THE CHANGING STRUCTURE OF CORPORATE PROFITS[*]

MURRAY Z. FRANK[†] JING GAO[‡]

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Abstract

What explains the evolution of corporate profits and firm dominance in the U.S economy over the past half century? We find that the median public firm assets grew at approximately 2% annually until the early 1990s and then accelerated to 5% annually. Large firms did particularly well. Firms in the top quintile became both larger and more profitable relative to the median firm. Using decomposition analysis and estimated transition matrices, we identify two main mechanisms that drive the growing large firm advantage. Large firms had greater reductions in flow costs than did the median firm. Large firms disproportionately benefited from the long term declining interest rates. We find no evidence of increased persistence among top firms. Approximately 30% of the top quintile firms exit within 5 years. That is true both in earlier decades and in recent decades. Exit rates and firm size transition rates have been quite stable. However, firm entry rates declined significantly since 2000. The evidence calls into question critical aspects of the claims of declining business dynamism and increasing entrenchment of large firms due to market power. A tractable heterogeneous firm model with state-dependent transitions is provided to give a unified account of our firm profit facts.

JEL codes: E2, G3, L2

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[†]Carlson School of Management, University of Minnesota (contact author, e-mail: murra280@umn.edu)

[‡]Carlson School of Management, University of Minnesota (e-mail: gao00268@umn.edu)

I. INTRODUCTION

Corporate profits play a critical role for understanding economic performance, resource allocation, and distributional outcomes. Despite the central role of profits, economists disagree not only about the main determinants, but even about basic trends over the decades. A number of papers have documented rising markups and increasing concentration, (Grullon et al. 2019, De Loecker et al. 2020). But other studies report declining corporate profits when measuring profits slightly differently (Davis et al. 2023). This conflict points to a fundamental open question. How has the structure of corporate profits actually evolved in recent decades, and what are the main factors responsible?

To answer this question, we analyze corporate profits of publicly traded US firms from 1971 to 2022. The main goal of this paper is to document the growing large firm advantage and how it relates to large firm persistence. In the data we find marked changes in the size distribution of firms and in their profits. From 1971 to the early 1990s the median public firm size (measured by book assets) grew at approximately 2% annually. That increased to about 5% annually through 2022. Firms in the top quintile gradually became larger and more profitable relative to the median firm. Importantly, this growing large firm advantage did not translate into increased firm persistence. Roughly 30% of the top quintile firms exit within five years. This is true in the earlier period and in more recent decades. This is directly contrary to the influential idea that firms have become increasingly entrenched in recent decades, perhaps due to changes in antitrust enforcement in the US (Covarrubias et al. 2020).

In order to understand these patterns in the data, we start with a simple model of Cobb-Douglas production with isoelastic demand (Hsieh & Klenow 2009, Haltiwanger et al. 2018, Kehrig & Vincent 2021), extended to include a non-zero opportunity cost of funds. Within this framework we explore three potential mechanisms: output market power, changing production technologies, and the impact of declining interest rates. We find that the growing large firm advantage is importantly affected by the large firm's better ability to cut flow costs, and the large firm's extra advantage from declining interest rates.

Our four main empirical results are not consistent with a common narrative regarding corporate profits. First, it is both true and well known that large firms have become more profitable. Not so well known is that firm size and profits are not good proxies for each other. Specifically, high firm profit is much less persistent than large firm size. Exceptional profitability has long been difficult to sustain and it still is.

Second, we find that changing cost structures are central to the diverging results between large firms and median firms. Firms of all sizes increased their total assets relative to sales. But large firms were able to obtain substantial reductions in flow costs that smaller firms were unable to match. At the same time the long-term decline in interest rates benefited large firms more strongly than smaller firms. There was always a gap in how the market treated large and small firms, but that gap got bigger.

The growing large firm advantage does not necessarily mean that specific dominant firms remain dominant as the years go by (Schumpeter 1942). Many once high profile firm names are barely remembered. Readers of a certain age may recall examples like Blackberry, Blockbuster, Borland, Circuit City, Gateway Computers, Howard Johnson's, Lotus 1-2-3, LTV Corporation, Palm, Polaroid, RadioShack, Saab, Sports Authority, Sears, or Westinghouse. Even among firms that do persist, unusually high profits seem to erode; think of Disney, General Electric, IBM, or Nokia for example. So we turn next to see if the growing large firm advantage is associated with greater large firm persistence.

Third, we show that there is more turnover among top firms than often recognized, and the turnover rates are rather stable. We use a version of the Expectation-Maximization method (Dempster et al. 1977, Moon 1996, Cappé et al. 2005) to estimate transition probabilities across firm states while accounting for firms that enter and exit our sample. The approach allows us to deal with the missing potential-firm problem. In other words we do not directly observe the number of firms that do not currently exist in our data, but might. Using the number of firms that enter, the number that exit, and consistency requirements, we can make reasonable inferences about the number of unobserved potential firms. Our estimates show that exit rates for firms in the top profit quintile have remained remarkably stable over the decades.

Fourth, not everything has remained stable. The most significant change in firm dynamics since 2000 is the sharp decrease in entry rates. However, exit rates and transition probabilities for existing firms across firm size quintiles remained stable. This evidence taken together is consistent with regulatory changes that increased the costs of going public such as Sarbanes-Oxley (Zhang 2007) and with improved financing options for private firms due to the National Securities Market Improvement Act (Ewens & Farre-Mensa 2020). It is inconsistent with theories emphasizing reduced competitive pressure on dominant firms. Within our extended model we estimate that the number of potential firms actually increased since 2000, so there may be more at work than just an increased regulatory cost of entry.

The evidence on firm transitions make it clear that our static single firm model is inadequate. So we extend the basic model to allow for firm transitions. To unify our findings, we develop a dynamic model with heterogeneous firms, with state-dependent transitions, and equilibrium price effects. Firms can enter, change sizes, and exit. Our framework accounts for the empirical patterns in a particularly simple manner, and it provides a foundation for policy counterfactual analysis.

To recap, relative to previous studies we provide novel evidence of the important role of firm inputs, the cost of capital, entry conditions, and firm persistence to account for the changing structure of profits observed at public firms over the decades. We document considerable churning among existing firms, consistent with perspectives such as Schumpeter (1942), Demsetz (1973). We estimate firm transitions using the Expectations-Maximization algorithm and this allows us to infer the impact of potential firms that do not currently exist, on the actions of firms that do exist. This is done by estimating their impact within a logically complete model. It should be stressed that we focus on establishing robust facts, the impacts of the technological change and the capital market conditions across firms. We do not explain why the technology changed nor do we explain why the capital market gradually started treating large firms better than other firms. A prominent idea is that computer-related technologies played a central role (Acemoglu 2002, Acemoglu & Restrepo 2022). The timing seems generally consistent with that idea. Furthermore, some of the cross-industry differences we document in the Internet Appendix also seem suggestive of that. However, we do not directly study that idea in this paper. Motivated by the debate in the literature, our focus is on establishing the facts themselves, and on providing a simple framework to interpret those facts.

Related Literature. We contribute to several inter-related strands of economic research on corporate profits, market structure, and firm dynamics. There are four main areas where we contribute. First, our evidence is directly related to the literature on the measurement and evolution of corporate profits. Recent papers have conflicting conclusions about profit trends. Some studies document rising margins and markups (De Loecker et al. 2020, Basu 2019, Grullon et al. 2019). Other studies find declining profitability (Davis et al. 2023). We show that this disagreement stems partly from measurement challenges. Traina (2018) and Syverson (2019) both highlight the fact that profit measures can be sensitive to expenditures like SG&A. They can produce significantly different conclusions for that reason. In Appendix A we show in detail how different accounting measures can generate the range of results observed in previous studies. Proper accounting for asset utilization costs is critical for understanding the true evolution of corporate profits.

Second, we extend the literature on large firm advantage and market concentration. Kwon et al. (2024) documents the growing importance of the top 1% of firms over the past century. Autor et al. (2020) emphasize the rise of superstar firms that capture increasing market share. The role of star firms is also important to Kehrig & Vincent (2021) and Ayyagari et al. (2024). Covarrubias et al. (2020) connects these trends to weakened antitrust enforcement, particularly after 2000. They explicitly say that dominant firms have become more entrenched. Our findings challenge this entrenchment idea. We demonstrate that despite their growing size and profitability, large firms have not had increased persistence. We provide novel transition probability estimates. Using those together with simpler descriptive measures, we show that exit rates for top firms have remained remarkably stable. Competitive forces apparently continue to constrain corporate dominance much as they did in earlier decades. This evidence complements work by Kehrig & Vincent (2021) and Haltiwanger et al. (2018) on firm heterogeneity, and at the same time we provide a new perspective on business dynamism.

Third, there is a literature devoted specifically to business dynamism (Luttmer 2007, Decker et al. 2016, 2020, Akcigit & Ates 2021). Luttmer (2010) stressed the importance of market entry in this context. We contribute to that literature by providing a new way to estimate the impact of changes to market entry conditions. We interpret the increase profit concentration as tying back to production technology. We also show the relative impact of the long run decline in interest rates on large firms relative to small, which seems to be inadequately recognized. Our analysis of firm transitions provides a distinctive perspective on business dynamism than seems to be common in the literature. We show that large firms have not become entrenched with protected monopoly rents. Instead, they remain just as likely to exit as previously. But the decline of new firm entry and changing capital market conditions play a more significant role in our interpretation of the changing firm dynamics.

Fourth, our evidence and our model contribute to understanding the mechanisms driving divergent firm performance by size. Much of the literature emphasizes output market power (De Loecker et al. 2020) or labor market monopsony (Berger et al. 2022). We provide evidence that two distinct mechanisms seem to be more important for understanding the evolution of corporate profits. First is the differential adaptation of cost structures. Second is the unequal benefits from declining interest rates. In documenting the importance of cost structure changes, we build on Lee et al. (2021), who show that high-Q firms increasingly focus on rewarding existing investors rather than pursuing investment opportunities. Our evidence that large firms benefited disproportionately from declining interest rates complements work by Liu et al. (2022) and Kroen et al. (2021). But we challenge the idea that these effects primarily operate through imperfect competition. Instead, our evidence is indicative of significant interactions between increasingly costly entry due to regulatory changes, and falling interest rates. This provides a consistent explanation for observed patterns.

We integrate these facts into a unified dynamic model. In that way our paper addresses a significant gap in the literature. Previous work has typically focused on the measurement of profits (Barkai 2020, Karabarbounis & Neiman 2014), the documentation of trends (Kwon et al. 2024), or theoretical mechanisms in isolation (Liu et al. 2022). Our approach combines measurement, a novel comprehensive empirical analysis of transitions, and simple model to provide a more complete account of the changing structure of corporate profits.

Outline of the paper The rest of the paper is organized as follows. Section II provides the static model that initially motivates our empirical work. Section III describes the data and provides descriptive statistics. Section IV shows the aggregate profit trends. Section V examines firm size differences. Section VI provides estimates of firm transition probabilities. To allow for the impact of firm size transitions, section VII extend the static model. Numerical results and counterfactual entry policy are considered. Section VIII discusses some alternative assumptions and extensions to the model. Section IX concludes. A number of Internet appendices are also provided. Appendix A is about measuring profits using accounting data. Appendix B provides algebraic detail for the simple model. Appendix C is about the high profile Magnificent Seven firms. Appendix D provides estimates from several other major countries. Appendix E is an overview of the estimation of transition probabilities. Appendix F is about using maximum likelihood/Expectations-Maximization to estimate the transitions probabilities. Appendix G provides algebraic details for the model with transitions.

II. MODEL

In this section we provide a model that focuses our attention on the firm's production as a way to understand the evolution of profits. Consider a firm that raises funds from investors and uses those funds to purchase flow inputs such as raw materials, labor and also productive assets like a machine. It uses these inputs to produce a good. It sells that good to generate sales revenue. Now suppose that we observe many such firms over a number of years. We find that the price-cost margin increased over time. What happened?

Sales revenue could have increased due to an increase in output market power (De Loecker et al. 2020, Covarrubias et al. 2020, Akcigit & Ates 2021). But it is also possible that at least some firms became better at using the inputs to produce output – improved productive technology (Ackerberg et al. 2015, De Loecker & Syverson 2021). How do the fund raising, the flow inputs and the productive assets connect to each other and to profitability?

To address this simply, consider a model of a firm that has Cobb-Douglas production function and isoelastic demand. This conceptual benchmark has been considered in a number of studies such as (Hsieh & Klenow 2009, Haltiwanger et al. 2018, Kehrig & Vincent 2021). We modify the model slightly by allowing for a non-zero opportunity cost of funds so that there is a role for the investor within the model. Then we use this model to derive the revenue share equations that we examine empirically. In section VII we extend this model to allow for transitions among firms of different sizes, entry and exit.

There is a single firm with a Cobb-Douglas production technology and isoelastic demand for its output. If the firm operates, it must pay a non-monetary fee κ. The firm chooses total assets A and flow inputs F to produce output Y using the production technology $Y = \theta A^{\alpha} F^{1-\alpha}$, where $\theta > 1$ and $\alpha \in (0,1)$. The prices of A and F are $p_A > 0$ and $p_F > 0$, respectively. Demand is given by an inverse demand curve $P = P(Y) = \phi Y^{-\mu}$ where $\phi > 0$ is a demand parameter and $\mu \in (0,1)$ reflects the firm's market power. Revenue $R(A,F)$ is output quantity times output price. The outside opportunity cost of funds is $\rho \in (0,1)$. An investor with wealth $W > 0$ chooses between investing in the firm or in an outside opportunity. The investor can be interpreted as the firm's owner and W might be interpreted as the firm's internal resources.

In the empirical work A will be interpreted as book total assets (AT) and F will be interpreted as the Cost of Goods Sold plus Selling, General and Administrative expenses, ($COGS+SGA$). That is why we use the notation A and F rather than K and L. This approach to using Compustat data is similar to De Loecker et al. (2020) but we use total assets rather than just property, plant and equipment. Firms need their assets in order to operate, not just PP&E; and all of those assets have an opportunity cost. The opportunity cost of funds will generally be proxied by the 10-year rate on US government bonds.

If the investor stays out, the payoff is $V^{out} = (1+\rho)W$. The alternative is to invest some of W in the firm to earn V^{In}. The firm solves $V = \max\{V^{out}, V^{In}\}$. How big is V^{In}? It is given by

$$V^{In} = \max_{\{A,F\}} R(A,F) - \kappa + (1+\rho)(W - p_A A - p_F F) \tag{1}$$

It is assumed that the inputs must be paid for in advance and so they reduce the amount of money that earns the outside return. κ prevents the firm from becoming arbitrarily small. For the firm to operate, it must cover the cost of κ and it must also be able to cover the opportunity cost of funds. These conditions are set out algebraically in the Appendix B. The firm's first order conditions are simple enough to be solved in closed form to get the optimal choices for A and F. These are also provided in the Appendix B.

Sales revenue shares provide an attractive way to understand the model. The sales revenue shares for flow cost share s_A, fixed cost share s_F, and revenue share s_Π are

$$s_A \equiv \frac{p_A A}{PY} = \frac{\alpha(1-\mu)}{1+\rho}, \tag{2}$$

$$s_F \equiv \frac{p_F F}{PY} = \frac{(1-\alpha)(1-\mu)}{1+\rho}, \tag{3}$$

$$s_\Pi \equiv \frac{PY - p_A A - p_F F - \kappa}{PY} = \frac{\mu}{1+\rho} + \frac{\rho}{1+\rho} - \frac{\kappa}{PY}, \tag{4}$$

The key implications follow directly. Increases in κ, p_A, or p_F make firm operation less likely. An increase in demand (ϕ) makes firm operation more likely. An increase in ρ leads to smaller operating firms and fewer firms operating overall. The Cobb-Douglas parameter α and market power μ play critical roles in determining revenue shares.

III. DATA DESCRIPTION

This section provides the data sources, sample construction, and key patterns observed in publicly traded firms from 1971-2022. Our main data source is Compustat. It covers U.S. publicly traded firms from 1971 to 2022. We begin in 1971 due to rather sparse Compustat coverage in earlier years. For international comparisons in Appendix D, we use Worldscope data for the G-7 countries spanning 1980-2021. All dollar values are converted to 2017 dollars using the GDP deflator from the Federal Reserve Economic Data (FRED) database. Following standard practice, we exclude regulated firms (utilities, railroads, and telecommunications) and financial firms due to their distinct operational, regulatory, and accounting characteristics. Our data cleaning methodology follows Frank & Goyal (2024), which provides detailed implementation procedures in Stata code format. Table A.1 maps our key variables to their corresponding Compustat codes and defines the financial ratios used in our analysis. For the aggregate trends we used aggregate data in FRED (https://fred.stlouisfed.org/) and compare it to what is observed overall for the publicly traded firms in Compustat.

Profit is conceptually defined as the money resulting from selling products and services, minus the cost of producing these products and services. Matching this concept to real world accounting data presents several challenges such as timing, accruals, allocations, and contingent events. Dechow et al. (2010) explain that even determining a theoretically correct approach to these issues is difficult. In Appendix A we show that different well known papers have adopted different accounting measures to call ‘profits’. This is part of the reason for different profit trends reported in various papers. In Appendix A we provide evidence on these alternatives and the extent to which they matter for inferences. They turn out to be critical to account for firms’ financing costs.

We define profits for firm i in year t as,

$$\Pi_{it} = \text{Sales}_{it} - \text{COGS}_{it} - \text{SGA}_{it} - T_{it} - \rho \text{AT}_{it} \tag{5}$$

where Π_{it} is the profit for firm i at date t, Sales_{it} is firm i's sales revenue in period t, COGS_{it} is cost of goods sold, SGA_{it} is selling, general and administrative costs, T_{it} is taxes, ρ is the opportunity cost of funds, AT_{it} is total assets. We refer to the sum of Cost of Goods Sold (COGS_{it}) plus Selling, General, and Administrative expenses (SGA_{it}) as 'flow costs'. Our profit definition is sometimes called Econmic Value Added. We also examining the impact of alternative definitions in Appendix A.

Some firm characteristics have changed over time. Table I presents both equal-weighted and book value-weighted statistics by decade, revealing significant changes over time. Mean total assets grew substantially from $764.69 million (1971-1979) to $3,534.20 million (2020-2022), as also illustrated in Figure I.

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Table I and Figure I about here.
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Equal-weighted average profits have been negative since the 1980s. This is because there were many small unprofitable public firms. Value-weighted profits remain positive. Median $\frac{\Pi}{AT}$ declined from 0.08 (1971-1979) to 0.06 (2020-2022). Figure II further illustrates these aggregate profit trends.

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Figure II about here.
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Mean $\frac{Debt}{AT}$ increased from 0.29 (1971-1979) to 0.37 (2020-2022). However, the interest-to-asset ratio remained stable under equal-weighting while declining for large firms when value-weighted, suggesting a competitive advantage for larger entities.

Figure III shows that there was a long-term change in the 10-year Treasury and BAA corporate bond yields. Following the sharp increases around 1981 during the Volcker-led inflation fight, rates declined gradually until reaching near-zero levels in 2010-2020. There has been some extra volatility post-2000, notably during the 2008 financial crisis and COVID-19 pandemic.

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Figure III about here.
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In the Appendix, Figure A.9 shows changes in firm profitability distribution at 10-year intervals. Several features jump out. There is a shift towards greater profits, particularly evident in 2011 and 2021. There is an increase in profit concentration in the highest profit bin, especially from 1991 onwards. There is a growing left tail, with an increased share of firms in the lowest profit bins. Again that is particularly marked in 2011 and 2021. Overall we see a widening disparity across firms. There is a growing gap between share of firms and share of profits in the highest profit bin from 1971 to 2021. Also in the Appendix, Figure A.10 compares distributions of profits, sales, and total assets between 1971 and 2021. They confirm a consistent rightward shift across all measures. This strengthens the idea that a fundamental economic transformation took place, and it was not just an accounting artifact. There is evidence of a significant break in trends that seems to have taken place around 2000.

These distributional shifts are not limited to just the top firms (Covarrubias et al. 2020, Kwon et al. 2024). The rightward shifts of the entire distributions make it less likely that they result solely from policies like antitrust changes favoring the largest firms. It makes it more likely that it is connected to the declining presence of small, unprofitable public firms as stressed by (Kahle & Stulz 2017). In summary, the data reveals a profound restructuring of the corporate sector over the past half-century, characterized by increasing firm size, growing profit concentration at the top firms, and a widening gap between the successful top firms and the median public firm.

IV. AGGREGATE PROFIT TRENDS

In this section we show the overall evolution of corporate profits at publicly traded US firms from 1971 to 2022. Figure II provides the foundation by illustrating the ratios of profits to total assets for both the median firm and the aggregate of all firms. For the median firm analysis the ratio of profits to total assets is constructed for each firm in the year, and then the median of that ratio is plotted. The aggregate instead adds up profits for the numerator and adds up total assets for the denominator. Then the ratio is formed for these aggregates. The aggregate is more reflective of large firms since they have a larger role to play in both the numerator and in the denominator.

From the start of our data in 1971 until the early 1980s the profitability of the median firm very closely tracked aggregate profitability. Starting in the early 1980s, a clear gap opens up and it never really closes. In fact since the financial crisis of 2007-2009 the bottom quartile of firms seems to have fallen further behind the median firm as their losses became larger.

To understand the overall evolution it helps to examine the profit quartiles in Figure II. Since 1980 there has been consistently negative profits in the lowest quartile of firms. A substantial fraction of the public firms operate at a loss each year. The difference between the first quartile and the third quartile has grown progressively larger over time. Top and bottom firms have pulled further away from each other.

Over a number of decades both the median and the aggregate profit ratios have drifted upwards. The large and the median firms have become more profitable. The bottom quartile of firms have not been able to keep up. As a result aggregate profits are increasingly concentrated in the hands of the larger firms. We explore this aspect in greater depth in section V.

These observations are robust to alternative profit measures, as detailed in Appendix A. Measures such as EBITDA, NOPAT, and EVA have quantitative differences. But they also exhibit the qualitative pattern of a gap between aggregate and median firm profits. It is noteworthy that profit measures that properly take into account the opportunity cost of capital, such as our baseline EVA measure, show a sharper upward trend in profits when compared to accounting measures that omit this aspect.

While these changes were taking place, major changes are also observed in the corporate use of finance. Table II shows that there was a declining interest cost for large firms when compared to smaller firms. This difference is particularly marked after 2000. There was a long-term decline in interest rates as shown in Figure III. Both Treasury yields and BAA corporate bond rates decline significantly after the early 1980s inflation problems. The decline is very prolonged. It happens faster for the higher credit bonds, represented here by Treasury rates. For less credit-worthy loans than those depicted the, decline was even slower. This trend is a noteworthy aspect to understand how corporate profit changed. The slowly dropping interest rates seem to have helped large firms more rapidly than they helped smaller firms, and this contributed to growing profit dispersion.

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Table II about here.

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Some studies (Covarrubias et al. 2020) focus on changes in US antitrust enforcement as potentially responsible for these profit trends. If it is US-specific regulation that is driving the data then as they argue, we should not see similar things taking place in other countries. So in Appendix D we examine the data from the G-7 countries. Fairly similar patterns are observed. The gap between aggregate and median profitability is also found in other major economies such as Canada, France, Germany, and Japan. There is of course significant heterogeneity in the magnitude and timing of these trends. Each country is unique in many ways. In this case it seems that 'Anglo-Saxon' economies exhibit a stronger gap than do the other European and Japanese firms.

This cross-country evidence indicates that we are documenting the impact of global forces affecting a variety of major countries, and not just the impact of US-specific government policies. In the rest of the paper we examine and interpret these forces in more detail with a specific focus on the gap between large and small firm outcomes.

V. THE LARGE FIRM-SMALL FIRM GAP

So far we have established that a wide gap that emerged between the aggregate profit and the median profits of publicly traded US firms starting in the early 1980s. Now we consider the mechanisms that drive this result. In this section we systematically examine why large firms became more profitable. We consider output market power, Superstar firms, and firm efficiency based interpretations.

V.A. Decomposing Aggregate Profits

The reported profit measurements are ratios. When observing properties of a ratio, it is natural to ask whether interesting properties are due to the numerator, the denominator or both jointly. Is the numerator (Π), or the denominator (AT) more responsible for the aggregate evolution?

If the numerator is more responsible, then high profit firms are largely responsible for the aggregate. Kehrig & Vincent (2021) call this the Superstar scenario in which superstar firms earn much of the aggregate profits. In that view what happens to superstar firms is of particular importance for the economy in the aggregate. It should be noted that the definition of a 'Superstar firm' does differ across papers. They can be defined by profitability, by productivity, or by still other metrics of outstanding performance. In this paper we follow the definition in Kehrig & Vincent (2021). Similar definitions are in papers such as Autor et al. (2020), Ayyagari et al. (2024). Some papers define a threshold such as the top 10% of firms (Ayyagari et al. 2024). Other papers focus on a more broadly based profit weighting (Kehrig & Vincent 2021).

If the denominator is more responsible, it is large firms that are primarily responsible for the aggregate. (Kehrig & Vincent 2021) call this the Big Player scenario. We refer to this as a Large Firm effect. In order to measure these effects, suppose that there are J firms. Define the asset weight of firm i on date t

as, $\omega_{it} = \frac{AT_{it}}{\sum_{j=1}^{J} AT_{jt}}$. For firm i define the profitability, or the return on assets (ROA), as $\pi_{it} = \frac{\Pi_{it}}{AT_{it}}$. Let ω_{it} be the asset-weighted value of firm i at date t. Summing across firms, the aggregate return on total assets is, $\pi_t = \frac{\Pi_t}{AT_t} = \sum_{i=1}^{J} \omega_{it}\pi_{it}$. For a given firm over a τ year horizon, the change in profits is $\Delta\pi_{it} = \pi_{it} - \pi_{it-\tau}$. Similarly the change in weights is $\Delta\omega_{it} = \omega_{it} - \omega_{it-\tau}$.

We begin by examining whether the observed aggregate trends are driven by exceptional performance among a small subset of superstar firms or by more systematic differences in the firm size distribution. Motivated by the decompositions in Olley & Pakes (1996), Foster et al. (2001), Kehrig & Vincent (2021), we decompose aggregate profitability as

$$\frac{\sum_i \Pi_{it}}{\sum_i AT_{it}} = \frac{1}{N_t}\sum_i \frac{\Pi_{it}}{AT_{it}} + \sum_i \left(\omega_{it} - \frac{1}{N_t}\right)\left(\frac{\Pi_{it}}{AT_{it}} - \overline{\left(\frac{\Pi}{AT}\right)_t}\right) \tag{6}$$

where the first term represents unweighted average profitability and the second term captures the covariance between firm size and profitability. N_t denotes the total number of firms on date t.

Following Kehrig & Vincent (2021) the decomposition can be rewritten as,

$$\Delta\pi_t = \underbrace{\sum_i \omega_{it-\tau}\Delta\pi_{it}}_{Within} + \underbrace{\sum_i \Delta\omega_{it}\pi_{it-\tau}}_{Between} + \underbrace{\sum_i \Delta\omega_{it}\Delta\pi_{it}}_{Interaction}. \tag{7}$$

Ignoring entry and exit they get,

$$\Delta Cov(\omega_{it}, \pi_{it}) = Cov(\Delta\omega_{it}, \pi_{it-1}) + Cov(\omega_{it-1}, \Delta\pi_{it}) + Cov(\Delta\omega_{it}, \Delta\pi_{it}).$$

To help fix ideas, the following labels are used to the current decomposition.

1. The Superstar effect, means that $Cov(\Delta\omega_{it}, \pi_{it-1}) > 0$ is the key driving force for aggregate profits. High profits π_{it-1} is associated with greater increase in the weighting in the aggregate profits $\Delta\omega_{it}$.

2. The Large Firm effect, means that $Cov(\omega_{it-1}, \Delta\pi_{it}) > 0$ is the key driving force for aggregate profits. High market share ω_{it-1} is associated with greater increase in profits $\Delta\pi_{it}$.

3. The Rising star effect, means that $Cov(\Delta\omega_{it}, \Delta\pi_{it}) > 0$ is the key driving force for aggregate profits. Increased market share $\Delta\omega_{it}$ and increased profits $\Delta\pi_{it}$ have a positive covariance.

Figure II shows that aggregate profits are rising. Which kind of firm effects are critical to that increase? Both superstar firms (Autor et al. 2020) and large firms (Kahle & Stulz 2017) have attracted particular attention in the literature. Rising stars and unicorn firms tend to be the subject of studies that are more narrowly focused on the entrepreneurship process itself (Ewens & Farre-Mensa 2022).

Turning to the data, we want to identify the relative importance of these components. So we construct counterfactual analyses presented in Figure IV for Superstar firms, and Figure V for Large firms. In Figure IV we present results from holding firm profitability fixed at its value in a baseline year (1981, 1991, 2001, or 2011) while allowing the firm size distribution to evolve as observed in the data. Specifically, we compute $\sum_i \omega_{it} \frac{\Pi_{i\hat{t}}}{AT_{i\hat{t}}}$ for each year t.

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Figures IV and V about here.
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The results are very sharp. The counterfactual series are very different from the realized aggregate profitability. Superstar firms are not responsible for the aggregate. When we defined Superstars based on their profitability in a particular baseline year, the subsequent trends do not match the aggregate. This is true no matte which year is picked as the base year.

Things are very different when we examine the impact of large firms. In Figure V we fix firms' weights at baseline year values while allowing profitability to evolve naturally ($\sum_i \omega_{i\hat{t}} \frac{\Pi_{it}}{AT_{it}}$). This time the counterfactual path very closely matches the aggregate profitability.

The power of large firms to track the aggregate is robust across all baseline years. Changes in the size distribution, not exceptional profitability for specific Superstar firms, drives the aggregate trend. A formal variance decomposition confirms this interpretation. Narrowing attention to the so-called Magnificent 7 firms reinforces the fact that it is large firms overall that matter, not just specific high profile firms. For details, see Appendix C.

V.B. Cost Structure Differences

How can we account for the superior profits at large firms? In light of the model in Section II we focus on financing advantages and operating efficiency advantages. In this section we consider operating cost, leaving financing to the next section.

Figure VI shows that both large firms and median firms have declining flow costs relative to assets since the later 1970s. There is some evidence that large firms were more successful at this. To show this we estimate

$$y_{it} = \beta_0 + \beta_1 I_{it}^{HIGH} + \gamma X_{it} + \varepsilon_{it} \tag{8}$$

where y_{it} represents in turn each of the components of profitability, and I_{it}^{HIGH} is an indicator for firms in the top quintile by size, while X_{it} includes industry and year fixed effects. We find that high-profit firms generate significantly higher value added relative to assets. The coefficient of I_{it}^{HIGH} is 0.085 with a standard error of 0.006. Further estimates show that this is achieved mostly by lowering flow costs. The coefficient estimates are tabulated in the Appendix Table A.5. The estimates show that this is achieved mostly by lowering flow costs.

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Figures VI about here.

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Next consider Figure VII panel c. It shows that the total asset share increased for both large and median firms. There was a widespread move towards greater asset intensity across public firms. It was not just limited to a handful. The combination of rising asset use and differences in the declining flow costs suggests large firms were more effective at taking advantage of the technological change to reduce operational costs. In effect there seem to have been growing economies of scale.

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Figures VII about here.
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V.C. The Large Firm Financing Advantage

Figure III shows that bond yields trended down starting in the mid-1980s, and that this happened faster for higher credit bonds. In this section we ask if this also affects publicly traded firms overall?

Table II provides detailed evidence relating the debt market to firm size and firm profits. Firms are sorted into quintiles based on profits and also based on size creating 25 cells. This is done for the whole time period as well as for the years starting in 2000. Within each cell we compute the average $\frac{Debt}{Sales}$ ratio to see if there are distinctions in the use of debt. Next we compute the $\frac{Interest}{Sales}$ ratio to see if various firms are paying out more interest than other firms. Finally we compute the $\frac{Interest}{Debt}$ ratios to see if the various firms are being charged different rates by the financial markets.

The results show a very clear large firm advantage. In Panel (a) we see that apart from the very lowest profit firms, large firms have debt-to-sales ratios that far exceed those of smaller firms. These firms are using more leverage. Despite those higher leverage ratios, Panel (b) shows that the large firms seem to payout roughly similar fractions of sales in the form of interest. In some categories they even payout less, despite having more debt. This suggests that they are getting more favorable terms.

Panel (c) looks at the terms directly. Large, highly profitable firms paid substantially lower interest rates (0.09) compared to small, high profit firms (0.18) over the entire 1971-2022 period. The large firm rate at high profit firms was half that at smaller high profit firms. Over the period 2000-2022 large firms paid less half the rate charged to small high profit firms 0.07 versus 0.19.

The capital market really favored the large high profit firms over the small ones. What about for low profit firms? In this case the comparison over the whole period is 0.26 versus 0.12, while in the years from 2000 it is 0.35 versus 0.10. So the large firm advantage became even stronger among the low profit firms. The results are so sharp that in the period 2000-2022 large unprofitable firms on average paid 0.10 while small high profit firms paid almost twice as much (0.19).

It should be stressed that these differences are not necessarily indicative of capital market failure. The large firms may be viewed differently by investors if the investor's taste for risk has changed. This paper is not about quantifying investor's taste for risk. But this evidence does reflect the interest costs that firms were actually paying. Those interest costs do show a shift that favored larger firms.

Compustat firms are publicly traded. To see if the overall trends apply to the overall economy including private firms, we use aggregate data from the Federal Reserve data at FRED. The aggregate data is similar to value-weighted Compustat in the sense that it does not distinguish individual firms. Very similar aggregate profit trends are found. As shown in Appendix Figure A.4, aggregate profit share has increased considerably since 1980s. This is driven by increasing financial profits and declining opportunity costs as shown in Appendix Figure A.5. Moreover, total asset share has increased from about 0.3 to over 0.4, accompanied by decreasing flow cost share as shown in Appendix Figure A.6.

VI. FIRM DYNAMICS AND PERSISTENCE

The facts documented so far are consistent with our cost-based interpretation. But they have not yet ruled out an output market power interpretation. To get at that important idea in this sections we investigate whether large, profitable firms have become more entrenched over time as suggested by theories of increasing market power (Covarrubias et al. 2020). We analyze firm transitions across states of profitability and size. In so doing we provide novel evidence on persistence, exit rates, and entry dynamics.

VI.A. Profit Persistence and Transitions

In Table III we provide descriptive tabulations of the profit transition matrices. These show how firms move among the profit quintiles over 5-year periods. For each panel the rows give the firm's profit quintile at an initial time t. The columns give its quintile five years later (t+5). The values within each cell are the transition probabilities. These give the percentage of firms starting in quintile i that end up in quintile j. Not all firms that existed at date t still exist five years later. Not all firms that exist five years later already existed at the earlier date. So we also tabulate alternatives. The 'exit' column measures firms that disappeared from the sample during the 5-year interval. The 'Bankruptcy' and 'M&A' columns decompose reasons for exit, representing the percentage of firms exiting due to bankruptcy/liquidation and merger and acquisition respectively. The 'enter' row measures the distribution of new entrants across profit quintiles. The column labeled '(mean)' gives the average profit-to-asset ratio for firms in each quintile. This serves to highlight the large differences between high and low profitability firms.

Panel (a) gives results for the full sample period (1971-2022). Panel (b) gives results for the more recent period (2000-2022). This is motivated by the idea (Covarrubias et al. 2020) that antitrust enforcement may have changed around the year 2000. If so we ought to observe a big change in the persistence among the most profitable firms. The transition matrix approach enables us to directly measure whether highly profitable firms have become more entrenched, as theories of increasing market power would suggest.

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Table III about here.

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Now consider the estimates in Table III. Panel (a) shows profit transitions during 1971-2022. Only 33% of high-profit firms (high quintile) are in the top quintile after five years. Among these firms 30% exited the sample entirely. The probability of transitioning from the top quintile to each lower quintile declines monotonically: 18% to the fourth quintile, 9% to the third, 7% to the second, and 4% to the bottom quintile.

Panel (b) provides corresponding numbers for the more recent period (2000-2022). The persistence rate of the top quintile group remains virtually unchanged at 33%. The firm exit rate is slightly higher at 32%. The actual stability of the transition probabilities is inconsistent with theories claiming reduced competitive pressure on top firms in recent decades. The mean profit-to-asset ratios reported in the rightmost column of Table III is also informative. Over the full sample period the top quintile firms average 0.15 profit/AT versus -0.56 for bottom quintile firms. Over the recent decades the corresponding numbers are 0.16 and -0.90. So there is little change in the top quintile and a marked drop in profit in the bottom quintile.

New entry shows notable change in the most recent decades. During the full period new entrants had significant negative profits (-0.21) but in the recent decades that became even larger (-0.41). The exit rate of new entrants increased from 0.44 to 0.55. This is consistent with a shift towards assets-intensive production technology that requires a longer build-up period, yields returns later, and potentially has relatively higher return uncertainty.

It is of interest to examine the other G-7 countries. These are tabulated in Appendix table A.3. Compared to these other countries US firms have somewhat less profit persistence and notably elevated exit rates. In Japan the top quintile firms remain there with a 41% chance while in the USA it is only 33% using the Worldscope data. Top profit quintile firms in Japan have a 12% exit rate over the five years. In the USA the top profit quintile firms have a 28% exit rate. It is hard to avoid the idea that firms in the US are less entrenched than top quintile firms in other G-7 countries.

VI.B. Size Persistence and Transitions

Large firm size persistence is a completely different story. Table IV Panel (a) shows that 70% of large firms, defined as those in the top quintile, remained in the top size quintile five years later, over the full 1971-

2022 time period. The exit rate for them was 21%. Turning to the recent decades (2000-2022) in Panel (b) we observe a very slight decline in the persistence rate to 67%. The exit rate remains unchanged at 21%. Stability of the exit probabilities among the large firms directly contradicts the idea that leading firms in the USA have become more entrenched. Entrenchment after 2000 is simply not what the data shows.

Since the quintiles are sorted on size, of course there are going to be differences. How large are are those differences? Table IV reports the mean values of total book assets by quintile measured in 2017 US dollars. Large firms averaged $6,611 million in assets compared to merely $9.4 million for small firms during 1971-2022. These differentials became even bigger changing to $10,220.8 million versus $10.8 million during 2000-2022. That is a massive difference.

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Table IV about here.
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A more formal test of the evolution of firm profitability using regressions is provided in Table A.7. The coefficient on $\beta_{+5} = -0.093$ (t-statistic $= -16.83$) for high-profit firms indicates significant mean reversion in profitability. Similarly, from the size evolution results in Table A.8, we find $\beta_{+5} = -0.489$ (t-statistic $= -13.82$) for large firms. Again there is substantial regression toward the mean.

VI.C. Structural Estimation of Transition Probabilities

The transition tabulations in Tables III and IV are informative. But they do not estimate the role of potential firms, and so they might bias our understanding of the firm dynamics. Of course potential firms are not observable. We can however infer their impacts using the Expectations-Maximization algorithm (Dempster et al. 1977, Cappé et al. 2005) to estimate a Hidden Markov Model of firm transitions. In this case we are estimating 1 year transitions, not 5 year transitions.

Table V gives these maximum likelihood estimates, using bootstrapped confidence intervals. Over the full time period 1971-2022 we find that big firms had a 91.0% probability of remaining large in consecutive years with a confidence interval of [90.8%, 91.2%]. For 2000-2022, this persistence rate is nearly identical at 90.1% with a confidence interval of [89.7%, 90.4%]. If you compound these 1 year transition rates they are very similar to the 5-year transitions provided earlier.

The most striking result in Table V is the sharp decline in entry rates. The transition probability from "out" to "small" decreased from 10.3% in the full sample to just 0.3% since 2000. This is a massive decline in the rate of entry.

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Table V about here.
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Our transition results could be due to just a few industries. To deal with that concern in Appendix Table A.9 we compute the transitions on major NAIC sectors. The differences seem reasonable. Manufacturing has the highest large-firm persistence at 92.4% [92.1%, 92.7%], while Professional Services exhibits more volatility with 84.5% [83.4%, 85.7%] persistence. While there is a fair bit of consistency, there really are important differences across industries.

VI.D. Discussion of Transition Results

A critical feature of the transition rates is the stability of exit rates for large and profitable firms along with the very sharp decline in entry rates. Accordingly it seems that regulatory changes and other aspects that directly affect entry costs are more plausible driving forces than reduced competitive pressure on large firms. Our findings are consistent with increased costs of going public following Sarbanes-Oxley (Zhang 2007) and the improved financing options for private firms due to the National Securities Market Improvement Act (Ewens & Farre-Mensa 2020).

Overall, our evidence on firm dynamics offers a more nuanced picture than a simple entrenchment claim.

There is evidence that large firms have become more profitable as we documented in section V. But there is no support for the idea that the large firms became better protected. The exit rates are unchanged over time. The most impactful shift in market structure is due to decreased entry, not due to greater persistence of dominant firms.

Where does this analysis leave the model we started with in section II? That is a static model, so by design it is not adequate to cope directly with our evidence. An extension of the model is required. To formally allow for transition dynamics, in section VII we provide a structural model that incorporates firm transitions. The model demonstrates how stable exit rates can coexist with declining entry rates and increasing concentration, without requiring assumptions about weakening competition.

VII. MODEL WITH TRANSITIONS

The evidence provides a set of facts that initially appear puzzling. Large firms have become more profitable over time. However, their exit rates have remained stable. This does not match the idea that firms have become increasingly entrenched. To understand this, in this section we provide a simple model that addresses this seeming inconsistency. To do that we focus on transition dynamics of firm across states. This highlights the impact of entry costs rather than weakened product market competition among leading firms.

The model will be developed in the following steps. First, we provide the model setup and the basic model assumptions. Second, the firm optimization problems are provided. Third, the value functions are derived. Fourth, the steady state equilibrium is defined. Fifth, we solve for the steady state value functions. Sixth, comparative statics are provided. Seventh, the model is used for counterfactuals.

VII.A. Setup

Consider an economy with firms each of which is in one of three possible states. The firm may be currently non-existent (State 0), small (State 1), or large (State 2). There is a Markov process that controls the firms transition among the states. All firms are endowed with wealth $W > 0$ and face perfect capital markets with a riskless asset offering a return of $1+\rho$.

The small and large firms are both price takers and for simplicity they operate in different markets. The firm in state $i \in \{1,2\}$ produces according to

$$Y_i = \theta_i A_i^{\alpha_i} F_i^{\beta_i} \tag{9}$$

where A_i is assets, F_i is flow inputs, and θ_i is productivity. Assume the firms have decreasing returns to scale ($0 < \alpha_i + \beta_i < 1$). We also assume that large firms are more productive ($\theta_2 > \theta_1$).

Each market has its own inverse demand function,

$$P_i = \left(\frac{Y_i^d}{D_i}\right)^{-\frac{1}{\varepsilon_i}}, \quad i \in \{1,2\}, \tag{10}$$

where D_i is a demand shifter and ε_i is the price elasticity of demand.

State 0 firms have an entry decision. They draw an entry cost $\kappa_i \sim U[\underline{\kappa}, \overline{\kappa}]$ and will only enter if the expected benefit exceeds the cost. Once they enter, they become State 1 (small) or State 2 (large) firms, which is determined by known exogenous probabilities as shown at the top node in Figure VIII. If they do not enter, they invest endowment W into the capital market and earn a return of $1+\rho$. State 1 and State 2 firms choose the amount of assets and flow inputs that they use in their production. They maximize expected profits taking prices as given.

At the end of each period, firms transition between states according to the Markov transition matrix $\Pi = [\pi_{ij}]$, where π_{ij} represents the probability of moving from state i to state j. Figure IX illustrates the sequence of events and possible transitions within a period. Figure VIII shows the feasible state transitions and the notation for each link.

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Figures VIII and IX about here.
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Firms in states 1 and 2 make production decisions. Using the production function and market structure, a firm in state $i \in \{1,2\}$ solves

$$\max_{A_i,F_i} R(A_i,F_i) - \kappa + (1+\rho)(W - p_A A_i - p_F F_i) \tag{11}$$

where p_A and p_F are the prices of assets and flow inputs, respectively. Take the first order conditions, and then solve these equations simultaneously, we get the optimal input choices

$$A_i^* = \phi_A(\alpha_i, \beta_i, p_A, p_F) \cdot (\frac{P_i\theta_i}{1+\rho})^{\frac{1}{1-\alpha_i-\beta_i}} \tag{12}$$

$$F_i^* = \phi_F(\alpha_i, \beta_i, p_A, p_F) \cdot (\frac{P_i\theta_i}{1+\rho})^{\frac{1}{1-\alpha_i-\beta_i}} \tag{13}$$

where ϕ_A and ϕ_F are functions of the parameters as shown in Appendix G. These expressions show that input use increases with productivity and with the output price, and decreases when input prices increase. In addition, input use decreases when the opportunity cost $1+\rho$ increases.

In each state the firm has a value function and they are linked. Let $\pi_i(A_i,F_i) = P_iY_i + (1+\rho)(W - p_A A_i - p_F F_i)$ be the period profit function, δ is the discount factor, and $V_0^{in} = \frac{\pi_{01}}{\pi_{01}+\pi_{02}}V_1 + \frac{\pi_{02}}{\pi_{01}+\pi_{02}}V_2$ is the expected value conditional on entry. Then the value functions can be written recursively.

$$V_0 = \mathbb{E}_{\kappa_i} \max\{W(1+\rho) + \delta V_0, (W - \kappa_i)(1+\rho) + \delta V_0^{in}\} \tag{14}$$

$$V_1 = \pi_1(A_1^*, F_1^*) + \delta(\pi_{10}V_0 + \pi_{11}V_1 + \pi_{12}V_2) \tag{15}$$

$$V_2 = \pi_2(A_2^*, F_2^*) + \delta(\pi_{20}V_0 + \pi_{21}V_1 + \pi_{22}V_2) \tag{16}$$

The value functions are the present discounted value of expected future profits for firms in each state. They include both immediate returns, and the value of possible transitions to other states. State 0 firms are potential firms that do not yet operate. Their value function includes the expected benefit from a decision to enter or to remain out. If they enter chance determines whether they become small (State 1) or large (State 2). If the State 0 firm stays out it earns the risk-free return on W. State 1 firms are small firms that operate. Their value includes current operating profits plus the discounted expected value from what happens subsequently. They might growing into a large firm, stay in the existing state, or exit altogether. State 2 firms are large. Again, their value function includes current operating profits plus the discounted expected value from what happens subsequently. They might stay in the current large state, get reduced to being small, or they might exit. These recursive relationships show that the firms' current decisions depend on expectations about future states. The interconnected nature of these value functions is crucial for understanding equilibrium firm behavior.

The entry threshold $\hat{\kappa}$ satisfies the natural indifference condition.

$$W(1+\rho)+\delta V_0 = (W-\hat{\kappa})(1+\rho)+\delta V_0^{in} \tag{17}$$

This means that the threshold is at,

$$\hat{\kappa} = \frac{\delta(V_0^{in} - V_0)}{1+\rho}. \tag{18}$$

In a steady state equilibrium we require a balanced flow in and out of each state. To be an equilibrium each firm is making optimal choices as shown above, and the market prices are such that each market clears. We also require that the equilibrium be recursive.

Recursive Steady State Equilibrium A recursive steady state equilibrium consists of the following: Time-invariant value functions $V_i(s)$ for $i \in \{0,1,2\}$, time-invariant policy functions $\sigma_i(s)$ for $i \in \{0,1,2\}$, constant transition probabilities π_{ij}, time-invariant distribution of firms across states λ^*, constant output prices P_1^*

and P_2^*, and constant number of firms in each state N_0^*, N_1^*, N_2^* that are consistent with the following: firms' optimal production and entry decisions maximize their value, output market clears, and the total number of firms as well as the distribution of firms is constant.

Accordingly, in a recursive steady state the distribution of firms across states remains constant. Let $\lambda = [\lambda_0, \lambda_1, \lambda_2]$ denote this distribution, where λ_i is the fraction of firms in state i. The steady state condition is

$$\lambda = \lambda \Pi \tag{19}$$

The market clearing conditions determine equilibrium prices.

$$P_i = \left[\frac{D_i}{N_i Y_i(A_i^*, F_i^*)} \right]^{\frac{1}{\varepsilon_i}} \tag{20}$$

where N_i is the number of firms in state i.

VII.B. Steady State Value Functions

With the model structure in hand, we now provide the equilibrium value functions that determine firm behavior in steady state. These value functions represent the present discounted value of expected profits across different firm states and show how firms weigh current operations against future transition possibilities. The steady state requires that all optimization conditions are satisfied while maintaining a constant distribution of firms across states. By solving the system of recursive equations and imposing the steady state condition $\lambda = \lambda \Pi$, we obtain closed-form expressions for the value functions as stated in Theorem VII.1

Theorem VII.1 (Steady State Value Functions)**.** *In a steady state equilibrium, the value functions for firms* *in each state are:*

$$V_0 = \left(\underbrace{W(1+\rho)}_{\textit{Outside investment value}} + \delta(1-\pi_{00})(\underbrace{\Gamma_1 v_1 + \Gamma_2 v_2}_{\textit{Expected production value}} - \underbrace{\frac{(1+\rho)(\hat{\kappa}+\underline{\kappa})}{2\delta}}_{\textit{Expected entry cost}}) \right) \underbrace{\Gamma_0}_{\textit{Adjustment factor}}$$

$$V_1 = \underbrace{\frac{1}{1-\delta(\pi_{11}+\frac{\delta\pi_{12}\pi_{21}}{1-\delta\pi_{22}})}}_{\textit{Persistence multiplier}} \cdot \left[\underbrace{v_1}_{\textit{Current period return}} + \underbrace{\delta\frac{\pi_{12}}{1-\delta\pi_{22}} v_2}_{\textit{Expected value from size transition}} + \underbrace{\delta\left(\pi_{10}+\frac{\delta\pi_{12}\pi_{20}}{1-\delta\pi_{22}}\right)V_0}_{\textit{Expected value from exit paths}} \right]$$

$$V_2 = \underbrace{\frac{1}{1-\delta(\pi_{22}+\frac{\delta\pi_{12}\pi_{21}}{1-\delta\pi_{11}})}}_{\textit{Persistence multiplier}} \cdot \left[\underbrace{v_2}_{\textit{Current period value}} + \underbrace{\delta\frac{\pi_{21}}{(1-\delta\pi_{11})} v_1}_{\textit{Expected value from downsizing}} + \underbrace{\delta(\pi_{20}+\frac{\delta\pi_{21}\pi_{10}}{1-\delta\pi_{11}})V_0}_{\textit{Expected value from exit paths}} \right]$$

where

$$\gamma \equiv \frac{\pi_{02}}{\pi_{01}+\pi_{02}}$$

$$\Gamma_0 \equiv \frac{1}{\underbrace{(1-\delta\pi_{00})}_{\textit{Stay out}} - \underbrace{\delta(1-\pi_{00})}_{\textit{Entry}} \underbrace{\frac{\delta}{1-\delta(\pi_{11}+\delta\frac{\pi_{12}\pi_{21}}{1-\delta\pi_{22}})}}_{\textit{Persistence multiplier}} \underbrace{\left(\pi_{10}+\delta\frac{\pi_{12}\pi_{20}}{1-\delta\pi_{22}}+\gamma\frac{(1-\delta)(\pi_{20}-\pi_{10})}{1-\delta\pi_{22}}\right)}_{\textit{Enter and then exit}}}$$

$$\Gamma_1 \equiv \underbrace{\frac{1}{1-\delta(\pi_{11}+\frac{\delta\pi_{12}\pi_{21}}{1-\delta\pi_{22}})}}_{\textit{Persistence multiplier}} (\underbrace{1-\gamma}_{\textit{Entry as small firm}} + \underbrace{\delta\frac{\gamma\pi_{21}}{1-\delta\pi_{22}}}_{\textit{Entry as large firm}})$$

$$\Gamma_2 \equiv \underbrace{\frac{1}{1-\delta(\pi_{22}+\delta\frac{\pi_{21}\pi_{12}}{1-\delta\pi_{11}})}}_{\textit{Persistence multiplier}} (\underbrace{\gamma}_{\textit{Entry as large firm}} + \underbrace{\delta\frac{(1-\gamma)\pi_{12}}{1-\delta\pi_{11}}}_{\textit{Entry as small firm}})$$

$\hat{\kappa}$ is the entry indifference value, and $\underline{\kappa}$ and $\overline{\kappa}$ are the lower and upper bounds of the entry cost distribution.

The value functions show a number of economic mechanisms that drive firm dynamics. We consider these in turn by category. The persistence multipliers show how the value of being in a particular state is enhanced by the probability of remaining in a profitable state over time. Larger values of π_{11} or π_{22} increase these multipliers due to the economic benefit of staying in a profitable state. This is similar to the way in which a lower discount rate increases present values in a standard net present value calculation. However it is slightly nuanced as it incorporates the transition matrix structure.

The state transition values are terms like $\delta \frac{\pi_{12}}{1-\delta\pi_{22}} v_2$. They are expected values from size transition. It gives the value of potentially moving to another state. These components show how firms in one state benefit from the possibility of transitioning to another state. For small firms, the possibility of becoming large creates additional value in addition to current operations. For a large firm it gives the probability of transiting to a smaller firm. These terms reflect changes that can happen along the firm's dynamic path.

The adjustment factors like Γ_0 modify how entry decisions are affected by outside opportunities and future transition possibilities. They have relatively complex structures as they include competing forces. For instance on the one hand there is the value of waiting to enter when conditions might improve as shown in the "Stay out" component. On the other hand there is the value of entering now to begin capturing operational profits as shown in the "Entry" component. The denominator incorporates future exit probabilities. This is in effect discounting the value of entry to also reflect the likelihood of subsequent market exit.

The path entry contributions are represented by Γ_1 and Γ_2. These factors show how the profits from small and large firms respectively contribute to the value of non-existent firms. These factors do more than just show the direct path of entering as a particular size. They also reflect the indirect paths where firms enter in one state but transition to another. So potential entrants value properly reflects more than just the immediate post-entry profits. It reflects the full distribution of potential paths for the firm throughout the time.

The conditional probability γ determines the initial distribution of entrants across firm size categories. It is conditioned by the decision of a state 0 firm to enter.

Consider the value function components for small firms. For State 1 (small) firms, the value function V_1 has three parts. First, the current profits v_1. Second, the expected value from potential transition to State 2. It is given by $\delta \frac{\pi_{12}}{1-\delta\pi_{22}} v_2$. Third, the expected value from potential market exit. This is given by $\delta\left(\pi_{10}+\frac{\delta\pi_{12}\pi_{20}}{1-\delta\pi_{22}}\right)V_0$. The persistence multiplier weighs these outcomes based on discount factors and transition probabilities. Suppose that State 1 were an absorbing state, that is $\pi_{11}=1$. Then $V_1=\frac{v_1}{1-\delta}$, which is just the present value of perpetual profits v_1.

An critical aspect of the model is that productivity improvements in one state create spillover effects in the other state. An increase in productivity parameter θ_i for either $i \in \{1,2\}$ increases both N_1 and N_2, the number of firms in States 1 and 2. This happens because of improved productivity in State i increases the value function V_i. That increases the entry threshold $\hat{\kappa}$ through the condition $\hat{\kappa}(1+\rho)=\delta[(1-\gamma)V_1+\gamma V_2-V_0]$. A higher $\hat{\kappa}$ means more firms find entry profitable. So there is an increased number of firms in both State 1 and State 2.

VII.C. Economic Mechanisms in the Model

There are a number of economic mechanisms that are at work in the model. Here we point out some important and relatively simple mechanisms, before turning to the complexity associated with an interest rate decline.

Empirically we found stable exit rates but declining entry rates. How are these consistent with the model? To see this note that the exit rates (π_{10} and π_{20}) are specified exogenously and they remain constant in steady state. Changes in entry costs affect the entry threshold $\hat{\kappa}$ without altering exit probabilities. So the number of exiting firms might change if the number of entering firms changes, but the exit rate does not change.

Market concentration can increase as a result of the changed market entry conditions, say it is now more expensive to enter. This does not require greater protection for the top firms. If fewer firms enter there are fewer firms in both State 1 and State 2. This will also result in greater profit for the large firms.

A productivity increase in State 2 has a larger impact on firm entry than does an equivalent increase in State 1. This is consistent with the disproportionate influence of large firms on market dynamics.

Productivity naturally spillovers across states in our model. Suppose that productivity θ_i increases in state i. That raises the value of firms in that state. That in turn makes entry more attractive. As a result the entry threshold $\hat{\kappa}$ increases. So more firms enter. By chance some of them wind up in State 1 while others wind up in State 2.

Now we turn to the greater complexity of the effect of an interest rate decline. In the data we documented that large firms received particularly favorable interest rates relative to the median firm in recent years. In effect large firms obtain a drop in ρ that is greater than the drop in ρ for the median firm. But in the model they have the same ρ. So the effects within the model omit that extra benefit for large firms in recent years. The question we try to address using the model is: how does a drop in interest rates affect firms in each state relative to each other? In the model, a drop in interest rates seems particularly beneficial to large firms because they have a larger production scale and thus use more funds at the opportunity cost of capital. But it turns out that the interest rate effects are more complex.

A drop in the interest rate ρ has several effects within the model. It has impacts on firms in each state that are not fully equivalent to each other. State 0 firms are affected directly by the impact on the entry decision threshold, $\hat{\kappa} = \frac{\delta(V_0^{in} - V_0)}{1+\rho}$. A drop in ρ decreases means that the denominator decreases. That directly increases $\hat{\kappa}$ and that encourages more entry. But that is not the whole story. It also depends on how the value difference $(V_0^{in} - V_0)$ is changed. To compute that requires the various value functions for each state since a firm that is operating may transition from one state to another.

Firms in State 1 and State 2 pick inputs A and F as shown in equations 12 and 13. Firms have decreasing returns to scale ($0 < \alpha_i + \beta_i < 1$), thus changes in ρ have meaningful effect on their production level. A decrease in ρ reduces the denominator in the last term. So both A_i^* and F_i^* increase, resulting in more output ceteris paribus. Decreasing returns to scale also implies that the impact of a decrease in gross return is not one-for-one, and the effect on output is amplified.[1]

How are the profits for these firms affected? Recall that the period profit function is $\pi_i(A_i, F_i) = P_i Y_i + (1+\rho)(W - p_A A_i - p_F F_i)$ consisting of the operating profits and the outside investment component. The operating profits part increases when ρ declines for the reasons just explained. But $(1+\rho)W$ decreases when ρ declines. These effects are not in the same direction. But even that is not complete. There is a further equilibrium effect is that operates through the output price P. It is also affected indirectly by ρ through entry rate. The changes in profits and outside investment revenue $(1+\rho)W$ affect firms' entry choices. A higher entry rate means more firms compete in the goods markets, which drives down goods prices and decreases profits.

Overall, some effects of a drop in ρ are formally signable, but other effects are not strictly signable. Of course, the drop in ρ automatically reduces the returns to wealth that is invested in the risk free asset. The optimal choice of inputs (A^*, F^*) by firms that are actually operating increase. That together with decreasing returns to scale means that output by firms in both states increase. The ratio of marginal products to input prices improves when ρ decreases, indicating more efficient production. For the entry threshold the denominator decreases for sure. But the numerator $\delta(V_0^e - V_0)$ depends on how firm values change, which is ambiguous.

The equilibrium effect of interest rate reduction on price deserves more discussion. This effect connects firm dynamics with the present value of profits in each state and the effects seem to have been largely ignored in the literature on interest rates and in the literature on dominant firms. Suppose that due to an interest rate cut, more firms enter and the existing firms produce more output. Then output market prices decline. This creates feedback effects that mitigate the initial impacts. In a market with more elastic demand (higher ε_i),

[1] To see this,

$$\frac{\partial A}{\partial(1+\rho)} = \phi_A (P\theta)^{\frac{1}{1-\alpha-\beta}} (1+\rho)^{-(1+\frac{1}{1-\alpha-\beta})} \left(-\frac{1}{1-\alpha-\beta}\right),$$

$$-\frac{\partial A}{\partial(1+\rho)} \cdot \frac{1+\rho}{A} = \frac{1}{1-\alpha-\beta} > 1.$$

the price adjustment from expanded output is smaller. This allows firms to keep more of the benefits from lower financing costs. In a market with less elastic demand, the price adjustments are bigger. That in turn reduces the net benefit to firms.

So the equilibrium output prices depend on the relative magnitude of increased output compared to the demand elasticities. How responsive will customers be to the increased output? The model treats that responsiveness as a given fact taken from outside the model. But it filters back to the profits of operating firms and to the entry decisions of potentially entering firms.

It seems plausible that large firms generally benefit more than small firms within the model. However a range of parameters matter opening the door to ambiguity. The impacts across firm states have major effects on market structure. When the interest rate falls a productivity gap between large and small firms tends to get bigger. This makes for increased market concentration. Notice that this is not caused by large firms become more protected from competition. It is caused by the fact that large firms can more effectively take advantage of cheaper financing to expand production. Importantly, by assumption the exit rates remain stable despite these changes. So the model can be consistent with our empirical evidence that large firms have not become more entrenched despite the that that they have become more profitable.

In our model, the total number of firms $N = N_0 + N_1 + N_2$ is exogenous. In reality, at times more people are likely to consider starting a firm if it looks as if the existing firms are particularly profitable. So N_0 might increase when firm profits are increased. Table V shows that the size of the pool of potential firms increased drastically despite low entry rate since 2000. A low observed entry rate is consistent with higher entry cost caused by regulation changes such as Sarbanes-Oxley Act. But if the only force at work was an increased cost of market entry cost, that alone seems unlikely to increase the pool of people considering starting a firm.

Suppose we consider extending our model to allow extra people to consider joining N_0 so that N can adjust. It seems plausible that such an effect will tend to happen when existing firms are observed to be particularly profitable. That suggests an addition natural economic explanation for our estimates that the pool of potential firms seems to have increased since 2000 despite the fact that we observe fewer actual market entrants.

VII.D. Numerical Example

In this section, we provide a numerical example of the model with transitions presented above. We also examine the impact of an entry subsidy on firm entry and the firm size distribution.

Baseline

To show how our model produces predictions about firm dynamics, we provide a baseline numerical example of the model using parameter values that roughly match empirical evidence on firm behavior. Table VI provides the parameter choices. The can be organized into four key categories. The categories are productivity parameters, production technology, market structure, and transition dynamics.

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Table VI about here.
****

Start with the firm productivity and returns to scale numbers. For State 1 (small) firms productivity is normalized to 1. For State 2 (large) firms productivity is set to 2. This reflects the substantial large firm productivity advantage documented in the empirical literature. Both firm types operate with decreasing returns to scale ($\alpha_i + \beta_i < 1$). Large firms have stronger decreasing returns ($\alpha_2 + \beta_2 < \alpha_1 + \beta_1$). This allows us to maintain bounded firm sizes while capturing the organizational challenges that typically accompany scaling operations.

Turning to the production technology, the capital shares are set to $\alpha_1 = 0.3$ and $\alpha_2 = 0.35$ which is reasonably consistent with standard estimates from production function literature. There is a slightly higher capital intensity for large firms because in reality large firms often seem better able to substitute capital for labor. Labor shares are $\beta_1 = 0.5$ and $\beta_2 = 0.35$. This results in returns to scale of 0.8 for small firms and 0.7 for large firms. The labor shares (β_1, β_2) are set higher than capital shares, consistent with empirical observations that labor costs typically exceed capital costs for most firms. These values seem to be in empirically plausible ranges.

The market structure has an annual real interest rate of $\rho = 0.04$. So the corresponding discount factor is $\delta = 0.96$. Market demand parameters are selected to ensure positive operational profits with demand shifters $D_1 = 10.0$ and $D_2 = 10.5$, and price elasticities $\varepsilon_1 = 4.5$ and $\varepsilon_2 = 3.5$. These elasticity values are reasonably consistent with firm-level demand estimates (Foster et al. 2008).

Next consider the input cost structure. Entry costs range from $\kappa_{min} = 0.5$ to $\kappa_{max} = 4.0$. The entry cost range is set to match the entry rates estimated in our empirical work. This wide range allows for heterogeneity in entry costs across potential firms. The price of flow inputs (p_F) is normalized to 1. Normalizing p_F to 1 simplifies without affecting the basic properties. The price of assets as inputs (p_A) is set to 0.05 so we get a capital-to-output ratio $(\frac{A}{Y})$ between 2 and 4, which is reasonably similar to values in macroeconomic literature.

The transition probabilities are of course, central to our analysis. They are chosen to match our empirical estimates. Persistence probabilities $\pi_{11} = \pi_{22} = 0.90$ mean that there is a strong tendency of firms to remain in their current state as we observe in the data. Exit probabilities $\pi_{10} = 0.05$ and $\pi_{20} = 0.02$ capture the distinct but stable exit rates difference small and large firms. Size transition probabilities $\pi_{12} = 0.05$ and $\pi_{21} = 0.08$ provide the observed patterns of firm growth and decline. Entry costs are uniformly distributed between $\underline{\kappa} = 0.5$ and $\overline{\kappa} = 4.0$. Those value produce an equilibrium entry rate that approximately matches our empirical estimates.

With these values in hand what does the model produce? The results are shown in Table VII. There are of course, only two sizes of firms that operate in the model. The distribution of firms by size is roughly 62%

small firms ($\frac{N_1}{N_1+N_2}$) and 38% large firms. The entry cost threshold is $\hat{\kappa} = 0.82$. This implies an entry rate $\pi_{00} = 0.093$, close to the entry rate of 0.112 that is estimated in Table V.

This numerical example reproduces several important features of the data. These features includ the following. The numerical predominance of small firms in the economy. The higher sales, profits, and valuations for large firms. We have fairly realistic entry rates. It produces stable exit rates across firm sizes. The baseline therefore provides a reasonable basis for simple counterfactual analyses examining how changes in the economic environment affect firm dynamics and market structure within the model.

An Entry Subsidy

An entry subsidy directly reduces the startup cost for potential entrants. It directly influences the firm creation process. With an entry subsidy of $\tau > 0$, the distribution of the entry fee becomes $\kappa_i \sim U[\underline{\kappa}(1-\tau), \overline{\kappa}(1-\tau)]$. This shifts the entire distribution of entry costs downward. Market entry is more attractive for all potential entrants.

Consider a subsidy rate of $\tau = 25\%$. Our simulation results show the entry rate increasing substantially from 0.093 in the baseline to 0.126 as shown in Panel (b) of Table VII. This is a 35% increase in entry. It shows that the subsidy policy is effective at stimulating firm creation. But the impact is not limited to just the number of firms. It affects the entire equilibrium including output and prices.

****
Table VII about here.
****

Notably, the value functions respond differently across firm states. Values for non-existent firms (State 0) and small firms (State 1) both increase relatively modestly. They go from 26.39 to 26.54 and from 27.09 to 27.12 respectively. This positive effect reflects the direct benefit of reduced entry costs. But the value of large firms (State 2) actually decreases slightly. This is because increased entry causes greater competition, driving down equilibrium prices and profit margins for operating firms. For State 1 firms this effect is also present, but it is numerically less important than the direct effect.

The economic mechanism at work shows that fixed entry costs affect firms of varying sizes differently. Of course, entry costs are modeled as a fixed expense. It does not scale with firm operations. So these costs are a relatively larger burden for smaller firms than for larger ones. The entry subsidies reduce this fixed cost component. So small firms experience a relatively larger benefit to their overall cost structure.

This counterfactual provides a somewhat nuanced policy implication. An entry subsidies can increase firm creation. At the same time is reduces the relative advantage held by large incumbent firms by reduces the burden on smaller firms.

VIII. ALTERNATIVE ASSUMPTIONS

Our model provides a basic set of observations and insights regarding firm dynamics in a model that focuses on entry decisions, size transitions, and the relationship between large and small firms. It has been intentionally kept very simple. Accordingly there are a large number of potential extensions that could be studied. In this section we comment on some of these. In particular we discuss financial frictions with size-dependent borrowing costs, endogenous transition probabilities, demand structures with possible strategic firm interactions, and heterogeneous productivity processes. Clearly the model can also be extended in other ways depending on the question of interest.

The baseline model assumes perfect capital markets. Introducing financial frictions would provide several aspects of potential interest. Size-Dependent borrowing constraints are possible. For instance that can be written as $A_i \leq \lambda_i W$, where λ_i is a leverage factor that might be higher for large firms. This could help strengthen the gap in capital costs observed empirically. It might further amplify the advantages of large firms during periods of rising interest rates.

In reality we observe different interest rates for firms of different sizes. We could add such differing interest rates into the model. Rather than having a single interest rate ρ, we could use $\rho_i = \rho + \omega_i$, where ω_i is the size-dependent discount rate. Such a direct approach would embed the empirical fact that large firms benefit disproportionately from declining interest rates. Of course it would require more algebraic complexity.

In our baseline model, transition probabilities π_{ij} are exogenous. A natural extension would make these probabilities dependent on firm decisions. For instance $\pi_{ij} = \pi_{ij}(I_i)$, where I_i represents investment in growth or productivity improvement. This would create a more realistic dynamic where firms actively influence their growth trajectories. The implications could be significant. The steady-state firm size distribution would now depend in a nontrivial way on the returns to investment. Entry decisions would account not just for the profits of alternative time paths, but for the option value of future investment decisions. Large and small firms might follow distinct investment strategies that reflect a range of factors that determine their state. Such an extension would allow us to distinguish whether declining entry rates are due to higher entry costs as in our model, or due to increased investment requirements for successful growth as technology changed. This is a particularly interesting question, but well beyond the scope of the current paper.

Among the transition probabilities the exit probability is of particular interest. We could make exit probabilities endogenous as in the classic Hopenhayn (1992). Firms could make optimal exit decisions by comparing continuation values to liquidation values: exit if $V_i < L_i$ where L_i represents the liquidation value. This would create a more explicit link between firm profitability and exit decisions. It seems unlikely to fundamentally change the main messages of our paper, so we opted for the simpler version. Furthermore the most common way that firms actually exit is through a merger or an acquisition process as documented in Frank & Goyal (2024). That makes sense since it permits poorly performing assets to be redeployed without the costs and delays of bankruptcy. But including mergers and acquisitions would call for an entirely different paper that would naturally focus on other issues than those we are studying.

The current model employs separate inverse demand functions for each firm type with constant elasticities. Alternative demand specifications include strategic interactions of product differentiation. Instead of assuming that firms as price takers, it is possible to model oligopolistic competition, $P_i = D_i - b\sum_{j \in J_i} Y_j$ where J_i represents firms competing in market i. This kind of strategic interactions would likely amplify the advantages of large firms through their greater ability to influence market prices. Introducing horizontal or vertical product differentiation where firms compete on quality as well as price would provide a richer framework, albeit one that focuses on a different set of questions. We could write, $P_i = P(Y_i, q_i; Y_{-i}, q_{-i})$ where q_i represents product quality. Suppose that large firms have advantages in quality production. This would provide another explanation for persistent profitability differences without requiring changes in market power. In subsequent work it might be interesting to test the importance of this distinction against the costly entry mechanism in our current model.

In short there are many ways to extend our model, and it will be interesting in the future to determine what more can be learned by exploring these alternatives in greater detail. We have presented a version of the model that is particularly simple since that seems to us as the appropriate starting point. Despite the model's simplicity it already offers a distinctive perspective and interpretation of the facts that we have documented.

IX. CONCLUSION

This paper has investigated the changing structure of corporate profits among U.S. firms over the past half century. The evidence calls into question some common ideas about corporate profits and market power. At the same time we offer a structured perspective on firm dynamics to account for the evidence.

First, we have documented that median public firm assets grew at approximately 2% annually until the early 1990s and then accelerated to 5% annually. Firms in the top quintile became both larger and more profitable relative to median firms during this period. Second, we identified two primary mechanisms driving this growing large firm advantage: greater reductions in flow costs than median firms achieved, and disproportionate benefits from long-term declining interest rates.

Third, contrary to theories claiming increased market power and entrenchment, we found no evidence of increased persistence among top firms. Approximately 30% of top quintile firms exit within 5 years; and this rate has remained remarkably stable across decades. Fourth, while transition rates between firm size categories have remained fairly steady, firm entry rates have declined significantly since 2000. So the attractiveness of entry to potential firms appears to have changed, but the pressure on incumbent firms has not gone away.

Our findings suggest a number of distinctive implications for our understanding of market competition, capital allocation, and policy design. The evidence of divergence between large and small firms appears to stem primarily from differential treatment in the capital markets, and from changes in technology rather than from and increasing product market power. Increased antitrust enforcement on large firms is unlikely to have much effect on firm entry or on the pricing of debt in the capital markets. Entry has changed and the capital markets do treat large firms better than they used to, but that had not translated in to greater firm entrenchment.

The impact of entry conditions on large firm profit is probably underappreciated in the literature. It is likely true that in the short run the impact is small. But over a few years the impact can really add up. After all in the US economy today many of the dominant firms have emerged from startups in very recent decades.

In the Internet Appendix we provided international evidence suggesting that generally similar patterns are found across developed economies. This again suggests that we are probably seeing the effects of global forces such as changing technology rather than U.S.-specific regulatory or antitrust policies.

Our evidence indicates that the evolution of corporate profits reflects a relatively complex interaction between technological change, capital market changes, and entry conditions. It is not just a simple increase in output or input market power. Policy responses to such forces would not be simple.

Author affiliation: Murray Z. Frank, Finance Department, University of Minnesota; and Jing Gao, Fi-

nance Department, University of Minnesota.

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Table I: Summary Statistics by Decade

This table presents summary statistics of main variables by each decade. All variables are measured in 2017 dollars deflated using the GDP deflator, and units are in millions of dollars except for ratio variables. All variables except for the weights ω_{sale} and ω_{AT} are winsorized at 1st and 99th percentiles. Averages are taken over the firm-year observations. In panel (a) observations are equally weighted and in panel (b) observations are value weighted. N is the number of firm-year observations, AT is the firm total assets deflated using the GDP deflator (GDPDEF from FRED). Sale is the firm sales revenue deflated by GDP deflator. $\frac{\Pi}{AT}$ is the average profit-asset ratio. med($\frac{\Pi}{AT}$) is the median value of the profit-asset ratio. $\frac{Debt}{AT}$ is the average debt-book asset ratio (book leverage). $\frac{Int}{AT}$ is the average interest to asset ratio. MTB is the market-to-book ratio and for this variable the number of observations for each decade are 25529, 39815, 48072, 41063, 29990, 8320. $s = \frac{COGS+SGA}{AT}$ is the flow cost to asset ratio. $\omega_{AT} = \frac{AT_i}{\sum_{j=1}^{J} AT_j} \times 1000$ is the asset concentration ratio multiplied by 1000. $\omega_{sale} = \frac{Sales_i}{\sum_{j=1}^{J} Sales_j} \times 1000$ is the sales concentration ratio multiplied by 1000.

(a) Equal weighted

Decade	N	AT	Sale	$\frac{\Pi}{AT}$	med($\frac{\Pi}{AT}$)	$\frac{Debt}{AT}$	$\frac{Int}{AT}$	MTB	s	ω_{AT}	ω_{Sale}
1971-1979	32980	764.69	974.53	0.07	0.08	0.29	0.03	1.25	1.50	0.27	0.27
1980-1989	45133	889.56	978.12	−0.05	0.01	0.30	0.04	1.78	1.33	0.22	0.22
1990-1999	53860	1114.03	1084.30	−0.06	0.03	0.30	0.03	2.26	1.28	0.19	0.19
2000-2009	44331	1782.46	1576.57	−0.14	0.03	0.33	0.04	2.75	1.31	0.23	0.23
2010-2019	31898	2913.34	2322.94	−0.13	0.05	0.34	0.04	3.07	1.18	0.31	0.31
2020-2022	8643	3534.20	2558.07	−0.08	0.06	0.37	0.03	3.13	0.91	0.35	0.35

(b) Asset Value weighted

Decade	N	AT	Sale	$\frac{\Pi}{AT}$	med($\frac{\Pi}{AT}$)	$\frac{Debt}{AT}$	$\frac{Int}{AT}$	MTB	s	ω_{AT}	ω_{Sale}
1971-1979	32980	10038.27	10023.87	0.09	0.09	0.26	0.02	1.22	1.17	3.61	2.84
1980-1989	45133	12349.14	11061.23	0.03	0.03	0.28	0.03	1.26	1.02	3.11	2.52
1990-1999	53860	14334.47	11471.01	0.05	0.05	0.31	0.03	1.81	0.86	2.43	1.99
2000-2009	44331	16043.32	12087.45	0.06	0.06	0.30	0.02	1.83	0.82	2.03	1.73
2010-2019	31898	17581.01	12570.93	0.08	0.07	0.34	0.02	1.89	0.75	1.89	1.70
2020-2022	8643	18249.69	12429.50	0.11	0.11	0.38	0.01	2.36	0.66	1.79	1.69

Table II: Debt and Interest Rates by Firm Size and Profitability

This table compares the debt financing costs for firms with different sizes and profitability. In each year, firms are sorted into quintiles based on their total assets AT_{it} and profitability $\frac{\Pi_{it}}{AT_{it}}$ separately. Panel IIa, IIb and IIc show the average $\frac{Debt}{Sale}$, $\frac{Interest}{Sale}$ and $\frac{Interest}{Debt}$ for the twenty five size-profitability groups respectively.

(a) $\frac{Debt}{Sale}$

	1971-2022					2000-2022				
	Small	2	3	4	Large	Small	2	3	4	Large
Low profitability	1.86	1.23	1.23	1.32	1.49	2.85	1.72	1.87	2.79	2.32
2	0.47	0.41	0.46	0.68	0.82	0.62	0.44	0.53	0.91	1.20
3	0.26	0.23	0.32	0.42	0.57	0.24	0.22	0.36	0.54	0.74
4	0.24	0.23	0.27	0.38	0.45	0.23	0.24	0.32	0.50	0.58
High profitability	0.23	0.21	0.29	0.35	0.41	0.21	0.22	0.38	0.45	0.48

(b) $\frac{Interest}{Sale}$

	1971-2022					2000-2022				
	Small	2	3	4	Large	Small	2	3	4	Large
Low profitability	0.31	0.15	0.13	0.13	0.13	0.52	0.22	0.18	0.22	0.23
2	0.05	0.04	0.04	0.06	0.07	0.08	0.05	0.04	0.07	0.09
3	0.03	0.02	0.03	0.03	0.04	0.02	0.02	0.03	0.04	0.04
4	0.02	0.02	0.02	0.03	0.03	0.02	0.02	0.02	0.03	0.03
High profitability	0.03	0.02	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.03

(c) $\frac{Interest}{Debt}$

	1971-2022					2000-2022				
	Small	2	3	4	Large	Small	2	3	4	Large
Low profitability	0.26	0.19	0.15	0.13	0.12	0.35	0.22	0.15	0.10	0.10
2	0.15	0.15	0.13	0.11	0.10	0.16	0.15	0.15	0.11	0.10
3	0.14	0.13	0.12	0.10	0.09	0.14	0.12	0.12	0.09	0.07
4	0.15	0.14	0.13	0.10	0.09	0.16	0.12	0.13	0.09	0.07
High profitability	0.18	0.17	0.15	0.12	0.09	0.19	0.15	0.13	0.10	0.07

Table III: Profit Transitions

This table presents transition probabilities based on simple tabulations. Within each year all existing firms are sorted into quintiles according to $\frac{\Pi}{AT}$. Five years later the quintile for each firm that still exists is identified. The use of a 5 year interval helps define medium run effects. The 'enter' category tabulates the pattern for firms that did not exist on date $t-1$ but did exist on date t. For each cell in 'enter' row, it represents the number of this group of new entry firms that fall in a given quintile (or 'exit' category) on date $t+5$ divided by the total number of new entry firms on date t. The 'exit' category tabulates the number of firms that existed on date t in a given quintile but did not exist on date $t+5$ divided by the total number of firms on date t. 'Bankruptcy' column presents the percentage of all firms in each category that went bankruptcy or liquidation (Compustat item $dlrsn = 02,\ 03$), while the 'M&A' column gives the percentage of all firms in each category that were merged or acquired ($dlrsn = 01$). For each year during the period from 1971 to 2017, we calculate the transition probability, then average across years. The 'mean' in the last row (column) of each panel displays the average $\frac{\Pi_{it}}{AT_{it}}$ of firms who belong to the 1st (low), 2nd, 3rd, 4th, 5th (high) quintile and 'exit' ('enter') category in year t+5 (t) respectively.

(a) 1971-2022

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	Bankruptcy	M&A	(mean)
Low_t	0.24	0.13	0.05	0.04	0.05	0.49	0.05	0.13	-0.56
2_t	0.09	0.22	0.16	0.09	0.06	0.38	0.03	0.19	-0.03
3_t	0.04	0.14	0.24	0.18	0.08	0.31	0.02	0.19	0.04
4_t	0.03	0.09	0.18	0.26	0.16	0.28	0.01	0.18	0.07
$High_t$	0.04	0.07	0.09	0.18	0.33	0.30	0.01	0.18	0.15
enter	0.14	0.12	0.10	0.09	0.11	0.44	0.03	0.17	-0.21
(mean)	-0.33	-0.02	0.03	0.05	0.06	-0.15			

(b) 2000-2022

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	Bankruptcy	M&A	(mean)
Low_t	0.27	0.10	0.02	0.01	0.03	0.56	0.03	0.13	-0.90
2_t	0.08	0.21	0.14	0.08	0.06	0.44	0.03	0.22	-0.05
3_t	0.02	0.13	0.26	0.18	0.08	0.33	0.01	0.20	0.04
4_t	0.01	0.08	0.18	0.27	0.17	0.30	0.01	0.19	0.08
$High_t$	0.02	0.07	0.09	0.17	0.33	0.32	0.01	0.18	0.16
enter	0.13	0.12	0.08	0.07	0.08	0.52	0.02	0.16	-0.41
(mean)	-0.65	-0.06	0.03	0.06	0.07	-0.25			

Table IV: Size Transitions

This table presents transition probabilities based on simple tabulations. Within each year all existing firms are sorted into quintiles according to AT. Five years later the quintile for each firm that still exists is identified. The use of a 5 year interval helps define medium run effects. The 'enter' category tabulates the pattern for firms that did not exist on date $t-1$ but did exist on date t. For each cell in 'enter' row, it represents the number of this group of new entry firms that fall in a given quintile (or 'exit' category) on date $t+5$ divided by the total number of new entry firms on date t. The 'exit' category tabulates the number of firms that existed on date t in a given quintile but did not exist on date $t+5$ divided by the total number of firms on date t. 'Bankruptcy' column presents the percentage of all firms in each category that went bankruptcy or liquidation (Compustat item $dlrsn = 02,\ 03$), while the 'M&A' column gives the percentage of all firms in each category that were merged or acquired ($dlrsn = 01$). For each year during the period from 1971 to 2017, we calculate the transition probability, then average across years. The 'mean' in the last row (column) of each panel displays the average AT of firms who belong to the 1st (small), 2nd, 3rd, 4th, 5th (big) quintile and 'exit' ('enter') category in year t+5 (t) respectively.

(a) 1971-2022

	$Small_{t+5}$	2	3	4	Big_{t+5}	exit	Bankruptcy	M&A	(mean)
$Small_t$	0.36	0.13	0.04	0.01	0.00	0.47	0.03	0.09	9.4
2_t	0.10	0.31	0.15	0.03	0.00	0.40	0.03	0.20	54.5
3_t	0.01	0.11	0.33	0.17	0.01	0.37	0.03	0.23	200.9
4_t	0.00	0.01	0.10	0.44	0.13	0.31	0.02	0.21	687.0
Big_t	0.00	0.00	0.00	0.08	0.70	0.21	0.01	0.14	6611.4
enter	0.13	0.12	0.12	0.11	0.07	0.44	0.03	0.17	485.5
(mean)	15.7	62.6	206.3	634.7	6002.1	704.9			

(b) 2000-2022

	$Small_{t+5}$	2	3	4	Big_{t+5}	exit	Bankruptcy	M&A	(mean)
$Small_t$	0.38	0.07	0.01	0.00	0.00	0.54	0.02	0.09	10.8
2_t	0.11	0.33	0.09	0.01	0.00	0.46	0.02	0.24	86.6
3_t	0.01	0.12	0.37	0.10	0.01	0.40	0.02	0.25	360.4
4_t	0.00	0.01	0.14	0.45	0.08	0.33	0.01	0.21	1257.5
Big_t	0.00	0.00	0.00	0.11	0.67	0.21	0.00	0.14	10220.8
enter	0.12	0.10	0.11	0.09	0.05	0.52	0.02	0.16	775.5
(mean)	20.8	110.3	410.1	1320.3	10150.9	1015.5			

Table V: Maximum Likelihood Estimated Transition Probabilities

This table presents the Maximum Likelihood estimates of the transition matrix. Since the underlying model is a Hidden Markov model, estimation was done using the Expectations Maximization algorithm. The transition probabilities are estimated together with the estimation of the number of firms that are not observed. The transition probabilities from a state at date t (Out, Small, or Big) to a state at date $t+1$ are reported. The state Out are firms that are not observed. The state Small is for the smallest 4 quintiles of firm total assets. The state Big is for the largest quintile of firms. Following each parameter estimate are two numbers in brackets. We bootstrapped the data with 500 replications. The reported numbers in brackets are the values at the 5th and the 95th percentiles. The upper panel reports results using data from 1971-2022. The lower panel reports results for just 2000-2022. *NObs* denotes the number of firm-year observations.

Period	State	Out_{t+1}	$Small_{t+1}$	Big_{t+1}
1971-2022	Out_t	0.888 [0.054,0.891]	0.103 [0.101,0.872]	0.009 [0.008,0.074]
	$Small_t$	0.110 [0.109,0.112]	0.876 [0.875,0.878]	0.013 [0.013,0.014]
	Big_t	0.052 [0.050,0.054]	0.038 [0.037,0.040]	0.910 [0.908,0.912]
	N Obs	191,925	171,554	42,861
2000-2022	Out_t	0.997 [0.996,0.997]	0.003 [0.003,0.004]	0.000 [0.000,0.000]
	$Small_t$	0.132 [0.130,0.134]	0.859 [0.857,0.861]	0.009 [0.009,0.010]
	Big_t	0.050 [0.047,0.052]	0.050 [0.047,0.052]	0.901 [0.897,0.904]
	N Obs	2,249,035	63,523	15,867

Table VI: Parameters for the Numerical Example

This table presents the parameter values used for our numerical analysis.

Parameter	Value(s)	Description
α_1	0.3	Capital share in production for State 1 firms
α_2	0.35	Capital share in production for State 2 firms
β_1	0.5	Labor share in production for State 1 firms
β_2	0.35	Labor share in production for State 2 firms
θ_1	1.0	Total factor productivity of State 1 firms
θ_2	2.0	Total factor productivity of State 2 firms
ρ	0.04	Annual real interest rate
κ_{min}	0.5	Lower bound of entry cost
κ_{max}	4.0	Upper bound of entry cost
D_1	10.0	Demand shifter of State 1 firms
D_2	10.5	Demand shifter of State 2 firms
ε_1	4.5	Price elasticity of demand of State 1 firms
ε_2	3.5	Price elasticity of demand of State 2 firms
W	1.0	Initial wealth (normalized)
p_A	0.05	Price of assets (assumed)
p_F	1.0	Price of flow inputs (normalized)
δ	0.96	Discount factor (assumed)
N	1000	Total number of firms
π_{10}	0.05	Transition probability from State 1 to State 0
π_{11}	0.90	Probability of State 1 firm to stay at State 1
π_{12}	0.05	Transition probability from State 1 to State 2
π_{20}	0.02	Transition probability from State 2 to State 0
π_{21}	0.08	Transition probability from State 2 to State 1
π_{22}	0.90	Probability of State 2 firm to stay at State 2
γ	0.20	Probability of an entry firm to become a State 2 firm $\frac{\pi_{02}}{\pi_{01}+\pi_{02}}$

Table VII: Numerical Analysis

This table presents the distribution of non-existent (State 0), small (State 1) and large firms (State 2) in the steady state, output Y, the amount of total assets A, the amount of flow input F, price P, firm values V, periodic profits v and entry rate. Panel (a) presents the moments using the baseline parameters in Table VI. In panel (b), we examine the potential effect of entry subsidy by decreasing the entry fee range from [0.5,4] to [0.375,3].

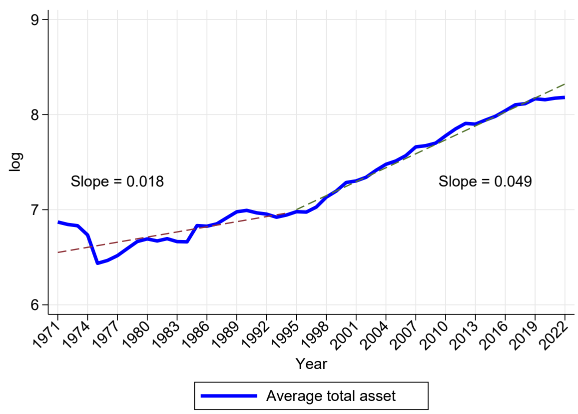
(a) Baseline

	Freq.	Y	A	F	P	V	profits
State 0	0.294	n.a.	n.a.	n.a.	n.a.	26.390	n.a.
State 1	0.435	0.259	0.871	0.073	0.584	27.087	1.070
State 2	0.272	1.930	4.249	0.212	0.327	28.061	1.229
Entry rate	0.093						

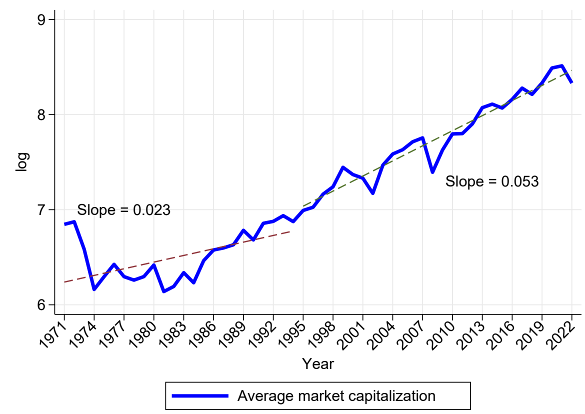
(b) Entry subsidy

	Freq.	Y	A	F	P	V	profits
State 0	0.234	n.a.	n.a.	n.a.	n.a.	26.542	n.a.
State 1	0.471	0.249	0.831	0.069	0.579	27.123	1.069
State 2	0.295	1.868	4.056	0.203	0.323	28.040	1.221
Entry rate	0.126						

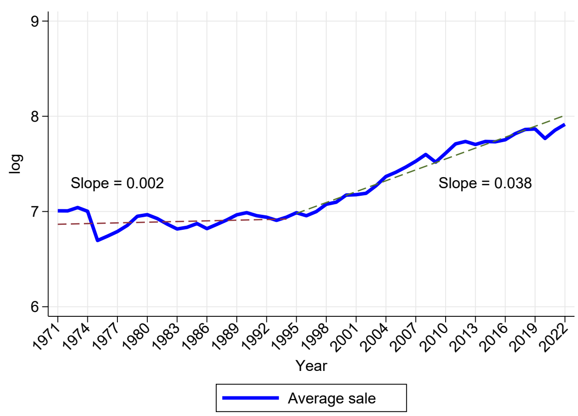
Figure I: Average Firm Size



(a) Total asset



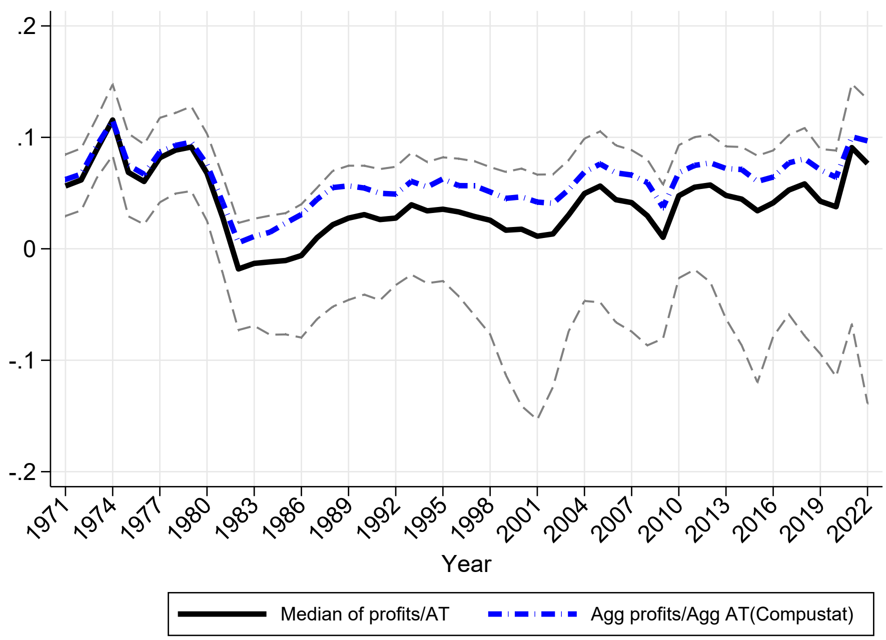
(b) Market capitalization



(c) Sale

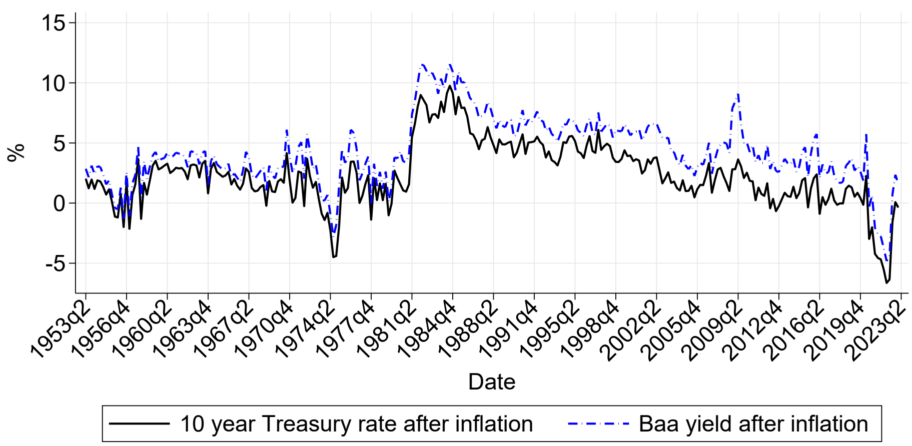
This figure depicts the evolution of average firm size and output over time. For each year, we calculate the average total asset, market capitalization and sale, i.e., $\sum_{i=1} \frac{y_{it}}{N_t}$, where y_{it} is firm i's total asset, market capitalization or sale, and N_t is the total number of firms on date t. All figures are plotted using logarithm scale on the yaxis.

Figure II: Aggregate Profits



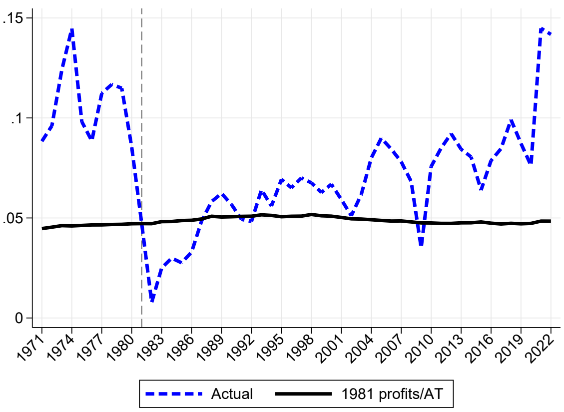
This figure depicts the trends of profits. Profit is defined as $\Pi_{it} = Sales_{it} - COGS_{it} - SGA_{it} - T_{it} - \rho AT_{it}$, where Sales is revenue, COGS is cost of goods sold, SGA is selling general and administrative expenses, T is tax, ρ is the outside rate of return and AT is total assets. The blue dash-dot line shows the ratio between aggregate profit and aggregate total assets ($\frac{\sum_i \Pi_{it}}{\sum_i AT_{it}}$) for each year t. The black solid line shows for each date t the median over firm i's $\frac{\Pi_{it}}{AT_{it}}$. The gray dash lines are the first and third quartiles over $\frac{\Pi_{it}}{AT_{it}}$ for each year t. The data are publicly traded U.S. firms from Compustat, excluding firms in financial and regulated industries.

Figure III: Key Bond Yields

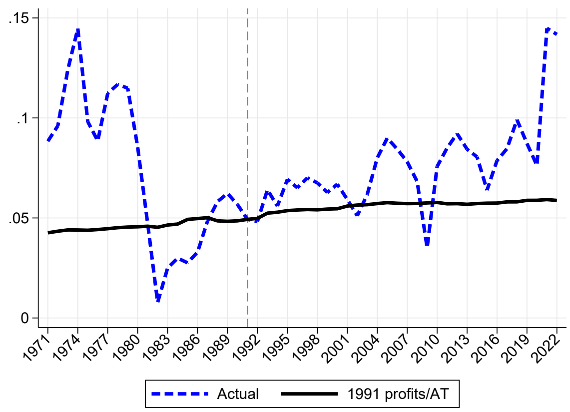


This figure plots the 10-year treasury rate and Baa yield from 1953 Q2 to 2023 Q1. The average value of 10 year treasury rate over the whole sample is 2.36%. From April 1953 to the end of 1980, the average was 1.45%. From January 1981 to March 2023, the average was 2.96%. Data is from https://fred.stlouisfed.org (GS10, BAA, and A191RI1Q225SBEA). The plot shows both series with inflation subtracted.

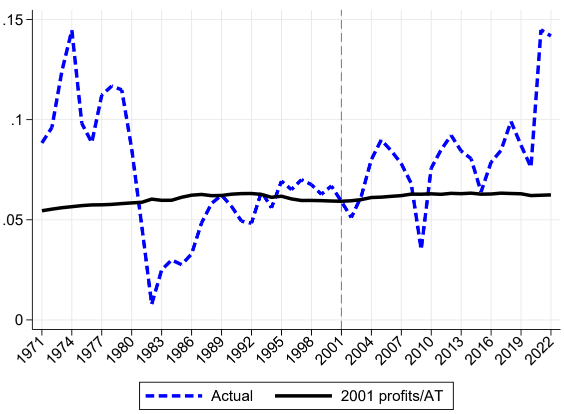
Figure IV: Superstar Firms



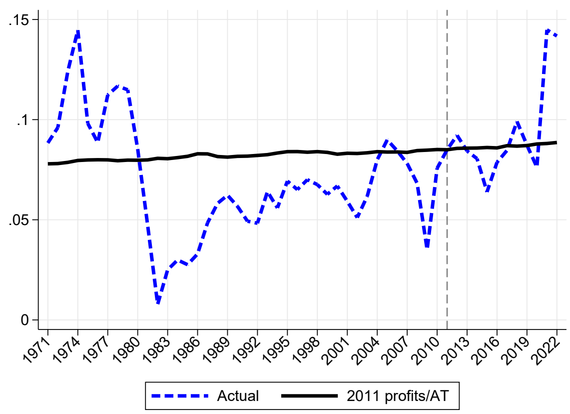
(a) Baseline year 1981



(b) Baseline year 1991



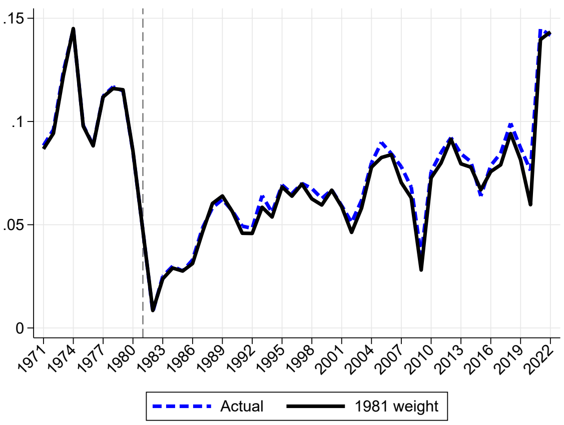
(c) Baseline year 2001



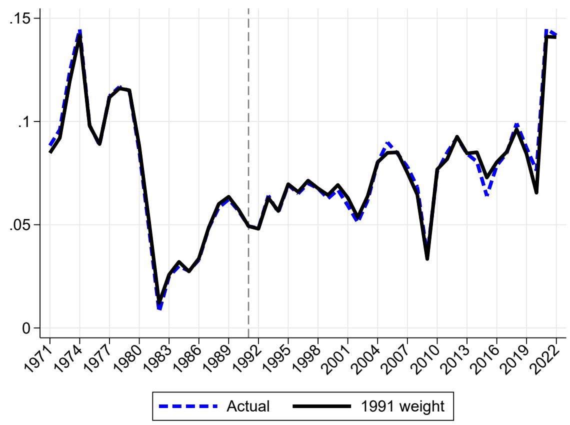
(d) Baseline year 2011

This figure depicts results for a balanced panel of annual data: only firms that exist for each year during 1971-2022 are included. In each panel the blue dashed line shows the aggregate profitability: $\frac{\sum_i \Pi_{it}}{\sum_i AT_{it}}$, where $t = 1971, \ldots, 2022$. Each black line fixes the profitability on a specific date $\hat{t}$, where $\hat{t} = 1981, 1991, 2001, 2011$ respectively. On a given t date, each firm is assigned its profitability at time $\hat{t}$: $\frac{\Pi_{i\hat{t}}}{AT_{i\hat{t}}}$. So for each date $t = 1971, \ldots 2022$ a counterfactual aggregate profitability ratio is $\sum \omega_{it} \frac{\Pi_{i\hat{t}}}{AT_{i\hat{t}}}$. Thus the observations are weighted to the fixed profitability for the $\hat{t}$ date. Firms that were very profitable on that date, are thus given a correspondingly large profitability value on every other date in the reweighted version. The result is robust for a full panel that includes firms that enter and exit.

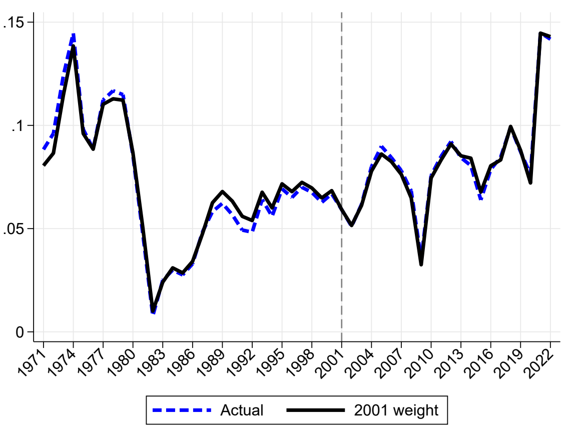
Figure V: Large Firms



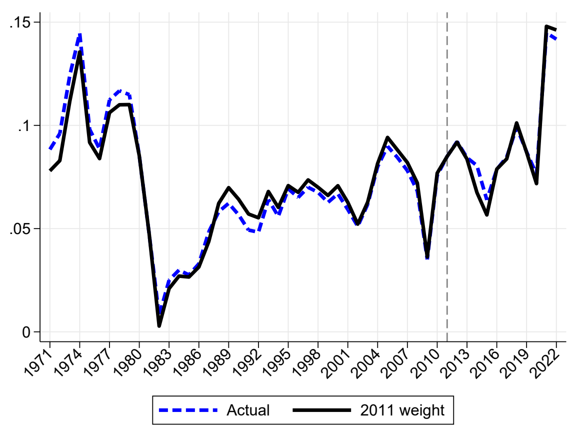
(a) Baseline year 1981



(b) Baseline year 1991



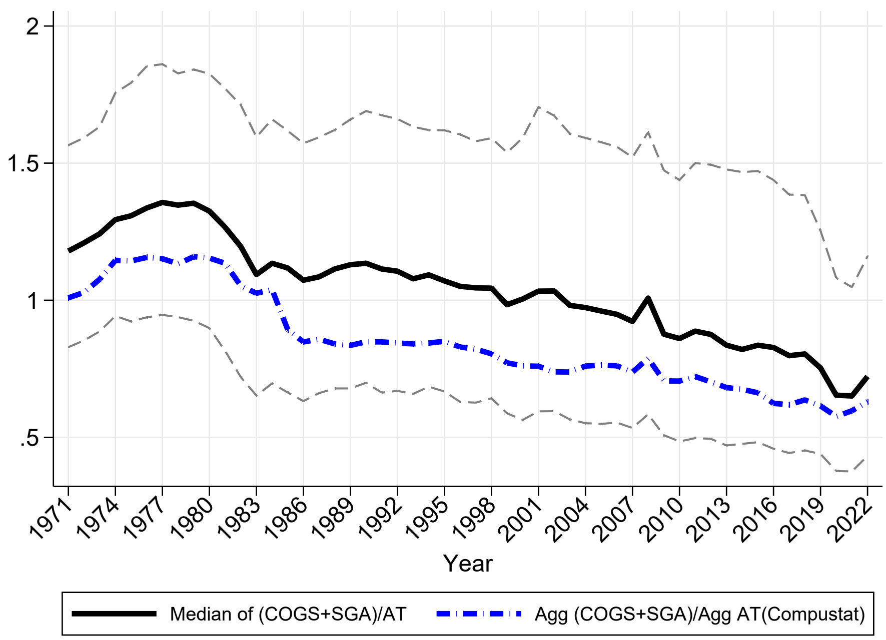
(c) Baseline year 2001



(d) Baseline year 2011

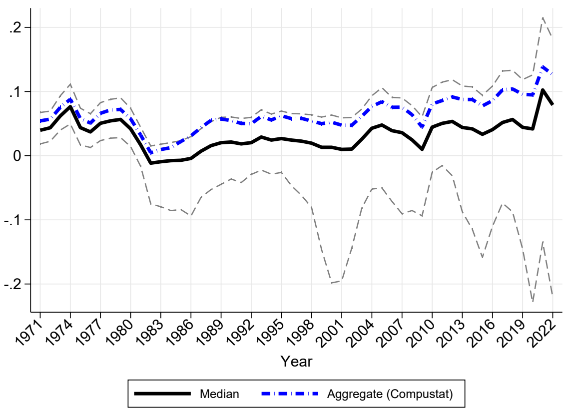
This figure depicts results for a balanced panel of annual data: only firms that exist for each year during 1971-2022 are included. This figure plots the actual profitability, $\sum \omega_{it} \frac{\Pi_{it}}{AT_{it}}$, against the counterfactual profitability under the Large firm scenario, $\sum \omega_{i\hat{t}} \frac{\Pi_{it}}{AT_{it}}$, where the weight $w_{i\hat{t}} = \frac{AT_{i\hat{t}}}{\sum AT_{i\hat{t}}}$. The weights are fixed at $\hat{t}$, where $\hat{t} = 1981, 1991, 2001$ or 2011 respectively, for the counterfactual throughout the whole sample period from 1971 to 2022. This figure shows that firms that are initially large drive most of the variation in overall firm profitability. The result is robust for a full panel that includes firms that enter and exit.

Figure VI: Aggregate Flow Cost

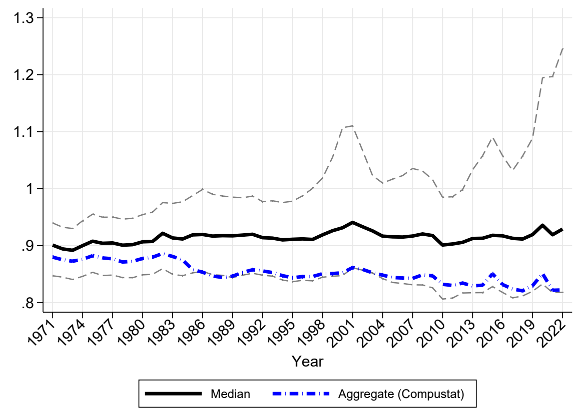


This figure depicts the trends of flow cost. The blue dash-dot line shows the ratio between aggregate flow costs and aggregate total assets ($\frac{\sum_i (COGS_{it}+SGA_{it})}{\sum_i AT_{it}}$) over time, the black solid line shows for each date t the median over firm i's $\frac{(COGS_{it}+SGA_{it})}{AT_{it}}$, and the gray dash lines are the first and third quartiles over $\frac{(COGS_{it}+SGA_{it})}{AT_{it}}$ for each date t. The sample used to calculate all these statistics includes U.S. firms from Compustat excluding those from financial and regulated industries. For the calculation of $\frac{\sum_i (COGS_{it}+SGA_{it})}{\sum_i AT_{it}}$, both $COGS_{it}+SGA_{it}$ and AT_{it} are winsorized at 1st and 99th percentiles across years.

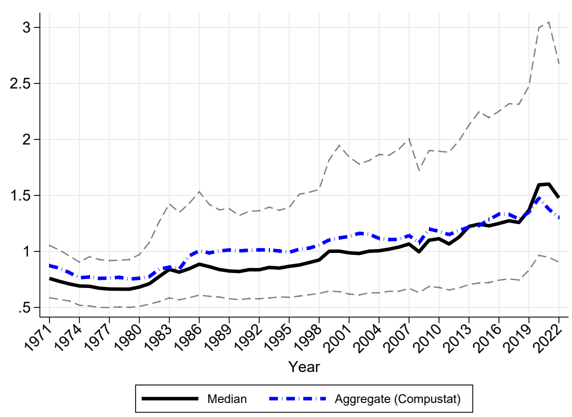
Figure VII: Sales Revenue Shares



(a) Profit share, s_{Π}



(b) Flow cost share, s_F



(c) Total asset share, s_A

This figure plots s_{Π}, s_F, s_A over time. Panel VIIa shows the profit share $\frac{\Pi}{Sales}$. Panel VIIb shows the flow cost share $s_F = \frac{COGS+SGA}{Sales}$. Panel VIIc shows the total asset share $s_A = \frac{AT}{Sales}$. In each panel the solid black line shows the median value among the firms in each year. The dashed blue line shows the aggregate across firms within a given year. Light grey lines show the locations of the first and third quartiles in each year. The data used to calculate all these measures is U.S. firms in Compustat, excluding those from financial and regulated industries. s_F, s_A and s_{Π} are all winsorized at 1st and 99th percentiles across years.

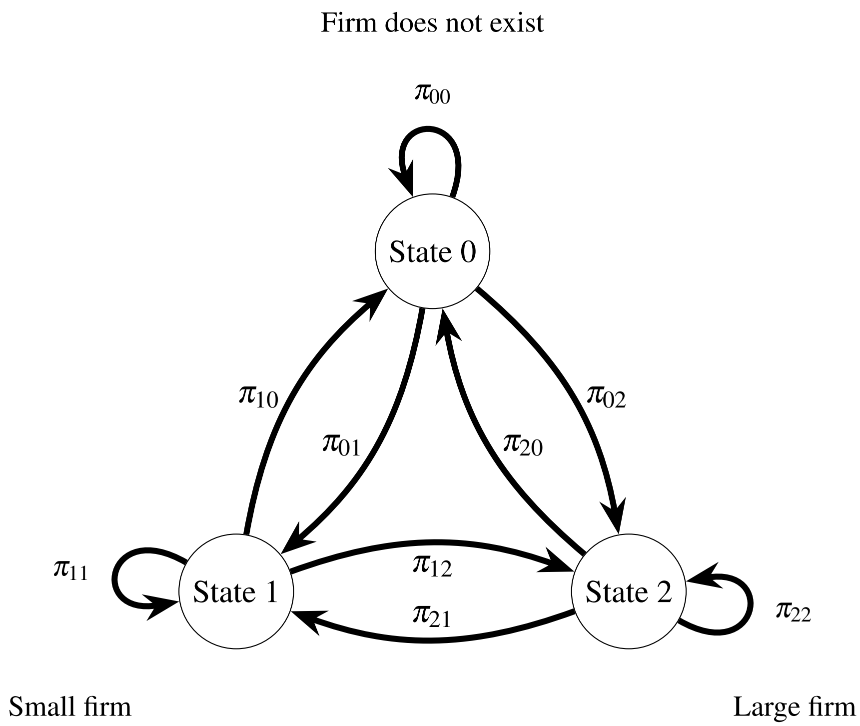
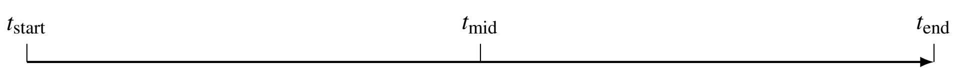


Figure VIII: Firm State Transitions.

π_{ij} is the transition probability from state i to state j. State 0: Non-existent firm; State 1: Small firm; State 2: Large firm.

Figure IX: Order of Events Within Period t



Start of Period
- Nature assigns κ_i to state 0 firms
- State 0 firms
  - Pay nothing, or pay κ_i
  - Invest remaining wealth (W or $W-\kappa_i$) at return ρ
- State 1 and 2 firms
  - Buy inputs A, F and start production
  - Invest remaining wealth $(W - p_A A - p_F F)$ at return ρ

During Period
- State 0 firms
  - Get return on financial investment
- Output market clears
- State 1 and 2 firms
  - Get operating profits
  - Get return on financial investment

End of Period
- State 0 firms that paid κ_i
  - Period $t+1$ state revealed: State 1 or 2
- State 1 and 2 firms
  - Period $t+1$ state revealed: State 0, 1, or 2

Internet Appendices for: The Changing Structure of Corporate Profits

Murray Z. Frank Jing Gao

Table A.1: Variable Definitions

Level variables	Compustat
AT	at
Debt	dltt+dlc
Interest	xint
Sale	sale
Tax	txt
CapEx	capx
COGS	cogs
SGA	xsga
EBITDA	oibdp
Π (EVA)	oibdp-txt-ρat
ρ	Baa corporate bond yield (BAA from FRED)
	deflated using the GDP deflator (GDPDEF from FRED)
VA (value added)	sale - cogs
NOPAT	oibdp-txt
OIADP	oiadp
NI (net income)	ni
DWC	-(recch+invch+apalch+txach+aoloch) if scf =7
	wcapch if scf=1, -wcapc if scf=2 or scf=3
FCF (free cash flow)	oibdp - txt - capx - DWC
FCFE (free cash flow to equity)	oibdp - txt - xint - capx - DWC + ΔDebt
Ratios	
s	$\frac{cogs+sga}{at}$
s_F	$\frac{cogs+sga}{sale}$
s_A	$\frac{at}{sale}$
s_Π	$\frac{oibdp-txt-\rho*at}{sale}$
ω_{AT}	$\frac{at_i}{\sum_{j=1}^{J} at_j}$
ω_{sale}	$\frac{sale_i}{\sum_{j=1}^{J} sale_j}$
MTB	$\frac{mva}{at}$, where mva =at+mve-seq,
	mve = prcc_f*csho,
	= mkvalt if prcc_f*csho is missing and mkvalt is not missing
	mva is set to be missing if mva$<$ 0 or mva $<$ Debt

I. MEASURING PROFIT USING ACCOUNTING DATA

The word profit has a clear meaning within a model. When it is applied to real firms, a variety of complications are unavoidable. As a result, there are quite a few closely related accounting measures that make a range of adjustments. Whether any particular adjustment is appropriate depends on the context and on the intended purpose. None of them are perfect matches for the theoretical concept of profits, in all settings. In academic papers, several different measures have been adopted (Mitton 2022). These papers typically contain very little discussion of the relative merits of the profit definition adopted.

It turns out that the choice of which accounting measure to define as profits is not innocuous. In this appendix we review a number of these measures and show that many common measures such as the popular EBITDA, ignore the cost to the firm of using assets. This turns out to matter for inferences. The accounting choice is normally given with little or no discussion or justification. Some parts of profits are easy to measure with reasonable reliability but not all. A sufficiently noisy measure may create more problems than it solves so a number of accounting measures choose to exclude certain aspects. Dropping an aspect can also alter the interpretation. Bushman et al. (2016) observe that accrual accounting is intended to smooth out the effect of noise on earnings. This implies that accruals and cash flows should be negatively correlated contemporaneously. This is observed, but it has sharply diminished over time. Rouen et al. (2021) consider alternative measures of corporate earnings. GAAP earnings focus attention on core earnings, but they find that non-core earnings are of growing importance. These papers help explain why so many adjustments to accounting data are observed in various papers.

In this section we consider the impact of a number of related measures. Since their definitions differ, the extent to which they generate different results can itself be informative. Often these are divided by total assets when used. The following measures are examined,

VA (Value Added). Formula: $VA_{it} = Sales_{it} - COGS_{it}$. VA is the value added by a company from core operations. It is the difference between its sales revenue and the cost of goods sold (COGS). Also called Gross Profit. Data from income statement. Example of use Kehrig & Vincent (2021).

EBITDA (Earnings Before Interest, Taxes, Depreciation, and Amortization). Formula: $EBITDA_{it} = Sales_{it} - COGS_{it} - SGA_{it}$. EBITDA is a measure of a company's operating performance before considering interest, taxes, depreciation, and amortization. It is the earnings generated from its core operations. Data from income statement. Example of use Davis et al. (2023).

NOPAT (Net Operating Profit After Tax). Formula: $NOPAT_{it} = EBITDA_{it} - T_{it}$. NOPAT is also called Earnings After Tax (EAT) and Operating Income Before Depreciation (OIBDP). It starts with EBITDA and subtracts taxes (T). Data from income statement. Example of use Grullon et al. (2019).

OIADP (Operating Income After Depreciation and Taxes). Formula: $OIADP_{it} = EBITDA_{it} - Depr_{it}$. OIADP represents a company's operating income after accounting for depreciation expenses. Data from income statement. Example of use Covarrubias et al. (2020).

NI (Net Income). Formula: $NI = EBITDA_{it} - T_{it} - Depr_{it} - Int_{it}$. NI represents a company's net income and accounts for taxes, depreciation, and interest expenses. Data from income statement. Example of use Kwon et al. (2024).

EVA (Economic Value Added). Formula: $EVA_{it} = EBITDA_{it} - T_{it} - \rho AT_{it}$. EVA measures a company's economic profit after deducting a charge for the use of assets (ρAT_{it}). It is EBITDA adjusted for tax and the cost of capital. Data from income statement, balance sheet, and the opportunity cost measure comes from from elsewhere. ρ is the opportunity cost of capital. Requires an empirical proxy for ρ such as the cost of debt or WACC (Frank & Shen 2016). Example of use Grant (2003).

FCF (Free Cash Flow). Formula: $FCF_{it} = EBITDA_{it} - T_{it} - CAPX_{it} - \Delta NWC_{it}$. FCF measures the cash generated or available for distribution to investors after considering taxes, capital expenditures (CAPX), and changes in net working capital (NWC). Data from income statement, balance sheet and statement of cash flows. Example of use Jensen (1986).

FCFE (Free Cash Flow to Equity). Formula: $FCFE_{it} = EBITDA_{it} - T_{it} - CAPX_{it} - \Delta NWC_{it} - Int_{it} + \Delta D_{it}$. FCFE is a measure of cash available to be distributed to equity holders after accounting for taxes, capital expenditures, changes in net working capital, interest, and changes in debt (D). Data from income statement, balance sheet and statement of cash flows. Example of use Damodaran (2007).

In the listed formulas the variable $Depr_{it}$ is the sum of depreciation and amortization. As with many other aspects of accounting, one could distinguish them and adjust the formulas accordingly. The listed definitions are standard.

Broadly speaking there we can distinguish two groups of profit definitions, depending on whether they subtract a cost of capital expense or not. First group: VA, EBITDA, NOPAT, OIADP, NI. This first group does not subtract the expense of obtaining and using assets. Second group: NI, FCF, FCFE. This second group does adjust to reflect the cost of using assets.

The first group generally shows profit declining over time. The second group generally show profit rising since about 1985. This difference suggests strongly that a key difference is that the cost of using assets has been declining for a number of decades. This makes sense since the interest rate environment has generally been declining over that same period.

Since accounting has several definitions, it is possible to start the calculations in different places and still get the same accounting measure in the end, provided the corresponding adjustments are used. For instance consider EBITDA. According to Mitton (2022) EBITDA is the most widely used measure of profits in the academic literature. Some papers use EBITDA as a measure of cash flows, but not necessarily as profits (Lian & Ma 2021). In this paper, following Compustat start with sales and then subtracts costs. But EBITDA is not a standardized measure. There are various ways to calculate it and in some cases various expenses or adjustments are used even beyond those listed here. Some common methods include: 1) Starting with Net Income (NI): EBITDA = Net Income (NI) + Interest + Taxes + Depreciation + Amortization; 2) Starting with Operating Income (OI): EBITDA = Operating Income (OI) + Depreciation + Amortization; 3)

Starting with Earnings Before Tax (EBT): EBITDA = Earnings Before Tax (EBT) + Depreciation + Amortization; 4) Starting with Earnings Before Interest (EBI): EBITDA = Earnings Before Interest (EBI) + Taxes + Depreciation + Amortization; 5) Starting with Gross Profit: EBITDA = Gross Profit + Operating Expenses excluding depreciation and amortization. So EBITDA described by one scholar may differ from EBITDA described by another. Many common measure of profits start with EBITDA. So variation in the method to calculate EBITDA will generate corresponding differences in the subsequent measures.

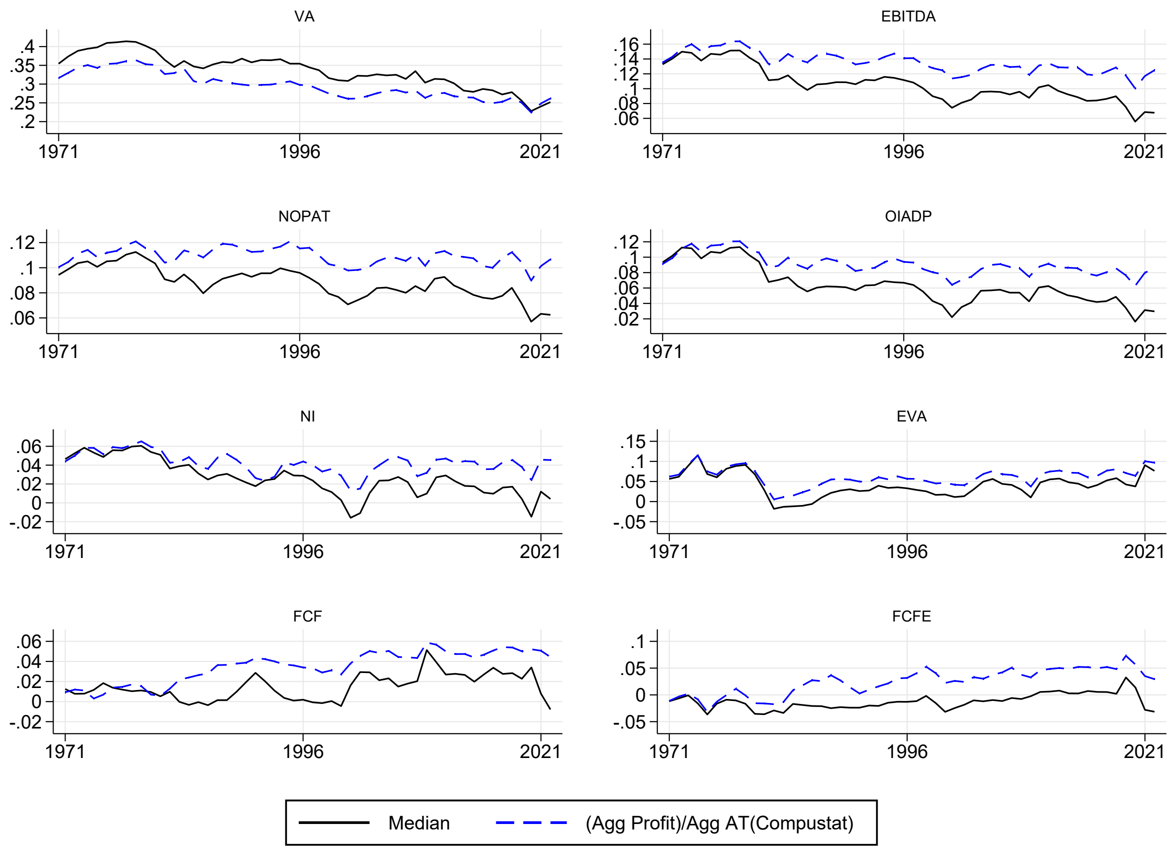
There is also some variation across accountants in how they interpret the instructions for how to apply the definitions to particular circumstances at particular firms. This can also create variability that is not readily measurable. All of the measures need to be inflation adjusted to a common year. The GDP deflator is used, based in 2017 dollars.

Table A.2: Correlation Among Profit Proxies

Profit definitions: $VA_{it} = Sales_{it} - COGS_{it}, EBITDA_{it} = Sales_{it} - COGS_{it} - SGA_{it}, NOPAT_{it} = EBITDA_{it} - T_{it}, OIADP_{it} = EBITDA_{it} - Depr_{it}, NI = EBITDA_{it} - T_{it} - Depr_{it} - Int_{it}, EVA_{it} = EBITDA_{it} - T_{it} - \rho AT_{it}, FCF_{it} = EBITDA_{it} - T_{it} - CAPX_{it} - \Delta NWC_{it}, FCFE_{it} = EBITDA_{it} - T_{it} - CAPX_{it} - \Delta NWC_{it} - Int_{it} + \Delta D_{it}$. For each firm in each year, each of the profit proxies are calculated. All firm years are equally weighted. So each firm/year observation has a weight $w_{it} = 1$. ρ is the Baa corporate bond yield after inflation (BAA and A191RI1Q225SBEA from FRED). In the Internet appendix, value (AT_{it}) weighted correlations among the measures are reported.

	VA	EBITDA	NOPAT	OIADP	NI	EVA	FCF	FCFE
VA	1.000							
EBITDA	0.945	1.000						
NOPAT	0.937	0.991	1.000					
OIADP	0.929	0.978	0.959	1.000				
NI	0.820	0.864	0.840	0.902	1.000			
EVA	0.883	0.940	0.949	0.923	0.832	1.000		
FCF	0.765	0.785	0.798	0.795	0.700	0.791	1.000	
FCFE	0.562	0.572	0.576	0.594	0.550	0.576	0.702	1.000

Figure A.1: Alternative Profit Measures



In this figure, each plot is a version of Figure II for a measure that has been used as a proxy for profits in previous academic studies. In each panel, the solid black line shows the median value across firms in each year. The dashed blue line shows the profits in a year aggregated across firms and dividend by total assets also aggregated across firms in that year. The data are for US firms in Compustat excluding financials and regulated firms. When calculating EVA, Moody's Seasoned Baa Corporate Bond Yield after inflation is used as a proxy for ρ. The measures are defined as follows: $VA_{it} = Sales_{it} - COGS_{it}, EBITDA_{it} = Sales_{it} - COGS_{it} - SGA_{it}, NOPAT_{it} = EBITDA_{it} - T_{it}, OIADP_{it} = EBITDA_{it} - Depr_{it}, NI = EBITDA_{it} - T_{it} - Depr_{it} - Int_{it}, EVA_{it} = EBITDA_{it} - T_{it} - \rho AT_{it}, FCF_{it} = EBITDA_{it} - T_{it} - CAPX_{it} - \Delta NWC_{it}, FCFE_{it} = EBITDA_{it} - T_{it} - CAPX_{it} - \Delta NWC_{it} - Int_{it} + \Delta D_{it}$.

II. MODEL CALCULATION DETAILS

Calculation of Optimal Inputs

Find the values of A and F that satisfy the first order conditions, $\frac{\partial V^{In}}{\partial A} = 0$, $\frac{\partial V^{In}}{\partial F} = 0$. Thus,

$$\frac{\partial V^{In}}{\partial A} = \frac{\partial P}{\partial A} Y + P \cdot MP_A - p_A(1+\rho) = 0 \tag{21}$$

$$\frac{\partial V^{In}}{\partial F} = \frac{\partial P}{\partial F} Y + P \cdot MP_F - p_F(1+\rho) = 0 \tag{22}$$

The first order conditions can also be expressed as, $(1-\mu)\alpha\frac{PY}{A} - (1+\rho)p_A = 0$ and $(1-\mu)\beta\frac{PY}{F} - (1+\rho)p_F = 0$. From these conditions, the revenue can instead be written as,

$$PY = \frac{(1+\rho)p_F F}{(1-\mu)\beta} = \frac{(1+\rho)p_A A}{(1-\mu)\alpha}. \tag{23}$$

Then,

$$\frac{p_A A}{p_F F} = \frac{\alpha}{\beta}. \tag{24}$$

Direct calculation gives the firm choices as,

$$A = \theta^{(1-\mu)/\mu} \left[\frac{p_A \beta}{p_F \alpha}\right]^{\beta(1-\mu)/\mu} \left[\frac{\phi\alpha(1-\mu)}{p_A(1+\rho)}\right]^{1/\mu}, \tag{25}$$

$$F = \theta^{(1-\mu)/\mu} \left[\frac{p_F \alpha}{p_A \beta}\right]^{\alpha(1-\mu)/\mu} \left[\frac{\phi\beta(1-\mu)}{p_F(1+\rho)}\right]^{1/\mu}, \tag{26}$$

Conditions of Firm Entry

For the firm to enter, the investor must have enough wealth to cover the firm's needs $p_A A + p_F F \leq W_0$. Recall that it is assumed that W_0 is very large, so $\frac{1-\mu}{1+\rho} PY \leq W_0$. That also means that, $PY \leq \frac{(1+\rho)W_0}{1-\mu}$.

Moreover, the firm profits must be large enough to cover their opportunity cost. That means, $V \geq (1+\rho)W_0$. From the first order conditions, $PY - (1+\rho)(p_A A + p_F F) = PY - (1+\rho)(\frac{\alpha(1-\mu)}{1+\rho} + \frac{\beta(1-\mu)}{1+\rho})PY$.

Thus, $PY-(1+\rho)(p_A A+p_F F)=\mu PY$. This in turn means that, $\mu PY+(1+\rho)W_0-\kappa \geq (1+\rho)W_0$. In other words, $\mu PY = PY-(1+\rho)(p_A A+p_F F)\geq \kappa$. That is, for firms to enter, the production revenue minus opportunity costs must be large enough so that the fixed cost is covered.

These expressions depend on PY. To get that, replace Y with Equations 24 and 26 to obtain,

$$PY = \phi(\frac{\alpha p_F}{\beta p_A})^{\alpha(1-\mu)}\theta^{\frac{(1-\mu)^2}{\mu}}\left[\frac{\alpha p_F}{\beta p_A}\right]^{\frac{\alpha(1-\mu)^2}{\mu}}\left[\frac{\phi\beta(1-\mu)}{p_F(1+\rho)}\right]^{\frac{1-\mu}{\mu}}. \tag{27}$$

Since ρ is the critical return parameter, rewrite the constraint to isolate it,

$$\rho \leq (1-\mu)\theta^{1-\mu}(\frac{\mu}{\kappa})^{\frac{\mu}{1-\mu}}\phi^{\frac{1}{1-\mu}}\left[\frac{\alpha}{p_A}\right]^{\alpha}\left[\frac{\beta}{p_F}\right]^{\beta}-1. \tag{28}$$

If inequality 28 does not hold, the firm does not enter. $A=F=\Pi=0$ and $V=(1+\rho)W_0$. The firm simply does not exist.

The Effect of an Increase in ρ

If the T-bill return is high enough then inequality 28 is violated. The investor puts all funds into T-bills and none into the firm. Then of course, an increase in ρ makes V bigger. Next, consider a firm that operates. Then A and F are both smaller due to ρ. The effect on V is more complex. In order to find the conditions we need for ρ, first combine terms in V,

$$V=(1+\rho)^{1-\frac{1}{\mu}}\theta^{\frac{1-\mu}{\mu}}(1-\mu)^{\frac{1}{\mu}}\phi^{\frac{1}{\mu}}\frac{\mu}{1-\mu}\left[\frac{\beta}{p_F}\right]^{\frac{\beta(1-\mu)}{\mu}}\left[\frac{\alpha}{p_A}\right]^{\frac{\alpha(1-\mu)}{\mu}}-\kappa+(1+\rho)W_0. \tag{29}$$

The first two terms (profits plus money from the investor) are decreasing in ρ. The last term is increasing in ρ. Therefore, in order for the total firm value V to be increasing ρ, the outside investment profit needs to be high enough.

$$\frac{\partial V}{\partial \rho} \geq 0 \Leftrightarrow \rho \geq (1-\mu)\theta^{1-\mu} W_0^{-\mu} \phi \left[\frac{1-\alpha}{p_F}\right]^{\beta(1-\mu)} \left[\frac{\alpha}{p_A}\right]^{\alpha(1-\mu)} - 1. \tag{30}$$

Will the firm actually operate when ρ changes? Recall from equation 28 that this requires,

$$\rho \leq (1-\mu)\theta^{1-\mu} (\frac{\mu}{\kappa})^{\frac{\mu}{1-\mu}} \phi^{\frac{1}{1-\mu}} \left[\frac{\alpha}{p_A}\right]^{\alpha} \left[\frac{\beta}{p_F}\right]^{\beta} - 1. \tag{31}$$

This can also be interpreted as a restriction that says κ is not too large,

$$\kappa \leq W_0^{1-\mu} \phi \left[\frac{1-\alpha}{p_F}\right]^{\beta(1-\mu)} \left[\frac{\alpha}{p_A}\right]^{\alpha(1-\mu)}. \tag{32}$$

III. MAGNIFICENT SEVEN FIRMS

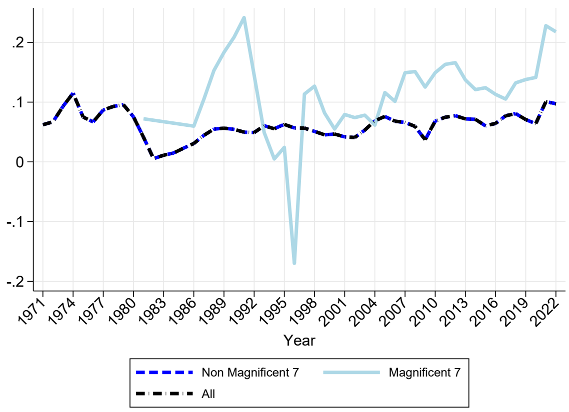
In the financial press it has become common to talk about the outsized impact of seven high profile firms. As a group these have come to be called the Magnificent Seven. "Big tech stocks have jumped 75% in 2023—and now make up about 30% of the S&P 500" [2] The seven firms are: Alphabet, Amazon, Apple, Meta, Microsoft, Nvidia, and Tesla. It is natural to wonder if these firms are also key to our results.

Figure A.2a plots the aggregate book profitability ($\frac{\sum_{i\in\Gamma_t}\Pi_{it}}{\sum_{i\in\Gamma_t}AT_{it}}$) for Magnificent Seven firms, and for the other firms excluding the 7 firms. Note that we do not winsorize total assets or profits for the Magnificent Seven firms as we do for other firms. This is because the winsorization directly affects the Magnificent Seven firms considering they are extremely large firms both in terms of total assets and market value. Prior to 2010 the Magnificent Seven were much more volatile. Since 2010 they are much less volatile. The average profits of the Magnificent Seven firms increased more relative to the other firms in the most recent decade.

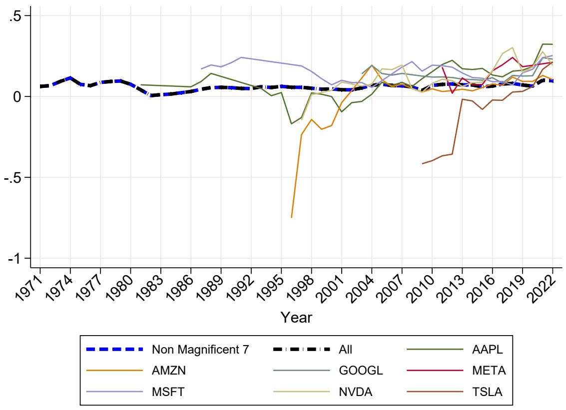
However, Magnificent Seven firms' stock market performance (rather than its accounting performance or the book values) is even more striking, which is why they are called Magnificent. Figure A.3 shows the

[2] https://www.wsj.com/finance/stocks/its-the-magnificent-sevens-market-the-other-stocks-are-just-living-in-it-5d212f95

Figure A.2: Magnificent Seven Firm Profits



(a) Aggregate



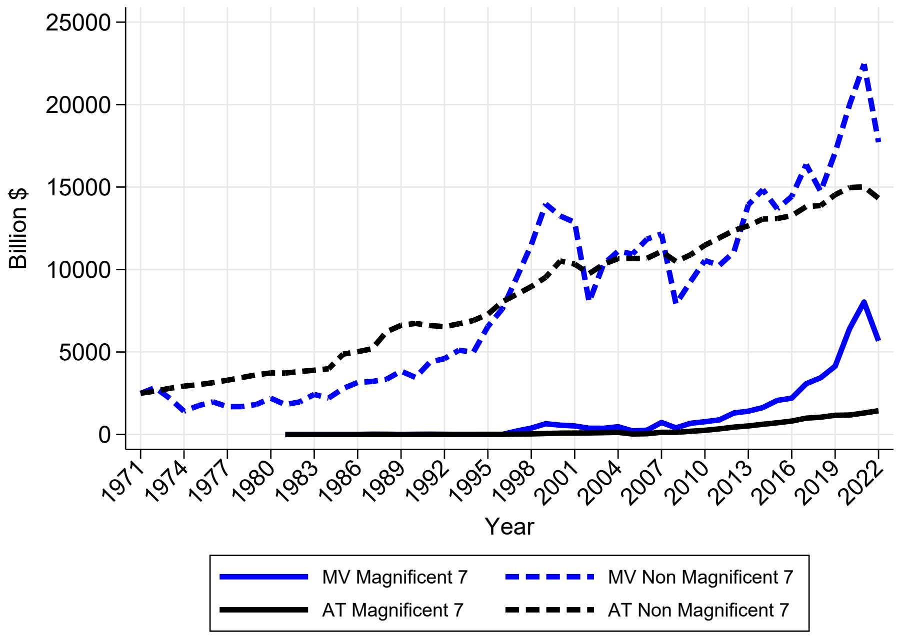
(b) Magnificent Seven

In panel (a), we plot the aggregate profitability for Magnificent Seven firms, the other firms and all firms separately. In panel (b), we plot the profitability for each of the Magnificent Seven firms separately. Profit is defined as $\Pi_{it} = Sales_{it} - COGS_{it} - SGA_{it} - T_{it} - \rho AT_{it}$, where Sales is revenue, COGS is cost of goods sold, SGA is selling general and administrative expenses, T is tax, ρ is the outside rate of return and AT is total assets. The light blue solid line shows the ratio between aggregate profit and aggregate total assets for Magnificent Seven firms ($\frac{\sum_{i\in\Gamma_t}\Pi_{it}}{\sum_{i\in\Gamma_t}AT_{it}}$) for each year t. Γ_t denotes the group of Magnificent Seven firms that include Nvidia, Meta, Amazon, Microsoft, Alphabet, Apple and Tesla. Among these seven firms, Apple entered the sample first in 1981, while Tesla first appeared in our sample in 2009. The blue dash line shows the ratio between aggregate profit and aggregate total assets for firms other than Magnificent Seven firms ($\frac{\sum_{i\notin\Gamma_t}\Pi_{it}}{\sum_{i\notin\Gamma_t}AT_{it}}$) for each year t. The black dot-dash line shows the ratio between aggregate profit and aggregate total assets for all firms ($\frac{\sum_i \Pi_{it}}{\sum_i AT_{it}}$) for each year t. The data are publicly traded U.S. firms from Compustat, excluding firms in financial and regulated industries.

evolution of total market value, and total book assets for the Magnificent Seven firms, and for the other firms. In 2022, the market value of these seven firms accounted for about 24% of the total market value of all firms in the sample (including Magnificent Seven and the other firms). Through the entire data period these firms accounted for roughly 7% of the total market value. That is massive growth. However, in 2022 the total book assets of the seven firms was only about 9% of the total assets of all firms.

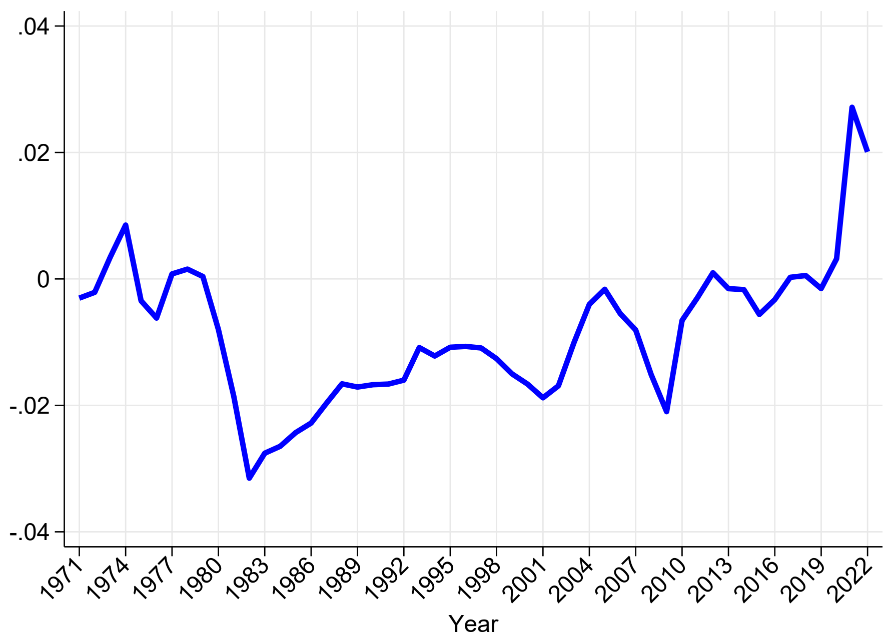
It is clear that the market value of these firms in recent years is widely regarded as remarkable. It also seems clear that these seven firms are not responsible the overall book profit trends that we document.

Figure A.3: Magnificent Seven Market Value and Total Assets



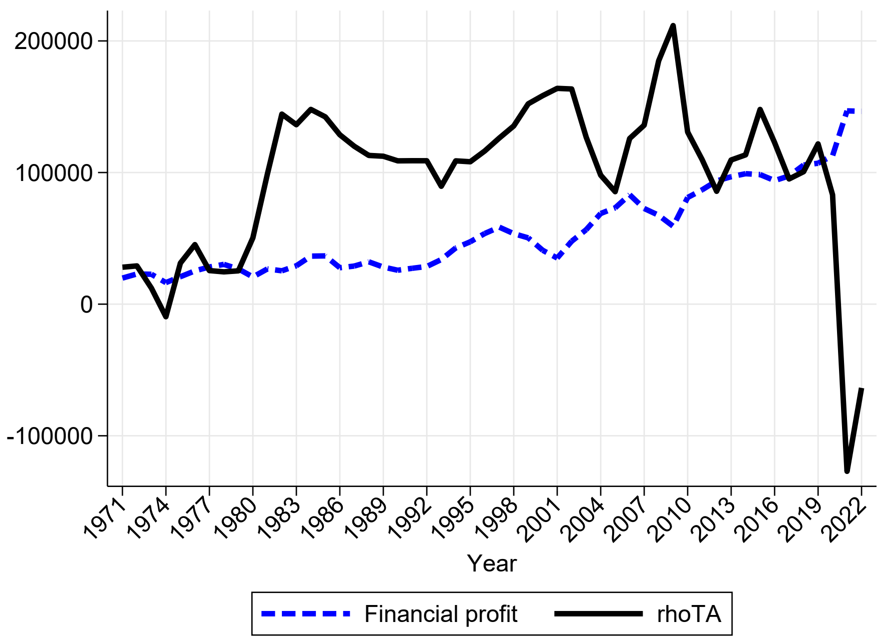
The blue solid line shows the total market value of Magnificent Seven firms ($\sum_{i \in \Gamma_t} MV_{it}$) for each year t (when any of these seven firms exists in the sample). Γ_t denotes the group of Magnificent Seven firms that include Nvidia, Meta, Amazon, Microsoft, Alphabet, Apple and Tesla. Among these seven firms, Apple entered the sample first in 1981, while Tesla first appeared in our sample in 2009. The black solid shows the total assets of these firms ($\sum_{i \in \Gamma_t} AT_{it}$). The blue dash line shows the total market value of non Magnificent Seven firms ($\sum_{i \notin \Gamma_t} MV_{it}$) for each year t, and the black dash line shows the total market value of non Magnificent Seven firms ($\sum_{i \notin \Gamma_t} AT_{it}$) for each year t. Note that we do not winsorize total assets or market value as we do for other results because the winsorization directly affects the Magnificent Seven firms considering they are extremely large firms both in terms of total assets and market value. The data are publicly traded U.S. firms from Compustat, excluding firms in financial and regulated industries.

Figure A.4: Profit share



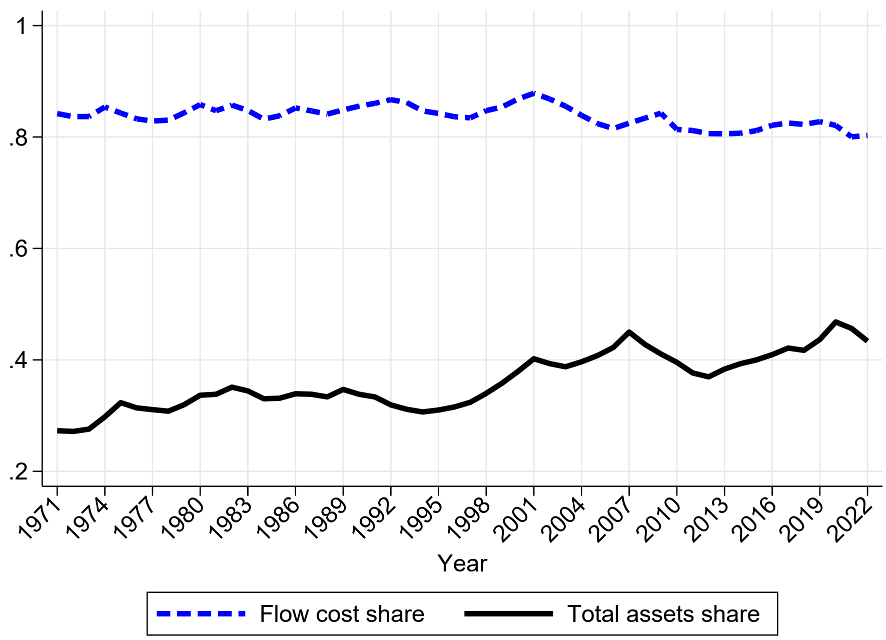
This figure presents the aggregated profit share $\frac{Profits}{Revenue}$ during 1971-2022. Profit is measured by nonfinancial corporate profits (W328RC1Q027SBEA from FRED) subtracted by $\rho * TA$, where TA is total assets (TABSNNCB from FRED). Revenue is non-financial corporate revenue (BOGZ1FA106030005Q from FRED). All variables are in millions of dollars (2017) and are deflated using GDP deflator (GDPDEF from FRED).

Figure A.5: Decompose profits



This figure plots the levels of non-financial corporate (financial) profits (blue dash line) and $\rho * TA$ (black solid line) separately. Financial profit is non-financial corporate profits (W328RC1Q027SBEA from FRED). TA is total assets (TABSNNCB from FRED). All variables are in millions of dollars (2017) and are deflated using GDP deflator (GDPDEF from FRED).

Figure A.6: Costs share



This figure plots the aggregate total assets and flow cost shares. Flow cost is measured by the cost of goods sold (BOGZ1FA106200005Q from FRED). Total asset is measured by TABSNNCB (FRED). All variables are in millions of dollars (2017) and are deflated using GDP deflator (GDPDEF from FRED).

IV. INTERNATIONAL EVIDENCE

Do the US corporate profit results generalize to other countries? Firm growth outside the USA is known to be more sensitive to current profits than are US firms (Frank & Sanati 2021). To examine the other G-7 countries we use Worldscope data for 1980-2021. Cross country comparisons that reported in the appendix could reflect data construction filters impose by Worldscope construction, or they could reflect actual differences across countries. Informal evidence suggests that the results are not just due to Worldscope. However, systematic investigation of that issue would go well beyond the scope of this appendix.

On average US firms are less profitable when they enter, and the have a higher exit rate within 5 years. This is consistent with the the widely held view that US financial markets have been relatively open to marginal firms. Among the G-7 countries Canada seems most similar to the USA in this respect.

Recall that Figure II shows that aggregate US profit is gradually increasing, and that the median firm is growing more slowly than the aggregate. Large firms are growing faster. That is for Compustat data. Figure A.7 provides corresponding plots for each of the G-7 countries. The results for USA firms using Worldscope data are very similar to the results for USA firms Using Compustat data.

Outside the USA there are gradual profit increases observed in Canada, France, Germany, and Japan. However neither Italy nor the UK show strong profit increases on average. Canada and France, like the USA show that larger firms are gradually growing more profitable than average firms. But for the other countries there is not such a clear gap according to firm size.

To dig more deeply into the structure of profits, recall figure VII . Keep in mind that these shares do not cover all components of the firm, and hence do not need to add up to 1. That figure shows that the for USA firms the profits and total asset shares are increasing over times. The flow cost share is largely flat for the median firm. Figure A.8 provides corresponding plots the profit share, flow cost share and total asset share for each of the G-7 countries. For the US firms profits and total asset shares are again observed to be rising over time.

For each of the G-7 countries the profit shares are gradually increasing. For each of the G-7 countries the total assets share is also gradually increasing. These are particularly marked after 2010. Many countries have a sharp increase in profits and total assets following Covid. The exact timing and magnitude varies across countries. But the essential share trends are not unique to US firms. The trends in flow costs varies somewhat from one country to another. In Canada flow cost shares increase while in Germany they are declining.

Overall, the shares over time seem fairly similar across the countries. Profits and total assets seem to be gradually increasing in recent decades. These trends are not unique to the USA.

Table A.3: Profit Transitions of Each of the G-7 Countries

Within each year all existing firms are sorted into quintiles according to $\frac{\Pi}{AT}$ for each of G-7 country excluding the US. Five years later the quintile for each firm that still exists is identified. The use of a 5 year interval helps define medium run effects. The 'enter' category tabulates the number of firms that did not exist on date $t-1$ but did exist on date t in a given quintile divided by the total number of new entry firms on date t. The 'exit' category tabulates the number of firms that existed on date t in a given quintile but did not exist on date $t+5$ divided by the total number of firms on date t. For each year during the period from 1980 to 2016 (the only exception is Japan whose sample starts from 1989), we calculate the transition probability, then average across years. The (mean) categories report the equally weighted average $\frac{\Pi}{AT}$ for all firm-years in that category. The 'mean' in the last row of each panel displays the average $\frac{\Pi_{it}}{AT_{it}}$ of firms who belong to the 1st, 2nd, 3rd, 4th, 5th quintile in year t+5 respectively.

(a) Canada

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	(mean)
Low_t	0.18	0.15	0.11	0.07	0.07	0.41	-0.95
2_t	0.11	0.16	0.21	0.11	0.08	0.34	-0.17
3_t	0.06	0.12	0.17	0.18	0.12	0.34	-0.02
4_t	0.04	0.08	0.13	0.25	0.18	0.32	0.06
$High_t$	0.04	0.05	0.07	0.17	0.34	0.32	0.18
enter	0.12	0.12	0.12	0.14	0.14	0.36	-0.34
(mean)	-0.42	-0.23	-0.11	0.01	0.05	-0.24	

(b) Germany

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	(mean)
Low_t	0.34	0.17	0.07	0.05	0.07	0.30	-0.14
2_t	0.16	0.27	0.20	0.11	0.06	0.20	0.04
3_t	0.09	0.19	0.25	0.19	0.09	0.19	0.07
4_t	0.07	0.13	0.21	0.27	0.17	0.16	0.10
$High_t$	0.06	0.07	0.11	0.22	0.36	0.18	0.18
enter	0.20	0.15	0.12	0.13	0.14	0.26	0.01
(mean)	-0.03	0.05	0.07	0.09	0.10	0.02	

Table A.3: Profit Transitions of Each of the G-7 Countries (continued)

(c) France

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	(mean)
Low_t	0.31	0.16	0.08	0.05	0.06	0.34	-0.13
2_t	0.16	0.26	0.19	0.10	0.06	0.24	0.03
3_t	0.10	0.19	0.26	0.17	0.07	0.21	0.06
4_t	0.08	0.11	0.20	0.28	0.14	0.19	0.09
$High_t$	0.06	0.08	0.11	0.20	0.32	0.24	0.15
enter	0.19	0.15	0.13	0.12	0.14	0.27	0.02
(mean)	-0.03	0.04	0.05	0.07	0.09	0.03	

(d) UK

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	(mean)
Low	0.26	0.17	0.09	0.07	0.05	0.36	-0.32
2_t	0.12	0.21	0.18	0.12	0.06	0.32	0.00
3_t	0.06	0.16	0.23	0.19	0.07	0.28	0.07
4_t	0.05	0.11	0.19	0.26	0.13	0.26	0.10
$High_t$	0.05	0.07	0.10	0.18	0.35	0.25	0.19
enter	0.14	0.14	0.11	0.13	0.11	0.37	-0.04
(mean)	-0.14	0.01	0.05	0.08	0.10	-0.03	

(e) Italy

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	(mean)
Low	0.32	0.16	0.10	0.05	0.04	0.34	-0.07
2_t	0.20	0.22	0.18	0.13	0.03	0.24	0.03
3_t	0.13	0.20	0.20	0.17	0.10	0.21	0.06
4_t	0.08	0.14	0.18	0.24	0.15	0.21	0.08
$High_t$	0.05	0.08	0.12	0.21	0.37	0.17	0.15
enter	0.18	0.13	0.15	0.14	0.19	0.21	0.06
(mean)	0.01	0.04	0.05	0.07	0.11	0.02	

Table A.3: Profit Transitions of Each of the G-7 Countries (continued)

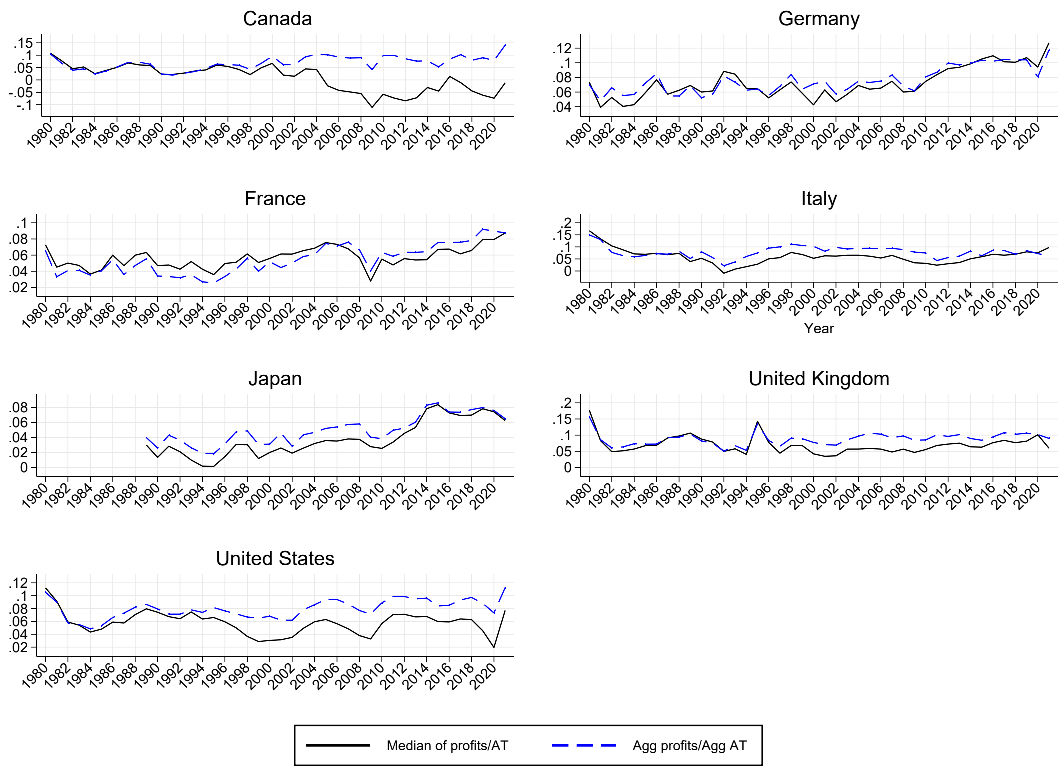
(f) Japan

	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	(mean)
Low_t	0.35	0.23	0.11	0.08	0.07	0.16	-0.04
2_t	0.24	0.31	0.19	0.10	0.05	0.11	0.02
3_t	0.14	0.23	0.27	0.19	0.07	0.10	0.04
4_t	0.10	0.12	0.23	0.29	0.16	0.11	0.06
$High_t$	0.08	0.06	0.10	0.23	0.41	0.12	0.11
enter	0.19	0.16	0.15	0.16	0.19	0.15	0.03
(mean)	0.01	0.02	0.04	0.05	0.07	0.02	

(g) USA

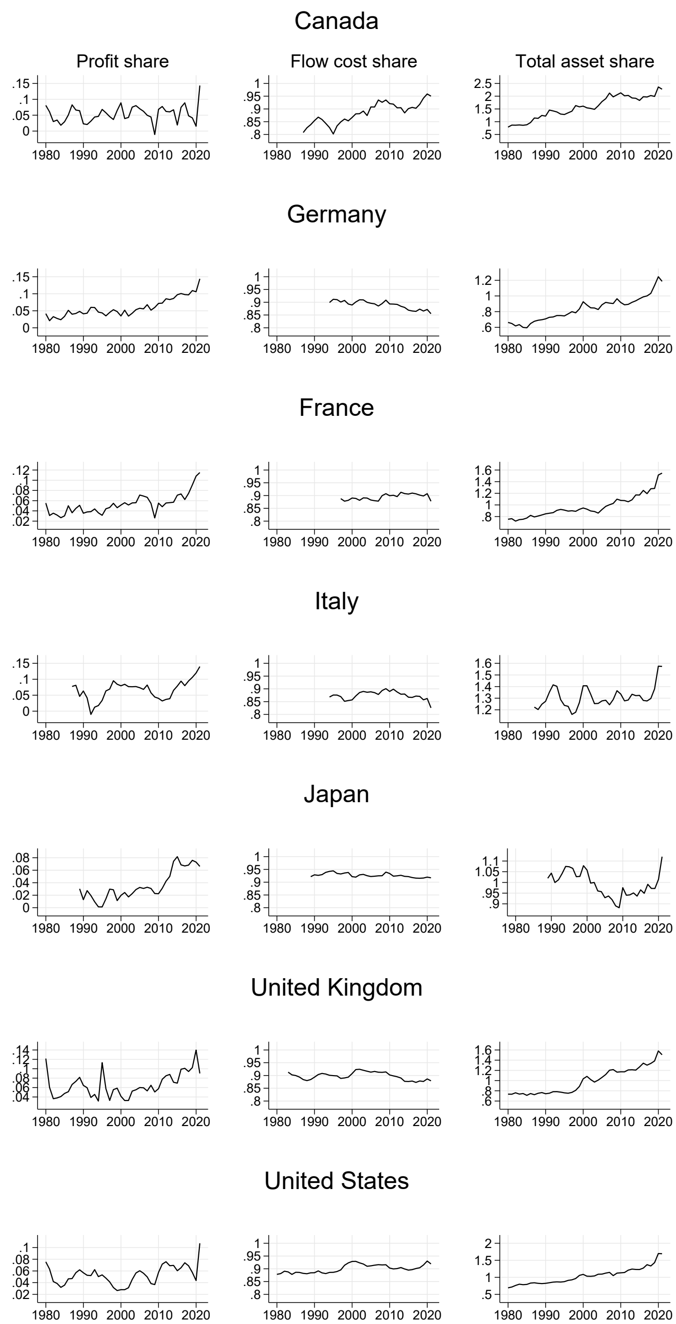
	Low_{t+5}	2_{t+5}	3_{t+5}	4_{t+5}	$High_{t+5}$	exit	(mean)
Low_t	0.22	0.15	0.08	0.05	0.06	0.44	-0.63
2_t	0.10	0.19	0.17	0.11	0.07	0.37	-0.05
3_t	0.05	0.13	0.24	0.20	0.09	0.29	0.05
4_t	0.04	0.09	0.17	0.26	0.17	0.26	0.10
$High_t$	0.05	0.07	0.10	0.18	0.33	0.28	0.18
enter	0.12	0.11	0.11	0.11	0.12	0.44	-0.22
(mean)	-0.31	-0.05	0.04	0.07	0.08	-0.16	

Figure A.7: International Aggregate Profits



This figure plots the evolution of the profits of G-7 countries using information from Worldscope. Profit is defined as $\Pi_{it} = Sales_{it} - COGS_{it} - SGA_{it} - T_{it} - \rho AT_{it}$, where Sales is revenue, COGS is cost of goods sold, SGA is selling general and administrative expenses, T is tax, ρ is the outside rate of return and AT is total assets. There into, ρ is measured using "Long-Term Government Bond Yields: 10-Year" from FRED for each of the G-7 countries. This time series only starts from 1991 for Italy, and for years before, we complement it using "Interest Rates, Government Securities, Government Bonds" series from FRED. The time series of Japan is shorter due to missing ρ. The blue dash-dot line shows the ratio between aggregate profit and aggregate total assets ($\frac{\sum_i \Pi_{it}}{\sum_i AT_{it}}$) for each year t. The black solid line shows for each date t the median over firm i's $\frac{\Pi_{it}}{AT_{it}}$. The data are firms from Worldscope, excluding firms in financial and regulated industries. Before calculating the aggregate ratio, profits and total assets have been converted to US dollar.

Figure A.8: International Revenue Shares



This figure plots s_F, s_A, s_Π over time for all G-7 countries. The first column shows the profit share $\frac{\Pi}{Sales}$. The second column shows the flow cost share $s_F = \frac{COGS+SGA}{Sales}$. The third column shows the total asset share $s_A = \frac{AT}{Sales}$. In each subfigure, the solid black line shows the median value among the firms in each country-year. To mitigate the impact of outliers, we remove country-years for which the number of firms with non-missing sale/input costs/total asset values is smaller than 100. The data used to calculate all these measures is from Worldscope, excluding those from financial and regulated industries. s_F, s_A and s_Π are all winsorized at 1st and 99th percentiles across years within each country.

V. TABULATING TRANSITION PROBABILITIES

Consider a Markov chain model where firms can be in one of three states. State 0 means the firm does not exist. State 1 means the firm is small. State 2 means the firm is large. The purpose is to estimate the transition probabilities between these states using observational data from a panel of firms. We want to estimate the transition matrix Π. It is defined as follows.

$$\Pi = \begin{pmatrix} \pi_{00} & \pi_{01} & \pi_{02} \\ \pi_{10} & \pi_{11} & \pi_{12} \\ \pi_{20} & \pi_{21} & \pi_{22} \end{pmatrix} \tag{33}$$

where π_{ij} is the transition probability from state i to state j.

For each time period t our available data consist of the following. $N_1(t)$ and $N_2(t)$ are the number of firms in state 1 and 2. $E_1(t)$ and $E_2(t)$ are the entries to states 1 and 2. $X_1(t)$ and $X_2(t)$ are the exits from states 1 and 2. $M_{12}(t)$ and $M_{21}(t)$ are the moves between states 1 and 2. The key challenge is sthat we cannot directly observe $N_0(t)$, the number of firms in state 0.

How do we calculate the transition probabilities? For directly observable transitions (π_{11}, π_{12}, π_{21}, π_{22}), we use

$$\hat{\pi}_{ij} = \frac{1}{T}\sum_{t=1}^{T} \frac{\text{Number of transitions from } i \text{ to } j \text{ at time } t}{\text{Total number in state } i \text{ at time } t}. \tag{34}$$

In other words,

$$\hat{\pi}_{11} = \frac{1}{T}\sum_{t=1}^{T}\frac{N_1(t)-E_1(t)-M_{21}(t)+M_{12}(t)-X_1(t)}{N_1(t)} \tag{35}$$

$$\hat{\pi}_{12} = \frac{1}{T}\sum_{t=1}^{T}\frac{M_{12}(t)}{N_1(t)} \tag{36}$$

$$\hat{\pi}_{21} = \frac{1}{T}\sum_{t=1}^{T}\frac{M_{21}(t)}{N_2(t)} \tag{37}$$

$$\hat{\pi}_{22} = \frac{1}{T}\sum_{t=1}^{T}\frac{N_2(t)-E_2(t)-M_{12}(t)+M_{21}(t)-X_2(t)}{N_2(t)} \tag{38}$$

These equations calculate the transition probabilities for each time period t separately, inside the summation. We then take the average of these time-specific probabilities over all T periods, as indicated by the $\frac{1}{T}\sum_{t=1}^{T}$ at the beginning of each equation. Each fraction inside the summation represents the probability for a specific transition at time t.

For transitions to state 0 we have, $\hat{\pi}_{10} = 1-\hat{\pi}_{11}-\hat{\pi}_{12}$, and $\hat{\pi}_{20} = 1-\hat{\pi}_{21}-\hat{\pi}_{22}$.

To estimate $N_0(t)$, π_{01}, and π_{02}, we depend on the steady state assumption. It says that the expected outflows from state 0 equals the expected inflows to state 0. Expected outflow from state 0 is $N_0(t)(\pi_{01}+\pi_{02})$. Expected inflow to state 0 is $N_1(t)\pi_{10}+N_2(t)\pi_{20}$. In a steady state these are equal, $N_0(t)(\pi_{01}+\pi_{02}) = N_1(t)\pi_{10}+N_2(t)\pi_{20}$. Then solving for $N_0(t)$ gives,

$$N_0(t) = \frac{N_1(t)\pi_{10}+N_2(t)\pi_{20}}{\pi_{01}+\pi_{02}} \tag{39}$$

The numerator $(N_1(t)\pi_{10}+N_2(t)\pi_{20})$ represents the total expected number of firms exiting the market. The denominator $(\pi_{01}+\pi_{02})$ represents the total probability of a non-existent firm entering the market. The ratio gives us the number of non-existent firms needed to maintain the balance between entries and exits. So the entry data is given by, $\hat{\pi}_{01} = \frac{\sum_{t=1}^{T}E_1(t)}{\sum_{t=1}^{T}N_0(t)}$, $\hat{\pi}_{02} = \frac{\sum_{t=1}^{T}E_2(t)}{\sum_{t=1}^{T}N_0(t)}$ and $\hat{\pi}_{00} = 1-\hat{\pi}_{01}-\hat{\pi}_{02}$.

The steady-state tabulation method is simple, computationally efficient, and easy to interpret. On the other hand maximum likelihood estimators are asymptotically efficient and thus might provide better estimates. Within MLE we can incorporate EM to handle unobserved states. MLE can be biased for small samples but is asymptotically unbiased. Count tabulation is unbiased but may have high variance for small samples. We use both methods and find essentially similar results.

VI. EM ESTIMATES OF TRANSITION PROBABILITIES

The likelihood function is a traditional approach to estimation that we can use to find the parameters of a Markov model. It represents the probability of observing the given data, given a set of model parameters. Let $L(\theta;X)$ be the likelihood function, with θ representing the parameters (transition probabilities) to be estimated, and X representing the observed data. The likelihood function is defined as,

$$L(\theta;X) = \prod_{t=1}^{T}\prod_{i=0}^{2}\prod_{j=0}^{2}\pi_{ij}^{n_{ij}}, \tag{40}$$

where π_{ij} is the transition probability from state i to state j, and n_{ij} is the number of observed transitions from state i to state j. This function calculates the probability of observing the entire sequence of state transitions in the data, given a particular set of transition probabilities. By maximizing this function, we can find the most likely set of transition probabilities that explain the observed data. For computational convenience, it is common to use the log-likelihood function,

$$\log L(\pi) = \sum_{i=0}^{S}\sum_{j=0}^{S} n_{ij}\log \pi_{ij}, \tag{41}$$

where S is the number of states.

The components of the Likelihood function are: $\prod_{t=1}^{T}$ which is the product over all time periods from 1 to T, $\prod_{i=0}^{2}$ which is the product over all starting states (0, 1, and 2), $\prod_{j=0}^{2}$ which is the product over all ending states (0, 1, and 2), and $\pi_{ij}^{n_{ij}}$ which is the probability of observing n_{ij} transitions from state i to state

j. In the likelihood function these are multiplied for standard reasons. First, each transition is assumed to be independent of others (given the current state), so their probabilities are multiplied. Second, according to the chain rule of probability, the likelihood of the entire sequence can be decomposed into a product of conditional probabilities. So, $P(X_1, X_2, ..., X_T) = P(X_1) \cdot P(X_2|X_1) \cdot P(X_3|X_2) \cdot ... \cdot P(X_T|X_{T-1})$. Third, the likelihood function represents the joint probability of all observed transitions, which is calculated by multiplying individual probabilities of independent events.

The approach rests on several assumptions. First is the Markov property which says that the transitions depend only on the current state, not on the history of states. Second it is assumed that each transition is independent of others. Third, it is assumed that all possible state-to-state transitions are represented. Fourth, the model properly reflects the time structure with potential transitions occurring at each time step. Fifth, many occurrences of the same transition type are represented by exponentiation.

VI.A. Expectations-Maximization

The likelihood estimation approach faces the same conceptual challenge as does the tabulation approach. State 0 is not directly observable. To deal with this we use the Expectations-Maximization approach (Dempster et al. 1977) which we explain here. Subsequently in Appendix B we provide a new proof that the parameters are identified. The proof is tailored to the simplicity of our structure and is intended to help explain why the approach works.

We use the EM algorithm to deal with the unobserved state 0 and together with the estimation of the transition probabilities. There are two steps. The E-step calculates the expected log-likelihood. The M-step maximizes the expected log-likelihood. The two steps are carried out iteratively until convergence.

Consider the E-Step. We calculate the expected value of the log-likelihood function, with respect to the conditional distribution of Z which represents the the unobserved transitions involving state 0 given X under the current estimate of the parameters $\theta^{(t)}$, i.e. $\pi^{(t)}$

$$Q(\theta|\pi^{(t)}) = E_{Z|X,\pi^{(t)}}[\log L(\pi;X,Z)] \tag{42}$$

In the M-step, we maximize $Q(\pi|\pi^{(t)})$ with respect to π. To maximize $Q(\pi|\pi^{t})$, we solve,

$$\frac{\partial Q}{\partial \pi_{ij}} = 0, \text{ subject to } \sum_{j=0}^{S} \pi_{ij} = 1 \text{ for all } i.$$

Using Lagrange multipliers from the maximization step, we get the updating formula,

$$\pi_{ij}^{t+1} = \frac{\sum_t E[n_{ij}(t)]}{\sum_k \sum_t E[n_{ik}(t)]}.$$

This formula updates the estimate of the probability of transitioning from state i to state j by dividing the expected total number of transitions from i to j by the expected total number of transitions out of state i.

The estimation of N_0, the number of potential firms, is related to the likelihood function through the expected number of transitions from state 0 to states 1 and 2

$$N_0 = \frac{E[n_{01}|X,\pi^{(t)}] + E[n_{02}|X,\pi^{(t)}]}{\pi_{01} + \pi_{02}} \tag{43}$$

The EM algorithm iterates between the E-step and M-step until convergence. In terms of the likelihood function, each iteration is guaranteed to increase the likelihood

$$L(\pi^{(t+1)};X) \geq L(\pi^{(t)};X) \tag{44}$$

The E-step and M-step are repeated iteratively. Each iteration improves the estimates of the transition probabilities, gradually maximizing the likelihood function. As the EM algorithm converges, it approaches the maximum likelihood estimates of the transition probabilities. The final estimates are those that maximize the likelihood of the observed data, taking into account the estimated unobserved data.

The EM algorithm provides a way to perform MLE in the presence of hidden data. It allows us to estimate the transition probabilities that maximize the likelihood of our observed data, even though we can't directly observe all the states and transitions in our Markov model. The E-step fills in the gaps in our data, while the M-step performs the maximum likelihood estimation based on this completed data.

VI.B. Identification

The mathematical justification for using the EM algorithm to estimate parameters of likelihood function with latent variables is well established[3]. The identification of the parameters π in our paper depends on the identification of maximum likelihood algorithm. Equation 41 gives the log-likelihood function we use to estimate π_{ij}, with the latent variable being the number of firms in State 0, n_{0j}. Note that this equation is a summation of terms that is monotonic in π_{ij}. Provided that n_{ij} does not equal 0 for all (i, j), then π_{ij} is identified by standard arguments.

Why does the EM approach work? How do we get the information that is actually missing from our data? Identification results for EM estimation can be found in a number of places such as Cappé et al. (2005) and Gassiat et al. (2016). They tend to study fairly general problems making the proofs relatively abstract. However, it seems clear that out setup could be interpreted as a simple special case. But that may not really explain why the approach works. Our simpler approach uses the overall structural consistency requirement together with ergodicity to allow us to infer the information about the missing state.

In essence, the proof works by showing that as we observe more firms (larger N), we gain enough information through the patterns of observed transitions to infer what must be happening with the unobserved state. Our approach uses the structure of the Markov process and the panel nature of the data to overcome

[3] For example, see Moon (1996), Cappé et al. (2005), or https://www.columbia.edu/~mh2078/MachineLearningORFE/EM_Algorithm.pdf.

the challenge of the hidden state.

We explain the idea first. Even though we can't directly observe state 0, we can infer information about it from the patterns we see in states 1 and 2 over time. The transitions between the observed states carry information about the hidden state. By looking at three consecutive time periods (t-1, t, t+1), we capture two transitions. These transitions, observed across many firms, give us indirect information about all possible state changes, including those involving the hidden state 0.

We have the observed data. For each time period, we observe the number of firms in states 1 and 2. Call the maximum number of firms we might observe in each state M. It grows with N. How many observations actually can exist? For three consecutive periods, we have $(M+1)^6$ possible combinations of observed data. That is because the model being estimated has two observed states, three time periods, and M+1 possible values including 0 for each observation. How many unknown parameters are there? There are 9 unknown transition probabilities reflecting the 3x3 matrix. So we can view this as a system of equations. Each of these $(M+1)^6$ possible observations gives us an equation relating the observed probabilities to the unknown transition probabilities. As N grows so does M. The number of unknowns is fixed at 9. But the number of equations ($(M+1)^6$) grows as the data increases. With more equations than unknowns, we have an overdetermined system. The fact that the system is overdetermined is crucial. It provides multiple ways to infer the same parameters. That permits us to uniquely pin down their values.

The proof is based on the idea that these equations are linearly independent. We get that due to the ergodicity assumption and non-degenerate initial distribution. Accordingly, each equation provides unique information, ensuring we can solve for all unknowns. As $N \to \infty$, we can estimate the probabilities of each possible three-period observation sequence with increasing precision. This allows us to set up a system of equations that, in the limit, exactly determines the transition probabilities.

Theorem F.1. *Consider a 3-state Markov chain with states {0, 1, 2} and transition probability matrix* $\Pi = (\pi_{i,j})_{i,j=0,1,2}$. *Suppose we observe a panel of N firms over T time periods, where for each time t we* *observe the number of firms in states 1 and 2, but not in state 0. Let* $\mathbf{Y} = (Y_t)_{t=1}^T$ *be the observed process, where* $Y_t = (Y_{1t}, Y_{2t})$ *represents the number of firms in states 1 and 2 at time* t*, respectively. Assume that the following conditions hold. 1. The Markov chain is ergodic (irreducible and aperiodic). 2.* $T \geq 3$ *and* N *is sufficiently large. 3. The initial distribution is not degenerate, so that it is not concentrated on a single state. Then, the transition probability matrix* Π *is point identified, and the EM algorithm converges to the true parameter values as* $N \to \infty$*.*

Proof. Our model is a Hidden Markov Model (HMM) where the hidden state is the number of firms in state 0, and the observed states are the numbers of firms in states 1 and 2. The key to identification is to show that the distribution of the observed process $\mathbf{Y}$ uniquely determines the transition probability matrix Π. Consider the joint distribution of three consecutive observations (Y_{t-1}, Y_t, Y_{t+1}). This joint distribution can be written as $P(Y_{t-1}, Y_t, Y_{t+1}) = \sum_{i,j,k} P(X_{t-1} = i, X_t = j, X_{t+1} = k, Y_{t-1}, Y_t, Y_{t+1})$, where X_t represents the full state (including the hidden state 0) at time t.

By the Markov property this can be factored as $P(Y_{t-1}, Y_t, Y_{t+1}) = \sum_{i,j,k} P(X_{t-1} = i) P(Y_{t-1} | X_{t-1} = i) \pi_{ij} P(Y_t | X_t = j) \pi_{jk} P(Y_{t+1} | X_{t+1} = k)$. Due to the assumption of ergodicity, $P(X_{t-1} = i)$ converges to the stationary distribution as t increases. The observation probabilities $P(Y_t | X_t = j)$ are thus deterministic given our observation model. Given a sufficiently large N, we can estimate $P(Y_{t-1}, Y_t, Y_{t+1})$ accurately for all possible values of Y_{t-1}, Y_t, Y_{t+1}. This gives us a system of equations in terms of the unknown transition probabilities π_{ij}.

The number of equations are determined by the possible values of Y_{t-1}, Y_t, Y_{t+1}. This grows as N increases. The number of unknown parameters (the transition probabilities) do not grow as N increases. Given the ergodicity assumption and the non-degenerate initial distribution, these equations will be linearly independent for sufficiently large N. Therefore, for sufficiently large N, we have a system of linearly independent equations that uniquely determine the transition probabilities π_{ij}.

The EM algorithm, when applied to this problem, maximizes the likelihood of the observed data. As $N \to \infty$, this likelihood converges to its population counterpart which is uniquely maximized at the true parameter values due to the above argument. Therefore, we conclude that the transition probability matrix Π is point identified, and the EM algorithm converges to the true parameter values as $N \to \infty$. $\square$

So the population log-likelihood has a unique maximum at the true parameter values. Two aspects are particularly critical. First, the ergodicity assumption ensures that different transition matrices lead to different stationary distributions. Second, the observed process over three or more time periods ($T \geq 3$) provides enough information to distinguish between different transition matrices that might lead to the same pairwise transition probabilities. We find that the transition probability matrix Π is point identified, and the maximum likelihood estimator as computed using the EM algorithm converges to the true parameter values as $N \to \infty$.

VI.C. Computing Standard Errors

A drawback to the EM algorithm is that it does not give us standard errors. There are several methods that can be used. One idea is to use the observed information matrix, which is the negative of the Hessian of the log-likelihood function evaluated at the MLE.[4] Bootstrapping is computationally expensive, but particularly simple to interpret, and it is familiar in finance. So we use this approach.

1. Generate B bootstrap samples by resampling with replacement from the original data.
2. Run the EM algorithm on each bootstrap sample to get B sets of parameter estimates.
3. Compute the standard deviation of each parameter across the B estimates to get its standard error.

Bootstrap Standard Error For parameter θ_i, the bootstrap standard error is:

$$SE_B(\hat{\theta}_i) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(\hat{\theta}_i^{(b)} - \bar{\theta}_i)^2} \tag{45}$$

[4] To use numerical approximations works as follows. 1) Use finite differences to approximate the Hessian of the log-likelihood. 2) Invert this numerical Hessian to get an approximation of the variance-covariance matrix. 3) Take square roots of the diagonal elements for standard errors. A useful discussion is in https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4173103/.

where $\hat{\theta}_i^{(b)}$ is the estimate from the b-th bootstrap sample and $\bar{\theta}_i$ is the mean across all bootstrap samples.

This set of steps should give us estimates for the transition probabilities Π as well as for the value of $N_0(t)$.

VII. MODEL WITH TRANSITIONS

In this section, we present the solutions of the model.

Lemma G.1 (Optimal Input Choices)**.** *In a steady state equilibrium with decreasing returns to scale, the optimal input choices for a firm in state $i \in \{1,2\}$ are:*

$$
\begin{aligned}
A_i &= \left(\frac{\beta_i}{p_F}\right)^{\beta_i} \left(\frac{\alpha_i}{p_A}\right)^{1-\beta_i} \left(\frac{P_i\theta_i}{1+\rho}\right)^{\frac{1}{1-\alpha_i-\beta_i}} \\
F_i &= \left(\frac{\beta_i}{p_F}\right)^{1-\alpha_i} \left(\frac{\alpha_i}{p_A}\right)^{\alpha_i} \left(\frac{P_i\theta_i}{1+\rho}\right)^{\frac{1}{1-\alpha_i-\beta_i}}
\end{aligned}
$$

where A_i is total assets, F_i is flow input, P_i is output price, θ_i is productivity, p_A is the price of assets, p_F is the price of flow inputs, and ρ is the outside return.

Proof. The production function is, $Y_i = \theta_i A_i^{\alpha_i} F_i^{\beta_i}$ and the firm's profit function is, $\pi_i = P_i Y_i - (1+\rho)(p_A A_i + p_F F_i)$.

The first-order conditions with respect to A_i and F_i are,

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial A_i} &= P_i\theta_i\alpha_i A_i^{\alpha_i-1} F_i^{\beta_i} - (1+\rho)p_A = 0 \\
\frac{\partial \pi_i}{\partial F_i} &= P_i\theta_i\beta_i A_i^{\alpha_i} F_i^{\beta_i-1} - (1+\rho)p_F = 0
\end{aligned}
$$

Accordingly,

$$P_i\theta_i\alpha_i A_i^{\alpha_i-1} F_i^{\beta_i} = (1+\rho)p_A \tag{46}$$

$$P_i\theta_i\beta_i A_i^{\alpha_i} F_i^{\beta_i-1} = (1+\rho)p_F. \tag{47}$$

So, $\frac{\alpha_i}{\beta_i} \cdot \frac{F_i}{A_i} = \frac{p_A}{p_F}$. This can be rewritten as $F_i = \frac{\beta_i p_A}{\alpha_i p_F} A_i$. Substitute this into equation 46 to get,

$$P_i \theta_i \alpha_i A_i^{\alpha_i - 1} \left(\frac{\beta_i p_A}{\alpha_i p_F} A_i \right)^{\beta_i} = (1+\rho) p_A.$$

This simplifies to, $P_i \theta_i \alpha_i \left(\frac{\beta_i p_A}{\alpha_i p_F} \right)^{\beta_i} A_i^{\alpha_i + \beta_i - 1} = (1+\rho) p_A$. Then solve for A_i to get,

$$A_i = \left[\frac{(1+\rho) p_A}{P_i \theta_i \alpha_i} \left(\frac{\alpha_i p_F}{\beta_i p_A} \right)^{\beta_i} \right]^{\frac{1}{\alpha_i + \beta_i - 1}}.$$

Algebraic simplification then gives the claimed expression for A_i.

Using the solved value of A_i and $F_i = \frac{\beta_i p_A}{\alpha_i p_F} A_i$ then gives the claimed expression for F_i. □

Lemma G.2 (Firm Output and Profit)**.** *Given the optimal input choices, the output and profit for a firm in state $i \in \{1,2\}$ are:*

$$Y_i = \theta_i \left[\left(\frac{\beta_i}{p_F} \right)^{\beta_i} \left(\frac{\alpha_i}{p_A} \right)^{\alpha_i} \left(\frac{1}{1+\rho} \right)^{\alpha_i + \beta_i} (P_i \theta_i)^{\alpha_i + \beta_i} \right]^{\frac{1}{1 - \alpha_i - \beta_i}}$$

$$v_i = (1 - \alpha_i - \beta_i) P_i Y_i + (1+\rho) W$$

where Y_i is output, v_i is profit, and W is initial wealth.

Proof. Again the production function is $Y_i = \theta_i A_i^{\alpha_i} F_i^{\beta_i}$. From the above Lemma substitute the expressions for A_i and F_i to get,

$$\begin{aligned} Y_i = \theta_i & \left[\left(\frac{\beta_i}{p_F} \right)^{\beta_i} \left(\frac{\alpha_i}{p_A} \right)^{1 - \beta_i} \left(\frac{P_i \theta_i}{1+\rho} \right)^{\frac{1}{1 - \alpha_i - \beta_i}} \right]^{\alpha_i} \\ & \times \left[\left(\frac{\beta_i}{p_F} \right)^{1 - \alpha_i} \left(\frac{\alpha_i}{p_A} \right)^{\alpha_i} \left(\frac{P_i \theta_i}{1+\rho} \right)^{\frac{1}{1 - \alpha_i - \beta_i}} \right]^{\beta_i} \end{aligned}$$

Algebraic simplification then gives the claimed expression for Y_i.

Next recall that the profit function is, $v_i = P_iY_i - (1+\rho)(p_AA_i + p_FF_i) + (1+\rho)W$. From the first-order conditions in Lemma G.1 we have, $p_AA_i = \frac{\alpha_i P_i Y_i}{1+\rho}$ and $p_FF_i = \frac{\beta_i P_i Y_i}{1+\rho}$. These are just substituted into the profit function.

$$
\begin{aligned}
v_i &= P_iY_i - (1+\rho)\left(\frac{\alpha_i P_i Y_i}{1+\rho} + \frac{\beta_i P_i Y_i}{1+\rho}\right) + (1+\rho)W \\
&= P_iY_i - (\alpha_i + \beta_i)P_iY_i + (1+\rho)W \\
&= (1-\alpha_i-\beta_i)P_iY_i + (1+\rho)W
\end{aligned}
$$

□

Lemma G.3 (Market Clearing Price). *In equilibrium, the market clearing price for firms in state i is:*

$$
P_i = \left[\frac{D_i}{N_i}\left(\frac{1}{\theta_i}\left(\frac{p_F}{\beta_i}\right)^{\beta_i}\left(\frac{p_A}{\alpha_i}\right)^{\alpha_i}(1+\rho)^{\alpha_i+\beta_i}\right)^{\frac{1}{1-\alpha_i-\beta_i}}\right]^{\frac{1}{\varepsilon_i+\frac{\alpha_i+\beta_i}{1-\alpha_i-\beta_i}}}
$$

where N_i is the number of firms in state i, D_i is a demand shift parameter, and ε_i is the price elasticity of demand.

Proof. The output demand function is, $Y_i^d = D_iP_i^{-\varepsilon_i}$. Because firms are symmetric, in equilibrium, output supply equals demand, $N_iY_i = Y_i^d$ so,

$$
N_iY_i = D_iP_i^{-\varepsilon_i} \tag{48}
$$

Use the expression for Y_i from Lemma G.2 to get

$$
N_i\theta_i\left[\left(\frac{\beta_i}{p_F}\right)^{\beta_i}\left(\frac{\alpha_i}{p_A}\right)^{\alpha_i}\left(\frac{1}{1+\rho}\right)^{\alpha_i+\beta_i}(P_i\theta_i)^{\alpha_i+\beta_i}\right]^{\frac{1}{1-\alpha_i-\beta_i}} = D_iP_i^{-\varepsilon_i}.
$$

Then solve for the market clearing output price P_i as,

$$P_i^{-\varepsilon_i - \frac{\alpha_i+\beta_i}{1-\alpha_i-\beta_i}} = \frac{N_i}{D_i}\left(\theta_i\left(\frac{\beta_i}{p_F}\right)^{\beta_i}\left(\frac{\alpha_i}{p_A}\right)^{\alpha_i}\left(\frac{1}{1+\rho}\right)^{\alpha_i+\beta_i}\right)^{\frac{1}{1-\alpha_i-\beta_i}}$$

$$P_i = \left[\frac{D_i}{N_i}\left(\frac{1}{\theta_i}\left(\frac{p_F}{\beta_i}\right)^{\beta_i}\left(\frac{p_A}{\alpha_i}\right)^{\alpha_i}(1+\rho)^{\alpha_i+\beta_i}\right)^{\frac{1}{1-\alpha_i-\beta_i}}\right]^{\frac{1}{\varepsilon_i+\frac{\alpha_i+\beta_i}{1-\alpha_i-\beta_i}}}$$

□

Theorem G.4 (Steady State Value Functions)**.** *In a steady state equilibrium, the value functions for firms in each state are:*

$$V_0 = \left(\underbrace{W(1+\rho)}_{\textit{Outside investment value}} + \delta(1-\pi_{00})(\underbrace{\Gamma_1 v_1 + \Gamma_2 v_2}_{\textit{Expected production value}} - \underbrace{\frac{(1+\rho)(\hat{\kappa}+\underline{\kappa})}{2\delta}}_{\textit{Expected entry cost}})\right)\underbrace{\Gamma_0}_{\textit{Adjustment factor}}$$

$$V_1 = \underbrace{\frac{1}{1-\delta(\pi_{11}+\frac{\delta\pi_{12}\pi_{21}}{1-\delta\pi_{22}})}}_{\textit{Persistence multiplier}} \cdot \left[\underbrace{v_1}_{\textit{Current period return}} + \underbrace{\delta\frac{\pi_{12}}{1-\delta\pi_{22}}v_2}_{\textit{Expected value from size transition}} + \underbrace{\delta\left(\pi_{10}+\frac{\delta\pi_{12}\pi_{20}}{1-\delta\pi_{22}}\right)V_0}_{\textit{Expected value from exit paths}}\right]$$

$$V_2 = \underbrace{\frac{1}{1-\delta(\pi_{22}+\frac{\delta\pi_{12}\pi_{21}}{1-\delta\pi_{11}})}}_{\textit{Persistence multiplier}} \cdot \left[\underbrace{v_2}_{\textit{Current period value}} + \underbrace{\delta\frac{\pi_{21}}{(1-\delta\pi_{11})}v_1}_{\textit{Expected value from downsizing}} + \underbrace{\delta(\pi_{20}+\frac{\delta\pi_{21}\pi_{10}}{1-\delta\pi_{11}})V_0}_{\textit{Expected value from exit paths}}\right]$$

where

$$\gamma \equiv \frac{\pi_{02}}{\pi_{01}+\pi_{02}}$$

$$\Gamma_0 \equiv \frac{1}{\underbrace{(1-\delta\pi_{00})}_{\textit{Stay out}} - \underbrace{\delta(1-\pi_{00})}_{\textit{Entry}} \underbrace{\frac{\delta}{1-\delta(\pi_{11}+\delta\frac{\pi_{12}\pi_{21}}{1-\delta\pi_{22}})}}_{\textit{Persistence multiplier}} \underbrace{\left(\pi_{10}+\delta\frac{\pi_{12}\pi_{20}}{1-\delta\pi_{22}}+\gamma\frac{(1-\delta)(\pi_{20}-\pi_{10})}{1-\delta\pi_{22}}\right)}_{\textit{Enter and then exit}}}$$

$$\Gamma_1 \equiv \underbrace{\frac{1}{1-\delta(\pi_{11}+\frac{\delta\pi_{12}\pi_{21}}{1-\delta\pi_{22}})}}_{\textit{Persistence multiplier}} \Big(\underbrace{1-\gamma}_{\textit{Entry as small firm}} + \underbrace{\delta\frac{\gamma\pi_{21}}{1-\delta\pi_{22}}}_{\textit{Entry as large firm}} \Big)$$

$$\Gamma_2 \equiv \underbrace{\frac{1}{1-\delta(\pi_{22}+\delta\frac{\pi_{21}\pi_{12}}{1-\delta\pi_{11}})}}_{\textit{Persistence multiplier}} \Big(\underbrace{\gamma}_{\textit{Entry as large firm}} + \underbrace{\delta\frac{(1-\gamma)\pi_{12}}{1-\delta\pi_{11}}}_{\textit{Entry as small firm}} \Big)$$

$\hat{\kappa}$ is the entry indifference value, and $\underline{\kappa}$ and $\overline{\kappa}$ are the lower and upper bounds of the entry cost distribution.

Proof. The Bellman equations for each state are,

$$V_0 = E[\max\{W(1+\rho)+\delta V_0, (W-\kappa)(1+\rho)+\delta[(1-\gamma)V_1+\gamma V_2]\}]$$

$$V_1 = v_1 + \delta[\pi_{10}V_0 + \pi_{11}V_1 + \pi_{12}V_2]$$

$$V_2 = v_2 + \delta[\pi_{20}V_0 + \pi_{21}V_1 + \pi_{22}V_2].$$

By definition $\hat{\kappa}$ is a value of κ_i such that the state 0 firm is just indifferent between paying the entry fee and not paying it. So, for V_0 the indifference condition at $\hat{\kappa}$ can be expressed as,

$$W(1+\rho)+\delta V_0 = (W-\hat{\kappa})(1+\rho)+\delta[(1-\gamma)V_1+\gamma V_2].$$

We can then solve this for V_0, $V_0 = -\frac{\hat{\kappa}(1+\rho)}{\delta} + (1-\gamma)V_1 + \gamma V_2$. For $\kappa < \hat{\kappa}$, firms enter. For $\kappa > \hat{\kappa}$, firms do not enter. So,

$$
\begin{aligned}
V_0 = & \int_{\underline{\kappa}}^{\hat{\kappa}} [(W-\kappa)(1+\rho) + \delta((1-\gamma)V_1 + \gamma V_2)] \frac{1}{\overline{\kappa} - \underline{\kappa}} d\kappa \\
& + \int_{\hat{\kappa}}^{\overline{\kappa}} [W(1+\rho) + \delta V_0] \frac{1}{\overline{\kappa} - \underline{\kappa}} d\kappa
\end{aligned}
$$

To obtain V_1 and V_2 we need to solve the system of equations,

$$
\begin{aligned}
V_1 &= v_1 + \delta[\pi_{10}V_0 + \pi_{11}V_1 + \pi_{12}V_2] \\
V_2 &= v_2 + \delta[\pi_{20}V_0 + \pi_{21}V_1 + \pi_{22}V_2].
\end{aligned}
$$

Using substitution and algebraic manipulation, we have

$$
\begin{aligned}
V_0 &= \left(W(1+\rho) \frac{(\overline{\kappa} - \underline{\kappa})}{\delta(\hat{\kappa} - \underline{\kappa})} + \Gamma_1 v_1 + \Gamma_2 v_2 - \frac{(1+\rho)(\hat{\kappa} + \underline{\kappa})}{2\delta} \right) \Gamma_0 \\
V_1 &= \frac{1}{(1-\delta\pi_{11}) - \delta\pi_{12}\frac{\delta\pi_{21}}{1-\delta\pi_{22}}} \left(v_1 + \frac{\delta\pi_{12}}{1-\delta\pi_{22}} v_2 + \left(\delta\pi_{10} + \frac{\delta\pi_{12}\delta\pi_{20}}{1-\delta\pi_{22}} \right) V_0 \right) \\
V_2 &= \frac{1}{(1-\delta\pi_{22})(1-\delta\pi_{11}-\delta\pi_{12}\frac{\delta\pi_{21}}{1-\delta\pi_{22}})} [\delta\pi_{21}v_1 + (1-\delta\pi_{11})v_2 + (\delta\pi_{20}(1-\delta\pi_{11}) + \delta\pi_{21}\delta\pi_{10})V_0]
\end{aligned}
$$

where $\gamma \equiv \frac{\pi_{02}}{\pi_{01}+\pi_{02}}$, $\Gamma_0 \equiv \frac{1}{\frac{\overline{\kappa}-\underline{\kappa}-\delta(\overline{\kappa}-\hat{\kappa})}{\delta(\hat{\kappa}-\underline{\kappa})} - \frac{\delta\pi_{10}(1-\delta\pi_{22})+\delta\pi_{12}\delta\pi_{20}+\gamma(1-\delta)\delta(\pi_{20}-\pi_{10})}{(1-\delta\pi_{22})(1-\delta\pi_{11}-\delta\pi_{12}\frac{\delta\pi_{21}}{1-\delta\pi_{22}})}}$, $\Gamma_1 \equiv \frac{(1-\delta\pi_{22})(1-\gamma)+\delta\pi_{21}\gamma}{(1-\delta\pi_{22})(1-\delta\pi_{11}-\delta\pi_{12}\frac{\delta\pi_{21}}{1-\delta\pi_{22}})}$, $\Gamma_2 \equiv \frac{\delta\pi_{12}(1-\gamma)+\gamma(1-\delta\pi_{11})}{(1-\delta\pi_{22})(1-\delta\pi_{11}-\delta\pi_{12}\frac{\delta\pi_{21}}{1-\delta\pi_{22}})}$. With further reorganization of the expressions, we obtain the claimed expressions for V_0, V_1 and V_2. $\square$

Corollary G.5 (Steady State Probabilities)**.** *The steady state probabilities of firms being in each state are:*

$$\pi_0^{ss} = \frac{\pi_{20}(1-\pi_{11})+\pi_{10}\pi_{21}}{\pi_{20}(1-\pi_{11})+\pi_{10}\pi_{21}+\pi_{01}(1-\pi_{22}+\pi_{12})+\pi_{02}(1-\pi_{11}+\pi_{21})}$$
$$\pi_1^{ss} = \frac{\pi_{01}(1-\pi_{22})+\pi_{21}\pi_{02}}{\pi_{20}(1-\pi_{11})+\pi_{10}\pi_{21}+\pi_{01}(1-\pi_{22}+\pi_{12})+\pi_{02}(1-\pi_{11}+\pi_{21})}$$
$$\pi_2^{ss} = \frac{\pi_{02}(1-\pi_{11})+\pi_{12}\pi_{01}}{\pi_{20}(1-\pi_{11})+\pi_{10}\pi_{21}+\pi_{01}(1-\pi_{22}+\pi_{12})+\pi_{02}(1-\pi_{11}+\pi_{21})}$$

Proof. By definition, in a steady state the probabilities must satisfy:

$$\pi_0^{ss} = \pi_0^{ss}\pi_{00}+\pi_1^{ss}\pi_{10}+\pi_2^{ss}\pi_{20}$$

$$\pi_1^{ss} = \pi_0^{ss}\pi_{01}+\pi_1^{ss}\pi_{11}+\pi_2^{ss}\pi_{21}$$

$$\pi_2^{ss} = \pi_0^{ss}\pi_{02}+\pi_1^{ss}\pi_{12}+\pi_2^{ss}\pi_{22}$$

By definition, probabilities must add up, so $\pi_0^{ss}+\pi_1^{ss}+\pi_2^{ss}=1$.

Substitute $\pi_{00}=1-\pi_{01}-\pi_{02}$ into the first equation, $\pi_0^{ss}=\pi_0^{ss}(1-\pi_{01}-\pi_{02})+\pi_1^{ss}\pi_{10}+\pi_2^{ss}\pi_{20}$. Then rearrange, $\pi_0^{ss}(\pi_{01}+\pi_{02})=\pi_1^{ss}\pi_{10}+\pi_2^{ss}\pi_{20}$. Then directly solve for π_0^{ss}, π_1^{ss}, and π_2^{ss} in terms of the transition probabilities. Algebraic simplification gives the claimed expressions. □

Corollary G.6 (Entry Probability)**.** *The probability of entry for a firm in state 0 is given by*

$$\pi_{00} = \frac{\hat{\kappa}-\underline{\kappa}}{\overline{\kappa}-\underline{\kappa}},$$

where $\hat{\kappa}$ is determined by the entry indifference condition.

Proof. The entry cost κ is uniformly distributed over $[\underline{\kappa},\overline{\kappa}]$. Firms enter if their entry cost is below $\hat{\kappa}$, and they do not entry otherwise. So the probability that a firm in state 0 does not enter (and so stays in state 0) is the probability that the entry cost is above $\hat{\kappa}$, $\pi_{00}=P(\kappa>\hat{\kappa})=\frac{\overline{\kappa}-\hat{\kappa}}{\overline{\kappa}-\underline{\kappa}}$; which gives us the claimed expression for π_{00}. □

Recall that $\hat{\kappa}$ is defined by the entry indifference condition for a state 0 firm considering entry. At $\hat{\kappa}$, a firm is indifferent between entering and not entering. Due to perfect competition, when choosing to enter or not, everything is constant except the realization of κ_i. So $\hat{\kappa}$ is a numerical comparison. Given the simplicity and monotonicity of the setup, it is obviously unique.

Table A.9: Maximum Likelihood Estimated Industry Transition Probabilities

This table presents the transition matrix across different states for each NAICS sector as defined by https://www.census.gov/programs-surveys/economic-census/year/2022/guidance/understanding-naics.html. For a sector to be included it must have at least 20 observations on average in each year. *NObs* denotes the number of firm-year observations. Estimates are based on the whole sample from 1971 to 2022. The parameters are estimated using Maximum Likelihood. Since the underlying model is a Hidden Markov model, estimation was done using the Expectations Maximization algorithm. The transition probabilities are estimated together with the estimation of the number of firms that are not observed. The transition probabilities from a state at date t (Out, Small, or Big) to a state at date $t+1$ are reported. The state Out are firms that are not observed. The state Small is for the smallest 4 quintiles of firm total assets. The state Big is for the largest quintile of firms. Following each parameter estimate are two numbers in brackets. We bootstrapped the data with 500 replications. The reported numbers in brackets are the values at the 5th and the 95th percentiles.

Industry	State	Out_{t+1}	$Small_{t+1}$	Big_{t+1}
Mining, Quarrying,	Out_t	0.034 [0.012,0.877]	0.877 [0.111,0.902]	0.089 [0.010,0.101]
Oil and Gas Extraction	$Small_t$	0.126 [0.121,0.131]	0.854 [0.849,0.860]	0.019 [0.017,0.022]
	Big_t	0.066 [0.058,0.074]	0.063 [0.055,0.072]	0.871 [0.860,0.882]
	N Obs	1354	9167	2266
Construction	Out_t	0.044 [0.008,0.975]	0.859 [0.023,0.904]	0.097 [0.002,0.119]
	$Small_t$	0.122 [0.112,0.132]	0.858 [0.847,0.869]	0.020 [0.016,0.024]

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Table A.9 – continued from previous page

Industry	State	Out_{t+1}	$Small_{t+1}$	Big_{t+1}
	Big_t	0.056 [0.041,0.070]	0.082 [0.064,0.102]	0.862 [0.839,0.883]
	N Obs	368	2598	626
Manufacturing	Out_t	0.028 [0.017,0.083]	0.901 [0.851,0.911]	0.071 [0.064,0.075]
	$Small_t$	0.098 [0.096,0.099]	0.890 [0.889,0.892]	0.012 [0.011,0.012]
	Big_t	0.044 [0.041,0.046]	0.032 [0.030,0.034]	0.924 [0.921,0.927]
	N Obs	9189	82133	20511
Wholesale Trade	Out_t	0.557 [0.016,0.940]	0.397 [0.054,0.886]	0.045 [0.006,0.110]
	$Small_t$	0.127 [0.121,0.132]	0.857 [0.851,0.862]	0.016 [0.014,0.019]
	Big_t	0.070 [0.061,0.079]	0.055 [0.046,0.063]	0.876 [0.864,0.888]
	N Obs	2655	8165	2019
Retail Trade	Out_t	0.892 [0.016,0.965]	0.100 [0.003,0.921]	0.007 [0.003,0.072]
	$Small_t$	0.110 [0.105,0.114]	0.877 [0.872,0.882]	0.013 [0.012,0.015]
	Big_t	0.043 [0.036,0.049]	0.043 [0.037,0.049]	0.915 [0.905,0.923]
	N Obs	11018	9878	2442
Transportation and Warehousing	Out_t	0.014 [0.006,0.902]	0.898 [0.089,0.919]	0.088 [0.008,0.108]
	$Small_t$	0.097 [0.089,0.104]	0.891 [0.883,0.898]	0.012 [0.010,0.015]
	Big_t	0.043 [0.032,0.053]	0.043 [0.033,0.053]	0.914 [0.901,0.928]
	N Obs	493	4522	1107
Information	Out_t	0.012 [0.009,0.019]	0.900 [0.889,0.908]	0.088 [0.080,0.095]

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Table A.9 – continued from previous page

Industry	State	Out_{t+1}	$Small_{t+1}$	Big_{t+1}
	$Small_t$	0.168 [0.164,0.172]	0.815 [0.810,0.820]	0.017 [0.015,0.019]
	Big_t	0.081 [0.074,0.088]	0.044 [0.039,0.050]	0.874 [0.865,0.883]
	N Obs	3185	16712	4154
Real Estate and	Out_t	0.049 [0.008,0.983]	0.856 [0.016,0.910]	0.096 [0.002,0.124]
Rental and Leasing	$Small_t$	0.156 [0.143,0.170]	0.821 [0.807,0.837]	0.023 [0.017,0.029]
	Big_t	0.079 [0.057,0.102]	0.094 [0.066,0.119]	0.827 [0.795,0.860]
	N Obs	273	1491	343
Professional, Scientific,	Out_t	0.025 [0.011,0.841]	0.875 [0.142,0.892]	0.100 [0.015,0.112]
and Technical Services	$Small_t$	0.164 [0.158,0.170]	0.816 [0.809,0.822]	0.020 [0.018,0.023]
	Big_t	0.096 [0.086,0.105]	0.059 [0.051,0.068]	0.845 [0.834,0.857]
	N Obs	1572	8183	2027
Waste Management and	Out_t	0.022 [0.009,0.855]	0.891 [0.133,0.912]	0.086 [0.012,0.102]
Remediation Services	$Small_t$	0.149 [0.140,0.157]	0.827 [0.818,0.836]	0.024 [0.020,0.028]
	Big_t	0.089 [0.077,0.103]	0.066 [0.054,0.080]	0.845 [0.826,0.861]
	N Obs	757	4344	1058
Health Care and	Out_t	0.014 [0.007,0.859]	0.912 [0.131,0.930]	0.074 [0.011,0.091]
Social Assistance	$Small_t$	0.138 [0.129,0.146]	0.839 [0.831,0.849]	0.023 [0.019,0.027]
	Big_t	0.087 [0.072,0.101]	0.048 [0.037,0.061]	0.866 [0.848,0.884]
	N Obs	606	3755	913

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Table A.9 – continued from previous page

Industry	State	Out_{t+1}	$Small_{t+1}$	Big_{t+1}
Arts, Entertainment,	Out_t	0.023 [0.007,0.973]	0.868 [0.023,0.901]	0.109 [0.003,0.141]
and Recreation	$Small_t$	0.135 [0.124,0.147]	0.841 [0.829,0.853]	0.024 [0.018,0.030]
	Big_t	0.077 [0.058,0.099]	0.093 [0.072,0.118]	0.830 [0.800,0.856]
	N Obs	278	1773	421
Accommodation and	Out_t	0.030 [0.009,0.965]	0.852 [0.031,0.882]	0.119 [0.004,0.138]
Food Services	$Small_t$	0.109 [0.103,0.116]	0.876 [0.868,0.883]	0.015 [0.012,0.018]
	Big_t	0.059 [0.049,0.070]	0.062 [0.050,0.074]	0.879 [0.865,0.894]
	N Obs	605	4738	1158
Other Services (except	Out_t	0.853 [0.007,0.991]	0.128 [0.008,0.885]	0.019 [0.001,0.154]
Public Administration)	$Small_t$	0.154 [0.135,0.170]	0.821 [0.804,0.840]	0.025 [0.017,0.035]
	Big_t	0.082 [0.053,0.110]	0.135 [0.096,0.178]	0.784 [0.739,0.828]
	N Obs	1109	952	208

Table A.4: Summary Statistics: All Years

All variables are deflated using the GDP deflator using the year 2017 as the base year, and units are in millions of dollars except for ratio variables. All variables, except for the weights ω_{sale} and ω_{AT} are then winsorized at 1st and 99th percentiles. Variable sources are in Table A.1. By definition, AT is total assets, MTB is market to book ratio, Sale is sales revenue, Π is profits, EBITDA is Earnings before interest and taxes depreciation added, Debt is long term debt plus short term debt (*dltt*+*dlc*), Δ is the change from year t-1 to year t, Tax is the taxed paid by a firm, Interest is the firm's interet expense, CapEx is the capital expenditure, $s_F = \frac{COGS+SGA}{AT}$, $\omega_{AT} = \frac{AT_i}{\sum_{j=1}^{J} AT_j}$, and $\omega_{sale} = \frac{Sales_i}{\sum_{j=1}^{J} Sales_j}$. The ω_{sale} and ω_{AT} are the original numbers multiplied by 1000.

	N	mean	median	sd	min	max	skewness	AR(1)
AT	216845	1511.97	136.68	4598.37	0.40	32011.03	5.04	0.99
MTB	192789	2.29	1.37	3.40	0.53	29.70	5.76	0.77
Sale	216845	1387.09	149.70	3868.59	0.07	25811.53	4.62	0.99
Sale/AT	216845	1.29	1.12	0.95	0.01	5.48	1.68	0.90
Π/AT	216845	-0.07	0.04	0.42	-3.22	0.30	-5.15	0.77
Π/Sale	216845	-0.85	0.03	5.29	-52.13	0.42	-8.25	0.71
EBITDA/AT	216845	0.01	0.11	0.44	-3.18	0.44	-4.96	0.78
Debt/AT	216845	0.31	0.24	0.36	0.00	2.55	3.31	0.82
ΔDebt/AT	216845	0.00	0.00	0.18	-0.83	0.64	-0.72	0.03
Tax/AT	216845	0.02	0.01	0.04	-0.08	0.17	0.97	0.63
Interest/AT	216845	0.03	0.02	0.06	0.00	0.49	5.36	0.77
Interest/Debt	194186	0.14	0.09	0.26	0.00	2.14	6.15	0.34
CapEx/AT	216845	0.06	0.04	0.07	0.00	0.44	2.51	0.65
s_F	216845	1.30	1.06	1.09	0.07	7.05	2.48	0.86
ω_{AT}	216845	0.24	0.03	0.74	0.00	13.67	6.56	0.99
ω_{Sale}	216845	0.24	0.03	0.67	0.00	9.61	5.09	0.99

Table A.5: High Profit Firms: Component Importance

This table reports the results from the following. Step 1, in each year sort firms into profit quintiles, and let I_{it}^{HIGH} be an indicator that a given firm-year observation is in the high profit quintile ($High$ in Table III). Step 2, define y_{it} to be $\frac{\Delta EBITDA_{it}}{AT_{it-1}}$, $\frac{\Delta T_{it}}{AT_{it-1}}$, $\frac{\Delta \rho AT_{it}}{AT_{it-1}}$, $\frac{\Delta VA_{it}}{AT_{it-1}}$, $\frac{\Delta Int_{it}}{AT_{it-1}}$, and $\frac{\Delta^2 D_{it}}{AT_{it-1}}$. Then run simple regressions, $y_{it} = \beta_0 + \beta_1 I_{it}^{HIGH} + \beta_2 X_{it} + \varepsilon_{it}$, where X_{it} is a vector of industry and year dummies used as controls. Panel A.5a shows the results for equally weighted firm/year regressions. Panel A.5b shows the results where each observation is weighted according to the share of total assets within the year.

(a) Equally weighted

	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{\Delta VA_{it}}{AT_{it-1}}$	$\frac{\Delta EBITDA_{it}}{AT_{it-1}}$	$\frac{\Delta T_{it}}{AT_{it-1}}$	$\frac{\Delta Int_{it}}{AT_{it-1}}$	$\frac{\Delta^2 D_{it}}{AT_{it-1}}$	$\frac{\Delta(\rho AT)_{it}}{AT_{it-1}}$
β_1	0.085***	0.083***	0.009***	-0.004***	-0.033***	-0.001
Mean	0.046	0.013	0.003	0.003	0.028	0.009
N	198305	198305	198302	195614	181020	198308
R^2	0.046	0.033	0.018	0.019	0.007	0.143

(b) Value weighted

	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{\Delta VA_{it}}{AT_{it-1}}$	$\frac{\Delta EBITDA_{it}}{AT_{it-1}}$	$\frac{\Delta T_{it}}{AT_{it-1}}$	$\frac{\Delta Int_{it}}{AT_{it-1}}$	$\frac{\Delta^2 D_{it}}{AT_{it-1}}$	$\frac{\Delta(\rho AT)_{it}}{AT_{it-1}}$
β_1	0.029***	0.028***	0.002*	-0.002***	-0.027***	-0.004***
Mean	0.022	0.010	0.002	0.002	0.017	0.004
N	198305	198305	198302	195614	181020	198308
R^2	0.077	0.073	0.033	0.060	0.020	0.359

Table A.6: Profit Decomposition Results Over Time

Does the importance of the various components change over time? This table reports the results from the following. Step 1, in each year sort firms into profit quintiles, and let I_{it}^{HIGH} be an indicator that a given firm-year observation is in the high profit quintile (*High* in Table III). Step 2, define y_{it} to be $\frac{\Delta VA_{it}}{AT_{it-1}}$, $\frac{\Delta EBITDA_{it}}{AT_{it-1}}$, $\frac{\Delta T_{it}}{AT_{it-1}}$, $\frac{\Delta Int_{it}}{AT_{it-1}}$, $\frac{\Delta^2 D_{it}}{AT_{it-1}}$, and $\frac{\Delta(\rho AT)_{it}}{AT_{it-1}}$. Then run simple regressions, $y_{it} = \beta_0 + \beta_1 I_{it}^{HIGH} + \beta_2 X_{it} + \varepsilon_{it}$, where X_{it} is a vector of industry and year dummies used as controls. Firm-years are equally weighted. Panel A.6a shows the results for 1971-1989. Panel A.6b shows the results for 1990-1999.Panel A.6c shows the results for 2000-2022.

(a) Results from 1971-1989

	(1) $\frac{\Delta VA_{it}}{AT_{it-1}}$	(2) $\frac{\Delta EBITDA_{it}}{AT_{it-1}}$	(3) $\frac{\Delta T_{it}}{AT_{it-1}}$	(4) $\frac{\Delta Int_{it}}{AT_{it-1}}$	(5) $\frac{\Delta^2 D_{it}}{AT_{it-1}}$	(6) $\frac{\Delta(\rho AT)_{it}}{AT_{it-1}}$
β_1	0.095***	0.080***	0.020***	-0.003***	-0.029***	0.001
Mean	0.042	0.016	0.004	0.002	0.026	0.010
N	61116	61116	61115	60598	55482	61119
R^2	0.067	0.045	0.054	0.031	0.004	0.196

(b) Results from 1990-1999

	(1) $\frac{\Delta VA_{it}}{AT_{it-1}}$	(2) $\frac{\Delta EBITDA_{it}}{AT_{it-1}}$	(3) $\frac{\Delta T_{it}}{AT_{it-1}}$	(4) $\frac{\Delta Int_{it}}{AT_{it-1}}$	(5) $\frac{\Delta^2 D_{it}}{AT_{it-1}}$	(6) $\frac{\Delta(\rho AT)_{it}}{AT_{it-1}}$
β_1	0.095***	0.099***	0.011***	-0.005***	-0.041***	-0.005**
Mean	0.064	0.012	0.005	0.002	0.037	0.016
N	53236	53236	53235	52598	47473	53236
R^2	0.068	0.048	0.019	0.039	0.008	0.080

(c) Results from 2000-2022

	(1) $\frac{\Delta VA_{it}}{AT_{it-1}}$	(2) $\frac{\Delta EBITDA_{it}}{AT_{it-1}}$	(3) $\frac{\Delta T_{it}}{AT_{it-1}}$	(4) $\frac{\Delta Int_{it}}{AT_{it-1}}$	(5) $\frac{\Delta^2 D_{it}}{AT_{it-1}}$	(6) $\frac{\Delta(\rho AT)_{it}}{AT_{it-1}}$
β_1	0.072***	0.077***	-0.000	-0.005***	-0.031***	-0.001**
Mean	0.039	0.011	0.002	0.003	0.025	0.003
N	83933	83933	83932	82399	78045	83933
R^2	0.030	0.024	0.004	0.011	0.006	0.161

Table A.7: Regression Analysis of Extreme Profit Evolution

This table presents the coefficient β estimated from the following OLS regressions $g_{it,t-5} = c_1 + \beta_{-5} I_{P_{it}} + \gamma X_{it} + \mu_{1it}$, and $g_{it,t+5} = c_2 + \beta_{+5} I_{P_{it}} + \gamma X_{it} + \mu_{2it}$. Profits is measured by $\frac{\Pi_{it}}{AT_{it}}$, and $I_{P_{it}}$ is a dummy variable. In the left panel the dummy is 1 if the firm is in the top quintile of profits. In the right panel the dummy is 1 if the firm is in the bottom quintile of profits. Change in firm profitability is measured as $g_{it,t-5} = \frac{\Pi_{it}}{AT_{it}} - \frac{\Pi_{it-5}}{AT_{it-5}}$, and $g_{it,t+5} = \frac{\Pi_{it+5}}{AT_{it+5}} - \frac{\Pi_{it}}{AT_{it}}$. X_{it} contains the year and industry (4-digit NAICS) fixed effects. Standard errors are clustered at the 4-digit NAICS industry level. In columns 1, 2, 5 and 6 firm-year observations are weighted equally. In columns 3, 4, 7 and 8 firm-year observations are value-weighted, with the weight being $\frac{AT_{it}}{\sum AT_{it}}$. The R^2 is the overall value.

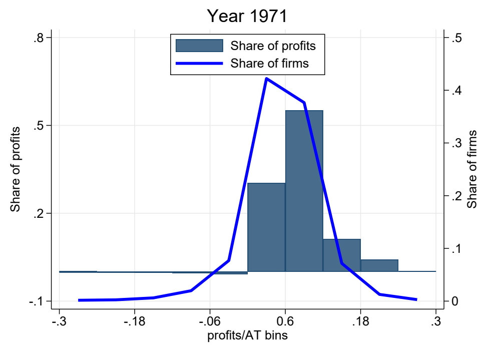
	High Profit Firms				Low Profit Firms			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable	$g_{it,t-5}$	$g_{it,t+5}$	$g_{it,t-5}$	$g_{it,t+5}$	$g_{it,t-5}$	$g_{it,t+5}$	$g_{it,t-5}$	$g_{it,t+5}$
β_{-5}	0.114***		0.048***		-0.239***		-0.137***	
	(15.96)		(19.42)		(-15.13)		(-5.85)	
β_{+5}		-0.093***		-0.043***		0.178***		0.082***
		(-16.83)		(-16.60)		(19.01)		(9.33)
Year fixed effects	yes	yes	yes	yes	yes	yes	yes	yes
Industry fixed effects	yes	yes	yes	yes	yes	yes	yes	yes
Weighting	equal	equal	value	value	equal	equal	value	value
N	122027	122111	122027	122111	122027	122111	122027	122111
Within R^2	0.020	0.014	0.069	0.055	0.061	0.037	0.048	0.027
R^2	0.045	0.038	0.260	0.259	0.085	0.060	0.243	0.236

Table A.8: Regression Analysis of Extreme Size Evolution

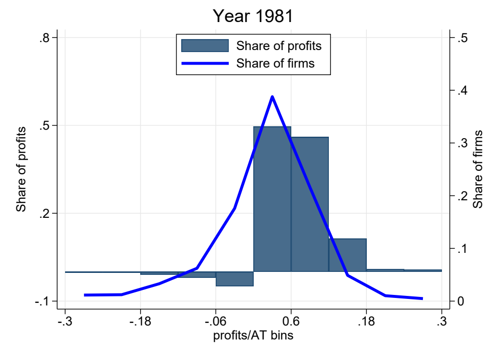
This table presents the coefficient β estimated from the following OLS regressions $g_{it,t-5} = c_1 + \beta_{-5} I_{P_{it}} + \gamma X_{it} + \mu_{1it}$, and $g_{it,t+5} = c_2 + \beta_{+5} I_{P_{it}} + \gamma X_{it} + \mu_{2it}$. Size is measured by the real total asset (winsorized at 1st and 99th percentiles), and $I_{P_{it}}$ is a dummy variable. In the left panel the dummy is 1 if the firm is in the top quintile of profits. In the right panel the dummy is 1 if the firm is in the bottom quintile of profits. Change in firm size is measured as $g_{it,t-5} = \frac{AT_{it} - AT_{it-5}}{AT_{it-5}}$, and $g_{it,t+5} = \frac{AT_{it+5} - AT_{it}}{AT_{it}}$. X_{it} contains the year and industry (4-digit NAICS) fixed effects. Standard errors are clustered at the 4-digit NAICS industry level. In columns 1, 2, 5 and 6 firm-year observations are weighted equally. In columns 3, 4, 7 and 8 firm-year observations are value-weighted, with the weight being $\frac{AT_{it}}{\sum AT_{it}}$. The R^2 is the overall value.

	Large Firms				Small Firms			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable	$g_{it,t-5}$	$g_{it,t+5}$	$g_{it,t-5}$	$g_{it,t+5}$	$g_{it,t-5}$	$g_{it,t+5}$	$g_{it,t-5}$	$g_{it,t+5}$
β_{-5}	0.222***		-0.090		-0.778***		-0.512***	
	(5.61)		(-1.13)		(-11.92)		(-7.35)	
β_{+5}		-0.489***		-0.396***		0.740***		0.680***
		(-13.82)		(-12.38)		(14.93)		(13.55)
Year fixed effects	yes	yes	yes	yes	yes	yes	yes	yes
Industry fixed effects	yes	yes	yes	yes	yes	yes	yes	yes
Weighting	equal	equal	value	value	equal	equal	value	value
N	122027	122111	122027	122111	122027	122111	122027	122111
Within R2	0.002	0.011	0.000	0.024	0.021	0.020	0.000	0.001
R2	0.052	0.063	0.125	0.083	0.070	0.071	0.125	0.061

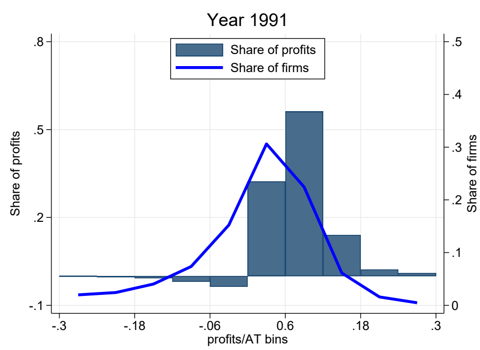
Figure A.9: Profit Distributions at Decade Intervals



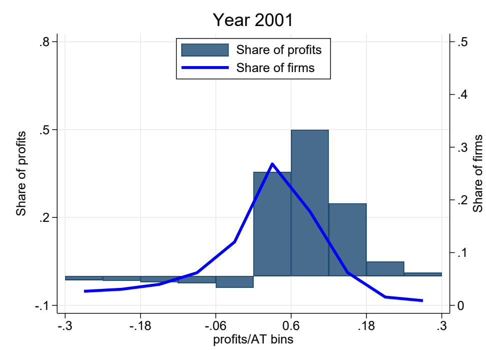
(a)



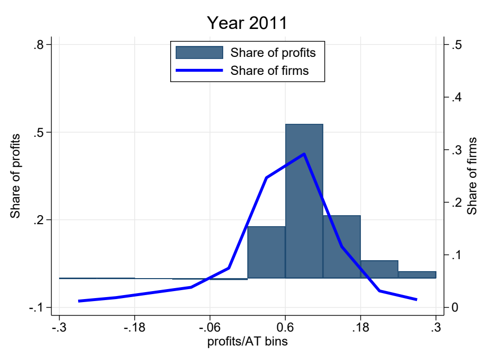
(b)



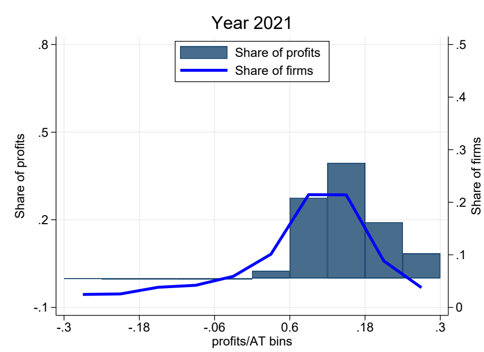
(c)



(d)



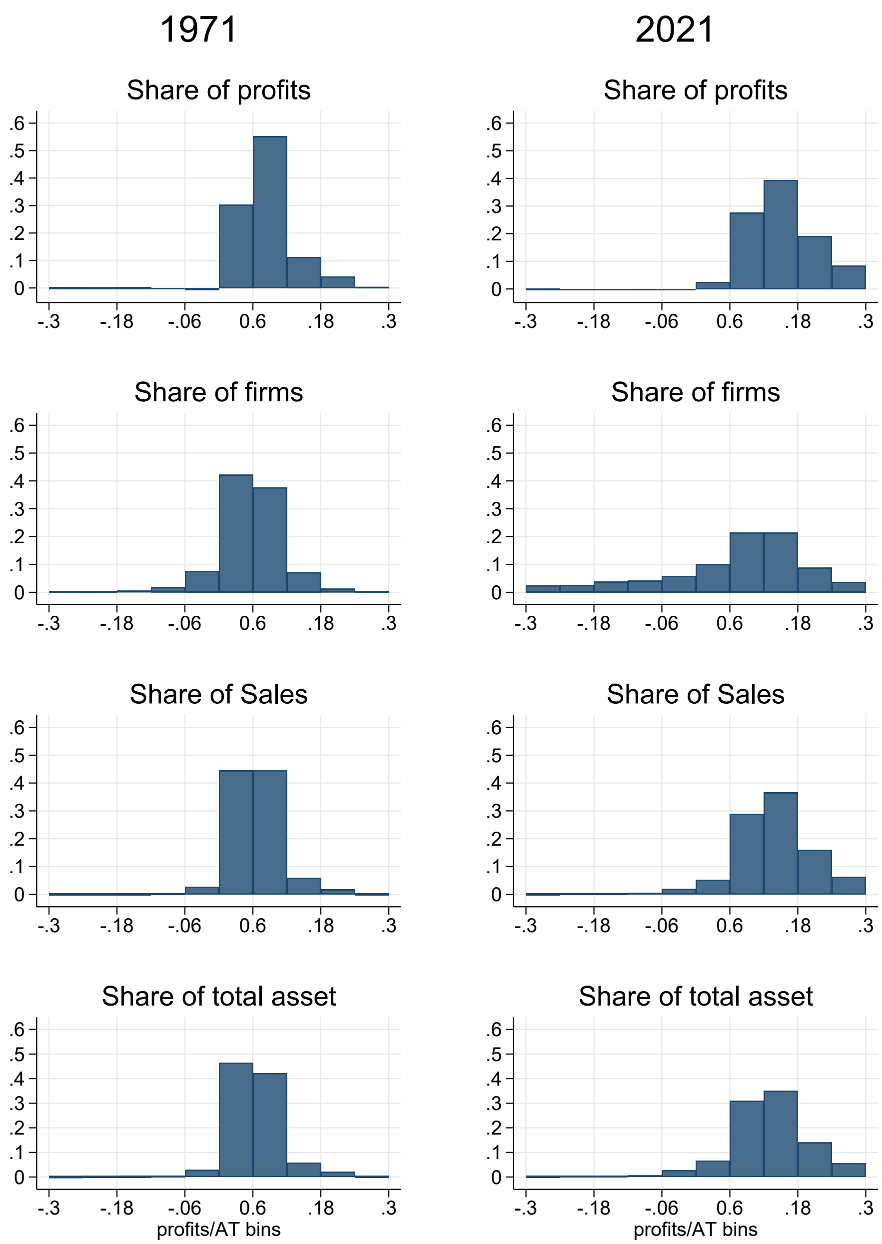
(e)



(f)

This figure presents the distribution of profits for each decade. Both profits and $\frac{\Pi}{AT}$ are winsorized at 1st and 99th percentiles across years. We divide [-0.3,0.3] into 10 bins. To calculate the share of profits, in each year we first calculate the total profits across all firms, then for each bin, we calculate the total profits for firms in that bin. Then the share of profits of each bin is the ratio of these two numbers. Similarly, for the share of firms, in each year we first calculate the total number of firms, then for each bin, we calculate the number of firms in that bin. Then the share of firms for each bin is the ratio of these two numbers.

Figure A.10: Firm Attributes Conditional on Profits



This figure compares the distributions of profits, number of firms, sales, and total assets over 10 profitability bins ($\frac{\Pi_{it}}{AT_{it}}$) in fiscal years 1971 (left column) and 2021 (right column). As before, profits, sales and assets are winsorized at 1st and 99th percentiles over years to avoid the impact of outliers.