The Cost of Intermediary Market Power for Distressed Borrowers

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Abstract

In the loan markets for distressed corporate borrowers, a few specialized lenders finance a large fraction of loans. Ultra-high yield spreads prevail even after removing the credit- and liquidity-risk component. Borrowers are in desperate need of financing but face limited funding options, while specialized lenders have repeated syndication relations with restrained participation. We develop and estimate a dynamic game-theoretic model, accounting for strategic competition, endogenous collusion capacity, endogenous participation, and latent heterogeneity. Lender market power accounts for 74 - 92% of the risk-adjusted yield spreads, with a significant fraction attributable to collusion. Smaller borrowers are more susceptible to lender market power. Importantly, both specialized lenders and distressed borrowers would be worse off if collusion is completely prohibited, suggesting that vigorous antitrust policies can be efficiency retarding.

Keywords: Collusion in Syndication, Antitrust, Intermediary Asset Pricing, Agency Conflicts, Likelihood-Based Estimation. (**JEL:** G12, G23, G30, L13)

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1 Introduction

It is well-known that repeated syndication interactions over time can facilitate coordination. When European Commission et al. (2019) published the long-awaited European Union (EU) report on the potential competition concerns of the loan syndication process,[1] regulators and corporations were clearly paying serious attention to the market power of syndication lenders in the loan creation process and expressing the concern of possible "club deals."

To understand the role of lender market power in determining the loan yield spread is essentially an asset pricing question. Traditional neoclassical asset pricing theories assume that a fully diversified representative investor can trade all assets freely and the asset markets are efficient, typically featuring perfect information, perfect competition, and absence of arbitrage (e.g., Fama, 1965; Ross, 1976). Thus, asset prices are unbiased estimates of the fundamental values of the assets in those theories, primarily determined by the compensations for aggregate fundamental risks borne by the representative investor in equilibrium (e.g., Sharpe, 1964; Merton, 1973). In reality, many important asset markets are, however, mainly intermediated by a relatively small number of highly specialized institutional investors. Market segmentations and the oligopolistic structures of these asset markets, due to high entry barriers, have important asset pricing implications beyond the traditional neoclassical theories. On the one hand, since the pioneered work by Shleifer and Vishny (1997), substantial progress has been made in understanding the unique role of intermediaries in connecting asset prices to economic quantities through the channels of funding liquidity risks, leverage constraints, and fund flow shocks.[2] On the other hand, imperfect competition among highly specialized institutional investors is expected to exert a significant impact on asset prices owing to the market power of these institutional investors. Yet, until recently, the effect of intermediary market power has been relatively understudied in the literature.[3]

[1]The report focuses on leveraged buyouts (LBOs) and project/infrastructure finance, across a sample of six member states: France, Germany, the Netherlands, Poland, Spain and the UK.

[2]Many theoretical studies have shown that intermediaries do not act simply as a sideshow in asset pricing (e.g., Shleifer and Vishny, 1997; Gromb and Vayanos, 2002; Gabaix et al., 2007; Brunnermeier and Pedersen, 2008; Basak and Pavlova, 2013; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Frazzini and Pedersen, 2014; Drechsler et al., 2018; Dou et al., 2022b).

[3]A few influential exceptions include studies that investigate how the market power of intermediaries shapes

This paper examines the role of imperfect competition among specialized institutional lenders in determining the prices of loans issued to distressed corporate borrowers in the primary market.[4] In particular, we document that, for these loans to distressed borrowers, a significant fraction of loan yield spreads cannot be explained by credit spreads or liquidity premia. Our estimates further show that the risk-adjusted yield spread is largely attributable to intermediary market power. More important, we further dissect intermediary market power in risk-adjusted yield spreads into two components. The first component captures the unilateral market power of specialized lenders that arises from their non-coordinated conduct.[5] This component mainly reflects the borrowers' price elasticity of demand for funding and the lenders' (endogenous) market concentration in syndication. The second component captures the collusive market power of specialized lenders, derived from their tacitly coordinated conduct.

Specifically, we find that, deviating from the traditional neoclassical asset pricing theories, the pricing behavior of lenders across two different primary loan markets for distressed corporate borrowers — the distressed loan market and the debtor-in-possession (DIP) loan market — is affected by imperfect competition that leads to large lenders' market power. Distressed loans are those issued to firms that are distressed but not yet bankrupt,[6] and DIP loans are those issued to firms that are already in Chapter 11 bankruptcy. One key empirical finding is that, after removing the credit- and liquidity-risk component, there is still a substantial unexplained yield spread, referred to as a "risk-adjusted yield spread," for both types of loans. Over 2001 – 2017, the yearly average risk-adjusted yield spread in the distressed loan market

the security prices in credit card markets (e.g., Knittel and Stango, 2003), certain OTC markets (e.g., Duffie et al., 2005, 2007), bank deposit markets (e.g., Drechsler et al., 2017; Egan et al., 2017), and life insurance markets (e.g., Koijen and Yogo, 2019).

[4]Institutional investors, as intermediaries, compete not only in the investment market of assets, but also in the market of delegation products. While this paper focuses on the former, it's worth noting that there have been recently an increasing number of studies that investigate the latter (e.g., Hortaçsu and Syverson, 2004; Hastings et al., 2017).

[5]The unilateral market power is the non-collusive (or non-coordinated) market power. Drawing from the Department of Justice (DOJ) and the Federal Trade Commission (FTC) horizontal merger guidelines, it is defined as the ability of an individual agent to act alone to set a price greater than the competitive price and make profits (e.g., Holt, 1989; Borenstein et al., 2002; Rassenti et al., 2003; Wolak, 2003).

[6]The definition of "distressed loans" is consistent with that of "distressed bonds" in bond markets. Specifically, unlike "leveraged loans" and "junk bonds," which include all debt securities with a credit rating of BB+ or lower, distressed debt securities (including loans and bonds) must have a credit rating of CCC+ or lower. The accurate definition of "distressed loans" for our analysis is elaborated in Section 4.1.

remained at a high level, fluctuating typically between 150 and 400 bps. Similarly, over 2002 – 2019, the yearly average risk-adjusted yield spread in the DIP loan market also remained at a high level, and it fluctuated in the range from 350 to 900 bps without obvious cyclical patterns or clear comovement with financial conditions of the economy. This pricing behavior contrasts sharply with that in corporate bond markets in which the credit- and liquidity-risk component accounts for almost all the bond yield spread (e.g., Longstaff et al., 2005; Blanco et al., 2005). In the cross-section of the syndicated loans for distressed corporate borrowers, the risk-adjusted yield spreads are larger for those loans with smaller distressed borrowers and those with fewer specialized lenders participating in the syndication.

This striking pricing behavior arises naturally from the oligopolistic structures of those loan markets in which a few specialized lenders finance a large fraction of loans and compete imperfectly. Intuitively, in a perfectly competitive financing market, lenders charge interest and fees that are commensurate with (i) the default risk of the borrower, (ii) the liquidity risk of the security, (iii) the ex-post re-contracting costs (Roberts and Sufi, 2009; Berg et al., 2016), and (iv) the marginal costs lenders incur in making debts to firms (such as the monitoring cost, the information generation cost, and the funding cost of lenders). However, the competitive loan pricing behavior is insufficient to account for the economically substantial risk-adjusted yield spreads observed in the market of distressed loans and that of DIP loans. We emphasize that the market power of specialized institutional lenders plays an essential role in explaining the financing cost of distressed corporate borrowers. This is not very surprising, because specialized lenders can possess strong market power over distressed firms for at least the following four reasons. First, distressed firms face a dire liquidity situation and are in desperate need to raise capital to survive with limited funding options, and thus their bargaining position is weak, reflected by their low price elasticity of demand for loans. Second, high entry barriers, caused by specialization and regulation, lead to segmented and concentrated markets in which specialized lenders can tacitly collude in the form of syndication. Third, the blocking power of a distressed firm's existing creditors prevents other specialized lenders from participating in a syndication deal. Existing creditors often hold a more favorable position in potential creditor conflicts, which further discourages other specialized lenders from participating in a syndication deal. Forth, syndicated loans have not been subject to registration requirements and anti-fraud provisions promulgated by the Securities and Exchange Commission ("SEC").

Dissecting the lender market power in these loan markets into the unilateral and the collusive components, however, is empirically challenging. First of all, the potential lender collusion, explicit or implicit, is unobservable to econometricians, and constructing empirical proxies to lender collusion is very difficult if not completely impossible. Secondly, we need to estimate the demand system and the supply structure, without observing actual costs. Particularly, the observed high loan spreads after risk adjustments may not necessarily reflect specialized lenders' market power; alternatively, they may reflect the substantial private costs incurred by lenders in making loans (such as the monitoring cost, the information generation cost, and the funding cost). Consistently estimating the demand curve is challenging due to the generic omitted variable issue (i.e., the endogeneity issue) — the correlation between loan prices and borrower-specific latent demand shocks, which are included in the econometric error term. Finally, it is highly endogenous whether the distressed borrower would end up getting the syndicated loan by specialized lenders or obtaining the funding from some alternative sources as lenders of last resort (i.e., outside options). Identifying and consistently estimating the size of the outside options is often a challenging task.

To overcome these challenges, we develop and estimate a new structural model that exploits the implications of endogenous (tacit) collusion to dissect the risk-adjusted yield spread into the components attributed to collusive versus unilateral market power of specialized institutional lenders. The model provides a unified description for both the distressed loan market and the DIP loan market. To address the first challenge, we model collusive and non-collusive equilibrium outcomes within a coherent and unified theoretical framework to ensure that the collusion capacity of specialized lenders is determined endogenously and co-vary coherently with other equilibrium outcomes such as the number of specialized lenders and the size of a syndicated loan, and that the collusive equilibrium outcomes that can never go beyond the monopoly or perfect collusion. By doing so, we improve the approach of Nevo (2001), who only considers a model with perfect collusion, even though it is probably not achievable under the estimated parameters. In fact, there have been important efforts made lately in the empirical industrial organization literature (e.g., Ciliberto and Williams, 2014; Miller and Weinberg, 2017) to improve the approach of Nevo (2001) by modeling the departure from non-collusive Nash equilibria exogenously as a parametric function of certain observed characteristics, such as the "multi-route contact" between carriers in Ciliberto and Williams (2014). Our paper takes one step further by endogenizing the relation of the departure from non-collusive Nash equilibrium to the structural parameters and borrowers' characteristics.

To address the second challenge of omitted latent variables in estimating demand and supply curves, a general strategy is to estimate the demand and then combine the estimated demand system with the specified pricing rules of the supply side to recover the latent costs and the markups. The first step of this strategy usually assumes that observed characteristics present a source of exogenous variation to the choice variable and there exists a vector of instrumental variables (IVs) that include characteristics and cost shifters (Berry, 1994; Berry, Levinsohn, and Pakes, 1995, hereafter referred to as BLP). Our methodology differs substantially from theirs. We develop a generic structural model that is analytically tractable. The closed-form solution allows us to estimate the demand system and the supply structure simultaneously using a likelihood-based estimation approach when latent demand curve shifts can be summarized by a relatively low-dimensional space.

Specifically, the structural model has two parts. The first part consists of the demand curves of distressed corporate borrowers, which specifies the risk-adjusted yield spread that the distressed borrower is willing to pay for a given amount of loan. The demand system specification is similar to that of Hendel (1999). The second part describes the supply side of the loan markets, and it is based on a repeated game model (e.g., Fudenberg and Maskin, 1986; Rotemberg and Saloner, 1986).[7] This theoretical model forms the basis for an econometric model of imperfect competition among a clique of specialized institutional lenders, in which, taking into

[7]Not many studies have been developed on the asset pricing and corporate finance implications of strategic competition in repeated games; a few exceptions include the recent works by Opp et al. (2014), Dou et al. (2021b,c), Chen et al. (2021), and Dou et al. (2022a).

account any externalities, these lenders endogenously choose whether to participate in a syndicated loan, and decide how to lend together, cooperatively or not, if joining the syndication. The repeated game model controls for the endogenous supply and competition of the lenders, eliminating inconsistency in the estimation of the demand system. Together, the two parts control for endogeneity for each other to achieve identification. This echoes the important insight that full information maximum likelihood estimator is equivalent to an instrumental variables estimator where the instruments embody all the over-identifying model-implied restrictions (e.g., Hausman, 1975).

We estimate the model using the data from the two markets separately, using the Markov Chain Monte Carlo (MCMC) estimator. The MCMC estimation methodology is a Bayesian approach and it computes the posterior distribution of the model parameters and latent variables conditioning on the observed quantities. Because the Bayesian inference via the MCMC approach can deal with the latent heterogeneity, if its dimensionality is relative low, through the Bayesian classification rules, valid IVs become unnecessary for identification and estimation. The MCMC methodology has seen a quick growth in finance studies (Sorensen, 2007; Johannes and Polson, 2010; Kortweg, 2010).

To tackle the third challenge of identifying outside options, we highlight that the process of making a distressed loan naturally follows a three-stage procedure: first, the distressed borrower negotiate with its existing lenders; second, if the borrower and its existing lenders fail to reach a deal, each specialized lender chooses whether and how to participate the syndicated loan; and third, if none of the specialized lenders is willing to make a loan, the distressed borrower has to turn to the lenders of last resort (usually hedge funds), which is effectively the outside option to the second stage. The lenders of last resort usually extract all the rents so that the distressed firms are indifferent from borrowing from them or not. The last-resort lending behavior can be well observed in the data, and thus it can be identified and consistently estimated.

We assemble a comprehensive dataset that contains 441 distressed loan facilities from 2001–2017 and 436 DIP loan facilities from 2002–2019. Relative to the literature, our sample has the most comprehensive coverage of information on these loans, including the loan size, spread, the number of specialized lenders in a syndication, participating lender identities, lender type, and characteristics of the borrowers. The estimates differ substantially across the two markets, which reveal distinct features that help explain the differentials in the risk-adjusted yield spread in the two loan markets albeit their similarities. Our estimation delivers a few novel findings. First, we find that lenders' tacit collusion contributes to a sizable component of the risk-adjusted yield spread, amounting to about 210 bps, in both markets. Second, borrowers in both markets exhibit similar and low price elasticity of demand. Taken together, the excess spread present in the DIP loan market cannot be explained by the differentials in collusion capacity or those in price elasticity across the two markets.

According to our estimates, the factors that set the two markets apart are the specialist lenders' participation cost and their variable cost of lending. Specifically, the participation cost accounts for 280 bps in the DIP loan spread but only 100 bps in the distressed loan spread. An important determinant of the participation cost is the existing lender's blocking power that prevents the borrowers borrowing from alternative lenders, and the other is the existing creditor's information advantage. Our results therefore suggest that existing lenders in the DIP-loan market have much stronger blocking power than those in the distressed loan market. Furthermore, we also find that lenders in the DIP loan market incur significantly higher variable cost of lending. The estimated variable cost is 180 bps in the DIP-loan market while only about 30 bps in the distressed loan market. This finding is quite intuitive since the DIP loan is expected to be more complex.

Our decomposition of the risk-adjusted yield spread in both market allows us to further analyze which types of borrowers are particularly susceptible to the market power of institutional lenders. We partition the borrowers in our sample into size quintiles and repeat our analyses above on the small (bottom quintile) and large (top quintile) borrowers. Our findings are striking: shutting down lender collusion reduces the loan yield spread by about 166 (126) bps for large borrowers and by 260 (255) bps for small borrowers in the distressed loan (the DIP loan) market. Our analysis thus suggests that small borrowers are more vulnerable to the market power of institutional lenders mainly because small borrower usually have lower price elasticity of loan demand and give the specialized lenders smaller incentives to deviate from the coordination. As a result, policies aiming at helping distressed companies in economic and financial crises should target more at small firms given their weak bargaining power and disadvantageous position in the distressed loan markets.

Lastly, we use the estimated model as a laboratory to examine the effect of a widely debated regulatory intervention — interest rate cap. We allow the regulator to impose a cap on the specialist lenders' markup in lending and thus restrict the loan spread that can be charged by these lenders. We analyze the specialist lenders' strategic responses to the rate-cap policy and the consequence on borrowers' welfare. Solving the model with rate-cap reveals a few intriguing implications. First, as loan spreads are capped, specialist lenders have less incentive to collude to a small loan size and thus they capture the residual profits by increasing the loan amount. Higher loan amount and lower spread improves the borrowers' welfare when they borrow from specialist lenders (i.e., a positive intensive margin). But meanwhile, rate-cap reduces the expected profits by specialist lenders and thus discourage their participation in the lending market. As fewer specialists are willing to lend in the market, the likelihood for the borrowers to borrow from the lenders of last-resort rises. Since loans made by lenders of last-resort are much more expensive, rate-cap generates an unintended consequence of reducing the depth of specialist lenders' market (i.e., a negative extensive margin). Combining the two effects, we demonstrate that the effect of rate-cap on borrower welfare is hump-shaped and there exists an optimal level of rate-cap. The optimal spread averages about 100 bps for distressed loans and 400 bps for DIP loans, compared with 340 bps and 710 bps observed in the data.

Related literature. Our paper contributes to the literature on loan pricing and contracting. Prior studies document the effect of information problems, lending relationship, syndicate structure, and lender specific characteristics on the pricing of corporate loans.[8] A few studies suggest that loans to bankrupt companies are not priced competitively and lenders can capture

[8]See, Rajan (1992); Petersen and Rajan (1994); Sufi (2007); Ivashina and Scharfstein (2010); Bharath, Dahiya, Saunders, and Srinivasan (2011); Chava and Purnanandam (2011) and Murfin (2012), among others.

economic rents (Hasan et al., 2019; Eckbo et al., 2020). Schwert (2020) shows that loan lenders earn a large premium relative to the bondholders of the same company, and documents that the premium is especially large for companies with high default risk. Cai et al. (2018) and Hatfield et al. (2020) suggest that price collusion by syndicate members may exist in syndicated loan markets.

Our study differs from prior papers on a few important fronts. First, we develop a theoretical model that takes into account the real-world frictions presented in the lending markets, including lenders' private funding costs, borrowers' downward sloping demand curve, and costs of collusion and monitoring. Second, fitting this model to our hand-collected novel dataset, we are able to quantify the margins earned by specialized lenders from distressed borrowers as a result of collusion versus private funding costs. More importantly, our study provides policy implications by showing that smaller borrowers are most susceptible to lender collusion and thus adds value to the recent policy debate on whether government should step in to finance bankrupt firms.[9]

2 Background and Motivating Evidence

2.1 Background

The loan markets for distressed corporate borrowers are among the few most essential asset markets in the economy because of its vital role in determining the ex-ante "financial distress cost" for the whole corporate sector, albeit not the largest by market value. These markets define the survival rate of financially distressed firms and the efficiency of bankruptcy processes. In particular, financially distressed firms seek urgent financing to support their working capital, investment, and debt repayment. Without such financing, their business operation would be in despair and may end up in a premature liquidation. Despite their desperation to fund

[9] In considering the difficulty of smaller distressed borrowers to access financing, the Consolidated Appropriations Act of 2021 ("CAA") that was signed into law on December 27, 2020 allows small debtors in bankruptcy to access the Paycheck Protection Program ("PPP") for loans. However, Small Business Administration ("SBA") holds a different view.

operations, financially distressed firms face severe financial constraints and other economic frictions in arranging financing.

We emphasize that the importance of an asset market is not equivalent to the size of the market. A useful analogy is the intensive care unit (ICU) in the healthcare and hospital system. The ICU is a small segment of the whole healthcare and hospital system because the ICU admission only accounts for less than 10% of the total hospital admission per annum and the ICU beds only account for about 10% of the hospital beds in total. However, the ICU has the first-order importance to ensure the whole healhcare and hospital system to operate properly. Similar to the ICU segment, the distressed loan and the DIP loan together account for about 10% of the outstanding leveraged loan per annum, but they account for over 45% of the outstanding leveraged loan in the year of 2009 (i.e., the Great Recession).[10] Despite the importance of this market to the survival of distressed firms, academic studies on the pricing of distressed loans are scarce. Our paper aims to fill this gap by quantifying the effect of lenders' market power on distressed loan pricing and suggesting implementable policies.

In the distressed loan market and the DIP loan market, distressed corporate borrowers deal with a clique of specialized lenders that finance a large fraction of loans. In Section 2.2, we present a set of novel evidence showing that loan spreads are ultra high even after removing the credit- and liquidity-risk components and that these specialized institutional lenders, as a clique of "market leaders," have substantial market power in these markets. In fact, not surprisingly, specialized lenders are expected to possess strong market power in financing distressed firms for at least the following three reasons.

First, the distressed borrowers' bargaining position is weak and their price elasticity of demand for loans is low. Distressed firms face a dire liquidity situation and are in desperate need to raise capital to survive. Importantly, these firms often have very limited "unencumbered" assets to pledge for additional secured credit, and very different from non-distressed ones, they

[10]Financing distressed companies has become an important concern for both policymakers and practitioners as the U.S. enters into a recession during a global pandemic and a record number of large companies filed for bankruptcy in 2020. Albeit not in bankruptcy, many firms suffered large operating losses and became financially distressed. Importantly, as emphasized by DeMarzo et al. (2020), there are a large number of firms that suffered from financial distress caused by a "pause" in cash flows.

hardly have any alternative external funding options such as equity, bonds, or unsecured lines of credit. Moreover, a distressed borrower is reluctant to approach a large number of lenders or even call for an open auction for getting the best priced loan due to concerns of information leakage that could adversely affects its security price, profit margin, customer base, and supply chain.

Second, high entry barriers lead to segmented loan markets in which a small clique of specialized lenders seize the majority of the market share and may tacitly collude. Lenders who are not familiar with or do not have expertise in restructuring of distressed firms and those with regulatory capital concerns choose not to enter the market.[11] Moreover, because specialized lenders have tight and repeated syndication relationships (Cai et al., 2018; Hatfield et al., 2020) and share a considerable amount of multi-market contact beyond the loan markets (Bernheim and Whinston, 1990; Evans and Kessides, 1994; Ciliberto and Williams, 2014), distressed borrowers typically find themselves facing only a clique of specialized lenders equipped with strong incentives and capacities to collude tacitly in making loans.

Third, even for the few specialized lenders, there are two additional forces preventing them from participating a loan syndication: one is existing creditors' blocking power, and the other is existing creditors' favorable position in potential creditor conflicts. As the first force, existing creditors of distressed borrowers have both the incentive and power to block competing outside lenders from participating. Specifically, they can, to certain extent, prevent outsider lenders from lending to the distressed borrowers due to their liens on the collateral, even when the borrowers would prefer to invite more outsider lenders to participate. Existing creditors have strong incentives to block outsider lenders' participation because they are not only reluctant to share the investment opportunity, but also concerned with ex-post coordination issues and creditor conflicts in bankruptcy processes (Gertner and Scharfstein, 1991; Morris and Shin, 2004; Brunner and Krahnen, 2008; Dou et al., 2021a). Importantly, existing creditors can have substantial control over the distressed borrower to ensure their blocking power. For example, rather than allow the distressed borrower to pursue a syndicated loan with a large number

[11] For example, the Basel-III framework requires financial institutions to set aside larger quantity of capital for financing companies with higher default risk because of the higher regulatory risk-weighting of high risk loans.

of outsider lenders, the existing creditors can impose a syndication with a small number of outsider lenders by threatening the distressed borrower of the firm of liquidation at the interim date (i.e., early liquidation).

As the second force, existing creditors' favorable position in potential creditor conflicts discourages the participation of competing outsider lenders who are concerned with information asymmetry and moral hazard problems. For example, the existing lenders can dictate an outcome that improves its position at the expense of other new creditors. One common practice is so-called "roll-up" in which the borrower pays off the earlier loan with proceeds of the newly-added DIP loan, ensuring that the earlier loan is paid in full even if it was not actually fully collateralized (e.g., Tung, 2020). Consequently, competing outside lenders are likely to be shied away from the deals, and then, existing creditors are allowed to hold up borrowers for ultra-high interest rates (e.g., Sharpe, 1990; Schenone, 2010; Santos and Winton, 2008).

2.2 Motivating Evidence

Concentrated markets dominated by a clique of specialized lenders. We first construct a comprehensive sample for large public borrowers in the distressed loan market from 2001 to 2017 and a comprehensive sample for large public borrowers in the DIP loan market from 2002 to 2019. We elaborate the details on the construction of the distressed loan sample and that of the DIP loan sample in Sections 4.1 and 4.2, respectively. In our sample, the average maturity of distressed loans is 49 months, and the average maturity of DIP loans is 11 months. The average loan-to-asset ratios are 13.0% and 10.7% for distressed loans and DIP loans, respectively. Moreover, the average loan yield spreads after removing the credit- and liquidity-risk component, i.e., the average risk-adjusted yield spreads, are 337 and 718 basis points for distressed and DIP loans, respectively. These spreads are excessively high, and these ultra-high risk adjusted spreads are the primary focus of this paper.

We first identify whether a lender of a distressed loan is a *major lender* following the procedure of Jiang et al. (2010) based on their titles (e.g., agent bank, lead arrangers), and identify whether a lender of a DIP loan is a *major lender* using the same procedure based on the DIP fi-

Table 1: Specialized lenders and market concentration

A. Names of specialized lenders

Rank	Distressed loan market Lender name	# of deals	DIP loan market Lender name	# of deals
1	Bank of America	188	Wells Fargo	96
2	JP Morgan Chase	182	Bank of America	88
3	Wells Fargo	124	JP Morgan Chase	88
4	Citigroup	107	GE Capital Corp	82
5	Credit Suisse	105	Citigroup	67
6	Deutsche Bank	102	Deutsche Bank	41
7	Goldman Sachs	60	Credit Suisse	31
8	GE Capital	58	Wachovia Bank	28
9	UBS	58	Wilmington Trust	27
10	Wachovia Bank	53	CIT Group	21

B. Three loan types

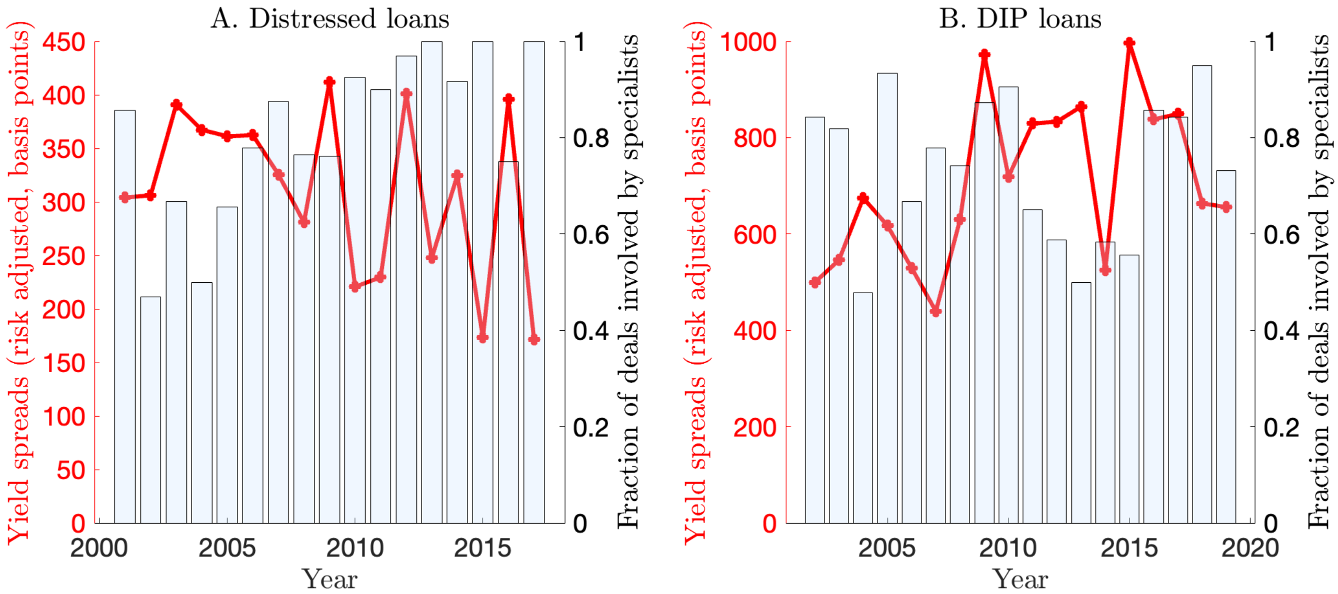
Lender type	Distressed loans				DIP loans			
	# of deals	# frac.	$ of deals	$ frac.	# of deals	# frac.	$ of deals	$ frac.
Type 1: Existing creditor	52	11.80%	13	5.65%	56	12.80%	5	4.90%
Type 2: Specialized lender	336	76.20%	208	90.40%	334	76.60%	94	92.16%
Type 3: Lender of last resort	53	12.00%	9	3.91%	46	10.60%	3	2.94%
Total	441	100%	230	100%	436	100%	102	100%

Note: The distressed loan sample is from 2001 to 2017, and the DIP loan sample is from 2002 to 2019. The loan size are measured by constant 2019 dollars and presented in the unit of billion dollars. The distressed loan sample includes only facilities after removing those with negative adjusted spreads.

nancing motions and master credit agreements. We then define a major lender to be a *specialized lender* in one loan market, if it is one of the top 10 lenders in the loan market ranked by the number of loans it financed as a major lender according to our sample, regardless of the number of major lenders in the loan syndicate. We can further define three types of loans for both the distressed loan and DIP loan markets. The first type contains loans that are provided by only one lender, who is an existing creditor but not a specialized lender. These loans are referred to as *existing-creditor loans* and denoted by *Type 1*. The second type contains loans that are provided by at least one specialized lender when there are more than one major lenders for a loan, and also contains loans that are provided by one specialized lender that is not an existing creditor. These loans are referred to as *specialized-lender loans* and denoted by *Type 2*. The third type contains loans that have over 50% of the major lenders as hedge funds and private equity funds. These loans are referred to as *last-resort loans* and denoted by *Type 3*.

Panel A of Table 1 presents the names of the specialized lenders in the distressed loan and DIP loan markets. Panel B of Table 1 reports the breakdown of the number of loans for each of the three types and shows their shares as a fraction of the total number of deals in our samples. There are several important points worth mentioning. First, the number of deals financed by the top intermediaries decays rather fast even among the specialized lenders — the involvement of the number 1 ranked specialized lender in the distressed loan market is about 4 times larger than that of the number 10 ranked specialized lender, and the same pattern exhibits in the DIP loan market. Second, the specialized-lender loan accounts for 76.20% (90.40%) and 76.60% (92.16%) fraction of the total distressed and DIP loan market, respectively, in terms of the number of deals (in terms of the loan size in dollars). Third, GE Capital Corp is the only specialized lender that is not a large bank with an investment banking arm. In fact, GE Capital Corp is oversight by the Federal Reserve as a "systemically important financial institution" like other large banks, and importantly, it has substantial multi-market contact with large banks in investment banking, real estate, commercial lending & leasing, and healthcare finance services, among others, which makes GE Capital Corp no ordinary shadow banking lender. Although it is unlikely for ordinary shadow banking lenders, such as hedge funds and private equity shops, to be included in the "club" as a reliable member in tacit coordination, GE Capital Corp is unique due to its substantial multi-market contact with large banks. Finally, the specialized lenders are largely overlapped (8 out of 10) between these two loan markets for distressed corporate borrowers, which is consistent with the hypothesis that it requires specialized skills in distress, restructuring, and bankruptcy to become a major player in the distressed and DIP loan markets. All these facts suggest that the loan markets for distressed corporate borrowers feature market segmentations and high concentration, dominated by a few specialized lenders.

Ultra-high risk-adjusted yield spreads. We measure the total cost of borrowers (TCB) by incorporating relevant fees paid to lenders, consistent with the literature (e.g., Ivashina, 2009; Lim et al., 2014; Berg et al., 2016). As emphasized by Berg et al. (2016), lenders do not use a single measure such as an interest rate spread to ensure an appropriate expected return, but rather use combinations of fees and interest rate spreads. It's important for us to take into



Note: This figure plots the risk-adjusted yield spread and the fraction of deals financed by specialized lenders in the distressed loan market and the DIP loan market over years. The curves represent the average risk-adjusted yield spread per year, and the bars represent the fraction of deals financed by the 10 specialized lenders per year.

Figure 1: Average risk-adjusted yield spreads and fraction of deals financed by specialized lenders

account the relevant fees for distressed borrowers in the calculation of a loan's return, because lenders with greater market power are likely to charge higher fees. Specifically, the TCB is measured in basis points, and it is defined as the interest rate spread of the loan facility over LIBOR for each dollar used under the loan commitment plus any fees that the borrower must pay to the lenders.

We consider three main types of fees paid by the borrower, including upfront fee, annual fee (or facility fee), and state-contingent fee such as default interests and commitment fee (or ticking fee).[12] Upfront fee is paid to the lenders at loan closing, annual fee is paid annually on the entire commitment amount, regardless of usage, and default interests can be viewed as a state-contingent fee paid by default borrowers ex post. In contrast, commitment fee is not incorporated in our TCB measure, although commitment fee is overall prevalent for revolving loans (e.g., Berg et al., 2016). It is a contingent fee and paid by borrowers on unused loan

[12]Other types of fees include utilization fee (or usage fee), cancellation fee (or termination fee), term-out fee, extension fee, and collateral monitoring fee. We exclude those fees from our TCB measure because they are not commonly used in syndicated loans (e.g., Berg et al., 2016).

commitments ex post. We exclude commitment fee because the distressed, even bankrupt, corporate borrowers almost always use the entire commitment amount in their credit lines. It's evident that an increase in expected default probability substantially increases the drawdown rate of credit lines (e.g., Jiménez et al., 2009). Taken together, for the distressed loans and DIP loans, we calculate the TCB over LIBOR as follows:

$$\begin{aligned}\text{TCB Spread} = \text{Interest Rate Spread} &+ \text{Annual Fee} \\ &+ \text{Upfront Fee/Risk Neutral Expected Loan Maturity in Years} \\ &+ \text{Risk Neutral Expected Annualized Default Fee,} \end{aligned} \tag{1}$$

where the details on the calculation of the risk-neutral expected loan maturity in years and the risk-neutral expected annualized default fee can be found in Appendix B.1.

The risk-adjusted yield spread is measured by the TCB spread net of the credit- and liquidity-risk component. We first estimate the credit spread component in the yield spread of distressed loans and that of DIP loans in our sample. For the DIP loan, we estimate the credit spread component by 20 bps. A DIP loan is a short-term loan with a maturity usually less than 1 year and secured with first lien. It is as safe as high investment-grade short-term loans. According to our own calculation using loan facilities in Dealscan and S&P issuer ratings, the median loan yield spread of syndicated loans with ratings of AA- and higher is about 20 bps. Therefore, we estimate the credit-risk component of each DIP loan by 20 bps.

For each distressed loan, we back out the implied risk-neutral default probability using the CDS written on the borrower and the estimated CDS recovery rate; subsequently, we estimate the credit-risk component using the implied risk-neutral default probability and the estimated recovery rate of the borrower's loans. Our approach closely follows the parity relation between CDS spreads and bond credit spreads derived by Duffie (1999).[13] The details are elaborated in Appendix B.1. To estimate the CDS recovery rate, we first identify the industry classification of the borrower and the loan issuance year. We then use the average recovery rate of senior

[13]Blanco et al. (2005) provide supporting evidence for the theoretical parity derived by Duffie (1999) among investment-grade bonds using the swap rate as the risk-free rate.

unsecured and subordinated bonds in this industry, adjusted for the year fixed effect and the loan type (revolving vs. term loans) fixed effect, to estimate the recovery rate of the CDS written on the borrower, and the pricing effect of investors' aversion against the uncertainty of recovery rates is captured by the implied risk-neutral default probability. We exert more caution in estimating the CDS recovery rate by accounting for the heterogeneity more carefully than the literature. For example, in the influential work of Longstaff et al. (2005), the CDS recovery rate is assumed to be constant at 50% throughout the estimation procedure. The historical information on the recovery rates of senior unsecured and subordinate bonds is provided by the NYU-Salomon Center Default database.[14] The average recovery rate of U.S. firms' senior unsecured and subordinate bonds in the NYU-Salomon sample is 34.39% from 2000 to 2019, in close proximity to 38.02%, the average recovery rate of corporate bonds with CCC+ or lower ratings in the year of 1997 used by Elton et al. (2001), and also close to 37%, the average debt recovery rate of U.S. large bankrupt firms from 1996 to 2014 estimated by Dou et al. (2021a). Similarly, to estimate the loan recovery rate, we first identify the industry classification of the borrower and the loan issuance year; then, we use the average loan recovery rate in this industry, adjusted for the year fixed effect and the loan type (revolving vs. term loan) fixed effect, to estimate the recovery rate of the loan issued by the borrower. The historical information on the loan recovery rates is obtained from Badoer et al. (2019), who use Moody's Default and Recovery Database to measure ultimate nominal recoveries at the completion of a debt restructuring to compile recovery statistics by industry and year.[15] Therefore, the average estimated credit-risk component of distressed loans is about 160 bps.

We then estimate the liquidity-risk component in the yield spread of distressed loans and DIP loans separately. For the DIP loan, we estimate the liquidity premium by 13 bps. First, a DIP loan is a short-term debt with a maturity usually less than 1 year, and it is a senior and secured loan with first and second lien. As a result, it unlikely cause liquidity concerns to its investors. Second, Longstaff et al. (2005) find that the credit spread component accounts for a

[14]For the senior unsecured and subordinate bonds, the distribution of recovery rates in each of the Fama-French 12 industries or each year from 2000 to 2019 is shown in Appendix.

[15]For the syndicated loans, the distribution of recovery rates in each of the Fama-French 12 industries or each year from 1987 to 2012 is shown in Appendix.

major portion of the bond yield spread for bonds. Moreover, Chen et al. (2007) find that the liquidity premium component alone explains about 15% of the bond yield spread for short-term bonds with maturities of 1 – 7 years and ratings of AA- or higher. Taken together, for the DIP loan, the liquidity premium component is unlikely higher than the credit spread component, estimated by 20 bps.

For the distressed loan, we estimate the liquidity premium by 20 bps. First, investors also trade distressed loans in secondary loan markets (e.g., Gande and Sauders, 2012). Particularly, Wittenberg-Moerman (2008) and Bushman et al. (2010) use Loan Trade Database to show that the secondary market of distressed loans is more liquid than that of less risky loans, opposite to the pattern in the secondary markets of bonds. These studies suggest that distressed loans syndicated by more reputable arrangers are likely traded with lower liquidity premia than the distressed bonds. Second, the estimates of Chen et al. (2007) suggest that the liquidity premium is about 34 bps for short-term bonds with maturities of 1 - 7 years and ratings of CCC+ or lower.

Panel A of Figure 1 plots the time series of the average distressed loan risk-adjusted yield spread per year and the fraction of distressed loans financed by specialized lenders per year. Panel B of Figure 1 plots the same yearly time series for the DIP loan market. There are several important points about Figure 1 that is worth mentioning. First, the average risk-adjusted yield spread after removing the credit spread and the liquidity premium component remain stable over years at a very high level. The average risk-adjusted yield spread of distressed loans fluctuates mainly between 150 and 400 bps, while that of DIP loans fluctuates mainly between 350 and 900 bps, without obvious cyclical patterns or clear comovement with financial conditions of the economy. Second, the fraction of loans financed by specialized lenders also stay high persistently, without obvious cyclical patterns or clear comovement with financial conditions of the economy. These facts suggest that the high concentration of the loan markets for distressed corporate borrowers is not due to high concentration clustered around a few point of time, but it is due to the persistent feature in the industry structure of loan markets for distressed corporate borrowers.

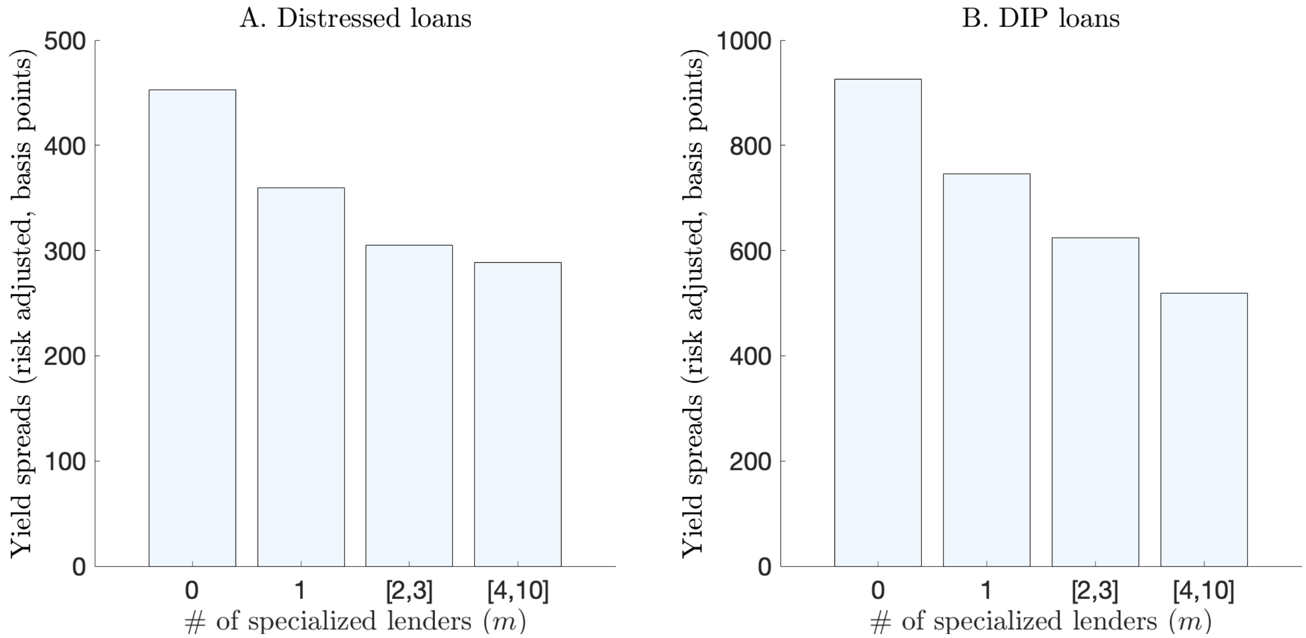


Figure 2: Lender market power.

Syndication concentration and loan yield spreads. We now investigate the relation between the number of specialized lenders in a loan syndication and the risk-adjusted loan spread. Figure 2 shows that the risk-adjusted loan spread monotonically decreases in the number of specialized lenders in a loan syndication, suggesting a strong effect of lenders' market power on the total cost of distressed borrowers.

3 Game-Theoretic Model for Distressed-Loan Markets

In this section, we build a novel and flexible game-theoretic model of distressed loan market competition. In the model, financially distressed firms borrow from the distressed loan market dominated in the hands of a few specialized lenders. On the loan demand side, heterogeneous borrowers with latent characteristics have downward-sloping demand curves of latent demand elasticity and level. On the loan supply side, a clique of specialized lenders can tacitly collude sustained by repeated syndication relations and optimally choose whether to enter a deal as a syndicate lender, leading to endogenous market concentration and market power. We first describe the model's setup, then explain the predictions that form the basis of our estimation.

3.1 Model Formulation

The model starts with M specialized major lenders in a specific loan market for distressed corporate borrowers. These lenders are referred to as "specialized lenders" throughout the paper. There are M_0 potential lenders, including non-specialized lenders, in the market with $M \ll M_0$. We emphasize that neither the exact value of M nor that of M_0 matters for our analysis because the effective market concentration in our model is endogenously determined by specialized lenders' optimal entry decisions to be syndication participants.

The specialized lenders have strong incentives to form tacit collusion and collaborate in syndicating the loans for distressed borrowers, because specialized lenders' dominant position is highly persistent and quite a few large deals arrive in the market every year. Importantly, "tacit coordination" needs not involve any explicit collusion with direct communication and agreement in the legal sense, and an interchangeable term is "tacit collusion" or "noncooperative collusion" or "conscious parallelism" (e.g., Martin, 2006; Ivaldi et al., 2007; Harrington, 2008; Green et al., 2014; Garrod and Olczak, 2018).

The loan deals arrive randomly. We assume that the arrivals of loan deals can be characterized by a Poisson process with intensity η. Each borrower can be characterized by its size, measured by the total asset value, denoted by A, and its demand curve type, denoted by $k \in \{1, \cdots, K\}$. As to be specified in the next section, our model allows heterogeneous demand curves across different borrowers, and thus the type k captures the demand curve a borrower is attached to. Even though size A is observable, the demand curve type k is seen only by the agents inside the model and is latent to the econometricians. We denote the fraction of type k borrowers in the population by π_k for any $k \in \{1, \cdots, K\}$. Each borrower of type k starts with trying to match with its existing creditors with a probability λ_k. If a borrower fails to reach an agreement with its existing creditors, it continues to approach the specialized lenders for funding. It is possible that no specialized lender is willing to participate in the syndicated loan deal, in which case the borrower has to find the last-resort non-specialized lender. We classify lenders into three types, denoted by $l \in \{1, 2, 3\}$. Particularly, we label the existing creditor by $l = 1$, the specialized lender by $l = 2$, and the last-resort non-specialized lender by $l = 3$. Before elaborating the timeline and sequence of lenders' decisions for each loan deal in Section 3.1.2, we first introduce the basics about borrowers and lenders in our model below in Sections 3.1.1 and 3.1.2.

3.1.1 Demand Specification

When a distressed borrower of type k and size A faces lenders of all types, the value of the borrower by optimally raising a loan of size L is

$$\max_{L>0} L^{\rho_k} S_k(A,z)^{1-\rho_k} - LR, \tag{2}$$

where $L^{\rho_k} S_k(A,z)^{1-\rho_k}$ captures the benefits derived from borrowing a loan of size L, and LR captures the costs incurred due to borrowing a loan of size L, which cannot be justified by the riskfree rate or the fair risk compensation. Here, the coefficient $\rho_k \in (0,1)$ captures the decreasing return to scale of the amount of loans raised by the borrower, $S_k(A,z)$ is the return shifter, a quantity that captures the effect of firm characteristics and incorporates the interactions between investor and asset characteristics (like in Hendel, 1999), and R captures the risk-adjusted loan yield spread after removing the riskfree rate and the credit- and liquidity-risk component. The return shifter $S_k(A,z)$ is specified as follows:

$$S_k(A,z) \equiv Ae^{\nu_k+\sigma z}, \tag{3}$$

where A is the total asset of the firm, ν_k captures the effect of the interactions between investor and asset characteristics, and z is a latent firm-specific demand shock.

The first-order condition for L leads to a demand curve of loan size for a type-k borrower as follows:

$$\ln\left(\frac{L}{A}\right) = \alpha_k - \varepsilon_k \ln(R) + \sigma z, \tag{4}$$

where

$$\alpha_k \equiv \nu_k + \frac{\ln \rho_k}{1-\rho_k}, \tag{5}$$

$$\varepsilon_k \equiv \frac{1}{1-\rho_k}. \tag{6}$$

The variation in the term $\alpha_k + \sigma z$ captures the demand curve shift across different firms in the population. The coefficient α_k captures the heterogeneous demand level depending on different borrower types k. The borrower type k is latent to the econometricians, whereas the lender type l is observable to the econometricians.

The demand function in equation (4) is a standard iso-elastic downward-sloping demand curve. The coefficient ε_k is effectively the price elasticity of demand. Consistent with the literature (e.g., Atkeson and Burstein, 2008), we assume that $\varepsilon_k > 1$.

We assume that z is i.i.d. distributed according to the standard normal distribution across different cases. The assumption that the demand curve depends on heterogeneous borrowers and differentiated lenders is a natural analog of that in the empirical IO literature emphasizing heterogeneous consumers and differentiated products (e.g., Berry et al., 1995). A higher α_k means that the borrower of type k has higher average demand of loans from the lenders.

Intuitively, charging higher spread R leads to smaller loan amount because $\varepsilon_k > 1$. The slope coefficient ε_k mainly captures the price elasticity of the type-k borrower's loan demand from the lenders. Similar in spirit to Koijen and Yogo (2019), the slope coefficient is determined by multiple important primitive characteristics of borrowers and lenders. Like the approach adopted by Koijen and Yogo (2015, 2019), among others, in their structural estimation, we start with specifying a flexible demand system on top of a structural agent-optimization model, rather than microfounding the equilibrium relations between the slope coefficient ε_k and various primitive characteristics of borrowers and lenders. Specifically, ε_k can reflect the bargaining power of the borrower and the lenders in the loan market, the marginal value of liquidity of the borrower, the intervention style of the lenders, and the non-pecuniary benefits to the firm from borrowing from the relationship lenders. With a larger ε_k, the amount of loans L the firm would like to borrow declines more with an increase in the risk-adjusted yield spread R required by the lender, meaning that the borrower's demand for loans is more elastic to the loan spread and thus the borrower has more bargaining power. Take an extreme case as an example to illustrate the role of the slope coefficient ε_k. When $\varepsilon_k \to +\infty$, the borrower has full bargaining power over the lenders since it is not willing to pay any spread on top of the risk-adjusted rate. In such an extreme case, a higher α_k obviously reflects a higher intrinsic loan demand from the borrower.

Demand curves can also vary by lender types. Classic banking theories suggest that banks act as traditional financial intermediaries to provide financing and acting as the delegated monitors (e.g. Diamond (1984, 1991)). Banks rarely seek direct control of the borrower through board representation and other mechanisms due to their concerns of legal liabilities (Fischel, 1989). The loan costs to the borrower are mostly reflected in spreads and fees to be paid to the lender. In contrast, alternative investors such as hedge funds and private equity firms use loan instruments as a tool to engage activism and seek control. They often adopt the "loan-to-own" strategies where they lend to firms with an intention to convert debt into ultimate equity ownership (Jiang et al., 2012). In addition to loan interest and fees, these alternative lenders impose strict covenants that may be tied to management changes and governance, which allow them to directly control the borrower's business operation. As a result, to borrowers, loans provided by traditional lenders and alternative lenders may be viewed as different products.

We normalize the loan size L by the total asset A in the modeling of the demand system for distressed loans in equation (4). The main reasons or motivations behind such modeling choice are threefold. First, the additional financial distress risk caused by the newly-added leveraged distressed loan L depends on the total asset of the borrower A. Second, the leveraged distressed loan L is mainly for covering working capital, which in turn is usually proportional to the firm size and thus the total asset level A. Third, in the data, we do find that a strong relationship between the normalized loan size L/A and the spread R within each type of deals.

3.1.2 Supply Specification

We first describe the market structure and "technology" of the lenders to make distressed loans. The supply side of distressed loans is characterized by Cournot competition of oligopolistic lenders similar to Berry et al. (1995). The oligopolies can tacit collude usually in the form of syndication, which goes beyond the non-collusive Nash behavior adopted by the BLP framework and captures the highly strategic competition behavior (i.e., tacit coordination). The literature shows that some credit markets are concentrated in the hands of a few leading financial institutions, and importantly, these institutional lenders compete highly strategically including competition in the form of tacit collusion. For example, Knittel and Stango (2003) documents micro-level evidence suggesting that tacit collusion at non-binding state-level ceilings was prevalent in the credit card lending market during the early 1980's. Moreover, on 5 April 2019, the European Commission published a report – prepared by Europe Economics at the request of the department for competition – on EU loan syndication and its impact on competition in credit markets. The commission had worried that syndicated loan coordination tended to occur tacitly, making anti-competitive collusion on interest rates easier to pull off. The worry is natural because syndicated loans naturally require coordination and communication. The report highlighted the risk that lenders involved in syndicating a loan together would promise future tacit cooperation in exchange for current concessions on limiting loan supply. Nocke and White (2007) and Hatfield et al. (2020) theoretically show that, under certain circumstances, tacit collusion can exist in syndicated markets with repeated interaction of lenders. Further, Carrasco and Manso (2006) theoretically show that syndication is the optimal response of colluding lenders to the communication costs resulting from the negotiations between them for a given loan. Our model builds on the important insights of loan syndication and potential tacit collusion of repeated interacted lenders in the distressed loan markets.

Costs of lending distressed loans. The lender incurs a fixed cost from learning the deal type and participating in lending, and it also incurs a variable cost that depends on the amount of loan made to the borrower L. Specifically, the lender incurs a one-time fixed cost of w if it decides to learn the deal type k and commit to participation. Fixed cost w is random and privately observed by the lender. However, the distribution of w is a common knowledge of all agents and is assumed to be the exponential distribution with mean μ with the following density function:

$$f(w;\mu) = \mu e^{-w/\mu}. \tag{7}$$

Fixed cost w captures both direct costs, such as compensation to talents and other labor costs, as well as indirect costs, such as the loss of other investment opportunities because of limited resources. In addition, the lender of type l incurs variable costs of $e^{\phi_l+\varsigma u}$ for each unit of lending, where u is a standard normal random variable that captures the deal-specific latent cost. Taken together, both latent characteristics z and u are deal specific (or firm specific). We denote $x \equiv (z,u)$. Variable cost $e^{\phi_l+\varsigma u}$ captures both direct costs, such as compensation, as well as indirect costs, such as marginal (shadow) costs of funding for the lender. Consistent with this assumption, we find that the lender explicitly charges proportional fees to cover the variable costs. For any deal, total costs from w and $e^{\phi_l+\varsigma u}$ are denoted by

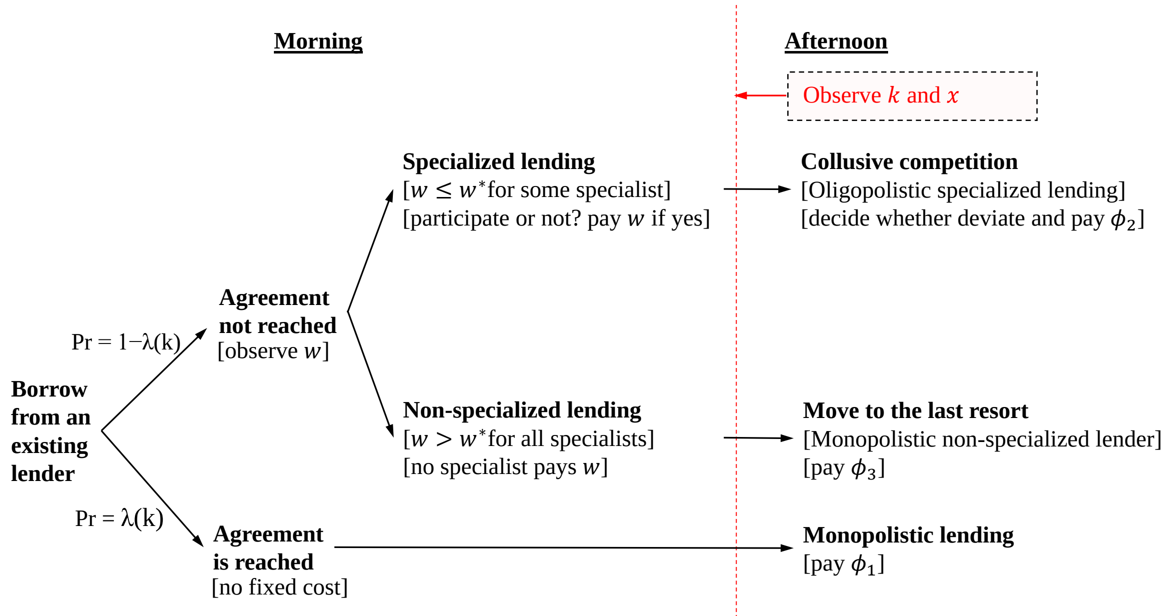
$$C_l(L) \equiv w\mathbf{1}_{\{L>0\}} + e^{\phi_l+\varsigma u}L. \tag{8}$$

Timing and sequence of lenders' decisions within each deal. Figure 3 illustrates lenders' choices and possible outcomes in each deal, including lending by existing lenders and last-resort lending possibilities. The time span for each deal is divided into two subperiods, "morning" and "afternoon." The shocks, such as whether to punish for deviation or not, the type of the deal k, and the private costs $(w_1,\cdots,w_M)$, are realized in the morning, while the lending decisions L and R are made in the afternoon. Importantly, specialists must make their decisions whether to participate in the syndicated lending by the end of the morning, and no specialist who has already committed can leave without participation after learning the number of participants m in the afternoon.

In the first step of "morning", the firm approaches the existing lender, who can be a specialized or non-specialized lender. We assume that each of the M_0 potential lenders can be the

Figure 3: Model timeline

This figure describes the timeline of the model. Each lending period is divided into two subperiods – morning and afternoon. The existing lender first decides whether to lend to the borrower with a probability $\lambda(k)$, if the existing lender does not lend to the borrower, the borrower approaches a group of specialist lenders who can decide whether to participate in a syndicate loan. The specialist lenders need to pay a fixed cost if they participate. If no specialist lenders are willing to participate, the borrower turns to the lenders of last-resort.



existing lender with equal chances (i.e., $1/M_0$). The lending agreement with this particular existing lender can only be achieved with a small probability λ_k which depends on case type k. If the lending agreement with the existing lender is reached, the monopolistic lender of type $l = 1$ chooses the optimal spread and the loan size according to the demand curve:

$$\Pi_1(A, k, x) = \max_L \left[\left(e^{\alpha_k + \sigma z} \frac{A}{L} \right)^{1/\varepsilon_k} - e^{\phi_1 + \varsigma u} \right] L, \tag{9}$$

where the default probability on the distressed loan does not show up since $R = (e^{\alpha_k + \sigma z} A/L)^{1/\varepsilon_k}$ is a risk-adjusted yield spread, and $x \equiv (z, u)$. It leads to the optimal monopolistic spread and loan size:

$$R_1(k, x) = \frac{\varepsilon_k}{\varepsilon_k - 1} e^{\phi_1 + \varsigma u} \quad \text{and} \quad L_1(A, k, x) = \left[1 - \frac{1}{\varepsilon_k} \right]^{\varepsilon_k} e^{\alpha_k - \varepsilon_k(\phi_1 + \varsigma u) + \sigma z} A, \quad \text{respectively.} \tag{10}$$

Therefore, the optimal profit is

$$\Pi_1(A,k,x) = \frac{1}{\varepsilon_k} R_1(k,x) L_1(A,k,x). \tag{11}$$

Loan markup is defined as $\left[R_1(k,x) - e^{\phi_1+\varsigma u}\right] / e^{\phi_1+\varsigma u}$. The risk-adjusted yield spread implies that the loan markup is $1/[\varepsilon_k - 1]$, suggesting that the markup ratio decreases with the elasticity coefficient ε_k. The optimal profit $\Pi_1(A,k,x)$ is $1/\varepsilon_k$ fraction of the revenue $R_1(k,x)L_1(A,k,x)$. That is, the profit margin is $1/\varepsilon_k$. When the elasticity coefficient ε_k is lower, the borrower's loan demand is effectively more urgent and pressing, leading to a higher markup and a higher profit margin. The detailed derivations of (10) and (11) are in the appendix.

The expected optimal profit is

$$\begin{aligned}
\Pi_1(A,k) &= \mathbb{E}^x\left[\Pi_1(A,k,x)\right] \\
&= \frac{1}{\varepsilon_k}\left[1 - \frac{1}{\varepsilon_k}\right]^{\varepsilon_k - 1} e^{\alpha_k - [\varepsilon_k - 1]\phi_1 + [\varepsilon_k - 1]^2 \varsigma^2/2 + \sigma^2/2} A.
\end{aligned}$$

The game moves to the second stage of "morning" if the firm fails to get a deal from the existing lender. In this case, the competition takes place among the M specialists (type $l = 2$). We consider tacit collusion of a few specialized lenders. Let $V^C(A,k,x,w,m;L^C)$ be the collusive value function of each specialist at the beginning of "afternoon", if there are m specialized lenders who decided to participate in the deal of syndicated loan, the case has the demand curve type k, the specialized lender has private cost w, the size of the borrower is A, the deal-specific characteristics x, and the agreed scheme on loan size is $L^C(A,k,x,m)$. There exists an equilibrium threshold w_C^* such that the specialized lender would participate in the syndicated loan if and only if $w \leq w_C^*$.

Note that the privately observed fixed cost w is a sunk cost when the lender chooses the loan amount L. Because $V^C(A,k,x,w,m;L^C)$ is value function of a specialized lender at the beginning of the "afternoon" when w, k, and x are already observed, the value function has the following functional form:

$$V^C(A,k,x,w,m;L^C) \equiv U^C(A,k,x,m;L^C) - w. \tag{12}$$

The value function $U^C(A,k,x,m;L^C)$ satisfies the following Bellman equation:

$$U^C(A,k,x,m;L^C) = \Pi_2(A,k,x,m;L^C) \quad + \quad \frac{W^C(L^C)}{1-\delta}, \quad \text{where} \tag{13}$$

$$W^C(L^C) = \mathbb{E}^{A',k'}\left\{\lambda_{k'}\frac{\Pi_1(A',k')}{M_0} + (1-\lambda_{k'})\,\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m';L^C) - w'\right)\mathbf{1}_{\{w'\leq w_C^*\}}\right]\right\}. \tag{14}$$

Here, the expected profit from participating the syndication in the next period is

$$\begin{aligned}&\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m';L^C) - w'\right)\mathbf{1}_{\{w'\leq w_C^*\}}\right] \\ &\quad = \sum_{m'=1}^{M} q(m'|w'\leq w_C^*)\left\{F(w_C^*)\mathbb{E}^{x'}\left[\Pi_2(A',k',x',m';L^C)\right] - \int_{w'\leq w_C^*} w' dF(w')\right\},\end{aligned} \tag{15}$$

the profit of the syndicated lending with tacit collusive loan size plan L^C is

$$\Pi_2(A',k',x',m';L^C) \equiv \left[\left(e^{\alpha_{k'}+\sigma z'}\frac{A'}{m'L^C(A',k',x',m')}\right)^{1/\varepsilon_{k'}} - e^{\phi_2+\varsigma u'}\right]L^C(A',k',x',m'), \tag{16}$$

and the conditional probability $q(m'|w'\leq w_C^*)$ is

$$\begin{aligned}q(m'|w'\leq w_C^*) &= \frac{\mathbb{P}\left\{\text{This specialist and other } m'-1 \text{ specialists participate the lending}\right\}}{\mathbb{P}\left\{\text{This specialist participates the lending}\right\}} \\ &= \binom{M-1}{m'-1}F(w_C^*)^{m'-1}\left[1-F(w_C^*)\right]^{M-m'}.\end{aligned}$$

The derivation of this Bellman equation is elaborated in detail in the appendix.

The game moves to the last-resort stage of "afternoon" if no specialists would like to participate the lending. In this case, the last resort (a non-specialized lender, i.e., the lender of type $i=3$) will lend to the firm as a monopoly with a fixed cost w and variable costs $e^{\phi_3+\varsigma u}$. The last-resort lender optimally chooses $L_3(k)$ to maximize the profit:

$$\Pi_3(A,k,x) = \max_L \left[\left(e^{\alpha_k + \sigma z} \frac{A}{L} \right)^{1/\varepsilon_k} - e^{\phi_3 + \varsigma u} \right] L. \tag{17}$$

It leads to the optimal monopolistic spread and loan size:

$$R_3(k,x) = \frac{\varepsilon_k}{\varepsilon_k - 1} e^{\phi_3 + \varsigma u} \quad \text{and} \quad L_3(A,k,x) = \left[1 - \frac{1}{\varepsilon_k} \right]^{\varepsilon_k} e^{\alpha_k - \varepsilon_k(\phi_3 + \varsigma u) + \sigma z} A, \text{ respectively.} \tag{18}$$

Therefore, the optimal profit is $1/\varepsilon_k$ fraction of the revenue $R_3(k,x)L_3(A,k,x)$ as follows:

$$\Pi_3(A,k,x) = \frac{1}{\varepsilon_k} R_3(k,x) L_3(A,k,x). \tag{19}$$

Similar to the case in which the existing lender supplies all the loan, lower elasticity ε_k leads to a higher markup $1/(\varepsilon_k - 1)$ and a higher profit margin $1/\varepsilon_k$. The detailed derivations of (18) and (19) are in the appendix.

3.1.3 Collusive Nash Equilibrium

The equilibrium outcomes in the lending of existing creditors and that of last-resort lenders can be explicitly characterized in (10) – (11) and (18) – (19), respectively. The equilibrium collusive loan amount is the central piece of model outcome that needs to be pinned down, which we explain in this subsection.

Optimal lending under tacit collusion and incentive compatibility constraint. We first denote the unconstrained optimal loan amount under tacit collusion by $L^C_{max}(A,k,x,m)$, which solves the following profit maximization problem:

$$L^C_{max}(A,k,x,m) \equiv \underset{L}{\operatorname{argmax}} \left[\left(e^{\alpha_k + \sigma z} \frac{A}{mL} \right)^{1/\varepsilon_k} - e^{\phi_2 + \varsigma u} \right] L. \tag{20}$$

The first-order condition gives the solution for $L^C_{max}(A,k,x,m)$ as follows:

$$L^C_{max}(A,k,x,m) \equiv \frac{1}{m}\left(1-\frac{1}{\varepsilon_k}\right)^{\varepsilon_k} e^{-\varepsilon_k\phi_2-\varepsilon_k\varsigma u} e^{\alpha_k+\sigma z} A, \tag{21}$$

with the corresponding unconstrained optimal risk-adjusted yield spread and lending profit under tacit collusion to be

$$R^C_{max}(k,x,m) = \frac{\varepsilon_k}{\varepsilon_k-1} e^{\phi_2+\varsigma u} \text{ and } \Pi^C_{max} = \frac{1}{\varepsilon_k} R^C_{max}(k,x,m) L^C_{max}(A,k,x,m), \text{ respectively.} \tag{22}$$

Comparing (21) – (22) with (10) – (11), the intuition behind the smallest possible collusive lending outcomes immediately follows. That is, the m syndication participants first form the strongest coalition that behaves as if it is a monopoly, and then they split the loan equally.

However, the unconstrained optimal collusive loan size $L^C_{max}(A,k,x,m)$, derived in (21), is usually unsustainable in the equilibrium because syndication participants would have strong incentives to deviate and reap additional profits by secretly supplying extra loans to the borrower. To sustain the tacit coordination among the specialized lenders as the club members who repeatedly participate in a particular market of distressed syndicated loans, the specialized lenders have an imperfect capacity of monitoring, communication, and ex-post punishment. Specifically, upon a deviation is detected, the specialized lenders will not tacitly collude anymore starting from the next period with a probability ξ as the punishment for deviation. This grim trigger punishment strategy is easy to implement and incentive compatible. There is a probability $1-\xi$ that the deviation will not be punished. Thus, the parameter ξ captures the tacit collusion capacity in a parsimonious way. A lower ξ reflects a lower tacit collusion capacity, which can be due to more costly monitoring, more costly communications, and higher chances of achieving successful ex-post renegotiation. In our theory and structural estimation, we capture and estimate the tacit collusion capacity by focusing the deep structural parameter ξ without specifying or estimating various possible economic mechanisms that micro-found the imperfect tacit collusion at a more granular level, which is beyond the scope of this paper.

Now we characterize the set of loan sizes that are sustainable under the tacit collusion scheme with collusion capacity captured by parameter ξ, namely, the set of loan sizes that satisfy the incentive-compatibility constraint. Intuitively, the loan size $L^C(A,k,x,m)$ under the tacit collusion cannot be overly small to ensure that the m participants will have no incentives to deviate. We denote by $\mathcal{L}^C(A,k,x,m)$ the set of loan sizes for each of the m syndication participants that satisfy the incentive-compatibility constraint, which can be expressed as follows:

$$\mathcal{L}^C(A,k,x,m) \equiv \left\{ L^C : \mathbb{E}^x \left[U^C(A,k,x,m;L^C) \right] \geq \mathbb{E}^x \left[U^D(A,k,x,m;L^C) \right] \right\}, \tag{23}$$

where $U^C(A,k,x,m;L^C)$ is the value function when all the m syndication participants stick to the tacit coordination scheme of specialized lenders in the market as club members, and $U^D(A,k,x,m;L^C)$ is the maximum value of the syndication participant that deviates from the given tacit coordination scheme $L^C(\cdot,\cdot,\cdot,\cdot)$, which is described in detail below.

Intuitively, the tacit coordination scheme on loan size $L^C(\cdot,\cdot,\cdot,\cdot)$ lies in the set $\mathcal{L}^C(A,k,x,m)$ if and only if the expected value of not deviating, $\mathbb{E}^x\left[U^C(A,k,x,m;L^C)\right]$, is not strictly dominated by that of deviating, $\mathbb{E}^x\left[U^D(A,k,x,m;L^C)\right]$. As explained in Figure 3, the deviation decision is made after the syndication participants learn their private lending cost w, borrower type k, and the number of syndication participants m, but before the latent case-specific characteristics x are learned. As a result, each syndication participant compares the two expected values contingent on not deviating or deviating.

Non-collusive Nash equilibrium and maximum value of deviation. To characterize the maximum value of deviation $U^D(A,k,x,m;L^C)$, we need to first characterize the phase of non-collusion competition among specialized lenders, where the outcome is described by the non-collusive Nash equilibrium. This is because the punishment for deviation is to shift into the phase of non-collusive competition.

The game moves to the second stage of "morning" if the firm fails to get a deal from the prepetition lender. In this case, the competition takes place among the M specialists (i.e. the lenders of type $l=2$). We consider the non-collusive Nash equilibrium in which the syndication participants never tacitly coordinate on making the loan. Let $V^N(A,k,x,w,m)$ be the non-collusive value function of each specialist at the end of "morning," a function of A, k, x, and m, there are m specialists who decided to participate in the deal of syndicated loan, the case is type k, and the specialist has private cost w. There exists an equilibrium threshold w_N^* such that the specialist would participate in the syndicated loan if and only if $w \leq w_N^*$.

Note that the privately observed fixed cost w is a sunk cost when the lender makes decision on L. Because $V^N(A,k,x,w,m)$ is value function of a specialist at the end of the morning when w is already privately observed, the value function has the following functional form:

$$V^N(A,k,x,w,m) \equiv U^N(A,k,x,m) - w. \tag{24}$$

The value function $U^N(A,k,x,m)$ prior to paying the fixed cost w and observing the deal-specific characteristics $x = (z,u)$ satisfies the following Bellman equation:

$$U^N(A,k,x,m) = \Pi_2(A,k,x,m;L^N) \quad + \quad \frac{W^N}{1-\delta}, \quad \text{with} \tag{25}$$

$$W^N = \mathbb{E}^{A',k'} \left\{ \lambda(k') \frac{\Pi_1(A',k')}{M_0} + \left[1 - \lambda(k')\right] \mathbb{E}^{w',m',x'} \left[\left(\Pi_2(A',k',x',m';L^N) - w' \right) \mathbf{1}_{\{w' \leq w_N^*\}} \right] \right\}. \tag{26}$$

Here, the expected profit from participating the syndication in the next period is

$$\begin{aligned} &\mathbb{E}^{w',m',x'} \left[\left(\Pi_2(A',k',x',m';L^N) - w' \right) \mathbf{1}_{\{w' \leq w_N^*\}} \right] \\ &= \sum_{m'=1}^{M} q(m'|w' \leq w_N^*) \left[F(w_N^*) \mathbb{E}^{x'} \left[\Pi_2(A',k',x',m';L^N) \right] - \int_{w' \leq w_N^*} w' dF(w') \right], \end{aligned} \tag{27}$$

the profit of the syndicated lending with non-collusive loan size plan L^N is

$$\Pi_2(A',k',x',m';L^N) \equiv \max_L \left[\left(e^{\alpha_{k'} + \sigma z'} \frac{A'}{L + (m'-1)L^N(A',k',m')} \right)^{1/\varepsilon_{k'}} - e^{\phi_2 + \varsigma u'} \right] L, \tag{28}$$

and the conditional probability $q(m'|w' \leq w_N^*)$ is

$$\begin{aligned} q(m'|w' \leq w_N^*) &= \frac{\mathbb{P}\left\{\text{This specialist and other } m'-1 \text{ specialists participate the lending}\right\}}{\mathbb{P}\left\{\text{This specialist participates the lending}\right\}} \\ &= \binom{M-1}{m'-1} F(w_N^*)^{m'-1} \left[1 - F(w_N^*)\right]^{M-m'}. \end{aligned}$$

The derivation of this Bellman equation is elaborated in detail in the appendix.

In the non-collusive equilibrium, the equilibrium loan size $L^N(A,k,x,m)$ is characterized as follows:

$$L^N(A,k,x,m) = \underset{L}{\text{argmax}} \left[\left(e^{\alpha_k + \sigma z} \frac{A}{L + (m-1)L^N(A,k,x,m)} \right)^{1/\varepsilon_k} - e^{\phi_2 + \varsigma u} \right] L, \tag{29}$$

which leads to

$$L^N(A,k,x,m) = \frac{1}{m} \left[1 - \frac{1}{m\varepsilon_k} \right]^{\varepsilon_k} e^{-\varepsilon_k \phi_2 - \varepsilon_k \varsigma u} e^{\alpha_k + \sigma z} A. \tag{30}$$

Therefore, the equilibrium revenue of a syndication participant has the following closed-form expression:

$$\Pi_2(A,k,x,m;L^N) = \frac{1}{m^2 \varepsilon_k} \left[1 - \frac{1}{m\varepsilon_k} \right]^{\varepsilon_k - 1} e^{-[\varepsilon_k - 1](\phi_2 + \varsigma u)} e^{\alpha_k + \sigma z} A. \tag{31}$$

Finally, we now characterize the maximum value of deviating from an agreed tacit coordination scheme $L^C(\cdot,\cdot,\cdot,\cdot)$. We denote by $V^D(A,k,x,w,m;L^C)$ the value function for deviation given a fixed tacit coordination scheme $L^C(\cdot,\cdot,\cdot,\cdot)$ and state variables A, k, m, and w. We define $V^D(A,k,x,w,m;L^C) \equiv U^D(A,k,x,m;L^C) - w$, and thus, the value function $U^D(A,k,x,m;L^C)$ satisfies the following Bellman equation:

$$U^D(A,k,x,m;L^C) = \Pi_2^D(A,k,x,m;L^C) + \frac{(1-\xi)W^C(L^C) + \xi W^N}{1-\delta}, \tag{32}$$

where $W^C(L^C)$ is the continuation value with the tacit coordination scheme $L^C(\cdot,\cdot,\cdot,\cdot)$, defined in (13), W^N is the continuation value in the phase of non-collusive competition, defined in (25), and the contemporaneous profit gained if the syndication participant deviate from the tacit coordination scheme $L^C(\cdot,\cdot,\cdot,\cdot)$ is

$$\Pi_2^D(A,k,x,m;L^C) \equiv \max_{L>0} \left[\left(e^{\alpha_k+\sigma z} \frac{A}{L+(m-1)L^C(A,k,x,m)} \right)^{1/\varepsilon_k} - e^{\phi_2+\varsigma u} \right] L. \tag{33}$$

Thus, according to (32) and (33), the set of loan size schemes $L^C(\cdot,\cdot,\cdot,\cdot)$ under tacit collusion competition that satisfy the incentive-compatibility constraint can be rewritten as

$$\mathcal{L}^C(A,k,x,m) \equiv \left\{ L^C : \frac{\xi[W^C(L^C)-W^N]}{1-\delta} \geq \mathbb{E}^x\left[\Pi_2^D(A,k,x,m;L^C)\right] - \mathbb{E}^x\left[\Pi_2(A,k,x,m;L^C)\right] \right\}. \tag{34}$$

3.1.4 Endogenous Participation Boundaries

The cutoff points w_N^* and w_C^* are determined as in Li and Zheng (2009). Like solving value functions and optimal policies, we solve w_N^*, then we solve w_C^*. We assume that w is first revealed and specialists choose whether to learn, then k and m are revealed.

We first solve w_N^* according to the condition:

$$\sum_{m=1}^{M} q(m|w=w_N^*, w^*=w_N^*)\mathbb{E}^{A,k}\left[U^N(A,k,m;w_N^*) - w_N^*\right] = \frac{W^N(w_N^*)}{1-\delta}, \tag{35}$$

meaning that the marginal specialist with $w = w_N^*$ is indifferent between participating and not participating the syndicated lending. In other words, the marginal specialist, on average, has zero gain or loss from participating the syndicated lending. Here, $W^N(w_N^*)$ is the continuation value if the cutoff is w_N^*, and $q(m|w = w_N^*, w^* = w_N^*)$ is the probability of m participants conditioning on the cost of the specialist is w_N^* and the cutoff w^* is w_N^*. $q(m|w = w_N^*, w^* = w_N^*)$ has the following expression:

$$q(m|w=w_N^*, w^*=w_N^*) = \binom{M-1}{m-1} F(w_N^*)^{m-1}\left[1-F(w_N^*)\right]^{M-m}. \tag{36}$$

The equality (35) can be written as

$$\sum_{m=1}^{M} q(m|w = w_N^*, w^* = w_N^*)\mathbb{E}^{A,k,x}\left[\Pi_2(A,k,x,m;L^N)\right] = w_N^*, \tag{37}$$

It can be numerically solved in the following steps:

(i) Solve $\mathbb{E}^{A,k,x}\left[\Pi_2(A,k,x,m;L^N)\right]$ for each m.

(ii) Guess an initial w_N^*. If $\sum_{m=1}^{M} q(m|w = w_N^*, w^* = w_N^*)\mathbb{E}^{A,k,x}\left[\Pi_2(A,k,x,m;L^N)\right] < w_N^*$, we should decrease w_N^*.

(iii) Iterate until the condition is satisfied.

Then, we solve w_C^* according to the condition:

$$\sum_{m=1}^{M} q(m|w = w_C^*, w^* = w_C^*)\mathbb{E}^{A,k,x}\left[U^C(A,k,x,m;w_N^*,w_C^*) - w_C^*\right] = \frac{W^C(w_N^*,w_C^*)}{1-\delta}, \tag{38}$$

where w_N^* is solved earlier. This means that the marginal specialist with $w = w_C^*$ is indifferent between participating and not participating the syndicated lending. In other words, the marginal specialist, on average, has zero gain or loss from participating the syndicated lending. Here, $W^C(w_N^*, w_C^*)$ is the continuation value if the cutoff points are w_N^* and w_C^* for non-collusive and collusive Nash equilibria, and $q(m|w = w_C^*, w^* = w_C^*)$ is the probability of m participants conditioning on the cost of the specialist is w_C^* and the cutoff w^* is w_C^*. $q(m|w = w_C^*, w^* = w_C^*)$ has the following expression:

$$q(m|w = w_C^*, w^* = w_C^*) = \binom{M-1}{m-1} F(w_C^*)^{m-1}\left[1 - F(w_C^*)\right]^{M-m}. \tag{39}$$

The equality (38) can be written as

$$\sum_{m=1}^{M} q(m|w = w_C^*, w^* = w_C^*)\mathbb{E}^{A,k,x}\left[\Pi_2(A,k,x,m;L^C)\right] = w_C^*, \tag{40}$$

It can also be numerically solved in the following steps:

(i) Solve $\mathbb{E}^{A,k,x}\left[\Pi_2(A,k,x,m;L^C)\right]$ for each m.

(ii) Guess an initial w_C^*. If $\sum_{m=1}^{M} q(m|w = w_C^*, w^* = w_C^*)\mathbb{E}^{A,k,x}\left[\Pi_2(A,k,x,m;L^C)\right] < w_C^*$, we should decrease w_C^*.

(iii) Iterate until the condition is satisfied.

3.2 Model Solution

There is a theoretical property of the model that can ensure the semi-closed-form solution and help immensely simplify the numerical analysis. Specifically, we show that the equilibrium loan sizes under collusive competition, non-collusive competition, and deviation have the following functional form:

$$L^j(A,k,x,m) \equiv \widehat{L}^j(k,m)e^{\alpha_k - \varepsilon_k \phi_l - \varepsilon_k \varsigma u + \sigma z}A, \quad \text{with } j \in \{C,N,D\} \text{ and } l \in \{1,2,3\}. \tag{41}$$

Therefore, all equilibrium outcomes only depend on the discrete state variables k and m in a nonparametric way, while their dependence on the continuous state variables in x has a closed form, which is already known.

We solve the model in three steps. We first solve for U^N and W^N, together with the endogenous cutoff of participation in a non-collusive equilibrium ω_N^*, which is chosen such that a specialist lender with a participation cost of ω_N^* is indifferent between participate or not. Then for any given level of colluded loan amount L^C, we can solve for U^C, W^C, and ω_C^* as functions of L^C, U^D, W^D, and ω_D^* can be solved accordingly, which are also functions of L^C. As the last step, we search for the equilibrium L^C that prevents deviation, as defined in Equation (34).

The model generates solutions for three observable variables of interest, including the number of participating specialist lenders m, the loan size L/A, and the DIP rate R. These variables are functions of the DIP deal type observed in the data (i.e., financed by a monopolistic prepetition lender, or by one or multiple specialist lenders, or by a lender of the last resort) as well as the model parameters.

Lastly, to bring the model to the data. We label the type of a borrower i by (A_i, k_i) with $A_i > 0$ and $k_i \in \{1, \cdots, K\}$. We specify the probability of a borrower i belonging to cluster k_i and having A_i as follows:

$$\begin{aligned}\pi\left(A_i, k_i\right) &= Prob(k_i|A_i) \times Prob(A_i) \\ &= \frac{e^{\gamma_{k_i}+\beta_{k_i}(ln(A_i)-\mathbb{E}[ln(A)])}}{1+\sum_{k=2}^{K} e^{\gamma_k+\beta_k(ln(A_i)-\mathbb{E}[ln(A)])}} \times h(A_i)\end{aligned}$$

where $h(A)$ is the PDF of borrower size and $\mathbb{E}\left[ln\left(A\right)\right]$ is the average log-size. In this specification, borrower size A_i is observable, and we assume that the likelihood of borrower i belonging to demand curve $k \in \{1, \cdots,, K\}$ follows the multinomial logistic distribution with parameter γ_k and β_k (and γ_1 and β_1 are normalized to 1). This specification allows the model to capture possible correlation between borrower size and the demand, as it is plausible that large borrowers and small borrowers differ in their overall demand for the loan and the price elasticity.[16]

4 Data Sample

Academics as well as practitioners commonly refer distressed loans as loans provided to firms that are either deeply distressed with the likely prospect of defaulting on its obligations or that are already bankrupt (Altman, Hotchkiss, and Wang, 2019). For our study, we construct a comprehensive data sample that consists of bank loans to both financially distressed but not yet bankrupt U.S. public firms from 2001–2017 and those public firms already in Chapter 11 bankruptcy from 2002–2019, using two different data filtering methods.

4.1 Loans to Distressed But Not Yet Bankrupt Firms

We first assemble a data set of loans to financially distressed firms that are not yet in bankruptcy. The academic literature has proposed a number of measures for identifying financially dis-

[16]The model can be solved with a continuous-valued size A and any number of demand curve group k. In estimation, we divide borrowers into deciles based on their size and assume K heterogeneous demand curve. The number of groups K will be determined by statistical tests.

tressed firms. The existing approaches are broadly categorized as accounting ratio based (e.g. Altman Z-score models) and market price based approaches. The advantage of the market price based measure over accounting ratios is that not only it is forward looking but more importantly, it directly measures how costly it is for a firm to raise financing in the financial markets. With fast development and high trading liquidity in the credit derivative markets, a few of recent studies use prices of credit default swaps (CDS) to take a cue on the financial health of firms (e.g., Hortacsu et al., 2013; Brown and Matsa, 2016). Given that credit ratings are also informative indicators for corporate default and many institutions rely on ratings for their investment decisions, for our study, we rely on both CDS prices and credit ratings to identify financially distressed firms.

We retrieve monthly five-year CDS prices on senior unsecured bonds of U.S. public issuers from IHS Markit database and monthly S&P Long-Term Domestic Issuer Ratings of U.S. public firms from Compustat for the period from 2000–2017.[17] We use two criteria to identify the beginning of a firm's distressed period, namely, whether the five-year CDS price first hits 1000 bps[18] or whether a firm's S&P rating drops to CCC+ or lower[19], whichever occurs first. After identifying the initial starting month of a firm's distress period, we trace the firm's CDS spreads and credit ratings until the end of 2017. For firms with CDS spreads, the distress period ends when their CDS spreads fall below 500 bps or when they file for bankruptcy.[20] For firms with credit ratings, the distress period ends when S&P ratings goes up to B- or higher or the firm defaults according to S&P ratings.

Consolidating the distress periods identified using CDS spreads and ratings, removing duplicated time periods, and combining two consecutive periods that have an in-between time gap of less than a year, and removing distressed periods that are shorter than 6 months to

[17]Our sample stops in 2017 because Compustat no longer provides S&P ratings of U.S. public firms after 2017.

[18]Prior studies refer bonds whose yield spread above riskfree rate is over 1000 bps as distressed bonds (Altman et al., 2019)

[19]Loans that are issued by firms with CCC+ or lower ratings can no longer be widely held by institutional investors such as the Collateralized Loan Obligations (CLOs), the most important type of investors in the leveraged loan market, because there are limitations on the amount of CCC-rated loans that can be included in the underlying collateral pool of CLOs (typically 7.5% of the collateral pool). If any, CLOs are net sellers of CCC-rated loans.

[20]Bankruptcy filings, including both Chapter 11 and Chapter 7 filings, by all U.S. public firms from 2001-2019 from obtained from New Generation Research's Bankruptcydata.com.

avoid capturing transitory periods, we have 637 distressed periods of 520 firms from 2001–2017. We merge the distressed periods with the Dealscan database using the link file by Chava and Roberts to identify loan facilities that have a start date falling into the distress period. After removing facilities that have missing information on lender identities or loan spreads measured by the all-in spread drawn (AISD), which is the sum of LIBOR spread and annual fee, and loans that are unsecured or subordinated, we have a final sample of 520 loan facilities (342 packages) in 185 distressed periods.

We consolidate all financial institutions to the parent company. For example, JP Morgan Securities would carry the same unique institution ID as JP Morgan & Co. Moreover, we consider institutions' M&As that occurred in our sample period and consolidate the target and the acquirer into one entity after the transaction. Moreover, we remove entities that are special purpose investment vehicles and structured products such as CLOs and CDOs, which have little direct involvement in the primary market of distressed loans.

Using historical loan issuance information in Dealscan, we are able to determine whether a major lender is an existing lender to a firm—that is, a lender of the distressed loan is also a lender in an earlier loan that has not yet matured. We also determine whether a lender is a private equity fund or hedge fund by searching the lender's website and industry publications (Jiang, Li, and Wang, 2012). Next, we identify the CDS prices at the end of the month immediately preceding the loan issuance date. We also collect Treasury Constant Maturity Rate of different maturities from the Federal Reserve at St. Louis and 3-month LIBOR rate from Bloomberg. Finally, we retrieve firms' key financial information immediately before loan issuance from Compustat.

We define three types of distressed loans: existing-creditor loans (type-1 loans), specialized-lender loans (type-2 loans), and last-resort loans (type-3 loans) (see Section 2.2). We remove 47 facilities that cannot be classified into any of the above three types. Among the 441 loan facilities in our sample before adjusting for credit- and liquidity-risk components from spreads and removing those with negative adjusted spreads, the type-2 loan is the most dominant type, accounting for 76.2% of the distressed loans, while the type-1 and type-3 loan account for 11.8%

Table 2: Summary Statistics

This table presents the summary statistics of our sample firms. Our sample consists of 441 distressed loans (in Panel A) and 436 DIP loans (in Panel B) to U.S. public firms between 2001 and 2019. All financial variables are taken from the last fiscal year reported immediately prior to loan initiation, retrieved from Compustat. Assets, Liabilities and Sales are book assets, book liabilities, and revenue measured in millions of dollars, respectively. Leverage is the ratio of book liabilities to book assets. ROA is EBITDA scaled by book assets. PP&E/assets is the ratio of net property, plant and equipment to book assets. Cash/assets is the ratio of cash and short-term securities to book assets. Loan amount (L) is in millions of dollars. L/A is the ratio of DIP amount to book assets. AISD (R) is all-in-spread drawn in basis points. Number of lenders is the number of unique institutions in a syndicate. Loan type 1, Loan type 2 and Loan type 3 are indicator variables for loans provided by an existing lender, specialist lenders, and lenders of last resort, respectively.

	N	Mean	Std	25%	Median	75%
Panel A: Distressed Loans						
Assets	441	7,762	11,661	548	2,593	10,292
Liabilities	441	7,772	13,123	557	2,589	9,951
Leverage	441	1.053	0.446	0.797	0.950	1.172
ROA	441	0.067	0.133	0.037	0.075	0.114
PP&E/assets	441	0.378	0.225	0.172	0.355	0.528
Cash/assets	441	0.061	0.066	0.011	0.035	0.086
Loan maturity	441	48.846	20.951	35.000	48.000	60.000
Loan amount (L)	441	430.552	631.420	70.000	200.000	500.000
L/A	441	0.130	0.142	0.033	0.084	0.178
Risk-adjusted spread	441	337.450	207.120	201.6	292.1	447.2
Number of lenders	441	3.789	2.855	2	3	5
Loan type 1	441	0.118	0.323	0	0	0
Loan type 2	441	0.762	0.426	1	1	1
Loan type 3	441	0.120	0.326	0	0	0
Loan type 1 (# of lenders)	52	1	0	1	1	1
Loan type 2 (# of lenders)	336	4.307	2.700	2	4	5
Loan type 3 (# of lenders)	53	3.245	3.491	2	2	2
Number of specialized lenders	441	2.351	1.678	1	2	3
Panel B: DIP Loans						
Assets	436	3,667	10,612	444	807	2,278
Liabilities	436	3,528	8,402	463	891	2,343
Leverage	413	1.096	0.493	0.834	0.999	1.236
ROA	412	0.040	0.183	-0.001	0.059	0.104
PP&E/assets	414	0.410	0.258	0.193	0.385	0.621
Cash/assets	414	0.051	0.065	0.012	0.029	0.071
Loan maturity	436	10.587	6.273	6.000	9.000	12.000
Loan amount (L)	436	194.821	446.744	35.000	75.000	186.900
L/A	436	0.107	0.106	0.034	0.076	0.136
Risk-adjusted spread	436	718.063	379.247	417.400	667.400	967.400
Number of lenders	436	2.287	1.926	1	2	3
Loan type 1	436	0.128	0.335	0	0	0
Loan type 2	436	0.766	0.424	1	1	1
Loan type 3	436	0.106	0.308	0	0	0
Loan type 1 (# of lenders)	56	1	0	1	1	1
Loan type 2 (# of lenders)	334	2.557	2.083	1	2	3
Loan type 3 (# of lenders)	46	1.891	0.994	1	2	3
Number of specialized lenders	436	1.330	1.036	1	1	2

and 12.0%, respectively. Panel A of Table 3 presents the summary statistics of the sample of distressed loans.

4.2 Loans to Chapter 11 Firms

A U.S. firm filing for Chapter 11 bankruptcy can obtain post-petition financing, commonly known as debtor-in-possession (DIP) financing, to support working capital and pay expenses in bankruptcy under Section 364 of the Bankruptcy Code. This law provision permits the bankrupt firm to arrange a DIP loan with super-priority status over all administrative expenses after notice and a court hearing. With existing lenders' court approval, a DIP loan can be secured by a senior or equal lien on a property that is already subject to a lien (i.e. the "priming lien" provision). These loan contracts are short term and typically contain extensive protective features such as restrictive covenants and milestones (Skeel, 2004; Ayotte and Elias, 2020; Eckbo et al., 2020). With such security and lender protection provision, the default risk of DIP loans are very low, comparable to that of high investment-grade loans as shown by Eckbo et al. (2020).

The advantages of including DIP loans for our analysis are threefold. First, given their minuscule default rates, we can use the all-in spreads over a high investment-grade benchmark directly to decompose the effect of lender power on loan pricing, compared to distressed loans for which we have to take out the default risk component of distressed loans using CDS spreads. Second, existing lenders' lien on distressed firms' assets and their private information about the borrower allow them to have strong bargaining power and even become monopolistic lenders, which is consistent with our model setup. Third, as Eckbo et al. (2020) suggest, the DIP-lending market is quite concentrated. The top 10 lenders financed more than three quarters of their sample firms. Moreover, the lending syndicate is smaller than that for distressed loans. The highly concentrated lending market and small lending syndicate together create an ideal environment for specialized lenders to collude, where monitoring among lenders can be less costly, making punishment on non-collusive lending a more credible threat.

Our initial study sample, similar to that used in Eckbo et al. (2020), includes all DIP loans to Chapter 11 filings by large U.S public firms (with assets above $100 million in constant 1980 dollars) from 2002–2019, compiled from the UCLA-LoPucki Bankruptcy Research Database, Bankruptcydata.com, the Public Access to Court Electronic Record (PACER), and the Dealscan. We calculate the weighted average LIBOR spread and AISD at the loan package level using facility amount as the weight.[21]

Similar to distressed loans, we define three types of DIP loans: existing-creditor loans (type-1 loans), specialized-lender loans (type-2 loans), and last-resort loans (type-3 loans). We remove 54 DIP loans that cannot be classified into any of the above three types.[22] Table 2, Panel B, presents the summary statistics of our sample firms which obtained DIP-loan packages. In the 436 loan facilities in our final sample, the type-2 loan is the most dominant type, accounting for 76.6% of all DIP loans, while type-1 and type-3 loans account for 12.8% and 10.6%, respectively. Table 2, Panel B, presents the summary statistics of our sample of DIP loans.

5 Estimation

5.1 Likelihood Function and Identification Strategy

Likelihood. Given a latent classification of all observations into different demand curves and size groups (i.e., give a realization of $\{k_i\}_{i=1}^N$), the observable variables include $y_i \equiv (m_i, l_i, \ln(L_i/A_i), \ln(R_i))$ for deals indexed by $i = 1, \cdots, N$. Here, l_i is the type of lenders in deal i, m_i is the number of specialized lenders, L_i/A_i is the loan amount normalized by the borrower's asset size, and R_i is the risk-adjusted loan yield spread. We denote by θ the parameter vector that contains all the model parameters $\{\alpha_k, \varepsilon_k, \lambda_k\}_{k=1}^K$, $\{\gamma_k, \beta_k\}_{k=2}^K$, σ, ς, ξ, μ, ϕ_1, ϕ_2, and ϕ_3.

Conditioning on a latent demand curve classification $\{k_i\}_{i=1}^N$, the model-implied log-likelihood

[21] We use loan packages for DIP loans because it is always the same group of lenders that provide different facilities in a DIP-loan package while for other distressed loans, different facilities could be provided by distinct groups of lenders.

[22] These loans include the case of General Motors as its DIP-loan, the largest size history ($33 billion), was provided by U.S. and Canadian governments with a large component of subsidy, and loans that are provided by potential acquirers that use the DIP loan to bridge takeover.

of these observable variables can be factorized as follows:

$$P\left(\{y_i\}_{i=1}^N|\theta, \{k_i\}_{i=1}^N\right) = \prod_{i=1}^{N} P\left(y_i|\theta, k_i\right), \tag{42}$$

$$\text{where } P\left(y_i|\theta, k_i\right) = P\left(l_i|\theta, k_i\right) \times P\left(m_i|\theta, l_i, k_i\right) \times P\left(\ln(L_i/A_i)|\theta, l_i, m_i, k_i\right) \times P\left(\ln(R_i)|\theta, \ln(L_i/A_i), l_i, m_i, k_i\right). \tag{43}$$

We summarize below the individual component of the likelihood function in Equation (43) for each observation. First, the likelihood of $\ln(R_i)$, conditioning on the parameters θ and the variables $\ln(L_i/A_i), l_i, m_i, k_i$, is a normal distribution with mean $\alpha_{k_i} - \varepsilon_{k_i}\ln(R_i)$ and variance σ^2. Thus, $P\left(\ln(R_i)|\theta, \ln(L_i/A_i), l_i, m_i, k_i\right)$ can be rewritten as $P\left(\ln(R_i)|\alpha_{k_i}, \varepsilon_{k_i}, \sigma, \ln(L_i/A_i), l_i, m_i, k_i\right)$, which identifies α_k and ε_k for $k \in \{1, \cdots, K\}$, as well as σ.

Second, the likelihood of $\ln(L_i/A_i)$, conditioning on the parameters θ and the variables l_i, m_i, k_i, is a normal distribution with mean $\ln \widehat{L}(k_i, m_i) + \alpha_{k_i} - \varepsilon_{k_i}\phi_{l_i}$ and variance ς^2. Thus, given that $\{\alpha_k, \varepsilon_k\}_{k=1}^K$ and σ, the conditional likelihood $P\left(\ln(L_i/A_i)|\theta, l_i, m_i, k_i\right)$ and its associated data identify ξ and $\{\phi_l\}_{l=1}^3$. Importantly, $\widehat{L}(k_i, m_i)$ is independent of the disutility of participating the syndication, w_i, given k_i and m_i, and thus, it doesn't depend on the parameter μ. Moreover, it is also worth noting that, ϕ_l does not show up in $\widehat{L}(k_i, m_i)$ but only shows up in the scaling factor (i.e., the exponential term $e^{\alpha_k - \varepsilon_k\phi_l - \varepsilon_k\varsigma u + \sigma z}$ in equation (41)). That means, ϕ_l does not affect the ratio, $\frac{E[L/A_{m>1}]}{E[L/A_{m=1}]}$, where $E[L/A_{m>1}]$ and $E[L/A_{m=1}]$ are the average relative loan size of the cases with $m > 1$ and those $m = 1$, respectively. This ratio is powerful in identifying ξ, because as we take the ratio, ϕ_l drops out as the scaling factor cancels out, and the ratio decreases as the collusion capacity ξ increases. The rationale of identifying ξ using this ratio is further discussed in Figures 4 and 5 below.

Third, the likelihood of m_i, conditioning on the parameters θ and the variables l_i and k_i, is a multinomial distribution. Specifically, when $l_i = 1$ or $l_i = 3$, it holds that $P\left(m_i = 0|\theta, l_i, k_i\right) = 1$; when $l_i = 2$, it holds that $P\left(m_i|\theta, l_i, k_i\right) = \begin{pmatrix} M \\ m_i \end{pmatrix} F(\omega_C^*)^{m_i}\left[1 - F(\omega_C^*)\right]^{M-m_i}$. Conditioning on the parameters $\{\alpha_k, \varepsilon_k\}_{k=1}^K$, $\{\phi_l\}_{l=1}^3$, and ξ, which have already been identified using other conditional likelihoods, and the variables l_i and k_i, the equilibrium cutoff point w_C^* only depends on the parameter μ. Therefore, $P\left(m_i|\theta, l_i, k_i\right)$ identifies μ.

Lastly, we know that

$$\begin{aligned}
P\left(l_i = 1|\theta, k_i\right) &= \lambda_{k_i}, \\
P\left(l_i = 2|\theta, k_i\right) &= \left(1 - \lambda_{k_i}\right)\left\{1 - \left[1 - F(\omega_C^*)\right]^M\right\}, \\
P\left(l_i = 3|\theta, k_i\right) &= \left(1 - \lambda_{k_i}\right)\left[1 - F(\omega_C^*)\right]^M.
\end{aligned}$$

Thus, it is clear that $P\left(l_i|\theta, k_i\right)$ identifies $\{\lambda_k\}_{k=1}^K$.

Bayesian estimation. Using the Baysian approach, we estimate the posterior distribution for the variables of interest, $P\left(\{k_i\}_{i=1}^N, \theta|\{y_i\}_{i=1}^N\right)$. This posterior distribution describes the estimate of model parameters and the augmented latent cluster variable of each deal based on the observed variables. Based on Hammersley-Clifford Theorem (Besag, 1974), this posterior distribution is fully characterized by two conditional distributions $P\left(\theta|\{y_i\}_{i=1}^N, \{k_i\}_{i=1}^N\right)$ and $P\left(\{k_i\}_{i=1}^N|\theta, \{y_i\}_{i=1}^N\right)$, which in turn can be broken down into more lower dimensional conditional distributions.

Conditioning on $\{k_i\}_{i=1}^N$, the distribution $P\left(\theta|\{y_i\}_{i=1}^N, \{k_i\}_{i=1}^N\right)$ is determined by the distribution $P\left(\theta|\{k_i\}_{i=1}^N\right)$ and the likelihood function, $P\left(\{y_i\}_{i=1}^N|\theta, \{k_i\}_{i=1}^N\right)$, implied by the model solution. Specifically,

$$P\left(\theta|\{y_i\}_{i=1}^N, \{k_i\}_{i=1}^N\right) \propto P\left(\theta|\{k_i\}_{i=1}^N\right) \times P\left(\{y_i\}_{i=1}^N|\theta, \{k_i\}_{i=1}^N\right) \tag{44}$$

Meanwhile, conditioning on θ, the distribution $P\left(\{k_i\}_{i=1}^N|\theta, \{y_i\}_{i=1}^N\right)$ is determined by $P\left(\{k_i\}_{i=1}^N|\theta\right)$ and the likelihood function $P\left(\{y_i\}_{i=1}^N|\theta, \{k_i\}_{i=1}^N\right)$:

$$P\left(\{k_i\}_{i=1}^N|\theta, \{y_i\}_{i=1}^N\right) \propto P\left(\{k_i\}_{i=1}^N|\theta\right) \times P\left(\{y_i\}_{i=1}^N|\theta, \{k_i\}_{i=1}^N\right) \tag{45}$$

5.2 MCMC Estimator

To implement MCMC estimator, for each loan i in the data, we define a vector π_i of dimension K (i.e., the number of possible demand curve) so that the kth element in the vector describes the probability of this deal belonging to demand curve k so that $\pi_{ik} \geq 0$ and $\sum_{k=1}^{K} \pi_{ik} = 1$. Then we apply the Metropolis-Hastings algorithm with the following procedure.

Step 1: In each iteration g, we first update $\pi_i^{(g)}$ for each deal i using the Bayes rule for classification based on outputs from the last iteration round $g-1$:

$$\pi_{ik}^{(g)} = \frac{\pi_{ik}^{(g-1)} P\left(y_i | \theta^{(g-1)}, k_i = k\right)}{\sum_{j=1}^{K} \pi_{ij}^{(g-1)} P\left(y_i | \theta^{(g-1)}, k_i = j\right)} \tag{46}$$

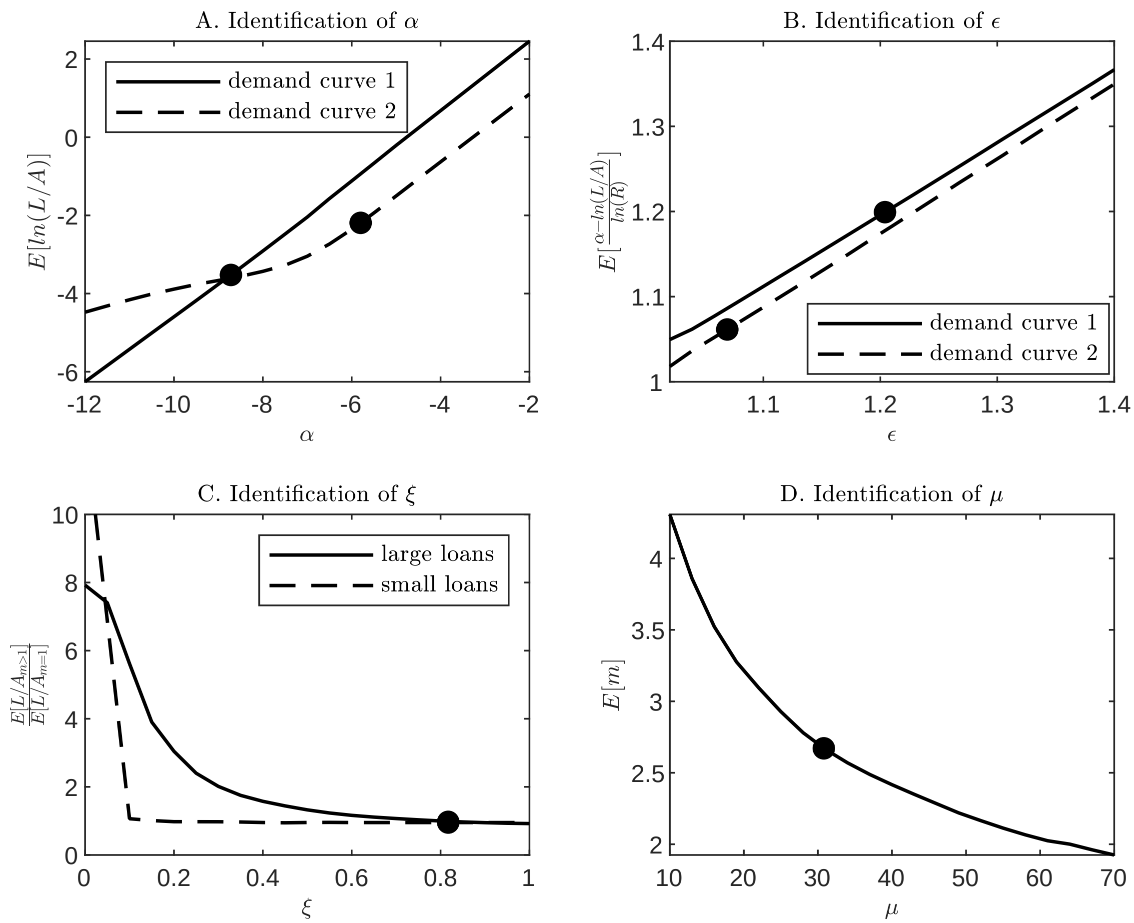
where $\pi_{ik}^{(g-1)}$ is the (posterior) probability of $k_i = k$, determined in the previous iteration (i.e., iteration $g-1$), and $P\left(y_i | \theta, k_i = k\right)$ is the likelihood function of observables evaluated at the parameter values simulated from the previous iteration (i.e., iteration $g-1$). We then follow the SEM algorithm (Celeux, 1985; Celeux and Diebolt, 1992) and draw the realization of $k_i^{(g)}$ from the multinomial distributions with weights in Equation (46). We throw out the draw and repeat this step if for any cluster k, the total number of deals assigned to this cluster is less than a cutoff (e.g., 3%).

Step 2: In the same iteration g, we then update other parameters in θ using the random walk Metropolis-Hastings algorithm based on the updated draws of $\{k_i^{(g)}\}_{i=1}^{N}$ from step 1. We compute the acceptance/rejection threshold:

$$\alpha\left(\theta^{(g)}, \theta^{(g-1)}\right) = min\left\{\frac{P\left(\theta^{(g)} | \{k_i^{(g)}\}_{i=1}^{N}\right) \prod_{i=1}^{N} P\left(y_i | \theta^{(g)}, k_i^{(g)} = k\right)}{P\left(\theta^{(g-1)} | \{k_i^{(g)}\}_{i=1}^{N}\right) \prod_{i=1}^{N} P\left(y_i | \theta^{(g)}, k_i^{(g)} = k\right)}, 1\right\}$$

where $\theta^{(g-1)}$ is the parameter vector from the last iteration, and $\theta^{(g)} = \theta^{(g-1)} + \Sigma\epsilon$ is the vector of proposed parameters.

Figure 4: Identification of parameters for the market of distressed loans



Step 3: In next iteration (i.e., iteration $g+1$), we repeat the procedure in step 1 and step 2 by simulating the classification realization $k_i^{(g+1)}$ according to the updated (posterior) probabilities $\pi_{ik}^{(g+1)}$ for all i, k and simulating the model parameters from the posterior distribution $P\left(\theta | \{y_i\}_{i=1}^N, \{k_i^{(g+1)}\}_{i=1}^N\right)$.

5.3 Identification of Parameters

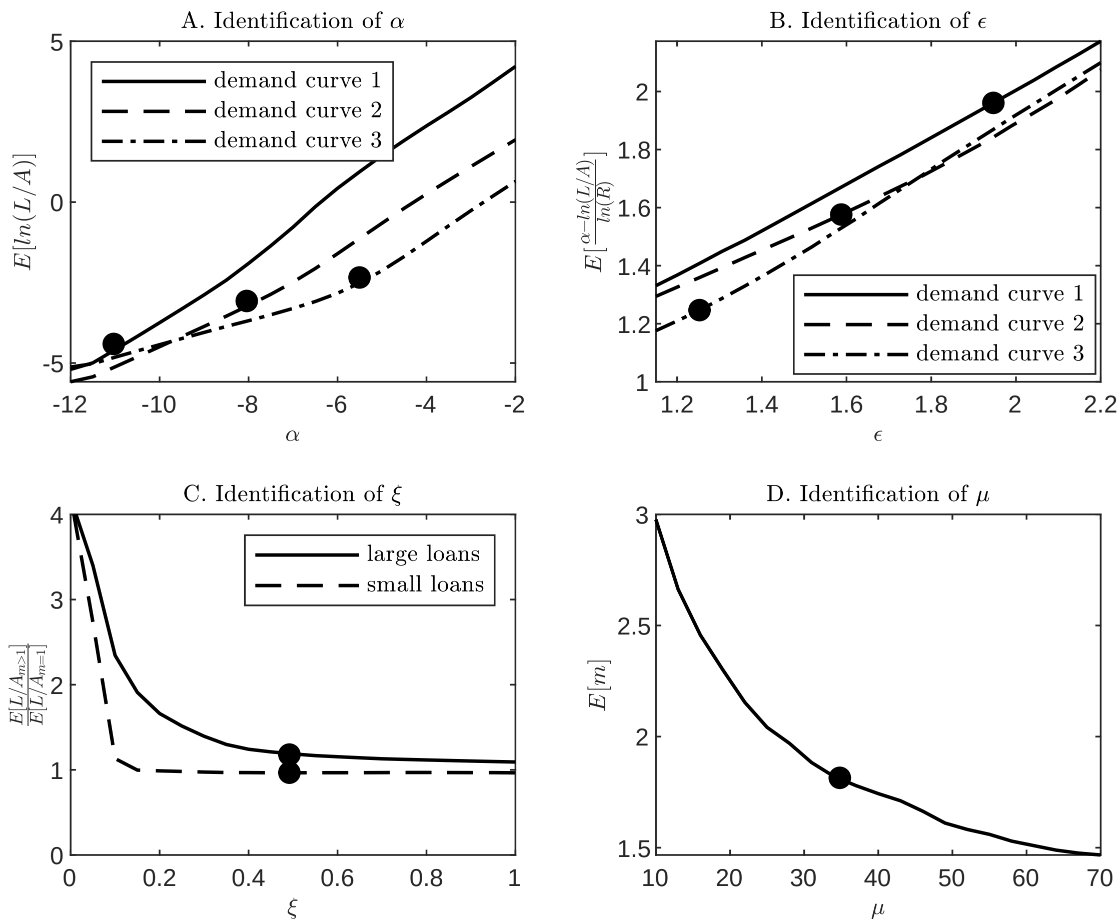
In this section, we first discuss what empirical patterns in the data help the MCMC estimator to identify model parameters. We then present the estimation results and our empirical findings for the two distressed loan markets.

Figures 4 and 5 explain the identification results for the distressed loans and DIP loans, respectively. The parameter α_k for $k = 1, \cdots, K$ controls the intercept of the demand curve k, and thus the average loan size associated with each demand curve is informative to identify this parameter. To illustrate this idea, we vary the value of α_k and solve the model for different values of α_k while keeping other model parameters at their baseline estimated values. We compute the model-implied average loan size, measured as the average logarithm of L/A, for each demand curve k, and examine how $\mathbb{E}[\ln(L/A)]$ varies with the value of α_k. Panel A of Figure 4 depicts the results for the distressed loan sample. For both demand curves, we observe that a higher α leads to a larger loan size, and the two dots represent the estimated value of α_k for $k \in \{1, \cdots, K\}$.

The parameter ε_k controls the elasticity of demand curve k, and the demand function specified in (4) suggests that this parameter is mainly identified by the slope of demand curve k. With the estimated intercept α_k, we compute the model-implied slope of demand curve k as $\mathbb{E}[\frac{\alpha_k - \ln(L/A)}{\ln(R)}]$, and we investigate how it varies as ε_k changes. Panel B of Figure 4 reveals that the model-implied demand curve slope monotonically increases with ε_k, showing that demand elasticity indeed rises as ε_k goes up.

A key parameter in our model, ξ, controls the tacit coordination among specialized lenders. In case of perfect collusion, lenders in a syndicate group coordinate as a cartel, and thus the total loan amount lent by the cartel equals the loan amount lent by a monopolistic lender. However, as collusion intensity falls, syndicated lenders lend more aggressively, leading to larger loan amount by the syndicate group than that by a monopolistic lender. As a result, the ratio of the average relative loan size made by a syndicate lender group with two or more specialized lenders to the average relative loan size made by a single monopolistic specialized lender, denoted by $\frac{\mathbb{E}[L/A_{m>1}]}{\mathbb{E}[L/A_{m=1}]}$, helps pin down ξ. Clearly, as ξ increases, this ratio declines. Two important things are worth noting here. First, when the profits from deviation are large enough, even the toughest punishment (i.e., $\xi = 1$) cannot fully restore perfect collusion, and therefore it is possible to observe the loan size ratio to remain above one even if $\xi = 1$. Second, collusion is harder to achieve in face of large borrowers, because profits from deviation increase as the demand grows. As a result, the model suggests that the loan size ratio is higher for large

Figure 5: Identification of parameters for the market of DIP loans



borrowers than for small borrowers given the level of ξ.

Panel C of Figure 4 illustrates how the loan size ratio varies with ξ for small borrowers by the dashed line and large borrowers by the solid line. With a ξ being around 0.2, the loan size ratio is already quite close to one, both in the model and in the data. We, however, observe that the loan size ratio is monotonically decreasing as ξ increases for loans made to large borrowers, and even with $\xi = 1$, the loan size ratio is still above one. The lines suggest that the parameter ξ is well identified in our estimation and the identification is mainly gained among larger borrowers. Since we estimate ξ using a likelihood-based approach, its estimated value is also affected by other observable variables such as the loan size, the risk-adjusted yield spread, and the number of specialized lenders, and thus the loan size ratio is not the only factor that pins down ξ.

Table 3: Parameter estimates.

Distressed Loans				DIP loans			
			General parameters				
ξ	σ	ς	μ	ξ	σ	ς	μ
0.817	0.923	0.607	30.79	0.492	0.664	0.430	34.81
(0.058)	(0.125)	(0.072)	(10.13)	(0.093)	(0.046)	(0.020)	(4.61)
			Lender-specific parameters				
	Existing creditor	Specialized lender	Last-resort lender		Existing creditor	Specialized lender	Last-resort lender
e^{ϕ_l}	0.0021	0.0022	0.0027	e^{ϕ_l}	0.0149	0.0158	0.0197
	(0.0010)	(0.0009)	(0.0012)		(0.0040)	(0.0040)	(0.0050)
			Borrower-specific Parameters				
	Demand curve 1	Demand curve 2			Demand curve 1	Demand curve 2	Demand curve 3
α_k	-8.718	-5.799		α_k	-11.031	-8.041	-5.505
	(1.225)	(0.308)			(1.504)	(0.534)	(0.131)
ε_k	1.204	1.069		ε_k	1.947	1.588	1.253
	(0.184)	(0.022)			(0.286)	(0.111)	(0.040)
λ_k	0.113	0.140		λ_k	0.036	0.150	0.119
	(0.069)	(0.024)			(0.043)	(0.034)	(0.026)
γ_k		1.213		γ_k		2.760	3.221
		(0.637)				(1.488)	(1.099)
β_k		-0.631		β_k		-1.011	-1.510

Note: This table reports the estimated model parameters together with the standard errors obtained from MCMC. The top panel shows the general model parameters including the likelihood of punishment on deviation ξ, the demand and supply shock σ and ς, the parameters that control the correlation between borrower size and the demand curve, γ and β, as well as the average participation cost μ. The mid panel shows the lender-specific model parameter ϕ that captures the lending costs for each type of lenders (i.e., existing lenders, the specialist lenders, and lenders of the last resort). The bottom panel shows the borrower-specific parameters including the demand curve coefficient α and ε and the likelihood for the existing lender to finance the loan λ.

The parameter μ determines the average disutility of participating in the syndication by specialized lenders. Intuitively, when μ is low, more specialized lenders find it profitable to participate in forming the syndication and thus we expect to observe a larger syndicate group, m, in the data. Panel D of Figure 4 shows that the average number of specialized lenders in a syndication monotonically decreases as the average disutility of participation, μ, rises.

The fraction of loan deals financed by the existsing lender helps identify the parameter λ_κ. This linkage is established exogenously in the model by assuming that there is a probability λ_κ that the existing lender has the exclusive right to provide the financing and thus block the firm from borrowing from outside lenders.

Different type of lenders (i.e., existing creditors, specialized lenders, and lenders of the last resort) may incur different variable costs. Part of the spread R is used to compensate for these variable costs. By comparing R across deals financed by different types of lenders, after controlling for the number of lenders involved, we can identify and estimate the parameter ϕ_l for existing creditors ($l = 1$), specialized lenders ($l = 2$), and lenders of the last resort ($l = 3$).

5.4 Estimation Results

MCMC estimator generates Markov chains for each model parameter and the augmented variables. We first present the estimation of demand curve classification. For each loan deal i in our sample, our estimator assigns it a vector $z_i^{(g)}$ that describes the likelihood of the deal belonging to each demand curve in the gth iteration on the MCMC chain, as specified in Equation (46). We take $z_i^{(g^*)}$ for the iteration g^* that generates the highest likelihood over the MCMC chain and classify loan i to the demand curve associated with the highest probability in the vector $z_i^{(g^*)}$. Based on the classification results, Figures 6 depicts the demand curves for the distressed loan sample and DIP loan sample, respectively. Each point in the figure represents an observation of loan, and the size of the points indicts the borrower's size. Points with the same color are classified as being on the same demand curve, and the straight lines depict the estimated demand curves. Our estimates suggest that, there are two statistically distinct demand curves in the distressed loan sample and three demand curves in the DIP loan sample. As we discussed above, the classification is performed based on the full likelihood of demand and supply in the model equilibrium and thus it differs from a simple classification based on the observed loan size $ln\left(\frac{L}{A}\right)$ and spread $ln\left(R\right)$. To illustrate this point, we also run an OLS regression by pooling together all observations on one demand curve. The dotted line plots the line of best-fit from the OLS regression. We find that the slope coefficient from OLS regression, which captures the elasticity of demand, is economically small and statistically insignificant from zero, which strongly contrasts the estimated elasticity of demand in our baseline results where classification of observations to different demand curves are jointly estimated with the demand curve coefficients. In both samples, borrowers on the demand curve 1 have a lower intercept of loan size but a similar (for distressed loans) or higher (for DIP loans) price elasticity of loan demand compared with those on demand curve 2 or 3. It indicates that borrowers on demand curve 1 have a lower level of overall demand for loan (relative to their size) and they are more sensitive to the loan price. Empirically, we find that borrowers on the bottom demand curve are also larger in size and have higher total revenues, which may explain partly the difference we identify in their demand curves.

Besides the classification of clusters and the associated demand curves, the MCMC estimator also delivers the posterior distribution of each model parameter. We report in Tables 3 the point estimate and the standard errors of each parameter for two samples respectively. The point estimate is taken as the parameter set that produces the highest likelihood along the chain, and the standard errors is computed as the standard deviation of the parameter draws along the chain.

The likelihood of punishment on deviation, ξ, is estimated to be 0.817 for distressed loans. It suggests that if a specialist lender deviates from the perceived cartel equilibrium, it will be pushed into a non-collusive equilibrium with 82% of chance. This parameter controls the collusion intensity in the model and a value of 0.817 implies a high level of tacit coordination among specialists. Our estimates show that the punishment on deviation is also sizeable for DIP loans with ξ estimated to be 0.492. ξ is estimated with relatively small standard errors in both samples, suggesting that the data pattern strongly rejects a model without tacit coordination among specialist lenders. As we discussed above, the ratio of loan size made by a syndicate group of lenders to the loan size made by a monopolistic lender disciplines the identification of tacit coordination among specialist lenders. Stronger coordination leads to a low loan size ratio, consistent with the data.

Our estimates of ς and σ show that unobservable heterogeneity, possibly driven by the lender-side and borrower-side shocks, is important in explaining the cross-sectional variation of the observed loan size and spread. A simple variance decomposition suggests that with the estimated unobservable heterogeneity, our model is able to explain 57% (40%) of variation in

$ln(\frac{L}{A})$ and 40% (34%) of variation in $ln(R)$ for the DIP loan sample (distressed loan sample).

It is also interesting to note that the lenders' participation cost, μ, is estimated to be higher in the DIP market. A crucial determinant of the participation cost is the blocking power by existing lenders (Eckbo et al., 2020): if the existing lender has a strong power in blocking the borrower from reaching out to other lenders even when it does not make the loan to the borrower, then such blocking power can manifest as a large estimated participation cost born by specialist lenders (i.e., it is more difficult for specialist lenders to participate).

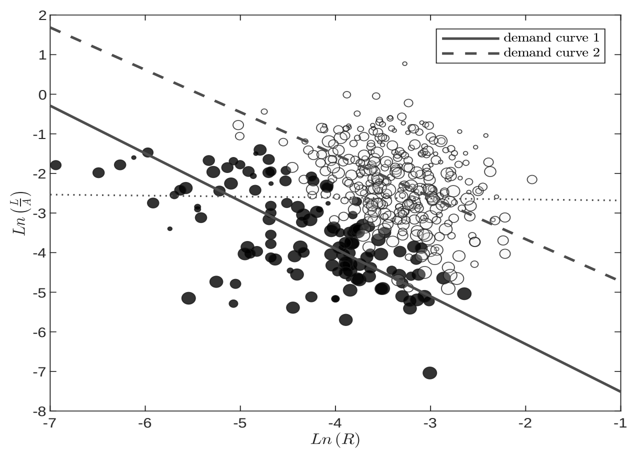
The mid panel of Tables 3 reports the lender-specific variable costs, which reflect the costs associated with lending to distressed firms. Monitoring cost and costs of lender participating in the restructuring process can be such variable costs. For example, as a distressed borrower approaches bankruptcy or is already in bankruptcy, lenders need to pay close attention to the borrower's business operations and legal challenges in the court process. The fact that the estimated variable costs of DIP loans are much higher than those for distressed loans is consistent with more uncertainties faced by lenders in bankruptcy court.

An interesting evidence that stands out in the DIP loan sample is that the estimated variable costs are about 150-160 bps for existing lenders and specialist lenders but about 200 bps for lenders of last-resort (mostly hedge funds). Unlike the existing lenders who possess information advantage or the specialist lenders who have expertise in this market, hedge funds are rarely frequent players in this market. Most of them appear only once or twice in our sample as the lenders of last-resort, and therefore it is plausible that they face higher variable costs. More importantly, hedge funds often play an active role in the governance of bankrupt firms such as seeking board representation and appointing managers in pursuing a "loan-to-own" strategy (Jiang et al., 2012; Li and Wang, 2016; Ayotte and Elias, 2020), which requires hedge funds' significant efforts and resources.

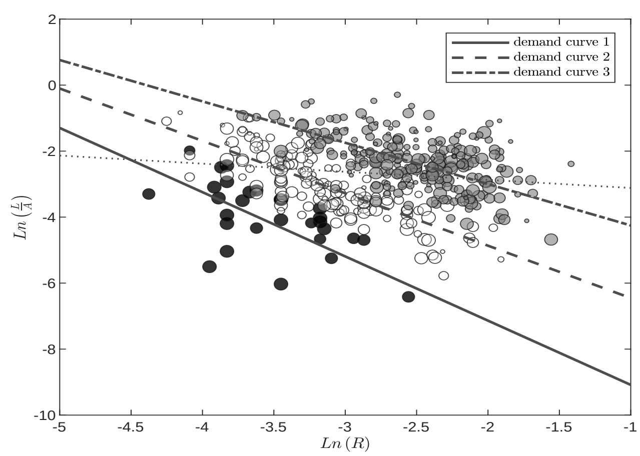
We report the parameters that govern the demand curves in the bottom panel of Tables 3. We estimate the level of demand, α_K, and the price elasticity of demand, ε_K. Within each market, the estimated intercept and elasticity just reiterate the demand curves shown in Figure 6. Across the two market, we find that DIP borrowers exhibit slightly higher elasticity of demand than

Figure 6: Demand Curves by Clusters

This figure shows the estimated demand curves and the classification of borrowers to these demand curves. Each point in the figure represents an observation in our sample, with points of the same color classified to the same demand curve. The straight lines are the estimated demand curves. The intercept and slope of the demand curves represent the constant term and the elasticity of the demand, specified in Equation 4. We choose the optimal number of demand curves to estimate following the Lo-Mendell-Rubin tests, and our estimates suggest that there are two statistically distinct demand curves in the distressed loan sample and three demand curves in the DIP loan sample. The dotted line depicts the line of best-fit from an OLS regression that pool together all observations on only one demand curve. Panel (a) depicts the classification in the distressed loan market and panel (b) depicts the classification in the DIP market.



(a) Distressed Loans



(b) DIP Loans

distressed loan borrowers. Our estimates, therefore, suggest that the strikingly high spread of DIP loans, compared with that of distressed loans, cannot be explained by the price elasticity of demand. In both markets, borrower size seems an important determinant of demand curve classification: large borrowers on average exhibit higher elasticity of demand and borrow less relative to their own size.

6 Decomposing Loan Yield Spread

Using the estimated model as a laboratory, we quantify the lenders' market power in the two separate distressed loan markets and decompose their market power into three components: (i) the potential collusion among specialists, (ii) limited participation by these lenders, and (iii) the oligopolistic structure in this market.

Both markets exhibit a few unique and interesting features. First, there is a club of specialist lenders that intermediate a large fraction of loan deals. Indeed, 83% of distressed loans and 70% of DIP deals in our estimated model are financed by the top 10 specialist lenders. This lending market, therefore, resembles an oligopolistic structure. To illustrate the dominant position of top lenders in the two loan markets, Figure 9 shows the loan market network for distressed loans and DIP loans, respectively. It is clear that the top lenders in each market form a strong network with other top lenders in syndicating the loans. Second, most deals are syndicated by a small group of specialist lenders as shown in Table 2. It implies limited participation in lending by specialists, which further reduces lender competition. Third, the lender clique may give rise to possible collusion among specialists that is hard to be detected by outsiders. In this section, we employ the estimated model to perform a few counterfactual benchmarks and decompose the market power of specialist lenders.

Starting from the baseline model with the estimated parameters, we first quantify the effect of lender collusion. To do so, we set the key parameter ξ to zero. In this counterfactual model, all specialist lenders deviate from collusion because no punishment is imposed, and as a result, a non-collusive equilibrium emerges. We keep other model parameters at their estimated values so that the non-collusive equilibrium still features limited participation by specialist lenders and an oligopolistic market structure. We examine how lender collusion affects the average loan yield spread, loan size, and the number of participating lenders by comparing the outputs from the two models.

Table 4 shows the results. Columns (1) and (5) report the baseline model predictions for the distressed loan sample and DIP sample respectively, and columns (2) and (6) report the non-collusive model outputs for the two samples. Notably, the average loan yield spread for specialist lenders declines by 211 bps (from 343 bps in the baseline model to 132 bps in the non-collusive model) for distressed loans, representing a 62% reduction in spread if specialist lenders compete in a non-collusive equilibrium. DIP loan yield spread declines by 212 bps (from 680 bps to 468 bps) as lender collusion is eliminated, representing a 31% decline. Without collusion, specialists compete more aggressively and lend a larger amount in aggregation. The average loan amount, relative to the borrowers' size, rises from 0.177 to 1.31 for distressed loans and from 0.114 to 0.271 for DIP loans. Intense competition has a moderate deterrence effect on lender participation, evident by a drop in the average number of syndicated lenders.

Next, we examine the effect of limited participation by specialist lenders. In our model, each lender incurs a stochastic participation cost that is assumed to follow an exponential distribution with the mean parameter μ. We construct a counterfactual benchmark of full participation by setting μ to zero. In this counterfactual model, we still keep $\xi = 0$ to measure the incremental effect of removing limited participation from the non-collusive model. The results for this counterfactual model are reported in columns (3) and (7) of Tables 4 for the two samples. The direct change is that all specialist lenders now participate in loan syndication and thus the number of lenders reaches the full capacity of 10. Full participation increases competition among specialist lenders, and the loan spread decreases by another 103 bps for distressed loans and 282 bps for DIP loans. This result suggests that limited participation by specialist lenders has a much more pronounced effect in DIP market than in distressed loan market. Because participation costs in our model incorporate both the costs of learning about a loan deal and the existing lenders' power on blocking other lenders from participating, our estimates indicate that such costs are higher for lenders in the DIP-loan markets. This is consistent with earlier studies that document that new DIP lenders cannot prime “liens" of existing lenders without their consent and thus “prepetition" lenders possess a strong bargaining position in deciding who can participate in a DIP loan syndicate (Skeel, 2004; Ayotte and Morrison, 2009). Moreover, because borrowers tend to be selective in which lenders to approach before bankruptcy filing, it can be costly for some lenders to learn about the deal type (Eckbo et al., 2020). This is generally less of a case for distressed loans.

As the last step, we quantify the effect of an oligopolisitc market structure. To turn an oligopolistic market into a competitive market, we increases the total number of specialist lenders from 10 to positive infinity. We build this counterfactual upon the non-collusive, full participation model to capture the incremental effect. In columns (4) and (8) of Tables 4, we find that the average loan yield spread drops by a negligible amount for both distressed loans and DIP loans financed by specialist lenders. Interestingly, this finding shows that the oligopolistic structure is not the main cause of high loan yield spread in either market, and the market would be competitive enough if all of the 10 specialist lenders can participate.

Table 4: Counterfactual Models and The Decomposition of Lender Market Power

This table reports the model implications for the estimated baseline model and a few counterfactual models. The Estimated model is the baseline model. In the non-collusive model, we shut down collusion by setting $\xi = 0$. In non-collusive, full participation model, we further zero out the participation cost for all specialist lenders and therefore all of them participate in a loan deal. In non-collusive, full participation, unlimited lenders model, we further increase the total number of specialist lenders to positive infinity so that it turns an oligopoly market to a competitive market.

	Distressed Loans				DIP Loans			
	Estimated Model	Non-collusive Model ($\xi = 0$)	Non-collusive, Full Participation ($\xi = 0, \mu = 0$)	Non-collusive, Full Participation, Unlimited Lenders ($\xi = 0, \mu = 0,$) ($M- >$ inf)	Estimated Model	Non-collusive Model ($\xi = 0$)	Non-collusive, Full Participation ($\xi = 0, \mu = 0$)	Non-collusive, Full Participation, Unlimited Lenders ($\xi = 0, \mu = 0,$) ($M- >$ inf)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Avg. Total Spread (bps)	344	182	68	68	702	566	235	228
Frac. Existing	0.128	0.128	0.128	0.128	0.113	0.113	0.113	0.113
Avg. Spread (bps)	334	334	334	334	618	618	618	618
Frac. Specialist	0.829	0.785	0.873	0.873	0.717	0.677	0.887	0.887
Avg. Spread (bps)	343	132	29	29	680	468	186	178
Frac. Last-Resort	0.044	0.088	0	0	0.171	0.211	0	0
Avg. Spread (bps)	391	407	NaN	NaN	852	853	NaN	NaN
Avg. m	2.67	2.22	10	Inf	1.81	1.72	10	Inf
Avg. L/A	0.177	1.310	2.764	2.972	0.114	0.271	0.677	0.722

Moving from the baseline model to the non-collusive, full participation, competitive model, we find that the lenders' market power accounts for about 92% of distressed loan's risk-adjusted yield spreads (i.e., $\frac{343-29}{343}$) and 74% of the DIP loan's risk-adjusted yield spreads (i.e., $\frac{680-178}{680}$). A further decomposition of the market power suggests that lender collusion contributes to 67% (i.e., $\frac{343-132}{343-29}$) of the market power for distressed loans and 42% of the market power for DIP loans (i.e., $\frac{680-468}{680-178}$), while limited participation contributes to 33% of the market power (i.e., $\frac{132-29}{343-29}$) for distressed loans and 56% of the market power (i.e., $\frac{468-186}{680-178}$) for DIP loans, Finally, the oligopolistic market structure contributes to the remaining negligible fraction of the market power to the two markets.

We summarize the decomposition of risk-adjusted yield spreads for the two markets in Figure 7. Specifically, we compare each component of the loan spread for distressed loans and DIP loans. This figure reveals two main conclusions that are central to our paper. First, tacit coordination among specialist lenders is prevalent in both markets, which allows the lender cartel to extract sizeable rent from borrowers (around 210 bps). Second, the excess loan spread in DIP market, compared with that in the distressed loan market, is mainly explained by the existing lenders' strong power in blocking the borrowers from reaching out to alternative lenders (i.e., high participation costs) and the high monitoring costs in DIP market (i.e., variable costs).

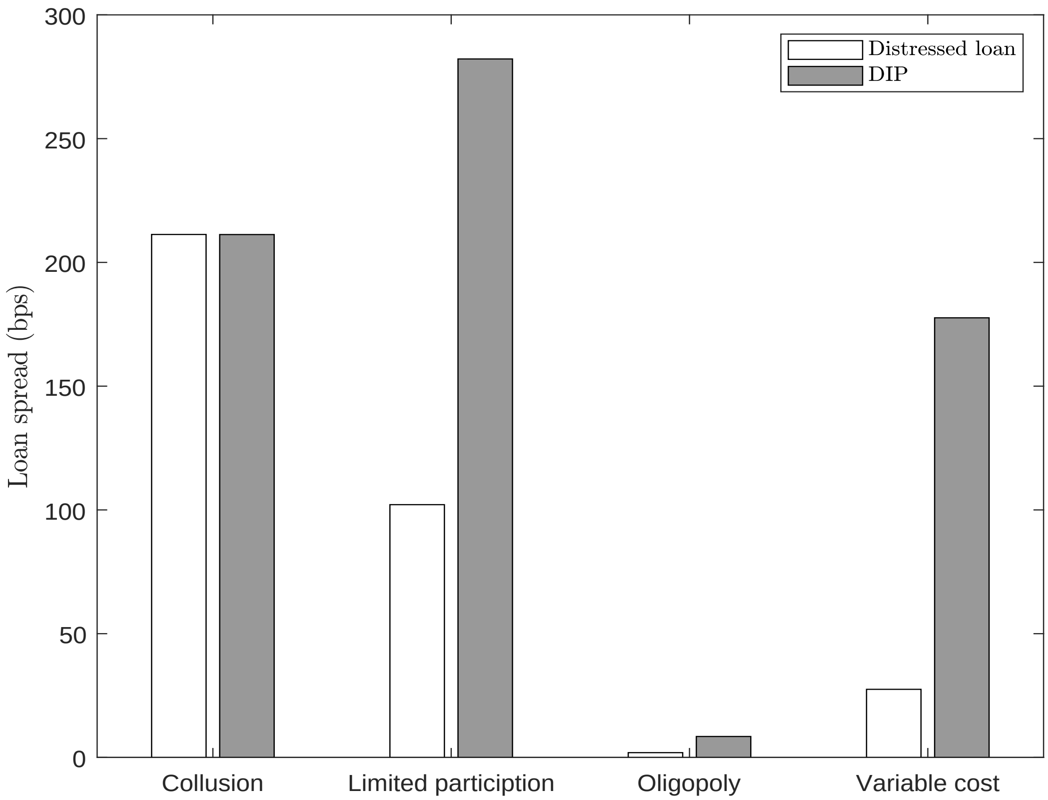
Among the different components that contribute to lender market power, lender tacit coordination often attracts most attention by regulators, and antitrust policies often target at breaking down such coordination. We then examine which borrowers are more susceptible to lender market power by measuring the effect of lender tacit coordination on borrowers of different size.

In the final part of the analysis, we partition the borrowers into size quintiles and repeat our analyses of the baseline model and the non-collusive model performed above. We report the changes in the average loan spread, loan size, and the number of participating specialist lenders as we shut down lender coordination in Table 5. We compare small borrowers (bottom size quintile) with large borrowers (top size quintile).

When lender coordination is eliminated, we find that small borrowers seem to benefit more. Specifically, the distressed loan spread drops by 260 bps (166 bps) for small (large) borrowers.

Figure 7: Spread Decomposition

This figure compares different components of risk-adjusted yield spreads in the distressed loan market and the DIP loan market. Collusion represents the component of risk-adjusted yield spreads that arises from the specialist lenders' tacit coordination in syndicated lending; Limited participation represents the component of risk-adjusted yield spreads that arises from the limited competition in small syndicated lending groups, which in turn is a consequence of high participation costs in the estimated model. Oligopoly represents the component of risk-adjusted yield spreads due to the oligopolistic market structure with a few concentrated lenders; and variable costs represent the component of risk-adjusted yield spreads used to compensate the lenders for their lending costs (not include funding costs) such as monitoring costs.



DIP loan spread drops 255 bps (126 bps) for small (large) borrowers. The increase in loan size due to the elimination of collusion is also more pronounced for small borrowers than large borrowers in both markets. Overall, our analyses here suggest that small borrowers in both markets are more susceptible to lender market power and the effect of collusion on smaller borrowers almost doubles that for large borrowers. Our findings therefore imply that antitrust policies that deter lender collusion may benefit small lenders more, and in economic or financial crises,

Table 5: Heterogeneous Effects of Lender Collusion

This table reports the effect of shutting down lender collusion on borrowers of different size. We divide borrowers into size quintiles by their book assets and compare counterfactuals between small borrowers (bottom quintile) and the large borrowers (top quintile). The estimated model is the baseline model, and in the non-collusive model, we shut down lender collusion by setting ξ to zero. The effect of shutting down collusion is calculated as the difference between the results from the non-collusive model and those from the baseline model.

		Distressed loans			DIP loans		
	Borrower size	Baseline	Non-collusive	change	Baseline	Non-collusive	change
R	Small	405	145	-260	767	512	-255
	Large	281	116	-166	512	386	-126
L/A	Small	0.200	1.704	1.504	0.124	0.322	0.198
	Large	0.135	0.829	0.694	0.087	0.158	0.071
m	Small	2.682	2.239	-0.443	1.799	1.732	-0.067
	Large	2.672	2.169	-0.503	1.768	1.646	-0.122

financial aids towards distressed companies should target at small firms. The implication is consistent with a recent policy proposal by legal scholars to encourage the U.S. government to provide direct funding to small firms that filed for bankruptcy during the COVID-19 pandemic.[23]

7 Policy Analysis

Our estimation and quantification results so far suggest that specialized lenders in the distressed loan market and DIP loan market extract rents from borrowers by charging excess risk-adjusted yield spreads above their marginal costs; moreover, a major part of these rents is made possible through the market power enjoyed by the few specialized lenders, especially through the tacit collusion in the form of syndication. For such implicit coordination is often hard to detect, let alone make prosecution and enforcement, a simple regulatory intervention is to impose a cap on interest rates. Our findings support the proposals on government intervention by disciplining both markets of distressed loans, reducing the borrowing costs and facilitating

[23] See "Use of Chapter 11 and Federal Lending to Help Small Businesses," by Kathryn Judge and Jared A. Ellias, July 27, 2020, Letter to the Office of Senator Sherrod Brown, as Ranking Member of the Committee on Banking, Housing, and Urban Affairs.

borrowers' credit accessibility, to mitigate the damage of inefficient bankruptcy waves owing to an economy-wide cash-flow pause (e.g., DeMarzo et al., 2020; Conti-Brown and Skeel, 2020). In this section, we focus on interest rate cap regulation (i.e., usury regulation), and use the estimated model to examine the effect of such policies.

Regulations and laws pertaining to interest rate caps have been one of the few most ubiquitous economic legislations historically and geographically (Blitz and Long, 1965). As of today, they still play a vital role in economic activities across different economies. Specifically, our setting is about commercial loans, defined as loans made primarily for business, commercial, investment, agricultural, or similar purposes, in contrast to consumer loans. For instance, in New York, corporations and limited liability companies (LLCs) cannot be charged more than 16% interest per annum, and specifically, loans to businesses under $2,500,000 are generally exempt from the 16% civil usury cap for consumer loans, but are subject to the 25% cap in 2021.

7.1 Formulation of Polices in the Model

The interest rate cap regulation can be implemented after conditioning on the characteristics of a borrower and adjusting the risk premium to better account for borrower risk heterogeneity and risk pricing. This can substantially improve the effectiveness of the interest rate cap policy because unsophisticated constant interest rate caps have severe limitations on balancing the tradeoff between borrower production and credit access (e.g., Cuesta and Sepúlveda, 2021). Assuming that the regulator can observe or estimate the marginal cost of providing distressed loans $e^{\phi+\varsigma u}$ and the risk premium of the loan, we can directly consider the interest rate cap that is imposed on the risk-adjusted spread R in the following form:

$$R_{max}(x) \equiv \mathcal{R}_{max} e^{\phi+\varsigma u}, \tag{47}$$

where $\mathcal{R}_{max}$ is a positive constant.

According to the demand system of the borrowers, the loan amount per specialized lender corresponding to the ceiling on the risk-adjusted spread is

$$L_{min}(k,x,m) = \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)} e^{[\alpha(k)-\varepsilon(k)\phi]-\varepsilon(k)\varsigma u+\sigma z} A,$$

where m is the number of specialized lenders in the syndication, k indicates the type of the borrower, and x contains the characteristics of the deal. In fact, under the interest rate cap specified in (47), the loan amount $L_{min}(k,x,m)$ is the minimum loan size each specialized lender will offer in equilibrium for the syndication characterized by (k,x,m).

Given that we focus on the risk-adjusted interest rate cap imposed on the spread, the optimal loan sizes for each specialized lender under non-collusive syndication, collusive syndication, and deviation have the following respective functional forms:

$$L^i(k,x,m;\mathcal{R}_{max}) \equiv \widehat{L}^i(k,m;\mathcal{R}_{max}) e^{[\alpha(k)-\varepsilon(k)\phi]-\varepsilon(k)\varsigma u+\sigma z} A, \quad \text{with } i \in \{N,C,D\}. \tag{48}$$

Non-collusive equilibrium with interest rate cap $\mathcal{R}_{max}$. Under the interest rate cap regulation, the value function prior to paying the fixed cost w and observing the deal-specific characteristics $x=(z,u)$, denoted by $U^N(k,x,m;\mathcal{R}_{max})$, satisfies the following Bellman equation:

$$U^N(k,x,m;\mathcal{R}_{max}) = \Pi_2(k,x,m;L^N,\mathcal{R}_{max}) \quad + \quad \frac{W^N(\mathcal{R}_{max})}{1-\delta}, \quad \text{where} \tag{49}$$

$$\begin{aligned} W^N(\mathcal{R}_{max}) &= \mathbb{E}^{k'}\left\{\lambda(k')\frac{\Pi_1(k';\mathcal{R}_{max})}{M_0}\right\} \\ &+ \mathbb{E}^{k'}\left\{[1-\lambda(k')]\sum_{m'=1}^{M} q(m'|w' \le w^*_{N,\mathcal{R}_{max}})\left[F(w^*_{N,\mathcal{R}_{max}})\Pi_2(k',m';L^N,\mathcal{R}_{max}) - \int_{w' \le w^*_{N,\mathcal{R}_{max}}} w' dF(w')\right]\right. \end{aligned}$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1,\cdots,K\}$ with probability weight $\pi(k')$ for each k', and the cutoff $w^*_{N,\mathcal{R}_{max}}$ is determined in the same way as w^*_N but they can be different in the equilibrium.

The symmetric non-collusive Nash equilibrium can be characterized by the following condition:

$$L^N(k,x,m;\mathcal{R}_{max}) = \underset{L\geq L_{min}(k,x,m)}{\text{argmax}} \left[\left(e^{\alpha(k)+\sigma z} \frac{A}{L+(m-1)L^N(k,x,m;\mathcal{R}_{max})} \right)^{1/\varepsilon(k)} - e^{\phi+\varsigma u} \right] L. \tag{50}$$

Plugging (48) into (50) results in the following relation:

$$\widehat{L}^N(k,m;\mathcal{R}_{max}) = \underset{\widehat{L}\geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}}{\text{argmax}} \left\{ \left[\widehat{L} + (m-1)\widehat{L}^N(k,m;\mathcal{R}_{max}) \right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}, \tag{51}$$

which leads to

$$\widehat{L}^N(k,m;\mathcal{R}_{max}) = \max \left\{ \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)},\ \frac{1}{m} \left[\frac{m\varepsilon(k)}{m\varepsilon(k)-1} \right]^{-\varepsilon(k)} \right\}. \tag{52}$$

Collusive equilibrium with interest rate cap $\mathcal{R}_{max}$. Under the interest rate cap regulation, the value function of a specialist at the beginning of the "afternoon" when w, k, and x are already observed, denoted by $V^C(k,x,w,m;L^C,\mathcal{R}_{max})$, has the following functional form:

$$V^C(k,x,w,m;L^C,\mathcal{R}_{max}) \equiv U^C(k,x,m;L^C,\mathcal{R}_{max}) - w. \tag{53}$$

The value function $U^C(k,x,m;L^C,\mathcal{R}_{max})$ satisfies the following Bellman equation:

$$U^C(k,x,m;L^C,\mathcal{R}_{max}) = \Pi_2(k,x,m;L^C,\mathcal{R}_{max}) \quad + \quad \frac{W^C(L^C,\mathcal{R}_{max})}{1-\delta}, \quad \text{where} \tag{54}$$
$$W^C(L^C,\mathcal{R}_{max}) = \mathbb{E}^{k'} \left\{ \lambda(k') \frac{\Pi_1(k',\mathcal{R}_{max})}{M_0} \right\}$$
$$+ \mathbb{E}^{k'} \left\{ \left[1-\lambda(k')\right] \sum_{m'=1}^{M} q(m'|w' \leq w^*_{C,\mathcal{R}_{max}}) \left[F(w^*_{C,\mathcal{R}_{max}})\Pi_2(k',m';L^C,\mathcal{R}_{max}) - \int_{w'\leq w^*_{C,\mathcal{R}_{max}}} w' dF(w') \right] \right\},$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1,\cdots,K\}$ with probability weight $\pi(k')$ for each k', and the cutoff $w^*_{C,\mathcal{R}_{max}}$ is determined in the same way as w^*_C but they can be different in the equilibrium.

For a given scheme of collusive loan size captured by $\widehat{L}^C(k,m)$, the optimal deviation in terms of loan size is the one that maximizes the expected deviation profit, characterized as follows:

$$\widehat{L}^D(k,m;\mathcal{R}_{max}) = \underset{\widehat{L}\geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}}{\operatorname{argmax}} \left\{ \left[\widehat{L} + (m-1)\widehat{L}^C(k,m;\mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}. \tag{55}$$

The benefit of deviation is the difference between the maximal expected deviation profit and the expected collusive profit without deviation, denoted by

$$\Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max}) \equiv \mathbb{E}^x\left[\Pi_2^D(k,x,m;\widehat{L}^C,\mathcal{R}_{max})\right], \text{ and} \tag{56}$$

$$\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max}) \equiv \mathbb{E}^x\left[\Pi_2(k,x,m;\widehat{L}^C,\mathcal{R}_{max})\right], \text{ respectively.} \tag{57}$$

Given the collusive scheme $\widehat{L}^C(\cdot,\cdot;\mathcal{R}_{max})$ and the interest rate cap regulation captured by $\mathcal{R}_{max}$, the maximal expected deviation profit $\Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max})$ is achieved at the optimal deviation $\widehat{L}^D(k,m;\mathcal{R}_{max})$, that is,

$$\begin{aligned}
&\Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max}) \\
&\quad = \left\{ \left[\widehat{L}^D(k,m;\mathcal{R}_{max}) + (m-1)\widehat{L}^C(k,m;\mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}^D(k,m;\mathcal{R}_{max}) \\
&\qquad \times \exp\left\{\alpha(k) + \frac{1}{2}\sigma^2 + [1-\varepsilon(k)]\phi + \frac{1}{2}[1-\varepsilon(k)]^2\zeta^2\right\} A.
\end{aligned} \tag{58}$$

Given the collusive scheme $\widehat{L}^C(\cdot,\cdot;\mathcal{R}_{max})$ and the interest rate cap regulation captured by $\mathcal{R}_{max}$, the expected collusive profit $\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max})$ is

$$\begin{aligned}
&\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max}) \\
&\quad = \left\{ \left[m\widehat{L}^C(k,m;\mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}^C(k,m;\mathcal{R}_{max}) \\
&\qquad \times \exp\left\{\alpha(k) + \frac{1}{2}\sigma^2 + [1-\varepsilon(k)]\phi + \frac{1}{2}[1-\varepsilon(k)]^2\zeta^2\right\} A.
\end{aligned}$$

We define the IC compatible set of functionals $\widehat{L}^C(\cdot, ; \mathcal{R}_{max})$ as follows:

$$\widehat{\mathcal{L}}^C(\mathcal{R}_{max}) \equiv \Bigg\{ \widehat{L}^C : \frac{\xi[W^C(\widehat{L}^C, \mathcal{R}_{max}) - W^N(\mathcal{R}_{max})]}{1-\delta} \geq \Pi_2^D(k, m; \widehat{L}^C, \mathcal{R}_{max}) - \Pi_2(k, m; \widehat{L}^C, \mathcal{R}_{max}),$$
$$\widehat{L}^C(k, m) \geq \frac{1}{m} \mathcal{R}_{max}^{-\varepsilon(k)}, \ \forall\, k, m \Bigg\}.$$

In the collusive Nash equilibrium we focus on, the loan size is

$$\widehat{L}^C(\cdot,\cdot) = \underset{\widehat{L} \in \widehat{\mathcal{L}}^C(\mathcal{R}_{max})}{\operatorname{argmax}} \ \mathbb{E}\left[U^C(k, x, m; \widehat{L}, \mathcal{R}_{max}) \right]. \tag{59}$$

7.2 The Effects of Interest Rate Cap

After we solve the model with interest rate cap, we investigate its effects on borrowers' welfare. In particular, we are interested in how this interest rate cap influences the loan spread that borrowers pay and the loan amount that they obtain. Since the effects of interest rate cap are similar qualitatively in both markets, we present the results for the DIP loan market as an example.

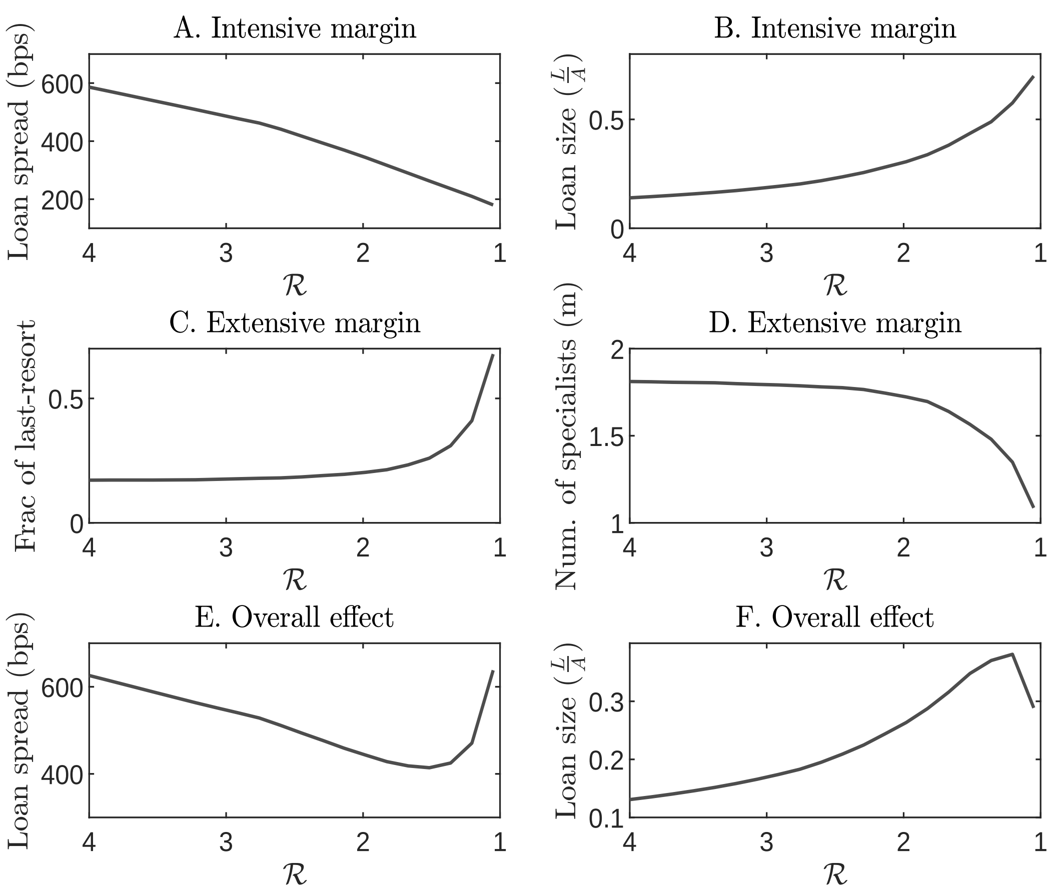
We start with demonstrating the intended consequences of the interest rate cap policy. In panels (a) and (b) of Figure 8, we plot the risk-adjusted yield spread (left) and loan amount (right) as a function of the tightness of interest rate cap controlled by the parameter $\mathcal{R}_{max}$. As $\mathcal{R}_{max}$ decreases, the lenders' markup declines and the equilibrium risk-adjusted yield spread drops, indicating a tight interest rate cap control.

As expected, a tighter cap on interest rate reduces the risk-adjusted yield spread almost mechanically. Meanwhile, since loan spread is capped, coordination among specialist lenders that aim at restricting the total loan amount for lifting loan spread becomes ineffective, and thus specialist lenders lend more aggressively in order to capture the profits from a larger loan size. Accordingly, we observe a sharp increase in the loan amount as interest rate cap tightens. Both the decline in risk-adjusted yield spread and increase in loan size by specialist lenders benefit the borrowers.

Even though interest rate cap improves borrower welfare through borrowing from the spe-

Figure 8: The Effects of Interest Rate Cap Policy

This figure shows the effects of the interest rate cap policy. The policy details are described in section 7. Panels (a) and (b) depict the intensive margin of rate-cap on reducing the risk-adjusted yield spread charged by specialist lenders and increasing the corresponding loan amount in the DIP-loan market; panel (c) and (d) show the extensive margin of rate-cap on reducing the number of participating specialist lenders and increasing the chance of the borrowers turning to lenders of last-resort; and panel (e) and (f) illustrate the overall effect of rate-cap on the average risk-adjusted yield spread and loan amount.



cialist lenders (i.e., the intensive margin), an unintended consequence of this policy is to further discourage the participation of specialist lenders and thus hinders the depth of this market (i.e., the extensive margin). Intuitively, specialist lenders decide to participate only when their expected profits are higher than the participation costs. Interest rate cap reduces the expected profits earned by specialist lenders and therefore excludes more specialist lenders from participating. If no specialist lenders are willing to participate in a specific deal, then the borrower is forced to borrow from the lenders of last-resort who are private investors in the market and may not be restricted by the interest rate cap. Lack of competition among the last-resort lenders and the high variable costs they bear make the loan very expensive. Panel (c) and (d) confirm this model prediction. As $\mathcal{R}_{max}$ declines and the spread-cap tightens, the likelihood for the borrowers to borrow from lenders of the last-resort climbs sharply from 10-20% to above 60%. Meanwhile, even if some borrowers can still borrow from specialist lenders, the average number of participating lenders becomes significantly smaller.

Combining the positive effect of interest rate cap on the intensive margin and its negative effect on the extensive margin, Panel (e) and (f) illustrate the net effect. The net effect captures the likelihood of the borrowers borrowing from different types of lenders and the risk-adjusted yield spread charged by these lenders. We observe a U-shaped relation between the average risk-adjusted yield spread paid by borrowers and the tightness of rate cap and a hump-shaped relation between the average loan amount obtained by these borrowers and the tightness of rate cap. These findings suggest that there exists an optimal level of rate cap that maximizes the borrowers' welfare. Specifically, the optimal interest rate cap maps to an average risk-adjusted yield spread of about 400 bps across all types of lenders in the DIP loan market, compared with the observed spread of 700 bps in the data.

8 Conclusions

The lending market for distressed loans features an oligopolistic structure, with a few specialist lenders financing a large fraction of loans. It raises the question of how lender market power drives loan pricing in this market. We develop a dynamic game-theoretic model of strategic competition in distress loan markets with endogenous entry. Our entry and competing model provide several novel implications, including the "entry effect" of collusion capacity on the number of potential specialists and thus the likelihood of ex-post inefficient last-resort lending. Taking into account collusive lending by specialists and latent heterogeneity, we then use a comprehensive data sample that contains both distressed loans to bankrupt firms and those not yet in bankruptcy to estimate the structural model. We find that lender market power accounts for more than 90% of the risk-adjusted yield spread of distressed loans and for up to two-thirds of the risk-adjusted yield spreads of DIP loans. More than half of the lender market power is attributed to collusive lending in distressed loans and limited participation, likely due to lenders' blocking power, in DIP loans. Smaller borrowers are particularly susceptible to lender market power than larger borrowers, calling for more attention from policy makers towards small borrowers in difficult times. Without lender collusion, a large fraction of distressed borrowers would switch from lenders of last resort such as hedge funds to specialized lenders and benefit from a significantly larger loan amount and a much lower loan spread. Our policy analysis on interest rate cap suggests that such policy has a hump-shaped effect on the borrowers' welfare due to a positive intensive margin counteracting a negative extensive margin. Specifically, there exists an optimal level of rate-cap that can be imposed by regulators.

Appendix

A Solution method:

We conjecture

$$L^C(k,x,m) \equiv \widehat{L}^C(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A \tag{60}$$

$$L^D(k,x,m) \equiv \widehat{L}^D(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A. \tag{61}$$

For each $\widehat{L}^C(k,m)$, we numerically solve

$$\widehat{L}^D(k,m) = \underset{\widehat{L}}{\operatorname{argmax}} \left[\left(\frac{1}{\widehat{L}+(m-1)\widehat{L}^C(k,m)}\right)^{1/\varepsilon_2(k)} - 1\right]\widehat{L}. \tag{62}$$

The first-order condition is the maximization problem above is

$$1 - \frac{1}{\varepsilon_2(k)} + \frac{1}{\varepsilon_2(k)}\frac{(m-1)\widehat{L}^C(k,m)}{\widehat{L}^D+(m-1)\widehat{L}^C(k,m)} = \left[\widehat{L}^D + (m-1)\widehat{L}^C(k,m)\right]^{1/\varepsilon_2(k)} \tag{63}$$

Thus, with the optimal deviation $\widehat{L}^D(k,m)$ pinned down in (63), the optimal deviation profit is

$$\mathbb{E}^x\left[\Pi_2^D(k,x,m;L^C)\right] = \left[\left(\frac{1}{\widehat{L}^D+(m-1)\widehat{L}^C}\right)^{1/\varepsilon_2(k)} - 1\right]\widehat{L}^D e^{\alpha_2(k)+\frac{1}{2}\sigma^2+(1-\varepsilon_2(k))\phi_2+\frac{1}{2}(1-\varepsilon_2(k))^2\zeta^2}A \tag{64}$$

and

$$\mathbb{E}^x\left[\Pi_2(k,x,m;L^C)\right] = \left[\left(\frac{1}{m\widehat{L}^C}\right)^{1/\varepsilon_2(k)} - 1\right]\widehat{L}^C e^{\alpha_2(k)+\frac{1}{2}\sigma^2+(1-\varepsilon_2(k))\phi_2+\frac{1}{2}(1-\varepsilon_2(k))^2\zeta^2}A. \tag{65}$$

We now try to get $\widehat{L}^C(k,m)$ for $L^C(k,x,m) \equiv \widehat{L}^C(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A$. We define the IC compatible set of functionals $\widehat{L}^C(\cdot,\cdot)$ as follows:

$$\widehat{\mathcal{L}}^C \equiv \left\{\widehat{L}^C:\ \frac{\xi(W^C(\widehat{L}^C)-W^N)}{1-\delta} \geq \mathbb{E}^x\left[\Pi_2^D(k,x,m;\widehat{L}^C)\right] - \mathbb{E}^x\left[\Pi_2(k,x,m;\widehat{L}^C)\right],\ \ \forall\ k,m\right\}. \tag{66}$$

In the collusive Nash equilibrium we focus on, the loan size is

$$\widehat{L}^C(\cdot,\cdot) = \underset{\widehat{L}\in\widehat{\mathcal{L}}^C}{\operatorname{argmax}}\, \mathbb{E}\left[U^C(k,x,m;\widehat{L})\right]. \tag{67}$$

Let $\widehat{L}^C_{max}(k,m) \equiv \frac{1}{m}\left[1-\frac{1}{\varepsilon_2(k)}\right]^{\varepsilon_2(k)}$. According to the definitions, it follows that $\widehat{L}^C_{max}(\cdot,\cdot)$ is the optimal loan size if $\widehat{L}^C_{max}(\cdot,\cdot) \in \widehat{\mathcal{L}}^C$.

B Estimation of Risk-Adjusted Yield Spreads

B.1 Estimation of Credit-Risk Components Based on CDS Prices

Under a CDS contract, the protection seller promises to buy the reference bond at its par value when a predetermined default event occurs. In return, the protection buyer makes periodic payments to the seller until the maturity date of the contract or until a credit event occurs. This periodic payment, which is usually expressed as a percentage (in basis points) of the notional value, is called the CDS spread. The reference bond is typically a senior unsecured bond. A CDS written on a particular reference bond normally provides coverage for all obligations of the reference entity that have equal or higher seniority.

Owing to the specification of CDS contracts and the liquid market for CDS transactions among sophisticated institutional investors, the CDS spread provides a pure measure of the credit risk component in the credit spread of the reference bond.

We observe T-year CDS premium, which is paid every period of Δ. The frequency $\Delta = 0.5$ is semiannual. We also have reliable estimation on the expected recovery rates δ and δ_L for the corporate bond and the loan, respectively. Recovery rate is the extent to which principal and accrued interest on defaulted debt can be recovered, expressed as a percentage of face value (i.e., par value). Moreover, we have the information on zero-coupon risk-free bond prices, denoted by $Z(0,i\Delta)$ with $i = 1,2,\cdots,T/\Delta$. Let $n = T/\Delta$ and $t_i = i\Delta$ for $i = 1,\cdots,n$.

Back out constant hazard rate p^*. If CDS only reflects credit risk, the non-arbitrage CDS premium formula is

$$s = \frac{1}{\Delta} \frac{(1-\delta) \sum_{i=1}^{n} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] Z(0, t_i)}{\sum_{i=1}^{n} P^*(0, t_i) Z(0, t_i)}, \tag{68}$$

where $P^*(0, t)$ is the risk-neutral probability of survival up to time t, modeled as

$$P^*(0, t) \equiv \exp(-p^* \times t). \tag{69}$$

We can estimate p^* using the equations (68) and (69) based on the data of s, δ, and $Z(0, t_i)$.

Risk-neutral expected loan maturity. Given the estimated risk-neutral hazard rate p^*, the risk-neutral expected maturity is

$$\begin{aligned}
Risk\ neutral\ expected\ maturity &= \sum_{i=1}^{\infty} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] \min\{t_i, t_n\} && (70) \\
&= \sum_{i=1}^{n} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] t_i + \sum_{i=n+1}^{\infty} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] t_n && (71) \\
&= \sum_{i=1}^{n} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] t_i + P^*(0, t_n) t_n && (72)
\end{aligned}$$

Risk-neutral expected annualized default fee. Suppose the default interest is D and the default borrower on average pays back in one year after the occurrence of default. The risk-neutral expected annualized default fee is

$$Risk\ neutral\ expected\ annualized\ default\ fee = \sum_{i=1}^{n} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] Z(0, t_i) D / t_i. \tag{73}$$

Estimate loan credit spreads. If a loan has maturity $T_L = t_m$, it holds that

$$1 = P^*(0, t_m) Z(0, t_m) + \sum_{i=1}^{m} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] Z(0, t_i) \delta_L + y\Delta \sum_{i=1}^{m} P^*(0, t_i) Z(0, t_i),$$

where y is the annualized loan yield. The equality above recognizes that convention that loans are sold at par, and that the loan yield is exactly equal to the coupon rate.

Then, the credit-risk component of the annualized 6-month loan yield is

$$y = \frac{1}{\Delta}\frac{1 - P^*(0,t_m)Z(0,t_m) - \sum_{i=1}^{m}\left[P^*(0,t_{i-1}) - P^*(0,t_i)\right]Z(0,t_i)\delta_L}{\sum_{i=1}^{m} P^*(0,t_i)Z(0,t_i)}. \tag{74}$$

Thus, the credit spread is $y - r_f$, where y is the annualized 6-month loan yield that is derived according to (74), and $r_f\Delta = 1/Z(0,1) - 1$ is the annualized 6-month (simple) risk-free rate.

C Government intervention

Suppose government sets up a special purpose vehicle (SPV) to participate the loan syndicate for distressed borrowers. For each borrower, there is a probability $\tau(A)$ with which government will join the loan syndicate. The probability $\tau(A)$ captures the intervention intensity, and depends on the size of the borrower A. The baseline model is a special case of the extended model with government intervention with $\tau(A) \equiv 0$. Particularly, we can specify the intervention intensity $\tau(A)$ as a logistic function of A:

$$\tau(A) \equiv \exp(\tau_0 + \tau_1 A) / \left[1 + \exp(\tau_0 + \tau_1 A)\right]. \tag{75}$$

We set $g = 1$ if and only if government intervenes, and we set $g = 0$ otherwise.

Compared with the baseline model, the value functions $U^C(A,k,x,m;L^C)$, $U^N(A,k,x,m)$, and $U^D(A,k,x,m;L^C)$ have the same functional forms as in (13), (25), and (32), respectively; the only exception is that the continuation values $W^C(L^C)$ and W^N are different. More specifically, the participation threshold with government intervention, denoted by w_G^*, is different from w_N^*.

First, given the threshold w_G^*, the continuation value in a collusive Nash equilibrium is

$$W^C(L^C) = \mathbb{E}^{A',k'}\left\{\lambda_{k'}\frac{\Pi_1(A',k')}{M_0}\right\} \tag{76}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\left[1-\tau(A')\right]\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m';L^C)-w'\right)\mathbf{1}_{\{w'\leq w_C^*\}}|g=0\right]\right\} \tag{77}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\,\tau(A')\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m'+1;L^N)-w'\right)\mathbf{1}_{\{w'\leq w_G^*\}}|g=1\right]\right\} \tag{78}$$

and the continuation value in a non-collusive equilibrium is

$$W^N = \mathbb{E}^{A',k'}\left\{\lambda_{k'}\frac{\Pi_1(A',k')}{M_0}\right\} \tag{79}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\left[1-\tau(A')\right]\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m';L^N)-w'\right)\mathbf{1}_{\{w'\leq w_N^*\}}|g=0\right]\right\} \tag{80}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\,\tau(A')\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m'+1;L^N)-w'\right)\mathbf{1}_{\{w'\leq w_G^*\}}|g=1\right]\right\} \tag{81}$$

Here, conditioning on the government intervention, the expected profit from participating the syndication in the next period is

$$\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m'+1;L^N)-w'\right)\mathbf{1}_{\{w'\leq w_G^*\}}|g=1\right] \tag{82}$$
$$=\sum_{m'=1}^{M} q(m'|w'\leq w_G^*, w^*=w_G^*, g=1)\left[F(w_G^*)\mathbb{E}^{x'}\left[\Pi_2(A',k',x',m'+1;L^N)\right]-\int_{w'\leq w_G^*} w' dF(w')\right],$$

the profit of the syndicated lending with non-collusive loan size plan L^N is

$$\Pi_2(A',k',x',m'+1;L^N) \equiv \max_L\left[\left(e^{\alpha_{k'}+\sigma z'}\frac{A'}{L+m'L^N(A',k',m'+1)}\right)^{1/\varepsilon_{k'}}-e^{\phi_2+\varsigma u'}\right]L, \tag{83}$$

and the conditional probability $q(m'|w' \leq w_G^*, g = 1)$ is

$$
\begin{aligned}
& q(m'|w' \leq w_G^*, w^* = w_G^*, g = 1) \\
&= \frac{\mathbb{P}\left\{\text{This specialist and other } m'-1 \text{ specialists participate the lending}|w^* = w_G^*, g = 1\right\}}{\mathbb{P}\left\{\text{This specialist participates the lending}|w^* = w_G^*, g = 1\right\}} \\
&= \binom{M-1}{m'-1} F(w_G^*)^{m'-1} \left[1 - F(w_G^*)\right]^{M-m'}.
\end{aligned} \tag{84}
$$

The following equality characterizes the cutoff w_G^*:

$$
\sum_{m=1}^{M} q(m|w = w_G^*, w^* = w_G^*, g = 1)\mathbb{E}^{A,k,x}\left[\Pi_2(A, k, x, m+1; L^N)\right] = w_G^*. \tag{85}
$$

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Figure 9: Loan Market Network

This figure describes the network of the distressed loan markets and the DIP loan market. Each circle represents one unique lender, with the size of the circle depending on the market share of the lender in the corresponding loan market. The tiny circles on the outer layer are for lenders whose market share is negligible. Each line between the lenders represents one coordination in a syndication.

A. Distressed Loans B. DIP Loans