

The Co-Pricing Factor Zoo^{*}

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Abstract

We analyze 18 quadrillion models for the joint pricing of corporate bond and stock returns. Only a handful of factors, behavioural and nontradable, are robust sources of priced risk. Yet, the true latent stochastic discount factor is *dense* in the space of observable factors. A Bayesian Model Averaging Stochastic Discount Factor (BMA-SDF), combining the corporate bond and stock factor zoos, explains risk premia better than all existing models, both in- and out-of-sample. We show that multiple factors are noisy proxies for common underlying sources of risk, and the BMA-SDF aggregates them optimally. The SDF, as well as its conditional mean and volatility, are persistent, track the business cycle and times of heightened economic uncertainty, and predict future asset returns. Finally, we show that stock factors price the credit component of corporate bond excess returns well, while the Treasury component is priced almost exclusively by the bond factors.

Keywords: Macro-finance, asset pricing, corporate bonds, bond-stock co-pricing, factor zoo, factor models, Bayesian methods.

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Wherever there is risk, it must be compensated to the lender by a higher premium or interest.

— J. R. McCullough (1830, pp. 508–9)

In their seminal paper, [Fama and French \(1993\)](#) set themselves to “examine whether variables that are important in bond returns help to explain stock returns, and vice versa.” Thirty years later, the equity literature has produced its own, independent, ‘factor zoo,’ as highlighted in [Cochrane \(2011\)](#), while the corporate bond literature has effectively returned to square one with [Dickerson, Mueller, and Robotti \(2023\)](#) showing that there is no satisfactory (observable) factor model for that asset class.¹ Hence, to date, a model for the *joint* pricing of corporate bonds and stocks has escaped discovery—we fill this gap.

We pick up the question asked in [Fama and French \(1993\)](#) and study which factors—bond, stock or nontradable—contain relevant information to explain the cross-section of corporate bond and stock returns. Leveraging recent advances in Bayesian econometrics, we comprehensively analyse all observable factors and models proposed to date in the bond and equity literature. Our method allows us to not only study models or factors in isolation, but also consider all of their possible combinations, resulting in over 18 quadrillion models stemming from the joint zoo of corporate bond and stock factors. And we do so while relaxing the cornerstone assumptions of previous studies: the existence of a unique, low-dimensional, correctly specified and well identified factor model. Ultimately, this allows us to pinpoint the robust sources of priced risk in both markets, and a novel benchmark Stochastic Discount Factor (SDF) that prices both asset classes, in- and out-of-sample, significantly better than all existing models.

First, we find that the ‘true’ latent SDF of bonds and stocks is *dense* in the space of observable bond and stock factors—literally dozens of factors, both tradable and nontradable, are necessary to span the risks driving asset prices. Yet, the SDF-implied maximum Sharpe ratio is not excessive, indicating that, as we confirm in our analysis, multiple bond and stock factors proxy for common sources of fundamental risk. Importantly, density of the SDF implies that the sparse models considered in the previous literature are affected by severe misspecification and, as we show, rejected by the data and outperformed by the most likely SDF components that we identify.

Second, a Bayesian Model Averaging Stochastic Discount Factor (BMA-SDF) over the space

¹More precisely, they document that all low dimensional linear factor models in the previous literature add little spanning to a simple *bond* version of the Capital Asset Pricing Model, the CAPMB. At the same time, they show that the CAPMB is in itself an unsatisfactory pricing model.

of all possible models (including bond, stock, and nontradable factors) explains (jointly and separately) corporate bond and equity risk premia better than all existing models and most likely factors, both in- and out-of-sample. Moreover, the BMA-SDFs conditional mean and volatility—hence, the conditional Sharpe ratio achievable in the economy—have clear business cycle patterns. In particular, the volatility of the SDF increases sharply at the onset of recessions and at times of heightened economic uncertainty. That is, the estimated SDF behaves as one would expect from the intertemporal marginal rate of substitution of an agent exposed to the risks arising from general economic conditions and market turmoil.

Third, the predictability of the first and second moments of the SDF suggests time-varying risk premia in the economy and predictability of asset returns with lagged SDF information. We verify this by running predictive regressions of future asset returns on the conditional variance of the BMA-SDF, alone and interacted with the conditional mean of the SDF, as implied by the [Hansen and Jagannathan \(1991\)](#) representation of the conditional SDF. We not only find that lagged SDF information is highly significant in predicting future asset returns, but also that the amount of explained time series variation in monthly and annual returns is much larger than what is achievable with canonical predictors. This result is remarkable for two reasons. First, the BMA-SDF is *not* by construction geared toward predicting future returns: it is instead identified only under the restriction that a valid SDF should explain the cross-section of risk premia—not the time series of returns. Second, it offers an important validation of our estimation of the SDF: if risk premia are time-varying, future returns should be predictable with lagged SDF information, and that is exactly what our BMA-SDF delivers.

Fourth, we shed light on which factors, and which types of risk, are reflected in the cross-section of bond and equity risk premia. We find that only a handful of factors should be in the SDF with high probability. In particular, two factors meant to capture the bond and stock post-earnings announcement drift anomalies, PEADB and PEAD, respectively, are very likely sources of priced risk in the joint cross-section of bond and stock returns.² In addition to these two behavioural sources of risk, the other most likely components of the SDF are all nontradable in nature, and are a proxy for the slope of the Treasury yield curve (YSP), the AAA/BAA yield spread (CREDIT), and the idiosyncratic equity volatility (IVOL). As we show, these factors

²The post-earnings announcement drift phenomenon is the observation, first documented in equity markets, that firms that experience positive earnings surprises subsequently earn higher returns than those with negative earnings surprises. See, e.g., [Hirshleifer and Teoh \(2003\)](#), [Della Vigna and Pollet \(2009\)](#), [Hirshleifer, Lim, and Teoh \(2011\)](#) and [Nozawa, Qiu, and Xiong \(2023\)](#) for the microfoundations of this phenomenon.

alone are enough to price the cross-section of bonds and stocks better than canonical observable factor models. Nevertheless, the BMA-SDF outperforms these most likely factors, both in- and (cross-sectionally) out-of-sample, as the true latent SDF is dense in the space of observable factors and demands sizeable compensations for risks that are not fully spanned by the most likely factors. Many nontradable, and both bond and stock tradable factors, are necessary—jointly—for an accurate characterization of the SDF, because they represent multiple noisy proxies for the same underlying sources of risk. Furthermore, we find that both discount rate and cash-flow news are sources of priced risk, and yield sizeable contributions (albeit larger for the former) to the Sharpe ratio of the latent SDF.

Fifth, we demonstrate that a portion of corporate bond risk premia serves as compensation for their implicit Treasury term structure risk. Once this component is removed, the factors proposed in the tradable bond factor zoo have very little residual information content for characterizing the SDF: in this case, a BMA-SDF constructed only with stock and nontradable factors can explain the joint cross-section of bonds and stocks as well as our full BMA-SDF. This finding extends and explains the result in [van Binsbergen, Nozawa, and Schwert \(2024\)](#), who show that once corporate bond returns are adjusted for duration risk, the equity CAPM has higher explanatory power for bond risk premia than benchmark bond models. Furthermore, we show that the empirical success of the bond factor zoo in the previous literature is largely driven by its ability to price the Treasury term structure risk—a component of bond risk premia that tradable stock factors do not capture.

Overall, our findings have first order implications for both theoretical and empirical analyses that aim to explain the risk compensation demanded by investors in corporate bonds, stocks and Treasury securities.

The remainder of the paper is organized as follows. Below we review the most closely related literature and our contribution to it. [Section 1](#) describes the data used in our analysis, while [Section 2](#) outlines the Bayesian SDF estimation method that we employ and its properties for inference, selection, and model aggregation. [Section 3](#) presents our empirical findings and [Section 4](#) concludes. Additional details and results are reported in the Appendix and the Internet Appendix.

Closely related literature. Our research contributes to the active and growing body of work that critically reevaluates existing findings in the empirical asset pricing literature using

robust inference methods. Following [Harvey, Liu, and Zhu \(2016\)](#), a large literature has tried to understand which existing factors (or their combinations) drive the cross-section of returns. In particular, [Gospodinov, Kan, and Robotti \(2014\)](#) develop a general method for misspecification-robust inference, while [Giglio and Xiu \(2021\)](#) exploit the invariance principle of PCA and recover the price of risk of a given factor from the projection on the span of latent factors driving a cross-section of returns. Similarly, [Dello Preite, Uppal, Zaffaroni, and Zviadadze \(2024\)](#) recover latent factors from the residuals of an asset pricing model, effectively completing the span of the SDF. [Feng, Giglio, and Xiu \(2020\)](#) combine cross-sectional asset pricing regressions with the double-selection LASSO of [Belloni, Chernozhukov, and Hansen \(2014\)](#) to provide valid inference on the selected sources of risk when the true SDF is sparse. [Kozak, Nagel, and Santosh \(2020\)](#) use a ridge-based approach to approximate the SDF and compare sparse models based on principal components of returns. Our approach instead identifies a dominant pricing model—if such a model exists—or a BMA across the space of all models, even if the true model is not sparse in nature, hence cannot be proxied by a small number of factors. Furthermore, and importantly, our work focuses on the *co-pricing* of corporate bond and stock returns, hence shedding light on both the common, as well as the market specific, sources of risk.

As [Harvey \(2017\)](#) stresses in his American Finance Association presidential address, the factor zoo naturally calls for a Bayesian solution—and we adopt one. In particular, we leverage the Bayesian method for model estimation, selection, and averaging developed in [Bryzgalova, Huang, and Julliard \(2023\)](#). Numerous strands of the literature rely on Bayesian tools for asset allocation, model selection, and performance evaluation. Our approach is most closely linked to [Pástor and Stambaugh \(2000\)](#) and [Pástor \(2000\)](#) in that we assign a prior distribution to the vector of pricing errors, and this maps into a natural and transparent prior for the maximal Sharpe ratio achievable in the economy. [Barillas and Shanken \(2018\)](#) also extend the prior formulation of [Pástor and Stambaugh \(2000\)](#) and provide a closed-form solution for the Bayes factors when all factors are tradable in nature. [Chib, Zeng, and Zhao \(2020\)](#) show that the improper prior formulation of [Barillas and Shanken \(2018\)](#) is problematic, and provide a new class of priors that leads to valid comparison for tradable factor models. As in these papers, our model and factor selection is based on posterior probabilities, but our method is designed to work with both tradable and *nontradable* factors—and as we show, the latter are a first order source of priced risk in the joint space of corporate bonds and stock returns. Most importantly, our approach can deal with a very large factor space, is not affected by the common

identification failures that invalidate inference in asset pricing, and provides an optimal method for aggregating the pricing information stemming from the joint zoo of corporate and equity factors.³

In the complete market benchmark, the pricing measure should be consistent across asset classes, and equilibrium models normally yield nontradable state variables. Therefore, we focus on the co-pricing of corporate bonds and stocks, and consider jointly a very broad collection of potential sources of risk that goes well beyond the set of bond and stock tradable factors that have been studied in isolation in the previous literature. Hence, our paper speaks to the large literature on co-pricing, which started with the seminal work of [Fama and French \(1993\)](#), and market segmentation of bonds and stocks (see, e.g., [Chordia, Goyal, Nozawa, Subrahmanyam, and Tong \(2017\)](#), [Choi and Kim \(2018\)](#), or [Sandulescu \(2022\)](#)).

In particular, our paper is related to the body of work that explores whether equity market risk proxies (see, e.g., [Blume and Keim \(1987\)](#) and [Elton, Gruber, Agrawal, and Mann \(2001\)](#)), equity volatilities (see, e.g., [Campbell and Taksler \(2003\)](#) and [Chung, Wang, and Wu \(2019\)](#)), and equity-based characteristics (see, e.g., [Fisher \(1959\)](#), [Giesecke, Longstaff, Schaefer, and Strebulaev \(2011\)](#), and [Gebhardt, Lee, and Swaminathan \(2001\)](#)) are likely drivers of corporate bond returns, and on the commonality of risks across markets (see, e.g., [He, Kelly, and Manela \(2017\)](#), [Lettau, Maggiori, and Weber \(2014\)](#), and [Chen, Roussanov, Wang, and Zou \(2024\)](#)). Overall, we find that factors in both the corporate bond and equity zoos are needed for the joint pricing of both asset classes, and stock factors do carry relevant information to explain bond returns. Yet, there is substantial overlap between the risks spanned by these two markets. That is, multiple bond and stock factors are noisy proxies for common underlying sources of risk. Nevertheless, as we show, corporate bond risk premia include an implicit compensation for Treasury term structure risk—a risk that the bond factor zoo, and nontradable factors proposed therein in particular, price very well, while equity factors do not. And once this term structure risk component is removed, tradable bond factors become largely unnecessary for the

³BMA is an optimal aggregation procedure for a very wide set of optimality criteria (see, e.g., [Raftery and Zheng \(2003\)](#) and [Schervish \(1995\)](#)). In particular, it is “optimal on average,” i.e., no alternative method can outperform the BMA for all values of the true unknown parameters. [Avramov, Cheng, Metzker, and Voigt \(2023\)](#) also propose a framework to integrate factor models via posterior probabilities in the presence of model uncertainty, but their approach is only appropriate for tradable factors and is not designed to be robust to the identification and inference problems arising from weak factors—problems that, as shown in [Bryzgalova, Huang, and Julliard \(2023\)](#), cannot be solved by simply projecting nontradable factors on the space of returns and then performing inference using the resulting mimicking portfolios. Furthermore, as shown in [Heyerdahl-Larsen, Illeditsch, and Walden \(2023\)](#), a BMA-SDF can be microfounded thanks to the equivalence between an economy populated by agents with heterogeneous beliefs, and a Bayesian representative agent setting.

joint pricing of bonds and stocks.

Several theoretical contributions stress that real economic activity and the business cycle should be among the drivers of bond risk premia (see, e.g., [Bhamra, Kuehn, and Strebulaev \(2010\)](#), [Khan and Thomas \(2013\)](#), [Chen, Cui, He, and Milbradt \(2018\)](#), and [Favilukis, Lin, and Zhao \(2020\)](#)). Echoing both the general equilibrium model predictions of [Gomes and Schmid \(2021\)](#) and the empirical findings of [Elton, Gruber, and Blake \(1995\)](#) and [Elkamhi, Jo, and Nozawa \(2023\)](#), we show that the BMA-SDF conditional first and second moments have a clear business cycle pattern and peak during recessions and at times of heightened economic uncertainty, and that nontradable factors (especially proxies of the economic cycle such as the slope of the yield curve), are salient components of the pricing measure.⁴ Furthermore, we show that the business cycle properties of the BMA-SDF and its volatility are predictable, and predict—as theory implies in this case—future asset returns, generating a substantial degree of time variation in conditional risk premia.

Our work also relates to behavioural biases and market frictions in asset pricing. In particular, complementing the evidence of [Daniel, Hirshleifer, and Sun \(2020\)](#) and [Bryzgalova, Huang, and Julliard \(2023\)](#) for the equity market, we show that the post earnings announcement drifts of both bonds (see [Nozawa, Qiu, and Xiong \(2023\)](#)) and stocks are extremely likely drivers of corporate bond and stock risk premia. Furthermore, we show that cash-flow and discount rate news (see, e.g., [Vuolteenaho \(2002\)](#), [Cohen, Gompers, and Vuolteenaho \(2002\)](#), [Zviadadze \(2021\)](#), and [De la O, Han, and Myers \(2023\)](#)) are both important drivers of risk premia in the joint cross-section of bonds and stocks, but the latter are responsible for a larger share of the volatility of the co-pricing SDF.

1 Data

Our analysis relies on a combination of corporate bond and equity data which we present in detail below.

⁴[Elton, Gruber, and Blake \(1995\)](#) show that adding fundamental macro-risk variables (such as GNP, inflation and term spread measures) significantly improves pricing performance relative to equity and bond market index models. [Elkamhi, Jo, and Nozawa \(2023\)](#) show that the long-run consumption risk measure of [Parker and Julliard \(2003\)](#) yields a one-factor model with significant explanatory power for corporate bonds, and such an SDF, as documented in [Parker and Julliard \(2005\)](#), has a very strong business cycle pattern.

Corporate bond data. We use the constituents of the corporate bond data set from the Bank of America Merrill Lynch (BAML) High Yield (H0A0) and Investment Grade (C0A0) indices made available via the Intercontinental Exchange (ICE), which starts in January 1997 and ends in December 2022 for the corporate bond-level data. For the period from January 1986 to December 1996 we rely on the Lehman Brothers Fixed Income Database (LHM). These data are then merged with the Mergent Fixed Income Securities Database (FISD), which contains additional bond characteristics. We follow [van Binsbergen, Nozawa, and Schwert \(2024\)](#) and begin the LHM sample in 1986.⁵ After merging the two data sets and applying the standard filters, our bond-level data spans 37 years over the period January 1986 to December 2022 for a total of 444 months. and comprises over 30,000 unique bonds. A detailed description of the databases and associated cleaning procedures is available in Section [IA.1](#) of the Internet Appendix.

We apply the following standard filters to the bond data: i) We remove bonds that are not publicly traded in the U.S. market. These include bonds issued through private placement, bonds issued under Rule 144A, bonds that are not traded in U.S. dollars, and bonds from issuers not based in the U.S. ii) We remove bonds that are classified as structured notes, mortgage- or asset-backed, agency-backed, equity-linked or convertible. iii) We exclude bonds that have a floating coupon rate. iv) Finally, we exclude bonds that have less than one year remaining until maturity.

Corporate bond returns. In the baseline analysis, we specify *excess* bond returns as the total bond return minus the one-month risk-free rate of return.⁶ In addition, we follow [van Binsbergen, Nozawa, and Schwert \(2024\)](#) and repeat our analysis with *duration-adjusted* returns, where the bond excess return is computed as the total bond return minus the return on a portfolio of duration-matched U.S. Treasury bonds. Details of the duration adjustment are provided in Appendix [C](#). We do not further winsorize, trim, or augment the underlying bond return data in any way, avoiding the biases that such procedures normally induce ([Duarte, Jones, Mo, and Khorram \(2024\)](#) and [Dickerson, Robotti, and Rossetti \(2024\)](#)).

⁵Prior to 1986, bonds in LHM are predominantly investment grade (91% of bonds) with 67% of all bonds priced with matrix pricing (i.e., the prices are not actual dealer quotes).

⁶We source the one-month risk-free rate from [Kenneth French's website](#).

Equity data. We rely on the publicly available stock anomaly data from [Chen and Zimmermann \(2022\)](#) and [Jensen, Kelly, and Pedersen \(2023\)](#). Full documentation and related data construction methods are detailed on the aforementioned authors’ websites.

The joint factor zoo. We use all factors in published papers for which a monthly time series matching our sample is publicly available. Our bond-specific factor zoo includes 16 tradable bond factors. From the equity literature, we include an additional 24 tradable factors. This set is smaller than the tradable equity zoo in [Bryzgalova, Huang, and Julliard \(2023\)](#) as for several of their 34 tradable factors an updated series is not publicly available, and we also exclude factors for which authors did not provide sufficient information for exact replication.⁷ Our nontradable zoo comprises 14 factors, and many of the nontradable factors used to analyse corporate bond returns have also been used to study stock returns. Overall, we consider a total of 54 factors, of which 40 are tradable and 14 are nontradable. We provide an overview of the factors in Table A.1 of Appendix B. All of the factors are publicly available from the authors’ personal websites, and public repositories, listed therein.⁸

Corporate bond and stock test asset portfolios. We construct a set of bond portfolios that are sorted on various bond characteristics. To ensure a broad enough cross-section for our in-sample (IS) estimation of the BMA, we use 50 bond portfolios. The first 25 portfolios are double-sorted on credit spreads and bond size. The remaining 25 portfolios are double-sorted on bond ratings and time-to-maturity. All portfolios are value-weighted by the market capitalization of the bond issue, defined as the bond dollar value multiplied by the number of outstanding units of the bond. These portfolios are publicly available at openbondassetpricing.com/corporate-bond-factor-zoo. For equity, we rely on a set of 33 portfolios and anomalies very similar to the one used in [Kozak, Nagel, and Santosh \(2020\)](#) and [Bryzgalova, Huang, and Julliard \(2023\)](#) which are publicly available from openassetpricing.com and jpkfactors.com.

The chosen characteristics yield a significant dispersion of average in-sample stock and bond portfolio returns. The inclusion of portfolios sorted on credit spreads is motivated by the work of [Nozawa \(2017\)](#) who finds that bond credit spreads are an important driver of the cross-sectional

⁷The twelve excluded factors are all among the *least* likely drivers of stock premia in [Bryzgalova, Huang, and Julliard \(2023\)](#).

⁸We make our 16 traded bond factors available on the companion website: openbondassetpricing.com

variation in excess corporate bond returns.⁹ Bond ratings are provided by Standard & Poors (S&P) and are a fundamental characteristic of bonds. They underpin most tradable bond factors, define institutional investment guidelines, and capture default risk. Bond maturity has been shown to be an important determinant of expected returns [Gebhardt, Hvidkjaer, and Swaminathan \(2005\)](#) and bond mutual fund holdings in recent work by [Bretscher, Schmid, and Ye \(2023\)](#). The chosen equity anomalies also capture a diverse array of possible risks and also have a large degree of dispersion in their average returns.

Finally, we include the tradable factors as additional test assets since, as emphasized in [Barillas and Shanken \(2016\)](#), factors included in a model should price any factor excluded from the model. This, along with the use of a nonspherical pricing error formulation (i.e., GLS) also imposes (asymptotically) the restriction of factors pricing themselves. For the estimation of the co-pricing BMA-SDF, we naturally include both stock and bond tradable factors. For the stock (bond) specific BMA-SDF, we only include the respective stock (bond) tradable factors. Overall, the cross-section contains a broad array of 50 bond and 33 stock portfolios, sorted on well-known bond or stock characteristics, and the underlying 40 tradable factors themselves.

Out-of-sample test assets. To test the out-of-sample (OS) asset pricing efficacy of the BMA-SDFs estimated on the IS test assets, we employ a broad cross-section of additional corporate bond, stock and U.S. Treasury bond portfolios.

For bonds, we use decile-sorted portfolios on: bond historical 95% value-at-risk, duration, bond value ([Houweling and Van Zundert \(2017\)](#)), bond book-to-market ([Bartram, Grinblatt, and Nozawa \(2020\)](#)), long-term reversals ([Bali, Subrahmanyam, and Wen \(2021\)](#)), momentum ([Gebhardt, Hvidkjaer, and Swaminathan \(2005\)](#)), as well as the bond version of the 17 Fama French industry portfolios, for a total of 77 bond-based portfolios.¹⁰

For stocks, we include decile-sorted portfolios on: earnings-to-price, momentum, long-term reversal, accruals, size (market capitalization), equity variance, and the equity version of the 17 Fama French industry portfolios (following [Lewellen, Nagel, and Shanken \(2010\)](#)), for a total of 77 equity-based portfolios all accessed from [Kenneth French’s webpage](#).

For U.S. Treasury bonds, we download monthly annualized continuously-compounded zero-coupon yields from [Jing Cynthia Wu’s webpage](#) ([Liu and Wu, 2021](#)). We price U.S. Treasury

⁹We follow the credit spread portfolio formation method in [Elkamhi, Jo, and Nozawa \(2023\)](#) and construct the portfolios based on the average bond credit spreads between months $t - 12$ and $t - 1$.

¹⁰All available at openbondassetpricing.com/corporate-bond-factor-zoo.

bonds each month using the yield curve data and then compute monthly discrete excess returns across the term structure as the total return in excess of the one-month Treasury Bill rate. Our set of OS U.S. Treasury portfolios consists of 29 portfolios, spanning 2-year Treasury notes up 30-year Treasury bonds in increments of one year.

Overall, our OS test assets comprise 154 stock and bond portfolios (77 each) from the 14 distinct cross-sections discussed above. We use these both as a joint cross-section and also to construct $2^{14} = 16,384$ possible unique combinations of OS cross-sections.¹¹

2 Econometric method

This section introduces the notation and summarises the methods employed in our empirical analysis. We consider linear factor models for the Stochastic Discount Factor and focus on the SDF representation since we aim to identify the factors that have pricing ability for the joint cross-section of corporate bond and stock returns.¹²

The returns of N test assets, which are long-short portfolios, are denoted by $\mathbf{R}_t = (R_{1t} \dots R_{Nt})^\top$, $t = 1, \dots, T$. We consider K factors, $\mathbf{f}_t = (f_{1t} \dots f_{Kt})^\top$, $t = 1, \dots, T$, that can be either tradable or nontradable. A linear SDF takes the form $M_t = 1 - (\mathbf{f}_t - \mathbb{E}[\mathbf{f}_t])^\top \boldsymbol{\lambda}_f$, where $\boldsymbol{\lambda}_f \in \mathbb{R}^K$ is the vector containing the market prices of risk associated with the individual factors. Throughout the paper, $\mathbb{E}[X]$ or μ_X denote the unconditional expectation of an arbitrary random variable X .

In the absence of arbitrage opportunities, we have that $\mathbb{E}[M_t \mathbf{R}_t] = \mathbf{0}_N$, hence expected returns are given by $\boldsymbol{\mu}_R \equiv \mathbb{E}[\mathbf{R}_t] = \mathbf{C}_f \boldsymbol{\lambda}_f$, where $\mathbf{C}_f$ is the covariance matrix between $\mathbf{R}_t$ and $\mathbf{f}_t$, and prices of risk, $\boldsymbol{\lambda}_f$, are commonly estimated via the cross-sectional regression

$$\boldsymbol{\mu}_R = \lambda_c \mathbf{1}_N + \mathbf{C}_f \boldsymbol{\lambda}_f + \boldsymbol{\alpha} = \mathbf{C} \boldsymbol{\lambda} + \boldsymbol{\alpha}, \quad (1)$$

where $\mathbf{C} = (\mathbf{1}_N, \mathbf{C}_f)$, $\boldsymbol{\lambda}^\top = (\lambda_c, \boldsymbol{\lambda}_f^\top)$, λ_c is a scalar average mispricing (equal to zero under the null of the model being correctly specified), $\mathbf{1}_N$ is an N -dimensional vector of ones, and $\boldsymbol{\alpha} \in \mathbb{R}^N$ is the vector of pricing errors in excess of λ_c (also equal to zero under the null of the model).

¹¹Further details about factors and in- and out-of-sample test assets, as well as links to the data sources, can be found in Internet Appendix [IA.1](#).

¹²Recall that a factor might have a significant risk premium even if it is not part of the SDF, just because it has non-zero correlation with the true latent SDF. Hence, in order to identify the pricing measure, focusing on the SDF representation is the natural choice.

Such models are usually estimated via GMM, MLE or two-pass regression methods (see, e.g., Hansen (1982), Cochrane (2005)). Nevertheless, as pointed out in a large literature, the underlying assumptions for the validity of these methods (see, e.g., Newey and McFadden (1994)), are often violated (see, e.g., Kleibergen and Zhan (2020) and Gospodinov and Robotti (2021)), and identification problems arise in the presence of a *weak* factor (i.e., a factor that does not have enough comovement with any of the assets, or has very little cross-sectional dispersion in this comovement, but is nonetheless considered a part of the SDF). These issues in turn lead to wrong inference for both weak and strong factors, erroneous model selection, and inflate the canonical measures of model fit.¹³

Albeit robust frequentist inference methods have been suggested in the literature for specific settings, our task is complicated by the fact that we want to parse the entire zoo of bond and stock factors, rather than estimate and test an individual model. Furthermore, we aim to identify the best specification—if a dominant model exist—or aggregate the information in the factor zoo into a single SDF if no clear best model arises. Therefore, we rely on the Bayesian method proposed in Bryzgalova, Huang, and Julliard (2023), since it is applicable to both tradable and nontradable factors, can handle the entire factor zoo, is valid under misspecification, and is robust to weak inference problems. This Bayesian approach is conceptually simple, since it leverages the naturally hierarchical structure of cross-sectional asset pricing, and restores the validity of inference using transparent and economically motivated priors.

Consider first the time-series layer of the estimation problem. Without loss of generality, we order the K_1 tradable factors first, $\mathbf{f}_t^{(1)}$, followed by K_2 nontradable factors, $\mathbf{f}_t^{(2)}$, hence $\mathbf{f}_t \equiv (\mathbf{f}_t^{(1),\top}, \mathbf{f}_t^{(2),\top})^\top$ and $K_1 + K_2 = K$. Denote by $\mathbf{Y}_t \equiv \mathbf{f}_t \cup \mathbf{R}_t$ the union of factors and returns, where $\mathbf{Y}_t$ is a p -dimensional vector.¹⁴ Modelling $\{\mathbf{Y}_t\}_{t=1}^T$ as multivariate Gaussian with mean $\boldsymbol{\mu}_Y$ and variance matrix $\boldsymbol{\Sigma}_Y$, and adopting the conventional diffuse prior $\pi(\boldsymbol{\mu}_Y, \boldsymbol{\Sigma}_Y) \propto |\boldsymbol{\Sigma}_Y|^{-\frac{p+1}{2}}$, yields the canonical Normal-inverse-Wishart posterior for the time series parameters $(\boldsymbol{\mu}_Y, \boldsymbol{\Sigma}_Y)$ in equations (A.4)-(A.5) of Appendix A.

The cross-sectional layer of the inference problem allows for misspecification of the factor model via the average pricing errors $\boldsymbol{\alpha}$ in equation (1). We model these pricing errors, as in the previous literature (e.g., (Pástor and Stambaugh, 2000) and (Pástor, 2000)), as $\boldsymbol{\alpha} \sim$

¹³These problems are common to GMM (Kan and Zhang, 1999a), MLE (Gospodinov, Kan, and Robotti, 2019), Fama-MacBeth regressions (Kan and Zhang (1999b), Kleibergen (2009)), and even Bayesian approaches with flat priors for risk prices (Bryzgalova, Huang, and Julliard, 2023).

¹⁴If one requires the tradable factors to price themselves, then $\mathbf{Y}_t \equiv (\mathbf{R}_t^\top, \mathbf{f}_t^{(2),\top})^\top$ and $p = N + K_2$.

$\mathcal{N}(\mathbf{0}_N, \sigma^2 \mathbf{\Sigma}_R)$, yielding the cross-sectional likelihood (conditional on the time series parameters)

$$p(\text{data} | \boldsymbol{\lambda}, \sigma^2) = (2\pi\sigma^2)^{-\frac{N}{2}} |\mathbf{\Sigma}_R|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2\sigma^2} (\boldsymbol{\mu}_R - \mathbf{C}\boldsymbol{\lambda})^\top \mathbf{\Sigma}_R^{-1} (\boldsymbol{\mu}_R - \mathbf{C}\boldsymbol{\lambda}) \right\},$$

where in the cross-sectional regression the ‘data’ are the expected risk premia, $\boldsymbol{\mu}_R$, and the factor loadings, $\mathbf{C} \equiv (\mathbf{1}_N, \mathbf{C}_f)$. The above likelihood can then be combined with a prior for risk prices (presented below) to obtain a posterior distribution and guide inference and model selection.

To handle model and factor selection we introduce a vector of binary latent variables $\boldsymbol{\gamma}^\top = (\gamma_0, \gamma_1, \dots, \gamma_K)$, where $\gamma_j \in \{0, 1\}$. When $\gamma_j = 1$, the j -th factor (with associated loadings $\mathbf{C}_j$) should be included in the SDF, and should be excluded otherwise.¹⁵ In the presence of potentially weak factors, and hence unidentified prices of risk, the posterior probabilities of models and factors are not well defined under flat priors. Hence, we introduce a (economically motivated) prior that, albeit not informative, restores the validity of posterior inference (see [Bryzgalova, Huang, and Julliard \(2023\)](#)). In particular, we model the uncertainty underlying the estimation and model selection problem with a (continuous spike-and-slab) mixture prior, $\pi(\boldsymbol{\lambda}, \sigma^2, \boldsymbol{\gamma}, \boldsymbol{\omega}) = \pi(\boldsymbol{\lambda} | \sigma^2, \boldsymbol{\gamma}) \pi(\sigma^2) \pi(\boldsymbol{\gamma} | \boldsymbol{\omega}) \pi(\boldsymbol{\omega})$, where

$$\lambda_j | \gamma_j, \sigma^2 \sim \mathcal{N}(0, r(\gamma_j) \psi_j \sigma^2).$$

Note the presence of three new elements, ψ_j , $r(\gamma_j)$, and $\pi(\boldsymbol{\omega})$, in the prior formulation.¹⁶

First, $r(\gamma_j)$ captures the ‘spike-and-slab’ nature of the prior formulation. When the factor should be included, we have $r(\gamma_j = 1) = 1$, and the prior, the ‘slab,’ is just a diffuse distribution centred at zero. When instead the factor should not be in the model, $r(\gamma_j = 0) = r \ll 1$, the prior is extremely concentrated—a ‘spike’ at zero. As $r \rightarrow 0$, the prior spike is just a Dirac distribution at zero, hence it removes the factor from the SDF.¹⁷

Second, we set

$$\psi_j = \psi \times \tilde{\boldsymbol{\rho}}_j^\top \tilde{\boldsymbol{\rho}}_j, \tag{2}$$

where $\tilde{\boldsymbol{\rho}}_j \equiv \boldsymbol{\rho}_j - \left(\frac{1}{N} \sum_{i=1}^N \rho_{j,i} \right) \times \mathbf{1}_N$, $\boldsymbol{\rho}_j$ is an $N \times 1$ vector of correlation coefficients between

¹⁵In the baseline analysis, we always include the common intercept in the cross-sectional layer, that is, $\gamma_0 = 1$. Nevertheless, we also consider $\gamma_0 = 0$, i.e., no common intercept, in the robustness analysis.

¹⁶For the cross-sectional variance scale parameter σ^2 we assume the customary diffuse prior $\pi(\sigma^2) \propto \sigma^{-2}$. As per Proposition 1 of [Chib, Zeng, and Zhao \(2020\)](#), since the parameter σ is common across models and has the same support in each model, the marginal likelihoods obtained under this improper prior are valid and comparable.

¹⁷We set $r = 0.001$ in our empirical analysis.

factor j and the test assets, and $\psi \in \mathbb{R}_+$ is a tuning parameter that controls the degree of shrinkage across all factors. That is, factors that have vanishing correlation with asset returns, or extremely low cross-sectional dispersion in their correlations (hence cannot help in explaining cross-sectional differences in returns), have a low value of ψ_j and are therefore endogenously shrunk toward zero. Instead, such prior has no effect on the estimation of strong factors since these have large and disperse correlations with the test assets, yielding a large ψ_j and consequently a diffuse prior. Pure ‘level’ factors—i.e., factors that have no explanatory power for cross-sectional differences in asset returns, but help in capturing the average level of risk premia across assets—can be accommodated removing the free intercept in the SDF (since it would be collinear with a pure level factor), and use simple correlations (instead of cross-sectionally demeaned ones) in equation (2), hence setting $\psi_j = \psi \times \boldsymbol{\rho}_j^\top \boldsymbol{\rho}_j$. We consider this particular case among our robustness exercises, and it leaves our main findings virtually unchanged.

Third, the prior $\pi(\boldsymbol{\omega})$ not only gives us a way to sample from the space of potential models, but also encodes belief about the sparsity of the true model using the prior distribution $\pi(\gamma_j = 1 | \omega_j) = \omega_j$. Following the literature on predictors selection, we set

$$\pi(\gamma_j = 1 | \omega_j) = \omega_j, \quad \omega_j \sim \text{Beta}(a_\omega, b_\omega).$$

Different hyperparameters a_ω and b_ω determine whether one a priori favors more parsimonious models or not. The prior expected probability of selecting a factor is simply $\frac{a_\omega}{a_\omega + b_\omega}$. We set $a_\omega = b_\omega = 1$ in the benchmark case, that is, we have a uniform (hence flat) prior for the model dimensionality and each factor has an ex ante expected probability of being selected equal to 50%.¹⁸

Note that the only free ‘tuning’ parameter in our setting, ψ in equation (2), has a straightforward economic interpretation, since the expected prior Sharpe ratio (SR) achievable with the factors is just $\mathbb{E}_\pi[SR_{\mathbf{f}}^2 | \sigma^2] = \frac{1}{2}\psi\sigma^2 \sum_{k=1}^K \tilde{\boldsymbol{\rho}}_k^\top \tilde{\boldsymbol{\rho}}_k$ as $r \rightarrow 0$.¹⁹ That is, in our empirical analysis we report results for various prior expectations of the Sharpe ratio achievable in the economy.²⁰

The above hierarchical system yields a well defined posterior distribution from which all

¹⁸However, we could set for instance, $a_\omega = 1$ and $b_\omega \gg 1$ to favor sparser models.

¹⁹Without a uniform prior for the SDF dimensionality the prior Sharpe ratio value becomes $\mathbb{E}_\pi[SR_{\mathbf{f}}^2 | \sigma^2] = \frac{a_\omega}{a_\omega + b_\omega} \psi \sigma^2 \sum_{k=1}^K \tilde{\boldsymbol{\rho}}_k^\top \tilde{\boldsymbol{\rho}}_k$ as $r \rightarrow 0$. Hence, beliefs about the prior Sharpe ratio and model dimensionality fully pin down our hyperparameters.

²⁰More precisely, we report results for different prior values of $\sqrt{\mathbb{E}_\pi[SR_{\mathbf{f}}^2 | \sigma^2]}$.

the unknown parameters and quantities of interest (e.g., R^2 , SDF-implied Sharpe ratio, and model dimensionality), can be sampled to compute posterior means and credible intervals via the Gibbs sampling algorithm in Appendix A. Most importantly, these posterior draws can be used to compute posterior model and factor probabilities, and, hence, identify robust sources of priced risk and—if such model exists—a dominant model for pricing assets.

Model and factor probabilities can also be used for aggregating optimally, rather than selecting, the pricing information in the factor zoo. For each possible model γ^m that one could construct with the universe of factors, we have the corresponding SDF: $M_{t,\gamma^m} = 1 - (\mathbf{f}_{t,\gamma^m} - \mathbb{E}[\mathbf{f}_{t,\gamma^m}])^\top \boldsymbol{\lambda}_{\gamma^m}$. Therefore, we construct a BMA-SDF by averaging all possible SDFs using as weights the posterior probability of each model:²¹

$$M_t^{BMA} = \sum_{m=1}^{\bar{m}} M_{t,\gamma^m} \Pr(\gamma^m | \text{data}),$$

where $\bar{m}$ is the total number of possible models.

The BMA aggregates information about the true latent SDF over the space of all possible models, rather than conditioning on a particular model. At the same time, if a dominant model exists (a model for which $\Pr(\gamma^m | \text{data}) \approx 1$), the BMA will use that model alone. Importantly, pricing with the BMA-SDF is robust to the problems arising from collinear loadings of assets on the factors, since any convex linear combination of factors with collinear loadings has exactly the same pricing implications. Moreover, the BMA-SDF can be microfounded, as in Heyerdahl-Larsen, Illeditsch, and Walden (2023), thanks to the equivalence of a log utilities and heterogenous beliefs economy with a representative agent using the Bayes rule. Furthermore, BMA aggregation is optimal under a wide range of criteria, but in particular, it is *optimal on average*: no alternative estimator can outperform it for all possible values of the true unknown parameters.²² Finally, since its predictive distribution minimizes the Kullback-Leibler information divergence relative to the true unknown data-generating process, the BMA aggregation delivers the most likely SDF given the data, and the estimated density is as close as possible to the true unknown one, even if all of the models considered are misspecified.

²¹See, e.g., Raftery, Madigan, and Hoeting (1997) and Hoeting, Madigan, Raftery, and Volinsky (1999).

²²See, e.g., Raftery and Zheng (2003) and Schervish (1995).

3 Estimation results

In this section, we apply the hierarchical Bayesian method to a large set of factors proposed in the previous bond and equity literature. Overall, we consider 40 tradable and 14 nontradable factors, yielding $2^{54} \approx 18$ quadrillion possible models for the combined bond and stock factor zoo. In Sections 3.1 and 3.3 we only consider returns for the bond portfolios in excess of the short term risk free rate (calculated as outlined in Section 1). In Section 3.2, we also use duration-adjusted excess returns, as well as U.S. Treasury portfolios, to disentangle the credit and Treasury term structure components of corporate bond returns.

3.1 Co-pricing bonds and stocks

We now consider the pricing power of the 54 factors to gauge to what extent the cross-section of corporate bond and stock returns is priced by the joint factor zoo. The IS test assets include the 50 bond and 33 stock portfolios described in Section 1 in addition to the 40 tradable factor portfolios ($N = 123$). Throughout, we use the continuous spike-and-slab approach described in Section 2. To report the results, we refer to the priors as a fraction of the ex post maximum Sharpe ratio in the data, which is equal to 5.4 annualized for the joint cross-section of portfolios, from a very strong degree of shrinkage (20%, i.e., a prior annualized Sharpe ratio of 1.0), to a very moderate one (80% or a prior annualized Sharpe ratio of 4.2). Given that the results demonstrate considerable stability across a wide range of prior Sharpe ratio values, we present selected findings for a prior set at 80% of the ex post maximum Sharpe ratio, as this choice tends to yield the best out-of-sample performance.²³

3.1.1 The co-pricing SDF

We start by asking which factors are likely components of the latent Stochastic Discount Factor in the economy. Figure 1 reports the posterior probabilities (given the data) of each factor (i.e., $\mathbb{E}[\gamma_j|\text{data}], \forall j$) for different values of the prior Sharpe ratio achievable with the linear SDF (expressed as a percentage of the ex post maximum Sharpe ratio). In Figure 2 we list all 54 factors in increasing order of posterior probabilities (top panel), for a prior Sharpe ratio of 80% of the maximum ex post Sharpe ratio, along with the corresponding annualised posterior means

²³Additional results for different values of the prior Sharpe ratio are reported in Table A.2 of the Appendix.

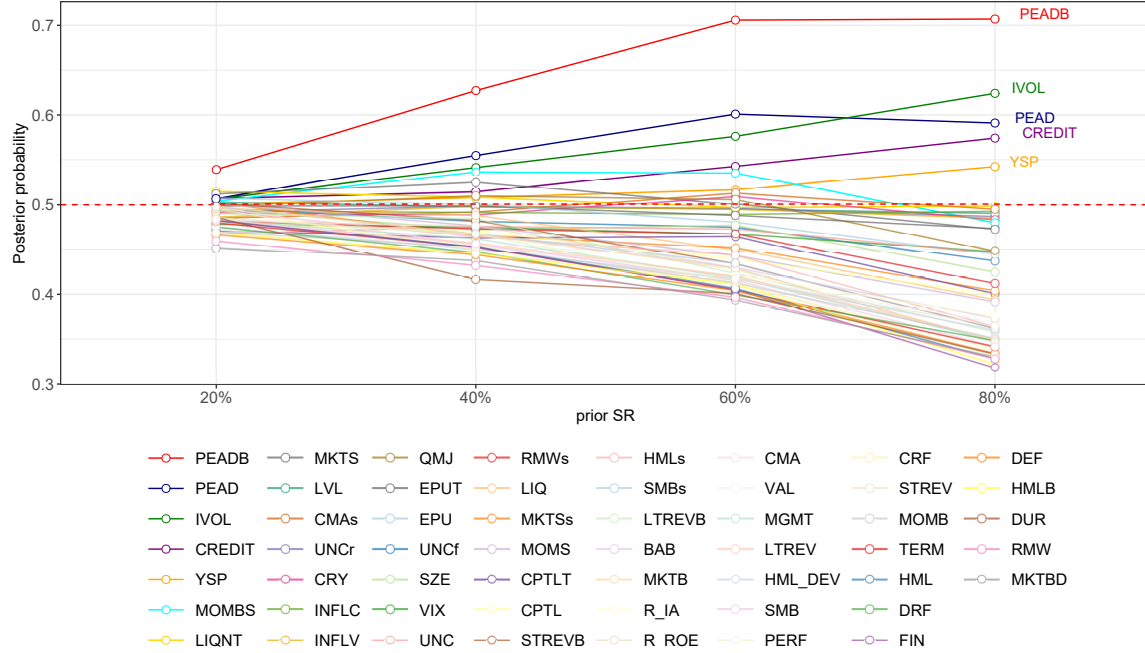


Figure 1: Posterior factor probabilities: co-pricing factor zoo.

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, of the 54 bond and stock factors described in Appendix B. The labels are ordered by each factor’s average posterior probability across the four levels of shrinkage (high to low). Test assets include the 83 bond and stock portfolios and the 40 tradable bond and stock factors ($N = 123$). The prior distribution for the j^{th} factor inclusion is a Beta(1, 1), yielding a 0.5 prior expectation for γ_j . Posterior probabilities for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_j^2 | \sigma^2]}$, set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the 83 bond and stock portfolios and tradable factors. Sample period: 1986:01 to 2022:12 ($T = 444$).

of the price of risk of the factors (i.e., $\mathbb{E}[\lambda_j|\text{data}], \forall j$, bottom panel).²⁴

Recall that we have a uniform (hence flat) prior for the model dimensionality and each factor has an ex ante expected probability of being selected equal to 50% (dashed horizontal lines in Figure 1 and top panel of Figure 2). Figure 1 illustrates that—with some notable exceptions—most factors proposed in the corporate bond and equity literatures have (individually) a posterior probability of being part of the SDF that is below its prior value of 50%.

Several observations are in order. First, given their posterior probabilities across the range of prior Sharpe ratios considered, there is strong evidence for including PEADB and PEAD (i.e., respectively, the bond and equity post-earnings announcement drift factors) as a source of

²⁴Posterior probabilities and market prices of risk for different priors are tabulated in Table A.2 of the Appendix. See Table A.1 in Appendix B for a detailed description of the factors.

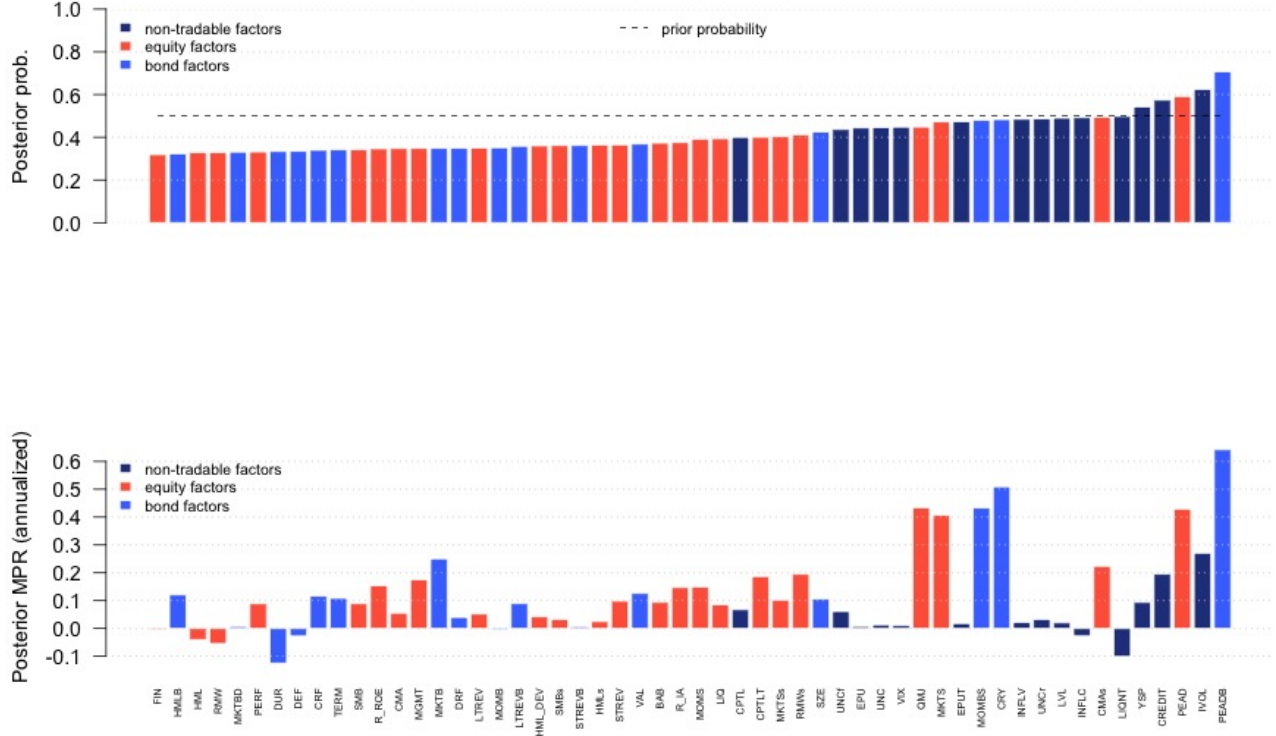


Figure 2: Posterior factor probabilities and market prices of risk – joint bond and stock factor zoo (excess bond returns).

Posterior factor probabilities (top panel), $\mathbb{E}[\gamma_j|\text{data}]$, and the corresponding posterior market prices of risk (bottom panel), $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix B. Test assets include the 83 bond and stock portfolios and the 40 tradable bond and stock factors ($N = 123$). The prior distribution for the j^{th} factor inclusion is a Beta(1, 1), yielding a 0.5 prior expectation for γ_j . The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 stock and bond portfolios and tradable factors. Sample period: 1986:01 to 2022:12 ($T = 444$).

priced risk in the SDF. Partially, this is a rather surprising result, as PEADB has not specifically been proposed as a priced risk factor in the previous literature. [Nozawa, Qiu, and Xiong \(2023\)](#) are the first to document a post-earnings announcement drift in corporate bond prices, and they rationalise their finding with a stylised model of disagreement. They also show that a strategy that purchases bonds issued by firms with high earnings surprises and sells bonds of firms with low earnings surprises generates sizeable Sharpe ratios and large risk-adjusted returns. On the other hand, [Bryzgalova, Huang, and Julliard \(2023\)](#) and [Avramov, Cheng, Metzker, and Voigt](#)

(2023) find strong evidence that the *stock market* post-earnings announcement drift (PEAD) factor of Daniel, Hirshleifer, and Sun (2020) exhibits a particularly strong posterior probability of being part of the SDF for equity returns. In fact, PEAD is the only other tradable factor with a posterior probability of being part of the SDF that prices the joint cross-section of corporate bond and stock returns that is above 50%. That is, the only two tradable factors with high posterior probabilities are the bond and stock versions of the post-earnings announcement drift. Note that, in equilibrium models in which rational agents with limited risk bearing capacity face behavioural asset demand, the drivers of the latter become part of the pricing measure—exactly as we find (see, e.g., De Long, Shleifer, Summers, and Waldman (1990)). Note also that, as shown in Tables IA.II and IA.III of the Internet Appendix, these are the tradable factors with the highest Sharpe ratio in our full sample. Moreover, PEADB has the highest Sharpe ratio among bond factors also in the first and second halves of the sample (1986:01 to 1999:12 and 2000:01 to 2022:12, respectively), while PEAD has the highest Sharpe ratio among equity factors in the first half, and one of the highest in the second half.

Second, the stock as well as the bond market factors (MKTS and MKTB, respectively) both exhibit posterior probabilities below 50% for the full range of prior Sharpe ratios for the joint cross-section of returns. Nevertheless, when separately pricing the cross-sections of bond and stock returns with only the factors in their respective zoo, both market indices become likely components of the SDF: for all prior levels in the MKTS case, and when sparse models are ex ante considered more likely in the MKTB case (see, respectively, Tables IA.IV and IA.V of the Internet Appendix). This confirms the findings that the equity market index contains valuable information for pricing stocks in an unconstrained SDF based on stock factors (as in (Bryzgalova, Huang, and Julliard, 2023)) and that the bond market index is a valuable factor when focusing on low dimensional bond models for pricing corporate bonds (as in (Dickerson, Mueller, and Robotti, 2023)). However, when the space of potential factors is expanded to include both bond and stock factors, without dimensionality restrictions on the SDF as we do in our baseline co-pricing exercise, models with MKTS and MKTB overall perform worse than denser models containing factors from both zoos. That is, the information in the two market indices appears to be spanned by the other factors in the zoos. Note that this finding is unlikely to be driven by the market indices acting as ‘level’ or ‘weak’ factors since asset returns display large and well dispersed loadings on these factors, the market prices of risk they command are substantial when included in the SDF (see bottom panel of Figure 2), and similar results hold

when omitting the common intercept from the SDF models considered (see Table IA.VI of the Internet Appendix).

Third, in Figure 1 there is a small number of *nontradable* factors that have posterior probabilities of being part of the SDF above 50% percent for all values of the prior Sharpe ratio: the slope of the Treasury yield curve (YSP, (Kojien, Lustig, and Van Nieuwerburgh, 2017)), the AAA/BAA yield spread (CREDIT, (Fama and French, 1993)), and idiosyncratic equity volatility (IVOL, (Campbell and Taksler, 2003)). Interestingly, the term premium and default risk factors had originally been suggested in Fama and French (1993) exactly for the purpose of co-pricing bonds and stocks.

Fourth, while there are a few factors for which the posterior probability is roughly equal to the prior (implying that at least some of these factors are likely to be weakly identified at best), there are a large set of factors that are unlikely to be *individually* part of the SDF pricing the joint cross-section of bond and stock returns. Specifically, besides PEADB and PEAD, the tradable bond and stock market factors are overall unlikely to be individually included in the SDF. For instance, with a prior Sharpe ratio set to 80% of the ex post maximum, the posterior probabilities of 29 of the 40 tradable bond and stock factors are below 40% (see top panel of Figure 2). Nevertheless, as shown and discussed extensively below, this does *not* imply that these factors, *jointly*, do not carry relevant information to characterise the true latent SDF.

Finally, as shown in Figure IA.2 and Tables IA.VI–IA.VIII of the Internet Appendix, removing the free intercept, and the consequent prior penalization of pure level factors, leaves all of the above results virtually unchanged.

The bottom panel of Figure 2 reports the posterior (annualized) market prices of risk of the factors (that are also tabulated, for different values of the prior Sharpe ratio, in Table A.2 of the Appendix). All five factors with posterior probabilities higher than their prior values (i.e., PEADB, IVOL, PEAD, CREDIT and YSP) command substantial market prices of risk. Out of the next fifteen factors with highest (individual) posterior probabilities, ten are also nontradable in nature. That said, the risk prices of many of these nontradable factors are small and in some cases effectively shrunk toward zero. This is due to the fact that these are likely *weak factors* in the joint cross-section of corporate bond and stock returns.²⁵ The occurrence of weak factors, which, in fact, is most common among the nontradable ones, causes identification failure and

²⁵That is, their correlations with the test assets are small and have little cross-sectional dispersion. See, e.g., Gospodinov, Kan, and Robotti (2019), Kleibergen (2009), and Bryzgalova, Huang, and Julliard (2023) for a formal definition of weak and level factors.

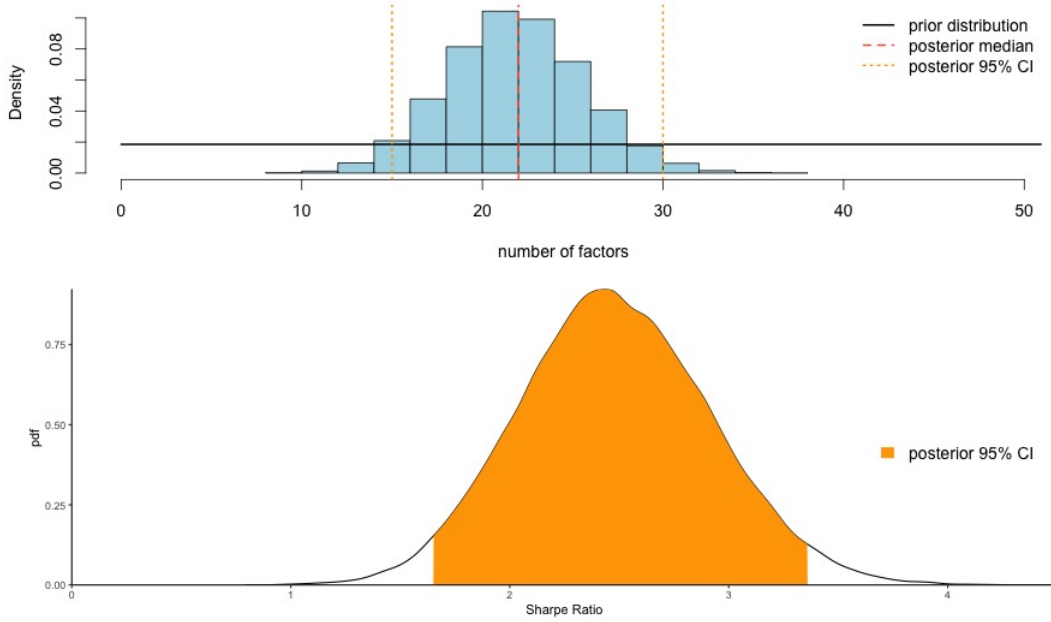


Figure 3: Posterior SDF dimensionality and Sharpe ratios – co-pricing factor zoo.

Posterior distributions of the number of factors to be included in the co-pricing SDF (top panel) and of the SDF-implied Sharpe ratio (bottom panel), computed using the 54 bond and stock factors described in Appendix B. The labels are ordered by each factor’s average posterior probability across the four levels of shrinkage (high to low). The prior distribution for the j^{th} factor inclusion is a Beta(1, 1), yielding a 0.5 prior expectation for γ_j . The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the bond and stock portfolios and tradable factors. Sample period: 1986:01 to 2022:12 ($T = 444$).

invalidates canonical estimation approaches (e.g., GMM, MLE, and two-pass regressions). This is *not* an issue for our Bayesian method, which restores inference, by design, by regularizing the marginal likelihood.

The above findings raise the question of whether the handful of most likely factors that we have identified are enough to characterise the true, latent, SDF that jointly prices bonds and stocks. Moreover, are the less likely to be included factors really devoid of useful pricing information? Since our method does not ex ante impose the existence of a unique, low dimensional, and correctly specified model (all assumptions that are needed with conventional frequentist asset pricing methods), we can formally answer these questions.

The top panel of Figure 3 reports the posterior dimensionality of the SDF in terms of observable factors to be included in it, and the bottom panel shows the posterior distribution of the Sharpe ratios achievable with such an SDF. It is evident that the low dimensional models

suggested in the previous corporate bond and equity literature have very weak support in the data, and are misspecified with very high probability as a substantial number of factors is needed to construct a likely SDF: the posterior median number of factor is 23 with a centered 95% coverage of 16 to 30 factors. In fact, the posterior probability of a model with less than 10 factors is virtually zero, indicating that the quest for a sparse, unique, SDF model among the observable factors in the joint bond and stock factor zoo is misguided at best.

The bottom panel of Figure 3 highlights that the ex post achievable Sharpe ratio given the SDF is not unrealistically large (recall that the ex post maximum Sharpe ratio in the data is 5.4), suggesting that many factors are likely to cover a lot of common risk. Furthermore, Table 1 shows that albeit the most likely (top five) factors to be included in the SDF are responsible for a substantial Sharpe ratio ($\mathbb{E}[SR_f|\text{data}]$, about 0.79 to 1.23 for a 60% to 80% prior in Panel A), the share of the SDF squared Sharpe ratio generated by these factors ($\mathbb{E}[\frac{SR_f^2}{SR_m^2}|\text{data}]$) is quite limited, highlighting that the less likely factors are needed, *jointly*, to provide an accurate characterisation of the risks priced by the true latent SDF. This feature of the data arises not only when jointly pricing bonds and stocks (Panel A), but also when separately focusing on the pricing of the two asset classes using their respective factor zoos (Panels B and C).

In Figure 4 we plot the cumulative SDF implied Sharpe ratio when subsequently adding factors ordered on their (individual) posterior probability. While the Sharpe ratio is monotonically increasing in the number of factors, some factors seem to add more to the implied Sharpe ratio than others. For example the factors ranked 8 to 11 (INFLC, LVL, UNCr, INFLV) do not seem to add much individually, while the Sharpe ratio increases distinctly once factor 12 (CRY) is added. This is due to the fact that many factors are potentially noisy proxies for the same fundamental sources of risk that are important for the SDF. All factors that are noisy proxies for a particular fundamental source of risk will display nonzero posterior probabilities and market prices of risk. However, the Sharpe ratio only jumps once the first of the factors spanning (at least partially) a common risk is included in the analysis. Instead, subsequent factors spanning the same risk generate a much smaller increase in the Sharpe ratio, due to the improved signal extraction of the common risk. Further examining the four factors in spots 8 to 11, these are all nontradable in nature and related to inflation, interest rates and uncertainty. Similarly, factors in spots 17 to 20 are all related to different measures of macroeconomic uncertainty. While it is important to include all of these factors in the SDF to increase the signal to noise ratio, their individual marginal contribution to the Sharpe ratio may be minimal as

Table 1: Most likely (top five) factor contribution to the SDF

	Total prior Sharpe ratio			
	20%	40%	60%	80%
Panel A: Co-pricing SDF				
$\mathbb{E}[SR_f \text{data}]$	0.16	0.42	0.79	1.23
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.01	0.04	0.13	0.28
Panel B: Bond SDF				
$\mathbb{E}[SR_f \text{data}]$	0.22	0.64	1.09	1.44
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.02	0.16	0.44	0.68
Panel C: Stock SDF				
$\mathbb{E}[SR_f \text{data}]$	0.17	0.41	0.72	1.12
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.01	0.07	0.20	0.43

Posterior mean of the implied Sharpe ratios, $\mathbb{E}[SR_f|\text{data}]$, and share of the SDF squared Sharpe ratio, $\mathbb{E}[SR_f^2/SR_m^2|\text{data}]$, of the top five factors. The subsets are split across the five most likely factors to be included in the BMA-SDF versus the remaining factors. The top five co-pricing factors are PEADB, IVOL, PEAD, CREDIT and YSP. The top five bond factors are PEADB, CREDIT, MOMBS, IVOL and YSP. The top five stock factors are PEAD, MKTS, IVOL, CMAs and EPUT. Panels A, B and C report results for the co-pricing, bond-only and stock-only BMA-SDFs, respectively, using the corresponding factor zoos.

they have common spanning of the underlying priced risk.

3.1.2 Which risks?

Next, we further decompose the posterior dimensionality of the SDF, and its implied Sharpe ratio, to better understand which types of risk are likely to be part of the true latent pricing measure, and to what extent different factors capture common information.

Table 2 presents the decomposition of the posterior SDF dimensionality and Sharpe ratio split between nontradable and tradable bond and stock factors, for different prior values. Panel A reports results for the pricing of the joint cross-section of stock and corporate bond returns using factors from both zoos to construct the SDF. Instead, Panels B and C focus, respectively, on the separate pricing of corporate bonds and stocks using only factors from their respective zoos. Several salient patterns are evident.

First, Panel A shows that an accurate characterisation of the pricing measure requires an SDF that is dense not only in the overall space of observable factors (as per top Panel of Figure

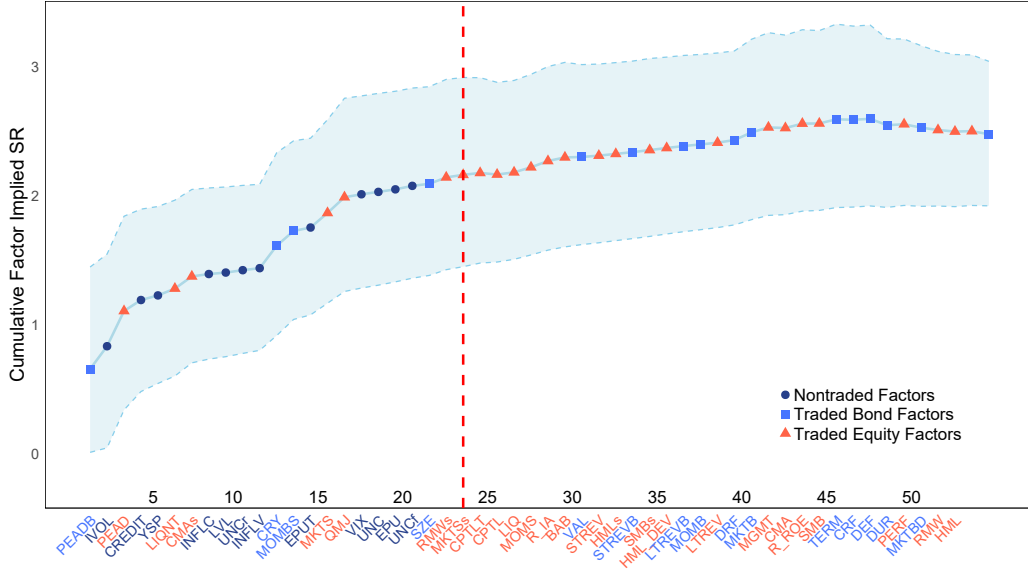


Figure 4: Cumulative co-pricing SDF implied Sharpe ratio.

We incrementally compute the implied Sharpe ratio of the SDF by including sequentially each of the 54 factors (including all factors up to that point) ordered by their posterior probability of inclusion (see top Panel of Figure 2). We estimate the factor implied Sharpe ratio as the annualized standard deviation of the SDF. The vertical red dashed line denotes the posterior median number of factors that should be included in the SDF. The light blue squares (red triangles) represent tradable bond (stock) factors. The dark blue circles represent nontradable factors. The light blue dashed lines and shaded area denote the centred 90% confidence interval. The 54 factors that comprise the co-pricing factor zoo are described in Appendix B. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 bond and stock portfolios and tradable factors. Sample period: 1986:01 to 2022:12 ($T = 444$).

3), but also over the individual subspaces of nontradable as well as bond and stock tradable factors: the posterior mean number of factors is about 7 for nontradable factors, 6 to 8 for bond, and 9 to 12 for stock tradable factors. Furthermore, this density of the SDF is not driven by the co-pricing task: even pricing only bonds (Panel B) or stocks (Panel C), requires about 7 nontradable factors, 6 to 8 for bond, and 10 to 12 stock tradable factors.

Second, each of the three categories of factors is *economically* important. Focusing on the moderate prior shrinkage case (i.e., 80% of the ex post achievable Sharpe ratio) in Panel A, the posterior mean of the (annualised) Sharpe ratio ascribable to the various types of factors ($\mathbb{E}[SR_f|\text{data}]$) is 1.12 for the nontradable ones, and 1.58 and 1.78, respectively, for the bond and stock tradable ones.

Third, there is substantial common priced information across the categories of factors, as

Table 2: SDF dimensionality and SR decomposition by type of factor

	Total prior SR				Total prior SR			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Co-pricing SDF								
	Nontradable factors				Tradable factors			
Mean	6.93	6.96	6.93	6.84	19.47	18.86	17.85	15.48
5%	4	4	4	4	14	14	13	10
95%	10	10	10	10	25	24	23	21
$\mathbb{E}[SR_f \text{data}]$	0.21	0.43	0.70	1.12	0.86	1.44	1.91	2.26
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.01	0.04	0.10	0.23	0.13	0.36	0.63	0.84
	Bond tradable				Stock tradable factors			
Mean	7.78	7.56	7.17	6.23	11.69	11.31	10.68	9.25
5%	5	4	4	3	8	7	7	5
95%	11	11	10	10	16	15	15	13
$\mathbb{E}[SR_f \text{data}]$	0.56	0.96	1.28	1.51	0.66	1.14	1.50	1.78
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.06	0.17	0.30	0.39	0.08	0.23	0.40	0.53
Panel B: Bond SDF								
	Nontradable factors				Tradable factors			
Mean	6.96	6.98	7.08	7.09	7.88	7.78	7.35	6.32
5%	4	4	4	4	5	5	4	3
95%	10	10	10	10	11	11	11	10
$\mathbb{E}[SR_f \text{data}]$	0.18	0.38	0.62	0.99	0.53	0.92	1.25	1.43
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.01	0.05	0.14	0.34	0.10	0.30	0.53	0.65
Panel C: Stock SDF								
	Nontradable factors				Tradable factors			
Mean	6.97	7.00	7.03	6.98	11.75	11.45	10.91	9.84
5%	4	4	4	4	8	7	7	6
95%	10	10	10	10	16	15	15	14
$\mathbb{E}[SR_f \text{data}]$	0.14	0.29	0.48	0.78	0.60	1.03	1.38	1.69
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.01	0.03	0.09	0.23	0.12	0.35	0.62	0.87

Posterior means of: number of factors (with 90% C.I.), implied Sharpe ratios, $\mathbb{E}[SR_f|\text{data}]$, and ratio of SR_f^2 to the total SDF Sharpe ratio, $\mathbb{E}[SR_f^2/SR_m^2|\text{data}]$, of a subset of factors. Subsets are split across tradable and nontradable factors, and within tradable factors we further separate bond and stock factors. Panels A, B and C report results for the co-pricing, bond-only and stock-only BMA-SDFs, respectively, using the corresponding factor zoos.

the sum of the Sharpe ratios generated by the three sets of factors ($1.12 + 1.51 + 1.78 = 4.41$ in Panel A) is much larger than the average posterior SDF implied Sharpe ratio (which is around 2.5 in the bottom panel of Figure 3). This overlap in risks captured by different types of factors is particularly strong among the tradable ones, where the sum of the Sharpe ratios of bond and stock factors in the SDF is $1.51 + 1.78 = 3.29$, while the posterior mean Sharpe ratio for all tradable factors jointly is about 2.26.

The degree of common spanning of priced risks can be formally assessed focusing on the estimated share of the squared Sharpe ratio of the SDF generated by the different types of factors, $\mathbb{E}[\frac{SR_f^2}{SR_m^2}|\text{data}]$. Summing the shares in Panel A ascribable to, respectively, nontradable (0.23) and tradable bond (0.39) and stock (0.53) factors yields a total of 1.15, i.e., more than 100%, indicating substantial commonality among the fundamental risks spanned by the different types of factors. Furthermore, the sum of the shares for bond and stock factors ($0.39 + 0.53 = 0.92$) is much larger than the share due to all tradable factors jointly (0.84). That is, tradable bond and stock factors capture, at least partially, the same underlying sources of priced risk. Similarly, summing the shares of squared Sharpe ratios ascribable to nontradable and tradable factors in Panels A to C yields 1.05, 0.99, and 1.1, indicating some common spanning between tradable and nontradable factors driven mostly by equity factors.

Given the saliency of tradable factors for the SDF outlined above, with their share of the squared Sharpe ratio of the SDF in the two thirds to four fifths ballpark, a natural question is what types of risks do these factors capture. Using the method pioneered by [Campbell and Shiller \(1988\)](#) and extended by [Vuolteenaho \(2002\)](#), we classify the tradable factors into those that relate more to discount rate news and those for which instead cash-flow news are more important.²⁶ Section IA.3 of the Internet Appendix details the empirical (VAR) methodology used for categorizing our 40 tradable stock and bond factors as (mostly) driven by either discount rate news or cash-flow news. The estimated positioning of the individual factors on the spectrum of discount rate and cash-flow news is summarized in Figure 5. Interestingly, the two most likely tradable components of the SDF, the post-earnings announcement drift factors in bonds and stocks, PEAD and PEADB, appear mostly driven by discount rate news.

Table 3 decomposes, for a range of prior values, the contribution to the SDF dimensionality and Sharpe ratio of tradable factors mostly related to discount rate and cash-flow news. Panel A reports results for the joint pricing of bonds and stocks with all factors, while Panels B

²⁶See [Kojen and Van Nieuwerburgh \(2011\)](#) and more recent work by [Zviadadze \(2021\)](#).

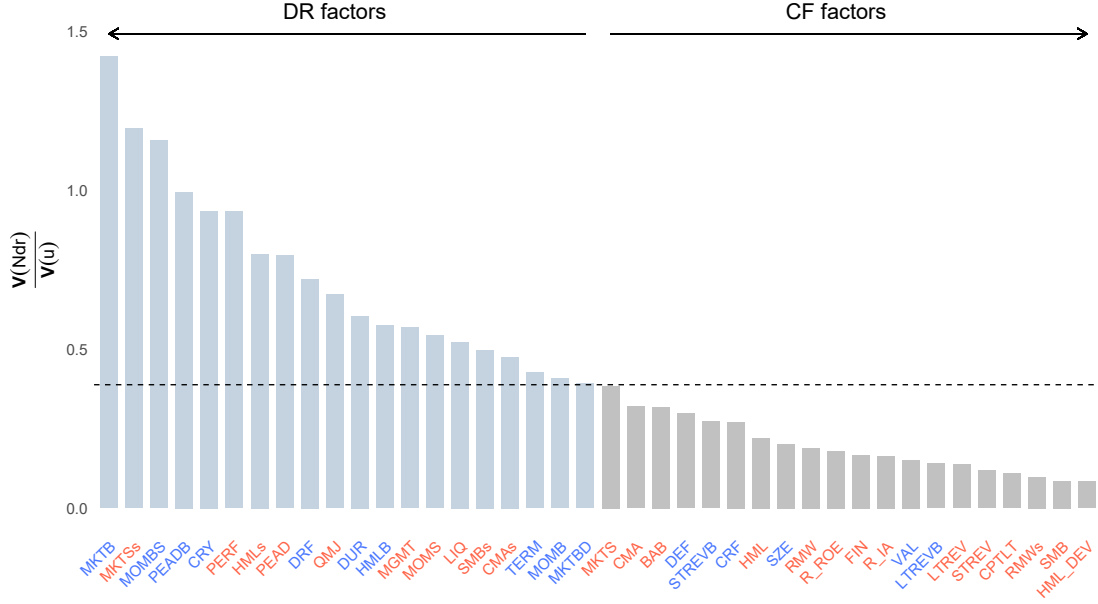


Figure 5: Tradable factor zoo decomposition – cash-flow and discount rate news.

The figure plots the ordered ratio of the variance of the discount rate news component to total variance of the residuals, $\mathbb{V}(Ndr)/\mathbb{V}(u)$, estimated using equation (IA.13) in Internet Appendix IA.3, for each bond and stock tradable factor. The dashed horizontal line corresponds to the median value of the ratio (0.39). The first 20 factors are associated (in a relative manner) with discount rate news, the latter 20 factors are associated more with cash-flow rate news. Bond factors are displayed in blue while stock factors are displayed in red on the x-axis.

and C focus on the two asset classes and factor zoos separately. The left (right) panels pertain to discount rate (cash-flow) news. First, discount rate news factors marginally dominate the composition of the co-pricing SDF in Panel A. The average factor implied Sharpe ratios, $\mathbb{E}[SR_f|\text{data}]$, of the discount rate news driven factors are always higher than the cash-flow driven counterparts. This translates into a much higher proportion of the total implied Sharpe ratio being driven by DR-related factors. For a prior level equal to 80% of the ex post achievable Sharpe ratio, DR driven factors comprise 75% of the total Sharpe ratio variance, compared to 56% for the CF driven factors. Second, in Panel B, when considering the corporate bond SDF, the total Sharpe ratio is predominantly driven by bond factors relating to DR news. The factor implied SR, $\mathbb{E}[SR_f|\text{data}]$ and $\mathbb{E}[\frac{SR_f^2}{SR_m^2}|\text{data}]$ for DR driven factors is close to double that of the CF driven factors. Finally, in Panel C, when considering only stock factors, both discount and cash-flow news seem to play an equally important role, providing very similar contributions to

Table 3: Discount rate or cash-flow news?

	Discount rate (DR) news				Cash-flow (CF) news			
	Total prior SR				Total prior SR			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Co-pricing SDF, stock & bond tradable factors								
Mean	9.81	9.62	9.27	8.18	9.66	9.25	8.58	7.31
5%	6	6	6	5	6	6	5	4
95%	13	13	13	12	13	13	12	11
$\mathbb{E}[SR_f \text{data}]$	0.65	1.19	1.70	2.10	0.60	1.06	1.46	1.77
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.08	0.26	0.52	0.75	0.07	0.21	0.39	0.56
Panel B: Bond SDF, bond tradable factors								
Mean	4.86	4.77	4.60	4.05	2.92	2.79	2.57	2.19
5%	2	2	2	2	1	1	1	0
95%	7	7	7	7	5	5	5	4
$\mathbb{E}[SR_f \text{data}]$	0.49	0.86	1.19	1.42	0.26	0.49	0.67	0.80
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.05	0.14	0.27	0.36	0.02	0.05	0.10	0.15
Panel C: Stock SDF, stock tradable factors								
Mean	4.94	4.85	4.67	4.13	6.74	6.46	6.01	5.12
5%	2	2	2	2	4	3	3	2
95%	8	7	7	7	10	10	9	8
$\mathbb{E}[SR_f \text{data}]$	0.41	0.81	1.18	1.48	0.52	0.91	1.23	1.49
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.04	0.13	0.28	0.41	0.05	0.16	0.29	0.41

The table reports the posterior means of the number of factors (with 90% C.I.), implied Sharpe ratios, $\mathbb{E}[SR_f|\text{data}]$, and the ratio of SR_f^2 to the total SDF Sharpe ratio, $\mathbb{E}[SR_f^2/SR_m^2|\text{data}]$, of a subset of factors. The subsets are split across factors which we classify as discount rate news (DR) driven or cash-flow (CF) news driven. The CF and DR decomposition follows Vuolteenaho (2002) and is detailed in Section IA.3 of the Internet Appendix. Panels A, B and C report results for the co-pricing, bond-only and stock-only BMA-SDFs, respectively, using the corresponding factor zoos.

the Sharpe ratio of the SDF.

3.1.3 Cross-sectional asset pricing

We now turn to the asset pricing performance of the BMA-SDF based on the joint cross-section and factor zoos, as well as based on bond and stock portfolios separately. In Table 4 we report results for in-sample cross-sectional pricing using various performance measures, while out-of-sample results are summarized in Table 5. The in-sample assets for the joint cross-section in Panel A of Table 4 are the 83 portfolios of stocks and bonds (described in Section 1) plus 40 tradable factors ($N = 123$). Panels B and C focus instead on only bonds (50 portfolios and

16 bond tradable factors, $N = 66$) and stocks (33 anomaly portfolios and 24 stock tradable factors, $N = 57$), respectively. The out-of-sample test assets in Table 5 comprise of 77 bond portfolios and 77 stock portfolios (described in Section 1) that are considered jointly in Panel A and separately in Panels B and C, respectively.

When assessing the pricing performance, we compare our BMA-SDF for different levels of prior Sharpe ratio with the performance of a number of benchmark models. In particular, we consider the bond CAPM (CAPMB), the stock CAPM, the Fama and French (1993) five-factor model (FF5), the intermediary asset pricing model of He, Kelly, and Manela (2017) (HKM), the PCs-based SDF of Kozak, Nagel, and Santosh (2020) (KNS) and the risk premia PCA approach of Lettau and Pelger (2020) (RPPCA).²⁷ In addition, since most of the previous literature has focused on selection, rather than aggregation, of pricing factors, we also include the respective ‘top factor’ models (Top) from our Bayesian analysis, that comprises only the factors with posterior probabilities exceeding the prior one (for the joint cross-section for example, this is a five-factor model with PEADB, IVOL, PEAD, CREDIT, and YSP). All the benchmark model SDFs are estimated via a GLS version of GMM.²⁸ Note that for the cross-sectional out-of-sample pricing we do not refit the BMA-SDF or the other benchmark models to the new data. Instead, we use the estimated parameters from the respective in-sample pricing exercises.

For the in-sample pricing in Table 4, a few observations are in order. First and foremost, the BMA-SDF using moderate shrinkage (80% of the prior Sharpe ratio) outperforms virtually all benchmark models on almost all dimensions considered, with the best alternative model being KNS. Second, all low dimensional models do not perform well. This should not come as a surprise given the discussion in Section 3.1.2 that implies that all low dimensional models are both misspecified with very high probability and strongly rejected by the data. In fact, the performance of both the bond and stock CAPM is rather underwhelming compared to the BMA-SDF. Moreover, popular models such as FF5 and HKM do not perform particularly well either. Third, the low dimensional Top factor model, albeit better performing than the low dimensional models from the literature, delivers worse pricing than the BMA-SDF with moderate shrinkage, once again pointing out that aggregation of factors, rather than selection, is preferred by the data. Or in other words, highlighting that just the most likely factors are not sufficient to provide an accurate characterization of the risks spanned by the true latent

²⁷The SDFs of both KNS and RPPCA are reestimated using our data and the methods proposed in the original papers. Details of the construction of all benchmark models are reported in Appendix D.

²⁸See, e.g., Cochrane (2005, pp. 256–258).

SDF. Fourth, the results are fairly consistent moving through the three panels. Apart from the BMA-SDF, KNS and (albeit to a lesser extent) RPPCA deliver consistently better in-sample pricing than the low dimensional models.

The co-pricing BMA-SDF performs exceptionally well out-of-sample (see Panel A of Table 5). While KNS is a close contender when it comes to the in-sample performance, the BMA-SDF strongly dominates KNS out-of-sample. The superiority of the BMA-SDF is less evident when separately considering the cross-sections of bonds and stocks, and SDFs based only on their specific factor zoos (see Panels B and C, respectively), once again highlighting the importance of both sets of factors for a complete characterisation of the SDF, but still performs better OS than all benchmark models we consider.

Obviously, there is a legitimate concern that the strong OS performance of the co-pricing BMA-SDF might be driven by the particular, yet rich, selection of test assets. To address this concern, we also consider the separate pricing of *all* the possible combinations of the 14 different cross-sections comprising our OS test assets. Figure 6 visualises the performance of the BMA-SDF vis-à-vis the best competitor, KNS, by depicting the distributions of different measures of fit across $2^{14} = 16,384$ OS cross-sections. For the cross-sectional R_{OLS}^2 , RMSE and MAPE, there is virtually no overlap in the distributions for the co-pricing BMA-SDF and KNS, with the former clearly besting the latter, implying that the Bayesian approach delivers strictly better OS pricing than its best competitor. There is only an overlap in the distribution when considering R_{GLS}^2 as the measure of fit, yet the BMA-SDF outperforms KNS in 96.6% of the OS cross-sections and its measure of fit concentrates on much higher values.

Given the findings in Tables 4 and 5 that bonds and stocks can be accurately priced *separately* with BMA-SDFs constructed based only on their respective factor zoos, a natural question is whether only bond or stock factors are enough to price *jointly* both asset classes. We answer this question in Figure 7 where we compare the OS pricing performance of the co-pricing BMA-SDF (in red, from Panel A of Table 4) to the one of BMA-SDFs constructed separately with only bond (in blue, from Panel B of Table 4) and stock (in yellow, from Panel C of Table 4) factors, respectively. As test assets we use once more the 16,384 combinations of our OS bond and stock cross-sections. Throughout, the co-pricing BMA-SDF exhibits much lower pricing errors and much higher R^2 s compared to the bond or stock only BMA-SDFs. That is, in order to price the joint cross-section of bond and stock excess returns, we need information from both factor zoos.

Table 4: In-sample cross-sectional asset pricing performance.

	BMA Prior Sharpe Ratio				CAPM	CAPMB	FF5	HKM	Top	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.214	0.203	0.186	0.168	0.260	0.278	0.258	0.259	0.230	0.166	0.201
MAPE	0.167	0.154	0.140	0.126	0.194	0.221	0.198	0.192	0.171	0.126	0.137
R_{OLS}^2	0.153	0.240	0.358	0.479	-0.244	-0.426	-0.233	-0.238	0.023	0.489	0.251
R_{GLS}^2	0.106	0.169	0.232	0.286	0.078	0.083	0.087	0.078	0.263	0.176	0.186
Panel B: Pricing bonds											
RMSE	0.178	0.146	0.121	0.103	0.209	0.213	0.202	0.206	0.159	0.195	0.189
MAPE	0.127	0.108	0.091	0.078	0.146	0.135	0.142	0.145	0.124	0.113	0.084
R_{OLS}^2	0.190	0.454	0.628	0.731	-0.107	-0.157	-0.038	-0.080	0.355	0.028	0.094
R_{GLS}^2	0.217	0.305	0.383	0.445	0.180	0.201	0.244	0.181	0.551	0.064	0.231
Panel C: Pricing stocks											
RMSE	0.230	0.242	0.237	0.220	0.292	0.264	0.275	0.292	0.365	0.162	0.230
MAPE	0.186	0.190	0.182	0.165	0.229	0.211	0.221	0.226	0.306	0.133	0.173
R_{OLS}^2	0.020	-0.080	-0.035	0.109	-0.570	-0.282	-0.392	-0.574	-1.451	0.515	0.022
R_{GLS}^2	0.145	0.213	0.285	0.355	0.120	0.118	0.130	0.121	0.301	0.311	0.315

In-sample asset pricing performance of the Co-pricing BMA-SDF (Panel A), the Bond BMA-SDF (Panel B) and the Stock BMA-SDF (Panel C), notable factor models, and the top five factors with an average posterior probability greater than 50%. In each panel, the models are estimated using the respective factor zoos. Bond returns are computed in excess of the one-month risk-free rate of return. We use GMM-GLS to estimate factor risk prices for CAPM, CAPMB, the and [Fama and French \(1992, 1993\)](#) model, which includes the MKTS, SMB, HML, DEF and TERM factors, and the tradable two-factor [He, Kelly, and Manela \(2017\)](#) model, HKM. KNS stands for the SDF estimation of [Kozak, Nagel, and Santosh \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. RPPCA is the Risk-Premia-PCA of [Lettau and Pelger \(2020\)](#), with five-factors and κ set to 20. In Panel A the models are estimated with the 83 bond and stock portfolios and the 40 tradable bond and stock factors ($N = 123$), Panel B (bond only) uses the 50 bond portfolios and 16 bond factors ($N = 66$), and Panel C (stock only) uses the 33 stock portfolios and 24 stock factors ($N = 57$) as described in Section 1. For the BMA-SDFs, we report results for a range of prior Sharpe ratio values that are set as 20%, 40%, 60% and 80% of the ex-post maximum Sharpe ratio of the relevant portfolios and factors. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).

Table 5: Out-of-sample cross-sectional asset pricing performance.

	BMA Prior Sharpe Ratio				CAPM	CAPMB	FF5	HKM	Top	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.114	0.102	0.095	0.090	0.224	0.154	0.139	0.223	0.171	0.160	0.166
MAPE	0.081	0.075	0.070	0.066	0.192	0.129	0.102	0.190	0.135	0.143	0.146
R_{OLS}^2	0.358	0.486	0.553	0.600	-1.478	-0.161	0.053	-1.444	-0.442	-0.268	-0.360
R_{GLS}^2	0.038	0.070	0.098	0.125	0.028	0.034	0.036	0.028	0.090	0.065	0.041
Panel B: Pricing bonds											
RMSE	0.123	0.116	0.110	0.105	0.130	0.128	0.139	0.133	0.102	0.112	0.085
MAPE	0.090	0.085	0.081	0.079	0.095	0.092	0.104	0.097	0.084	0.081	0.062
R_{OLS}^2	0.050	0.156	0.235	0.305	-0.062	-0.028	-0.221	-0.107	0.342	0.205	0.540
R_{GLS}^2	0.019	0.055	0.080	0.101	-0.006	0.022	-0.032	-0.007	0.100	0.068	0.069
Panel C: Pricing stocks											
RMSE	0.105	0.088	0.078	0.070	0.123	0.119	0.116	0.124	0.158	0.078	0.124
MAPE	0.078	0.067	0.062	0.057	0.089	0.085	0.082	0.091	0.123	0.060	0.096
R_{OLS}^2	0.292	0.502	0.614	0.683	0.032	0.099	0.136	0.019	-0.606	0.613	0.014
R_{GLS}^2	0.089	0.158	0.223	0.280	0.103	0.065	0.099	0.107	0.141	0.207	-0.011

Out-of-sample asset pricing performance of the co-pricing BMA-SDF (Panel A), the bond BMA-SDF (Panel B) and the btock BMA-SDF (Panel C), notable factor models, and the top five factors with an average posterior probability greater than 50%. Bond returns are computed in excess of the one-month risk-free rate of return. We use GMM-GLS to estimate factor risk prices for CAPM, CAPMB, the and [Fama and French \(1992, 1993\)](#) model, which includes the MKTS, SMB, HML, DEF and TERM factors, and the tradable two-factor [He, Kelly, and Manela \(2017\)](#) model, HKM. KNS stands for the SDF estimation of [Kozak, Nagel, and Santosh \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. RPPCA is the Risk-Premia-PCA of [Lettau and Pelger \(2020\)](#), with five-factors and κ set to 20. In Panel A the models are estimated with the 83 bond and stock portfolios and the 40 tradable bond and stock factors ($N = 123$), Panel B (bond only) uses the 50 bond portfolios and 16 bond factors ($N = 66$), and Panel C (stock only) uses the 33 stock portfolios and 24 stock factors ($N = 57$). Out-of-sample (OS) test assets include 154 bond and stock portfolios (Panel A), and 77 bond (stock) portfolios in Panel B (C) as described in Section 1. The models are first estimated using the baseline IS test assets and the resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. For the BMA-SDFs, we report results for a range of prior Sharpe ratio values that are set as 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio relevant portfolios and factors. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).

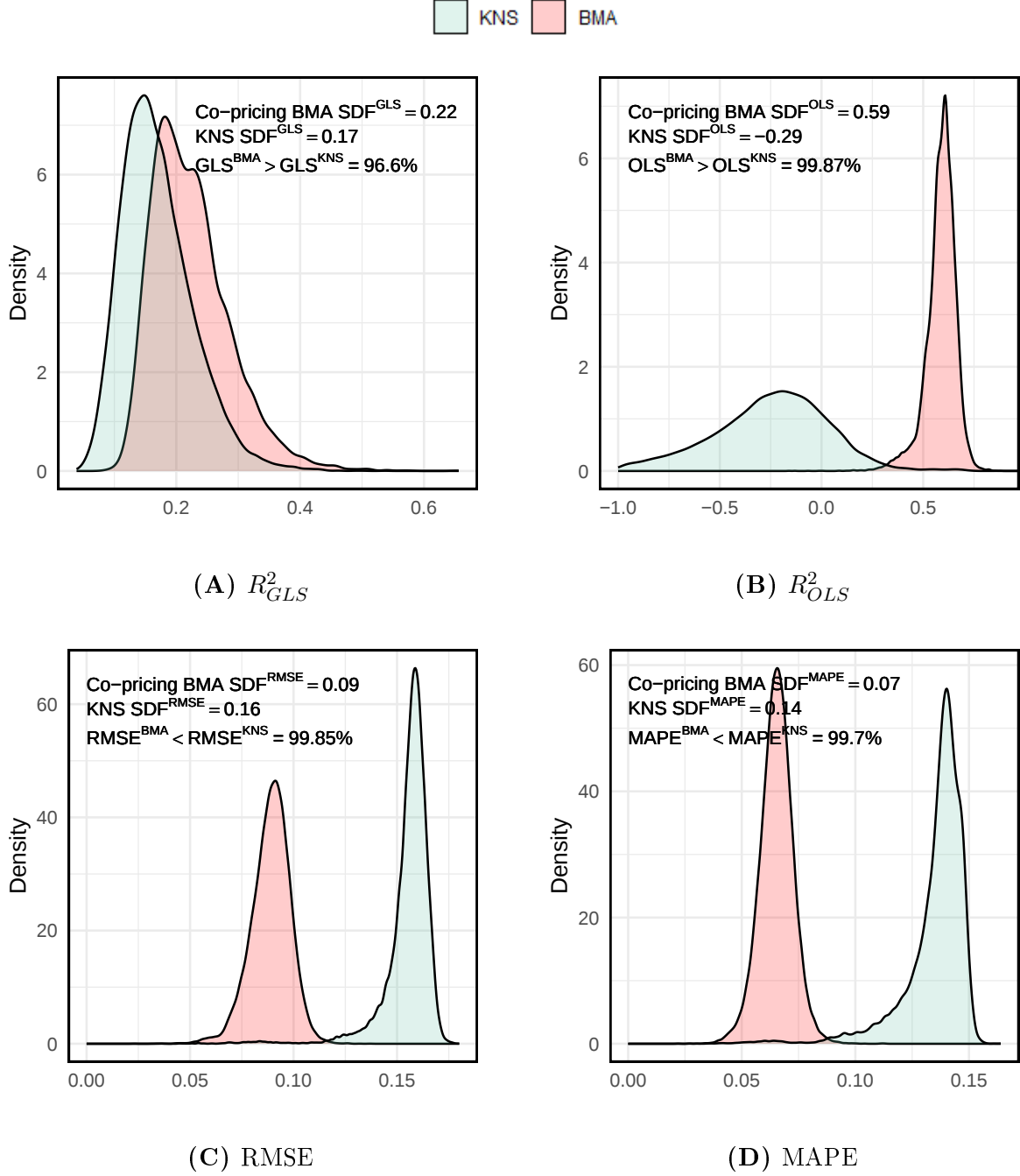


Figure 6: Pricing out-of-sample stocks and bonds with BMA-SDF and KNS.

This figure plots the distributions of R_{GLS}^2 , R_{OLS}^2 , RMSE and MAPE in Panels A, B, C and D, respectively, across 16,384 possible OS bond and stock cross-sections using the 14 sets of bond and stock test assets ($2^{14} = 16,384$). KNS stands for the SDF estimation of [Kozak, Nagel, and Santosh \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. The models are first estimated using the baseline IS test assets and the resulting SDF is then used to price (with no additional parameter estimation) each set of the 16,384 OS combinations of test assets. The BMA-SDF is computed with a prior Sharpe ratio value set to 80% of the ex-post maximum Sharpe ratio of the IS test assets. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).

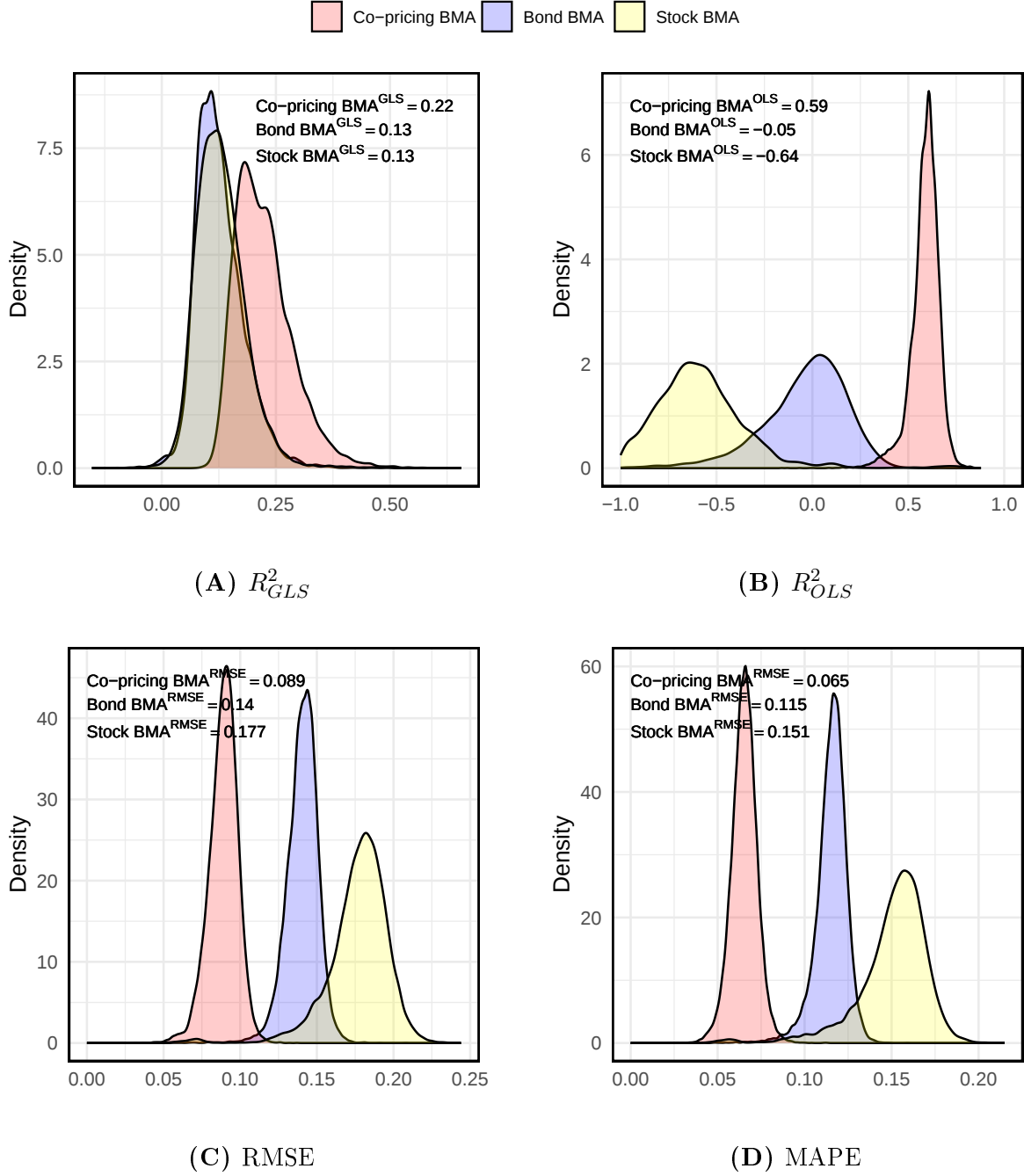


Figure 7: Pricing out-of-sample stocks and bonds with different BMA-SDFs.

This figure plots the distributions of R_{GLS}^2 , R_{OLS}^2 , RMSE and MAPE in Panels A, B, C and D respectively across 16,384 possible bond and stock cross-sections using the 14 sets of stock and bond test assets ($2^{14} = 16,384$) priced using the respective BMA-SDF. The models are first estimated using the baseline set of IS test assets and then used to price (with no additional parameter estimation) each set of the 16,384 OS combinations of test assets. The red distributions corresponds to the pricing performance of the co-pricing BMA-SDF. The blue (yellow) distributions corresponds to the pricing performance of the bond (stock) only BMA-SDF. The BMA-SDFs are computed with a prior Sharpe ratio value set to 80% of the ex-post maximum Sharpe ratio of the IS test assets. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).

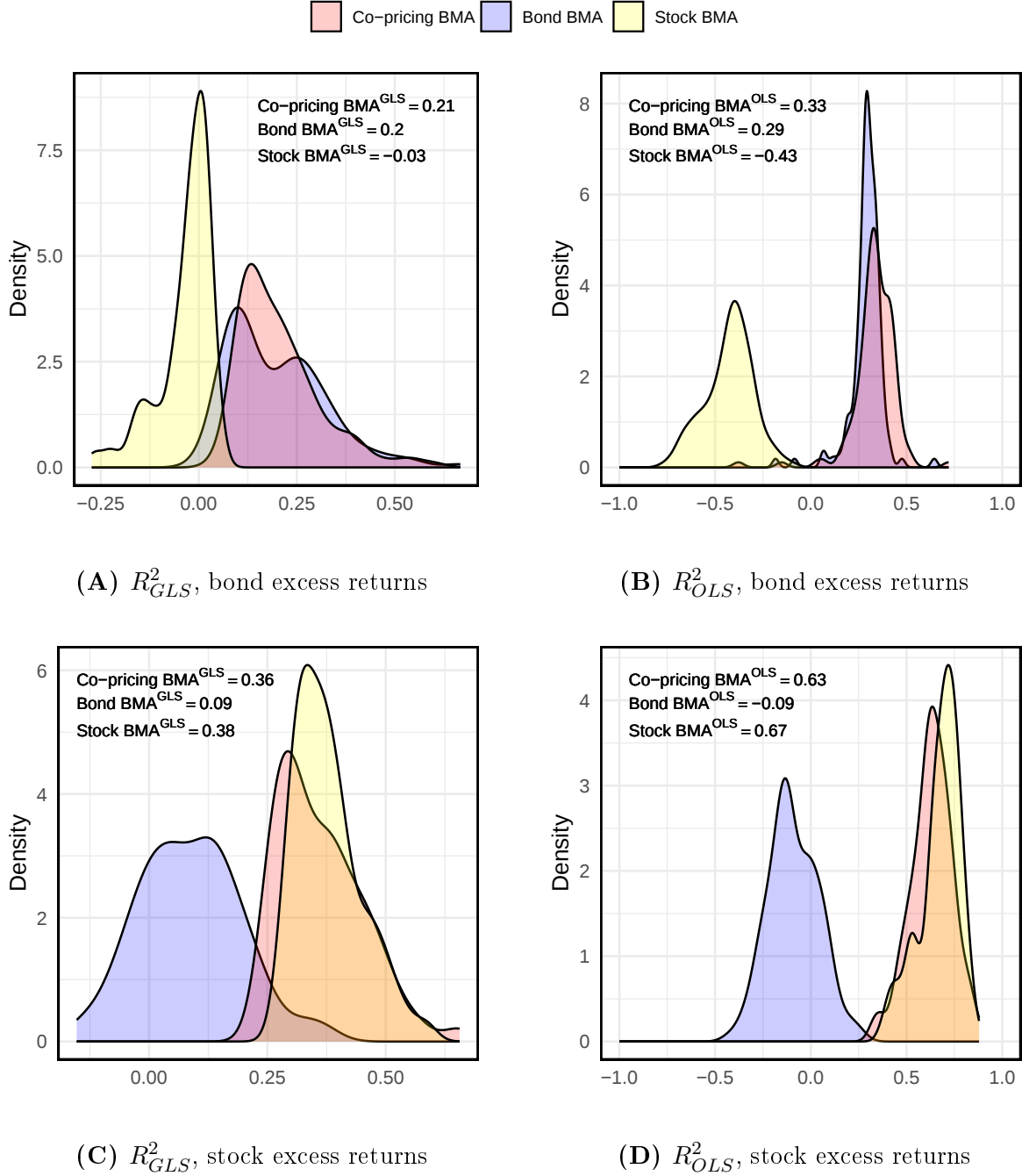


Figure 8: Separate out-of-sample pricing of bond and stock excess returns

This figure plots the distributions of R^2_{GLS} , R^2_{OLS} , RMSE and MAPE in Panels A, B, C and D respectively across 128 possible bond cross-sections using the 7 sets of bond test assets ($2^7 = 128$) priced using the respective BMA-SDF. The models are first estimated using the baseline set of IS test assets and then used to price (with no additional parameter estimation) each set of the 128 OS combinations of test assets. The red distributions corresponds to the pricing performance of the co-pricing BMA-SDF. The blue (yellow) distribution corresponds to the pricing performance of the bond (stock) only BMA-SDF. The BMA-SDFs are computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).

But, can the co-pricing BMA-SDF also price well bonds and stocks separately? In Figure 8 we report OS R_{GLS}^2 and R_{OLS}^2 for the separate pricing of these two asset classes using the $2^7 = 128$ possible combinations of our OS corporate bond portfolios in Panels A and B, and the same number of combinations of OS stock portfolios in Panels C and D. Clearly, the co-pricing BMA-SDF can individually price the respective bond and stock cross-sections well, implying that the superior performance of the co-pricing BMA-SDF is not due to the fact that it prices one cross-section better than the other. Nevertheless, the asset-class-specific BMA-SDFs price the respective cross-sections very well. That is, using only information from the bond market factor zoo delivers a pricing performance for the cross-section of bond excess returns that is only marginally worse than the one achievable with the co-pricing BMA-SDF. Similarly, the stock-only BMA-SDF does price stock returns very well OS, confirming the findings in Bryzgalova et al. (2023). Yet, clearly, the information in the bond factor zoo alone is insufficient to price the cross-section of stock returns and, vice versa, information from the stock market is not sufficient to price the cross-section of corporate bond excess returns.

3.2 The information content of the two factor zoos

As shown in Section 3.1.3 (see Figure 8 and Tables 4 and 5), albeit one can construct well performing BMA-SDFs to price bonds and stocks separately only using the information in their respective zoos, the joint pricing of these assets requires information from both sets of factors (see Figure 7). In this section we show that this result arises from the fact that corporate bond returns reflect not only a component related to credit risk compensation, but also a *Treasury term structure* risk premium.

To illustrate this point, we now turn our focus to bond returns in excess of a duration-matched portfolio of U.S. Treasuries. More precisely, for every bond i we construct the following duration-adjusted return

$$\underbrace{R_{bond\,i,t} - R_{dur\,bond\,i,t}^{Treasury}}_{\text{Duration-adjusted return}} = \underbrace{R_{bond\,i,t} - R_{f,t}}_{\text{Excess return}} - \underbrace{\left(R_{dur\,bond\,i,t}^{Treasury} - R_{f,t}\right)}_{\text{Treasury component}} \quad (3)$$

where $R_{bond\,i,t}$ is the return of bond i at time t , $R_{f,t}$ denotes the short term risk free rate, and $R_{dur\,bond\,i,t}^{Treasury}$ denotes the return on a portfolio of Treasury securities with the same duration as bond i (constructed as in van Binsbergen, Nozawa, and Schwert (2024)). As is obvious from the right hand side of equation (3), the duration adjustment removes the implicit Treasury compo-

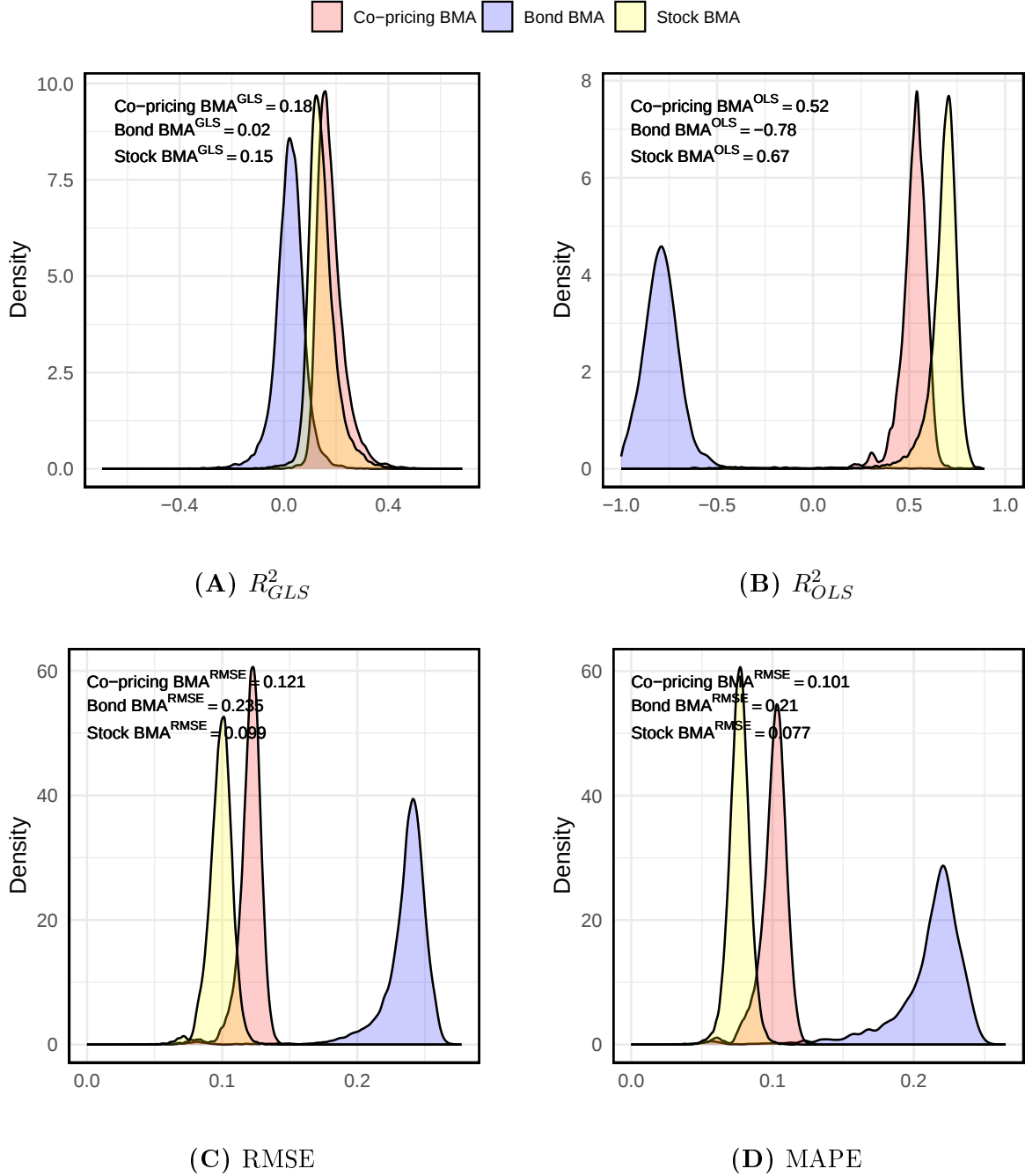


Figure 9: Pricing the joint cross-section of stock and duration-adjusted bond returns.

This figure plots the distributions of R_{GLS}^2 , R_{OLS}^2 , RMSE and MAPE in Panels A, B, C and D respectively across 16,384 possible bond and stock cross-sections using the 14 sets of stock and bond test assets ($2^{14} = 16,384$). All bond test assets and factors are formed with duration-adjusted returns described in Appendix C. The BMA-SDFs are first estimated using the baseline IS test assets and then used to price (with no additional parameter estimation) each set of the 16,384 OS combinations of test assets. The red distributions corresponds to the pricing performance of the co-pricing BMA-SDF. The blue (yellow) distributions corresponds to the pricing performance of the bond (stock) only BMA-SDF. The BMA-SDFs computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. Data are standardized, i.e., pricing errors are in Sharpe ratio units. Sample: 1986:01–2022:12 ($T = 444$).

ment from the bond excess return, hence isolating the remaining sources of risk compensation that investing on a given bond entails.

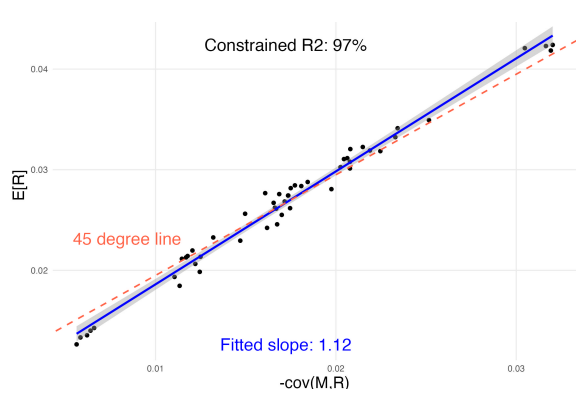
Figure 9 reports the distribution of OS measures of fit (R_{GLS}^2 , R_{OLS}^2 , RMSE and MAPE) across 16,384 possible bond and stock cross-sections using the 14 sets of stock and bond test assets for three different BMA-SDFs based on (i) bond factors only, (ii) stock factors only, and (iii) both bond and stock factors. The contrast with Figure 7 is stark: once bond returns are adjusted for duration, the BMA-SDF based only on equity information prices jointly bonds and stocks as well as the co-pricing BMA-SDF that additionally includes bond factors. That is, the information content of the bond factor zoo becomes largely irrelevant for co-pricing once the Treasury component of bond returns is removed.

This last finding raises a natural question: why do we need the bond factors for co-pricing assets in the absence of the duration adjustment? As we are about to show, bond factors price the Treasury component of corporate bond returns.

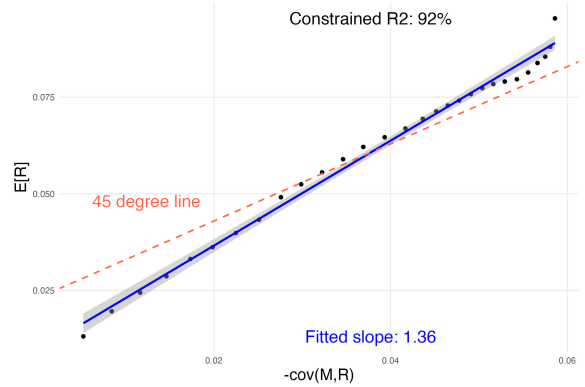
Panel A of Figure 10 depicts the IS pricing of the Treasury component of corporate bond returns using the BMA-SDF based only on the bond factor zoo. Remarkably, the pricing is almost perfect with a cross-sectional (constrained) R_{OLS}^2 of about 97%. Similarly, Panel B shows that the OS pricing of a cross-section of Treasury excess returns using the same BMA-SDF is also almost perfect, with a (constrained) R_{OLS}^2 of 92% and average excess returns and SDF-implied risk premia lining up closely around the 45 degrees line. In contrast, Panels C and D of Figure 10 report the same cross-sectional pricing exercises performed using a BMA-SDF based only on stock factors. Clearly, those are not able to price the Treasury component of corporate bond returns neither in- nor out-of-sample, yielding extremely low measures of fit (6% to 14% R_{OLS}^2) and slope coefficients very far from the theoretical value.

The above highlights that the bond factor zoo is necessary for co-pricing bonds and stocks because the factors proposed in the corporate bond literature price extremely well the Treasury component implicit in corporate bond returns—a component that stock factors fail to price. But once this component is accounted for—as in the case of duration-adjusted bond returns—co-pricing can effectively be achieved using only equity information.

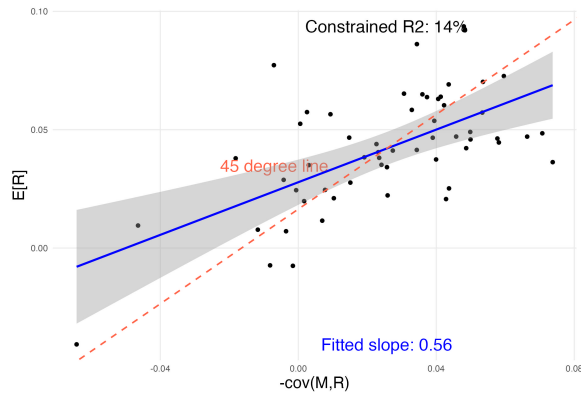
Moreover, this Treasury component is economically important. The ex post (annualized) maximum Sharpe ratio achievable with the excess returns on the duration-matched Treasury portfolios in equation (3) is about 1.48. And, as shown in the bottom panel of Figure 11, this is roughly the mode of the Sharpe ratio achievable with an SDF that prices this Treasury



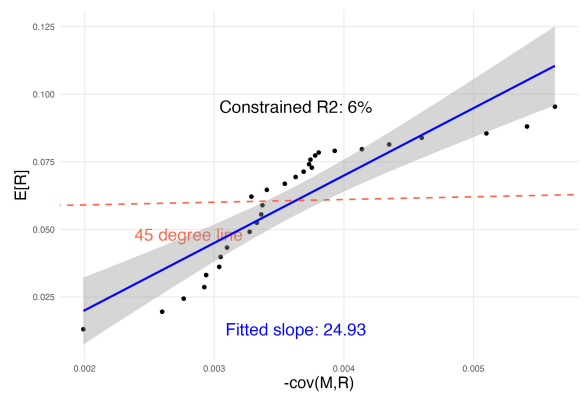
(A) In-sample with bond BMA-SDF



(B) Out-of-sample with bond BMA-SDF



(C) In-sample with stock BMA-SDF



(D) Out-of-sample with stock BMA-SDF

Figure 10: Pricing the Treasury component of corporate bond returns.

Sample average (y -axis) vs. bond BMA-SDF-implied (x -axis) risk premia. Panel A: $R_{dur\ bond\ i,t}^{Treasury} - R_{f,t}$ described in Appendix C. Panel B: monthly returns in excess of the risk free rate for 29 U.S. Treasury portfolios (two- to 30-year maturity in increments of 1-year). Constrained OLS R^2 s computed setting the slope coefficient to 1. Sample: 1986:01–2022:12 ($T = 444$)

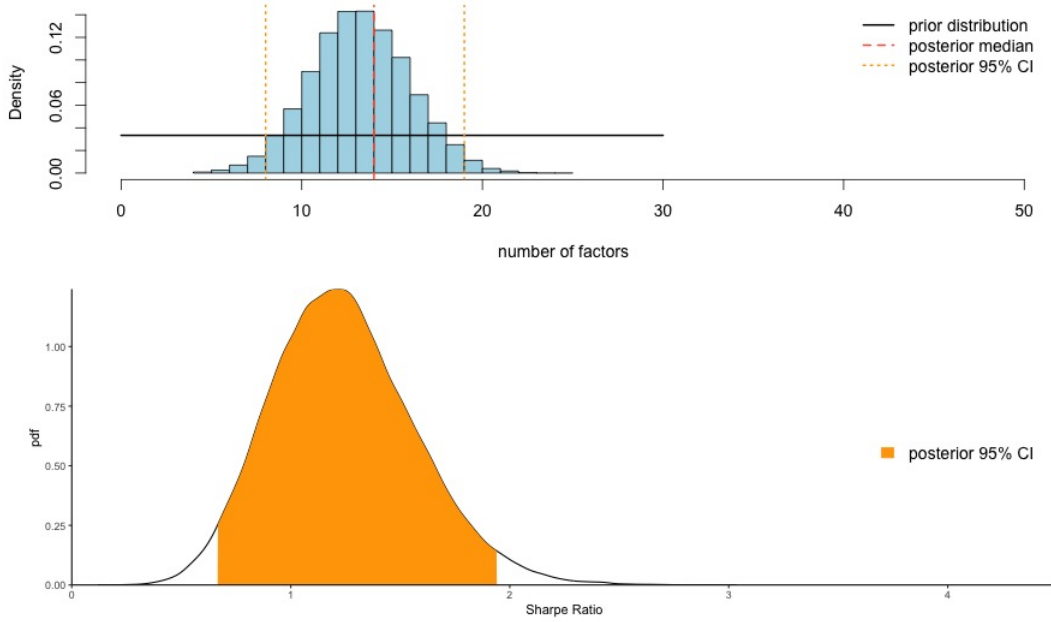


Figure 11: Posterior factor dimensionality and SR of the SDF that prices the Treasury component of corporate bond returns using only the bond factor zoo.

Posterior distributions of the number of factors to be included in the bond SDF (top panel) and of the SDF-implied Sharpe ratio (bottom panel), computed using the 30 nontradable and tradable bond factors described in Appendix B. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the U.S. Treasury returns component, in excess of the one-month risk-free rate, of the 50 corporate bond portfolios and 16 bond tradable factors. Sample period: 1986:01 to 2022:12 ($T = 444$).

component with only the factors in the bond factor zoo. Note also that, as shown in the top panel of the figure, even for pricing just this component, the required SDF is quite dense, with a median number of factors equal to 14 and a posterior 95% C.I. ranging from 8 to 19 factors. Furthermore, as shown in Table 6, pricing this component requires a dense SDF in the space of both nontradable and tradable factors, with posterior mean numbers of factors being 7 and 8, respectively, across the prior range, and with roughly equal contribution to the Sharpe ratio of the SDF from the two types of factors.

Mirroring the analysis in Section 3.1, we can assess which factors are more likely to price this Treasury component individually. Figure 12 reports the posterior factor probabilities and market prices of risk implied by the pricing of the Treasury component of corporate bond returns using the corporate bond factor zoo (the prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio). Overwhelmingly, the most likely factors are nontradable: five out

Table 6: SDF dimensionality, and SR decomposition by type of factor, for pricing the Treasuries component of corporate bond returns

	Total prior SR				Total prior SR			
	20%	40%	60%	80%	20%	40%	60%	80%
	Nontradable factors				Tradable factors			
Mean	6.91	6.94	6.91	6.91	7.90	7.79	7.90	7.90
5%	4	4	4	4	5	5	5	5
95%	10	10	10	10	11	11	11	11
$\mathbb{E}[SR_f \text{data}]$	0.15	0.32	0.52	0.83	0.28	0.64	0.64	0.80
$\mathbb{E}[\frac{SR_f^2}{SR_m^2} \text{data}]$	0.02	0.09	0.24	0.54	0.08	0.36	0.36	0.51

Posterior means of: number of factors (with 90% C.I.), implied Sharpe ratios, $\mathbb{E}[SR_f|\text{data}]$, and ratio of SR_f^2 to the total SDF Sharpe ratio, $\mathbb{E}[SR_f^2/SR_m^2|\text{data}]$, of a subset of factors. Subsets are split across tradable and nontradable factors, and within tradable factors we further separate bond and stock factors. The results are reported for the U.S. Treasury SDF which is estimated with the corporate bond factor zoo. The in-sample test assets are the U.S. Treasury returns component, in excess of the one-month risk-free rate, of the 50 corporate bond portfolios.

of the six factors with posterior probability higher than the prior value are nontradable, with the only exception being PEADB—the most likely tradable factor in our co-pricing analysis. Furthermore, largely, the factors are the same as those that appear most likely when co-pricing bonds and stocks, with the top three being YSP, IVOL and CREDIT—exactly the three most likely nontradable factors in our co-pricing BMA-SDF—followed by a term structure level factor, LVL, and a factor based on unexpected shocks to core inflation, INFLC. Moreover, these nontradable factors command large mark market prices of risk and the probability of no nontradable factor being in the BMA-SDF that prices the Treasury component of corporate bond returns is virtually zero (or 0.018%).

3.3 The economic properties of the co-pricing SDF

We now turn to assessing the economic properties of the co-pricing BMA-SDF. Figure 13 depicts the time series of the BMA-SDF posterior mean, along with its conditional time series mean (estimated using an ARMA(3,1) model selected via BIC). The ARMA specification is selected based on both the Akaike and Bayesian Information Criteria. Both the SDF and its conditional mean exhibit a clear business cycle behaviour as they increase during expansions and tend to peak right before recessions, being substantially reduced during economic contractions. Moreover, as highlighted in Panel A of Figure 14, the BMA-SDF is highly predictable: virtually

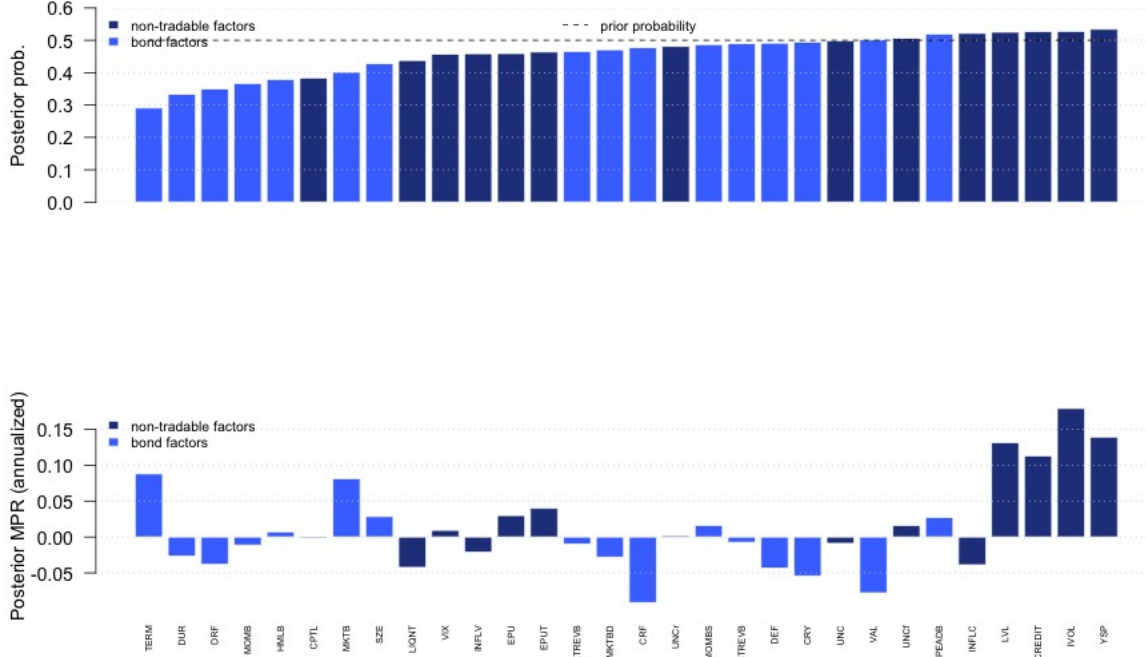


Figure 12: Posterior factor probabilities and market prices of risk – pricing the Treasury component of corporate bond returns using the bond factor zoo.

Posterior factor probabilities (top panel), $\mathbb{E}[\gamma_j|\text{data}]$, and the corresponding posterior market prices of risk (bottom panel), $\mathbb{E}[\lambda_j|\text{data}]$, of the 30 nontradable and tradable bond factors described in Appendix B. Test assets are the U.S. Treasury returns component, in excess of the one-month risk-free rate, of the 50 corporate bond portfolios. The prior distribution for the j^{th} factor inclusion is a Beta(1, 1), yielding a 0.5 prior expectation for γ_j . The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the portfolios and tradable factors. Sample period: 1986:01 to 2022:12 ($T = 444$).

all of its autocorrelation coefficients are statistically significant at the 1% level up to 20 months ahead, and the p -value of the [Ljung and Box \(1978\)](#) test of joint significance is zero at this horizon. Additionally, about one fifth of its time series variance is explained by its own lags (23% for the best AR specification and 19% for the best ARMA specification according to the BIC).

Note also that, as shown in Figure [IA.6](#) of the Internet Appendix, none of the other celebrated SDF models come close to displaying such a level of business cycle variation and persistency: the KNS SDF has about 11% of its time series variation being predictable by its own history, while this number drops to 4% for RPPCA, and its only 2% to 3%, for FF5 and CAPMB, and zero for HKM and CAPM. Remarkably, as shown in Panel A of Table [IA.XI](#) of

the Internet Appendix, the SDFs with a higher degree of persistency, KNS and RPPCA, are exactly the ones with the highest degree of correlation with the BMA-SDF (0.78 and 0.55, respectively), and are the closest competitors for the BMA-SDF in the pricing exercises in Section 3.1. Instead, SDFs that perform significantly worse in cross-sectional pricing have both little time series persistency and correlations with the BMA-SDF in the 0.16 to 0.29 range.

Obviously, the nontradable factors in the BMA-SDF play an important role in generating a pronounced business cycle pattern and a high degree of predictability. Nevertheless, even when removing the nontradable factors from the BMA-SDF, the resulting SDF remains predictable, with 5% to 10% of its time series variation explained by its own lags, and a highly significant [Ljung and Box \(1978\)](#) test statistic even up to 20 months ahead. Furthermore, note that the five most likely factors in the SDF (PEAD, IVOL, PEADB, CREDIT, YSP) explain only about 47% of the time series variation of the BMA-SDF, further confirming the dense nature of the pricing kernel. Individually, only PEADB and IVOL explain marginally more than 20% of the time series variation of SDF, with the other factors accounting individually for 3% to 7%.

Recall that the variance of the SDF is equal to the squared Sharpe ratio achievable in the economy. Hence, whether our filtered SDF implies time-varying compensation for risk can be elicited by analyzing the predictability of its volatility. As pointed out in [Engle \(1982\)](#), the presence of volatility clustering can be assessed, without taking a parametric stance on the variance process, by simply analyzing the serial correlation of the squared one-step-ahead forecast errors, since these are consistent (yet noisy) estimates of the latent conditional variance. Note that, for instance, such a variance proxy has been used extensively in the macro-finance literature (see, e.g., [Bansal, Khatchatrian, and Yaron \(2005\)](#), [Bansal, Kiku, and Yaron \(2012\)](#), [Beeler and Campbell \(2012\)](#), and [Chen \(2017\)](#)), and squared forecast errors of returns are a commonly used as proxy of the latent conditional variances.

Panel B of Figure 14 reports the empirical autocorrelation function of the squared forecast errors of the BMA-SDF. Most of the autocorrelation coefficients are statistically significant at the 1% level up to seven months ahead. Moreover, the [Ljung and Box \(1978\)](#) test strongly rejects the joint null of zero autocorrelations up to 20 months into the future (the p -value of the test is zero). That is, not only the first moment of our filtered SDF exhibits substantial predictability, but so does its second moment, suggesting time-varying risk compensation in the economy.

To tackle the question whether the SDF-implied time variation in risk compensation (i.e.,

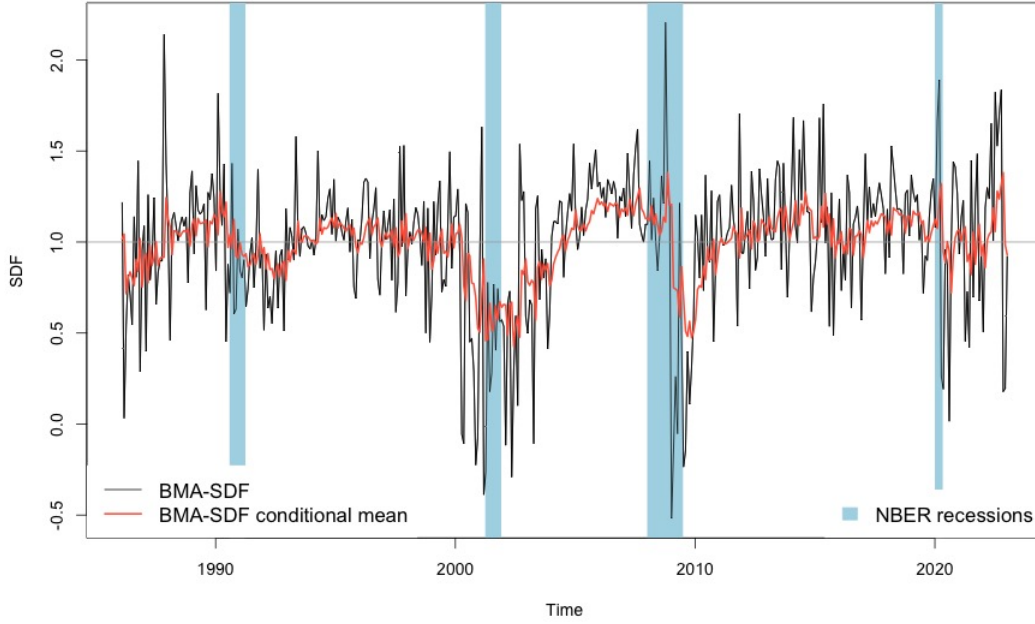


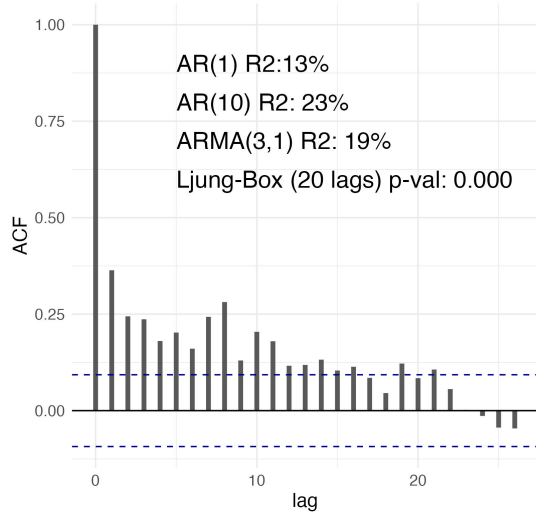
Figure 13: The co-pricing SDF and its conditional mean

Time series of the (posterior mean of) the co-pricing BMA-SDF and its conditional mean. The conditional mean is obtained by fitting an ARMA(3,1) to the BMA-SDF. The order of the ARMA specification is selected using the Bayesian Information Criterion. Shaded areas denote NBER recession periods. The sample period is 1986:01–2022:12 ($T = 444$).

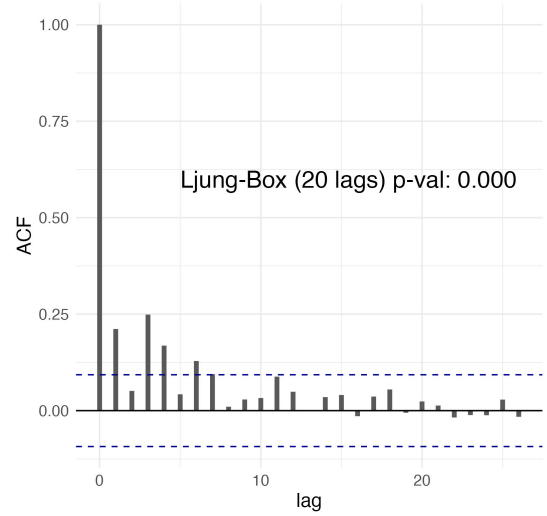
the economy-wide conditional Sharpe ratio) that we uncover makes economic sense, we fit a simple GARCH(1,1) (see [Bollerslev \(1986\)](#)) process to our BMA-SDF.²⁹ Figure 15 presents the estimated conditional volatility of the SDF, revealing striking results. The implied conditional Sharpe ratio is not only highly countercyclical but also exhibits pronounced spikes during periods of market turbulence and heightened economic uncertainty. These include Black Monday, the Asian financial crisis, the burst of the dot-com bubble, the 9/11 terrorist attacks, the Iraq invasion, the great financial crisis, the Greek default and subsequent Eurozone debt crisis, the COVID-19 pandemic, and the aftermath of Russia’s invasion of Ukraine. Note that the estimated GARCH coefficients imply a highly persistent conditional volatility, with deviations from the mean exhibiting a half-life of approximately 16.6 months.³⁰

²⁹We estimate the process based on the posterior mean of the BMA-SDF. Ideally, one would estimate the volatility process for each draw of the SDF and for each possible model, and then compute the posterior average of these ‘draws’ for the volatility process. Nevertheless, since GARCH estimation requires numerical optimisation, the ideal approach is unfeasible in our model space with quadrillions of models.

³⁰Recall that the half-life of a GARCH(1,1) process is defined as $1 + \frac{\ln(1/2)}{\ln(\alpha + \beta)}$ where α and β denote, respectively,



(A) Co-pricing BMA-SDF predictability



(B) Squared forecast errors of BMA-SDF

Figure 14: autocorrelation functions of co-pricing BMA-SDF and its squared forecast errors

Autocorrelation coefficients of co-pricing BMA-SDF, in Panel A, and its squared forecast errors, in Panel B. ARMA(3,1) conditional mean process selected via BIC. Sample: 1986:01–2022:12 ($T = 444$).

As per Panel A in Table 2, nontradable factors account for about a quarter of the SDF variance. Hence, a natural question is whether the SDF volatility pattern depicted in Figure 15 is simply due to tradable factors. We evaluate this conjecture by removing all tradable factors from the BMA-SDF and re-estimating the volatility process of this nontradable-only SDF. We find that the resulting volatility process remains very persistent (with a half-life of 12.3 months), with pronounced business cycle variation and reaction to periods of heightened economic uncertainty (see Figure IA.8 of the Internet Appendix). Moreover, it has a correlation with the volatility of the BMA-SDF in Figure 15 of about 62%. That is, both tradable and nontradable components of the BMA-SDF are characterised by a very persistent volatility with a clear business cycle pattern.

But is the strong countercyclical behaviour of the BMA-SDF volatility, and its sharp increase during periods of economic uncertainty, just a mechanical byproduct of it loading on several tradable factors? Figure 15 suggests that this is not the case. Focusing on the celebrated Fama-French five factor model, and the bond CAPM (the best model for pricing corporate bonds in Dickerson, Mueller, and Robotti (2023)), we apply the same procedure of estimating their SDF

the coefficients on lagged squared error and variance.

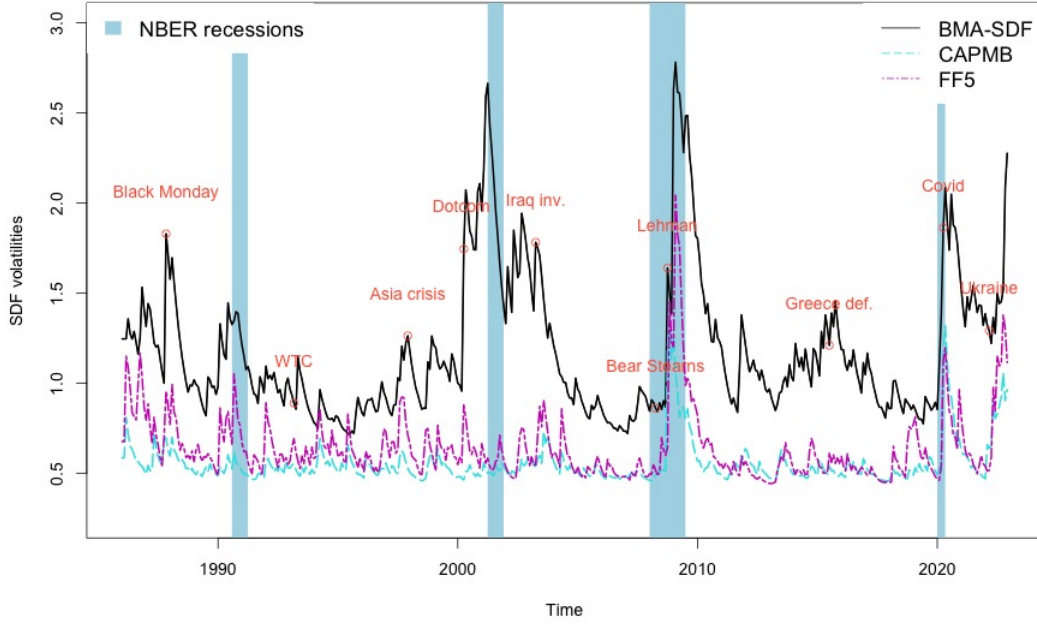


Figure 15: Volatility of the co-pricing BMA-SDF

Annualized volatility of the co-pricing BMA-SDF as well as CAPMB and FF5 SDFs. Shaded areas denote NBER recession periods. The volatility of the BMA-SDF is obtained fitting a ARMA(3,1)-GARCH(1,1) to the posterior mean of the co-pricing BMA-SDF (specification selected via BIC). The GARCH Quasi-maximum likelihood coefficient estimates are:

	$\sigma_{t+1}^2 = \omega + \alpha\epsilon_t^2 + \beta\sigma_t^2$		
	ω	α	β
Estimate	0.01	0.15	0.81
Robust SE	0.00	0.04	0.06

The volatilities of the CAPMB and FF5 SDFs are also computed as GARCH(1,1) estimates after selecting an ARMA mean process using the BIC. Sample: 1986:01–2022:12 ($T = 444$).

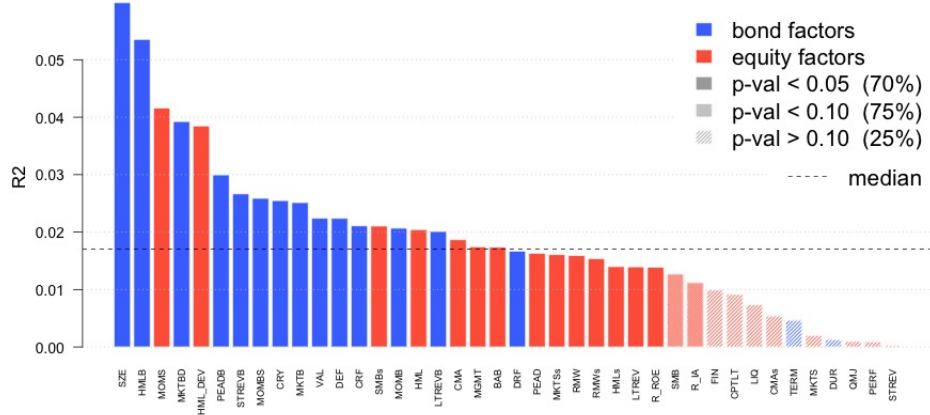
coefficients and computing the implied conditional volatilities using a GARCH specification (after fitting a mean model based on AIC). The estimated volatilities for these two SDF models in Figure 15 make clear that the use of tradable factors in the SDF does not mechanically deliver our findings for the BMA-SDF: both the cyclical pattern and the reaction to periods of heightened economic uncertainty is much less pronounced for the FF5 model, and even more so for the CAPMB. This is formally measured in Figure IA.7 of the Internet Appendix that shows that the half-life of volatility shocks to the FF5 SDF model is only 4.21 months, and for the CAPMB it is just 3 months. Finally, Figure IA.9 of the Internet Appendix depicts the residual of the linear projection of the BMA-SDF estimated volatility on the estimated

volatilities of the KNS, RPPCA, CAPM, CAPMB, HKM and FF5, with the residual showing a strong business cycle pattern and being particularly large and positive during periods of high economic uncertainty, suggesting that these alternative SDF models do not sufficiently capture these states despite being based on tradable factors.

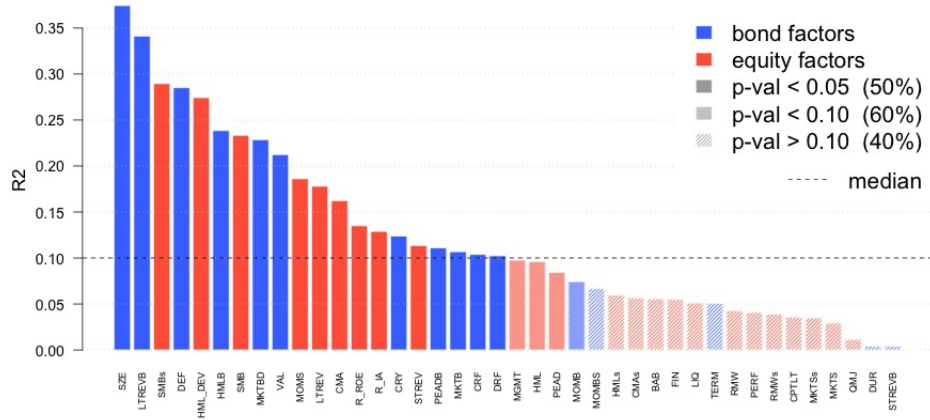
The observed business cycle variations and predictability in both the first and second moments of the SDF would imply, within a structural model, time-varying and predictable risk premia for tradable assets. Therefore, we now turn to testing this *time series* prediction of our BMA-SDF identified from *cross-sectional* pricing.

The precise functional form of the predictive relation between current SDF moments and future asset returns does depend on the postulated model. Nevertheless, as shown in [Bryzgalova, Huang, and Julliard \(2024\)](#), the [Hansen and Jagannathan \(1991\)](#) conditional SDF projections on the space of returns imply a (log) linear SDF driven by two factors: the innovations to the SDF and the product of the conditional mean of the SDF and the same innovations. Therefore, assuming a contemporaneous linear relationship between asset returns and the SDF yields a simple linear dependence of conditional risk premia on two variables: (i) the conditional variance of the SDF and (ii) the product of this conditional variance with the conditional mean of the SDF. Leveraging this insight, we run predictive regressions of asset (log) returns between time $t - 1$ and t , as well as cumulated (log) returns between $t - 1$ and $t + 12$, on SDF information observed at time $t - 1$: $\mathbb{E}_{t-1}[M_t] \times \mathbb{E}_{t-1}[\sigma_t^2]$ and $\mathbb{E}_{t-1}[\sigma_t^2]$, where the conditional mean is constructed by fitting an ARMA(3,1) process (the preferred specification according to the BIC) of the BMA-SDF, and the conditional variance is obtained from the GARCH(1,1) estimates depicted in Figure 15 (and also selected via BIC). As test assets to be predicted we employ the bond and stock factors used in our cross-sectional analysis since these are generally hard to predict and should, according to the previous literature, demand sizable risk premia.

Figure 16 summarizes the predictability results. In Panel A we report the R^2 values for the one-month-ahead and in Panel B the same for the cumulative twelve-month-ahead predictive regressions. We encode, via shading, the joint statistical significance of the regressors as implied by an F -test of the regression coefficients. The results are striking. For the majority of test assets, we find that information embedded in the lagged SDF significantly predicts future asset returns. At the monthly horizon shown in Panel A, this predictability is statistically significant in 75% of cases at the 10% confidence level and in 70% of cases at the 5% significance level. Second, the amount of predictability is economically large, albeit not unrealistically so: for



(A) Predictability of monthly log returns



(B) Predictability of twelve months cumulative log returns

Figure 16: Predictability of tradable factors with lagged SDF information

R^2 s of the predictive regressions of factor returns on the previous month estimates of the co-pricing BMA-SDF conditional variance and conditional variance interacted with the conditional mean of the BMA-SDF. Process estimated as QMLE ARMA(3,1)-GARCH(1,1) and selected via BIC. Panel A: monthly log returns. Panel B: twelve months log returns. To account for the overlapping nature of the observations in Panel B we construct robust standard errors using a Bartlett kernel (Newey and West (1987)) kernel with 15 lags, b) a sandwich estimate of the covariance matrix, and c) applying a degrees of freedom correction. The 40 predicted tradable factors are described in Appendix B.

the statistically significant cases it ranges from 1.1% to 6% at the monthly horizon (Panel A). As shown in Martin (2017), these R^2 s are extremely high relative to the predictive ability of valuation ratios and even the SVIX, and such scale of predictability yields very high Sharpe ratios when used to inform market timing strategies. At the twelve month horizon (Panel B) the median R^2 is about 10%, with many factors having more than one fifth of their time series variation being predictable. Moreover, even with an extremely conservative approach to

constructing the covariance matrix, the F -test yields statistically significant results in 60% of cases at the 10% level and 50% of cases at the 5% level.³¹

4 Conclusion

We apply a Bayesian approach to the analysis of 18 quadrillion linear factor models for the joint pricing of corporate bond and stock returns.

We find that the latent SDF is *dense* in the space of observable nontradable and tradable bond and stock factors. This implies that all low dimensional observable factor models proposed to date are affected by severe misspecification and rejected by the data.

Individually, only very few factors should be included in the SDF with a high probability. Most notably, two behavioural factors capturing the post earnings announcement drift in bonds and stocks exhibit posterior probabilities above 50%, along with nontradable factors such as the slope of the Treasury yield curve, the AAA/BAA yield spread, and the idiosyncratic equity volatility. But these factors capture only a fraction of the risks priced in the joint cross-section of bonds and stocks, and literally dozens of other factors, both tradable and nontradable, are necessary—jointly—to span the risks driving asset prices.

Nevertheless, the SDF-implied maximum Sharpe ratio is not extreme because the many factors necessary for an accurate characterization of the SDF are multiple noisy proxies for common underlying sources of risk.

A Bayesian Model Averaging over the space of all possible Stochastic Discount Factor models aggregates this diffuse pricing information optimally and outperforms, in- and out-of-sample, all existing models in explaining—jointly and individually—the cross-section of corporate bond and stock returns.

The BMA-SDF has a distinctive business cycle behaviour, and persistent and cyclical first and second moments. Furthermore, its volatility increases sharply during recessions and at times of heightened economic uncertainty, suggesting time variation in conditional risk premia. And indeed, we find that lagged BMA-SDF information is a strong and significant predictor of future asset returns.

³¹We construct conservative standard errors by *a*) using a Bartlett kernel (Newey and West (1987)) with 15 lags, *b*) constructing a sandwich estimate of the covariance matrix, and *c*) applying a degrees of freedom correction to account for the relatively small sample of independent observations. For comparison, OLS standard errors yield a statistically significant F -test, at the 5% level, in 92.5% of the cases, and similar results arise using simple bootstrap.

Decomposing bond excess returns into their credit and a Treasury components, we find that nontradable and *stock* tradable factors are sufficient for the pricing of the former, while nontradable and *bond* tradable factors are necessary for the pricing of the latter—a component that stock tradable factors do not explain.

Overall, our results have first order implications for theoretical and empirical asset pricing models that aim to explain jointly the cross-sections of corporate bond, stock, and Treasury returns.

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Appendix

A Posterior sampling

The posterior of the time series parameters follows the canonical Normal-inverse-Wishart distribution (see, e.g., [Bauwens et al. 1999](#)) given by

$$\boldsymbol{\mu}_Y | \boldsymbol{\Sigma}_Y, \mathbf{Y} \sim \mathcal{N}(\hat{\boldsymbol{\mu}}_Y, \boldsymbol{\Sigma}_Y/T), \quad (\text{A.4})$$

$$\boldsymbol{\Sigma}_Y | \mathbf{Y} \sim \mathcal{W}^{-1}\left(T-1, \sum_{t=1}^T (\mathbf{Y}_t - \hat{\boldsymbol{\mu}}_Y)(\mathbf{Y}_t - \hat{\boldsymbol{\mu}}_Y)^\top\right), \quad (\text{A.5})$$

where $\hat{\boldsymbol{\mu}}_Y \equiv \frac{1}{T} \sum_{t=1}^T \mathbf{Y}_t$, $\mathcal{W}^{-1}$ is the inverse-Wishart distribution, $\mathbf{Y} \equiv \{\mathbf{Y}_t\}_{t=1}^T$, and note that the covariance matrix of factors and test assets, $\mathbf{C}_f$, is contained within $\boldsymbol{\Sigma}_Y$.

Define $\mathbf{D}$ as a diagonal matrix with elements $c, (r(\gamma_1)\psi_1)^{-1}, \dots, (r(\gamma_K)\psi_K)^{-1}$. Hence, in matrix notation, the prior for $\boldsymbol{\lambda}$ in equation (2) is $\boldsymbol{\lambda} | \sigma^2, \boldsymbol{\gamma} \sim \mathcal{N}(0, \sigma^2 \mathbf{D}^{-1})$. It then follows that, given our prior formulations, the posterior distributions of the parameters in the cross-sectional layer $(\boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\omega}, \sigma^2)$, conditional on the draws of $\boldsymbol{\mu}_R$, $\boldsymbol{\Sigma}_R$, and $\mathbf{C}$ from the time series layer, are (see [Bryzgalova et al. 2023](#) for a formal derivation):

$$\boldsymbol{\lambda} | \text{data}, \sigma^2, \boldsymbol{\gamma}, \boldsymbol{\omega} \sim \mathcal{N}(\hat{\boldsymbol{\lambda}}, \hat{\sigma}^2(\hat{\boldsymbol{\lambda}})), \quad (\text{A.6})$$

$$\frac{p(\gamma_j = 1 | \text{data}, \boldsymbol{\lambda}, \boldsymbol{\omega}, \sigma^2, \boldsymbol{\gamma}_{-j})}{p(\gamma_j = 0 | \text{data}, \boldsymbol{\lambda}, \boldsymbol{\omega}, \sigma^2, \boldsymbol{\gamma}_{-j})} = \frac{\omega_j}{1 - \omega_j} \frac{p(\lambda_j | \gamma_j = 1, \sigma^2)}{p(\lambda_j | \gamma_j = 0, \sigma^2)}, \quad (\text{A.7})$$

$$\omega_j | \text{data}, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \sigma^2 \sim \text{Beta}(\gamma_j + a_\omega, 1 - \gamma_j + b_\omega), \quad (\text{A.8})$$

$$\sigma^2 | \text{data}, \boldsymbol{\omega}, \boldsymbol{\lambda}, \boldsymbol{\gamma} \sim \mathcal{IG}\left(\frac{N + K + 1}{2}, \frac{(\boldsymbol{\mu}_R - \mathbf{C}\boldsymbol{\lambda})^\top \boldsymbol{\Sigma}_R^{-1}(\boldsymbol{\mu}_R - \mathbf{C}\boldsymbol{\lambda}) + \boldsymbol{\lambda}^\top \mathbf{D}\boldsymbol{\lambda}}{2}\right), \quad (\text{A.9})$$

where $\hat{\boldsymbol{\lambda}} = (\mathbf{C}^\top \boldsymbol{\Sigma}_R^{-1} \mathbf{C} + \mathbf{D})^{-1} \mathbf{C}^\top \boldsymbol{\Sigma}_R^{-1} \boldsymbol{\mu}_R$, $\hat{\sigma}^2(\hat{\boldsymbol{\lambda}}) = \sigma^2(\mathbf{C}^\top \boldsymbol{\Sigma}_R^{-1} \mathbf{C} + \mathbf{D})^{-1}$ and $\mathcal{IG}$ denotes the inverse-Gamma distribution.

Hence, posterior sampling is achieved with a Gibbs sampler that draws sequentially the time series layer parameters $(\boldsymbol{\mu}_R, \boldsymbol{\Sigma}_R, \text{ and } \mathbf{C})$ from equations (A.4)-(A.5), and then, conditional on these realizations, draws sequentially from equations (A.6)-(A.9).

B The factor zoo

Traded and nontraded factors We present the 54 bond, equity and nontraded factors used in Table A.1 including a detailed description of their construction, associated reference, and data source.

Table A.1: List of factors for cross-sectional asset pricing. This table presents the list of tradable bond, equity and nontradable factors used in the main paper. For each of the factors, we present their identification index (Factor ID), a description of the factor construction, and the source of the data for downloading and/or constructing the time series.

Factor ID	Factor name and description	Reference	Source	
Panel A: Traded corporate bond factors				
CRF	Credit risk factor. Equally-weighted average return on two ‘credit portfolios’: CRF_{VaR} , and CRF_{REV} . CRF_{VaR} is the average return difference between the lowest-rating (i.e., highest credit risk) portfolio and the highest-rating (i.e., lowest credit risk) portfolio across the VaR95 portfolios. CRF_{REV} is the average return difference between the lowest-rating portfolio and the highest-rating portfolio across quintiles sorted on bond short-term reversal.	Bai et al. (2019)	Open Bond Pricing	Source Asset
CRY	Bond carry factor. Independent sort (5×5) to form 25 portfolios according to ratings and bond credit spreads (CS). For each rating quintile, calculate the weighted average return difference between the highest CS quintile and the lowest CS quintile. CRY is computed as the average long-short portfolio return across all rating quintiles.	Houweling and Van Zundert (2017)	Open Bond Pricing	Source Asset
DEF	Bond default risk factor. The difference between the return on the market portfolio of long-term corporate bond returns (the Composite portfolio on the corporate bond module of Ibbotson Associates) and the long-term government bond return.	Fama (1992) and French	Amit Goyal website	
DRF	Downside risk factor. Independent sort (5×5) to form 25 portfolios according to ratings and 95% value-at-risk (VaR95). For each rating quintile, calculate the weighted average return difference between the highest VaR5 quintile and the lowest VaR5 quintile. DRF is computed as the average long-short portfolio return across all rating quintiles.	Bai et al. (2019)	Open Bond Pricing	Source Asset
DUR	Bond duration factor. Independent sort (5×5) to form 25 portfolios according to ratings and bond duration (DUR^B). For each rating quintile, calculate the weighted average return difference between the highest DUR^B quintile and the lowest DUR^B quintile. DUR is computed as the average long-short portfolio return across all rating quintiles.	Dang et al. (2023)	Open Bond Pricing	Source Asset
HMLB	Bond book-to-market factor. Independent sort (2×3) to form 6 portfolios according to bond size and bond book-to-market (BBM), defined as bond principal value scaled by market value. For each size portfolio, calculate the weighted average return difference between the lowest BBM tercile and the highest BBM tercile. HMLB is computed as the average long-short portfolio return across the two size portfolios.	Bartram et al. (2020)	Open Bond Pricing	Source Asset

LTREVB	Bond long-term reversal factor. Dependent sort ($3 \times 3 \times 3$) to form 27 portfolios according to ratings, maturity, and the 48-13 cumulative previous bond return ($LTREV^B$). For each rating quintile, the factor is computed as the average return differential between the portfolio with the lowest $LTREV^B$ and the one with the highest $LTREV^B$ within the rating and maturity portfolios. LTREVB is computed as the average long-short portfolio return across the nine rating-maturity terciles.	Bali et al. (2021)	Open Bond Pricing	Source Asset
MKTB	Corporate Bond Market excess return. Constructed using bond returns in excess of the one-month risk-free rate of return.	Dickerson et al. (2023)	Open Bond Pricing	Source Asset
MKTBD	Corporate Bond Market duration adjusted return. Constructed using bond returns in excess of their duration-matched U.S. Treasury bond rate of return.	van Binsbergen et al. (2024)	Open Bond Pricing	Source Asset
MOMB	Bond momentum factor formed with bond momentum. Independent sort (5×5) to form 25 portfolios according to ratings and the 12-2 cumulative previous bond return (MOM). For each rating quintile, calculate the weighted average return difference between the highest MOM quintile and the lowest MOM quintile. MOMB is computed as the average long-short portfolio return across all rating quintiles.	Gebhardt et al. (2005)	Open Bond Pricing	Source Asset
MOMBS	Bond momentum factor formed with equity momentum. Independent sort (5×5) to form 25 portfolios according to ratings and the 6-1 cumulative previous equity return (MOMs). For each rating quintile, calculate the weighted average return difference between the highest MOMs quintile and the lowest MOMs quintile. MOMBS is computed as the average long-short portfolio return across all rating quintiles.	Dang et al. (2023)	Open Bond Pricing	Source Asset
PEADB	Bond earnings announcement drift factor. Independent sort (2×3) to form 6 portfolios according to market equity and earnings surprises (CAR), computed according to Chan et al. (1996) . For each firm size portfolio, calculate the weighted average return difference between the highest CAR terciles and the lowest CAR tercile. PEADB is computed as the average long-short portfolio return across the two firm size portfolios.	Nozawa et al. (2023)	Open Bond Pricing	Source Asset
STREVB	Bond short-term reversal factor. Independent sort (5×5) to form 25 portfolios according to ratings and the prior month's bond return (REV). For each rating quintile, calculate the weighted average return difference between the lowest REV quintile and the highest REV quintile. STREVB is computed as the average long-short portfolio return across all rating quintiles.	Bali et al. (2021)	Open Bond Pricing	Source Asset
SZE	Bond size factor. Dependent sort (3×3) to form 3 portfolios according to ratings and then with each rating tercile another 3 portfolios on bond size (SIZE). Bond size is defined as bond price multiplied by issue size (amount outstanding). For each rating tercile, calculate the weighted average return difference between the lowest SIZE tercile and the highest SIZE tercile. SZE is computed as the average long-short portfolio return across all rating terciles.	Houweling and Van Zundert (2017)	Open Bond Pricing	Source Asset
TERM	Bond term structure risk factor. The difference between the monthly long-term government bond return and the one-month T-Bill rate of return.	Fama and French (1992)	Amit Goyal website	
VAL	Bond value factor. Independent sort (2×3) to form 6 portfolios according to bond size and bond value (VAL^B). VAL^B is computed via cross-sectional regressions of credit spreads on ratings, maturity, and the 3-month change in credit spread. The percentage difference between the actual credit spread and the fitted ('fair') credit spread for each bond is the VAL^B characteristic. For each size portfolio, calculate the weighted average return difference between the highest VAL^B tercile and the lowest VAL^B tercile. VAL is computed as the average long-short portfolio return across the two size portfolios.	Houweling and Van Zundert (2017)	Open Bond Pricing	Source Asset

Panel B: Nontraded corporate bond and equity factors

CPTL	Intermediary capital nontraded risk factor. Constructed using AR(1) innovations to the market-based capital ratio of primary dealers, scaled by the lagged capital ratio.	He et al. (2017)	Zhiguo He website
CREDIT	Bond credit risk factor. Difference between the yields of BAA and AAA indices.	Fama and French (1993)	Zhiguo He website
EPU	Economic Policy Uncertainty. First difference in the economic policy uncertainty index.	Dang et al. (2023)	FRED
EPUT	Economic Tax Policy Uncertainty. First difference in the economic tax policy uncertainty index.	Dang et al. (2023)	FRED
INFLC	Shocks to core inflation. Unexpected core inflation component captured by an ARMA(1,1) model. Monthly core inflation is calculated as the percentage change in the seasonally adjusted Consumer Price Index for All Urban Consumers: All Items Less Food and Energy which is lagged by one-month to account for the inflation data release lag.	Fang et al. (2022)	FRED
INFLV	Inflation volatility. Computed as the 6-month volatility of the unexpected inflation component captured by an ARMA(1,1) model. Monthly inflation is calculated as the percentage change in the seasonally adjusted Consumer Price Index for All Urban Consumers (CPI) which is lagged by one-month to account for the inflation data release lag.	Kang and Pflueger (2015) and Ceballos (2023)	FRED
IVOL	Idiosyncratic equity volatility factor. Cross-sectional volatility of all firms in the CRSP database in each month t .	Campbell and Taksler (2003)	CRSP
LVL	Level term structure factor. Constructed as the first principal component of the one- through 30-year CRSP Fixed Term Indices U.S. Treasury Bond yields.	Koijen et al. (2017)	CRSP Indices
LIQNT	Liquidity factor, computed as the average of individual-stock measures estimated with daily data (residual predictability, controlling for the market factor)	Pástor and Stambaugh (2003)	Robert Stambaugh website
UNC	First difference in the Macroeconomic uncertainty index, which is lagged by one-month to align the forecast to the returns observed in month t .	Koijen et al. (2017)	Sydney Ludvigsson website
UNCF	First difference in the Financial economic uncertainty index, which is lagged by one-month to align the forecast to the returns observed in month t .	Koijen et al. (2017)	Sydney Ludvigsson website
UNCr	First difference in the Real economic uncertainty index, which is lagged by one-month to align the forecast to the returns observed in month t .	Koijen et al. (2017)	Sydney Ludvigsson website
VIX	First difference in the CBOE VIX.	Chung et al. (2019)	FRED
YSP	Slope term structure factor. Constructed as the difference in the five and one-year U.S. Treasury Bond yields.	Koijen et al. (2017)	CRSP Indices

Panel C: Traded equity factors

BAB	Betting-against-beta factor, constructed as a portfolio that holds low-beta assets, leveraged to a beta of 1, and that shorts high-beta assets, de-leveraged to a beta of 1	Frazzini and Pedersen (2014)	AQR library
CMA	Investment factor, constructed as a long-short portfolio of stocks sorted by their investment activity	Fama and French (2015)	Ken French website
CMAs	CMA with a hedged unpriced component	Daniel et al. (2020)	Kent website Daniel website
CPTLT	The value-weighted equity return for the New York Fed's primary dealer sector not including new equity issuance	He et al. (2017)	Zhiguo He website
FIN	Long-term behavioral factor, predominantly capturing the impact of share issuance and correction	Daniel et al. (2020)	Kent website Daniel website
HML	Value factor, constructed as a long-short portfolio of stocks sorted by their book-to-market ratio	Fama and French (1992)	Ken French website
HML_DEV	A version of the HML factor that relies on the current price level to sort the stocks into long and short legs	Asness and Frazzini (2013)	AQR library
HMLs	HML with a hedged unpriced component	Daniel et al. (2020)	Kent website Daniel website
LIQ	Liquidity factor, constructed as a long-short portfolio of stocks sorted by their exposure to LIQ_NT	Pástor and Stambaugh (2003)	Robert Stambaugh website
LTREV	Long-term reversal factor, constructed as a long-short portfolio of stocks sorted by their cumulative return accrued in the previous 60-13 months	Jegadeesh and Titman (2001)	Ken French website
MGMT	Management performance mispricing factor	Stambaugh and Yuan (2017)	Global factor data website
MKTS	Market excess return	Sharpe (1964) and Lintner (1965)	Ken French website
MKTSs	Market factor with a hedged unpriced component	Daniel et al. (2020)	Kent website Daniel website

MOMS	Momentum factor, constructed as a long-short portfolio of stocks sorted by their 12-2 cumulative previous return	Carhart (1997), Jegadeesh and Titman (1993)	Ken French website
PEAD	Short-term behavioral factor, reflecting post-earnings announcement drift	Daniel et al. (2020)	Kent website Daniel
PERF	Firm performance mispricing factor	Stambaugh and Yuan (2017)	Global factor data website
QMJ	Quality-minus-junk factor, constructed as a long-short portfolio of stocks sorted by the combination of their safety, profitability, growth, and the quality of management practices	Asness et al. (2019)	AQR data library
RMW	Profitability factor, constructed as a long-short portfolio of stocks sorted by their profitability	Fama and French (2015)	Ken French website
RMWs	RMW with a hedged unpriced component	Daniel et al. (2020)	Kent Daniel website
R_IA	Investment factor, constructed as a long-short portfolio of stocks sorted by their investment-to-capital	Hou et al. (2015)	Lu Zhang website
R_ROE	Profitability factor, constructed as a long-short portfolio of stocks sorted by their return on equity	Hou et al. (2015)	Lu Zhang website
SMB	Size factor, constructed as a long-short portfolio of stocks sorted by their market cap	Fama and French (1992)	Ken French website
SMBs	SMB with a hedged unpriced component	Daniel et al. (2020)	Kent Daniel website
STREV	Short-term reversal factor, constructed as a long-short portfolio of stocks sorted by their previous month return	Jegadeesh and Titman (1993)	Ken French website

C Duration-adjusted bond returns

Duration-adjusted returns are computed for each bond i at each time t such that the resultant bond return is in ‘excess’ of a portfolio of duration-matched U.S. Treasury Bond returns (van Binsbergen, Nozawa, and Schwert (2024), Andreani, Palhares, and Richardson (2023)). The total return for corporate bond i in month t is,

$$R_{it} = \frac{B_{it} + AI_{it} + Coupon_{ijt}}{B_{it-1} + AI_{it-1}} - 1,$$

where B_{it} is the clean price of bond i in month t , AI_{it} is the accrued interest, and $Coupon_{it}$ is the coupon payment, if any.

The bond credit excess return (‘duration adjusted return’) is the total bond return minus a hedging portfolio of U.S Treasury Bonds that have the same duration as the bond in month t . The duration-adjusted return isolates the portion of a bonds performance that is attributed to the credit risk of each bond (or other non-interest rate related risks). The duration-adjusted return is defined as $R_{it} - R_{durbond,i,t}^{Treasury}$, where $R_{durbond,i,t}^{Treasury}$ is a portfolio of treasury securities that matches the duration of the cash-flows of of bond i . We use the duration-adjusted return to re-compute the traded bond factor returns and underlying bond portfolios (basis assets).

D Benchmark asset pricing models

We benchmark the performance of the BMA-SDF against several frequentist asset pricing models as well as other latent factor models:

CAPM and CAPMB: The single-factor equity CAPM and the bond-equivalent CAPMB. The CAPM is the value-weighted equity market factor from [Kenneth French’s webpage](#). The bond CAPM (CAPMB) is the value-weighted corporate bond market factor.

FF5: The original five-factor model of [Fama and French \(1993\)](#) that includes the MKTS, SMB and HML factors from [Fama and French \(1992\)](#) and the default (DEF) and term structure (TERM) factors introduced in [Fama and French \(1993\)](#).

HKM: The intermediary capital two-factor asset pricing model of [He et al. \(2017\)](#). Includes the MKTS factor from [Fama and French \(1992\)](#) and the value-weighted (traded version) of the intermediary capital factor, CPTLT.

KNS: The latent factor model approach of [Kozak et al. \(2020\)](#). For each in-sample bond, stock or co-pricing cross-section, we select the optimal shrinkage level and number of factors chosen by twofold cross-validation. Given our data has a time-series length of $T = 444$, the first sample is simply 1986:01 to 2004:06 and the second sample is 2004:07 to 2022:12.

RP-PCA: The risk premia-principal components methodology of [Lettau and Pelger \(2020\)](#). In our main results, we set $\gamma = 20$. Changing this parameter to 10, or a lower value does not quantitatively affect pricing performance. We use 5 principal components.

E Additional table

Table A.2: Posterior factor probabilities and risk prices for the co-pricing factor zoo

Factors:	Factor prob., $\mathbb{E}[\gamma_j \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \text{data}]$			
	Total prior		Sharpe ratio		Total prior		Sharpe ratio	
	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.539	0.627	0.706	0.707	0.053	0.214	0.444	0.642
PEAD	0.506	0.555	0.601	0.591	0.034	0.136	0.287	0.429
IVOL	0.506	0.541	0.576	0.624	0.011	0.044	0.112	0.270
CREDIT	0.507	0.514	0.542	0.574	0.008	0.033	0.084	0.195
YSP	0.501	0.508	0.516	0.542	0.003	0.014	0.036	0.094
MOMBS	0.504	0.536	0.535	0.480	0.060	0.208	0.361	0.433
LIQNT	0.514	0.508	0.496	0.498	-0.003	-0.015	-0.041	-0.101
MKTS	0.512	0.524	0.500	0.473	0.056	0.177	0.292	0.407
LVL	0.503	0.497	0.495	0.490	0.001	0.003	0.008	0.021
CMA _s	0.485	0.491	0.512	0.495	0.015	0.060	0.136	0.223
UNC _r	0.491	0.498	0.495	0.487	0.001	0.004	0.012	0.032
CRY	0.491	0.488	0.509	0.483	0.052	0.166	0.339	0.509
INFLC	0.493	0.491	0.489	0.492	-0.001	-0.004	-0.011	-0.027
INFLV	0.486	0.498	0.494	0.485	0.002	0.007	0.014	0.022
QMJ	0.498	0.510	0.506	0.448	0.073	0.197	0.338	0.434
EPUT	0.498	0.498	0.488	0.473	0.002	0.008	0.014	0.018
EPU	0.507	0.500	0.480	0.444	0.001	0.003	0.005	0.007
UNC _f	0.496	0.481	0.475	0.437	-0.003	-0.002	0.016	0.061
SZE	0.495	0.472	0.477	0.425	0.006	0.026	0.064	0.106
VIX	0.478	0.475	0.467	0.448	0.000	0.003	0.006	0.011
UNC	0.468	0.476	0.473	0.446	-0.001	-0.001	0.003	0.013
RMW _s	0.482	0.473	0.467	0.412	0.024	0.073	0.137	0.195
LIQ	0.495	0.488	0.450	0.394	0.006	0.026	0.057	0.085
MKTS _s	0.502	0.466	0.452	0.404	0.014	0.035	0.063	0.102
MOMS	0.501	0.476	0.443	0.392	0.021	0.060	0.099	0.149
CPTLT	0.471	0.463	0.464	0.401	0.024	0.072	0.133	0.187
CPTL	0.479	0.470	0.439	0.400	0.015	0.042	0.066	0.068
STREVB	0.501	0.482	0.434	0.362	0.003	0.007	0.010	0.007
HML _s	0.492	0.465	0.443	0.364	0.004	0.014	0.023	0.025
SMB _s	0.497	0.468	0.434	0.362	0.004	0.016	0.028	0.033
LTREVB	0.490	0.484	0.423	0.358	0.016	0.052	0.078	0.090
BAB	0.485	0.467	0.430	0.373	0.021	0.052	0.075	0.094
MKT _B	0.498	0.467	0.428	0.348	0.087	0.181	0.236	0.250
R_IA	0.484	0.454	0.421	0.376	0.034	0.081	0.119	0.147
R_ROE	0.482	0.476	0.429	0.346	0.050	0.106	0.141	0.154
CMA	0.497	0.455	0.420	0.348	0.029	0.061	0.074	0.055
VAL	0.472	0.454	0.418	0.369	0.017	0.058	0.099	0.126
MGMT	0.483	0.462	0.418	0.348	0.058	0.124	0.161	0.175
LTREV	0.481	0.458	0.420	0.350	0.008	0.028	0.049	0.052
HML_DEV	0.476	0.450	0.417	0.360	0.001	0.003	0.019	0.042
SMB	0.483	0.456	0.415	0.342	0.010	0.047	0.082	0.089
PERF	0.485	0.467	0.413	0.332	0.047	0.104	0.112	0.089
CRF	0.495	0.450	0.411	0.340	0.015	0.052	0.091	0.116
STREV	0.469	0.445	0.417	0.365	0.009	0.030	0.066	0.099
MOMB	0.470	0.453	0.413	0.351	-0.003	-0.007	-0.007	-0.003
TERM	0.480	0.453	0.405	0.342	0.027	0.060	0.087	0.109
HML	0.482	0.453	0.406	0.329	0.003	-0.017	-0.040	-0.040
DRF	0.475	0.447	0.399	0.349	0.040	0.071	0.072	0.040
FIN	0.482	0.452	0.405	0.319	0.036	0.040	0.016	-0.004
DEF	0.466	0.444	0.404	0.335	0.000	-0.007	-0.020	-0.027
HMLB	0.469	0.445	0.411	0.323	0.040	0.102	0.141	0.121
DUR	0.486	0.416	0.401	0.334	0.012	-0.017	-0.075	-0.125
RMW	0.459	0.432	0.397	0.329	0.027	0.021	-0.022	-0.054
MKTBD	0.452	0.437	0.394	0.331	0.013	0.028	0.025	0.007

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and equity factors described in Appendix B. The factors are ordered (high to low) by each factors average posterior probability across the four levels of shrinkage. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Test assets include the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$). Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_F^2 | \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Sample period: 1986:01 to 2022:12 ($T = 444$).

Internet Appendix for:
The Co-Pricing Factor Zoo

Abstract

This Internet Appendix provides additional tables, figures, information and results supporting the main text.

IA.1 Detailed data and variables construction

The following sections describe the various databases that we use in the paper. Across all databases, we filter out bonds which have a time-to-maturity of less than 1-year. Furthermore, for consistency, across all databases, we define bond ratings as those provided by Standard & Poors (S&P). We include the full spectrum of ratings (AAA to D), but exclude bonds which are unrated. For each database that we consider, we (the authors) *do not* winsorize or trim bond returns in any way.

IA.1.1 Corporate bond databases

IA.1.1.1 Mergent Fixed Income Securities Database (FISD) database

Mergent Fixed Income Securities Database (FISD) for academia is a comprehensive database of publicly offered U.S. bonds, research market trends, deal structures, issuer capital structures, and other areas of fixed income debt research. We apply to the FISD data the standard filters used in the previous literature:

1. Only keep bonds that are issued by firms domiciled in the United States of America, `COUNTRY_DOMICILE == 'USA'`.
2. Remove bonds that are private placements, `PRIVATE_PLACEMENT == 'N'`.
3. Only keep bonds that are traded in U.S. Dollars, `FOREIGN_CURRENCY == 'N'`.
4. Bonds that trade under the 144A Rule are discarded, `RULE_144A == 'N'`.
5. Remove all asset-backed bonds, `ASSET_BACKED == 'N'`.
6. Remove convertible bonds, `CONVERTIBLE == 'N'`.
7. Only keep bonds with a fixed or zero coupon payment structure, i.e., remove bonds with a floating (variable) coupon, `COUPON_TYPE != 'V'`.
8. Remove bonds that are equity linked, agency-backed, U.S. Government, and mortgage-backed, based on their `BOND_TYPE`.

9. Remove bonds that have a “non-standard” interest payment structure or bonds not caught by the variable coupon filter (`COUPON_TYPE`). We remove bonds that have an `INTEREST_FREQUENCY` equal to -1 (N/A), 13 (Variable Coupon), 14 (Bi-Monthly), and 15 and 16 (undocumented by FISD). Additional information on `INTEREST_FREQUENCY` is available on page 60 to 67 of the FISD Data Dictionary 2012 document.

IA.1.1.2 Bank of America Merrill Lynch (BAML) database

The BAML data is made available by the Intercontinental Exchange (ICE) and provides daily bond price quotes, accrued interest, and a host of pre-computed corporate bond characteristics such as the bond option-adjusted credit spread (OAS), the asset swap spread, duration, convexity, and bond returns in excess of a portfolio of duration-matched Treasuries. The ICE sample spans the time period 1997:01 to 2021:09 and includes constituent bonds from the ICE Bank of America High Yield (H0A0) and Investment Grade (C0A0) Corporate Bond Indices.

ICE bond filters. We follow [van Binsbergen et al. \(2024\)](#) and take the last quote of each month to form the bond-month panel. We then merge the ICE data to the filtered Mergent FISD database. The following ICE-specific filters are then applied:

1. Only include corporate bonds, `Ind_Lvl_1 == 'corporate'`
2. Only include bonds issued by U.S. firms, `Country == 'US'`
3. Only include corporate bonds denominated in U.S. Dollars, `Currency == 'USD'`

BAML/ICE bond returns. Total bond returns are computed in a standard manner in ICE, and no assumptions about the timing of the last trading day of the month are made because the data is quote based, i.e., there is always a valid quote at month-end to compute a bond return. This means that each bond return is computed using a price quote at exactly the end of the month, each and every month. This introduces homogeneity into the bond returns because prices are sampled at exactly the same time each month. ICE only provides bid-side pricing, meaning bid-ask bias is inherently not present in the monthly sampled prices, returns and credit spreads. The monthly ICE return variable is (as denoted in the original database) `trr_mtd_loc`, which is the month-to-date return on the last business day of month t .

IA.1.1.3 Lehman Brothers (LHM) database

The Lehman Brothers Bond database holds monthly price data for corporate (and other) bonds from January 1973 to December 1997. The database categorizes the prices as either quote or matrix prices and identifies whether the bonds are callable or not. However, as per the findings of [Chordia, Goyal, Nozawa, Subrahmanyam, and Tong \(2017\)](#), the difference between quote and matrix prices or callable and non-callable bonds does not have a material impact on cross-sectional return predictability. Hence, we include both types of observations. In addition, the Lehman Brothers data provides key bond details such as the amount outstanding, credit rating, offering date, and maturity date. For the main results, we use the LHM data from 1986:01 to 1996:12.

LHM filters. As for the other databases, we merge the LHM data to the pre-filtered Mergent database and then apply the following LHM-specific filters following [Elkamhi, Jo, and Nozawa \(2023\)](#):

1. Only include corporate bonds classified as ‘industrial’, ‘telephone utility’, ‘electric utility’, ‘utility (other)’, and ‘finance’, as per the LHM industry classification system, `icode == {3 | 4 | 5 | 6 | 7}`.
2. Remove the following dates for which there are no observations or valid return data, `date == {1975-08 | 1975-09 | 1984-12 | 1985-01}`.

LHM returns. The LHM bond database includes corporate bond returns that have been pre-computed. The accuracy of the LHM return computation has been verified empirically by [Elkamhi et al. \(2023\)](#).

LHM additional filters. We follow [Bessembinder et al. \(2008\)](#) and [Chordia et al. \(2017\)](#) and apply the following filters to the LHM data to account for potential data errors:

1. Remove observations with large return reversals, defined as a 20% or greater return followed by a 20% or greater return of the opposite sign.
2. Remove observations if the prices appear to bounce back in an extreme fashion relative to preceding days. Denote R_t as the month t return, we exclude an observation at month t if $R_t \times R_{t-k} < -0.02$ for $k = 1, \dots, 12$.

3. Remove observations if prices do not change for more than three months, i.e., $\frac{P_t}{P_{t-3}} - 1 \neq 0$, where P is the quoted or matrix price.

IA.1.2 Combined data

For our main results, we rely on the data set that combines the LHM, and ICE data sets over the sample period 1986:01–2022:12. The data is spliced together as follows:

1. From 1986:01–1996:12 we use the LHM data.
2. From 1997:01–2022:12 we use the ICE data.

IA.1.3 In-sample and out-of-sample test assets

Corporate bond, stock and U.S. Treasury bond data We present descriptions of the in-sample and out-of-sample portfolio and anomaly data we use to estimate and test the BMA-SDFs and other asset pricing models we consider along with the associated reference and source.

Table IA.I: List of corporate bond, stock and U.S. Treasury bond test assets. This table presents the list of in-and-out-of-sample test assets used in the main results of the paper. For each test asset, we present their identification (Asset ID), a description of their construction, and the source of the data for downloading and/or constructing the time series. Panel A describes the IS corporate bond portfolios. Panel B describes the IS stock anomalies/portfolios. Panel C describes the OS corporate bond portfolios. Panel D describes the OS stock portfolios. Panel E describes the OS U.S. Treasury portfolios.

Asset ID	Name and description	Reference	Source	
Panel A: In-sample bond portfolios/anomalies				
25 spread/size bond portfolios	5 Bond credit spread $\times$ 5 bond market capitalization double sorted portfolios.	Nozawa (2017) and Elkamhi et al. (2023)	Open Bond Pricing	Source Asset
25 rating/maturity bond portfolios	5 Bond rating $\times$ 5 bond time to maturity double sorted portfolios.	Gebhardt et al. (2005) and others	Open Bond Pricing	Source Asset
Panel B: In-sample stock portfolios/anomalies				
cash_at	CashAssets. Cash and Short Term Investments scaled by Assets.	Palazo (2012)	Global Data	Factor
ope_be	FCFBook. Operating Profit to Equity scaled by BE.	Thesmar (2019)	Global Data	Factor
ocf_me	CFPrice. Operating Cash Flow scaled by ME.	Desai et al. (2004)	Global Data	Factor
at_turnover	Asset Turnover. Sales scaled by average of total assets.	Haugen and Baker (1996)	Global Data	Factor
capx_gr2	CapIntens. CAPX 2 year growth.	Anderson and Garcia-Feijoo (2006)	Global Data	Factor
div12m_me	DP tr. Dividend to Price - 12 Months.	Litzenberger and Ramaswamy (1979)	Global Data	Factor
ppeinv_gr1a	PPE delta. Change in Property, Plant and Equipment Less Inventories scaled by lagged Assets.	Lyandres et al. (2008)	Global Data	Factor
sale_me	SalesPrice	Barbee et al. (1996)	Global Data	Factor
ret_12_7	IntermMom. Momentum 7-12 Months.	Novy-Marx (2012)	Global Data	Factor

prc_highprc_252d	YearHigh. Current price to high price over last year.	George and Hwang (2004)	Global Data	Factor
ni_me	PE tr. Net Income scaled by ME.	Basu (1983)	Global Data	Factor
bidaskhl_21d	BidAsk. 21 Day Bid-Ask High-Low. High-low bid ask estimator created using code from Corwin and Schultz (2012).	Corwin and Schultz (2012)	Global Data	Factor
dolvol_126d	Volume. Dollar trading volume.	Brennan et al. (1998)	Global Data	Factor
dsale_dsga	SGASales. Change Sales minus Change SG&A.	Abarbanell and Bushee (1998)	Global Data	Factor
cop_atl1	Cash Based Operating Profitability scaled by lagged Assets.	Ball et al. (2016)	Global Data	Factor
ivol_capm_252d	iVolCAPM. Idiosyncratic volatility from the CAPM (252 days).	Ali et al. (2003)	Global Data	Factor
ivol_ff3_21d	iVolFF3. Idiosyncratic volatility from the Fama-French 3-factor model.	Ang et al. (2006)	Global Data	Factor
rvol_21d	Max Return to Volatility.	Ang et al. (2006)	Global Data	Factor
ebit_sale	ProfMargin. Operating Profit Margin after Depreciation.	Soliman (2008)	Global Data	Factor
ocf_at	PriceCostMargin. Operating Cash Flow scaled by Assets.	Bouchard et al. (2019)	Global Data	Factor
opex_at	OperLev. Operating Leverage.	Novy-Marx (2011)	Global Data	Factor
lnoa_gr1a	NetSalesNetOA. Change in Long-Term NOA scaled by average Assets.	Fairfield et al. (2003)	Global Data	Factor
oaccruals_at	Operating Accruals.	Sloan (1996)	Global Data	Factor
at_gr1	Asset growth. Asset Growth 1yr.	Cooper et al. (2008)	Global Data	Factor
eqnpo_12m	Net Equity Payout - 12 Month.	Daniel and Titman (2006)	Global Data	Factor
gp_at	Gross Profit scaled by Assets.	Novy-Marx (2013)	Global Data	Factor
capex_abn	Abnormal Corporate Investment.	Titman et al. (2004)	Global Data	Factor
noa_at	NetOA. Net Operating Asset to Total Assets.	Hirshleifer et al. (2004)	Global Data	Factor
o_score	Ohlson O-score.	Dichev (1998)	Global Data	Factor
niq_at	ROA. Quarterly return on assets.	Balakrishnan et al. (2010)	Global Data	Factor
chcsho_12m	Net stock issues.	Pontiff and Woodgate (2008)	Global Data	Factor
LRreversal	Long-run reversal.	De Bondt and Thaler (1985)	Open Asset Pricing	
Lev	Market leverage.	Bhandari (1988)	Open Asset Pricing	

Panel C: Out-of-sample bond portfolios

10× VaR portfolios	Decile sorted bond portfolios sorted on 24-month rolling 95% historical value-at-risk (VaR) defined as the second lowest return value in the rolling period.	Bai et al. (2019)	Open Bond Pricing	Source Asset
10× duration portfolios	Decile sorted bond portfolios sorted on bond duration.	Gebhardt et al. (2005)	Open Bond Pricing	Source Asset
10× bond value portfolios	Decile sorted bond portfolios sorted on bond market capitalization defined as bond price multiplied by bond amount outstanding.	Houweling and Van Zundert (2017)	Open Bond Pricing	Source Asset
10× bond BTM portfolios	Decile sorted bond portfolios sorted on bond book-to-market (BTM) defined as the market value of the bond scaled by the par value.	Bartram et al. (2020)	Open Bond Pricing	Source Asset
10× bond LTREV portfolios	Decile sorted bond portfolios sorted on bond long-term reversal defined as the sum of the bond returns from t-12 to t-48.	Bali et al. (2021)	Open Bond Pricing	Source Asset
10× bond MOM portfolios	Decile sorted bond portfolios sorted on bond momentum defined as the sum of the bond returns from t-6 to t-1.	Gebhardt et al. (2005)	Open Bond Pricing	Source Asset
17× bond FF17 portfolios	17 Fama-French industry portfolios computed with bond returns.	Kelly et al. (2023)	Open Bond Pricing	Source Asset

Panel D: Out-of-sample stock portfolios/anomalies

10× E/P portfolios	Decile sorted stock portfolios sorted on the earning-to-price ratio (E/P).	Fama & French	Kenneth French's page	web-
10× MOM portfolios	Decile sorted stock portfolios sorted on equity momentum.	Fama & French	Kenneth French's page	web-

10× LTREV portfolios	Decile sorted stock portfolios sorted on stock long-term reversals.	Fama & French	Kenneth French's page	web-
10× accruals portfolios	Decile sorted stock portfolios sorted on equity accruals.	Fama & French	Kenneth French's page	web-
10× size portfolios	Decile sorted stock portfolios sorted on firm size (market capitalization).	Fama & French	Kenneth French's page	web-
10× variance portfolios	Decile sorted stock portfolios sorted on the earning-to-price ratio (E\ P).	Fama & French	Kenneth French's page	web-
17× stock FF17 portfolios	17 Fama-French industry portfolios computed with stock returns.	Fama & French	Kenneth French's page	web-
Panel E: Out-of-sample Treasury portfolios				
29× Treasury portfolios	Monthly excess U.S. Treasury bond returns computed across the term structure using annualized continuously-compounded zero coupon yields computed as in Liu and Wu (2021). We price the U.S. Treasury Bonds each month using the yield-curve data and then compute monthly discrete excess returns across the term structure as the total return in excess of the one-month Treasury Bill rate. The portfolios span from the 2-year T Bond up until the 30-year T-Bond in increments of 1-year.	Liu and Wu (2021)	Jing Wu's webpage	Cynthia

IA.2 CAPMB: Two-pass regression risk premium vs. SDF-based market price of risk

In this section we report two-pass regression estimates of the risk premium attached to MKTB as sole factor as well as linear SDF estimates of the market price of risk in the CAPMB model used to price our baseline cross-section of corporate bonds and bond tradable factors.

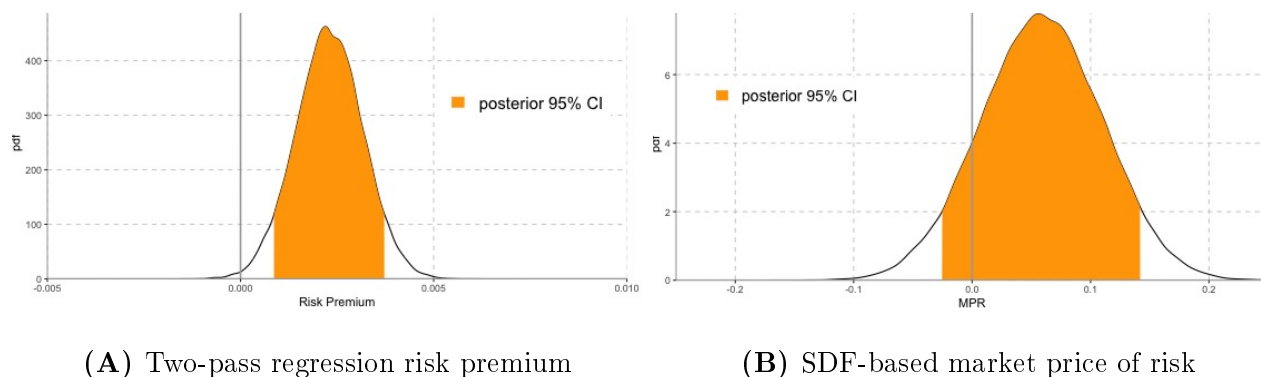
To understand why the two types of estimations can lead to very different outcomes, let's consider a simple example with two (demeaned) tradable risk factors only, i.e. $\mathbf{f}_t = [f_{1,t}, f_{2,t}]^\top$, and suppose for simplicity that their covariance matrix is

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

Suppose further that only the first factor is part of the SDF, and has a market price of risk equal to κ . That is

$$M_t = 1 - \mathbf{f}_t^\top \boldsymbol{\lambda}_f = 1 - [f_{1,t}, f_{2,t}]^\top \begin{bmatrix} \kappa \\ 0 \end{bmatrix} = 1 - f_{1,t}\kappa$$

Figure IA.1: CAPMB: two pass-regression risk premium, and market price of risk



The figure plots the posterior distributions of the two-pass regression ex post risk premium, left panel, and SDF-based market price of risk, right panel, of a model with MKTB as the only risk factor, i.e. CAPMB. Test assets include the baseline cross-section of corporate bond returns, and the corporate bond tradable factors, described in Section 1. The prior Sharpe ratio does not impose any shrinkage, being set to the ex post Sharpe ratio of the MKTB factor. Sample period: 1986:01 to 2022:12 ($T = 444$).

Denoting with $\boldsymbol{\mu}_{RP} = [\mu_{RP,1}, \mu_{RP,2}]^\top$ the vector of risk premia of the factors, applying the fundamental asset pricing equation to the returns generated by the factors we have

$$\boldsymbol{\mu}_{RP} = \Sigma \boldsymbol{\lambda}_f = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} \kappa \\ 0 \end{bmatrix} = \begin{bmatrix} \kappa \\ \rho \kappa \end{bmatrix}.$$

That is, the second factor, that is *not* part of the SDF, commands nevertheless a non-zero risk premium (equal to $\rho \kappa$) as long as the factor has non-zero correlation (i.e., as long as $\rho \neq 0$) with the true risk factor—the one that is part of the SDF. This also implies that a two-pass regression method that uses the second factor as the sole driver of a cross-section of asset returns will estimate its ex post risk premium as being non-zero – in fact, the estimated risk premium for the second factor will be inflated relative to its true value. This is due to the fact that the estimated betas of f_2 will be, in population, smaller than the ones of f_1 by a factor equal to ρ . Hence, in population, the two pass regression will yield an estimated risk premium for f_2 equal to $\rho^{-1} \kappa$ (where $|\rho| \leq 1$).

To estimate the SDF of the CAPMB model we rely on the Bayesian-SDF estimator in Definition 1 of [Bryzgalova et al. \(2023\)](#). This is equivalent to the method presented in Section 2 under the null that MKTB is the only factor in the SDF with probability 1 and that the model is true. To put the comparison of MRP and ex post risk premia estimates on the same

footing, we estimate the two pass regression using the Bayesian implementation of the [Fama and MacBeth \(1973\)](#) method in [Bryzgalova et al. \(2022\)](#). Posterior distributions of the two-pass regression ex post risk premium and SDF-based market price of risk are plotted, respectively, in panels (a) and (b) of Figure [IA.1](#). The estimates suggests that, albeit MKTB carries a sizable and significant risk premium, it is very unlikely that the data are generated by a “true” latent SDF with MKTB as the only factor—the (Bayesian) p -value of its market price of risk being equal to zero is about 52.34%.

IA.3 Cash-flow and discount rate news decomposition

IA.3.1 Tradable factor return decomposition

[Vuolteenaho \(2002\)](#), [Cohen et al. \(2002\)](#), and others decompose unexpected asset returns into an expected-return (discount) component and a cash-flow component:

$$r_{t+1} - E_t r_{t+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j e_{t+1+j} - \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1+j}, \quad (\text{IA.10})$$

where ΔE_{t+1} denotes the change in expectations from t to $t+1$ (i.e., $E_{t+1}(\cdot) - E_t(\cdot)$), e_{t+1} the aggregate return on equity (ROE), and r_{t+1} the log asset return. ρ is determined by the data, and in our setting is equal to 0.979, although any value between 0.95 and 1.00 makes an immaterial difference to the results. We define the two return components as cash-flow news (N_{cf}) and discount rate news (N_r):

$$N_{cf,t+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j e_{t+1+j}, \quad N_{r,t+1} = \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1+j}. \quad (\text{IA.11})$$

As argued by [Vuolteenaho \(2002\)](#), using ROE as the measure of firm cash-flows is more appropriate in our case since we are dealing with both debt and equity-based traded factors and many firms do not pay cash-based dividends.

IA.3.2 Implementation using the VAR methodology

To empirically estimate equation [IA.10](#), we implement a parsimonious vector autoregression (VAR). The behaviour of the traded factors is captured by a vector, $z_{i,t}$ of state variables. The

first variable is always the traded stock or bond factor, whilst the remaining variables could be any set of predictors that are associated with future stock or bond returns. In this respect, we use predictors that are standard in the literature. We define the vector, $z_t = [r_t, roe_t, bm_t, gz_t]$, where r_t is the traded factor return, roe_t is the log of aggregate return on equity (ROE), bm_t is the log of the aggregate book-to-market ratio, and gz_t is the first difference of the log of the [Gilchrist and Zakrajšek \(2012\)](#) aggregate credit spread. Aggregate ROE is the equally-weighted averaged of firm-level net income (NI) scaled by one-quarter lagged book equity. Aggregate book-to-market is from Amit Goyal's data repository available [here](#). The GZ credit spread is computed as in [Gilchrist and Zakrajšek \(2012\)](#).¹

The vector of state variables, z_t is assumed to follow a first order VAR,

$$z_{t+1} = Az_t + u_{t+1}$$

From the VAR, we estimate DR news as,

$$Ndr_{t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} = e'_1 \sum_{j=1}^{\infty} \rho^j A^j u_{t+1} \quad (\text{IA.12})$$

$$= e'_1 \rho A (I - \rho A)^{-1} u_{t+1} = \lambda' u_{t+1}, \quad (\text{IA.13})$$

where $\lambda' = e'_1 \rho A (I - \rho A)^{-1}$ and e_1 is a vector whose first element is equal to 1 and zero otherwise. The CF news component is computed as the residual of the total unexpected factor return and DR news,

$$Ncf_{t+1} = r_{t+1} - E_t r_{t+1} + Ndr = (e'_1 + \lambda') u_{t+1}. \quad (\text{IA.14})$$

IA.3.3 Factor decomposition

We now implement the VAR and decompose each traded factor into the component related to discount rate news (DR) and cash-flow news (CF). Following [Vuolteenaho \(2002\)](#) and [Cohen et al. \(2002\)](#) we compute the variance of the discount rate news component, $\mathbb{V}(Ndr)$ and the ratio of the discount rate news variance to total unexpected factor return variance $\frac{\mathbb{V}(Ndr)}{\mathbb{V}(u)}$. To pin down a relative classification of the factors into a DR or CF-based category, we use the

¹We thank Yoshio Nozawa for making this data available to us.

median level of $\frac{\mathbb{V}(Ndr)}{\mathbb{V}(u)}$ as a break-point. Factors above the break-point, are classified (relatively) as more likely to capture discount-rate news as opposed to cash-flow rate news.

We present the results of the decomposition in Figure 5. The y -axis of the figure shows the proportion of residual variance of each factor estimated from the VAR model that represents discount rate news. Overall, 10 of the 16 bond factors (62%) are driven relatively more by discount rate news as opposed to cash-flow news shocks. In contrast, slightly more equity factors (14/26=53%) are driven by cash-flow shocks

The two most likely factors that ought to be included in the co-pricing BMA-SDF (both PEAD and PEADB) are driven relatively more by discount rate news as opposed to cash-flow news. Most other behavioural-linked factors such as MOMBS (bond factor formed with equity momentum), PERF and MGMT (equity and management performance factor of [Stambaugh and Yuan \(2017\)](#)), are also classified as relatively more discount rate news based.

IA.4 Additional tables

Table IA.II: Traded factor performance statistics – full sample

	SR	IR	μ	t -stat.	α	t -stat.
Panel A: Corporate Bond Factors						
CRF	0.04	0.04	0.08	[0.75]	0.08	[0.69]
CRY	0.13	0.02	0.23	[2.21]	0.03	[0.41]
DEF	0.02	−0.03	0.03	[0.39]	−0.05	[−0.56]
DRF	0.12	−0.09	0.27	[2.35]	−0.09	[−1.88]
DUR	0.08	−0.15	0.14	[1.66]	−0.14	[−2.51]
HMLB	0.14	0.06	0.21	[2.44]	0.09	[1.19]
LTREVB	0.11	0.12	0.09	[2.09]	0.11	[1.97]
MKTB	0.19	-	0.30	[3.55]	-	-
MKTBD	0.06	−0.01	0.08	[1.05]	−0.02	[−0.20]
MOMB	−0.00	0.03	−0.01	[−0.10]	0.04	[0.53]
MOMBS	0.19	0.26	0.18	[3.69]	0.23	[4.36]
PEADB	0.36	0.40	0.13	[7.17]	0.14	[6.88]
STREVB	0.04	0.00	0.07	[0.95]	0.00	[−0.07]
SZE	0.09	0.11	0.07	[1.78]	0.08	[2.30]
TERM	0.12	0.01	0.36	[2.50]	0.03	[0.23]
VAL	0.06	0.06	0.07	[1.16]	0.07	[0.94]
Panel B: Stock Factors						
BAB	0.20	0.23	0.74	[3.52]	0.84	[3.55]
CMA	0.14	0.20	0.29	[2.55]	0.40	[3.45]
CMA _s	0.16	0.19	0.20	[3.24]	0.24	[3.77]
CPTLT	0.11	−0.02	0.75	[2.21]	−0.08	[−0.42]
FIN	0.14	0.23	0.59	[2.78]	0.86	[4.25]
HML	0.06	0.08	0.18	[1.02]	0.25	[1.26]
HML_DEV	0.04	0.04	0.16	[0.81]	0.14	[0.68]
HML _s	0.06	0.07	0.10	[1.01]	0.12	[1.19]
LIQ	0.08	0.06	0.29	[1.52]	0.24	[1.24]
LTREV	0.06	0.05	0.17	[1.16]	0.14	[0.86]
MGMT	0.18	0.26	0.52	[3.37]	0.70	[4.33]
MKTS	0.15	-	0.69	[3.22]	-	-
MKTS _s	0.17	0.12	0.56	[3.39]	0.34	[2.27]
MOMS	0.11	0.15	0.51	[2.3]	0.66	[3.36]
PEAD	0.26	0.28	0.53	[5.4]	0.56	[5.98]
PERF	0.17	0.24	0.52	[3.4]	0.66	[4.93]
QMJ	0.19	0.32	0.47	[3.45]	0.69	[6.44]
RMW	0.15	0.20	0.38	[2.95]	0.48	[3.81]
RMW _s	0.21	0.20	0.31	[4.67]	0.31	[4.46]
R_IA	0.14	0.20	0.31	[2.72]	0.42	[3.55]
R_ROE	0.18	0.24	0.49	[3.58]	0.62	[5.35]
SMB	0.02	−0.01	0.06	[0.45]	−0.03	[−0.25]
SMB _s	0.03	0.04	0.06	[0.58]	0.08	[0.72]
STREV	0.07	0.02	0.24	[1.69]	0.06	[0.45]

Corporate bond and stock traded factor performance statistics. SR is the Sharpe ratio, IR is the Information ratio, μ is the average return, and α is the single-factor MKTB (MKTS) alpha. The sample is from 1986:01 to 2022:12. All statistics are reported monthly. μ and α are reported in percent. t -statistics are reported in square brackets with Newey-West standard errors computed with four lags.

Table IA.III: Traded factor performance statistics – subsamples

	1986:01–1999:12						2000:01–2022:12					
	SR	IR	μ	t -stat.	α	t -stat.	SR	IR	μ	t -stat.	α	t -stat.
Panel A: Corporate Bond Factors												
CRF	0.10	0.22	0.08	[1.26]	0.16	[2.31]	0.03	0.02	0.08	[0.47]	0.05	[0.29]
CRY	0.25	0.24	0.18	[2.94]	0.18	[2.63]	0.12	-0.02	0.26	[1.59]	-0.02	[-0.26]
DEF	-0.05	0.06	-0.05	[-0.73]	0.05	[0.89]	0.04	-0.04	0.08	[0.70]	-0.07	[-0.62]
DRF	0.12	-0.24	0.17	[1.48]	-0.11	[-3.05]	0.13	-0.06	0.33	[1.93]	-0.07	[-0.94]
DUR	0.09	-0.24	0.12	[1.12]	-0.13	[-2.81]	0.07	-0.13	0.15	[1.28]	-0.14	[-1.68]
HMLB	0.22	0.11	0.18	[2.48]	0.07	[1.32]	0.13	0.06	0.23	[1.74]	0.10	[0.89]
LTREVB	0.12	0.33	0.07	[1.37]	0.15	[3.37]	0.11	0.10	0.11	[1.66]	0.09	[1.27]
MKTB	0.21	-	0.29	[2.43]	-	-	0.18	-	0.31	[2.67]	-	-
MKTBD	0.06	0.12	0.05	[0.72]	0.09	[1.53]	0.06	-0.04	0.11	[0.88]	-0.06	[-0.47]
MOMB	-0.08	-0.13	-0.09	[-1.04]	-0.14	[-1.60]	0.02	0.08	0.04	[0.38]	0.14	[1.23]
MOMBS	0.33	0.36	0.11	[3.79]	0.12	[3.77]	0.19	0.27	0.21	[2.89]	0.29	[3.64]
PEADB	0.41	0.41	0.08	[4.89]	0.08	[5.09]	0.38	0.42	0.17	[6.07]	0.18	[5.85]
STREVB	-0.04	-0.03	-0.05	[-0.50]	-0.04	[-0.43]	0.07	0.02	0.13	[1.40]	0.03	[0.36]
SZE	0.08	0.13	0.03	[0.91]	0.05	[1.55]	0.10	0.11	0.09	[1.56]	0.10	[1.92]
TERM	0.14	-0.12	0.37	[1.73]	-0.14	[-1.58]	0.11	0.03	0.35	[1.84]	0.10	[0.49]
VAL	-0.01	0.24	-0.01	[-0.12]	0.14	[2.44]	0.10	0.04	0.12	[1.39]	0.05	[0.57]
Panel B: Stock Factors												
BAB	0.18	0.18	0.60	[1.72]	0.60	[1.67]	0.21	0.25	0.82	[3.40]	0.93	[3.33]
CMA	0.09	0.26	0.18	[1.12]	0.42	[3.07]	0.16	0.19	0.36	[2.25]	0.41	[2.51]
CMAs	0.22	0.31	0.27	[2.78]	0.36	[3.72]	0.13	0.14	0.16	[2.06]	0.18	[2.25]
CPTLT	0.16	-0.05	1.08	[2.10]	-0.20	[-0.75]	0.08	-0.01	0.55	[1.21]	-0.04	[-0.15]
FIN	0.16	0.32	0.53	[1.93]	0.90	[3.65]	0.14	0.20	0.62	[2.15]	0.83	[3.06]
HML	0.03	0.15	0.07	[0.30]	0.33	[1.47]	0.07	0.07	0.25	[1.03]	0.26	[0.95]
HML_DEV	-0.04	0.06	-0.13	[-0.48]	0.16	[0.67]	0.08	0.06	0.34	[1.25]	0.23	[0.85]
HMLs	0.11	0.19	0.17	[1.25]	0.28	[2.04]	0.03	0.03	0.05	[0.40]	0.05	[0.37]
LIQ	0.05	0.07	0.17	[0.62]	0.22	[0.86]	0.09	0.07	0.36	[1.42]	0.29	[1.15]
LTREV	0.11	0.11	0.26	[1.26]	0.26	[1.17]	0.04	0.03	0.12	[0.60]	0.09	[0.39]
MGMT	0.18	0.36	0.41	[2.21]	0.68	[4.28]	0.18	0.23	0.58	[2.70]	0.71	[3.18]
MKTS	0.23	-	1.00	[3.04]	-	-	0.11	-	0.50	[1.74]	-	-
MKTSs	0.24	0.14	0.74	[2.72]	0.37	[1.46]	0.14	0.10	0.45	[2.30]	0.30	[1.79]
MOMS	0.32	0.28	0.99	[3.59]	0.86	[3.08]	0.04	0.09	0.22	[0.73]	0.43	[1.73]
PEAD	0.57	0.55	0.92	[8.06]	0.87	[7.68]	0.13	0.16	0.29	[2.29]	0.35	[2.79]
PERF	0.19	0.17	0.42	[2.32]	0.37	[1.86]	0.17	0.26	0.57	[2.64]	0.75	[4.54]
QMJ	0.27	0.38	0.45	[2.90]	0.60	[3.95]	0.17	0.30	0.49	[2.46]	0.68	[5.11]
RMW	0.16	0.18	0.25	[1.71]	0.28	[2.05]	0.16	0.20	0.46	[2.62]	0.57	[3.48]
RMWs	0.21	0.20	0.28	[2.80]	0.27	[2.68]	0.21	0.21	0.34	[3.88]	0.33	[3.70]
R_IA	0.17	0.35	0.31	[2.04]	0.54	[4.28]	0.13	0.16	0.31	[1.94]	0.37	[2.20]
R_ROE	0.38	0.36	0.73	[4.99]	0.68	[4.66]	0.11	0.19	0.34	[1.76]	0.51	[3.42]
SMB	-0.09	-0.13	-0.26	[-1.11]	-0.35	[-1.44]	0.08	0.06	0.26	[1.53]	0.17	[1.02]
SMBs	-0.07	-0.07	-0.14	[-0.95]	-0.13	[-0.81]	0.08	0.09	0.18	[1.31]	0.19	[1.39]
STREV	0.09	0.03	0.21	[1.07]	0.07	[0.41]	0.07	0.03	0.25	[1.34]	0.10	[0.51]

Corporate bond and stock traded factor performance statistics. SR is the Sharpe ratio, IR is the Information ratio, μ is the average return, and α is the single-factor MKTB (MKTS) alpha. The sample is split into two subperiods following [van Binsbergen et al. \(2024\)](#). The first sample is from 1986:01 to 1999:12, and the second sample is from 2000:01 to 2022:12. All statistics are reported monthly. μ and α are reported in percent. t -statistics are reported in square brackets with Newey-West standard errors computed with four lags.

Table IA.IV: Posterior factor probabilities and risk prices for the corporate bond factor zoo

Factors:	Factor prob., $\mathbb{E}[\gamma_j \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.588	0.767	0.827	0.822	0.089	0.357	0.632	0.802
MOMBS	0.542	0.616	0.621	0.582	0.078	0.279	0.469	0.563
CREDIT	0.501	0.538	0.617	0.674	0.008	0.034	0.095	0.243
IVOL	0.515	0.536	0.551	0.573	0.005	0.019	0.048	0.119
YSP	0.492	0.513	0.532	0.564	0.003	0.014	0.036	0.100
INFLV	0.493	0.498	0.508	0.518	0.004	0.018	0.040	0.081
UNCf	0.516	0.502	0.508	0.478	-0.009	-0.028	-0.054	-0.084
INFLC	0.500	0.490	0.489	0.511	0.000	-0.001	-0.003	-0.008
CRY	0.488	0.492	0.516	0.485	0.037	0.125	0.274	0.417
EPU	0.498	0.497	0.492	0.485	0.003	0.008	0.014	0.022
UNC	0.504	0.508	0.496	0.465	-0.005	-0.014	-0.019	-0.022
LIQNT	0.494	0.494	0.484	0.489	-0.002	-0.006	-0.014	-0.030
EPUT	0.486	0.490	0.496	0.485	0.004	0.013	0.028	0.058
LVL	0.492	0.480	0.489	0.494	0.000	0.000	0.000	0.003
MKTB	0.504	0.510	0.488	0.433	0.066	0.173	0.271	0.347
VIX	0.490	0.486	0.483	0.455	0.000	-0.001	-0.006	-0.017
UNCr	0.478	0.477	0.466	0.462	0.000	0.001	0.005	0.012
CPTL	0.501	0.477	0.465	0.435	0.003	0.012	0.035	0.072
SZE	0.497	0.486	0.432	0.376	0.012	0.048	0.084	0.104
HMLB	0.509	0.485	0.437	0.344	0.036	0.096	0.133	0.119
DRF	0.496	0.461	0.430	0.372	0.027	0.054	0.059	0.026
STREVB	0.486	0.458	0.421	0.335	0.001	0.004	0.008	0.004
MKTBD	0.466	0.449	0.407	0.347	0.017	0.041	0.051	0.050
DEF	0.473	0.445	0.404	0.332	-0.007	-0.023	-0.051	-0.075
DUR	0.467	0.425	0.403	0.356	0.009	-0.018	-0.083	-0.158
VAL	0.465	0.436	0.410	0.337	0.018	0.064	0.111	0.117
MOMB	0.477	0.455	0.401	0.307	-0.005	-0.010	-0.005	0.007
LTREVB	0.485	0.455	0.402	0.299	0.025	0.061	0.073	0.055
TERM	0.466	0.419	0.380	0.305	0.039	0.064	0.070	0.062
CRF	0.469	0.419	0.370	0.291	0.011	0.047	0.079	0.089

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_j|\text{data}]$, of the 16 traded bond and 14 nontraded factors described in Appendix B. The factors are ordered (high to low) by each factors average posterior probability across the four levels of shrinkage. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Test assets include the 50 bond portfolios and the 16 traded bond factors ($N = 66$). Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\mathcal{F}}^2 | \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Sample period: 1986:01 to 2022:12 ($T = 444$).

Table IA.V: Posterior factor probabilities and risk prices for the stock factor zoo

Factors:	Factor prob., $\mathbb{E}[\gamma_j \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \text{data}]$			
	Total prior		Sharpe ratio		Total prior		Sharpe ratio	
	20%	40%	60%	80%	20%	40%	60%	80%
PEAD	0.530	0.601	0.672	0.725	0.034	0.145	0.332	0.570
MKTS	0.503	0.546	0.554	0.568	0.039	0.155	0.298	0.468
CMAAs	0.508	0.520	0.531	0.520	0.021	0.078	0.175	0.289
LVL	0.522	0.521	0.508	0.498	0.000	0.002	0.005	0.014
IVOL	0.497	0.496	0.501	0.549	0.004	0.016	0.043	0.123
EPUT	0.508	0.501	0.516	0.510	0.001	0.005	0.014	0.036
INFLV	0.514	0.514	0.510	0.497	0.000	-0.001	-0.002	-0.004
LIQNT	0.505	0.505	0.516	0.507	0.000	-0.002	-0.009	-0.036
VIX	0.503	0.509	0.505	0.496	-0.001	-0.002	-0.006	-0.016
YSP	0.499	0.493	0.512	0.507	0.001	0.003	0.009	0.026
CREDIT	0.505	0.509	0.498	0.491	0.000	-0.001	-0.001	-0.001
RMWs	0.500	0.503	0.503	0.489	0.032	0.099	0.186	0.274
UNCr	0.493	0.503	0.498	0.495	0.001	0.004	0.010	0.028
UNC	0.494	0.499	0.494	0.496	0.000	0.002	0.007	0.018
INFLC	0.489	0.497	0.500	0.485	0.000	0.001	0.001	0.002
UNCf	0.488	0.484	0.491	0.498	0.001	0.003	0.011	0.038
QMJ	0.484	0.487	0.486	0.477	0.048	0.129	0.239	0.387
EPU	0.473	0.478	0.494	0.484	-0.001	-0.005	-0.012	-0.030
CPTL	0.481	0.491	0.484	0.465	0.017	0.058	0.107	0.155
CPTLT	0.496	0.481	0.469	0.436	0.018	0.061	0.106	0.136
MKTSs	0.489	0.508	0.465	0.404	0.016	0.049	0.071	0.081
BAB	0.499	0.498	0.464	0.397	0.027	0.077	0.121	0.156
LIQ	0.492	0.479	0.458	0.404	0.006	0.022	0.049	0.072
STREV	0.492	0.465	0.455	0.413	0.008	0.030	0.072	0.123
MGMT	0.498	0.473	0.441	0.395	0.056	0.127	0.194	0.246
PERF	0.505	0.473	0.442	0.381	0.039	0.092	0.129	0.134
R_ROE	0.495	0.475	0.426	0.347	0.040	0.083	0.098	0.091
R_IA	0.482	0.457	0.428	0.372	0.028	0.065	0.101	0.126
MOMS	0.478	0.463	0.429	0.367	0.014	0.037	0.051	0.056
SMBs	0.475	0.459	0.425	0.368	0.004	0.020	0.039	0.056
LTREV	0.494	0.464	0.416	0.344	0.007	0.022	0.032	0.033
CMA	0.485	0.451	0.418	0.361	0.027	0.055	0.072	0.069
HMLs	0.480	0.448	0.420	0.349	0.005	0.014	0.017	0.011
HML_DEV	0.475	0.435	0.407	0.368	0.005	0.019	0.056	0.123
SMB	0.472	0.447	0.407	0.346	0.011	0.050	0.088	0.109
HML	0.468	0.441	0.405	0.354	0.002	-0.031	-0.082	-0.134
FIN	0.481	0.443	0.399	0.324	0.032	0.035	0.014	-0.004
RMW	0.470	0.434	0.389	0.330	0.033	0.042	0.018	-0.021

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_j|\text{data}]$, of the 24 traded stock and 14 nontraded factors described in Appendix B. The factors are ordered (high to low) by each factors average posterior probability across the four levels of shrinkage. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Test assets include the 33 stock portfolios and the 24 traded stock factors ($N = 57$). Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\mathbf{f}}^2 | \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Sample period: 1986:01 to 2022:12 ($T = 444$).

Table IA.VI: Posterior factor probabilities and risk prices for the co-pricing factor zoo – excluding the intercept

Factors:	Factor prob., $\mathbb{E}[\gamma_j \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.546	0.646	0.708	0.698	0.060	0.238	0.471	0.653
IVOL	0.516	0.579	0.628	0.729	0.021	0.091	0.222	0.510
PEAD	0.540	0.589	0.619	0.617	0.045	0.171	0.336	0.479
CREDIT	0.492	0.523	0.544	0.562	0.010	0.042	0.103	0.214
MOMBS	0.497	0.538	0.558	0.484	0.071	0.242	0.424	0.470
QMJ	0.508	0.533	0.522	0.485	0.084	0.235	0.390	0.517
YSP	0.498	0.497	0.501	0.532	0.004	0.017	0.043	0.110
CMA _s	0.506	0.507	0.527	0.485	0.017	0.065	0.146	0.222
MKTS	0.487	0.505	0.516	0.470	0.061	0.189	0.328	0.437
INFLC	0.488	0.490	0.498	0.490	-0.001	-0.004	-0.011	-0.028
LVL	0.479	0.492	0.496	0.498	0.001	0.005	0.014	0.038
UNC _r	0.490	0.485	0.490	0.491	0.001	0.004	0.012	0.032
LIQNT	0.481	0.482	0.491	0.485	-0.004	-0.015	-0.045	-0.105
INFLV	0.490	0.489	0.491	0.466	0.002	0.006	0.011	0.008
EPUT	0.498	0.481	0.481	0.454	0.002	0.007	0.014	0.017
UNC _f	0.508	0.489	0.468	0.438	-0.010	-0.021	-0.013	0.020
VIX	0.483	0.479	0.482	0.456	-0.003	-0.008	-0.017	-0.035
CRY	0.469	0.474	0.487	0.460	0.062	0.193	0.379	0.531
LIQ	0.501	0.494	0.465	0.408	0.008	0.032	0.068	0.102
EPU	0.478	0.473	0.462	0.444	0.001	0.002	0.004	0.005
CPTLT	0.478	0.482	0.472	0.422	0.030	0.087	0.150	0.211
UNC	0.486	0.477	0.459	0.429	-0.003	-0.008	-0.009	-0.011
SZE	0.484	0.475	0.466	0.419	0.007	0.028	0.066	0.099
MOMS	0.497	0.472	0.458	0.408	0.025	0.070	0.125	0.182
CPTL	0.481	0.470	0.463	0.409	0.020	0.054	0.076	0.064
RMW _s	0.482	0.468	0.457	0.414	0.027	0.078	0.143	0.201
MKTS _s	0.507	0.472	0.440	0.385	0.022	0.052	0.083	0.129
SMB _s	0.506	0.480	0.444	0.367	0.006	0.023	0.035	0.038
LTREVB	0.484	0.486	0.441	0.372	0.017	0.057	0.086	0.095
LTREV	0.509	0.471	0.438	0.357	0.011	0.036	0.055	0.052
STREV	0.495	0.461	0.428	0.366	0.012	0.044	0.084	0.118
HML_DEV	0.487	0.451	0.420	0.379	0.005	0.010	0.040	0.080
R_ROE	0.489	0.464	0.432	0.343	0.054	0.115	0.157	0.168
VAL	0.466	0.450	0.440	0.369	0.018	0.063	0.118	0.140
HML _s	0.481	0.457	0.420	0.359	0.007	0.019	0.031	0.037
R_IA	0.479	0.456	0.417	0.358	0.037	0.087	0.127	0.149
CRF	0.478	0.456	0.423	0.354	0.018	0.063	0.106	0.139
BAB	0.484	0.455	0.417	0.350	0.029	0.062	0.081	0.093
CMA	0.492	0.455	0.412	0.345	0.034	0.070	0.080	0.062
SMB	0.499	0.459	0.407	0.335	0.016	0.061	0.097	0.101
STREVB	0.489	0.456	0.411	0.342	0.005	0.014	0.023	0.024
PERF	0.473	0.476	0.420	0.329	0.051	0.117	0.126	0.095
HMLB	0.499	0.468	0.416	0.315	0.055	0.128	0.163	0.124
MGMT	0.474	0.461	0.405	0.339	0.062	0.129	0.163	0.171
DEF	0.479	0.444	0.397	0.340	0.003	0.000	-0.009	-0.015
MOMB	0.462	0.443	0.406	0.341	0.001	0.004	0.009	0.016
FIN	0.488	0.436	0.390	0.317	0.041	0.040	0.012	-0.013
MKTBD	0.474	0.423	0.403	0.325	0.018	0.032	0.031	0.007
TERM	0.474	0.427	0.393	0.325	0.039	0.082	0.126	0.156
MKTB	0.475	0.445	0.393	0.302	0.115	0.207	0.238	0.205
HML	0.471	0.430	0.390	0.311	0.005	-0.013	-0.036	-0.038
RMW	0.461	0.427	0.391	0.313	0.033	0.023	-0.024	-0.073
DRF	0.471	0.427	0.380	0.312	0.053	0.077	0.068	0.029
DUR	0.455	0.413	0.364	0.293	0.014	-0.025	-0.079	-0.095

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and equity factors described in Appendix B. All models are estimated without an intercept. The factors are ordered (high to low) by each factors average posterior probability across the four levels of shrinkage. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Test assets include the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$). Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_f^2] | \mathcal{A}}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Sample period: 1986:01 to 2022:12 ($T = 444$).

Table IA.VII: Posterior factor probabilities and risk prices for the corporate bond factor zoo – excluding the intercept

Factors:	Factor prob., $\mathbb{E}[\gamma_j \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.613	0.766	0.797	0.754	0.125	0.431	0.664	0.759
MOMBS	0.587	0.701	0.741	0.600	0.174	0.549	0.827	0.735
CREDIT	0.526	0.575	0.664	0.703	0.029	0.111	0.264	0.506
IVOL	0.518	0.580	0.599	0.656	0.024	0.095	0.208	0.456
YSP	0.502	0.516	0.523	0.554	0.006	0.023	0.055	0.139
INFLC	0.512	0.515	0.507	0.505	-0.002	-0.008	-0.018	-0.046
VIX	0.504	0.497	0.509	0.488	-0.007	-0.030	-0.073	-0.135
INFLV	0.485	0.506	0.494	0.494	0.005	0.017	0.026	0.019
LVL	0.491	0.490	0.482	0.503	0.001	0.003	0.012	0.041
UNCf	0.510	0.503	0.500	0.446	-0.033	-0.098	-0.168	-0.224
EPUT	0.497	0.497	0.480	0.480	0.003	0.011	0.022	0.055
UNCr	0.498	0.489	0.479	0.478	0.000	0.001	0.003	0.011
CRY	0.494	0.496	0.519	0.427	0.089	0.274	0.497	0.492
EPU	0.490	0.487	0.476	0.464	0.002	0.002	-0.002	-0.006
LIQNT	0.473	0.484	0.468	0.460	-0.003	-0.013	-0.027	-0.053
UNC	0.495	0.477	0.464	0.435	-0.010	-0.025	-0.036	-0.054
SZE	0.491	0.469	0.437	0.351	0.016	0.054	0.085	0.079
CPTL	0.476	0.461	0.424	0.348	-0.007	-0.016	-0.017	-0.010
LTREVB	0.507	0.462	0.411	0.310	0.033	0.074	0.084	0.062
VAL	0.461	0.461	0.431	0.335	0.032	0.109	0.172	0.157
HMLB	0.486	0.460	0.388	0.277	0.083	0.171	0.185	0.124
MOMB	0.494	0.437	0.375	0.303	0.006	0.023	0.040	0.057
MKTB	0.509	0.446	0.384	0.269	0.200	0.310	0.328	0.234
STREVB	0.488	0.441	0.384	0.288	0.006	0.022	0.036	0.029
CRF	0.461	0.438	0.387	0.313	0.026	0.085	0.129	0.147
TERM	0.476	0.441	0.386	0.287	0.071	0.129	0.176	0.169
DEF	0.477	0.435	0.384	0.286	-0.004	-0.016	-0.035	-0.043
MKTBD	0.469	0.432	0.385	0.291	0.020	0.049	0.059	0.041
DRF	0.446	0.392	0.338	0.247	0.051	0.049	0.020	-0.006
DUR	0.437	0.387	0.343	0.235	-0.007	-0.092	-0.149	-0.079

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_j|\text{data}]$, of the 16 traded bond and 14 nontraded factors described in Appendix B. All models are estimated without an intercept. The factors are ordered (high to low) by each factors average posterior probability across the four levels of shrinkage. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Test assets include the 50 bond portfolios and the 16 traded bond factors ($N = 66$). Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\mathbf{f}}^2 | \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Sample period: 1986:01 to 2022:12 ($T = 444$).

Table IA.VIII: Posterior factor probabilities and risk prices for the stock factor zoo – excluding the intercept

Factors:	Factor prob., $\mathbb{E}[\gamma_j \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \text{data}]$			
	Total prior		Sharpe ratio		Total prior		Sharpe ratio	
	20%	40%	60%	80%	20%	40%	60%	80%
PEAD	0.527	0.607	0.675	0.725	0.037	0.159	0.359	0.607
MKTS	0.501	0.550	0.585	0.562	0.051	0.191	0.365	0.517
IVOL	0.498	0.524	0.538	0.594	0.005	0.024	0.064	0.181
QMJ	0.505	0.517	0.524	0.517	0.064	0.173	0.328	0.504
YSP	0.515	0.513	0.513	0.500	0.001	0.003	0.010	0.027
CMA _s	0.486	0.495	0.509	0.523	0.022	0.085	0.186	0.318
UNC	0.510	0.508	0.496	0.485	0.000	0.001	0.003	0.009
INFLC	0.501	0.500	0.498	0.498	0.000	0.001	0.002	0.003
CPTL	0.498	0.506	0.508	0.486	0.020	0.070	0.127	0.189
CREDIT	0.496	0.504	0.501	0.495	0.000	-0.001	-0.002	-0.004
LVL	0.491	0.490	0.495	0.513	0.001	0.004	0.010	0.028
LIQNT	0.494	0.505	0.499	0.490	0.000	-0.002	-0.012	-0.044
EPUT	0.499	0.483	0.507	0.498	0.002	0.006	0.019	0.049
INFLV	0.500	0.496	0.503	0.486	0.000	-0.001	-0.002	-0.003
UNC _r	0.495	0.497	0.487	0.495	0.001	0.003	0.010	0.028
RMW _s	0.486	0.490	0.497	0.468	0.038	0.115	0.208	0.284
EPU	0.485	0.490	0.487	0.478	-0.001	-0.005	-0.011	-0.023
VIX	0.489	0.479	0.481	0.483	-0.001	-0.003	-0.007	-0.019
UNC _f	0.483	0.489	0.476	0.473	0.000	0.000	0.004	0.022
CPTLT	0.484	0.472	0.489	0.446	0.023	0.073	0.128	0.160
STREV	0.497	0.480	0.465	0.429	0.010	0.041	0.087	0.148
MKTS _s	0.500	0.494	0.461	0.403	0.019	0.054	0.077	0.090
LIQ	0.478	0.487	0.452	0.410	0.007	0.029	0.060	0.090
BAB	0.507	0.484	0.438	0.372	0.040	0.097	0.138	0.161
MOMS	0.499	0.475	0.444	0.380	0.018	0.046	0.068	0.083
SMB _s	0.494	0.468	0.447	0.351	0.008	0.029	0.057	0.072
HML_DEV	0.467	0.453	0.431	0.386	0.009	0.031	0.086	0.175
SMB	0.494	0.458	0.428	0.347	0.018	0.071	0.119	0.132
MGMT	0.486	0.455	0.421	0.355	0.072	0.147	0.213	0.240
PERF	0.477	0.467	0.418	0.354	0.044	0.106	0.141	0.145
CMA	0.487	0.468	0.411	0.346	0.035	0.070	0.081	0.074
HML _s	0.479	0.455	0.417	0.347	0.007	0.019	0.029	0.015
R_IA	0.473	0.456	0.415	0.353	0.039	0.083	0.118	0.140
R_ROE	0.492	0.462	0.410	0.332	0.051	0.096	0.109	0.102
LTREV	0.482	0.457	0.411	0.346	0.011	0.029	0.039	0.039
HML	0.481	0.430	0.406	0.349	0.003	-0.035	-0.095	-0.146
FIN	0.477	0.416	0.382	0.305	0.040	0.035	0.009	-0.013
RMW	0.470	0.419	0.379	0.306	0.044	0.046	0.014	-0.026

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_j|\text{data}]$, of the 24 traded stock and 14 nontraded factors described in Appendix B. All models are estimated without an intercept. The factors are ordered (high to low) by each factors average posterior probability across the four levels of shrinkage. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Test assets include the 33 stock portfolios and the 24 traded stock factors ($N = 57$). Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\mathbf{f}}^2 | \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Sample period: 1986:01 to 2022:12 ($T = 444$).

Table IA.IX: In-sample cross-sectional asset pricing performance – BMA-SDFs estimated without the intercept.

	BMA Prior Sharpe Ratio				CAPM	CAPMB	FF5	HKM	Top	KNS	RP ^{PCA}
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.209	0.201	0.184	0.165	0.260	0.278	0.258	0.259	0.233	0.166	0.201
MAPE	0.158	0.149	0.135	0.120	0.194	0.221	0.198	0.192	0.176	0.126	0.137
R^2_{OLS}	0.193	0.254	0.376	0.499	-0.244	-0.426	-0.233	-0.238	0.001	0.489	0.251
R^2_{GLS}	0.051	0.129	0.205	0.266	0.078	0.083	0.087	0.078	0.244	0.176	0.186
Panel B: Pricing bonds											
RMSE	0.171	0.130	0.104	0.090	0.209	0.213	0.202	0.206	0.162	0.195	0.189
MAPE	0.116	0.093	0.078	0.068	0.146	0.135	0.142	0.145	0.128	0.113	0.084
R^2_{OLS}	0.278	0.578	0.733	0.798	-0.107	-0.157	-0.038	-0.080	0.347	0.028	0.094
R^2_{GLS}	0.096	0.239	0.338	0.393	0.180	0.201	0.244	0.181	0.549	0.064	0.231
Panel C: Pricing stocks											
RMSE	0.240	0.258	0.250	0.231	0.292	0.264	0.275	0.292	0.194	0.162	0.230
MAPE	0.192	0.202	0.190	0.172	0.229	0.211	0.221	0.226	0.154	0.133	0.173
R^2_{OLS}	-0.063	-0.229	-0.155	0.018	-0.570	-0.282	-0.392	-0.574	0.306	0.515	0.022
R^2_{GLS}	0.059	0.146	0.234	0.317	0.120	0.118	0.130	0.121	0.424	0.311	0.315

In-sample asset pricing performance of the Co-pricing BMA-SDF (Panel A), the Bond BMA-SDF (Panel B) and the Stock BMA-SDF (Panel C), notable factor models, and the top five factors with an average posterior probability greater than 50%. The BMA-SDFs are estimated without an intercept. In each panel, the model is estimated with bonds and stocks (A), bonds only (B) and stocks only (C). Bond returns are computed in excess of the one-month risk-free rate of return. We use GMM-GLS to estimate factor risk prices for CAPM, CAPMB, the and [Fama and French \(1992, 1993\)](#) model, which includes the MKTS, SMB, HML, DEF and TERM factors, and the traded two-factor [He et al. \(2017\)](#), HKM. KNS stands for the SDF estimation of [Kozak et al. \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. RP^{PCA} is the Risk-Premia-PCA of [Lettau and Pelger \(2020\)](#), with five-factors and κ set to 20. In Panel A the models are estimated with the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$), Panel B (bond only) uses the 50 bond portfolios and 16 bond factors ($N = 66$), and Panel C (stock only) uses the 33 stock portfolios and 24 stock factors ($N = 57$). For the BMA-SDFs, we report results for a range of prior Sharpe ratio values that are set as 20%, 40%, 60% and 80% of the ex-post maximum Sharpe ratio of the relevant portfolios and factors. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).

Table IA.X: Out-of-sample cross-sectional asset pricing performance – BMA-SDFs estimated without the intercept.

	BMA Prior Sharpe Ratio				CAPM	CAPMB	FF5	HKM	Top	KNS	RP ^{PCA}
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.111	0.101	0.093	0.088	0.224	0.154	0.139	0.223	0.158	0.160	0.166
MAPE	0.080	0.075	0.069	0.064	0.192	0.129	0.102	0.190	0.130	0.143	0.146
R_{OLS}^2	0.390	0.500	0.572	0.617	-1.478	-0.161	0.053	-1.444	-0.226	-0.268	-0.360
R_{GLS}^2	0.031	0.070	0.103	0.134	0.028	0.034	0.036	0.028	0.100	0.065	0.041
Panel B: Pricing bonds											
RMSE	0.120	0.110	0.105	0.101	0.130	0.128	0.139	0.133	0.102	0.112	0.085
MAPE	0.087	0.080	0.077	0.076	0.095	0.092	0.104	0.097	0.084	0.081	0.062
R_{OLS}^2	0.090	0.238	0.302	0.363	-0.062	-0.028	-0.221	-0.107	0.342	0.205	0.540
R_{GLS}^2	0.056	0.107	0.134	0.158	-0.006	0.022	-0.032	-0.007	0.101	0.068	0.069
Panel C: Pricing stocks											
RMSE	0.102	0.087	0.078	0.072	0.123	0.119	0.116	0.124	0.064	0.078	0.124
MAPE	0.077	0.068	0.063	0.059	0.089	0.085	0.082	0.091	0.051	0.060	0.096
R_{OLS}^2	0.330	0.511	0.608	0.666	0.032	0.099	0.136	0.019	0.734	0.613	0.014
R_{GLS}^2	0.053	0.132	0.205	0.262	0.103	0.065	0.099	0.107	0.246	0.207	-0.011

Out-of-sample asset pricing performance of the Co-pricing BMA-SDF (Panel A), the Bond BMA-SDF (Panel B) and the Stock BMA-SDF (Panel C), notable factor models, and the top five factors with an average posterior probability greater than 50%. The BMA-SDFs are estimated without an intercept. Bond returns are computed in excess of the one-month risk-free rate of return. We use GMM-GLS to estimate factor risk prices for CAPM, CAPMB, the and [Fama and French \(1992, 1993\)](#) model, which includes the MKTS, SMB, HML, DEF and TERM factors, and the traded two-factor [He et al. \(2017\)](#), HKM. KNS stands for the SDF estimation of [Kozak et al. \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. RP^{PCA} is the Risk-Premia-PCA of [Lettau and Pelger \(2020\)](#), with five-factors and κ set to 20. In Panel A the models are estimated with the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$), Panel B (bond only) uses the 50 bond portfolios and 16 bond factors ($N = 66$), and Panel C (stock only) uses the 33 stock portfolios and 24 stock factors ($N = 57$). Out-of-sample (OS) test assets include 154 bond and stock portfolios (Panel A), and 77 bond (stock) portfolios in Panel B (C). All defined in Section 1. The models are first estimated using the baseline IS test assets and then used to price (with no additional parameter estimation) each set of the OS assets. For the BMA-SDFs, we report results for a range of prior Sharpe ratio values that are set as 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio relevant portfolios and factors. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).

Table IA.XI: Correlation of SDF levels and volatilities

	KNS	RPPCA	CAPM	CAPMB	FF5	HKM
Panel A: SDF levels						
BMA	0.78	0.55	0.16	0.28	0.29	0.16
KNS		0.85	0.11	0.46	0.32	0.13
RPPCA			0.09	0.35	0.18	0.11
CAPM				0.42	0.70	0.98
CAPMB					0.70	0.41
FF5						0.66
Panel B: SDF estimated volatilities						
BMA	0.76	0.70	0.74	0.52	0.56	0.74
KNS		0.71	0.64	0.55	0.55	0.65
RPPCA			0.54	0.18	0.24	0.56
CAPM				0.57	0.61	0.98
CAPMB					0.75	0.57
FF5						0.58

Correlation of the SDFs (Panel A) and their filtered volatilities (Panel B) estimated in Panel A of Table 4 with the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$). BMA-SDF obtained with 80% prior Sharpe ratio. Bond returns are computed in excess of the one-month risk-free rate of return. The ARMA mean process for each model is selected using the BIC. Volatilities are estimated using a GARCH(1,1). The [Ljung and Box \(1978\)](#) p -value tests the null of squared autocorrelations being equal to zero. We use GMM-GLS to estimate factor risk prices for CAPM, CAPMB, the and [Fama and French \(1992, 1993\)](#) model, which includes the MKTS, SMB, HML, DEF and TERM factors, and the traded two-factor [He et al. \(2017\)](#), HKM. KNS stands for the SDF estimation of [Kozak et al. \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. RP^{PCA} is the Risk-Premia-PCA of [Lettau and Pelger \(2020\)](#), with five-factors and κ set to 20. Sample period 1986:01–2022:12 ($T = 444$).

IA.5 Additional figures

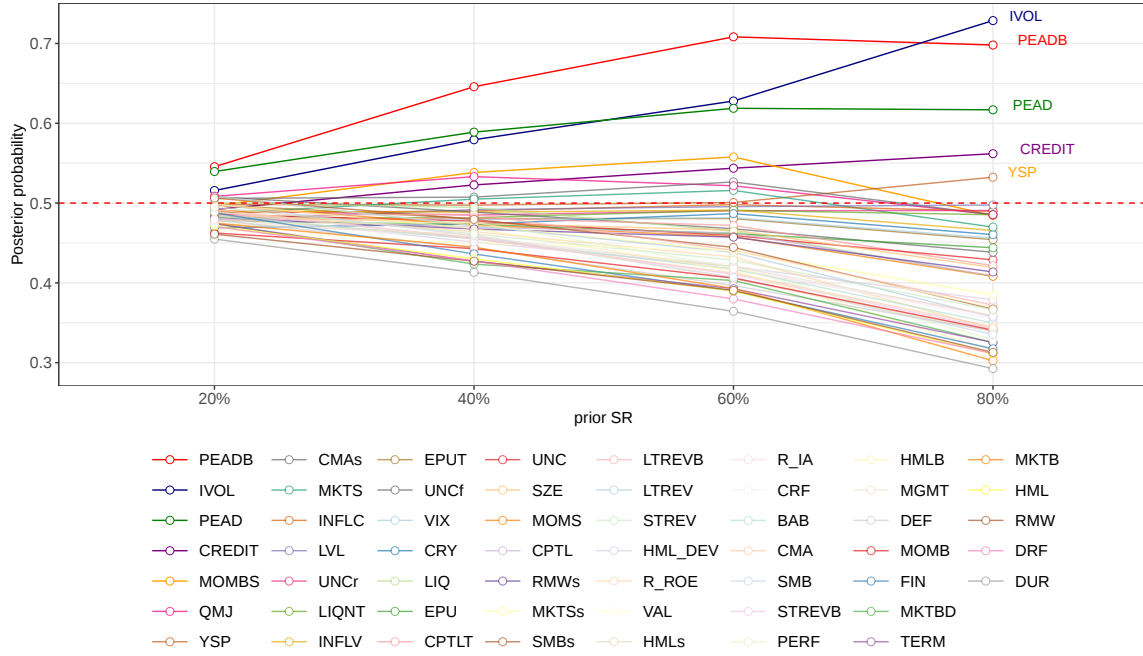


Figure IA.2: Posterior factor probabilities – co-pricing factor zoo – excluding the intercept.

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, of the 54 stock and bond factors described in Appendix B. All models are estimated without an intercept. The legend factor labels are ordered (high to low) by each factors average posterior probability across the four levels of shrinkage. Test assets include the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$). The prior distribution for the j^{th} factor inclusion is a Beta(1, 1), yielding a 0.5 prior expectation for γ_j . Posterior probabilities for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_{\pi}[SR_{\mathcal{F}}^2 | \sigma^2]}$, set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the 83 stock and bond portfolios and traded factors. Sample period: 1986:01 to 2022:12 ($T = 444$).

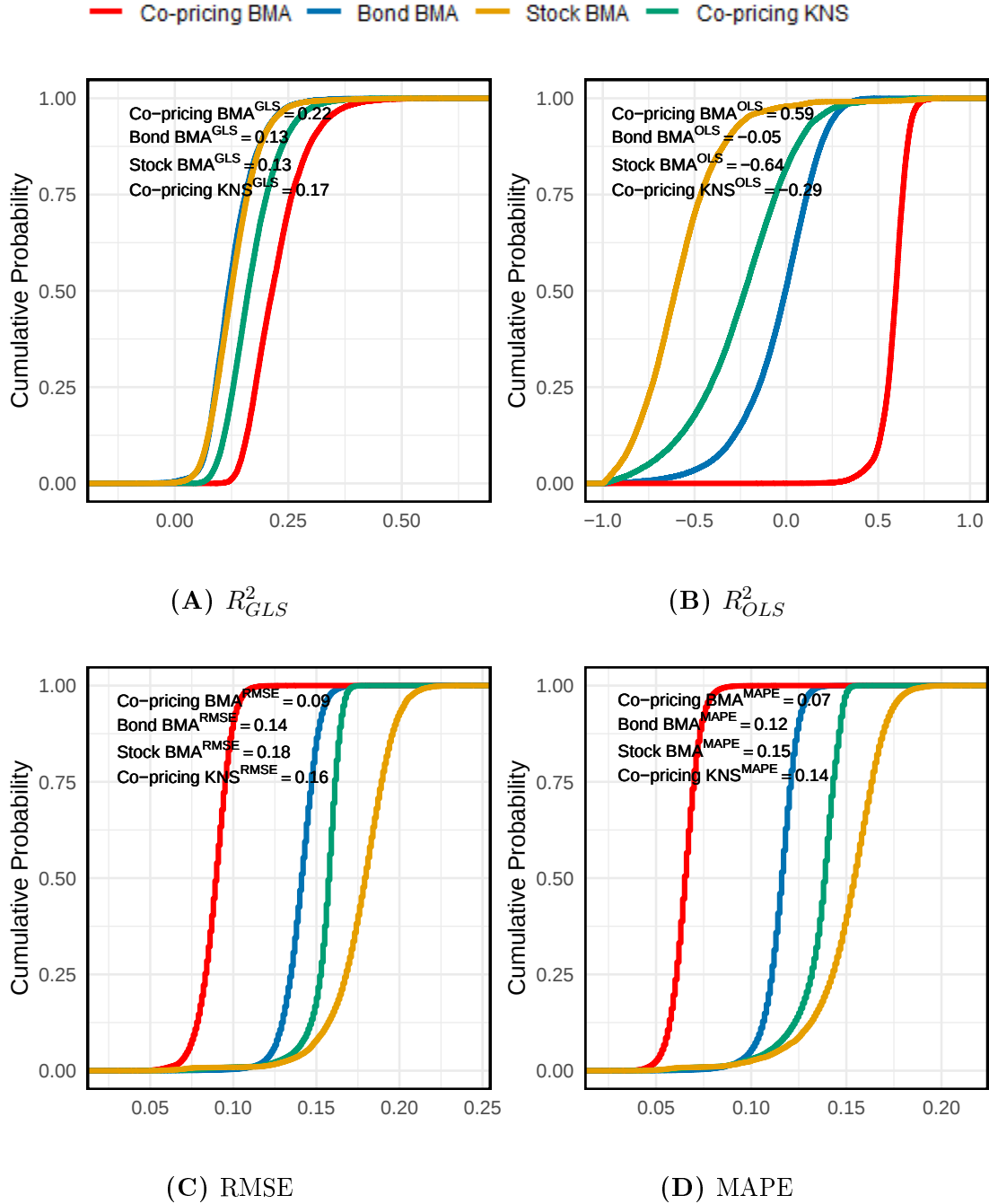
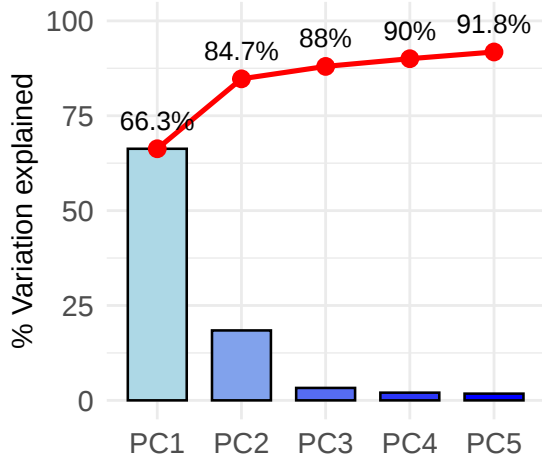
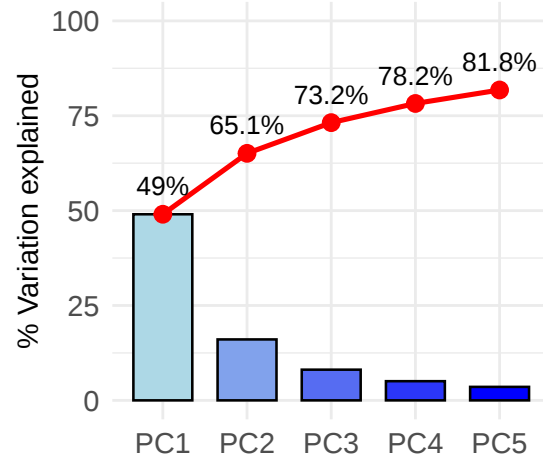


Figure IA.3: Pricing out-of-sample stocks and bonds with different BMA-SDFs and KNS-SDF.

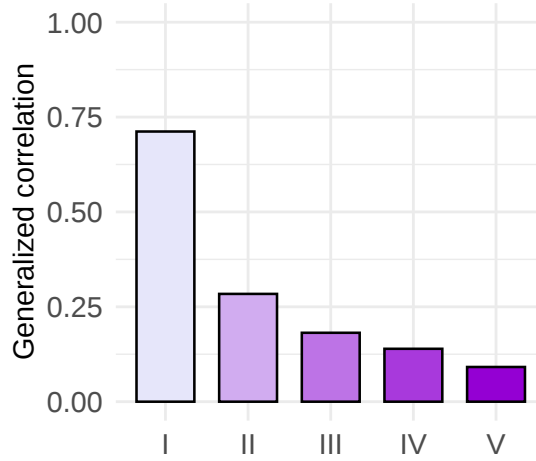
This figure plots the empirical CDFs of R_{GLS}^2 , R_{OLS}^2 , RMSE and MAPE in Panels A, B, C and D respectively across 16,384 possible bond and stock cross-sections using the 14 sets of stock and bond test assets ($2^{14} = 16,384$) priced using the respective BMA-SDF and the co-pricing KNS-SDF respectively. The models are first estimated using the baseline set of IS test assets and then used to price (with no additional parameter estimation) each set of the 16,384 OS combinations of test assets. The red distributions corresponds to the pricing performance of the co-pricing BMA-SDF. The blue (yellow) distributions corresponds to the pricing performance of the bond (stock) only BMA-SDF. The green distributions correspond to the pricing performance of the co-pricing KNS-SDF. The BMA-SDF (BMA) is computed with a prior Sharpe ratio value set to 80% of the ex-post maximum Sharpe ratio of the IS test assets. All data is standardized, that is, pricing errors are in Sharpe ratio units and span the sample period 1986:01–2022:12 ($T = 444$).



(A) Corporate bonds



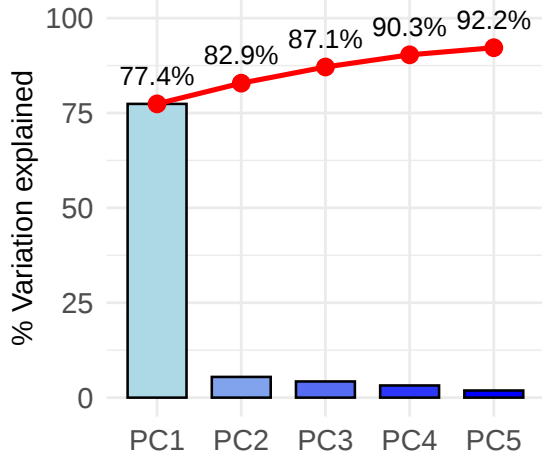
(B) Stocks



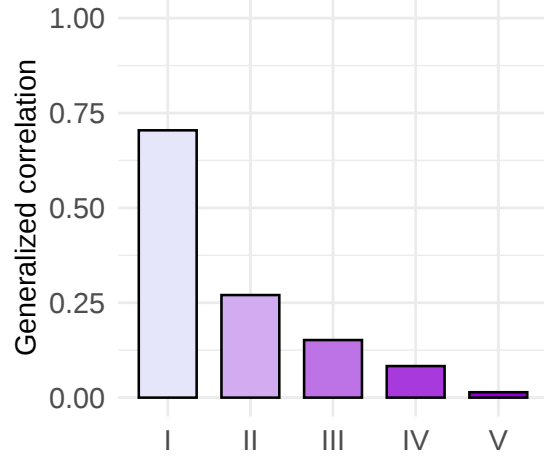
(C) Corporate bond-stock correlation

Figure IA.4: How (dis)integrated are equity and corporate bond markets?

This figure plots the amount of variation explained by the principal components of the in-sample bond and stock portfolios and their generalized (canonical) correlations. Panels (a) and (b) reports the percentage variation explained by the first five PCs of the corporate bond and stock test assets. Panel (c) reports the generalized correlations. Define $\hat{v}_t^B$ and $\hat{v}_t^S$ as the top five PCs of the corporate bond and stock in-sample test assets. The generalized correlations between $\hat{v}_t^B$ and $\hat{v}_t^S$ are defined as the squared root of the eigenvalues of $\text{cov}(\hat{v}_t^B, \hat{v}_t^S)^\top \text{cov}(\hat{v}_t^B)^{-1} \text{cov}(\hat{v}_t^B, \hat{v}_t^S) \text{cov}(\hat{v}_t^S)^{-1}$. The stock test assets comprise 33 portfolios and the 24 traded factors ($N = 57$), the bond test assets comprise the 50 portfolios and 16 traded factors ($N = 66$). The sample spans the period 1986:01–2022:12 ($T = 444$).



(A) Bonds (duration-adjusted)



(B) Bond (duration-adj.) - stock correlation

Figure IA.5: How (dis)integrated are equity and corporate bond markets? Duration-adjusted bond returns.

This figure plots the amount of variation explained by the principal components of the in-sample bond (duration-adjusted) portfolio returns and their generalized (canonical) correlations with the stock portfolio returns. Bond returns are duration-adjusted as described in Section C of the Appendix. Panel (a) reports the percentage variation explained by the first five PCs of the corporate bond test assets. Panel (b) reports the generalized correlations. Define $\hat{v}_t^B$ and $\hat{v}_t^S$ as the top five PCs of the corporate bond and stock in-sample test assets. The generalized correlations between $\hat{v}_t^B$ and $\hat{v}_t^S$ are defined as the squared root of the eigenvalues of $\text{cov}(\hat{v}_t^B, \hat{v}_t^S)^\top \text{cov}(\hat{v}_t^B)^{-1} \text{cov}(\hat{v}_t^B, \hat{v}_t^S) \text{cov}(\hat{v}_t^S)^{-1}$. The stock test assets comprise 33 portfolios and the 24 traded factors ($N = 57$), the bond test assets comprise the 50 portfolios and 16 traded factors ($N = 66$). The sample spans the period 1986:01–2022:12 ($T = 444$).

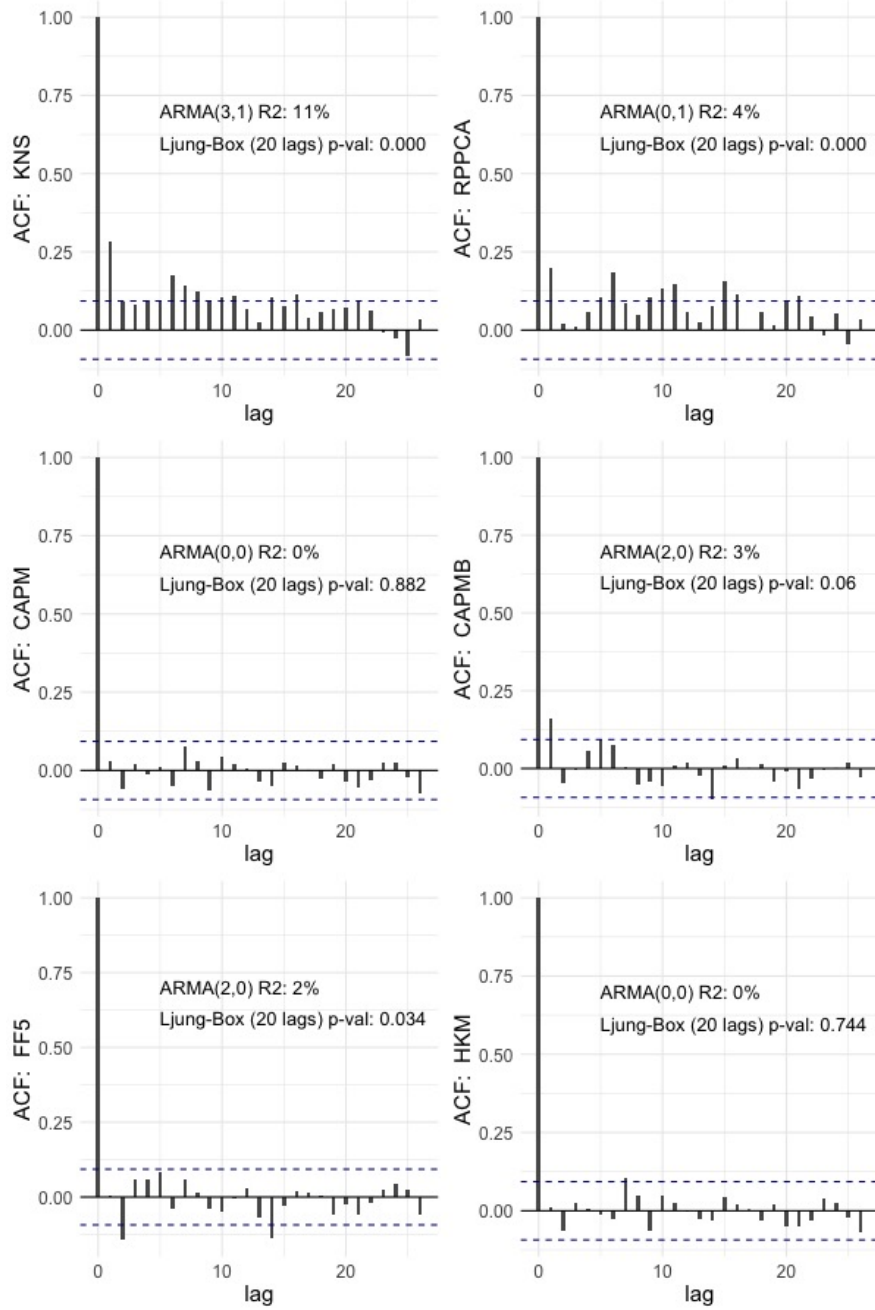


Figure IA.6: Autocorrelations of SDF levels

Autocorrelation of the notable SDF estimated in Panel A of Table 4 with the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$). Bond returns are computed in excess of the one-month risk-free rate of return. The ARMA mean process for each model is selected using the BIC. The [Ljung and Box \(1978\)](#) p -value tests the null of squared autocorrelations being equal to zero. We use GMM-GLS to estimate factor risk prices for CAPM, CAPMB, the and [Fama and French \(1992, 1993\)](#) model, which includes the MKTS, SMB, HML, DEF and TERM factors, and the traded two-factor [He et al. \(2017\)](#), HKM. KNS stands for the SDF estimation of [Kozak et al. \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. RP^{PCA} is the Risk-Premia-PCA of [Lettau and Pelger \(2020\)](#), with five-factors and κ set to 20. Sample period 1986:01–2022:12 ($T = 444$).

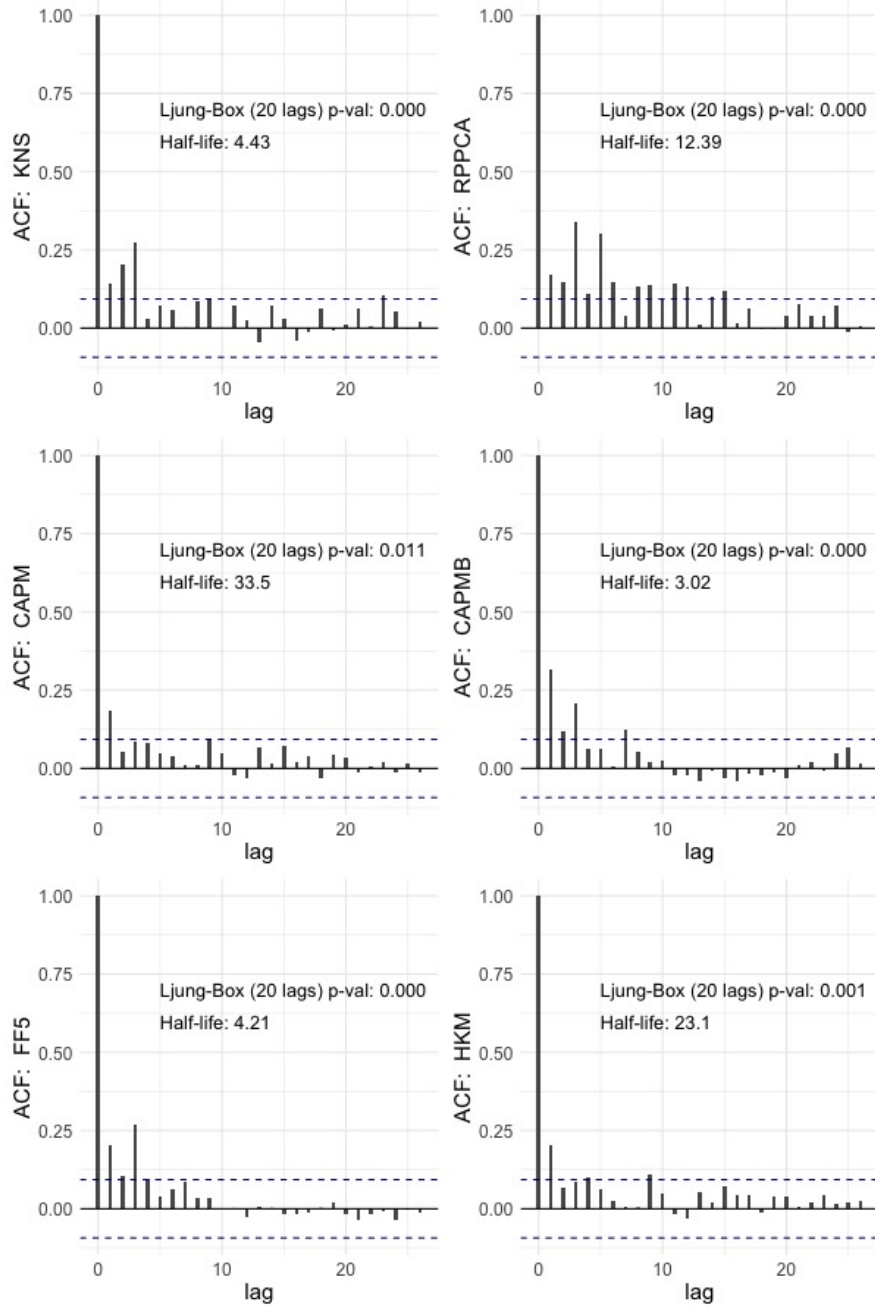


Figure IA.7: Autocorrelations of SDF squared residuals

Autocorrelation of the squared residuals of notable SDF estimated in Panel A of Table 4 with the 83 bond and stock portfolios and the 40 traded bond and stock factors ($N = 123$). Bond returns are computed in excess of the one-month risk-free rate of return. The ARMA mean process for each model is selected using the BIC and reported in Table IA.6. The [Ljung and Box \(1978\)](#) p -value tests the null of squared autocorrelations being equal to zero. We use GMM-GLS to estimate factor risk prices for CAPM, CAPMB, the and [Fama and French \(1992, 1993\)](#) model, which includes the MKTS, SMB, HML, DEF and TERM factors, and the traded two-factor [He et al. \(2017\)](#), HKM. KNS stands for the SDF estimation of [Kozak et al. \(2020\)](#), with tuning parameter and number of factors chosen by twofold cross-validation. RP^{PCA} is the Risk-Premia-PCA of [Lettau and Pelger \(2020\)](#), with five-factors and κ set to 20. Sample period 1986:01–2022:12 ($T = 444$).

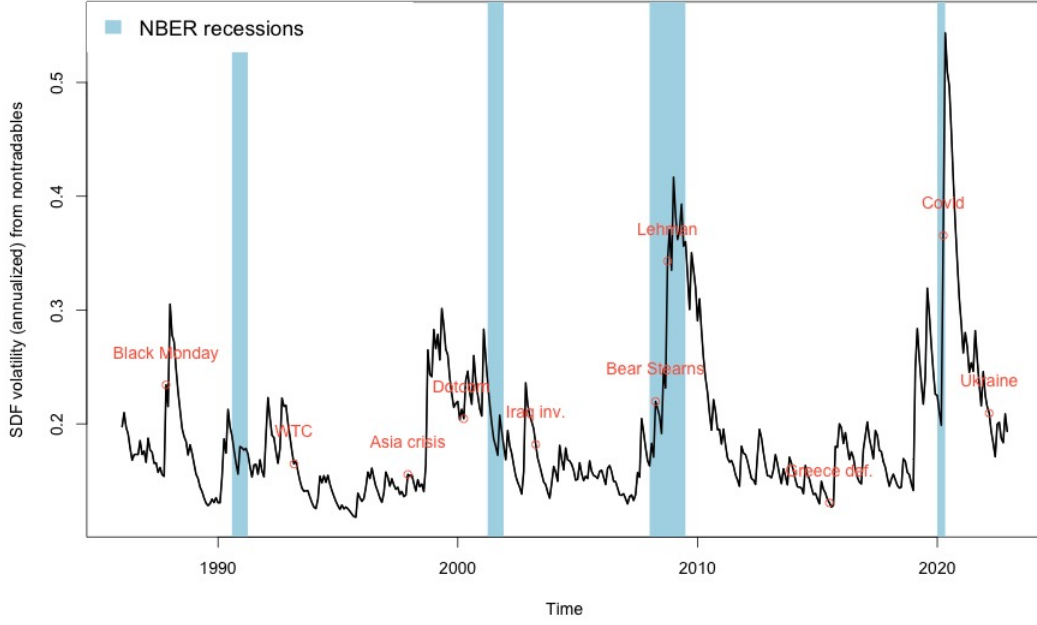


Figure IA.8: Volatility of the co-pricing BMA-SDF with only nontradable factors

Annualized volatility of the co-pricing BMA-SDF with only non-tradable factors. Shaded areas denote NBER recession periods. Volatility is obtained fitting a ARMA(3,1)-GARCH(1,1) to the posterior mean of the co-pricing BMA-SDF (specification selected via BIC). The GARCH Quasi-maximum likelihood coefficient estimates are:

$$\sigma_{t+1}^2 = \omega + \alpha \epsilon_t^2 + \beta \sigma_t^2$$

	ω	α	β
Estimate	0.000202	0.142293	0.798533
Robust SE	0.000090	0.052041	0.047567

Sample: 1986:01–2022:12 ($T = 444$).

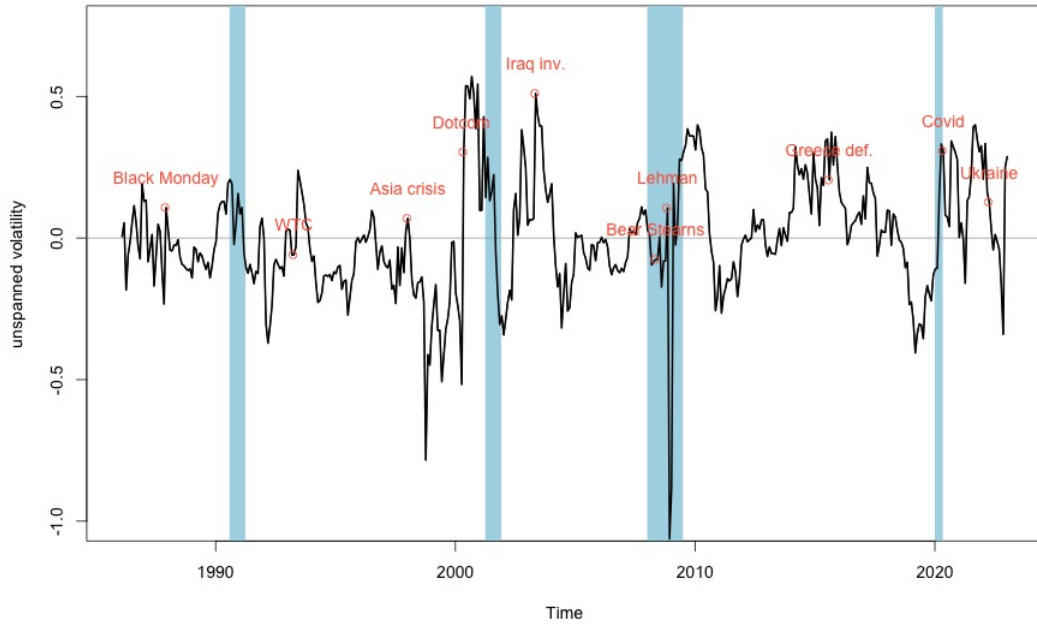


Figure IA.9: Residual volatility of the co-pricing BMA-SDF

Residuals of the linear projection of the BMA-SDF estimated volatility on the volatilities of CAPM, CAPMB, KNS, RPPCA, FF5 and HKM SDFs. Sample: 1986:01–2022:12 ($T = 444$).