The Long and Short of Financial Development

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Abstract

Financial development helps a producer raise capital to fund long term complex investments. Consequently, it should increase output and welfare. However, our analysis suggests this is not always so. We consider a simple economy where producers and consuming/financing households are distinct agents, where producers lack sufficient capital, and where households care about both pledgeable returns and liquidity. In this economy, the producer's greater ability to pledge long-term project earnings to financiers can reduce long term production and welfare, even though it makes financing more accessible. Our results have implications for why economies face impediments to financial development and overall growth, especially when producer capital is scarce. The competitive equilibrium is constrained efficient only when producer capital is low and pledgeability constraints lead more productive assets to offer lower pledegable returns than less productive assets .

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1 Introduction

A fundamental challenge in development is transitioning from simple, quick economic production processes with low returns to more complex, longer-term production that generate higher returns. Financing such production is particularly difficult. Producers must offer investors financial claims with attractive returns. However, conflicts of interest, moral hazard, and low transparency can limit producers' ability to pledge future output from production especially when production processes are long and complex. Financial development, for instance, through improved corporate governance, should increase the financeability of such projects by enhancing the pledgeability of output. This, in turn, should increase high-return complex production and foster economic growth. Yet the impediments to financial development seem more than simply a lack of awareness of its benefits. What might they be?

We consider economic situations with three characteristics. First, competitive and homogeneous producers can choose between simple short but low return production and complex long but high return production. Specifically, they can undertake either short-term lower-return investments making tradeable goods using simple, transparent methods (such as planting seeds for fresh vegetables, mining for silver or gold, or holding inventories of commodities to trade them) or higher-return complex investment with an extended duration between input and final output (such as building a factory to produce canned tomato paste or bicycles). Producers value consumption at any time equally, caring only about their overall returns.

Second, each of these investments has an associated pledgeability — defined as the share of output that can be committed to be paid to outside investors. For inventory investment, think of greater pledgeability as more effective and easily monitored warehousing technology that ensures the pledged inventory is available to support any lender's efforts to collect promised payment. Long pledgeability is similarly defined as the share of output from the long term investment that can be paid out, reflecting for instance the quality of corporate governance, which ensures the long term projects are managed in the interests of investors. With the quick turnaround from input to output on short investment, short pledgeability is typically higher than long pledgeability (though see Myers and Rajan 1998). We term increases in short pledgeability *credit development* and increases in long pledgeability *financial development*.

Producers can secure funding for a portion of their real investments by issuing financial claims to households, who have some capital. Funding is limited to the present value of the pledgeable portion of their production output. So when pledgeability is low, producers must co-invest their own capital to make up the difference between required investment and available external funds. Consequently, production is limited by producers' capital. Low producer capital and low pledgeability of production also mean that producers can only offer low rates of return to households on their claims, with the remaining return accruing as rents to producers. These "rents from financing" accrue despite producers being competitive – because producers cannot commit to pay more. They may differ across financial assets and are critical in the analysis for they alter the producer's return from investment.

Third, financing households are also consumers (which is what we will call them from now on). They can finance producers but cannot save elsewhere (though access to low return storage is easily accommodated). Consumers are also uncertain about when they need to consume – liquidity needs because of unmodeled income shocks or emergency spending needs like healthcare are an important concern, especially in more volatile developing economies. Therefore, consumers will value the liquidity of financial claims, defined as the return they can obtain at an early date, in addition to valuing long-term returns. These three elements are crucial to our analysis and results.

We assume a competitive financial market on each date. This market allows competing producers to issue financial claims to consumers initially and later allows consumers to trade financial claims with each other. Competition among producers (all with access to the same technologies) requires them to pass through to consumers as much of the output produced as is pledgeable.

Because producers can undertake either short or long term investment and can raise funding in a competitive market, producer returns on either investment, including the rents from financing, must be equal if both investments are undertaken; else, only the investment with the higher return to producers will be undertaken. The rates of return available to consumers on short term and long term financial claims depend on the degree of pledgeability of output from each maturity as well as on the endogenous market prices for those claims when issued or resold. Importantly, long-term claims are illiquid if they resell at interim dates for low prices, so those holding short claims can then buy the cheap long claim to obtain higher long term returns. This will be important in determining ex ante household allocations to claims.

Our analysis will be in three parts. First, we will determine the decentralized competitive equilibrium and how financial development affects it. Whereas higher pledgeability increases the outside capital that can be raised, it could reduce the producer's financing rents and consequently the attractiveness of producing more of the asset. Consumer returns from buying financial claims on the asset move in the opposite direction to producer returns, which also affects consumer allocations to claims. This implies that an increase in the pledgeability of the long asset, that is, *financial development*, does not always increase producer production or consumer financing of the long asset, unlike what a partial-equilibrium analysis might suggest. In the second part, we will ask what a social planner faced with similar pledgeability constraints might be able to achieve given the weights they place on consumer and producer welfare. Third, we will ask how the political economy of financial development may play out, given what we learn from the first two parts. It will turn out that financial development is indeed difficult because of the incentive distortions created by rents, and the associated political economy forces.

Some examples of the competitive outcomes may help fix ideas. Start with the case when assets are fully pledgeable. In that case, competitive producers will pledge all the returns from externally-financed projects to consumers (so producers get no rents from financing), and the producers do not need to make up financing shortfalls in any asset with their own capital. They will invest their own capital in higher return long production for their own consumption. Consumers allocate their capital by trading off the higher return from long-term claims and the liquidity offered by short-term claims.

Now consider lower levels of asset pledgeability. Start first with the case where producers have large amounts of capital relative to consumers, and so can co-invest as needed. Producers pay out up to the pledgeable portion of output, but they have to raise only a small fraction of the investment needed in each project from consumers, co-investing the rest. Producer competition will ensure that the rents from financing the long asset are driven to zero, and consumers are paid the return on their (small) holdings of long claims they would get if the long asset were fully pledgeable. Consumers will get higher returns from the long claim, once again with the liquidity benefits from the short financial claim motivating them to hold both claims in equilibrium.

By way of contrast, consider the case where producers have no capital. In that case, the output that will accrue to consumers is only what is pledgeable. Since the consumer has to put up all the funds for investment, he might allocate them to financing only the short asset if the pledgeable returns from the short asset exceed the pledgeable returns on the long asset. In this case, pledgeability determines what is produced, and the lower pledgeability of the long asset may cause it to be dominated. However, producers make substantial "rents from financing" since they pay out only the pledgeable part of any output, retaining the rest of the output for themselves despite not investing a cent, and despite financial markets being competitive. The rents stem from the producers' monopoly over production, with the lack of pledgeability and shortage of producer capital effectively limiting competition.

The main focus of this paper is on what happens when neither long pledgeability nor producer capital are at extremes. We will see that the level of long pledgeability affects how increases in it (financial development) play out. A critical level is when the pledgeable return of the high return long term project just equals the pledgeable return on the more pledgeable low return short term project. *Ceteris paribus*, above this level of long pledgeability, project pledgeability and project returns are *aligned*, that is, higher return projects generate more pledgeable output, while they are *misaligned* at levels below, in that the lower return project generates more pledgeable output.

At very low levels of long pledgeability, returns and pledgeability are grossly misaligned, and only the short term project will be undertaken. Financial development over a range has no effect on project choice or output. The outcomes here are reminiscent of primitive economies where the accent is on simple subsistence production and commodity trade.

At higher levels of long pledgeability, while returns and pledgeability are still misaligned, we will see financial development helps increase producer and consumer allocations to the long asset. However, producers get a disproportionate share of the additional returns, so much so that consumers are worse off. So in this region, consumers would not support financial development.

Matters change considerably when long pledgeability increases further, aligning returns with pledgeability, so that higher return projects also have more pledgeable output. Intriguingly, consumers' liquidity concerns now make their capital allocations across financial claims insensitive to financial development. So financial development results in a higher consumer return on long financial claims (because a greater share of output flows to long claims without any increase in their capital allocation), and thus lower rents from financing to the producer. Producers will have incentives to tilt towards production that is less pledgeable, that is, the short term lower return asset. This contradicts the partial-equilibrium intuition that an increase in pledgeability of an asset, and thus an increase in the financing available for it, should increase its production. Over a range, financial development reduces the share of aggregate capital that is devoted to long projects, and reduces producer welfare, as well as overall output, while enhancing consumer returns. Consequently, producer lobbies have an incentive to oppose financial development in this region, akin to a middle-development trap.

Finally, at very high levels of long pledgeability, the elimination of rents from financing longs will make producers abandon opposition to financial development. Conflicts of interest over greater pledgeability dissipate.[1]

Consider now a social planner who faces the same pledgeability constraints, but can

[1]We also examine the effects of increases in short asset pledgeability, that is, *credit development*. We find that it makes the consumer better off, and makes the producer (weakly) worse off. The effects on measures of social welfare are, once again, more ambiguous.

choose what financial claims to issue and in what amounts, as well as how much to allocate to short and long investment. The preferred outcomes for the social planner will depend, of course, on the relative weight they place on consumers and producers, as well as on consumer preferences. We will see that competitive outcomes differ from the socially optimal for three reasons. First, producers are usually forced by competition to pay out the maximum pledgeable amount on claims, the social planner need not be so bound. Second, consumer allocations to the issued claims typically are not socially optimal because of a pecuniary externality associated with trading in an incomplete market. The competitive solution will have too little investment in the short term asset than socially optimal if consumers are sufficiently risk averse and too much if risk aversion is low. This demand side effect mirrors that in Jacklin (1987) and Allen and Gale (2004). Third, on the supply side, producers allocate capital to the two assets based on the rent augmented non-pledgeable return they get (as discussed above, competition for rents forces the pledgeable return to be promised away to consumers) rather than the total returns available on each asset. Each of these effects can push decentralized competitive outcomes away from the socially optimal.

Interestingly, at low levels of long pledgeability, when only the short asset is financed in the competitive case, the producer has no ability to determine investment allocations. It turns out then that the competitive outcome is constrained socially optimal; First because only one financial claim is produced, there is no trading and no associated externality: And second, the producer has no choice to make (she is forced to produce only short assets) and therefore there are no associated allocation distortions because of producer rents. At somewhat higher levels of long pledgeability, both long and short assets are financed, but the forces of competition lead to financing and production of very few long assets. We show that this again turns of to lead to constrained efficient allocations of capital, given the levels of pledgeability. This suggests that poor developing countries might be doing the best they can given levels of financial and credit development, even if they make few long term complex investments in, say, manufacturing.

Conversely, economies at higher levels of financial development, where large fractions of both assets are financed and produced, may be quite far from the constrained social optimal. Consumer allocations are distorted by the pecuniary externality, while producer allocations are distorted by rents.

Finally, we examine how a social planner, whose preferences may be entirely for consumers (a "democratic" administration) or entirely for producers (an "oligarchic" administration) might affect financial development (assuming they can do so in incremental steps). The important finding is that neither a producer-oriented nor consumer-favoring planner has an incentive to favor financial development at every level of long pledgeability. The planner will turn against financial development eventually. The sobering message is that conflicts of interest over further development dissipate only when long pledgeability is at a high level or when producers are well capitalized. This suggests a version of what is termed the Matthew effect ("to everyone who has will more be given,...") may apply to financial development also. It also suggests why in a developing economy, an initial increase in inequality, with producers obtaining relatively more capital, may be associated with more growth, as Simon Kuznets observed.

Our paper also explains how producer capital is not just useful in facilitating producer investment (as in the literature on intermediary capital, see He and Krishnamurthy (2013); Holmstrom and Tirole (1997); Rampini and Viswanathan (2019)), but also the allocation of overall investment. Indeed, since risk-bearing producer capital can shrink relative to consumer capital n times of economic adversity, while expanding in booms, our model has implications for business cycles, which we explain later in the paper.

The rest of the paper is as follows. In section 2, we present the model, and analyze equilibria for various parameters in section 3. In section 4, we study the social planner's problem under different constraints. In section 5, we examine incentives for financial development given the comparative statics of various equilibria if decision making is in different hands, and relate our work to the literature in section 6. We conclude in section 7.

2 Model

2.1 Agents and Preferences

Consider an economy with three dates $t = 0, 1, 2$ and total capital endowment normalized to one. There are two categories of agents: consumers and producers.

Let $\eta \in [0, 1]$ be the total capital owned by consumers at $t = 0$, with each consumer owning 1 unit.[2] With *i.i.d.* probability $1 - q$, a consumer turns out to be early, denoted by e, with probability q, he turns out to be late, denoted by l. An early consumer only cares about consumption at $t = 1$, so his utility function is $U(C_1^e)$, whereas a late consumer's utility function is $U(C_1^l + C_2^l)$. Consumer type (early or late) is the private information of each consumer. The uncertainty about the desired timing of consumption, a form of liquidity shock, leads to a demand for asset liquidity. We assume that the utility function $U(.)$ is increasing and weakly concave. A straightforward case is when consumers' preferences are

[2]Given the total capital owned by either consumers or producers, their individual size is not critical. One interpretation is that consumers have a total measure of η, with each owning one unit of capital. Alternatively, consumers have a total measure of one, with each owning η amount of capital. The results remain the same in both scenarios.

linear and thus risk-neutral. Other than allowing a possible lower degree of concavity, these preferences are identical to those in Diamond and Dybvig (1983). Our positive results, such as resource allocation and equilibrium prices, remain unchanged as one varies consumers' risk aversion.

Producers are endowed with total capital $1-\eta$ at $t=0$, with each owning 1 unit. They can consume at both $t=1$ and $t=2$, and their utility is linear and equal to $\Pi_1+\Pi_2$, where Π_t is their payoff at date t.

2.2 Assets, Claims and Pledgeability

Producers can invest in two types of real assets (using their capital and the funding raised from consumers) at date 0. Both assets are constant returns to scale investments available to all producers, but only to them. One is a short-term asset (henceforth short asset) with a return per unit invested of $R \geq 1$ at $t=1$. The output of this investment should be thought of as a tradeable consumption good. The second asset is a long-term one (henceforth long asset) with a return of $X > R$ at $t=2$ but zero return if liquidated early at $t=1$. This asset could be thought of as a sophisticated asset, that is, a project or firm that pays off in the long run.

Producer investments are made with the producers' own capital as well as the resources they raise from consumers. We refer to the financial contracts offered by producers when they raise funding as financial claims (or just claims). Not all of the asset returns can be used to back and eventually pay claims. In the case of the short asset, the producer may need to retain some "skin in the game" upfront to assure consumers that they will get their promised share of output. This is especially the case if the production process requires effort. An alternative interpretation is that there are defects in the production process, implying that only a fraction of the short asset's output is consumable or exchangeable by consumers, while the rest can only be consumed by the producer (think of the producer producing misshapen or unattractive vegetables that are intrinsically edible but are unacceptable to consumers because they are uncertain about quality). We do not differentiate between these different microfoundations and assume that only a positive fraction γ_S of the short-term asset's output is payable to consumers. We refer to γ_S as *short pledgeability*, and we term increases in it *credit development*. Better banks, more reliable warehouses where inventory can be stored and monitored, better enforcement of collateral pledges, etc., would all contribute to higher short pledgeability.

Similarly, we assume only a positive fraction γ_L of the long-term asset's output at $t=2$ is pledgeable, where γ_L *is long pledgeability*. The reasons only a portion is pledgeable could be similar to those for the short asset. In addition, though, long assets require greater probity of, and incentives for, the producer since she has more time and cover (because of the more complex nature of the asset) to steal output, or shirk. In that sense, long pledgeability proxies for the external governance exercised over the long term asset. Improvements in accounting standards, corporate disclosure, transparency, and corporate governance, etc would all contribute to higher long pledgeability, that is, *financial development*.

For now, we assume both production technologies are only available at $t = 0$. In other words, there is no other means for consumers to save from $t = 1$ to $t = 2$. However, our assumption that late consumers value consumption equally on both date 1 and 2, i.e., $U(C_1^l + C_2^l)$, is equivalent to having them value only date 2 consumption, while being able to store pledgeable consumption goods between those dates at a zero net rate of return.

2.3 Financial Market and Rates of Return on Claims

Financial markets open at $t = 0$ and $t = 1$. In the $t = 0$ financial market, the producer can sell financial claims against the pledgeable output produced by the real assets. Let consumers investing at 0 receive promised gross rates of return, $r_{0\tau}^a$, between dates 0 and τ for claim $a \in \{S, L\}$, where S denotes the claim against the short asset and L the claim on the long asset. In addition, because consumers can trade in a competitive market at date 1, there will be an endogenous market clearing rate of return (a price) to sell the long claim for the repayment obtained on short claims. We define this rate of return between dates 1 and 2 as r_{12}^L, and it is earned by every unit of short claim repayment used to buy long claims. Consequently the return of a late consumer per unit invested in short claims at $t = 0$ is $r_{01}^S r_{12}^L$.

Similarly, a long claim of one unit maturing at date 2 sells for $\frac{1}{r_{12}^L}$ and as a result a unit of long claim investment at date 0, worth r_{02}^L at date 2, sells for $\frac{r_{02}^L}{r_{12}^L} \equiv r_{01}^L$ on date 1. This is the rate of return at date 1 to an early consumer from a unit invested in the long claim. These rates of return depend on the supply of claims offered by producers and consumers' demand for them. Atomistic consumers will take returns as given, so we begin by looking at consumer decisions for given rates of return.

2.4 Consumer Demand for Claims

The role of a short-term financial claim, which pays out at date 1, is two fold. First, the payout can be used for immediate consumption. Second, when consumers are late, the payout can be used to buy long-term financial claims.

Let the representative consumer invest share θ and $1-\theta$ of their capital at date 0 in long claims and short claims, respectively. Once the uncertainty about when they will consume is resolved, some consumers will gain from trading in the $t=1$ financial market, where only consumers can trade. Early consumers will sell all of their long claims to late consumers at any positive price. Clearly, only late consumers want to buy the claim. Late consumers get an endogenous rate of return between dates 1 and 2 of r^L_{12} from the purchase. Because late consumers care only about the sum of date 1 and 2 consumption , they will only be willing buy long claims at date 1 if $r^L_{12} \geq 1$. At any lower return, they would consume date-1 payout rather than buy long claims.

After trading, early consumer consumption, C^e_1, is

$$C^e_1 = \underbrace{\theta \frac{r^L_{02}}{r^L_{12}}}_{\text{sell long-financial}} + \underbrace{(1-\theta) r^s_{01}.}_{\text{consume short-financial}}$$

Late consumers will use all their short claims to buy longs sold if $r^L_{12} > 1$. If $r^L_{12} = 1$, they are indifferent between consuming immediately and using some of their early claims to buy longs. Market clearing requires $r^L_{12} \geq 1$. As a result, the consumption of late consumers, $C^l_1 + C^l_2$, is:

$$C^l_1 + C^l_2 = \underbrace{\theta r^L_{02}}_{\text{consume long-financial}} + \underbrace{(1-\theta) r^S_{01} r^L_{12}.}_{\text{buy long-financial using payoff from short-financial or consume short financial}}$$

On inspection, $C^l_1 + C^l_2 = C^e_1 \cdot r^L_{12}$. Consumers choose their date 0 holdings to solve

$$\max_{\theta \in [0,1]} \; (1-q)\, U\left(C^e_1\right) + qU\left(C^l_1 + C^l_2\right),$$

and the first-order condition with respect to θ becomes

$$\left[(1-q)\, U'\left(C^e_1\right) \frac{1}{r^L_{12}} + qU'\left(C^l_1 + C^l_2\right)\right] \left(r^L_{02} - r^L_{12} r^S_{01}\right). \tag{1}$$

Due to anticipated liquidity shocks, there is a demand for liquid claims, and the resale prices of long claims (equivalently, the future rates of return offered in trading the claims) impact the demand for claims at date 0. There is a unique level of r^L_{12} consistent with consumers being willing to hold both assets at date 0, which we specify in Lemma 1. At other levels of r^L_{12}, one asset will dominate another. This result about the effects of liquidity demand is present in Jacklin (1987) and Allen and Gale (2004), where consumers have direct access to real assets (and thus no need for production of claims by producers).

Lemma 1. *If $r_{02}^L \geq r_{01}^S$, consumers hold both claims, choosing $\theta \in (0,1)$ in equilibrium. The endogenous rate of return between dates 0 and 1 is equal for short and long claims:* $r_{01}^S = r_{01}^L \equiv \dfrac{r_{02}^L}{r_{12}^L}$.

Proof. The one period return from date 0 to 1 (which is the return of an early consumer) of a long claim is $\frac{r_{02}^L}{r_{12}^L}$ and the one period return of a short claim over those dates is r_{01}^S. An increase in θ increases early consumer consumption by: $\frac{\partial C_1^e}{\partial \theta} = \frac{r_{02}^L}{r_{12}^L} - r_{01}^S$ and increases late consumer consumption by: $\frac{\partial C_1^l + C_2^l}{\partial \theta} = r_{02}^L - r_{01}^S r_{12}^L$. Increasing θ increases both early and late consumption if $r_{12}^L < \frac{r_{02}^L}{r_{01}^S}$ and decreases both if the reverse inequality holds. Either is inconsistent with an interior choice of θ. So an interior choice, which requires both derivatives to be zero, occurs iff $r_{12}^L = \frac{r_{02}^L}{r_{01}^S} \equiv r_{01}^L$. If $r_{02}^L > r_{01}^S$, this means $r_{12}^L > 1$. If $r_{02}^L = r_{01}^S$, then $r_{12}^L = 1$. □

Consumers hold both claims only if they offer the same one period return because the ability to trade at date 1 induces them to hold the asset portfolio with the highest one period payoff (obviously early consumers prefer this, but it also positions them to get the highest two period return payoff if they happen to be late). Lemma 1 shows that only if each asset provides the same return from date 0 to 1 will the consumers hold both assets at date 0.[3] . Lemma 2 uses this result to determine the capital allocation consumers at date 0, based on the liquidity demand from trade at date 1.

Lemma 2. *If $r_{02}^L > r_{01}^S$, then date-1 market clearing condition implies $\theta = q < 1$. If $r_{02}^L = r_{01}^S$, then $r_{12}^L = 1$ and consumers optimally choose any $\theta \in [0,1)$. If claims are produced, then $\theta < 1$.*

Proof. If $r_{02}^L > r_{01}^S$ and thus $r_{12}^L > 1$, then all long payouts held by early consumers ($(1-q)\theta r_{02}^L$), are bought with all short payouts held by late consumers ($q(1-\theta)r_{01}^S$) and thus $r_{12}^L = \dfrac{r_{02}^L}{r_{01}^S} = \dfrac{(1-q)\,\theta r_{02}^L}{q(1-\theta) r_{01}^S}$, or $(1-q)\,\theta = q(1-\theta)$, implying $\theta = q$. If $r_{02}^L = r_{01}^S$, then any $\theta < 1$ is individually optimal. If $\theta = 1$, then no consumer has any short claims to buy longs sold by the early consumers and they sell for a price of zero at date 1. Market clearing would imply that $r_{12}^L \to \infty$, a contradiction. □

Lemma 2 will be very important when we discuss welfare for it suggests consumer allocations do not necessarily equate the ratio of the consumer's marginal utilities of consumption

[3]If long claims have higher returns to maturity than short, they would dominate shorts if their period 0 to 1 return when sold at date 1 was also higher than the return from a short-term asset. Conversely, shorts would dominate longs if the return from holding a short term asset used to buy a long at date 1 exceeded the return on a long held to maturity. This would occur if the one period return on the long sold at date 1 was below the return on the short.

to the marginal rate of transformation (see Jacklin (1987) and Allen and Gale (2004)). Note also that Lemma 2 also implies that not all claims can be long ($\theta = 1$ is not possible), otherwise holding some shorts would allow a late consumer to buy longs for free, implying shorts could be issued at a very high price, and producers would do so. However, it is possible for only short claims to be issued.

Lemma 3. If $r_{02}^L < r_{01}^S$ then long term claims are dominated and consumers invest only in short term claims ($\theta = 0$).

Proof. If $r_{02}^L < r_{01}^S$, then from $r_{12}^L \geq 1$, we have $\frac{\partial C_1^e}{\partial \theta} < 0$ and $\frac{\partial C_1^l + C_2^l}{\partial \theta} < 0$, implying $\theta = 0$ and long claims are dominated by shorts. □

Lemma 3 implies that producers can issue long-term claims only if they can offer returns at least equal to short claims issued. This constraint of undominated long claims will be an important consideration.

Before turning to the producers' problem, it is worth noting that in a partial equilibrium setup where the returns $\left\{r_{02}^L, r_{01}^S, r_{12}^L\right\}$ are fixed, changes in short- and long pledgeability will not affect consumers' allocation and welfare. In partial equilibrium, higher pledgeability would help producers make more profitable choices. This is not the case when we endogenize the returns after introducing the producers' problem and analyze the solution in the general equilibrium setup.

2.5 Producer Supply of Financial Claims

If consumers could invest directly in constant returns to scale real assets without going through producers, then $r_{01}^S = R$ and $r_{02}^L = X > R$. In general, producer rents associated with financing will imply consumers get a lower return investing in financial claims.

Producers invest in real assets and can issue financial claims out of the pledgeable fraction of asset returns. They must retain the remaining non-pledgeable fraction. Let p_a be the quantity of date-0 capital consumers contribute to buy a financial claim written against all of the pledgeable fraction, γ_a, of one unit of investment in asset a. The quantity p_a is both the fraction of consumer capital in each unit of investment and the date-0 price of a claim on the pledgeable fraction of the asset's payoff. So a unit of capital invested in a long asset ($a = L$) delivers cash flows $\gamma_L X$ at $t = 2$ that can be pledged to the consumer, and p_L is its date-0 price. Similarly, p_S is the price of a short claim delivering cash flows $\gamma_S R$ at $t = 1$. Given this, the return to maturity is $r_{02}^L = \frac{\gamma_L X}{p_L}$ and $r_{01}^S = \frac{\gamma_S R}{p_S}$ on the respective claims.

If $p_a < 1$, an asset is produced with a fraction p_a of consumer capital and $1 - p_a$ of producer capital. If the producer has sufficient capital, she may also self-fund an asset entirely and retain all of its return. For the rest of this paper, we also refer to p_S and p_L as the *financeability* of the short- and long-term asset, respectively.

Let b_F be the endogenous date-1 price per unit of a long financial claim (that is, a claim on $\gamma_L X$). If the long claim is bought at this price at date 1, it offers a rate of return between dates 1 and 2 of $r_{12}^L = \frac{\gamma_L X}{b_F}$. Since $r_{12}^L \geq 1$, $b_F \leq \gamma_L X$.

Producers are competitive and take the claim prices p_S and p_L as given. A representative producer allocates a fraction y_L of producer capital to the production of the partly externally financed long asset, y_S to producing the short asset, and $1 - y_L - y_S$ to long asset production that she self-finances entirely and whose payoffs she consumes entirely. Consumers will buy all of the financial claims issued. Producers retain the non-pledgeable fraction of each asset's payoffs. Note that the producer never entirely self-finances any short production, because long investments are more productive, $X > R$, and she values cash flows equally at both $t = 1$ and $t = 2$. Then the economy is characterized by six unknowns $\{\theta, y_L, y_S, p_S, p_L, b_F\}$.

A producer's payoff then is

$$\Pi = \max_{y_L, y_S} y_S \underbrace{\frac{(1-\gamma_S) R}{1-p_S}}_{\text{non-pledgeable short return}} + y_L \underbrace{\frac{(1-\gamma_L) X}{1-p_L}}_{\text{non-pledgeable long return}} + (1 - y_L - y_S) X.$$

Note that due to producer competition neither p_L nor p_S can be greater than 1 for that would mean the consumer entirely finances investment and more, so every producer would compete the relevant price down to 1, given they have no personal cost of production.

From consumer demand, we know that it is never the case that only long claims are produced. For competitive producers to produce some claims that consumers will hold, it must be that producers earn no less from allocating their capital to producing short claims as long. This leads to the following, which is also their first order condition (FOC).

$$\frac{(1-\gamma_S) R}{1-p_S} \geq \frac{(1-\gamma_L) X}{1-p_L}. \tag{2}$$

If both claims are produced, the FOC holds with equality. In this case, note that the rent the producer obtains from financing the long asset is

$$\frac{y_L (1-\gamma_L) X}{1-p_L} - y_L X = \frac{y_L X (p_L - \gamma_L)}{1-p_L}.$$

So the rent from financing comes from the producer's ability to sell γ_L of financial claims on the long asset for $p_L > \gamma_L$, and similarly for the short asset. Rents exist despite producers being competitive because assets have limited pledgeability and producers have limited capital. Finally, note the producer will not self finance long assets when she earns rents on them – externally financing them will earn a higher return on producer capital.

2.6 Market Clearing

Consumers possess a fraction η of total capital and invest a fraction θ in long claims, and fraction $1-\theta$ in short claims. Producers possess a fraction $1-\eta$ of capital and invest a fraction y_L in producing long claims and y_S in producing short claims, in equilibrium with market clearing. As a result, the date-0 claim prices (or equivalently, the share consumers invest in each asset) are given by:[4]

$$p_L = \frac{\theta\eta}{\theta\eta + (1-\eta)y_L} \tag{3}$$

$$p_S = \frac{(1-\theta)\eta}{(1-\theta)\eta + (1-\eta)y_s}. \tag{4}$$

.

A competitive equilibrium requires market clearing, and price taking optimization by consumers and producers.

3 Decentralized Market Equilibrium Outcome

3.1 Simple Benchmarks

Let us start with some simple benchmark cases.

Full pledgeability, $\gamma_L = \gamma_S = 1$

Full pledgeability combined with competitive producers with constant returns to scale investments immediately implies that all of the output from capital invested by consumers must accrue to consumers (as in the case when consumers can invest directly). That is, the zero excess profit condition for producers immediately implies that $r^S_{01} = R$, $r^L_{02} = X$, and $p_L = p_S = 1$. Consumers provide all of the capital for production of financial claims when there is full pledgeability. Since the producer does not have to make up any capital shortfall after issuing financial claims and gets no rents from issuing financial claims, she will invest

[4]Equivalently: $\underbrace{\eta\frac{\theta}{p_L}}_{\text{demand for long financial}} = \underbrace{(1-\eta)\frac{y_L}{1-p_L}}_{\text{supply of long financial}}$ *and* $\underbrace{\eta\frac{1-\theta}{p_S}}_{\text{demand for short financial}} = \underbrace{(1-\eta)\frac{y_S}{1-p_S}}_{\text{supply of short financial}}$

her own capital in long assets and consume the output. From Lemma 2, and $R < X$, each consumer invests a fraction $\theta = q$ in long claims.

Producers have no capital (implying $\eta \to 1$).

If producers have no capital of their own and assets are not fully pledgeable, consumers provide all the capital for investments, and thus $p_S = p_L = 1$. The returns offered to consumers are $r_{01}^S = \gamma_S R$ and $r_{02}^L = \gamma_L X$, leaving unavoidable rents to producers. As in the case of full pledgeability, the rates of return offered to consumers are set directly by technology and competition. It is possible also that the pledgeable return on shorts exceeds that on longs, or $\gamma_S R > \gamma_L X$, so long claims are not attractive to consumers and the long claim is not produced in equilibrium.

Producers have all the capital (implying $\eta \to 0$).

When producers have essentially all the capital, they can co-invest with consumers as needed. Producer competition will ensure that the rents from producing long financial claims are driven to zero, and consumers are paid $r_{02}^L = X$, the return on their (small) holdings of long claims they would get if the long asset were fully pledgeable.

Note that the direct consumer return on the short claim differs from the full-pledgeability benchmark. Full-pledgeability and competition across producers leads to a full pass-through of short returns to consumers. With limited pledgeability, some producer capital must back financial claims. Because producer's opportunity return from production is X, they must earn at least this from producing short claims. Because the return on short assets is only $R < X$, a more than proportional share of the output from the short asset must go to producers relative to their capital investment so that they earn a return of X from investing in short assets. Equivalently, short financial claims will directly yield r_{01}^S less than R to consumers. For the consumer to be induced to invest in both claims then, it must be that long claims trade at a discount at date 1 so that $r_{12}^L > 1$ and also $\theta = q$, to clear the market.[5]

3.2 Limited Pledgeability and Equilibrium Regions

When pledgeability is limited and producers have some capital, they compete for consumer funding by investing some of their own capital to offer consumers a higher return for a given investment. In this case, the incentives of consumers and producers interact to determine the returns available on financial claims.

[5] We will see in 3.3.4 that consumers earn less than R on short claims even when producers earn no rents.

Higher pledgeability of an asset has two important effects. First, it increases the rate of return that claims offer consumers for a given allocation of capital. Second, greater pledgeability usually (but importantly, not always) reduces the rate of return for producers, because they retain the shrunken non-pledgeable portion of output and compete down financing rents when selling claims on the now-expanded pledgeable portion to consumers. Thus changes in pledgeability also affect the *incentive* of producers to produce that asset. The relative scarcity of producer capital, represented by the ratio of consumer to producer capital, also makes a difference.

We have argued that short pledgeability is naturally likely to be higher than long pledgeability, that is, $\gamma_S > \gamma_L$. In institutionally underdeveloped economies, it is possible that long pledgeability is so low that $\gamma_S R > \gamma_L X$. In such a situation, pledgeable returns are *misaligned* with underlying asset returns because less productive assets are more pledgeable. Of course, at high levels of long pledgeability, *ceteris paribus* , $\gamma_S R \leq \gamma_L X$, and pledgeable returns and underlying asset returns will be *aligned*.

Figure 1 anticipates our general results on pledgeability, where we plot the equilibrium regions as a function of γ_L and γ_S. Next, we explain the four equilibrium regions.

3.3 Variation in the long pledgeability

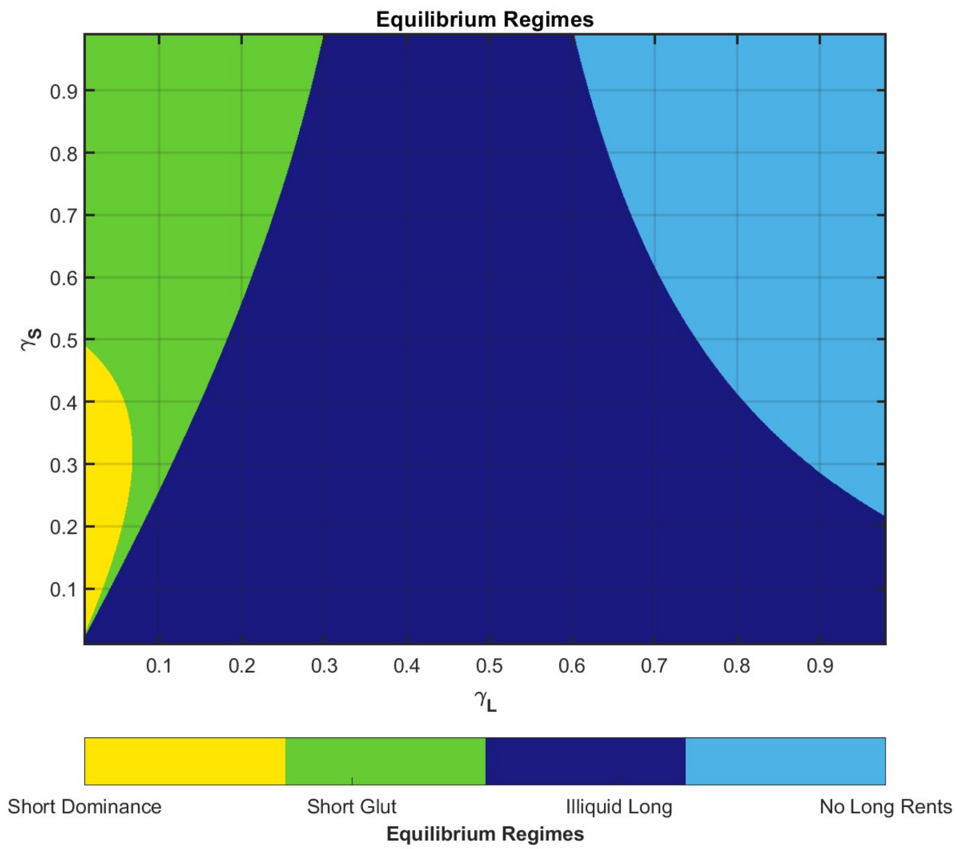
In describing the equilibrium regions, we first hold the pledgeability of the short asset constant at $\gamma_s \in (0, 1)$ and vary the pledgeability of the long asset (that is, we move from left to right in parallel to the x axis). The regions are

1. *Short dominance (yellow) :* At very low levels of γ_L, producers cannot raise sufficient financing for the long asset, and will find its returns dominated by investing solely in the short asset and issuing short claims. This resembles a primitive economy where short production dominates.

2. *Short glut (green):* When γ_L increases sufficiently, producers will see their return on the production of long assets rise to their return on the production of short assets and a small number of long assets and financial claims will start getting produced. At date 1, there will consequently be a glut of short claims sold relative to long, ensuring the scarce long financial claim will be liquid in that it sells for full face value at date 1. This resembles a developing economy with the beginnings of complex long production.

3. *Illiquid long (dark blue)*: When γ_L increases further, and sufficient producer and consumer capital shifts to long production, long financial claims offer higher returns to

Figure 1: Equilibrium Regions as a function of γ_L and γ_S



This figure plots equilibrium regions when γ_L and γ_S vary. The parameters are: $X = 2$, $R = 1$, $q = 0.5$ and $\eta = 0.75$.

maturity than short and have an interim price b_F less than $\gamma_L X$, and hence are illiquid. The equilibrium moves from short glut to illiquid long, conditions resembling an emerging economy.

4. *No long rent (light blue)*: When γ_L is higher still, the date-0 price of the long financial claim is driven down to the point that producers offer consumers the full rate of return available from long production and there are no rents associated with externally financed production. The conditions here are consistent with a developed economy, with long production not distorted by financing rents.

Let us now be more specific about the regions, leaving questions about the constrained social optimal to the next section.

3.3.1 Short dominance

If $\gamma_L \leq \frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}$, we will show that given the shadow prices it is unprofitable for the producer to produce the long asset or the consumer to invest in the associated financial claim. In such an equilibrium, $y_L = 0$ and $\theta = 0$. All of consumer capital goes into short claims. We will show the producer will not self-finance long assets so all her resources are devoted to producing the short asset and $y_S = 1$. If so, $p_S = \eta$. The producer must prefer producing short assets to producing and retaining long so $\frac{(1-\gamma_S)R}{1-p_S} \geq X \Rightarrow (1-\gamma_S)R \geq (1-\eta)X$, a necessary condition for the short dominance region to exist.

When all claims are short, any early consumer who deviated and had a long to sell would obtain the full date 2 value $b_F = \gamma_L X$ from a late buyer with short claims. That is, the shadow $b_F = \min\left\{\frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}}, \gamma_L X\right\} = \gamma_L X$.[6] Consumers prefer short claims weakly over longs if $r_{01}^S \geq r_{02}^L$. When all capital goes into short assets so $p_s = \eta$,the shadow p_L must satisfy :

$$\frac{\gamma_L X}{p_L} \leq \frac{\gamma_S R}{p_S} \Rightarrow p_L \geq \frac{\gamma_L X}{\gamma_S R}p_S => p_L \geq \underline{p}_L \equiv \frac{\gamma_L X}{\gamma_S R}\eta$$

In words, for consumers to shun long claims which pay $\gamma_L X$, the fraction of their own capital that needs to go into each unit of long must be so high as to depress the returns below what they can get from investing in shorts.

Equally, it must be that the producer finds it less profitable to produce the long asset rather than the short, so

$$\frac{(1-\gamma_L)X}{1-p_L} \leq \frac{(1-\gamma_S)R}{1-p_S} \Rightarrow 1-p_L \geq \frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1-p_S) \Rightarrow p_L \leq \bar{p}_L \equiv 1-\frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1-\eta).$$

Put differently, the return from from producing longs per unit of producer capital that must be deployed is dominated by the return available on shorts.

The set of p_L satisfying both constraints for no long claims to be held or long assets

[6] The reason is if so, it must be that $\frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}}$ is finite. Since $\theta = 0$, this implies $p_L \to 0$. However, consumer FOC implies

$$q\frac{1-\theta}{\theta}\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L} \leq (1-q)\frac{\gamma_S R}{p_S} + (1-q)\frac{\theta}{1-\theta}\frac{\gamma_L X}{p_L}$$
$$\underbrace{q\frac{\gamma_L X}{p_L}\left[1-\frac{1-q}{q}\frac{\theta}{1-\theta}\right]}_{+\infty} + \underbrace{q\frac{1-\theta}{\theta}\frac{\gamma_S R}{p_S}}_{\to+\infty} \leq (1-q)\frac{\gamma_S R}{p_S}$$

which is impossible. Therefore, it cannot be that $p_L \to 0$ and it must be that $b_F = \gamma_L X$.

produced, is non-empty if

$$\underline{p}_L \leq \bar{p}_L \Rightarrow \frac{\gamma_L X}{\gamma_S R}\eta \leq 1 - \frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1-\eta) \Rightarrow \gamma_L \leq \frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}. \tag{5}$$

In this equilibrium, consumer welfare is $U(\frac{\gamma_S R}{\eta})$, and producer profits are $\Pi = \frac{(1-\gamma_S)R}{1-\eta}$. The short asset dominates because, given low long pledgeability and the shadow price acceptable to consumers, far too much producer capital is required to be allocated to long assets for them to offer producers the same return as short assets. Conversely, the implied shadow price of the long financial claim acceptable to producers is too high for consumers to prefer them to the short claim. With limited producer capital relative to consumer capital ($\frac{\eta}{(1-\eta)}$ is large), producers find it more profitable to produce short assets exclusively.

3.3.2 Short glut ($b_F = \gamma_L X$)

As γ_L rises further and $\gamma_L \in [\frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}, \underline{\gamma}_L]$, some long externally financed assets will be produced.[7]

An increase in γ_L increases the share of long asset output that can be pledged to households and thus the fraction of each unit of long investment that can come from households, p_L.while remaining competitive with short claims. With lower investment $(1-p_L)$ per unit required from producers, producer returns from longs will match returns on shorts, so that $\frac{(1-\gamma_L)X}{1-p_L} = \frac{(1-\gamma_S)R}{1-p_S}$ and both assets will start getting produced. Nevertheless, in this region, given how much producer capital each long asset needs, the producer can produce only a relatively small amount of the long asset. Since consumers mainly hold short claims, not all of those will be used to buy the longs sold at date 1, so the the date 1 to 2 gross interest rate bottoms out at $r_{12}^L = 1$.

With the increase in long pledgeability, long returns can now match that on short claims and $\frac{\gamma_L X}{p_L} = \frac{\gamma_S R}{p_S}$ (the shadow long return was below short returns in the dominance region) while at the same time, p_S rises to make producers indifferent between assets so that $\frac{(1-\gamma_L)X}{1-p_L} = \frac{(1-\gamma_S)R}{1-p_S}$.

Substituting $\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$ into the producer's indifference condition and rearranging, we get the prices where producers are indifferent about assets produced and consumers are

[7] $\underline{\gamma}_L$ solves $X(\eta(1-q)-\gamma_S)\gamma_L^2 + \gamma_S(R(\eta q - 1 + \gamma_S) + (1-\eta+\eta q)X)\gamma_L - qR\eta\gamma_S^2 = 0$. This equation is derived from three conditions: the long asset is liquid, i.e., $b_F = \gamma_L X$; consumers put exactly a fraction $\theta = q$ of their endowments to into long financial assets; and there is no glut of short financial asset so all short output in hands of late consumers is used to purchase long claims .

indifferent about claims held as:

$$p_S = \frac{\gamma_S}{X}\frac{(X(1-\gamma_L) - R(1-\gamma_S))}{(\gamma_S - \gamma_L)}$$
$$p_L = \frac{\gamma_L}{R}\frac{(X(1-\gamma_L) - R(1-\gamma_S))}{(\gamma_S - \gamma_L)}.$$

In this short glut region, $\frac{\gamma_S R}{p_S} = \frac{\gamma_L X}{p_L}$ holds with equality. Using (3) and (4), this becomes

$$\frac{\gamma_L X}{\gamma_S R} = \frac{1 + \frac{(1-\eta)(1-y_S)}{\eta(1-\theta)}}{1 + \frac{(1-\eta)y_L}{\eta\theta}} \tag{6}$$

This sets the ratios of producer to consumer capital in each asset so that both financial claims are attractive to consumers. Early and late consumers get the same consumption of $\frac{\gamma_L X}{p_L} = \frac{\gamma_S R}{p_S}$, and consumer welfare is $U(\frac{\gamma_L X}{p_L})$.

Comparative Statics with respect to γ_L Recognize that if $\gamma_L X < \gamma_S R$ (which we will see in Proposition 1 is true in this region), it must be that $(1-\gamma_S)R < (1-\gamma_L)X$ so from producer indifference it must be that $1 - p_S < 1 - p_L$. So producers put more capital per unit of long than per unit of short in this region. Using these inequalities, we show

Lemma 4. *In the short glut equilibrium, as γ_L increases: y_L increases, θ increases, $\frac{y_L}{\theta}$ decreases, $\frac{1-y_L}{1-\theta}$ increases, p_S increases, p_L increases, $\frac{\gamma_L}{p_L}$ decreases, consumer welfare $U(\frac{\gamma_L X}{p_L})$ decreases, producer profits Π increases,*

As γ_L rises in this region, more of the return from long assets can be paid out through financial claims. With more consumer financing per unit of long (that is, p_L rises), and with the producer payoff per unit of capital invested in long claim still exceeding that on short claims so that $(1-\gamma_S)R < (1-\gamma_L)X$, the producer would want to shift capital to producing longs, which means she produces more units of them. From condition (6), given higher γ_L,the ratio of producer to consumer capital in longs must fall relative to shorts. This can only happen if the consumer also shifts his allocation towards longs, which is required to fund additional long production.

Since $1 - p_L > 1 - p_S$, the capital-constrained producer can produce less than one unit of long asset for every unit reduction of short asset, and because long assets are less pledgeable (that is, $\gamma_L X < \gamma_S R$), the aggregate future payouts that can be pledged to consumers fall. Given fixed consumer capital up front, and equal returns across financial claims, it must be that consumer returns fall and consumers are worse off as they shift capital to longs.

By contrast, producers benefit from this change because they produce more long assets and receive higher prices for their issued financial claims, increasing their profitability. From an aggregate perspective, since more long assets are produced from the available resources, total output (and hence aggregate consumption) increases.

Essentially, in this region, greater long pledgeability enhances long financeability without diminishing producer incentives to produce long – because consumers shift allocations to longs, thereby increasing producer returns also. Financial development increases overall output. We will see this is no longer the case as we move into the illiquid long region.

3.3.3 Illiquid Long

As we have seen, with an increase in γ_L in the short glut region, more units of long assets are produced relative to short assets. Eventually, sufficient long financial claims are issued relative to short that the late consumer's funds from short holdings obtained at date 1 is less than the future payout on long claims sold by early consumers, so that $b_F = \frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}} < \gamma_L X$. Now the date-1 price on the long is less than face value, which means longs are illiquid. Recall that the first period return on longs and short claims are always equal when both are held. So held to maturity, longs return more than shorts. Also, the consumer's asset allocations are now set anticipating their liquidity demand and their date 1 trades, which implies it is only when $\theta = q$ that neither claim dominates the other, as we have explained in subsection 2.5. Consumer allocations to each asset do not vary with γ_L in this region. Given so, $p_L = \frac{q\eta}{q\eta+(1-\eta)y_L}$ and $p_S = \frac{\eta(1-q)}{\eta(1-q)+(1-\eta)y_s}$, and prices are fully determined by producer allocations.[8]

Comparative Statics with respect to γ_L

Lemma 5. *In the illiquid long with rent equilibrium, as γ_L* increases*: y_L decreases, p_S decreases, p_L increases, and $\frac{\gamma_L}{p_L}$ increases. Consumer returns to both early and late consumers increase and thus consumer welfare increases, producer profits Π decreases, and total output (and consumption) decreases with γ_L.*

The key difference here from the short glut region is that consumer allocations to claims do not change with γ_L. Producer allocations are therefore dispositive here. So when γ_L goes up, non-pledgeable producer output share on long assets falls and producer investment in the long asset, y_L, must go down. Intuitively, to moderate producer dis-incentives to invest

[8] Further substituting these prices into (2), the producer's FOC, we get a quadratic in y_L, $\frac{(1-\gamma_L)X}{\frac{\eta}{1-\eta}(1-q)+(1-y_L)}(1-y_L) = \frac{(1-\gamma_S)R}{\frac{\eta}{1-\eta}q+y_L}y_L$.

in the long asset, it must be that $p_L(=\frac{q\eta}{q\eta+(1-\eta)y_L})$ increases, which given constant consumer investment can only be if the producer invests less in the long asset, that is, y_L falls.

Consequently, $p_S = \frac{\eta(1-q)}{\eta(1-q)+(1-\eta)y_s}$ falls (since the producer invests more in the short), so that consumer returns from shorts, $\frac{\gamma_S R}{p_S}$, increases with γ_L. For both claims to be held, this must imply that the consumer's return from holding long $\frac{\gamma_L X}{p_L}$ increases (and p_L increases by less than γ_L). So in the new equilibrium, the producer's return from producing shorts, $\frac{(1-\gamma_S)R}{1-p_S}$, falls, so too must the producer's return from producing longs, $\frac{(1-\gamma_L)X}{1-p_L}$(despite the increase in p_L).

Note that different from the short glut region, consumer returns from both claims increase – the long claim because it becomes more pledgeable so larger payoffs offering higher returns are available for sale, reducing producer rents from financing and increasing consumer returns, and the short claim because the producer shifts to producing more of it, reducing prices per pledgeable payoff (given the consumer does not shift allocations). Overall output is fully determined by producer allocations, and total output and consumption fall since long production falls. Since consumer returns increase on both claims and the consumer's allocations do not change, consumer welfare increases.

Importantly, an increase in the pledgeability of any asset in this region tends to reduce producer returns, and pushes the producer to produce more of the other asset in order to limit the fall in producer returns. This seemingly counter-intuitive effect of higher pledgeability on an asset's production is because the possibility of interim trade means that consumer allocations are based on the known (and constant) distribution of types to prevent arbitrage. Consequently, since consumer allocations do not shift towards the more pledgeable asset to enhance its price, higher pledgeability for an asset directly reduces the producer's return from producing the asset.

3.3.4 No Long Rent ($p_L = \gamma_L$)

As γ_L rises further in the illiquid long with rent region, p_L rises but at a slower rate and eventually meets γ_L from above. At this point, the rent from financing the long asset falls to zero because consumers contribute the fraction of long asset return that is pledgeable (the price at which the long claim is sold to consumers is exactly equal to its long pledgeable output) – so the rate of return X is passed through to the consumer (that is, $p_L = \gamma_L \Rightarrow \frac{\gamma_L X}{p_L} = X$) . Similarly, for the producer,

$$p_L = \gamma_L \Rightarrow \frac{(1-\gamma_L)\,X}{1-p_L} = X.$$

Since the producer's return on the long asset is X, the producer's FOC requires this to be the return on producing the short asset whenever $\gamma_S < 1$, which implies

$$p_S = 1 - (1 - \gamma_S)\frac{R}{X}.$$

It is easily checked that $r_{01}^S < R$, while $r_{02}^L = X$. Yet the consumer holds both claims because the long claim is illiquid and resells for less than X at date 1. In this region, only changes in short pledgeability can change the rate of return available to consumers. Note that $y_S + y_L \leq 1$ and the producer invests $1 - y_S - y_L$ in self-financed and retained longs. The consumer again invests $\theta = q$ to avoid arbitrage profits from trade at date 1. Market clearing implies that

$$y_L = \frac{\eta}{1-\eta}\frac{q(1-\gamma_L)}{\gamma_L}, \qquad y_S = \frac{\eta}{1-\eta}\frac{(1-q)}{1-(1-\gamma_S)\frac{R}{X}}(1-\gamma_S)\frac{R}{X}, \qquad b_F = \frac{\gamma_L\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}.$$

Comparative Statics with respect to γ_L

Lemma 6. *In the no long rent region, y_L decreases with γ_L, y_S is unchanged with γ_L so producer retention goes up with γ_L. θ and p_S are independent of γ_L, p_L increases with γ_L, and $\frac{\gamma_L}{p_L}$ is unchanged with γ_L. Consumer welfare U, producer profits Π, and thus total output are all unchanged with γ_L.*

In the no long rent region, the limited pledgeability of the long asset does not constrain the pricing or production of long financial claims. Furthermore, the rate of return on producer capital invested in the short asset is also fixed to equal that of producing the long asset, X. That is, the producer earns no rent on producing short claims and short claims have consumer returns below R only because consumers will pay for liquidity benefits, while producers will find that the added return on shorts allows it to match their opportunity return on longs. Since an increase in the pledgeability of the long asset only reduces producer allocation to externally financed production but not overall production of the long asset, it has no effect on producer consumption. The consumer's allocations are also fixed, and her return on the long claim is fixed. So overall output does not change with long pledgeability.

3.3.5 Discussion

The first two regions, short dominance and short glut, where short assets predominate, seem more consistent with economic underdevelopment, where complex long production is scarce. Indeed we have

Proposition 1. *If returns and pledgeability are aligned so that $\gamma_S R \leq \gamma_L X$, then short dominance and short glut are impossible.*

Conversely, all four cases are possible when returns are misaligned ($\gamma_S R > \gamma_L X$).[9] The related literature (see, for example, Ebrahimy (2022) and Matsuyama (2007)) has focused on the case of misaligned returns, with assets of equal maturity. In their work, only the more productive asset is produced when returns are aligned with pledgeability. However, when assets are of different maturities with the longer term asset more productive, as in this paper, both assets will be produced even when returns are aligned because of the short asset's liquidity benefits.

We conclude this subsection by validating the existence and uniqueness of equilibrium. The proof is in Appendix A.1.

Proposition 2. *There exists a unique equilibrium.*

3.4 Credit development

An increase in short pledgeability should ordinarily (though not always) increase consumer allocations to the short claim issued, increasing the producer's incentive to produce more of it. At the same time, an increase in short pledgeability will reduce a producer's financing *rents,* by reducing the fraction of short assets that are not pledgeable. Ordinarily (though not always), this should reduce the producer's incentive to produce more of it. Outcomes depend on how financeability trades off against rents.

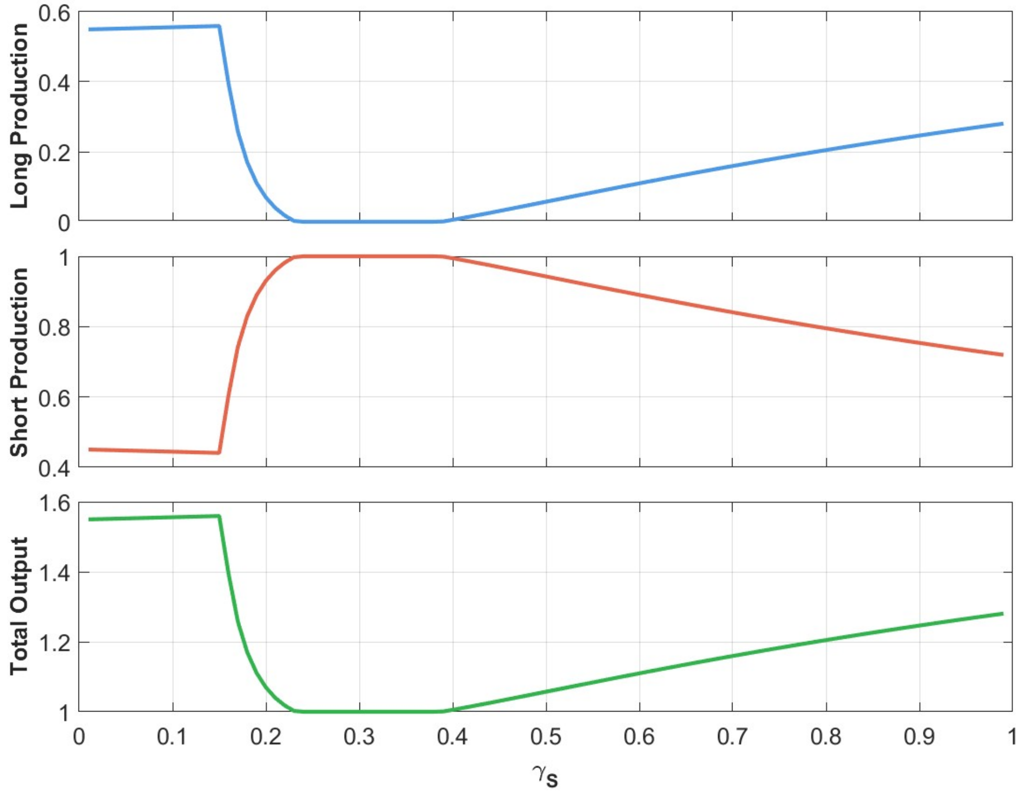
We will see that increased short pledgeability always makes the consumer better off, and makes the producer (weakly) worse off. The effects on total output are, once again, more ambiguous. An example may be useful to set ideas.

We focus on scenarios where returns may be misaligned, i.e., γ_L is relatively low. As illustrated in Figure 1, as γ_S increases (a movement up, parallel to the y axis), the equilibrium progresses through several stages: it moves from an illiquid long region to a short glut, then to short dominance, and finally returns to the short glut region. Figure 2 describes the amount of long and short assets, as well as the total output being produced.

In this example, the decentralized equilibrium is in the *illiquid long* region when γ_S is below 0.14. Since consumers do not reallocate in this region (consumer's allocation stays unchanged at $\theta = q$), the producer shifts allocations toward the long asset following an increase in short pledgeability γ_S since the loss in producer rents on shorts dominates incentives. As γ_S rises above 0.14, the equilibrium shifts to *short glut.* Both producer and consumer

[9] A proof is in Appendix A.1.

Figure 2: Production and Output under different γ_S



This figure plots equilibrium production and output when γ_S varies. The parameters are: $X = 2$, $R = 1$, $q = 0.5$, $\eta = 0.75$, and $\gamma_L = 0.06$.

allocations to long assets fall with γ_S until they reach zero, at which point the equilibrium enters the short dominance region.[10] Here, increases in γ_S push the consumer return on short assets up, reducing producer returns, and making long asset production increasingly attractive for the producer. So finally, as γ_S increases further, the equilibrium returns to the short glut region. Because as γ_S increases, consumers finance most of short investment, producers move their capital to long investment. The bottom panel of Figure 2 shows that total output can change non-monotonically with γ_S: it first increases, then drops abruptly with the shift to only short production, and then increases again as γ_S gets sufficiently high and almost all short claims are financed by consumer capital.

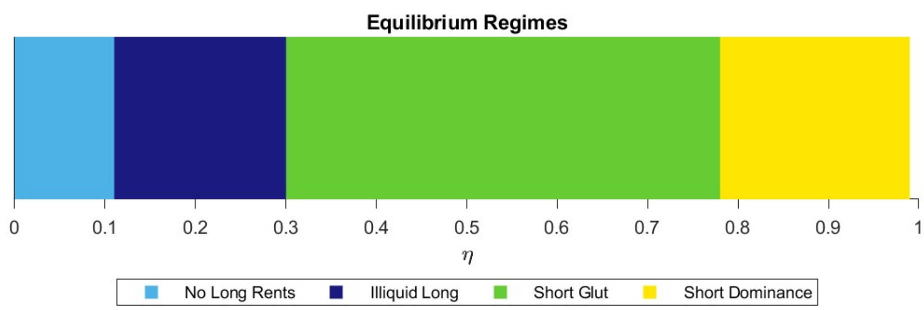
We present the formal results on comparative statics with respect to γ_S in Appendix A.3

[10] In the short glut region, producer and consumer allocations to long assets are in general non-monotonic in γ_S.

3.5 Initial Capital Distribution

Let us turn finally to changes in the amount of consumer capital relative to producers. Figure 3 plots the equilibrium region for our example as η varies from 0 to 1. We consider parameters where returns are misaligned and $\gamma_L R > \gamma_L X$. The light blue region is the illiquid long no rent, dark blue is illiquid long with rent, green is short glut, and yellow region is the dominant short asset region. Clearly, as η increases so that the producers have relatively less and less capital, the equilibrium moves from the no long rents region to illiquid long, short glut and eventually to the short dominance region. In an example with aligned returns, the short glut and short dominance regions do not exist and an increased fraction of consumer capital would move the equilibrium from the no long rent region to the illiquid long region.

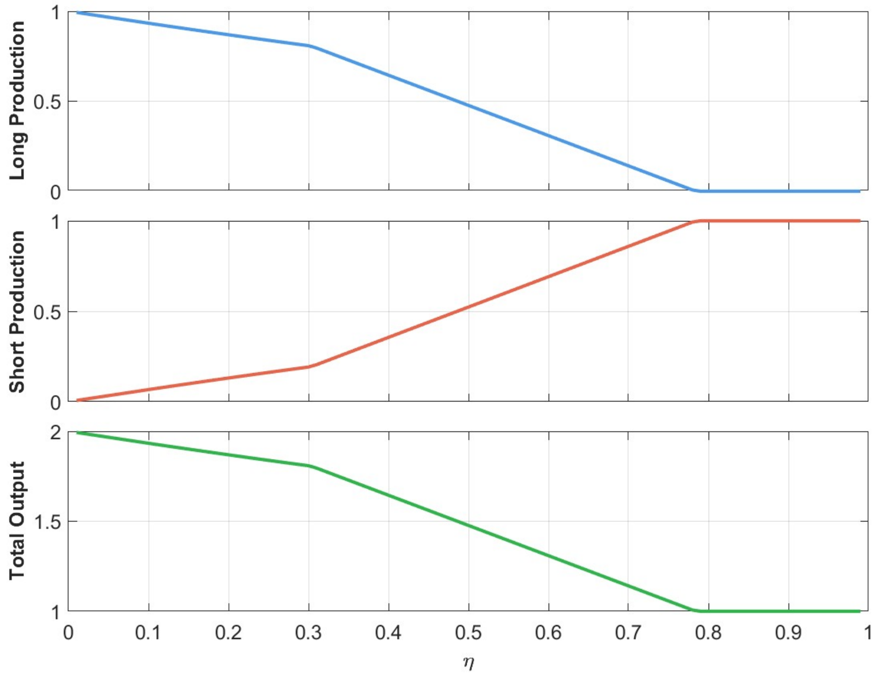
Figure 3: Equilibrium Cases as a function of η, when returns are misaligned



This figure plots equilibrium regions when η varies. The parameters are: $X = 2$, $R = 1$, $q = 0.5$, $\gamma_L = 0.06$ and $\gamma_S = 0.5$.

Figure 4 shows that as η increases, the amount of long production goes down, short production goes up, and the total output goes down. We supplement the formal results on comparative statics with respect to η in Appendix A.4.

Figure 4: Production and Output as a function of η



This figure plots equilibrium production and output when η varies. The parameters are: $X = 2$, $R = 1$, $q = 0.5$, $\gamma_L = 0.06$ and $\gamma_S = 0.5$.

3.5.1 Discussion

An increase in η could represent business cycle downturn, a financial crisis, or a trade shock where producer capital, which is relatively more risk exposed, falls in comparison to consumer capital. This immediately means that if returns are misaligned with pledgeability, we get relatively less production of the high return long asset, and more of the short asset (also see Matsuyama (2007)). Thus productivity falls in downturns, as noted by Eisfeldt and Rampini (2008). Furthermore, consumer returns fall, not just because of the adverse economic outcome, but because the producer's rents to financing go up. Interestingly, these "business cycle" effects would be more muted in a primitive economy with short dominance, so long as changes in producer capital do not take us out of the region – for instance, a hit to producer capital would not alter the productivity of investment, since it continues to be entirely invested in shorts.

4 Efficient Allocations: The Planner's Problem

4.1 The Planner's Problem

We now describe the efficient allocations that would be selected by a benevolent social planner subject to various constraints. The demand for claims comes from consumers who trade claims once they learn their need for liquidity. The planner may prefer allocations that differ from that in a competitive economy. The market is incomplete because consumers have privately-observed shocks to their need for liquidity. The price at which a claim is traded can differ from its value for consumption. As a result, in a decentralized competitive economy, trading may lead to pecuniary externalities, influencing date-0 claim issue prices, and producer choices between short and long assets. Depending on the value of short claims to society, the market may induce producers to produce too much or too little of them, compared to a planner who need not respond to market incentives. In addition to the possible pecuniary externalities due to consumers trading assets, limited pledgeability and associated rents from financing directly influence producer incentives and allocations in a way that can differ from what a planner would choose.

The economy has η consumers and $1-\eta$ producers, i.e., 1 in total. We can substitute out the budget constraint and take note of the fact that early consumers do not value late consumption, implying $C_2^e = 0$, and write producers' total payoff as

$$\begin{aligned} & z_S R + (1-z_S) X - \eta \left[(1-q) C_1^e + q \left(C_1^l + C_2^l \right) \right] \\ = & z_S (R - X) + X - \eta \left[(1-q) C_1^e + q \left(C_1^l + C_2^l \right) \right], \end{aligned}$$

where z_S is social planner's allocation of total capital to the production of the short asset.

The social planner's basic problem becomes:

$$\begin{aligned} \max_{z_S \in [0,1], C_1^e \geq 0, C_1^l \geq 0, C_2^l \geq 0} \; & \beta\eta \left[(1-q) U \left(C_1^e \right) + qU \left(C_1^l + C_2^l \right) \right] \\ & + (1-\beta) \left[z_S (R - X) - \eta \left[(1-q) C_1^e + q \left(C_1^l + C_2^l \right) \right] \right] \\ \text{Subject to:} & \\ PLS: & \eta \left[(1-q) C_1^e + qC_1^l \right] - z_S \gamma_S R \leq 0 \\ PLL: & \eta q C_2^l - (1-z_S) \gamma_L X \leq 0 \\ ICe: & C_1^l - C_1^e \leq 0 \\ ICl: & C_1^e - \left(C_1^l + C_2^l \right) \leq 0, \end{aligned}$$

where β and $1-\beta$ are the welfare weights on the consumer and producers, respectively.

PLS and PLL are pledgeability constraints limiting consumer long and short claims respectively, which also impose non negativity of producer consumption because γ_L and γ_S do not exceed 1. The ICe and ICl constraints are self-selection constraints of early and late consumers, imposed because consumers have private information about their types at date 1. With only this set of constraints, the planner can choose the allocation of total capital, z_S.

When the planner cares only about producers ($\beta = 0$), then only long assets are produced (because $X > R$) and all long output goes to producers. When the planner cares only about consumers ($\beta = 1$), then the planner's problem becomes equivalent to one where consumers have direct access to production assets with short and long returns of $\gamma_S R$ and $\gamma_L X$ respectively, implying a social marginal rate of transformation of early to late consumption of $\frac{\gamma_L X}{\gamma_S R}$. Lower levels of β increase the perceived social cost of early consumption due to the higher welfare weight on producer consumption. In the limit where $\beta \to 0$, pledgeability constraints are slack (consumer consumption approaches zero) and consumers get more date 2 consumption relative to date 1. Lemma 7, Proposition 3 and Proposition 4 describe the results from the planner's problem analyzed Appendix A.5 and A.6 .

Lemma 7. *In the social planner's problem, the pledgeability constraint PLS always binds. Moreover, z_S and therefore total date-1 consumption increase with β.*

4.1.1 Linear utility where early and late consumer consumption levels are perfect substitutes.

Let us start by analyzing the planner's problem when consumers have linear utility of consumption $U(C_1^e) = C_1^e$ and $U\left(C_1^l + C_2^l\right) = C_1^l + C_2^l$. The consumption of early and late consumers are now perfect substitutes and each consumer's ex-ante welfare goal is to maximize expected consumption, $q(C_1^l + C_2^l) + (1-q)C_1^e$. Recall producers have linear utility of consumption as well. If there were no other constraints, including none on pledgeability, the consumption which is the least costly to produce would maximize social welfare. Long assets have a higher return than short, so a planner without other constraints would choose only the more productive long asset. Pledgeability constraints limit the consumption of consumers. When the planner puts sufficient weight on consumer welfare and returns are misaligned , the planner will choose to produce only short assets and claims and give all of their pledgeable return to consumers. More generally, except in a knife edge case, either only short or only long assets are produced. This is described in Proposition 3

Proposition 3. *When consumers have linear utility of consumption, and the planner chooses assets and consumption:*

- If $\beta > \frac{1}{2}$, then:
  - If $\beta\gamma_S R + (1-\beta)(1-\gamma_S) R > \beta\gamma_L X + (1-\beta)(1-\gamma_L) X$, (which requires misaligned returns $\gamma_S R - \gamma_L X > 0$) [11] then $z_S = 1$ and $C_1^e = C_1^l = \frac{\gamma_S R}{\eta}$,
  - If $\beta\gamma_S R + (1-\beta)(1-\gamma_S) R < \beta\gamma_L X + (1-\beta)(1-\gamma_L) X$, then $z_S = 0$. In this case, $C_1^e = C_1^l = 0$, $C_2^l = \frac{\gamma_L X}{\eta q}$,
  - If $\beta\gamma_S R + (1-\beta)(1-\gamma_S) R = \beta\gamma_L X + (1-\beta)(1-\gamma_L) X$, then any z_S and z_L satisfy $z_S + z_L = 1$ is a solution and $\eta((1-q)C_1^e + qC_1^l) = z_S\gamma_S R$, and $\eta q C_2^l = (1-z_S)\gamma_L X$.
- If $\beta = \frac{1}{2}$, then the objective function becomes $z_S(R-X)$ so that $z_S = 0$ and $z_L = 1$. In this case, $C_1^e = C_1^l = 0$, and $\forall C_2^l \leq \gamma_L X$ is a solution.
- If $\beta < \frac{1}{2}$, then $z_S = 0$ and $C_1^e = C_1^l = C_2^l = 0$, and only the producers consume.

When early and late consumer consumption are perfect substitutes, there is no special demand for short term claims. Short claims and assets are only produced when it is difficult to pledge long term payments to consumers and when the planner cares more about consumers than producers. Of course, if some level of date 1 consumption by early consumers is more valuable ex-ante than consumption by late consumers at date 2, implying that they are imperfect substitutes, there is a social value to producing short claims. Concavity of the consumer's utility $U(\cdot)$ will generate such imperfect substitutability As $U(\cdot)$ becomes more concave, early and late consumption are less substitutable. We now turn to this.

4.1.2 The General Case with Concave Utility and Inada conditions

When consumer utility is concave and Inada conditions, $U'(C) \to \infty$ as $C \to 0$ and $U'(C) \to 0$ as $C \to \infty$ hold, then if $\beta > 0$, optimal consumption sets the consumer marginal rate of substitution equal to the the planer's social marginal rate of transformation (which incorporates all of the constraints). Proposition 4 describes the allocations.

Proposition 4. *When consumer utility is concave and Inada conditions, $U'(C) \to \infty$ as $C \to 0$ and $U'(C) \to 0$ as $C \to \infty$ hold, and the planner chooses asset allocations and consumption:*

1. *If only consumer welfare matters and $\beta = 1$:*
   (a) *If returns are misaligned, the solution is all short assets and claims: $C_1^e = C_1^l = \frac{\gamma_S R}{\eta}, C_2^l = 0, z_S = 1$.*
   (b) *If aligned, the solution is: $C_1^e > 0, C_1^l = 0, C_2^l > C_1^e, z_S \in (0,1)$*

[11] This can easily seen by rearranging this inequality, which becomes $(2\beta-1)(\gamma_S R - \gamma_L X) > (1-\beta)(X-R)$.

2. *If only producer welfare matters and* $\beta = 0$*, the solution is:*
   $C_1^e = C_1^l = C_2^l = 0, z_S = 0.$

3. *If both matter to welfare and* $\beta \in (0, 1)$*, then:*

   (a) *If returns are misaligned, solutions are:*

      i. $C_1^e = C_1^l = \frac{\gamma_S R}{\eta}, C_2^l = 0, z_S = 1$*, (if long pledgeability binds more than short)*[12]*,*

      ii. $C_1^e > 0, C_1^l > 0, C_2^l > 0$*,* $C_1^l + C_2^l = C_1^e, z_S \in (0, 1)$*,(if long and short pledgeability both bind similarly), or*

      iii. $C_1^e > 0, C_1^l = 0, C_2^l > C_1^e, z \in (0, 1)$*. (if short pledgeability binds more than long).*

   (b) *If returns are aligned,*$\gamma_S R - \gamma_L X < 0$*, then*
      $C_1^e > 0, C_1^l = 0, C_2^l > C_1^e, \qquad z_S \in (0, 1).$

Proof. See Appendix A.6. □

There are several notable features of the planner's solution. The pledgeability constraint (PLS) on date 1 consumption (short claims) always binds with $\beta > 0$, because the Inada condition guarantees some consumption by early consumers and producers always prefer lower cost late consumption. Proposition 4 shows that consumer allocations are of one of three types: 1) all short: $C_1^e = C_1^l > 0$ and no date 2 consumption, 2) equal total consumption of early and late consumers, with $C_1^e > 0, C_1^l > 0, C_2^l > 0$, $C_1^l + C_2^l = C_1^e$ and 3) higher total consumption of late consumers and no early consumption of late consumers, $C_1^e > 0, C_1^l = 0, C_2^l > C_1^e$. The planner can chose z_S to implement each of these consumption levels.

Intuitively, the only reason to produce short assets is to produce short claims and only if the welfare weight on consumers is positive. If all the welfare weight is on consumers, then the production of assets is all short if returns are misaligned. If aligned and all weight on consumers, all of the pledgeable output of short assets goes to early consumers and some or all of the pledgeable output of long assets funds consumption by late consumers. Thus the pledgeable output of short assets is used to finance late consumer consumption ($C_1^l > 0$) only if returns are misaligned.

It turns out, as we show in Appendix A.7, that these allocations, including the ones with linear preferences, can be implemented by the planner offering consumers traded financial

[12] Implying the shadow price of the PLL constraint is higher than that of the PLS constraint – see proof in appendix.

claims, assigning returns on each claim (that might differ from producer-chosen returns in the competitive case) but allowing consumers to choose allocations at date 0 and trade at date 1. The only exception is the extreme case of only long asset production and claims (which required linear preferences). Implementation is still possible approximately here by allocating consumers an infinitesimal amount of short financial claim. This suggests that it is not consumer trading per se that creates a wedge between the decentralized and the socially optimal outcomes, but producer incentives to invest in assets as well as create associated financial claims that maximize their own utility.

4.1.3 The competitive outcome and its relationship to the planner's solution

There are then three sources of differences between the competitive outcomes and the planner's solution. First, producers choose how much of each asset produced to allocate to consumers when issuing claims, and competition forces them to to promise all the pledgeable part of any asset produced whenever they earn rents. This choice may differ from what the planner would choose. Second, and relatedly, in the competitive case the prospect of date-1 trading influences consumer allocations to claims and thus post-trade consumption, as well as producer allocations to assets, that will typically further differ from the planner's solution. The effect of trading is similar to that in Jacklin (1987), Allen and Gale (2004) and Farhi et al. (2009) with the additional twist that it also further affects producer allocations. Finally, production is determined by producer incentives in the competitive case. While the planner solution is obtained respecting pledgeability constraints, the weight put on consumers determines whether the planner cares more about pledgeable output relative to total output (including non-pledgeable output). In the competitive solution, upfront financing allows producers, who determine allocations, to partly internalize non-pledgeable output, though financing rents distorts this choice. Let us now examine the situations where the two solutions are similar and where they are different.

If returns are aligned, then the capital allocation at date 0 by competing producers given the demand for claims by consumers almost surely does not implement the planner's optimum. To see this, when returns are aligned consumers are always offered higher returns to maturity on long claims than on short, and market clearing leads to $\theta = q$ (each consumer buys a fraction q of long claims). This is necessary for all of the short claims to go to early consumers and all the longs to late consumers when they later trade. ‘

The distortion in consumer demand with aligned returns is similar to that in Jacklin (1987), describing the effects on trading on implementing the optimal allocations of Diamond and Dybvig (1983), where consumers have direct access to claims with $r^S_{01} = R$ and $r^L_{02} = X$. In that case, consumers can trade at date 1 and each will choose the portfolio of claims at date 0 with the highest market value at date 1. They will then trade at date 1 to get short claims if early and long claims if late, with long asset buyers getting a date-1 return of $r_{12}^L = \dfrac{X}{R} > 1$ when consumers optimally choose $\theta = q$ at date 0. This implies (as we showed earlier) that the ratio of late to early consumer consumption will be $\dfrac{X}{R}$. The market sets the ratio of the consumptions of late to early consumers equal to the marginal rate of transformation while a planner would equate consumers' marginal rate of substitution,$\dfrac{U'(C_1^e)}{U'(C_2^l)}$,to it. These are identical only if $U'(C) = \dfrac{1}{C}$, or $U(C) = log(\mathrm{C})$. In our model where competing producers produce claims, the outcome is identical to Jacklin (1987) if both long and short pledgeability are equal to one. In this case, if short and long term assets are very close substitutes (consumers have close to linear utility of consumption), the competitive solution offers too much investment in short-term assets and if they are very poor substitutes (consumers have high concavity) the market offers too little.

Given consumer demand for financial claims and the pledgeability of returns, producers choose an asset allocation to maximize their profit and this is the same for all consumer preferences. As a result, the competitive choice of z_S does not depend on consumer concavity but the socially optimal allocations do.

Unlike Jacklin (1987), when pledgeabilities are less than one, then it is not just consumer demand, but also producer incentives to supply claims, driven by the non-pledgeable return of each asset that affects the equilibrium.

When returns are aligned, we have seen that competitive producers' incentives in both the illiquid long region and the no long rents region do not depend on the concavity of consumer preferences but on the fixed consumer allocation and relative pledgeabilities, which are independent of preferences. Almost surely, this means that for a fixed weight on consumer welfare, there is at most one level of concavity where these are the socially optimal levels of consumption and production when the planner can choose production. The planner's problem cannot therefore be implemented in the decentralized solution.

Interestingly, when returns are misaligned, it is possible that competitive allocations are also constrained socially optimal. For sufficiently high planner weight on consumers, the planner then chooses all short assets and claims for all levels of concavity. This is also the competitive allocation when there is short dominance, which itself requires misaligned returns. The other competitive case requiring misaligned returns is the short glut, where both long and short claims offer the same returns to consumers. This is not the planner's choice if the welfare weight on consumers is sufficiently high because consumers prefer all short assets and claims. However, for higher weight on producers, this is the planner's optimal allocation.

Developing economies, where returns are misaligned, therefore in many cases have Pareto efficient allocations because the pledgeability constraints themselves dictate the allocations (essentially, supply constraints on claims are binding). Trade in assets at date 1 is not driven by pecuniary externalities: short assets are held at the margin for their value in use rather than for a different value in trade. This is true when the competitive allocations are short dominance or short glut. No intervention is needed in short dominance. If the planner puts enough weight on producers, the short glut is efficient (and is Pareto optimal in any case, as it sits on the frontier).

In other cases, including all cases with aligned returns, short assets are valued for their use in trade at the margin, and the prospect of trade requires that the date 1 market value of the consumption bundle of early consumers equals that of late consumers. Here, as we have argued above, the competitive outcome can differ substantially from the social optimal. Finally, for enough weight on producer welfare, there is overinvestment in shorts in both illiquid long and no long rents, because the planner desires almost exclusively longs.[13]

4.1.4 Effects of Consumer outside options, or changes in pledgeability

We assume that consumers cannot produce assets, and their outside option at date 0 is zero. If instead, they had an outside option to invest at a positive return below that of producers, then producers could benefit from increased pledgeability whenever they cannot produce claims that match a consumer's outside option. In circumstances where consumer outside options do not bind (and competition leads to higher returns), then our previous results apply. In particular, producers can only benefit from higher pledgeability in the case of a short glut (where long claims need more capital to compete with shorts).

Finally, suppose that the planner can change pledgeability. In the basic planner's problem where the planner can choose the allocation of investment between maturities, increasing the pledgeability of an asset can never hurt consumers.[14] The more interesting question is what would happen if a planner or government could choose pledgeability, while leaving the

[13]The basic planner's problem gives producers an outside option of zero. If we impose a producer outside option of using their own endowment and not creating financial claims, a payoff of $(1-\eta)X$, and this binds, this forces the planner to choose a set of allocations and production decisions which would be chosen with a higher welfare weight on producers (a lower β). Because producers are not subject to pledgeability constraints for their own consumption, this is the only effect. This leads to more long production and lower returns to consumers. The competitive solution always give producers at least this outside option.

[14]This can be easily proved by contradiction. An increase in pledgeability still means the old allocations are feasible. If consumers are strictly worse off, it must be that producers are strictly better off. But producers consumption from short remains unchanged if the increased pledgeability was from long or decreases if the increased pledgeability was from short. This implies that producer consumption from long must increase, which, implies that the PLL must be slack before the increased pledgeability. According to Proposition 4, it must be that $C_1^l = 0$, and $u'(C_2^l) = (1-\beta)/\beta$, a contradiction.

decentralized outcome to then emerge. We explore that shortly.

5 The Political Economy of Financial and Credit Development

Institutional developments, including improvements in pledgeability, are often seen as universally beneficial, providing society with more tools, contractability, and commitment ability, thus enhancing economic growth and well-being. With competitive producers choosing capital allocation and returns, changes in pledgeability may affect producer choices and the returns offered to consumers, as we have seen. In this context, financial and credit development may not benefit everyone or even society as a whole. The preferences of the planner/government who chooses financial and credit development, whether they are oligarchic and producer friendly (β =0) or democratic and consumer friendly (β =1), matter. [15] Importantly, any government with fixed preferences might foster or hinder financial development at various stages, suggesting no smooth path to financial development.

5.1 Technologies: short term vs long term

Table 1: Effects of Increases in Long- and Short-term Pledgeability

(a) Long Pledgeability

$\gamma_L \uparrow$	Consumer	Producer	Long Production	Total Consumption
Short Dominance	0	0	0	0
Short Glut	−	+	+	+
Illiquid Long	+	−	−	−
No rents	0	0	0	0

(b) Short Pledgeability

$\gamma_S \uparrow$	Consumer	Producer	Long Production	Total Consumption
Short Dominance	+	−	0	0
Short Glut	+	−	depends	depends
Illiquid Long	+	−	+	+
No rents	+	0	+	+

Table 1 compares the various cases. An improvement in short pledgeability (credit development) always increases consumer welfare while decreasing or leaving unchanged producer

[15]There is a literature on the political economy of financial development (see, for example, Haber (1997); La Porta et al. (1998); Roe (1996); Rajan and Zingales (2003); Rajan (2009)).

welfare, hence is always favored by democratic governments. Outside the short glut region, this typically leads to an increase in total output (and therefore overall welfare). The main effect is that producers can allocate more capital to higher-return long assets, economizing on capital for the lower-return short assets.

In contrast, improved long pledgeability often reduces producers' incentive to create long, output-enhancing assets. It also decreases the amount of producer capital needed per unit of long asset. This creates a tradeoff between rents and financeability, which typically reduces or leaves unchanged producer welfare. However, there's an exception in the short glut region. Here, increasing long pledgeability from low levels can benefit producers by allowing larger consumer allocations to long claims and making long asset production more attractive to producers. The bottom line, however, from Table 1 is that if financial development requires small incremental steps, no government with fixed preferences ($\beta = 0$ or $\beta = 1$) will favor it through all regions.

5.1.1 Short dominance: Primitive economy and the possibility of development traps

In underdeveloped or primitive societies, short-term pledgeability typically greatly exceeds long-term pledgeability. This misalignment often leads to an economy dominated by short-term production, focusing on low-return primary sector goods. The appropriable returns from long-term investment appear relatively low for both consumers and producers. This situation is more pronounced when producers have little capital compared to consumers. The low returns from short-term production make it difficult for producers to accumulate capital, even in a dynamic setting. Moreover, the absence of long-term production provides little incentive to improve corporate governance and long-term pledgeability.

The path of institutional development in this scenario depends on who holds power. In an oligarchy controlled by producers, development may stagnate. In a consumer-led democracy, development might focus solely on enhancing short-term credit, potentially creating a skewed system.

These implications align with historical observations (see, for example, Braudel (1986)). Early Western capitalism, for instance, saw entrepreneurs concentrating on trading short-term production rather than investing in capital-intensive, long-term projects. Similarly, in underdeveloped economies, entrepreneurs often gravitate towards lower-return commerce instead of complex manufacturing, reflecting an environment of low producer capital and minimal long-term pledgeability.[16] Apart from technological development, our model suggests

[16]Of course, institutions can also be weak on the real side. Long, high return production may suffer from a lack of property rights enforcement – complex fixed assets may need more security – which may reduce

the shift from commerce toward manufacturing required (1) producers to become relatively richer (for instance, as a result of the steady accumulation of business profits or as a result of windfalls that benefited the adventurous producer class) (2) the relative pledgeability of long versus short assets to increase, say as a result of other institutional developments.

5.1.2 Short glut region: Developing country and oligarchic development

In developing economies with higher potential returns from long-term investment but low long pledgeability and moderate short-term pledgeability, both forms of production coexist in a "short glut" region. Increasing long pledgeability here improves overall welfare by boosting long-term production. This occurs because more consumer capital is drawn to long-term financing, enhancing producer rents from both long and short production.

However, producers and consumers have opposing views on increasing long-term pledgeability. Producers favor it as they can sell more financial claims at higher prices, while consumers dislike it due to lower returns. The opposite is true for increases in short-term pledgeability.

The type of government significantly influences development in this region. An oligarchy, controlled by producers, is likely to enhance long pledgeability, increasing long-term production and producer rents at the expense of consumers. In contrast, a consumer-oriented democracy tends to boost short pledgeability, potentially reducing overall output but benefiting consumers.

5.1.3 Illiquid long region with producer rents: The Middle Income Trap

As long pledgeability increases, moving the economy into the "illiquid long with rent" region, producers lose interest in further pledgeability improvements of either type. Consumer allocations become fixed, eliminating the financing benefits of enhanced pledgeability for producers while still reducing their financing rents. This situation can lead to a "middle income trap" if producers control the government, halting further financial and credit development.

Consumers, however, would still benefit from greater pledgeability. In a democracy, they might implement such changes, but this could reduce overall welfare if producers shift away from long-term production. This scenario suggests that financing rents, in addition to other monopoly rents, contribute to producer opposition towards reforms in middle-income economies.

their returns relative to short duration production.

A transition from oligarchy to democracy in this economic state would likely boost financial and credit development, benefiting consumers at the expense of producers. The impact on overall output would depend on which type of pledgeability is enhanced: negative for long pledgeability increases, but positive for short pledgeability improvements.

5.1.4 No long rent region: The absence of conflicts

When both long-term and short pledgeability reach high levels, pushing the economy into the "no long rent" region, the dynamics of financial and credit development change significantly. In this region, further increases in long pledgeability have no effect on consumer, producer, or overall welfare. However, improvements in short pledgeability continue to enhance both consumer and total welfare. Increased short pledgeability results in a Pareto superior outcome because it allows shorts to be funded with a bigger fraction of consumer capital. Consumers are the ones who benefit from short-term claims.

The key feature of this region is the reduction of conflicts of interest over financial development. No group opposes higher pledgeability, regardless of its type. This harmony occurs because the distortionary financing rents, which previously influenced allocations and rent-sharing, are eliminated in the " no long rent" region.

5.1.5 Finally...

When producer capital significantly outweighs consumer capital, producers invest enough in each asset to reduce financing rents. Their production choices then primarily reflect intrinsic returns and consumer preferences, even with modest financial development. In this scenario, all economic agents become more supportive of increased pledgeability.

This analysis suggests that financial development becomes easier for more developed countries for two main reasons. First, wealthier producers compete away financing rents. Second, beyond a certain threshold, financial development itself reduces financing rents and associated conflicts of interest, moving the system into a "no rent" equilibrium.

However, transitioning to this region from other regions is challenging. Our model highlights the complex interplay between economic development, wealth distribution, and financial structures, underscoring the difficulties countries face in achieving sustainable financial progress.

6 Related Literature

There is a large literature on limited pledgeability and the role of the net worth of producers in facilitating investment. Important studies include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Hart and Moore (1994) and Holmström and Tirole (1998). A bit closer to our model is the literature on financial intermediary capital, where some assets are best held by financial intermediaries and their net worth determines if they are able to hold the asset which helps determine the asset's price. Key work in this area includeHe and Krishnamurthy (2013), Holmstrom and Tirole (1997), and Rampini and Viswanathan (2019). These models focus on how low intermediary capital prevents an institution from providing its important service (monitoring or superior collateralization). Our focus, instead, is on the impact of low intermediary capital (our producers are best thought of as a fusion of producer and financial intermediary) on the relative profitability of assets with different horizons, which could be thought of as the vehicles to provide the services.

In prior work, we allow pledgeability to be an endogenous choice of corporations, and study how industry liquidity can affect it (Diamond et al., 2020a,b, 2022) . Our focus here is on how economy wide changes in pledgeability affect outcomes, and hence the incentives to change it.

Most closely related are previous studies that examine investment in assets which vary in their pledgeability but have identical maturity. Our model has similarities to Matsuyama (2007), who examines an economy where indivisible projects have misaligned returns – higher productivity projects have lower pledgeability. Producer capital really matters now, since projects need more own-financing to be undertaken. When producer capital is low, more pledgeable but low return projects are undertaken because they require less producer capital, but this ensures producer capital does not grow, suggestive of a poverty trap. Conversely, a producer with more capital can undertake more productive projects, funding the shortfall given their low pledgeability with own capital, generating higher future capital. Higher producer capital therefore implies higher productivity and growth. In Matsuyama (2007), the most attractive project, taking into account both productivity and pledgeability, attracts all the funding. So undoubtedly, an improvement in the pledgeability of the most productive project must improve its chances of being undertaken, and hence overall productivity. However, an improvement in the pledgeability of less productive projects can also improve their chances of being undertaken, in this case reducing productivity. So financial development is not always good.

Unlike Matsuyama (2007), we allow for both types of projects to be undertaken simultaneously, and for project maturity to also matter. We show that high productivity long term projects with higher-than-short pledgeability may still coexist with short projects, with the latter valued for liquidity. Unlike Matsuyama (2007), we also show that an increase in the pledgeability of the high productivity long project can reduce welfare because producers produce less of it given their diminished rents from financing. Conversely, an increase in the pledgeability of the lower productivity short project can improve welfare because the economy can generate the needed liquidity with fewer low productivity projects. The difference in our results derives, of course, from differences in our models.

In a dynamic model which shares features with ours, Ebrahimy (2022) examines the choice of producer investment when producers have the choice between high return low pledgeability projects and low return high pledgeability projects. Unlike us, he does not allow investors to differ in their consumption preferences, or for projects to differ by maturity, and hence for investors to have a choice between claims of different maturity. Ebrahimy (2022) shows that an increase in the pledgeability of the low return project, a form of financial development, can move the economy away from the social optimum, as more is invested in the more pledgeable but lower return project. However, an increase in the pledgeability of the high return project tends to attract more investment to it, which is the case in our model only when the returns to maturity on long and short financial claims are equal (short glut region).

We examine the market supply of financial claims when consumers have a demand for liquidity. The decentralized market allows claims to be reallocated when consumers learn their individual need for liquidity. We view trade in markets as fundamental to this reallocation and the incentives of differentiated producers and assets with limited pledgeability as essential. Our model is related to Diamond and Dybvig (1983) which assumed that there is a mechanism (interpreted as a bank) which can interact with consumers at all dates without the need for trade in the market. Diamond and Dybvig (1983) assume that assets are fully pledgeable and that competition forces the optimal bank to pass through all of the returns from investment to consumers. Because utility of consumption is sufficiently concave and long assets return more than short ($X > R$ in their and our notation), there is always a mix of short and long claims offered to allow liquidity insurance to consumers with private information about their type. In addition, there is no need for trade in markets (no need for trade and a mechanism that can interact with all consumers on each date) and the pecuniary externalities of such trade impose constraints on what bank can offer. This constraint imposed by possible trade was first pointed out by Jacklin (1987) and more completely characterized in Allen and Gale (2004). Farhi et al. (2009) characterize interventions (required minimum holdings of short assets) which overcome the distortions on liquidity insurance imposed by the prospect of trading assets). They assume that assets can be only be held by those who are subject to the intervention, allowing the aggregate holding of short assets to be regulated.

Our analysis differs from these four studies in several dimensions. Most importantly, limited pledgeability and the limits this imposes on the passing through of returns to consumers introduces a supply constraint on claim production. This allows the possibility of misaligned returns, where less profitable shorts offer higher pledgeable returns to consumers for a given allocation of capital. In addition, we view the possibility of trade as essential to the reasons that short claims are beneficial, because they can be used for other trades (similar to a means of payment) and they can be acquired by selling long claims. Rather than an added restriction on the creation of liquid claims, we view the possibility of trade is part of the benefit of short and liquid claims.

7 Conclusion

This paper examines how financial and credit development, through improved pledgeability of returns, affects production decisions and welfare in a market economy with distinct producer and consumer groups. We find that increased pledgeability does not always lead to higher output or welfare. In certain equilibrium regions, improving long pledgeability can actually reduce long-term production and welfare. The effects of financial development depend critically on the existing level of development and the relative scarcity of producer capital.

Our model implies important conflicts of interest over financial development between producers and consumers. This dynamic helps explain why economies may face impediments to financial development and growth, especially when producer capital is scarce. Interestingly, our results suggest that financial development becomes easier and faces less opposition at higher levels of development. This is partly because financing rents diminish and conflicts of interest abate as the economy progresses, creating a form of virtuous cycle in advanced stages of development.

For given levels of financial and credit development, the nature of possible inefficiencies from competitive choices of real assets and financial claims differ depending on the level of development. At low levels of development leading to misaligned returns, the competitive allocations are efficient, but the economy is not very productive. The way forward is improved development, but the conflicts of interest due to its redistributive effects are a major impediment. At high levels of development, the problem becomes possibly inefficient allocation of capital resulting from the profitability of producing claims priced for their value from trading profits rather than their value in consumption. By providing a more nuanced understanding of the complex relationships between pledgeability, production decisions, and welfare, this paper contributes to ongoing debates about the role of financial development in economic growth.

There is ample scope for further research. For instance, is there scope to improve the decentralized outcome for developing countries using financial institutions like banks and by postponing the emergence of financial markets (or increasing the transaction costs of accessing them)? What should pledgeabilities be set at in such situations?

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A Appendix

A.1 Conditions for all cases

We derive conditions for the various regions to exist if γ_L is allowed to vary.

1. $X < \frac{1-\eta q}{1-\eta}(1-\gamma_S)R$. There is not a *no long rent* region.
   (a) $\gamma_L \in [0, \frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}]$: short dominance
   (b) $\gamma_L \in [\frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}, \underline{\gamma}_L]$: short glut
   (c) $\gamma_L \in [\underline{\gamma}_L, 1]$: illiquid long.
2. $X > \frac{1}{1-\eta}(1-\gamma_S)R$. There is not a *short dominance region*
   (a) $\gamma_L \in [0, \underline{\gamma}_L]$: short glut
   (b) $\gamma_L \in [\underline{\gamma}_L, \frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta+\eta q)X-(1-\gamma_S)R}]$: illiquid long
   (c) $\gamma_L \in [\frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta+\eta q)X-(1-\gamma_S)R}, 1]$: no long rent
3. $\frac{1-\eta q}{1-\eta}(1-\gamma_S)R < X \leq \frac{1}{1-\eta}(1-\gamma_S)R$. All four regions exist
   (a) $\gamma_L \in [0, \frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}]$: short dominance
   (b) $\gamma_L \in [\frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}, \underline{\gamma}_L]$: short glut
   (c) $\gamma_L \in [\underline{\gamma}_L, \frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta+\eta q)X-(1-\gamma_S)R}]$: illiquid long
   (d) $\gamma_L \in [\frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta+\eta q)X-(1-\gamma_S)R}, 1]$: no long rent

First, we prove equilibrium existence and uniqueness. Second, we establish conditions for the existence of each case. These two steps finish the proofs for Proposition 1 and 2.

Short Dominance

1. The price $p_S = \eta \geq 1-(1-\gamma_S)\frac{R}{X}$ implies that
$$\frac{(1-\gamma_S)R}{1-\eta} \geq X. \tag{7}$$
   Note that this condition is sufficient to guarantee that $p_S \geq \gamma_S$.
2. The condition of a shadow p_L requires
$$\frac{\gamma_L X}{\gamma_S R/p_S} \leq 1 - \frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1-p_S) \Rightarrow \gamma_L \leq \frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}.$$

Short glut

We know from Proposition 1 that $X(1-\gamma_L) > R(1-\gamma_S)$ must hold so that

$$\gamma_L < 1 - \frac{R}{X}(1-\gamma_S)$$

This condition implies $\gamma_L < \gamma_S$, $p_L < p_S$, and $\gamma_L X < \gamma_S R$. These results come from

$$\frac{(1-\gamma_L)X}{1-p_L} = \frac{(1-\gamma_S)R}{1-p_S}.$$

The condition $X(1-\gamma_L) > R(1-\gamma_S)$ implies $1-p_L > 1-p_S$ and equivalently $p_S > p_L$. Note that $\frac{p_S}{p_L} = \frac{\gamma_S R}{\gamma_L X}$, so that $\gamma_L X < \gamma_S R$. Given $X > R$, it must be that $\gamma_L < \gamma_S$.

Moreover, we know

$$p_S = \frac{\gamma_S}{X} \frac{X(1-\gamma_L) - R(1-\gamma_S)}{\gamma_S - \gamma_L}$$
$$p_L = \frac{\gamma_L}{R} \frac{X(1-\gamma_L) - R(1-\gamma_S)}{\gamma_S - \gamma_L}.$$

1. $\theta = \frac{p_L(p_S-\eta)}{\eta(p_S-p_L)} \in [0,1]$ requires $\eta \le p_S$ and $\eta \ge p_L$.

2. $y_L = \frac{\eta\theta(1-p_L)}{(1-\eta)p_L} \in [0,1]$, which requires $\frac{1-\eta}{\eta} \ge \frac{1-p_L}{p_L}\theta$ and $p_L < 1$. The first condition becomes

$$\frac{1-\eta}{\eta} \ge \frac{(1-p_L)(p_S-\eta)}{\eta(p_S-p_L)}$$

   which simplifies to $\eta \ge p_L$. This is redundant given the first constraint. The second constraint becomes
$$\frac{\gamma_L}{R} \frac{X(1-\gamma_L) - R(1-\gamma_S)}{\gamma_S - \gamma_L} < 1 \Rightarrow \gamma_L X < \gamma_S R$$
   which always holds under Proposition 1.

3. $p_L \ge \gamma_L$ (and $p_S \ge 1-(1-\gamma_S)\frac{R}{X}$). The first simplifies into

$$(X-R)(1-\gamma_L) \ge 0,$$

   which always holds. The second simplifies into

$$\gamma_S \ge \gamma_S - \gamma_L,$$

   which also always holds.

4. $\frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}} \ge \gamma_L X$, which becomes $\theta \le q$. This is stronger than the first condition.

To summarize, beyond $R(1-\gamma_S) < X(1-\gamma_L)$, we only need conditions such that $\theta \in [0,q]$, which becomes

1) $\eta \leq p_S$ and 2) $\frac{1-\eta}{\eta} \leq q\frac{1-p_L}{p_L} + (1-q)\frac{1-p_S}{p_S}$. We know

$$\frac{1-p_S}{p_S} = \frac{1-\gamma_S}{\gamma_S}\frac{\gamma_S R - \gamma_L X}{X(1-\gamma_L) - R(1-\gamma_S)}$$
$$\frac{1-p_L}{p_L} = \frac{1-\gamma_L}{\gamma_L}\frac{\gamma_S R - \gamma_L X}{X(1-\gamma_L) - R(1-\gamma_S)}$$

The first condition simplifies to

$$\gamma_S(X-R) \geq \left[\frac{\eta}{1-\eta}(1-\gamma_S) - \gamma_S\right](\gamma_S R - X\gamma_L).$$

- If $\frac{\eta}{1-\eta}(1-\gamma_S) - \gamma_S \leq 0 \Rightarrow \gamma_S \geq \eta$, this condition is redundant.
- If $\frac{\eta}{1-\eta}(1-\gamma_S) - \gamma_S < 0 \Rightarrow \gamma_S < \eta$, then we need

$$\gamma_L \geq \frac{\gamma_S}{X}\left(R - \frac{(X-R)}{\frac{\eta}{1-\eta} - (\frac{1}{1-\eta})\gamma_S}\right) \Rightarrow \gamma_L \geq \frac{\gamma_S}{X}\frac{(1-\gamma_S)R - (1-\eta)X}{\eta - \gamma_S}$$

  We can show that the RHS falls below $1 - \frac{R}{X}(1-\gamma_S)$. Note that if $(\frac{1}{1-\eta})(1-\gamma_S)R < X$ holds, so that (7) is violated, then the condition above is redundant.

The second condition simplifies to

$$X\left(\eta(1-q) - \gamma_S\right)\gamma_L^2 + \gamma_S\left(R(\eta q - 1 + \gamma_S) + (1-\eta+\eta q)X\right)\gamma_L - qR\eta\gamma_S^2 \leq 0$$

We know the LHS is negative for $\gamma_L = 0$. If we evaluate the LHS at $\gamma_L = 1 - \frac{R}{X}(1-\gamma_S)$, we get

$$\frac{\frac{\eta}{1-\eta}(X-R)(1-\gamma_S)\left((1-q)(X-R) + R\gamma_S\right)}{X} > 0.$$

If we evaluate the LHS at $\gamma_L = \frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}$, we get

$$-\frac{\eta q(X-r)^2(1-\gamma_S)\gamma_S^2}{X(\eta-\gamma_S)^2} < 0.$$

Define $\underline{\gamma}_L \in \left(\frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}, 1 - \frac{R}{X}(1-\gamma_S)\right)$ be the unique root that solves

$$X\left(\eta(1-q) - \gamma_S\right)\gamma_L^2 + \gamma_S\left(R(\eta q - 1 + \gamma_S) + (1-\eta+\eta q)X\right)\gamma_L - qR\eta\gamma_S^2 = 0.$$

- If $(\frac{1}{1-\eta})(1-\gamma_S)R < X$, then we need
$$\gamma_L \in [0, \underline{\gamma}_L].$$
- Otherwise, we need
$$\gamma_L \in \left[\frac{\gamma_S}{X}\frac{(1-\gamma_S)R - (1-\eta)X}{\eta-\gamma_S}, \underline{\gamma}_L\right].$$

Illiquid long with rent

Simple calculation shows that the equilibrium reduces to a quadratic equation on y_L:

$$(X(1-\gamma_L) - R(1-\gamma_S))\, y_L^2 + \left[\frac{\eta}{1-\eta}\left(qX(1-\gamma_L) + (1-q)R(1-\gamma_S)\right) - \left(X(1-\gamma_L) - R(1-\gamma_S)\right)\right] y_L - \frac{\eta}{1-\eta} qX(1-\gamma_L) = 0.$$

In equilibrium, both $(1-\gamma_L)X > (1-\gamma_S)R$ and $(1-\gamma_L)X < (1-\gamma_S)R$ can hold. In the first case, $p_L < p_S$, and $y_L > q$. In the second case, $p_L > p_S$, and $y_L < q$. By evaluating the LHS of the above equation, we know that the value is negative at $y_L = 0$. At $y_L = 1$, the value is

$$\frac{\eta}{1-\eta}(1-q)R(1-\gamma_S) > 0.$$

Therefore, there exists a unique y_L that solves this equation.

1. $y_L \in [0,1]$. This is obviously satisfied.
2. $\theta = q \in [0,1]$ is always satisfied
3. $b_F = \frac{p_L \gamma_S R}{p_S} \leq \gamma_L X$, $p_L = \frac{q}{q + \frac{y_L(1-\eta)}{\eta}}$ and $p_S = \frac{1-q}{1-q+\frac{(1-y_L)(1-\eta)}{\eta}}$. The first condition simplifies into

$$y_L \geq \frac{q\left[\gamma_S R(1-\eta q) - (1-q)\eta\gamma_L X\right]}{(1-\eta)\left[(1-q)\gamma_L X + q\gamma_S R\right]}.$$

- If $\gamma_S R(1-\eta q) - (1-q)\eta\gamma_L X < 0 \Rightarrow \gamma_L > \frac{\gamma_S R(1-\eta q)}{X\eta(1-q)}$ so that the RHS is negative, this condition is redundant.
- If $\gamma_S R(1-\eta q) - (1-q)\eta\gamma_L X > 0$, then there are two cases:
  - If $X(1-\gamma_L) - R(1-\gamma_S) > 0$, then we need to plug in $\frac{q[\gamma_S R(1-\eta q)-(1-q)\eta\gamma_L X]}{(1-\eta)[(1-q)\gamma_L X + q\gamma_S R]}$ into the equation and the resulting number is negative.
  - If $X(1-\gamma_L) - R(1-\gamma_S) \leq 0$, then we also need to plug in $\frac{q[\gamma_S R(1-\eta q)-(1-q)\eta\gamma_L X]}{(1-\eta)[(1-q)\gamma_L X + q\gamma_S R]}$ into the equation and the resulting number is negative.
  - In both cases, when we plug in, we get the sign is equal to the sign of

    $$-\frac{1}{1-\eta}\left\{\gamma_L\gamma_S\left[R(\eta q - 1) + \eta q X + R\gamma_S + (1-\eta)X\right] - X\gamma_L^2(\eta(q-1)+\gamma_S) - \eta q R\gamma_S^2\right\},$$

    which is the same one as the short glut case. In order for this to be negative, we need

    $$\frac{1}{1-\eta}\left\{\gamma_L\gamma_S\left[R(\eta q - 1) + \eta q X + R\gamma_S + (1-\eta)X\right] - X\gamma_L^2(\eta(q-1)+\gamma_S) - \eta q R\gamma_S^2\right\},$$

    which requires $\gamma_L \geq \underline{\gamma}_L$.
- Combining the previous two cases, all we need is to have $\gamma_L \geq \min\{\underline{\gamma}_L, \frac{\gamma_S R(1-\eta q)}{X\eta(1-q)}\}$. We evaluate the LHS of the equation above at $\frac{\gamma_S R(1-\eta q)}{X\eta(1-q)}$ and the sign is the same as $\eta(1-q)X - (1-\eta q)\gamma_S R$. We know that the above equation is positive whenever $\gamma_S R(1-\eta q) - (1-q)\eta\gamma_L X < 0$, which implies $\underline{\gamma}_L = \min\{\underline{\gamma}_L, \frac{\gamma_S R(1-\eta q)}{X\eta(1-q)}\}$. Therefore, this case needs $\gamma_L \geq \underline{\gamma}_L$.

4. $p_S \geq 1-(1-\gamma_S)\frac{R}{X}$, $p_L \geq \gamma_L$. The two conditions become:

$$y_L \geq \frac{X-(1-\gamma_S)R\left[\frac{\eta}{1-\eta}(1-q)+1\right]}{X-(1-\gamma_S)R}$$

and

$$\frac{q\frac{\eta}{1-\eta}}{q\frac{\eta}{1-\eta}+y_L} \geq \gamma_L \Rightarrow y_L \leq q\frac{\eta}{1-\eta}\frac{1-\gamma_L}{\gamma_L}.$$

When we evaluate the LHS of the equation at $\frac{X-(1-\gamma_S)R\left[\frac{\eta}{1-\eta}(1-q)+1\right]}{X-(1-\gamma_S)R}$, we need it to be negative. When we evaluate the LHS of the equation at $q\frac{\eta}{1-\eta}\frac{1-\gamma_L}{\gamma_L}$, we need it to be positive. It turns out that both equations reduce to

$$\frac{\eta}{1-\eta}q\left(X-R(1-\gamma_S)\right) > \gamma_L\left((1+\frac{\eta}{1-\eta}q)X-(\frac{1}{1-\eta})(1-\gamma_S)R\right)$$
$$\Rightarrow \gamma_L < \frac{\eta q\left(X-(1-\gamma_S)R\right)}{(1-\eta)X+\eta qX-(1-\gamma_S)R}.$$

- If $X-(1-\gamma_S)R\left[\frac{\eta}{1-\eta}(1-q)+1\right] < 0$, the first condition is not needed, and $\frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta)X+\eta qX-(1-\gamma_S)R} > 1$. In this case, no further condition is needed.
- If $X-(1-\gamma_S)R\left[\frac{\eta}{1-\eta}(1-q)+1\right] \geq 0$, then we need $\gamma_L < \frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta)X+\eta qX-(1-\gamma_S)R}$.

To summarize, this case needs $\gamma_L > \underline{\gamma}_L$. If in addition,

$$(1-\eta q)(1-\gamma_S)R < (1-\eta)X$$

this case also needs

$$\gamma_L < \frac{\eta q\left(X-(1-\gamma_S)R\right)}{(1-\eta)X+\eta qX-(1-\gamma_S)R}.$$

Illiquid long no rent

We know in equilibrium $\theta = q$, $y_L = \frac{\frac{\eta}{1-\eta}q(1-\gamma_L)}{\gamma_L}$, $y_S = \frac{\frac{\eta}{1-\eta}(1-q)}{1-(1-\gamma_S)\frac{R}{X}}(1-\gamma_S)\frac{R}{X}$ and $b_F = \frac{\gamma_L\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}$.

1. $\theta \in [0,1]$ is always guaranteed.
2. $b_F \leq \gamma_L X$ can be shown simplified into $R \leq X$ so always holds.
3. $y_S \in [0,1]$, $y_L \in [0,1]$ and $y_S + y_L \in [0,1]$. $y_S \in [0,1]$ becomes

$$(1-\eta q)(1-\gamma_S)R < (1-\eta)X.$$

Note this condition does not require γ_L. $y_L \in [0,1]$ is less stringent than $y_L \leq 1-y_S$, which becomes

$$\gamma_L > \frac{\eta q}{\eta q+(1-\eta)-\frac{\eta(1-q)(1-\gamma_S)\frac{R}{X}}{1-(1-\gamma_S)\frac{R}{X}}} = \frac{\eta q\left(X-(1-\gamma_S)R\right)}{(1-\eta)X+\eta qX-(1-\gamma_S)R}. \quad (8)$$

We want to show that all four cases are possible when returns are misaligned ($\gamma_S R > \gamma_L X \Rightarrow \gamma_L < \gamma_S\frac{R}{X}$). It is sufficient to show the No Long Rent case exists, i.e, under $\gamma_L < \gamma_S\frac{R}{X}$, it can be that $\gamma_L \geq \frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta+\eta q)X-(1-\gamma_S)R}$.

This is equivalent to showing

$$\frac{\eta q\,(X-(1-\gamma_S)R)}{(1-\eta+\eta q)X-(1-\gamma_S)R} < \gamma_S\frac{R}{X},$$

which, after some simplification, becomes

$$(1-\gamma_S)\gamma_S\frac{R^2}{X} < (1-\eta)\gamma_S R - \eta q(X-R).$$

If $\eta \to 0$, this becomes

$$(1-\gamma_S)\gamma_S\frac{R^2}{X} < \gamma_S R \Rightarrow (1-\gamma_S)R < X,$$

which always holds. If $\eta \to 1$, this is always violated.

A.2 Comparative Statics with respect to γ_L

Proof of Proposition 1

Proof. In the case of short glut, we just showed

$$\begin{aligned} p_S &= \frac{\gamma_S}{X}\frac{(X(1-\gamma_L)-R(1-\gamma_S))}{(\gamma_S-\gamma_L)} \\ p_L &= \frac{\gamma_L}{R}\frac{(X(1-\gamma_L)-R(1-\gamma_S))}{(\gamma_S-\gamma_L)}. \end{aligned}$$

A producer's return must also be strictly above X, the return from retention. Therefore, $\frac{(1-\gamma_L)X}{1-p_L} > X \Rightarrow p_L > \gamma_L$. Therefore

$$p_L = \frac{\gamma_L}{R}\frac{(X(1-\gamma_L)-R(1-\gamma_S))}{(\gamma_S-\gamma_L)} > \gamma_L,$$

which is true only if $\gamma_S > \gamma_L$.[17] Given $\gamma_S > \gamma_L$, it must be that:

$$\begin{aligned} &(1-\gamma_S) < (1-\gamma_L) \\ \Rightarrow\ &(1-\gamma_S)R < (1-\gamma_L)X. \end{aligned}$$

Then, from producer indifference $\frac{1-p_L}{1-p_S} = \frac{(1-\gamma_L)X}{(1-\gamma_S)R}$, we know it must be that

$$1-p_S < 1-p_L \Rightarrow p_L < p_S.$$

Then, from consumer indifference $\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$, we know it must be that pledgeability and total returns are misaligned so $\gamma_L X < \gamma_S R$. In the short dominance region, γ_L is even lower so $\gamma_L X < \gamma_S R$ and $(1-\gamma_S)R < (1-\gamma_L)X$ must also hold in that case.

□

[17] If $\gamma_S < \gamma_L$, then cross-multiplying, it must be that $R(\gamma_S-\gamma_L) > X(1-\gamma_L)-R(1-\gamma_S)$ or $R > X$,which is impossible

Proof of Lemma 4

Proof. We know that

$$\frac{\partial p_S}{\partial \gamma_L} = \frac{-(1-\gamma_S)\gamma_S(R-X)}{X(\gamma_L-\gamma_S)^2}.$$

Given $R - X < 0$, we know that

$$\frac{\partial p_S}{\partial \gamma_L} > 0.$$

Because $\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$, this immediately implies that $\frac{\partial p_L}{\partial \gamma_L} > 0$, and also p_L must increase more than proportionately with γ_L for the equality to hold, so that $\frac{\partial(\gamma_L/p_L)}{\partial \gamma_L} < 0$. Given that

$$\theta = \frac{\frac{1-\eta}{\eta} - \frac{1-p_S}{p_S}}{\frac{1-p_L}{p_L} - \frac{1-p_S}{p_S}} \in (0,1),$$

we know that if p_L stays unchanged, the RHS would increase in γ_L. Now that $\frac{1-p_L}{p_L}$ decreases with γ_L, we know θ must increase in γ_L. The market clearing condition implies

$$y_S = \frac{\frac{\eta}{1-\eta}(1-\theta)(1-p_S)}{p_S}$$

must decrease in γ_L, implying that y_L increases in γ_L.

Both sides of the producer's equilibrium condition

$$\frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

go up with γ_L, given that p_S increases. Therefore, producer's profits Π increases with γ_L. We know that consumer welfare is

$$U = \frac{\gamma_S R}{p_S}$$

which decreases with γ_L. Finally, turning to total welfare, we can write

$$W = (\frac{\eta}{1-\eta}\theta + y_L)X + (\frac{\eta}{1-\eta}(1-\theta) + (1-y_L))R,$$

which increases in γ_L given both y_L and θ increase in γ_L.

□

Proof of Lemma 5

Proof. (i) From

$$\frac{(1-\gamma_L)X}{\frac{\eta}{1-\eta}(1-q) + (1-y_L)}(1-y_L) = \frac{(1-\gamma_S)R}{\frac{\eta}{1-\eta}q + y_L}y_L,$$

we know that when γ_L goes up, producer investment in the long asset, y_L, must go down. Consequently, $p_S = \frac{\frac{\eta}{1-\eta}(1-q)}{\frac{\eta}{1-\eta}(1-q)+(1-y_L)}$ falls with γ_L so that $\frac{\gamma_S R}{p_S}$ increases with γ_L. Now, let us turn to $\frac{\gamma_L X}{p_L}$. We are going to show this also increases. If p_S goes down with γ_L,the producer's cum-financing return on the short asset falls (the LHS of the producer's FOC below), so the cum-financing return on the long asset should also fall (the RHS of the FOC below).

$$\frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

This implies

$$\frac{d\frac{(1-\gamma_L)}{1-p_L}}{d\gamma_L} < 0 \Rightarrow -(1-p_L) + (1-\gamma_L)\frac{dp_L}{d\gamma_L} < 0 \Rightarrow \frac{dp_L}{d\gamma_L} < \frac{1-p_L}{1-\gamma_L}.$$

Meanwhile,

$$\frac{d\frac{\gamma_L}{p_L}}{d\gamma_L} = \frac{p_L - \gamma_L \frac{dp_L}{d\gamma_L}}{\gamma_L^2} > \frac{p_L - \gamma_L \frac{1-p_L}{1-\gamma_L}}{\gamma_L^2} > \frac{\frac{p_L}{\gamma_L} - \frac{1-p_L}{1-\gamma_L}}{\gamma_L} > 0.$$

The last inequality holds because $p_L \geq \gamma_L$. Therefore, both $\frac{\gamma_S R}{p_S}$ and $\frac{\gamma_L X}{p_L}$ increase with γ_L.

(ii) Consumer welfare is given by $U = (1-q)\,\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L}$, which clearly increases in consumer returns $\frac{\gamma_S R}{p_S}$ and $\frac{\gamma_L X}{p_L}$, and hence increases with γ_L. Turning to producer profits: $\Pi = \frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$ which falls in γ_L since the cum financing producer returns fall on either asset. Finally, total welfare

$$\eta U + (1-\eta)\Pi = X y_L + R(1-y_L) + \frac{\eta}{1-\eta}(qX + (1-q)R),$$

increases in y_L, and hence falls in γ_L.

□

Proof of Lemma 6

Proof. The other expressions are obvious. We supplement the expressions for welfare here. consumer welfare is

$$U = (1-q)\,\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L} = (1-q)\,\frac{\gamma_S R}{1-(1-\gamma_S)\,\frac{R}{X}} + qX.$$

Producer profits are $\Pi = X$.

□

A.3 Comparative Statics with respect to γ_S

Suppose that one is in the short glut region. For consumers to hold both claims after an increase in γ_S, $\frac{p_L}{p_S}(= \frac{\gamma_L X}{\gamma_S R})$, the ratio of fractions of consumer capital in longs relative to shorts, should fall. Think of this as relative financeability. At the same time, from the producer's perspective, $\frac{1-p_L}{1-p_S}(= \frac{(1-\gamma_L)X}{(1-\gamma_S)R})$ should increase. Think of this as relative producer rents. Both conditions can be met with a fall in p_L and a rise in p_S as γ_S rises.

If γ_S is low relative to γ_L (recall it cannot be too low for the economy to be in the region), an increase in γ_S will have more effect on relative financeability and little effect on relative producer rents. It makes sense for the producer to shift to producing more short assets, with consumers allocating more capital to short claims, away from long claims. Given that each unit of long releases more producer capital than each unit of short requires (recall $1-p_L > 1-p_S$ in this region), and vice versa for consumer capital, it must be that a disproportionate amount of consumer capital leaves longs, pushing down p_L. So returns to consumers from holding longs will increase in the new equilibrium. Of course, for producers to see a financing reason to shift allocations, it must be that p_S rises. Since in equilibrium, the consumer returns to holding shorts must rise to equal the returns to holding longs, $\frac{\gamma_S}{p_S}$ increases with γ_S.

As γ_S rises further, an increase in γ_S reduces relative producer rents significantly while not increasing relative financeability as much. The trade-off shifts. This is when the producer starts increasing long production with further increases in γ_S, which is why total welfare is non monotonic. So while each unit of short not produced allows less than one unit of long to be produced because the latter needs more producer capital, the released consumer capital has to pay both for the more pledgeable remaining short claims and the additional long claims. Given the limited consumer capital, consumer returns continue rising, as is true in the entire region.

Lemma 8. *In the short glut equilibrium, p_L decreases with γ_S, and $\frac{\gamma_S}{p_S}$ increases with γ_S, consumer welfare U increases with γ_S, producer profits Π decrease with γ_S. Total welfare $\eta U + (1-\eta)\Pi$ is non-monotonic in γ_S.*

Proof of Lemma 8

Proof. We know that

$$\frac{\partial p_L}{\partial \gamma_S} = -\frac{(1-\gamma_L)\,\gamma_L(X-R)}{R\,(\gamma_L-\gamma_S)^{\,2}} < 0.$$

Therefore, $\frac{\gamma_L X}{p_L}$ goes up, which implies $\frac{\gamma_S R}{p_S}$ also goes up. consumer welfare $U = \frac{\gamma_L X}{p_L}$ goes up. Producer's profits $\Pi = \frac{(1-\gamma_L)X}{1-p_L}$ go down.

□

Lemma 9. *In the illiquid long asset region, y_L increases with γ_S, p_S increases with γ_S, and p_L decreases with γ_S. Consumer welfare U increases with γ_S, producer profits Π decreases with γ_S, and total welfare $\eta U + (1-\eta)\Pi$ increases with γ_S.*

Proof. From

$$\frac{(1-\gamma_L)X}{\frac{\eta}{1-\eta}(1-q)+(1-y_L)}(1-y_L) = \frac{(1-\gamma_S)R}{\frac{\eta}{1-\eta}q+y_L}y_L,$$

we know that when γ_S goes up, y_L must go up. If y_L goes down, the RHS goes down, whereas the LHS goes up so that the equation cannot hold. Given this result, the total welfare $\eta U + (1-\eta)\Pi$ goes up. Also $p_L = \frac{q\frac{\eta}{1-\eta}}{q\frac{\eta}{1-\eta}+y_L}$ goes down and $p_S = \frac{\frac{\eta}{1-\eta}(1-q)}{\frac{\eta}{1-\eta}(1-q)+(1-y_L)}$ goes up. Coming to consumer welfare

$$U = (1-q)\,\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L}.$$

Clearly, $\frac{\gamma_L X}{p_L}$ goes up. We show $\frac{\gamma_S R}{p_S}$ also goes up. Specifically, we know

$$\frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

both go down. This implies

$$\frac{d\frac{(1-\gamma_S)}{1-p_S}}{d\gamma_S} < 0 \Rightarrow -(1-p_S)+(1-\gamma_S)\frac{dp_S}{d\gamma_S} < 0 \Rightarrow \frac{dp_S}{d\gamma_S} < \frac{1-p_S}{1-\gamma_S}.$$

Meanwhile,

$$\frac{d\frac{\gamma_S}{p_S}}{d\gamma_S} = \frac{p_S-\gamma_S\frac{dp_S}{d\gamma_S}}{\gamma_S^2} > \frac{p_S-\gamma_S\frac{1-p_S}{1-\gamma_S}}{\gamma_S^2} > \frac{\frac{p_S}{\gamma_S}-\frac{1-p_S}{1-\gamma_S}}{\gamma_S} > 0.$$

The last inequality holds because $p_S > \gamma_S$. Therefore, consumer welfare goes up. Finally, producer profits are:

$$\Pi = \frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

Given that p_L goes down, Π also goes down.

$\square$

Lemma 10. *In the no long rent region, y_L is unchanged with γ_S, and y_S decreases with γ_S so producer self-financed long goes up with γ_S. θ and p_L are independent of γ_S. p_S increases with γ_S, and $\frac{\gamma_S}{p_S}$ increases with γ_S. Consumer welfare increases with γ_S, producer profits Π are independent of γ_S. Total welfare $\eta U + (1-\eta)\Pi$ increases with γ_S.*

Proof. In the no long rent region, an increase in short pledgeability allows the producer to allocate more to the self-funded long asset. So her allocation to short production falls. The consumer's allocations are fixed at $\theta = q$, and his return on the long claim is fixed. With the increase in short pledgeability, the price of the short claim rises but by less than the increase in γ_S, so consumer returns rise. As a result, the consumer is better off – essentially her gains come from the greater overall allocation to the higher return long asset, away from the more pledgeable short asset.

$\square$

A.4 Comparative Statics with respect to η

We supplement the analysis on how the thresholds in γ_L for different regions vary. By taking first-order derivatives, it is easily verified that both $\frac{\gamma_S}{X}\frac{(1-\gamma_S)R-(1-\eta)X}{\eta-\gamma_S}$ and $\frac{\eta q(X-(1-\gamma_S)R)}{(1-\eta)X+\eta qX-(1-\gamma_S)R}$ increase with η. To study $\underline{\gamma}_L$, let us rewrite the equation that solves $\underline{\gamma}_L$:

$$X\left(\frac{\eta}{1-\eta}(1-q)-(\frac{1}{1-\eta})\gamma_S\right)\gamma_L^2+\gamma_S\left(R(\frac{\eta}{1-\eta}(q-1)-1)+\frac{\eta}{1-\eta}qX+(\frac{1}{1-\eta})R\gamma_S+X\right)\gamma_L-qR\frac{\eta}{1-\eta}\gamma_S^2=0$$

$$\frac{\eta}{1-\eta}\left\{X\left((1-q)-\gamma_S\right)\gamma_L^2+\gamma_S\left(R((q-1))+qX+R\gamma_S\right)\gamma_L-qR\gamma_S^2\right\}+X\left(-\gamma_S\right)\gamma_L^2+\gamma_S\left(-R+R\gamma_S+X\right)\gamma_L=0$$

$$\frac{\eta}{1-\eta}\left\{X\left((1-q)-\gamma_S\right)\gamma_L^2+\gamma_S\left(R((q-1))+qX+R\gamma_S\right)\gamma_L-qR\gamma_S^2\right\}+\left[X\left(1-\gamma_L\right)-R(1-\gamma_S)\right]\gamma_S\gamma_L=0$$

Given that $X(1-\gamma_L)-R(1-\gamma_S)>0$ holds on $(\underline{\gamma}_L-\varepsilon, \underline{\gamma}_L+\varepsilon)$ for ε sufficiently small, we know that the coefficient in front of $\frac{\eta}{1-\eta}$ must satisfy

$$X\left((1-q)-\gamma_S\right)\gamma_L^2+\gamma_S\left(R((q-1))+qX+R\gamma_S\right)\gamma_L-qR\gamma_S^2<0.$$

Therefore, the solution $\underline{\gamma}_L$ must increase in η.

Lemma 11. *In the short glut region, y_L decreases with η, θ decreases with η, p_S and p_L are independent of η. Consumer welfare U and producer profits Π are independent of η.*

Proof. Clearly, the closed-form solutions for the fractions of consumer capital backing each asset, p_S and p_L, derived in section 3.3.2 show that both are independent of η. From $\theta = \frac{\frac{1-\eta}{\eta}-\frac{1-p_S}{p_S}}{\frac{1-p_L}{p_L}-\frac{1-p_S}{p_S}}$, we know that θ

decreases with η. From $y_S = \frac{\frac{\eta}{1-\eta}(1-\theta)(1-p_S)}{p_S}$, we know y_S must increase with η, so that $y_L = 1 - y_s$ decreases with η. Consumer welfare $U = \frac{\gamma_S R}{p_S}$, producer profits $\Pi = \frac{1-\gamma_L}{1-p_L} X$ are both independent of η.

□

Lemma 12. *In the illiquid long region, y_L increases with η if and only if $(1-\gamma_S)R > (1-\gamma_L)X$. θ is independent of η.*

Proof. We can rewrite the equation that determines y_L as

$$\begin{aligned}
(1-\gamma_L)X \frac{1-y_L}{\frac{\eta}{1-\eta}(1-q)+(1-y_L)} &= (1-\gamma_S)R\frac{y_L}{\frac{\eta}{1-\eta}q+y_L} \\
\Rightarrow \frac{(1-\eta)y_L(1-y_L)+\eta q(1-y_L)}{(1-\eta)(1-y_L)y_L+\eta(1-q)y_L} &= \frac{(1-\gamma_S)R}{(1-\gamma_L)X}.
\end{aligned}$$

We differentiate both sides and get:

$$\underbrace{\left[\frac{\eta}{1-\eta}\frac{1-q}{(1-y_L)^2}\frac{(1-\gamma_S)R}{(1-\gamma_L)X} + \frac{\eta}{1-\eta}\frac{q}{y_L^2}\right]}_{>0}\frac{dy_L}{d\frac{\eta}{1-\eta}} = \frac{q}{y_L} - \frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}.$$

Therefore, the sign of $\frac{dy_L}{d\frac{\eta}{1-\eta}}$ depends on the sign of $\frac{q}{y_L} - \frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}$. Clearly,

$$\begin{aligned}
\text{sign}\left(\frac{q}{y_L} - \frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}\right) &= \text{sign}\left(\frac{\frac{\eta}{1-\eta}q}{y_L} - \frac{\eta}{1-\eta}\frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}\right) \\
&= \text{sign}\left(\frac{(1-\gamma_S)R}{(1-\gamma_L)X} - 1\right),
\end{aligned}$$

where the last inequality follows from

$$\begin{aligned}
\frac{1+\frac{\eta}{1-\eta}q/y_L}{1+\frac{\eta}{1-\eta}(1-q)/(1-y_L)} &= \frac{(1-\gamma_S)R}{(1-\gamma_L)X} \\
\Rightarrow 1 + \frac{\frac{\eta}{1-\eta}q}{y_L} &= \frac{(1-\gamma_S)R}{(1-\gamma_L)X} + \frac{(1-\gamma_S)R}{(1-\gamma_L)X}\frac{\eta}{1-\eta}(1-q)/(1-y_L) \\
\Rightarrow \frac{\frac{\eta}{1-\eta}q}{y_L} - \frac{\eta}{1-\eta}\frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X} &= \frac{(1-\gamma_S)R}{(1-\gamma_L)X} - 1.
\end{aligned}$$

□

Lemma 13. *In the illiquid long no rent equilibrium, both y_L and y_S increases with η. θ, p_L, and p_S are independent of η.*

Proof. Obvious from the solutions.

□

A.5 Social Planner's Problem under Linear Utility

First-best allocation

Let us assume the social-welfare function takes the form

$$\alpha\eta U + (1-\eta)\Pi = \alpha\eta\left((1-q)C_1^e + q(C_1^l + C_2^l)\right) + (1-\eta)\left(\Pi_1 + \Pi_2\right).$$

Implicitly, we assume the welfare function has equal weights within consumers. The resource constraint is

$$\eta\frac{(1-q)C_1^e}{R} + \eta\frac{qC_1^l}{R} + \eta\frac{qC_2^l}{X} + (1-\eta)\frac{\Pi_1}{R} + (1-\eta)\frac{\Pi_2}{X} = 1.$$

Our next result describes the first-best allocation. The proof is obvious and therefore omitted.

Lemma 14. *In the first-best allocation, it is without loss of generality to let $C_1^l = 0$, $C_1^e = 0$ and $\Pi_1 = 0$. Moreover,*

1. If $\alpha > 1$, then $\Pi_2 = 0$, and $C_2^l = \frac{X}{\eta q}$.
2. If $\alpha < 1$, then $C_2^l = 0$, and $\Pi_2 = X\left(\frac{1}{1-\eta}\right)$.
3. If $\alpha = 1$, then any combination of C_2^l and Π_2 that satisfies $\frac{\frac{\eta}{1-\eta}qC_2^l}{X} + \frac{\Pi_2}{X} = \frac{1}{1-\eta}$ attains first-best allocation.

Pledgeability-Constrained Allocation

Let z_S and z_L be the total resources allocated to short and long-term production at $t = 0$. Clearly, we have $z_S + z_L = \frac{1}{1-\eta}$. Moreover, the pledgeability constraint implies that consumer's consumption on both dates are constrained by the pledgeable cash flows generated from the assets, i.e.

$$\begin{aligned}\eta(1-q)C_1^e + \eta qC_1^l &\leq (1-\eta)z_S\gamma_S R\\ \eta qC_2^l &\leq (1-\eta)z_L\gamma_L X,\end{aligned}$$

and producers' profits are bounded below by the non-pledgeable cash flows from producing the two types of assets

$$\begin{aligned}z_S R \geq \Pi_1 &\geq z_S(1-\gamma_S)R\\ z_L X \geq \Pi_2 &\geq z_L(1-\gamma_L)X.\end{aligned}$$

Finally, we introduce the resource constraints at both $t = 1$ and $t = 2$

$$\begin{aligned}\eta(1-q)C_1^e + \eta qC_1^l + (1-\eta)\Pi_1 &= (1-\eta)z_S R\\ \eta qC_2^l + (1-\eta)\Pi_2 &= (1-\eta)z_L X.\end{aligned}$$

Our next result summarizes the pledgeability constrained-optimal allocation.

Lemma 15. *In the pledgeability constrained-optimal allocation, we have*

1. If $\alpha > 1$,

   - If $\alpha\gamma_S R + (1-\gamma_S)\, R > \alpha\gamma_L X + (1-\gamma_L)\, X$, then $z_S = \frac{1}{1-\eta}$ and $z_L = 0$. In this case, $\eta(1-q)C_1^e + \eta q C_1^l = \gamma_S R$, $C_2^l = 0$, $\Pi_1 = \left(\frac{1}{1-\eta}\right)(1-\gamma_S)\, R$, and $\Pi_2 = 0$.
   - If $\alpha\gamma_S R + (1-\gamma_S)\, R < \alpha\gamma_L X + (1-\gamma_L)\, X$, then $z_S = 0$ and $z_L = \frac{1}{1-\eta}$. In this case, $C_1^e = C_1^l = 0$, $C_2^l = \frac{\gamma_L X}{\eta q}$, $\Pi_1 = 0$, and $\Pi_2 = \left(\frac{1}{1-\eta}\right)(1-\gamma_L)\, X$.
   - If $\alpha\gamma_S R + (1-\gamma_S)\, R = \alpha\gamma_L X + (1-\gamma_L)\, X$, then any z_S and z_L satisfy $z_S + z_L = \frac{1}{1-\eta}$ is a solution. In this case, $\eta(1-q)C_1^e + \eta q C_1^l = (1-\eta)z_S\gamma_S R$, and $C_2^l = \frac{1-z_S+z_S\eta}{\eta q}\gamma_L X$.

2. If $\alpha = 1$, then $z_S = 0$ and $z_L = \frac{1}{1-\eta}$. In this case, $C_1^e = C_1^l = 0$, and $\forall C_2^l \leq (\frac{1}{1-\eta})\gamma_L X$ is a solution.

3. If $\alpha < 1$, then $z_S = 0$ and $z_L = \frac{1}{1-\eta}$. In this case, $C_1^e = C_1^l = C_2^l = 0$, $\Pi_1 = 0$, and $\Pi_2 = \left(\frac{1}{1-\eta}\right) X$.

Proof. Let z_S and z_L be the allocation to short and long-term production at $t = 0$. The problem becomes

$$\begin{aligned}
\max_{z_S, z_L \in [0,1]} \; & \alpha\eta \left[(1-q)C_1^e + q(C_1^l + C_2^l)\right] + (1-\eta)(\Pi_1 + \Pi_2) \\
s.t.\; & z_S + z_L = \frac{1}{1-\eta} \\
& z_S R \geq \Pi_1 \geq z_S(1-\gamma_S)R \\
& z_L X \geq \Pi_2 \geq z_L(1-\gamma_L)X \\
& \eta(1-q)C_1^e + \eta q C_1^l \leq (1-\eta)z_S\gamma_S R \\
& \eta q C_2^l \leq (1-\eta)z_L\gamma_L X \\
& \eta(1-q)C_1^e + \eta q C_1^l + (1-\eta)\Pi_1 = (1-\eta)z_S R \\
& \eta q C_2^l + (1-\eta)\Pi_2 = (1-\eta)z_L X.
\end{aligned}$$

After the resource constraint, the first four are pledgeability constraints; the last two resource constraints. To solve this problem, let $\frac{\eta}{1-\eta}(1-q)C_1^e + \frac{\eta}{1-\eta}qC_1^l = \tilde{C}_1$, and $\frac{\eta}{1-\eta}qC_2^l = \tilde{C}_2$. We can rewrite the problem as

$$\begin{aligned}
\max_{z_S, z_L \in [0,1]} \; & \alpha\left[\tilde{C}_1 + \tilde{C}_2\right] + \Pi_1 + \Pi_2 \\
s.t.\; & z_S + z_L = \frac{1}{1-\eta} \\
& z_S R \geq \Pi_1 \geq z_S(1-\gamma_S)R \\
& z_L X \geq \Pi_2 \geq z_L(1-\gamma_L)X \\
& \tilde{C}_1 \leq z_S\gamma_S R \\
& \tilde{C}_2 \leq z_L\gamma_L X \\
& \tilde{C}_1 + \Pi_1 = z_S R \\
& \tilde{C}_2 + \Pi_2 = z_L X,
\end{aligned}$$

which further becomes

$$\max_{z_S, z_L \in [0,1]} \alpha \left[\tilde{C}_1 + \tilde{C}_2\right] + \left(z_S R - \tilde{C}_1\right) + \left((\frac{1}{1-\eta} - z_S)X - \tilde{C}_2 \right)$$
$$s.t. 0 \le \tilde{C}_1 \le z_S \gamma_S R$$
$$0 \le \tilde{C}_2 \le (\frac{1}{1-\eta} - z_S)\gamma_L X.$$

The objective function is equivalent to

$$\left[(2\beta-1)\tilde{C}_1 + (2\beta-1)\tilde{C}_2\right] + (1-\beta) z_S \left(R - X\right)$$

The solution is

□

- If $\beta > \frac{1}{2}$, then $\tilde{C}_1 = z_S \gamma_S R$ and $\tilde{C}_2 = (1-z_S)\gamma_L X$, $\Pi_1 = z_S\left(1-\gamma_S\right)R$, and $\Pi_2 = (1-z_S)\left(1-\gamma_L\right)X$. The objective function is equivalent to

$$\left[(2\beta-1)(\gamma_S R - \gamma_L X) + (R - X)\right] z_S = \left\{\left[\alpha\gamma_S R + \left(1-\gamma_S\right)R\right] - \left[\alpha\gamma_L X + \left(1-\gamma_L\right)X\right]\right\} z_S$$

  - If $\alpha\gamma_S R + (1-\gamma_S)R > \alpha\gamma_L X + (1-\gamma_L)X$, then $z_S = \frac{1}{1-\eta}$ and $z_L = 0$. In this case, $\frac{\eta}{1-\eta}((1-q)C_1^e + qC_1^l) = \left(\frac{1}{1-\eta}\right)\gamma_S R$, $C_2^l = 0$, $\Pi_1 = \left(\frac{1}{1-\eta}\right)(1-\gamma_S)R$, and $\Pi_2 = 0$.
  - If $\alpha\gamma_S R + (1-\gamma_S)R < \alpha\gamma_L X + (1-\gamma_L)X$, then $z_S = 0$ and $z_L = \frac{1}{1-\eta}$. In this case, $C_1^e = C_1^l = 0$, $C_2^l = \frac{\gamma_L X}{\eta q}$, $\Pi_1 = 0$, and $\Pi_2 = \left(\frac{1}{1-\eta}\right)(1-\gamma_L)X$.
  - If $\alpha\gamma_S R + (1-\gamma_S)R = \alpha\gamma_L X + (1-\gamma_L)X$, then any z_S and z_L satisfy $z_S + z_L = \frac{1}{1-\eta}$ is a solution. In this case, $\frac{\eta}{1-\eta}((1-q)C_1^e + qC_1^l) = z_S\gamma_S R$, and $\frac{\eta}{1-\eta}qC_2^l = (\frac{1}{1-\eta} - z_S)\gamma_L X$
- If $\alpha = 1$, then the objective function becomes $z_S(R-X)$ so that $z_S = 0$ and $z_L = \frac{1}{1-\eta}$. In this case, $C_1^e = C_1^l = 0$, and $\forall C_2^l \le (\frac{1}{1-\eta})\gamma_L X$ is a solution.
- If $\alpha < 1$, then $\tilde{C}_1 = 0$ and $\tilde{C}_2 = 0$. The objective function becomes

$$z_S R + (\frac{1}{1-\eta} - z_S)X,$$

  in which case, the optimal is always $z_S = 0$ and $z_L = \frac{1}{1-\eta}$. In this case, $C_1^e = C_1^l = C_2^l = 0$, $\Pi_1 = 0$, and $\Pi_2 = \left(\frac{1}{1-\eta}\right)X$.

Pledgeability- and Private Information-Constrained Allocation

When the consumer type is private information, two additional constraints are needed to get types to select the consumption for their type: $C_1^e \ge C_1^l$ to get the early to self select and $C_1^l + C_2^l \ge C_1^e$ for the late. Note that the allocations in Lemma 15 satisfy the two constraints.

A.6 Social Planner's Problem under Concave Utility and Inada conditions

This subsection solves the problem under Concave Utility and Inada conditions and proves Lemma 7 and Proposition 4.

We start by formulating Lagrangian to the social planner's problem:[18]

$$\begin{aligned}
PIR &: \mu_1 : -z_S(R-X) + \eta\left[(1-q)C_1^E + q\left(C_1^L + C_2^L\right)\right] - X \le 0\\
PLS &: \mu_2 : \eta\left[(1-q)C_1^E + qC_1^L\right] - z_S\gamma_S R \le 0\\
PLL &: \mu_3 : \eta q C_2^L - (1-z_S)\gamma_L X \le 0\\
ICe &: \mu_4 : C_1^L - C_1^E \le 0\\
ICl &: \mu_5 : C_1^E - \left(C_1^L + C_2^L\right) \le 0,
\end{aligned}$$

which leads to the following FOCs:

$$\begin{aligned}
FOC_{C_1^E} &\beta\eta(1-q)u'\left(C_1^E\right) - (1-\beta)(1-q)\eta - \mu_1\eta(1-q) - \mu_2\eta(1-q) + \mu_4 - \mu_5 = 0\\
FOC_{C_1^L} &: \beta\eta q u'\left(C_1^L + C_2^L\right) - (1-\beta)\eta q - \mu_1\eta q - \mu_2\eta q - \mu_4 + \mu_5 \le 0\\
FOC_{C_2^L} &: \beta\eta q u'\left(C_1^L + C_2^L\right) - (1-\beta)\eta q - \mu_1\eta q - \mu_3\eta q + \mu_5 \le 0\\
FOC_{z_S} &: (1-\beta)(R-X) + \mu_1(R-X) + \mu_2\gamma_S R - \mu_3\gamma_L X.
\end{aligned}$$

We then get the complementary slack conditions are:

$$\begin{aligned}
PIR_{\mu_1} &: \mu_1\left[-z_S(R-X) + \eta\left[(1-q)C_1^E + q\left(C_1^L + C_2^L\right)\right] - X\right] = 0\\
PLS_{\mu_2} &: \mu_2\left[\eta\left[(1-q)C_1^E + qC_1^L\right] - z_S\gamma_S R\right] = 0\\
PLL_{\mu_3} &: \mu_3\left[\eta q C_2^L - (1-z_S)\gamma_L X\right] = 0\\
ICe_{\mu_4} &: \mu_4\left[C_1^L - C_1^E\right] = 0\\
ICl_{\mu_5} &: \mu_5\left[C_1^E - \left(C_1^L + C_2^L\right)\right] = 0.
\end{aligned}$$

Multipliers satisfy:

$$\mu_1 \ge 0, \quad \mu_2 \ge 0, \quad \mu_3 \ge 0, \quad \mu_4 \ge 0, \quad \mu_5 \ge 0.$$

Clearly, PLS and PLL imply PIR, $\mu_1 = 0$. In the following, we first solve the model by assuming $\mu_4 = \mu_5 = 0$, after which we consider the cases with $\mu_4 > 0$ or $\mu_5 > 0$. We discuss the solutions case by case.

1. $\mu_1 = 0, 0 < \mu_2 < \mu_3$. In this case, we know $C_1^L > 0$, $C_2^L = 0$, and $C_1^E = C_1^L$.

   From PLL, we get $z_S = 1$. By PLS, we derive $C_1^E = C_1^L = \frac{\gamma_S R}{\eta}$. From $FOC_{C_1^E}$, we have

   $$\mu_3 > \mu_2 = \beta u'\left(\frac{\gamma_S R}{\eta}\right) - (1-\beta) > 0, \qquad \longrightarrow \qquad u'\left(\frac{\gamma_S R}{\eta}\right) > \frac{1-\beta}{\beta}.$$

   The choice of μ_3 is irrelevant. Next, we need that the FOC on z_S is positive; that is

   $$FOC_{z_S} : (1-\beta)(R-X) \ge \mu_3\gamma_L X - \mu_2\gamma_S R,$$

   $$\longrightarrow \qquad FOC_{z_S} : 0 > (1-\beta)(R-X) \ge \mu_3\gamma_L X - \mu_2\gamma_S R \ge \mu_2\left(\gamma_L X - \gamma_S R\right),$$

[18] Note that we show that the PIR constraint of non negative payoff of producers never binds, so we omitted it from the statement of the planner's problem stated in the main text above.

which implies $\gamma_L X - \gamma_S R < 0$. Finally, from $FOC_{C_2^L}$ and FOC_{z_S}, we get

$$\beta u'\left(C_1^E\right) - (1-\beta) \leq \mu_3, \qquad \frac{1}{\gamma_L X}\left[(1-\beta)(R-X) + \mu_2\gamma_S R\right] \geq \mu_3,$$

giving

$$\beta u'\left(C_1^E\right) - (1-\beta) \leq \mu_3 \leq \frac{1}{\gamma_L X}\left[(1-\beta)(R-X) + \mu_2\gamma_S R\right].$$

Plugging C_1^E and μ_2 into above relationship produces

$$\beta u'\left(\frac{\gamma_S R}{\eta}\right) - (1-\beta) \leq \frac{1}{\gamma_L X}\left[(1-\beta)(R-X) + \left(\beta u'\left(\frac{\gamma_S R}{\eta}\right) - (1-\beta)\right)\gamma_S R\right],$$

$$\longrightarrow \qquad u'\left(\frac{\gamma_S R}{\eta}\right) \geq \frac{1-\beta}{\beta}\frac{R(1-\gamma_S) - X(1-\gamma_L)}{\gamma_L X - \gamma_S R}.$$

2. $\mu_1 = 0, 0 < \mu_3 < \mu_2$. We get $FOC_{C_1^L} < 0$, $FOC_{C_2^L} = 0$, $C_1^L = 0$ and $C_2^L > 0$.
By PLS and PLL, we get $C_1^E = \frac{z_S\gamma_S R}{\eta(1-q)}$ and $C_2^L = \frac{(1-z_S)\gamma_L X}{\eta q}$. From $FOC_{C_1^E}$ and $FOC_{C_2^L}$, we have

$$\mu_2 = \beta u'\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right) - (1-\beta), \qquad \mu_3 = \beta u'\left(\frac{(1-z_S)\gamma_L X}{\eta q}\right) - (1-\beta).$$

Next, by PLS and PLL, we get $0 < z_S < 1$, which implies

$$FOC_{z_S} : (1-\beta)(R-X) + \mu_2\gamma_S R - \mu_3\gamma_L X = 0.$$

This FOC derives the following equation which determines z_S:

$$(1-\beta)\left[R(1-\gamma_S) - X(1-\gamma_L)\right] + \gamma_S R\beta u'\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right) - \gamma_L X\beta u'\left(\frac{(1-z_S)\gamma_L X}{\eta q}\right) = 0.$$

Define the function $h(z)$ as

$$h(z) := (1-\beta)\left[R(1-\gamma_S) - X(1-\gamma_L)\right] + \gamma_S R\beta u'\left(\frac{z\gamma_S R}{\eta(1-q)}\right) - \gamma_L X\beta u'\left(\frac{(1-z)\gamma_L X}{\eta q}\right).$$

We can derive that $h(z_S) = 0$ and

$$h'(z) := \beta\frac{(\gamma_S R)^2}{\eta(1-q)}u''\left(\frac{z\gamma_S R}{\eta(1-q)}\right) + \beta\frac{(\gamma_L X)^2}{\eta q}u''\left(\frac{(1-z)\gamma_L X}{\eta q}\right) < 0.$$

From ICl, we have $C_1^E < C_2^L$, which implies $z_S < \frac{(1-q)\gamma_L X}{q\gamma_S R + (1-q)\gamma_L X}$. Since $h'(z) < 0$, we get $h\left(\frac{(1-q)\gamma_L X}{q\gamma_S R + (1-q)\gamma_L X}\right) < h(z_S) = 0$, i.e.,

$$(1-\beta)\left[R(1-\gamma_S) - X(1-\gamma_L)\right] + \beta(\gamma_S R - \gamma_L X)u'\left(\frac{\gamma_L X\gamma_S R}{\eta\left[q\gamma_S R + (1-q)\gamma_L X\right]}\right) < 0.$$

Since $\beta \in [0,1]$, we have $\gamma_S R - \gamma_L X < 0$ and $R(1-\gamma_S) - X(1-\gamma_L) < 0$. In addition, plugging C_1^E and C_2^L into PIR, we get $z_S \leq \frac{X(1-\gamma_L)}{X(1-\gamma_L) - R(1-\gamma_S)}$. Finally, since $\mu_3 > 0$ and $z_S < \frac{(1-q)\gamma_L X}{q\gamma_S R + (1-q)\gamma_L X}$, we obtain $u'\left(\frac{\gamma_L X\gamma_S R}{\eta[q\gamma_S R + (1-q)\gamma_L X]}\right) > \frac{1-\beta}{\beta}$.

3. $\mu_1 = 0, 0 = \mu_2 < \mu_3$. We get $FOC_{C_1^L} = 0$, $FOC_{C_2^L} < 0$, $C_1^L > 0$, and $C_2^L = 0$.

   Then FOC_{z_S} becomes $(1-\beta)(R-X) - \mu_3\gamma_L X < 0$, which implies $z_S = 0$. However, by PLL, we get $(1-z_S)\gamma_L X = 0$, which implies $z_S = 1$. We get a contradiction.

4. $\mu_1 = 0, 0 = \mu_3 < \mu_2$. We have $FOC_{C_1^L} < 0$, $FOC_{C_2^L} = 0$, $C_1^L = 0$, and $C_2^L > 0$.

   Then by PLS and PLL, we get

$$0 < \eta(1-q)C_1^E = z_S\gamma_S R, \qquad 0 < \eta q C_2^L \leq (1-z_S)\gamma_L X,$$

   which implies $0 < z_S < 1$. So we have $FOC_{z_S} = 0$, which gives $\mu_2 = \frac{(1-\beta)(X-R)}{\gamma_S R}$. Next, from $FOC_{C_2^L}$ we get $u'\left(C_2^L\right) = \frac{1-\beta}{\beta}$. In addition, by $FOC_{C_1^E}$, we have $\beta u'\left(C_1^E\right) = \mu_2 + (1-\beta)$. Combined with $\mu_2 = \frac{(1-\beta)(X-R)}{\gamma_S R}$, we get $u'\left(C_1^E\right) = \frac{1-\beta}{\beta}\frac{X-R(1-\gamma_S)}{\gamma_S R}$. Finally, PLS implies $z_S = \frac{\eta(1-q)}{\gamma_S R}C_1^E$.

5. $\mu_1 = 0, 0 < \mu_2 = \mu_3$. Let $\mu_2 = \mu_3 = \mu$. It must be that $C_1^L > 0$ and $C_2^L > 0$

   This implies

$$PLS : \mu_2 : \eta\left[(1-q)C_1^E + qC_1^L\right] = z_S\gamma_S R > 0,$$

$$PLL : \mu_3 : \eta q C_2^L = (1-z_S)\gamma_L X > 0.$$

   So we get $0 < z_S < 1$. Then $FOC_{z_S} = 0$ gives $\mu = \frac{(1-\beta)(X-R)}{\gamma_S R - \gamma_L X} > 0$, which implies $\gamma_S R - \gamma_L X > 0$. Next, $FOC_{C_1^E}$ gives

$$u'\left(C_1^E\right) = \frac{1-\beta}{\beta}\frac{X(1-\gamma_L) - R(1-\gamma_S)}{\gamma_S R - \gamma_L X} > 0, \qquad \longrightarrow \qquad X(1-\gamma_L) - R(1-\gamma_S) > 0.$$

   In addition, PLL gives $C_2^L = (1-z_S)\frac{\gamma_L X}{\eta q}$. Combined with $C_1^L = C_1^E - C_2^L$ and PLS, we obtain $z_S = \frac{\eta C_1^E - \gamma_L X}{\gamma_S R - \gamma_L X}$.

6. $\mu_1 = 0, 0 = \mu_2 = \mu_3$. We get $FOC_{z_S} : (1-\beta)(R-X) < 0$.

   This implies $z_S = 0$. Then $PLS : \mu_2$ implies $(1-q)C_1^E + qC_1^L \leq 0$, which is impossible.

Next, we show that the ICe and ICl are always redundant.

1. $\mu_4 = 0, \mu_5 > 0$

   From ICl, we obtain $C_1^E = C_1^L + C_2^L$. $FOC_{C_1^E}$ becomes

$$FOC_{C_1^E} - (1-\beta)(1-q)\eta - \mu_1\eta(1-q) - \mu_2\eta(1-q) = \mu_5 - \beta\eta(1-q)u'\left(C_1^E\right).$$

By $FOC_{C_1^L}$,

$$\begin{aligned}FOC_{C_1^L}:&\beta\eta(1-q)u'\left(C_1^L+C_2^L\right)-(1-\beta)(1-q)\eta-\mu_1\eta(1-q)-\mu_2\eta(1-q)+\frac{1-q}{q}\mu_5\\&=\beta\eta(1-q)u'\left(C_1^L+C_2^L\right)+\frac{1-q}{q}\mu_5+\mu_5-\beta\eta(1-q)u'\left(C_1^E\right)\\&=\beta\eta(1-q)\left[u'\left(C_1^L+C_2^L\right)-u'\left(C_1^E\right)\right]+\frac{1}{q}\mu_5\\&=\frac{1}{q}\mu_5\le 0.\end{aligned}$$

We get a contradiction.

2. $\mu_4>0,\mu_5=0$

We have $FOC_{C_1^E}:\ \beta\eta u'\left(C_1^E\right)-(1-\beta)\eta-\mu_1\eta-\mu_2\eta=-\frac{\mu_4}{1-q}<0$, and $FOC_{C_1^L}$ and $FOC_{C_2^L}$ become

$$\begin{aligned}FOC_{C_1^L}:&\beta\eta qu'\left(C_1^L+C_2^L\right)-(1-\beta)\eta q-\mu_1\eta q-\mu_2\eta q-\mu_4\\&\le q\left(\beta\eta u'\left(C_1^E\right)-(1-\beta)\eta-\mu_1\eta-\mu_2\eta\right)-\mu_4=-q\frac{\mu_4}{1-q}-\mu_4<0,\\FOC_{C_2^L}:&\beta\eta qu'\left(C_1^L+C_2^L\right)-(1-\beta)\eta q-\mu_1\eta q-\mu_3\eta q=0.\end{aligned}$$

Then we get $C_1^L=0$ and $C_2^L>0$, which contradicts with $ICe:\mu_4\ C_1^L=C_1^E>0$.

3. $\mu_4>0,\mu_5>0$

In this case, we have $C_2^L=0$ and $C_1^E=C_1^L>0$. Then $FOC_{C_1^L}$ and $FOC_{C_2^L}$ become

$$\begin{aligned}FOC_{C_1^L}:&\beta\eta qu'\left(C_1^L\right)-(1-\beta)\eta q-\mu_1\eta q-\mu_2\eta q-\mu_4+\mu_5=0,\\FOC_{C_2^L}:&\beta\eta qu'\left(C_1^L\right)-(1-\beta)\eta q-\mu_1\eta q-\mu_3\eta q+\mu_5\le 0.\end{aligned}$$

Note that $FOC_{C_1^L}$ implies $\beta\eta qu'\left(C_1^L\right)-(1-\beta)\eta q-\mu_1\eta q+\mu_5=\mu_2\eta q+\mu_4>0$. Then $FOC_{C_2^L}$ gives $0<\beta\eta qu'\left(C_1^L\right)-(1-\beta)\eta q-\mu_1\eta q+\mu_5\le\mu_3\eta q$, meaning $\mu_3>0$. So we get $z_S=1$ from PLL. In addition, $FOC_{C_1^E}$ produces $\beta\eta qu'\left(C_1^E\right)-(1-\beta)\eta q-\mu_1\eta q-\mu_2\eta q=-\frac{q(\mu_4-\mu_5)}{1-q}$, then $FOC_{C_1^L}$ becomes

$$FOC_{C_1^L}:\beta\eta qu'\left(C_1^L\right)-(1-\beta)\eta q-\mu_1\eta q-\mu_2\eta q-\mu_4+\mu_5=\frac{q\left(\mu_5-\mu_4\right)}{1-q}+\mu_5-\mu_4=0.$$

So we get $\mu_4=\mu_5>0$. Next, from FOC_{z_S}, we derive

$$(1-\beta)(R-X)+\mu_1(R-X)+\mu_2\gamma_S R\ge\mu_3\gamma_L X>0,\quad\longrightarrow\quad\mu_2\gamma_S R>(1-\beta+\mu_1)(X-R)>0,$$

which implies $\mu_2>0$. Then by PLS, we get $C_1^E=C_1^L=\frac{\gamma_S R}{\eta}$. Finally, we analyze the parameters conditions. Note that $FOC_{C_1^E}$ implies $\mu_2=\beta u'\left(\frac{\gamma_S R}{\eta}\right)-(1-\beta)>0$, then we get $u'\left(\frac{\gamma_S R}{\eta}\right)>\frac{1-\beta}{\beta}$. By FOC_{z_S} and $FOC_{C_2^L}$, we get

$$\begin{aligned}FOC_{z_S}:&\frac{1}{\gamma_L X}\left[(1-\beta)(R-X)+\mu_2\gamma_S R\right]\ge\mu_3>0,\\FOC_{C_2^L}:&0<\mu_5\le\mu_3\eta q+(1-\beta)\eta q-\beta\eta qu'\left(C_1^L\right)\longrightarrow\qquad\beta u'\left(C_1^L\right)-(1-\beta)<\mu_3,\end{aligned}$$

which gives

$$\beta u'\left(C_1^L\right) - (1-\beta) < \mu_3 \leq \frac{1}{\gamma_L X}\left[(1-\beta)(R-X) + \mu_2\gamma_S R\right].$$

After simplifications, we get the conditions

$$\beta u'\left(\frac{\gamma_S R}{\eta}\right)(\gamma_S R - \gamma_L X) > (1-\beta)\left(X(1-\gamma_L) - R(1-\gamma_S)\right),$$

which implies $\gamma_S R - \gamma_L X > 0$. The solutions and parameters conditions are the same as the case with $\mu_1 = 0, 0 < \mu_2 < \mu_3$.

Therefore, we can conclude in all cases, ICe and ICl are always satisfied.

Comparative static analysis As discussed above, only four cases are possible: 1) if $\mu_1 = 0, 0 < \mu_2 < \mu_3$, 2) if $\mu_1 = 0, 0 < \mu_3 < \mu_2$, 3) if $\mu_1 = 0, 0 = \mu_3 < \mu_2$, and 4) if $\mu_1 = 0, 0 < \mu_2 = \mu_3$. We analyze these four cases respectively.

1. If $\mu_1 = 0, 0 < \mu_2 < \mu_3$, we have

$$FOC_{z_S} : (1-\beta)(R-X) + \mu_2\gamma_S R - \mu_3\gamma_L X \geq 0, \qquad \longrightarrow \qquad \gamma_S R > \gamma_L X.$$

   So $\gamma_S R > \gamma_L X$ is necessary for $z_S = 1$.

2. If $\mu_1 = 0, 0 < \mu_3 < \mu_2$, first note that

$$\gamma_S R u'\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right) - \gamma_L X u'\left(\frac{(1-z_S)\gamma_L X}{\eta q}\right) = \left(\frac{1}{\beta} - 1\right)\left[X(1-\gamma_L) - R(1-\gamma_S)\right] > 0,$$

   and the left-hand side decreases with z_S, so when β increases, z_S also increases. Second, taking derivative of z_S w.r.t. γ_S in both sides gives

$$\frac{\partial z_S}{\partial \gamma_S} = R\frac{\frac{1-\beta}{\beta} - \left[u'\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right) + \frac{z_S\gamma_S R}{\eta(1-q)}u''\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right)\right]}{u''\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right)\frac{(\gamma_S R)^2}{\eta(1-q)} + u''\left(\frac{(1-z_S)\gamma_L X}{\eta q}\right)\frac{(\gamma_L X)^2}{\eta q}}$$

   If $cu'(c)$ is a decreasing function of c, then

$$u'(c) + cu''(c) < 0, \qquad \to \qquad u'\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right) + \frac{z_S\gamma_S R}{\eta(1-q)}u''\left(\frac{z_S\gamma_S R}{\eta(1-q)}\right) < 0.$$

   Then we get $\frac{\partial z_S}{\partial \gamma_S} < 0$, implying z_S decreases with γ_S. Moreover, we evaluate the equation that determines z_S and have that the left-hand side decreases with z_S. By evaluating this at $z_S = 1-q$, we get

$$(1-\beta)\underbrace{\left[R(1-\gamma_S) - X(1-\gamma_L)\right]}_{<0} + \eta\beta\left[\frac{\gamma_S R}{\eta}u'\left(\frac{\gamma_S R}{\eta}\right) - \frac{\gamma_L X}{\eta}u'\left(\frac{\gamma_L X}{\eta}\right)\right].$$

   Regarding the second term, we know that if $cu'(c)$ is a decreasing function, the second term is positive because $\gamma_L X > \gamma_S R$. Both $z_S < 1-q$ and $z_S > 1-q$ are possible.

3. If $\mu_1 = 0, 0 = \mu_3 < \mu_2$, first note that when β increases, C_1^E increases, and thus z_S also increases. Numerical experiments show that when this case occurs, both $\gamma_S R > \gamma_L X$ and $\gamma_S R < \gamma_L X$ are

possible. Second, from $z_S = \frac{\eta(1-q)}{\gamma_S R} C_1^E$, taking derivative of z_S w.r.t. γ_S gives

$$\frac{\mathrm{d}z_S}{\mathrm{d}\gamma_S} = \frac{\eta(1-q)}{\gamma_S R}\left(\frac{\mathrm{d}C_1^E}{\mathrm{d}\gamma_S} - \frac{C_1^E}{\gamma_S}\right).$$

By $u'\left(C_1^E\right) = \left(\frac{1}{\beta}-1\right)\left(\frac{X-R}{\gamma_S R}+1\right)$, we get $\frac{\mathrm{d}C_1^E}{\mathrm{d}\gamma_S} = -\frac{u'\left(C_1^E\right)-\left(\frac{1}{\beta}-1\right)}{\gamma_S u''\left(C_1^E\right)}$. Then we derive

$$\frac{\mathrm{d}z_S}{\mathrm{d}\gamma_S} = -\frac{\eta(1-q)}{(\gamma_S)^2 R}\frac{\left[u'\left(C_1^E\right)+C_1^E u''\left(C_1^E\right)\right]-\left(\frac{1}{\beta}-1\right)}{u''\left(C_1^E\right)}.$$

If $cu'(c)$ is a decreasing function of c, then $u'\left(C_1^E\right)+C_1^E u''\left(C_1^E\right) < 0$, and thus $\frac{\mathrm{d}z_S}{\mathrm{d}\gamma_S} < 0$, meaning z_S decreases with γ_S. Third, C_2^L does not change with risk aversion or pledgeabilities. Fourth, by the equations that determines C_1^E and z_S, when $u'()$ gets lower, then C_1^E needs to be higher, in which case z_S will increase. Finally, we know

$$\frac{\mathrm{d}}{\mathrm{d}\gamma_S}\left[\left(\frac{1}{\beta}-1\right)\left(\frac{X-R}{\gamma_S R}+1\right)\right] < 0,$$

so that an increase in γ_S will increase C_1^E. We compare z_S with $1-q$: $\frac{z_S}{1-q} = \frac{C_1^E}{\gamma_S R/\eta}$. Therefore, it is equivalent to comparing C_1^E with $\gamma_S R/\eta$, which, in turn, is to compare $u'(\gamma_S R/\eta)$ with $\left(\frac{1}{\beta}-1\right)\left(\frac{X-R}{\gamma_S R}+1\right)$. Depending on parameters, both $z_S > 1-q$ and $z_S < 1-q$ can occur.

4. If $\mu_1 = 0, 0 < \mu_2 = \mu_3$, first note that when β increases, C_1^E increases, and thus z_S also increases. Next, from

$$u'\left(C_1^E\right) = \left(\frac{1}{\beta}-1\right)\frac{X(1-\gamma_L)-R(1-\gamma_S)}{\gamma_S R-\gamma_L X},$$

we know that as $u'()$ decreases, C_1^E needs to be higher, implying that $z_S = \frac{\eta C_1^E-\gamma_L X}{\gamma_S R-\gamma_L X}$ also increases. This implies that $C_2^L = \frac{\gamma_L X}{\eta q}(1-z_S)$ will decrease. If $z_S = 1-q$, then we have $C_1^E = \frac{(1-q)\gamma_S R+q\gamma_L X}{\eta}$ and $C_2^L = \frac{\gamma_L X}{\eta}$. In this case, we need to compare $u'\left(C_1^E\right) = u'\left(\frac{(1-q)\gamma_S R+q\gamma_L X}{\eta}\right)$ with $\left(\frac{1}{\beta}-1\right)\frac{X(1-\gamma_L)-R(1-\gamma_S)}{\gamma_S R-\gamma_L X}$. If $u'\left(\frac{(1-q)\gamma_S R+q\gamma_L X}{\eta}\right)$ is higher, this implies z_S is higher than $1-q$. Otherwise, z_S is lower than $1-q$.

Special case: $\beta = 1$ and $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$. When $\beta = 1$, only two cases are possible: $\{\mu_1 = 0, 0 < \mu_2 < \mu_3\}$ and $\{\mu_1 = 0, 0 < \mu_3 < \mu_2\}$. Under CRRA, we have $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$. We analyze the case with $\{\mu_1 = 0, 0 < \mu_3 < \mu_2\}$. The solutions becomes

$$C_1^L = 0, \qquad z_S = \frac{1}{1+\left(\frac{\gamma_L X}{\gamma_S R}\right)^{\frac{1-\sigma}{\sigma}}\frac{q}{1-q}}, \qquad C_1^E = \frac{z_S\gamma_S R}{\eta(1-q)}, \qquad C_2^L = (1-z_S)\frac{\gamma_L X}{\eta q},$$

and we must have $\gamma_L X > \gamma_S R$. We can derive following results:

1. By $\gamma_L X > \gamma_S R$, we know that if σ increases, $\left(\frac{\gamma_L X}{\gamma_S R}\right)^{\frac{1-\sigma}{\sigma}}$ decreases, and thus z_S increases.

2. By $\gamma_L X > \gamma_S R$ and $\frac{1-q}{z_S}\frac{1-z_S}{q} = \left(\frac{\gamma_L X}{\gamma_S R}\right)^{\frac{1-\sigma}{\sigma}}$, we know that if $\sigma > 1$, we get $\frac{1-\sigma}{\sigma} < 0$ and $\left(\frac{\gamma_L X}{\gamma_S R}\right)^{\frac{1-\sigma}{\sigma}} < 1$, then $\frac{1-q}{z_S}\frac{1-z_S}{q} < 1$, which implies $z_S > 1-q$. If $\sigma < 1$, we get $z_S < 1-q$.

3. Letting $\epsilon \in (0,1)$ and $z_S < 1-\epsilon$, we get $\ln\left(\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right) < \frac{1-\sigma}{\sigma}\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)$. Since $\gamma_L X > \gamma_S R$, we get $\frac{\ln\left(\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right)}{\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)} + 1 < \frac{1}{\sigma}$.

   (a) If $\frac{\ln\left(\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right)}{\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)} + 1 \leq 0$, i.e., $\epsilon \leq \frac{1}{1+\frac{\gamma_L X}{\gamma_S R}\frac{1-q}{q}}$, then the inequality $\frac{\ln\left(\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right)}{\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)} + 1 < \frac{1}{\sigma}$ holds for $\sigma > 0$, which implies that for all $\sigma > 0$, we have $z_S < 1-\epsilon$.

   (b) If $\epsilon > \frac{1}{1+\frac{\gamma_L X}{\gamma_S R}\frac{1-q}{q}}$, we have $\frac{\ln\left(\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right)}{\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)} + 1 > 0$, then $\frac{\ln\left(\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right)}{\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)} + 1 < \frac{1}{\sigma}$ implies $\sigma < \frac{\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)}{\ln\left(\frac{\gamma_L X}{\gamma_S R}\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right)}$, i.e., for $\sigma < \frac{\ln\left(\frac{\gamma_L X}{\gamma_S R}\right)}{\ln\left(\frac{\gamma_L X}{\gamma_S R}\frac{\epsilon}{1-\epsilon}\frac{1-q}{q}\right)}$, we have $z_S < 1-\epsilon$.

A.7 Implementation with simple traded financial assets

When consumers have concave preferences, Proposition 4 shows that consumer allocations are of one of three types: 1) all short: $C_1^e = C_1^l > 0$ and no date 2 consumption, 2) equal total consumption of early and late consumers, with $C_1^e > 0, C_1^l > 0, C_2^l > 0$, $C_1^l + C_2^l = C_1^e$ and 3) higher total consumption of late consumers and no early consumption of late consumers, $C_1^e > 0, C_1^l = 0, C_2^l > C_1^e$. The planner can chose z_S to implement each of these consumption levels. We now show that each can be implemented with claims where consumers are free to trade at date 1. Consider the first one. The planner can assign the claims of $C_1^e = C_1^l$ to each consumer. There are no gains from trade at date 1, so this implements the allocation. Consider the second one. The planner can offer each consumer short claims $(1-q) + q(\dfrac{C_1^l}{C_1^l + C_2^l})$ and long claims of $q(1 - (\dfrac{C_1^l}{C_1^l + C_2^l}))$, where each maturity receives the value C_1^e per unit invested. Because late consumers have more short claims than needed to buy all of the long claims (because long claim holdings are less than q) from early consumers at date 1 price $r_{12}^l = 1$, they will not pay a higher price and will retain some to consume. The third case is implemented by assigning a fraction q of long claims worth C_2^l and $1-q$ short claims worth C_1^e to each consumer at date 0. If all of the long claims held by early consumers are sold for all or the short claims held by the late consumers, the price is $r_{12}^L = \dfrac{q(1-\theta)C_2^l}{(1-q)\theta C_1^e} = \dfrac{C_2^l}{C_1^e} > 1$, and the market clears.

When the two consumptions are perfect substitutes (linear utility), and $\beta > \frac{1}{2}$, then only one maturity asset is produced. When this is only short assets the implementation is as case 1 in the previous paragraph. If only longs are produced, then implementation with traded claims is not possible because there is no way for late consumers to pay for the long claims held by the early consumers. However, producing a short claim with an arbitrarily small but positive return removes the problem. The implementation is identical to case 2 above. When $\beta < \frac{1}{2}$, the planner assigns all consumption to producers and no trade is needed.

The previous paragraphs assumed that the planner assigns claims to consumers at date 0. However, given the returns that those claims offer, we now show that this is not required and we can set returns on short and long claims at date 0 and allow consumers to allocate their holdings in these proportions. The planner sets returns r_{01}^S and r_{02}^L and endogenous market clearing trade on date 1 will trade r_{12}^L long claims per short claim.

In case 1, with all short claims, the planner set $r_{01}^S = C_1^e = C_1^l$ and consumers will invest in them and must choose $\theta = 0$. In case 3, where $C_1^e > 0, C_1^l = 0, C_2^l > C_1^e$, the planner sets $r_{01}^S = C_1^e$ and $r_{02}^L = C_2^l$, and by Lemma 2, consumers set $\theta = q$ and trade at date 1 with $r_{12}^L = \dfrac{C_2^l}{C_1^e} > 1$ and all date 1 claims go to early consumers and all date 2 claims to late consumers after trade. In case 2, the planner offers claims of $r^S_{01} = r^L_{02} = C^e_1$. Consumers choose the fraction of each claim to hold at date 0, and must set $\theta = q(1-(\frac{C^l_1}{C^l_1 + C^l_2})) < q$ to clear the market at date 1. By Lemma 2, consumers will choose such a portfolio with at date 0 only if $r^S_{01} = r^L_{02}$, and setting these equal to C^e_1 implements the allocation. Market clearing at date 1, where late investors retain some of their early claims and acquire all the late claims, requires $r^L_{12} = 1$. This implements case 2.

When consumers have linear preferences, the all short allocation requires no trade and is implemented with all short claims with return $r^S_{01} = C^e_1 = C^l_1$. The all long allocation has the problem mention above with ex post trade where late consumers have no short claim to buy longs, but can be approximately implemented with a very small allocation to short with $r^S_{01} = \epsilon > 0$ and $r^L_{12} = C^l_2 - \delta$ for a small $\delta > 0$. Consumers will then choose $\theta = q$ and trade allows all long claims go to late consumers.

This shows that the possible differences between the competitive outcome and the planner's problem are due to the incentives of producers to produce claims that are chosen by producers.