be absorbed by the informed investors. As a result, the impact of noise traders' liquidity demand on the asset price becomes larger, and the liquidity decreases.

Related literature. Our paper belongs to the literature on financial price informativeness and investors' information acquisition decisions. Asset prices aggregate and reveal dispersed information (Hayek (1945); Grossman and Stiglitz (1980); Hellwig (1980)), and price informativeness measures the ability of an asset's price to aggregate and reveal information. Han and Yang (2013) find that the development of social networks can harm price informativeness when information is endogenous. Dávila and Parlatore (2021) find that the effect of transaction costs on price informativeness depends on the source of informational heterogeneity of investors. Among the papers in this strand of literature, ours is closely related to those that develop dynamic noisy rational expectations equilibrium models. Brunnermeier et al. (2022) argue that government intervention can divert investors' attention away from fundamentals and reduce price informativeness. Goldstein and Yang (2022) show that commodity futures' price informativeness is increasing in the mass of speculators and is decreasing in the mass of hedgers. Cai (2019) shows that more uncertainty can reduce investors' incentive to acquire information because it makes future investors trade less aggressively and lowers the sensitivity of asset price to information. Farboodi and Veldkamp (2020) show that as financial data technology advances, investors' learning about the fundamentals first increases and then decreases.

This paper also contributes to the literature on the relationship between real efficiency and financial market efficiency. It is natural to think that real efficiency and financial market efficiency are positively related. Benhabib et al. (2019) show that a higher financial price informativeness increases firms' expected profits. However, extant literature has shown that a higher financial price informativeness does not necessarily lead to a higher real efficiency (see, e.g., Dow and Gorton (1997), Goldstein and Yang (2014), and Bond et al. (2012)). There are also papers showing that a higher real efficiency can reduce financial market efficiency. For example, Peress (2010) finds that a more competitive product market (which indicates a higher real efficiency) decreases firms' ability to hedge against productivity shocks, increasing the firms' risks, reducing informed trading, and finally lowering financial price informativeness. We complement this strand of literature by showing that a more efficient production technology decreases financial market efficiency.

Our paper is also related to the literature on the negative impacts of public information disclosure. Goldstein and Yang (2019) show that when a firm discloses information that the real decision maker knows little about to the financial market, price informativeness may decrease. Goldstein and

Yang (2017) shows that when information production is endogenous, public disclosure can crowd out information production and reduce price informativeness. Amador and Weill (2010) demonstrate that releasing information about productivity may reduce the informativeness of the real price system. Qin (2013) and Christensen and Qin (2014) show that public information disclosure can be welfare-decreasing when investors have heterogeneous prior beliefs. Our paper complements this strand of literature by revealing another channel through which public information disclosure can harm financial market efficiency. Our result that price informativeness is decreasing in disclosure does not rely on the assumptions that private information acquisition is endogenous as in Goldstein and Yang (2017) or that the firm discloses something they know little about as in Goldstein and Yang (2019).

Our paper is organized as follows. Section 2 presents the dynamic model. Section 3 analyzes a benchmark case in which information is exogenous. Section 4 analyzes the full model in which information production and information extraction are endogenous. Section 5 provides some empirical evidence for our theoretical results. Section 6 concludes.

2. The model

In this section, we develop an overlapping-generation model of financial market with a firm in the real economy, by extending the dynamic framework in Farboodi and Veldkamp (2020) to a setting with a production function which is similar to that in Benhabib et al. (2019). We also provide a sketch of the approach to solving the model.

2.1. Model setup

In our model, a firm produces and sells a real product in each period. At the beginning of a period, a continuum of investors is born. They trade a risky financial asset whose dividend is contingent on the firm's income. At the end of the period, they receive the dividend payoff from the financial asset, resell their asset holdings to the next generation of investors, and consume their wealth.

2.1.1. Firm in the real economy

At the beginning of each period t, a firm decides how much capital it will invest to produce its products. The production will be finished and the products will be sold at the end of the period.

The production function of the firm is

$$Y_{t+1} = \bar{Z} A_{t+1} K_t^{\eta_t}, \tag{1}$$

where Y_{t+1} is the output (i.e., the quantity of the product) of the firm at the end of period t, constant $\bar{Z}$ represents the common productivity, A_{t+1} is the firm's specific productivity shock that realizes at the end of period t, K_t is the firm's capital input at the beginning of period t, and $\eta_t = \frac{dY_{t+1}/Y_{t+1}}{dK_t/K_t}$ is the output elasticity of capital in period t. The entire sequence of the output elasticities $\{\eta_t\}_{t=0}^{\infty}$ is common knowledge.

The market demand for the firm's product is

$$Y_{t+1} = \left(\frac{1}{P_{t+1}}\right)^{\theta} \epsilon_{t+1} \bar{Y}, \tag{2}$$

where P_{t+1} is the price of the firm's product sold at the end of period t, ϵ_{t+1} is the idiosyncratic shock to the demand for the firm's product at the end of period t, $\bar{Y}$ is a constant representing aggregate output, and $\theta = -\frac{dY_{t+1}/Y_{t+1}}{dP_{t+1}/P_{t+1}}$ is the absolute value of the price elasticity of demand for the product.

2.1.2. Financial market and investors

A risky financial asset whose dividend is contingent on the firm's income is traded in the financial market. At the end of period t, each unit of the asset pays a dividend d_{t+1} that follows

$$d_{t+1} - d_t = (1 - G)(\mu - d_t) + \alpha v_{t+1}, \tag{3}$$

where $\mu \geq 0$, $\alpha > 0$, and $G \in [0, 1)$ are constants. The shock to the dividend growth, $v_{t+1} = \ln(P_{t+1} Y_{t+1})$, is set as an increasing function of the firm's income at the end of period t. There is also a risk-free asset with a deterministic gross rate of return $R_f \geq 1$.

Remark 1 (Dividend process)**.** Our assumption on the dividend is for tractability.[1] It can be considered as an extension of the dividend process in Farboodi and Veldkamp (2020) to a setting with endogenous "dividend innovation". Moreover, this assumption does not conflict with reality,

[1]Ad hoc assumptions for tractability are usually inevitable in theoretical models. For example, in investment-based asset pricing models (see, e.g., Li et al. (2009)), while the dividend process is fully endogenized, the financial market is simply captured by an exogenously given stochastic discount factor process. In contrast, in our model (as well as in most of the REE financial market models), the investors' decisions and the asset price are fully endogenized.

as it at least captures the fact that companies tend to increase their dividend payouts if their incomes become higher. When $G > 0$ and $\mu > 0$, our assumption on the dividend also captures the "dividend smoothing" phenomenon observed in data (see, e.g., Leary and Michaely (2011)). Our qualitative results are unaffected if we set $G = \mu = 0$.

At the beginning of period t, a measure-one continuum of investors is born. Each investor $i \in [0, 1]$ is endowed with initial wealth $W_{i,t}$ and decides how much of this endowment will be invested in the risky financial asset and how much will be invested in the risk-free asset. At the end of the period t, the investor receives payoff from the assets and then sells all her holdings to the next generation of investors. Then she consumes all her resources. The budget constraint for investor i is

$$c_{i,t+1} = (W_{i,t} - m_{i,t}q_t)R_f + m_{i,t}(d_{t+1} + q_{t+1}), \tag{4}$$

where $c_{i,t+1}$ is the investor's consumption in period $t + 1$, $m_{i,t}$ is the investor's risky asset holding in period t, and q_t is the price of this financial asset at the beginning of period t.

2.1.3. Uncertainties and information

Assume that the firm-specific productivity shocks are log-normally distributed,

$$a_{t+1} = \ln A_{t+1} \sim N\left(-\frac{1}{2}\tau_a^{-1}, \tau_a^{-1}\right).$$

At the beginning of period t, the firm receives a noisy signal about the current period's productivity shock, $s_t = a_{t+1} + e_t$, where $e_t \sim N(0, \tau_{s,t}^{-1})$. The firm will disclose s_t immediately to the financial market after receiving it. We also assume that the entire sequence of public signal precision $\{\tau_{s,t}\}_{t=0}^{\infty}$ is common knowledge.

Assume that the real demand shocks are also log-normally distributed,

$$\varepsilon_{t+1} = \ln \epsilon_{t+1} \sim N\left(-\frac{1}{2}\tau_\varepsilon^{-1}, \tau_\varepsilon^{-1}\right).$$

Investor i can observe a signal about the real demand shock, $x_t^i = \varepsilon_{t+1} + \mathcal{Q}_t^i$, where $\mathcal{Q}_t^i \sim N(0, \tau_{xi,t}^{-1})$. Noise traders also participate in financial trading, and their financial demand in period t is $n_{t+1} \sim N(0, \tau_n^{-1})$. Informed investor i can also observe a signal about the noise traders' order flow, $g_t^i = n_{t+1} + \mathcal{Q}_{n,t}^i$, where $\mathcal{Q}_{n,t}^i \sim N(0, \tau_{gi,t}^{-1})$. We assume that $(\{a_t\}, \{\varepsilon_t\}, \{n_t\}, \{e_t\}, \{\mathcal{Q}_t^i\}, \{\mathcal{Q}_{n,t}^i\})$ are mutually independent. Define $I_t^- = I_{t-1} \cup \{d_t, \varepsilon_t, a_t, n_t\}$, $F_{i,t} = I_t^- \cup \{q_t, s_t, x_t^i, g_t^i\}$, and $I_t = \cup_{i \in [0,1]} F_{i,t}$. Intuitively, I_t^- is all information available before the beginning of period t, $F_{i,t}$

is investor i's information set when making investment decision in period t, and I_t contains all information in the financial market when investors are trading in period t.

2.2. Financial market equilibrium

Now we consider the equilibrium with exogenous information. This equilibrium consists of a real market equilibrium in which the firm maximizes its expected profit and the product market clears, and a financial market equilibrium in which investors maximize their expected utility and the financial market clears.

2.2.1. Firm's problem

The problem faced by the firm in period t is to maximize its expected profit in the current period conditional on information about its productivity shock and real demand shock,

$$\max_{K_t} E\left[P_{t+1}Y_{t+1} - R_f K_t | s_t, q_t\right], \tag{5}$$

subject to the production function (1) and real demand function (2). The financial asset price is a source of information for the firm because it aggregates investors' dispersed information about the product demand.

2.2.2. Investors' portfolio selection

Investor i's portfolio choice problem in period t is to maximize the expected utility of end-of-period consumption conditional on the information set $F_{i,t}$,

$$\max_{m_{i,t}} E\left[U(c_{i,t+1})|F_{i,t}\right], \tag{6}$$

subject to the budget constraint (4). We assume that investors have constant absolute risk aversion (CARA) utility , i.e., $U(c) = -e^{-\gamma c}$, where γ is the risk aversion coefficient.

2.2.3. Definition of the financial market equilibrium

Definition 1 (Financial market equilibrium)**.** *A financial market equilibrium is a sequence of financial prices* $\{q_t\}$*, investors' portfolio choices* $\{m_{i,t}\}$*, the firm's investment decision* $\{K_t\}$*, and the real product prices* $\{P_t\}$*, such that in each period* t*,*

(i) investors and the firm use Bayes' law to update their beliefs with their available information;

(ii) each investor i *chooses the risky asset holding* $m_{i,t}$ *to solve for problem* (6)*, taking financial*

price q_t and choices of other investors as given, subject to budget constraint (4)*;*
(iii) the financial asset price q_t clears the financial market, i.e.,

$$\int_0^1 m_{i,t}di + n_{t+1} = 1; \tag{7}$$

(iv) the firm's investment decision K_t solves the expected profit maximization problem (5)*;*
(v) the real product price P_t clears the real market by equating the production function (1) *and the real demand function* (2)*.*

Although the equilibrium includes both the real product price and the financial asset price, our main focus is the financial asset price. Therefore, we call this equilibrium with exogenous information as the "financial market equilibrium".

2.3. Solving the model

We briefly discuss our approach to solving the model in this subsection. Our approach generalizes the approach in Farboodi and Veldkamp (2020) and involves four steps. In the first step, we clear the real product market and solve for the firm's value and the financial asset's dividend shock. In the second step, we solve for the firm's investment decision. In the third step, we solve for the investors' portfolio choices. In the fourth step, we clear the financial market. See Appendix A for a detailed approach to solving for the equilibrium.

2.3.1. Dividend

From the real production function (1) and the real demand function (2), we can solve for the logarithm of the real product's price at the end of period t,

$$p_{t+1} = -\frac{1}{\theta}(a_{t+1} + \eta_t k_t - \varepsilon_{t+1} + \bar{z} - \bar{y}), \tag{8}$$

where $\bar{z} = \ln \bar{Z}$, $\bar{y} = \ln \bar{Y}$, and $k_t = \ln K_t$. Moreover, the stochastic part of the dividend (or the logarithm of the firm's income) at the end of period t is

$$v_{t+1} = \frac{1}{\theta}\varepsilon_{t+1} + \left(1 - \frac{1}{\theta}\right) a_{t+1} + \eta_t \left(1 - \frac{1}{\theta}\right) k_t + \frac{1}{\theta}\bar{y} + \left(1 - \frac{1}{\theta}\right) \bar{z}. \tag{9}$$

2.3.2. Firm's investment decision

Solving for firm's investment problem (5), we can derive that the firm's investment decision at the beginning of period t is

$$K_t = K\left(s_t, q_t\right) = \left(\frac{1}{R_f}\right)^{\Theta_t} \left[\eta_t \left(1 - \frac{1}{\theta}\right) \bar{Y}^{\frac{1}{\theta}} \bar{Z}^{1-\frac{1}{\theta}}\right]^{\Theta_t} \left(E\left[A_{t+1}^{1-\frac{1}{\theta}} \epsilon_{t+1}^{\frac{1}{\theta}} | s_t, q_t\right]\right)^{\Theta_t}, \tag{10}$$

where $\Theta_t = -\frac{1}{\eta_t(1-\frac{1}{\theta})-1}$. Conjecture that the financial price at period t is

$$q_t = \beta_{0,t} + \beta_{1,t}(\varepsilon_{t+1} + \beta_{2,t}s_t + \beta_{3,t}n_{t+1}) + \beta_{4,t}(d_t - \mu), \tag{11}$$

where $\beta_{l,t}, \quad l = 0, 1, 2, 3, 4$ are deterministic coefficients. The price signal observed by the firm is

$$\hat{q}_t = \frac{q_t - \beta_{0,t} - \beta_{1,t}\beta_{2,t}s_t - \beta_{4,t}(d_t - \mu)}{\beta_{1,t}} = \varepsilon_{t+1} + \beta_{3,t}n_{t+1}, \tag{12}$$

where the signal precision is $\tau_{\hat{q},t} = (Var[\hat{q}_t|\varepsilon_{t+1}])^{-1} = \beta_{3,t}^{-2}\tau_n$. Using the Bayes' law, we can express the logarithm of the firm's optimal investment decision $k_t = \ln K_t$ as a linear combination of the productivity signal s_t, the price signal $\hat{q}_t$, and a non-random process ϕ_t.

2.3.3. Investors' portfolio choices

Solving for the portfolio choice problem for investor i (6), we have the optimal holding of risky asset in period t as

$$m_{i,t} = \frac{E\left[d_{t+1} + q_{t+1}|F_{it}\right] - R_f q_t}{\gamma Var\left[d_{t+1} + q_{t+1}|F_{it}\right]}. \tag{13}$$

The investors' perceived payoff variance can be expressed as

$$Var\left[d_{t+1} + q_{t+1}|F_{it}\right] = (1 + 2\beta_{4,t+1})Var\left[d_{t+1}|F_{it}\right] + Var\left[q_{t+1}|F_{it}\right]. \tag{14}$$

Equation (14) shows that the investors care about not only the risk of the dividend they receive from the asset, but also the risk of the asset price when they resell their holdings to investors of the next generation. This is an important difference of our dynamic model from static models in which investors only care about firms' liquidation values.

From the financial price formula (11) we can derive that the price signal for investor i is

$$\begin{aligned}\tilde{q}_t^i &= \frac{q_t - \beta_{0,t} - \beta_{1,t}\beta_{2,t}s_t - \beta_{1,t}\beta_{3,t}E[n_{t+1}|F_{i,t}] - \beta_{4,t}(d_t - \mu)}{\beta_{1,t}} \\ &= \varepsilon_{t+1} + \beta_{3,t}(n_{t+1} - E[n_{t+1}|g_t^i]),\end{aligned} \tag{15}$$

where the signal precision is $\tau_{qi,t} = \beta_{3,t}^{-2}(\tau_n + \tau_{gi,t})$. Notice that the signal about the noise traders' order flow is used to filter out the noise in the price signal. With the price signal $\tilde{q}_t^i$, the productivity signal s_t, the signal about real demand shock x_t^i, and the signal about noise traders' order flow g_t^i, we can calculate the conditional expectation and conditional variance in (13).

2.3.4. Clearing the financial market

Since all informed investors are ex ante identical, we focus on the symmetric equilibrium where for all $i \in [0,1]$, $\tau_{xi,t} = \tau_{xt}$, $\tau_{gi,t} = \tau_{gt}$, and $\tau_{qi,t} = \tau_{qt}$. Therefore, all investors' perceived payoff variance in period t are the same. Denote $Var\left[d_{t+1} + q_{t+1} | F_{it}\right]$ by V_{Ft}. Substituting the investors' optimal risky asset holdings (13) into the market clearing condition (7), and rearranging terms, we can express the asset price as

$$q_t = R_f^{-1} \left[\int_0^1 E[d_{t+1} + q_{t+1} | F_{i,t}] di + \gamma V_{Ft}(n_{t+1} - 1) \right]. \tag{16}$$

Further calculations show that the right-hand side of Eq. (16) is linear in the product demand shock ε_{t+1}, the public signal s_t, the noise traders' order flow n_{t+1}, and the dividend d_t. Comparing the implied price function (16) and the conjectured price function (11), we can derive a system of difference equations that characterizes the equilibrium, given signal precisions τ_{xt} and τ_{gt}. We summarize the above discussion in Proposition 1.

Proposition 1 (Equilibrium asset price)**.** *The equilibrium financial asset price can be expressed as*

$$q_t = \beta_{0,t} + \beta_{1,t}(\varepsilon_{t+1} + \beta_{2,t} s_t + \beta_{3,t} n_{t+1}) + \beta_{4,t}(d_t - \mu). \tag{17}$$

The financial price coefficients must satisfy the following system of difference equations,

$$\begin{aligned}
\beta_{1,t} &= \frac{\alpha}{R_f}(1 + \beta_{4,t+1}) \left[\left(\frac{1}{\theta}\right) \frac{\tau_{xt} + \tau_{qt}}{\tau_\varepsilon + \tau_{xt} + \tau_{qt}} + \left(1 - \frac{1}{\theta}\right) \eta_t \left(\frac{\Theta_t}{\theta}\right) \frac{\tau_{\hat{q}t}}{\tau_\varepsilon + \tau_{\hat{q}t}} \right], \\
\beta_{2,t} &= \frac{\alpha}{R_f}(1 + \beta_{4,t+1}) \left[\frac{\Theta_t}{\beta_{1,t}} \left(1 - \frac{1}{\theta}\right) \frac{\tau_{st}}{\tau_a + \tau_{st}} \right], \\
\beta_{3,t} &= \frac{1}{\alpha(1 + \beta_{4,t+1})} \left[\frac{\gamma V_{Ft} \theta (\tau_\varepsilon + \tau_{xt} + \tau_{qt})(\tau_n + \tau_{gt})}{(\tau_{xt} + \tau_{qt})(\tau_n + \tau_{gt}) - \tau_{qt}\tau_n} \right], \\
\beta_{4,t} &= \frac{G}{R_f}(1 + \beta_{4,t+1}),
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
V_{Ft} = (1 + \beta_{4,t+1})^2 \alpha^2 & \left[\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon + \tau_{xt} + \tau_{qt}} + \left(1 - \frac{1}{\theta}\right)^2 \frac{1}{\tau_a + \tau_{s,t}} \right] \\
& + \beta_{1,t+1}^2 \left[\frac{1}{\tau_\varepsilon} + \beta_{2,t+1}^2 \left(\frac{1}{\tau_a} + \frac{1}{\tau_{s,t+1}} \right) + \beta_{3,t+1}^2 \frac{1}{\tau_n} \right]
\end{aligned} \tag{19}$$

is the variance of the asset payoff $Var\left[d_{t+1} + q_{t+1} | F_{it}\right]$. *The equation for* $\beta_{4,t}$ *has a stationary solution,*

$$\beta_{4,t} = \beta_4 = \frac{G}{R_f - G}, \qquad \forall t. \tag{20}$$

The expression for $\beta_{0,t}$ *is provided in Appendix A.*

Proof. See Appendix A. □

3. Analysis of the financial market equilibrium

In this section, we analyze the financial market equilibrium with exogenous information precision. Specifically, we investigate the impacts of output elasticity of capital and public information disclosure on financial price informativeness. For simplicity and clarity we assume that there is no information extraction, i.e., $\tau_{gt} = 0$, $\forall t$. We will allow for information extraction in Section 4. For this section we also assume that information production is constant over time, i.e., $\tau_{xt} = \tau_x$, $\forall t$.

Corollary 1 (Characterization of the financial market equilibrium)**.** *Assume that* $\tau_{gt} = 0$ *and* $\tau_{xt} = \tau_x$*,* $\forall t$*. The equilibrium price informativeness* $1/\beta_{3,t}$ *is determined by the following equation,*

$$\left(\frac{1}{\beta_{3,t}}\right)^3 \left[\frac{\gamma\alpha R_f}{R_f - G}\left(1 - \frac{1}{\theta}\right)^2 \frac{\tau_n}{\tau_a + \tau_{st}} + \gamma Z_t \tau_n\right] + \left(\frac{1}{\beta_{3,t}}\right)\left[\frac{\gamma\alpha R_f}{R_f - G}\left(\frac{1}{\theta^2} + \left(1 - \frac{1}{\theta}\right)^2 \frac{\tau_\varepsilon + \tau_x}{\tau_a + \tau_{st}}\right) + \gamma Z_t(\tau_\varepsilon + \tau_x)\right] = \frac{\tau_x}{\theta}, \tag{21}$$

where

$$Z_t = \frac{R_f - G}{\alpha R_f}\beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon} + \beta_{2,t+1}^2\left(\frac{1}{\tau_a} + \frac{1}{\tau_{s,t+1}}\right) + \beta_{3,t+1}^2\frac{1}{\tau_n}\right]. \tag{22}$$

Proof. See Appendix B. □

As is standard in the noisy rational expectations equilibrium literature (see, e.g., Farboodi and Veldkamp (2020)), we define the financial price informativeness in period t as the asset price's signal-to-noise ratio $1/\beta_{3,t} = \beta_{1,t}/(\beta_{1,t}\beta_{3,t})$. Notice that the precision $\tau_{\hat{q},t} = \beta_{3,t}^{-2}\tau_n$ of the price signal $\hat{q}_t$ defined in (12) is increasing in the price informativeness. Therefore, the price informativeness measures how efficiently the financial asset price aggregates investors' private information.

Corollary 1 shows that the equilibrium financial price informativeness in period t is determined by both the current period's public disclosure $\tau_{s,t}$ and the next period's price coefficients $\beta_{1,t+1}$, $\beta_{2,t+1}$, $\beta_{3,t+1}$, and public disclosure $\tau_{s,t+1}$. Note that all the variables of period $t+1$ affect the price informativeness of period t through Z_t. Based on Proposition 1 and Corollary 1 we can investigate the impacts of output elasticity of capital and public disclosure on the equilibrium financial price informativeness.

3.1. Output elasticity of capital and price informativeness

Proposition 2 (Effects of output elasticity on financial price informativeness)**.** *The current period's equilibrium price informativeness,* $\frac{1}{\beta_{3,t}}$*, is* ***(i)*** *not affected by the current period's output elasticity of capital* η_t*, and* ***(ii)*** *decreasing in the next period's output elasticity of capital* η_{t+1}*, i.e.,* $\frac{\partial(1/\beta_{3,t})}{\partial \eta_t} = 0$ *and* $\frac{\partial(1/\beta_{3,t})}{\partial \eta_{t+1}} < 0$.

Proof. See Appendix B. □

Proposition 2 shows that an increase in the current period's output elasticity of capital does not affect the current period's financial price informativeness. However, an increase in the next period's output elasticity of capital lowers the current period's financial price informativeness. To understand the results, we delve deeper into how the output elasticity of capital affects the investors' perceived asset payoff risk.

Proposition 3 (Effects of output elasticity on investors' perceived variances)**.** ***(i)*** *An increase in the current period's output elasticity of capital* η_t *does not affect investors' perceived dividend variance* $Var[d_{t+1}|F_{it}]$*, future price variance* $Var[q_{t+1}|F_{it}]$*, and payoff variance* $Var[d_{t+1}+q_{t+1}|F_{it}]$*.* ***(ii)*** *The investors' perceived dividend variance* $Var[d_{t+1}|F_{it}]$*, future price variance* $Var[q_{t+1}|F_{it}]$*, and payoff variance* $Var[d_{t+1} + q_{t+1}|F_{it}]$ *in the current period are increasing in the next period's output elasticity of capital* η_{t+1}*.*

Proof. See Appendix B. □

Proposition 3 shows that an increase in the current period's output elasticity of capital does not affect investors' perceived payoff variance in the current period, while an increase in the next period's output elasticity of capital increases investors' perceived payoff variance in the current period. To explain the intuition of Proposition 3, notice that Eq. (9) shows that the output elasticity of capital is positively related to the sensitivity of the firm's income to the capital input. The firm's capital investment decision in period t, k_t, is made based on the public signal s_t and the financial price q_t. Since the investors can also observe the public signal s_t and the financial price q_t, they know exactly the realization of the current period's capital investment k_t. Therefore, an increase in the current period's output elasticity of capital η_t does not affect investors' perceived risk of the firm's income, $Var[v_{t+1}|F_{it}]$, and investors' perceived risk of the dividend, $Var[d_{t+1}|F_{it}]$. Notice that the current period's output elasticity of capital η_t does not affect the next period's equilibrium price coefficients. As a result, investors' perceived future price variance, $Var[q_{t+1}|F_{it}]$, and investors' perceived payoff variance, $Var[d_{t+1} + q_{t+1}|F_{it}]$, are also not affected by the current period's output elasticity of capital η_t.

In contrast, since the investors born at the beginning of period t cannot observe the next period's public signal s_{t+1} and financial price q_{t+1}, they are uncertain about the firm's capital investment k_{t+1} in the next period. An increase in the next period's output elasticity of capital η_{t+1} also amplifies the impact of the uncertain capital investment k_{t+1} on the firm's future income v_{t+2} and dividend $d_{j,t+2}$, which increases the next period's dividend variance $Var[d_{t+2}|F_{it}]$ and financial price variance $Var[q_{t+1}|F_{it}]$ perceived by the investors of period t. The increase in the investors' perceived future price variance $Var[q_{t+1}|F_{it}]$ also leads to a higher payoff variance $Var[d_{t+1} + q_{t+1}|F_{it}]$.

Now we can explain the intuition of Proposition 2. Since an increase in the next period's output elasticity η_{t+1} increases investors' perceived payoff variance $Var[d_{t+1} + q_{t+1}|F_{it}]$, the risk-averse informed investors trade less aggressively with lower confidence in their prediction on the asset payoff. Therefore, less private information is injected into the current period's asset price, and the current period's financial price informativeness $1/\beta_{3,t+1}$ decreases. In contrast, an increase in the current period's output elasticity η_t does not change the informed investors' perceived payoff risk, and therefore the investor's trading behavior is not affected. As a result, the amount of information injected into the price is also not affected.

3.2. Public disclosure and financial price informativeness

Proposition 4 (Effects of public information disclosure on financial price informativeness)**.** *The current period's equilibrium price informativeness, $1/\beta_{3t}$, is increasing in the current period's disclosure τ_{st}, i.e., $\frac{\partial(1/\beta_{3t})}{\partial\tau_{st}} > 0$. Moreover, the sensitivities of the next period's asset price to the next period's shocks are increasing in the next period's disclosure $\tau_{s,t+1}$, i.e., $\frac{\partial\beta_{1,t+1}}{\partial\tau_{s,t+1}} > 0$ and $\frac{\partial(\beta_{1,t+1}\beta_{2,t+1})}{\partial\tau_{s,t+1}} > 0$.*

Proof. See Appendix B. □

Proposition 4 shows that the current period's financial price informativeness $1/\beta_{3t}$ is increasing in the current period's public signal precision τ_{st}. The reason is that, when more information about the current period's productivity a_{t+1} is disclosed to the investors, their perceived dividend variance $Var[d_{t+1}|F_{it}]$ and payoff variance $Var[d_{t+1} + q_{t+1}|F_{it}]$ decrease. Therefore, the investors trade more aggressively and inject more information about the current period's demand shock ε_{t+1} into the asset price, increasing the financial price informativeness.

Proposition 4 also shows that, the sensitivity of the next period's asset price to the next period's real demand shock, $\partial q_{t+1}/\partial \varepsilon_{t+2} = \beta_{1,t+1}$, and the sensitivity of the next period's asset price to the next period's public information, $\partial q_{t+1}/\partial s_{t+1} = \beta_{1,t+1}\beta_{2,t+1}$, are increasing in the next period's public signal precision $\tau_{s,t+1}$. In period $t+1$, if the public signal precision $\tau_{s,t+1}$ increases, investors will rely more on the public signal s_{t+1}, and the asset price will also be more sensitive to the public signal. Moreover, since an increase in $\tau_{s,t+1}$ also lowers the next period's investors' perceived payoff variance $Var[d_{t+2} + q_{t+2}|F_{i,t+1}]$, the investors will trade more aggressively, and thus the asset price q_{t+1} will also be more sensitive to the investors' private information and to the real demand shock ε_{t+2}. For investors in the current period (i.e., period t), since they cannot learn any information about the next period's real demand shock ε_{t+2} and productivity shock a_{t+2}, their perceived resale price variance, $Var[q_{t+1}|F_{it}]$, may increase with $\tau_{s,t+1}$ due to the increase in $\beta_{1,t+1}^2 Var[\varepsilon_{t+2}|F_{it}]$ and $(\beta_{1,t+1}\beta_{2,t+1})^2 Var[s_{t+1}|F_{it}]$. Therefore, the current period's investors' perceived payoff variance $Var[d_{t+1}+q_{t+1}|F_{it}]$ can be increasing in the next period's public disclosure $\tau_{s,t+1}$, and the current period's financial price informativeness $\beta_{1,t}/\beta_{3,t}$ can be decreasing in the next period's public disclosure $\tau_{s,t+1}$.

4. The model with endogenous information acquisition

In this section we extend the baseline model in Section 3 to allow for endogenous information production and information extraction. Then we investigate how the output elasticity of capital and the public information disclosure affect the price informativeness, the investors' information choice, and the firm's profit.

4.1. Information choice problem

Assume that at the beginning of each period t, the precision of the signal about the real demand shock $\tau_{xi,t}$ (information production) and the precision about the noise traders' order flow $\tau_{gi,t}$ (information extraction) can be chosen by each investor i subject to the data capacity H_t before making investment decision. Investor i's information choice problem is to maximize the expected utility conditional on information set I_t^-,

$$\max_{\tau_{xi,t},\tau_{gi,t}} E\left[U(c_{i,t+1})|I_t^-\right], \tag{23}$$

subject to information constraint,

$$\tau_{xi,t}^2 + \chi\tau_{gi,t}^2 \leq H_t, \tag{24}$$

where $H_t > 0$ is the data capacity (financial data technology) for every investor in period t, and $\chi > 0$ is a parameter that determines whether information production is relatively easier or harder than information extraction. This information constraint is similar to that in Farboodi and Veldkamp (2020). Assume that the sequence of data capacity $\{H_t\}_{t=0}^{\infty}$ is known in period 0: $\{H_t\}_{t=0}^{\infty} \subset I_0$.

We can show that maximizing an investor's ex ante expected utility is equivalent to minimizing the investor's perceived payoff variance $Var\left[d_{t+1} + q_{t+1}|F_{it}\right]$. It is intuitive that a risk-averse investor tries to make the uncertainty as low as possible. Minimizing the investor's perceived payoff variance can further be converted to the following problem,

$$\begin{aligned} &\max_{\tau_{xit},\tau_{git}} \tau_{xit} + \left(\frac{1}{\beta_{3,t}}\right)^2 \tau_{git}, \\ \text{s.t. } \; &\tau_{xit}^2 + \chi\tau_{git}^2 \leq H_t, \quad \tau_{xit} \geq 0, \quad \tau_{git} \geq 0. \end{aligned} \tag{25}$$

This problem can be solved using Lagrange's method of multipliers. Notice that we focus on the symmetric equilibrium where all investors choose the same levels of information production and information extraction. We have the following proposition that characterizes the information acquisition decisions of the investors.

Proposition 5 (Information acquisition)**.** *Investors' information production decision (i.e., the precision of the signal about the real demand shock) at the beginning of period t can be expressed as*

$$\tau_{xt} = \sqrt{\frac{H_t\chi}{\chi + (\frac{1}{\beta_{3,t}})^4}}, \tag{26}$$

and investors' information extraction decision (i.e., the precision of the signal about the noise traders' order flow) at the beginning of period t can be expressed as

$$\tau_{gt} = \left(\frac{1}{\beta_{3,t}}\right)^2 \frac{1}{\chi}\sqrt{\frac{H_t\chi}{\chi + (\frac{1}{\beta_{3,t}})^4}}. \tag{27}$$

Proof. See Appendix B. □

Combining Proposition 1 and Proposition 5, we can solve for the full equilibrium of the economy. We focus on the stationary equilibrium where all deterministic variables do not vary with time. Particularly, we assume that $\tau_{st} = \tau_s$, $\eta_t = \eta$, and $H_t = H$, $\forall t$. Accordingly, the endogenous price coefficients and signal precisions are also constant over time, i.e., $\beta_{l,t} = \beta_l$, $l = 0, 1, 2, 3, 4$, and $\tau_{xt} = \tau_x$, $\tau_{gt} = \tau_g$, $\forall t$.

4.2. Impacts of output elasticity of capital

Proposition 6 (The impacts of output elasticity on financial price informativeness and information acquisition)**.** *A higher output elasticity decreases financial price informativeness, increases information production, and decreases information extraction, i.e.,* $\frac{\partial(1/\beta_3)}{\partial\eta} < 0$, $\frac{\partial\tau_x}{\partial\eta} > 0$, *and* $\frac{\partial\tau_g}{\partial\eta} < 0$.

Proof. See Appendix B. □

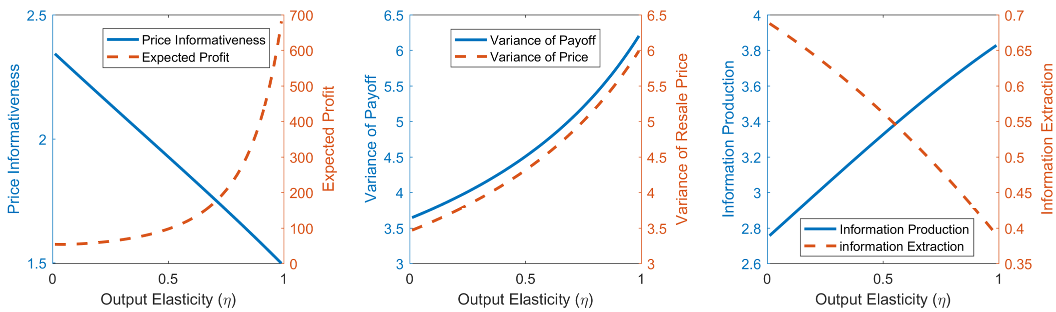

Figure 1. Impacts of output elasticity of capital. This figure plots price informativeness $1/\beta_3$, ex ante expected profit $E[P_{t+1}Y_{t+1} - R_f K_t]$, conditional variance of financial asset payoff $Var[d_{t+1} + q_{t+1}|F_{it}]$, conditional variance of resale price $Var[q_{t+1}|F_{it}]$, information production τ_x, and information extraction τ_g as functions of output elasticity of capital η. Parameter values: $\tau_a = 100$, $\tau_\varepsilon = 80.08$, $\tau_n = 19.75$, $\gamma = 0.05$, $\chi = 22$, $\alpha = 1$, $R_f = 1.02$, $G = 0.98$, $\theta = 2$, $H = 18.0036$, $\tau_s = 10$, $\bar{z} = 4$, $\bar{y} = 4$.

Proposition 6 and the left panel of Figure 1 show that the financial price informativeness is decreasing in the output elasticity of capital, and the firm's expected profit is increasing in the output elasticity of capital. This result clearly shows the misalignment between financial market efficiency and real efficiency: a more efficient production technology that can improve the firm's profit is associated with a lower financial price informativeness. The intuition for this result is consistent with that for Proposition 2 and 3. Recall that the sensitivity of the firm's income to its capital input is increasing in the output elasticity of capital. Since the investors are uncertain about the firm's capital input decision in the next period, their perceived resale price risk and asset payoff risk are also increasing in the output elasticity of capital (see the middle panel of Figure 1). Therefore, when the output elasticity of capital increases, the investors are less confident about their predictions on the asset payoff, and trade less aggressively. Consequently, less private information about the real product demand is injected into the asset price from the investors, resulting in a lower price informativeness.

In the full model, the investors can acquire the information about both the product demand and the noise traders' order flow, using the financial data technology. We call the investors' learning about the product demand as "information production", because the investors inject the information about the product demand that they learned into the asset price through their trading, making the asset price informative about the product demand. Since the asset price is informative, the investors can also extract the information about the product demand from the price. However, the information contained in the asset price is contaminated by the noise traders' order flow. Knowing more about the noise traders' order flow helps the investors better filter out the noise (see Equation 15) and extract more information about the product demand from the asset price. Therefore, we call investors' learning about the noise traders' order flow as "information extraction".

The right panel of Figure 1 shows that the investors' information production (i.e., learning about the real product demand) is increasing in the output elasticity of capital, and their information extraction (i.e., learning about the noise traders' order flow) is decreasing in the output elasticity of capital. When the output elasticity becomes higher, the amount of information about the firm's product demand that can be learned from the asset price decreases. Therefore, the investors optimally choose to produce more information about the product demand by themselves using the financial data technology, in order to acquire enough information about the to make the best investment decision. Correspondingly, since the data capacity is limited, investors reduce their learning about the noise traders' order flow in response to a lower financial price informativeness. Notice that although a higher output elasticity of capital increases the information production, which in turn tends to increase the price informativeness, this positive effect is too small to offset the negative direct impact of the output elasticity on the price informativeness. Moreover, the decrease in information extraction also tends to decrease the price informativeness.[2] Therefore, the equilibrium price informativeness is decreasing in the output elasticity of capital.

[2]Price informativeness is increasing in information extraction. With the information about noise trading, the informed investors can profit from trading against the noise traders, which decreases the impact of noise trading on the asset price. Learning about noise trading is first proposed and analyzed by Ganguli and Yang (2009). See page 2496 of Farboodi and Veldkamp (2020) for a detailed discussion about the information on noise traders' order flow.

4.3. Output elasticity of capital and asset pricing properties

Last subsection explores the impact of the output elasticity of capital on the informational content of the asset price. In this section, we investigate how the output elasticity of capital affects other traditional asset pricing properties, including risk premium, return volatility, Sharpe ratio, liquidity, and trading volume. Note that the investors in our model have constant absolute risk aversion (CARA) preference, so they care about the magnitude of the return instead of the rate of return. Therefore, only the absolute returns are meaningful in our model. Other papers that investigate asset returns with CARA frameworks include Mondria et al. (2022).

At the beginning of period t, if an investor spends q_t to invest in one unit of the risky asset, she will receives $d_{t+1} + q_{t+1}$ at the end of the period if she resells the asset, and the return on the risky asset is $d_{t+1} + q_{t+1} - q_t$. Alternatively, if she invests q_t into the risk-free asset for the period, she will receive $R_f q_t$ at the end of the period, and the return on the risk-free asset is $R_f q_t - q_t$. Therefore, the excess return is $(d_{t+1} + q_{t+1} - q_t) - (R_f q_t - q_t) = d_{t+1} + q_{t+1} - R_f q_t$. We provide the expression for the unconditional expected excess return (i.e., the risk premium) in the following proposition.

Proposition 7 (Expected dividend and risk premium)**.** *Let* $(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$ *be a stationary solution to the system of equations* (18)*,* (26)*, and* (27)*. Given the initial dividend* d_0*, the unconditional expectation of the dividend can be expressed as* $E[d_{t+1}] = G^{t+1}[d_0 - \mu - \alpha\bar{v}/(1-G)] + \mu + \alpha\bar{v}/(1-G)$*, where* $\bar{v} = E[v_{t+1}]$*. The expression for* $\bar{v}$ *is provided in Appendix B. Notably, if* $d_0 = \mu + \alpha\bar{v}/(1-G)$*, then the unconditional expectation of the dividend in any period is*

$$E[d_{t+1}] = \mu + \frac{\alpha\bar{v}}{1-G}, \quad \forall t. \tag{28}$$

Therefore, the unconditional expectation of the asset price, $E[q_{t+1}]$*, is also constant over time, i.e.,*

$$E\left[q_{t+1}\right] = \beta_0 + \beta_1\left[-\frac{1}{2}\tau_\varepsilon^{-1} + \beta_2\left(-\frac{1}{2}\tau_a^{-1}\right)\right] + \beta_4\frac{\alpha\bar{v}}{1-G}, \quad \forall t. \tag{29}$$

Moreover, the expected risky asset return in excess of the risk-free return (i.e., the risk premium) can be expressed as

$$E[d_{t+1} + q_{t+1}] - R_f E[q_t] = E[d_{t+1}] - (R_f - 1)E[q_t], \tag{30}$$

because $E[q_{t+1}] = E[q_t]$*. Note that the risk premium is also constant over time.*

Proof. See Appendix B. □

Proposition 7 provides a sufficient condition such that the expectation of the excess return (i.e., the risk premium) does not vary with time, which simplifies our analysis. In this subsection, we always assume that this condition holds (i.e., $d_0 = \mu + \alpha\bar{v}/(1-G)$). We plot the risky asset's (il)liquidity, trading volume, risk premium, return volatility, Sharpe ratio, and expected dividend as functions of the output elasticity of capital η in Figure 2.

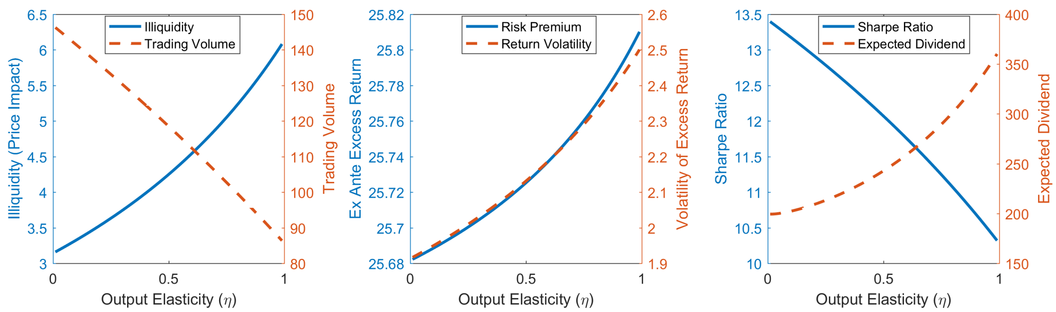

Figure 2. Output elasticity of capital and asset pricing properties. This figure plots illiquidity (i.e., price impact $\beta_1\beta_3$), expected trading volume $E[\int_0^1 |m_i|di]$, risk premium $E[d_{t+1}+q_{t+1}]-R_fE[q_t]$, return volatility $\sqrt{Var[d_{t+1}+q_{t+1}-R_fq_t|I_t^-]}$, Sharpe ratio $E[d_{t+1}+q_{t+1}-R_fq_t|I_t^-]/\sqrt{Var[d_{t+1}+q_{t+1}-R_fq_t|I_t^-]}$, and expected dividend $E[d_{t+1}]$ as functions of the output elasticity of capital η. Parameter values: $\tau_a = 100$, $\tau_\varepsilon = 80.08$, $\tau_n = 19.75$, $\gamma = 0.05$, $\chi = 22$, $\alpha = 1$, $R_f = 1.02$, $G = 0.98$, $\theta = 2$, $H = 18.0036$, $\eta = 0.8$, $\bar{z} = 4$, $\bar{y} = 4$, $d_0 = \mu + \alpha\bar{v}/(1-G)$.

As is standard in the rational expectations equilibrium literature (see, e.g., Kyle (1985), Han and Yang (2013), and Farboodi and Veldkamp (2020)), we define the risky asset's illiquidity as the noise trader's price impact $\beta_1\beta_3$. The left panel of Figure 2 shows that the price impact is increasing in the output elasticity of capital (i.e., the liquidity is decreasing in the output elasticity of capital), and the informed investors' trading volume $E[\int_0^1 |m_i|di]$ is increasing in the output elasticity of capital. The intuition is that, when the output elasticity increases, investors become more uncertain about the asset payoff (see the middle panel Figure 1) and trade less aggressively with less confidence, reducing the trading volume. Since the trading becomes less active, noise traders' liquidity demands are more difficult to be satisfied, increasing their price impact (or decreasing the liquidity).

The middle panel of Figure 2 shows that both the risk premium of the risky asset and the return volatility (i.e., the volatility of the excess return) are increasing in the output elasticity of capital. When the output elasticity of capital increases, the firm's value is more sensitive to the firm's capital input decision, so the asset payoff becomes more volatile and more risky, which increases the return volatility and the risk premium. Moreover, recall that the price informativeness is decreasing in the output elasticity of capital (see the left panel of Figure 1). In other words, a higher output elasticity of capital reduces the information about the asset payoff that can be extracted from the asset price, which further increases the risk faced by the investors and thus the risk premium.

The right panel of Figure 2 shows that the Sharpe ratio of the risky asset is decreasing in the output elasticity of capital, and the expected dividend is increasing in the output elasticity of capital. Although both the risk premium and the return volatility increase with the output elasticity of capital, the return volatility increases faster. Therefore, the Sharpe ratio, which is the ratio of the risk premium to the return volatility, is decreasing in the output elasticity of capital. In other words, a more efficient production technology, indicated by a higher output elasticity of capital, which increases the expected profit (see the left panel of Figure 1) and dividend payout, reduces the risk adjusted return of the financial asset.

4.4. Impacts of public information disclosure

To investigate the impacts of the firm's disclosure of information about its productivity, we plot the financial price informativeness, the firm's ex ante expected profit, the investors' perceived variances of asset payoff and resale price, and the investors' information production and extraction as functions of the firm's information disclosure (i.e., the precision of the public signal) in Figure 3.

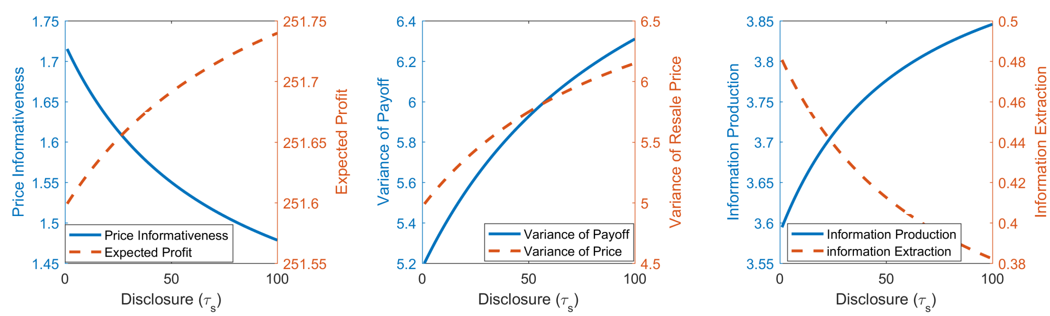

Figure 3. Impacts of firm's information disclosure. This figure plots price informativeness $1/\beta_3$, ex ante expected profit $E[P_{t+1}Y_{t+1} - R_f K_t]$, conditional variance of financial asset payoff $Var[d_{t+1} + q_{t+1}|F_{it}]$, conditional variance of resale price $Var[q_{t+1}|F_{it}]$, information production τ_x, and information extraction τ_g as functions of the firm's information disclosure τ_s. Parameter values: $\tau_a = 100$, $\tau_\varepsilon = 80.08$, $\tau_n = 19.75$, $\gamma = 0.05$, $\chi = 22$, $\alpha = 1$, $R_f = 1.02$, $G = 0.98$, $\theta = 2$, $H = 18.0036$, $\eta = 0.8$, $\bar{z} = 4$, $\bar{y} = 4$.

The left panel of Figure 3 shows that the financial price informativeness is decreasing in information disclosure, while the firm's expected profit is increasing in information disclosure. Notice that in our model, information disclosure is equivalent to the firm's information acquisition. With a more precise signal about its productivity, the firm can make a better capital input decision, which increases its profit. However, as is shown in the middle panel of Figure 3, investors become more uncertain about the resale price and the asset payoff when the disclosure increases. The intuition is consistent with that of Proposition 4. If the firm discloses a more precise signal about its productivity to the financial market, investors will rely more on this public signal when trading the asset, increasing the sensitivity of the asset price to the public signal. Since investors in the current period does not know the realization of the public signal in the next period, an increase in the sensitivity of the asset price to the public signal makes them more uncertain about the resale price. Therefore, the investors trade less aggressively with lower confidence in their prediction on the asset payoff. Consequently, less private information is injected into the asset price from the investors, lowering the financial price informativeness. With less information about the product demand contained in the asset price, investors optimally choose to learn more about the product demand by themselves, which occupies more data capacity, resulting in a decrease in information extraction (see the right panel of Figure 3).

5. Suggestive evidence: descriptive statistical analysis

In this section, we provide some suggestive evidence for Proposition 2 and the left panel of Figure 1 (i.e., the inconsistency between production efficiency and financial market efficiency). We show that the sensitivity of income growth to new capital investment[3], which corresponds to the parameter η in our model, is negatively related to the stock price informativeness that we estimate using the method proposed by Dávila and Parlatore (2018).

5.1. Data

We consider all U.S. listed firms included in the Center for Research in Security Prices (CRSP) and CRSP/Compustat Merged (CCM) database between 1973 and 2022. To estimate stock price

[3]We use new capital investment instead of capital stock because in our model the firm has no capital stock, i.e., the new capital invested at the beginning of a period fully depreciates at the end of the period. Therefore, new investment in reality is corresponding to the capital input K in our model.

informativeness, we use earnings per share (EPSFXQ) from "Fundamentals Quarterly" of CCM, stock prices (PRC) and shares outstanding (SHROUT) from "Monthly Stock File" of CRSP,[4] and price index for personal consumption expenditures from Bureau of Economic Analysis. To calculate the sensitivity of income growth to new investment, we use operating income (OIBDP), number of employees (EMP), capital stock (property, plant, and equipment, PPEGT), and depreciation (DP) from "Fundamentals Annual" of CCM, average wage index from Social Security Administration, and price index for GDP from the Bureau of Economic Analysis.

5.2. Stock price informativeness

We employ the method proposed by Dávila and Parlatore (2018) to estimate stock price informativeness. Market capitalization M, calculated by multiplying stock price (PRC) and shares outstanding (SHROUT), is used to proxy for the stock price. Earnings EN is calculated by multiplying earnings per share (EPSFXQ) and shares outstanding (SHROUT). Both the earnings and the market capitalization are deflated by the price index for personal consumption expenditures. As in Dávila and Parlatore (2018), we match the earnings in a quarter with the market capitalization one quarter forward. Then for each firm j that has more than 80 observations, we run the following two time-series regressions,

$$M_{j,t} = b_{j,0} + b_{j,1} \times EN_{j,t} + b_{j,2} \times EN_{j,t+1} + \xi_{j,t}, \tag{31}$$

and

$$M_{j,t} = b'_{j,0} + b'_{j,1} \times EN_{j,t} + \xi'_{j,t}. \tag{32}$$

Denote the estimated R-squared of equations (31) and (32) by $R^2_{j,1}$ and $R^2_{j,2}$, respectively. Then the stock price informativeness can be calculated as

$$INFO_j = \frac{R^2_{j,1} - R^2_{j,2}}{1 - R^2_{j,1}}, \tag{33}$$

where $INFO_j$ is firm j's stock price informativeness.

[4] Note that although we download the monthly data, we use the stock price and shares outstanding data at a quarterly frequency (i.e., we remove the first three observations of a quarter).

5.3. Sensitivity of income growth to new investment

We use the sensitivity of income growth to new investment to proxy for the parameter η in our model. As in İmrohoroğlu and Tüzel (2014), the operating income (OI) is calculated as operating income before interests and depreciation (OIBDP) plus labor expenses, where the labor expenses is equal to average wage multiplied by number of employees (EMP). Then the income growth IG of firm j in year t is

$$IG_{j,t} = \frac{OI_{j,t} - OI_{j,t-1}}{OI_{j,t-1}}. \tag{34}$$

Firm j's new investment in year t, $INV_{j,t}$ is calculated as $INV_{j,t} = KS_{j,t} + DP_{j,t} - KS_{j,t-1}$, where $KS_{j,t}$ is the capital stock (PPEGT) of year t, and $DP_{j,t}$ is the depreciation (DP) of year t. The investment rate IR of firm j in year t, which is the ratio of new investment in current year to the capital stock of the last year, is

$$IR_{j,t} = \frac{INV_{j,t}}{KS_{j,t-1}}. \tag{35}$$

Note that both the operating income and investment are deflated by the price index for GDP. Then we run the following time-series regression at annual frequency for each firm j with more than 20 observations,

$$IG_{j,t} = c_{j,0} + c_{j,1} \times IR_{j,t} + \xi''_{j,t}. \tag{36}$$

Then firm j's sensitivity of income growth to new investment, SEN_j, is equal to the estimated regression coefficient $c_{j,1}$. Intuitively, when $SEN_j = 0.5$, if the new investment rate increases 1% in this year, then the income growth rate tends to increase 0.5%. A higher SEN_j means that the same ratio of new investment to capital stock is associated with a higher income growth rate, indicating a higher production efficiency.

5.4. Production efficiency and stock price informativeness

After the above estimation procedures, we have a cross-sectional sample of 2694 firms. We winsorize the dependent variable SEN and the independent variable $INFO$ at 5%. We also winsorize the market capitalization at 5%. Table 1 presents the summary statistics. Notice that the sensitivities of income growth to new investment of a very small proportion of firms are negative. In the following analysis, we replace these negative sensitivities with their absolute values.

To provide suggestive evidence for our theoretical prediction that production efficiency is negatively related to financial market efficiency (see Proposition 2 and the left panel of Figure 1), we plot

Table 1: Summary statistics

Statistic	N	Mean	St. Dev.	Min	P25	P50	P75	Max
Price Informativeness	2694	0.14	0.15	0.00	0.02	0.09	0.20	0.60
Production Efficiency	2694	0.46	0.81	-2.18	0.15	0.45	0.76	2.85
Market Cap. (10^9 \$)	2694	3.31	6.68	0.02	0.16	0.72	2.74	32.61

Note: This table presents summary statistics for the variable used in the simple empirical analysis. It provides information on the sample size, sample mean, standard deviation, minimum, 25th percentile (P25), median (P50), 75th percentile (P75), and the maximum. The price informativeness ($INFO$) is estimated from Eq. (33). The production efficiency (i.e., the sensitivity of income growth to new investment, SEN) is estimated from Eq. (36). The market capitalization (expressed in billion U.S. dollars) is calculated by averaging each firm's market capitalization across time, at a quarterly frequency.

the data points in the Figure 4. We also regress each firm's stock price informativeness ($INFO$) on its sensitivity of income growth to new investment (SEN),

$$INFO_j = \alpha_0 + \alpha_1 \times SEN_j + \delta \times X_j + \zeta_j, \tag{37}$$

where X_j is the possible control variable and ζ_j is the error term. The results of the regression are reported in Table 2.

Column (1) of Table 2 reports the baseline univariate regression result with ordinary standard error. The estimated coefficient $\hat{\alpha}_1$ is -0.03, and the corresponding t-statistic is -6.46. Intuitively, the sensitivity of income growth to new investment is negatively related to the stock price informativeness, indicating a negative relationship between production efficiency and financial market efficiency. Figure 4 also shows that the estimated function (37) is downward sloping. This result is consistent with our theoretical prediction in Proposition 2 and Figure 1. We also include the market capitalization as a control variable in column (3) of Table 2. After considering the market capitalization, the coefficient of the sensitivity of income growth to new investment (SEN) is still significantly negative. Moreover, consistent with the results in the Figure 5 of Dávila and Parlatore (2018), column (3) of Table 2 shows that the stock price informativeness is positively correlated

Table 2: Production efficiency and market efficiency

	(1)	(2)	(3)	(4)
Production Efficiency	-0.03	-0.03	-0.03	-0.03
	(-6.46)	(-7.95)	(-6.26)	(-7.72)
Market Capitalization			0.01	0.01
			(12.57)	(10.77)
Constant	0.16	0.16	0.14	0.14
	(37.22)	(37.53)	(31.80)	(32.66)
R-squared	0.01	0.01	0.06	0.06
Heteroskedasticity-Robust Standard Error	No	Yes	No	Yes

Note: This table presents the results of the cross-sectional regression (37). The dependent variable is stock price informativeness ($INFO$), calculated by Eq. (33). The sensitivity of income growth to new investment (SEN) is calculated by Eq. (36). The market capitalization (M) is expressed in billion dollars. The t-statistics are shown in parentheses. Column (1) presents the baseline univariate regression result with ordinary standard error. In columns (3) and (4), we include the time-series average market capitalization as a control variable. In columns (2) and (4), the standard errors are adjusted for conditional heteroskedasticity.

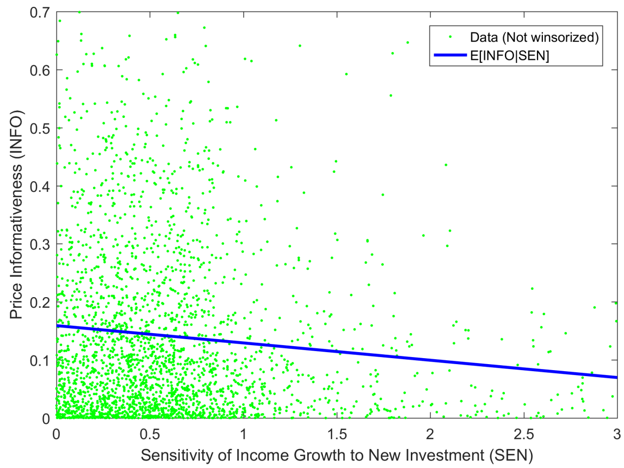

Figure 4. Production efficiency and market efficiency: suggestive evidence. This figure plots the data point of each firm $(SEN_j, INFO_j)$ and the estimation result of equation (37), $\hat{INFO}_j = \hat{\alpha}_0 + \hat{\alpha}_1 \times SEN_j$, where SEN_j is firm j's sensitivity of income growth to new investment, and $INFO_j$ is firm j's stock price informativeness.

with the market capitalization. Columns (2) and (4) of Table 2 show that our results are robust to the heteroskedasticity-adjusted standard error estimator.

6. Conclusion

This paper develops a dynamic noisy rational expectations equilibrium (NREE) model of a financial market and a firm. The firm has a decreasing-returns-to-scale production technology with capital as the factor of production. We show that the price informativeness of a financial asset contingent on the firm's value is decreasing in the firm's output elasticity of capital. In other words, as the firm's production technology gets closer to a constant-returns-to-scale technology, its financial price informativeness decreases. This result indicates that a higher real efficiency can lead to a lower financial market efficiency. We provide suggestive evidence of the negative relationship between production efficiency and financial market efficiency.

We also find that more information disclosure by the firm can decrease the financial price informativeness. In our model, what the firm discloses is the information about its productivity, which by assumption cannot be privately acquired by the investors. In other words, before the disclosure, the firm knows better about its own productivity than the investors do. Moreover, as is implied by our analytical benchmark model, the firm's information disclosure can reduce the financial price informativeness when the investors' private signal precision is exogenously given. Therefore, the negative impact of disclosure on market efficiency in our model is through a new channel that does not require the assumptions in previous literature that information acquisition is endogenous or that the firm is disclosing the information which they know little about.

In our dynamic model, both the output elasticity of capital and the information disclosure negatively affect the price informativeness through increasing the investors' perceived risk of the asset's resale price. This mechanism cannot exist in static models in which investors only care about the firm's liquidation value.

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Appendix

A. Solving the model

A.1. Dividend

From the real production function (1) and the real demand function (2), we can solve for the logarithm of the real product's price at the end of period t,

$$p_{t+1} = -\frac{1}{\theta}(\bar{z} + a_{t+1} + \eta_t k_t - \varepsilon_{t+1} - \bar{y}). \tag{38}$$

Moreover, the dividend innovation (or the logarithm of firm's income) at the end of period t is

$$v_{t+1} = \frac{1}{\theta}\varepsilon_{t+1} + \left(1 - \frac{1}{\theta}\right) a_{t+1} + \eta_t \left(1 - \frac{1}{\theta}\right) k_t + \frac{1}{\theta}\bar{y} + \left(1 - \frac{1}{\theta}\right) \bar{z}. \tag{39}$$

A.2. Firm's investment decision

Using the real production function (1) and the real demand function (2), we can calculate that

$$E\left[P_{t+1}Y_{t+1} - R_f K_t | s_t, q_t\right] = \bar{Z}^{1-\frac{1}{\theta}} K_t^{\eta_t(1-\frac{1}{\theta})} \bar{Y}^{\frac{1}{\theta}} E\left[A_{t+1}^{1-\frac{1}{\theta}} \epsilon_{t+1}^{\frac{1}{\theta}} | s_t, q_t\right] - R_f K_t. \tag{40}$$

Therefore, the first-order condition for the firm's investment problem (5) is

$$\bar{Z}^{1-\frac{1}{\theta}} \eta_t \left(1 - \frac{1}{\theta}\right) K_t^{\eta_t(1-\frac{1}{\theta})-1} \bar{Y}^{\frac{1}{\theta}} E\left[A_{t+1}^{1-\frac{1}{\theta}} \epsilon_{t+1}^{\frac{1}{\theta}} | s_t, q_t\right] - R_f = 0, \tag{41}$$

and the firm's optimal investment decision at period t is

$$K_t = K\left(s_t, q_t\right) = \left(\frac{1}{R_f}\right)^{\Theta_t} \left[\eta_t \left(1 - \frac{1}{\theta}\right) \bar{Y}^{\frac{1}{\theta}} \bar{Z}^{1-\frac{1}{\theta}}\right]^{\Theta_t} \left(E\left[A_{t+1}^{1-\frac{1}{\theta}} \epsilon_{t+1}^{\frac{1}{\theta}} | s_t, q_t\right]\right)^{\Theta_t}, \tag{42}$$

where $\Theta_t = -\frac{1}{\eta_t(1-\frac{1}{\theta})-1}$. Therefore, the equilibrium conditional expected profit for the firm is

$$\begin{aligned} & E\left[P_{t+1}Y_{t+1} - R_f K\left(s_t, q_t\right) | s_t, q_t\right] \\ = & \left[1 - \eta_t \left(1 - \frac{1}{\theta}\right)\right] \left[\eta_t \left(1 - \frac{1}{\theta}\right)\right]^{\Theta_t - 1} \left[\frac{\bar{Y}^{\frac{1}{\theta}} \bar{Z}^{1-\frac{1}{\theta}}}{R_f}\right]^{\Theta_t} \left(E\left[A_{t+1}^{1-\frac{1}{\theta}} \epsilon_{t+1}^{\frac{1}{\theta}} | s_t, q_t\right]\right)^{\Theta_t}. \end{aligned} \tag{43}$$

Conjecture that financial price at period t is

$$q_t = \beta_{0,t} + \beta_{1,t}(\varepsilon_{t+1} + \beta_{2,t}s_t + \beta_{3,t}n_{t+1}) + \beta_{4,t}(d_t - \mu). \tag{44}$$

The price signal observed by the firm is

$$\hat{q}_t = \frac{q_t - \beta_{0,t} - \beta_{1,t}\beta_{2,t}s_t - \beta_{4,t}(d_t - \mu)}{\beta_{1,t}} = \varepsilon_{t+1} + \beta_{3,t}n_{t+1}, \tag{45}$$

where the signal precision is $\tau_{\hat{q},t} = \beta_{3,t}^{-2}\tau_n$. Then the logarithm of the optimal investment decision is

$$\begin{aligned} k_t = & -\Theta_t \ln R_f + \Theta_t ln\left[\eta_t\left(1 - \frac{1}{\theta}\right)\bar{Y}^{\frac{1}{\theta}}\bar{Z}^{1-\frac{1}{\theta}}\right] + \Theta_t\left[\frac{1}{\theta}E\left[\varepsilon_{t+1}|s_t, q_t\right] + \right. \\ & \left.\left(1 - \frac{1}{\theta}\right)E\left[a_{t+1}|s_t, q_t\right] + \frac{1}{2\theta^2}Var\left[\varepsilon_{t+1}|s_t, q_t\right] + \frac{1}{2}\left(1 - \frac{1}{\theta}\right)^2 Var\left[a_{t+1}|s_t, q_t\right]\right] \\ = & \ \phi_t - \Theta_t \ln R_f + \left(\frac{\Theta_t}{\theta}\right)\frac{\tau_{\hat{q},t}}{\tau_\varepsilon + \tau_{\hat{q},t}}\hat{q}_t + \Theta_t\left(1 - \frac{1}{\theta}\right)\frac{\tau_{st}}{\tau_a + \tau_{st}}s_t, \end{aligned} \tag{46}$$

where

$$\begin{aligned} \phi_t = & \ \Theta_t \ln\left[\eta_t\left(1 - \frac{1}{\theta}\right)\right] + \frac{\Theta_t}{\theta}\bar{y} + \left(1 - \frac{1}{\theta}\right)\Theta_t\bar{z} + \frac{\Theta_t}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_{\hat{q},t}}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)\right] \\ & + \Theta_t\left(1 - \frac{1}{\theta}\right)\left[\frac{\tau_a}{\tau_a + \tau_{st}}\left(-\frac{1}{2}\tau_a^{-1}\right)\right] + \frac{1}{2}\Theta_t\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon + \tau_{\hat{q},t}} + \frac{1}{2}\Theta_t\left(1 - \frac{1}{\theta}\right)^2\frac{1}{\tau_a + \tau_{st}}. \end{aligned} \tag{47}$$

A.3. Investors' portfolio choices

Solving for the portfolio choice problem for investor i (6), we have the optimal holding of risky asset in period t as

$$m_{i,t} = \frac{E\left[d_{t+1} + q_{t+1}|F_{it}\right] - R_f q_t}{\gamma Var\left[d_{t+1} + q_{t+1}|F_{it}\right]}. \tag{48}$$

Conditional variance. The variance of the asset payoff at the end of period t conditional on the information available at the beginning of period t, $Var\left[d_{t+1} + q_{t+1}|F_{it}\right]$, can be decomposed as

$$Var\left[d_{t+1} + q_{t+1}|F_{it}\right] = Var\left[d_{t+1}|F_{it}\right] + Var\left[q_{t+1}|F_{it}\right] + 2Cov\left[d_{t+1}, q_{t+1}|F_{it}\right]. \tag{49}$$

The conditional covariance between the dividend and the financial price at the end of period t can be expressed as

$$\begin{aligned} Cov\left[d_{t+1}, q_{t+1}|F_{it}\right] &= Cov\left[d_{t+1}, \Lambda_{t+2} + \beta_{4,t+1}(d_{t+1} - \mu)|F_{it}\right] \\ &= Cov\left[d_{t+1}, \beta_{4,t+1}d_{t+1}|F_{it}\right] \\ &= \beta_{4,t+1}Var\left[d_{t+1}|F_{it}\right], \end{aligned} \tag{50}$$

where $\Lambda_{t+2} = \beta_{0,t+1} + \beta_{1,t+1}(\varepsilon_{t+2} + \beta_{2,t+1}s_{t+1} + \beta_{3,t+1}n_{t+2})$ is not correlated with d_{t+1} conditional on the information available at the beginning of period t. Then we can show that the conditional variance of the asset payoff is determined by the conditional variance of the dividend and the conditional variance of the future price, i.e.,

$$Var\left[d_{t+1}+q_{t+1}|F_{it}\right]=(1+2\beta_{4,t+1})Var\left[d_{t+1}|F_{it}\right]+Var\left[q_{t+1}|F_{it}\right]. \tag{51}$$

From the financial price function (11) we can derive that the price signal for investor i is

$$\begin{aligned}\tilde{q}_t^i &= \frac{q_t-\beta_{0,t}-\beta_{1,t}\beta_{2,t}s_t-\beta_{1,t}\beta_{3,t}E[n_{t+1}|F_{i,t}]-\beta_{4,t}(d_t-\mu)}{\beta_{1,t}} \\ &= \varepsilon_{t+1}+\beta_{3,t}(n_{t+1}-E[n_{t+1}|F_{i,t}]),\end{aligned} \tag{52}$$

where the signal precision is $\tau_{qi,t}=\beta_{3,t}^{-2}(\tau_n+\tau_{gi,t})$. Using Eqs. (3), (9), and Bayes' rule for normal variables, we can calculate that

$$\begin{aligned}Var\left[d_{t+1}|F_{it}\right] &= Var\left[\mu+G(d_t-\mu)+\alpha v_{t+1}|F_{it}\right] \\ &= \alpha^2 Var\left[\frac{1}{\theta}\varepsilon_{t+1}+\left(1-\frac{1}{\theta}\right)a_{t+1}+\eta\left(1-\frac{1}{\theta}\right)k_t+\frac{1}{\theta}\bar{y}+\frac{1}{\theta}\bar{z}\middle| F_{it}\right] \\ &= \alpha^2\left[\left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_{t+1}|F_{it}\right]+\left(1-\frac{1}{\theta}\right)^2 Var\left[a_{t+1}|F_{it}\right]\right] \\ &= \alpha^2\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_{xit}+\tau_{qit}}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_{st}}\right].\end{aligned} \tag{53}$$

Notice that the information available at the beginning of period t, F_{it}, has no predictive power on ε_{t+2}, s_{t+1}, and n_{t+2}. Therefore, we have

$$\begin{aligned}&Var\left[q_{t+1}|F_{it}\right] \\ &= \beta_{1,t+1}^2\left(Var\left[\varepsilon_{t+2}\right]+\beta_{2,t+1}^2 Var\left[s_{t+1}\right]+\beta_{3,t+1}^2 Var\left[n_{t+2}\right]\right)+\beta_{4,t+1}^2 Var\left[d_{t+1}|F_{it}\right] \\ &= \beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon}+\beta_{2,t+1}^2\left(\frac{1}{\tau_a}+\frac{1}{\tau_{s,t+1}}\right)+\beta_{3,t+1}^2\frac{1}{\tau_n}\right]+\beta_{4,t+1}^2 Var\left[d_{t+1}|F_{it}\right],\end{aligned} \tag{54}$$

where the second equality also uses the fact that $s_{t+1}=a_{t+2}+e_{t+1}$. Finally, we can express the conditional variance of the time $t+1$ asset payoff perceived by an investor at the beginning of period t as

$$
\begin{aligned}
&Var\left[d_{t+1}+q_{t+1}|F_{it}\right] \\
&=(1+2\beta_{4,t+1})Var\left[d_{t+1}|F_{it}\right]+Var\left[q_{t+1}|F_{it}\right] \\
&=(1+2\beta_{4,t+1}+\beta_{4,t+1}^2)Var\left[d_{t+1}|F_{it}\right]+\beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon}+\beta_{2,t+1}^2\left(\frac{1}{\tau_a}+\frac{1}{\tau_{s,t+1}}\right)+\beta_{3,t+1}^2\frac{1}{\tau_n}\right] \\
&=(1+\beta_{4,t+1})^2\alpha^2\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_{xit}+\tau_{qit}}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_{s,t+1}}\right] \\
&\quad+\beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon}+\beta_{2,t+1}^2\left(\frac{1}{\tau_a}+\frac{1}{\tau_{s,t+1}}\right)+\beta_{3,t+1}^2\frac{1}{\tau_n}\right].
\end{aligned}
\tag{55}
$$

Conditional expectation. The expected financial price can be expressed as

$$
\begin{aligned}
&E\left[q_{t+1}|F_{it}\right] \\
&=\beta_{0,t+1}+\beta_{1,t+1}\left(E\left[\varepsilon_{t+2}\right]+\beta_{2,t+1}E\left[s_{t+1}\right]+\beta_{3,t+1}E\left[n_{t+2}\right]\right)+\beta_{4,t+1}\left(E\left[d_{t+1}|F_{it}\right]-\mu\right) \\
&=\beta_{0,t+1}+\beta_{1,t+1}\left[\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\beta_{2,t+1}\left(-\frac{1}{2}\tau_a^{-1}\right)+0\right]+\beta_{4,t+1}\left(E\left[d_{t+1}|F_{it}\right]-\mu\right),
\end{aligned}
\tag{56}
$$

and the expected dividend can be expressed as

$$
\begin{aligned}
&E\left[d_{t+1}|F_{it}\right] \\
&=\mu+G(d_t-\mu)+\alpha\left\{\frac{1}{\theta}E\left[\varepsilon_{t+1}|F_{it}\right]+\left(1-\frac{1}{\theta}\right)E\left[a_{t+1}|F_{it}\right]+\eta_t\left(1-\frac{1}{\theta}\right)k_t+\frac{1}{\theta}\bar{y}+\left(1-\frac{1}{\theta}\right)\bar{z}\right\} \\
&=\mu+G(d_t-\mu)+\alpha\left[\left(\frac{1}{\theta}\right)\frac{\tau_\varepsilon(-\frac{1}{2}\tau_\varepsilon^{-1})+\tau_{xit}x_t^i+\tau_{qit}\tilde{q}_t^i}{\tau_\varepsilon+\tau_{xit}+\tau_{qit}}+\left(1-\frac{1}{\theta}\right)\frac{\tau_a(-\frac{1}{2}\tau_a^{-1})+\tau_{st}s_t}{\tau_a+\tau_{st}}\right] \\
&\quad+\alpha\left[\eta_t\left(1-\frac{1}{\theta}\right)k_t+\frac{1}{\theta}\bar{y}+\left(1-\frac{1}{\theta}\right)\bar{z}\right].
\end{aligned}
\tag{57}
$$

Therefore, the expected asset payoff is

$$\begin{aligned}
&E\left[d_{t+1}+q_{t+1}|F_{it}\right] \\
&= E\left[d_{t+1}|F_{it}\right]+E\left[q_{t+1}|F_{it}\right] \\
&= E\left[d_{t+1}|F_{it}\right]+\beta_{0,t+1}+\beta_{1,t+1}\left[\left(-\frac{1}{2}\tau_{\varepsilon}^{-1}\right)+\beta_{2,t+1}\left(-\frac{1}{2}\tau_{a}^{-1}\right)\right]+\beta_{4,t+1}\left(E\left[d_{t+1}|F_{it}\right]-\mu\right) \\
&= \beta_{0,t+1}+\beta_{1,t+1}\left[\left(-\frac{1}{2}\tau_{\varepsilon}^{-1}\right)+\beta_{2,t+1}\left(-\frac{1}{2}\tau_{a}^{-1}\right)\right]+(1+\beta_{4,t+1})E\left[d_{t+1}|F_{it}\right] \\
&= \beta_{0,t+1}+\beta_{1,t+1}\left[\left(-\frac{1}{2}\tau_{\varepsilon}^{-1}\right)+\beta_{2,t+1}\left(-\frac{1}{2}\tau_{a}^{-1}\right)\right]+(1+\beta_{4,t+1})\times \\
&\quad \left\{\mu+G(d_t-\mu)+\alpha\left[\left(\frac{1}{\theta}\right)\frac{\tau_{\varepsilon}(-\frac{1}{2}\tau_{\varepsilon}^{-1})+\tau_{xit}x_t^i+\tau_{qit}\tilde{q}_t^i}{\tau_{\varepsilon}+\tau_{xit}+\tau_{qit}}+\left(1-\frac{1}{\theta}\right)\frac{\tau_a(-\frac{1}{2}\tau_a^{-1})+\tau_{st}s_t}{\tau_a+\tau_{st}}\right]\right. \\
&\quad \left.+\alpha\left[\eta_t\left(1-\frac{1}{\theta}\right)k_t+\frac{1}{\theta}\bar{y}+\left(1-\frac{1}{\theta}\right)\bar{z}\right]\right\}.
\end{aligned} \tag{58}$$

A.4. Clearing the financial market

Recall that we focus on the symmetric equilibrium where for all $i \in [0,1]$, $\tau_{xi,t}=\tau_{xt}$, $\tau_{gi,t}=\tau_{gt}$, and $\tau_{qi,t}=\tau_{qt}$, $\forall t$. Therefore, all investors' perceived payoff variance at period t are the same, i.e.,

$$\begin{aligned}
Var\left[d_{t+1}+q_{t+1}|F_{it}\right] &= (1+\beta_{4,t+1})^2\alpha^2\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_{\varepsilon}+\tau_{xt}+\tau_{qt}}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_{st}}\right] \\
&\quad +\beta_{1,t+1}^2\left[\frac{1}{\tau_{\varepsilon}}+\beta_{2,t+1}^2\left(\frac{1}{\tau_a}+\frac{1}{\tau_{s,t+1}}\right)+\beta_{3,t+1}^2\frac{1}{\tau_n}\right].
\end{aligned} \tag{59}$$

Denote $Var\left[d_{t+1}+q_{t+1}|F_{it}\right]$ by V_{Ft}. The market clearing condition can be written as

$$\frac{1}{\gamma V_{Ft}}\int_0^1\left(E\left[d_{t+1}+q_{t+1}|F_{it}\right]-R_f q_t\right)di+n_{t+1}=1. \tag{60}$$

Consider the integration of individual-specific signals x_t^i and $\tilde{q}_t^i$. We can calculate that

$$\int_0^1 x_t^i di=\varepsilon_{t+1}+\int_0^1 \mathcal{Q}_t^i di=\varepsilon_{t+1}, \tag{61}$$

and

$$\begin{aligned}
\int_0^1 \tilde{q}_t^i di &= \int_0^1 \varepsilon_{t+1}+\beta_{3,t}(n_{t+1}-E[n_{t+1}|F_{i,t}])di \\
&= \varepsilon_{t+1}+\beta_{3,t}n_{t+1}-\beta_{3,t}\int_0^1 E[n_{t+1}|F_{i,t}]di \\
&= \varepsilon_{t+1}+\beta_{3,t}n_{t+1}-\beta_{3,t}\frac{\tau_{gt}}{\tau_n+\tau_{gt}}\int_0^1 \tilde{g}_t^i di \\
&= \varepsilon_{t+1}+\beta_{3,t}\frac{\tau_n}{\tau_n+\tau_{gt}}n_{t+1}.
\end{aligned} \tag{62}$$

Therefore, the integration of the expected asset payoff is

$$
\begin{aligned}
&\int_0^1 E\left[d_{t+1}+q_{t+1}|F_{it}\right]di \\
&=\beta_{0,t+1}+\beta_{1,t+1}\left[\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\beta_{2,t+1}\left(-\frac{1}{2}\tau_a^{-1}\right)\right]+(1+\beta_{4,t+1})\times \\
&\left\{\mu+G(d_t-\mu)+\alpha\left[\left(\frac{1}{\theta}\right)\frac{(-\frac{1}{2})+(\tau_{xt}+\tau_{qt})\varepsilon_{t+1}+\beta_{3,t}\frac{\tau_{qt}\tau_n}{\tau_n+\tau_{gt}}n_{t+1}}{\tau_\varepsilon+\tau_{xt}+\tau_{qt}}+\left(1-\frac{1}{\theta}\right)\frac{(-\frac{1}{2})+\tau_{st}s_t}{\tau_a+\tau_{st}}\right]\right. \\
&\left.+\alpha\left[\eta_t\left(1-\frac{1}{\theta}\right)\left(\phi_t-\Theta_t\ln R_f+\left(\frac{\Theta_t}{\theta}\right)\frac{\tau_{\hat{q},t}\hat{q}_t}{\tau_\varepsilon+\tau_{\hat{q},t}}+\Theta_t\left(1-\frac{1}{\theta}\right)\frac{\tau_{st}s_t}{\tau_a+\tau_{st}}\right)+\frac{1}{\theta}\bar{y}+\left(1-\frac{1}{\theta}\right)\bar{z}\right]\right\},
\end{aligned}
\tag{63}
$$

where (46) is also used in the equation. Combining the above equation with the market clearing condition (7), and rearranging terms, we have the following linear combination of ε_{t+1}, s_t, n_{t+1}, $d_t-\mu$, q_t, and a constant:

$$0=C_{0,t}+C_{1,t}\varepsilon_{t+1}+C_{2,t}s_t+C_{3,t}n_{t+1}+C_{4,t}(d_t-\mu)+C_{5,t}q_t, \tag{64}$$

where

$$
\begin{aligned}
C_{1,t}&=\frac{(1+\beta_{4,t+1})\alpha}{\gamma V_{Ft}}\left[\left(\frac{1}{\theta}\right)\frac{\tau_{xt}+\tau_{qt}}{\tau_\varepsilon+\tau_{xt}+\tau_{qt}}\right], \\
C_{2,t}&=\frac{(1+\beta_{4,t+1})\alpha}{\gamma V_{Ft}}\left(1-\frac{1}{\theta}\right)\left\{\frac{\tau_{st}}{\tau_a+\tau_{st}}+\eta_t\left[\Theta\left(1-\frac{1}{\theta}\right)\frac{\tau_{st}}{\tau_a+\tau_{st}}-\left(\frac{\Theta}{\theta}\right)\frac{\tau_{\hat{q}t}\beta_{2,t}}{\tau_\varepsilon+\tau_{\hat{q}t}}\right]\right\}, \\
C_{3,t}&=\frac{(1+\beta_{4,t+1})\alpha}{\gamma V_{Ft}}\left[\left(\frac{1}{\theta}\right)\left(\frac{\tau_{qt}}{\tau_\varepsilon+\tau_{xt}+\tau_{qt}}\right)\frac{\tau_n\beta_{3,t}}{\tau_n+\tau_{gt}}\right]+1, \\
C_{4,t}&=\frac{(1+\beta_{4,t+1})}{\gamma V_{Ft}}\left[G-\alpha\left(1-\frac{1}{\theta}\right)\left(\frac{\eta_t\Theta_t}{\theta}\right)\left(\frac{\tau_{\hat{q}t}}{\tau_\varepsilon+\tau_{\hat{q}t}}\right)\frac{\beta_{4,t}}{\beta_{1,t}}\right], \\
C_{5,t}&=\frac{1}{\gamma V_{Ft}}\left[-R_f+(1+\beta_{4,t+1})\alpha\left(1-\frac{1}{\theta}\right)\left(\frac{\eta_t\Theta_t}{\theta}\right)\left(\frac{\tau_{\hat{q}t}}{\tau_\varepsilon+\tau_{\hat{q}t}}\right)\frac{1}{\beta_{1,t}}\right], \\
C_{0,t}&=\frac{1}{\gamma V_{Ft}}\left\{\beta_{0,t+1}+\beta_{1,t+1}\left[\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\beta_{2,t+1}\left(-\frac{1}{2}\tau_a^{-1}\right)\right]\right. \\
&\quad+(1+\beta_{4,t+1})\left[\mu+\left(\frac{\alpha}{\theta}\right)\frac{(-\frac{1}{2})}{\tau_\varepsilon+\tau_{xt}+\tau_{qt}}+\alpha\left(1-\frac{1}{\theta}\right)\left(\bar{z}+\frac{(-\frac{1}{2})}{\tau_a+\tau_s}+\frac{\alpha}{\theta}\bar{y}\right)\right] \\
&\quad\left.-\beta_{4,t+1}\mu-\gamma V_{Ft}+(1+\beta_{4,t+1})\alpha\left(1-\frac{1}{\theta}\right)\eta_t\left(\phi_t-\Theta_t\ln R_f-\left(\frac{\Theta_t}{\theta}\right)\left(\frac{\tau_{\hat{q}t}}{\tau_\varepsilon+\tau_{\hat{q}t}}\right)\frac{\beta_{0,t}}{\beta_{1,t}}\right)\right\}.
\end{aligned}
\tag{65}
$$

A little bit transformation of the financial price function (11) yields

$$0=C_{1,t}\left[\varepsilon_{t+1}+\beta_{2,t}s_t+\beta_{3,t}n_t+\frac{\beta_{4,t}}{\beta_{1,t}}(d_t-\mu)-\frac{1}{\beta_{1,t}}q_t+\frac{\beta_{0,t}}{\beta_{1,t}}\right]. \tag{66}$$

Matching coefficients of ε_{t+1}, s_t, n_{t+1}, $d_t-\mu$, and q_t, we can derive Eq. (18), and

$$\beta_{0,t}=\frac{B_{t+1}\beta_{1,t}}{(1+\beta_{4,t+1})\alpha(1-\frac{1}{\theta})\eta_t(\frac{\Theta_t}{\theta})\frac{\tau_{\hat{q}t}}{\tau_\varepsilon+\tau_{\hat{q}t}}+\gamma V_{Ft}C_{1,t}}, \tag{67}$$

where

$$\begin{aligned}B_{t+1}=&\ \beta_{0,t+1}+\beta_{1,t+1}\left[\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\beta_{2,t+1}\left(-\frac{1}{2}\tau_a^{-1}\right)\right]\\&+(1+\beta_{4,t+1})\left[\mu+\left(\frac{\alpha}{\theta}\right)\frac{(-\frac{1}{2})}{\tau_\varepsilon+\tau_{xt}+\tau_{qt}}+\alpha\left(1-\frac{1}{\theta}\right)\left(\bar{z}+\frac{(-\frac{1}{2})}{\tau_a+\tau_s}+\frac{\alpha}{\theta}\bar{y}\right)\right]\\&-\beta_{4,t+1}\mu-\gamma V_{Ft}+(1+\beta_{4,t+1})\alpha\left(1-\frac{1}{\theta}\right)\eta_t(\phi_t-\Theta_t\ln R_f).\end{aligned} \tag{68}$$

B. Proofs of Propositions

Proof of Corollary 1

Proof. From Proposition 1 we know that $\beta_4=G/(R_f-G)$. Therefore, we have

$$\frac{1}{1+\beta_4}=\frac{R_f-G}{R_f}. \tag{69}$$

By assumption, $\tau_{gt}=0$ and $\tau_{xt}=\tau_x$. We can calculate that

$$\beta_{3,t}=\frac{R_f-G}{R_f}\frac{1}{\alpha}\frac{\gamma V_{Ft}\theta(\tau_\varepsilon+\tau_x+\tau_{qt})\tau_n}{(\tau_x+\tau_{qt})\tau_n-\tau_{qt}\tau_n}=\frac{R_f-G}{R_f}\frac{1}{\alpha}\frac{\gamma V_{Ft}\theta(\tau_\varepsilon+\tau_x+\tau_{qt})}{\tau_x}. \tag{70}$$

Therefore, we have

$$\frac{1}{\beta_{3,t}}\gamma V_{Ft}(\tau_\varepsilon+\tau_x+\tau_{qt})\frac{R_f-G}{\alpha R_f}=\frac{\tau_x}{\theta}. \tag{71}$$

Substituting in (19) to the above equation, we can calculate that

$$\begin{aligned}&\frac{1}{\beta_{3,t}}\gamma\Bigg((\frac{\alpha R_f}{R_f-G})\left[\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x+\tau_{qt}}{\tau_a+\tau_{s,t}}\right]\\&\quad+\frac{R_f-G}{\alpha R_f}\beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon}+\beta_{2,t+1}^2(\frac{1}{\tau_a}+\frac{1}{\tau_{s,t+1}})+\beta_{3,t+1}^2\frac{1}{\tau_n}\right](\tau_\varepsilon+\tau_x+\tau_{qt})\Bigg)=\frac{\tau_x}{\theta}.\end{aligned} \tag{72}$$

Define

$$Z_t=\frac{R_f-G}{\alpha R_f}\beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon}+\beta_{2,t+1}^2(\frac{1}{\tau_a}+\frac{1}{\tau_{s,t+1}})+\beta_{3,t+1}^2\frac{1}{\tau_n}\right]. \tag{73}$$

Note that $\tau_{gt}=0$ and $\tau_{qt}=(\frac{1}{\beta_{3,t}})^2(\tau_n+\tau_{gt})=(\frac{1}{\beta_{3,t}})^2\tau_n$. Substituting in the expression of τ_q and (22) to (72) and collecting terms, we have

$$\begin{aligned}&(\frac{1}{\beta_{3,t}})^3\left[\gamma\frac{\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{\tau_a+\tau_{st}}+\gamma Z_t\tau_n\right]+\\&\quad(\frac{1}{\beta_{3,t}})\left[\gamma\frac{\alpha R_f}{R_f-G}\left(\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x}{\tau_a+\tau_{s,t}}\right)+\gamma Z_t(\tau_\varepsilon+\tau_x)\right]=\frac{\tau_x}{\theta}.\end{aligned} \tag{74}$$

$\square$

Proof of Proposition 2

Proof. Since η_t does not enter (21) and (22), we can conclude that $\partial(\frac{1}{\beta_{3,t}})/\partial\eta_t = 0$. Taking derivative with respect to Z_t on both sides of (21), we have

$$
\begin{aligned}
&3(\frac{1}{\beta_{3,t}})^2\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial Z_t}\left[\gamma\frac{\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{\tau_a+\tau_{s,t}}+\gamma Z_t\tau_n\right]+(\frac{1}{\beta_{3,t}})^3\gamma\tau_n \\
&\quad+\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial Z_t}\left[\gamma\frac{\alpha R_f}{R_f-G}\left(\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x}{\tau_a+\tau_{s,t}}\right)+\gamma Z_t(\tau_\varepsilon+\tau_x)\right]+(\frac{1}{\beta_{3,t}})\gamma(\tau_\varepsilon+\tau_x)=0.
\end{aligned}
\tag{75}
$$

Collecting terms, we have

$$
\begin{aligned}
&\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial Z_t}\Bigg[3(\frac{1}{\beta_{3,t}})^2\left(\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{\tau_a+\tau_{s,t}}+\gamma Z_t\tau_n\right) \\
&\quad+\frac{\gamma\alpha R_f}{R_f-G}\left(\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x}{\tau_a+\tau_{s,t}}\right)+\gamma Z_t(\tau_\varepsilon+\tau_x)\Bigg]+(\frac{1}{\beta_{3,t}})\gamma\left[\tau_\varepsilon+\tau_x+(\frac{1}{\beta_{3,t}})^2\tau_n\right]=0.
\end{aligned}
\tag{76}
$$

From (22) we know that $Z_t > 0$. By assumption, $\theta > 1$ $R_f \geq 1$, and $G < 1$. Therefore, we have

$$
\begin{aligned}
&\Bigg[3(\frac{1}{\beta_{3,t}})^2\left(\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{\tau_a+\tau_{s,t}}+\gamma Z_t\tau_n\right) \\
&\quad+\frac{\gamma\alpha R_f}{R_f-G}\left(\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x}{\tau_a+\tau_{s,t}}\right)+\gamma Z_t(\tau_\varepsilon+\tau_x)\Bigg]>0.
\end{aligned}
\tag{77}
$$

It's also obvious that

$$(\frac{1}{\beta_{3,t}})\gamma\left[\tau_\varepsilon+\tau_x+(\frac{1}{\beta_{3,t}})^2\tau_n\right]>0. \tag{78}$$

Therefore, we must have

$$\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial Z_t}<0 \tag{79}$$

for (76) to establish. Notice that $\Theta_{t+1} = -\frac{1}{\eta_{t+1}(1-\frac{1}{\theta})-1}$ and $\partial\Theta_{t+1}/\partial\eta_{t+1} > 0$. From (18) we can calculate that

$$\beta_{1,t+1}=\frac{\alpha}{R_f-G}\left[\frac{1}{\theta}\frac{\tau_x+\tau_{q,t+1}}{\tau_\varepsilon+\tau_x+\tau_{q,t+1}}+(\Theta_{t+1}-1)\frac{1}{\theta}\frac{\tau_{\hat{q},t+1}}{\tau_\varepsilon+\tau_{\hat{q},t+1}}\right], \tag{80}$$

because

$$1+\beta_{4,t+2}=1+\frac{G}{R_f-G}=\frac{R_f}{R_f-G}, \tag{81}$$

and

$$(1-\frac{1}{\theta})\eta_{t+1}\Theta_{t+1} = \left[(1-\frac{1}{\theta})\eta_{t+1}-1\right]\Theta_{t+1}+\Theta_{t+1} = \frac{-1}{\Theta_{t+1}}\Theta_{t+1}+\Theta_{t+1} = \Theta_{t+1}-1. \tag{82}$$

Recall that we have proved that $\partial(\frac{1}{\beta_{3,t+1}})/\partial\eta_{t+1} = 0$. By definition and the assumption that $\tau_{g,t+1} = 0$, we have $\tau_{q,t+1} = \tau_{\hat{q},t+1} = (\frac{1}{\beta_{3,t+1}})^2\tau_n$. Therefore, $\partial\tau_{q,t+1}/\partial\eta_{t+1} = \partial\tau_{\hat{q},t+1}/\partial\eta_{t+1} = 0$, and

$$\frac{\partial\beta_{1,t+1}}{\partial\eta_{t+1}} = \frac{\alpha}{R_f-G}\frac{1}{\theta}\frac{\partial\Theta_{t+1}}{\partial\eta_{t+1}} > 0. \tag{83}$$

From (18) we can also calculate that

$$\beta_{1,t+1}\beta_{2,t+1} = \frac{\alpha}{R_f-G}(1-\frac{1}{\theta})\Theta_{t+1}\frac{\tau_{s,t+1}}{\tau_a+\tau_{s,t+1}}, \tag{84}$$

and

$$\frac{\partial(\beta_{1,t+1}\beta_{2,t+1})}{\partial\eta_{t+1}} = \frac{\alpha}{R_f-G}(1-\frac{1}{\theta})\frac{\tau_{s,t+1}}{\tau_a+\tau_{s,t+1}}\frac{\partial\Theta_{t+1}}{\partial\eta_{t+1}} > 0. \tag{85}$$

Since $\beta_{3,t+1}$ is not affected by η_{t+1}, we have

$$\frac{\partial(\beta_{1,t+1}\beta_{3,t+1})}{\partial\eta_{t+1}} = \beta_{3,t+1}\frac{\partial\beta_{1,t+1}}{\partial\eta_{t+1}} > 0. \tag{86}$$

Therefore,

$$\begin{aligned}\frac{\partial Z_t}{\partial\eta_{t+1}} = \frac{R_f-G}{\alpha R_f}&\left[\frac{1}{\tau_\varepsilon}2\beta_{1,t+1}\frac{\partial\beta_{1,t+1}}{\partial\eta_{t+1}}+(\frac{1}{\tau_a}+\frac{1}{\tau_{s,t+1}})2\beta_{1,t+1}\beta_{2,t+1}\frac{\partial(\beta_{1,t+1}\beta_{2,t+1})}{\partial\eta_{t+1}}\right.\\ &\left.+2\beta_{1,t+1}\beta_{3,t+1}\frac{\partial(\beta_{1,t+1}\beta_{3,t+1})}{\partial\eta_{t+1}}\right] > 0,\end{aligned} \tag{87}$$

and

$$\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial\eta_{t+1}} = \frac{\partial(\frac{1}{\beta_{3,t}})}{\partial Z_t}\frac{\partial Z_t}{\partial\eta_{t+1}} < 0. \tag{88}$$

□

Proof of Proposition 3

Proof. From Appendix A we know that

$$Var\left[d_{t+1}|F_{it}\right] = \alpha^2\left[\frac{1}{\theta^2}\frac{1}{\tau_\varepsilon+\tau_x+\tau_{qt}}+(1-\frac{1}{\theta})^2\frac{1}{\tau_a+\tau_{st}}\right], \tag{89}$$

where by the assumption that $\tau_{gt} = 0$ we have $\tau_{qt} = (\frac{1}{\beta_{3,t}})^2\tau_n$. We can see that

$$\frac{\partial Var\left[d_{t+1}|F_{it}\right]}{\partial(\frac{1}{\beta_{3,t}})} < 0. \tag{90}$$

From Proposition 2 we know that $\partial(\frac{1}{\beta_{3,t}})/\partial\eta_t = 0$ and $\partial(\frac{1}{\beta_{3,t}})/\partial\eta_{t+1} < 0$. Therefore,

$$\frac{\partial Var\left[d_{t+1}|F_{it}\right]}{\partial\eta_t} = \frac{\partial Var\left[d_{t+1}|F_{it}\right]}{\partial(\frac{1}{\beta_{3,t}})}\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial\eta_t} = 0, \tag{91}$$

and

$$\frac{\partial Var\left[d_{t+1}|F_{it}\right]}{\partial\eta_{t+1}} = \frac{\partial Var\left[d_{t+1}|F_{it}\right]}{\partial(\frac{1}{\beta_{3,t}})}\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial\eta_{t+1}} > 0. \tag{92}$$

From Appendix A we also know that

$$Var\left[q_{t+1}|F_{it}\right] = \beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon} + \beta_{2,t+1}^2\left(\frac{1}{\tau_a} + \frac{1}{\tau_{s,t+1}}\right) + \beta_{3,t+1}^2\frac{1}{\tau_n}\right] + \beta_{4,t+1}^2 Var\left[d_{t+1}|F_{it}\right]. \tag{93}$$

From (18) we know that η_t does not affect future price coefficients $\beta_{1,t+1}$, $\beta_{2,t+1}$, $\beta_{3,t+1}$, and $\beta_{4,t+1}$. Moreover, we have proved that η_t does not affect $Var\left[d_{t+1}|F_{it}\right]$. Therefore, η_t does not affect $Var\left[q_{t+1}|F_{it}\right]$ and $Var[d_{t+1}+q_{t+1}|F_{it}]$. Note that $Var\left[q_{t+1}|F_{it}\right]$ can also be expressed as

$$Var[q_{t+1}|F_{it}] = \frac{\alpha R_f}{R_f - G}Z_t + (\frac{G}{R_f - G})^2 Var\left[d_{t+1}|F_{it}\right]. \tag{94}$$

From (87) we know that $\frac{\partial Z_t}{\partial\eta_{t+1}} > 0$. We have also proved that $\frac{\partial Var[d_{t+1}|F_{it}]}{\partial\eta_{t+1}} > 0$. Therefore, $Var[q_{t+1}|F_{it}]$ is increasing in η_{t+1}, and $Var[d_{t+1}+q_{t+1}|F_{it}]$ is also increasing in η_{t+1}. □

Proof of Proposition 4

Proof. We first investigate $\partial(\frac{1}{\beta_{3,t}})/\partial\tau_{st}$. Taking derivative with respect to τ_{st} on both sides of (21), we have

$$\begin{aligned}
&3(\frac{1}{\beta_{3,t}})^2\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial\tau_{st}}\left[\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{\tau_a+\tau_{st}} + \gamma Z_t\tau_n\right] + (\frac{1}{\beta_{3,t}})^3\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{-\tau_n}{(\tau_a+\tau_{st})^2}\\
&+\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial\tau_{st}}\left[\gamma\frac{\alpha R_f}{R_f-G}\left(\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x}{\tau_a+\tau_{s,t}}\right)+\gamma Z_t(\tau_\varepsilon+\tau_x)\right]\\
&+(\frac{1}{\beta_{3,t}})\gamma\frac{\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2(\tau_\varepsilon+\tau_x)\frac{-1}{\tau_a+\tau_{s,t}} = 0.
\end{aligned} \tag{95}$$

Collecting terms, we have

$$\begin{aligned}
&\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial\tau_{st}}\left[3(\frac{1}{\beta_{3,t}})^2\left(\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{\tau_a+\tau_{st}}+\gamma Z_t\tau_n\right)+\right.\\
&\left.\gamma\frac{\alpha R_f}{R_f-G}\left(\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x}{\tau_a+\tau_{s,t}}\right)+\gamma Z_t(\tau_\varepsilon+\tau_x)\right]\\
&=(\frac{1}{\beta_{3,t}})^3\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{(\tau_a+\tau_{st})^2}+(\frac{1}{\beta_{3,t}})\gamma\frac{\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2(\tau_\varepsilon+\tau_x)\frac{1}{\tau_a+\tau_{s,t}}.
\end{aligned} \tag{96}$$

Since

$$\left[3(\frac{1}{\beta_{3,t}})^2\left(\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{\tau_a+\tau_{st}}+\gamma Z_t\tau_n\right)+\right.$$
$$\left.\gamma\frac{\alpha R_f}{R_f-G}\left(\frac{1}{\theta^2}+(1-\frac{1}{\theta})^2\frac{\tau_\varepsilon+\tau_x}{\tau_a+\tau_{s,t}}\right)+\gamma Z_t(\tau_\varepsilon+\tau_x)\right]>0, \tag{97}$$

and

$$(\frac{1}{\beta_{3,t}})^3\frac{\gamma\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2\frac{\tau_n}{(\tau_a+\tau_{st})^2}+(\frac{1}{\beta_{3,t}})\gamma\frac{\alpha R_f}{R_f-G}(1-\frac{1}{\theta})^2(\tau_\varepsilon+\tau_x)\frac{1}{\tau_a+\tau_{s,t}}>0, \tag{98}$$

we must have

$$\frac{\partial(\frac{1}{\beta_{3,t}})}{\partial\tau_{st}}>0 \tag{99}$$

for (96) to establish.

Recall that by assumption $\tau_{g,t}=0$, we have $\tau_{qt}=\tau_{\hat{q}t}=(\frac{1}{\beta_{3,t}})^2\tau_n$. Moreover, we have proved that $\partial(\frac{1}{\beta_{3,t}})/\partial\tau_{st}>0$. Therefore, $\frac{\partial\tau_{qt}}{\partial\tau_{st}}>0$. From (18) we know that $\frac{\partial\beta_{1,t}}{\partial\tau_{qt}}>0$. Therefore, we have

$$\frac{\partial\beta_{1,t+1}}{\partial_{\tau_{s,t+1}}}=\frac{\partial\beta_{1,t+1}}{\partial\tau_{q,t+1}}\frac{\partial\tau_{q,t+1}}{\partial\tau_{s,t+1}}>0. \tag{100}$$

We can calculate that

$$\beta_{1,t+1}\beta_{2,t+1}=\frac{\alpha}{R_f-G}(1-\frac{1}{\theta})\Theta_{t+1}\frac{\tau_{s,t+1}}{\tau_a+\tau_{s,t+1}}. \tag{101}$$

Therefore, we have $\frac{\partial(\beta_{1,t+1}\beta_{2,t+1})}{\partial\tau_{s,t+1}}>0$. □

Proof of Proposition 5

Proof. The consumption of an investor born at the beginning of period t is

$$\begin{aligned} c_{i,t+1}&=R_fW_{i,t}+m_{i,t}(d_{t+1}+q_{t+1}-R_fq_t)\\ &=R_fW_{i,t}+\frac{E\left[d_{t+1}+q_{t+1}|F_{it}\right]-R_fq_t}{\gamma Var\left[d_{t+1}+q_{t+1}|F_{it}\right]}(d_{t+1}+q_{t+1}-R_fq_t), \end{aligned} \tag{102}$$

so the ex post utility is

$$\begin{aligned} E\left[U(c_{i,t+1})|F_{i,t}\right]&=-E\left[e^{-\gamma\left[R_fW_{i,t}+\frac{E[d_{t+1}+q_{t+1}|F_{it}]-R_fq_t}{\gamma Var[d_{t+1}+q_{t+1}|F_{it}]}(d_{t+1}+q_{t+1}-R_fq_t)\right]}|F_{i,t}\right]\\ &=-e^{-\gamma R_fW_{i,t}-\frac{1}{2}\frac{E^2[d_{t+1}+q_{t+1}-R_fq_t|F_{i,t}]}{V_{Ft}}}. \end{aligned} \tag{103}$$

By law of iterated expectation, the ex ante utility is

$$
\begin{aligned}
E\left[U(c_{i,t+1})|I_t^-\right] &= E\left[E\left[U(c_{i,t+1})|F_{i,t}\right]|I_t^-\right] \\
&= E\left[-e^{-\gamma R_f W_{i,t}-\frac{1}{2}\frac{E^2[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}]}{V_{Ft}}}|I_t^-\right],
\end{aligned} \tag{104}
$$

so maximizing the ex ante utility is equivalent to minimizing

$$
E\left[e^{-\frac{1}{2}\frac{E^2[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}]}{V_{Ft}}}|I_t^-\right], \tag{105}
$$

subject to the information constraint (24). $E\left[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}\right]|I_t^-$ is normally distributed and we denote its expectation and variance as μ_t and V_t respectively. Then $\frac{E^2[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}]}{V_t}$ is a noncentral chi-squared distributed variable conditional on I_t^- with unit degree of freedom and a non-centrality parameter that equals $\frac{\mu_t^2}{V_t}$. Rearranging (105) and using the moment generating function of noncentral chi-squared distribution, we have

$$
\begin{aligned}
E\left[e^{-\frac{1}{2}\frac{E^2[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}]}{V_{Ft}}}|I_t^-\right] &= E\left[e^{-\frac{1}{2}\frac{V_t}{V_{Ft}}\frac{E^2[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}]}{V_t}}|I_t^-\right] \\
&= \frac{1}{(1+\frac{V_t}{V_{Ft}})^{\frac{1}{2}}} e^{-\frac{1}{2}\frac{\mu_t^2}{V_{Ft}}(1+\frac{V_t}{V_{Ft}})^{-1}},
\end{aligned} \tag{106}
$$

so minimizing (105) is now converted to maximizing

$$
\frac{1}{2}\ln(1+\frac{V_t}{V_{Ft}}) + \frac{1}{2}\frac{1}{1+\frac{V_t}{V_{Ft}}}\frac{\mu_t^2}{V_{Ft}}, \tag{107}
$$

subject to (24). We can calculate that

$$
\mu_t = E\left[E\left[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}\right]|I_t^-\right] = E\left[d_{t+1}+q_{t+1}-R_f q_t|I_t^-\right], \tag{108}
$$

which is not related to investor i's information choice τ_{xit}, τ_{git}, and

$$
\begin{aligned}
V_t &= Var\left[E\left[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}\right]|I_t^-\right] \\
&= Var\left[d_{t+1}+q_{t+1}-R_f q_t|I_t^-\right] - E\left[Var\left[d_{t+1}+q_{t+1}-R_f q_t|F_{i,t}\right]|I_t^-\right] \\
&= Var\left[d_{t+1}+q_{t+1}-R_f q_t|I_t^-\right] - V_{Ft}.
\end{aligned} \tag{109}
$$

Since $V_t = Var\left[d_{j,t+1}+q_{j,t+1}-R_f q_{j,t}|I_t^-\right]$ is not affected by a given investors' information choice, τ_{xit} and τ_{git} only enter V_t through V_{Ft}. Equation (107) can also be written as

$$
\frac{1}{2}ln(1+\frac{V_t-V_{Ft}}{V_{Ft}}) + \frac{1}{2}\frac{1}{1+\frac{V_t-V_{Ft}}{V_{Ft}}}\frac{\mu_t^2}{V_{Ft}} = \frac{1}{2}\ln(\frac{V_t}{V_{Ft}}) + \frac{1}{2}\frac{\mu_t^2}{Vt}, \tag{110}
$$

which is an decreasing function of V_{Ft}, so maximizing (107) is equivalent to minimizing V_{Ft}. After some transformation we can write the information choice problem for investor i as

$$\begin{aligned} &\max_{\tau_{xit},\tau_{git}} \tau_{xit} + (\frac{1}{\beta_{3,t}})^2 \tau_{git}, \\ s.t.\ \ &\tau_{xit}^2 + \chi\tau_{git}^2 \leq H_t, \quad \tau_{xit} \geq 0, \quad \tau_{git} \geq 0. \end{aligned} \tag{111}$$

Using Lagrange's method of multipliers, it's easy to find that the optimal information choice for investor i is

$$\begin{aligned} \tau_{xit} &= \sqrt{\frac{H_t\chi}{\chi + (\frac{1}{\beta_{3,t}})^4}}, \\ \tau_{git} &= (\frac{1}{\beta_{3,t}})^2 \frac{1}{\sqrt{\chi}} \sqrt{\frac{H_t}{\chi + (\frac{1}{\beta_{3,t}})^4}}. \end{aligned} \tag{112}$$

Combining (18) and (112), we have the system of difference equations that characterizes the full equilibrium. □

Proof of Proposition 6

Proof. From Eq. (18) and (112) we know that the stationary equilibrium financial price informativeness $\xi = \frac{1}{\beta_3}$ is determined by the equation $\xi = h(\xi, \eta)$.[5] The function $h(\xi, \eta)$ is defined as

$$h(\xi,\eta) = \left(\frac{\alpha R_f}{R_f - G}\right) \frac{(\tau_x(\xi) + \tau_q(\xi))(\tau_n + \tau_g(\xi)) - \tau_q(\xi)\tau_n}{\gamma\theta f(\xi,\eta)(\tau_\varepsilon + \tau_x(\xi) + \tau_q(\xi))(\tau_n + \tau_g(\xi))}, \tag{113}$$

where $\tau_q(\xi) = \xi^2(\tau_n + \tau_g(\xi))$. From Proposition 5 we have $\tau_x(\xi) = \sqrt{\frac{H\chi}{\chi+\xi^4}}$ and $\tau_g(\xi) = \frac{1}{\chi}\xi^2\tau_x(\xi)$. Moreover, the function $f(\xi, \eta)$ is defined as

$$\begin{aligned} f(\xi,\eta) = &\left(\frac{\alpha R_f}{R_f - G}\right)^2 \left[\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon + \tau_x(\xi) + \tau_q(\xi)} + \left(1 - \frac{1}{\theta}\right)^2 \frac{1}{\tau_a + \tau_s}\right] \\ &+ [\rho(\xi,\eta)]^2 \frac{1}{\tau_\varepsilon} + [\psi(\xi,\eta)]^2 \left(\frac{1}{\tau_a} + \frac{1}{\tau_{s,t+1}}\right) + [\rho(\xi,\eta)\xi]^2 \frac{1}{\tau_n}, \end{aligned} \tag{114}$$

where

$$\rho(\xi,\eta) = \frac{1}{R_f}\left(\frac{\alpha R_f}{R_f - G}\right)\left[\left(\frac{1}{\theta}\right)\frac{\tau_x(\xi) + \tau_q(\xi)}{\tau_\varepsilon + \tau_x(\xi) + \tau_q(\xi)} + \left(1 - \frac{1}{\theta}\right)\eta\left(\frac{\Theta}{\theta}\right)\frac{\tau_{\hat{q}}(\xi)}{\tau_\varepsilon + \tau_{\hat{q}}(\xi)}\right], \tag{115}$$

$$\psi(\xi,\eta) = \frac{1}{R_f}\left(\frac{\alpha R_f}{R_f - G}\right)\Theta\left(1 - \frac{1}{\theta}\right)\frac{\tau_s}{\tau_a + \tau_s}, \tag{116}$$

[5]If there are multiple equilibria, we always focus on the equilibrium with the lowest price informativeness.

and $\tau_{\hat{q}}(\xi) = \xi^2\tau_n$. Recall that $\Theta = -[\eta(1-\frac{1}{\theta}) - 1]^{-1} > 0$, $\frac{\partial\Theta}{\partial\eta} > 0$, and $(1-\frac{1}{\theta})\eta\Theta = \Theta - 1$. Therefore, we have $\frac{\partial\rho(\xi,\eta)}{\partial\eta} > 0$, $\frac{\partial\psi(\xi,\eta)}{\partial\eta} > 0$, and thus $\frac{\partial f(\xi,\eta)}{\partial\eta} > 0$. It immediately follows that $\frac{\partial h(\xi,\eta)}{\partial\eta} < 0$. We can also calculate that

$$h(0,\eta) = \left(\frac{\alpha R_f}{R_f - G}\right)\frac{\sqrt{H}\tau_n}{\gamma\theta f(\xi,\eta)(\tau_\varepsilon + \sqrt{H})\tau_n} > 0. \tag{117}$$

Let $\eta' > \eta > 0$. Let $\xi(\cdot)$ be the function implied by $\xi = h(\xi,\cdot)$. Since $\frac{\partial h}{\partial\eta} < 0$, we have $h(\xi(\eta),\eta') < h(\xi(\eta),\eta) = \xi(\eta)$. Equivalently, we have $h(\xi(\eta),\eta') - \xi(\eta) < 0$. Recall that $h(0,\eta') - 0 > 0$. By the intermediate value theorem, we know that there exists a $\zeta \in (0,\xi(\eta))$, such that $h(\zeta,\eta') - \zeta = 0$, or equivalently, $\zeta = h(\zeta,\eta')$. Notice that $\zeta = \xi(\eta')$. Therefore, we have $\xi(\eta') < \xi(\eta)$. Since the selection of $\eta' > \eta$ is arbitrary, we know that $\xi'(\eta) < 0$. Intuitively, the price informativeness is decreasing in output elasticity.

Recall that $\tau_x(\xi) = \sqrt{\frac{H\chi}{\chi+\xi^4}}$, so by inspection we have $\tau_x'(\xi) < 0$. By the chain rule, we have $\frac{\partial\tau_x}{\partial\eta} = \tau_x'(\xi)\xi'(\eta) > 0$. By the information constraint, we have $\tau_g(\xi) = \sqrt{\frac{H-[\tau_x(\xi)]^2}{\chi}}$, so $\tau_g'(\xi) > 0$. Therefore, we have $\frac{\partial\tau_g}{\partial\eta} = \tau_g'(\xi)\xi'(\eta) < 0$. Intuitively, the information production is increasing in output elasticity, and the information extraction is decreasing in output elasticity. □

Proof of Proposition 7

Proof. The dividend process (3) can be written as $d_{t+1} = Gd_t + (1-G)\mu + \alpha v_{t+1}$. We can also calculate that the unconditional expectation of v_{t+1} is

$$\begin{aligned} E[v_{t+1}] = &\frac{1}{\theta}(-\frac{1}{2}\tau_\varepsilon^{-1}) + (1-\frac{1}{\theta})(-\frac{1}{2}\tau_a^{-1}) + \eta(1-\frac{1}{\theta})[\phi - \Theta ln R_f \\ &+ \frac{\Theta}{\theta}\frac{\tau_{\hat{q}}}{\tau_\varepsilon + \tau_{\hat{q}}}(-\frac{1}{2}\tau_\varepsilon^{-1}) + \Theta(1-\frac{1}{\theta})\frac{\tau_s}{\tau_a+\tau_s}(-\frac{1}{2}\tau_a^{-1})] + \frac{1}{\theta}\bar{y} + (1-\frac{1}{\theta})\bar{z}. \end{aligned} \tag{118}$$

Notice that $E[v_{t+1}]$ is constant over time. Let $\bar{v} = E[v_{t+1}]$. The unconditional expectation of dividend is

$$\begin{aligned} E[d_{t+1}] &= GE[d_t] + (1-G)\mu + \alpha\bar{v} \\ &= G(GE[d_{t-1}] + (1-G)\mu + \alpha\bar{v}) + (1-G)\mu + \alpha\bar{v} \\ &= G^2E[d_{t-1}] + [(1-G)\mu + \alpha\bar{v}](1+G^1) \\ &= \cdots \\ &= G^{t+1}d_0 + [(1-G)\mu + \alpha\bar{v}](1+G^1+\cdots+G^t). \end{aligned} \tag{119}$$

Using the sum formula for a geometric sequence, we can calculate that $1 + G^1 + \cdots + G^t = (1 - G^{t+1})/(1 - G)$, so

$$E[d_{t+1}] = G^{t+1}d_0 + [(1-G)\mu + \alpha\bar{v}]\frac{1-G^{t+1}}{1-G} = G^{t+1}(d_0 - \mu - \frac{\alpha\bar{v}}{1-G}) + \mu + \frac{\alpha\bar{v}}{1-G}. \tag{120}$$

Therefore, when $d_0 = \mu + \alpha\bar{v}/(1-G)$, we have Equation (28). Based on Equation (28), the derivation of Equations (29) and (30) is obvious. Note that if $d_t = d_0 = \mu + \alpha\bar{v}/(1-G)$, then

$$\begin{aligned} E[d_{t+1}|I_t^-] &= Gd_t + (1-G)\mu + \alpha\bar{v} \\ &= G\mu + \frac{G\alpha\bar{v}}{1-G} + \mu - \mu G + \alpha\bar{v} \\ &= \mu + \frac{\alpha\bar{v}}{1-G}, \end{aligned} \tag{121}$$

so $E[d_{t+1}|I_t^-] = E[d_{t+1}]$. Therefore, we also have $E[d_{t+1} + q_{t+1} - R_f q_t|I_t^-] = E[d_{t+1} + q_{t+1} - R_f q_t]$ when $d_t = d_0 = \mu + \alpha\bar{v}/(1-G)$.

The expression for return volatility. The variance of the excess return conditional on the information set I_t^-, $Var[d_{t+1} + q_{t+1} - R_f q_t|I_t^-]$, can be expressed as

$$\begin{aligned} &Var[d_{t+1} + q_{t+1} - R_f q_t|I_t^-] \\ = &Var[D_1\varepsilon_{t+1} + D_2 a_{t+1} + D_3 n_{t+1} + D_4 e_t + D_5\varepsilon_{t+2} + D_6 a_{t+2} + D_7 e_{t+1} + D_8 n_{t+2}], \end{aligned} \tag{122}$$

where

$$\begin{aligned} D_1 &= (1+\beta_4)\alpha\left[\frac{1}{\theta} + \eta(1-\frac{1}{\theta})\frac{\Theta}{\theta}\frac{\tau_{\hat{q}}}{\tau_\varepsilon + \tau_{\hat{q}}}\right] - R_f\beta_1, \\ D_2 &= (1+\beta_4)\alpha\left[(1-\frac{1}{\theta}) + \eta(1-\frac{1}{\theta})^2\Theta\frac{\tau_s}{\tau_a + \tau_s}\right] - R_f\beta_1\beta_2, \\ D_3 &= (1+\beta_4)\alpha\left[\eta(1-\frac{1}{\theta})\frac{\Theta}{\theta}\frac{\tau_{\hat{q}}}{\tau_\varepsilon + \tau_{\hat{q}}}\beta_3\right] - R_f\beta_1\beta_3, \\ D_4 &= (1+\beta_4)\alpha\left[\eta(1-\frac{1}{\theta})^2\Theta\frac{\tau_s}{\tau_a + \tau_s}\right] - R_f\beta_1\beta_2, \end{aligned} \tag{123}$$

and $D_5 = \beta_1$, $D_6 = D_7 = \beta_1\beta_2$, and $D_8 = \beta_1\beta_3$. Also notice that (ε_{t+1}, a_{t+1}, n_{t+1}, e_t, ε_{t+2}, a_{t+2}, e_{t+1}, n_{t+2}) are mutually independent. So the return volatility, $\sqrt{Var[d_{t+1} + q_{t+1} - R_f q_t|I_t^-]}$, can be calculated using the variances of these independent random variables. □

C. Additional Results: Impacts of the Growth of Financial Data Technology

In this section we analyze a special case where there is only demand shock in the real economy and investigate the impact of growth of financial data technology on price informativeness, information production, and information extraction. We also assume that the level of financial data technology is time varying. Letting $\tau_a \to +\infty$, we can derive the following corollary of Proposition 1 and Proposition 5.

Corollary 2. *The equilibrium financial asset price in the model where there is only the real demand shock is*

$$q_t = \beta_{0,t} + \beta_{1,t}(\varepsilon_{t+1} + \beta_{3,t} n_{t+1}) + \beta_{4,t}(d_t - \mu), \tag{124}$$

where

$$\begin{aligned}
\beta_{1,t} &= \frac{1}{R_f}(1+\beta_{4,t+1})\alpha\left[\frac{1}{\theta}\frac{\tau_{xt}+\tau_{qt}}{\tau_\varepsilon+\tau_{xt}+\tau_{qt}} + (1-\frac{1}{\theta})\eta\frac{\Theta}{\theta}\frac{\tau_{\hat{q}t}}{\tau_\varepsilon+\tau_{\hat{q}t}}\right],\\
\beta_{3,t} &= \frac{1}{1+\beta_{4,t+1}}\frac{1}{\alpha}\frac{\gamma V_{Ft}\theta(\tau_\varepsilon+\tau_{xt}+\tau_{qt})(\tau_n+\tau_{gt})}{(\tau_{xt}+\tau_{qt})(\tau_n+\tau_{gt})-\tau_{qt}\tau_n},\\
\beta_{4,t} &= \frac{G}{R_f}(1+\beta_{4,t+1}),\\
\tau_{xt} &= \sqrt{\frac{H_t\chi}{\chi+(\frac{1}{\beta_{3,t}})^4}},\\
\tau_{gt} &= (\frac{1}{\beta_{3,t}})^2\frac{1}{\sqrt{\chi}}\sqrt{\frac{H_t}{\chi+(\frac{1}{\beta_{3,t}})^4}},
\end{aligned} \tag{125}$$

and

$$V_{Ft} = (1+\beta_{4,t+1})^2\alpha^2\left[\frac{1}{\theta^2}\frac{1}{\tau_\varepsilon+\tau_{xt}+\tau_{qt}}\right] + \beta_{1,t+1}^2\left[\frac{1}{\tau_\varepsilon} + \beta_{3,t+1}^2\frac{1}{\tau_n}\right] \tag{126}$$

is the variance of the asset payoff $d_{t+1} + q_{t+1}$ conditional on the average investor's information set F_t with average signal realizations and average precision. The equation for $\beta_{4,t}$ has a stationary solution which is

$$\beta_{4,t} = \beta_{4,t+1} = \beta_4 = \frac{G}{R_f - G}, \qquad \forall t. \tag{127}$$

We analyze the model using numerical examples. In these examples we set $\tau_\varepsilon = 80.08$, $\tau_n = 19.75$, $\gamma = 0.05$, $\chi = 22$, $\alpha = 1$, $R_f = 1.02$, $G = 0.98$. We also assume that the financial data technology follows a deterministic time series

$$H_t = 0.00095 \times 2^{0.49\times(t-1)}, \quad t = 1, 2, 3, \cdots. \tag{128}$$

The computation process of this model is also similar to that in Farboodi and Veldkamp (2020). We first compute the steady state of the model when $H_t \to +\infty$, and use this steady state solution as the equilibrium at the terminal date $T = 150$, then calculate backward the equilibria from $t = 149$ to $t = 1$.

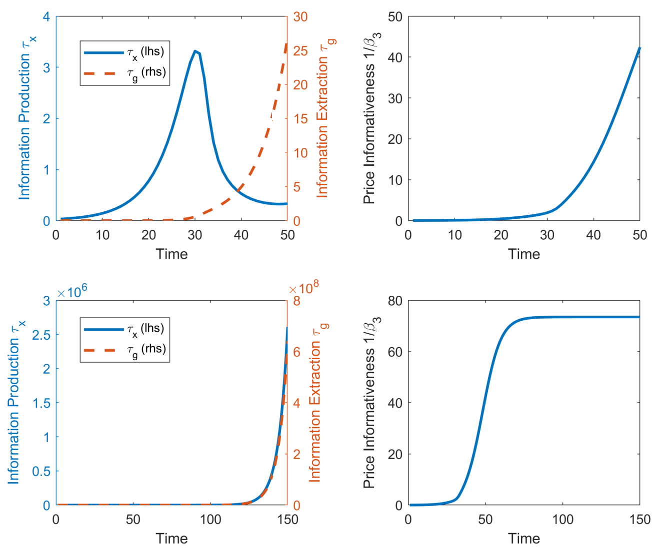

Figure 5. Impacts of financial data technology. This figure plots information production, information extraction, and financial price informativeness as functions of time. Note that the financial data technology evolves according to (128). Parameter values: $\tau_\varepsilon = 80.08$, $\tau_n = 19.75$, $\gamma = 0.05$, $\chi = 22$, $\alpha = 1$, $R_f = 1.02$, $G = 0.98$, $\theta = 2$, $H = 18.0036$, $\tau_s = 10$ $\eta = 0.5$.

The upper left panel of Figure 5 shows that when time increases from 0 to 50, information production first increases and then decreases, while information extraction always increases. The upper right panel of Figure 5 shows that when time increases from 0 to 50, the price informativeness also increases. The lower left panel of Figure 5 shows that after $t \approx 125$ both information production and information extraction grow very fast. The lower right panel of Figure 5 shows that as time evolves, the price informativeness first increases slowly, and then it increases very fast, and finally it keeps at a high level. Figure 5 indicates that the dynamics of information production, information extraction, and price informativeness are consistent with those in Farboodi and Veldkamp (2020).