

Tech Dollars: Technological Innovation and Exchange Rates*

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Abstract

We document a robust positive correlation between U.S. innovation and the appreciation of the real dollar. Periods of high innovation coincide with rising foreign capital inflows, both in the aggregate and toward innovative U.S. firms. These facts motivate a simple general equilibrium model in which technological progress drives dollar demand. We depart minimally from the standard endowment economy by allowing innovation-driven productivity gains to accrue disproportionately to a subset of agents—entrepreneurs. Crucially, this structure gives foreign investors a direct channel to participate in U.S. innovation via equity markets. As a result, the dollar appreciates not merely as a safe asset but as a claim on the benefits of technological innovation. A calibrated version of the model matches key empirical patterns, including the joint dynamics of the dollar, innovation, equity returns, inequality, output growth, and trade flows. The analysis highlights a novel channel linking innovation, global capital flows, and the valuation of the U.S. dollar.

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How do technological innovations and productivity shocks affect exchange rates? Over the recent decades, many significant innovations have originated in the US. Importantly, periods of significant technological breakthroughs in the U.S.—such as the rise of personal computers in the 1980s, the emergence of internet companies around 2000, and recent advancements in artificial intelligence and large language models—have coincided with times of a strong dollar. Yet, standard theory suggests that innovation-driven booms should lower marginal utility and lead to real exchange rate depreciation. In a complete markets world, good news for the U.S. means wealth transfers to the rest of the world, and a weaker dollar. Yet this prediction is at odds with the data: empirical evidence shows a weak, sometimes even positive, correlation between macro fundamentals and exchange rates (Backus and Smith, 1993; Kollmann, 1995; Obstfeld and Rogoff, 2001; Chahrour, Cormun, De Leo, Guerrón-Quintana, and Valchev, 2024).

In this paper, we document a strong positive correlation between US innovation and the growth of the real dollar index. We provide a theoretical model that rationalizes this positive comovement. The key assumption in the model is incomplete markets: the gains from innovation in the US are not shared equally as they cannot be fully appropriated by investing in the shares of US firms. Part of these gains accrue to innovators (entrepreneurs) or other workers that are key in the production of innovation. As a result, increases in the rate of US innovation lead to an increase in the wealth share of the US economy, an increase in aggregate consumption growth, but also to an increase in the average marginal utility for US households through a Constantinides and Duffie (1996) effect—and therefore to an appreciation of the real exchange rate. Overall, the model is consistent with the joint dynamics of exchange rates, wealth shares, output and consumption growth, income inequality, and stock returns. The model generates a novel mechanism for holding dollars: investing in the shares of US innovative firms. Consistent with this prediction, we show that US innovation is associated with an increase in foreign capital inflows at both the aggregate and firm levels.

We begin by analyzing the empirical relationship between U.S. innovation and real dollar appreciation. Our annual measure of innovation is the log ratio of total patent value each year (Kogan, Papanikolaou, Seru, and Stoffman, 2017) to aggregate stock market capitalization. Focusing on data from the post-Bretton Woods era, we document a robust positive correlation: a one-standard-deviation rise in innovation predicts approximately 3 to 4 log points of real exchange rate appreciation per year. Furthermore, this measure of innovation strongly correlates with U.S. productivity growth, as captured by utilization-adjusted total factor productivity (TFP) from Fernald (2014). From the perspective of complete markets, this positive correlation presents a puzzle: if high innovation in the US represents favorable states for US households, the US would transfer wealth to the rest of the world, leading to a depreciation of the real exchange rate and a reduction in US relative wealth.

We next present evidence consistent with imperfect international risk-sharing of innovation shocks. Specifically, we show that higher U.S. innovation is associated with an increase in the relative

wealth of U.S. households compared to the rest of the world, as well as rising income inequality within the U.S. Additionally, innovation shocks coincide with significant foreign capital inflows at both aggregate and firm levels. Aggregate measures reveal a clear positive correlation between U.S. innovation and inflows of foreign direct investment and portfolio equity to the US. At the firm level, we find that U.S. companies experiencing an innovation shock—measured by the grant of valuable or significant new patents—subsequently attract greater foreign institutional ownership.

Our goal is to rationalize these patterns through a simple economic framework. To clarify the economic mechanism, we start with a set of minimal ingredients: a two-country endowment economy with log utility and home-bias in consumption. In addition to standard aggregate endowment shocks, each country experiences displacement shocks designed to capture key aspects of creative destruction. Economic growth arises partly from new, innovative projects (firms) that potentially displace incumbents. This structure serves as a reduced-form representation of more elaborate models featuring endogenous growth and creative destruction, in which periods of rapid innovation coincide naturally with substantial reallocation of economic activity (e.g., [Aghion and Howitt, 1992](#); [Kogan, Papanikolaou, and Stoffman, 2020](#)). Crucially, markets are incomplete: new projects are not owned by existing shareholders but rather randomly allocated to a small subset of agents. Households cannot trade away their future claims to these potential innovations. Thus, shocks that increase the relative profitability of new firms redistribute wealth from incumbent owners to new entrepreneurs, amplifying consumption inequality. Because marginal utility is convex, the increase in inequality following the displacement shock raises the stochastic discount factor as in [Constantinides and Duffie \(1996\)](#). As a result, the real exchange rate appreciates. This real exchange rate appreciation enhances the value of domestic assets and thereby increases the country’s aggregate wealth, consistent with findings in [Dahlquist, Heyerdahl Larsen, Pavlova, and Penasse \(2023\)](#).

Our theoretical model delivers a number of predictions that are consistent with the data. First, it sheds new light on the [Backus and Smith \(1993\)](#) puzzle. Specifically, bilateral exchange rate growth depends not only on the relative consumption growth between home and foreign countries but also crucially on shifts in their relative wealth. This prediction is consistent with the data: increases in a country’s relative wealth coincide systematically with real exchange rate appreciation. Moreover, once we account for changes in relative wealth, the correlation between exchange rate movements and standard macroeconomic fundamentals turns negative and statistically significant—precisely as standard complete-market logic would suggest.

Second, a positive innovation (displacement) shock in the model leads to an increase in income inequality, consistent with the evidence in [Aghion, Akcigit, Bergeaud, Blundell, and Hemous \(2018\)](#). As a result, the model implies a positive correlation between exchange rates and changes in (relative) income inequality, which is consistent with the findings in [Kocherlakota and Pistaferri \(2008\)](#). Replicating their analysis in our sample of eleven countries covering the post-Bretton Woods era, we

find a positive and economically significant correlation between changes in bilateral exchange rates and changes in relative income inequality. For instance, focusing on the coefficients from the pooled regression, a one-standard deviation increase in income inequality in a foreign country relative to the United States is associated with a 1.7 log point appreciation of its currency relative to the US dollar.

Third, the model suggests a direct proxy for the displacement shock that follows the logic in [Gârleanu, Panageas, Papanikolaou, and Yu \(2016\)](#). In particular, the difference between the aggregate market capitalization growth and the returns from holding the market portfolio is related to the realizations of the displacement shock in the model. This gap emerges because market investors must continuously liquidate existing positions to finance purchases of shares in new, entering firms, ensuring the portfolio remains self-financing. Consequently, as new firms enter the market, returns on the market portfolio consistently lag behind aggregate market capitalization growth. Using this insight, we construct an empirical displacement shock measure for the U.S. and find that it is positively correlated with the appreciation of the real US dollar. Notably, periods characterized by significant innovation and the entry of new firms systematically coincide with real dollar appreciation.

We then explore the ability of our mechanism to quantitatively account for the key correlations in the data. To this end, we introduce a few additional elements: we allow for recursive preferences over relative consumption and relax the assumption of extreme inequality—a positive measure set of households receive new projects. In addition, we allow for the distribution of the displacement shock to vary over time. Though these modifications are not needed to qualitatively explain the key patterns in the data, they help the model deliver realistic quantitative predictions.

Our model successfully replicates the joint dynamics of innovation, exchange rates, consumption growth, and capital flows, while generating low and relatively smooth risk-free rates. Importantly, our model quantitatively reproduces the three key ‘anomalies’ in the exchange rate literature: the volatility puzzle of [Brandt, Cochrane, and Santa-Clara \(2006\)](#), the [Backus and Smith \(1993\)](#) correlation puzzle, and the violation of the uncovered interest rate parity (UIP). The key to replicating the failure of the UIP is the time-varying distribution of the displacement shock.

The quantitative success of the model does not come at the cost of unrealistic parameters. In terms of preference parameters, the model calibration requires a degree of relative risk aversion of 6.2 and an elasticity of inter-temporal substitution (EIS) equal to 1.6, which are largely in line with the literature. Further, the model requires a highly persistent and risk-skewed displacement shock. To ensure that the magnitudes of displacement shocks are realistic, we discipline the distribution of the displacement shock using the moments of income inequality in the data.

Our work contributes to several strands of the literature. Over the last several decades, a voluminous literature has studied the determination of exchange rates in two-country equilibrium

models.¹ Similar to most of these models, our model does require a high degree of home bias in household preferences to generate sufficiently volatile exchange rates given the high level of correlation in consumption growth across countries. Our key innovation relative to these papers, however, is that exchange rates are pro-cyclical in our model, since increases in technological innovation are positively correlated with future growth in consumption, productivity and output.

In this regard, our work is highly complementary to the findings of [Chahrour et al. \(2024\)](#), who document a correlation between the dollar exchange rate and subsequent movements in total factor productivity. Specifically, [Chahrour et al. \(2024\)](#) identify a shock to future TFP growth in the US using VARs, and show that this shock leads to an appreciation of the dollar over the short run. These patterns are hard to reconcile with models with complete markets (for example [Colacito and Croce, 2013](#)), in which positive news on future productivity would lead to lower marginal utility of US investors and therefore to a depreciation of the dollar. By contrast, our model is both qualitatively, but also quantitatively, consistent with this pattern: a positive innovation shock in the US leads to both higher productivity and output growth in the US, but also to an appreciation of the dollar.

More broadly, our work addresses the ‘exchange rate disconnect’ puzzle—the persistently weak correlation between exchange rates and fundamentals.² While this puzzle has motivated various models featuring segmented or incomplete asset markets, our mechanism is distinct.³ Existing models typically assume exogenous capital flow shocks that are orthogonal to fundamentals, and study their implications for exchange rates under imperfect risk sharing (e.g., [Gabaix and Maggiori, 2015](#); [Itskhoki and Mukhin, 2021](#)). By contrast, our model’s market incompleteness arises endogenously from imperfect risk-sharing of agents’ endowment shocks. Capital flows are endogenously determined in equilibrium.

Our model generates a positive correlation between capital inflows and currency appreciation, consistent with the data ([Hau and Rey, 2006](#)). In the model, a positive innovation shock in the home country is associated with the creation of new firms (projects) which are initially owned by a small subset of households (entrepreneurs). Entrepreneurs sell their shares to diversify their holdings and foreign investors buy some of these shares to rebalance their portfolio as the share of the home country in the world market portfolio increases. The net effect is that the home country experiences net capital inflows and its currency appreciates.

Our work also connects to the voluminous literature that aims to rationalize the failure of

¹See, for example, [Alvarez, Atkeson, and Kehoe \(2002, 2009\)](#); [Pavlova and Rigobon \(2007, 2008, 2010, 2011\)](#); [Verdelhan \(2010\)](#); [Colacito and Croce \(2011, 2013\)](#); [Colacito, Croce, Gavazzoni, and Ready \(2018\)](#); [Gavazzoni and Santacreu \(2020\)](#); [Kekre and Lenel \(2024b\)](#).

²See, for instance [Meese and Rogoff \(1983\)](#); [Obstfeld and Rogoff \(2001\)](#); [Yu \(2013\)](#); [Colacito, Croce, Liu, and Shaliastovich \(2021\)](#); [Chernov, Haddad, and Itskhoki \(2023\)](#); [Colacito, Croce, Liu, and Shaliastovich \(2023\)](#).

³An incomplete list includes [Bacchetta and Van Wincoop \(2006\)](#); [Hau and Rey \(2006\)](#); [Farhi and Werning \(2014\)](#); [Itskhoki and Mukhin \(2021\)](#); [Lustig and Verdelhan \(2019\)](#); [Greenwood, Hanson, Stein, and Sunderam \(2020\)](#); [Gourinchas, Ray, and Vayanos \(2020\)](#); [Camanho, Hau, and Rey \(2020\)](#); [Fang and Liu \(2021\)](#); [Fang \(2021\)](#); [Fukui, Nakamura, and Steinsson \(2023\)](#); [Jiang, Krishnamurthy, and Lustig \(2023b\)](#); [Kekre and Lenel \(2024a\)](#).

uncovered interest parity (UIP) in the data (Fama, 1984). Overall, the literature suggests that the UIP puzzle can be explained either by time variation in higher-order moments of the stochastic discount factor or by cross-country differences in economic environments.⁴ More recently, Hassan, Mertens, and Wang (2024) argue that the puzzle is still unresolved, as many of these existing models imply a counterfactually high predictability of exchange rates by interest rate differentials. Their critique does not apply to our work: the UIP fails to hold in our model even as exchange rates are essentially unpredictable by interest rate differentials.

Our work contributes to the growing body of literature that studies the special role of dollar assets in international markets. Previous studies have focused on the US’s role as a global insurance provider and its exorbitant privilege, the convenience yield of holding US assets, and more recently the impact of US fiscal policy.⁵ Our work offers a novel perspective: a key motive for investors to hold dollars is to invest in innovative technology firms in the US. Thus, part of the strong demand for dollar assets is driven by the increased rates of technological innovation in the US relative to the rest of the world.

The existence of common risk factors in exchange rates has been the subject of considerable debate (Lustig, Roussanov, and Verdelhan, 2011; Verdelhan, 2018). Recent work emphasizes the importance of international trade linkages in generating comovement across currencies (Richmond, 2019; Lustig and Richmond, 2019; Jiang and Richmond, 2019). Our model has implications for the joint dynamics of exchange rates and trade flows. First, both in the model and in the data, net exports are counter cyclical. Moreover, in our model, displacement shocks lead to both rising income inequality and a deterioration in the current account balance. Consistent with this mechanism, we find a negative correlation across countries between changes in top income shares and changes in current account balances.

At a broader level, our mechanism is closely related to the Balassa (1964); Samuelson (1964) effect. The traditional Balassa-Samuelson logic posits that productivity gains in the tradable sector lead to real exchange rate appreciation as workers experiencing a positive wealth shock due to higher wages increase their demand for non-tradable goods, and therefore to an increase in the price and the real exchange rate. A key ingredient in this logic is incomplete markets: if markets were complete, relative wealth, and therefore relative prices, would remain unchanged. Our mechanism is related, but distinct: technological innovation leads to an increase in within-country inequality and an increase in marginal utility through the Constantinides and Duffie (1996) effect.⁶ Absence of

⁴See, for instance, Backus, Foresi, and Telmer (2001); Verdelhan (2010); Martin (2011); Hassan (2013); Colacito and Croce (2013); Ready, Roussanov, and Ward (2017); Colacito et al. (2018); Richmond (2019); Wiriadinata (2021).

⁵See, for example, Gourinchas and Rey (2007a,b); Gourinchas, Rey, and Govillot (2010); Atkeson, Heathcote, and Perri (2022); Dahlquist et al. (2023); Sauzet (2023); Jiang, Krishnamurthy, and Lustig (2021); Jiang, Krishnamurthy, Lustig, and Sun (2021); Atkeson et al. (2022); Koijen and Yogo (2020); Chen, Jiang, Lustig, Nieuwerburgh, and Xiaolan (2023); Jiang, Krishnamurthy, and Lustig (2023a); Van Nieuwerburgh, Jiang, Lustig, and Xiaolan (2021); Jiang (2021).

⁶The increase in marginal utility in response to innovation shocks is also present in Papanikolaou (2011); Gârleanu et al. (2016); Kogan et al. (2020); Huang, Kogan, and Papanikolaou (2023). Kogan et al. (2020) build a general equilibrium model with capital embodied technology shocks in which benefits of innovation are distributed asym-

arbitrage and complete international financial markets imply that the exchange rates appreciate.

1 Dollar Exchange Rates and US Innovation

We begin by documenting a set of stylized facts regarding the joint dynamics of US innovation, the real exchange rate, capital flows, and the distribution of income and wealth. Below, we briefly discuss the sources of main variables in our analysis. Appendix A contains additional details.

1.1 Data Sources

Our sample consists of a combination of G-10 currency countries and G-7 countries. Specifically, it includes Australia, Canada, Germany, Japan, Norway, New Zealand, Sweden, Switzerland, United Kingdom, France, Italy and the United States. We take the domestic country to be the United States and define the exchange rate as the units of foreign currency per dollar. The sample period covers the post-Bretton Woods era (1974 to 2022).

We obtain end-of-year nominal exchange rates from the IMF. The real exchange rate is calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country. We calculate the growth of the real dollar index as the equal-weighted average of the log growth rates of the real dollar exchange rates against the currencies in our sample. The level of the dollar index is obtained by accumulating these growth rates over the past years.

We obtain data on consumption, GDP, and net exports from the World Development Indicators published by the World Bank. We use household final consumption expenditure for consumption series, and the difference between the indices of export of goods and imports of goods and services as our net export series. Both consumption and GDP are real. We measure income inequality using the top 1% income share and obtain data on the top income share from the World Inequality Database. The World Inequality Database also provides each country's total net wealth (code = mnweal) in local currency. To convert each country's wealth into US dollars, we multiply the total net wealth by the corresponding exchange rate.

We measure the annual US innovation level as the log of the ratio of the total economic value of patents (Kogan et al., 2017) each year to the total stock market capitalization at the end of the year. Data on aggregate foreign direct investment and portfolio equity flows are obtained from the World Bank. Data on interest rates comes from Global Financial Data. Real interest rates are constructed using three-month T-bills yields from the Global Financial Data, adjusting for realized

metrically across the economy. They show that the resulting displacement risk can lead to increased demand for insurance (an increase in the stochastic discount factor) and can help rationalize certain cross-sectional features of asset returns. Huang et al. (2023) examine this mechanism in a multi-region model of a monetary union and study its implications for regional inflation dynamics. Gârleanu et al. (2016) embed a reduced-form of this mechanism in a standard endowment model and study its implications for the equity risk premium. Kogan, Papanikolaou, Schmidt, and Song (2020) present complementary evidence that surges in innovation correlate with higher labor income risks for incumbent workers, leading to a stronger demand for insurance.

inflation using annual changes in CPI. Data on equity index returns (MSCI series) are obtained from Datastream.

1.2 Innovation and the US Dollar

We begin by documenting a positive correlation between US innovation and the growth of the real dollar exchange rate. We estimate the following specification

$$\Delta \log e_{t+1}^{USD} = \beta \text{Inno}_{US,t+1} + c \mathbf{Z}_t + \varepsilon_{t+1} \quad (1)$$

The dependent variable is the growth in the (log) dollar index level from t to $t + 1$. The independent variable is the US innovation at year $t + 1$. Depending on the specification, we include the lagged dollar index level, lagged US innovation, or both as control variables. We also control for lagged US output growth. Standard errors are calculated using the Newey-West procedure, with a bandwidth of one year.

Table 1 reports the estimates from (1). We see an economically and statistically significant positive correlation between the level of US innovation and the growth of the US dollar index. A one-standard-deviation increase in US innovation is associated with approximately 3 to 4 log points of exchange rate appreciation at the annual level. Figure 1 plots the innovations in the series of real dollar index growth and US innovation, orthogonalized with respect to lagged US innovation and the lagged dollar index. The specification corresponds to column (3) in Table 1. Examining the figure, we observe that, first, the two series are highly correlated, and, second, during periods of significant innovation, such as the internet boom around the 2000s and the AI boom during the 2020s, US innovation is high, and the dollar appreciates in real terms.

This positive correlation between the US dollar index and US innovation is robust to alternative specifications. Specifically, Appendix Table A.1 shows that our results remain qualitatively similar when using the trade-weighted dollar index from the Fed. In addition, we define annual US innovation as the log of the ratio of the total real economic value of patents (Kogan et al., 2017) to the total number of patents. Appendix Table A.2 presents the results, showing that this alternative measure is positively correlated with the growth of the real dollar index. Last, we also examine the correlation between US innovation and the US dollar in a panel regression,

$$\Delta \log e_{F,t+1} = \beta \text{Inno}_{US,t+1} + c \mathbf{Z}_t + \varepsilon_{F,t+1}. \quad (2)$$

Compared with (1), the dependent variable now is the growth in the bilateral exchange rate e between the foreign country F and the US. The advantage of doing so is that now we can include several country-level controls that can account for the variation in bilateral exchange rates. The control variables $\mathbf{Z}_t$ include the lagged innovation level, lagged growth of output ratio, and the lagged exchange rate level. We now include country fixed effects, with standard errors computed

using the [Driscoll and Kraay \(1998\)](#) methodology to account for heteroskedasticity, serial correlation, and cross-sectional dependence due to the fact that all the bilateral exchange rates are against the US dollar.

We report the estimates from (2) in the last column of Table 1. Examining the estimated coefficient β , we note that its point estimate is similar in magnitude with the response of the US dollar index: a one-standard-deviation increase in the dollar value of innovation is associated with a 2.6 to 3.9 log-point appreciation of the dollar over next year. Last, Appendix Table A.4 reports results separately for individual countries. Even though the standard errors are larger due to the smaller number of observations, we see that the point estimates are positive in 10 out of the 11 cases.

Overall, we show that the level of innovation in the US is strongly associated with appreciation in the US dollar over short horizons. Here, we note that our results are largely in line with the recent literature emphasizing the importance of news about TFP shocks as a key driver of exchange rates ([Nam and Wang, 2015](#); [Chahrour et al., 2024](#)). In particular, [Chahrour et al. \(2024\)](#) argue, using a vector auto-regression, that news shocks about future TFP are the dominant driver of exchange rates. Given that our measure innovation is positively correlated with future TFP growth ([Kogan et al., 2017](#)), our results are highly complementary to [Chahrour et al. \(2024\)](#). More specifically, [Kogan et al. \(2017\)](#) show that the measure of innovation that we use is positively correlated with future productivity and output growth in the US. For completeness, we replicate their results in Appendix Figure A.1. In terms of magnitudes, a one-standard-deviation increase in US innovation is associated with an annual growth of 0.3 to 0.4 log points in US TFP.

1.3 Innovation, Income and Wealth

Next, we provide evidence consistent with incomplete markets. Specifically, we examine the extent to which increases in the rate of innovation in the United States are unlikely to be shared across households, both across but also within countries.

First, we examine the response of countries' relative wealth in response to innovation shocks. To this end, we re-estimate equation (2), but now use the change in the wealth ratio between the US and foreign countries as the dependent variable. As before, estimating this relation in a panel allows us to partial out global fluctuations in wealth that are unrelated to innovation. As our baseline, we measure country wealth using the World Inequality Database, which provides each country's total net wealth in local currency. To convert each country's wealth into US dollars, we multiply the total net wealth by the corresponding exchange rate. Columns (1) to (4) of Table 2 present the results.

Examining Table 2, we see that increases in innovation are associated with an increase in US wealth relative to foreign countries. A one standard deviation increase in US innovation is associated with a 3 log point increase in the share of US wealth relative to foreign countries. Overall, we see that an increase in US innovation is associated with a net transfer of wealth from foreign countries

to the US. Put differently, the gains from US innovation accrue disproportionately more to US rather than foreign households. This fact is consistent with incomplete markets—imperfect risk sharing across foreign and US investors.

Second, we examine the extent to which US innovation is associated with reallocation of wealth within a country. Given the difficulty of measuring wealth inequality across countries, we focus on income inequality instead. We examine the response of country level income inequality to US Innovation. Specifically, we re-estimate equation (2), but now replace the dependent variable with the growth in the income inequality in the United States relative to foreign countries. As before, estimating this relation using a panel specification allows to parcel out common movements in inequality across countries. Columns (5) through (8) of Table 2 report our estimates. We see that an increase in the rate of innovation in the United States is also associated with an increase in income inequality, which again indicates imperfect risk sharing, but now within rather than simply across countries.

1.4 Innovation and Capital Flows

In a world where countries share the benefits of US innovation, the arrival of innovation shocks in the US should translate into capital flows from the US to the rest of the world. By contrast, in a world where innovation shocks are imperfectly shared, we would expect to see a capital inflow from the rest of the world to the US as US firms become more innovative.

We first examine the relationship between US innovation, US foreign direct investment (FDI) inflows, and US portfolio equity inflows at the aggregate level. Panel A of Figure 2 shows the trends in US innovation intensity and FDI inflows over time. The correlation between US innovation and FDI inflows is approximately equal to 31 percent. US innovation experienced a significant boom around 2000, driven by advancements in internet technology, and this period was associated with a large increase in FDI inflows. Panel B plots the relationship between US innovation and aggregate portfolio equity flows. We observe a significant positive correlation between the two time series (approximately 28 percent). Moreover, periods of higher innovation intensity are associated with increased portfolio equity inflows.

A key advantage of our innovation measure is its granularity: we can observe the value of innovation performed by a given firm in a given year. This granularity allows for a sharper test of our hypothesis: does foreign capital flow into innovative firms? To explore this, we next shift our focus to portfolio equity flows at the firm level. We then examine how foreign institutional capital responds to firm-level innovation shocks,

$$\Delta \text{ForeignInstOwn}_{i,t+1} = \beta \log(\text{inno})_{i,t} + \gamma \mathbf{Z}_{i,t} + \varepsilon_{i,t+1} \quad (3)$$

Where the dependent variable is the change in foreign institutional ownership for firm i between t

and $t + 1$. The foreign institutional ownership data are from the FactSet Lionshare database.

The main independent variable is the (log) number of important patents granted to firm i in the previous year t , according to three innovation measures to capture patent quality. First, we adjust the number of patents based on their number of forward citations. Second, we adjust patents for their economic value, according to [Kogan et al. \(2017\)](#). Breakthrough patents are defined as being in the top 20% based on citations or economic value in each year. Lastly, we use the patent breakthrough series of [Kelly, Papanikolaou, Seru, and Taddy \(2021\)](#).⁷ When *inno* is equal to zero, we replace $\log(\textit{inno})$ by zero and add a dummy equal to one if *inno* is equal to 0, thereby preventing the removal of the observation from the data. The vector of controls $X_{i,t}$ includes foreign institutional ownership at time t , along with firm and year fixed effects. We also estimate a specification where we add the firm’s sales (log) and size at time t as additional controls. The sample covers 2000-2017, allowing patents a five-year window for citation accumulation.

Table 3 presents the estimated coefficient β from equation (3). Examining the table, we see that foreign institutional ownership increases after firms are granted novel patents. In terms of magnitude, a one-standard deviation in important patent grants is associated with approximately a 0.2 percentage point increase in the ownership of foreign institutional investors. Moreover, these estimates remain robust after controlling for firm size and revenue (Columns 4 to 6 of Table 3), as well as for foreign institutions’ time-varying preferences for specific sectors (Columns 7 to 9 of Table 3). These findings are consistent with the notion that technological innovations attract international equity capital flows. The heightened demand for US stocks, which embody frontier technology, drives up the demand for dollars.

2 Model

So far, we have documented that increases in the rate of innovation in the US result in appreciation of the dollar and capital inflows, together with an increase in the wealth share of the US and income inequality. The response of capital flows, relative wealth and income inequality strongly suggest that an incomplete market for innovation shocks is a key ingredient. That is, it appears to be the case that households, either in the home or foreign country, cannot purchase Arrow-Debreau securities whose payoff is contingent on the realization of innovation shocks. If households cannot commit ex-ante to sharing the gains from innovation with others, then the arrival of innovation shocks will coincide with wealth reallocation, both within but also across countries, consistent with our findings in the previous section.

Why would innovation shocks lead to a dollar appreciation? To fix ideas, consider first a closed economy. If innovation shocks are imperfectly shared across households within the innovative

⁷[Kelly et al. \(2021\)](#) use textual analysis to identify significant novel patents; we define breakthrough patents as those that are in the top 20% of the distribution in terms of their backward and forward similarities, using a five-year window.

country, then there is a wedge between the average marginal utility of individual households, and the marginal utility of a representative agent that consumes the aggregate endowment (Kogan et al., 2020; Gârleanu et al., 2016). Thus, it is possible that higher innovation increases both aggregate consumption but also leads to higher average marginal utility across households. Since the absence of arbitrage implies that the real exchange rate is related to the ratio of marginal utilities across countries, this mechanism could rationalize why innovation shocks in the US lead to an appreciation of the dollar.

In what follows, we embed this mechanism in a two country model that introduces a minimal departure from the standard models. Households in both the foreign and home country can trade goods and can invest in stocks and bonds issued by firms in either country. The only market incompleteness is that the gains from innovation are imperfectly shared: part of the benefits of innovation accrue to new firms, which are randomly allocated to a small fraction of existing households in the economy. Once these new firms enter the market, other households (domestic or foreign) purchase shares as part of holding a diversified portfolio. To simplify the exposition, we start with a simple endowment economy model that provides analytic expressions. Section 2.5 provides explicit microfoundations for our key assumptions. Section 3 embeds our mechanism in a more general model that can be calibrated to quantitatively fit the data.

2.1 Setup

The economy consists of two countries, home (H) and foreign (F), and two goods, Y_H and Y_F produced by each country. Time is discrete and is indexed by t .

Production

There is a continuum of firms in each country that produce output. Firms in each respective country only produce the local good. That is, the firms in the home country only produce the Y_H good, while foreign firms only produce the Y_F good. There is an expanding measure of firms in each country, indexed by (i, s, c) where s denotes the date at which the firm is created, $i \in [0, 1]$ denotes the index of the firm within its cohort in each country, and $c \in \{H, F\}$ denotes the country.

A firm characterized by (i, s, c) produces a flow of output $y_{t,s}^{i,c}$ at time t , according to

$$y_{t,s}^{i,c} = a_{t,s}^{i,c} Y_{c,t} \quad (4)$$

Here, $a_{t,s}^{i,c} \in [0, 1]$ denotes the fraction of aggregate output accruing to a firm i located in country c . By construction, these shares add to one

$$\sum_{s \leq t} \left(\int_{i \in [0,1]} a_{t,s}^{i,c} di \right) = 1, \quad c \in \{H, F\} \quad (5)$$

The model has an element of creative destruction, in which new productive units displace existing ones. We model this in reduced form, following [Gârleanu et al. \(2016\)](#), but provide an explicit microfoundation in Appendix B.2. Each period a new set of firms arrive exogenously in each country. These new firms, indexed by $i \in [0, 1]$, are heterogeneous in their productivity. The productivity of a newly arriving firm i in country $c \in \{H, F\}$ satisfies

$$a_{t,t}^{i,c} = (1 - e^{-u_t^c}) dL_t^{i,c} \quad (6)$$

where u_t^H, u_t^F are random, non-negative, shocks in home and foreign countries, affecting all firms in each country at time t . The components $L_t^{i,H}, L_t^{i,F}$ denotes cross-sectional measures and its increment $dL_t^{i,H}, dL_t^{i,F}$ are random, non-negative, idiosyncratic productivity components, which are determined at time t and satisfies $\int_{i \in [0,1]} dL_t^{i,H} = 1$ and $\int_{i \in [0,1]} dL_t^{i,F} = 1$. It follows that the total output produced by the cohort of firms born at time t is equal to

$$\int_{i \in [0,1]} y_{t,t}^{i,c} = (1 - e^{-u_t^c}) Y_{c,t}. \quad (7)$$

Examining (7), we see that the innovation shocks u_t^c reallocate revenue from incumbents to new entrants. Hence, we will also term them displacement shocks throughout the paper. Collectively, the fraction of output produced by existing firms in country c is $e^{-u_t^c}$. Specifically, the output share of an incumbent firm created at a time $s < t$ in country $c \in \{H, F\}$ is given by

$$a_{t,s}^{i,c} = a_{s,s}^{i,c} \exp \left(- \sum_{n=s+1}^t u_n^c \right) \quad (8)$$

Aggregate output in each country evolves according to

$$\Delta \log Y_{c,t+1} = \mu + \varepsilon_{t+1}^c + \delta u_{t+1}^c. \quad (9)$$

Note that output in each country c is driven by two country-specific shocks, ε and u . The first shock, ε , affects the output (and dividends) of all firms symmetrically. The second shock, u , is the innovation, or displacement, shock discussed above, which reallocates market share from incumbents to new firms. We allow this shock to affect aggregate output and parameterize its impact by $\delta \in (0, 1)$. Importantly, equation (9) can be explicitly derived from a relatively standard quality ladder model of endogenous growth. We provide such a microfoundation in Section 2.5 below.

Households

Each country is populated by a unit measure of infinitely-lived agents, indexed by (i, c) where $i \in [0, 1]$ and $c \in \{H, F\}$ denotes their country. At time zero, households are equally endowed with all firms in existence at that time. Households have access to the financial market and maximize

their expected utility of consumption

$$U_{i,t}^c = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \log(C_{i,s}^c). \quad (10)$$

Household consumption C_t^c is an aggregate of the two goods produced by the home country and the foreign country. Importantly, households exhibit ‘home bias’, that is, they tilt their consumption basket to the domestically produced good. That is, the consumption basket of each household living in country $c \in \{H, F\}$ at date t is given by

$$C_t^H = (x_{H,t}^H)^\alpha (x_{F,t}^H)^{1-\alpha}, \quad C_t^F = (x_{H,t}^F)^{1-\alpha} (x_{F,t}^F)^\alpha \quad \text{with } \alpha \in \left(\frac{1}{2}, 1\right) \quad (11)$$

Here, $x_{H,t}^c$ and $x_{F,t}^c$ denote the consumption of the home and foreign good by country $c \in \{H, F\}$ at date t . The parameter $\alpha \in (\frac{1}{2}, 1)$ captures the degree of home bias in household preferences.

Households have finite lives: at each date t , a mass ξ of agents, chosen randomly, die, and a mass of ξ of new agents are born, so that the population remains constant. As in [Blanchard \(1985\)](#), households can hedge their mortality risk using a competitive annuity market. Households are risk averse, hence they all purchase annuities. The annuity issuer collects the wealth of deceased households ξW and distributes the proceeds to the surviving population and the newly born agents.

Last, we normalize the price of the home consumption good (the numeraire) to one; hence,

$$\alpha p_{h,t} + (1 - \alpha) p_{f,t} = 1 \quad (12)$$

where $p_{h,t}$ and $p_{f,t}$ are the price of the home and foreign goods, respectively.

Creative Destruction and New Firms

Each period, households innovate with some probability. Successful innovation leads to the creation of new firms. The key feature of the model is that households cannot share this risk ex-ante, that is, they cannot sell claims against their future endowment of these new firms, as in [Kogan et al. \(2020\)](#). As a result, a shock to the relative profitability of new firms u leads to the redistribution of wealth from the owners of existing firms to the new entrepreneurs.

In particular, at time zero, agents are equally endowed with all the firms in existence at that time. From that point onward, agent (i, c) where $i \in [0, 1]$ and $c \in \{H, F\}$ receives firm (i, t, c) at time t , i.e., a new firm with productivity proportional to $a_{t,t}^{i,c}$. For tractability, we closely follow [Gârleanu et al. \(2016\)](#) and focus on the limiting case in which firm creation generates extreme inequality. Specifically, we assume that only a set of measure zero of firms manages to produce non-zero profits; by contrast, the vast majority of new firms are worthless.⁸ Consequently, when

⁸More formally, we assume that, for every t , the distribution of idiosyncratic productivity $dL_t^{i,c}$ consists exclusively of point masses. That is, we assume that L_t^c is a discrete measure on $[0, 1]$, so that it is an increasing, right-continuous,

making consumption and saving decisions, households attach zero probability to the event that they receive a profitable firm.⁹

Financial Markets

Households can trade a complete set of securities contingent on the realization of aggregate shocks. That is, they can trade equity claims on existing firms and risk-less, zero-net-supply bonds in either country. Effectively, consumers can trade a full set of contingent claims on the realizations of the displacement shocks (u^H, u^F) and the neutral shocks $(\varepsilon_H, \varepsilon_F)$ that drive output growth.

Importantly, however, a key market is missing: consumers cannot enter contracts that are contingent on the realized value of their future endowments of new firms. This market incompleteness is a key part of the mechanism, as it introduces a wedge between aggregate consumption growth and the marginal utility of the average investor.

2.2 Equilibrium

Our definition of equilibrium is standard. An equilibrium is a set of price processes, consumption choices, and asset allocations such that (a) consumers maximize expected utility over consumption and asset choices subject to their dynamic budget constraint, (b) all asset and goods markets clear.

Markets are incomplete, hence households' marginal utilities are not equalized across states. To solve for the competitive equilibrium, we construct a representative agent whose preferences are a weighted average of household utilities in each country

$$\max_{\{x_{c,t}^H, x_{c,t}^F\}} \sum_t \beta^t (\log C_t^H + \lambda_t \log C_t^F) \quad (13)$$

This representative agent maximizes her utility subject to the resource constraints,

$$x_{c,t}^H + x_{c,t}^F = Y_{c,t}, \quad c \in \{H, F\}, \quad (14)$$

along with the consumption aggregator in (11).

Importantly, the Pareto-Negishi weight λ_t is not a constant but depends on the state of the economy. In particular, λ_t is the time-varying ratio of marginal utilities of either good in the two countries at time t . It is equal to the wealth ratio between the two countries, and it varies over time

left-limits process that is constant on $[0, 1]$ except on a countable set, where it is discontinuous. Both the magnitudes of the jumps of L_t , and the locations of the points of discontinuity are random. This assumption ensures that only a set of measure zero of consumers obtains the profitable new firms.

⁹More precisely, what matters for household portfolio decisions is the probability of obtaining a new firm times the marginal utility of consumption in that state. Not only is the probability of receiving a new firm equal to zero, but also is the marginal utility of wealth (and consumption) since each firm is extremely valuable.

as the result of market incompleteness

$$\lambda_t = \frac{W_{F,t}}{W_{H,t}}, \quad W_{c,t} \equiv \int_{i \in [0,1],c} w_t^{i,c} \quad (15)$$

where $W_{c,t}$ is the total wealth of households in country $c \in \{H, F\}$.¹⁰

In equilibrium, the ratio of wealth λ_t between the foreign and the home country affects both real allocations as well as the terms of trade. For example, the relative price of the foreign good Y in units of the domestic good X is equal to

$$p_t \equiv \frac{p_{f,t}}{p_{h,t}} = \frac{Y_{H,t}}{Y_{F,t}} \frac{1 - \alpha + \alpha \lambda_t}{\alpha + (1 - \alpha) \lambda_t}, \quad (16)$$

and depends not only on aggregate quantities $Y_{c,t}$, but also on the countries' relative wealth λ_t .

2.3 Displacement Risk and the SDF

The presence of displacement risk introduces a wedge between aggregate consumption growth and the stochastic discount factor. To understand why this is the case, note that, because of incomplete markets, the marginal utility of the 'representative' household is not only determined by aggregate consumption, but also by the realization of the displacement shock.

To see this, consider the following simplified version of the model, in which a) households have extreme home bias preferences $\alpha = 1$ (or equivalently, the single-country version of the model) and b) the value of all new firms is equally and randomly allocated to a measure π of the population. In this case, we can divide all households at each point in time into two groups, those that receive profitable new firms and those that do not. Agents have a constant consumption to wealth ratio, hence their consumption process is directly linked to the dividends of the firms they own. Hence, the equilibrium stochastic discount factor can be written as

$$\frac{M_{t+1}^c}{M_t^c} = \beta \left(\frac{Y_{c,t+1}}{Y_{c,t}} \right)^{-1} \left((1 - \pi) e^{u_{t+1}^c} + \pi \left(\frac{1 - e^{-u_{t+1}^c}}{\pi} \right)^{-1} \right). \quad (17)$$

Recall that we have assumed that income inequality is extreme, that is, L_t^i is comprised of point masses or equivalently $\pi \rightarrow 0$. In this case, the expression for the SDF simplifies to

$$\frac{M_{t+1}^c}{M_t^c} = \beta \left(\frac{Y_{c,t+1}}{Y_{c,t}} \right)^{-1} e^{u_{t+1}^c}. \quad (18)$$

¹⁰Here, we note that even though households in both countries are heterogeneous in their wealth, consumption-wealth ratios are equalized within each country which facilitates aggregation. Hence, the representative consumer in each country solves the same optimization problem. That said, it is important to emphasize that even though we construct the preferences of each representative household as a function of the country-level consumption variables C_t^H and C_t^F , no household actually consumes that amount. Given our assumption, the effect of market incompleteness collapses into a scaling factor λ_t^c —and without loss of generality we have normalized $\lambda_t^H = 1$. See Appendix B.1 for more details.

Examining equation (18), we see that incomplete markets introduce a wedge between our stochastic discount factor and the one arising in a standard, Lucas-tree endowment economy. This additional term, given by $e^{u_{t+1}}$ adjusts for the fact that not all households experience the same growth rate in consumption; a set of measure zero experiences a dramatic increase as they receive new firms. Since marginal utility is a convex function of consumption, an increase in the dispersion of consumption growth raises the level of the stochastic discount factor, as in [Constantinides and Duffie \(1996\)](#).

Given the above, the dynamics of the stochastic discount factor in each country are given by

$$\frac{M_{t+1}^H}{M_t^H} = \beta \frac{C_t^H}{C_{t+1}^H} \frac{1}{b_{H,t+1}} \quad \text{and} \quad \frac{M_{t+1}^F}{M_t^F} = \beta \frac{C_t^F}{C_{t+1}^F} \frac{1}{b_{F,t+1}}, \quad (19)$$

where $b_{H,t+1}$ and $b_{F,t+1}$ are the wealth shares of the people in home and foreign country who did not receive profitable firms at $t + 1$,

$$b_{H,t+1} = \frac{\int_{i \in [0,1], a_{t+1,t+1}^{i,H}=0} w_{t+1}^{i,H}}{\int_{i \in [0,1]} w_t^{i,H}} \quad \text{and} \quad b_{F,t+1} = \frac{\int_{i \in [0,1], a_{t+1,t+1}^{i,F}=0} w_{t+1}^{i,F}}{\int_{i \in [0,1]} w_t^{i,F}} \quad (20)$$

The difference between (19) and equation (18) above is due to the fact that households own both domestic as well as foreign stocks, which implies that $b_{H,t+1}$ depends on both the domestic as well as the foreign displacement shocks u_H and u_F . That said, the relationship between b and u depends on the state of the economy, specifically, the relative wealth of the two countries, as captured by λ . For instance, when λ is high then country F is richer than country H . In this case, a small u_H shock will likely lead to a larger change in b_H than would be the case if country H were richer than F —since the new trees created in country H constitute a large share of wealth relative to the wealth of H households.

2.4 Exchange Rates

We next characterize the behavior of the real exchange rate in the model. Because financial markets are integrated between the two countries, the absence of arbitrage implies that the exchange rate is equal to the ratio of the two countries stochastic discount factors,

$$e_t = \frac{M_{t+1}^H}{M_{t+1}^F}. \quad (21)$$

The change in the exchange rate (in logs) can be written as

$$\begin{aligned} \Delta \log e_{t+1} &= \Delta \log C_t^F - \Delta \log C_t^H + \underbrace{\Delta \log W_{H,t+1} - \Delta \log W_{F,t+1}}_{-\Delta \lambda_{t+1}} \\ &= \Delta \log C_{t+1}^F - \Delta \log C_{t+1}^H + \log b_{F,t+1} - \log b_{H,t+1}. \end{aligned} \quad (22)$$

Equation (22) summarizes the main result in this paper. Given that households have log utility, if markets were complete, the ratio of home to foreign wealth in equation (15) would be a constant. In that case, bilateral exchange rate movements are purely determined by movements in the relative consumption growth between the home and foreign country. More generally in the case where the coefficient of relative risk aversion was different from one, the ratio of wealth could vary over time, but its movements would still be determined by movements in relative consumption growth (either in the short run or in the long run). As a result, these models imply that exchange rates are *counter-cyclical*: an economic boom in the home country (an increase in $Y_{H,t}$ and thus, due to home-bias, C_t^H) leads to a decline in e , that is, a *depreciation* of the home currency relative to the foreign currency.

By contrast, in our model, there is an additional factor in play that arises due to market incompleteness: displacement risk, which is captured by $b_{H,t+1}$ and $b_{F,t+1}$ defined in equation (20). To obtain some intuition for this result, we can approximate the evolution of λ_t in equation (15) around its long-run mean using a first-order Taylor expansion,

$$\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) = \Delta \log\left(\frac{W_{F,t+1}}{W_{H,t+1}}\right) = \log\left(\frac{b_{H,t+1}}{b_{F,t+1}}\right) \approx u_{t+1}^F - u_{t+1}^H. \quad (23)$$

See Appendix B.1 for more details on the derivation of (23).

Examining (23), we see that a positive realization of u_{t+1}^F implies that a measure-zero of households in the foreign country receive claims to new firms. Since there are no securities whose payoff is contingent on which household receives claims to these new firms, these households are not able to share the benefits ex-ante with the other households in either the home or foreign country. As a result, the relative wealth of the foreign country λ_t rises.

Overall, the log growth rate of the exchange rate can be approximated as

$$\begin{aligned} \Delta e_{t+1} &\approx \Delta c_{t+1}^F - \Delta c_{t+1}^H + u_{t+1}^H - u_{t+1}^F \\ &\approx \underbrace{(2\alpha - 1)(1 - \delta)}_{> 0} (u_{t+1}^H - u_{t+1}^F) + (1 - 2\alpha)(\varepsilon_{t+1}^H - \varepsilon_{t+1}^F). \end{aligned} \quad (24)$$

Consistent with the discussion so far, a positive displacement shock u_{t+1}^H will lead to an appreciation of the exchange rate, while a positive ‘neutral’ shock ε_{t+1}^H will cause the exchange rate to depreciate. Since country output and consumption depend on both shocks, exchange rates in the model can be either positively or negatively correlated with consumption or output growth.

Our model can generate pro-cyclical exchange rates. To see this, consider the log growth in the relative country output,

$$\Delta \log Y_{H,t+1} - \Delta \log Y_{F,t+1} = \delta (u_{t+1}^H - u_{t+1}^F) + \varepsilon^H - \varepsilon^F \quad (25)$$

which is increasing in both u_{t+1}^H and ε_{t+1}^H . Similarly, the growth in relative consumption can be written as

$$\Delta c_{t+1}^H - \Delta c_{t+1}^F \approx (1 - 2\alpha) \left((1 + \delta - 2\alpha) (u_{t+1}^H - u_{t+1}^F) - (\varepsilon_{t+1}^H - \varepsilon_{t+1}^F) \right). \quad (26)$$

In what follows, we will assume that,

$$\delta < 2\alpha - 1, \quad (27)$$

which implies that the aggregate consumption growth in the home country is positively correlated with the displacement shock in that country, u_{t+1}^H .

In brief, we see that the presence of the displacement shock u induces a positive correlation between exchange rates and the growth in aggregate consumption, output, and productivity. By contrast, the presence of the neutral shock ε tends to make exchange rates counter-cyclical, just like the standard models in the literature. As a result, the unconditional correlation (disconnect) between exchange rates, country output and consumption depends on model parameters, for instance, the relative variance of the two aggregate shocks.

2.5 A Microfoundation Based on a Model of Creative Destruction

Here, we provide a microfoundation for the model in the previous section based on [Aghion and Howitt \(1992\)](#). Firms compete on a fixed set of product lines. Innovation by new entrants takes the form of quality improvements in across different product lines. The displacement shock u drives the measure of varieties that experience innovation. The parameter δ is related to the size of the productivity improvement from each innovation. To conserve space, we briefly detail the outline of the model. Appendix [B.2](#) contains all details.

Output in country $c \in \{H, F\}$ is produced as a CES aggregate of a continuum of varieties indexed by j ,

$$Y_{c,t} = Z_{c,t} \left(\int_0^1 [x_{c,t}(j)]^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \quad (28)$$

where

$$\Delta \log Z_{c,t+1} = \mu + \varepsilon_{c,t+1} \quad (29)$$

is an exogenous productivity shock, and $x_{c,t}(j)$ denotes the quantity of variety j produced at time t , and σ is the elasticity of substitution across varieties. Intermediate goods are produced using a linear technology with land as the only input:

$$x_{c,t}(j) = A_{c,t}(j) l_{c,t}(j), \quad (30)$$

where $A_{c,t}(j)$ denotes the productivity of the leading producer in variety j and $l_{c,t}(j)$ is the land

input used in production. Land is in fixed supply,

$$\int_0^1 l_{c,t}(j) = 1. \quad (31)$$

Claims on land are traded in international markets, and both foreign and domestic households can own these claims.

At the beginning of each period t , a displacement (innovation) shock $u_{c,t}$ is realized in each country. This displacement shock raises the probability that each product line is challenged by a new entrant,

$$m_{c,t} = 1 - e^{-u_{c,t}}. \quad (32)$$

If an entrant arrives at line j , the entrant has a productivity advantage: it can produce in variety j with productivity $e^\delta A_{c,t-1}(j)$, where $\delta > 0$ and $A_{c,t-1}(j)$ is the productivity of the incumbent firm. New entrants and incumbents compete under Bertrand competition. As a result, the entrant undercuts the incumbent and takes over the product line.

The assumptions above yield the key equations in the model above. First, note that, each period t , a fraction $m_{c,t}$ of incumbents are displaced and replaced by new entrants. In each displaced line, the old firm's profits fall to zero, and the entrant captures the full profit stream. Thus, the total fraction of output that is produced by new firms at time t is equal to $m_{c,t} Y_{c,t}$, which gives equation (7) above. Second, aggregate output in country c is given by

$$\begin{aligned} \Delta \log Y_{c,t+1} &= \mu + \varepsilon_{c,t+1} + \frac{1}{\sigma - 1} \log \left[e^{-u_{c,t+1}} + (1 - e^{-u_{c,t+1}}) e^{\delta(\sigma-1)} \right] \\ &\approx \mu + \varepsilon_{c,t+1} + \delta u_{c,t+1}, \end{aligned} \quad (33)$$

which is approximately equal to equation (9) as $u_{c,t+1}$ becomes small.

In brief, the setup illustrates how the innovation shock $u_{c,t+1}$ can serve to both increase aggregate output by δ , but also serve to displace incumbent firms by reallocating profits to existing firms to new entrants. This setup therefore captures the key intuition of [Aghion and Howitt \(1992\)](#), that economic growth takes place through a process of creative destruction. Here, note that, for simplicity, we have assumed that all innovation takes place by new entrants. This assumption is not crucial for the qualitative predictions of the model. As in [Huang et al. \(2023\)](#), we could also allow incumbent firms to innovate. This would imply that a fraction of the output $m_{c,t} Y_{c,t}$ accrues to incumbent firms, with the remainder to new entrants. As long as new entrants appropriate some of the benefits of innovation, and crucially, households cannot buy claims to future new firms, the main qualitative predictions of our setup remain.

2.6 Testable Predictions

Our model has predictions that are directly testable in the data. We explore these next.

Exchange rates and relative consumption growth

Our model has direct implications for the [Backus and Smith \(1993\)](#) puzzle. Importantly, equation (22) suggests that, the bilateral exchange rate should be a function of not only relative consumption growth between the two countries, but also relative wealth. Importantly, once we control for wealth fluctuations between countries, we should be able to observe a negative correlation between consumption growth and the exchange rate growth, as predicted by risk-sharing.

To explore this implication in the data, we estimate the response of the bilateral exchange rate between country c and the US using the following specification,

$$\Delta \log e_{F,t+1} = \beta_1 (g_{us,t+1} - g_{c,t+1}) + \beta_2 (R_{US,t+1} - R_{F,t+1}) + c \mathbf{Z}_t + \varepsilon_{F,t+1}. \quad (34)$$

Here, $g_{c,t+1}$ refers to consumption or output growth and $R_{c,t+1}$ is the change in log nominal wealth of the US or foreign country F in dollars. The vector of controls $\mathbf{Z}_t$ includes lagged dependent and independent variables: the level of lagged exchange rate, the lagged level of relative consumption or output, and the lagged level of relative wealth.

Panel A of Table 4 reports the estimated coefficients β_1 and β_2 from equation (34). Examining the table, we can see that the estimated slope coefficient β_2 is positive in both the individual country regressions and the panel regression. A positive estimate of β_2 implies that an increase in a country's relative wealth is associated with appreciation of its currency. More importantly, after controlling for changes in a country's relative wealth, the coefficient β_1 on consumption growth is both negative and statistically significant—which is consistent with theory. Focusing on the panel regression result, a one-standard deviation increase in consumption growth differentials is associated with a 2 log point depreciation of its currency against the dollar. Examining the results for individual countries, we note that the correlation is negative in 10 out of the 11 countries in the sample. Panel B of Table 4 shows that the estimates for output are similar. The estimated coefficient β_2 is always positive. In the panel regression, the estimates of β_1 on relative output is equal to -0.016 with a [Driscoll and Kraay \(1998\)](#) standard error of 0.004. When estimating (34) separately for each country, we again see that the point estimates β_1 on relative output growth are all negative.

The fact that the estimated coefficient β_2 is positive and statistically significant aligns with the findings in [Dahlquist et al. \(2023\)](#), who document that a country's currency appreciation typically increases the dollar value of its domestic assets, which results in an increase in its relative wealth. Importantly, a country's wealth denominated in local currency does not adjust enough to offset the movement from the nominal exchange rate. Consequently, fluctuations in the nominal exchange rate lead to significant changes in a country's wealth measured in dollars, which is thus reflected in the high R-squared values when estimating (34), which range from 78 to 95 percent. Naturally, this raises the question of whether the high R-squared values from estimating equation (34), simply reflect the low sensitivity of wealth in local currency to nominal exchange rates. That may indeed

be possible; however, a high correlation between nominal exchange rates and wealth ratios would not necessarily a negative estimate of β_1 .

In brief, after modifying the Backus-Smith specification, we show that, after conditioning on these imperfectly shared wealth fluctuations, there is a (re)connection between exchange rate and macroeconomic fundamentals. That is, the ratio of marginal utilities (the real exchange rate) appreciates when the relative fundamentals are weaker. This result complements recent work by [Aguiar, Itskhoki, and Mukhin \(2024\)](#), which shows that risk-sharing across countries is better than implied by the Backus-Smith correlation when analyzed through consumption allocation and trade shares, as these are less influenced by financial market imperfections. Unlike [Aguiar et al. \(2024\)](#), who use trade shares to circumvent imperfectly shared shocks and test risk-sharing through quantities, our method accounts for these imperfectly shared shocks by conditioning on relative wealth across countries.

A measure of displacement based on the value of new firms

Here, we construct a proxy for the displacement shock that follows directly from our theoretical model. Similar to [Gârleanu et al. \(2016\)](#), the displacement shock in our model is related to the difference between the aggregate market capitalization growth and the returns from holding the market portfolio. To see this, consider the value of existing firms in each country (the stock market) in country $c \in \{H, F\}$,

$$S_t^c = D_{c,t} + E_t \left[\frac{M_{t+1}^c}{M_t^c} \left(S_{t+1}^c e^{-u_{t+1}^c} \right) \right] = D_{c,t} \left(1 + pd_t^c \right), \quad (35)$$

where pd_t^c is the price-dividend ratio in country c , and

$$D_{c,t} \equiv p_{c,t} Y_{c,t} \quad (36)$$

are the total dividends (the value of output) in country c .

Given the above, the log return, in local currency and excluding dividends, of holding the market portfolio of country c is

$$\begin{aligned} r_{t+1}^c &= \log \left(\frac{D_{c,t+1}}{D_{c,t}} \frac{1 + pd_{t+1}^c}{pd_t^c} e^{-u_{t+1}^c} \right) - \log \left(\frac{p_{c,t+1}}{p_{c,t}} \right) \\ &= \mu + (\delta - 1) u_{t+1}^c + \varepsilon_{t+1}^c + \log \left(\frac{1 + pd_{t+1}^c}{pd_t^c} \right) \end{aligned} \quad (37)$$

Equation (37) highlights an important feature of our model: the distinction between aggregate dividend growth $p_{c,t} Y_{c,t}$ and the growth of dividends that accruing to incumbent firms (the market portfolio). The reason for this distinction is that aggregate dividends do not constitute the gains from holding the stock market: investing in the stock market at time t only generates $p_{c,t} Y_{c,t+1} e^{-u_{t+1}^c}$

dividends at $t + 1$. A positive displacement shock increases aggregate dividends by introducing new firms, but also dilutes the shares of the existing firms. On the other hand, following a positive displacement shock the price-dividend ratio also decreases. As a result, a positive displacement shock leads to a decline in the stock market returns of incumbent .

Equation (37) provides guidance on how to construct estimate the realizations of the displacement shock u from the difference between the aggregate market capitalization growth and the returns from holding the market portfolio. In particular, the difference between the growth in the aggregate market capitalization $S_{c,t}$ and the return of the market portfolio r_{t+1}^c can be written as

$$\log \left(\frac{D_{c,t+1}}{D_{c,t}} \frac{1 + pd_{t+1}^c}{pd_t^c} \right) - \log \left(\frac{D_{c,t+1}}{D_{c,t}} \frac{1 + pd_{t+1}^c}{pd_t^c} e^{-u_{t+1}^c} \right) = u_{t+1}^c \quad (38)$$

The difference between the growth of aggregate market capitalization and the returns on the market portfolio in (38) is equal to the innovation or displacement shock. This difference arises because an investor holding the market portfolio must pay to acquire new firms entering the market. To maintain the self-financing nature of the strategy, the investor must continually liquidate some of the shares she holds to purchase shares of new firms. Since new firms enter the market each period, the growth of the market portfolio falls short of the growth of the aggregate market capitalization.

We next use this result to construct a direct proxy for the displacement shock in the data. Each month, we calculate the value of a portfolio that holds the entire market, excluding all dividend payments but adjusted for stock splits (CRSP item RETX). We then compare this with the aggregate market capitalization of the same portfolio at the end of each month. The log difference between the two values is due to newly listed firms, and can be interpreted as the realization of the displacement shock u for that month. We aggregate all monthly displacement shocks over a year to obtain the annual displacement series. We aggregate these monthly displacement shocks over a year to construct the annual displacement series.¹¹

Figure 3 plots the displacement shock against the growth of the real US dollar index. We observe that the correlation between the displacement shocks and the US dollar index is positive and approximately equal to 35 percent. Notably, during periods of significant innovation, many new firms emerge, and these periods coincide with periods that the dollar appreciates in real terms.

Using this direct proxy for the displacement shock, we revisit our motivating evidence in Sections 1.2 and 1.3. As we see in Tables 5 and 6 we obtain qualitatively and quantitatively similar estimates as in Tables 1 to 2. In particular, a one-standard deviation increase in our displacement measure leads to a 3.6 to 4.4 percentage point appreciation in the dollar, a 4.9 to 5.3 percentage

¹¹Here, there are two issues that are related to timing that need to be discussed. First, it is possible that the actual displacement occurs when the new firms are created, rather than when the firms are listed in the stock market. If that is the case, then the displacement shock that we are measuring is likely to be a moving average of the true displacement shocks in the past. In this case, it would make sense to include as controls lags of the displacement series. That said, given that shares become liquid only after the IPO, it is also possible that the reallocation in wealth is realized then. However, these wealth effects may be delayed if there are lock-up periods post IPO. Thus, to account for the possibility of a lock-up period, we lag the annual displacement series by one year.

point increase in the relative wealth of the US, and a 1.6 to 1.8 percentage point increase in income inequality in the US compared to the rest of the world. Appendix Table A.3 shows that the estimated correlation between our displacement shocks and the bilateral dollar exchange rate are uniformly positive across all individual countries, and, in most cases, statistically different from zero.

Exchange rates and the growth of top incomes

Our notion of the displacement shock u captures the idea that the benefits of economic growth are not shared equally. In the model, u directly affects the wealth reallocation between owners of existing firms and entrepreneurs who create new firms. These entrepreneurs consist of a very small fraction of the population (measure zero), but they receive a significant share of overall output. If we were to treat this transfer as capital income, fluctuations in u would translate into fluctuations into income inequality in the model.

Here, we develop this idea further and connect the displacement shock u in the model to an observable quantity, the top 1% share of income. In particular, the top 1% income consists of two groups of households. The first group consists of the households that receive new firms in the current period. The total capital income from new firms in country c is given by

$$I_{c,t}^{ent} = S_t^c (1 - e^{-u_t^c}). \quad (39)$$

The second group consists of households that have had previously received new firms in the past and consequently earn a large capital income on from their wealth. These households derive capital income equal to

$$I_{c,t}^{inc} = \xi W_{c,t} + \left(D_{H,t} e^{-u_t^H} + D_{F,t} e^{-u_t^F} \right) \frac{W'_{c,t}}{W'_{H,t} + W'_{F,t}}, \quad (40)$$

where $W'_{H,t}$ and $W'_{F,t}$ are the total wealth of the two countries excluding new projects,

$$W'_{c,t} = W_{c,t} - (1 - e^{-u_t^c}) S_t^c, \quad c \in (H, F).$$

These capital gains in (40) are distributed to all households in direct proportion to their wealth.

Contrasting equations (39) and (40), we see that the size of the u shock at time t determines the amount of wealth that is transferred from the shareholders of existing firms to the owners of new firms. The value of these new firms constitutes a capital gain for the successful entrepreneurs, and they are randomly distributed to a small part of the population. Hence, some of it is part of the income share of the top 1%.

The above discussion illustrates how these model-implied joint dynamics of income inequality and exchange rates are informative about the significance of the model's mechanism in the data. In particular, recall equation (22), which states that exchange rate growth is determined by relative consumption growth and changes in the wealth share of households that are displaced in each

country b_H and b_F , which are primarily driven by the displacement shock u_H and u_F , respectively. To the extent that income inequality is a useful proxy for the u shock in the model, the correlation between exchange rates and income inequality would reveal the importance of the displacement shock u as a driver of exchange rates.

To explore this idea further, we estimate the following specification,

$$\Delta X_{t+1,F} = \beta \left(\log \frac{\theta_{t+1}^{US}}{\theta_t^{US}} - \log \frac{\theta_{t+1}^F}{\theta_t^F} \right) + c \mathbf{Z}_{F,t} + \varepsilon_{F,t+1}. \quad (41)$$

Here, the dependent variable $X_{t,F}$ is equal to either the growth in the bilateral exchange rate between the US and country F . The independent variable θ_t^c is the level of income inequality in country c at time t . As our baseline case, we measure ψ using the income share of the top 1%. We control for the level of the exchange rate and the level of inequality at time t .

Table 7 presents the estimated coefficients from the panel regression (41), along with country-by-country estimates. Focusing on estimates of β from the panel specification in the first column, we see that the estimated coefficient β is positive and both economically and statistically significant. In particular, increases in income inequality in the foreign country are associated with an appreciation of its currency relative to the US. The magnitudes are not small: a one-standard deviation increase in income inequality in the foreign country is associated with a 1.7 log point appreciation of its currency relative to the US dollar. Examining the country-level regressions, we observe a consistent pattern: the estimated coefficients β are generally positive, though not always statistically significant.

3 Quantitative Implications

So far, we have presented a stylized model that allows us to highlight the key mechanism in the paper. Though transparent, however, the model is not rich enough to quantitatively capture all the interesting aspects of the data. Here, we introduce several additional features that allow for a full quantitative exploration of the mechanism.

3.1 Modifications to the Baseline Model

To conserve space, we only highlight the differences with the simpler model in the previous section. Appendix B.3 contains all details and derivations.

We make two changes relative to the previous setup. First, we modify household preferences. Agents have non-time separable preferences; in addition, they care about both their own absolute level of consumption but also their consumption relative to an index. In particular, households'

continuation utility at time t is of the Epstein-Zin form,

$$U_{i,t}^c = \left((1 - \beta)(\hat{C}_{i,t}^c)^{1-\frac{1}{\psi}} + \beta \mathbb{E}_t[(U_{i,t+1}^c)^{1-\gamma}]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \quad (42)$$

The parameters γ and ψ measure the relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS), respectively. The coefficient β is the effective time-preference parameter, which also incorporates the probability of death, that is, $\beta = \tilde{\beta}(1 - \xi)$ where ξ is the probability of death and $\tilde{\beta}$ is the households' subjective time discount factor.

In equation (42), we allow for preferences over relative consumption: $\hat{C}_{i,t}^c$ refers to a composite good that depends both on the households' own consumption $C_{i,t}^c$ but also its level relative to the aggregate level $\bar{C}_t^c$ of consumption in their country,

$$\hat{C}_{i,t}^c = (C_{i,t}^c)^h \left(\frac{C_{i,t}^c}{\bar{C}_t^c} \right)^{1-h}. \quad (43)$$

Here, $C_{i,t}^c$ is the agent i 's own consumption bundle in country $c \in \{H, F\}$ —defined in (11)—which is comprised of both home and foreign goods. The parameter h denotes the strength of the relative preference effect. When $h = 1$, these preferences specialize to the standard Epstein-Zin preferences. In general, for $h \in [0, 1]$ agents place a weight h on their own consumption and a weight $1 - h$ on their consumption relative to average consumption in country $c \in \{H, F\}$.

Second, we relax the assumption of extreme inequality, by assuming that the measure of population that receives the value of new firms is non-negligible, that is, $\pi > 0$. Though this modification makes the model significantly less tractable, it helps the model match the observed patterns of inequality in the data.

Given our assumptions, the stochastic discount factor in country c is now given by

$$\frac{M_{t+1}^c}{M_t^c} = \beta \left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}} \right)^{-\frac{h}{\psi} + h - 1} \tilde{b}_{c,t+1} \left(\frac{U_{c,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{c,t+1}^{1-\gamma}]} \right)^{\frac{1/\psi - \gamma}{1-\gamma}} \quad (44)$$

where now the share of wealth of incumbent households at $t + 1$ is given by

$$\tilde{b}_{c,t+1} = \pi \left(\frac{1 - (1 - \pi) b_{c,t+1}}{\pi} \right)^{-\frac{1}{\psi} + \frac{1/\psi - \gamma}{1-\gamma}} + (1 - \pi) b_{c,t+1}^{-\frac{1}{\psi} + \frac{1/\psi - \gamma}{1-\gamma}}, \quad (45)$$

where $b_{c,t+1}$ is given by equation (20).

As before, exchange rates are determined by the absence of arbitrage (21). The exchange rate

between the home and foreign country is now given by

$$\begin{aligned} \Delta \log e_{t+1} = & \left(\frac{h}{\psi} + 1 - h \right) (\Delta \log C_{t+1}^F - \Delta \log C_{t+1}^H) + \left(\log(\tilde{b}_{F,t+1}) - \log(\tilde{b}_{H,t+1}) \right) \\ & + \frac{1/\psi - \gamma}{1 - \gamma} \left(\log \frac{U_{H,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{H,t+1}^{1-\gamma}]} - \log \frac{U_{F,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{F,t+1}^{1-\gamma}]} \right) \end{aligned} \quad (46)$$

Examining equation (46), we note the similarities with the log utility case—equation (22) in the baseline model. Specifically, exchange rate dynamics are still driven by relative consumption growth in the two countries, as well as the relative degree of displacement in the current period given by equation (45). The key difference with the baseline case with log utility is that, the shocks to the future distribution of these variables matters also matter for exchange rate dynamics, which are reflected in households' continuation utilities $U_{H,t+1}$ and $U_{F,t+1}$.

3.2 Calibration

In this section, we describe how we calibrate the model to the data. Given the degree of non-linearity in our model, solution methods that are based on log-linearizations around the steady state are not necessarily reliable. As such, we solve for the global solution of the model by discretizing the state-space and using a combination of value and policy function iteration. See Appendix B.4 for a brief description of our numerical procedure.

Aggregate Shocks

First, we make distributional assumptions about the shocks driving output dynamics. We allow for the displacement shocks in each country to be correlated, possibly due to technology spillovers. That is, the effective displacement shock u_t^c in each country is a weighted average of each country's idiosyncratic displacement shock $\bar{u}$,

$$\begin{aligned} u_{t+1}^H &= (1 - \rho_u) \bar{u}_{t+1}^H + \rho_u \bar{u}_{t+1}^F \\ u_{t+1}^F &= (1 - \rho_u) \bar{u}_{t+1}^F + \rho_u \bar{u}_{t+1}^H. \end{aligned} \quad (47)$$

We parameterize the distribution of the idiosyncratic displacement shocks in each country $\bar{u}_t^c, c \in \{H, F\}$ as a Markov chain with three states $[u_1, u_2, u_3]$ and transition matrix T

To reduce the number of parameters, we make simplifying restrictions on the dynamics of u shocks. Specifically, we assume that the matrix T corresponds to the transition matrix of a discretized AR(1) process, so that it could be parameterized by only two parameters p and q , such that

$$T = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix}. \quad (48)$$

Second, even though we allow for three possible states for u , we calibrate the model so that $u_2 \approx u_1$. Therefore, a transition from u_1 to u_2 mostly affects the future distribution of u (as the transition probabilities change), rather than the current level of displacement. We interpret the transition from u_1 to u_2 as representing the early stages of a potential technological revolution, in which the benefits have yet to materialize.

Last, we assume that the neutral shocks are i.i.d. and jointly normally distributed $[\varepsilon^h, \varepsilon^f]$ with standard deviation σ_e . We allow for the correlation ρ_e between the neutral shocks in each country to be positive.

Parameter Choice

The model has a total of 16 parameters. We choose the probability of household death $\xi = 1/40$, which corresponds to an average working life of 40 years. To conserve space, we briefly describe the calibration procedure here. Appendix B.5 contains all details.

To calibrate the model we choose a set of target moments, reported in the first column of Table 8. Our list of calibration targets includes a combination of first and second moments of aggregate quantities, asset prices and exchange rates. In addition, we also target the comovement of key variables that are informative about the relative importance of the two shocks in the model. Specifically, the relative importance of the neutral and the displacement shocks are important for the cyclicity of exchange rates and relative wealth ratios. Thus, we target the degree of comovement between wealth changes and consumption, and output. We also target the correlation between dollar index growth and our model-implied proxy for displacement shocks in Section 2.6 and presented in Table 5 and 6. To account for leverage, we consider the stock market as a levered claim on a claim on domestic consumption by a factor of two.

Table 9 reports our choice of parameters. In terms of preference parameters, the calibration requires a moderate level of relative risk aversion (6.2) and an elasticity of inter-temporal substitution (1.6). The coefficient of relative risk aversion γ is identified from the mean and volatility of stock returns, as well as the volatility of the risk-free rate. Similarly, the EIS affects the volatility of interest rates and hence is primarily identified by the volatility of excess returns. In line with most equilibrium models of exchange rates (for instance, Colacito and Croce, 2013), the model requires a very high level of home bias (0.989) in order to generate volatile exchange rates. In addition, the preference weight on relative consumption is moderately high (0.81), which is in line with Kogan et al. (2020). This parameter is identified by the volatility of exchange rate and the correlation between wealth and consumption growth. As h falls, households place a higher emphasis on relative consumption, thereby increasing the significance of the displacement shock. The subjective discount factor β is mainly identified by the mean of the risk-free rate.

In terms of the parameters of technology shocks, the mean $\mu = 0.012$ and volatility $\sigma_e = 0.013$ of the neutral shock is identified by the first two moments of consumption and output growth. The

distribution of the displacement shock u is primarily identified by the volatility of exchange rate; the stock market (since the spread between u_1 and u_3 affects the volatility of the SDF in the model); and the correlation of u with wealth changes and exchange rates growth. The two parameters governing the correlation between the home/foreign shocks (ρ_e and ρ_u) are primarily identified by the correlation of home and foreign consumption growth, output, and the stock market. The persistence of the displacement shock u is primarily identified by the equity premium and volatility of stock returns, since the u shock is a key driver of stock returns. In addition, its level of skewness helps the model replicate the failure of the UIP in the data. Last, the parameter driving the spillover of displacement shocks to aggregate output δ is primarily identified by the correlation between wealth and output, and the volatility of exchange rates, since it determines the joint dynamics of the SDF and output growth.

3.3 Model Fit

Table 8 shows that the baseline model fits data reasonably well. Most of the empirical moments are close to their model counterparts and fall within the 5th to 95th intervals from simulations. Our model reproduces the realist patterns of both aggregate consumption and output growth. On the asset pricing side, the model generates the realistic levels of equity risk premium and volatility of the stock market. The volatility of the realized interest rate in the data is more volatile than the simulated data, but this may be largely driven by the high inflation around 1980s.

On the international side, our model successfully replicates the three key anomalies in the literature: the volatility puzzle of [Brandt et al. \(2006\)](#), the Backus-Smith correlation puzzle, and the violation of the UIP. Moreover, the model generates positive correlation between wealth changes and consumption. This is because the u shock is not only positively correlated to the aggregate consumption and output, and is also associated with significant wealth transfer due to imperfect risk sharing. That is, our model can replicate the pattern in the data where shocks that drive up wealth ratios are positive supply shocks. The key to the replication of the UIP anomaly is the time-varying volatility—more precisely, the time-varying distribution of the effective size of the u -shock—that endogenously arises in the equilibrium. Despite the fact that consumption, output and stock market are highly correlated, the exchanges rate in our model is as volatile as in the data due to a high level of home-bias. Finally, net exports in our model are counter-cyclical, as in the data.

In addition, the replication of international puzzles does not require a unrealistic magnitude of displacement shocks. Targeting the moments of income inequality helps impose some discipline in the calibration of the displacement shocks. As we can see in Table 8, our model generates a realistic level of income inequality. The correlation between wealth and inequality in the data falls within the 5th and 95th percentiles of the simulation intervals. Furthermore, our model reproduces the estimated coefficients of the bivariate regression of exchange rate growth on wealth changes, consumption, and output in Table 4. That is, the model can quantitatively replicate the observed

joint dynamics between exchange rates, wealth, and consumption or output growth.

4 Model Mechanisms

Here, we focus on the key mechanisms in the model and their implications for the data.

4.1 The Backus-Smith Puzzle

Figure 4 presents the response of key model quantities to the two shocks in the model. Panel A shows the response to the displacement shock u , and Panel B shows the response to the neutral shock ε . The first two columns of Figure 4 show the response of the exchange rate and consumption growth to the two shocks. As we can see, a positive ε shock in the home country leads to a depreciation of the currency and an increase in consumption growth. This is the standard shock in most models and the reason why exchange rates are counter-cyclical. By contrast, we see that a positive u shock leads to an appreciation of the exchange rate as well as an increase in consumption growth.

The next two columns of Figure 4 illustrate why the exchange rate appreciates in response to a positive u shock. Columns three and four of Figure 4 illustrate how the last two terms of equation (46) respond to the shocks in the model. We see that an increase in the level of displacement in the home country u_H leads to a decline in the wealth share of the owners of incumbent firms in the home country b_H . In addition, we see that this increase in u_H leads to a decline in the continuation utility of households in the home country U_H . Both of these forces lead the home currency to appreciate, since they both lead to an increase in the level of the stochastic discount factor (the equivalent of marginal utility in the log utility case) as we see in equation (44).

This figure illustrates why the model is able to generate a positive correlation between countries' differences in consumption growth, and exchange rate growth, resolving the Backus-Smith anomaly. The displacement shocks produce a positive comovement between consumption and exchange rate, while neutral shocks generates a negative correlation between two variables. To resolve the Backus-Smith puzzle, it needs to be that the impact of displacement shock in driving this correlation is larger than the impact of the neutral shock. This magnitude depends on the calibration of its displacement effect δ and households' preference h over relative consumption, as well as the relative magnitude between two shocks.

4.2 Productivity Shocks and Exchange Rates

Our model delivers a positive correlation between exchange rates and future productivity growth, consistent with the evidence in Chahrouh et al. (2024). In particular, Chahrouh et al. (2024) revisit the exchange rate disconnect puzzle using a vector auto-regression to argue that news about future US productivity drives exchange rate movements. They show that shocks to expectations about future US productivity explain a large fraction of the variation in both exchange rates and real

macroeconomic quantities, though at different horizons. Despite both focusing on medium-to-long-term productivity growth, these authors argue that two-country models with long-run productivity shocks (for instance, [Colacito and Croce, 2013](#)) are at odds with their findings.

By contrast, our model is consistent with the evidence in [Chahrour et al. \(2024\)](#). The reason is that the displacement shock u generates a positive correlation between the exchange rate and subsequent growth in consumption and output, as we saw in Panel A of Figure 4. To the extent that the displacement shock is persistent, an increase in the displacement shock can also increase the likelihood of future displacement shocks, which will be reflected in increases in future output and productivity. Put differently, the displacement shock u in the model is also a driver of future productivity growth. To see this, we replicate the VAR analysis in [Chahrour et al. \(2024\)](#) in simulated data from the model.¹²

Figure 5 shows that our model is quantitatively consistent with these facts. In particular, we plot the median impulse response across all simulations (black line) along with the empirical impulse response and 90th percent confidence intervals from [Chahrour et al. \(2024\)](#). Examining the figure, we see that, in both the model and the data, the main FX shock identified by the VAR is strongly positively related to subsequent productivity and consumption growth.

4.3 The Forward Premium Anomaly

The Uncovered interest rate parity (UIP) states that the expected change in exchange rates should be equal to the interest rate differential between two countries, and that the currency with lower interest rate tends to appreciate. Therefore, the regression coefficient of future exchange rates growth on interest rate differential should be equal to one. Empirically, the coefficient is much smaller than one and even negative, which is referred to as the forward premium puzzle ([Fama, 1984](#)).

We next show that in our model the failure of the UIP is an endogenous equilibrium outcome in our model. In particular, we next estimate the standard UIP regression in simulated data.¹³ Figure 9 displays the distribution of UIP coefficients. Examining the figure, we observe that UIP is largely violated in the model: the sample average of the UIP coefficient closely aligns with its counterpart in the data. To illustrate why UIP fails in our model, Figure 6 shows the responses of home and foreign interest rate differentials, the exchange rate, and the difference in volatility of the log-SDF to positive displacement shocks. The top panel of Figure 6 shows the response when the economy

¹²Specifically, we consider a vector of variables that includes the real exchange rate, home consumption, foreign consumption, home TFP, and interest rate differentials between the home and foreign countries. We estimate the VAR with one lag. Following [Chahrour et al. \(2024\)](#), we apply the methodology in [Uhlig \(2004\)](#) to extract the “main exchange rate” (main FX) shock, which accounts for the largest share of variation in the real exchange rate. We then calculate the impulse response functions of the real exchange rate, consumption, and productivity (i.e. output) based on the model’s simulation. Specifically, we simulate the model for 100 periods, estimate the impulse response, and repeat the process 10,000 times.

¹³We initialize the model at the symmetric steady state and is simulated for 150 periods, repeated 10000 times. We use the last 50 periods for each simulated sample to perform the UIP regression.

transitions from u_1 to u_3 , while the lower panel shows the response when the economy transitions from u_1 to u_2 .

In the first column of Figure 6, we see that following a positive displacement shock u to the home country, the home interest rate falls relative to the foreign country. If UIP were to hold, this would imply an appreciation of the home currency in the future—which is indicated by the dotted red line in the second column. However, we see in the second column of the figure that the home currency depreciates in the near future in response to the shock. The reason why this pattern occurs in the model is due to time variation in the higher order moments of consumption growth in equilibrium. Specifically, the third column of Figure 6 shows that after the transitions $u_1 \rightarrow u_3$ and $u_1 \rightarrow u_2$, the volatility of the domestic log-SDF is higher than that of the foreign SDF in the subsequent periods. This greater volatility, which represents an increase in the market price of risk, causes the interest rate in the home country to decline.¹⁴

Replicating the failure of UIP has been a major focus of the literature. Indeed, models with habit preferences or long run risk can also reproduce the failure of the UIP (Verdelhan, 2010; Colacito and Croce, 2013). However, recent work has criticized these models on two grounds. First, Hassan et al. (2024) argue that the difference in currency returns should arise mostly from interest rate differentials, as exchange rates are notoriously difficult to predict empirically Meese and Rogoff (1983). They point out that the inverse relationship between the mean and higher moments of log-SDFs in the international finance literature is inconsistent with the observed unpredictability of exchange rates. Second, Chahrour et al. (2024), argue that shocks to the expectation of future TFP changes are actually associated with exchange rate appreciation, rather than depreciation as implied by models with long run risk.

Our model is robust to both of these criticisms. First, in our model, exchange rates are essentially unpredictable based on interest rate differentials: a regression of exchange rate growth on interest rate differentials has a median R^2 of 0.02 across model simulations, which is in line with empirical estimates (Fama, 1984; Lustig, Stathopoulos, and Verdelhan, 2019). The reason why exchange rate movements are not predictable, even though the conditional moments of the stochastic discount factor vary over time, is that the distribution of log-SDFs is highly skewed. In particular, our parameterization of the displacement shock u in our calibration implies that the states u_1 and u_2 differ mainly in their expectations of future innovation shocks, which could reflect early-stage learning during a technological revolution. As a result, even though the two states have different mean values of log-SDFs, exchange rate growth is still not predictable in our model because these shocks are infrequent. Second, as we saw in Figure 5, our model is in fact able to quantitatively reproduce the positive relation between exchange rates and future TFP growth documented by Chahrour et al. (2024).

¹⁴This mechanism is consistent with prior empirical evidence on the relation between declining real interest rates and rising risk premia across various markets (see, for example Fama and French, 1989; Bekaert, Engstrom, and Xing, 2009).

4.4 Trade and Financial Flows

Our model has implications for both output growth and international financial flows. In the model, the home and foreign country's net exports as a fraction of total output are

$$\begin{aligned}\frac{NX_t^H}{Y_{H,t}} &\equiv \frac{p_{h,t}Y_{H,t} - p_{h,t}x_{H,t}^H - p_{f,t}x_{F,t}^H}{p_{h,t}Y_{H,t}} = 1 - \frac{1}{\alpha + (1 - \alpha)\lambda_t} \\ \frac{NX_t^F}{Y_{F,t}} &\equiv \frac{p_{f,t}Y_{F,t} - p_{f,t}x_{F,t}^F - p_{h,t}x_{H,t}^F}{p_{f,t}Y_{F,t}} = 1 - \frac{\lambda_t}{1 - \alpha + \alpha\lambda_t}\end{aligned}\tag{49}$$

Examining equation (49), we see that fluctuations in λ_t drive fluctuations in trade flows.

The flip side of trade flows are portfolio flows. The net international investment position (NIIP) of country c scaled by the country's wealth is equal to

$$\frac{A_t^c}{W_t^c} \equiv \frac{W_t^c - S_t^c}{W_t^c},\tag{50}$$

that is, it is equal to the wealth of the country minus the value of the firms located in that country.

To see how these quantities respond to the shocks in the model, Figure 7 plots impulse responses for consumption share λ_t , wealth share w_t , output growth, net export scaled by output and net international investment position scaled by country's wealth. The top panel shows the response to the displacement shock u in the home country while the bottom panel shows the response to the neutral shock ε . The first two figures show that a positive displacement shock in the home country increases its relative wealth and consumption share (first panel) while its output grows relative to the foreign country.

The third and the fourth column of Figure 7 show that the dynamics of the international flows are mostly driven by the displacement shocks. Specifically, the third column shows that following a positive displacement shock in the home country, net exports to the foreign country declines as the country becomes richer, domestic households want to consume more, and therefore imports rise. Overall, the home country exports less of domestic goods and imports more of the foreign goods. The increase in the trade deficit is mirrored by capital inflows. Each period, foreign and domestic investors who hold the market portfolio need to acquire the new firms that enter the market in order to remain diversified. When the home country receives a larger displacement shock than the foreign country, there are more new firms in home country than the foreign. Households receiving these new firms (entrepreneurs) sell their shares to other households (domestic and foreign). The net result is that foreign demand for home assets increases relative to home demand for foreign assets, and therefore the home country experiences net capital inflows as its wealth increases.

Overall, the model is able to reproduce the positive correlation between exchange rate appreciation and capital inflows (Hau and Rey, 2006). As we contrast the top and bottom panel of Figure 7, we see that this pattern arises purely due to the innovation shock in the model. The neutral

productivity shock simply leads to an increase in output but no changes in the country’s trade balance and capital flows.

The model’s predictions about capital flows are consistent with the evidence in Section 1.4. Recall that, at the aggregate level, there is a strong correlation between US foreign direct investment inflows correlates positively with US innovation intensity (Figure 2). At the firm level, in Table 3 we saw that firms which innovate attract foreign capital inflows to these firms, leading to an increase in foreign ownership. These firm-level results are consistent with the spirit of the model. That is, our model is stylized in that only new entrants innovate. We could extend the model along the lines of Huang et al. (2023) and allow for innovation by existing firms. In this case, innovation by existing firms would increase their size—or the number of varieties they are the leading producer in the microfoundation in Section 2.5.

Last, we also provide some cross-country evidence that is consistent with the model’s implication about innovation and the trade deficit. This evidence is indirect due to the absence of measures of innovation or displacement for countries other than the US. However, we can exploit the fact that, in the model, the displacement shock is strongly related to income inequality. Thus, we can examine the relation between inequality and current account deficits at the country level. Using data on Current Account balances from the World Economic Outlook database, we see in Figure 8 that countries which experience larger increases in current account deficits also experience greater inequality growth. In terms of magnitudes, an increase of one percentage point in the top 1% income share over the period corresponds to a deterioration of the current-account-to-GDP ratio by 1.7 percentage points.

5 Conclusion

In this paper, we first document a positive correlation between US innovation and the growth of real dollar index. We show that exchange rate movements reconnect with relative changes in aggregate quantities, such as consumption and output growth, once wealth changes are accounted for. Moreover, we find that countries’ wealth fluctuations are positively correlated with macroeconomic fundamentals, indicating that the underlying shocks driving these fluctuations are positive productivity shocks. Finally, we show that foreign institutional ownership of firms increases following the granting of significant patents.

We provide a quantitative general equilibrium model that successfully replicates these patterns, as well as the joint dynamics of exchange rates, consumption growth, trade flows, and stock returns. We introduce a minimal deviation to the standard endowment economy model: in addition to the standard endowment shock in each country, countries can now each experience displacement shocks that reallocate output among agents. This minimal deviation from the standard model is sufficient to generate the documented patterns.

Our calibrated model successfully replicates the first two moments of aggregate consumption

and output growth, exchange rates, and stock returns while generating low and relatively smooth risk-free rates. Our model replicates the three key ‘anomalies’ in the exchange rate literature: the volatility puzzle of [Brandt et al. \(2006\)](#), the Backus-Smith correlation puzzle, and the violation of the uncovered interest rate parity (UIP).

References

- Aghion, P., U. Akcigit, A. Bergeaud, R. Blundell, and D. Hémous (2018, 06). Innovation and Top Income Inequality. *The Review of Economic Studies* 86(1), 1–45.
- Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica* 60(2), 323–351.
- Aguiar, M., O. Itskhoki, and D. Mukhin (2024). How Good is International Risk Sharing. Nber working paper.
- Alvarez, F., A. Atkeson, and P. J. Kehoe (2002). Money, interest rates, and exchange rates with endogenously segmented markets. *Journal of Political Economy* 110(1), 73–112.
- Alvarez, F., A. Atkeson, and P. J. Kehoe (2009). Time-varying risk, interest rates, and exchange rates in general equilibrium. *The Review of Economic Studies* 76(3), 851–878.
- Atkeson, A., J. Heathcote, and F. Perri (2022). The end of privilege: A reexamination of the net foreign asset position of the united states. Working paper.
- Bacchetta, P. and E. Van Wincoop (2006, June). Can information heterogeneity explain the exchange rate determination puzzle? *American Economic Review* 96(3), 552–576.
- Backus, D. K., S. Foresi, and C. I. Telmer (2001). Affine term structure models and the forward premium anomaly. *The Journal of Finance* 56(1), 279–304.
- Backus, D. K. and G. W. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics* 35(3), 297–316.
- Balassa, B. (1964). The purchasing-power parity doctrine: A reappraisal. *Journal of Political Economy* 72(6), 584–596.
- Bekaert, G., E. Engstrom, and Y. Xing (2009). Risk, uncertainty, and asset prices. *Journal of Financial Economics* 91(1), 59–82.
- Björkman, M. and K. Holmström (2000, 12). Global optimization of costly nonconvex functions using radial basis functions. *Optimization and Engineering* 1, 373–397.
- Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy* 93(2), 223–47.
- Brandt, M. W., J. H. Cochrane, and P. Santa-Clara (2006). International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics* 53(4), 671–698.
- Camanho, N., H. Hau, and H. Rey (2020). Global portfolio rebalancing and exchange rates. Technical report.

- Chahrour, R., V. Cormun, P. De Leo, P. A. Guerrón-Quintana, and R. Valchev (2024, June). Exchange rate disconnect revisited. Working Paper 32596, National Bureau of Economic Research.
- Chen, Z., Z. Jiang, H. Lustig, S. V. Nieuwerburgh, and M. Xiaolan (2023). Exorbitant privilege gained and lost: Fiscal implications.
- Chernov, M., V. Haddad, and O. Itkhoki (2023). What do financial markets say about the exchange rate?
- Colacito, R. and M. M. Croce (2011). Risks for the long run and the real exchange rate. *Journal of Political Economy* 119(1), 153–181.
- Colacito, R. and M. M. Croce (2013). International asset pricing with recursive preferences. *The Journal of Finance* 68(6), 2651–2686.
- Colacito, R., M. M. Croce, F. Gavazzoni, and R. Ready (2018). Currency risk factors in a recursive multicountry economy. *The Journal of Finance* 73(6), 2719–2756.
- Colacito, R., M. M. Croce, Y. Liu, and I. Shaliastovich (2021, 09). Volatility Risk Pass-Through. *The Review of Financial Studies* 35(5), 2345–2385.
- Colacito, R., M. M. Croce, Y. Liu, and I. Shaliastovich (2023). Volatility (Dis)Connect in International Markets. *working paper*.
- Constantinides, G. M. and D. Duffie (1996). Asset pricing with heterogeneous consumers. *Journal of Political Economy* 104(2), 219–240.
- Dahlquist, M., C. Heyerdahl Larsen, A. Pavlova, and J. Penasse (2023). International capital market and wealth transfers.
- Driscoll, J. C. and A. C. Kraay (1998). Consistent covariance matrix estimation with spatially dependent panel data. *The Review of Economics and Statistics* 80(4), 549–560.
- Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics* 14(3), 319–338.
- Fama, E. F. and K. R. French (1989). Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics* 25(1), 23–49.
- Fang, X. (2021, 09). Intermediary Leverage and the Currency Risk Premium. *working paper* 35(5), 2345–2385.
- Fang, X. and Y. Liu (2021). Volatility, intermediaries, and exchange rates. *Journal of Financial Economics* 141(1), 217–233.
- Farhi, E. and I. Werning (2014). Dilemma not trilemma? capital controls and exchange rates with volatile capital flows. *IMF Economic Review* 62(4), 569–605.
- Fernald, J. (2014). A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. Federal reserve bank of san francisco working paper.
- Fukui, M., E. Nakamura, and J. Steinsson (2023, May). The macroeconomic consequences of exchange rate depreciations. Working Paper 31279, National Bureau of Economic Research.

- Gabaix, X. and M. Maggiori (2015, 03). International Liquidity and Exchange Rate Dynamics *. *The Quarterly Journal of Economics* 130(3), 1369–1420.
- Gârleanu, N., S. Panageas, D. Papanikolaou, and J. Yu (2016). Drifting apart: The pricing of assets when the benefits of growth are not shared equally. Working paper, University of California, Berkeley.
- Gavazzoni, F. and A. M. Santacreu (2020). International r&d spillovers and asset prices. *Journal of Financial Economics* 136(2), 330–354.
- Gourinchas, P. and H. Rey (2007a). International financial adjustment. *Journal of Political Economy* 115(4), 665–703.
- Gourinchas, P.-O., W. Ray, and D. Vayanos (2020). A preferred-habitat model of term premia and currency risk. Technical report.
- Gourinchas, P.-O. and H. Rey (2007b). From World Banker to World Venture Capitalist: US External Adjustment and the Exorbitant Privilege. In *G7 Current Account Imbalances: Sustainability and Adjustment*, NBER Chapters, pp. 11–66. National Bureau of Economic Research, Inc.
- Gourinchas, P.-O., H. Rey, and N. Govillot (2010, August). Exorbitant Privilege and Exorbitant Duty. IMES Discussion Paper Series 10-E-20, Institute for Monetary and Economic Studies, Bank of Japan.
- Greenwood, R., S. G. Hanson, J. C. Stein, and A. Sunderam (2020, July). A quantity-driven theory of term premia and exchange rates. Working Paper 27615, National Bureau of Economic Research.
- Hassan, T., T. M. Mertens, and J. Wang (2024). A currency premium puzzle. Working paper.
- Hassan, T. A. (2013). Country size, currency unions, and international asset returns. *The Journal of Finance* 68(6), 2269–2308.
- Hau, H. and H. Rey (2006). Exchange rates, equity prices, and capital flows. *Review of Financial Studies* 19(1), 273–317.
- Huang, Q., L. Kogan, and D. Papanikolaou (2023). Productivity shocks and inflation in incomplete markets. Working paper.
- Itskhoki, O. and D. Mukhin (2021). Exchange rate disconnect in general equilibrium. *Journal of Political Economy* 129(8), 2183–2232.
- Jiang, Z. (2021). Us fiscal cycle and the dollar. *Journal of Monetary Economics* 124, 91–106.
- Jiang, Z., A. Krishnamurthy, and H. Lustig (2021). Foreign safe asset demand and the dollar exchange rate. *The Journal of Finance* 76(3), 1049–1089.
- Jiang, Z., A. Krishnamurthy, and H. Lustig (2023a, 11). Dollar Safety and the Global Financial Cycle. *The Review of Economic Studies*, rdad108.
- Jiang, Z., A. Krishnamurthy, and H. Lustig (2023b, November). Implications of asset market data for equilibrium models of exchange rates. Working Paper 31851, National Bureau of Economic Research.

- Jiang, Z., A. Krishnamurthy, H. Lustig, and J. Sun (2021, July). Beyond Incomplete Spanning: Convenience Yields and Exchange Rate Disconnect. Research Papers 3964, Stanford University, Graduate School of Business.
- Jiang, Z. and R. J. Richmond (2019). Origins of international factor structures. *working paper*.
- Kekre, R. and M. Lenel (2024a). Exchange rates, natural rates, and the price of risk. Working paper.
- Kekre, R. and M. Lenel (2024b, June). The flight to safety and international risk sharing. *American Economic Review* 114(6), 1650–91.
- Kelly, B., D. Papanikolaou, A. Seru, and M. Taddy (2021, September). Measuring technological innovation over the long run. *American Economic Review: Insights* 3(3), 303–20.
- Kocherlakota, N. and L. Pistaferri (2008). Inequality and real exchange rates. *Journal of the European Economic Association* 6(2-3), 597–608.
- Kogan, L., D. Papanikolaou, L. D. W. Schmidt, and J. Song (2020, April). Technological innovation and labor income risk. Working Paper 26964, National Bureau of Economic Research.
- Kogan, L., D. Papanikolaou, A. Seru, and N. Stoffman (2017). Technological Innovation, Resource Allocation, and Growth. *The Quarterly Journal of Economics* 132(2), 665–712.
- Kogan, L., D. Papanikolaou, and N. Stoffman (2020). Left behind: Creative destruction, inequality, and the stock market. *Journal of Political Economy* 128(3).
- Koijen, R. S. J. and M. Yogo (2020, June). Exchange rates and asset prices in a global demand system. Working Paper 27342, National Bureau of Economic Research.
- Kollmann, R. (1995, April). Consumption, real exchange rates and the structure of international asset markets. *Journal of International Money and Finance* 14(2), 191–211.
- Lee, B.-S. and B. F. Ingram (1991). Simulation estimation of time-series models. *Journal of Econometrics* 47(2), 197–205.
- Lustig, H. and R. J. Richmond (2019, 09). Gravity in the Exchange Rate Factor Structure. *The Review of Financial Studies* 33(8), 3492–3540.
- Lustig, H., N. Roussanov, and A. Verdelhan (2011, 08). Common Risk Factors in Currency Markets. *The Review of Financial Studies* 24(11), 3731–3777.
- Lustig, H., A. Stathopoulos, and A. Verdelhan (2019, December). The term structure of currency carry trade risk premia. *American Economic Review* 109(12), 4142–77.
- Lustig, H. and A. Verdelhan (2019, June). Does incomplete spanning in international financial markets help to explain exchange rates? *American Economic Review* 109(6), 2208–44.
- Martin, I. (2011, November). The forward premium puzzle in a two-country world. Working Paper 17564, National Bureau of Economic Research.
- Meese, R. A. and K. Rogoff (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics* 14(1), 3–24.

- Nam, D. and J. Wang (2015). The effects of surprise and anticipated technology changes on international relative prices and trade. *Journal of International Economics* 97(1), 162–177.
- Obstfeld, M. and K. Rogoff (2001). The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual* 15, 339–390.
- Papanikolaou, D. (2011). Investment shocks and asset prices. *Journal of Political Economy*, forthcoming.
- Pavlova, A. and R. Rigobon (2007, 01). Asset Prices and Exchange Rates. *The Review of Financial Studies* 20(4), 1139–1180.
- Pavlova, A. and R. Rigobon (2008). The role of portfolio constraints in the international propagation of shocks. *The Review of Economic Studies* 75(4), 1215–1256.
- Pavlova, A. and R. Rigobon (2010). An asset-pricing view of external adjustment. *Journal of International Economics* 80(1), 144–156. Special Issue: JIE Special Issue on International Macro-Finance.
- Pavlova, A. and R. Rigobon (2011). Equilibrium Portfolios and External Adjustment under Incomplete Markets. Technical report.
- Ready, R., N. Roussanov, and C. Ward (2017). Commodity trade and the carry trade: A tale of two countries. *The Journal of Finance* 72(6), 2629–2684.
- Richmond, R. J. (2019). Trade network centrality and currency risk premia. *The Journal of Finance* 74(3), 1315–1361.
- Samuelson, P. A. (1964). Theoretical notes on trade problems. *The Review of Economics and Statistics* 46(2), 145–154.
- Sauzet, M. (2023). Asset prices, global portfolios, and the international financial system.
- Uhlig, H. (2004, August). What moves GNP? Econometric Society 2004 North American Winter Meetings 636, Econometric Society.
- Van Nieuwerburgh, S., Z. Jiang, H. Lustig, and M. Xiaolan (2021, April). The U.S. Public Debt Valuation Puzzle. CEPR Discussion Papers 16082, C.E.P.R. Discussion Papers.
- Verdelhan, A. (2010). A habit-based explanation of the exchange rate risk premium. *The Journal of Finance* 65(1), 123–146.
- Verdelhan, A. (2018). The share of systematic variation in bilateral exchange rates. *The Journal of Finance* 73(1), 375–418.
- Wiriadinata, U. (2021). External debt, currency risk, and international monetary policy transmission. Working paper.
- Yu, J. (2013). A sentiment-based explanation of the forward premium puzzle. *Journal of Monetary Economics* 60(4), 474–491.

Figures and Tables

Table 1: Dollar Index growth and U.S. Innovation

Growth in Real Dollar	Time Series Estimate (EW Dollar Index)				Panel Estimate
	(1)	(2)	(3)	(4)	(5)
KPSS/MKT	0.026** (0.011)	0.026** (0.012)	0.036** (0.017)	0.039** (0.015)	0.033** (0.016)
Lagged Dollar Index	Y	Y		Y	Y
Lagged Output growth		Y	Y	Y	Y
Lagged Innovation			Y	Y	Y
Observations	49	49	49	49	454
R2	0.175	0.175	0.099	0.199	0.139

Notes: The table reports regression results of the growth of log dollar index on U.S. innovation:

$$\Delta \log e_{t+1}^{USD} = \alpha + \beta_1 Inno_{US,t+1} + \beta_2 X_t + \varepsilon_{t+1}$$

The sample period is 1974-2022. U.S. innovation is measured by the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Control variable X_t includes lagged innovation, lagged output growth and lagged Dollar Index level at t . Both series are in logs. The last column present estimates from the following panel regression:

$$\Delta \log e_{t+1}^{US,f} = \alpha + \beta_1 Inno_{US,t+1} + \beta_2 X_t^{US,f} + \varepsilon_{t+1}$$

where $X_t^{US,f}$ accounts for the lagged exchange rate level, lagged innovation, and the lagged growth of the output ratio between the U.S. and each foreign country. The sample consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, United Kingdom, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. Standard errors (in parentheses) are obtained using Newey-West with one period lag. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2: U.S. Innovation and Imperfect Risk Sharing

Growth	US relative wealth					Rel. Inequality		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
KPSS/MKT	0.028*	0.035**	0.029**	0.031**	0.011*	0.012**	0.012**	0.014**
	(0.015)	(0.015)	(0.014)	(0.015)	(0.006)	(0.006)	(0.006)	(0.006)
Lagged innovation	Y	Y	Y	Y	Y	Y	Y	Y
Lagged output growth	Y	Y	Y	Y	Y	Y	Y	Y
Lagged wealth/inequality ratio		Y	Y	Y		Y	Y	Y
Lagged 2-year wealth/inequality ratio			Y	Y			Y	Y
Lagged 2-year output growth				Y				Y
Observations	412	412	409	401	481	481	475	470
R2	0.033	0.106	0.170	0.179	0.032	0.102	0.102	0.103

Notes: Columns 1-4 of this table reports regression results of the growth of wealth ratio on US innovation:

$$\Delta \log W_{t+1}^{US}/W_{t+1}^f = \alpha + \beta_1 \text{Inno}_{US,t+1} + \beta_2 X_t + \varepsilon_{t+1}$$

The sample period is 1974–2022. The dependent variable is the growth of wealth ratio of the U.S. to a foreign country. U.S. innovation is measured as the logarithm of the ratio of patent value (KPSS) to total market capitalization. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, United Kingdom, Switzerland, France and Italy. Control variable X_t includes lagged innovation, lagged wealth ratio, and the lagged growth of the output ratio between the U.S. and each foreign country. Independent variables are standardized to unit standard deviation using unconditional moments. Column 5-8 reports regression results where the dependent variable is replaced with the relative inequality growth, defined as the difference in log top 1 percent income share between the US and foreign countries. The panel regressions include country fixed effects, and standard errors are reported in parentheses using the [Driscoll and Kraay \(1998\)](#) method. Data on income inequality and wealth is from World Inequality Database. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3: Innovation and Foreign Institutional Ownership in the US

Dependent variable = Change in Foreign Institutional ownership									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
KPSS (Top 20%)	0.182*** (0.061)			0.138** (0.062)			0.159** (0.065)		
Cites (Top 20%)		0.221*** (0.045)			0.190*** (0.049)			0.210*** (0.056)	
KPST(Top 20%)			0.192** (0.067)			0.170** (0.071)			0.186** (0.066)
Firm Controls	No	No	No	YES	YES	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES	No	No	No
Industry $\times$ Year	No	No	No	No	No	No	YES	YES	YES
Observations	67986	67986	67986	65761	65761	65761	64723	64723	64723
Adj R2	0.788	0.787	0.787	0.795	0.794	0.794	0.796	0.796	0.796
Within R2	0.386	0.384	0.384	0.399	0.398	0.398	0.368	0.367	0.367

Notes: This table reports regression coefficients (times 100) of :

$$\Delta \text{IO_FOR}_{i,t+1} = \beta \log(\text{inno})_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t+1}. \quad (51)$$

The dependent variable $\Delta \text{IO_FOR}_{i,t+1}$ is the change in foreign institutional ownership for firm i between t and $t + 1$. The foreign institutional ownership data are from FactSet Lionshare database. The independent variable is the (log) number of important patents granted to firm i in the last year t , according to various innovation measures. When inno is equal to zero, we replace $\log(\text{inno})$ with zero and add a dummy equal to one if inno is equal to 0, thereby preventing the removal of the observation from the data. The vector of control $X_{i,t}$ includes foreign institutional ownership at time t , $\text{IO_FOR}_{i,t}$, firm and year fixed effects. In columns (4)-(6), we add firm size and sales (log) at t as additional controls. In columns (7)-(9), we replace year fixed effects with industry $\times$ year fixed effects. Independent variables are standardized to unit standard deviation using unconditional moments. Standard errors in parentheses are clustered at the SIC industry and year level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 4: Exchange Rate Growth, Wealth Growth and Consumption and Output Growth

Panel A. Real Exchange Rate Growth, Consumption Growth and Wealth Changes												
	Panel	AUS	CAN	CHE	DEU	FRA	GBP	ITA	JPN	NOR	NZL	SWE
Wealth change	0.107*** (0.004)	0.103*** (0.007)	0.099*** (0.009)	0.108*** (0.010)	0.131*** (0.009)	0.106*** (0.006)	0.116*** (0.008)	0.094*** (0.010)	0.101*** (0.006)	0.101*** (0.007)	0.082*** (0.015)	0.114*** (0.007)
Consumption growth	-0.020*** (0.003)	-0.010 (0.007)	-0.015* (0.008)	-0.027** (0.010)	-0.024** (0.009)	-0.005 (0.008)	-0.027*** (0.008)	-0.001 (0.009)	-0.032*** (0.008)	-0.011 (0.008)	0.003 (0.023)	-0.022** (0.008)
Observations	420	49	49	31	25	25	49	25	49	42	27	49
R2	0.840	0.860	0.779	0.870	0.950	0.961	0.870	0.893	0.881	0.867	0.843	0.882
Panel B. Real Exchange Rate Growth, Output Growth and Wealth Changes												
	Panel	AUS	CAN	CHE	DEU	FRA	GBP	ITA	JPN	NOR	NZL	SWE
Wealth change	0.103*** (0.004)	0.103*** (0.007)	0.096*** (0.008)	0.105*** (0.010)	0.129*** (0.008)	0.105*** (0.006)	0.109*** (0.008)	0.095*** (0.011)	0.100*** (0.007)	0.099*** (0.007)	0.102*** (0.014)	0.106*** (0.007)
GDP growth	-0.016*** (0.004)	-0.011* (0.006)	-0.015* (0.008)	-0.039*** (0.013)	-0.019** (0.007)	-0.007 (0.005)	-0.014* (0.008)	-0.001 (0.010)	-0.026*** (0.007)	-0.010 (0.010)	-0.036 (0.021)	-0.007 (0.008)
Observations	420	49	49	31	25	25	49	25	49	42	27	49
R2	0.832	0.865	0.811	0.862	0.950	0.965	0.849	0.878	0.874	0.863	0.860	0.856

Notes: Panel A of the table reports regression results of the growth of log exchange rate on log wealth ratio and log consumption growth ratio:

$$\Delta \log e_{t+1} = \alpha + \beta_1 \Delta \log W_{t+1} + \beta_2 \Delta \log C_{t+1} + \gamma X_t + \varepsilon_{t+1}$$

where the vector of controls X_t includes lagged relative levels of exchange rates, consumption and relative wealth. The sample period is 1974-2022. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, United Kingdom, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with five periods lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Panel B repeats the analysis, replacing consumption with the country's GDP. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 5: Model-implied Displacement shock and US Dollar

Growth in Real Dollar	Time Series Estimate				Panel Estimate
	(1)	(2)	(3)	(4)	(5)
Displacement	0.036*** (0.008)	0.037*** (0.007)	0.044*** (0.013)	0.042*** (0.012)	0.038*** (0.012)
Lagged Dollar Index	Y	Y		Y	Y
Lagged Output growth		Y	Y	Y	Y
Lagged Innovation			Y	Y	Y
Observations	49	49	49	49	467
R2	0.266	0.266	0.187	0.277	0.202

Notes: The table reports regression results of the growth of log dollar index on U.S. innovation:

$$\Delta \log e_{t+1}^{USD} = \alpha + \beta_1 \text{Displacement}_{US,t} + \beta_2 X_t + \varepsilon_{t+1}$$

The sample period is 1974-2022. U.S. displacement is measured as described in section 2.6. The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Control variable X_t includes lagged innovation, lagged output growth and lagged Dollar Index level at t . Both series are in logs. The last column present estimates from the following panel regression:

$$\Delta \log e_{t+1}^{US,f} = \alpha + \beta_1 \text{Displacement}_{US,t} + \beta_2 X_t^{US,f} + \varepsilon_{t+1}$$

where $X_t^{US,f}$ accounts for the lagged exchange rate level, lagged innovation, and the lagged growth of the output ratio between the U.S. and each foreign country. The sample consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, United Kingdom, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. Standard errors (in parentheses) are obtained using Newey-West with one period lag. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 6: Model-implied Displacement Shock and Imperfect Risk Sharing

Growth	US relative wealth				Rel. Inequality			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Displacement	0.049*** (0.014)	0.055*** (0.013)	0.053*** (0.013)	0.053*** (0.013)	0.018** (0.008)	0.016** (0.007)	0.017** (0.007)	0.016** (0.007)
Lagged displacement	Y	Y	Y	Y	Y	Y	Y	Y
Lagged output growth	Y	Y	Y	Y	Y	Y	Y	Y
Lagged wealth/inequality ratio		Y	Y	Y		Y	Y	Y
Lagged 2-year wealth/inequality ratio			Y	Y			Y	Y
Lagged 2-year output growth				Y				Y
Observations	412	412	409	401	481	481	475	470
R2	0.117	0.215	0.274	0.282	0.072	0.138	0.138	0.139

Notes: Columns 1-4 of this table reports regression results of the growth of wealth ratio on US innovation:

$$\Delta \log W_{t+1}^{US}/W_{t+1}^f = \alpha + \beta_1 \text{Displacement}_{US,t} + \beta_2 X_t + \varepsilon_{t+1}$$

The sample period is 1974–2022. The dependent variable is the growth of wealth ratio of the U.S. to a foreign country. U.S. displacement is measured as described in section 2.6. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, United Kingdom, Switzerland, France and Italy. Control variable X_t includes lagged innovation, lagged wealth ratio, and the lagged growth of the output ratio between the U.S. and each foreign country. Independent variables are standardized to unit standard deviation using unconditional moments. Column 5-8 reports regression results where the dependent variable is replaced with the relative inequality growth. The panel regressions include country fixed effects, and standard errors are reported in parentheses using the [Driscoll and Kraay \(1998\)](#) method. Data on income inequality and wealth is from World Inequality Database. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: Inequality growth and exchange rate

<i>Exchange rate and inequality growth</i>												
	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Inequality growth	0.017*** (0.005)	0.065*** (0.021)	-0.007 (0.017)	0.015 (0.023)	0.058 (0.064)	0.027 (0.029)	0.037** (0.017)	0.093** (0.038)	0.023 (0.022)	-0.004 (0.013)	0.008 (0.012)	0.014 (0.019)
Observations	418	49	49	42	18	25	42	18	42	42	49	42
R2	0.124	0.412	0.138	0.189	0.202	0.176	0.302	0.385	0.080	0.094	0.161	0.114

Notes: Panel A of the table reports regression results of the growth of log exchange rate on log income inequality growth ratio.

$$\Delta \log e_{t+1} = \alpha + \beta \Delta \log I_{t+1} + \gamma X_t + \varepsilon_{t+1}$$

where $\Delta \log I_{t+1}$ is the growth of the ratio of top 1% income share between t and $t + 1$. The sample period is 1974-2022. Control vector X_t includes lagged inequality level, lagged exchange rate. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, United Kingdom, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and we report [Driscoll and Kraay \(1998\)](#) standard errors in parentheses. Exchange rate, consumption, and GDP data are from the World Bank and the IMF. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Moments used in Model Estimation

	Data	Model		
		Median	5%	95%
<i>A. Aggregate Quantities</i>				
Consumption growth, mean	0.014	0.012	0.004	0.018
Consumption growth, volatility	0.022	0.018	0.016	0.057
Output growth, mean	0.014	0.012	0.008	0.018
Output growth, volatility	0.021	0.017	0.016	0.020
<i>B. Asset prices</i>				
Risk-free rate, mean	0.016	0.021	-0.028	0.032
Risk-free rate, volatility	0.031	0.014	0.005	0.056
Excess stock returns, mean	0.037	0.038	0.016	0.139
Excess stock returns, volatility	0.252	0.121	0.059	0.321
Exchange rate, volatility	0.115	0.080	0.037	0.207
<i>C. Comovement of Key Variables</i>				
<i>i. Univariate Regression Slopes</i>				
Consumption growth and wealth growth	0.008	0.008	0.001	0.089
Output growth and wealth growth	0.005	0.002	-0.003	0.005
Exchange rate growth and inequality growth	0.017	0.046	-0.010	0.122
US displacement shocks and US wealth ratio growth	0.053	0.038	-0.099	0.149
<i>ii. Bi-variate Regression Slopes</i>				
Exchange rate and				
—relative c-growth	-0.020	-0.014	-0.064	-0.007
—relative wealth growth	0.107	0.090	0.038	0.262
Exchange rate and				
—relative output-growth	-0.016	-0.010	-0.015	-0.003
—relative wealth growth	0.103	0.081	0.036	0.207
<i>iii. Pairwise Correlations</i>				
Consumption growth (H and F)	0.454	0.763	0.146	0.911
Output growth (H and F)	0.596	0.849	0.718	0.952
Stock Returns (H and F)	0.598	0.207	-0.171	0.667
Trade surplus (as % of output) growth and c-growth	-0.299	-0.162	-0.935	0.238
<i>D. Other</i>				
Uncovered Interest Parity Slope	-0.215	-0.179	-4.743	1.929
Dollar index growth and US displacement, correlation	0.350	0.396	-0.644	0.743

Notes: This table reports both empirical moments computed using the G-7 & G-10 data set and simulated moments from the model. All the parameters are estimated as in Table 9.

Table 9: Parameter Estimates

Description	Symbol	Value
<i>Preferences:</i>		
Home bias	α	0.989
Preference for own consumption	h	0.195
Subjective discount rate	β	0.996
Risk aversion	γ	6.226
Elasticity of intertemporal substitution	ψ	1.629
<i>Endowments:</i>		
Displacement shock productivity	δ	0.207
Measure of projects-receiver	π	0.101
Mean of output growth	μ	0.010
Displacement shock low and middle state	$u_1 = u_2$	0.001
Displacement shock high state	u_3	0.146
Persistence of displacement shock		
— persistence in low state	p	0.936
— persistence in high state	q	0.841
Volatility of neutral shock	σ_e	0.016
Technology spillover	ρ_u	0.711
Correlation of neutral shock	ρ_e	0.859

Notes: This table reports the estimated parameters of the model. See the main text and the Appendix [B.5](#) for details on the estimation of the model.

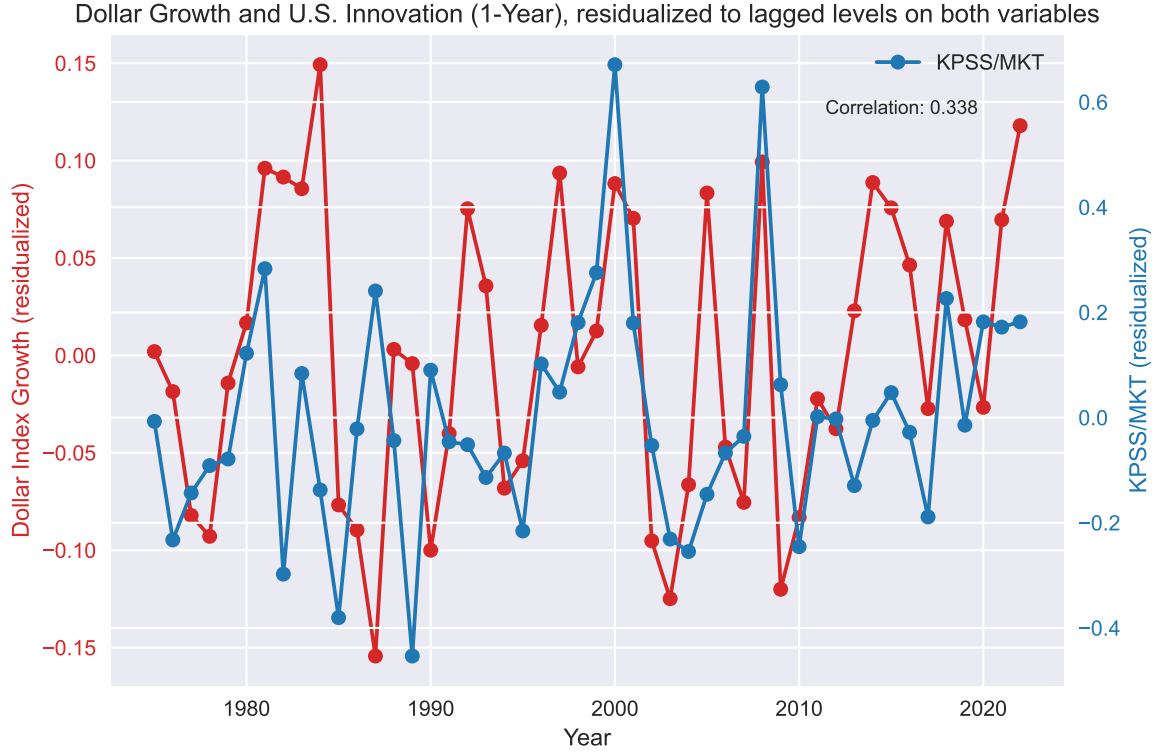
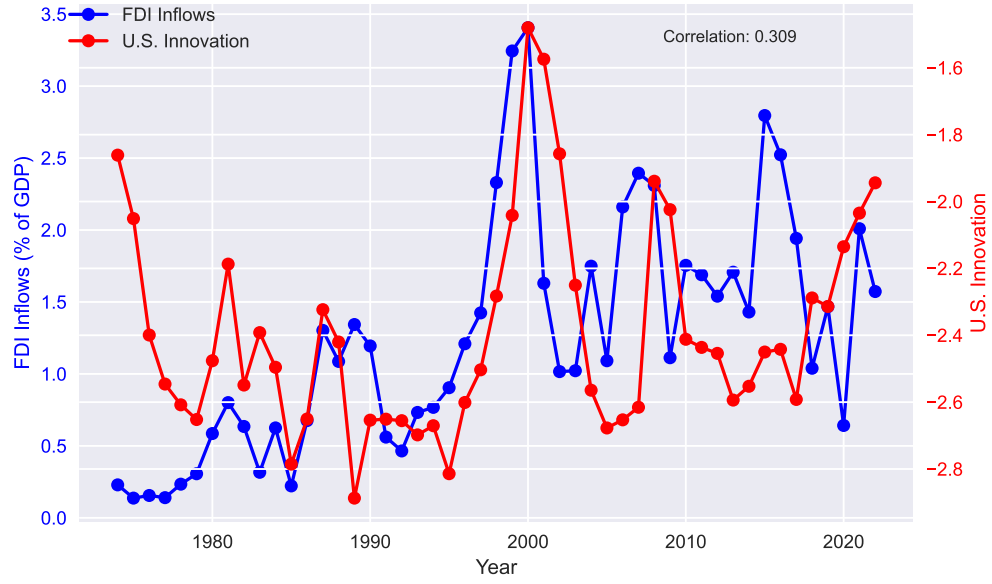


Figure 1: This figure plots the dollar index growth against U.S. innovation from 1974-2022. U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Both series are in logs. The log growth is computed over 1-year. Both series are residualized to the lagged levels, corresponding to the regression specification in (1).

Panel A. U.S. Innovation and U.S. FDI Inflows



Panel B. U.S. Innovation and U.S. Portfolio Equity Inflows

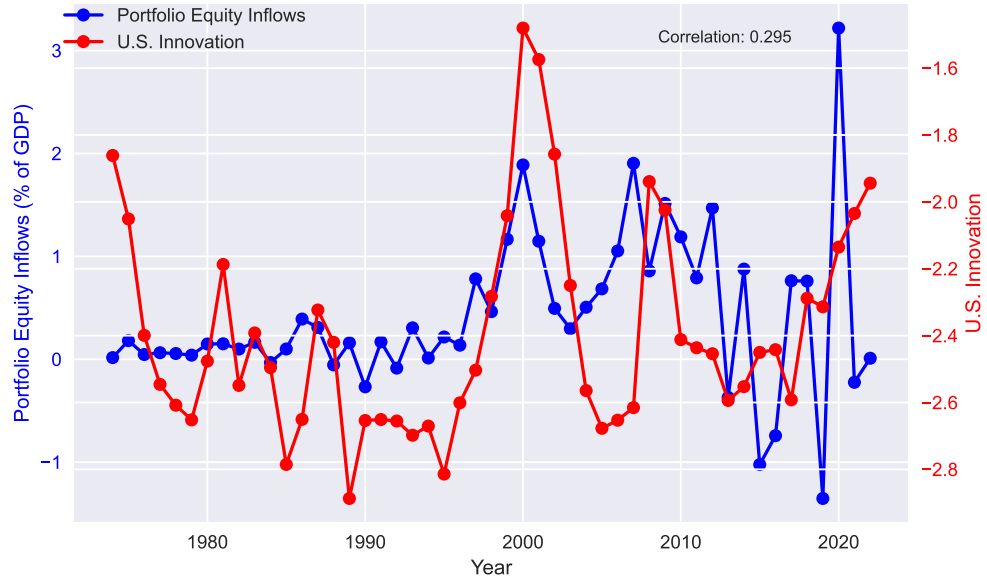


Figure 2: This figure plots the U.S. innovation index and foreign direct investment inflows (in Panel A) and portfolio equity inflows (in Panel B) in the U.S. The U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The aggregate FDI inflows and portfolio equity inflows are obtained from the World Bank. See A for details.

The real dollar index and the US innovation (Alt. Displacement shocks)

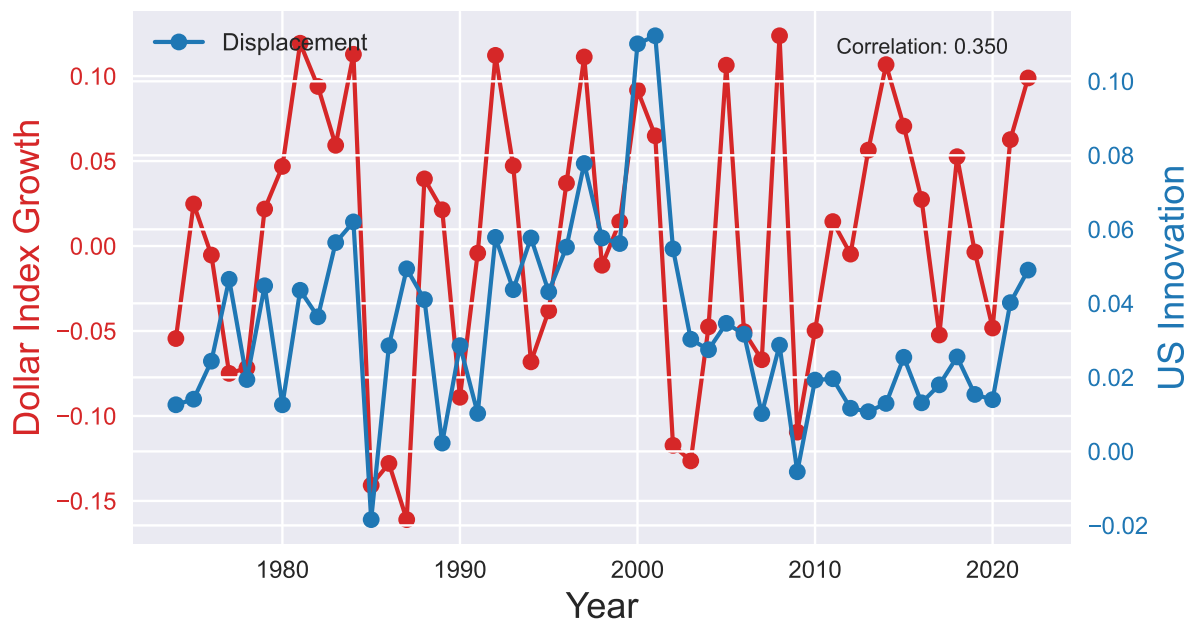
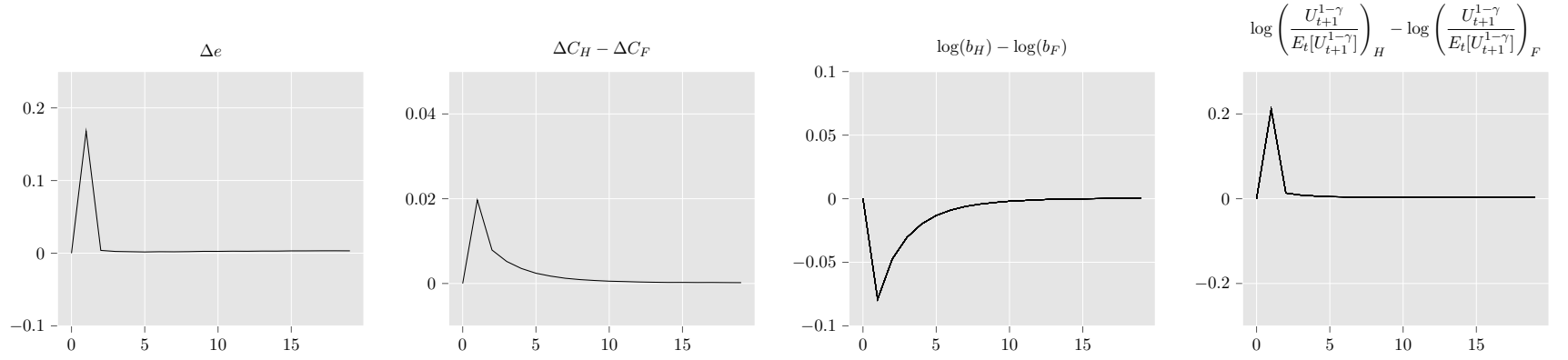


Figure 3: This figure plots the growth of the real dollar index (red) and the alternative US displacement series (blue) constructed in Section 2.6 that is based on the difference between the aggregate market capitalization growth and the returns from holding the market portfolio. The dollar index in red is the equal-weighted real dollar indexes.

The Backus-Smith Correlation

A. Response to Displacement Shock ($u : u_2 \rightarrow u_3$)



B. Response to Neutral Shock (ε)

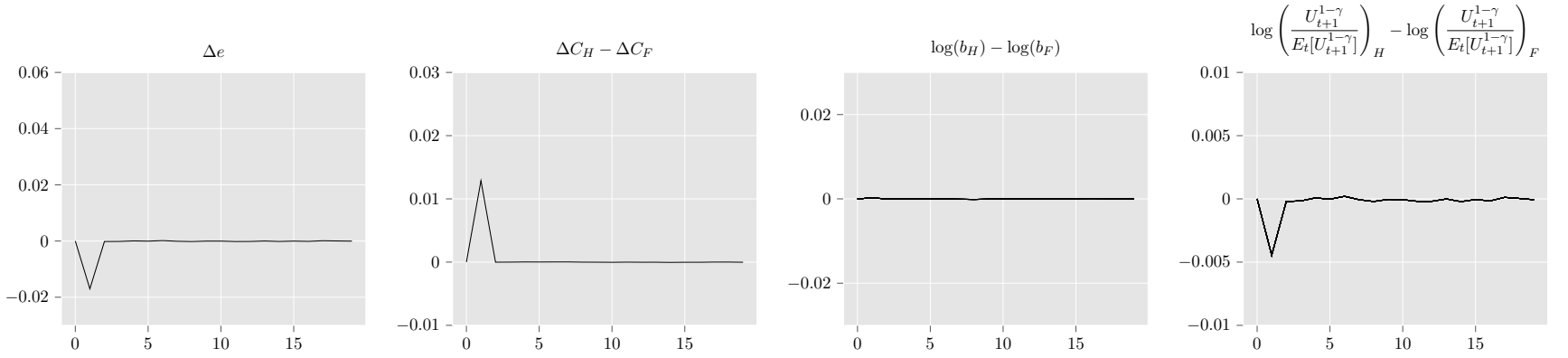


Figure 4: This figure plots the impulse response of the difference between the home and foreign countries' variables in response to a shock to the home country (u in Panel A and ε in Panel B). All parameters are calibrated to the values reported in Table 9. We construct the impulse responses by introducing an additional one-standard deviation shock at time $t=1$ without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

Model-implied Impulse Response to the ‘FX shock’ from [Chahrour et al. \(2024\)](#)

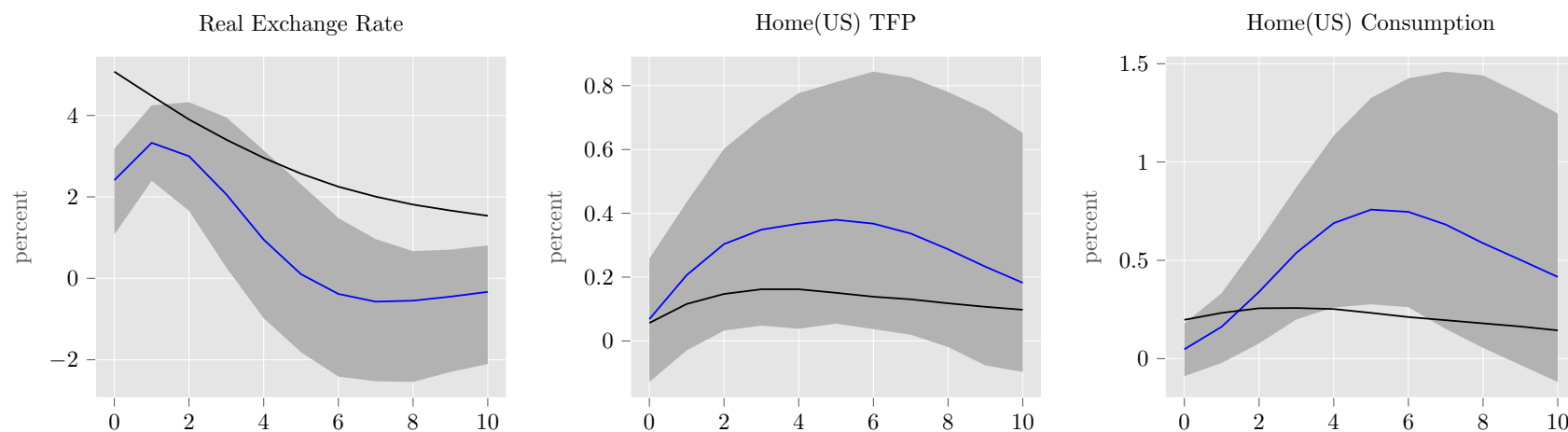
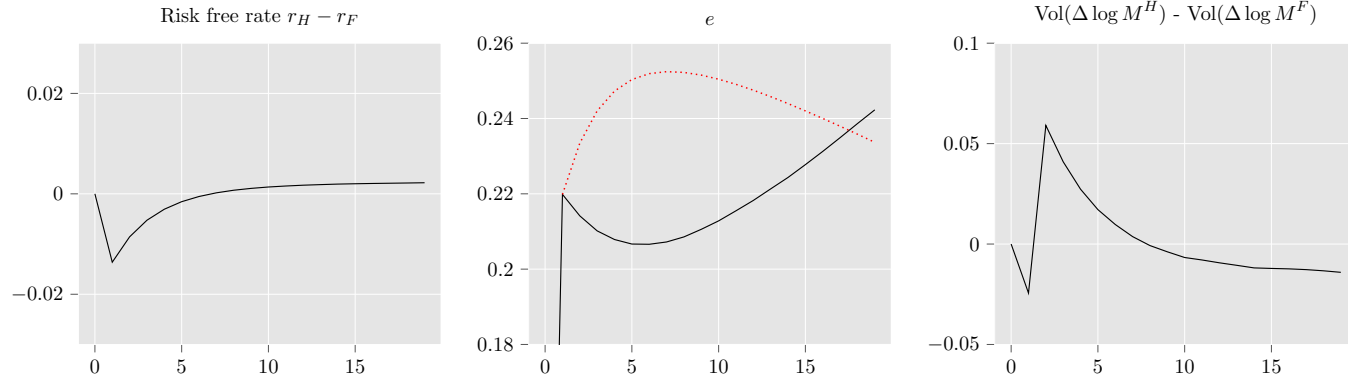


Figure 5: This Figure replicates the VAR analysis in [Chahrour et al. \(2024\)](#) in simulated data from the model. The first figure plots the extracted ‘FX shock’, the second figure plots the response of productivity (output) in the US and the third figure plots the response of US consumption. The black line plots the median impulse response in simulated data. The blue line represents the impulse responses to the main exchange rate shock in the data along with 90% confidence intervals. All parameters are calibrated to the values reported in [Table 9](#)

The Forward Premium Puzzle (UIP)

A. Response to Displacement Shock ($u : u_1 \rightarrow u_3$)



B. Response to Displacement Shock ($u : u_1 \rightarrow u_2$)

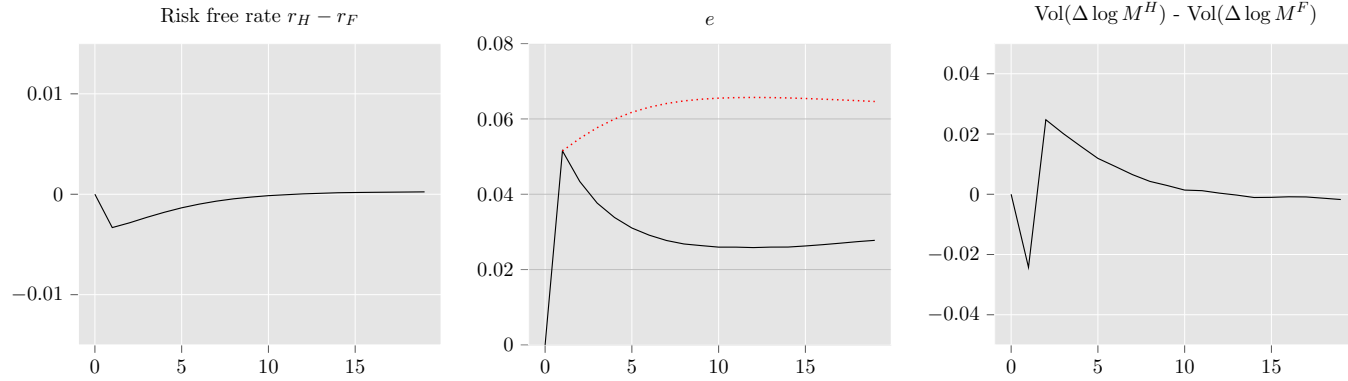


Figure 6: This figure plots the impulse responses of key variables to a shock to the home country ($u_1 \rightarrow u_3$ in Panel A and $u_1 \rightarrow u_2$ in Panel B). The red dotted line in the middle column shows the path that would obtain if Uncovered Interest Parity (UIP) were to hold. All parameters are calibrated to the values reported in Table 9. We construct the impulse responses by introducing an additional one-standard deviation shock at time $t=1$ without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

Output and Trade Flows

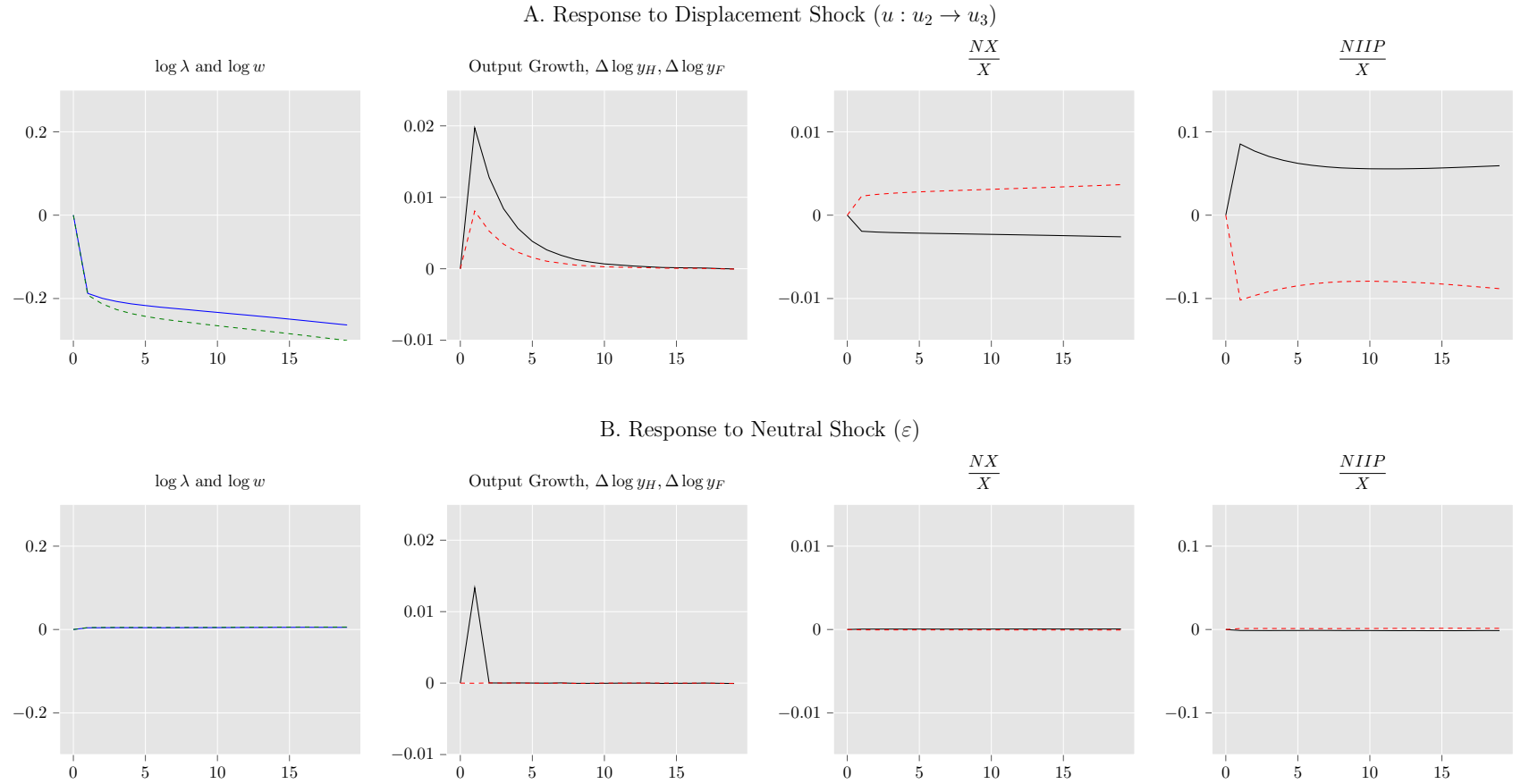


Figure 7: This figure plots the impulse response of variables to a shock to the home country (u in Panel A and ε in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 9. We construct the impulse responses by introducing an additional one-standard deviation shock at time $t=1$ without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock. In the first column, the solid blue line represents the log consumption-expenditure ratio $\log \lambda$, and the dashed green line represents the log wealth ratio $\log w$.

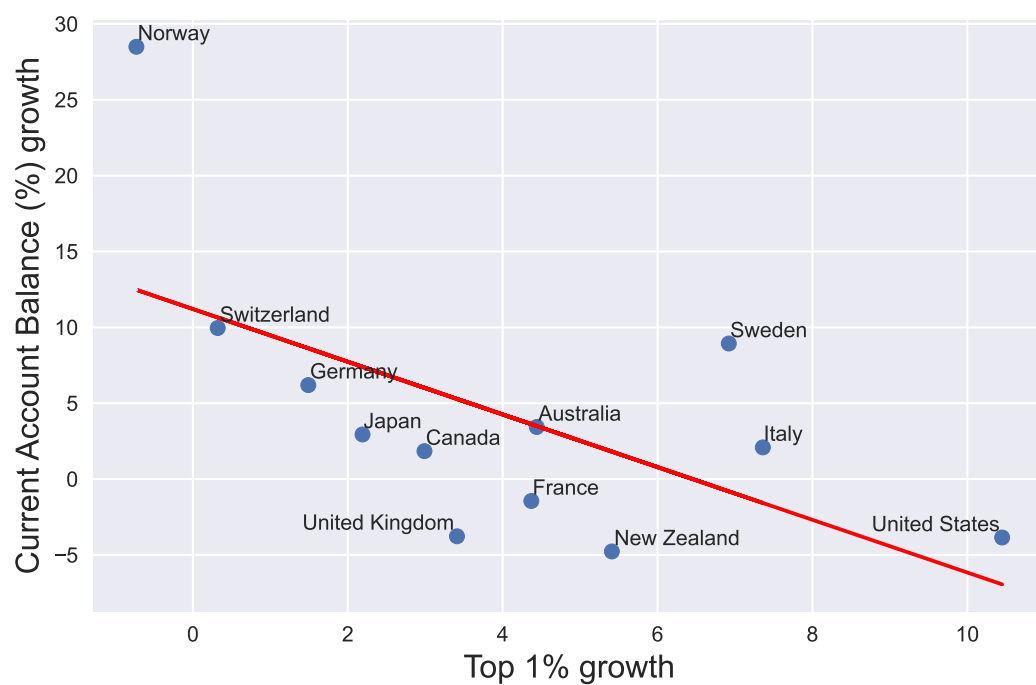


Figure 8: This figure plots the changes between top 1% income share and current account balance (%) from 1980-2022. Income inequality data is from the world inequality database, and the current account balance data is from the world economic outlook database.

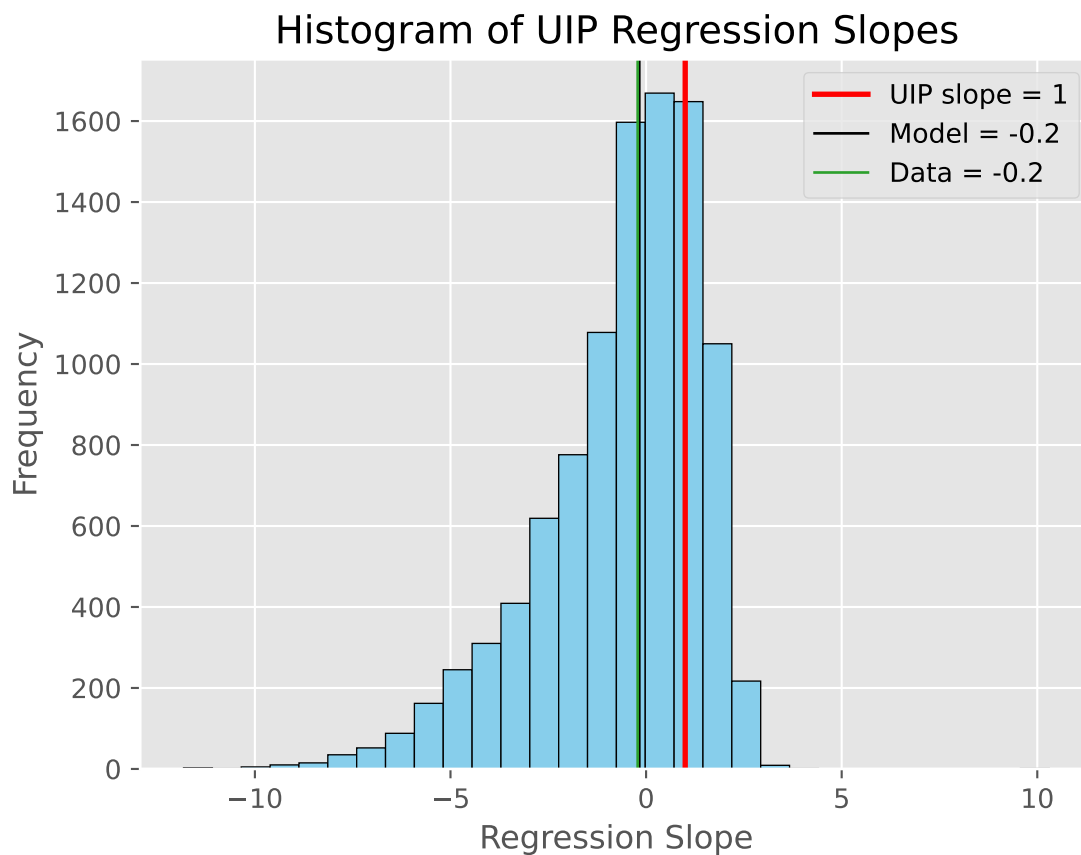


Figure 9: This figure shows the histogram of UIP regression slopes from individual simulations. The calibrated model is simulated 10,000 times, each spanning 150 periods, with UIP regressions performed using the last 50 years of each simulation. The black line represents the mean of the estimated UIP coefficients across all simulations, while the blue line represents the estimate from the data.

Appendix

A Data Appendix

The consumption, GDP and net export data come from the World Bank. We use households final consumption expenditure for consumption series, and the difference between the indices of export of goods and imports of goods and services as our net export series. Both consumption and GDP are real. We use end-of-period exchange rate data from the International Financial Statistics (IMF). Sample period is 1974-2022.

Inflation rates are calculated using Consumer Price Index (CPI) from world bank. The real exchange rate are calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country.

Real interest rates are constructed using three-month T-bills yields from the Global Financial Data, adjusting for realized inflation using annual changes in CPI. The interest rates series for New Zealand and Switzerland starts from 1978 and 1980, respectively. For the rest, the sample period is 1974-2022.

Data on equity index returns (MSCI series) is obtained from Datastream. Equity returns data for New Zealand starts from 1980. Data on top 1% (0.1%) percentage income share and country's total wealth is from World Inequality Database. To calculate each country's wealth in dollars, we multiply the total wealth (data code = mnweal) from the World Inequality Database, denominated in local currency, by the corresponding nominal exchange rate.

The current account data is from the world economic outlook database, spanning from 1980-2022. Data on foreign direct investment net inflows (as a percentage of GDP) and portfolio equity investment net inflows are obtained from the World Development Indicators (World Bank)¹⁵. We divide portfolio equity net inflows by the corresponding year's GDP to measure equity inflows. The foreign institutional ownership is from the Factset Lionshare database. Patents data is from [Kogan et al. \(2017\)](#). U.S. firms fundamentals are from Compustat.

B Model Appendix

Here, we discuss the derivation of the model.

B.1 Simplified Model

The representative agent in each country

First, we show that within a country, finding optimal solutions for heterogeneous agents is equivalent to finding optimal solution for a representative agent.

¹⁵Data is from World Development Indicators. Data code for foreign direct investment is BX.KLT.DINV.WD.GD.ZS; Data code for portfolio equity net inflows is BX.PEF.TOTL.CD.WD.

In each country, even though agents are heterogeneous in their wealth, because of homothetic preference consumption-wealth ratios are equalized.

Consider H country for example, we define the representative agent as

$$U_t^H = \int_{i \in [0,1]} U_{i,t} w_t^{i,H}$$

where $U_{i,t}$ and $w_t^{i,t}$ are the utility and wealth share of household i . That is, the representative agent takes the country-level endowment and the wealth distribution as given and maximizes the wealth-weighted utility.

Because all agents within a country are solving the same optimization problem up to their wealth, so is the wealth-weighted representative agent. Put differently, the representative agent behaves the same way as the individual agent, but scaled up to a wealth that is equal to the country's aggregate wealth. Thus, solving for the equilibrium solutions for heterogeneous agents within a country is equivalent to finding the optimal solution for the representative agent.

Denote C_t^c as the country-level aggregate consumption, the utility of representative agent can be written as

$$U_t^H = \lambda_t^H U(C_t^H)$$

Where $U(x)$ is the utility function for individual household – $U(x) = \log(x)$ in this case. That is, the utility of representative agent is proportional to an fictitious agent who consumes country-level aggregate consumption. The time-varying scaling factor λ_t^H reflects the change of wealth distribution $w_t^{i,c}$ within the country. If market is complete, wealth distribution is invariant and λ_t^H would be a constant.

Now the equilibrium allocation problem reduces to a problem with two (representative) agents and incomplete markets.

Aggregation with log preference

The H's representative agent's utility can be written as

$$U_t^H = \sum_{s=t}^{\infty} \beta^{s-t} \log C_s^H$$

With incomplete markets, the usual construction of a planner's utility as a weighted sum, with constant weights, of individual representative utility function is not possible. Instead, we are going to employ a fictitious planner with stochastic weights.

This fictitious representative agent maximizes his utility subject to the resource constraints:

$$\begin{aligned} \max_{\{x_{H,t}^H, x_{F,t}^H, x_{H,t}^F, x_{F,t}^F\}, t=0,1,2,\dots} \quad & \sum_t \beta^t (\log C_t^H + \lambda_t \log C_t^F) \\ \text{s.t.} \quad & x_{H,t}^H + x_{H,t}^F = Y_{H,t} \end{aligned}$$

$$\begin{aligned}
x_{F,t}^H + x_{F,t}^F &= Y_{F,t} \\
C_t^H &= (x_{H,t}^H)^\alpha (x_{F,t}^H)^{1-\alpha} \\
C_t^F &= (x_{H,t}^F)^{1-\alpha} (x_{F,t}^F)^\alpha
\end{aligned}$$

where we have normalized the weight on the Home representative agent to be equal to one and assigned the weight λ to the foreign representative agent. λ_t is the marginal utilities of either good of the two countries.

Allocations

For concreteness, we focus on the exposition on the Home consumer. First, at each t , we derive the consumer's demands for Y_H and Y_F goods, keeping overall consumption expenditure $\mathcal{C}_H$ fixed.

$$\max_{\{x_{H,t}^H, x_{F,t}^H\}} \alpha \log x_{H,t}^H + (1 - \alpha) \log x_{F,t}^H \quad (52)$$

$$\text{s.t.} \quad p_{h,t} x_{H,t}^H + p_{f,t} x_{F,t}^H = \mathcal{C}_H \quad (53)$$

We obtain the following demands

$$x_{H,t}^H = \frac{\alpha \mathcal{C}_H}{p_{h,t}}, \quad x_{F,t}^H = \frac{(1 - \alpha) \mathcal{C}_H}{p_{f,t}} \quad (54)$$

The indirect utility function defined as $U_H(\mathcal{C}_H, p_{h,t}, p_{f,t})$ is then given by

$$U_H(\mathcal{C}_H, p_{h,t}, p_{f,t}) = \log(\mathcal{C}_H) + F(p_{h,t}, p_{f,t}) \quad (55)$$

Function F depends only on variables that are exogenous from the viewpoint of the consumer and therefore, because of the separability, it drops out the portfolio choice.

Hence, the optimization problem of consumer is equivalent to the single-good consumption-investment problem, with consumption expenditure $\mathcal{C}_H$ replacing the consumption. Importantly, it implies that the prices of individual goods $p_{h,t}, p_{f,t}$ do not pose a risk that the consumer desires to hedge.

With log preference, consumers have constant consumption-to-wealth ratio. Thus, the Pareto weights λ_t is equal to the consumption expenditure ratio, which in turn is equal to the wealth ratio between two countries $\lambda_t = \frac{W_{F,t}}{W_{H,t}}$. Substituting the demand functions in the budget constraints, we get the allocations (??)-(??).

$$x_{H,t}^H = \frac{\alpha}{\alpha + (1 - \alpha)\lambda_t} Y_{H,t} \quad (56)$$

$$x_{H,t}^F = \frac{(1 - \alpha)\lambda_t}{\alpha + (1 - \alpha)\lambda_t} Y_{H,t} \quad (57)$$

$$x_{F,t}^H = \frac{1 - \alpha}{1 - \alpha + \alpha\lambda_t} Y_{F,t} \quad (58)$$

$$x_{F,t}^F = \frac{\alpha \lambda_t}{1 - \alpha + \alpha \lambda_t} Y_{F,t}. \quad (59)$$

SDFs and Asset Prices

SDF. Let $\mathcal{N}_{t+1}^c$ denote the set of all indices of agents in country c who receive worthless ideas at time $t + 1$. In what follows, we will focus on the exposition on the Home consumer. By definition,

$$\frac{M_{t+1}^H}{M_t^H} = \beta \mathbb{E} \left(\frac{c_{t+1}^{i,H}}{c_t^{i,H}} \right)^{-1} = \beta \left(\frac{\int_{i \in \mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i \in \mathcal{N}_{t+1}^H} dC_t^{i,H}} \right)^{-1} \quad (60)$$

where the first equation follows from the consumer's Euler equation and the second equation follows from the probability of receiving a profitable firm being zero. As a result, households' anticipated consumption growth coincides with the consumption growth of the cohort $\mathcal{N}_{t+1}^H$. Market clearing implies:

$$C_{t+1}^H = \int_{i \in \mathcal{N}_{t+1}^H} dC_{t+1}^{i,H} + \int_{i \notin \mathcal{N}_{t+1}^H} dC_{t+1}^{i,H} \quad (61)$$

Note that $\mathbf{1}_{i \notin \mathcal{N}_{t+1}^H} \times \mathbf{1}_{i \notin \mathcal{N}_t^H} = 0$ almost surely, so

$$\int_{i \in \mathcal{N}_{t+1}^H} dC_t^{i,H} = C_t^H \quad (62)$$

Combining (60)-(62) along with the allocation rules (56)-(59) we have that

$$\frac{M_{t+1}^H}{M_t^H} = \beta \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right)^{-\alpha} \left(\frac{Y_{F,t+1}}{Y_{F,t}} \right)^{\alpha-1} \left(\frac{\alpha + (1-\alpha)\lambda_t}{\alpha + (1-\alpha)\lambda_{t+1}} \right)^{-\alpha} \left(\frac{\alpha\lambda_t + 1 - \alpha}{\alpha\lambda_{t+1} + 1 - \alpha} \right)^{\alpha-1} \left(1 - \frac{\int_{i \notin \mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i \in [0,1]} dC_{t+1}^{i,H}} \right)^{-1} \quad (63)$$

Note that with log preference, consumption bundles are proportional to consumption expenditure, which in turn is proportional to wealth. Therefore the last term can be written as

$$b_{H,t+1} = \frac{\int_{i \in \mathcal{N}_{t+1}^H} w_{t+1}^{i,H}}{\int_{i \in [0,1]} w_{t+1}^{i,H}} = \frac{\int_{i \in \mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i \in [0,1]} dC_{t+1}^{i,H}} = 1 - \frac{\int_{i \notin \mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i \in [0,1]} dC_{t+1}^{i,H}} \quad (64)$$

Substituting back we obtain (19). Similarly, we can derive the SDF for foreign consumers.

$$\frac{M_{t+1}^F}{M_t^F} = \beta \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right)^{\alpha-1} \left(\frac{Y_{F,t+1}}{Y_{F,t}} \right)^{-\alpha} \frac{1}{b_{F,t+1}} \frac{\lambda_t}{\lambda_{t+1}} \left(\frac{\alpha + (1-\alpha)\lambda_t}{\alpha + (1-\alpha)\lambda_{t+1}} \right)^{\alpha-1} \left(\frac{\alpha\lambda_t + 1 - \alpha}{\alpha\lambda_{t+1} + 1 - \alpha} \right)^{-\alpha} \quad (65)$$

Asset Prices. Let us first focus on the stock market in Home country. The SDF can be used to price the risky stocks by no arbitrage:

$$S_t^H = p_{h,t}Y_{H,t} + E_t \left[\sum_{s=t+1}^T \frac{M_s^H}{M_t^H} p_{h,s}Y_{H,s} e^{-\sum_{n=t+1}^s u_n^H} \right] \quad (66)$$

Note that M_t^H is the SDF using consumption bundles of the home country; if we define ζ_t^H as the SDF using local goods of home country, then we have

$$M_t^H p_{h,t} = \zeta_t^H$$

Note that the first-order condition of Y_H -good for consumers gives:

$$\zeta_s^H = \beta^{s-t} \frac{\alpha}{c_{H,s}^H} \quad (67)$$

where $c_{H,s}^H$ is the total consumption of Y_H goods by the households who have not received any profitable firms between $t+1$ and s , which has a probability of one. Therefore,

$$c_{H,s}^H = \frac{\alpha}{\alpha + (1-\alpha)\lambda_s} Y_{H,s} \Pi_{t+1}^s b_{H,s} \quad (68)$$

Substituting (67) and (68) into (66), we have

$$S_t^H = p_{h,t}Y_{H,t} E_t \left[\sum_{s=t+1}^T \beta^{s-t} \frac{\Pi_{t+1}^s \frac{1}{b_{H,s}} (\alpha + (1-\alpha)\lambda_s)}{\frac{1}{b_{H,t}} (\alpha + (1-\alpha)\lambda_t)} e^{-\sum_{n=t+1}^s u_n^H} \right] + p_{h,t}Y_{H,t} \quad (69)$$

The derivation for the foreign country's stock market is similar.

The Change of Wealth Distribution

In B.1 we show that the optimization of consumer is equivalent to the single-good consumption-investment problem, with consumption expenditure $\mathcal{C}$ replacing the consumption. Moreover, the consumers do not hedge the prices of individual goods $p_{x,t}, p_{y,t}$.

This implies that the consumers in home and foreign are solving the same portfolio-choice problem. As a result, their optimal portfolios and wealth growth are the same across different states. Hence, the wealth ratio at $t+1$ is given by

$$\lambda_{t+1} = \frac{\int_{i \in [0,1]} w_{t+1}^{i,F}}{\int_{i \in [0,1]} w_{t+1}^{i,H}} = \frac{\int_{i \in \mathcal{N}_{t+1}^F} w_{t+1}^{i,F} + \int_{i \notin \mathcal{N}_{t+1}^F} w_{t+1}^{i,F}}{\int_{i \in \mathcal{N}_{t+1}^H} w_{t+1}^{i,H} + \int_{i \notin \mathcal{N}_{t+1}^H} w_{t+1}^{i,H}} \quad (70)$$

Note that the total value of profitable firms at $t+1$ is related to the displacement shocks u_{t+1}^H, u_{t+1}^F .

From 7 and ?? it follows that the total value of new firms are:

$$\int_{i \notin \mathcal{N}_{t+1}^H} w_{t+1}^{i,H} = S_{t+1}^H (1 - e^{-u_{t+1}^H}) \quad (71)$$

$$\int_{i \notin \mathcal{N}_{t+1}^F} w_{t+1}^{i,F} = S_{t+1}^F (1 - e^{-u_{t+1}^F}) \quad (72)$$

And the total value of old firms is

$$\int_{i \in \mathcal{N}_{t+1}^F} w_{t+1}^{i,F} + \int_{i \in \mathcal{N}_{t+1}^H} w_{t+1}^{i,H} = S_{t+1}^H e^{-u_{t+1}^H} + S_{t+1}^F e^{-u_{t+1}^F} \quad (73)$$

Because the consumers in home and foreign hold the same portfolio, the wealth ratio for the households that do not receive new firms are the same at t and $t + 1$. Hence,

$$\lambda_t = \frac{\int_{i \in [0,1]} w_t^{i,F}}{\int_{i \in [0,1]} w_t^{i,H}} = \frac{\int_{i \in \mathcal{N}_{t+1}^f} w_t^{i,F}}{\int_{i \in \mathcal{N}_{t+1}^h} w_t^{i,H}} \quad (74)$$

Combining (70)-(74) we obtain

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t} \left(S_{t+1}^H e^{-u_{t+1}^F} + S_{t+1}^F e^{-u_{t+1}^H} \right) + \frac{1}{\lambda_t} S_{t+1}^F (1 - e^{-u_{t+1}^F})}{\frac{1}{1+\lambda_t} \left(S_{t+1}^H e^{-u_{t+1}^F} + S_{t+1}^F e^{-u_{t+1}^H} \right) + S_{t+1}^H (1 - e^{-u_{t+1}^H})} \quad (75)$$

Approximate solution

We now derive the approximate analytical solutions near the long-term steady state. That is, when $\lambda_t = 1$ and when u_{t+1}^H, u_{t+1}^F are small.

By symmetry, when $\lambda_t = 1$ the price-dividend ratio of the stock markets are the same. Let us denote this ratio as C_{pd} , i.e.,

$$\left(\frac{S_t^H}{p_{h,t} Y_{H,t}} \right)_{\lambda_t=1} = \left(\frac{S_t^F}{p_{f,t} Y_{F,t}} \right)_{\lambda_t=1} = C_{pd} \quad (76)$$

Using the price ratio relation given by (16), we can rewrite (75) as

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t} \left(e^{-u_{t+1}^H} pd_{t+1}^H + \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} e^{-u_{t+1}^F} pd_{t+1}^F \right) + \frac{1}{\lambda_t} \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} (1 - e^{-u_{t+1}^F})}{\frac{1}{1+\lambda_t} \left(e^{-u_{t+1}^H} pd_{t+1}^H + \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} e^{-u_{t+1}^F} pd_{t+1}^F \right) + (1 - e^{-u_{t+1}^H}) pd_{t+1}^H} \quad (77)$$

where pd_{t+1}^c is the price-dividend ratio of the stock market in country $c \in \{H, F\}$ at $t + 1$. To further simplify, we use the fact that u_{t+1}^H, u_{t+1}^F are small so that $pd_{t+1}^c \approx C_{pd}$ for $c \in \{H, F\}$. Denote the total wealth of the stock market as $\bar{W} = W_H + W_F$, we make the following observation:

$$\bar{W} = S_{t+1}^H + S_{t+1}^F \quad (78)$$

$$\frac{S_{t+1}^F}{S_{t+1}^H} \approx \frac{1 - \alpha + \alpha\lambda_{t+1}}{\alpha + (1 - \alpha)\lambda_{t+1}} \quad (79)$$

The second equation is because $pd_{t+1}^c \approx C_{pd}$. It follows that

$$S_{t+1}^H = \frac{\alpha + (1 - \alpha)\lambda_{t+1}}{1 + \lambda_{t+1}} \bar{W}, \quad S_{t+1}^F = \frac{1 - \alpha + \alpha\lambda_{t+1}}{1 + \lambda_{t+1}} \bar{W} \quad (80)$$

The dynamics of wealth distribution can thus be written as

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t} \left(e^{-u_{t+1}^H} (\alpha + (1 - \alpha)\lambda_{t+1}) + (1 - \alpha + \alpha\lambda_{t+1}) e^{-u_{t+1}^F} \right) + \frac{1}{\lambda_t} (1 - \alpha + \alpha\lambda_{t+1}) (1 - e^{-u_{t+1}^F})}{\frac{1}{1+\lambda_t} \left(e^{-u_{t+1}^H} (\alpha + (1 - \alpha)\lambda_{t+1}) + (1 - \alpha + \alpha\lambda_{t+1}) e^{-u_{t+1}^F} \right) + (\alpha + (1 - \alpha)\lambda_{t+1}) (1 - e^{-u_{t+1}^H})} \quad (81)$$

Denote the common terms in both the numerator and denominator as

$$B = \left(e^{-u_{t+1}^H} (\alpha + (1 - \alpha)\lambda_{t+1}) + (1 - \alpha + \alpha\lambda_{t+1}) e^{-u_{t+1}^F} \right) \quad (82)$$

Some algebra gives

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t} B + \frac{1-\alpha}{\lambda_t} (1 - e^{-u_{t+1}^F})}{\frac{1}{1+\lambda_t} B + \alpha (e^{-u_{t+1}^F} - e^{-u_{t+1}^H}) + (1 - \alpha) \lambda_{t+1} (1 - e^{-u_{t+1}^H})} \quad (83)$$

To progress further, we use the result from B.1 that consumers in both countries have the same portfolios and therefore the same wealth growth. At $t + 1$, the wealth of households in both countries who do not receive profitable firms is

$$\int_{i \in N_{t+1}^H} w_{t+1}^{i,H} + \int_{i \in N_{t+1}^F} w_{t+1}^{i,F} = \left(e^{-u_{t+1}^H} (\alpha + (1 - \alpha)\lambda_{t+1}) + (1 - \alpha + \alpha\lambda_{t+1}) e^{-u_{t+1}^F} \right) \quad (84)$$

$$\frac{1}{1 + \lambda_{t+1}} \bar{W} \quad (85)$$

Because consumers hold the same portfolio, we have

$$\frac{\int_{i \in N_{t+1}^H} w_{t+1}^{i,H}}{\int_{i \in N_{t+1}^F} w_{t+1}^{i,F}} = \frac{\int_{i \in [0,1]} w_t^{i,H}}{\int_{i \in [0,1]} w_t^{i,F}} = \frac{1}{\lambda_t} \quad (86)$$

Therefore

$$\int_{i \in N_{t+1}^H} w_{t+1}^{i,H} = \frac{1}{1 + \lambda_t} \left(e^{-u_{t+1}^H} (\alpha + (1 - \alpha)\lambda_{t+1}) + (1 - \alpha + \alpha\lambda_{t+1}) e^{-u_{t+1}^F} \right) \frac{1}{1 + \lambda_{t+1}} \bar{W} \quad (87)$$

On the other hand, by definition

$$\int_{i \in [0,1]} w_{t+1}^{i,H} = \int_{i \in N_{t+1}^H} w_{t+1}^{i,H} + \int_{i \notin N_{t+1}^H} w_{t+1}^{i,H} \quad (88)$$

so that

$$\begin{aligned}\int_{i \in N_{t+1}^H} w_{t+1}^{i,H} &= \int_{i \in [0,1]} w_{t+1}^{i,H} - \int_{i \notin N_{t+1}^H} w_{t+1}^{i,H} \\ &= \frac{1}{1 + \lambda_{t+1}} \bar{W} - (1 - e^{-u_{t+1}^H}) \frac{\alpha + (1 - \alpha)\lambda_{t+1}}{1 + \lambda_{t+1}} \bar{W}\end{aligned}\quad (89)$$

Substituting (87) and (89) into (83), after some algebra we get

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1 - \alpha + \alpha e^{-u_{t+1}^H} + (1 - \alpha)(1 - e^{-u_{t+1}^F}) \frac{1}{\lambda_t}}{1 - \alpha + \alpha e^{-u_{t+1}^F} + (1 - \alpha)\lambda_t(1 - e^{-u_{t+1}^H})}\quad (90)$$

Using the fact that $e^x \approx 1 + x$ and $\lambda_t = 1$, we have

$$\begin{aligned}\Delta \log \lambda_{t+1} &= \log \frac{1 - \alpha + \alpha e^{-u_{t+1}^H} + (1 - \alpha)(1 - e^{-u_{t+1}^F}) \frac{1}{\lambda_t}}{1 - \alpha + \alpha e^{-u_{t+1}^F} + (1 - \alpha)\lambda_t(1 - e^{-u_{t+1}^H})} \\ &\approx u_{t+1}^F - u_{t+1}^H\end{aligned}\quad (91)$$

To get the approximate expression for log growth of consumption ratio, substituting (56)-(59) into (11), we have

$$\Delta c_{t+1}^H - \Delta c_{t+1}^F = (2\alpha - 1) \left[\Delta \log Y_{H,t+1} - \Delta \log Y_{F,t+1} + \Delta \log \frac{1 - \alpha + \alpha \lambda_{t+1}}{\alpha + (1 - \alpha)\lambda_{t+1}} \right] - \Delta \log \lambda_{t+1}\quad (92)$$

note that $\lambda_{t+1} \approx 1 + \Delta \log \lambda_{t+1}$, so we have

$$\Delta \log \frac{1 - \alpha + \alpha \lambda_{t+1}}{\alpha + (1 - \alpha)\lambda_{t+1}} \approx (2\alpha - 1) \Delta \log \lambda_{t+1}\quad (93)$$

substituting back we get (26). The derivation for log growth of output ratio is straightforward from definitions.

B.2 A micro-foundation for the displacement shock

We consider a continuum of varieties indexed by $j \in [0, 1]$ in each sector $S \in \{H, F\}$. Each variety is produced by a monopolist who faces CES (constant elasticity of substitution) demand and sets its price accordingly. The final good in sector S at time t , denoted by $Y_{S,t}$, is aggregated from these varieties as:

$$Y_{c,t} = Z_{c,t} \left(\int_0^1 [x_{c,t}(j)]^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1,\quad (94)$$

where

$$\Delta \log Z_{c,t+1} = \mu + \varepsilon_{c,t+1}\quad (95)$$

and $x_{S,t}(j)$ denotes the quantity of variety j consumed at time t , and σ is the elasticity of substitution across varieties.

Intermediate goods are produced using a linear technology with land as the only input:

$$x_{S,t}(j) = A_{S,t}(j) l_{S,t}(j), \quad (96)$$

where $A_{S,t}(j)$ denotes productivity and $l_{S,t}(j)$ is the land input. Given the rental rate of land R_t , cost minimization implies:

$$l_{S,t}(j) = \frac{x_{S,t}(j)}{A_{S,t}(j)}, \quad \text{MC}_{S,t}(j) = \frac{R_t}{A_{S,t}(j)}. \quad (97)$$

Each intermediate variety is demanded by final-good producers who aggregate across varieties using a CES aggregator. This implies that the monopolist producing variety j faces the isoelastic demand curve:

$$x_{S,t}(j) = \left(\frac{p_{S,t}(j)}{P_{S,t}} \right)^{-\sigma} Y_{S,t}, \quad (98)$$

where $P_{S,t}$ is the CES price index and $\sigma > 1$ is the elasticity of substitution across varieties.

Given this demand and marginal cost $\text{MC}_{S,t}(j) = R_t/A_{S,t}(j)$, the monopolist sets its price as a constant markup over marginal cost:

$$p_{S,t}(j) = \frac{\sigma}{\sigma - 1} \cdot \frac{R_t}{A_{S,t}(j)}. \quad (99)$$

And price index is

$$P_{S,t} = \left(\int_0^1 p_{S,t}(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (100)$$

Displacement via Entry and Bertrand Competition

At the beginning of period $t+1$, a displacement (or innovation) shock $u_{S,t+1}$ is realized. This shock raises the probability that each product line is challenged by a new entrant,

$$m_{S,t+1} = 1 - e^{-u_{S,t+1}}, \quad 0 < m_{S,t+1} < 1. \quad (101)$$

If an entrant arrives at line j , it competes with the incumbent under Bertrand pricing. The entrant operates with a productivity advantage: it can produce with productivity $e^\delta A_{S,t}(j)$, where $\delta > 0$. As a result, the entrant undercuts the incumbent and takes over the product line. The productivity in line j at time $t+1$ is therefore:

$$A_{S,t+1}(j) = \begin{cases} e^\delta A_{S,t}(j), & \text{with probability } m_{S,t+1}, \\ A_{S,t}(j), & \text{otherwise.} \end{cases} \quad (102)$$

A fraction $m_{S,t+1}$ of incumbents are therefore displaced and replaced by more productive entrants. In each displaced line, the old firm's profits fall to zero, and the entrant captures the full profit

stream.

Aggregate Productivity and Displacement

The final good at $t + 1$ is a CES aggregator over varieties. Using the demand structure, the sectoral price index at $t + 1$ becomes:

$$P_{S,t+1} = \frac{\sigma}{\sigma - 1} R_{t+1} \left(\int_0^1 A_{S,t+1}(j)^{\sigma-1} dj \right)^{-1/(\sigma-1)}. \quad (103)$$

Define the effective productivity aggregator:

$$\tilde{A}_{S,t} \equiv \left(\int_0^1 A_{S,t}(j)^{\sigma-1} dj \right)^{\frac{1}{\sigma-1}}. \quad (104)$$

Using the updating rule in (102), and noting that displacement occurs independently across product lines, we obtain:

$$\begin{aligned} \tilde{A}_{S,t+1}^{\sigma-1} &= (1 - m_{S,t+1}) \tilde{A}_{S,t}^{\sigma-1} + m_{S,t+1} e^{\delta(\sigma-1)} \tilde{A}_{S,t}^{\sigma-1} \\ &= \left[(1 - m_{S,t+1}) + m_{S,t+1} e^{\delta(\sigma-1)} \right] \tilde{A}_{S,t}^{\sigma-1}, \end{aligned}$$

so

$$\tilde{A}_{S,t+1} = \tilde{A}_{S,t} \left[(1 - m_{S,t+1}) + m_{S,t+1} e^{\delta(\sigma-1)} \right]^{\frac{1}{\sigma-1}}. \quad (105)$$

Taking logs,

$$\Delta \log \tilde{A}_{S,t+1} = \frac{1}{\sigma - 1} \log \left[e^{-u_{S,t+1}} + (1 - e^{-u_{S,t+1}}) e^{\delta(\sigma-1)} \right]. \quad (106)$$

Since output is proportional to $\tilde{A}_{S,t}$ and $Z_{c,t}$, we obtain:

$$\Delta \log Y_{S,t+1} = \mu + \varepsilon_{S,t+1} + \frac{1}{\sigma - 1} \log \left[e^{-u_{S,t+1}} + (1 - e^{-u_{S,t+1}}) e^{\delta(\sigma-1)} \right]. \quad (107)$$

For small $u_{S,t+1}$, we have:

$$e^{-u_{S,t+1}} \approx 1 - u_{S,t+1}, \quad e^{\delta(\sigma-1)} \approx 1 + \delta(\sigma - 1), \quad (108)$$

so

$$\Delta \log Y_{S,t+1} \approx \mu + \varepsilon_{S,t+1} + \delta u_{S,t+1}. \quad (109)$$

Summary. This setup captures how an exogenous displacement shock $u_{S,t+1}$ can simultaneously (i) raise aggregate productivity by δ for a subset $m_{S,t+1}$ of product lines, and (ii) reallocate profits from incumbents to new entrants.

- **Creative destruction:** The fraction $m_{S,t+1}$ of varieties that is displaced each period embodies

Schumpeterian creative destruction. Newer, more productive firms replace old incumbents in exactly those lines.

- **Reallocation:** Ownership of that fraction of product lines switches to new entrants. Old incumbents lose the revenue from those lines; new entrepreneurs gain it.
- **Unspanned risk:** If households *cannot* buy ex ante claims on “who will be the successful innovator,” this yields incomplete markets and idiosyncratic risk over who gains or loses in each period’s displacement.

B.3 Full Model

With Epstein-Zin preference, we can construct the representative agent because the aggregation property only depends on the homotheticity of the preference. So in this case, the representative agent constructed above behaves the same as an individual agent in a country but scaled up to the country-level wealth.

Dynamics of the Consumption Ratio

Denote $W_t^c = W(\hat{C}_t^c, U_{t+1}^c)$ as the utility of the representative agent of country c . Denote the partial derivatives with respect to composite consumption and continuation utility as $W_{1,t}^c, W_{2,t}^c$, we have

$$\begin{aligned}\frac{\partial W_t^c}{\partial \bar{C}_t^c} &= \frac{\partial W_t^c}{\partial \hat{C}_t^c} \frac{\partial \hat{C}_t^c}{\partial \bar{C}_t^c} = W_{1,t}^c (\bar{C}_t^c)^{h-1} \\ \frac{\partial W_t^c}{\partial U_{t+1}^c} &= W_{2,t}^c\end{aligned}$$

The intertemporal marginal rate of substitution of representative agent of country c is

$$M_{t,t+1}^c = \frac{\frac{\partial W_{t+1}^c}{\partial U_{t+1}^c} \frac{\partial W_t^c}{\partial \bar{C}_{t+1}^c}}{\frac{\partial W_t^c}{\partial \bar{C}_t^c}} = \frac{W_{2,t+1}^c W_{1,t}^c}{W_{1,t}^c} \left(\frac{\bar{C}_{t+1}^c}{\bar{C}_t^c} \right)^{h-1} \quad (110)$$

International trade of Y_H good implies that the marginal utilities of good Y_H for $t = 1, 2, \dots$ in each possible state is

$$\left(\prod_{j=0}^{t-1} W_{2,j}^H \right) W_{1,t}^H \bar{C}_t^H \frac{\alpha}{x_{H,t}^H} (\bar{C}_t^H)^{h-1} = (\bar{C}_t^F)^{h-1} \frac{1-\alpha}{x_{H,t}^F} \bar{C}_t^F W_{1,t}^F \left(\prod_{j=0}^{t-1} W_{2,j}^F \right) \quad (111)$$

Define the date- t Pareto weights as

$$\begin{aligned}\Lambda_t^c &= \Lambda_0^c \left(\prod_{j=0}^{t-1} W_{2,j}^c \right) W_{1,t}^c \bar{C}_t^c (\bar{C}_t^c)^{h-1} \\ &= \Lambda_{t-1}^c W_{2,t-1}^c \frac{W_{1,t}^c}{W_{1,t-1}^c} \left(\frac{\bar{C}_t^c}{\bar{C}_{t-1}^c} \right)^{h-1} \frac{\bar{C}_t^c}{\bar{C}_{t-1}^c} = \Lambda_{t-1}^c M_{t-1,t}^c \exp(\Delta c_t^c)\end{aligned}$$

Since the economy starts with a symmetric setup, $\Lambda_0^H = \Lambda_0^F$. We can rewrite (111) as

$$\Lambda_t^H \frac{\alpha}{x_{H,t}^H} = \frac{1-\alpha}{x_{H,t}^F} \Lambda_t^F$$

Denote $\lambda_t = \frac{\Lambda_t^F}{\Lambda_t^H}$ as the ratio of Pareto weights. The optimality condition can be written as

$$\lambda_t = \frac{\alpha x_{H,t}^F}{(1-\alpha)x_{H,t}^H} \quad (112)$$

Similar to the log case, note that with Cobb-Douglas preference over different goods, households' consumption expenditure share for each good is fixed. That is, foreign households spend $1-\alpha$ on Y_H -good and home households spend α on Y_H -good. Therefore, (112) shows that λ_t is also the consumption expenditure between foreign and home. That is, $\lambda_t = \frac{C_{F,t}}{C_{H,t}}$. Also, we have that

$$\lambda_{t+1} = \lambda_t \frac{M_{t,t+1}^F e^{\Delta c_{t+1}^F}}{M_{t,t+1}^H e^{\Delta c_{t+1}^H}} \quad (113)$$

Allocations and Exchange Rate

Similar to the log case, since the ratio of consumption expenditure is $\lambda_t = \frac{C_{F,t}}{C_{H,t}}$, we have

$$\begin{aligned} x_{H,t}^H &= \frac{\alpha C_{H,t}}{p_{h,t}}, & x_{F,t}^H &= \frac{(1-\alpha)C_{H,t}}{p_{f,t}}, \\ x_{H,t}^F &= \frac{(1-\alpha)C_{F,t}}{p_{h,t}}, & x_{F,t}^F &= \frac{\alpha C_{F,t}}{p_{f,t}} \end{aligned}$$

Substituting these demands into the resource constraints, we get the allocations (56), (57), (58), (59). Given these allocations, we can calculate the consumption bundles:

$$\bar{C}_{H,t} = (x_{H,t}^H)^\alpha (x_{F,t}^H)^{1-\alpha} \quad (114)$$

$$\bar{C}_{F,t} = (x_{H,t}^F)^{1-\alpha} (x_{F,t}^F)^\alpha \quad (115)$$

We can also compute the price of consumption bundles in home and foreign countries:

$$p_t^H = \frac{p_{h,t}x_{H,t}^H + p_{f,t}x_{F,t}^H}{C_{H,t}} \quad (116)$$

$$p_t^F = \frac{p_{h,t}x_{H,t}^F + p_{f,t}x_{F,t}^F}{C_{F,t}} \quad (117)$$

Note that the relative price of good Y_F in terms of good Y_H is

$$p_t = \frac{Y_{H,t}}{Y_{F,t}} \frac{1-\alpha + \alpha\lambda_t}{\alpha + (1-\alpha)\lambda_t} \quad (118)$$

By definition, the exchange rate is the ratio of prices of consumption bundles:

$$E_t = \frac{p_t^H}{p_t^F} = \frac{\bar{C}_{F,t}}{\bar{C}_{H,t}} \frac{1}{\lambda_t} \quad (119)$$

The exchange rate growth is

$$\frac{E_{t+1}}{E_t} = \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{C}_{F,t+1}/\bar{C}_{F,t}}{\bar{C}_{H,t+1}/\bar{C}_{H,t}} \quad (120)$$

Note that (113) and (120) show that in our model exchange rate growth is equal to the growth of SDF, as the model has integrated financial markets.

SDF

Let us focus on the home country. The derivation for foreign country is similar. Since preference is homothetic, consumption is proportional to wealth. To calculate the SDF of the representative agent, we need to consider two groups of population: the population that receive the new firms in the current period (with measure π , denote as N); and the population that does not receive the new firms in the current period (with measure $1 - \pi$, denote as O).

To this end, first note that $b_{i,t+1}$ is the fraction of wealth account for by the cohort that does not receive profitable projects from period t to $t + 1$ in country i . The wealth shares of these two groups within the home country are

$$b_{H,t}(1 - \pi), b_{H,t}\pi + 1 - b_{H,t}$$

The consumption growth and relative consumption growth for group O are $\frac{\bar{C}_{t+1}}{\bar{C}_t} b_{H,t+1}$ and $b_{H,t+1}$. And the consumption growth and relative consumption growth for group N are $\frac{b_{H,t+1}\pi + 1 - b_{H,t+1}}{\pi} \frac{\bar{C}_{t+1}}{\bar{C}_t}$ and $\frac{b_{H,t+1}\pi + 1 - b_{H,t+1}}{\pi}$. Therefore, the growth in the composite consumption for two groups {O, N} are (we omit the country index H from now on)

$$\begin{aligned} \frac{\hat{C}_{t+1}}{\hat{C}_t}_O &= \left(\frac{\bar{C}_{t+1} b_{t+1}}{\bar{C}_t} \right)^h (b_{t+1})^{1-h} = \frac{\bar{C}_{t+1}^h}{\bar{C}_t^h} b_{t+1} \\ \frac{\hat{C}_{t+1}}{\hat{C}_t}_N &= \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi} \right)^h \left(\frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi} \right)^{1-h} \\ &= \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^h \frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi} \end{aligned}$$

Similarly, we can derive the growth in continuation utility for these two groups. Since the consumption to utility ratio are equalized across two groups, we have

$$\frac{\left(\frac{U_{O,t+1}^{1-\gamma}}{E_t(U_{t+1}^{1-\gamma})} \right)}{\left(\frac{U_{N,t+1}^{1-\gamma}}{E_t(U_{t+1}^{1-\gamma})} \right)} = \frac{b_{t+1}}{\frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi}} \quad (121)$$

The growth of the SDF of these two groups can be written as $(M_{x,t,t+1} = \frac{M_{x,t+1}}{M_{x,t}}, x \in \{O, N\})$

$$M_{O,t,t+1} = \frac{M_{O,t+1}}{M_{O,t}} = \beta \left(\frac{\hat{C}_{t+1}}{\hat{C}_t} \right)_O^{-\frac{1}{\psi}} \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{h-1} \left(\frac{U_{O,t+1}^{1-\gamma}}{\mathbb{E}_t(U_{t+1}^{1-\gamma})} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

$$M_{N,t,t+1} = \frac{M_{N,t+1}}{M_{N,t}} = \beta \left(\frac{\hat{C}_{t+1}}{\hat{C}_t} \right)_N^{-\frac{1}{\psi}} \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{h-1} \left(\frac{U_{N,t+1}^{1-\gamma}}{\mathbb{E}_t(U_{t+1}^{1-\gamma})} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

Note that the time $t + 1$ state used to define the intertemporal marginal rates of substitution above includes the investor type, N or O, which is idiosyncratic and becomes known only at time $t + 1$. Because each agent is assigned new projects in proportion to their current wealth level and becoming an innovator in each period is independent of all other shocks in the economy, the equilibrium SDF in this economy (which prices claims contingent on aggregate states) can be expressed as the conditional expectation of the inter-temporal marginal rate of substitution of any agent, which is conditioned on the aggregate state at time $t + 1$. Hence the cross-sectional average of investors' inter-temporal marginal rates of substitution is a stochastic discount factor. That is,

$$M_{t,t+1} = \frac{M_{t+1}}{M_t} = (1 - \pi)M_{O,t,t+1} + \pi M_{N,t,t+1}$$

$$= \beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\frac{h}{\psi} + h - 1} \left(\pi \left(\frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi} \right)^{-\frac{1}{\psi}} \left(\frac{U_{N,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{t+1}^{1-\gamma}]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}} + (1 - \pi) b_{t+1}^{-\frac{1}{\psi}} \left(\frac{U_{O,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{t+1}^{1-\gamma}]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}} \right)$$

Combining with (121), we have (44).

Wealth ratio

First, note that the marginal utility of consumption of the representative agent in each country is

$$\frac{\partial \tilde{U}}{\partial C} = (1 - \tilde{\beta}) \tilde{U}^{\frac{1}{\psi}} \hat{C}^{-\frac{1}{\psi}} \bar{C}^{h-1}$$

we can compute the wealth of households who didn't receive projects at t , in units of local consumption bundles:

$$\begin{aligned} \hat{W}_H &= \frac{\tilde{U}}{\frac{\partial \tilde{U}}{\partial C_H}} \\ &= \frac{1}{1 - \beta} (\tilde{U})^{1-1/\psi} (\hat{C}_{H,t})^{\frac{1}{\psi}} \bar{C}_{H,t}^{1-h} \\ &= \frac{1}{1 - \beta} \left(\frac{\tilde{U}_{H,t}}{\hat{C}_{H,t}} \right)^{1-1/\psi} \hat{C}_{H,t} \bar{C}_{H,t}^{1-h} \\ &= \frac{1}{1 - \beta} \left(\frac{\tilde{U}_{H,t}}{\hat{C}_{H,t}} \right)^{1-1/\psi} \bar{C}_{H,t} \end{aligned}$$

Similarly we can derive the wealth for foreign country,

$$\hat{W}_F = \frac{1}{1-\beta} \tilde{U}_{F,t}^{1-1/\psi} \hat{C}_{F,t}^{1/\psi} \bar{C}_{F,t}^{1-h} = \frac{1}{1-\beta} \left(\frac{\tilde{U}_{F,t}}{\hat{C}_{F,t}} \right)^{1-1/\psi} \bar{C}_{F,t} \quad (122)$$

Note that the wealth above are calculated in the units of local consumption bundles, so the ratio of two countries' wealth should be adjusted by the price of their respective consumption bundles

$$\frac{W_F}{W_H} = \frac{\hat{W}_F}{\hat{W}_H} \frac{p_F}{p_H} = \left(\frac{\frac{U_{F,t}}{\hat{C}_{F,t}}}{\frac{U_{H,t}}{\hat{C}_{H,t}}} \right)^{1-1/\psi} \lambda_t \quad (123)$$

The second equation comes from the fact that $\lambda = \frac{p_F \bar{C}_F}{p_H \bar{C}_H}$ (Recall (112)).

Asset Prices

Similar to the log case, we have

$$\begin{aligned} S_t^H &= p_{h,t} Y_{H,t} + E_t \left[\frac{M_{t+1}^H}{M_t^H} S_{t+1}^H \right] \\ pd_t^H &= E_t \left[\frac{M_{t+1}^H}{M_t^H} \frac{p_{h,t+1} Y_{H,t+1}}{p_{h,t} Y_{H,t}} (1 + pd_{t+1}^H) e^{-u_{t+1}^H} \right] \\ S_t^F &= p_{f,t} Y_{F,t} + E_t \left[\frac{M_{t+1}^F}{M_t^F} S_{t+1}^F \right] \\ pd_t^F &= E_t \left[\frac{M_{t+1}^F}{M_t^F} \frac{p_{f,t+1} Y_{F,t+1}}{p_{f,t} Y_{F,t}} (1 + pd_{t+1}^F) e^{-u_{t+1}^F} \right] \end{aligned}$$

Trade and Capital Flows

The net export as a fraction of total output is

$$\frac{NX_t^H}{Y_{H,t}} = \frac{p_{h,t} Y_{H,t} - p_{h,t} x_{H,t}^H - p_{f,t} x_{F,t}^H}{p_{h,t} Y_{H,t}} = 1 - \frac{1}{\alpha + (1-\alpha)\lambda_t} \quad (124)$$

$$\frac{NX_t^F}{Y_{F,t}} = \frac{p_{f,t} Y_{F,t} - p_{f,t} x_{F,t}^F - p_{h,t} x_{H,t}^F}{p_{f,t} Y_{F,t}} = 1 - \frac{\lambda_t}{1 - \alpha + \alpha\lambda_t} \quad (125)$$

The net international investment position scaled by country's wealth is

$$\frac{A_t^H}{p_{h,t} Y_{H,t}} = \frac{W_t^H - S_t^H}{W_t^H} \quad (126)$$

$$\frac{A_t^F}{p_{f,t} Y_{F,t}} = \frac{W_t^F - S_t^F}{W_t^F} \quad (127)$$

B.4 Numerical Procedure

Here, we briefly describe the numerical procedure for solving the model.

The equilibrium is obtained by jointly solving the set of non-linear equations that describe the equilibrium conditions: (9), (11), (16), (20), (35), (56), (57), (58), (59), (123), (44). We put all the equations here, as below:

On the aggregate level, we have

$$d \log Y_{H,t} = \mu + \delta u_{H,t} + \varepsilon_{H,t}$$

$$d \log Y_{F,t} = \mu + \delta u_{F,t} + \varepsilon_{F,t}$$

For each country's allocation we have (56)-(59).

$$x_{H,t}^H = \frac{\alpha}{\alpha + (1 - \alpha)\lambda_t} Y_{H,t} \quad (128)$$

$$x_{H,t}^F = \frac{(1 - \alpha)\lambda_t}{\alpha + (1 - \alpha)\lambda_t} Y_{H,t} \quad (129)$$

$$x_{F,t}^H = \frac{1 - \alpha}{1 - \alpha + \alpha\lambda_t} Y_{F,t} \quad (130)$$

$$x_{F,t}^F = \frac{\alpha\lambda_t}{1 - \alpha + \alpha\lambda_t} Y_{F,t} \quad (131)$$

The displacement effect:

$$b_{H,t+1} = 1 - \frac{(1 + pd_{H,t+1})(1 - e^{-u_{H,t+1}})}{\left(1 + pd_{H,t+1} + (1 + pd_{F,t+1}) \frac{p_{f,t+1} Y_{F,t+1}}{p_{h,t+1} Y_{H,t+1}}\right) \frac{1}{1+w_{t+1}}} \quad (132)$$

$$b_{F,t+1} = 1 - \frac{(1 + pd_{F,t+1})(1 - e^{-u_{F,t+1}})}{\left((1 + pd_{H,t+1}) \frac{p_{h,t+1} Y_{H,t+1}}{p_{f,t+1} Y_{F,t+1}} + (1 + pd_{F,t+1})\right) \frac{w_{t+1}}{1+w_{t+1}}} \quad (133)$$

where the dividend ratio is

$$\frac{p_{f,t} Y_{F,t}}{p_{h,t} Y_{H,t}} = \frac{1 - \alpha + \alpha\lambda_t}{\alpha + (1 - \alpha)\lambda_t}$$

Post-Dividend price-dividend ratios are given by

$$pd_t^H = E_t \left[\frac{M_{t+1}^H}{M_t^H} \frac{p_{h,t+1} Y_{H,t+1}}{p_{h,t} Y_{H,t}} (1 + pd_{t+1}^H) e^{-u_{t+1}^H} \right] \quad (134)$$

$$pd_t^F = E_t \left[\frac{M_{t+1}^F}{M_t^F} \frac{p_{f,t+1} Y_{F,t+1}}{p_{f,t} Y_{F,t}} (1 + pd_{t+1}^F) e^{-u_{t+1}^F} \right] \quad (135)$$

Aggregate consumption is given by

$$\bar{C}_t^H = (x_{H,t}^H)^\alpha (x_{F,t}^H)^{1-\alpha} \quad (136)$$

$$\bar{C}_t^F = (x_{H,t}^F)^{1-\alpha} (x_{F,t}^F)^\alpha \quad (137)$$

The two SDFs are given by (44),

$$\frac{M_{t+1}^c}{M_t^c} = \beta \left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}} \right)^{-\frac{h}{\psi} + h - 1} b_{o,c,t+1}^{-\frac{1}{\psi}} \left(\frac{U_{o,c,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{c,t+1}^{1-\gamma}]} \right)^{\frac{1/\psi - \gamma}{1-\gamma}} \quad (138)$$

$$\frac{M_{t+1}^c}{M_t^c} = \beta \left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}} \right)^{-\frac{h}{\psi} + h - 1} b_{n,c,t+1}^{-\frac{1}{\psi}} \left(\frac{U_{n,c,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{c,t+1}^{1-\gamma}]} \right)^{\frac{1/\psi - \gamma}{1-\gamma}} \quad (139)$$

where

$$\begin{aligned} b_n &= \frac{b\pi + 1 - b}{\pi} \\ b_o &= b \\ \frac{U_{n,t+1}}{C_t} &= \frac{U_{n,t+1}}{C_{n,t+1}} \frac{\bar{C}_{t+1}}{\bar{C}_t} \left(\frac{b\pi + 1 - b}{\pi} \right) \\ \frac{U_{o,t+1}}{C_t} &= \frac{U_{o,t+1}}{C_{o,t+1}} \frac{\bar{C}_{t+1}}{\bar{C}_t} b \end{aligned}$$

We use cross-sectional average as the aggregate SDF:

$$\frac{M_{t+1}}{M_t} = \beta \left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}} \right)^{-\frac{h}{\psi} + h - 1} \left(\pi \left(\frac{b_{c,t+1}\pi + 1 - b_{c,t+1}}{\pi} \right)^{-\frac{1}{\psi} + \frac{1/\psi - \gamma}{1-\gamma}} + (1 - \pi) b_{c,t+1}^{-\frac{1}{\psi} + \frac{1/\psi - \gamma}{1-\gamma}} \right) \left(\frac{\bar{U}_{c,t+1}^{1-\gamma}}{\mathbb{E}_t[U_{c,t+1}^{1-\gamma}]} \right)^{\frac{1/\psi - \gamma}{1-\gamma}}$$

and the wealth ratio is given by (123) and the lambda ratio by (113).

B.5 Calibration Details

The model has a total of 16 parameters. We choose the probability of household death $\xi = 1/40$, which corresponds to an average working life of 40 years. We put two restrictions on the dynamics of u shocks to reduce the number of parameters. First, we assume that $u_1 = u_2$. Hence, a transition from u_1 to u_2 only affects the future distribution of u (as the transition probabilities change) rather than the current level of displacement. Second, we assume that the matrix T corresponds to transition matrix of a discretized AR(1) process, so that it could be parameterized by only two parameters—the corresponding autocorrelation parameter p and q . Specifically, we assume that the transition matrix has the following form

$$T = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix} \quad (140)$$

Where p^2 is the probability of staying in the lowest state once already there and q^2 is the probability of staying in the highest state once there. We estimate the remaining parameters of the model using a simulated minimum distance method [Lee and Ingram \(1991\)](#). Specifically, given a vector of X of

target statistics in the data, we obtain parameter estimates by

$$\hat{p} = \arg \min_{p \in \mathcal{P}} \left(X - \frac{1}{S} \sum_{i=1}^S \hat{X}_i(p) \right)' W \left(X - \frac{1}{S} \sum_{i=1}^S \hat{X}_i(p) \right) \quad (141)$$

Where $\hat{X}_i(p)$ is the vector of statistics computed in one simulation of the model. Our choice of weighting matrix $W = \text{diag}(XX')^{-1} I_W$ penalizes proportional deviations of the model statistics from their empirical counterparts. I_W is a diagonal matrix that adjusts for the relative importance of the statistics in our estimation. We apply a factor of 10 on the equity risk premium, the volatility of exchange rate and the UIP slope. The rest elements on the diagonal of I_W are normalized to one.

We use different weights on the diagonal of I_W to reflect the relative importance of the following moments: equity risk premium, the volatility of exchange rate and the UIP slope. We do this because the magnitude of these moments are relatively well documented in the literature, and also speaks directly to the model's mechanism.

Our calibration targets are reported in the first column of Table 8. They include a combination of first and second moments of aggregate quantities, asset prices and exchange rates. They are defined as follows:

1. **Consumption, output and net export.** Output is gross domestic product. Consumption is households and NPISHs final consumption expenditure (private consumption). Net export is the exports of goods and services minus the imports of good and services.
2. **Standard deviation of aggregate quantities.** We first calculate the standard deviation for each US-foreign country pair, and then we take the average and use that as our target.
3. **Bilateral correlations between aggregate quantities.** Similar to the standard deviation, we first calculate the correlation for each US-foreign country pair, and then we take the average and use that as our target.
4. **Real exchange rate.** Inflation rates are calculated using Consumer Price Index (CPI) from world bank. The real exchange rate are calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country.
5. **Risk free rate and Stock market returns.** Risk free rate is constructed using three-month T-bills yield, adjusting for realized inflation using annual changes in CPI. Stock market returns are obtained using MSCI indexes from Datastream.
6. **UIP coefficient.** for each US-foreign country pair, we regress the exchange rate growth from t to $t + 1$ on the interest rate differentials at t :

$$\Delta e_{US,F,t,t+1} = \alpha_F + \beta_{UIP,F}(r_{F,t} - r_{US,t}) + \varepsilon_{F,t}$$

Then we take an average of the estimated $\beta_{UIP,F}$ across all countries F in our sample.

7. **Inequality, wealth and the coefficients in regression.** Income inequality and wealth data is from World Inequality Database, the top 1% income share including capital income. We use the estimated coefficients of the panel regression with country fixed effects, as in Table 4 and Table 7. In these regressions, independent variables are standardized using unconditional moments.
8. **Dollar and U.S displacement.** The correlation between the equal-weighted dollar index and the proxy for U.S. displacement as described in section 2.6.
9. **U.S. displacement and wealth ratios.** The correlation between the proxy for U.S. displacement and the growth of wealth ratios.
10. **Coefficients of bi-variate regressions.** The regression slopes that correspond to the bi-variate regression in Table 4.

In addition to these standard international moments in the literature, we also target a set of correlations. The neutral shock and displacement shock have different implications for the cyclicalities of the exchange rates. Thus, the set of correlation between exchange rates and consumption, output and stock market, together with the set of bilateral correlations, are informative about the relative magnitude of these two shocks. We also target the estimated coefficients of regressions (41).

We simulate the model at annual frequency. For each simulation, we first simulate 100 years data as burn-in, to remove the samples' dependencies on the initial condition. Then, we simulate the data for 50 years – the same length as our empirical sample. The simulation starts with the symmetric steady state where the displacement shocks are at the middle state and $\lambda = 1$. In each iteration we simulate 10000 samples, and simulate pseudo-random variables using the same seed in each iteration.

Solving each iteration of the model is costly, and thus computing the minimum (141) using standard methods is infeasible. We therefore use the Radial Basis Function (RBF) algorithm in Björkman and Holmström (2000). The Björkman and Holmström (2000) algorithm first fits a response surface to data by evaluating the objective function at a few points. Then, it searches for a minimum by balancing between local and global search in an iterative fashion. We use a commercial implementation of the RBF algorithm that is available through the TOMLAB optimization package.

Appendix Tables and Figures

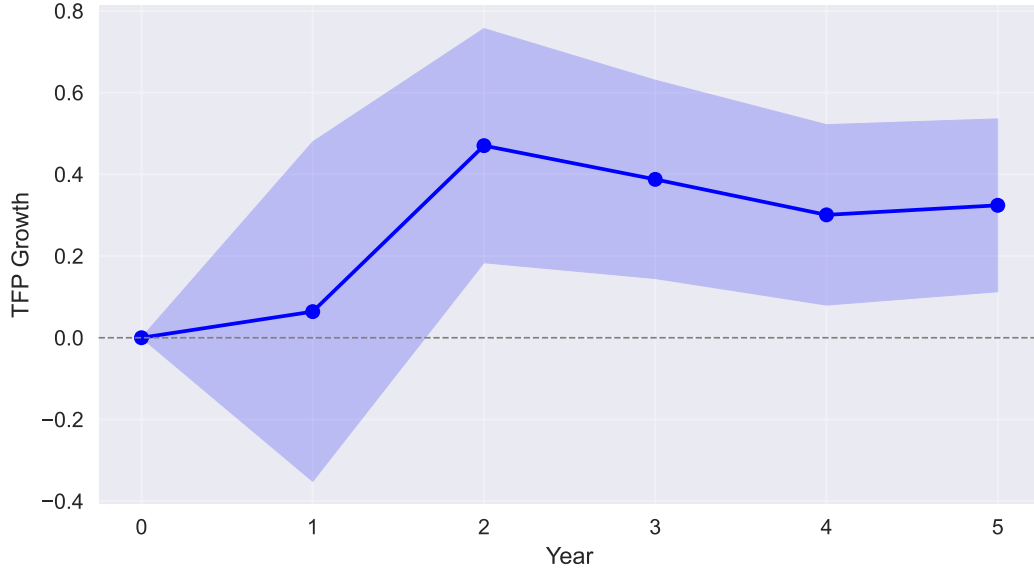
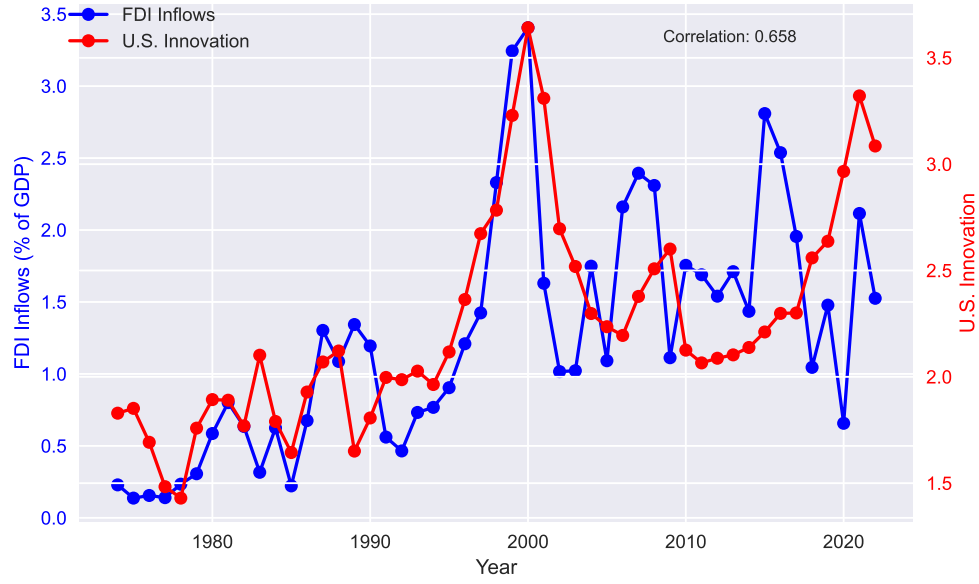


Figure A.1: Innovation and Total Factor Productivity. This figure plots the coefficients β_s (for $s = 1, 2, 3, 4, 5$) from the following specification, $\Delta A_{t,t+s} = \alpha + \beta_s \text{Inno}_{t+1} + \beta_c X_t + \varepsilon_{t+s}$. US innovation is measured as the log of the ratio of the total value of patents each year from [Kogan et al. \(2017\)](#) to the total market value. TFP growth is based on annual utilization-adjusted US TFP from [Fernald \(2014\)](#). Vector of controls includes the lagged level of TFP and lagged level of innovation.

Panel A. US Innovation and US FDI Inflows



Panel B. US Innovation and US Portfolio Equity Inflows

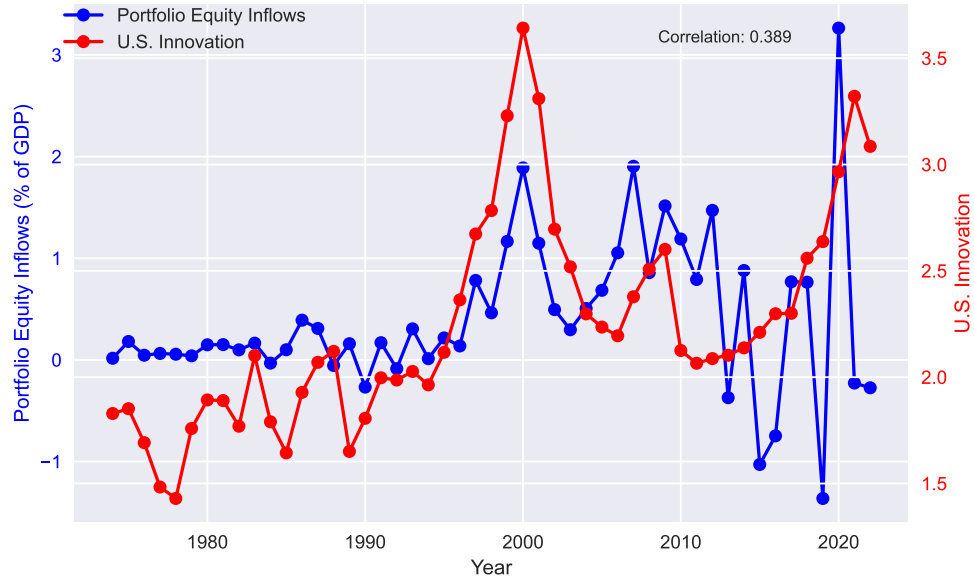


Figure A.2: This figure plots the US innovation index and foreign direct investment inflows (in Panel A) and portfolio equity inflows (in Panel B) in the US. The US innovation is measured as the annual average real value of patents (Kogan et al. (2017)). The aggregate FDI inflows and portfolio equity inflows are obtained from the World Bank. See Appendix A for details.

Table A.1: Dollar Index growth and U.S. Innovation

Time Series Estimate (TW Dollar Index)						
	1-Year	1-Year	1-Year	3-Year	3-Year	3-Year
KPSS/MKT	0.018* (0.009)	0.024* (0.014)	0.027** (0.012)	0.042** (0.018)	0.047** (0.019)	0.058*** (0.019)
Lagged Output growth	YES	YES	YES	YES	YES	YES
Lagged Dollar Index	YES	NO	YES	YES	NO	YES
Lagged Innovation	NO	YES	YES	NO	YES	YES
Observations	49	49	49	47	47	47
R2	0.127	0.080	0.146	0.404	0.253	0.479

Notes: The table reports regression results of the growth of log dollar index on U.S. innovation:

$$\Delta \log e_{t,t+s}^{USD} = \alpha + \beta_1 Inno_{US,t,t+s} + \beta_2 X_t + \varepsilon_t$$

The sample period is 1974-2022, $s = 1, 3$. U.S. innovation is measured by the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The dollar Index is an trade-weighted average real value of the US dollar index, obtained from the Fed. Control variable X_t includes lagged innovation, lagged output growth and lagged Dollar Index level at $t - s$. Both series are in logs. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.2: Dollar Index growth and U.S. Innovation

Depende Variable = Dollar Index Growth						
	1-Year	1-Year	1-Year	3-Year	3-Year	3-Year
KPSS(avg)	0.028** (0.012)	0.038 (0.025)	0.031 (0.023)	0.051** (0.022)	0.114*** (0.021)	0.106*** (0.017)
Lagged Dollar Index	YES	NO	YES	YES	NO	YES
Lagged Innovation	NO	YES	YES	NO	YES	YES
Observations	49	49	49	47	47	47
R-squared	0.188	0.053	0.188	0.493	0.239	0.581

Notes: The table reports regression results of the growth of log dollar index on U.S. innovation:

$$\Delta \log e_{t,t+s}^{USD} = \alpha + \beta_1 Inno_{US,t,t+s} + \beta_2 X_t + \varepsilon_{t+s}$$

The sample period is 1974-2022. U.S. innovation is measured as the log of the ratio of the average real value of patent each year (Kogan et al. (2017)) . The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Control variable X_t includes lagged innovation and lagged Dollar Index level at t . Both series are in logs. The sample consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. Standard errors (in parentheses) are obtained using Newey-West with one/three period lag. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.3: Exchange rate growth and U.S. Innovation

	Panel	AUS	CAN	CHE	DEU	FRA	GBP	ITA	JPN	NOR	NZL	SWE
Displacement	0.039*** (0.011)	0.047*** (0.017)	0.030** (0.011)	0.029* (0.017)	0.038 (0.028)	0.051* (0.026)	0.034* (0.018)	0.042 (0.026)	0.031 (0.021)	0.050*** (0.016)	0.057*** (0.019)	0.063*** (0.019)
Observations	467	49	49	49	25	25	49	25	49	49	49	49
R2	0.194	0.297	0.248	0.286	0.250	0.297	0.222	0.262	0.105	0.237	0.278	0.230

Notes: The table reports regression results of the growth of log exchange rate on displacement shocks:

$$\Delta \log e_{t+1} = \alpha + \beta_1 Displacement_t + \beta_2 X_t + \varepsilon_t$$

The sample period is 1974-2022. U.S. displacement is measured as described in section 2.6. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, United Kingdom, Switzerland, France and Italy. The vector of controls X_t includes lagged displacement and lagged exchange rate at t . Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Exchange rate, consumption and GDP data are from the World Bank and the IMF. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.4: Exchange rate growth and U.S. Innovation

	Dependent Variable = FX Growth										
	AUS	CAN	CHE	DEU	FRA	GBP	ITA	JPN	NOR	NZL	SWE
KPSS/MKT	0.046** (0.020)	0.041*** (0.014)	0.017 (0.023)	0.019 (0.040)	0.003 (0.041)	0.030 (0.022)	0.012 (0.041)	0.002 (0.025)	0.045** (0.022)	0.061** (0.025)	0.049* (0.026)
Observations	49	49	41	25	25	49	25	49	49	44	49
R-squared	0.277	0.271	0.205	0.223	0.234	0.221	0.266	0.112	0.160	0.250	0.138

Notes: The table reports regression results of the growth of log dollar exchange rate on U.S. innovation:

$$\log e_{t+1} - \log e_t = \alpha + \beta_1 \text{Inno}_{US,t+1} + \beta_2 X_t + \varepsilon_{t+1}$$

The sample period is 1974-2022. U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. X_t account for lagged exchange rate, lagged U.S. innovation, and the lagged growth of the output ratio between the U.S. and each foreign country. The sample consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Control variable X_t includes lagged innovation and lagged exchange rate at t . Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.5: Inequality growth and exchange rate

<i>Panel A. Exchange rate and inequality growth</i>												
	Panel	AUS	CAN	CHE	DEU	FRA	GBP	ITA	JPN	NOR	NZL	SWE
Inequality growth	0.019*** (0.005)	0.056*** (0.017)	-0.010 (0.017)	0.023 (0.025)	0.049 (0.062)	0.043 (0.028)	0.039** (0.017)	0.097** (0.040)	0.025 (0.023)	-0.002 (0.014)	0.013 (0.013)	0.010 (0.017)
Observations	418	49	49	42	18	25	42	18	42	42	49	42
R2	0.131	0.399	0.132	0.203	0.188	0.233	0.303	0.405	0.084	0.093	0.152	0.122
<i>Panel B. Wealth change and inequality growth</i>												
	Panel	AUS	CAN	CHE	DEU	FRA	GBP	ITA	JPN	NOR	NZL	SWE
Inequality growth	0.022*** (0.006)	0.064*** (0.018)	-0.013 (0.019)	0.016 (0.027)	0.035 (0.057)	0.070** (0.030)	0.037* (0.019)	0.069 (0.046)	0.045* (0.025)	0.002 (0.015)	0.019 (0.020)	0.003 (0.020)
Observations	396	49	49	42	18	25	42	18	42	42	27	42
R2	0.107	0.416	0.124	0.108	0.164	0.355	0.142	0.331	0.198	0.154	0.122	0.112

Notes: Panel A of the table reports regression results of the growth of log exchange rate on log income inequality growth ratio.

$$\log e_{t+1} - \log e_t = \alpha + \beta \Delta \log I_{t+1} + \gamma X_t + \varepsilon_{t+1}$$

where $\Delta \log I_{t+1}$ is the growth of the ratio of top 0.1% income share. The sample period is 1974-2022. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and we report [Driscoll and Kraay \(1998\)](#) standard errors in parentheses. Panel B repeats the analysis with the dependent variable equal to the growth of wealth ratios. Data on income inequality and wealth is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Exchange rates, Productivity Growth and Consumption Growth (Both Countries)

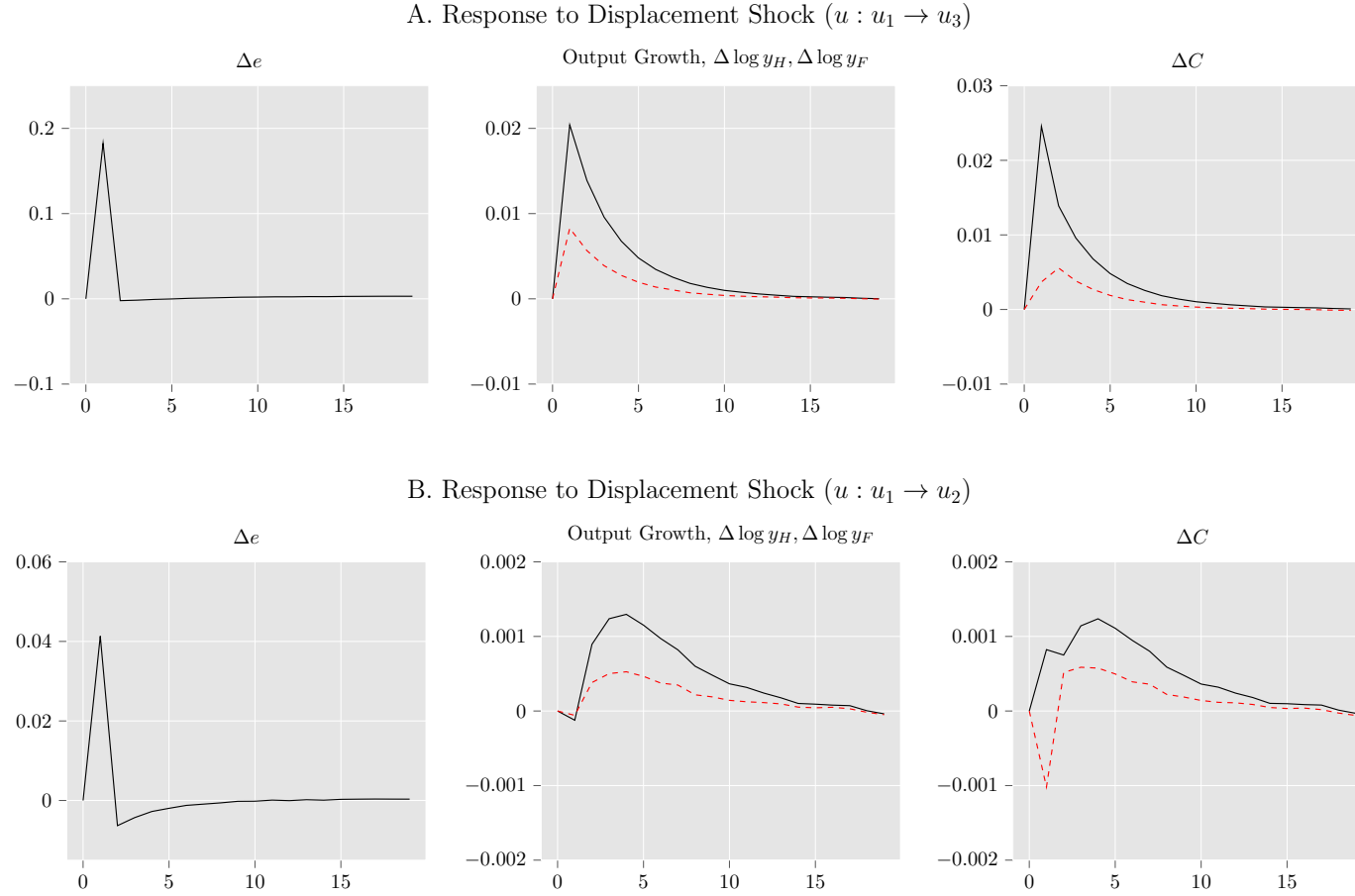
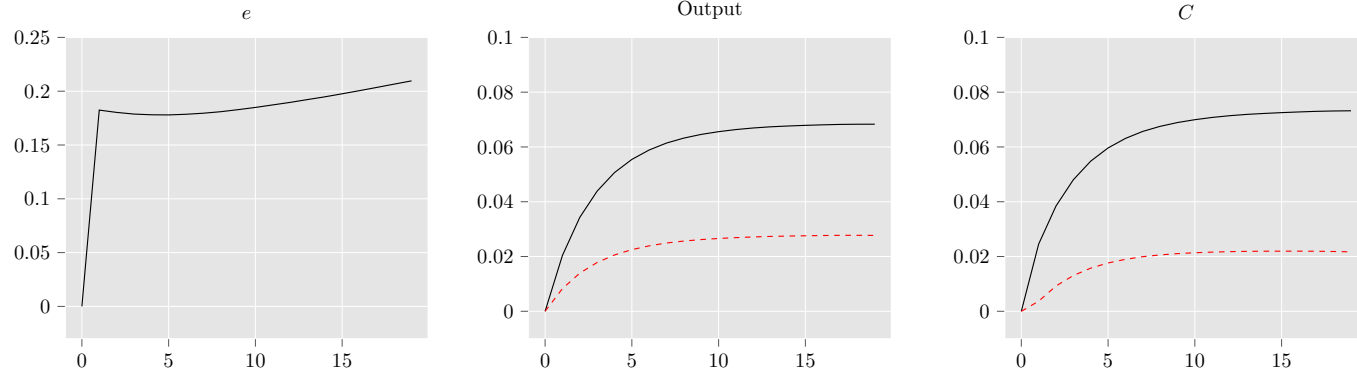


Figure A.3: This figure plots the impulse response of variables to a shock to the home country ($u_1 \rightarrow u_3$ in Panel A and $u_1 \rightarrow u_2$ in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 9. We construct the impulse responses by introducing an additional one-standard deviation shock at time $t=1$ without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

Exchange rates, Productivity Growth and Consumption Growth (Both Countries, Level)

A. Response to Displacement Shock ($u : u_1 \rightarrow u_3$)



B. Response to Displacement Shock ($u : u_1 \rightarrow u_2$)

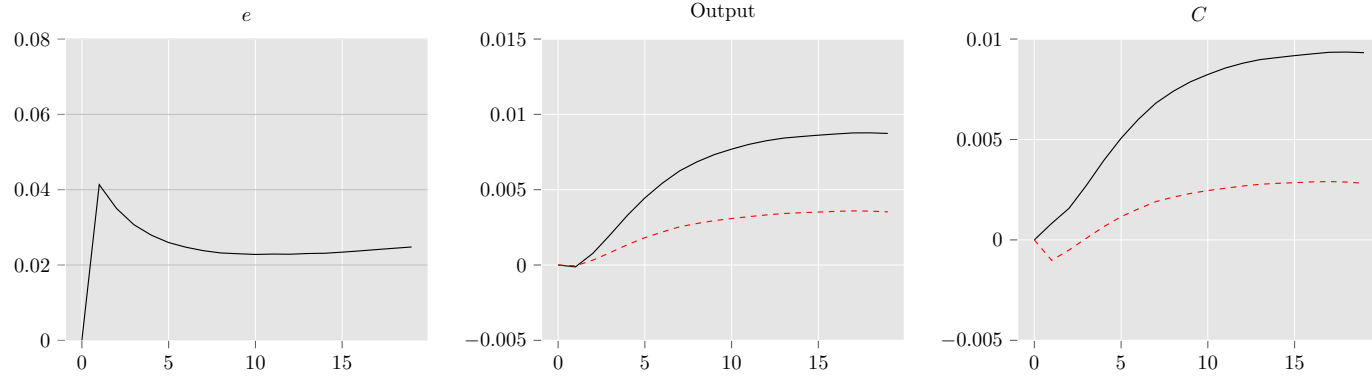


Figure A.4: This figure plots the impulse response of variables to a shock to the home country ($u_1 \rightarrow u_3$ in Panel A and $u_1 \rightarrow u_2$ in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 9. We construct the impulse responses by introducing an additional one-standard deviation shock at time $t=1$ without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

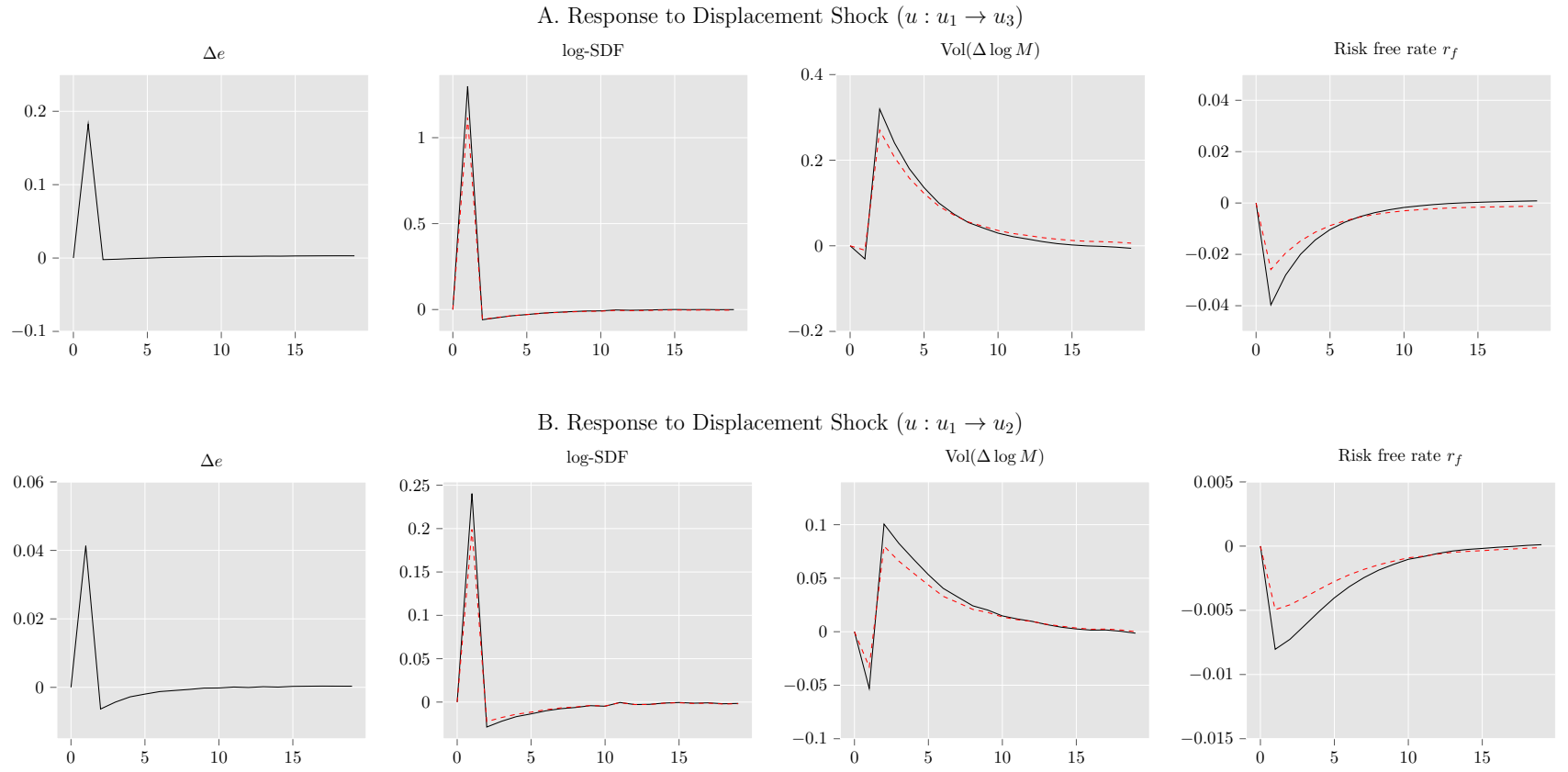


Figure A.5: This figure plots the impulse response of variables to a shock to the home country ($u_1 \rightarrow u_3$ in Panel A and $u_1 \rightarrow u_2$ in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 9. We construct the impulse responses by introducing an additional one-standard deviation shock at time $t=1$ without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

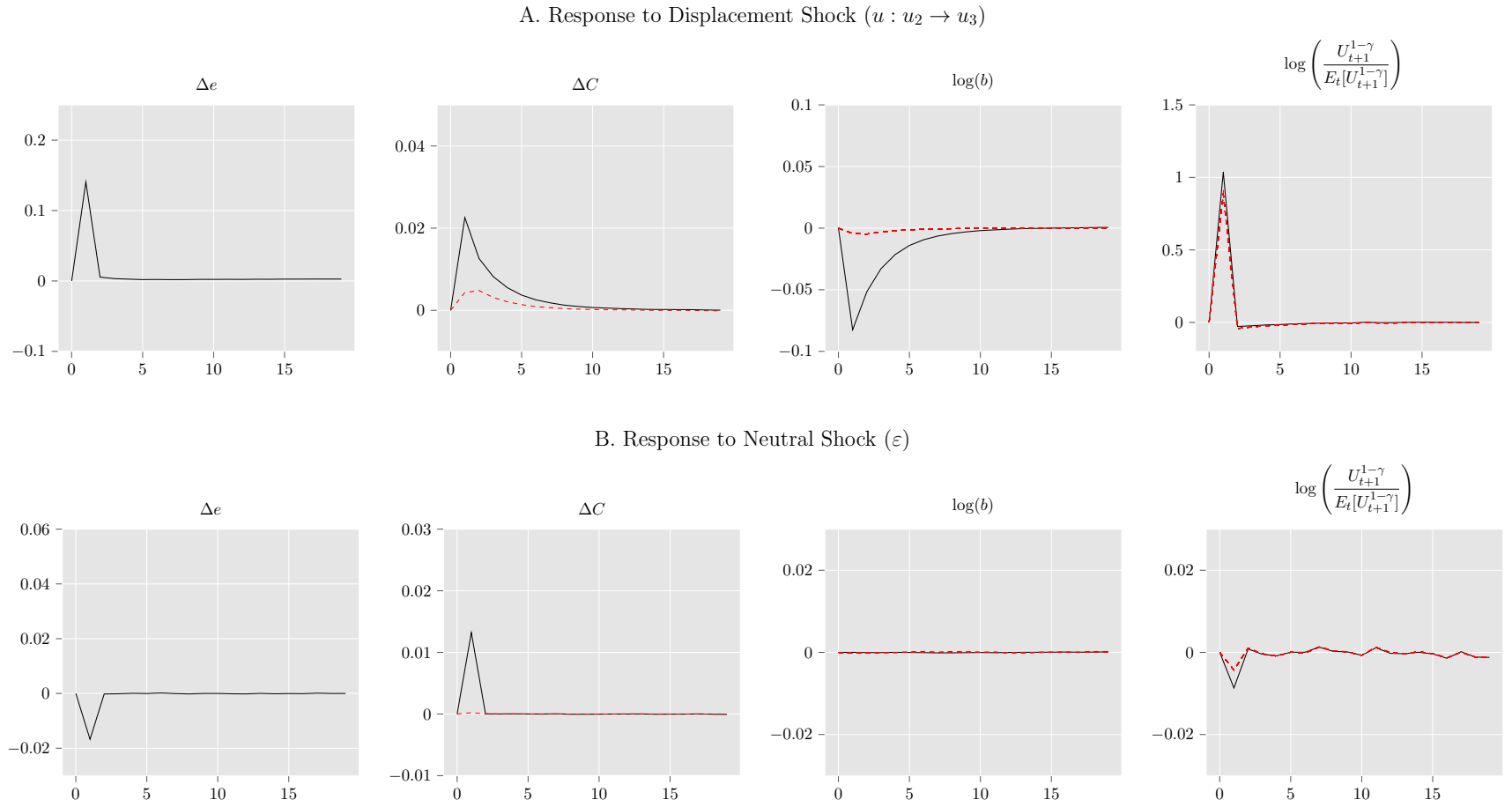


Figure A.6: This figure plots the impulse response of variables to a shock to the home country (u in Panel A and ε in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 9. We construct the impulse responses by introducing an additional one-standard deviation shock at time $t=1$ without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.