Equity Valuation Without DCF*

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Abstract

We introduce *discounted alphas*, a novel framework for equity valuation. Our approach circumvents the need for stock-level cost-of-equity estimates required in discounted cash flow (DCF) valuation and identifies economically important variation in fundamental value not captured by best-in-class DCF methods. We find that discretionary buy-and-hold funds tilt toward characteristics that predict underpricing but not short-term alphas and that private equity funds appear to capture substantial CAPM misvaluation, both initially at buyout and subsequently at exit. However, despite these pockets of misvaluation, we find that firm equity values are "almost efficient" by Black's (1986) definition.

Keywords: equity valuation, fundamental value, price-level mispricing, DCF, market efficiency, discretionary investing, private equity

JEL classification: G12, G14, G32

Every day, investors collectively commit billions of dollars to the stock market based on a deceptively simple question: What is a company truly worth? These perceptions of fundamental value drive the actions of not only discretionary buy-and-hold investors but also sell-side analysts and firm managers, influencing market prices, capital allocation, and firm behavior.[1] Since equity valuation—estimating a stock's buy-and-hold cash-flow value (henceforth "fundamental value")—is the cornerstone of many key financial decisions, effective valuation methods are critical in order to analyze these actions and guide those agents toward more efficient outcomes.

However, measuring fundamental value is challenging. Discounted cash flow (DCF) models are sensitive to stock-specific cost of equity estimates, which prior research has found to be "distressingly imprecise" (Fama and French, 1997). Moreover, estimating future stock-level cash flows is not only demanding but also allows those who use DCF to be able to rationalize a wide range of desired values. Finally, valuation using price multiples, a widely-used alternative to DCF, can also be misleading, as low multiples may reflect distress risk (Fama and French, 1993) or low future cash flows (Cohen, Polk, and Vuolteenaho, 2003) rather than undervaluation. As a consequence, equity valuation remains frustratingly elusive, plaguing financial decision making at all levels, from individual investors to corporate boardrooms.[2]

Our contribution is to introduce *discounted alphas* as a novel approach to equity valuation. This approach values stocks as their current price plus the present value of all future (buy-and-hold) alphas, drawing on a novel asset-pricing identity (Cho and Polk, 2024). Our novel technique eliminates the need for stock-specific cost-of-equity estimates, as required in DCF models, and enables us to leverage existing research measuring alphas. We operationalize the approach for the valuation of individual stocks and use these estimates to generate new empirical insights into stock valuations and market efficiency.

Our approach is based on an identity expressing the fundamental value of equity as the

[1]Berkshire Hathaway and Capital Group are major discretionary investors that buy perceived undervalued stocks. Sell-side analysts prioritize fundamental value over short-term returns. Perceived misvaluations can drive takeovers, M&As, share issuance or buybacks, and real investment (Edmans, Goldstein, and Jiang (2012); Dessaint, Olivier, Otto, and Thesmar (2021); Graham and Harvey (2001); Brav, Graham, Harvey, and Michaely (2005); Polk and Sapienza (2009); Dessaint, Foucault, Frésard, and Matray (2019)).

[2]The difficulty of measuring fundamental value has been recognized as early as Graham (1949).

current price plus a discounted sum of future alphas from a buy-and-hold strategy:

$$V_{i,t} \equiv \sum_{\tau=0}^{\infty} E_t[\widetilde{M}_{t,t+\tau} D_{i,t+\tau}] \tag{1}$$

$$= P_{i,t} + \sum_{\tau=1}^{\infty} E_t[\widetilde{M}_{t,t+\tau} P_{i,t+\tau-1} \alpha_{i,t+\tau-1}]. \tag{2}$$

The first equation defines $V_{i,t}$, the fundamental value of stock i at time t, as the present value of dividends, $\{D_{i,t+\tau}\}$, discounted by a model-specific cumulative discount factor, $\{\widetilde{M}_{t,t+\tau}\}$.[3] The second equation is an identity that says $V_{i,t}$ also equals the sum of the current price, $P_{i,t}$, and the present value of future (buy-and-hold) alphas, $\{\alpha_{i,t+\tau-1}\}$, discounted by the price-weighted cumulative discount factor.[4] Consequently, stock characteristics associated with persistent future alphas predict that fundamental value exceeds the current price.

Cho and Polk (2024) were the first to derive this intuitive identity; our contribution is to provide a novel way to operationalize the conditional valuation of individual stocks via discounted alphas, even in real time.[5] In particular, we first express equation (2) as a two-period identity for the value-to-price ratio, $\frac{V}{P}$:

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \frac{\alpha_{i,t}}{1 + R_{f,t}} + E_t\left[\widetilde{M}_{t+1} \frac{P_{i,t+1}}{P_{i,t}} \left(\frac{V_{i,t+1}}{P_{i,t+1}} - 1\right)\right]. \tag{3}$$

This identity tells us that if a stock is overpriced ($\frac{V}{P} > 1$), it must be expected to either generate abnormal returns next period ($\alpha > 0$) or become even more overpriced.

We then estimate $\frac{V}{P}$ based on a simple idea: Since $\frac{V}{P}$ appears on both sides of equation (3), for t and $t+1$, we can identify the model of $\frac{V}{P}$ that is consistent not only with a model of short-horizon alpha but also with the above law of motion for $\frac{V}{P}$.

Specifically, our model of stock-level $\frac{V}{P}$ is the projection of $\frac{V}{P}$ on a vector of stock charac-

[3] In the spirit of Hansen and Jagannathan (1991, 1997), $\widetilde{M}$ is the candidate pricing model implied by the factor model used to evaluate equity values and is likely not be the true stochastic discount factor.

[4] The second equation is an identity, as it holds under the mild assumption of no explosive bubble. Importantly, it holds even when the fundamental value and the market price are expected to remain dislocated in the future.

[5] The approximate loglinear identity of Campbell and Shiller (1988) contains a related idea, but the potentially large volatility correction required to go from the expected log return to the expected simple return, to which risk adjustment can be applied, poses issues. See Cho and Polk (2024) for more details.

teristics that results in the best fit of equation (3) across all stocks. This estimation takes two steps.

1. Model and estimate stock-level alphas, $\alpha_{i,t}$, as linear in stock characteristics and an error term as done, for example, in Lewellen (2015) and Kelly, Pruitt, and Su (2019). Similarly model and estimate the processes for stock-specific capital gains and the evolution of stock characteristics.

2. Model $\frac{V}{P}$ as linear in stock characteristics and an error term. Estimate the coefficients in that model such that the resulting fitted $\frac{V_{i,t}}{P_{i,t}}$ is most consistent with
   (i) the stock-level estimates of alphas, $\alpha_{i,t}$, and
   (ii) the ($\widetilde{M}$-and-capital-gain) discounted value of the next-period $\frac{V_{i,t+1}}{P_{i,t+1}}$ implied by the evolution of stock characteristics.

Since price is observed, estimates of $\frac{V}{P}$ produce stock-level estimates of both fundamental value per share ($V = \frac{V}{P} \times P$) and total firm equity ($V\times$ shares outstanding).[6]

We emphasize that our novel estimates of fundamental value are always with respect to a specific information set and a specific factor model of priced risks. For the former, we use a set of seven stock characteristics: book-to-market, profitability, beta, size, investment, net issuance, and past return (momentum).[7] For the latter, we consider three alternative models of risk: the CAPM, the three-factor model of Fama and French (1993), and the five-factor model of Fama and French (2015).[8]

We use our approach to estimate out-of-sample fundamental values for approximately two million stock-month observations over 1953m6–2023m12, as illustrated in Figures 1 (for Apple and Tesla) and 2 (for the 10 largest stocks as of December 2023). To validate these estimates, we show that our out-of-sample $\frac{V}{P}$ with respect to a factor model generates large and persistent differences in post-formation alphas (and other measures of misvaluation)

[6] Although we estimate $\frac{V}{P}$ with a linear projection, our framework can easily accommodate a nonlinear approach as well.

[7] Note that our approach can nest other ways to estimate fundamental value; one can simply add the estimates from that alternative to the vector of stock characteristics predicting $\frac{V}{P}$. Indeed, we can feed our approach a much larger information set, if so desired, as long as we also add an appropriate shrinkage method.

[8] We add time fixed effects to our regressions to focus on the cross-sectional component of model-specific misvaluation.

with respect to the same factor model. As further validation, our out-of-sample estimates detect the relative underpricing (overpricing) of stocks at the bottom (top) of the Russell 1000 large-cap index (Russell 2000 small-cap index) (Chang, Hong, and Liskovich (2015)).

Based on these estimates, we document six new empirical findings.

1. Profitable firms with low market beta that trade cheap (i.e., high book-to-market equity) tend to be the most undervalued with respect to the CAPM, consistent with the present-value identity of Vuolteenaho (2002) and the *adjusted value* metric of Cho and Polk (2024). This economically important variation in fundamental value is not captured by best-in-class DCF methods such as Gonçalves and Leonard (2023).

2. Nevertheless, measures of misvaluation such as Gonçalves and Leonard (2023), Stambaugh and Yuan (2017), or Asness, Frazzini, and Pedersen (2019) do contain useful incremental information about CAPM-implied equity values beyond what the econometrician can detect based on our baseline set of stock characteristics.

3. Discretionary buy-and-hold funds tend to pick stocks that are significantly underpriced relative to the CAPM but do not generate CAPM alpha in the short run. These funds prefer to hold stocks whose price has not risen strongly over the past year, which helps avoid overpriced stocks but bets against momentum.

4. Private equity funds capture substantial CAPM misvaluation, purchasing stocks at roughly 7% below fundamental value and subsequently selling them at around 16% above it.

5. Despite these pockets of misvaluation, the price levels of individual stocks are overall "almost efficient" with respect to the CAPM based on the price-level criteria proposed by Black (1986). Thus, though the stock market may not be efficient with respect to the CAPM based on short-horizon return tests (Fama, 1970), we find that prices are almost right with respect to that model (Cohen, Polk, and Vuolteenaho, 2009).

6. Implementing our approach with respect to an "excess-return" model (i.e., one without any risk adjustment) reveals economically large and statistically significant variation in long-term discount rates across stocks that is much greater than that found in Keloharju,

Linnainmaa, and Nyberg (2021). Moreover, once one controls for this discount-rate effect, the value spread then strongly forecasts future cash-flow growth, consistent with Cohen et al. (2003) and in stark contrast to the claim in De La O, Han, and Myers (2023) that cash-flow *growth* is not predictable.

Related literature

The asset pricing literature on valuations is thin compared to the vast literature on short-term expected returns. Ohlson (1995), Frankel and Lee (1998), Dechow, Hutton, and Sloan (1999), Lee, Myers, and Swaminathan (1999) explored variants of the DCF approach, including the residual income model. However, these approaches require stock-level cost of equity estimates that tend to be imprecise (Fama and French, 1997). Because of this, the vector autoregression (VAR) approach to DCF taken by Gonçalves and Leonard (2023) avoids risk adjusting cash flows altogether, applying a single discount rate across all stocks. Relatedly, Stambaugh and Yuan (2017), Bartram and Grinblatt (2018), and Gerakos and Linnainmaa (2018) and Golubov and Konstantinidi (2019) use, respectively, a composite signal, an "agnostic" regression, and an accounting-based approach to provide metrics of stock misvaluation.

Our approach complements these existing approaches by connecting equity valuation to the vast literature on short-horizon returns. Cho and Polk (2024) use a version of the identity utilized in this paper to estimate, at the portfolio level, the average percentage deviation of price from the factor-model-implied fundamental value over a long sample of portfolio formation periods. Our empirical approach shares similarities with the Fama-MacBeth regression approach to αs in Lewellen (2015) and the instrumented principal component analysis approach of Kelly, Pruitt, and Su (2019, 2020) in modeling stock-level alpha as a function of multiple stock characteristics.

Organization of the paper

Section 1 develops our estimation approach. Sections 2 and 3 describe the data, estimate stock-level fundamental values in-sample and out-of-sample, and validate the estimates in various ways. Section 4 analyzes and interprets our findings in the context of the existing literature on equity valuations, institutional investors, and market efficiency. Section 5 implements a risk-neutral version of our approach to present new findings on firm-level cost of equity and price multiples. Section 6 concludes.

1 Fundamental Values via Discounted Alphas

1.1 Asset pricing environment and definitions

An asset generates a stream of cash flows (dividends), $\{D_{i,t+\tau}\}_{\tau=1}^{\infty}$, where i and t index asset and time, respectively. $\{\widetilde{M}_{t,t+\tau}\}_{\tau=1}^{\infty}$ is a candidate cumulative stochastic discount factor. Define *fundamental value* as the buy-and-hold value of the asset's cash flows, discounted according to the candidate asset pricing model.

Definition 1 (**Fundamental value and the value-to-price ratio**). *Fundamental value of asset i at time t, denoted $V_{i,t}$, is the buy-and-hold value of all future cash flows discounted with the candidate SDF:*

$$V_{i,t} \equiv \sum_{\tau=1}^{\infty} E_t \left[\widetilde{M}_{t,t+\tau} D_{i,t+\tau} \right]. \tag{4}$$

The value-to-price ratio, denoted $\frac{V}{P}$, is the fundamental value divided by the market price. Here, the candidate SDF is a pricing model an econometrician uses to evaluate asset prices in the sense of Hansen and Jagannathan (1991, 1997) and may not be the true SDF.

We emphasize three aspects of this definition of fundamental value. First, fundamental value is the asset's buy-and-hold cash flow value rather than a buy-and-sell value that depends on the terminal selling price—i.e., it takes the perspective of a long-term buy-and-hold investor rather than a short-term dynamic trader.[9] Second, fundamental value is subject to the joint hypothesis problem emphasized by Fama (1970): fundamental value may not equal the price either because the assumed factor model of risk does not correctly measure the true model of market equilibrium or becasue there is genuine misvaluation. Thus, fundamental value is specific to the assumed model of risk and can vary across different risk models. Third, fundamental value is specific to the econometrician's information set, which we assume to

[9] $V_{i,t}^{ST} \equiv E_t \left[\widetilde{M}_{t,t+1}(D_{i,t+1} + P_{i,t+1}) \right]$, where $V_{i,t} \neq V_{i,t}^{ST}$ is allowed when the candidate SDF is not the true SDF. That is, fundamental value is "fundamental" in the sense that it evaluates the value of all future buy-and-hold cash flows rather than a future selling price.

include all historical data on returns and a set of stock characteristics up to that point.

To work with a stationary variable, we scale fundamental value by the market price, which we call the value-to-price ratio (denoted $\frac{V}{P}$). $\frac{V}{P} > 1$ means that the asset is underpriced from the perspective of an econometrician using the particular candidate pricing model, and $\frac{V}{P} < 1$ if it is overpriced. The range of values $\frac{V}{P}$ can take is $[0, \infty)$, which is the range of returns $([-1, \infty))$ shifted to the right by one. Since price and shares outstanding are observed, estimating a stock's $\frac{V}{P}$ is equivalent to estimating its fundamental value per share (V) or the fundmamental value of total equity ($V\times$ Shares Outstanding).[10]

1.2 The discounted-alphas identity

An exact identity links an asset's value-to-price ratio to a sequence of future abnormal returns.

Lemma 1 (The discounted-alphas identity). *As an identity, an asset's (centered) value-to-price ratio, $\frac{V}{P}$, equals the sum of a discounted next-period α and a discounted next-period (centered) $\frac{V}{P}$, where the $\frac{V}{P}$'s and the α are with respect to the same risk model:*

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \frac{\alpha_{i,t}}{1 + R_{f,t}} + E_t \left[\widetilde{M}_{t+1} \frac{P_{i,t+1}}{P_{i,t}} \left(\frac{V_{i,t+1}}{P_{i,t+1}} - 1 \right) \right], \tag{3}$$

where $\alpha_{i,t}$ is the $\widetilde{M}$-implied conditional abnormal return and $\frac{P_{i,t+1}}{P_{i,t}}$ is capital gain. Under the transversality condition, the identity can be iterated forward to a discounted-alphas expression:

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \sum_{\tau=1}^{\infty} E_t \left[\widetilde{M}_{t,t+\tau} \frac{P_{i,t+\tau-1}}{P_{i,t}} \alpha_{i,t+\tau-1} \right]. \tag{5}$$

Proof. Appendix B in the Internet Appendix provides the proof as well as derivations for all subsequent results. □

[10] $\frac{V}{P}$ is a simple linear transformation of abnormal price in Cho and Polk (2024), denoted δ:

$$\delta_{i,t} \equiv \frac{P_{i,t} - V_{i,t}}{P_{i,t}} = 1 - \frac{V_{i,t}}{P_{i,t}}$$

Cho and Polk then derive a closely related identity on δ. We choose to work with $\frac{V}{P}$ rather than δ in this paper, since our focus is on stock-level fundamental values rather than (portfolio-level) abnormal price as in Cho and Polk.

The identity states that undervaluation with respect to a risk model ($\frac{V}{P} > 1$) forecasts future buy-and-hold alphas with respect to the same risk model ($\alpha > 0$). The one-period version of the identity, which we use in our estimation, states that an undervalued stock either generates a positive next-period alpha or continues to be undervalued next period (or both). Being an identity, these relations do not rely on assumptions about investor behavior or the market environment, requiring only that there is an asset with zero abnormal return with respect to $\widetilde{M}$ (e.g., a risk-free asset that is priced in a manner consistent with the interest rate component of $\widetilde{M}$).

Suppose that a stock at time t is underpriced to an econometrician using the CAPM as a pricing model. If price appreciates to undo part of the underpricing, the capital gain component of time $t+1$ return will be abnormally high. If instead the asset remains underpriced forever, which our identity also allows for, the dividend yield component of time $t+1$ return will still be abnormally high, since time $t+1$ dividend will appear too high relative to the (abnormally low) time t price. In both cases, time t CAPM underpricing gets revealed by a time $t+1$ CAPM alpha.[11]

The identity allows us to solve for a model of stock-level $\frac{V}{P}$ given a model of stock-level α. The idea is simple. Since $\frac{V}{P}$ appears on both sides of equation (3), for time t and for time $t+1$, we can find a model of stock-level $\frac{V}{P}$ that is consistent with our model of stock-level alphas and the law of motion.

To solve for $\frac{V}{P}$ as a function of stock characteristics, first write $\frac{V}{P}$ as linear in stock characteristics $z_{i,t}$:[12]

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \gamma_V z_{i,t} + u_{i,t} \tag{6}$$

where γ_V and $u_{i,t}$ are the slope coefficients and projection error, respectively. Plugging this

[11]There is yet a third case in which price depreciates further to deepen the underpricing. In the event of a sufficiently large price depreciation, time t underpricing may not generate a time $t+1$ alpha immediately but a deepened underpricing at time $t+1$, which will be detected through a larger subsequent alpha. For an underpricing to be never revealed through future alphas, price depreciation needs to occur persistently and sufficiently to the extent of disconnecting the price from the dividend process, which we rule out through the no-explosive-bubble condition. This is a mild assumption and is not restrictive, as it allows for most patterns of mispricing, including permanent mispricing. See Cho and Polk (2024) for further discussion.

[12]This linearity is not crucial for our approach, and the general idea is to write $\frac{V_{i,t}}{P_{i,t}} - 1 = h(z_{i,t}; \gamma_\delta) + u_{i,t}$, where h can be nonlinear.

into the one-period identity in equation (3), we obtain the γ_V vector as the slope coefficients from regressing the panel of stock-level αs on a panel of stock-specific vectors measuring how quickly each stock characteristic decays (the expression in outer parantheses):

$$\frac{\alpha_{i,t}}{1+R_{f,t}} = \gamma_V \left(z_{i,t} - E_t\left[\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\right] E_t\left[z_{i,t+1}\right] - Cov_t\left(\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}, z_{i,t+1}\right)\right) + u_{i,t}, \quad (7)$$

where $R_{f,t}$ is the risk-free rate from time t to $t+1$.[13]

To understand this regression approach, think of α as a "flow" of abnormal return paid out from a "stock" of misvaluation generated by a bundle of characteristics, z. Equation (7) shows that a characteristic predicts a large "stock" of misvaluation (i.e., the characteristic has a large γ_V) if

(i) it predicts a large alpha (the left-hand side is large);

(ii) it decays slowly ($z_{i,t} - E_t\left[\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\right] E_t\left[z_{i,t+1}\right]$ is small); or

(iii) it decays less in more important states ($Cov_t\left(\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}, z_{i,t+1}\right)$ is large).

The regression in equation (7) estimating γ_V takes all of these effects into consideration.

1.3 Specifying asset returns, evolution of stock characteristics, and risk factors

To rewrite equation (7) in terms of known quantities, we specify the model of asset returns, capital gain, the evolution of asset characteristics, and risk factors.

Returns, capital gain, and characteristics. Without loss of generality, write excess return as a projection on risk factors in the candidate discount factor model $\widetilde{M}$:

$$R^e_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}, \quad (8)$$

where $\alpha_{i,t}$ is an asset-specific intercept, $\beta_{i,t}$ is an asset-specific K-row vector, f_{t+1} is a K-column vector of candidate risk factors, and $\epsilon_{i,t+1}$ is a stock-specific projection error such

[13] If we allow for u to be persistent so that $E_t\left[u_{i,t+1}\right] \neq 0$, we must add $E_t\left[\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\right] E_t\left[u_{i,t+1}\right]$ to the error term.

that $E_t[\epsilon_{i,t+1}] = E_t[\epsilon_{i,t+1} f_{t+1}] = 0$.[14] Similarly, write excess capital gain defined as a capital gain above the risk-free rate of return as a projection on the candidate risk factors:

$$G^e_{i,t+1} \equiv G_{i,t+1} - R_{f,t} = \alpha_{G,i,t} + \beta_{G,i,t} f_{t+1} + \epsilon_{G,i,t+1} \tag{9}$$

where $\alpha_{G,i,t}$ is interpreted as a "capital-gain alpha," the component of return alpha produced by capital gain, $E_t[\epsilon_{G,i,t+1}] = E_t[\epsilon_{G,i,t+1} f_{t+1}] = 0$, and all other quantities are defined analogously as they are for excess returns. Finally, write a vector of characteristics as a projection on the candidate risk factors:

$$z_{i,t+1} = \alpha_{z,i,t} + \beta_{z,i,t} f_{t+1} + \epsilon_{z,i,t+1}, \tag{10}$$

where $E_t[\epsilon_{z,i,t+1}] = E_t[\epsilon_{z,i,t+1} f_{t+1}] = 0$ but $\sigma_{G,z,i,t} \equiv E_t[\epsilon_{G,i,t+1} \epsilon_{z,i,t+1}]$ can be nonzero. Although the characteristics (usually) are not returns and there is no notion of an "alpha," we use the alpha and beta notation so that it is easier for the reader to see the symmetry with the specification of returns and of capital gains. Since $z_{i,t+1}$ is an L-column vector, $\alpha_{z,i,t}$ and $\epsilon_{z,i,t+1}$ are also L-column vectors, and $\beta_{z,i,t}$ an L-by-K matrix.

One may worry that stocks in different industries follow different processes. Although our approach allows for such an extension, it is reasonable to postulate the same process across different industries, as the characteristics of different firms eventually converge to the same steady state (Keloharju et al., 2021).

Following Lewellen (2015) and Kelly et al. (2019), we specify the αs, βs, and $\sigma_{G,z}$ in equations (8), (9), and (10) to be linear in the stock characteristics:

$$\begin{aligned} \alpha_{i,t} &= \gamma_R z_{i,t}, & \beta'_{i,t} &= \Gamma_R z_{i,t}, \\ \alpha_{G,i,t} &= \gamma_G z_{i,t}, & \beta'_{G,i,t} &= \Gamma_G z_{i,t}, \\ \alpha_{z,i,t} &= \gamma_z z_{i,t}, & \beta'_{z,i,t} &= \begin{pmatrix} \beta'_{z,1,i,t} & \dots & \beta'_{z,L,i,t} \end{pmatrix}, \\ \beta'_{z,l,i,t} &= \Gamma_{z,l} z_{i,t}, & \sigma_{G,z,i,t} &= \Gamma_{G,z} z_{i,t}, \end{aligned} \tag{11}$$

[14] Researchers have modeled expected returns as a function of characteristics since at least Fama and MacBeth (1973) and factor loadings as a function of characteristics since at least Shanken (1990).

where γ_R (γ_G) is an L-row vector, Γ_R (Γ_G) is an K-by-L matrix, γ_z ($\Gamma_{G,z}$) an L-by-L matrix, and $\Gamma_{z,l}$ is a K-by-L matrix for each $l = 1, ..., L$. Although our approach allows these quantities to be nonlinear in the characteristics, the linearity we assume is not particularly restrictive, since it can include the polynomials of the variables as well as their interactions.

Candidate risk factors. Finally, we require that the candidate risk factors explain their own returns as well as the risk-free rate proxied by the Treasury bill rate, an assumption maintained in the conventional expected short-horizon return analysis:

$$E_t\left[\widetilde{M}_{t+1}\right] = \frac{1}{1+R_{f,t}} \tag{12}$$

$$Cov_t\left(\widetilde{M}_{t+1}, f_{t+1}\right) = \frac{1}{1+R_{f,t}}\lambda_t, \tag{13}$$

where $\lambda_t \equiv E_t\left[f_{t+1}\right]$ is the vector of conditional factor risk premia. In words, expected excess returns on candidate risk factors only come from risk premia. Not having to specify the exact functional form of the candidate SDF is an important strength of our approach; in contrast, the portfolio-level misvaluation estimator of Cho and Polk (2024) requires specifying a functional form of the candidate SDF (e.g., exponentially linear in the factors). Our risk-neutral $\frac{V}{P}$ analysis in Section 5 interprets λ_t simply as expected excess returns, dropping the relation to risk.

1.4 Estimating fundamental values via discounted alphas

The model in Section 1.3 reduces equation (7), the discounted-alphas regression for fundamental values, to a simpler expression containing quantities we can estimate.

Remark 1 (Asset-level $\frac{V}{P}$ via discounted alphas). *Let γ_V be the coefficients from projecting asset-level value-to-price ratio, $\frac{V}{P}$, on a vector of cross-sectionally demeaned asset characteristics, z. Given the model of excess returns, capital gain, and characteristics in Section 1.3, the regression approach in equation (7) simplifies to*

$$\alpha_{i,t} = \gamma_V\left[(1+R_{f,t})(z_{i,t}-\alpha_{z,i,t}) - \alpha_{G,i,t}\alpha_{z,i,t} - \sigma_{G,z,i,t}\right] + \widetilde{u}_{i,t}, \tag{14}$$

where $\widetilde{u}_{i,t} \equiv (1+R_{f,t})u_{i,t}$ is an error term. That is, γ_V is the slope parameter in a population *regression of asset-level α on the expression inside the square bracket.*

We estimate $\frac{V}{P}$ in two steps:

(i) Estimate equations (8), (9), and (10) in a weighted least squares panel regression. Based on the residuals from these regressions, we regress $\widehat{\epsilon}_{G,i,t+1}\widehat{\epsilon}_{z,l,i,t+1}$ on $z_{i,t}$ for each $l = 1, ..., L$ to obtain $\widehat{\sigma}_{G,z,i,t}$, the last term in the square bracket in equation (14).

(ii) Regresses $\widehat{\alpha}_{i,t}$ on the L-vector of regressors,

$$(1 + R_{f,t})(z_{i,t} - \widehat{\alpha}_{z,i,t}) - \widehat{\alpha}_{G,i,t}\widehat{\alpha}_{z,i,t} - \widehat{\sigma}_{G,z,i,t}, \tag{15}$$

where $\widehat{\alpha}_{i,t}$, $\widehat{\alpha}_{z,i,t}$, $\widehat{\alpha}_{G,i,t}$, and $\widehat{\sigma}_{G,z,i,t}$ are estimated from the first step and we include time fixed effects to estimate the cross-sectional $\frac{V}{P}$.[15]

We use one year as the interval of time between t and $t+1$ in equations (8), (9), and (10) but estimate all regressions using overlapping monthly observations.[16] We advocate using value-weighted least squares to prevent small stocks with outlier values of some characteristics from driving the results.[17] We provide t-statistics and confidence intervals on γ_V and stock-specific $\frac{V}{P}$ estimates based on a bootstrap that corrects for the cross-sectional and time-series uncertainty in our two-stage estimation. We multiply the stock-specific $\frac{V}{P}$ by the price to get stock-specific fundamental values, $V_{i,t}$.

To summarize, our approach measures stock-level value-to-price ratios from predictable patterns in abnormal stock returns, departing from the DCF approach that projects the stock's future cash flows and discounts them using stock-specific cost of equity estimates.

[15] The l'th regressor in the L-vector of regressors equals $(1 + R_{f,t})(z_{l,i,t} - \widehat{\alpha}_{z,l,i,t}) - \widehat{\alpha}_{G,i,t}\widehat{\alpha}_{z,l,i,t} - \widehat{\sigma}_{G,z,l,i,t}$. For the constant term, $z_{z,1,i,t} = 1$ (when $l = 1$), the regressor value reduces to $(1 + R_{f,t})$, which then gets absorbed by the time fixed effects included in the regression.

[16] There is some discretion over what time interval one uses as one period (t). Monthly is too short to capture how accounting-based characteristics evolve over time, but using a time interval that is too long results in an inaccurate estimation of alpha, since over such a long period a significant part of the return comes from dividends that gets paid out at different points in time. We measure one period to be a year and use annual data to estimate the first- and second-stage coefficients. Two-year or three-year intervals could also be reasonable alternatives if one wants to capture longer-horizon dynamics of characteristics.

[17] For example, we use $\widetilde{w}_{i,t} = \frac{1}{1+R_{f,t}} \frac{MktCap_{i,t}}{\sum_j MktCap_{j,t}}$ as the weight on asset i at time t. The risk-free rate adjustment here is quantitatively unimportant but ensures that our regression minimizes the weighted sum of squared u rather than $\widetilde{u}$.

By directly using abnormal returns that are already risk-adjusted, our approach capitalizes on decades of research on short-horizon abnormal returns and avoids the need to risk- and time-adjust future cash flows with stock-specific costs of equity, which Fama and French (1997) describe as "distressingly imprecise."

Our approach is flexible in that it can be deployed using nonlinear projections, a large information set containing a large number of signals (which then calls for shrinkage regressions instead of the traditional least squares approach), or a step that also extracts factor models of price levels as done for returns in Kelly et al. (2020). Our method allows orthogonal information contained in other fundamental value estimates to improve our estimate; we can simply add that to the vector of characteristics in our information set.

1.5 Alternative approaches to estimating fundamental values

Other than DCF and our proposed discounted-alphas approach, what are some other potential approaches to estimating fundamental values?

1.5.1 $\widetilde{M}$*-discounted dividends is discounted alphas*

What if we start with the definition of fundamental value but write a one-period law of motion in terms of discounted dividends, not discounted alphas? We show below that this approach of discounting dividends with state-specific $\widetilde{M}$ recovers our discounted-alphas approach.

To see this, rewrite the definition of fundamental value, $V_{i,t} = \sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{i,t+j} D_{i,t+j}\right]$, as a one-period law of motion:

$$\frac{V_{i,t}}{P_{i,t}} = E_t\left[\widetilde{M}_{t+1}\frac{D_{i,t+1}}{P_{i,t}}\right] + E_t\left[\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\frac{V_{i,t+1}}{P_{i,t+1}}\right]. \tag{16}$$

But since $\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\frac{V_{i,t+1}}{P_{i,t+1}} = \widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}} + \widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}(\frac{V_{i,t+1}}{P_{i,t+1}} - 1)$, the law of motion can be rewritten as

$$\frac{V_{i,t}}{P_{i,t}} = \underbrace{E_t\left[\widetilde{M}_{t+1}\left(\frac{D_{i,t+1}}{P_{i,t}} + \frac{P_{i,t+1}}{P_{i,t}}\right)\right]}_{=1+\frac{\alpha_{i,t}}{1+R_{f,t}}} + E_t\left[\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\left(\frac{V_{i,t+1}}{P_{i,t+1}} - 1\right)\right]. \tag{17}$$

Intuitively, when dividends are modeled jointly with capital gains, it brings us back to a model of returns or, equivalently, a model of alphas.

1.5.2 *Campbell-Shiller approximation*

What if we design a discounted-alphas approach to equity valuation using the approximate loglinear identity of Campbell and Shiller (1988) rather than the exact identity from Cho and Polk (2024)? It turns out, such a loglinear identity is trickier to use in equity valuation, since risk adjusting expected log returns requires a Jensen's correction term.

To see this, begin with the law of motion from the loglinear decomposition of Campbell and Shiller (1988):

$$v_{i,t} - d_{i,t} = k + E_t \Delta d_{i,t+1} - E_t \widetilde{r}_{i,t+1} + \rho E_t[v_{i,t+1} - d_{i,t+1}]. \tag{18}$$

Here, $v_{i,t} - d_{i,t} \equiv \log(V_{i,t}) - \log(D_{i,t})$ is the log fundamental-value-to-dividend ratio, k is a constant, $\Delta d_{i,t+1}$ is the log dividend growth, ρ is a constant that is less than but close to one, and $\widetilde{r}_{i,t+1} \equiv \log(1 + \widetilde{R}_{i,t+1})$ with $\widetilde{R}_{i,t+1}$ representing return at time $t+1$ what return would be if the candidate risk model were the true model.

Nevertheless, not only is the log zero-mispricing-return $\widetilde{r}_{i,t+1}$ unobserved in reality, but also is its volatility, whose deviation from the observed volatility necessitates a Jensen's correction of unknown amount (see the internet appendix to Cho and Polk (2024) for an in-depth analysis of these concerns). An added issue is that the log dividend growth or the value-to-dividend ratio is undefined for many stocks with zero dividend.[18]

1.5.3 *A simple discounted sum of future alphas*

Our proposal is to consider a broad class of discounted-alphas approaches to valuation, not just one specific implementation. For instance, while our baseline approach accounts for how differences in the stocks' cash-flow duration or in how their conditional alphas covary with the cumulative discount factor implied by the factor model, these sources of heterogeneity

[18] The loglinear firm-level identity of Cho, Kremens, Lee, and Polk (2024), which extends the one in Vuolteenaho (2002) to allow for the role of investment, does not fix the problem of unobserved $\widetilde{r}_{i,t+1}$, although it does address the problem of zero dividends.

can be shut down to generate a simpler approach.

One such simplification is to ignore the covariance component of equation (7) and simply discount future alphas with a time discount factor that differs across stocks. Table A1, which repeats the validation exercise done in Table 4's first row in Panel B, shows that this approach still leads to a reasonable estimate, despite some noticeable loss in accuracy. Further simplifications are likely to lead to a poorer outcome. Hence, we advocate using our baseline approach whenever possible.

1.5.4 *Why not estimate portfolio abnormal price and project those onto characteristics?*

Another approach could be to estimate portfolio abnormal price as in Cho and Polk (2024) and use this to understand how a multivariate set of characteristics may map to those portfolio abnormal prices, generating firm-level estimates of abnormal price. An earlier draft of Cho and Polk as well as van Binsbergen et al. (2023) have used this approach. However, it is difficult to use this approach to generate reliable out-of-sample fundamental values based on shorter historical samples, since estimates of portfolio abnormal price—the key ingredient in such an approach—requires a long sample period.

In any case, our method is not necessarily meant to drive out these existing approaches but is meant to complement them. The availability of different approaches to fundamental value could uncover orthogonal knowledge that could help further advance our understanding of fundamental values and long-run discount rates.

2 Data and Variables

We combine monthly stock price data from the Center for Research in Security Prices (CRSP), annual accounting data from CRSP/Compustat Merged (CCM), and the pre-Compustat book equity data from Davis, Fama, and French (2000) to create our monthly stock-level dataset. We obtain factor data from Kenneth French's data library, including the risk-free rate proxied by the one-month Treasury bill rate. We proxy for the annual risk-free rate by rolling over the one-month Treasury bill rates over the year.

Our analysis focuses on seven stock-level characteristics used in Cho and Polk (2024): book-to-market (*BM*), profitability (*Prof*), and market beta (*Beta*) are characteristics that, together could proxy for CAPM mispricing according to a present-value identity (Cho and Polk, 2024). Market equity (*ME*) is a potential proxy for overpricing if fundamental value does not rise in lockstep with market value (Berk, 1995). Investment (*Inv*) and net issuance (*NetIss*) may signal overpricing if firm managers time these decisions partly on perceived mispricing of the firm. Momentum (*Ret*) defined as the 12-month return from month -12 to month 0 would signal misvaluation if it arises through either price underreaction or price overreaction. Besides these seven, part of our baseline analysis considers lagged momentum (*LagRet*), defined as the 12-month return from month -24 to month -12, to ensure that we capture richer dynamics in $\frac{V}{P}$ arising from past returns.

Following Kelly et al. (2020), we work with cross-sectional ranks of these characteristics, with the exception of return, which is included after a cross-sectional demeaning to ensure that a covariance between capital gain and the projection error does not bias the coefficients (the last paragraph of Appendix B.3).[19] These variables are cross-sectionally standardized with value weights. We report the cross-sectional correlations and the time-series (cross) autocorrelations of these characteristics in Table 1.

Overall, we have a stock-month panel spanning from June 1939 to December 2023, with the first lagged characteristics beginning in June 1938.[20] Using a moving window of 50 years (with a minimum of 15 years), we estimate out-of-sample fundamental values for approximately 2.4 million stock-month observations from June 1953 to December 2023.[21] For comparison, we also present in-sample estimates for June 1953 to December 2023. We consider three alternative factor models of risk: the CAPM, the three-factor model of Fama and French (1993), and the five-factor model of Fama and French (2015). CAPM fundamental values are especially interesting to estimate and will be analyzed more extensively, since

[19]By doing this, we ensure that $Cov_t(G_{i,t+1}, u_{i,t+1}) \approx 0$.

[20]We choose to start our sample in 1938, as Cohen et al. (2003) argue that before 1938, accounting practices were still converging to full compliance with the reporting requirements of the 1934 Securities Exchange Act.

[21]Fundamental value reflects how stock characteristics relate to a firm's long-term prospects, so conservative estimates require a longer moving window than typical short-horizon analyses. A 40-year window yields stronger validation but leads to estimates that suggest larger misvaluations in recent prices. An extension of our method could use cross-validation to determine the optimal window length or parameter for an exponentially weighted moving average.

surveys of CFOs suggest that the CAPM is the most popular model used in firms' actual capital budgeting decisions (Graham and Harvey, 2001).[22]

3 Estimating Stock-level Fundamental Values

We use the two-step regression approach to discounted alphas, explained in Section 1.4, to estimate how the ratio of model-specific fundamental value to price loads on stock characteristics:

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \gamma_V z_{i,t} + u_{i,t}, \tag{6}$$

where $z_{i,t}$ is the vector of stock i's characteristics at time t and $u_{i,t}$ is a projection error. We do this with respect to three candidate factor models (CAPM, FF3, and FF5). We first present in-sample estimates (Section 3.1) and then out-of-sample estimates based on a moving window (Section 3.2). We finish the section by validating our estimates of fundamental value (Section 3.3).

3.1 In-sample estimation and incremental predictors of stock misvaluation

Table 2 reports the in-sample estimates of γ_V (along with t-statistics), indicating which characteristics serve as an incremental predictor of under- or over-valuation with respect to a factor model of risk. Different from the previous analysis in the literature, we study the incremental effect of each characteristic in a multi-characteristic setting.[23]

[22] Recent evidence also shows that the size and value factor exposures affect the costs of capital firms report in their earnings announcements (Gormsen and Huber, 2023). On the one hand, further refinements of the three-factor model such as the five-factor model of Fama and French (2015) or the four-factor model of Hou, Xue, and Zhang (2015) are recent developments and are likely to have been less relevant for decision makers during most of our sample period that begins in 1939. On the other hand, these patterns may reflect economic forces present throughout the 20th century. Regardless of one's view, we report estimates with respect to the five-factor model to illustrate how the fundamental value estimates might change with these refinements.

[23] Cho and Polk (2024) and van Binsbergen, Boons, Opp, and Tamoni (2023) link characteristics to model-specific misvaluation in a univariate setting. An earlier draft of Cho-Polk and van Binsbergen et al. project their portfolio misvaluations on a vector of stock characteristics. Both of these analyses find an important incremental role of book-to-market but do not detect how profitability and beta play prominent roles, controlling for book-to-market.

3.1.1 *Book-to-market, profitability, and beta*

When it comes to $\frac{V}{P}$ with respect to the CAPM, three characteristics comprising book-to-market, profitability, and beta carry coefficients that are an order of magnitude larger than others. Their coefficients in percentage units are 9.3, 12.5, and -14.8, respectively, interpreted as a one-standard-deviation increase in the cross-sectional rank raising the stock's value-to-price ratio by those percentage points. The coefficients are statistically significant for profitability and beta but borderline insignificant for book-to-market in the in-sample analysis based on a long sample period; however, a moving-window analysis in Figure 3 and Table 3 shows that the coefficient on book-to-market is significant over the majority of the sample period including the most recent sample window.

The prominence of *BM*, *Prof*, and *Beta* for CAPM underpricing is interesting in light of the present-value identity of Vuolteenaho (2002), which implies that cheap stocks (high book-to-market equity ratio) that are nonetheless profitable (high future clean-surplus ROEs) and have low risk (low market beta) are likely to be underpriced (high value-to-price ratio):

$$\log\left(\frac{V_{i,t}}{P_{i,t}}\right) = \underbrace{bm_{i,t}}_{\text{Book-to-Market}} + \underbrace{\sum_{\tau=0}^{\infty}\rho^{\tau}E_t roe_{i,t+1+\tau}}_{\text{Profitability}} - \underbrace{\sum_{\tau=0}^{\infty}\rho^{\tau}E_t\widetilde{r}_{i,t+1+\tau}}_{\text{Beta}}, \tag{19}$$

where v is log fundamental value, p is log price, bm is the log book-to-market ratio, ρ is a constant close to but less than one, roe is the log return on (book) equity, and $\widetilde{r}$ is the log of the return that prevails if the candidate pricing model were the true model. In Section 4, we relate the prominence of these three characteristics to the *Adjusted Value* metric of Cho and Polk (2024).

Profitability and beta continue to be important for predicting three-factor $\frac{V}{P}$. Controlling for RMW in the five-factor model also leaves the importance of profitability unchanged, since we measure profitability with gross profitability, which has a relatively low correlation with operating profitability.

3.1.2 *Investment and net issuance*

Investment and net equity issuance contain statistically important information about stock price levels not contained in other signals. They both predict the stock to be overvalued with respect to the CAPM and FF3, although the effect is not statistically significant with respect to FF5. In terms of the magnitude, a firm whose rank of investment (net issuance) rises by one standard deviation in the cross-section of firms is associated with a 2.0 (3.0) percentage-points rise in overvaluation with respect to the CAPM. In Section 4, we interpret this finding in the context of informational asymmetry between firm managers and financial market participants.

3.1.3 *Size and momentum*

Despite being a persistent characteristic, size (market equity) adds little incremental information about CAPM or FF3 misvaluation beyond what the other stock characteristics provide. Interestingly, however, larger stocks are estimated to be overpriced (lower $\frac{V}{P}$) with respect to the five-factor benchmark with a coefficient of -6.2%.

Adding lagged past return (in addition to past return) to the econometrician's information set shows that, with respect to both the CAPM and FF3, stocks are almost correctly priced at the time of entering into the momentum category but become overpriced over the subsequent year (i.e., the coefficient on lagged past return is negative and statistically significant). This observation, however, rests critically on the assumed risk model, since momentum stocks appear underpriced with respect to FF5. This adds more nuance to the finding in the literature based on a univariate analysis that momentum is likely an outcome of investor overreaction (Cho and Polk, 2024; van Binsbergen et al., 2023).

3.2 Out-of-sample estimates from moving windows

Our in-sample estimates in the previous subsection assume that the value-to-price ratios have fixed loadings on stock characterisics over time. The present subsection allows the loadings to change over time using 50-year moving estimation window (with a minimum of 15 years).

We make three observations from the time-series variation in the multivariate CAPM $\frac{V}{P}$ coefficients for the eight characteristics (Figure 3) and from the most recent moving window coefficients (Table 3). First, the γ_V coefficients vary substantially over time, making the selection of the estimation window an important decision for those estimating fundamental values. Second, the coefficient on book-to-market is economically large and statistically significant for the majority of the sample but has declined, to some degree, over the last few decades. Despite the close relation between book-to-market and lagged return (return reversal) for short-horizon returns, we find that both signals contain orthogonal information about the deviation of CAPM fundamental values from prices over the majority of the sample. Third, the magnitude of the coefficients on profitability, investment, and net issuance have risen over time. We will see in our discussion of the distribution of stock-level $\frac{V}{P}$ that these facts have led to greater CAPM misvaluations in recent years.

We use the coefficients from these moving windows to construct out-of-sample estimates of fundamental value. For example, the coefficients estimated over the 1940m7-1990m6 window is multiplied by the vector of stock characteristics as of 1990m6 to produce an out-of-sample estimate of $\frac{V}{P}$ as of 1990m6. Figure 2 plots the CAPM- and FF3-implied fundamental equity values ($V \times$ Shares Outstanding) for the 10 largest stocks as of December 2023.

What do these out-of-sample estimates say about the high market valuations of tech stocks in recent years? Figure 2 shows that the answer tends to depend on the factor model of risk. Relative to the CAPM, the answer as of December 2023 is mixed—Nvidia appears overpriced, whereas the other tech stocks tend to be either correctly priced or slightly underpriced. Relative to FF3, however, most stocks appear underpriced, including Nvidia. One reason for this difference is the book-to-market characteristic. The out-of-sample coefficients in Table 3 show that the $\frac{V}{P}$ coefficient on book-to-market switches sign from positive to negative as we go from the CAPM to the three-factor benchmark. Relative to the three-factor benchmark, being a growth stock means mild underpricing, not overpricing. That profitability is a stronger predictor of underpricing relative to the three-factor benchmark also contributes to Nvidia's apparent underpricing relative to the benchmark.

3.3 Validating the fundamental value estimates

How should one validate stock-level estimates of fundamental value? Our discounted-alphas identity in equation (5) provides concrete guidance: Sorting stocks on a valid measure of model-specific misvaluation ($\frac{V}{P}$) should generate persistent long-horizon differences in alphas with respect to the same risk model. Hence, we sort stocks on our estimated $\frac{V}{P}$ and check if it leads to large and persistent differences in alphas. We focus on validating our out-of-sample estimates and report the results for in-sample estimates in the appendix.

Prior to formal tests, we show in Figure 4 that the stocks sorted on our in-sample or out-of-sample CAPM value-to-price ratio indeed generate persistent differences in CAPM alphas, whereas stock-level out-of-sample estimates of CAPM alpha lead to faster-declining post-formation alphas. We find similar results for FF3 and FF5, although the distinction between the $\frac{V}{P}$ sort and the α sort is less pronounced.

3.3.1 *Post-formation alphas: 5-year CAR*

A simple way to aggregate future buy-and-hold alphas is to add them over 5 years to form cumulative abnormal returns (CARs). Using a calendar-time approach that addresses the overlapping-samples issue, Table 4 Panel A shows that out-of-sample $\frac{V}{P}$ with respect to a factor model generates large differences in 5-year CARs with respect to the same model. Moreover, model-specific $\frac{V}{P}$ exhibits less predictability of CARs with respect to another factor model, suggesting that our estimates capture valuation information specific to each model of risk.

3.3.2 *Average portfolio* $\frac{V}{P}$

Although the CAR presents consistent evidence, our discounted-alphas identity in equation (5) shows that today's $\frac{V}{P}$ is more strongly related to more recent alphas arising in more important (high cumulative $\widetilde{M}$ states. The average portfolio $\frac{V}{P}$ estimator of Cho and Polk (2024) is similar to CAR but applies to correct weights to realized post-formation alphas when adding them up to arrive at the average formation-period value-to-price ratio.

Table 4 Panel B shows that indeed, the estimated *conditional* CAPM $\frac{V}{P}$ at the stock level generates monotonic and statistically significant variation in *average* portfolio $\frac{V}{P}$.[24] We find similar results for FF3, but the short sample over which out-of-sample FF5 $\frac{V}{P}$ is available means that we cannot reliably estimate the portfolio average $\frac{V}{P}$ with respect to FF5.

[24]More detailed estimation results are available as Tables A3 and A2 in the Internet Appendix.

3.3.3 *Russell index constituents*

To further validate our estimates, we exploit the fact that, because of the way those indices are constructed, stocks at the bottom of Russell 1000 large-cap index (top of Russell 2000 small-cap index) receive disproportionately large capital (Chang et al., 2015). Hence, a reliable measure of model-specific misvaluation should ideally pick up the valuation effect of such price pressure.

Table 5 shows that the bottom 150 stocks in Russell 1000 are 5.0% underpriced from a CAPM investor's perspective, controlling for their inclusion in the index itself, whereas the top 150 stocks in Russell 2000 stocks are 7.9% overpriced. Hence, our out-of-sample estimates capture these demand-induced variation in the market value of equity.

4 Applications and Interpretations

4.1 Do existing measures of misvaluation add information?

The previous literature has suggested measures of stock misvaluations. Do these contain incremental information about misvaluation with respect to factor models beyond what we extract from the vector of seven or eight stock characteristics?

4.1.1 *The DCF-based signal of Gonçalves and Leonard (2023)*

Gonçalves and Leonard (2023) forecast future cash flows using a Vector Autoregressive (VAR) model of firm-level variables to obtain firm-level ratio of fundamental value to price, which they call the fundamental-to-market ratio (FE/ME). They avoid the problem of having to estimate stock-specific costs of equity by applying the same discount rate to all stocks, the rate that makes the market's fundamental value equal to its price.

Table 6 shows strong evidence that this dividend-based measure contains incremental information about the deviation of CAPM fundamental value from prices. FE/ME carries an economically large coefficient of 8.14 that is also statistically significant; i.e., controlling for the other characteristics, a one-standard-deviation increase in the rank of FE/ME is associated with a 8.14% point rise in the CAPM-implied value-to-price ratio. Comparing the coefficients on the other characteristics in the first column to those from the second column of Table 2, we find that the incremental explanatory power of this measure draws partly from driving out the explanatory power of gross profitability, investment, and lagged return (long-term reversal) characteristics. However, this fact does not seem to explain the large magnitude of its coefficient, which means that dividend-based measures of value-to-price likely contain information orthogonal to our baseline $\frac{V}{P}$ estimates.

4.1.2 *The characteristic-based signals of Stambaugh and Yuan (2017) and Asness et al. (2019)*

Stambaugh and Yuan (2017) and Asness et al. (2019) take a different approach to proxying for mispricing in the price level, which is to combine several characteristics likely to proxy for mispricing into a composite signal. Stambaugh and Yuan (2017) do this to generate two "mispricing" factors, management (*Mgmt*) and performance (*Perf*), whereas Asness et al. (2019) do this to generate quality (*Quality*). Columns 2 and 3 of Table 6 show that these signals also contain incremental information about mispricing. The mispricing factors of Stambaugh and Yuan appear to drive out the explanatory power of investment and net issuance, which is expected from *Mgmt* containing measures of investment, although *Perf* may contain more orthogonal information. *Quality* also contains incremental information about CAPM fundamental values and seems to do so without substantially weakening the coefficients on other characteristics. Hence, although Cho and Polk (2024) find *Quality* to be a weak univariate signal of mispricing in the price level, this result shows that it may work in a multivariate setting that controls for the effect of other characteristics on prices.

4.1.3 *What does this imply?*

These results imply that existing misvaluation measures contain orthogonal information about stock-level misvaluation. These results—especially that on the DCF-based signal—also suggest that the more traditional DCF estimates based on qualitative research, as done in discretionary mutual funds and sell-side analysts, could contain orthogonal information about fundamental values.

In other words, our discounted-alphas approach and the traditional DCF approach could be complementary, and expanding the information set beyond the vector of seven or eight stock-level characteristics that we assume can help capture additional variation in $\frac{V}{P}$. For instance, adding the rank of *FE/ME* to our existing model could generate more powerful estimates of fundamental value. Indeed, being able to add other signals of misvaluation as additional elements in the characteristic vector is an important advantage of our approach.

4.2 Do discretionary buy-and-hold investors chase underpricing or alpha?

Some discretionary investors may approach security selection from a long-term buy-and-hold perspective. If so, their objective is then to look for stocks that are significantly underpriced, even if those stocks might not deliver the highest short-term alphas. We ask if the holdings of four of the largest, most famous discretionary investors of this type in our sample—Berkshire Hathaway, Tiger Management (Julian Robertson), Capital Group, and Dodge & Cox—demonstrate this investment philosophy.

Table 7 shows that stocks held by these discretionary investors tend to be significantly underpriced (Panel A). A typical stock held by Berkshire Hathaway is around 9.0% underpriced relative to the CAPM (4.9% with value weights), whereas a typical stock held by this group of discretionary investors, which includes Berkshire, is 3.3% underpriced (4.2% with value weights).

Interestingly, however, these stocks do not deliver positive alphas in the short run, except for a small positive alpha associated with Berkshire Hathaway's holdings in the equal weight specification (Panel B). In fact, a typical stock held by this group of discretionary investors is predicted to deliver a negative monthly alpha of −4.7 basis points.

Panel C shows that this disparity between underpricing and short-term alphas of stocks held by these discretionary investors arises from their contrarian behavior. These investors tend to hold stocks with a negative momentum characteristic, which our analysis above has shown is associated with underpricing, but hurts their short-term alpha performance. The same panel also shows that it is these funds' negative bets on beta, investment, net issuance, and lagged return (long-term reversal) that result in their tilt towards underpriced stocks. They do not, however, appear to tilt strongly toward profitable firms.

An implication of these findings is that the short-term alphas that investors in these funds earn may not accurately measure the welfare contributions of these discretionary funds. By identifying and holding underpriced stocks, discretionary investors contribute to long-term price discovery and, ultimately, efficient capital allocation.

4.3 Private equity funds buy low and sell high

A related topic is how private equity (PE) funds trade equity shares. Table 8 shows that PE funds buy stocks that are around 3.2 to 8.9% cheaper than other stocks from the perspective of the CAPM and sell at prices that are around 12.5 to 15.9% higher than other stocks. Overall, holding the stocks' fundamental CAPM value fixed, PE fnds appear to raise the market value of the stocks by more than 20% points (the last column of Panel A).

Interestingly, Panel B shows that the sign of the characteristics that PE funds look for in a stock buyout—previously documented in Stafford (2022)—exactly coincides with those that predict CAPM underpricing (Column (2) in Table 2). The stocks that they sell tend to have the opposite sign of the characteristics.

Overall, these results are in line with the view that PE funds are sophisticated investors that trade stocks based on their valuation levels. That is, independent of their ability to improve the fundamental value of their portfolio firms, PE funds appear to be the canonical long-term arbitrageur of valuation levels.

4.4 Price multiple analysis using *adjusted value*

Raw price multiples—such as the raw market-to-book equity or the price-to-earnings ratio—are problematic to use in comparable analysis, since a low price multiple could signal low expected cash-flow growth or high future risk, not just current undervaluation (Cohen et al., 2003). As a simple remedy, Cho and Polk (2024) propose *adjusted value* as a simple predictor of CAPM undervaluation:

$$Adjusted\,Value = z(B/M) + z(Prof) - z(Beta),$$

where z is the z-score of the cross-sectional rank. The metric adjusts the traditional value signal (book-to-market equity ratio) by awarding more points to stocks with a low market value compared to the book value despite being profitable and low-beta.

Our finding that book-to-market, profitability, and beta are the most prominent predictors of CAPM-implied $\frac{V}{P}$ (Sections 3.1 and 3.2) is consistent with the rationale and the ad-hoc formula behind *adjusted value*. Figure 5 plots the time-series of R^2 for how well unadjusted value (book-to-market) and adjusted value explain the cross-sectional variation in out-of-sample CAPM $\frac{V}{P}$. It shows that the portion of CAPM $\frac{V}{P}$ explained by *adjusted value* is large at around 80% and that this R^2 has been increasing over time last decade. This contrasts sharply with the fraction explained by raw book-to-market, which has plummeted over the last two decades as observed by several others.

4.5 Are the prices of individual stocks efficient? Revisiting Fama (1970) and Black (1986)

Fama (1970) define an efficient capital market as one in which the firms can make production-investment decisions and the investors can make portfolio decisions on the basis of the *level* of security prices (p.383):

> "A market in which firms can make production-investment decisions, and investors can choose among the securities that represent ownership of firms' activities under the assumption that security prices at any time "fully reflect" all available information."

Despite this emphasis on the level, Fama goes on to test capital market efficiency in terms of the *change* in prices as revealed by short-horizon returns.

On the other hand, in his "Noise" address to the American Finance Association, Fischer Black (1986) defines an efficient market as on in which the level of security prices does not deviate from the fundamental value by more than a factor of two:

> "However, we might define an efficient market as one in which price is within a factor of 2 of value, i.e., the price is more than half of value and less than twice value. . . . By this definition, I think almost all markets are efficient almost all of the time. "Almost all" means at least 90%" (p.533).

Although this definition of market efficiency directly uses the level of prices, it is harder to test empirically, so Black proceeds to conjecture that the market is efficient based on his definition.

Though arbitrary, as Black grants, the use of a factor of two as a rule-of-thumb to identify genuine misvaluation is pervasive, with Warren Buffett quoted as advocating a similar margin of safety on fundamental value estimates. How common are such opportunities of potential misvaluation? Are those occurrences indeed sufficiently infrequent to conform to Black's intuition that 90% of the stock market is efficient at least 90% of the time?

Figure 6 plots the distribution of our out-of-sample value-to-price ratio (Panel A). It shows that large mispricing opportunities are rare, even with respect to the CAPM. The 5% and 95% values for the $\frac{V}{P}$ ratio occur when stocks CAPM fundamental value is 27% below price or 29% above it ($\frac{V}{P} < 0.73$ or $\frac{V}{P} > 1.29$), respectively. The 1% and 99% values are when CAPM fundamental value is 40% below or 42% above ($\frac{V}{P} > 0.60$ or $\frac{V}{P} < 1.42$).

Although our estimates reveal pockets of misvaluations, we find the overall stock market to be 'almost efficient' by Black's (1986) definition. Panel B of Figure 6 shows that an estimated CAPM misvaluation greater than -50% or +50% occurs in less than 5% of the market capitalization at all times. Indeed, even misvaluations for only 2% or greater of the stock market rarely happen: the Kennedy Slide of 1962, the dot-com episode of the late 1990s to 2000, and, interestingly, in recent years.[25] Besides our estimates, the value spread and the Baker and Wurgler (2002) sentiment index also point to the possibility of greater misvaluations in recent years. Note, however, that the apparent misvaluation we identify in recent years could either indicate that CAPM-implied misvaluations are more common today or that the long memory (moving window) we assume for how characteristics relate to misvaluation fails to capture the way characteristics relate to valuations in today's market.

[25] Note that an undervaluation by a factor two would be a value-to-price ratio of 2, not 1.5 that we examine. However, such a misvaluation of such a magnitude does not arise in our estimates.

5 Further Applications Based on 'Risk-Neutral' $\frac{V}{P}$

In this section, we use our approach to examine cross-sectional variation in firms' costs of equity as well as to more effectively separate the information in valuation ratios related to future cash-flow growth rates versus discount rates.

5.1 The excess-return-model $\frac{V}{P}$

Our discounted alphas approach to valuation also allows us to estimate the fundamental value of a stock to an investor who evaluates assets based on their average excess returns. Appendix B.5 explains how to adapt our approach for such a model.

The excess-return-model (ERM) $\frac{V}{P}$ is interesting to estimate, since it represents the pure discount-rate effect in prices. Furthermore, dividing the ERM-implied V by an accounting quantity such as book equity allows us to isolate the cash-flow information in a valuation ratio based on the accounting measure.

We estimate the ERM-implied V out-of-sample for each stock over 1953m–2023m12 using the eight-characteristic model. We apply these estimates to answer three questions in asset pricing.

5.2 Can we predict persistent differences in average returns?

Before we study firm-level costs of equity, we first examine whether our methods can identify persistent differences in average returns, a question asked earlier by Keloharju et al. (2021). Repeating the analysis done in their paper but sorting stocks based on our ERM-based out-of-sample $\frac{V}{P}$ measure, which captures the pure discount-rate effect in prices, we find that average return differences between high $\frac{V}{P}$ and low $\frac{V}{P}$ do tend to come down over time but at a much slower rate than shown in Keloharju et al. (Figure 7). Furthermore, the figure shows a persistent component of roughly 0.2 to 0.3% per month in the cross-section that remains statistically significant at the seven-year horizon (and close to statistically significant even 10 years post portfolio formation). We conclude that average return differences can be more persistent than previously understood.

5.3 Does the cost of equity vary across firms and why?

Understanding firm-level variaion in the cost of equity is of critical importance, and the starting point is to ask how much variation in cost of equity there is across stocks. However, estimating a measure of cost of equity has been challenging both conceptually (due to the lack of a consensus on how to define it) and statistically (due to the difficulty of working with long-horizon returns), as Fama and French (1997) highlight in the context of industry portfolios.

Since ERM-implied $\frac{V}{P}$ captures the pure discount-rate effect in prices, it measures the cost of equity at the firm level. Different from the internal rate of return (IRR) also used to describe cost of equity, our measure accounts for how a unit change in IRR has greater impact on the stock prices of high-duration firms.

We document three main findings, without corresponding tabulated results.

1. There is substantial cross-sectional variation in cost of equity, with the firm-level ERM-implied $\frac{V}{P}$ having a cross-sectional spread of 16.6%.

2. The 49 industries of Fama and French explain only around 13.4% of the cross-sectional variation in costs of equity with only 11 industries having a cost of equity that is significantly different from that of the market. This finding implies that there is substantial intra-industry variation in cost of equity that should be important to explore in future research and that estimating cost of equity solely at the level of industry misses important aspects of a firm's cost of capital.

3. Risk adjustment through CAPM betas can exacerbate rather than explain the variation in cost of equity across firms; i.e., the inverted security market line, which relates beta and short-horizon stock returns, also applies to costs of equity. The CAPM risk adjustment leads to a higher cross-sectional standard deviation in unexplained costs of equity (CAPM-implied $\frac{V}{P}$) of 17.3%.

5.4 Do valuation ratios predict cash-flow growth?

Equipped with the pure discount-rate component of prices, the ERM-implied $\frac{V}{P}$, we revisit the finding of De La O et al. (2023) that valuation ratios do not strongly forecast future cash-flow growth.

We form 25 portfolios by independently sorting stocks on size and the market-to-book equity ratio, as typically done in a portfolio analysis in the present-value literature (e.g., Cohen et al. (2003)). We find that earnings are negative around 10% of the time for these portfolios, which makes coming up with a definition of earnings growth challenging. Instead, we ask if dividend growth can be forecasted using a valuation ratio.[26]

We consider two alternative valuation ratios. The first is the market-to-book ratio, which can reflect both cash-flow and discount-rate information. But if firms with high expected dividend growth also tend to have higher discount rates, the two effects may cancel each other out and market-to-book equity variation can cease to forecast future dividend growth. By computing the value-to-book ratio wth respect to the excess return model, we isolate the dividend-growth information and discount-rate information in the market-to-book ratio:

$$\frac{M}{B} = \underbrace{\frac{V}{B}}_{\text{cash flow}} \Big/ \underbrace{\frac{V}{P}}_{\text{discount rate}} ,$$

where V here is the ERM-implied value.

Figure 8 Panel A confirms the finding that the market-to-book ratio does not predict future log dividend growth. However, the ERM-implied $\frac{V}{B}$ ratio does forecast large differences in future log dividend growth. Furthermore, part of this predictability arises through the CAPM-implied fundamental value, implying that differences in market risk alone does not explain why cash-flow growth information is hidden in the market-to-book ratio. Panel B shows that the other component of the market-to-book ratio, the ERM-implied $\frac{V}{P}$, predicts future returns more strongly than the unadjusted market-to-book ratio.

[26] This is the cash-flow growth term in the Campbell and Shiller (1988) identity.

6 Conclusion

We develop a novel way to estimate stock-level fundamental values by simply estimating linear regressions. The flexible nature of our methodology allows researchers to use their own inputs and favourite asset-pricing model to come up with bespoke but rigorous estimates of fundamental value, not only for stocks but also for other assets.

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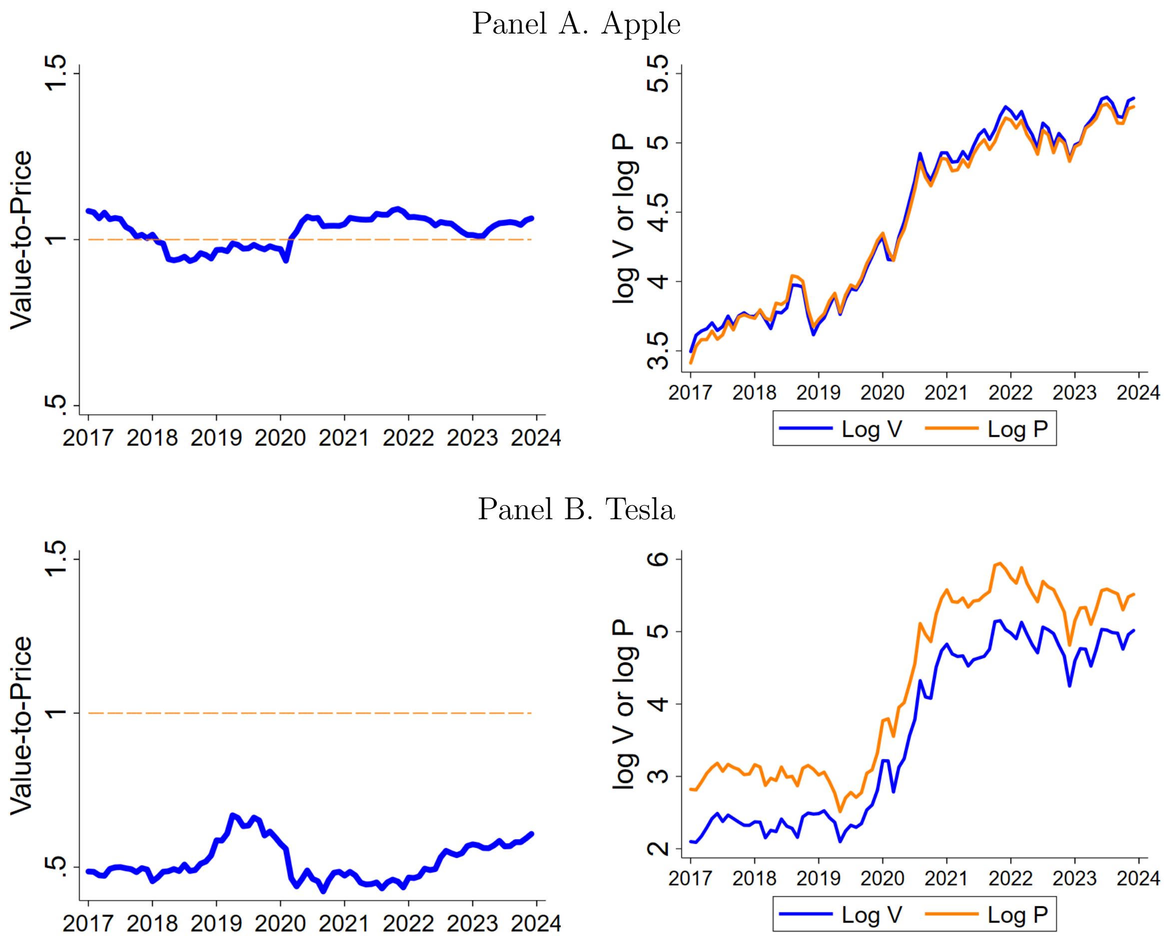

Figure 1: **CAPM-Implied Fundamental Values (Out-of-Sample Estimates)**

The figure plots out-of-sample estimates of fundamental value over the 2017m1–2023m12 subsample for Apple (Panel A) and Tesla (Panel B). The left plot in both panels shows the CAPM value-to-price ratio, and the right plot in both panels shows the log components of that ratio. We estimate those fundamental values using the paper's discounted alphas approach and the specification in Table 3 Column (2).

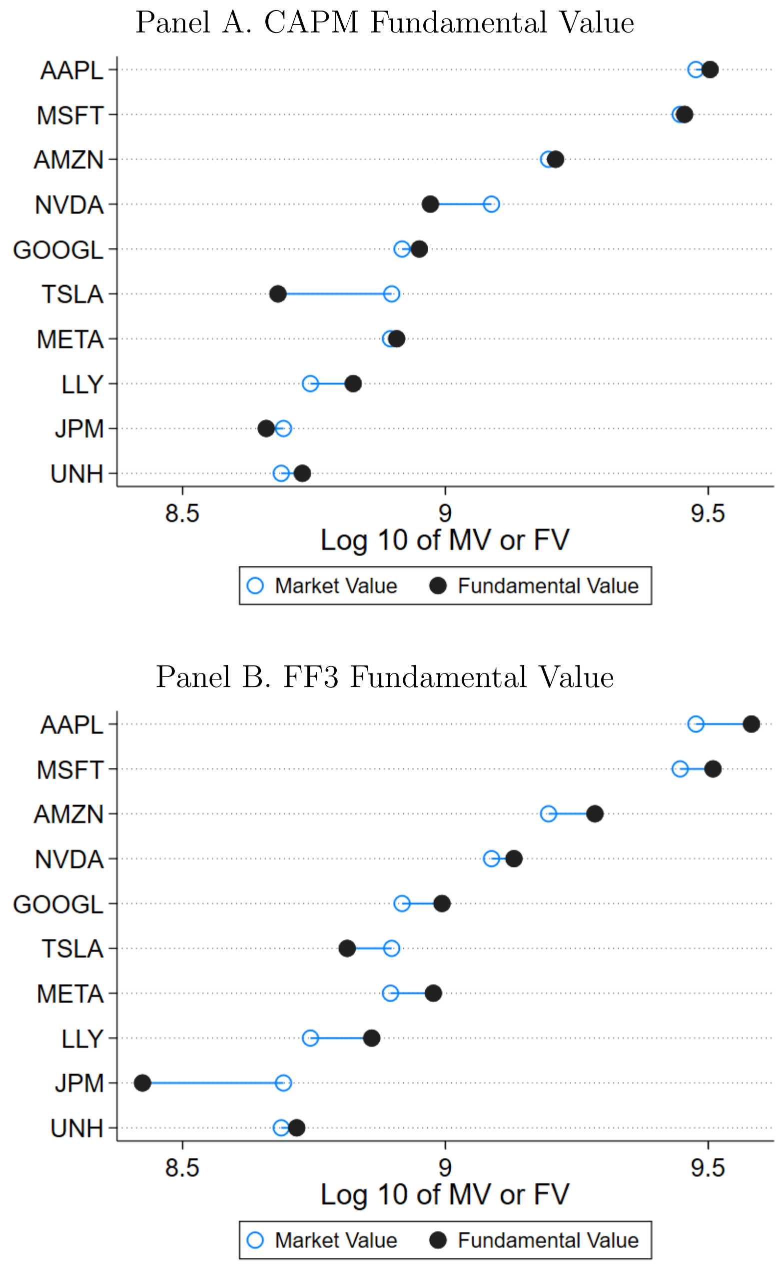

Figure 2: **Fundamental Equity Values of Top Companies (December 2023)**

This figure compares the market value of the 10 largest US stocks as of the end of December 2023 to their fundamental value implied by either the CAPM (Panel A) or the Fama and French (1993) three-factor model.

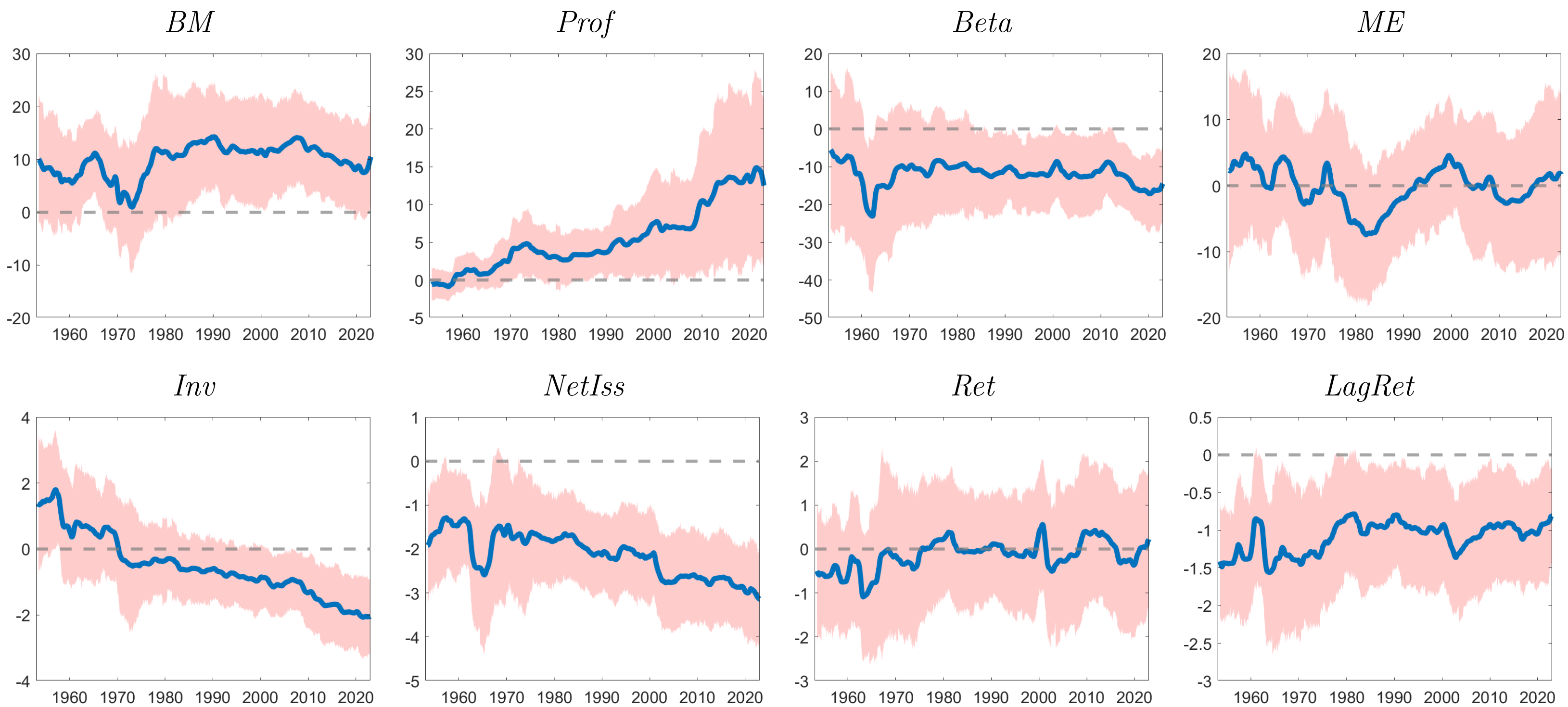

Figure 3: **Moving-Window Multivariate Coefficients of Stock-Level CAPM $\frac{V}{P}$ on Characteristics**

The figure reports the multivariate projection coefficients, γ_V, linking stock-level CAPM $\frac{V}{P}$ to stock characteristics. We estimate these coefficients in rolling windows that cover 50 years (with 15 years as a minimum window size at the beginning of the sample period) over the period 1953m6–2023m12. The shaded area represents the 95% bootstrap confidence interval.

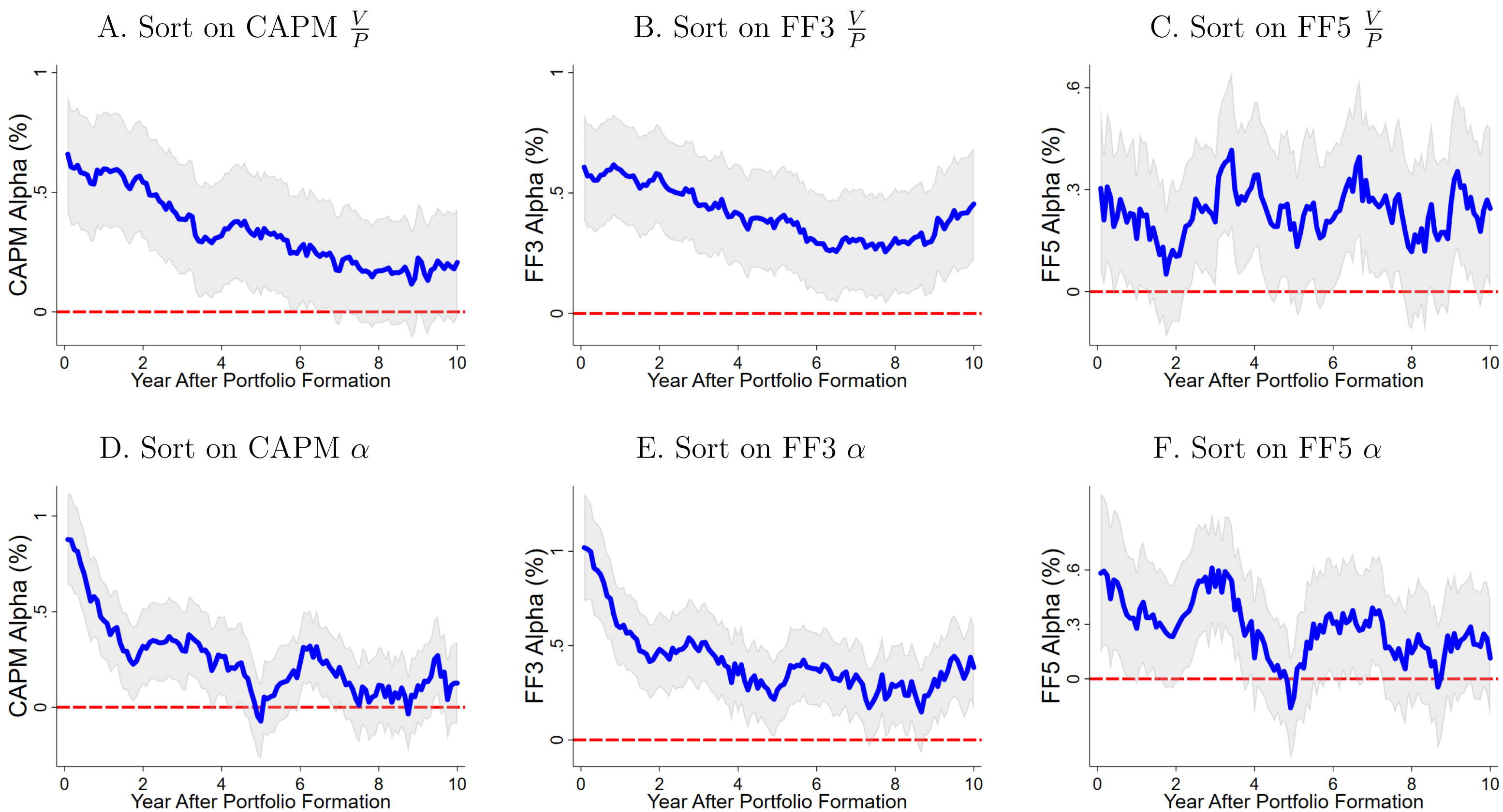

Figure 4: **Post-Formation Alphas on Portfolios Sorted on Out-of-Sample V/P**

The figure reports the evolution of alpha on long-short quintile portfolios formed by sorting on out-of-sample model-specific V/P. The bottom row repeats the analysis using portfolios sorted on the corresponding out-of-sample estimates of one-month α. Across all panels, the gray shaded area represents the 95% bootstrap confidence interval. The sample period is 1953m6–2023m12 for the CAPM and the Fama and French (1993) three-factor model and 1979m6–2023m12 for the Fama and French (2015) five-factor model.

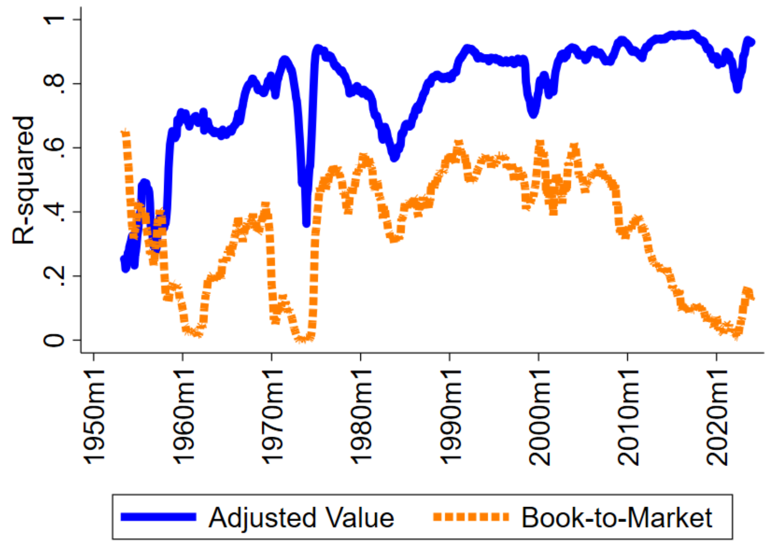

Figure 5: **Adjusted Value as a Proxy for CAPM-Implied Misvaluations: Comparison to Simple Book-to-Market Equity**

The figure plots the time-series variation in the cross-sectional R^2 from regressing out-of-sample CAPM $\frac{V}{P}$ on either the *adjusted value* metric of Cho and Polk (2024) (blue solid line) or the book-to-market equity ratio (orange dashed line). Adjusted value is defined as $z(B/M) + z(Prof) - z(Beta)$, where z is the standardized cross-sectional rank score of the characteristic. The sample period is 1953m6–2023m12.

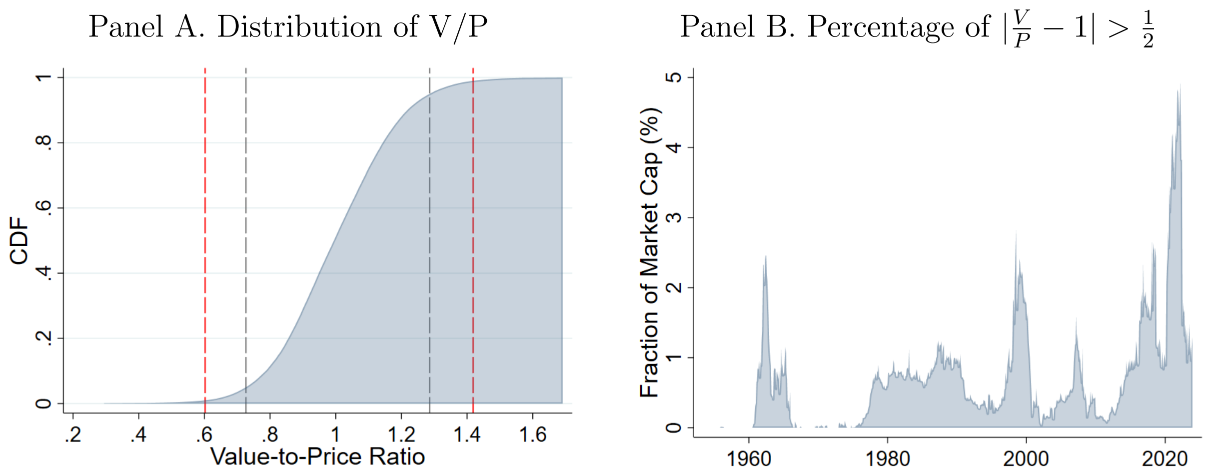

Figure 6: **Distribution of CAPM-Implied Value-to-Price Ratio**

These figures show aspects of the distribution of stock-level $\frac{V}{P}$, generated from the out-of-sample implementation of our procedure. Panel A plots the distribution of $\frac{V}{P}$ estimated with respect to the CAPM. We value-weight across firms and equal-weight across time when measuring that distribution. The gray lines denote the 5% and 95% cutoff values and the red lines denote the 1% and 99% cutoff values. Panel B plots the fraction of total market capitalization that has a $\frac{V}{P}$ estimate falling outside of the range of 0.5 to 1.5. The sample period is 1953m6–2023m12.

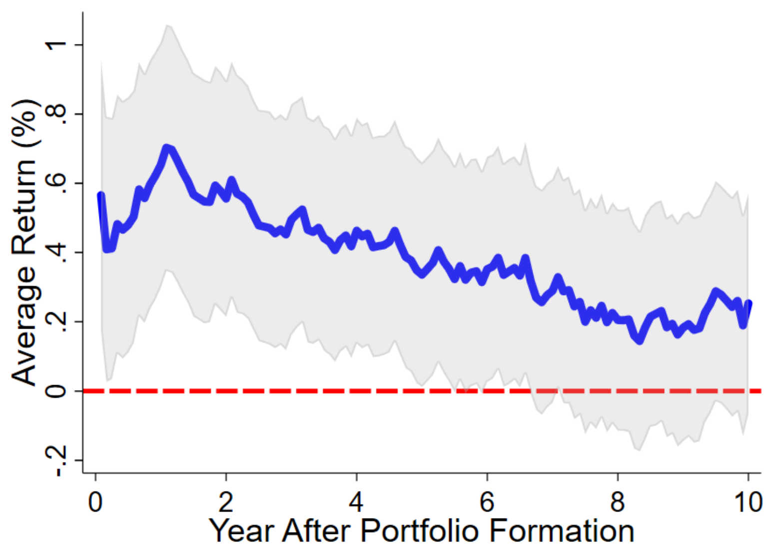

Figure 7: **Post-Formation Return Differences: High vs. Low Excess-Return $\frac{V}{P}$**

The figure plots long-term average return differences across extreme decile portfolios sorted on our out-of-sample estimated excess-return $\frac{V}{P}$. The gray shaded area represents the 95% bootstrap confidence interval. So that our results are directly comparable to those of Keloharju et al. (2021), we limit our analysis to the sample period of 1963m6–2018m12.

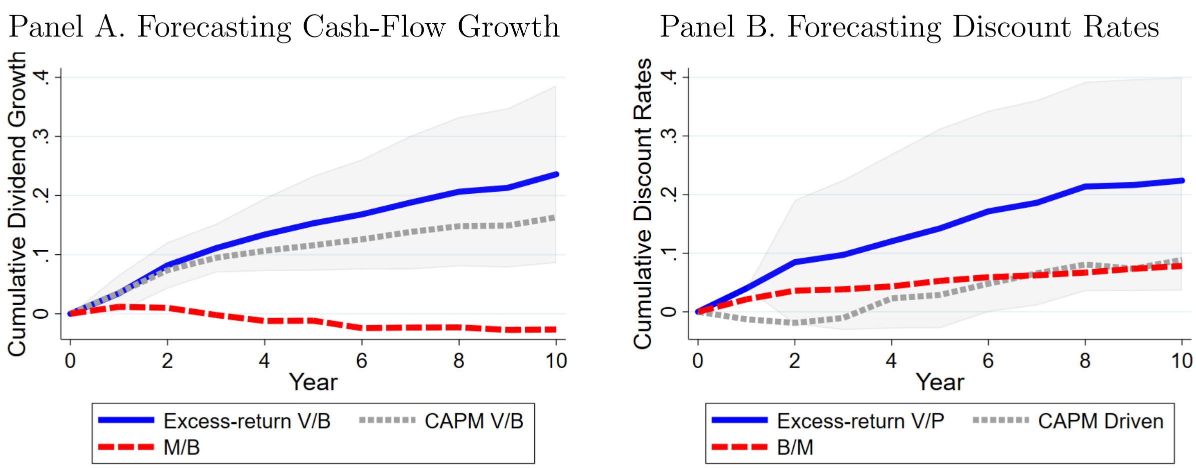

Figure 8: **A Valuation Ratio Predicts Future Cash-Flow Growth**

Panel A of the figure plots regression coefficients from predicting N-year post-formation cumulative dividend growth rates on the 25 Fama-and-French (1993) size and book-to-market portfolios using their market-to-book ratio (M/B) (red dashed line), CAPM $\frac{V}{B}$ (gray dotted line), or excess-return $\frac{V}{B}$ (blue solid line). Panel B of the figure then forecasts N-year cumulative future stock returns using either the book-to-market ratio (B/M) (red dashed line), CAPM $\frac{V}{P}$ (gray dotted line), or excess-return $\frac{V}{P}$ (blue solid line). In both plots, the gray shaded area represents the 95% bootstrap confidence interval.

Table 1: **Descriptive Statistics**

The table reports the correlation (Panel A) and autocorrelation (Panel B) matrix for the eight main characteristics used in the paper. We cross-sectionally rank-transform the first six characteristics and then standardize all variables by their cross-sectional value-weight standard deviation. The sample period is 1953m6–2023m12.

A. Cross-Sectional Correlations

	BM	*Prof*	*Beta*	*ME*	*Inv*	*NetIss*	*Ret*	*LagRet*
BM	1.00	-0.27	-0.18	-0.36	-0.23	-0.17	-0.29	-0.21
Prof	-0.27	1.00	0.05	0.07	0.13	-0.05	0.09	0.11
Beta	-0.18	0.05	1.00	0.31	0.08	0.19	-0.03	0.02
ME	-0.36	0.07	0.31	1.00	0.23	-0.14	0.17	0.16
Inv	-0.23	0.13	0.08	0.23	1.00	0.19	0.07	0.26
NetIss	-0.17	-0.05	0.19	-0.14	0.19	1.00	-0.00	0.06
Ret	-0.29	0.09	-0.03	0.17	0.07	-0.00	1.00	0.01
LagRet	-0.21	0.11	0.02	0.16	0.26	0.06	0.01	1.00

B. Autoregressive Coefficients

	12-Month Lag							
	BM	*Prof*	*Beta*	*ME*	*Inv*	*NetIss*	*Ret*	*LagRet*
BM	0.83	-0.03	-0.01	-0.00	0.05	0.01	0.02	0.01
Prof	-0.05	0.89	0.00	-0.01	-0.05	-0.01	0.02	-0.01
Beta	-0.01	-0.01	0.90	0.01	0.01	0.04	0.03	0.02
ME	0.03	0.02	0.00	0.99	-0.02	-0.03	0.02	0.00
Inv	-0.20	0.04	-0.05	0.04	0.17	0.10	0.13	0.06
NetIss	-0.14	-0.08	0.10	-0.10	0.01	0.42	0.03	-0.00
Ret	0.11	0.08	-0.01	-0.01	-0.06	-0.08	0.04	0.01
LagRet	0.00	-0.00	0.00	-0.00	-0.00	0.00	0.97	-0.00

Table 2: **Full-Sample Estimates of Stock-level $\frac{V}{P}$**

Each column reports, in percentage units, the estimates ($\widehat{\gamma}_V$) linking characteristics (z) to a stock's value-to-price ratio ($\frac{V}{P}$):

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \gamma_V z_{i,t} + u_{i,t},$$

where $V_{i,t} \equiv \sum_{s=1}^{\infty} E_t \left[\widetilde{M}_{t,t+s} D_{i,t+s} \right]$ is the fundamental cash-flow value of stock i at time t, $\widetilde{M}_{t,t+s}$ is a candidate cumulative discount factor that depends on the factor model of risk, $P_{i,t}$ is the market price, and $u_{i,t}$ is a projection error. Columns report estimates with respect to different factor models (the CAPM, the three-factor model of Fama and French (1993), the five-factor model of Fama and French (2015)) or different sets of characteristics. We report coefficients in percentage units and bootstrap absolute t statistics in parentheses. Estimates are based on value-weight stock-level panel regressions over the full sample period of 1953m6–2023m12.

	Factor Model					
	CAPM		Three-factor		Five-factor	
Characteristic	(1)	(2)	(3)	(4)	(5)	(6)
BM	9.30	6.95	-3.90	-5.50	0.07	0.37
	(1.91)	(1.50)	(1.71)	(2.44)	(0.03)	(0.14)
Prof	12.53	12.41	19.00	18.73	18.68	18.55
	(2.75)	(2.83)	(4.84)	(4.87)	(4.65)	(4.46)
Beta	-14.83	-14.03	-10.57	-9.96	-0.52	-0.92
	(3.09)	(3.00)	(2.34)	(2.22)	(0.12)	(0.21)
ME	1.15	1.13	-0.55	-0.63	-6.20	-6.14
	(0.24)	(0.26)	(0.15)	(0.19)	(3.09)	(3.09)
Inv	-2.04	-1.80	-2.21	-1.96	-0.55	-0.56
	(3.59)	(3.41)	(3.99)	(3.69)	(1.08)	(1.11)
NetIss	-3.07	-2.85	-2.26	-2.06	-0.61	-0.66
	(5.65)	(5.47)	(4.51)	(4.33)	(1.06)	(1.22)
Ret	0.83	-0.19	1.32	0.39	2.72	2.87
	(1.58)	(0.27)	(2.45)	(0.55)	(3.63)	(3.29)
LagRet		-1.05		-0.99		0.09
		(3.14)		(2.81)		(0.21)

Table 3: **Out-of-Sample Estimates in 2023m12 of Stock-level $\frac{V}{P}$**

Each column reports, in percentage units, the estimates ($\widehat{\gamma}_V$) linking characteristics (z) to a stock's value-to-price ratio ($\frac{V}{P}$):

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \gamma_V z_{i,t} + u_{i,t},$$

where $V_{i,t} \equiv \sum_{s=1}^{\infty} E_t \left[\widetilde{M}_{t,t+s} D_{i,t+s} \right]$ is the fundamental cash-flow value of stock i at time t, $\widetilde{M}_{t,t+s}$ is a candidate cumulative discount factor that depends on the factor model of risk, $P_{i,t}$ is the market price, and $u_{i,t}$ is a projection error. Columns report estimates with respect to different factor models (the CAPM, the three-factor model of Fama and French (1993), the five-factor model of Fama and French (2015)) or different sets of characteristics. We report coefficients in percentage units and bootstrap absolute t statistics in parentheses. Estimates are based on value-weight stock-level panel regressions over the 50-year moving window of 1974m1–2023m12, providing out-of-sample estimates for 2023m12.

	Factor Model					
	CAPM		Three-factor		Five-factor	
Characteristic	(1)	(2)	(3)	(4)	(5)	(6)
BM	11.92	10.09	-1.78	-2.76	2.89	3.30
	(2.27)	(2.01)	(0.83)	(1.24)	(1.07)	(1.11)
Prof	12.99	12.97	23.41	23.32	15.06	15.01
	(2.08)	(2.12)	(4.73)	(4.69)	(2.98)	(3.05)
Beta	-13.58	-12.96	-7.10	-6.92	-1.90	-2.43
	(2.64)	(2.61)	(1.67)	(1.63)	(0.36)	(0.45)
ME	0.11	0.25	-2.11	-2.08	-6.08	-5.89
	(0.02)	(0.04)	(0.46)	(0.49)	(2.10)	(2.23)
Inv	-2.35	-2.16	-2.78	-2.59	-0.27	-0.30
	(3.47)	(3.46)	(4.94)	(4.99)	(0.49)	(0.61)
NetIss	-3.33	-3.16	-2.59	-2.47	-0.63	-0.70
	(5.55)	(5.54)	(4.57)	(4.60)	(1.09)	(1.25)
Ret	1.09	0.29	1.72	1.10	2.43	2.69
	(1.85)	(0.34)	(2.77)	(1.45)	(3.16)	(2.91)
LagRet		-0.78		-0.63		0.17
		(1.97)		(1.60)		(0.39)

Table 4: **Post-Formation Mispricing Measures of $\frac{V}{P}$-Sorted Portfolios**

The table reports, in percentage units, the five-year model-specific cumulative abnormal returns (CARs) and average post-formation Cho-Polk portfolio-level $\frac{V}{P}$ for the extreme quintile portfolios sorted on stock-level out-of-sample $\frac{V}{P}$ with respect to various factor models. For both CAR and Cho-Polk average $\frac{V}{P}$, we exploit a calendar-time approach where, for instance, the CAR is the sum of alphas on 60 portfolios formed in each of the preceding 60 months based on portfolio sorts on $\frac{V}{P}$ estimated at that point in time. In Panel B, we do not report the results for FF5 V/P, since the limited sample period prevents estimating the Cho-Polk average $\frac{V}{P}$. We bold the diagonal elements as those estimates as we expect those estimates to be economically and statistically significant.

Panel A. Five-year Cumulative Abnormal Return (CAR)

	5-Year CAR		
Sorting Variable	CAR_{CAPM}	CAR_{FF3}	CAR_{FF5}
CAPM V/P	**26.58**	19.11	2.95
	(4.09)	(2.95)	(0.43)
FF3 V/P	23.94	**28.01**	15.49
	(4.24)	**(4.93)**	(2.58)
FF5 V/P	2.79	11.53	**13.42**
	(0.29)	(2.57)	**(2.87)**

Panel B. Portfolio Average $\frac{V}{P}$ (Cho and Polk, 2024)

	Model-Specific Average $\frac{V}{P}$		
Sorting Variable	Avg CAPM $\frac{V}{P}$	Avg FF3 $\frac{V}{P}$	Avg FF5 $\frac{V}{P}$
CAPM V/P	**50.06**	28.60	23.83
	(2.40)	(2.00)	(2.16)
FF3 V/P	62.04	**59.57**	18.83
	(2.76)	**(2.77)**	(2.38)

Table 5: **Fundamental Value of Russell 1000/2000 Constituents**

This table reports estimates of regressions of fundamental value on the Russell 1000/2000 constituent effect of Chang et al. (2015). In particular, we regress out-of-sample CAPM $\frac{V}{P}$ in percentage units on four indicator variables for the bottom of Russell 1000 (bottom 150 stocks in the index), top of Russell 2000 (top 150 stocks in the index), Russell 1000, and Russell 2000. We report t-statistics based on standard errors that are robust to both time and stock-level clustering. The sample period is 1987–2019.

Dependent Variable: Out-of-Sample CAPM $\frac{V}{P}$			
Bottom of Russell 1000	5.00		5.01
	(4.71)		(4.72)
Top of Russell 2000		-7.87	-7.91
		(9.17)	(9.11)
Russell 1000	-3.90		-3.59
	(2.84)		(2.02)
Russell 2000		3.39	1.29
		(4.12)	(1.42)
Fixed effect	Yes	Yes	Yes

Table 6: **Incremental Information in Misvaluation Measures for Stock-level $\frac{V}{P}$**

The table reports, in percentage units, the in-sample estimated projection coefficients (γ_V) of stock-level value-to-price ratio ($\frac{V}{P}$) on a vector of stock characteristics (z) that includes (an) existing measure(s) of misvaluation:

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \gamma_V z_{i,t} + u_{i,t},$$

where $V_{i,t} \equiv \sum_{s=1}^{\infty} E_t\left[\widetilde{M}_{t,t+s} D_{i,t+s}\right]$ is the buy-and-hold fundamental value of future dividends of stock i at time t discounted with respect to a candidate stochastic discount factor $\widetilde{M}$, $P_{i,t}$ is the market price, and $u_{i,t}$ is a projection error. In each regression, we add one of the following misvaluation measures to the specification considered in Table 2 COlumn (2): the fundamental-to-market ratio (*FE/ME*) of Gonçalves and Leonard (2023) (GL), the composite mispricing measures of Stambaugh and Yuan (2017) (SY), and the quality metric of Asness et al. (2019) (AFP). We report coefficients in percentage units and bootstrap absolute t statistics in parentheses. The $\frac{V}{P}$'s are estimated with respect to the CAPM.

	FE/ME (GL)		*Mispricing Factors* (SY)		*Quality* (AFP)	
BM	11.02	(1.71)	9.77	(2.20)	8.59	(1.92)
Prof	10.46	(1.61)	11.58	(2.38)	12.98	(2.41)
Beta	-15.95	(2.37)	-12.49	(2.70)	-12.72	(2.77)
ME	0.86	(0.12)	0.43	(0.86)	-1.79	(0.33)
Inv	-1.10	(1.51)	-0.31	(0.67)	-1.69	(3.33)
NetIss	-2.89	(4.22)	-0.31	(1.05)	-1.69	(5.44)
Ret	-0.56	(0.54)	-1.20	(1.69)	0.36	(0.51)
LagRet	-0.52	(0.99)	-1.04	(3.06)	-1.01	(2.82)
FE/ME	8.14	(3.22)				
Mgmt			3.26	(4.26)		
Perf			4.06	(3.12)		
Quality					2.57	(3.02)
Sample period	1975–2018		1953–2023		1953–2023	

Table 7: **Do Discretionary Buy-and-Hold Investors Tilt Towards $\frac{V}{P}$ or Alpha?**

We regress out-of-sample CAPM $\frac{V}{P}$ (Panel A, in % units) or out-of-sample one-month CAPM alpha (Panel B, in % units) on an indicator variable for whether the stock is held by Berkshire Hathaway (Warren Buffett) or a broader group of discretionary buy-and-hold investors (Berkshire Hathaway, Tiger Management (closed in 2001), Capital Group, and Dodge & Cox). Panel C regresses the cross-sectionally standardized characteristic ranks of the portfolio on the indicator variables. All regressions assign the same weight to all time periods by deflating all variables by the number of stocks in that month. Equal weight (EW) gives the same weight to all stocks in the cross-section, whereas value weight (VW) uses the market capitalization as the cross-sectional weight. All regressions include a time fixed effect and size control. We report t-statistics based on standard errors that are robust to both time and stock-level clustering. The sample period is 1980–2023.

Panel A. Underpricing (LHS: Out-of-Sample CAPM $\frac{V}{P}$)

Buffett	8.97		6.89	4.91		4.11
	(5.20)		(3.95)	(2.09)		(1.76)
Discretionary		3.29	2.91		4.17	3.74
		(6.29)	(5.58)		(4.59)	(4.39)
Regression weight	EW	EW	EW	VW	VW	VW

Panel B. Short-term Alpha (LHS: Out-of-Sample CAPM One-Month α)

Buffett	0.053		0.090	0.041		0.037
	(2.23)		(3.75)	(1.20)		(1.07)
Discretionary		-0.047	-0.052		0.026	0.022
		(5.67)	(6.24)		(1.85)	(1.68)
Regression weight	EW	EW	EW	VW	VW	VW

Panel C. Characteristics of Stock Holdings

LHS:	BM	Prof	Beta	Inv	NetIss	Ret	LagRet
Buffett	-0.160	0.067	-0.336	-0.154	-0.241	-0.080	-0.067
	(1.16)	(0.40)	(2.59)	(1.63)	(2.58)	(2.19)	(1.94)
Discretionary	0.099	0.038	-0.155	-0.204	-0.242	-0.222	-0.223
	(1.95)	(0.68)	(3.24)	(6.31)	(6.44)	(12.17)	(10.54)
Regression weight	VW	VW	VW	VW	VW	VW	VW

Table 8: **Private Equity Funds Buy Low and Sell High**

This table shows that stocks delisted due to private equity buyout tend to be significantly underpriced (relative to the CAPM), whereas those sold publicly by private equity funds tend to be significantly overpriced according to our estimates. Interestingly, the characteristics private equity funds look for when buying or selling coincide with the characteristics our model shows predict CAPM misvaluation. The sample period is 1981 to 2023.

Panel A. Equity Valuation Relative to the CAPM

	Dependent Variable: Out-of-Sample CAPM $\frac{V}{P}$				
PE Buyout	8.87	3.19			6.74
	(11.78)	(34.61)			(33.15)
PE Exit (Sale)			-14.36	-12.50	-15.92
			(25.05)	(21.39)	(16.76)
Sample	Delisting stocks	All	IPO stocks	All	All

Panel B. Characteristics of Stocks: Buyout

BM	Prof	Size	Beta	Inv	NetIss	Ret	LagRet
0.211	0.536	-0.116	-0.310	-0.135	-0.143	-0.270	-0.078
(5.36)	(15.80)	(2.89)	(3.93)	(3.64)	(4.77)	(4.28)	(1.63)

Panel C. Characteristics of Stocks: Exit (Sale)

BM	Prof	Size	Beta	Inv	NetIss	Ret	LagRet
-0.427	0.350	0.981	1.172	0.039	0.338	0.064	0.050
(14.05)	(11.63)	(27.83)	(22.09)	(1.03)	(15.88)	(1.29)	(1.04)

A Additional Results

	Panel A. Value-to-Price (γ_V)	Panel B. Monthly Alpha (γ_α^{1m})
BM	9.3	0.17
Prof	12.5	0.21
Beta	-14.8	-0.16
ME	1.2	0.01
Inv	-2.0	-0.03
NetIss	-3.1	-0.10
Ret	0.8	0.23

Figure A1: **Multi-Characteristic Model of Stock-Level $\frac{V}{P}$ and One-Month α: CAPM Benchmark**

These figures compare coefficients linking stock characteristics to $\frac{V}{P}$ to corresponding estimates linking those characteristics to α. We describe the former set of estimates in Table 2; we estimate the latter set of estimates in a value-weight stock-level panel regression over 1953m6–2023m12

Panel A. Fundamental Investing (OOS $\frac{V}{P}$ Sort) Panel B. Alpha Maximizing (OOS α^{1m} Sort)

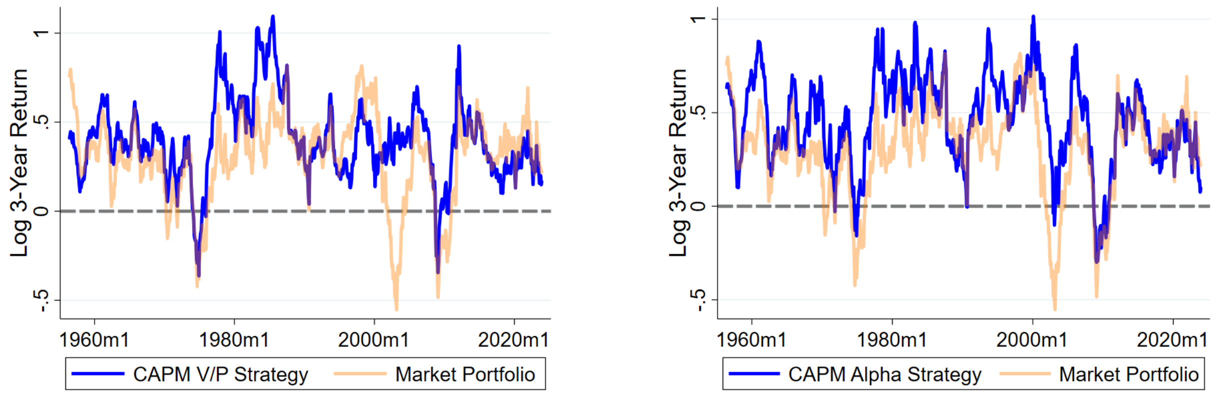

Figure A2: **3-Year Cumulative Returns: Stocks with High OOS CAPM V/P vs. with High OOS CAPM 1-Month Alpha**

The figure reports three-year log cumulative returns on stocks in either the top out-of-sample (OOS) CAPM $\frac{V}{P}$ decile (left plot) or in the top OOS one-month CAPM α decile (right plot). We plot 3-year log cumulative returns on the market portfolio in light orange for comparison. The sample period is 1953m6–2023m12.

Table A1: **Post-Formation Alphas on $\frac{V}{P}$-Sorted Portfolios: Comparison to the Simple Discounted Sum of Alphas Approach**

The table reports the average post-formation Cho-Polk portfolio $\frac{V}{P}$ across quintile portfolios sorted on stock-level out-of-sample CAPM $\frac{V}{P}$, but estimated using a simple discounted sum of alphas approach that ignores the covariance component of discounted alphas.

Portfolio Average $\frac{V}{P}$ (Cho and Polk, 2024)

	Model-Specific Average $\frac{V}{P}$		
Sorting Variable	Avg CAPM $\frac{V}{P}$	Avg FF3 $\frac{V}{P}$	Avg FF5 $\frac{V}{P}$
CAPM V/P (Simple Discounted Alphas)	**47.20**	21.26	22.14
	(2.29)	(1.20)	(1.99)

Table A2: **Testing Out-of-Sample Stock-Level $\frac{V}{P}$ with Average Portfolio $\frac{V}{P}$**

The table reports average portfolio $\widehat{\frac{V}{P}}$ based on the methodology of Cho and Polk (2023). The different columns specify the factor model with respect to which the stock-level $\frac{V}{P}$ is estimated. The different rows indicate the risk model used to estimate the Cho-Polk $\frac{V}{P}$. The reported coefficients are the estimated in-sample average (as opposed to conditional stock-level) $\frac{V}{P}$ and their difference between the two extreme quintile value-weighted portfolio formed based on monthly NYSE cutoff values of out-of-sample estimated stock-level $\frac{V}{P}$. This result is for the eight-characteristic version of our implementation. The result for the 5-factor, 5-factor version is inaccurate, since there is not enough sample years to reliably estimate the candidate SDF loadings on five different factors.

Stock $\frac{V}{P}$ Model,	Portfolio Average $\frac{V}{P} - 1$						
Portfolio $\frac{V}{P}$ Model	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
CAPM, CAPM	-18.76	0.73	10.92	30.92	31.30	50.06	0.016
	(-2.37)	(0.17)	(2.05)	(2.80)	(2.22)	(2.40)	
CAPM, 3-Factor	-11.58	-1.03	5.56	19.30	17.02	28.60	0.046
	(-1.87)	(-0.22)	(0.85)	(2.34)	(1.95)	(2.00)	
CAPM, 5-Factor	-5.64	-4.13	3.68	14.59	18.19	23.83	0.031
	(-1.74)	(-1.05)	(1.32)	(2.32)	(1.97)	(2.16)	
3-Factor, 3-Factor	-34.51	-13.88	6.82	21.83	25.07	59.57	0.006
	(-2.80)	(-2.38)	(1.45)	(3.37)	(2.42)	(2.77)	
3-Factor, CAPM	-31.41	-10.40	7.37	20.53	30.64	62.04	0.006
	(-2.25)	(-2.00)	(1.57)	(3.02)	(2.95)	(2.76)	
3-Factor, 5-Factor	-12.52	4.09	0.97	6.57	6.30	18.83	0.017
	(-1.88)	(0.69)	(0.23)	(1.47)	(1.33)	(2.38)	
5-Factor, 5-Factor	-0.36	-0.63	1.59	1.61	4.08	4.44	0.953
	(-0.02)	(-0.95)	(0.09)	(0.03)	(0.07)	(0.06)	
5-Factor, CAPM	-33.55	16.90	22.97	32.43	39.34	72.90	0.340
	(-1.07)	(1.37)	(0.89)	(0.87)	(0.80)	(0.95)	
5-Factor, 3-Factor	-17.14	12.86	7.30	10.29	12.17	29.32	0.377
	(-0.91)	(1.01)	(0.63)	(0.73)	(0.74)	(0.88)	

Table A3: **Validating Stock-Level $\frac{V}{P}$ Estimates with Average Portfolio $\frac{V}{P}$**

The table reports average portfolio $\frac{V}{P}$ based on the methodology of Cho and Polk (2023). The different columns specify the factor model with respect to which the stock-level $\frac{V}{P}$ is estimated. The different rows indicate the risk model used to estimate the Cho-Polk $\frac{V}{P}$. The reported coefficients are the estimated in-sample average (as opposed to conditional stock-level) $\frac{V}{P}$ and their difference between the two extreme quintile value-weighted portfolio formed based on monthly NYSE cutoff values of out-of-sample estimated stock-level $\frac{V}{P}$. This result is for the seven-characteristic version of our implementation.

Stock $\frac{V}{P}$ Model,	Portfolio Average $\frac{V}{P} - 1$						
Portfolio $\frac{V}{P}$ Model	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
CAPM, CAPM	-23.11	-7.01	8.52	24.49	35.49	58.60	0.001
	(-2.86)	(-1.50)	(1.51)	(3.18)	(3.11)	(3.44)	
CAPM, 3-Factor	-19.63	-8.69	4.64	16.42	24.28	43.91	0.023
	(-2.41)	(-1.71)	(0.78)	(2.06)	(2.05)	(2.27)	
CAPM, 5-Factor	-7.78	-4.85	-3.97	12.22	12.43	20.22	0.018
	(-2.49)	(-1.33)	(-1.15)	(1.86)	(1.95)	(2.36)	
3-Factor, 3-Factor	-26.15	-6.42	2.22	6.76	15.25	41.40	0.039
	(-2.13)	(-1.97)	(0.46)	(1.45)	(1.56)	(2.06)	
3-Factor, CAPM	-19.38	2.09	6.85	1.40	10.89	30.27	0.352
	(-1.05)	(0.19)	(0.74)	(0.17)	(0.68)	(0.93)	
3-Factor, 5-Factor	-11.40	1.26	-1.03	8.20	5.68	17.09	0.029
	(-1.46)	(0.37)	(-0.30)	(1.10)	(1.65)	(2.18)	
5-Factor, 5-Factor	-6.57	7.37	-2.37	6.29	5.81	12.38	0.042
	(-1.12)	(0.88)	(-0.75)	(2.39)	(1.41)	(2.03)	
5-Factor, CAPM	-41.63	0.57	23.32	24.10	50.99	92.63	0.216
	(-1.26)	(0.05)	(1.12)	(1.41)	(1.16)	(1.24)	
5-Factor, 3-Factor	-19.83	1.90	10.21	13.81	23.06	42.89	0.078
	(-1.51)	(0.23)	(1.43)	(2.10)	(1.80)	(1.76)	

Table A4: **Risk and Returns to a Fundamental Investing Strategy**

This table shows that a strategy that bets on high out-of-sample (OOS) estimates of CAPM $\frac{V}{P}$ deliver strong CAPM alphas along with low volatility and turnover. The strategy forms a monthly-rebalanced portfolio of the highest-decile OOS CAPM $\frac{V}{P}$. For comparison, we report the results from betting on the highest-decile out-of-sample CAPM one-month α^{1m}. The returns and volatilities are in percentages; we compute idiosyncratic volatility with respect to the three-factor model. Retention refers to the value-weighted probability that a stock in the portfolio remains in the portfolio after either one year or five years.

	High OOS $\frac{V}{P}$	High OOS α^{1m}
$\overline{R}^e$	0.84	1.04
$\sigma(R^e)$	3.88	4.60
α_{CAPM}	0.42	0.49
	(4.47)	(5.25)
β_{CAPM}	0.65	0.86
α_{FF3}	0.25	0.44
	(2.94)	(4.92)
$\beta_{FF3,MKT}$	0.67	0.81
$\beta_{FF3,SMB}$	0.17	0.34
$\beta_{FF3,HML}$	0.44	0.12
σ_{idio}	2.31	2.42
1-Yr Retention	73%	26%
5-Yr Retention	55%	21%

B Theory Appendix

B.1 The value-to-price identity (Lemma 1)

The definition of $V_{i,t}$ in equation (4) (Definition 1) and the law of iterated expectations imply that the fundamental asset pricing equation holds for $V_{i,t}$ with respect to $\widetilde{M}$:

$$V_{i,t} = E_t\left[\widetilde{M}_{t+1}\left(D_{i,t+1} + V_{i,t+1}\right)\right], \tag{20}$$

where $\widetilde{M}_{t+1}$ is the one-period candidate SDF. Dividing both sides by $P_{i,t}$ and doing some add-and-subtract gives

$$\frac{V_{i,t}}{P_{i,t}} = E_t\left[\widetilde{M}_{t+1}(\frac{D_{i,t+1}}{P_{i,t}} + \frac{P_{i,t+1}}{P_{i,t}} - \frac{P_{i,t+1}}{P_{i,t}} + \frac{V_{i,t+1}}{P_{i,t}}\right] + \underbrace{1 - E_t\left[\widetilde{M}_{t+1}(1+R_{f,t})\right]}_{=0 \text{ if } \widetilde{M} \text{ explains the risk-free rate}} \tag{21}$$

Next, rearrange the terms to get the law of motion for $\frac{V}{P}$:

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \underbrace{\frac{\alpha_{i,t}}{1+R_{f,t}}}_{=E_t\left[\widetilde{M}_{t+1}R^e_{i,t+1}\right]} + E_t\left[\widetilde{M}_{t+1}\frac{P_{i,t+1}}{P_{i,t}}\left(\frac{V_{i,t+1}}{P_{i,t+1}} - 1\right)\right]. \tag{3}$$

B.2 An approximate value-to-price identity

Up to a small approximation error, we can re-express the term inside the expectation in equation (3) in terms of the first and second moments of $\frac{V}{P}$ at time $t+1$:

$$\begin{aligned}\frac{V_{i,t}}{P_{i,t}} - 1 &= \frac{\alpha_{i,t}}{1+R_{f,t}} + E_t\left[\widetilde{M}_{t+1}(1+G_{i,t+1})\right] E_t\left[\frac{V_{i,t+1}}{P_{i,t+1}} - 1\right] \\ &\quad + (1+E_t\left[G_{i,t+1}\right])\, Cov_t\left(\widetilde{M}_{t+1}, \frac{V_{i,t+1}}{P_{i,t+1}}\right) + \frac{1}{1+R_{f,t}} Cov_t\left(G_{i,t+1}, \frac{V_{i,t+1}}{P_{i,t+1}}\right),\end{aligned} \tag{22}$$

where $1 + G_{i,t+1} \equiv \frac{P_{i,t+1}}{P_{i,t}}$ denotes capital gain.

The restated identity, which simplifies the exact law of motion in equation (3), further elucidates the intuition behind the identity. If $E_t\left[\widetilde{M}_{t+1}(1+R_{i,t+1})\right]$ equals one on average, $E_t\left[\widetilde{M}_{t+1}(1+G_{i,t+1})\right]$ is less than one on average. Hence, the term acts as a time discount on the conditional next-period $\frac{V}{P}$. The next two covariance terms shows that $\frac{V}{P}$ that occurs in a high-$\widetilde{M}$ state or a high-capital-gain state matters more. Furthermore, having expressed the identity in terms of the first two moments is useful, since these moments are easier to relate to the existing asset pricing literature on short-horizon expected returns than the third moment.

To see how we arrive at the approximate identity, The definitions of covariance and coskewness tell us that for any random variables A, B, C with standard deviations σ_A, σ_B, σ_C:

$$E(ABC) = E(AB)E(C) + E(A)Cov(B,C) + E(A)cov(B,C) + Coskew(A,B,C)\sigma_A\sigma_B\sigma_C$$

Where:

$$Coskew(A,B,C) = \frac{E[(A - E(A))(B - E(B))(C - E(C))]}{\sigma_A\sigma_B\sigma_C}$$

If we apply this identity to the product of $\widetilde{M}_{t+1}$, $\frac{P_{i,t+1}}{P_{i,t}}$, and$\left(\frac{V_{i,t+1}}{P_{i,t+1}} - 1\right)$ and define $G_{i,t+1} = \frac{P_{i,t+1}}{P_{i,t}} - 1$, then the law of motion in equation (3) becomes:

$$\frac{V_{i,t}}{P_{i,t}} - 1 = \frac{\alpha_{i,t}}{1 + R_{f,t}} + E_t\left[\widetilde{M}_{t+1}\left(1 + G_{i,t+1}\right)\right] E_t\left[\frac{V_{i,t+1}}{P_{i,t+1}} - 1\right] + \left(1 + E_t\left[G_{i,t+1}\right]\right) Cov_t\left(\widetilde{M}_{t+1}, \frac{V_{i,t+1}}{P_{i,t+1}}\right)$$

$$+ E_t\left[\widetilde{M}_{t+1}\right] Cov_t\left(G_{i,t+1}, \frac{V_{i,t+1}}{P_{i,t+1}}\right) + Coskew_t(\widetilde{M}_{t+1}, G_{i,t+1}, \frac{V_{i,t+1}}{P_{i,t+1}})\sigma_t\left(\widetilde{M}_{t+1}\right)\sigma_t\left(G_{i,t+1}\right)\sigma_t\left(\frac{V_{i,t+1}}{P_{i,t+1}}\right)$$

The last coskewness term is small compared to the terms involving a covariance. To see this, the bound on the correlation implies

$$\left|\left(1 + E_t\left[G_{i,t+1}\right]\right) Cov_t\left(\widetilde{M}_{t+1}, \frac{V_{i,t+1}}{P_{i,t+1}}\right)\right| \leq \left(1 + E_t\left[G_{i,t+1}\right]\right)\sigma_t\left(\widetilde{M}_{t+1}\right)\sigma_t\left(\frac{V_{i,t+1}}{P_{i,t+1}}\right)$$

$$\left|E_t\left[\widetilde{M}_{t+1}\right] Cov_t\left(G_{i,t+1}, \frac{V_{i,t+1}}{P_{i,t+1}}\right)\right| \leq E_t\left[\widetilde{M}_{t+1}\right]\sigma_t\left(G_{i,t+1}\right)\sigma_t\left(\frac{V_{i,t+1}}{P_{i,t+1}}\right)$$

$$\left|Coskew_t(\widetilde{M}_{t+1}, G_{i,t+1}, \frac{V_{i,t+1}}{P_{i,t+1}})\sigma_t\left(\widetilde{M}_{t+1}\right)\sigma_t\left(G_{i,t+1}\right)\sigma_t\left(\frac{V_{i,t+1}}{P_{i,t+1}}\right)\right| \leq$$

$$\left|Coskew_t(\widetilde{M}_{t+1}, G^e_{i,t+1}, \frac{V_{i,t+1}}{P_{i,t+1}})\right|\sigma_t\left(\widetilde{M}_{t+1}\right)\sigma_t\left(G_{i,t+1}\right)\sigma_t\left(\frac{V_{i,t+1}}{P_{i,t+1}}\right)$$

The coskewness term involves the product of three standard deviation, whereas the covariance terms only involve two. For a realistic assumptions and calibrations, the coskewness is of an order not greater than 1. For example, under joint lognormality a reasonable calibration gives a maximum absolute value coskewness of around 1.3. Hence, compared to the covariance terms, the coskewness term is smaller by the factor of the candidate SDF volatility or the capital gain volatility, which are substantially less than 1.

To confirm that the coskewness is not large without making any assumptions on the distribution of returns, we can calculate the empirical coskewness for all stocks with at least 10 annual observations in our dataset, using proxies for $\widetilde{M}_{t+1}$ and $\frac{V_{i,t+1}}{P_{i,t+1}}$. If we assume that $\widetilde{M}_{t+1} = 1 - R_{m,t+1}$ (i.e. CAPM), and proxy $\frac{V_{i,t+1}}{P_{i,t+1}}$ with market to book, then the median absolute value of empirical coskew is 0.17 and the largest for any stock is 2.2. If we instead assume $\widetilde{M}_{t+1} = R^{-1}_{m,t+1}$ so that the SDF is the marginal utility of a log investor fully invested in the market, then the median is 0.17 and the largest is 1.6. If we let $\widetilde{M}_{t+1} = R^{-20}_{m,t+1}$, allowing the marginal investor to have CRRA utility with an implausibly high level of risk aversion, than the median is still just 0.13 and the largest is 2.7. Thus any empirical estimation of the quantities involved in calculating $\frac{V}{P}$ is unlikely to place a large weight on the coskew term.

For this reason, we focus on estimating the covariance components and treat coskewness as part of the residual term of the regression that uncovers a value projection.

B.3 Linear projection in the approximate $\frac{V}{P}$ identity

We model $\frac{V}{P}$ as a linear projection on stock characteristics: $\frac{V_{i,t}}{P_{i,t}} - 1 = h(z_{i,t};\gamma_\delta) + u_{i,t}$. Plugging this in, equation (22) simplifies to

$$\begin{aligned} \frac{\alpha_{i,t}}{1+R_{f,t}} \quad = \quad & \gamma_V [z_{i,t} - E_t\left[\widetilde{M}_{t+1}(1+G_{i,t+1})\right] E_t\left[z_{i,t+1}\right] \\ & - \left(1+E_t\left[G_{i,t+1}\right]\right) Cov_t\left(\widetilde{M}_{t+1}, z_{i,t+1}\right) \\ & - \frac{1}{1+R_{f,t}} Cov_t\left(G_{i,t+1}, z_{i,t+1}\right)] + u_{i,t}, \end{aligned} \tag{23}$$

In deriving the last equation, we assume that the covariances involving $u_{i,t+1}$ are small: $Cov_t\left(\widetilde{M}_{t+1}, u_{i,t+1}\right) \approx 0$ and $Cov_t\left(G_{i,t+1}, u_{i,t+1}\right) \approx 0$. One may worry that capital gain and $u_{i,t+1}$, the unexplained part of abnormal price, would covary if a price run-up is a signal of overpricing, for instance. However, such an effect would be absorbed by the characteristic vector $z_{i,t+1}$ if it includes the return characteristic (momentum).

To understand the regression model in equation (23), note that the expression in square brackets is approximately equal to $E_t\left[z_{i,t} - \widetilde{M}_{t+1}\left(1+G_{i,t+1}\right)z_{i,t+1}\right]$.[27] In other words, it is the expected change in the discounted, price-adjusted level of the characteristics. If we think of the characteristics as a stock of value that gives us a flow of alphas, then this metric shows how much of the characteristic stock is "paid out" to deliver the alphas for the period. Regressing this metric on the alphas tells us how much alpha is delivered by each increment of the characteristic, which is what the slope coefficient γ_V measures. Multiplying this coefficient by the characteristic's level tells us how much discounted cumulative alpha we should expect to realize as the whole characteristic level is paid out—i.e., $\frac{V}{P}-1$.

For example, suppose momentum is associated with positive alphas but decays fast (i.e. $E_t\left(z_{i,t+1}\right)$ is close to 0). Then $z_{i,t} - E_t\left[\widetilde{M}_{t+1}\left(1+G_{i,t+1}\right)z_{i,t+1}\right]$ is close to $z_{i,t}$, and so the delta coefficient is almost the same as the one period alpha coefficient. Whereas if, for example, the book-to-market-equity ratio decays very slowly, $E_t\left[z_{i,t} - \widetilde{M}_{t+1}\left(1+G_{i,t+1}\right)z_{i,t+1}\right]$ is close to 0. So any ability of the book-to-market-equity ratio to predict alpha will be associated with large $\frac{V}{P}-1$.

B.4 Estimating $\frac{V}{P}$ via discounted alphas

Applying the model in Section 1.3 to equation (23),

$$\begin{aligned}
\frac{1}{1+R_{f,t}}\alpha_{i,t} \quad = \quad & \gamma_V[z_{i,t} - \left(1+E_t\left[\widetilde{M}_{t+1}G^e_{i,t+1}\right]\right)E_t\left[z_{i,t+1}\right] \\
& -\left(1+R_{f,t}+E_tG^e_{i,t+1}\right)Cov_t\left(\beta_{z,i,t}f_{t+1},\widetilde{M}_{t+1}\right) \\
& -\frac{1}{1+R_{f,t}}Cov_t\left(\beta_{z,i,t}f_{t+1}+\epsilon_{z,i,t+1},f'_{t+1}\beta'_{G,i,t}+\epsilon_{G,i,t+1}\right)] + u_{i,t}.
\end{aligned}$$

[27] The approximation employed in equation (22) links this expression to the version in equation (23)

Since we require $E_t\left[\widetilde{M}_{t+1}f_{t+1}\right]=0$,

$$\begin{aligned}
E_t\left[\widetilde{M}_{t+1}G^e_{t+1}\right]=E_t\left[\widetilde{M}_{t+1}(\alpha_{G,i,t}+\beta_{G,i,t}f_{t+1})\right] &= \frac{1}{1+R_{f,t}}\alpha_{G,i,t}\\
E_tG^e_{i,t+1} &= \alpha_{G,i,t}+\beta_{G,i,t}\lambda_t\\
Cov_t\left(\beta_{z,i,t}f_{t+1},\widetilde{M}_{t+1}\right)=\underbrace{E_t\left[\widetilde{M}_{t+1}f_{t+1}\right]}_{=0}\beta'_{z,i,t}-\frac{\lambda_t\beta'_{z,i,t}}{1+R_{f,t}} &= -\frac{\lambda_t\beta'_{z,i,t}}{1+R_{f,t}}\\
Cov_t\left(\beta_{z,i,t}f_{t+1}+\epsilon_{z,i,t+1},f'_{t+1}\beta'_{G,i,t}+\epsilon_{G,i,t+1}\right) &= \beta_{G,i,t}\Sigma_{f,t}\beta'_{z,i,t}+\sigma_{G,z,i,t},
\end{aligned}$$

where $E_tf_{t+1}=\lambda_t$, $\Sigma_{f,t}\equiv Var_t(f_{t+1})$, and $\sigma_{G,z,i,t}\equiv Cov_t\left(\epsilon_{G,i,t+1},\epsilon_{z,i,t+1}\right)$. Hence, the equation at the top becomes

$$\begin{aligned}
\alpha_{i,t} &= \gamma_V[(1+R_{f,t})z_{i,t}-(1+R_{f,t}+\alpha_{G,i,t})\,E_tz_{i,t+1}\\
&\quad +\beta_{z,i,t}\lambda_t\left(1+R_{f,t}+\alpha_{G,i,t}+\lambda'_t\beta'_{G,i,t}\right)-\beta_{G,i,t}\Sigma_{f,t}\beta'_{z,i,t}-\sigma_{G,z,i,t}]+\widetilde{u}_{i,t},
\end{aligned}$$

where $\widetilde{u}_{i,t}\equiv(1+R_{f,t})u_{i,t}$. Rewriting,

$$\begin{aligned}
\alpha_{i,t} &= \gamma_V[\underbrace{(1+R_{f,t})z_{i,t}}_{\text{1. Alpha effect}}-\underbrace{(1+R_{f,t}+\alpha_{G,i,t})\,(\alpha_{z,i,t}+\beta_{z,i,t}\lambda_t)}_{\text{2. Expected discounted decay effect}}\\
&\quad +\underbrace{(1+R_{f,t}+\alpha_{G,i,t})\,\beta_{z,i,t}\lambda_t-\beta_{G,i,t}(\Sigma_{f,t}-\lambda_t\lambda'_t)\beta'_{z,i,t}}_{\text{3a. Systematic covariance effect}}-\underbrace{\sigma_{G,z,i,t}}_{\text{3b. Idiosyncratic covariance effect}}]+\widetilde{u}_{i,t}.
\end{aligned}$$

To see where these terms come from, recall that applying the covariance rule to the value-to-price identity in equation (3) gives

$$\frac{V_{i,t}}{P_{i,t}}-1=\underbrace{\frac{\alpha_{i,t}}{1+R_{f,t}}}_{\text{1. Alpha effect}}+\underbrace{E_t\left[\widetilde{M}_{t+1}\left(1+G_{i,t+1}\right)\right]E_t\left[\frac{V_{i,t+1}}{P_{i,t+1}}-1\right]}_{\text{2. Expected discounted decay effect}}+\underbrace{Cov_t\left(\widetilde{M}_{t+1}\left(1+G_{i,t+1}\right),\frac{V_{i,t+1}}{P_{i,t+1}}\right)}_{\text{3. Covariance effect}},$$

which implies that a characteristic has a higher γ_V coefficient if the characteristic (1) has a large alpha, (2) decays slowly, or (3) covaries with the price-augmented-$\widetilde{M}$ in a way that magnifies the γ_V. Applying this to the previous regression-style equation, for a given level of α, the coefficient γ_V is larger if

(1) z is small (so that the given α is generated from a small characteristic deviation);

(2) z decays slowly (i.e., $E_t\left[z_{i,t+1}\right]=\alpha_{z,i,t}+\beta_{z,i,t}\lambda_t$ is close to z, which means the given level of α is generated despite a small expected decay in $\frac{V}{P}$); or

(3) z covaries with $\widetilde{M}_{t+1}(1+G_{i,t+1})$ in a way that magnifies the overall effect by diminishing the expected risk-and-capital-gain-adjusted decay of z.

Crossing out the term $(1+R_{f,t}+\alpha_{G,i,t})\,\beta_{z,i,t}\lambda_t$ from the regression-style equation above,

$$\alpha_{i,t} \;\;=\;\; \gamma_V[(1+R_{f,t})z_{i,t}-(1+R_{f,t}+\alpha_{G,i,t})\,\alpha_{z,i,t}-\beta_{z,i,t}\,(\Sigma_{f,t}-\lambda_t\lambda_t')\,\beta_{G,i,t}'-\sigma_{G,z,i,t}]+\widetilde{u}_{i,t}.$$

The cancellation of the time-discounted risk premia term, $(1+R_{f,t}+\alpha_{G,i,t})\,\beta_{z,i,t}\lambda_t$, is revealing. The time-discounted next-period $\frac{V}{P}$ contains the effect of some characteristics having a higher conditional mean because of the characteristics' factor exposures. But that effect cancels out with the way next-period $\frac{V}{P}$ covaries with $\widetilde{M}_{t+1}$. For instance, if B/M has a negative market beta, it implies B/M will be lower in times of high market risk premia. So the "mean" part of next-period $\frac{V}{P}$ would suggest that high-B/M stocks are less underpriced in times of high market risk premia. But if high-B/M stocks are indeed less underpriced in times of high market risk premia (high marginal value of wealth), this lowers the contribution of next-period $\frac{V}{P}$ on high-B/M stocks (i.e., making it associated with more underpricing). So the two effects cancel each other out.

Hence, the equation states

$$\alpha_{i,t} \;\;=\;\; \gamma_\delta\left[(1+R_{f,t})(z_{i,t}-\alpha_{z,i,t})-\alpha_{G,i,t}\alpha_{z,i,t}-\underbrace{\beta_{z,i,t}\,(\Sigma_{f,t}-\lambda_t\lambda_t')\,\beta_{G,i,t}'}_{\text{“balanced-out term”}}-\sigma_{G,z,i,t}\right]+\widetilde{u}_{i,t}.$$

The presence of what we call the *balanced-out term*, $\beta_{z,i,t}\,(\Sigma_{f,t}-\lambda_t\lambda_t')\,\beta_{G,i,t}'$, means that estimation in the first stage requires additionally estimating conditional factor moments. However, this term is very small in practice such that dropping this term from the equation makes very little difference to the estimated coefficients $\widehat{\gamma_\delta}$. Hence, our simple regression model in equation (14) drops this term, which leads to our final expression:

$$(1+R_{f,t})(z_{i,t}-\alpha_{z,i,t})-\alpha_{G,i,t}\alpha_{z,i,t}-\sigma_{G,z,i,t}.$$

The simple model is useful, as it eliminates the need for estimating the conditional first and second moments of the factors, $\Sigma_{f,t}$ and $\lambda_{f,t}$.

Conceptually, when asset-level instruments $z_{i,t}$ are cross-sectionally demeaned to generate cross-sectional estimates of δ_t, which is the case in this paper, the balanced-out term is small for two reasons. First, the cross-sectionally demeaned instruments tend to have small exposures to aggregate factors, leading to small $\beta_{z,i,t}$ and $\beta_{G,i,t}$. The balanced-out term involves the product of the two β terms, which makes the component even smaller. Second, the conditional variance ($\Sigma_{f,t}$) and the squared conditional mean ($\lambda_t \lambda_t'$) of factors are around the same order of magnitude, which makes their difference small. This leads to the simple regression model of conditional δ.

We estimate γ_δ with the balanced-out term included to show that the results are very similar to our "simple" baseline approach that drops the balanced-out term. We estimate conditional factor premia by regressing annual realized factor returns (from June to next June) on market-wide book-to-market ratio and net issuance signals (e.g., Cohen et al. (2003); Greenwood and Hanson (2012); Cho et al. (2024)). We estimate the conditional variances and covariances of log factor realizations by first estimating realized annual return variance from daily return data and then obtaining fitted (conditional) values in a first-order autoregressive model. See Appendix C.3.1 for more details.

B.5 Excess-return-model $\frac{V}{P}$ via discounted alphas

Section 5 studies how much variation in price is accounted for by variation in discount rates. To answer this question, we measure what the approximate value-to-price ratios would be if all stocks were discounted at the same rate.

To do so, we consider the value of a stock to a hypothetical risk-neutral buy-and-hold investor for whom the market is correctly priced. Because this investor is risk-neutral, she applies the same discount rate to all stocks:

$$\widetilde{M}_{t+1} = \frac{1}{1 + E_t R_{m,t+1}}$$

Where $R_{m,t+1}$ is the return on the market

Note that the risk-free rate is not correctly priced to such a a risk-neutral investor (i.e. $E\left(\widetilde{M_{t+1}}(1 + R_t^f)\right) \neq 1$. Hence instead of using returns in excess of the risk-free rate in the approximate value-to-price identity, we will use returns in excess of the market.[28]

[28] Deriving the approximate identity using the mispriced risk-free-rate yields exactly the same expression, with a few extra steps of algebra

The value to price identity, after using a constant value for $\widetilde{M}$ and using excess-of-market returns instead of excess-of-risk-free becomes:

$$
\begin{aligned}
\frac{V_{i,t}^{RN}}{P_{i,t}} - 1 \quad = \quad & \frac{E_t(R_{i,t+1} - R_{m,t+1})}{1 + E_t R_{m,t+1}} + \left(1 + \frac{E_t(G_{i,t+1} - R_{m,t+1})}{1 + E_t R_{m,t+1}}\right) E_t \left[\frac{V_{i,t+1}^{RN}}{P_{i,t+1}} - 1\right] \\
& + \frac{1}{1 + E_t R_{m,t+1}} Cov_t \left(G_{i,t+1}, \frac{V_{i,t+1}^{RN}}{P_{i,t+1}}\right)
\end{aligned}
$$

Because $\widetilde{M}_{t+1}$ is no longer stochastic, there are no coskewnwess terms and this form of the identity holds exactly, not just approximately.

We apply the model of returns in section 1.3 to this identity to derive an empirical implementation of risk-neutral value. We will assume that the first of the k-factors is the return on the market minus the risk-free rate, as in all of our empirical tests.

Hence we have:

$$
\begin{aligned}
1 + E_t R_{m,t+1} \quad & = \quad 1 + R_{f,t} + \lambda_{1,t} \\
E_t(R_{i,t+1} - R_{m,t+1}) \quad & = \quad \alpha_{i,t} + \beta_{i,t}\lambda_t - \lambda_{1,t} \\
E_t(G_{i,t+1} - R_{m,t+1}) \quad & = \quad 1 + R_{f,t} + \alpha_{G,i,t} + \beta_{G,i,t}\lambda_t - \lambda_{1,t} \\
E_t z_{i,t+1} \quad & = \quad \alpha_{z,i,t} + \beta_{z,i,t}\lambda_t \\
Cov_t(z_{i,t+1}, G_{i,t+1}) \quad & = \quad \beta_{G,i,t}\Sigma_{f,t}\beta'_{z,i,t} + \sigma_{G,z,i,t}.
\end{aligned}
$$

And the projection form of the approximate identity becomes:

$$
\begin{aligned}
\alpha_{i,t} + \beta_{i,t}\lambda_t - \lambda_{1,t} \quad = \quad & \gamma_V [(1 + R_{f,t} + \lambda_{1,t}) z_{i,t} - (1 + R_{f,t} + \alpha_{G,i,t} + \beta_{G,i,t}\lambda_t)(\alpha_{z,i,t} + \beta_{z,i,t}\lambda_t) \\
& - \beta_{G,i,t}\Sigma_{f,t}\beta'_{z,i,t} - \sigma_{G,z,i,t}] + u_{i,t},
\end{aligned}
$$

We cannot drop as many factor loading terms as in the expression with risk because there is no longer a "balancing-out" effect as in B.4. The contribution of a characteristic to expected returns through factor-loadings now enters into the identity the same way that contributions through non-systematic "alphas" enter.

C Empirical Appendix

C.1 Additional empirical findings

C.1.1 Do firm mangers have private information about firm values?

Arguably the most important characteristics to study in our price-level context are investment and equity issuance, given the potential link between misvaluation and the allocation of capital by firms to real investment projects.[29] The empirical literature that finds real investment and equity issuance to be associated with stock overvaluation often interprets this evidence as firm managers having superior information about the firm's fundamental value.[30]

Both the in-sample results in Section 3.1 and the moving-window results in 3.2 show that net issuance and investment contain statistically significant incremental information about share misvaluation. In fact, net issuance has by far the largest t-statistics of around 4.5 to 5.5 in predicting CAPM or FF3 misvaluation ($\frac{V}{P}$).[31]

These findings are consistent with the survey evidence that firm CFOs tend to use the CAPM and that they respond to perceived under- or over-valuation of their shares by repurchasing or issuing equity shares (Graham and Harvey, 2001; Brav et al., 2005). However, the relatively modest degree of misvaluation implied by the coefficient on net issuance suggests that these actions generate only modest gains for shareholders. Hence, though firms know more about their valuation, the degree of informational asymmetry may not be striking, in line with the view that firms also learn from the market about their prospects (e.g., Dow and Gorton (1997), Edmans et al. (2012), Edmans, Goldstein, and

[29] That link may occur indirectly, through the equity issuance decision (Stein, 1996; Baker and Wurgler, 2002; Baker, Stein, and Wurgler, 2003), or directly, through catering by the firm to investor sentiment (Polk and Sapienza, 2009).

[30] Ikenberry, Lakonishok, and Vermaelen (1995) use a simple univariate sort and 4-year buy-and-hold returns to provide evidence that equity issuance is associated with share overpricing. Morck, Shleifer, Vishny, Shapiro, and Poterba (1990) is an example of earlier work linking stock prices and corporate investment. Baker, Ruback, and Wurgler (2007) reviews the literature linking stock prices with share issuance and repurchase.

[31] One may worry that simply looking at the coefficient on net issuance or investment could be misleading, since a typical firm engaging in issuance or investment may have a variety of motivations for doing so other than perceived share overvaluation. We find that using a composite measure of financial constraint (the average z-scores of ranks based on Kaplan and Zingales (1995), Whited and Wu (2006), and Hadlock and Pierce (2010)) to isolate firms whose net issuance decision is more purely motivated by market timing makes little difference to the results.

Jiang (2015)).

The coefficients on both investment and net issuance are no longer economically or statistically significant when modeling FF5 misvaluation. Of course, since one of Fama and French's factors is an investment factor, it is not surprising that the coefficient on investment is subsumed. The fact that net issuance no longer plays a role in FF5 misvaluation is consistent with Fama and French (2016), who show that their five-factor model explains the repurchase / issuance anomaly.[32]

C.1.2 Fundamental investing

Farboodi and Veldkamp (2020) estimate that, despite the growth of quantitative investing, more than half of active capital is devoted to fundamental investing. Suppose an investor had our machinery and went for the highest decile CAPM $\frac{V}{P}$ portfolio each month. We treat this portfolio as a proxy for the portfolio traded by a fundamental investor and study what the investment returns look like.

Figure A2 in the Internet Appendix shows that the fundamental investing strategy, compared to the strategy of maximizing out-of-sample alpha, has less volatile returns and almost completely avoids a drawdown during the dot com episode. On the other hand, fundamental investing seems to suffer at least as much—and sometimes more—in market crashes that are arguably caused at least in part by an aggregate cash-flow event (the 1973–1974 stock market crash and the 2007–2008 global financial crisis). This finding connects to earlier research that connects value investing to aggregate cash-flow risk (Campbell and Vuolteenaho, 2004) and suggests that fundamental investors require strong conviction about future cash-flow patterns.

Table A4 shows that the main advantage of fundamental investing might be the resulting low volatility and low turnover. The lower volatility seems to arise from a lower market beta, since idiosyncratic volatility after controlling for the three factors is roughly similar between the two strategies. These results may suggest that fundamental investing might be subject to unique risk that is not diversifiable to those who engage in fundamental investing. The low volatility could also imply that fundamental investing generates higher

[32]Of course, their finding does not mean that the pattern necessarily reflects systematic risk; however, it does mean that there is a "shared" story across this anomaly and the many other anomalies that their five-factor model explains.

long-term (log) returns for the same level of arithmetic average return, through the Jensen's correction term.[33] The low turnover is expected but still interesting, since it confirms that fundamental investors end up behaving like a long-term buy-and-hold investor even if they did not intend to. Since misvaluation tends to persist, an investor who keeps rebalancing to the most underpriced decile of stocks ends up not having to trade as much as one might think.

C.2 Further details on data and variables

C.2.1 Data sources and basic adjustments

We use domestic common stocks (CRSP share code *SHRCD* 10 or 11) listed on the three major exchanges (CRSP exchange code *EXCHCD* 1, 2, or 3). We replace missing prices with the average bid-ask price when available and drop observations with missing share or price information in the previous month. We code missing returns as zero returns and add delisting returns to returns. If delisting returns ($DLRET$) are missing, but the CRSP delisting code ($DLSTCD$) is 500 or between 520 and 584, we use -35% (-55%) as the delisting returns for NYSE and AMEX stocks (for NASDAQ stocks) (Shumway, 1997; Shumway and Warther, 1999). We compute capital gains $RETX$ in CRSP.

To compute stock characteristics, we use Compustat Quarterly, Compustat Annual, and the book equity data of Davis et al. (2000), in a descending order of preference. For Compustat, we use the CRSP/Compustat Merged Database. We assume that Compustat Quarterly information is available to investors 4 months after the month in which *DATA-DATE* falls in. We assume that Compustat Annual information for accounting year y is available to investors at the end of June of calendar year $y + 1$. We exclude stocks with less than two years of data to be able to compute characteristics that use accounting data or past returns.

C.2.2 Stock-level characteristics

Since our goal is to estimate real-time stock-level $\frac{V}{P}$, we use most up-to-date accounting information rather than stale information to compute the signals. We do this by using quarterly accounting data when available to compute annual quantities (e.g., compute an-

[33]However, the difference in the volatility here is small enough that it is not enough to bridge the difference in the average arithmetic return.

nual gross profits as the sum of the last four quarterly gross profits).Whenever possible, we use the raw characteristic rather than the rank to preserve the time variation in the characteristic's cross-sectional dispersion. However, since we rely on book equity information to compute profitability and investment in the pre-Compustat period, we use the rank for those two characteristics to make the quantity comparable between the pre-Compustat book equity values and post-Compustat book asset values, which allows us to use an estimation window that includes the two types of data. We also use the rank to combine information in the two alternative measures of net equity issuance.

Book-to-market (BM) is the monthly-updated log of book value of equity in the most recent quarter divided by the current month's market value. Quarterly book equity is calculated as the stockholder's equity ($SEQQ$ when available and ATQ minus LTQ otherwise) plus the deferred taxes and investment tax credit ($TXDITCQ$ when available and zero otherwise) minus preferred stock ($PSTKQ$ when available and zero otherwise). If the quarterly Compustat is unavailable, we compute BM as of June of calendar year y as the book equity in fiscal year $y-1$ divided by the current month's market value. Annual book equity is defined as stockholders' equity SEQ plus balance sheet deferred taxes and investment tax credit $TXDITC$ minus book value of preferred stock ($BE = SEQ + TXDITC - BPSTK$). Book value of preferred stock $BPSTK$ equals the preferred stock redemption value $PSTKRV$, preferred stock liquidating value $PSTKL$, preferred stock $PSTK$, or zero depending on data availability. If SEQ is unavailable, we set it equal to total assets AT minus total liabilities LT. If $TXDITC$ is unavailable, it is assumed to be zero. In the pre-Compustat period, we use the book equity data from Davis et al. (2000). We treat zero or negative book values as missing. Following Fama and French (2015), when computing the ratio of book value to market value, we adjust the book value for the changes in the number of shares outstanding between the time in which the book value is reported and the time in which the market value is computed by deflating market equity by the growth of shares between the two time periods.[34] Doing so leads to a substantial fall in the number of extreme outliers in the book-to-market figure due to a mismatch in the shares outstanding used to compute the book equity and the market equity. We further adjust for the cases that a firm has multiple common equity share classes, since not doing the adjustment may make the book-to-market of each individual

[34]See, e.g., the description in Table 1 of their paper (p.3).

share class seem unusually high.

Profitability (*Prof*) is the monthly-updated cross-sectional rank of gross profitability over assets defined as the trailing 4-quarter sum of quarterly gross profitability. Quarterly gross profitability is defined as sales minus cost of goods sold over the quarter divided by the most recent quarter's asset. When quarterly gross profit data are unavailable, we use annual gross profitability computed each June of calendar year y as sales minus cost of goods sold in fiscal year $y-1$ divided by total assets in fiscal year $y-1$. When neither quarterly or annual gross profit is unavailable, as is the case with pre-Compustat era, we use the rank of return on equity computed based on either Compustat data or the Davis-Fama-French book equity data.

Market beta (*Beta*) is the monthly-updated trailing 4-year market beta (minimum of 2 years) calculated based on overlapping 3-day returns. We winsorize the beta cross-sectionally at 1% and 99% to ensure that the beta better reflects the firm's market exposure over a long run. Size (*ME*) is the monthly-updated log market equity. Investment (*Inv*) is the cross-sectional rank of asset growth when available (computed using the quarterly Compustat if available and annual Compustat otherwise) and the rank of book equity otherwise (based on either Compustat data or the Davis-Fama-French book equity data). Net issuance (*NetIss*) is the average of the cross-sectional z-scores on two competing measures of net issuance: the 12-month growth in shares outstanding (Pontiff and Woodgate, 2008) and the 12-month equity net payout (Daniel and Titman, 2006). Return (*Ret*) is the cumulative gross return over the previous 12 months. Lagged return (*LagRet*) is the cumulative gross return from month -24 to month -12.

C.3 The making of our simple approach: the balanced-out term

We derive the simple regression approach to estimating stock-level $\frac{V}{P}$ and hence the fundamental values of individual stocks through three modeling assumptions:

1. Dropping the coskewness term to approximate the mispricing identity as equation (22).
2. Dropping the balanced-out term from the regressor in equation (14) and putting it in the projection error. (The argument for this choice is towards the end of Appendix B in the Internet Appendix.)

3. Using linear projection as opposed to a nonlinear projection (e.g., spectral projection).

We examine the effect of the second assumption by repeating the in-sample estimation in Table 2 with the "balanced-out term" included as a regressor in the second stage. To do this, we need time-series estimates of conditional first and second moments of the candidate risk factors, which we estimate as explained below. We find that none of the projection coefficients are dramatically affected by leaving out the balanced-out term and that the stock-level $\frac{V}{P}$'s estimated with these two alternative methods have a correlation of around 99.9%.

C.3.1 Estimating conditional factor moments

Extending our baseline approach to include the balanced-out term in equation (14) requires estimating both $\lambda_t = E_t\left[f_{t+1}\right]$ and $\Sigma_{f,t} = Var_t\left(f_{t+1}\right)$. Below, we explain how we estimate these terms.

Conditional factor means. We obtain conditional one-year factor means by projecting them on the value (book-to-market) spread (Cohen et al., 2003) and the net issuance spread (Greenwood and Hanson, 2012; Cho et al., 2024). We use the spread in the rank of book-to-market and of net issuance to prevent outliers from driving these estimates. We measure net eqiuty issuance as the one-year growth in common shares outstanding.

Conditional factor variances and covariances. We obtain conditional factor second moments (variances and covariances) from the first-order autoregressive model. We estimate realized factor second moments by annualizing the daily variances and covariances of log factor returns as explained below.

Conditional simple variance of a factor from log second moments.

$$Var_t\left(R_{L,t+1} - R_{S,t+1}\right) = Var_t\left(1 + R_{L,t+1}\right) + Var_t\left(1 + R_{S,t+1}\right) - 2Cov_t\left(1 + R_{L,t+1}, 1 + R_{S,t+1}\right)$$

$$= Var_t\left(\exp\left(r_{L,t+1}\right)\right) + Var_t\left(\exp\left(r_{S,t+1}\right)\right) - 2Cov_t\left(\exp\left(r_{L,t+1}\right), \exp\left(r_{S,t+1}\right)\right)$$

where

$$Var_t\left(\exp\left(r_{p,t+1}\right)\right) = \exp\left(Var_t\left(r_{p,t+1}\right) - 1\right)\exp\left(2E_t\left[r_{p,t+1}\right] + Var_t\left(r_{p,t+1}\right)\right)$$

Since $E_t\left[\exp\left(r_{p,t+1}\right)\right] = \exp\left(E_t\left[r_{p,t+1}\right] + \frac{1}{2}Var_t\left(r_{p,t+1}\right)\right)$, it follows that $\log\left(1 + E_t\left[R_{p,t+1}\right]\right) = E_t\left[r_{p,t+1}\right] + \frac{1}{2}Var_t\left(r_{p,t+1}\right)$, which means $2\log\left(1 + E_t\left[R_{p,t+1}\right]\right) = 2E_t\left[r_{p,t+1}\right] + Var_t\left(r_{p,t+1}\right)$. Hence,

$$Var_t\left(\exp\left(r_{p,t+1}\right)\right) = \left(\exp\left(Var_t\left(r_{p,t+1}\right)\right) - 1\right)\exp\left(2\log\left(1 + E_t\left[R_{p,t+1}\right]\right)\right)$$

$$= \left(\exp\left(Var_t\left(r_{p,t+1}\right)\right) - 1\right)\left(1 + E_tR_{p,t+1}\right)^2$$

Next,

$$Cov_t\left(\exp\left(r_{L,t+1}\right), \exp\left(r_{S,t+1}\right)\right) = \left(\exp\left(Cov_t\left(r_{L,t+1}, r_{S,t+1}\right)\right) - 1\right)\left(1 + E_tR_{L,t+1}\right)\left(1 + E_tR_{S,t+1}\right)$$

Conditional simple covariance from log second moments.

$$Cov_t\left(R_{1,L,t+1} - R_{1,S,t+1}, R_{2,L,t+1} - R_{2,S,t+1}\right) = Cov_t\left(R_{1,L,t+1}, R_{2,L,t+1}\right) - Cov_t\left(R_{1,L,t+1}, R_{2,S,t+1},\right)$$

$$-Cov_t\left(R_{1,S,t+1}, R_{2,L,t+1}\right) + Cov_t\left(R_{1,S,t+1}, R_{2,S,t+1}\right)$$

where covariance is computed similarly to the formula above.

C.4 Confidence interval on stock-level $\frac{V}{P}$

Since a stock's estimated $\frac{V}{P}$ is $\widehat{\gamma}_V z_{i,t}$, it follows that $Var(\widehat{\gamma}_V z_{i,t}) = z_{i,t}' Var(\widehat{\gamma}_V) z_{i,t}$.