How Do Government Guarantees Affect Deposit Supply?

Thomas Flanagan* Edward T. Kim† Shohini Kundu‡

Amiyatosh Purnanandam§

First Draft: March 2025

Abstract

The market value of deposit insurance changes over time and across banks as the value of the underlying put option changes, but the premium they pay for the insurance does not adjust to completely capture this variation. As a result, the effective subsidy that banks enjoy from deposit insurance changes over time and across banks, affecting their incentive to supply deposits. Factors that change the market value of insurance, such as asset risk and interest rates, move the supply curve. Consistent with this idea, we show that the deposit supply curve shifts outward during periods of high risk and for riskier banks. The effect is more pronounced for insured deposits. Our findings uncover a novel channel of deposit supply, with immediate implications for the transmission of monetary policies and existing research on "deposit channel of monetary policy" and "reaching-for-yield" literature.

For helpful comments, the authors thank brownbag participants at the University of Michigan.
*Fisher College of Business, The Ohio State University, e-mail: flanagan.224@osu.edu
†Stephen M. Ross School of Business, University of Michigan. e-mail: etkim@umich.edu
‡UCLA Anderson School of Management and CEPR. e-mail: shohini.kundu@anderson.ucla.edu
§Stephen M. Ross School of Business, University of Michigan. e-mail: amiyatos@umich.edu

1 Introduction

Bank deposits are central to economic growth and liquidity creation. It is not surprising that a large literature in banking and economics studies forces that shape the demand and supply of deposits, and consequently their impact on economic outcomes. Economic shocks, policy choices, consumer preference, and banking market structure have all been shown to affect the demand and supply of deposits in the prior literature.[1] In this paper, we uncover a new channel that shifts the supply curve of bank deposits: variation in the effective subsidy that banks enjoy from deposit insurance.

Bank deposits are insured by regulators in almost all major economies. In the United States, the Federal Deposit Insurance Corporation (FDIC) guarantees deposits up to a certain amount in the event of a bank's failure. Banks pay a premium to the FDIC, called the Deposit Insurance Premium (DIP), to avail of this guarantee. FDIC's guarantee gives the insured bank a put option that can be exercised by them in the event of default. Therefore, the market value of deposit insurance varies with factors that change the value of a put option, such as the riskiness of the underlying assets. However, the premium paid by banks to the FDIC does not adjust fully to reflect changes in its market value for four key, not mutually exclusive, reasons: (a) banks are classified into coarse risk categories for the assessment of FDIC insurance premium, (b) they are often subsidized, (c) premiums change infrequently, and (d) premiums typically depend on the actuarial value of insurance and not on the risk-neutral distribution of losses.[2] Therefore, the market value of insurance changes over time and across banks but the fees paid by banks do not change by the same amount. Consequently, the government guarantee in the form of deposit insurance creates variation in the effective subsidy across time and across banks.

The incentive of banks to supply deposits changes as the subsidy changes. When the risk of a bank increases, the value of the subsidy becomes higher because it is still paying low fees.

[1]Black (1975) and Fama (1985) focus on the effect of reserve requirements. Gorton and Pennacchi (1990) highlight the role of bank deposits in alleviating information frictions faced by savers. Stein (1998) teases out the role of adverse selection on the demand and supply of deposits. Kashyap and Stein (2000) focuses on the effect of policy shocks on banks' ability to raise outside funding and its impact on loan supply. Stein (2012) discusses the goals and methods of financial stability policy in an economy with privately-created money such as bank deposits. Gilje et al. (2016) relate exogenous supply shocks to bank deposits to lending markets. Drechsler et al. (2017) document the importance of bank's market power on the supply of deposits. There are numerous other valuable contributions in this field as sumamrized in survey articles such as Gorton and Winton (2003) and Bhattacharya and Thakor (1993).

[2]While there has been a move towards risk-based pricing of deposit insurance in the recent past, it still is not fully risk-sensitive due to the above mentioned reasons. See Duffie et al. (2003); Ronn and Verma (1986); Marcus and Shaked (1984) for studies on the pricing of deposit insurance premium.

Therefore, they expand the supply curve. They are now more likely to issue insured deposits and pay a higher rate for them. A contraction occurs when banks enjoy a lower subsidy, as in periods of low risk. Therefore, factors that influence the riskiness of banks over time and in the cross section influence the supply of deposits. We formalize these intuitions in a standard model of insurance pricing, i.e. Merton (1977), where deposit insurance is priced as a put option owned by insured banks. Banks face a convex cost function in raising deposits and the subsidy obtained through the underpriced insurance premium influences their decision to supply deposits. The model shows that the deposit supply curve shifts outwards when a bank's asset risk increases. In addition, the supply curve shifts outward in the low-interest rate regime for two reasons. Since policy rates are often lowered in bad economic times, the Fed interest rates are negatively correlated with bank risk, a fact that we confirm in the data. The negative correlation between interest rate and volatility leads to an outward shift in the supply curve when interest rates are low. Independently, lower interest rates move the supply curve outward in an economy where consumers are willing to pay more for liquidity benefits of deposits in periods with lower liquidity in the system.

We test these implications using detailed data on the quantity and pricing of deposits for all the U.S. banks covered in the Call Reports from 1986 to 2023. Our goal is to empirically establish a link between deposit insurance subsidy that a riskier bank enjoys and its deposit supply. In our main tests, we use the non-performing loans to asset ratio of a bank in a given quarter as our key measure of asset risk. Since asset risk is an unobserved quantity, using the NPL ratio as a proxy for asset risk has several benefits. The measure is an indicator of the performance of the bank's asset, mapping directly to the model primitive. It captures the riskiness of a bank's asset through the deterioration in its lending portfolio, allowing us to measure the time-varying nature of a bank's asset risk (Nagel and Purnanandam, 2020). It is available for the entire sample of banks, unlike some market-based risk measures that are only available for publicly traded banks. Finally, we are able to obtain this measure for the entire sample, i.e., for almost four decades, providing us with substantial variation in monetary policy shocks and bank risk.

Our main empirical specification uses a panel data regression, using bank-quarter observations over a period of almost four decades, to establish a link between asset risk and deposit supply. We show that banks with one standard deviation higher NPL ratio have 1.7% higher deposits. We include bank fixed effects to soak away the effect of factors such as the bank's management style, franchise value, and geographical presence on deposit supply. We include time-fixed effects to soak away aggregate risk factors. In later analyses, we present time-series evidence to shed light on the aggregate dynamics of bank risk and deposit supply.

A key identification challenge for establishing a link between changes in subsidy and deposit supply is reverse causality: if banks with higher deposit insurance subsidy make riskier investments, then we are likely to find a positive correlation between measures of bank risk and deposits. It is well known that subsidized insurance can increase a bank's incentive to make riskier loans. Note that if a bank increases its asset risk in a given quarter, it will take several quarters for the loans to turn into an NPL. Hence, the use of NPL as a measure of asset risk ameliorates this concern because higher levels of NPL in a given quarter are a reflection of riskier portfolio choices made by the banks in the past.

We directly address this endogeneity concern with an instrumental variable strategy using a Bartik instrument. For each bank-quarter in the sample, we obtain the bank's loan portfolio composition across five different asset classes, namely, real estate loans, C&I loans, household & consumer loans, agricultural loans, and loans to other financial firms, three years ago. Fixing their portfolio weight at this point, we obtain the predicted value of their non-performing loans ratio based on the aggregate default rate of each category over time. Thus, we obtain an instrument that depends on the cross-sectional differences in a bank's initial portfolio decision and the subsequent economy-wide shocks to the respective asset categories. Asset decisions are fixed three years ago in this methodology, i.e., sufficiently before the deposit supply decision we study, and therefore, by design, they are not influenced by the reverse causality concerns. Further, the allocation of asset portfolio three years ago is unlikely to be influenced by the knowledge of future default rates of these categories, which occur years later in the sample. These arguments form the basis of our identifying assumption.

The instrument varies by bank based on their lagged asset allocation and over quarters based on the economy-wide shocks to each of the asset classes. In the first stage regression, we find a statistically strong coefficient on the portfolio weight instrument. The instrument is strong: even within bank and time fixed effects, it explains almost 10% of the variation in the realized NPL ratio of banks in the entire sample. The F-statistics of the instrument is over 100. Using the predicted value of NPL ratios, we find a positive and strong coefficient on the amount of deposits of the bank. One standard deviation higher NPL ratio causes an increase of 16.1% in the quantity of deposits.

We present two additional results on the quantity of deposits before turning to our results on deposit pricing. We show that as the NPL ratio increases, the fraction of insured deposits at a bank also increases. This finding is consistent with our channel that banks supply higher amounts of deposits when the value of government subsidy goes up, and that insured deposits directly benefit the most from this government subsidy. In our second test, we estimate the model using branch-level data on deposit quantities. A key advantage of this specification is that we include both the branch fixed effects, as well as county $\times$ time fixed effects, to separate out the impact of local economic conditions, demand for deposits, and banking market competition from the effect we capture from the subsidy effect. The branch-level regression model rules out the possibility that distressed banks are located in areas with different compositions of depositor base since we are able to include county fixed effects in these models to account for local depositor base and economic conditions. Our results remain strong in these specifications.

Does the increase in the quantity of deposits represent a supply shift instead of a demand shift? To establish the supply side effects, in our next test, we analyze the pricing of deposits in response to an increased value of the insurance subsidy that the banks enjoy. If banks increase the supply of insured deposits in response to an increase in subsidy, then we expect interest rates offered by them to increase. A demand side effect produces just the opposite prediction. Banks provide interest rates on deposits of different denominations. We focus on two denominations for our analysis: the wholesale insured rates, defined as rates offered on deposits that exploit the insured limit to the maximum, i.e., on deposits of $100,000 until 2010 and $250,000 after that. We find that banks with a one standard deviation higher NPL ratio offer interest rates on these products that are 1% higher relative to the mean. The results are similar for the 2SLS regression model with a larger economic magnitude, with an 11% increase. In addition, we also analyze interest rates on denominations just below the maximum insured limit: $50,000 before 2010 and $200,000 after that point. Our results remain similar.

In sum, banks with higher NPL ratios have higher amounts of deposits, and they pay higher rates on them: the combined effect of an increase in quantity and a decrease in price is consistent with our assertion that banks shift the supply curve outward when the value of deposit insurance subsidy goes up. Our results so far exploit variation across banks over time. We now present some aggregate effects using time-series analysis.

Figures 2 plots the aggregate changes in deposits in the banking sector during a quarter, along with changes in NPL over a long time series from 1986 to 2021. The positive correlation between the two time series is striking. The bottom panel presents the plot for only the insured deposits: the pattern is even more striking. We estimate a time-series regression using the growth rate of aggregate deposits as the dependent variable and the growth rate in NPL ratio as the explanatory variable. One standard deviation increase in NPL growth is associated with a 1.1% higher deposit growth rate. Consistent with our channel, the NPL growth rate is associated with an increase in the fraction of insured deposits in the system. Finally, the deposit spread, defined as the difference between the Federal Funds rate and the rates offered by banks, decreases with the increase in the NPL ratio. Together these results show that as the banking sector enjoys higher government subsidy, it increases the supply of deposits.

The value of deposit insurance subsidy can change not only with the volatility of the asset but also with the interest rate in the economy. We find a strong negative correlation between the NPL ratios and the Fed Funds rates. As interest rates fall, the value of the deposit insurance subsidy goes up, and the banking sector's willingness to increase the supply of deposits goes up. Our insight, therefore, has implications for the relation between interest rates and deposit supply as documented by the deposit channel of monetary policy (Drechsler et al., 2017). In Drechsler et al. (2017) the mechanism driving the connection between interest rates and deposit supply is the market power that the banks enjoy. In our channel, it is the value of the deposit insurance subsidy that affects the supply of deposits. These channels are not mutually exclusive; instead they can reinforce each other. In our next test, we replicate the key findings of the deposit channel literature, and then add NPL ratio as an additional explanatory variable in the regression model. Specifically, we estimate our branch level regression model for deposit quantity with the inclusion of changes in Fed Funds rate and the HHI of the local deposit market as in the deposit channel literature, and then include our measure of bank risk as an additional explanatory variable. The impact of NPL ratio on deposit quantity remains strong, suggesting that the effect we document is not simply a reflection of bank market power. In addition, we show that the inclusion of the NPL ratio in the model explains about one-half of the effect of changes in Fed Funds rate on deposit quantity estimated by the earlier literature. In sum, the subsidy channel provides an independent explanation of movement in the deposit supply curve in response to interest rate changes.

Our study also has implications for the literature on reaching-for-yield in financial intermediation (Rajan, 2006; Acharya and Naqvi, 2019). This behavior has been generally described as the propensity to invest in riskier assets in low interest rate environment to achieve higher yields. Managerial incentive to target a desired nominal return, regardless of risk, is a common friction that can explain this behavior. Our channel that subsidy increases during low interest rate environment provides an independent explanation of this behavior.

Our paper is related to Billett et al. (1998) who show that deposit insurance shields banks from market discipline. They find that banks substitute towards insured deposits, away from uninsured ones, as they get downgraded by Moody's, making market discipline less effective. One of our findings that the fraction of insured deposits go up in response to the increase in asset risk is consistent with this finding. However, the economic mechanism behind of our study is not rooted in market discipline that they focus on. More importantly, we tease out the expansion in supply curve, not merely a substitution across insured and uninsured deposits. Further, our study traces out the supply curve using both the quantity and pricing data, unlike their study that predominantly focuses on equity market returns of banks.

2 History of Deposit Insurance Premiums

The Banking Act of 1933 led to the creation of the FDIC as the guarantor of bank deposits in the United States. Since then, bank deposits have been fully insured up to a certain limit, most recently up to $250,000 per depositor per bank. In the early years of the formation of the FDIC, the deposit insurance premium that the banks paid was a flat amount, i.e., every bank paid the same amount to the FDIC every year for obtaining deposit insurance. After the Savings & Loans Crisis of 1980s, Congress passed a legislation in 1989 requiring the FDIC to maintain reserves of at least 1.25% of insured deposits in its Deposit Insurance Fund.[3] A flat-rate schedule was approved to achieve the target level of reserves.[4] The FDIC's Deposit Insurance Fund is maintained by quarterly premiums (called assessments) that are calculated as each bank's assessment base multiplied by an assessment rate.

A later regulation in 1991 required the FDIC to charge the insurance premium based on the riskiness of the bank. The first risk-based assessment went into effect on a transitional basis in 1993, and became permanent in 1994. Banks were categorized into three groups based on their capitalization ratio and further divided into three groups based on their supervisory ratings, providing a total of nine groups of banks for deposit fee assessment. However, there was little variation in premiums paid by banks in practice: the premium varied between 23 basis point to 31 basis points from the least risky to the most risky group.

By 1996, when the deposit insurance fund had reached its target of 1.25%, 95% banks paid

[3]See https://www.minneapolisfed.org/article/1998/a-brief-history-of-reserves-and-premiums#1

[4]See https://www.fdic.gov/analysis/cfr/staff-studies/2020-01.pdf for a detailed history of deposit insurance premium in the United States.

no insurance premium at all, providing no variation in their fees.[5] This system continued between 1996 and 2006, where most banks paid no assessment fees for accessing deposit insurance. FDIC continued to categorize banks into 9 categories, in a 3x3 matrix based on capitalization and supervisory rating, but banks in the top-most category, i.e., those assessed as well capitalized and with healthy supervisory rating paid zero assessment fees. In terms of the amount (number) of deposits more than 96% (92%) of banks fell under this safest category, effectively making the system risk-insensitive (Duffie et al., 2003).

FDIC undertook a serious attempt to move towards risk-based pricing in 2007. Banks were now categorized into four categories based on their capital ratio and supervisory rating. The assessment rate varied from 5 to 43 basis points across banks based on this method. However, a majority of banks were still under the safest category and there was little variation across banks in a category. In 2009, in the immediate aftermath of the global financial crisis, the assessment method was significantly revised, taking into consideration not only the bank's overall capital and supervisory rating, but also on the composition of their debt (e.g., the extent of unsecured debt or brokered deposits). Some distinctions were made based on whether a bank falls under the large or small bank category as well, as determined by a $10 billion asset threshold. As a result, the assessment fees varied considerably across the four risk categories and it also varied within a risk-category depending on the liability composition of the bank. The net result of this modified system was a variation in deposit insurance premium between 7 to 77.5 basis points across banks. Finally, in 2016, the assessment base was further changed, providing a range of 1.5 to 40 basis points across banks.

How large is the magnitude of deposit insurance subsidy? A quantitative assessment is beyond the scope of this paper. The prior literature, however, provides some estimates based on historical data. Duffie et al. (2003) use a reduced form credit risk model to estimate the fair value of market price of deposit insurance premium. In an approximate sense, they show that the fair market price equals the bank's short-maturity credit spread multiplied by the ratio of expected losses to the insurer in the even of failure to expected fractional loss on bank debt. They provide estimates on the CDS spreads of banks and their corresponding insurance premium in 2002. Based on reasonable assumption on loss given default, the fair market premium can be as high as 64 basis points compared to 0-3 basis points that most banks paid at the time.

[5]See Federal Deposit Insurance Corporation. Press Release 87-96: "Under the existing rate schedule in effect since January of 1996, institutions in the lowest risk category will continue to pay no premiums during the first half of 1997. A total of 9,538, or 94.4 percent of all BIF-insured institutions, are in the lowest risk category."

Overall, the history of deposit insurance premium suggests a flat, risk-insensitive premium for most of the history of the U.S. banking. The premiums have become more risk-sensitive after 2007, though not yet fully risk-sensitive since banks are still categorized in coarse rating buckets, and the premium does not adjust with the state price, as a market-based insurer would require.

3 Model

We formalize our intuition in a model similar to Merton (1977) in this section. We consider a bank that issues D dollars of insured deposit and funds a loan of value D+E where E is the equity value. Banks fund a fraction of their loans with equity such that $L = \frac{D}{w}$ so that $E = \frac{D}{w} - D$. w is the leverage ratio. We assume that there are no uninsured deposits in the bank's liability mix. In the baseline specification, deposits provide no liquidity or convenience service and therefore they simply earn the risk-free rate because they are insured: $r_d = r$, where r is the risk-free rate. We assume a maturity date of T for both loans and deposits to keep our analysis focused on our channel.

The loan market is assumed to be perfectly competitive, so banks simply maximize the value of the put option of deposit insurance net of any deposit insurance premium and the convex cost of raising deposits. Raising deposits incurs costs such as branch network operations, ATM network, maintenance cost, and staff cost; together, they are convex in the amount of deposits raised. Deposit insurance premium is paid today at a flat rate of $\bar{p}$ per unit of deposit. Although the deposit insurance premium is flat and constant over time, factors that move its market value such the volatility of bank assets, σ or r are not. They change with macroeconomic conditions, policy choices, as well as bank-specific factors.

Banks optimization problems are as follows:

$$\max_{\{D\}} P[L, K, \sigma, r, T] - \frac{1}{2}cD^2 - \bar{p}D \tag{1}$$

where, $P[L, K, \sigma, r, T]$ denotes the value of a European put option on the underlying asset L with strike price K, the face value of deposits. Therefore,

$$\begin{aligned}
P[L,K,\sigma,r,T] &= Ke^{-rT}\Phi(-d_2) - L\Phi(-d_1) \\
d_1 &= \frac{ln(\frac{L}{K}) + (r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}; \\
d_2 &= d_1 - \sigma\sqrt{T}; \\
K &= D\cdot e^{rT}; \\
\frac{L}{K} &= \frac{D}{wD\cdot e^{rT}} = \frac{1}{w\cdot e^{rT}}.
\end{aligned} \tag{2}$$

Simplification of the above expressions lead to the following:

$$\begin{aligned}
P[L,K,\sigma,r,T] &= D\Phi(-d_2) - \frac{D}{w}\Phi(-d_1); \\
d_1 &= \frac{-ln(w)}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}; \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned} \tag{3}$$

The first order condition of the bank's optimization problem in Equation 1 can be written as follows:

$$\frac{\partial P[L,K,\sigma,r,T]}{\partial D} = cD + \bar{p} \tag{4}$$

Simplifying further:

$$\begin{aligned}
\frac{\partial P[L,K,\sigma,r,T]}{\partial D} &= \Phi(-d_2) + D.\frac{\partial\Phi(-d_2)}{\partial D} - \frac{\Phi(-d_1)}{w} - \frac{D}{w}\frac{\partial\Phi(-d_1)}{\partial D} \\
&= \Phi(-d_2) - D\phi(-d_2)\frac{\partial d_2}{\partial D} - \frac{\Phi(-d_1)}{w} + \frac{D}{w}\phi(-d_1)\frac{\partial d_1}{\partial D} \\
&= \Phi(-d_2) - \frac{\Phi(-d_1)}{w}
\end{aligned}$$

Therefore, the optimal quantity of deposits is given by the following equation:

$$\Phi(-d_2) - \frac{\Phi(-d_1)}{w} = cD + \bar{p}$$

$$D^* = \frac{1}{c}\{\Phi(-d_2) - \frac{\Phi(-d_1)}{w} - \bar{p}\} \tag{5}$$

Optimal supply of deposits decreases when marginal cost c is higher; and when deposit insurance premium paid, $\bar{p}$ is higher. It increases when the default option has a higher value.

Optimal supply as volatility changes: Now, using the envelope theorem, we find the sensitivity of optimal quantity of deposits with respect to asset volatility σ:

$$\begin{aligned}
\frac{\partial D^*}{\partial \sigma} &= \frac{1}{c}\frac{\partial}{\partial \sigma}\{\Phi(-d_2) - \frac{\Phi(-d_1)}{w}\} \\
&= \frac{1}{c}\{-\phi(-d_2).\frac{\partial d_2}{\partial \sigma} + \frac{1}{w}\phi(-d_1).\frac{\partial d_1}{\partial \sigma}\}
\end{aligned}$$

Note:

$$\phi(-d_2) = \frac{1}{w}\phi(d_1) \tag{6}$$

Therefore,

$$\begin{aligned}
\frac{\partial D^*}{\partial \sigma} &= \frac{1}{c}\{-\frac{1}{w}\phi(d_1).\frac{\partial d_2}{\partial \sigma} + \frac{1}{w}\phi(-d_1).\frac{\partial d_1}{\partial \sigma}\} \\
&= \frac{\phi(d_1)}{cw}\{-\frac{\partial d_2}{\partial \sigma} + \frac{\partial d_1}{\partial \sigma}\} \\
&= \frac{\phi(d_1)}{cw}\sqrt{T} > 0 \qquad (7)
\end{aligned}$$

Therefore, banks supply more deposits when volatility goes up. Periods with higher macroeconomic volatility or bank-specific losses are likely to be associated with a higher supply of deposits.

3.1 Optimal supply as ‘r’ changes:

Interest rates can affect deposit insurance value through two potential channels. First, policy rates are often lowered in bad economic times, producing a negative correlation between ‘r’ and σ. Second, interest rates can affect the price of liquidity and therefore the premium depositors are willing to pay for liquidity benefits of deposits. We first present a model where ‘r’ and σ are negatively correlated. Then we generalize the model to a setting with liquidity benefits.

3.1.1 Policy Rates

Interest rates in the economy follow a negative relation with the observed volatility of the banking sector. This is consistent with the idea that policy interventions often happen during bad economic times, when rates are lowered. We capture that intuition with a simple linear relationship between volatility and interest rate as follows:

$$\sigma = \hat{b} - \hat{a} * r \tag{8}$$

It follows that (Proof in the Appendix):

$$\begin{aligned} \frac{\partial D^*}{\partial r} &= \frac{1}{c}\frac{\partial}{\partial r}\{\Phi(-d_2) - \frac{\Phi(-d_1)}{w}\} \\ &= -\phi(d_1)\{\frac{\hat{a}\sqrt{T}}{cw}\} < 0 \end{aligned} \tag{9}$$

Therefore, when interest rates go up, optimal deposit financing comes down. The sensitivity of optimal deposit supply to interest rate is high when a is high, i.e., when asset volatility is more sensitive to r. If we set this parameter to zero, we obtain Merton (1977) that the value of deposit insurance put option is insensitive to interest rates.

3.1.2 Liquidity Benefits of Deposits

So far, in our model, deposits are priced at the risk-free rate. We now extend our model to include the liquidity benefits of deposits that the consumers enjoy. On average, deposits pay lower interest rates than the risk-free rate because they come with liquidity benefits. In periods of scarce liquidity in the aggregate financial system, the value of liquidity provided by the deposit contracts is likely to be relatively higher compared to periods with abundant liquidity. Periods of low liquidity are characterized in our model as periods of high interest rates. Therefore, we now assume that the interest rate on deposits (r_d) is given by the following schedule that accounts for higher liquidity premium in high interest rate regime:

$$r_d = r(1 - \alpha) - \beta, \tag{10}$$

where $0 < \alpha < 1$ and β are positive numbers. As shown in the Appendix:

$$\frac{\partial P[.]}{\partial r} = -\alpha D T e^{-(\alpha r+\beta)T}\Phi(-d2) < 0 \tag{11}$$

Therefore, the value of the put option decreases when interest rates are high. Consequently, the supply of deposits comes down in a high interest rate regime. As shown in the appendix, we get the following relationship between optimal deposit quantity supplied by the banks and interest rates:

$$c\frac{\partial D^*}{\partial r} = -\alpha T e^{-(\alpha r+\beta)T}\Phi(-d_2) < 0. \tag{12}$$

Therefore, the optimal quantity of deposits comes down when interest rates go up. The effect is stronger when parameter 'c' is smaller, i.e., for banks that are likely to face a lower marginal cost of raising deposits. Similarly, the effects are stronger for banks that have depositors who value liquidity more (the α parameter).

4 Data and Descriptive Statistics

We collect comprehensive data on bank balance sheet items from the quarterly Call Reports from the FFIEC. Our main outcome variables of interest are total deposits, the ratio of estimated insured deposits to total deposits, and deposit rates. We incorporate information on bank size (total assets) and leverage (total equity capital) as control variables. Additionally, we study branch-level deposit quantities using the FDIC summary of deposits dataset.

To examine how changes in the government subsidy affects deposit supply, we measure the value of deposit insurance as the volatility of bank assets (Merton, 1977). Our main measure of bank risk is the quarterly non-performing loan ratio, which is computed using Call Reports data and defined as nonaccruing loans and accruing loans that are 90 days or more past due divided by total assets.[6]

For our instrumental variables analysis, we use each bank's historical asset holdings across the following five categories: Real estate loans, C&I loans, household & consumer loans, agricul-

[6]In the Call Reports, these measures are RCFD1403 and RCFD1407. When these variables are missing from the Call Reports, we construct it by adding up the NPL from their subcomponents as defined in the Call Reports.

tural loans, and loans to other financial firms. Specifically, we compute a measure of estimated NPL ratio weighted by historical asset composition (portfolio weight for each bank in each asset class from three years ago.

To study the deposit insurance subsidy over time, we augment our baseline dataset with quarterly information on FDIC assessment fees available in the Call Reports. Additionally, we use branch-level deposit rate data from the S&P Global's RateWatch database to measure interest rates, specifically for new insured wholesale deposits (accounts with minimim size at or above the FDIC deposit insurance limit) and submarginal deposits (accounts just below the limit). The RateWatch panel is available from January 2001 to December 2020.

The sample period for our baseline analysis spans 1986 to 2023. Table 1 reports the descriptive statistics of key variables for the full sample period. We winsorize all variables at the 1% level to remove the effect of extreme outliers.

5 Empirical Results

We begin our analysis by using a panel data model to empirically establish a causal link between the deposit insurance subsidy and deposit supply. Subsequently, we present graphical evidence on the correlation between deposit insurance subsidy and deposits in the banking sector in aggregate.

5.1 Panel Data Analysis

5.1.1 Deposit Quantity

We empirically test the implications of changes in subsidy on the supply of deposits using the following empirical model estimated with bank-quarter panel data:

$$d_{b,t} = \alpha_b + \mu_t + \beta\sigma_{b,t} + \Sigma X_{b,t} + \epsilon_{b,t}. \tag{13}$$

where $d_{b,t}$ is a measure of deposit quantities in quarter t for bank b. We use two main measures of deposit quantities in the paper: the log of total deposit quantity and the fraction of insured to total deposit of a bank. α_b and μ_t are banks and year-quarter fixed effects. $X_{b,t}$ is a vector of control variables, such as capitalization ratio as measured by equity-to-asset ratio of bank b in quarter t. The variable of interest is $\sigma_{b,t}$, a measure of bank b's risk in quarter t. We use the NPL ratio of a bank scaled by total assets for the main analysis.

Column (1) of Table 3 presents the regression results using the OLS method. We find a positive and significant coefficient on the NPL ratio: one standard deviation higher NPL ratio is associated with 1.7% higher deposits. The inclusion of bank and time fixed effects ensures that our results are not driven by macroeconomic shocks or bank-specific factors such as their market power or skill.

We use an instrumental variable model to causally identify the effect of asset risk on deposits. Our instrument is constructed as follows: for every bank in quarter t, we obtain their asset holdings in quarter $t-12$, i.e., three years ago, across the following five broad categories: Real Estate loans, C&I loans, household & consumer loans, agricultural loans, and loans to other financial firms. Since the data on agriculture loans became available after 1991, the regression model with the IV design is estimated on a slightly smaller time period from 1994 till 2023. For each bank-quarter observation, we compute a measure of their estimated NPL ratio by multiplying the portfolio weight for each bank in each asset class from three years, $w_{b,i,t-12}$ with realized NPL shocks $s_{i,t}$, the average fraction of NPL in each category in quarter t:

$$\text{Portfolio Weight IV}_{b,t} = \sum_i w_{b,i,t-12} * s_{i,t}. \tag{14}$$

This instrument, therefore, estimates the NPL the bank would have if (1) it did not change its portfolio weights and (2) it did not have idiosyncratic shocks NPL shocks to its loan portfolio. The instrument, called the *PortfolioWeightIV* in the rest of the paper, gives us a measure of NPL ratio based on the initial investment decision of the firm and aggregate shocks to these asset categories. By construction, it eliminates the reverse causality bias that we are trying to overcome in our analysis because the shares cannot be manipulated ex-post. We control for the equity capitalization ratio of banks to ensure that we isolate the effect of increased leverage on deposit financing in our model.

Column (2) presents the first-stage regression result. *PortfolioWeightIV* is a strong predictor of a bank's NPL ratio three years later. Our instrument allows us to capture the variation in asset risk that comes from a bank's historical investment decisions. The estimate of 0.612 indicates that one standard deviation higher value of *PortfolioWeightIV* is associated with 0.6 higher standard deviation of NPL ratio. The instrument is strong, with t-statistics of over 50.

Column (3) presents the reduced form estimate. One standard deviation higher *PortfolioWeightIV* is associated with 9.8% higher deposits. The 2SLS result, presented in Column (4), shows that banks with higher NPL ratio have significantly higher deposits. One standard deviation higher NPL ratio results in 16.1% higher deposits based on the IV regression model estimates.

When a bank becomes riskier, it can increase the value of its deposit insurance subsidy by increasing specifically insured deposits. Therefore, we expect banks to increase the share of insured deposits to total deposits in response to an increase in the NPL ratio. We test this hypothesis in Table 3, where we use the fraction of insured deposit as the dependent variable. We also control for the bank's size, measured as the log of asset value, in this model to soak away the differential reliance on insured deposits across banks of different sizes. In the OLS model in Column (1), we find that one standard deviation higher NPL ratio is associated with a 1.1% higher fraction of insured deposit. The 2SLS regression coefficient, using *PortfolioWeightIV* as the instrument, is presented in Column (3). One standard deviation higher NPL results in 3% higher fraction of insured deposit in this model.

Branch-level regression:

So far, our results relate banks' aggregate deposits to their NPL ratio. Deposit data is also available at the branch level beginning in 1994 with the FDIC summary of deposit data, which is the same time that our instrument becomes available. We now estimate a model similar to equation 14 on branch-level deposits. We include branch fixed effect to soak away branch specific characteristics, and we also include fixed effects for either *State* $\times$ *Time* or *County* $\times$ *Time* to isolate the effect of time-varying local economic conditions such as employment opportunities or demand for deposits in the local market, from the subsidy channel that we are interested in.

The estimation results are provided in Table 4. Columns (1) and (2) show that branches affiliated with banks with higher NPL ratios increase their deposits by 2.2-2.6%, depending on the model. In the 2SLS framework, the effect of NPL ratio on branch level deposits is significantly higher at 18.5%-20.6%.

5.1.2 Pricing

Table 5 presents results on interest rates offered by banks when the value of subsidy goes up. The model is estimated at the branch level using data from 2001 to 2020. We use the interest rate offered on 12-month CDs of denomination $100,000 until 2010 and $250,000 after that to capture the rate that was offered by the bank on wholesale insured deposits. We call the variable the

Wholesale Insured Rate. At these amounts, the bank is able to raise large denomination insured CDs. Banks with one standard deviation higher NPL ratio offer about 0.6 basis points more on these CDs. We include branch and *County* $\times$ *Time* fixed effects to isolate the effect of factors such as the bank's ATM network or local demand conditions from the effect of NPL ratio.

Column (2) presents the second-stage estimate of the IV regression model. The estimate is economically large and estimated precisely. Banks with one standard deviation higher NPL ratio offer 7.4 basis points higher interest rate on their insured deposits.

We also present the analyses with rates on CDs that are slightly smaller than the wholesale insured rate. For periods before 2010, we use CDs of denomination $50,000 and after 2010, we use CDs of denomination $200,000 for this analysis. Banks with higher NPL ratios offer higher interest rates on these CDs as well, consistent with a shift in the supply curve.

6 Time-series Evidence:

Figure 2 plots the time series of the quarterly YoY change in the total banking sector deposits as well as the change in the total banking sector NPL from 1986 to 2021. The figure graphically shows a strong correlation between these measures. Deposit growth is higher when growth in NPL is higher, and the correlation roughly 30%. The bottom panel similarly plots the time series of the growth of insured deposits. The growth driven specifically by insured deposits supports our channel, which banks have incentives to expand insured deposits to maximize the value of the government subsidy. This means that economy-wide variation that affect the overall demand for deposits and the need for funding based on lending opportunities cannot explain our results. Further, figure 3 plots how the deposit spread (fed funds minus deposit rates) varies with the growth in NPL. The figure shows a strikingly negative correlation. When NPL growth is high, deposit spreads are lower. Table 6 tabulates these associations with time series regression.

Together, these findings are consistent with our mechanism, in which periods of higher asset risk drive an increase in deposit supply. These results indicate that the mechanism operates not only in the time series within banks but also at an aggregate level.

6.1 Bank Risk and Interest Rate Environment

The value of deposit insurance subsidy can change not only with the volatility of the asset but also with the interest rate in the economy. The bottom panel of Figure 3 plots the change in federal funds rates against the change in the aggregate NPL in the banking sector. We find a strong negative relationship between the change in fed funds and the change in NPL, with a correlation of approximately -.5. This observation implies that the changes in the Fed Funds rate will also be correlated with the value of the deposit insurance subsidy that banks enjoy.

To empirically show this, we re-estimate our baseline specification in Table 7 using the change in Fed Funds as our bank explanatory variable without using any quarter-year fixed effects. The estimates show that when the Fed Funds rate increases, banks expand deposit supply and insured deposits, in line with our model predictions in section 3.1.1.

To further show that our channel is independent of previously documented monetary transmission channels, in Table 8, we replicate the key findings of the deposit channel literature and then add NPL ratio as an additional explanatory variable in the regression model. Specifically, we estimate our branch level regression model for deposit quantity with the inclusion of changes in Fed Funds rate and the HHI of the local deposit market as in the deposit channel literature, and then include the change in our measure of bank risk as an additional explanatory variable. The impact of NPL ratio on deposit quantity remains strong and leads to a greater deposit growth, suggesting that the effect we document is not simply a reflection of bank market power. In addition, we show that the inclusion of the NPL ratio in the model explains about one-half of the effect of changes in the Fed Funds rate on deposit quantity estimated by the earlier literature.

Together, these findings show that the subsidy channel provides an independent explanation of movement in the deposit supply curve in response to interest rate changes.

7 Conclusion

We show that banks increase the supply of deposits in response to the value of guarantee they receive from the FDIC for their insured deposits. Since the premium paid by banks to the FDIC does not fully adjust when the market value of insurance changes, the effective subsidy that a bank enjoys changes with parameters that change the market value of the FDIC insurance guarantee. Using various measures of banks' risk, we show that increased risk results in higher deposits, most notably higher insured deposits.

References

Acharya, V. and Naqvi, H. (2019). On reaching for yield and the coexistence of bubbles and negative bubbles. *Journal of Financial Intermediation*, 38:1–10.

Bhattacharya, S. and Thakor, A. V. (1993). Contemporary banking theory. *Journal of financial Intermediation*, 3(1):2–50.

Billett, M. T., Garfinkel, J. A., and O'Neal, E. S. (1998). The cost of market versus regulatory discipline in banking. *Journal of Financial Economics*, 48(3):333–358.

Black, F. (1975). Bank funds management in an efficient market. *Journal of Financial Economics*, 2(4):323–339.

Drechsler, I., Savov, A., and Schnabl, P. (2017). The deposits channel of monetary policy. *The Quarterly Journal of Economics*, 132(4):1819–1876.

Duffie, D., Jarrow, R., Purnanandam, A., and Yang, W. (2003). Market pricing of deposit insurance. *Journal of Financial Services Research*, 24:93–119.

Fama, E. F. (1985). What's different about banks? *Journal of monetary economics*, 15(1):29–39.

Gilje, E. P., Loutskina, E., and Strahan, P. E. (2016). Exporting liquidity: Branch banking and financial integration. *The Journal of Finance*, 71(3):1159–1184.

Gorton, G. and Pennacchi, G. (1990). Financial intermediaries and liquidity creation. *The Journal of Finance*, 45(1):49–71.

Gorton, G. and Winton, A. (2003). Financial intermediation. In *Handbook of the Economics of Finance*, volume 1, pages 431–552. Elsevier.

Kashyap, A. K. and Stein, J. C. (2000). What do a million observations on banks say about the transmission of monetary policy? *American Economic Review*, 90(3):407–428.

Marcus, A. J. and Shaked, I. (1984). The valuation of fdic deposit insurance using option-pricing estimates. *Journal of Money, Credit and Banking*, 16(4):446–460.

Merton, R. C. (1977). An analytic derivation of the cost of deposit insurance and loan guarantees an application of modern option pricing theory. *Journal of banking & finance*, 1(1):3–11.

Nagel, S. and Purnanandam, A. (2020). Banks' risk dynamics and distance to default. *The Review of Financial Studies*, 33(6):2421–2467.

Rajan, R. G. (2006). Has finance made the world riskier? *European financial management*, 12(4):499–533.

Ronn, E. I. and Verma, A. K. (1986). Pricing risk-adjusted deposit insurance: An option-based model. *The Journal of Finance*, 41(4):871–895.

Stein, J. C. (1998). An adverse-selection model of bank asset and liability management with implications for the transmission of monetary policy. *The Rand Journal of Economics*, pages 466–486.

Stein, J. C. (2012). Monetary policy as financial stability regulation. *The Quarterly Journal of Economics*, 127(1):57–95.

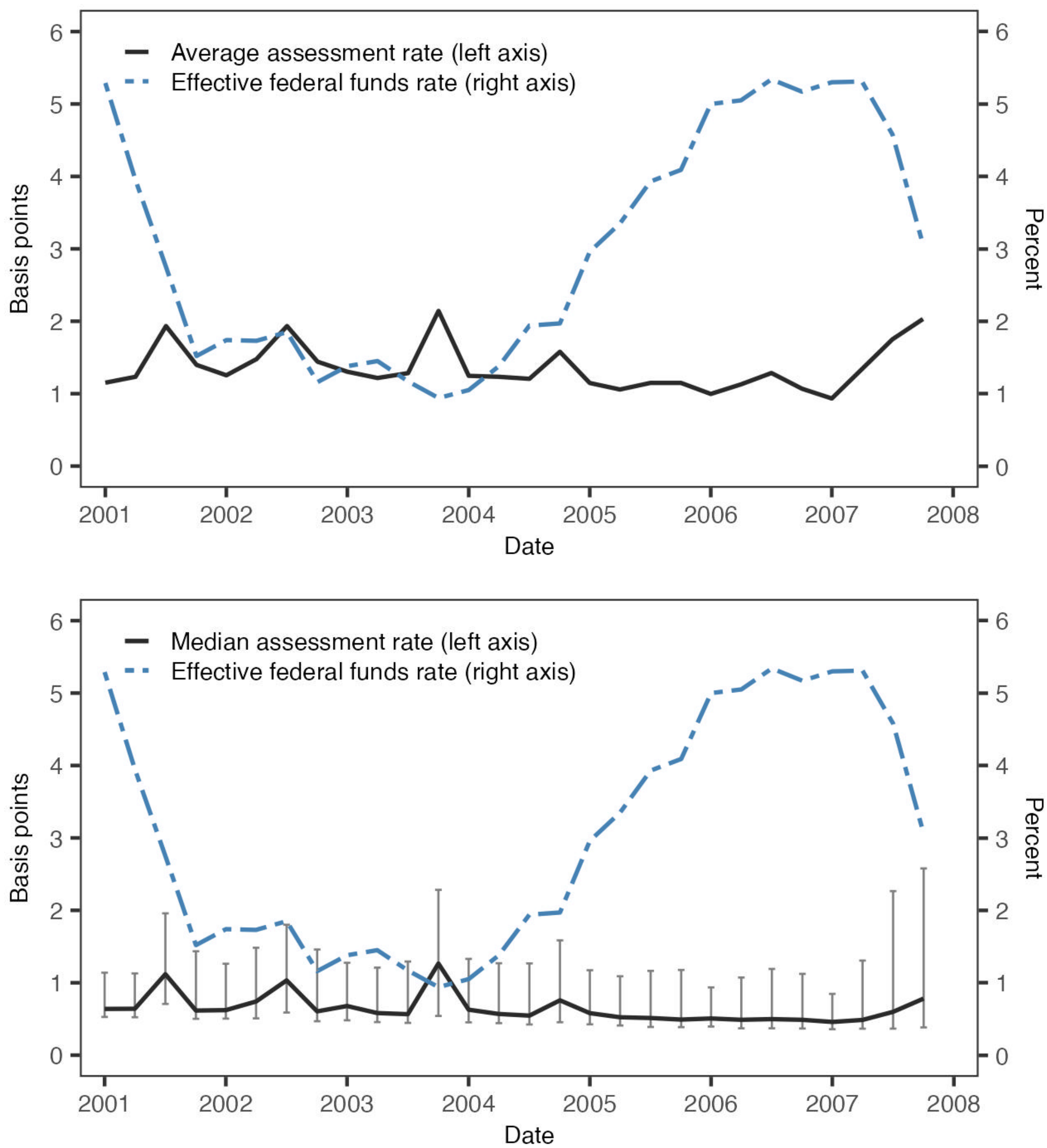

Figure 1: FDIC Assessment Fees Across Time

Notes: This figure plots the relationship between average FDIC assessment rates and the effective federal funds rate between 2001Q1 and 2007Q4. The top panel plots the average assessment rate and bottom panel plots the median assessment rate with 25th and 75th percentile bars.
Source: Call Reports.

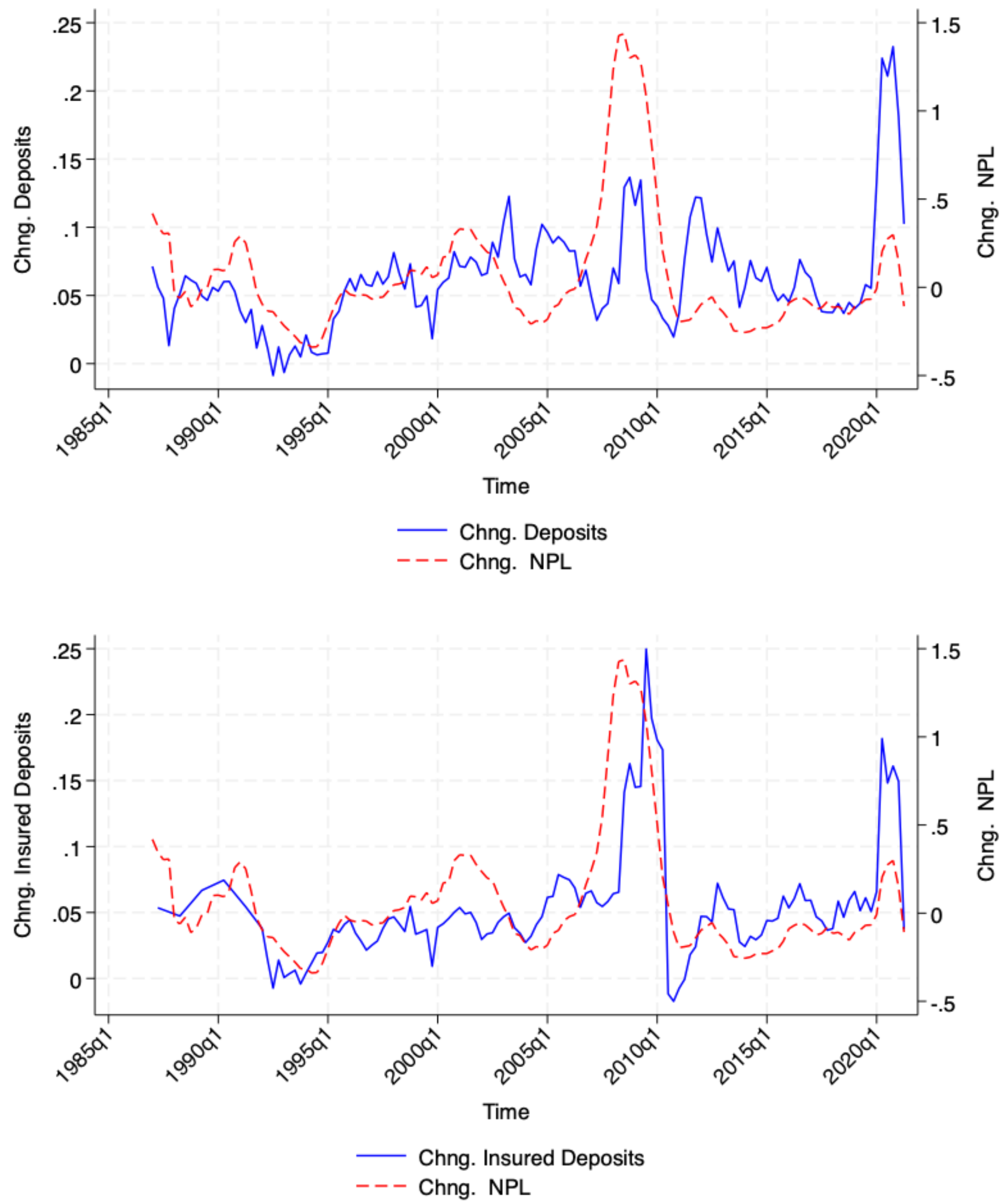

Figure 2: Deposits and NPL Growth

Notes: This figure plots the relationship between quarterly deposit growth rates and the aggregate change in the NPL. The top panel plots the growth of total deposits and the bottom panel plots the growth of insured deposits.
Source: Call Reports.

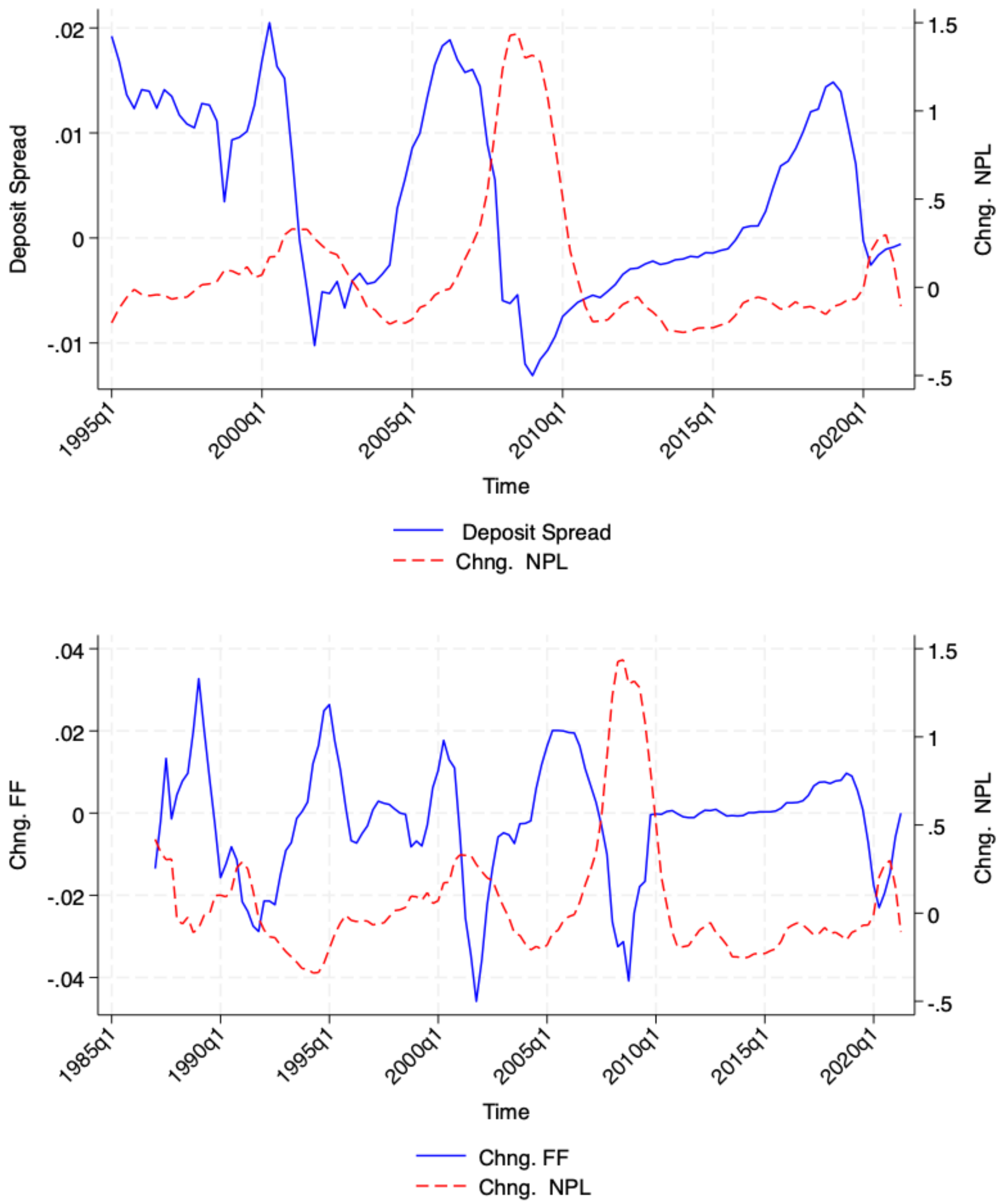

Figure 3: Deposit Spreads, Interest Rates, and NPL Growth

Notes: This figure plots the relationship between interest rates and the change in the NPL ratio. The top panel plots the deposit rate spread and the bottom panel plots the effective federal funds rate, in relation to the change in the NPL ratio.
Source: Call Reports.

8 Tables

Table 1: Summary Statistics

	Mean	SD	Min	P25	P50	P75	Max	N
Implied σ	0.337	0.24	0.029	0.232	0.280	0.365	8.486	25,756
Portfolio Weight IV	0.012	0.01	0.000	0.005	0.007	0.015	0.079	732,677
NPL Ratio	0.009	0.01	0.000	0.002	0.005	0.012	0.073	1,173,296

Notes: Implied σ is rolling one-year implied volatility of sample banks' equity returns using Optionsmetric. NPL ratio is defined as the sum of nonaccrual loans and accruing loans that are past due by more than 90 days. PortfolioWeightIV is the Bartik instrument constructed by multiplying a bank's 3-year lagged portfolio weights across five asset classes by the aggregate NPL levels in those asset classes.
Source: OptionMetrics, Call Reports

Table 2: Deposit Put Channel: Deposit Quantities

	Log(Deposits)	NPL Ratio	Log(Deposits)	
	(1)	(2)	(3)	(4)
NPL Ratio	0.017***			
	(7.11)			
$\widehat{\text{NPL Ratio}}$				0.161***
				(12.80)
Portfolio Weight IV		0.612***	0.098***	
		(50.34)	(13.20)	
Book Equity to Book Assets	-0.254***	-0.086***	-0.131***	-0.117***
	(-48.92)	(-12.56)	(-15.29)	(-13.72)
Year-Quarter FE	✓	✓	✓	✓
Bank FE	✓	✓	✓	✓
Observations	1,172,151	732,120	732,120	732,120
R^2	0.9138	0.4791	0.9365	-0.0471

Notes: This table documents the relationship between a bank's NPL Ratio and deposit quantities. Log(Deposits) is the log of total deposits. $\widehat{\text{NPL Ratio}}$ is the NPL Ratio instrumented by the lagged portfolio weights Bartik instrument. Heteroskedasticity-robust t-statistics are shown in parentheses. *, **, and *** represent statistical significance at the 10%, 5%, and 1% levels, respectively.
Source: Call Reports.

Table 3: Deposit Put Channel: Insured Fraction

	Insured Fraction		
	(1)	(2)	(3)
NPL Ratio	0.011***		
	(28.80)		
$\widehat{\text{NPL Ratio}}$			0.030***
			(18.19)
Portfolio Weight IV		0.018***	
		(17.28)	
Book Equity to Book Assets	-0.005***	0.002**	0.004***
	(-8.00)	(2.18)	(5.01)
Bank Size	-0.049***	-0.058***	-0.058***
	(-23.88)	(-19.12)	(-19.55)
Year-Quarter FE	✓	✓	✓
Bank FE	✓	✓	✓
Observations	973,702	732,115	732,112
R^2	0.7706	0.7987	0.0104

Notes: This table documents the relationship between the NPL Ratio of a bank and the fraction of insured deposits. "Insured Fraction" is the fraction of deposits at the bank that are insured. $\widehat{NPLRatio}$ is the NPL Ratio instrumented by the lagged portfolio weights Bartik instrument.
Source: Call Reports.

Table 4: Deposit Put Channel: Branch Deposit Quantities

	Log(Branch Deposits)			
	(1)	(2)	(3)	(4)
NPL Ratio	0.022***	0.026***		
	(5.42)	(4.99)		
$\widehat{\text{NPL Ratio}}$			0.185**	0.206**
			(2.33)	(2.09)
Book Equity to Book Assets	-0.061***	-0.063***	-0.040***	-0.041***
	(-5.97)	(-6.13)	(-5.54)	(-5.39)
No. Branches	-0.061***	-0.073***	-0.126***	-0.139***
	(-3.13)	(-3.85)	(-5.24)	(-5.57)
Bank Size	0.019**	0.012	0.002	-0.001
	(1.97)	(1.38)	(0.27)	(-0.16)
Year-Quarter FE				
Branch FE	✓	✓	✓	✓
State × Year-Quarter FE	✓		✓	
County × Year-Quarter FE		✓		✓
Observations	1,917,159	1,911,341	1,342,116	1,337,414

Notes: This table documents the relationship between a bank's NPL Ratio and branch level deposit quantities, measured as log total deposits. $\widehat{\text{NPL Ratio}}$ is the NPL Ratio instrumented by the lagged portfolio weights Bartik instrument. *Source:* Call Reports, Summary of Deposits.

Table 5: Deposit Put Channel: Rates Branch level

	Wholesale Insured Rate		Submarginal Insured Rate	
	(1)	(2)	(3)	(4)
NPL Ratio	0.006**		0.011***	
	(2.26)		(2.65)	
$\widehat{\text{NPL Ratio}}$		0.074***		0.066**
		(3.65)		(2.24)
Bank Size	-0.007	0.008	-0.002	-0.002
	(-1.08)	(1.02)	(-0.21)	(-0.18)
Book Equity to Book Assets	-0.008	0.008	-0.003	0.003
	(-1.13)	(0.73)	(-0.34)	(0.25)
No. Branches	-0.014*	-0.025	0.006	0.051**
	(-1.77)	(-1.27)	(0.30)	(2.49)
County $\times$ Year-Quarter FE	✓	✓	✓	✓
Branch FE	✓	✓	✓	✓
Observations	250,298	133,231	198,297	76,305

Notes: This table documents the relationship between a bank's NPL Ratio and the interest rate on insured deposits. $\widehat{\text{NPL Ratio}}$ is the NPL Ratio instrumented by the lagged portfolio weights Bartik instrument. "Wholesale Insured Rate" is the 12M CD rate on 100K deposits from 2001 to 2010 and the 12M CD rate on 250K deposits from 2010 onwards. "Submarginal Insured Rate" is the 12M CD rate on 50K deposits from 2001 to 2010 and the 12M CD rate on 200K deposits from 2010 onwards.
Source: Call Reports, RateWatch.

Table 6: Deposit Put Channel: Time Series NPL Ratio

	Deposit Growth	Chng Insured Fraction	Deposit Spread
	(1)	(2)	(3)
NPL Growth	0.011***	0.008***	-0.003***
	(3.81)	(3.02)	(-4.91)
Observations	138	118	106
R^2	0.0866	0.1412	0.1139

Notes: This table documents the time series correlation between a bank's NPL Ratio and its deposit growth, quarterly change in the fraction of insured deposits, and the deposit spread computed as the federal funds rate minus the average deposit rate.
Source: Call Reports, RateWatch.

Table 7: Deposits and Fed Funds Rate: 1991Q1-2021Q4

	Δln(Dep)	Δln(Ins. Dep)	Ins. Share
	(1)	(2)	(3)
Δ FF Rate	-0.0064***	-0.0051***	0.0015***
	(-29.4344)	(-25.5186)	(18.7566)
ln(Assets)	-0.0102***	-0.0112***	-0.0532***
	(-16.7383)	(-18.2216)	(-55.7977)
Equity/Assets	0.3943***	0.4875***	-0.2313***
	(9.5522)	(12.4971)	(-9.8562)
Bank FE	✓	✓	✓
Observations	1,002,858	990,356	1,002,957
R^2	0.0774	0.0855	0.7401

Notes: This table documents the relationship between the quarterly total deposit growth, quarterly insured deposit growth, quarterly the fraction of insured deposits and quarterly changes in the Federal Funds Effective Rate. Control variables include size and leverage.
Source: Call Reports.

Table 8: Deposit Put Channel: Deposit Growth

	Deposit Growth					
	(1)	(2)	(3)	(4)	(5)	(6)
Δ FF	-0.403***	-0.278***	-0.194*			
	(-4.44)	(-2.76)	(-1.86)			
Δ NPL Ratio		0.010***	0.008***	0.005**	0.006***	0.006***
		(3.71)	(3.01)	(2.49)	(3.13)	(3.01)
Bank Size			-0.025***	-0.010***	-0.009***	-0.009**
			(-8.54)	(-3.16)	(-2.85)	(-2.43)
Book Equity to Book Assets			-0.022***	-0.010***	-0.010***	-0.009***
			(-8.13)	(-3.20)	(-3.50)	(-2.89)
HHI			0.061***	0.048**	0.034**	
			(3.06)	(2.36)	(2.06)	
Δ FF $\times$ HHI			-1.420***	-1.113***	-1.262***	
			(-3.60)	(-3.06)	(-4.56)	
Year-Quarter FE				✓		
Branch FE	✓	✓	✓	✓	✓	✓
State $\times$ Year-Quarter FE					✓	
County $\times$ Year-Quarter FE						✓
Observations	1,701,731	1,691,307	1,691,307	1,691,307	1,691,281	1,686,338
R^2	0.1375	0.1379	0.1447	0.1532	0.1612	0.1940

Notes: This table replicates the DSS effect on interest rates and documents the relationship between the change in NPL ratio of a bank and deposit growth. Δ NPL Ratio is the one-year change in a bank's NPL Ratio. Δ FF is the one-year change in the federal funds rate. "HHI" is the county-level Herfindahl–Hirschman index of bank branch deposiots.
Source: Summary of Deposits, Call Reports.

Appendix

8.1 Sensitivity of Market Value of Deposit Insurance to r

$$\begin{aligned}
P[L,K,\sigma,r,T] &= Ke^{-rT}\Phi(-d_2) - L\Phi(-d_1) \qquad (15)\\
d_1 &= \frac{ln(\frac{L}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}};\\
d_2 &= d_1 - \sigma\sqrt{T};\\
K &= D \cdot e^{rT};\\
\frac{L}{K} &= \frac{D}{wD \cdot e^{rT}} = \frac{1}{w \cdot e^{rT}}.\\
\sigma &= \hat{b} - \hat{a} * r
\end{aligned}$$

$$\begin{aligned}
P[L,K,\sigma,r,T] &= D\Phi(-d_2) - \frac{D}{w}\Phi(-d_1)\\
d_1 &= \frac{-ln(w \cdot e^{rT}) + (r + \frac{(\hat{b}-\hat{a}*r)^2}{2})T}{(\hat{b} - \hat{a} * r)\sqrt{T}}\\
d_1 &= \frac{-ln(w) - rT + (r + \frac{(\hat{b}-\hat{a}*r)^2}{2})T}{(\hat{b} - \hat{a} * r)\sqrt{T}}\\
d_1 &= \frac{-ln(w) + (\frac{(\hat{b}-\hat{a}*r)^2}{2})T}{(\hat{b} - \hat{a} * r)\sqrt{T}}\\
d_1 &= \frac{-ln(w)}{(\hat{b} - \hat{a} * r)\sqrt{T}} + \frac{(\hat{b} - \hat{a} * r)\sqrt{T}}{2}\\
d_2 &= d_1 - (\hat{b} - \hat{a} * r)\sqrt{T} \qquad (16)
\end{aligned}$$

It follows that:

$$\frac{1}{D}\frac{\partial P[L,K,\sigma,r,T]}{\partial r} = \frac{\partial \Phi(-d_2)}{\partial r} - \frac{1}{w}\frac{\partial \Phi(-d_1)}{\partial r} \qquad (17)$$

$$\begin{aligned}
\frac{\partial \Phi(-d_1)}{\partial r} &= \frac{\partial \Phi(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial r}\\
&= -\phi(-d_1).\frac{\partial}{\partial r}\{\frac{-ln(w)}{(\hat{b} - \hat{a} * r)\sqrt{T}} + \frac{(\hat{b} - \hat{a} * r)\sqrt{T}}{2}\}\\
&= -\phi(-d_1)\{\frac{-\hat{a}.ln(w)}{(\hat{b} - \hat{a} * r)^2\sqrt{T}} - \frac{\hat{a}\sqrt{T}}{2}\}\\
&= \phi(-d_1)\{\frac{\hat{a}.ln(w)}{(\hat{b} - \hat{a} * r)^2\sqrt{T}} + \frac{\hat{a}\sqrt{T}}{2}\} \qquad (18)
\end{aligned}$$

Now,

$$\begin{aligned}
\phi(-d_2) &= \phi(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}} = \frac{1}{\sqrt{2\pi}}e^{-\frac{(d_1-\sigma\sqrt{T})^2}{2}} = \frac{1}{\sqrt{2\pi}}e^{-\frac{(d_1^2-2d_1\sigma\sqrt{T}+\sigma^2T)}{2}} \\
&= \phi(d_1)e^{-\frac{(-2d_1\sigma\sqrt{T}+\sigma^2T)}{2}} = \phi(d_1)e^{-\frac{(-2(ln\frac{L}{K}+(r+\frac{\sigma^2}{2})T)+\sigma^2T)}{2}} \\
&= \phi(d_1)e^{ln(\frac{L}{K})+rT} = \phi(d_1)e^{ln(\frac{1}{w.e^{rT}})+rT} \\
&= \phi(d_1)e^{-ln(w)} = \frac{1}{w}\phi(d_1) \qquad (19)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial\Phi(-d_2)}{\partial r} &= -\phi(-d_2)\frac{\partial d_2}{\partial r} \\
&= -\phi(-d_2)\{\frac{\partial d_1}{\partial r} + \hat{a}\sqrt{T}\} \\
&= -\frac{1}{w}\phi(d_1)\{\frac{-\hat{a}.ln(w)}{(\hat{b}-\hat{a}*r)^2\sqrt{T}} + \frac{\hat{a}\sqrt{T}}{2}\} \qquad (20)
\end{aligned}$$

Combining with equation 14 above:

$$\begin{aligned}
\frac{1}{D}\frac{\partial P[L,K,\sigma,r,T]}{\partial r} &= \frac{\partial\Phi(-d_2)}{\partial r} - \frac{1}{w}\frac{\partial\Phi(-d_1)}{\partial r} \\
&= -\frac{1}{w}\phi(d_1)\{\frac{-\hat{a}.ln(w)}{(\hat{b}-\hat{a}*r)^2\sqrt{T}} + \frac{\hat{a}\sqrt{T}}{2}\} - \frac{1}{w}\phi(-d_1)\{\frac{\hat{a}.ln(w)}{(\hat{b}-\hat{a}*r)^2\sqrt{T}} + \frac{\hat{a}\sqrt{T}}{2}\} \\
&= (\frac{1}{w}-\frac{1}{w})\phi(d_1)\{\frac{\hat{a}.ln(w)}{(\hat{b}-\hat{a}*r)^2\sqrt{T}}\} + \phi(d_1)(-\frac{1}{w}-\frac{1}{w})\{\frac{\hat{a}\sqrt{T}}{2}\} \\
&= \phi(d_1)(-\frac{2}{w})\{\frac{\hat{a}\sqrt{T}}{2}\} = -\phi(d_1)\{\frac{\hat{a}\sqrt{T}}{w}\} \qquad (21)
\end{aligned}$$

Since $w > 0$, the above value is negative for all parameter values.

8.2 Bank's Optimization Problem

$$\max_{\{D\}} P[L,K,\sigma,r,T] - \frac{1}{2}cD^2 - \bar{p}D$$

FOC:

$$\frac{\partial P[L,K,\sigma,r,T]}{\partial D} = cD + \bar{p}$$

Note:

$$P[L,K,\sigma,r,T] = D\Phi(-d_2) - \frac{D}{w}\Phi(-d_1) \tag{22}$$
$$d_1 = \frac{-ln(w)}{(\hat{b}-\hat{a}*r)\sqrt{T}} + \frac{(\hat{b}-\hat{a}*r)\sqrt{T}}{2}$$
$$d_2 = d_1 - (\hat{b}-\hat{a}*r)\sqrt{T}$$

$$\begin{aligned} \frac{\partial P[L,K,\sigma,r,T]}{\partial D} &= \Phi(-d_2) + D.\frac{\Phi(-d_2)}{\partial D} - \frac{\Phi(-d_1)}{w} - \frac{D}{w}\frac{\partial\Phi(-d_1)}{\partial D} \\ &= \Phi(-d_2) - D\phi(-d_2)\frac{\partial d_2}{\partial D} - \frac{\Phi(-d_1)}{w} + \frac{D}{w}\phi(-d_1)\frac{\partial d_1}{\partial D} \\ &= \Phi(-d_2) - \frac{\Phi(-d_1)}{w} \end{aligned} \tag{23}$$

Therefore, optimal quantity of deposits is given by the following equation:

$$\Phi(-d_2) - \frac{\Phi(-d_1)}{w} = cD + \bar{p}$$
$$D^* = \frac{1}{c}\{\Phi(-d_2) - \frac{\Phi(-d_1)}{w} - \bar{p}\} \tag{24}$$

Optimal supply of deposits decreases when marginal cost c is higher; and when deposit insurance premium paid, $\bar{p}$ is higher. It increases when the default option has higher value.

Now using the envelope theorem, let's find the sensitivity of optimal D w.r.t. 'r'

$$\begin{aligned} \frac{\partial D^*}{\partial r} &= \frac{1}{c}\frac{\partial}{\partial r}\{\Phi(-d_2) - \frac{\Phi(-d_1)}{w}\} \\ &= -\phi(d_1)\{\frac{\hat{a}\sqrt{T}}{cw}\} \end{aligned} \tag{25}$$

Therefore, when interest rate goes up, optimal deposit financing comes down.

8.3 Model with liquidity benefits of deposits:

Suppose a bank makes a loan of value L with a maturity of T_L at an interest rate r_L. Risk-free rate is r and $r_L > r$. Loan is funded with insured deposit D and equity capital of E. For simplicity assume that the deposits are issued as zero coupon bond with maturity T and a promised rate of return of r_D. Therefore, deposit has a face value of $D.e^{r_d T} \equiv K$. Rate of return on deposits is below the market rate of return r, specifically, $r_d = r.(1-\alpha) - \beta$, where $0 < \alpha < 1$, and $\beta > 0$. This is a flexible parametrization that captures both a fixed amount and a variable amount of liquidity premium a depositor is willing to pay.

Deposits are insured by the FDIC at a fixed rate of c per unit of the market value of a deposit. The market value of the deposit insurance, P, can be obtained by a standard put option formula on the asset value of the bank with the face value of a deposit as the strike price.

$$
\begin{aligned}
P[L,K,\sigma,r,T] &= Ke^{-rT}\Phi(-d_2) - L\Phi(-d_1) && (26)\\
d_1 &= \frac{ln(\frac{L}{K}) + (r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$

The amount of subsidy that a bank enjoys on its insured deposit is the difference between the market value of deposit insurance and the premium paid. The subsidy changes as interest rates change because the fair market valuation of deposit insurance changes with interest rate. When interest rates are high, market value comes down and therefore the subsidy enjoyed by banks comes down. Proof below:

$$
\begin{aligned}
\frac{\partial P[L,K,\sigma,r,T]}{\partial r} &= \frac{\partial Ke^{-rT}\Phi(-d_2)}{\partial r} - \frac{\partial L\Phi(-d_1)}{\partial r} && (27)\\
&= \frac{\partial De^{r_d T}e^{-rT}\Phi(-d_2)}{\partial r} - \frac{\partial L\Phi(-d_1)}{\partial r}\\
&= \frac{\partial De^{(-\alpha r-\beta)T}\Phi(-d_2)}{\partial r} - \frac{\partial L\Phi(-d_1)}{\partial r}\\
&= \{-\alpha TDe^{-(\alpha r+\beta)T}\Phi(-d2)\} + \frac{De^{-(\alpha r+\beta)T}\partial\Phi(-d_2)}{\partial r} - \frac{\partial L\Phi(-d_1)}{\partial r} && (28)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\Phi(-d_1)}{\partial r} &= \frac{\partial\Phi(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial r}\\
&= -\phi(-d_1).\frac{\partial}{\partial r}\frac{ln(\frac{L}{K}) + (r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\\
&= -\phi(-d_1).\frac{1}{\sigma\sqrt{T}}\{\frac{\partial}{\partial r}(-lnK) + T\}\\
&= -\phi(-d_1).\frac{1}{\sigma\sqrt{T}}\{\frac{\partial}{\partial r}(-ln(De^{r_d T})) + T\}\\
&= -\phi(-d_1).\frac{1}{\sigma\sqrt{T}}\{\frac{\partial}{\partial r}(-r_d T) + T\}\\
&= -\phi(-d_1).\frac{1}{\sigma\sqrt{T}}\{(-(1-\alpha)T) + T\}
\end{aligned}
\tag{29}
$$

$$\begin{aligned}
&= -\phi(-d_1).\frac{1}{\sigma\sqrt{T}}\alpha T \\
\frac{\partial\Phi(-d_2)}{\partial r} &= -\phi(-d_2)\frac{\partial d_2}{\partial r} = -\phi(-d_2)\frac{\partial d_1}{\partial r} = -\phi(-d_2)\frac{1}{\sigma\sqrt{T}}\alpha T \\
\frac{\partial P[.]}{\partial r} &= \{-\alpha T D e^{(-\alpha r-\beta)T}\Phi(-d2)\} \\
&\quad -De^{-\alpha r T}\phi(-d_2).\frac{1}{\sigma\sqrt{T}}\alpha T + L\phi(-d_1).\frac{1}{\sigma\sqrt{T}}\alpha T
\end{aligned} \tag{30}$$

$$\begin{aligned}
\phi(-d_2) &= \phi(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}} = \frac{1}{\sqrt{2\pi}}e^{-\frac{(d_1-\sigma\sqrt{T})^2}{2}} = \frac{1}{\sqrt{2\pi}}e^{-\frac{(d_1^2-2d_1\sigma\sqrt{T}+\sigma^2T)}{2}} \\
&= \phi(d_1)\frac{1}{\sqrt{2\pi}}e^{-\frac{(-2d_1\sigma\sqrt{T}+\sigma^2T)}{2}} \\
&= \phi(d_1)e^{ln(\frac{L}{K})+rT} = \phi(d_1)e^{ln(\frac{L}{D.e^{r_dT}})+rT} \\
&= \phi(d_1)e^{rT}\frac{L}{D.e^{r_dT}} = \phi(d_1)e^{(\alpha r+\beta)T}\frac{L}{D}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P[.]}{\partial r} &= \{-\alpha T D e^{-(\alpha r+\beta)T}\Phi(-d2)\} \qquad (31)\\
&\quad -De^{-(\alpha r+\beta)T}\phi(d_2).\frac{1}{\sigma\sqrt{T}}\alpha T + L\phi(d_1).\frac{1}{\sigma\sqrt{T}}\alpha T \\
&= \{-\alpha T D e^{-(\alpha r+\beta)T}\Phi(-d2)\} \qquad (32)\\
&\quad -De^{-(\alpha r+\beta)T}\phi(d_1)e^{(\alpha r+\beta)T}\frac{L}{D}.\frac{1}{\sigma\sqrt{T}}\alpha T + L\phi(d_1).\frac{1}{\sigma\sqrt{T}}\alpha T \\
&= -\alpha D T e^{-(\alpha r+\beta)T}\Phi(-d2) < 0 \qquad (33)
\end{aligned}$$

Note from the first order condition:

$$\frac{\partial P[L,K,\sigma,r,T]}{\partial D} = cD + \bar{p} \tag{34}$$

$$\begin{aligned}
\frac{\partial P[L,K,\sigma,r,T]}{\partial D} &= \frac{\partial K e^{-rT}\Phi(-d_2)}{\partial D} - \frac{\partial L\Phi(-d_1)}{\partial D} \qquad (35)\\
&= \frac{\partial D e^{r_dT}e^{-rT}\Phi(-d_2)}{\partial D} - \frac{\partial D\Phi(-d_1)}{w\partial D} \\
&= \frac{\partial D e^{(-\alpha r-\beta)T}\Phi(-d_2)}{\partial D} - \frac{\partial D\Phi(-d_1)}{w\partial D} \\
&= e^{-(\alpha r+\beta)T}\Phi(-d2) + \frac{De^{-(\alpha r+\beta)T}\partial\Phi(-d_2)}{\partial D} - \frac{\Phi(-d_1)}{w} - \frac{D\partial\Phi(-d_1)}{w\partial D} \\
&= e^{-(\alpha r+\beta)T}\Phi(-d2) - De^{-(\alpha r+\beta)T}\phi(-d_2)\frac{\partial d_2}{\partial D} - \frac{\Phi(-d_1)}{w} + \frac{D}{w}\phi(-d_1)\frac{\partial d_1}{\partial D} \\
&= e^{-(\alpha r+\beta)T}\Phi(-d_2) - \frac{\Phi(-d_1)}{w} \qquad (36)
\end{aligned}$$

Optimal deposit qunatity is given by:

$$\begin{aligned} e^{-(\alpha r+\beta)T}\Phi(-d_2) - \frac{\Phi(-d_1)}{w} &= cD^* + \bar{p} \\ D^* &= \frac{1}{c}\{e^{-(\alpha r+\beta)T}\Phi(-d_2) - \frac{\Phi(-d_1)}{w} - \bar{p}\} \end{aligned}$$

There, the sensitivity of deposit supply to interest rate is given by the following:

$$\begin{aligned} c\frac{\partial D^*}{\partial r} &= -\alpha T e^{-(\alpha r+\beta)T}\Phi(-d_2) + e^{-(\alpha r+\beta)T}\frac{\partial \Phi(-d_2)}{\partial r} - \frac{1}{w}\frac{\partial \Phi(-d_1)}{\partial r} \\ &= -\alpha T e^{-(\alpha r+\beta)T}\Phi(-d_2) - e^{-(\alpha r+\beta)T}\phi(-d_2)\frac{\partial d_2}{\partial r} + \frac{1}{w}\phi(-d_1)\frac{\partial d_1}{\partial r} \\ &= -\alpha T e^{-(\alpha r+\beta)T}\Phi(-d_2) - e^{-(\alpha r+\beta)T}\phi(-d_2)\frac{\alpha T}{\sigma\sqrt{T}} + \frac{1}{w}\phi(-d_1)\frac{\alpha T}{\sigma\sqrt{T}} \\ &= -\alpha T e^{-(\alpha r+\beta)T}\Phi(-d_2) - e^{-(\alpha r+\beta)T}.e^{(\alpha r+\beta)T}\frac{1}{w}\phi(d_1)\frac{\alpha T}{\sigma\sqrt{T}} + \frac{1}{w}\phi(d_1)\frac{\alpha T}{\sigma\sqrt{T}} \\ &= -\alpha T e^{-(\alpha r+\beta)T}\Phi(-d_2) < 0. \end{aligned} \tag{37}$$