Disagreement, Skewness, and Asset Prices

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We present a frictionless theory addressing the seemingly unrelated puzzles that disagreement and skewness both negatively predict stock returns. Our key insight is that skewness leads to nonlinear demand curves that prevent trades due to disagreement from "canceling out." The model predicts that expected returns are governed by an interaction between disagreement and skewness; each variable amplifies the impact of the other. This interaction further has an "invariant" relationship with Sharpe ratios, independent of the level of volatility, showing how the effect of disagreement is distinct from that of uncertainty. We provide robust empirical support for these predictions.

KEYWORDS: disagreement, divergence of opinions, analyst forecast dispersion, skewness, asset pricing, nonlinear demand, heterogeneous agents, beliefs, expectations
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1 Introduction

Investor disagreement (i.e., divergence of opinions) is a pervasive element in financial markets that, intuitively, is essential for trade. However, its impact on asset prices is less obvious. A large literature starting with Diether et al. (2002) proxies for disagreement using analyst forecast dispersion and finds it is *negatively* related to expected stock returns. These papers typically cite as explanation the argument of Miller (1977) that disagreement leads to inflated asset prices since short-sales frictions hinder the expression of pessimists' beliefs. Yet, in the agency mortgage-backed security (MBS) market, which is argued to be free of "significant short-sale constraints, illiquidity, or other frictions," Carlin et al. (2014) find that analyst forecast dispersion is positively related to expected MBS returns. Güntay and Hackbarth (2010) obtain similar findings in the corporate bonds. Consistent with this evidence in fixed income markets, Abel (1989) shows that disagreement lowers prices (and increases expected returns) due to reduced risk-sharing even without trading frictions.

In this paper, we develop a simple theory that rationalizes all of the above empirical findings: belief heterogeneity affects equilibrium price via demand curvature, which can arise frictionlessly when payoffs are skewed. Moreover, our theory is a bridge between these papers and the seemingly unrelated finding that skewness negatively predicts stock returns (Boyer et al., 2010, among others). Finally, the theory generates novel predictions about the interaction of disagreement and skewness, for which we provide robust empirical support.

We analyze the asset pricing implications of disagreement in a parsimonious neoclassical framework with essentially no parametric assumptions on utility or payoff distributions. Our model considers a two-period financial market with trading in a risk-free asset and a single risky asset, which pays an uncertain liquidating dividend. The risky asset is in zero net supply with its equilibrium price determined by standard market clearing.[1] There is

[1] In Section 3.4, we show our results are robust to non-zero supply. The risk-free asset has infinitely elastic supply with price and payoff normalized to unity.

a continuum of investors who are price takers, maximize expected utility, and have a common utility function with the following standard properties: investors strictly prefer more to less, are strictly risk averse, and have non-increasing absolute risk aversion (NARA).[2] NARA ensures that a risky asset with a positive expected return is not an inferior good; i.e., wealthier investors allocate weakly more money to the risky asset. Arrow (1971) argues these are properties of any reasonable utility function. We make no parametric assumptions on utility, but note that the NARA class nests both the constant absolute relative risk aversion (CARA) and constant relative risk aversion (CRRA) preference classes. Though seemingly obvious and innocuous, NARA is in fact quite powerful. Just as risk-aversion implies a distaste for variance, NARA implies a preference for positive skewness (Arditti, 1967).

Our main theoretical result is that an asset's skewness interacts with disagreement to determine equilibrium price through its impact on the curvature of investors' demand schedules. Via Taylor polynomials of investor demands and the market-clearing price, we obtain the following pricing equations:

$$\text{expected excess return} \propto -\text{volatility} * \text{skewness} * \text{Var(beliefs)}, \tag{1}$$

$$\text{Sharpe ratio} \propto -\text{skewness} * \text{Var(beliefs)}. \tag{2}$$

Equation (1) predicts an interaction effect: skewness controls the sign and magnitude of the impact of disagreement on asset prices. Because empirically most stocks have positively skewed returns (Boyer et al., 2010) and MBS payoffs are decidedly negatively skewed, this interaction implies a negative relationship between disagreement and expected returns in equities (the "disagreement effect," as found by Diether et al., 2002) but a positive relationship in MBS (as found by Carlin et al., 2014). Because variance is non-negative, equation (1) also predicts that expected returns are decreasing in skewness (the "skewness effect," as

[2] In Section 3.2, we show our results are robust to investors having heterogeneous utility functions satisfying these properties.

found by Boyer et al., 2010). Finally, we obtain a "null" result: when skewness is approximately zero, the effect of disagreement on expected returns should be negligible. Likewise, when disagreement is approximately zero, the effect of skewness on returns should be negligible. Moreover, equation (2) predicts that the relationship between skewness, disagreement, and Sharpe ratios should be invariant to the level of volatility. This result supports the conclusion in Diether et al. (2002) that forecast dispersion does not proxy for risk and further shows how the disagreement effect in stocks can be separated from the idiosyncratic volatility puzzle (Falkenstein, 1994).

In Section 4, we provide economically and statistically significant empirical support for the interaction effect predicted by equation (1) and the invariance prediction of equation (2) within the universe of U.S. equities. We first use an independent double sort to construct a portfolio which is both skew- and disagreement-neutral but exploits the interaction effect. It has an economically significant annualized CAPM alpha of 7.3% with a statistically significant t-statistic greater than 3. Fama and French (2015) and Daniel et al. (2020) alphas exhibit comparable metrics. We next use a semi-parametric Fama-Macbeth regression to show that, indeed, the disagreement effect disappears when skewness is near zero and, similarly, the skewness effect disappears when disagreement is negligible. Finally, we repeat the exercise on scaled returns (realized returns divided by ex-ante option implied volatility) and confirm the invariance prediction of equation (2).

To understand the mechanism in our model that generates these effects, first note that NARA utility implies that an investor's demand schedule is convex in a neighborhood around the investor's subjective expected payoff, $p \approx \mu_i$, if and only if the payoff skewness is positive. Further, convexity is locally proportional to skewness. These results obtain from a second-order approximation of an investor's demand schedule at the candidate price $p = \mu_i$. Figure 1 provides a graphical intuition for the relationship between skewness and demand convexity. Start with the demand function for an asset with arbitrary payoff distribution, given by the thinner curve.[3] Now consider an otherwise identical asset with higher skewness, whose demand is given by the thicker curve.[4] Since a NARA investor has a preference for skewness, demand for this new asset must be weakly higher at any price. Because of risk aversion, however, when price, p, equals the subjective expected payoff, μ_i, demand for any asset is zero.[5] As a result, the thicker curve is more "curved" than the thinner curve. Accordingly, increasing skewness locally increases the demand function's convexity.

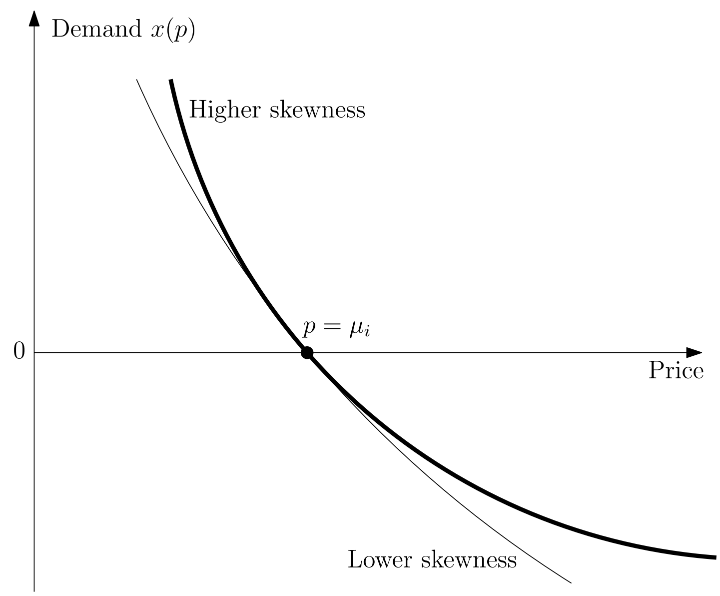


Figure 1: Higher Skewness Leads to a More Convex Demand Curve

Note: This figure illustrates that ceteris paribus, higher skewness leads to a more convex demand curve. The thicker curve represents the demand associated with a higher skewness, while the thinner curve represents the demand associated with a lower skewness. Since a NARA investor has a preference for skewness, the thicker curve is weakly above the thinner curve for any price. Furthermore, when price equals the subjective expected payoff (μ_i), demand is precisely zero, regardless of skewness. Put differently, both curves must cross the price axis at the same point, where they are tangent. As a result, the thicker curve is more "curved" than the thinner curve in the neighborhood around the expected payoff.

[3]As drawn, its second derivative is positive, but the argument works for any value.

[4]We cautiously note that it is generally impossible to change skewness without affecting other moments unless the distribution is unbounded. However, as we show in the proof of Lemma 1, the graphical illustration is correct in a neighborhood around the subjective expected payoff, as the curvature of the demand function locally does not depend on moments higher than the third.

[5]This is because a risk-averse agent rejects any mean-zero lotteries.

We model differences of opinion in our baseline setup by separating investors into two types: "optimists" (positive type, denoted as +) and "pessemists" (negative type, denoted as −). [6] Investors disagree about the mean of the dividend distribution but agree about its shape, and, hence, agree about volatility and all higher-order central moments.[7] Letting μ and σ be the objective mean and volatilty of the payoff, respectively, the beliefs of the two types are $\mu_+ = \mu + \sigma\delta$ and $\mu_- = \mu - \sigma\delta$ so the average belief is correct and δ parameterizes the level of disagreement (per unit of volatility). Note that this structure of beliefs implies that investors disagree about the asset's Sharpe ratio by $\pm\delta$. Figure 2 plots their demand schedules for a positively skewed asset. Due to agreement on higher moments, the curves are locally parallel.

With this backdrop, consider a candidate equilibrium price equal to the average belief: $p = \mu$. At this price, the positive-type investor perceives the asset as under-priced and would go long while the negative-type investor perceives it as over-priced and would go short. Can the market clear? The answer is no, because at this price there would be excess demand because buying a positively skewed asset entails more desirable upside risk whereas shorting the asset involves more downside risk. Convexity implies the positive type would want to buy more shares of the asset than those shorted by the negative type. To clear the market, the equilibrium price must be higher than μ. In summary, the equilibrium price will be higher than the expected payoff for a positively skewed asset. Similarly, the equilibrium price will be lower than the expected payoff for a negatively-skewed asset since negative skew implies concave demand schedules. When the payoff distribution is symmetric, demand is linear and price equals the average belief.

We note that our proposed theory is complementary (rather than contradictory) to existing explanations of the relationship between disagreement and expected returns. In a simple setting with frictionless trading, zero net supply, and linear demand curves (as in

[6] In Section 3.1, we show our results are robust to an arbitrary number of investor types.

[7] In Section 3.3, we show our results are robust to alternative structures of disagreement such as differences of opinion about variance and skewness.

the case of a CARA-normal environment or approximate mean-variance preferences), disagreement has no pricing implications: price equals the average belief. Short-sale frictions can be viewed as another source of demand convexity. When short-sale costs are proportional to quantity shorted, demand is kinked at $p = \mu_i$, with a flatter slope in the short-selling region. The demand schedule is piecewise linear, but convex.[8] Overall, our graphical intuition highlights the importance of nonlinear demand curves and extends applicability beyond traditional frameworks with short-selling constraints.

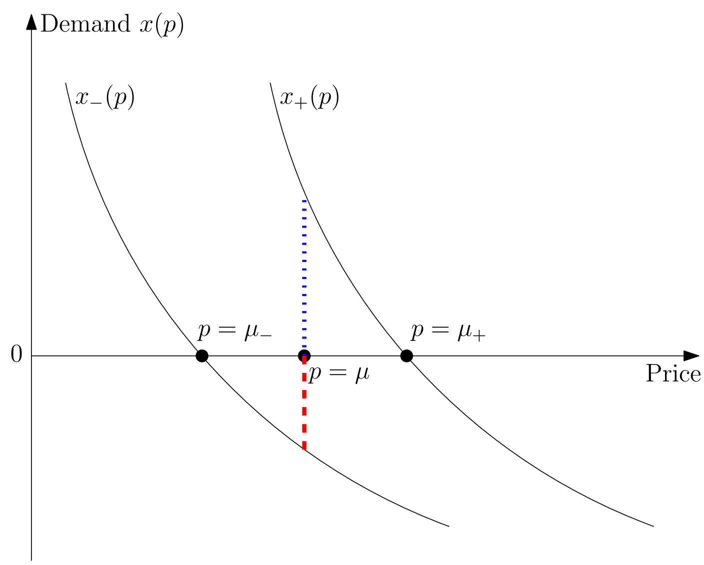


Figure 2: Disagreement Lowers Returns for a Positively Skewed Asset

Note: This figure illustrates the intuition that disagreement lowers returns for a positively skewed asset. Because of positive skewness, an investor's demand is convex in the neighborhood of the asset's expected payoff under the investor's belief. The two curves represents the demand schedules of a positive-type investor and a negative-type investor, respectively. For example, the demand of a positive-type investor crosses zero at price $\mu + \sigma\delta$, which is the risky asset's expected payoff under a positive type's belief. Now suppose the price is equal to μ. The blue (upper) dotted line segment represents the shares that a positive-type investor would buy, while the red (lower) dashed line segment represents the shares that a negative-type investor would sell. Because of demand convexity, the length of the dotted-blue line is greater than that of the dashed-red line. In other words, at price μ, there is excess demand. To clear the market, the equilibrium price must be greater than μ.

[8] When short sales are not allowed, demand is zero for $p \geq \mu_i$. Demand is still convex. In fact, demand convexity obtains if short-sale cost is weakly convex in quantity shorted.

Related Literature

Disagreement is an important basis for trade in financial markets and the subject of a large theoretical literature motivating empirical exploration.[9] Several studies since Diether et al. (2002) have proposed various explanations for the "disagreement effect" in equities, the negative relationship between analyst forecast dispersion and expected returns.[10] Unique to the literature is (Johnson, 2004), which puts forth a fully rational explanation that further predicts a relationship between dispersion and firm leverage. Avramov et al. (2009), however, contend such a relationship is not supported in the data. Our theory offers a simple frictionless explanation for the dispersion effect via the interaction of disagreement and skewness. Further, our model is consistent with the findings in Carlin et al. (2014) and Güntay and Hackbarth (2010) that forecast dispersion is positively related to expected bond returns.

Several recent empirical studies have documented the skewness effect, a negative relationship between ex-ante (idiosyncratic) skewness and expected returns (Boyer et al., 2010; Conrad et al., 2013; Amaya et al., 2015; Boyer and Vorkink, 2014). Existing theoretical explanations include optimistic beliefs (Brunnermeier and Parker, 2005; Brunnermeier et al., 2007), cumulative prospect theory preferences (Barberis and Huang, 2008), or heterogeneous preferences and underdiversification (Mitton and Vorkink, 2007).[11] Goulding et al. (2023) show that NARA utility and the presence of noise traders generate the skewness effect. In contrast, our theory not only generates the skewness effect under the mild assump-

[9] Miller (1977), Jarrow (1980), Diamond and Verrecchia (1987), and Chen et al. (2002) explore static models while Harrison and Kreps (1978), Scheinkman and Xiong (2003), Hong et al. (2006), and Martin and Papadimitriou (2022) study the dynamics of speculative bubbles. On issues related to trading volumes, price volatility, prices co-movement, and informed trading, see Harris and Raviv (1993), Kandel and Pearson (1995), Cao and Ou-Yang (2008), Dumas et al. (2009), Banerjee and Kremer (2010), Ottaviani and Sørensen (2015), Atmaz and Basak (2018), Banerjee et al. (2018), and Chabakauri and Han (2020), among others. See Bielecki et al. (2004), Hong and Stein (2007), Curcuru et al. (2010), and Xiong (2013) for surveys of this large literature.

[10] See Sadka and Scherbina (2007), Yu (2011), Ali et al. (2019); Barinov (2013); Hong and Sraer (2016); Daniel et al. (2023), among others, for studies that use this measure. See Chang et al. (2022) for an overview of this literature.

[11] Mitton and Vorkink (2007) show that if some investors have mean-variance preferences and others have mean-variance-skew preferences (as in Kraus and Litzenberger, 1976), the idiosyncratic skewness phenomenon obtains even without trading frictions or disagreement.

tion of NARA utility and without appealing to noise traders, but also generates distinct predictions for which we provide robust empirical support.

Our paper is in the same spirit of Yan (2010), who points out that individual biases may not necessarily cancel out when demand is nonlinear. However, that paper takes demand non-linearity as an assumption. Our paper provides a microfoundation for non-linear demand, relying on only the mild NARA utility assumption. Moreover, our theory leads to joint pricing implications of disagreement and skewness, which are absent from Yan (2010).

Finally, our paper is related to the literature that explores the pricing impacts of skewness with differences of opinion (driven by different priors or information differences). First, on different priors, Martin and Papadimitriou (2022) use a binomial tree setting with log-utility investors and no trading frictions to show that heterogeneous priors can drive sentiment that leads to an inflated price bubble for a skewed asset. Banerjee et al. (2022) and Banerjee et al. (2022) among others demonstrate that skewness can affect expected returns for CARA investors who agree to disagree with each others. Second, several papers have explored asymmetric payoffs and belief dispersion in a noisy rational expectations setting. Goulding (2015) generates an interactive pricing effect of payoff skewness and investor belief dispersion driven by private signals and public information in analyst forecasts and price in a framework with binary payoffs and CARA preferences. Applying various parametric payoff assumptions, Breon-Drish (2015), Chabakauri et al. (2022), and Cianciaruso et al. (2023) demonstrate pricing effects of payoff asymmetry and belief dispersion driven by private signals and price under CARA preferences. Albagli et al. (2024) derive an interaction effect of skewness and belief dispersion driven by private signals and price that applies to a larger collection of payoff distributions and investor preferences without parametric assumptions.

Our approach differs from these studies in its use of dogmatic beliefs, its transparency regarding exogenous primitives, its flexibility for analyzing pricing effects, and its distinct quantitative predictions. Albagli et al. (2024) rely on restrictions on the endogenous distribution of a sufficient statistic, which cannot be translated into conditions on exogenous primitives. In contrast, we work within a dogmatic beliefs framework, which allows us to derive asset pricing predictions entirely from *exogenous* primitives. Moreover, by directly characterizing how skewness drives nonlinear demand schedules without parametric assumptions on payoffs or preferences, we provide simple yet general graphical explanations for the intuition behind the equilibrium pricing mechanism as well as flexibly accommodate features that directly influence demand schedules, such as short-sales frictions. Furthermore, our paper generates *quantitative* pricing predictions for skewness and disagreement, as outlined in equations (1) and (2), including the distinct quantitative prediction that the interaction between skewness and disagreement has an "invariant" relationship with the Sharpe ratio, for which we provide consistent empirical evidence.

Road Map

The paper is organized as follows. In Section 2, we present a simple baseline model to derive the main results. In Section 3, we show our main results hold in several extensions of the baseline model, including disagreement with arbitrary types (Sec 3.1), heterogeneous preferences (Sec 3.2), disagreement on higher-order moments (Sec 3.3), and non-zero aggregate supply (Sec 3.4). In Section 4, we present the empirical method, data sample, and empirical analyses. We conclude the main text in Section 5. Proofs and various robustness tests are in the Appendix.

2 Model

In this section, we present and analyze a model of investor disagreement in a financial market for a risky asset that can have a skewed payoff distribution. In Section 3, we discuss a variety of model extensions to demonstrate how the insights presented here generalize. All proofs are in Appendix A.

2.1 Financial Market

We study a two-period financial market, with dates $t \in \{0,1\}$, and a continuum of investors, indexed by $i \in [-1,1]$. Each investor has common utility function $u(w)$ and initial wealth w_0.[12] The function $u(\cdot)$ is thrice continuously differentiable and exhibits the following properties of any reasonable specification of preferences (Arrow, 1971): investors strictly prefer more to less, which implies $u'(w) > 0$; investors are strictly risk averse, which implies $u''(w) < 0$; and an investor's absolute risk aversion does not increase in wealth—non-increasing absolute risk aversion, or NARA utility—which implies $u'''(w) > 0$. NARA ensures that a risky asset with a positive expected return is not an inferior good; i.e., wealthier investors allocate weakly more money to the risky asset. Note that the NARA utility class nests widely-used parametric utility functions such as constant absolute risk aversion (CARA) utility and constant relative risk aversion (CRRA) utility classes.

Investors trade in two assets: a risk-free asset, with both its price and payoff normalized to one, and a single risky asset, with date-0 price p and date-1 payoff $\widetilde{\theta}$. Let $F(\theta) = \Pr[\widetilde{\theta} \leq \theta]$ denote the cumulative distribution function (CDF) of the payoff $\widetilde{\theta}$ and $\mu = \mathrm{E}[\widetilde{\theta}]$, $\sigma^2 = \mathrm{Var}[\widetilde{\theta}]$, and $s = \mathrm{E}[(\widetilde{\theta} - \mu)^3]/\sigma^3$ denote its mean, variance, and skewness, respectively. Similarly, investor i's beliefs are given by μ_i, σ_i^2, and s_i. Let $E_i[\cdot]$ denote her subjective expectation operator. For now, we leave investor beliefs unspecified. Let $\widetilde{r} = (\widetilde{\theta} - p)/p$ denote the net return, $\sigma_r^2 = \mathrm{Var}[\widetilde{r}]$ denote the return variance, and $\mathrm{Sharpe}[\widetilde{r}] = \mathrm{E}[\widetilde{r}]/\sigma_r$ denote the Sharpe ratio.[13] Note that $\widetilde{\theta}$ is denominated in units of currency, $\widetilde{r}$ is unitless, and $\sigma = \sigma_r p$.

[12] In Section 3.2, we study heterogeneous utility functions as well as heterogeneous initial wealth.

[13] We assume μ is finite and $\sigma_r > 0$ to avoid a trivial solution. Note that net return is well defined if the equilibrium price is positive, which holds, for example, for stocks whose payoffs cannot be negative because of limited liability. Note also that excess return and return are equal because the risk-free asset is normalized to have zero return.

2.2 Investor Demand

Investor i submits a demand schedule $x_i(p)$, which specifies how many shares of the risky asset she would buy or sell at price p. We further assume that the supply of the risky asset is zero, so the market's clearing condition is $\int_{-1}^{1} x_i(p)\,di = 0$, which determines the equilibrium price p.[14] An equilibrium consists of $(\{x_i(\cdot) : i \in [-1,1]\}, p)$ such that the demand schedule $x_i(p)$ solves investor i's expected utility maximization problem and the price p clears the market.

We start with an investor's problem. Formally, the demand schedule $x_i(p)$ solves the following program:

$$x_i(p) = \arg\max_x E_i[u(w_0 + x_i(\widetilde{\theta} - p))], \tag{3}$$

where the expectation is taken under investor i's beliefs. Since u is strictly concave and thrice continuously differentiable, the necessary and sufficient condition which determines $x_i(p)$ is given by the first-order condition (FOC):

$$E_i[u'(w_0 + x_i(p)(\widetilde{\theta} - p))(\widetilde{\theta} - p)] = 0. \tag{4}$$

Note that when the price equals the expected payoff under investor i's belief, $p = E_i[\widetilde{\theta}]$, zero demand ($x_i = 0$) solves the FOC. It follows that $x_i(\mu_i) = 0$, given that the strict concavity of u guarantees a unique solution of the FOC. We now analyze the properties of optimal demand when p is close to μ_i via a Taylor expansion of $x_i(p)$ at the point $p = \mu_i$. Doing so, we obtain the following lemma:

Lemma 1. *Investor i's demand schedule is given by*

$$x_i(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_i^2}(p - \mu_i) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s_i}{\sigma_i^3}(p - \mu_i)^2 + o(1)(p - \mu_i)^2, \tag{5}$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $p \to \mu_i$.

[14] In Section 3.4, we discuss the case of non-zero aggregate supply, from which we reach similar conclusions.

The slope of the demand function at $p = \mu_i$ is given by $x_i'(\mu_i) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_i^2}$, which is negative because $u'' < 0$. Consider a special case when the utility function is given by a CARA utility function and the payoff of the underlying risky asset $\widetilde{\theta}$ is normally distributed. In this case, skewness is zero, and the quadratic term in equation (5) is zero, consistent with the well-known result that the demand function is linear under CARA and normality.

The quadratic term in equation (5) captures the curvature of the demand function at $p = \mu_i$. It is clear that the sign of $x_i''(\mu_i)$ solely depends on s_i, since the utility function has a negative second derivative and a positive third derivative. Specifically, if $s_i > 0$, then $x_i''(\mu_i) > 0$; if $s_i < 0$, then $x_i''(\mu_i) < 0$. Lemma 1 states that a NARA investor's demand of a positively (negatively) skewed risky asset is strictly convex (concave) at $p = \mu_i$. Figure 3 offers a graphical intuition for the result. Recall that a NARA investor has a preference for positive skewness. This fact implies that when we increase the skewness of the underlying asset, holding everything else equal, the investor would demand weakly more shares of the risky asset, i.e., the demand function is weakly higher, consistent with the observation that the solid curve is above the dashed line in Figure 3.[15] Furthermore, demand has to be zero when $p = \mu_i$ because of risk aversion. As a result, the "curvature" of the demand function of a positively skewed asset at $p = \mu_i$ has to be greater than the "curvature" of a straight line. Since the curvature of a straight line is 0, the curvature of the solid curve at $p = \mu_i$ has to be positive.

2.3 Investor Beliefs

We allow investors to disagree about the date-1 payoff of the risky asset. Each investor's belief corresponds to a horizontal shift of the payoff CDF, $F(\cdot)$, which implies that investors disagree about the mean (and also the Sharpe ratio) but agree about higher central moments

[15] We cautiously note that it is generally impossible to increase skewness without affecting other moments. However, as we show in the proof of Lemma 1, the graphical illustration is correct in the neighborhood of $p = \mu_i$, as the curvature of the demand function at $p = \mu_i$ does not depend on moments higher than the third.

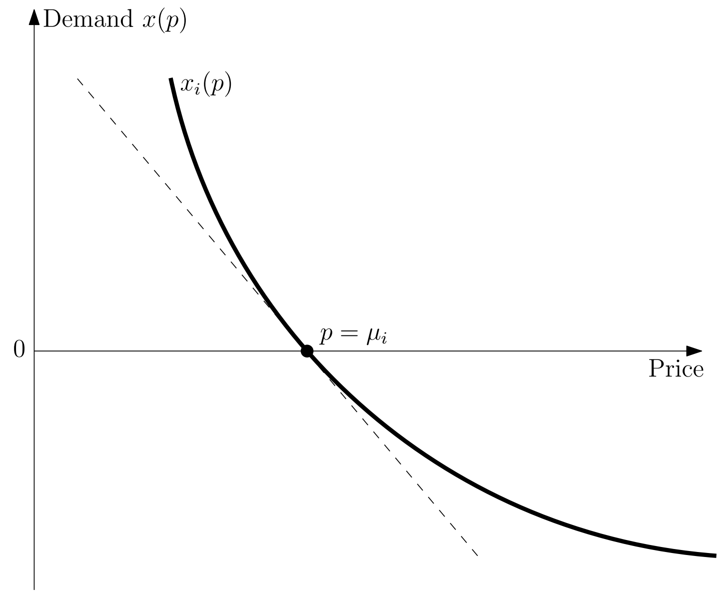


Figure 3: The Demand Schedule of a Positively Skewed Asset

Note: This figure illustrates the shape of the demand schedule. The dashed line is linear demand, which obtains under CARA and normality. The solid curve is the demand schedule when the underlying risky asset is positively skewed. The solid curve and the dashed line are tangent at the point $(p,x) = (\mu_i, 0)$. Since the solid curve has to be weakly higher than the dashed line, the curvature of the solid curve at $p = \mu_i$ has to be positive, consistent with Lemma 1 that the demand function of a positively skewed asset is convex at $p = \mu_i$.

such as variance, skewness, etc., because the shape of the distribution is common.[16] We use the notation F_z to reflect a horizontal shift of z units; i.e., $F_z(\theta) = F(\theta - \sigma z), \forall \theta$. Note that z is unitless.

For simplicity of presentation we consider two types of investors, but we show in Section 3.1 that our results generalize to arbitrary number of types. Each investor can be either a positive "optimist" type or a negative "pessimist" type: investors from $(0,1]$ are of positive type, and investors from $[-1,0]$ are of negative type. We assume a positive-type investor believes that the date-1 payoff is drawn from CDF F_δ, while a negative-type investor believes the payoff is drawn from CDF $F_{-\delta}$, such that a higher level of the horizontal shift parameter $\delta \geq 0$ corresponds to a wider dispersion of beliefs between the two investor

[16] In Section 3.3, we discuss disagreement about other moments of the distribution.

types.[17]

Let $E_+[\cdot]$ and $E_-[\cdot]$ denote the expectation operator under positive and negative type beliefs, respectively. From the horizontal shift assumption, we obtain immediately $\mu_+ = \mathrm{E}_+[\widetilde{\theta}] = \mu + \sigma\delta$ and, likewise, $\mu_- = \mu - \sigma\delta$. Accordingly, the two types of investors have mean payoff beliefs corresponding to $\mu \pm \sigma\delta$, where $(+)$ is for the positive type of investors and $(-)$ is for the negative type. Since a horizontal shift in the CDF doesn't affect centered moments, we also have $\sigma_+ = \sigma_- = \sigma$ and $s_+ = s_- = s$; investors agree on the variance and skewness of the payoff distribution.

Using the identity $\sigma = \sigma_r p$, subtracting p, dividing the result by p, and noting that $\mathrm{E}[\widetilde{r}] = \frac{\mu - p}{p}$, we can recast investor disagreement as disagreement about expected returns $\mathrm{E}[\widetilde{r}] \pm \sigma_r \delta$. Further dividing through by return volatility, our model of investor disagreement simplifies to disagreement about the asset's Sharpe ratio: $\mathrm{Sharpe}[\widetilde{r}] \pm \delta$.

Using Lemma 1 combined with agreement about centered moments, the demand schedules of positive and negative-type investors are given by

$$x_+(p) = \frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu_+) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}(p-\mu_+)^2 + o(1)(p-\mu_+)^2, \tag{6}$$

$$x_-(p) = \frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu_-) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}(p-\mu_-)^2 + o(1)(p-\mu_-)^2, \tag{7}$$

respectively, where the little-o notation $o(1)$ is an unknown function that converges to 0 as $p \to \mu_i$.

[17] We remark that this structure of disagreement may not be realistic if the dividend distribution is bounded and investors can write side contracts. This is because the upper bound of a positive type's perceived distribution is outside the support of the perceived distribution from a negative type. If side contracts are allowed, investors would bet an infinite amount for a contract that only pays outside the support of their respective perceived distribution. In our baseline model, one way to justify our assumption is that side contracts are not allowed or sufficiently costly to write, which is to say that the market is incomplete. We can also justify our assumption by assuming the support of the dividend distribution is the entire real line in which case the above concern is absent. Overall, we choose the simple structure in our baseline model to highlight the mechanism through which disagreement affects asset prices. As we show in Sections 3.1 and 3.3, our results are robust to general structures of disagreement.

2.4 Equilibrium Price and Expected Return

We use the market-clearing condition to solve for the equilibrium price. Setting the aggregate demand to be zero, price solves the following equation.

$$x_+(p)+x_-(p)=0. \tag{8}$$

Proposition 1. *There exists a $\overline{\delta}>0$ such that if $\delta<\overline{\delta}$, then the equilibrium price is given by the following equation:*

$$p=\mu+\overline{u}s\sigma\delta^2+o(1)\delta^2, \tag{9}$$

where

$$\overline{u}=\frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} \tag{10}$$

is a positive constant and where the little-o notation $o(1)$ is an unknown function that converges to 0 as $\delta\to 0$.

Bounded disagreement ensures that demand convexity does not change signs between μ_- and μ_+. From zero net supply, the equilibrium price must lie in this interval. The quantity $\overline{u}$ is always positive because the NARA assumption implies $u'''>0$. Suppose the risky asset payoff is positively skewed. Proposition 1 states that the equilibrium price will be higher than μ, which is the price when there is no disagreement, i.e., $\delta=0$, or when the marginal investor has the average belief. Proposition 1 implies that a higher level of disagreement biases the equilibrium price upward. Moreover, Proposition 1 provides a functional form for the deviation of the equilibrium price from μ.

Figure 2 provides the intuition. Suppose the underlying risky asset is positively skewed so that an investor's demand is convex in the neighborhood of the asset's expected payoff (Lemma 1). The two curves represent the demand schedules of a positive-type investor and a negative-type investor, respectively. For example, the demand of a positive-type investor crosses zero at price $\mu+\sigma\delta$, which is the risky asset's expected payoff under a positive type's belief. Now suppose the price is equal to μ. The dotted blue line represents the shares that a positive-type investor would buy, while the dashed red line represents the shares that a negative-type investor would sell. Because of convexity, the length of the blue line is greater than that of the red line. Put differently, since buying a positively skewed asset entails more desirable upside risk whereas shorting the asset involves more downside risk, the positive type would buy more shares of the asset than those shorted by the negative type. So, at a candidate equilibrium price μ, we have excess demand for the asset. To clear the market, the equilibrium price must be higher than μ.

From Proposition 1 and the identity $\sigma = \sigma_r p$, we obtain the following expressions for the dominant terms of the equilibrium expected return and Sharpe ratio.

Corollary 2. *Using the second-order Taylor polynomial for equilibrium price in* (9)*, the corresponding expected return and Sharpe ratio can be expressed as follows:*

$$\mathrm{E}[\tilde{r}] \approx -\overline{u} s \sigma_r \delta^2, \tag{11}$$

$$\mathrm{Sharpe}[\tilde{r}] \approx -\overline{u} s \delta^2. \tag{12}$$

Because $\overline{u} > 0$, for ex-ante positively skewed assets, Corollary 2 indicates that the expected return and Sharpe ratio decrease in the level of disagreement δ. This negative predictive relationship between disagreement and expected returns of individual stocks is consistent with Diether et al. (2002) and a large literature in empirical asset pricing. We arrive at our prediction, however, in a novel way. Rather than appealing to short-sales frictions—the leading explanation for the disagreement effect—our model is frictionless and appeals instead to the implications of asset skewness for demand convexity, which governs the relationship between investor disagreement and expected returns. Moreover, for negatively skewed assets such as MBS, Corollary 2 predicts a positive relationship between disagreement and expected returns, consistent with Carlin et al. (2014).

Note that because of the simplifying standard assumption of zero supply, expected returns (11) are zero when there is no skewness. More generally, if net supply were positive, then a positive risk premium would be part of the expected return to induce offsetting investor demand in equilibrium. Therefore, we can interpret our results as the isolated effect of disagreement on expected returns net of the risk premium.[18]

2.5 Comparative Statics

Corollary 2 allows us to further analyze how various parameters affect the expected return and Sharpe ratio predictions.

Proposition 3. *The expected return and Sharpe ratio expressions of Corollary 2 satisfy the following properties.*

1. *(The skewness effect): Holding disagreement fixed, expected return and Sharpe ratio are decreasing in skewness. When disagreement is zero, the skewness effect should be negligible.*

2. *(The disagreement effect): Holding skewness fixed, expected return and Sharpe ratio are decreasing in disagreement if and only if skewness is positive. If skewness is zero, the disagreement effect is negligible.*

3. *(The interaction effect): Higher skewness amplifies the disagreement effect on expected return and Sharpe ratio and higher disagreement amplifies the skewness effect on expected return and Sharpe ratio.*

Proposition 3 offers empirical predictions that we test in Section 4. The proposition offers predictions on two well-known anomalies: (1) the negative relationship between ex-ante return skewness and expected equity returns and (2) the negative relationship between disagreement and expected equity returns. Moreover, to the best of our knowledge, the interaction effect from our theory is novel to the theoretical asset pricing literature.

[18] In Section 3.4, we show the interactive effect of skewness and disagreement still obtains in the case of non-zero aggregate supply.

Furthermore, our model predicts that the analogous three effects also hold for the Sharpe ratio and, therefore, the effects are invariant to the level of ex-ante return volatility. These Sharpe ratio results affirm and expand on the argument in the literature that disagreement is distinct from risk (Diether et al., 2002).

3 Discussion

Section 2 presents a simple two-type disagreement model of frictionless trading to highlight how skewness mediates the impact of disagreement on expected returns. In this section, we relax the simplifying assumptions of Section 2 and show that our main results are robust to several extensions.

3.1 Disagreement with Arbitrary Types

The restriction in Section 2 to two types of investors is not an essential assumption. In this subsection, we extend our results to an arbitrary number of types.[19] We assume investor i believes that the date-1 payoff is drawn from CDF F_{δ_i}. The unitless bias parameter δ_i is drawn from a bounded mean-zero random variable $\widetilde{\delta}$, whose variance is denoted as $\mathrm{Var}(\widetilde{\delta})$.[20] In particular, $\widetilde{\delta}$ does not have to be symmetric. In this extended setup, $\mathrm{Var}(\widetilde{\delta})$ captures relative disagreement across investors. Note that the model of Section 2 is a special case in which the random variable $\widetilde{\delta}$ is binary, i.e., only takes two possible values ($\pm\delta$), and relative disagreement reduces to $\mathrm{Var}(\widetilde{\delta}) = \delta$. Proposition 4 summarizes the result.

Proposition 4. *There exists a $\overline{\delta} > 0$ such that if $Var(\widetilde{\delta})$ is bounded by $\overline{\delta}$, then the equilibrium price is given by the following equation.*

$$p = \mu + \overline{u}s\sigma Var(\widetilde{\delta}) + o(1)Var(\widetilde{\delta}), \tag{13}$$

[19] For technical convenience, we restrict attention to a finite number of types. It is straightforward to extend to countable or even uncountable types at the cost of cumbersome notation.

[20] Because of the boundedness assumption, the variance is finite.

where $\overline{u}$ is a positive constant defined in (10) and the little-o notation $o(1)$ is an unknown function that converges to 0 as $Var(\widetilde{\delta}) \to 0$.

From Proposition 4 and the identity $\sigma = \sigma_r p$, we obtain the following expressions for the dominant terms of the equilibrium expected return and Sharpe ratio, consistent with the implications of the main model.

Corollary 5. *Using the second-order Taylor polynomial for equilibrium price in (9), the corresponding expected return and Sharpe ratio can be expressed as follows:*

$$\mathrm{E}[\widetilde{r}] \approx -\overline{u} s \sigma_r Var(\widetilde{\delta}), \tag{14}$$

$$\mathrm{Sharpe}[\widetilde{r}] \approx -\overline{u} s Var(\widetilde{\delta}). \tag{15}$$

3.2 Heterogeneous Utility Functions

So far, our analysis assumes homogeneous utility functions. In this subsection, we show that this assumption does not drive our results. To simplify the analysis, we assume each investor has one of the two utility functions: $u_1(\cdot)$ and $u_2(\cdot)$, both in the NARA class. We return to our baseline model and assume that each investor can be either a positive "optimist" type or a negative "pessimist" type.

First, we consider the case that each investor can be one of the following four types with equal probability: u_1 and optimist; u_2 and optimist; u_1 and pessimist; u_2 and pessimist. Put differently, utility function being u_1 or u_2 and belief being positive or negative are independent. In this case, it is clear that the equilibrium price is higher than the expected payoff under the average belief, μ. This is because at the price μ, there is excess demand among the two belief types with the same utility function: u_k and pessimist, and u_k and optimist, for $k = 1,2$. Summing over k still gives excess aggregate demand. In all, the equilibrium price has to be higher than μ to clear the market.

Second, we consider the case in which each investor can be one of the following two types with equal probability: u_1 and optimist; u_2 and pessimist. In other words, investors with positive belief necessarily have the utility function u_1, while investors with negative belief necessarily have u_2. We define the benchmark price as the price that obtains when investors have mean-variance preferences, in which case demand schedules are linear. We denote the benchmark price as p_0, which is given by the following equation

$$p_0 = \frac{\frac{u_1'(w_0)}{-u_1''(w_0)}\mu_+ + \frac{u_2'(w_0)}{-u_2''(w_0)}\mu_-}{\frac{u_1'(w_0)}{-u_1''(w_0)} + \frac{u_2'(w_0)}{-u_2''(w_0)}}. \tag{16}$$

That is, the benchmark price is the weighted average of the expected payoff under each type's belief, where the weight is given by the reciprocal of each type's absolute risk aversion. Intuitively, a less risk-averse investor, trading more aggressively, gets a larger weight in determining the benchmark price.

We are now ready to present the result.

Proposition 6. *There exists a $\overline{\delta} > 0$ such that if $\delta \leq \overline{\delta}$, then the equilibrium price is greater than the benchmark price p_0 if and only if $s > 0$.*

We make a final remark. Section 2 assumes that all investors have the same initial wealth w_0. If investors have different initial wealth, effectively, investors have different risk aversion and prudence. As we can see from the analysis in this subsection, Proposition 6 still holds. Hence, the same initial wealth assumption is not crucial.[21]

3.3 Disagreement On Other Moments

Thus far, we have assumed that investors disagree about the mean payoff but agree on the variance and skewness. In this subsection, we discuss the possibility of disagreement on other moments. For the ease of exposition, we assume there are two types of investors, type A and type B. The probability of each type is one-half. The two types of investors

[21] Our benchmark price p_0 is consistent with Yan (2010).

can disagree about the distribution of the time-1 payoff, but have common utility function $u(w)$. We assume that a type j investor believes that the expected payoff is μ_j, the standard deviation is σ_j, and the skewness is s_j, for $j = A, B$.

First, we note that if $\mu_A = \mu_B = \mu$, then the equilibrium price is also $p = \mu$. This is because each type investor demands zero shares of the asset so the market is cleared at $p = \mu$, given our zero aggregate supply assumption. Therefore we assume $\mu_A \neq \mu_B$ so that disagreement "matters."

Now, we can write each type of investors' demand schedule in a neighborhood around μ_j as the following.

$$x_A(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_A^2}(p - \mu_A) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_A}{\sigma_A^3}(p - \mu_A)^2 + o(1)(p - \mu_A)^2, \tag{17}$$

$$x_B(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_B^2}(p - \mu_B) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_B}{\sigma_B^3}(p - \mu_B)^2 + o(1)(p - \mu_B)^2, \tag{18}$$

where $x_j(p)$ denotes type j's demand, for $j = A, B$.

We define the benchmark price as the price that obtains when investors have mean-variance preferences, in which case the demand schedules are linear. Given zero net supply, the benchmark price is given by the following expression

$$p_0 = \frac{\frac{\mu_A}{\sigma_A^2} + \frac{\mu_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}}. \tag{19}$$

Intuitively, the benchmark price is the weighted average of the expected payoff under each type's belief, where the weight is given by the reciprocal of each type of investor's belief about the variance of the payoff. This intuition holds because an investor who believes the variance is smaller would trade more aggressively, implying a higher weight in determining the benchmark price. We are now ready to present the result.

Proposition 7. *There exists a $\bar{\epsilon} > 0$ such that if $|\mu_A - \mu_B| \leq \bar{\epsilon}$, then the equilibrium price is greater than the benchmark price p_0 if $(\sigma_A s_A + \sigma_B s_B)/(\sigma_A + \sigma_B) > 0$; the equilibrium price is* *lower than the benchmark price p_0 if $(\sigma_A s_A + \sigma_B s_B)/(\sigma_A + \sigma_B) < 0$.*

When $\sigma_A s_A + \sigma_B s_B = 0$, it is ambiguous whether the equilibrium price is higher or lower than the benchmark price. Proposition 7 shows that the volatility-weighted average subjective skewness, $(\sigma_A s_A + \sigma_B s_B)/(\sigma_A + \sigma_B)$, is a "sufficient statistic" that plays the role of objective skewness in Proposition 1.

3.4 Non-Zero Aggregate Supply

So far our analysis assumes zero aggregate supply. In this subsection, we discuss the case when aggregate supply is not zero. Denote the aggregate supply as χ. To be clear, we return to the baseline model in Section 2 and only change the aggregate supply from zero to χ. In particular, we still assume investors have the same utility function and there are positive-type and negative-type investors.

It is important to discuss what the benchmark price would be in this case. Ideally, the benchmark price should approach the non-disagreement price as the level of disagreement approaches zero. Define $x_0(p)$ as the demand schedule for an investor with objective beliefs, given by $F(\cdot)$. As a result, both $x_+(\cdot)$ and $x_-(\cdot)$ approach to $x_0(\cdot)$ as $\delta \to 0$. We define the benchmark price as the price that obtains when $\delta = 0$. That is, the benchmark price p_0 solves the following equation.[22]

$$2x_0(p_0) = \chi. \tag{20}$$

Proposition 8. *Suppose the risky asset's payoff, $\widetilde{\theta}$, is bounded under both types of investor beliefs, and there is non-zero aggregate supply χ. Let p_0 denote the benchmark price as defined in (20). Then, there exists thresholds $\overline{\delta} > 0$, $\overline{s} > \underline{s}$, such that the following properties*

[22] We remark that the benchmark price defined here is consistent with the one defined in the previous subsections where we used the mean-variance preference. Specifically, mean-variance preference implies a linear demand function that passes through the point defined in (20). Despite disagreement on the first moment, the aggregate demand function is still a straight line that passes through the point defined in (20). So the equilibrium price associated with mean-variance preferences is given by p_0.

hold.

- *If $s > \overline{s}$ and $\delta < \overline{\delta}$, the equilibrium price is greater than p_0.*
- *If $s < \underline{s}$ and $\delta < \overline{\delta}$, the equilibrium price is smaller than p_0.*

4 Empirical Tests

In this section, we test the prediction that skewness and disagreement have an interactive pricing effect, summarized in Section 2.5, on a sample of U.S. stocks.

4.1 Data

Our sample is formed as the intersection of the Center for Research in Security Prices (CRSP) and Institutional Brokers Estimate System (IBES) universes. From CRSP we obtain monthly stock returns for U.S. firms (adjusted for delisting bias[23]), prices, volume, and shares outstanding. We drop firm-month observations with missing prior-month market capitalization (price×shares outstanding) or where the closing price one week ago is less than \$5/share.

Analysts' forecasts data are taken from the Unadjusted U.S. Statistics Summary History dataset of IBES. Our primary proxy for forecast dispersion, DISP, is essentially the same as that used by Diether et al. (2002) and many others: the month-end standard deviation of current-fiscal-year earnings per share (EPS) estimates scaled by the absolute value of the mean forecast across analysts tracked by IBES, from December 1983 to December 2023:

$$\mathrm{DISP} := \frac{\mathrm{stdF}_t}{|\mathrm{meanF}_t|}, \tag{21}$$

where stdF_t is the month-end standard deviation of next-fiscal-year earnings estimates across analysts tracked by IBES, and meanF_t is the average of those forecasts. We define

[23] This adjustment is suggested by Shumway (1997). Our results are not sensitive to this adjustment.

the next fiscal year as the closest upcoming within 170 to 550 days of the IBES statistical period (date of aggregation). We exclude firm-months where $\text{meanF}_t < 0.01$, which is a small number of instances. We also exclude firm-months where the price/earnings ratio, $P_t/\text{meanF}_t < 1$. This exclusion affects a small number of instances and likely data errors. In order to compute stdF_t, there must be at least two analyst forecasts. Firms with only one forecast are typically micro-caps. Note that DISP is a unitless measure of disagreement that corresponds to our modeled unitless relative disagreement δ in Section 2.

Our primary proxy for skewness, SKEW, is the monthly expected idiosyncratic skewness measure for each stock as in Boyer et al. (2010), provided by the authors from July 1969 to December 2023. Using total skewness gives essentially identical results. Expected (or ex-ante) skewness is difficult to measure. As opposed to means, variances and covariances, skewness is not stable over time. Moreover, lagged skewness alone does not adequately forecast skewness (Harvey and Siddique, 1999; Boyer et al., 2010). Instead, Boyer et al. (2010) (hereafter BMV) use firm-level variables to predict skewness (following the approach of Chen et al., 2001). Specifically, BMV develop measures of skewness each month that predict skewness of the return distribution over the next 60 months, based on firm characteristics in the prior 60 months, including lagged skewness, idiosyncratic volatility, momentum, turnover, size, exchange, and industry. BMV point out that although other variables, such as accounting variables, could be useful in predicting skewness, limiting variables to this collection allows the measure to be computed for every stock in CRSP with available history. As a result, using BMV's proxies for skewness maintains a large sample. Moreover, BMV demonstrate that their measures of skewness exhibit a negative cross-sectional relationship with expected returns—the skewness effect. This measure of skewness implicitly requires at least 250 days of daily returns in the prior 60-month period. In our sample, the 5th and 95th percentiles of expected idiosyncratic skewness are 0.03 and 1.67, respectively; it is typically positive. Note that SKEW is a measure of return skewness that corresponds to our modeled return skewness s in Section 2.

After merging and applying filters, our sample consists of 922,989 firm-months. There are 10,637 unique firms, for an average of 1,712 firms per month. Our data sample is comparable to those used in prior studies of the forecast dispersion effect, with the main innovation being the inclusion of a measure of skewness.

Finally, we obtain monthly "factor" returns for the Fama and French (2015) (FF5) model from WRDS (Wharton Research Data Service) and for the Daniel et al. (2020) (DMRS) model from Kent Daniel's website.[24] The DMRS data ends in March 2023. Therefore, our sample covers all months in the period from December 1983 through March 2023.

4.2 Double Sort

Our primary test of the model implications is via an independent double-sort. To eliminate the effects of firm size (and other characteristics correlated with size), we first sort observations each month into five quintiles based on prior month market capitalization. Within each quintile, we then perform an independent two-way sort on SKEW and DISP. Finally, we collapse the size dimension, yielding 25 portfolios.

Table 1: Average SKEW by Portfolio

	LowSkew	2	3	4	HighSkew
LowDisp	0.38	0.61	0.74	0.92	1.19
2	0.38	0.60	0.75	0.92	1.18
3	0.37	0.60	0.75	0.93	1.20
4	0.37	0.60	0.77	0.93	1.21
HighDisp	0.35	0.60	0.77	0.95	1.26

Note: This table presents the average idiosyncratic skewness deviation by portfolio. For each stock, each month, we obtain the predicted skewness of the Fama-French three-factor residuals from Boyer et al. (2010). Each month, we compute the median across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

First, we show that our independent double-sort conforms to desirable properties. Each month, we compute the median of SKEW and DISP across stocks in each portfolio. We then

[24]http://www.kentdaniel.net/data.php

Table 2: Average DISP by Portfolio

	LowSkew	2	3	4	HighSkew
LowDisp	0.02	0.02	0.02	0.02	0.02
2	0.04	0.04	0.03	0.04	0.04
3	0.06	0.06	0.06	0.06	0.06
4	0.09	0.09	0.10	0.10	0.10
HighDisp	0.23	0.24	0.25	0.25	0.26

Note: This table presents the average forecast dispersion by portfolio. For each stock, each month, we obtain the analyst forecast dispersion as described in Section 4.1. Each month, we compute the median across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

average these values across months. We report the resulting values for SKEW and DISP in Table 1 and Table 2, respectively. A "test" of a good independent sort is cross-sectional variation in each characteristic that is approximately orthogonal with the other: for a given quintile of SKEW, SKEW itself should be approximately constant across quintiles of DISP, which obtains in Table 1; and, similarly, for a given quintile of DISP, DISP itself should be approximately constant across quintiles of SKEW, which obtains in Table 2. Moreover, sorting produces large spreads in SKEW and DISP, which are approximately orthogonal to firm size. Furthermore, Table 1 indicates that the average level of SKEW is positive in all 25 portfolios, which supports our focus in Section 2 on results pertaining to the case of positive skewness, $s > 0$. Also, Table 2 indicates that the average level of DISP in the highest DISP quintile is well below one.

Next, we adapt predictions from Corollary 2 (and the more general analogous predictions from Corollary 5) to form the following hypotheses:

H1: When DISP$\approx$0, the effect of SKEW on expected returns should be negligible.

H2: The effect of SKEW on expected returns should be increasing (in magnitude) in DISP.

H3: When SKEW$\approx$0, the effect of DISP on expected returns should be negligible.

H4: The effect of DISP on expected returns should be increasing (in magnitude) in SKEW.

Table 3 gives annualized average excess returns by portfolio as well as high–low spreads (with t-statistics in parentheses computed using the method of Newey and West, 1987). The top row shows that ignoring DISP, our sample displays the unconditional SKEW effect: monotonically lower expected returns for higher quintiles of skewness. The first column likewise shows the unconditional DISP effect: monotonically lower expected returns for higher DISP quintiles. Turning to the predictions of the model, we first look at DISP spreads (last row) and obtain the predicted pattern: the DISP spread in returns is substantially larger for high SKEW assets (**H2**). Moreover, the spread in average returns is essentially zero for lowest quintile of SKEW stocks (**H1**). Looking at SKEW spreads (last column) we observe a similar pattern: the skewness spread is larger for high dispersion assets (**H4**). Moreover, the spread in average returns is essentially zero for the lowest quintile of DISP stocks (**H3**). At least optically, the model's predictions hold in the data.

Table 3: Average Annualized Return by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	7.7	7.4	6.8	6.0	4.8	-2.9
		(2.65)	(2.60)	(2.39)	(2.11)	(1.56)	(1.75)
LowDisp	8.2	7.7	9.1	8.2	7.9	8.2	0.5
	(3.39)	(3.14)	(3.62)	(3.23)	(3.31)	(2.96)	(0.34)
2	7.0	7.0	7.2	6.6	7.6	6.6	-0.4
	(2.70)	(2.61)	(2.63)	(2.41)	(2.86)	(2.35)	(0.25)
3	7.2	8.5	8.2	7.7	6.1	5.5	-3.0
	(2.56)	(2.83)	(2.81)	(2.69)	(2.14)	(1.76)	(1.68)
4	6.2	7.5	7.2	6.5	5.7	4.1	-3.4
	(2.03)	(2.31)	(2.29)	(2.14)	(1.85)	(1.28)	(1.69)
HighDisp	4.0	7.8	5.6	4.5	3.0	1.1	-6.8
	(1.15)	(2.09)	(1.62)	(1.25)	(0.84)	(0.28)	(2.64)
H-L	-4.1	0.1	-3.5	-3.7	-4.9	-7.1	-7.3
	(2.37)	(0.06)	(1.82)	(1.84)	(2.46)	(3.47)	(3.29)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

Finally, the bottom-right corner is calculated as the return on the long-short portfolio (S5D5+S1D1-S1D5-S5D1). Essentially, it is the average main-diagonal return minus the average off-diagonal return. Equivalently, it is the High-Low DISP spread for HighSkew (-7.1%) minus the High-Low DISP spread for LowSkew (0.1%). Therefore, it tests whether the dispersion effect depends on skewness. Furthermore, it is the High-Low SKEW spread for HighDisp (-6.8%) minus the High-Low SKEW spread for LowDisp (0.5%). Therefore, it tests whether the skewness effect spread depends on dispersion. Hence, it is simultaneously a test of **H2** and **H4**. As the model predicts, this return is negative, with economically and statistically significant magnitude.

Our theory is about alphas, not simply excess returns. Patterns of factor loadings could possibly overturn the conclusions drawn from Table 3. However, this brings up the age-old "joint-hypothesis problem" of which factors to include? Assuming no arbitrage, there exists some factor model that "explains" all patterns in expected returns. Besides the CAPM, most common factor models are loosely (if at all) microfounded. Fama and French (2015) argue that their five factor model (FF5) is almost tautological given the present value relation and clean surplus accounting. Still, it is commonly used in current research and does "absorb" substantial cross-sectional variation in average returns. Additionally, Daniel et al. (2020) argue that the FF5 model can be improved by their DMRS procedure to "remove unpriced risk" from characteristic-sorted factors "using covariance information estimated from past returns." Their procedure nearly doubles the Sharpe ratio of the resulting mean-variance efficient combination. Using time-series regressions, we compute alphas relative to these three factor models: CAPM, FF5, DMRS. These results are presented in Tables 4, 5, and 6, respectively. The results presented above based on excess returns are essentially unchanged. The monotone patterns and interaction effect all persist with similar economic and statistical significance.

In our main specification, we do not filter on market capitalization and equal weight

Table 4: Average Annualized CAPM α by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	-1.0	-1.7	-2.4	-3.3	-4.5	-3.5
		(0.51)	(1.07)	(1.54)	(2.49)	(3.24)	(2.01)
LowDisp	0.6	0.3	1.3	0.5	0.4	0.5	0.2
	(0.42)	(0.19)	(0.78)	(0.31)	(0.25)	(0.34)	(0.13)
2	-1.2	-0.7	-1.3	-1.9	-1.0	-1.6	-0.8
	(0.84)	(0.38)	(0.74)	(1.17)	(0.79)	(0.96)	(0.50)
3	-1.8	-0.1	-1.0	-1.5	-3.0	-3.6	-3.4
	(1.23)	(0.08)	(0.59)	(0.90)	(2.09)	(2.19)	(1.89)
4	-3.6	-1.9	-2.7	-3.2	-4.0	-5.6	-3.7
	(2.64)	(0.88)	(1.84)	(1.95)	(2.68)	(3.73)	(1.67)
HighDisp	-6.8	-2.6	-4.9	-6.1	-7.9	-9.7	-7.1
	(3.87)	(0.95)	(2.15)	(3.03)	(4.15)	(5.22)	(2.63)
H-L	-7.4	-2.9	-6.2	-6.6	-8.3	-10.2	-7.3
	(5.27)	(1.39)	(3.43)	(3.72)	(4.99)	(5.63)	(3.17)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

the portfolios. There are two potential concerns with this approach. First, the documented effects may be driven by very small stocks where market frictions like short-selling constraints are more likely to matter. Second, "idiosyncratic" skewness is hard to interpret for mega-cap stocks, especially when adusting returns using a linear factor model. Indeed, Kraus and Litzenberger (1976) derive an extension of the CAPM incorporating co-skewness. For these reasons, we now repeat the above exercise but drop the smallest and largest quintiles of stocks each month based on market capitalization. We also value-weight the resulting portfolios. The results are presented in Tables 14, 15, 16, and 17 in Appendix B. Comparing the interaction effect (bottom-right corner), with equal-weight results, the magnitudes are all slightly larger and t-statistics slightly smaller; the value-weighted estimates are all statistically significant at conventional levels.

Table 5: Average Annualized FF5 α by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	-2.9	-3.8	-4.3	-4.7	-5.4	-2.4
		(2.60)	(4.13)	(4.89)	(4.79)	(4.34)	(1.33)
LowDisp	-2.2	-2.4	-1.8	-2.5	-2.2	-1.5	0.9
	(2.64)	(2.15)	(1.61)	(2.23)	(1.96)	(1.18)	(0.54)
2	-3.9	-3.3	-4.1	-4.7	-3.2	-4.1	-0.8
	(4.51)	(2.72)	(3.66)	(4.03)	(2.95)	(3.26)	(0.48)
3	-4.0	-2.2	-3.4	-3.7	-5.1	-5.8	-3.6
	(4.76)	(1.67)	(3.37)	(3.44)	(4.00)	(4.28)	(1.82)
4	-4.4	-3.1	-3.7	-4.4	-4.6	-5.8	-2.7
	(5.02)	(2.08)	(3.43)	(3.78)	(3.68)	(3.85)	(1.14)
HighDisp	-6.7	-3.4	-5.6	-6.6	-7.9	-8.9	-5.5
	(6.28)	(1.68)	(3.73)	(5.00)	(6.09)	(5.10)	(2.05)
H-L	-4.5	-1.0	-3.8	-4.1	-5.7	-7.4	-6.4
	(3.88)	(0.48)	(2.21)	(2.56)	(3.99)	(4.72)	(2.87)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

4.3 Fama-Macbeth Regressions

The above two-way portfolio sorts are essentially a non-parametric regression of returns on the Cartesian product of SKEW (S) and DISP (D) quintiles, $(1:5)\times(1:5)$. Such a method is quite flexible in modeling the surface $\mathrm{E}[\tilde{r}] = f(S,\ D)$, but this flexibility may come at the cost of reduced power. We now test the model predictions using a semi-parametric Fama-Macbeth procedure. The bottom line is, interpretations are unchanged.

Before outlining the method, we first note that average returns for univariate sorts on SKEW and DISP are approximately linear in quintile number, as shown in Figure 4. Therefore, we treat quintile numbers as essentially cardinal in what follows, though strictly speaking they are ordinal. For ease of numerical interpretation, we remap quintile numbers $(1:5)$ into the unit interval, $(0, 0.25, 0.50, 0.75, 1)$. Hence, $S = 1$ indicates a stock is in the highest quintile of SKEW that month and similarly $D = 1$ indicates a stock is in the

Table 6: Average Annualized DMRS α by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	2.0	1.5	0.9	-0.0	-0.9	-3.0
		(0.58)	(0.49)	(0.29)	(0.01)	(0.30)	(1.60)
LowDisp	1.5	0.8	2.4	1.5	1.5	1.9	1.1
	(0.60)	(0.27)	(0.93)	(0.62)	(0.63)	(0.76)	(0.65)
2	0.8	0.6	0.9	0.5	1.5	0.7	0.0
	(0.31)	(0.22)	(0.31)	(0.20)	(0.55)	(0.25)	(0.03)
3	1.0	3.0	2.1	1.4	-0.6	-0.8	-3.8
	(0.33)	(0.81)	(0.62)	(0.46)	(0.20)	(0.26)	(1.78)
4	0.8	2.4	1.8	0.9	0.3	-1.2	-3.7
	(0.24)	(0.59)	(0.51)	(0.27)	(0.07)	(0.38)	(1.47)
HighDisp	-0.6	3.3	1.2	-0.2	-2.1	-4.2	-7.5
	(0.16)	(0.71)	(0.31)	(0.04)	(0.50)	(1.07)	(2.67)
H-L	-2.1	2.5	-1.2	-1.7	-3.6	-6.1	-8.6
	(1.11)	(1.03)	(0.58)	(0.76)	(1.62)	(2.70)	(3.67)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

highest quintile of DISP that month.

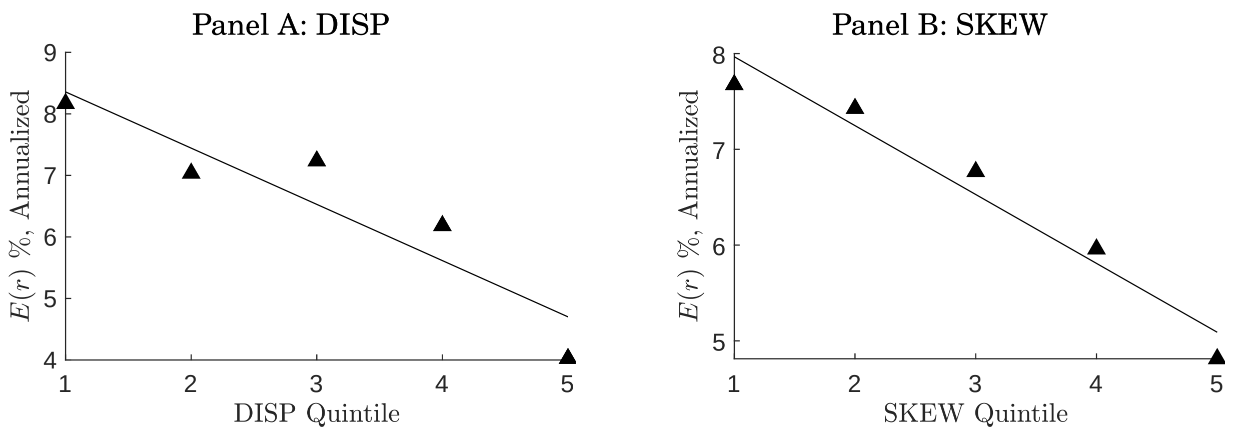
In our baseline specification, we estimate a cross-sectional Fama-Macbeth regression each month of realized excess returns on S, D, and their interaction, $S \times D$, which we abbreviate by SD:

$$r_{i,t} = \gamma_{0,t} + \gamma_{S,t} S_{i,t} + \gamma_{D,t} D_{i,t} + \gamma_{SD,t} SD_{i,t} + \epsilon_{i,t}. \tag{22}$$

The resulting γ coefficients are realized returns on managed portfolios. Under the null in which expected returns are decreasing in skewness and dispersion, but there is no interaction effect, we should obtain $E\left(\gamma_{S,t}\right) < 0$, $E\left(\gamma_{D,t}\right) < 0$, and $E\left(\gamma_{SD,t}\right) = 0$. Under our model alternative, $E\left(\gamma_{S,t}\right) = 0$, $E\left(\gamma_{D,t}\right) = 0$, and $E\left(\gamma_{SD,t}\right) < 0$. Since the estimated γs are portfolio returns, we can also compute αs relative to factor models. This procedure allows us to "control" for factor exposures without estimating noisy factor loadings for individual stocks.

The results are presented in the first three columns of Table 7. The first row displays

Figure 4: Quintiles vs Average Returns



Note: This figure displays average returns vs quintile number for univariate sorts based in on DISP and SKEW in Panels (A) and (B), respectively.

results for average returns, while the remaining rows control for factor exposures using a time-series regression of estimated $\gamma_{\cdot,t}$ on the factor returns. The null hypothesis is soundly rejected. Estimates of $E\left(\gamma_{S,t}\right)$ and $E\left(\gamma_{D,t}\right)$ are economically and statistically non-different from zero, whereas $E\left(\gamma_{SD,t}\right)$ is significantly negative under all specifications. Interestingly, the estimated $E\left(\gamma_{SD,t}\right)$ coefficients are quantitatively similar (both in magnitude and statistical significance) to the corresponding bottom-right estimates in Tables 3, 4, 5, and 6. The last column displays estimates of $E\left(\gamma_{SD,t}\right)$ from the restricted model, $r_{i,t} = \gamma_{0,t} + \gamma_{SD,t} SD_{i,t}$. Estimates are similar to the baseline model, further evidence that SKEW and DISP only impact returns through their interaction.

One may be concerned that our baseline specification generates a spurious interaction effect by not allowing for non-linearity in the SKEW and DISP effects. Therefore we also estimate the expanded model,

$$r_{i,t} = \gamma_{0,t} + \gamma_{S,t} S_{i,t} + \gamma_{D,t} D_{i,t} + \gamma_{S2,t} S_{i,t}^2 + \gamma_{D2,t} D_{i,t}^2 + \gamma_{SD,t} SD_{i,t} + \epsilon_{i,t}. \tag{23}$$

The estimates are presented in Table 8. The values for γ_{SD} are essentially unchanged compared to the baseline linear design (see Table 7). The last column gives p-values from a Wald test of the model-implied hypothesis, $\gamma_S = \gamma_D = \gamma_{S2} = \gamma_{D2} = 0$. Even accounting for non-

Table 7: Fama-Macbeth Estimates

	$[S$	D	$S \times D]$	$[S \times D]$
Excess	0.7	0.1	-6.9	-6.2
	(0.55)	(0.07)	(3.23)	(3.24)
CAPM	0.5	-2.8	-7.0	-8.8
	(0.38)	(1.61)	(3.14)	(4.90)
FF5	0.9	-0.6	-6.1	-5.9
	(0.60)	(0.36)	(2.68)	(3.45)
DMRS	1.3	2.8	-8.4	-5.2
	(0.86)	(1.24)	(3.59)	(2.68)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and regressions are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

linearities, there is essentially *no* evidence for separate SKEW or DISP effects on average returns after controlling for the interaction. For robustness, we present value-weighted regressions in Tables 18 and 19 in Appendix B.

Table 8: Fama-Macbeth Estimates

	$[S$	D	S^2	D^2	$S \times D]$	Wald p
Excess	2.1	3.1	-1.5	-3.0	-6.7	54.5
	(0.79)	(1.51)	(0.64)	(1.53)	(3.14)	
CAPM	0.5	0.5	0.0	-3.4	-6.9	100.0
	(0.17)	(0.27)	(0.02)	(1.72)	(3.12)	
FF5	0.5	0.5	0.5	-1.0	-6.4	100.0
	(0.16)	(0.22)	(0.18)	(0.50)	(2.80)	
DMRS	2.0	4.3	-0.7	-1.7	-8.2	100.0
	(0.63)	(1.90)	(0.28)	(0.79)	(3.54)	

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and regressions are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

4.4 Volatility

Thus far in our empirical analyses, we have ignored return volatility (σ_r), essentially treating it as cross-sectionally independent of SKEW and DISP. However, volatility varies with DISP. As a simple measure of volatility, we use the at-the-money 91-day implied volatility from the Options Metrics implied volatility files. This, of course, restricts our sample to stocks with traded options and pushes up the start date of the sample to 1996. As shown in Table 9, volatility is increasing in DISP. Indeed, high DISP stocks have 30-40% higher implied volatility compared to low DISP stocks. Note that volatility exhibits little to no correlation with SKEW in our sample.

Table 9: Average Implied Volatility by Portfolio

	LowSkew	2	3	4	HighSkew
LowDisp	10.0	10.1	10.0	10.1	10.4
2	10.4	10.5	10.7	10.8	11.2
3	11.1	11.3	11.5	11.6	12.1
4	12.0	12.0	12.4	12.5	13.2
HighDisp	13.4	13.3	13.8	14.2	15.1

Note: This table presents the average option implied volatility by portfolio. Each month, we compute the mean across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

Recall that our model gives an explicit function for the impact of cross-sectional variation in skewness and disagreement on Sharpe ratios (expected returns scaled by volatility), $\text{Sharpe}[\tilde{r}] = -\overline{u}s\delta^2$, in terms of $\overline{u}$, a positive constant reflecting the shape of the utility function; s, the ex ante skewness corresponding to our empirical proxy SKEW; and δ, a unitless measure of relative disagreement corresponding to our empirical proxy DISP—see equation (12). Therefore, we should not expect variation in Sharpe ratios with respect to SKEW and DISP to materially differ for stocks with low volatility versus stocks with high volatility. We now take this prediction seriously.

Before turning our attention to comparisons between low versus high volatility, we report the cross-sectional evidence for the variation in Sharpe ratios (expected returns scaled by volatility) across different levels of SKEW and DISP for all stocks. We draw similar conclusions as we did with the evidence for expected returns. Table 10 is a repeat of Table 3 except that instead of simple excess returns, we use scaled returns

$$\widetilde{r}_{i,t} = \frac{\frac{0.4}{\sqrt{12}} r_{i,t}}{\sigma_{i,t}}, \tag{24}$$

where $\sigma_{i,t}$ is the option implied volatility. The $\frac{0.4}{\sqrt{12}}$ scaling (0.4 annualized) is arbitrary, but set near the median implied volatility in the sample. The unconditional DISP and SKEW effects as well as the interaction effect are present when predicting Sharpe ratios, consistent with hypotheses **H2** and **H4**. Further, looking at the LowSkew column and LowDisp row, we find evidence consistent with hypotheses **H1** and **H3**. The DISP effect is essentially zero for low-SKEW stocks and similarly, the SKEW effect is essentially zero for low-DISP stocks.

Now we turn to our model's prediction regarding the consistency of the relationship between Sharpe ratios and SKEW×DISP (the interaction effect) for different levels of volatility. We sort stocks each month into two groups, LowVol and HighVol. Using the same breakpoints for DISP and SKEW quintiles, we separately sort each group into 25 portfolios (as earlier) and test the interaction effect. Tables 11 and 12 give the average implied volatility by portfolio for these two sorts. First note that, compared to Table 9, the cross-sectional variation in implied volatility is dramatically reduced. However, Tables 20 and 21 in Appendix B show that the average DISP by portfolio is essentially unchanged compared to the full-sample results shown in Table 2. Tables 22 and 23 (also in Appendix B) show the same holds true for SKEW. Hence, the subsamples of LowVol and HighVol stocks are comparable in terms of DISP and SKEW, but vary substantially in terms of implied volatility.

To save space, in Table 13 we present the interaction effect for All stocks and separately for the LowVol and HighVol subsamples. For example, the first entry of -7.7 corresponds to the interaction effect for all stocks shown in the bottom-right corner of Table 9. As in the

Table 10: Average Annualized Scaled Return by Portfolio (All)

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	11.5	9.9	9.5	7.8	6.6	-4.9
		(3.10)	(2.66)	(2.67)	(2.24)	(1.77)	(2.52)
LowDisp	11.8	11.5	12.1	12.4	11.8	10.3	-1.3
	(3.36)	(3.06)	(3.27)	(3.38)	(3.52)	(2.57)	(0.58)
2	9.9	10.2	9.4	10.5	9.6	9.6	-0.6
	(2.73)	(2.69)	(2.40)	(2.84)	(2.75)	(2.53)	(0.28)
3	9.5	11.8	9.8	9.1	9.0	7.7	-4.1
	(2.62)	(3.07)	(2.54)	(2.48)	(2.54)	(1.95)	(1.76)
4	8.1	11.5	9.2	8.0	6.6	5.8	-5.7
	(2.20)	(2.92)	(2.36)	(2.17)	(1.72)	(1.54)	(2.39)
HighDisp	6.4	12.0	7.9	7.6	3.7	3.0	-9.0
	(1.75)	(3.02)	(2.07)	(2.02)	(0.97)	(0.78)	(3.28)
H-L	-5.4	0.5	-4.2	-4.8	-8.1	-7.3	-7.7
	(3.04)	(0.19)	(1.70)	(2.08)	(3.76)	(3.17)	(2.68)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

full sample, we also adjust for the CAPM, FF5, and DMRS factor models. The remarkable takeaway is that for scaled returns (Sharpe ratios), the interaction effect is essentially the same across subsamples and is further unaffected by controlling for factor exposures.

Table 11: Average Implied Volatility by Portfolio (LowVol)

	LowSkew	2	3	4	HighSkew
LowDisp	8.7	8.8	8.7	8.6	8.3
2	8.9	9.1	9.1	9.0	8.8
3	9.1	9.4	9.4	9.3	9.2
4	9.5	9.7	9.8	9.7	9.5
HighDisp	9.8	10.2	10.2	10.2	10.1

Note: This table presents the average option implied volatility by portfolio. Each month, we compute the mean across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

Table 12: Average Implied Volatility by Portfolio (HighVol)

	LowSkew	2	3	4	HighSkew
LowDisp	13.6	13.3	13.3	13.2	13.7
2	13.5	13.3	13.4	13.5	13.7
3	13.7	13.6	13.8	13.8	14.3
4	14.2	14.0	14.3	14.4	15.0
HighDisp	15.1	14.8	15.3	15.8	16.3

Note: This table presents the average option implied volatility by portfolio. Each month, we compute the mean across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

Table 13: Interaction Effect on Scaled Returns

	Excess	CAPM	FF5	DMRS
All	-7.7	-7.5	-7.4	-8.1
	(-2.68)	(-2.49)	(-2.47)	(-2.74)
LowVol	-8.4	-7.1	-8.0	-7.2
	(-2.12)	(-1.73)	(-2.01)	(-1.95)
HighVol	-7.7	-7.6	-7.0	-7.8
	(-2.21)	(-2.16)	(-1.97)	(-2.15)

Note: This table presents the average monthly "interaction effect" for All stocks and separately for Low and High implied volatility subsamples. Returns are annualized and portfolios are equal-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

5 Conclusion

In this paper, we present a simple frictionless microfoundation for the conflicting empirical findings of (i) the negative relationship between dispersion in financial analysts' forecasts and expected equity returns (Diether et al., 2002), (ii) the positive relationship between disagreement and mortgage-backed security (MBS) returns Carlin et al. (2014), as well as (iii) the seemingly unrelated negative relationship between skewness and expected returns (Boyer et al., 2010). Our model is parsimonious yet flexible, and is amenable to intuitive graphical interpretations of its mechanisms. We also demonstrate the robustness of our theory through several extensions.

Moreover, we demonstrate, both theoretically and empirically, a novel interaction effect between disagreement and skewness on expected returns. On a sample of U.S. stocks, our empirical tests reveal that a portfolio which is skew- and disagreement-neutral but exploits the interaction effect has an economically significant average annualized CAPM alpha of 7.3% with a statistically significant t-statistic greater than 3. Fama and French (2015) and Daniel et al. (2020) alphas exhibit comparable metrics. The theoretical interaction effect carries over to Sharpe ratios, with the additional prediction of invariance to the level of volatility, for which we provide empirical support.

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Appendix for

"Disagreement, Skewness, and Asset Prices"

by Goulding, Santosh, and Zhang

Appendix A Proofs

Proof of Lemma 1. This is Lemma 1 of Goulding et al. (2023) generalized to arbitrary subjective beliefs. Recall, an investor's FOC is given by

$$E[u'(w_0 + x_i(p)(\tilde{\theta} - p))(\tilde{\theta} - p)] = 0. \tag{A1}$$

Our goal is to derive $x_i(p)$ when p is close to μ_i. By Taylor expansion,

$$x_i(p) = x_i(\mu_i) + x_i'(\mu_i)(p - \mu_i) + \frac{x_i''(\mu_i)}{2}(p - \mu_i)^2 + o((p - \mu_i)^2).$$

In the following, we solve all the coefficients, $x(\mu_i)$, $x_i'(\mu_i)$, and $x_i''(\mu_i)$ explicitly.

First, from the FOC (A1), $x_i(\mu_i) = 0$ clearly solves the FOC. Together with the fact that the utility function is concave, which implies the solution to (A1) is unique, we conclude that $x_i(\mu_i) = 0$.

Second, taking the derivative with respect to p on both sides of (A1) gives the following equation:

$$0 = E\left[[u''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i'(p)(\tilde{\theta} - p) - x_i(p)](\tilde{\theta} - p)\right] \tag{A2}$$

$$- E[u'(w_0 + x_i(p)(\tilde{\theta} - p))]. \tag{A3}$$

Plugging in $p = \mu_i$ and using the fact that $x_i(\mu_i) = 0$, we derive

$$x_i'(\mu_i) = \frac{u'(w_0)}{u''(w_0)\sigma^2}. \tag{A4}$$

Third, taking the derivative with respect to p on both sides of (A2) gives to the following equation:

$$\begin{aligned}
0 =& E[u'''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i'(p)(\tilde{\theta} - p) - x_i(p)]^2(\tilde{\theta} - p) \\
&+ E[u''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i''(p)(\tilde{\theta} - p) - 2x_i'(p)](\tilde{\theta} - p) \\
&- E[u''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i'(p)(\tilde{\theta} - p) - x_i(p)] \\
&- E[u''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i'(p)(\tilde{\theta} - p) - x_i(p)].
\end{aligned} \tag{A5}$$

Plugging in $p = \mu_i$, we derive

$$x_i''(\mu_i) = -\frac{u'''(w_0)E(\tilde{\theta} - \mu_i)^3 x_i'(\mu_i)^2}{u''(w_0)\sigma^2} = -\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}, \tag{A6}$$

where the last equality uses Equation (A4).

So we obtain the Taylor series of $x(p)$ around $p = \mu_i$:

$$x_i(p) = \frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p - \mu_i) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}(p - \mu_i)^2 + o(1)(p - \mu_i)^2, \tag{A7}$$

where the little-o notation $o(1)$ is an unknown function that converges to zero when $p \to \mu$.

Proofs of Proposition 1 and Proposition 4. Since Proposition 1 is a special case of Proposition 4 when the random variable $\widetilde{\delta}$ is binary, we proceed to prove the more general case, i.e., Proposition 4.

Since $\widetilde{\delta}$ is discrete, we write the possible realizations as δ_i for $i = 1, \cdots, n$ with the probability of δ_i being q_i. With some abuse of notations, we write $x_{\delta_i}(p)$ as the demand schedule for the investor with δ_i. Following the same argument in Lemma 1, we have the Taylor expansion for $x_{\delta_i}(p)$ as given by the following equation.

$$x_{\delta_i}(p) = \frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu_{\delta_i}) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}(p-\mu_{\delta_i})^2 + \epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2, \tag{A8}$$

where $\epsilon_{\delta_i}(p)$ is a function that converges to 0 as $p \to \mu_{\delta_i}$.

The market's clearing condition states that

$$\sum_{i=1}^{n} q_i x_{\delta_i}(p) = 0. \tag{A9}$$

Substituting $x_{d_i}(p)$ from equation (A8), it leads to the following equation.

$$\begin{aligned}\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}\sum_{i=1}^{n} q_i(p-\mu_{\delta_i}) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}\sum_{i=1}^{n} q_i(p-\mu_{\delta_i})^2 \\ + \sum_{i=1}^{n} q_i\epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2 = 0.\end{aligned} \tag{A10}$$

Notice that $\sum_{i=1}^{n} q_i(p-\mu_{\delta_i}) = \sum_{i=1}^{n} q_i(p-\mu-\delta_i\sigma) = p-\mu$, where the last equality uses the fact that $\widetilde{\delta}$ is a mean-zero random variable, and $\sum_{i=1}^{n} q_i(p-\mu_{\delta_i})^2 = \sum_{i=1}^{n} q_i(p-\mu-\delta_i\sigma)^2 = \sum_{i=1}^{n} q_i[(p-\mu)^2 - 2(p-\mu)\delta_i\sigma + \delta_i^2\sigma^2] = (p-\mu)^2 + Var(\widetilde{\delta})\sigma^2$. We write $v^2 := Var(\widetilde{\delta})$. It follows that (A10) is equivalent to the following equation.

$$\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}\left((p-\mu)^2 + v^2\sigma^2\right) + \sum_{i=1}^{n} q_i\epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2 = 0. \tag{A11}$$

We vary v to study the behavior of the equilibrium price when v is small. To emphasize that the price changes when v changes, we write the price as a function of v: $p(v)$. By Taylor expansion, it follows that $p(v) = p(0) + p'(0)v + p''(0)/2v^2 + o(1)v^2$ for small v.

First, $p(0) = \mu$. This can be seen by setting $v = 0$ in (A11). Second, taking derivative with respect to v on both sides in (A11) gives to

$$\frac{\partial(p-\mu)}{\partial v} = \frac{1}{2}\frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2}\frac{s}{\sigma}\left[2(p-\mu)\frac{\partial(p-\mu)}{\partial v} + 2v\sigma^2\right] - \frac{\partial\mathscr{E}(v)}{\partial v}, \tag{A12}$$

where

$$\mathscr{E}(v) = \frac{u''(w_0)\sigma^2}{u'(w_0)} \sum_{i=1}^{n} q_i \epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2. \tag{A13}$$

We claim that $\frac{\partial \mathscr{E}(v)}{\partial v}|_{v=0} = 0$. It is sufficient to show that $\frac{\partial \mathscr{E}_{\delta_i}(v)}{\partial v}|_{v=0} = 0$, where $\mathscr{E}_{\delta_i}(v) = \epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2$. Note that $\frac{\partial \mathscr{E}_{\delta_i}(v)}{\partial v} = \epsilon'_{\delta_i}(p)p'(v)(p-\mu-v\delta_i)^2 + \epsilon_{\delta_i}(p)2(p-\mu-v\delta_i)(p'(v)-\delta_i)$, which goes to zero as $v \to 0$.

Evaluating equation (A12) at $v = 0$ implies that $\frac{\partial(p-\mu)}{\partial v}|_{v=0} = 0$, i.e., $p'(0) = 0$. Taking derivative with respect to v on both sides in (A12) gives to

$$\frac{\partial^2(p-\mu)}{\partial v^2} = \frac{1}{2}\frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2}\frac{s}{\sigma}\left[2\left(\frac{\partial(p-\mu)}{\partial v}\right)^2 + 2(p-\mu)\frac{\partial^2(p-\mu)}{\partial v^2} + 2\sigma^2\right] - \frac{\partial^2 \mathscr{E}(v)}{\partial v^2}. \tag{A14}$$

We claim that $\frac{\partial^2 \mathscr{E}(v)}{\partial^2 v}|_{v=0} = 0$. It is sufficient to show that $\frac{\partial^2 \mathscr{E}_{\delta_i}(v)}{\partial v^2}|_{v=0} = 0$. Note that $\frac{\partial^2 \mathscr{E}_{\delta_i}(v)}{\partial v^2} = \epsilon''_{\delta_i}(p)p'(v)^2(p-\mu-v\delta_i)^2 + \epsilon'_{\delta_i}(p)p''(v)(p-\mu-v\delta_i)^2 + \epsilon'_{\delta_i}(p)p'(v)2(p-\mu-v\delta_i)(p'(v)-\delta_i^*) + \epsilon'_{\delta_i}(p)p'(v)2(p-\mu-v\delta_i)(p'(v)-\delta_i) + \epsilon_{\delta_i}(p)2(p'(v)-\delta_i)(p'(v)-\delta_i) + \epsilon_{\delta_i}(p)2(p-\mu-v\delta_i)p''(v)$, which goes to zero as $v \to 0$ because every single term goes to zero as $v \to 0$. Evaluating (A14) at $v = 0$ implies that

$$\left.\frac{\partial^2(p-\mu)}{\partial v^2}\right|_{v=0} = \frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2} s\sigma. \tag{A15}$$

Using Taylor expansion, we have

$$p-\mu = \frac{1}{2}\left.\frac{\partial^2(p-\mu)}{\partial v^2}\right|_{v=0} v^2 + o(1)v^2 = \frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} s\sigma Var(\tilde{\delta}) + o(1)Var(\tilde{\delta}), \tag{A16}$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $v \to 0$.

Proof of Proposition 3.

It is straightforward to see that $\frac{\partial \mathrm{E}[\tilde{r}]}{\partial s} = -\overline{u}\sigma\delta^2 < 0$, $\frac{\partial \mathrm{E}[\tilde{r}]}{\partial \delta} = -2\overline{u}s\sigma\delta < 0 \Leftrightarrow s > 0$, and $\frac{\partial^2 \mathrm{E}[\tilde{r}]}{\partial s \partial \delta} = -2\overline{u}\sigma\delta < 0$. The analogous results for the Sharpe ratio hold because the Sharpe ratio is the expected return divided by a positive constant, σ_r.

Proof of Proposition 6. Given the two types of investors and Lemma 1, we can write the market's clearing condition as the following equation.

$$\begin{aligned}
&\frac{u_1'(w_0)}{u_1''(w_0)}\frac{1}{\sigma^2}(p-\mu_+)-\frac{1}{2}\frac{u_1'''(w_0)}{u_1''(w_0)}\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2\frac{s}{\sigma^3}(p-\mu_+)^2+o(1)(p-\mu_+)^2\\
&+\frac{u_2'(w_0)}{u_2''(w_0)}\frac{1}{\sigma^2}(p-\mu_-)-\frac{1}{2}\frac{u_2'''(w_0)}{u_2''(w_0)}\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2\frac{s}{\sigma^3}(p-\mu_-)^2+o(1)(p-\mu_-)^2=0. \qquad \text{(A17)}
\end{aligned}$$

With loss, we assume $s>0$. (When $s<0$, all analysis carries over). In order to show that the equilibrium price is greater than the benchmark price p_0, we need to show that at price p_0, there is excess demand. In other words, we need to show that the left-hand-side of the above market's clearing condition is strictly positive when $p=p_0$.

Note that because

$$p_0=\frac{\frac{u_1'(w_0)}{-u_1''(w_0)}\mu_+ + \frac{u_2'(w_0)}{-u_2''(w_0)}\mu_-}{\frac{u_1'(w_0)}{-u_1''(w_0)}+\frac{u_2'(w_0)}{-u_2''(w_0)}}, \qquad \text{(A18)}$$

by algebraic manipulation, it is straightforward to see that

$$\frac{u_1'(w_0)}{u_1''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_+)+\frac{u_2'(w_0)}{u_2''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_-)=0. \qquad \text{(A19)}$$

In order to show there is excess demand p_0, it is sufficient to show that

$$-\frac{1}{2}\frac{u_1'''(w_0)}{u_1''(w_0)}\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_+)^2-\frac{1}{2}\frac{u_2'''(w_0)}{u_2''(w_0)}\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_-)^2>0. \qquad \text{(A20)}$$

Plugging into p_0, note that

$$\begin{aligned}
&-\frac{1}{2}\frac{u_1'''(w_0)}{u_1''(w_0)}\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_+)^2-\frac{1}{2}\frac{u_2'''(w_0)}{u_2''(w_0)}\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_-)^2\\
&=\frac{1}{2}\frac{(\mu_+-\mu_-)^2}{\left(\frac{u_1'(w_0)}{-u_1''(w_0)}+\frac{u_2'(w_0)}{-u_2''(w_0)}\right)^2}\left[\frac{u_1'''(w_0)}{-u_1''(w_0)}+\frac{u_2'''(w_0)}{-u_2''(w_0)}\right]\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2\frac{s}{\sigma^3}, \qquad \text{(A21)}
\end{aligned}$$

which is positive as long as $s>0$. This completes the proof.

Proof of Proposition 7. we can write the market's clearing condition as the following equation.

$$
\begin{aligned}
&\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma_A^2}(p-\mu_A)-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_A}{\sigma_A^3}(p-\mu_A)^2+o(1)(p-\mu_A)^2\\
&+\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma_B^2}(p-\mu_B)-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_B}{\sigma_B^3}(p-\mu_B)^2+o(1)(p-\mu_B)^2=0.
\end{aligned}
\tag{A22}
$$

Similar to the proof of Proposition 6, we need to show that there is excess demand at price p_0 if and only if $\sigma_A s_A+\sigma_B s_B>0$.

Note that when

$$
p_0=\frac{\frac{\mu_A}{\sigma_A^2}+\frac{\mu_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2}+\frac{1}{\sigma_B^2}},
\tag{A23}
$$

it follows that

$$
\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_A)+\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_B)=0,
\tag{A24}
$$

and

$$
\begin{aligned}
&-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_A}{\sigma_A^3}(p_0-\mu_A)^2-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_B}{\sigma_B^3}(p_0-\mu_B)^2\\
&=\frac{1}{2}\frac{u'''(w_0)}{-u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{(\mu_A-\mu_B)^2}{\left(\frac{1}{\sigma_A^2}+\frac{1}{\sigma_B^2}\right)^2}\frac{\sigma_A s_A+\sigma_B s_B}{\sigma_A^4\sigma_B^4}
\end{aligned}
\tag{A25}
$$

which is positive if and only if $\sigma_A s_A+\sigma_B s_B>0$.

Proof of Proposition 8. The proof follows from the proof of Proposition 3 in Goulding et al. (2023), in which they show that the demand function $x(p)$ is convex for a sufficiently large skewness. Similar to that proof, the demand function is concave for a sufficiently small (potentially negative) skewness.

Appendix B Supplementary Tables

Table 14: Average Annualized Return by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	7.4	7.4	7.1	5.3	4.6	-2.7
		(2.58)	(2.63)	(2.56)	(1.93)	(1.49)	(1.45)
LowDisp	7.3	6.4	8.0	8.1	7.7	6.4	0.0
	(3.07)	(2.67)	(3.03)	(3.18)	(3.24)	(2.15)	(0.02)
2	6.2	6.3	7.3	5.2	6.1	5.8	-0.5
	(2.45)	(2.44)	(2.61)	(1.94)	(2.29)	(1.95)	(0.26)
3	6.8	8.3	6.4	8.3	5.6	4.8	-3.5
	(2.43)	(2.69)	(2.18)	(2.77)	(1.97)	(1.51)	(1.70)
4	6.4	7.6	7.0	8.0	4.7	5.9	-1.6
	(2.16)	(2.27)	(2.33)	(2.61)	(1.53)	(1.80)	(0.66)
HighDisp	5.0	9.7	9.1	4.9	2.9	0.7	-9.0
	(1.47)	(2.43)	(2.56)	(1.40)	(0.81)	(0.19)	(2.65)
H-L	-2.3	3.3	1.1	-3.2	-4.8	-5.7	-9.0
	(1.28)	(1.19)	(0.47)	(1.41)	(2.28)	(2.35)	(2.76)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are value-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

Table 15: Average Annualized CAPM α by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	-1.2	-1.9	-2.1	-4.0	-4.7	-3.5
		(0.63)	(1.25)	(1.44)	(3.14)	(3.33)	(1.79)
LowDisp	-0.3	-0.7	-0.0	0.2	0.1	-1.3	-0.6
	(0.20)	(0.39)	(0.02)	(0.14)	(0.10)	(0.69)	(0.33)
2	-2.1	-1.3	-1.3	-3.3	-2.6	-2.6	-1.3
	(1.46)	(0.72)	(0.69)	(1.94)	(1.86)	(1.36)	(0.63)
3	-2.3	-0.2	-3.0	-1.1	-3.6	-4.3	-4.0
	(1.70)	(0.12)	(1.91)	(0.59)	(2.26)	(2.38)	(1.85)
4	-3.4	-1.8	-2.8	-1.9	-5.0	-3.8	-2.1
	(2.51)	(0.75)	(1.86)	(1.13)	(3.54)	(2.22)	(0.79)
HighDisp	-5.9	-1.0	-1.6	-5.8	-8.2	-10.1	-9.1
	(3.39)	(0.32)	(0.65)	(2.75)	(3.92)	(5.07)	(2.55)
H-L	-5.7	-0.4	-1.6	-6.0	-8.3	-8.8	-8.4
	(3.76)	(0.13)	(0.69)	(2.79)	(4.36)	(3.82)	(2.44)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are value-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

Table 16: Average Annualized FF5 α by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	-2.7	-3.6	-4.1	-5.2	-5.8	-3.1
		(2.05)	(3.62)	(4.68)	(4.90)	(4.41)	(1.51)
LowDisp	-2.8	-3.0	-2.7	-2.4	-2.1	-3.4	-0.4
	(3.12)	(2.11)	(2.13)	(1.83)	(1.65)	(1.97)	(0.19)
2	-4.6	-3.5	-3.7	-6.5	-4.7	-5.2	-1.6
	(5.03)	(2.68)	(2.58)	(4.68)	(3.71)	(3.29)	(0.79)
3	-4.3	-1.7	-4.9	-3.5	-5.6	-6.5	-4.8
	(5.25)	(1.14)	(4.57)	(2.77)	(3.93)	(4.36)	(2.30)
4	-4.0	-2.3	-3.7	-3.2	-5.5	-4.3	-2.0
	(4.51)	(1.18)	(3.03)	(2.57)	(4.23)	(2.48)	(0.72)
HighDisp	-5.6	-1.4	-1.5	-5.8	-7.9	-9.8	-8.3
	(4.86)	(0.58)	(0.72)	(3.50)	(4.52)	(4.50)	(2.26)
H-L	-2.9	1.5	1.2	-3.4	-5.8	-6.4	-7.9
	(2.08)	(0.58)	(0.48)	(1.60)	(3.22)	(2.88)	(2.34)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are value-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

Table 17: Average Annualized DMRS α by Portfolio

	All	LowSkew	2	3	4	HighSkew	H-L
All	-	2.5	1.7	1.3	-0.8	-0.8	-3.3
		(0.69)	(0.53)	(0.42)	(0.25)	(0.25)	(1.55)
LowDisp	1.0	0.1	1.8	1.7	1.6	0.9	0.8
	(0.41)	(0.05)	(0.64)	(0.68)	(0.66)	(0.35)	(0.39)
2	0.2	0.9	1.3	-1.4	-0.0	0.0	-0.8
	(0.09)	(0.31)	(0.44)	(0.51)	(0.01)	(0.01)	(0.42)
3	0.7	3.5	0.4	1.8	-1.5	-1.2	-4.7
	(0.22)	(0.93)	(0.12)	(0.54)	(0.49)	(0.36)	(2.01)
4	1.5	3.3	1.8	2.9	-0.4	0.9	-2.4
	(0.43)	(0.76)	(0.53)	(0.80)	(0.12)	(0.25)	(0.83)
HighDisp	0.6	5.6	5.2	0.4	-2.6	-4.2	-9.8
	(0.15)	(1.06)	(1.22)	(0.09)	(0.63)	(1.08)	(2.46)
H-L	-0.4	5.5	3.4	-1.3	-4.2	-5.1	-10.6
	(0.20)	(1.65)	(1.35)	(0.56)	(1.78)	(1.99)	(3.05)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and portfolios are value-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

Table 18: Average Annualized Return (or α) by Portfolio

	$[S$	D	$S \times D]$	$[S \times D]$
Excess	1.1	3.1	-8.6	-5.1
	(0.73)	(1.33)	(3.05)	(2.53)
CAPM	0.7	-0.1	-8.4	-7.9
	(0.42)	(0.05)	(2.77)	(3.90)
FF5	0.6	2.2	-8.0	-5.7
	(0.37)	(0.98)	(2.43)	(2.75)
DMRS	1.4	5.7	-10.4	-4.6
	(0.86)	(2.07)	(3.21)	(2.14)

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and regressions are value-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

Table 19: Average Annualized Return (or α) by Portfolio

	$[S$	D	S^2	D^2	$S \times D]$	Wald p
Excess	3.2	3.9	-1.9	-0.7	-8.9	36.0
	(1.00)	(1.44)	(0.68)	(0.30)	(3.14)	
CAPM	0.8	1.4	0.0	-1.4	-8.8	100.0
	(0.25)	(0.51)	(0.02)	(0.59)	(2.90)	
FF5	0.4	0.7	0.5	1.7	-8.6	100.0
	(0.11)	(0.26)	(0.14)	(0.73)	(2.67)	
DMRS	0.5	4.4	1.0	1.2	-10.6	100.0
	(0.13)	(1.51)	(0.32)	(0.46)	(3.29)	

Note: This table presents the average monthly percent excess returns by portfolio. Returns are annualized and regressions are value-weighted. Heteroskedasticity and autocorrelation consistent t-statistics are in parentheses.

Table 20: Average DISP by Portfolio (LowVol)

	LowSkew	2	3	4	HighSkew
LowDisp	0.02	0.02	0.01	0.01	0.01
2	0.03	0.03	0.03	0.03	0.03
3	0.05	0.05	0.05	0.05	0.04
4	0.08	0.08	0.08	0.08	0.08
HighDisp	0.21	0.23	0.23	0.22	0.23

Note: This table presents the average forecast dispersion by portfolio. For each stock, each month, we obtain the analyst forecast dispersion as described in Section 4.1. Each month, we compute the median across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

Table 21: Average DISP by Portfolio (HighVol)

	LowSkew	2	3	4	HighSkew
LowDisp	0.02	0.02	0.02	0.02	0.02
2	0.03	0.03	0.03	0.03	0.03
3	0.05	0.05	0.05	0.05	0.05
4	0.08	0.08	0.08	0.08	0.08
HighDisp	0.21	0.22	0.24	0.25	0.25

Note: This table presents the average forecast dispersion by portfolio. For each stock, each month, we obtain the analyst forecast dispersion as described in Section 4.1. Each month, we compute the median across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

Table 22: Average SKEW by Portfolio (LowVol)

	LowSkew	2	3	4	HighSkew
LowDisp	0.27	0.48	0.62	0.77	0.98
2	0.27	0.49	0.63	0.77	1.01
3	0.28	0.49	0.65	0.79	1.02
4	0.29	0.53	0.69	0.80	1.05
HighDisp	0.34	0.65	0.76	0.88	1.16

Note: This table presents the average idiosyncratic skewness deviation by portfolio. For each stock, each month, we obtain the predicted skewness of the Fama-French three-factor residuals from Boyer et al. (2010). Each month, we compute the median across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.

Table 23: Average SKEW by Portfolio (HighVol)

	LowSkew	2	3	4	HighSkew
LowDisp	0.30	0.50	0.66	0.81	1.08
2	0.28	0.48	0.64	0.80	1.06
3	0.27	0.47	0.64	0.80	1.07
4	0.25	0.47	0.65	0.80	1.09
HighDisp	0.22	0.47	0.66	0.82	1.15

Note: This table presents the average idiosyncratic skewness deviation by portfolio. For each stock, each month, we obtain the predicted skewness of the Fama-French three-factor residuals from Boyer et al. (2010). Each month, we compute the median across all stocks in a portfolio. Finally, this monthly portfolio value is averaged over the entire sample.