

Financing Small Investments

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October 20, 2025

Abstract

Most financing theories predict that external funding should take the form of debt, especially when the funding requirement is small. We show, in a standard moral hazard model, that a risk-averse entrepreneur will optimally not use debt to meet a sufficiently low funding need. Debt financing causes the entrepreneur to bear the firm's entire risk; while it also maximizes effort incentives, this benefit is second-order if the amount raised is small. This rationalizes the use of equity-based funding for start-ups with small funding needs, and suggests a greater use of non-debt financing in developing countries.

Small investments, such as those undertaken by start-ups, are often financed by equity. This involves seed funding, angel finance, and investments from friends and relatives. In developing countries, the latter is part of “informal finance”, and a substantial source of funding for small firms (Beck, Demirgüç-Kunt, Maksimovic (2008)).

However, most financing theories predict that external financing should be raised through debt. This includes models based on moral hazard, whether with unobservable cash flows (Hart and Moore (1998)), cash flows observable at a cost (Townsend (1979), Gale and Hellwig (1985)), and contractible cash flows (Innes (1990)).¹ Other theories argue that debt should be used when financing needs are small, but equity should be used when they are sufficiently large. The trade-off theory of capital structure predicts that firms should first issue debt to benefit from tax shields, but when they are close to exhausting their tax shields and bankruptcy risk rises, they should switch to equity. The pecking order theory (Myers, 1984) predicts that a firm should first issue debt and switch to equity when it has exhausted its debt capacity.

In this paper, we rationalize equity funding for small investments in a simple model of financing with moral hazard. We consider a standard Innes (1990) model, with the only departure being that the entrepreneur is risk averse. In Innes (1990), the optimal financing contract is debt so that the entrepreneur is the residual claimant, thus maximizing his effort incentives. It may seem that the Innes (1990) logic continues to apply under risk aversion if risk aversion is sufficiently low, and so risk-sharing appears secondary to providing effort incentives. Indeed, the textbook of Tirole (2006) states that: “when an entrepreneur’s borrowing needs are relatively small and there is enough guaranteed future income (collateral, or certain cash flow) to repay the corresponding debt ... issuing debt to investors implies that any increase in the firm’s profit goes to the entrepreneur. Put

¹Debt remains optimal in some broader moral hazard frameworks. When the entrepreneur and investors can renegotiate the contract after effort, the optimal contract is debt because it maximizes the entrepreneur’s effort incentives (Hermalin and Katz (1991); Dewatripont, Legros, and Matthews (2003)). When the manager can affect the dispersion of output in addition to its mean, the optimal contract is debt because it is the least risky security (Hébert (2018)).

differently, the entrepreneur fully internalizes the increase in profit brought about by her actions, and so faces the “right incentives” to minimize cost and maximize profit.”

In contrast, we show that, even if the entrepreneur’s risk aversion is low, the optimal financing contract is not debt for small investments. Indeed, the contract is the polar opposite of debt: the investor receives levered equity. The intuition is that, with small financing needs, the entrepreneur’s effort incentives are very similar to when he owns the whole firm. As a result, the contract is primarily designed for risk-sharing purposes. Issuing debt to the investor exposes the entrepreneur to the firm’s entire risk and is thus suboptimal.

It may seem that there is a countervailing force: when the amount raised is small, the risk-sharing benefit should also be small, and so the entrepreneur does not lay off much risk by issuing an equity (or other non-debt) claim. However, the first effect dominates, and the intuition is as follows. When there is no need for external financing, the entrepreneur receives the entire output and his effort is first-best optimal. When there is a small financing need and so the entrepreneur only needs to give the investor a small share of output, the change in effort induced by the contract has a second-order effect on the entrepreneur’s objective function, since effort was first-best optimal without the financing need. In contrast, there is no risk-sharing without the financing need, and so the benefits of sharing risk with a risk-neutral investor are first-order.

Issuing levered equity to the investor leads to the greatest risk sharing under contracting constraints. Risk sharing involves flattening the entrepreneur’s payoff across states. For low outputs, the entrepreneur receives the highest possible payoff (the entire output), and so the investor receives zero – the lowest possible under investor limited liability. For high outputs, levered equity means that the entrepreneur payoff is capped and the investor is residual claimant. We obtain this result in a general setting that does not assume specific functional forms for the entrepreneur’s utility of wealth, cost of effort, or the probability distribution of cash flows.

The optimality of levered equity arises if two conditions are true. The first is that the entrepreneur is risk averse. Entrepreneurs are less wealthy than typical CEOs, particu-

larly in developing countries where most jobs are created and operated by self-employed entrepreneurs (Cho and Honorati (2014)), which suggests that risk aversion is a first-order consideration. The second is that the amount of financing raised is small. This holds in several common cases. One is start-ups in most countries. Recent evidence suggests that many start-ups do not need much external funding at early stages. According to the U.S. Small Business Administration, the starting cost of a typical microbusiness is around \$3,000. Cloud computing allowed entrepreneurs to replace large fixed investments in hardware by cloud-based expenses (Ewens et al. (2018)), which “unleashed a wave of new entrepreneurs starting firms with much less capital” (Nanda and Phillips (2023)). Likewise, generative AI is reducing the cost of starting businesses.² In developing countries, smartphones lowered the cost of setting up start-ups (Asongu, Nwachukwu, and Orim (2018)). The second case is micro, small, and medium enterprises (“MSMEs”) in developing countries. World Bank data suggests that the average amount of finance raised is \$22,840; the median is likely lower if the distribution of external funding is positively skewed.³ Indeed, amounts raised are often much smaller. On the crowdfunding platform Kiva, which provides millions of loans to borrowers in developing countries, the average loan amount is \$927.⁴

To highlight the role of small financing needs in the optimality of levered equity, we next consider the case of very large financing needs, and show that the optimal contract is quite different. Since the investor needs to be given high payoffs to ensure her participation, the constraint that now binds is entrepreneur limited liability. As a result, for outputs below a threshold, the entrepreneur is paid zero and the investor receives the full output. This contrasts the case of small financing needs, where it is the entrepreneur who receives the entire output below a threshold. With small financing needs, the entrepreneur has debt (or any debt-like payoff that has priority over equity, such as a wage); with large financing needs, the investor has debt. Above the threshold, the entrepreneur and investor split the

²*The Economist* (August 11, 2025) remarks that “Some hope that generative AI will make starting a business so cheap and hassle-free that anyone will be able to become an entrepreneur much as anyone can become a YouTuber”.

³Source: MSME Finance Gap, report by the International Finance Corporation, 2017.

⁴Source: Kiva 2023 Annual Report.

additional output to share risk while incentivizing effort. Under some conditions (e.g. log utility and normally distributed output), the contract is linear above the threshold with a slope less than 1, i.e. both the investor and the entrepreneur have levered equity. Thus, the investor receives both debt and levered equity, compared to the case of low financing needs where she receives only levered equity.

For all utility functions and output distributions, the contracting party who is paid first is the entrepreneur when the financing need is small and the investor when it is large. The party who is paid first receives a debt-like payoff, since the debtholder receives the entire output when output is low, plus something else in some cases, e.g. levered equity for the investor when the financing need is large. The other party is the one whose limited liability constraint binds. For example, when the financing need is small, the entrepreneur is paid first and the investor is paid zero; the investor is only paid if output is sufficiently large.

Having explored the optimal contract, we finally study the link between the amount of financing raised and the entrepreneur's optimal effort level. We show numerically that effort can be a non-monotonic function of the amount of external funding. When this funding requirement is low, the entrepreneur's payoff is high, so that his risk aversion is low and the forces in the Innes (1990) model dominate: an increase in external funding means that the entrepreneur is less of a residual claimant, reducing his effort incentives. On the other hand, an increase in external funding reduces the entrepreneur's payoff and increases his marginal utility, due to his concave utility function, thus increasing his effort incentives. If the entrepreneur has constant relative risk aversion ("CRRA"), the second effect dominates for intermediate levels of external funding. This is because an entrepreneur with CRRA utility has decreasing absolute risk aversion ("DARA"), and is therefore prudent, i.e. his utility function has a high curvature at low levels of wealth. As a result, decreases in the entrepreneur's payoff (from further increases in external funding needs) have a particularly positive effect on his marginal utility and thus effort incentives. Finally, for high levels of external funding, the entrepreneur is already paid close to zero for low outputs, so that increases in external funding needs do not substantially increase his marginal utility, and

the skin-in-the-game effect again dominates.

Some theories based on asymmetric information can explain equity-based investments when some investors have better information about firm value (Boot and Thakor (1993), Axelson (2007)), better information about its growth opportunities (Fulghieri, García, and Hackbarth (2020)), or can invest resources to produce better information (Fulghieri and Lukin (2001)). However, investors do not have strong incentives to acquire costly information when investing small amounts. Moreover, there is evidence that angel investors are neither highly sophisticated (Yimfor (2021)) nor very responsive to financial returns (Denes et al. (2023)), so that Nanda and Phillips (2023) argue that they are “more likely to suffer adverse selection.” Furthermore, angel investments are often done with minimal due diligence on the part of investors, which suggests that they do not have an informational advantage (Edelman, Manolova, and Brush (2017); Huang et al. (2023)). In sum, theories based on asymmetric information which predict that firms facing informed investors will issue equity seem to be less relevant when financing needs are small.

This paper is related to research exploring when optimal financing contracts under moral hazard are not debt. When the entrepreneur or the investor is ambiguity averse and the output distribution is uncertain (rather than risky), the optimal contract can include an equity component (Lee and Rajan (2020), Malenko and Tsoy (2023)). Their focus on model uncertainty may be more relevant for start-up with disruptive or transformational investments rather than those with a low need for external funding that we focus on. Chaigneau, Edmans, and Gottlieb (2024) give conditions for the manager to be the residual claimant in a compensation model where the owner offers a contract to the manager and the manager’s participation constraint is non-binding. As a result, there are no risk-sharing considerations. In our financing model, the entrepreneur offers a contract to the investor who must break even. Her participation constraint always binds, which introduces risk-sharing concerns. Indeed, they are the key force in our model, and for small investments the investor is the residual claimant. In addition, we show how the size of the investment affects the optimality of debt and effort level exerted by the entrepreneur, but it plays no role in Chaigneau,

1 The Model

An entrepreneur (“he”) must raise funding $I > 0$ from a risk-neutral investor (“she”) at $t = 0$ to invest in a project that produces a stochastic payoff q at $t = 1$. After doing so, the entrepreneur exerts unobservable effort $e \in \mathbb{R}_+$ at $t = 0$. Effort is any action that improves the output distribution but is privately costly to the entrepreneur, such as working rather than shirking, choosing projects that generate cash rather than private benefits, or not extracting rents.

The entrepreneur’s cost of effort $C(\cdot)$ is strictly increasing, strictly convex, twice continuously differentiable, with $C(0) = C'(0) = 0$ and $\lim_{e \rightarrow \infty} C'(e) = \infty$. He is risk averse with utility function $u(\cdot)$ which is strictly increasing, strictly concave, and twice continuously differentiable. Denote by $A(w)$ the coefficient of absolute risk aversion. He has outside wealth $\bar{W} > 0$ such that $u'(\bar{W})$ is a finite constant.⁵

Effort affects the probability distribution of contractible output q . Output is continuously distributed with full support on the bounded interval $[0, \bar{q}]$. The density function of output is denoted by $f(q|e)$. We assume that it is continuously differentiable in both q and e , and uniformly bounded away from zero: $f(q|e) \geq \delta > 0$ for some $\delta > 0$. For a given effort e , the likelihood ratio of q is defined as:

$$\frac{\frac{\partial f}{\partial e}(q|e)}{f(q|e)}. \quad (1)$$

We assume that the likelihood ratio of output, $\frac{\frac{\partial f}{\partial e}(q|e,s)}{f(q|e,s)}$, is strictly increasing in output q (“MLRP”).

We assume bilateral limited liability: both the entrepreneur and the investor are pro-

⁵We implicitly assume that this is the portion of his wealth that the entrepreneur cannot invest in the firm. For example, the entrepreneur might have illiquid wealth, or must keep a minimum level of wealth for consumption.

tected by limited liability. As in Innes (1990), we assume a monotonicity constraint: a dollar increase in output cannot increase the entrepreneur's payoff by more than a dollar, otherwise he would inject his own money into the firm to increase output.

The entrepreneur has full bargaining power and offers the investor a contract with a payoff of $w(q)$.

2 Analysis

2.1 Optimal Contract

Define:

$$H_e \equiv \int_0^{\bar{q}} \frac{\partial^2 f}{\partial e^2}(q|e) dq, \quad \text{and} \quad K_e \equiv \int_0^{\bar{q}} q \max \left\{ \frac{\partial^2 f}{\partial e^2}(q|e), 0 \right\} dq.$$

We derive a condition to guarantee the validity of the first-order approach (FOA) to the entrepreneur's effort choice problem.⁶

Lemma 1 *Suppose that $u(\bar{W})H_e + u'(\bar{W})K_e < C''(e) \forall e \in \mathbb{R}_+$. Then the FOA is valid.*

We henceforth assume that the condition in Lemma 1 is satisfied.

Let e^* be implicitly defined as

$$\int u(q) \frac{\partial f}{\partial e}(q|e^*) dq = C'(e^*).$$

That is, e^* is the effort that the entrepreneur chooses when he self-finances the project.

The contracting problem is:

$$\max_{w_I(\cdot), e_I} \int_0^{\bar{q}} u(w_I(q)) f(q|e) dq - C(e) \tag{2}$$

⁶This condition is similar to the condition for the validity of the FOA in Chaigneau, Edmans, and Gottlieb (2022), where it was derived without an upper bound on the agent's payment but with an upper bound on the utility function.

subject to

$$\int_0^{\bar{q}} u(w_I(q)) f_e(q|e_I) dq = C'(e) \quad (\text{IC})$$

$$\int_0^{\bar{q}} [q - w_I(q)] f(q|e_I) dq - I = 0 \quad (\text{IR})$$

$$0 \leq w_I(q) \leq 1 \quad \forall q \quad (\text{Mon})$$

$$0 \leq w_I(q) \leq q \quad (\text{LL})$$

where f_e denotes the first derivative of f with respect to e . The I subscript in e_I and w_I highlights how the entrepreneur's effort and payment depend on the amount of financing raised.

We now study the optimal financing contract when the financing need is small. This applies both to small investments and to large investments which require a small amount of external financing to complement the firm's internal financing. A debt contract is defined by the debt repayment q^* , such that the investor's payoff is $\min\{q, q^*\}$ when output is q .

Proposition 1 *There exists $\bar{I} > 0$ such that for every $I < \bar{I}$, the optimal financing contract is*

$$w_I(q) = \min\{q, q_I^*\}.$$

The key to this result is the low financing need I . This matters for two reasons. First, it reduces the disincentives that arise from the entrepreneur needing to give a share of the output to the investor. Without a financing need, the ex-ante optimal effort (that the entrepreneur would like to commit to at the contracting stage) and the ex-post optimal effort (that the entrepreneur will optimally choose after the contract has been written) automatically coincide because the entrepreneur fully internalizes the effect of his effort. With a financing need, these two levels of effort no longer coincide: the entrepreneur would like to commit to a high effort level at the time of contracting, which would increase expected output and allow the entrepreneur to offer the investor a lower share of output to meet her participation constraint. However, when choosing his effort, after contracting, the en-

trepreneur does not internalize the effect of his effort on the investor's claim and therefore chooses a lower ex-post level of effort. For a small financing need, this distortion is also small: the entrepreneur is close to the residual claimant and the ex-post and ex-ante effort choices are similar. As a result, effort incentives are a second-order driver of the contract.

Instead, the contract is primarily designed to achieve optimal risk sharing given the constraints on contracting. The investor receives levered equity, which is the riskiest contract feasible with limited liability: the investor is only paid after the entrepreneur receives his full payment. This is the polar opposite to the debt contract derived in Innes (1990) under risk aversion. Second, a low financing need means that the investor's payoffs across states are low and so her limited liability constraint binds. It is optimal for this constraint to bind (and her to receive zero) for low outputs, as this maximizes the payments to the risk-averse entrepreneur for low outputs when his marginal utility is high. As a result, the optimal contract is levered equity.

To highlight the role of low I in delivering a levered equity contract for the investor, we now study the case of very high financing needs. We show that if I is sufficiently large, the optimal contract pays zero to the entrepreneur if output is low enough – in contrast to the case of low I where the entrepreneur receives the full output. To demonstrate this result, intuition might suggest that we consider the program above, evaluating the solution at I close to the maximum amount possible. The problem with this specification is that, unlike in the case of a small funding need (where $I = 0$ is the lower bound regardless of effort), the maximum amount of funding depends on effort, which is endogenous to the amount of funding.⁷

To overcome the endogeneity of effort, we consider the contract that maximizes the

⁷For example, holding effort fixed, the entrepreneur cannot pledge more than his entire payment w . Therefore,

$$I \leq \int qf(q|e)dq,$$

which is attained by setting $w(q) = 0$ (a.s.). However, in this case, the entrepreneur chooses to exert zero effort (by IC). It is generally possible to finance amounts greater than $E[q|e = 0]$ with an increasing contract.

investor's profits – the dual problem. By the investor's breakeven constraint, the solution gives the highest amount that the investor would be willing to provide. Formally, we consider the program:

$$\max_{w(\cdot), e} \int_0^{\bar{q}} [q - w(q)] f(q|e) dq \quad \text{subject to (IC), (Mon), and (LL).}$$

Letting w^* and e^* denote solutions to this program, $I^* = \int [q - w^*(q)] f(q|e^*) dq$ is the highest amount that an entrepreneur would be able to raise. Therefore, this program considers the opposite case of a very high financing need, so the entrepreneur needs to give as much of the output as possible to the investor. Proposition 2 shows that the solution to this program always features $w(q) = 0$ for q small enough:

Proposition 2 *There exists $\varepsilon > 0$ such that $w(q) = 0$ for all $q \leq \varepsilon$.*

The entrepreneur's payoff is equity-like, as he receives zero for low outputs. The investor's payoff includes debt, as she receives the entire output for low outputs, in contrast to the case of low I where her claim is levered equity. The key to this result is the high financing need I , and it matters for two reasons. First, the entrepreneur is now far from the residual claimant, and so effort is far from the first-best level. A first-order determinant of the contract is ensuring that the entrepreneur can credibly commit to a high level of effort, which is achieved by paying the entrepreneur zero for low output. Second, high I means that the investor receives high payoffs across states. It is now the entrepreneur's limited liability constraint that binds, and so she receives zero for some outputs. To provide effort incentives, the outputs for which she receives a zero payoff are low outputs. Both considerations mean that the investor receives the entire output for low output levels.

We can derive additional results on the form of the optimal contract in particular cases. The numerical computations in Appendix A show that, with log utility and output that follows a truncated normal distribution, the optimal contract is linear whenever limited liability constraints do not bind. It can therefore be written as: $w(q) = \max \{ \min \{ b(q - y) + y, q \}, 0 \}$ for two constants $b \leq 1$ and y . This also allows us to derive

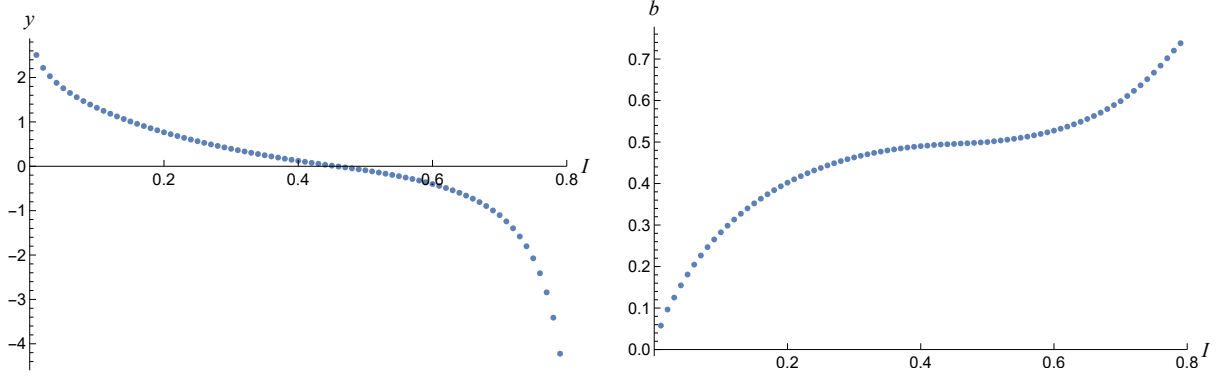


Figure 1: Threshold y (left panel) and slope b (right panel) as a function of I for a contract of the form $w(q) = \max \{ \min \{ b(q - y) + y, q \}, 0 \}$, as derived numerically for $u(w) = \ln(w)$, $C(e) = c\frac{e^2}{2}$, with $c = \frac{1}{2}$ and $\bar{W} = 1$, and a truncated normal output distribution on $[0, 30]$ with location and scale parameters e and $\sigma = 1$.

the optimal contract for all values of I , i.e. not only small and large I . The left-hand side of Figure 1 depicts the threshold y for different values of the investment I . If and only if this threshold is positive, the optimal contract gives the investor levered equity only. If this threshold is negative, the investor receives both debt and levered equity. The right-hand side shows how the pay-performance sensitivity b varies with I . As I approaches zero, the manager's equity stake approaches zero because the investor receives levered equity.

The resulting contracts are depicted in Figure 2. The entrepreneur's payoff is given by a solid line, the investor's by a dashed line. The graph with $I = 0.75$ depicts the case of an entrepreneur with a high need for external funding (relative to expected output). In this case, the entrepreneur's limited liability constraint $w(q) \geq 0$ binds at low outputs. The entrepreneur receives a levered equity claim on output, similarly to the Innes (1990) model, but with the difference that the slope is less than 1 to achieve risk-sharing. The investor receives debt plus levered equity, as opposed to debt in Innes (1990). Thus, her payoff initially has a slope of 1 due to her debt. After q crosses the face value of debt, the slope falls below 1 as she shares the residual with the entrepreneur. The payoff structure is consistent with Figure 1: for $I = 0.75$, $y < 0$ and so the investor receives debt and equity.

The three other graphs display the optimal contract when the funding need is progres-

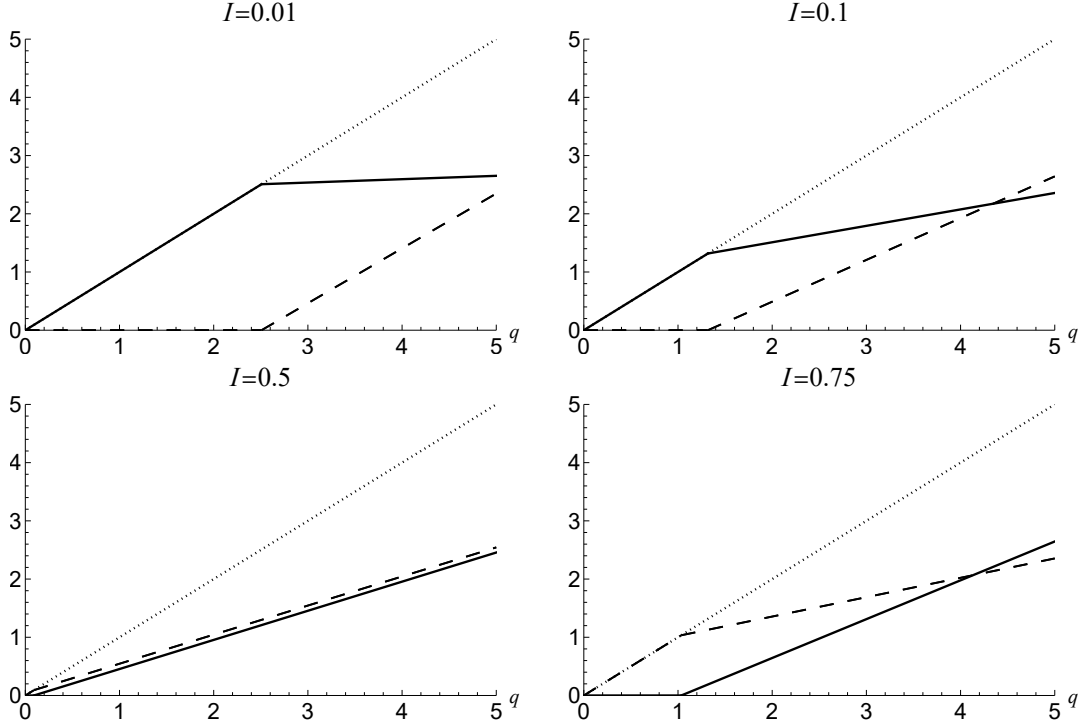


Figure 2: Each panel depicts the optimal payoff of the entrepreneur ($w(q)$) as a solid line and the optimal payoff of the investor ($q - w(q)$) as a dashed line, as a function of q . The dotted line is the 45 degree line. The optimal contract is derived numerically for $u(w) = \ln(w)$ and $C(e) = c\frac{e^2}{2}$, with $c = \frac{1}{2}$ and $\bar{W} = 1$, and a truncated normal output distribution on $[0, 30]$ with location and scale parameters e and $\sigma = 1$.

sively reduced. For $I = 0.5$, y is close to 0 and so we are close to the crossover point where neither the investor nor the entrepreneur hold debt; both have levered equity. For $I = 0.1$ and 0.01, Figure 1 shows that $y > 0$ and so the investor receives levered equity only. Her limited liability constraint $w(q) \leq q$ binds at low and intermediate outputs; the entrepreneur receives debt and levered equity.

In Innes (1990), since there is no risk-sharing motive, the optimal contract is driven by productive efficiency, i.e. it maximizes effort incentives. A higher need for external funding reduces the equilibrium effort, because the entrepreneur internalizes less of its effects. We now show with counterexamples that this result does not extend to the case of a risk-averse entrepreneur.

Figure 3 plots the entrepreneur's effort as a function of external financing needs. It shows

that, for the functional forms and parameter values considered, effort is non-monotonic in the funding need. The intuition is as follows. When external financing needs are low, the entrepreneur's expected payoff is high, and so he is on the less concave part of his utility function. Thus, the standard "skin-in-the game" effect of Innes (1990) dominates: as funding needs rise, the entrepreneur is less of a residual claimant and exerts less effort.

In contrast, when external financing needs are intermediate, the chosen effort level can be increasing in the amount of external funding. Intuitively, an increase in the need for external funding reduces the entrepreneur's payoff across most levels of output, increasing his marginal utility. When the entrepreneur is risk averse and prudent (as is the case for any utility function with CARA or DARA, including CRRA), this increase is especially pronounced when his payoff is low, because this is where marginal utility is the steepest and most sensitive to wealth. This "marginal utility" effect increases effort incentives.⁸

Finally, when external financing needs are high, the chosen effort level is again decreasing in the amount of external funding. In this case, increases in I no longer substantially reduce the entrepreneur's payoff, because he is already paid close to zero for low outputs, and the sensitivity of pay to performance for high outputs is increasing in I (see Figure 1). Thus, the contract is primarily designed to provide effort incentives: the entrepreneur has levered equity, similar to Innes (1990). As a result, the standard "skin-in-the game" effect again dominates.

In sum, the marginal utility effect can dominate the skin-in-the-game effect for intermediate amount of external funding when the entrepreneur is sufficiently risk averse. Because effort is costly to him and because his payoff is zero when output is low (because of imperfect risk sharing), he is worse off than an otherwise identical entrepreneur who needs less external funding, but his firm is substantially more productive due to his greater effort.⁹

⁸With a prudent agent ($u''' > 0$), u' is convex, so that marginal utility decreases with wealth at a decreasing rate. This means that marginal utility will be steepest for low levels of wealth.

⁹When the funding need is too high, the optimization program does not have a solution: there is no contract and associated incentive-compatible effort such that the investor breaks even.

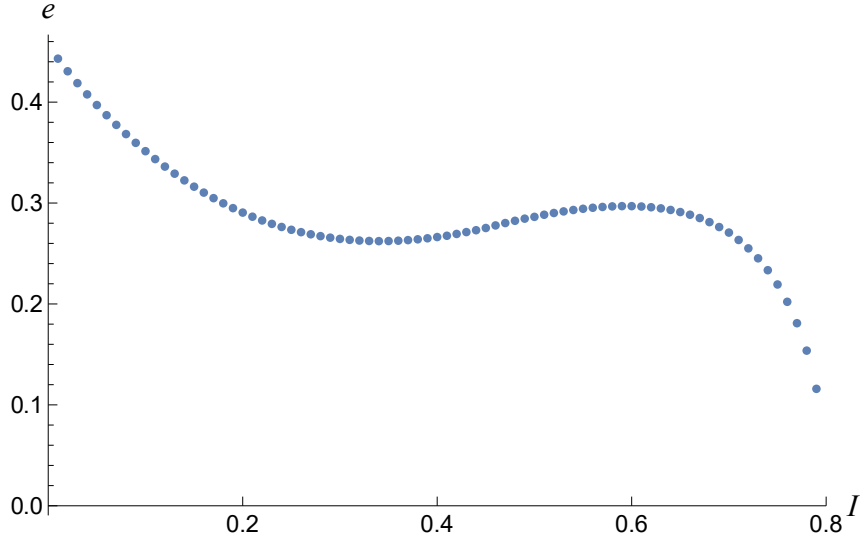


Figure 3: Effort e optimally chosen by the entrepreneur as a function of external funding I , as computed numerically for $u(w) = \ln(w)$, $C(e) = c\frac{e^2}{2}$, with $c = \frac{1}{2}$ and $\bar{W} = 1$, and a truncated normal output distribution on $[0, 30]$ with location and scale parameters e and $\sigma = 1$.

3 Conclusion

Financing models are often criticized for their inability to explain the variety of financing instruments used by firms in practice. In the moral hazard model of Innes (1990), with a risk-neutral entrepreneur, debt is optimal regardless of the amount to be raised externally or the cash flow distribution (as long as the monotone likelihood ratio property is satisfied). By letting the entrepreneur be risk averse, we show that debt is no longer optimal for small amounts of external funding. This justifies the use of alternative financing instruments, including equity-based financing, in these instances.

An important application of these results is to MSME financing in developing countries. Access to finance is ranked by firms in developing countries as one of the main obstacles to operation and growth, and it is by far ranked as the main obstacle in sub-Saharan Africa (Beck and Cull (2014)). Beck and Cull (2014) note that Africa’s financial system heavily relies on banks, and that “there are few if any instruments and vehicles for equity finance available in most African countries.” Investments by the World Bank mostly take the form of

debt financing.¹⁰ A normative implication of our model is that other financing instruments should be considered, especially if new technologies such as automated invoicing and bank reconciliation algorithms improve cash flow verifiability.

¹⁰Through the International Finance Corporation (IFC), the World Bank's investment commitments included \$12.0bn in short-term finance, \$11.2bn in trade finance, and \$2.2bn in MSME (Micro, Small and Medium Enterprises) loans. Source: <https://www.ifc.org/en/insights-reports/annual-report>. In response to financing needs, development banks also provide loans to small firms in developing countries (Beck, Demirgüç-Kunt, Maksimovic (2008)).

Appendix

A Numerical examples

Assume $u(w) = \ln(w)$ and $\tilde{q}$ follows a truncated normal distribution on $[0, \infty)$ with location parameter e and scale parameter σ . Supposing for now that the solution of the relaxed problem without the monotonicity constraint satisfies the monotonicity constraint, the Lagrangian of the optimization problem in equations (2)-(LL) with log utility and a truncated normal distribution is:

$$\begin{aligned} \mathcal{L} = & \int \ln(\bar{W} + w(q)) f(q|e) dq - C(e) + \mu \left(\int \ln(\bar{W} + w(q)) \frac{\partial f}{\partial e}(q|e) dq - C'(e) \right) \\ & + \lambda \left(\int (q - w(q)) f(q|e) dq - I \right) + \int (\kappa_{LL}(q)w(q) + \kappa_{UL}(q)(q - w(q))) dq \end{aligned} \quad (3)$$

where $\kappa_{LL}(q)$ is the Lagrange multiplier associated with the constraint $w(q) \geq 0$ for a given q , and $\kappa_{UL}(q)$ is the Lagrange multiplier associated with the constraint $w(q) \leq q$ for a given q . With log utility, $u'(w) = 1/w$. Let ϕ and Φ denote respectively the PDF and CDF of the standard normal distribution. The likelihood ratio of a truncated normal distribution on $[0, \bar{q}]$ is $\frac{\frac{\partial f}{\partial e}(q|e)}{f(q|e)} = \frac{q-e}{\sigma^2} - \frac{\phi(e/\sigma) - \phi((e-\bar{q})/\sigma)}{\sigma(\Phi(e/\sigma) - \Phi((e-\bar{q})/\sigma))}$, which is an affine function of q . Denote the second term on the RHS of this equation, which is not a function of q , as δ . The FONC with respect to $w(q)$ can be rearranged as:

$$w(q) = \frac{1 + \mu \left(\frac{q-e}{\sigma^2} - \delta \right)}{\lambda - \hat{\kappa}_{LL}(q) + \hat{\kappa}_{UL}(q)} - \bar{W} \quad (4)$$

where $\frac{\mu}{\lambda} \frac{1}{\sigma^2} > 0$, $\hat{\kappa}_{LL}(q) \equiv \frac{\hat{\kappa}_{LL}(q)}{f(q|e)}$ and $\hat{\kappa}_{UL}(q) \equiv \frac{\hat{\kappa}_{UL}(q)}{f(q|e)}$, where $\kappa_{LL}(q) > 0$ if $w(q) = 0$ and $\kappa_{LL}(q) = 0$ otherwise, and $\kappa_{UL}(q) > 0$ if $w(q) = q$ and $\kappa_{UL}(q) = 0$ otherwise. The FONC for an interior solution ($w(q) \in (0, q)$) is simply:

$$w(q) = \frac{1}{\lambda} + \frac{\mu}{\lambda} \left(\frac{q-e}{\sigma^2} - \delta \right) - \bar{W} \quad (5)$$

The monotonicity constraint is automatically satisfied at corner solutions, and it is satisfied at interior solutions if $\frac{\mu}{\lambda} \frac{1}{\sigma^2} \leq 1$. In sum, the optimal contract as a function of q can be described by two constants a and b such that:

$$w(q) = \max \{ \min \{ a + bq, q \}, 0 \} \quad (6)$$

We solve numerically for the effort e and the contract variables a and b that solve the optimization problem in equations (2)-(IR) with the optimal contract as in equation (6) and given parameter values. Finally, we verify that the solution is such that $b \leq 1$, in which case the monotonicity constraint is indeed satisfied.

B Proofs

Proof of Lemma 1

The FOA is valid if the following objective function is concave in e :

$$\int_0^{\bar{q}} u(\bar{W} + w(q)) f(q|e) dq - C(e).$$

Thus, the FOA is valid if:

$$\int_0^{\bar{q}} u(\bar{W} + w(q)) \frac{\partial^2 f}{\partial e^2}(q|e) dq < C''(e) \quad \forall e. \quad (7)$$

From bilateral limited liability, $w(q) \in [0, q]$ for all q . We have:

$$\begin{aligned} & \int_0^{\bar{q}} u(\bar{W} + w(q)) \frac{\partial^2 f}{\partial e^2}(q|e) dq \\ & \leq \int_0^{\bar{q}} \max \left\{ (u(\bar{W}) + u'(\bar{W}) \max\{q, 0\}) \frac{\partial^2 f}{\partial e^2}(q|e), u(\bar{W}) \frac{\partial^2 f}{\partial e^2}(q|e) \right\} dq \end{aligned}$$

since u is monotonically increasing and nonconvex. Therefore, a sufficient condition for

equation (7) is:

$$u(\bar{W}) \int_0^{\bar{q}} \frac{\partial^2 f}{\partial e^2}(q|e) dq + u'(\bar{W}) \int_0^{\bar{q}} q \max \left\{ \frac{\partial^2 f}{\partial e^2}(q|e), 0 \right\} dq < C''(e) \quad \forall e. \quad (8)$$

Rewriting using the definition of H_e and K_e gives Lemma 1. ■

Proof of Proposition 1:

We follow a relaxed approach, ignoring monotonicity for now (and verifying later). Set up the Lagrangian:

$$\begin{aligned} \ell = & \int u(w_I(q)) f(q|e) dq - C(e) + \lambda_{IC}^I \left[\int u(w_I(q)) f_e(q|e) dq - C'(e) \right] \\ & + \lambda_{IR}^I \left[\int [q - w_I(q)] f(q|e) dq - I \right] + \lambda_{LLD}(q) w_I(q) + \lambda_{LLU}(q) (q - w_I(q)) \end{aligned}$$

We will write (w_I, e_I) to denote the solution parametrized by $I > 0$.

The FOC w.r.t. e is:

$$\underbrace{\int u(w_I(q)) f_e(q|e) dq - C'(e)}_{0 \text{ by IC}} + \underbrace{\lambda_{IC}^I \left[\int u(w_I(q)) f_{ee}(q|e) dq - C''(e) \right]}_{<0 \text{ by FOA}} + \underbrace{\lambda_{IR}^I \left[\int [q - w_I(q)] f_e(q|e) dq \right]}_{\geq 0 \text{ by (Mon) and MLRP}} = 0$$

where f_{ee} denotes the second derivative of f with respect to e , FOA refers to the concavity of the entrepreneur's objective function with respect to e from Lemma 1, and $\int [q - w_I(q)] f_e(q|e) dq \geq 0$ follows by Chebyshev inequality (see the proof of Claim 1 for details).

$$\therefore \lambda_{IC}^I \left[\int u(w_I(q)) f_{ee}(q|e_I) dq - C''(e_I) \right] + \lambda_{IR}^I \left[\int [q - w_I(q)] f_e(q|e_I) dq \right] = 0. \quad (9)$$

Note that since the terms inside brackets have opposite signs, the multipliers have the same sign.

For the moment, suppose λ_{IR}^I remains bounded as $I \searrow 0$. We verify that this assumption

must hold later. Then:

$$\lambda_{IC}^I \underbrace{\left[\int u(w_I(q)) f_{ee}(q|e_I) dq - C'''(e_I) \right]}_{<0 \ \forall e} + \underbrace{\lambda_{IR}^I}_{Bounded} \underbrace{\left[\int [q - w_I(q)] f_e(q|e_I) dq \right]}_{\rightarrow 0} = 0 \therefore \lim_{I \searrow 0} \lambda_{IC}^I = 0.$$

For the first term in brackets, we use the condition for the FOA from Lemma 1. For the second term in brackets, we use the fact that $q - w_I(q) \searrow 0$ almost surely as $I \searrow 0$ and the assumption that output has bounded support and $f(\cdot|\cdot)$ is continuously differentiable w.r.t. both arguments, which implies that $f_e(q|e)$ is bounded $\forall q, e$.

Next, consider the pointwise FOC w.r.t. $w_I(\cdot)$:

$$u'(w_I(q)) f(q|e_I) - \lambda_{IR}^I f(q|e_I) + \lambda_{IC}^I u'(w_I(q)) f_e(q|e_I) + \lambda_{LLD}(q) - \lambda_{LLU}(q) = 0.$$

Since $\lim_{I \searrow 0} \lambda_{IC}^I = 0$, the derivative on the LHS converges to

$$[u'(w_I(q)) - \lambda_{IR}^I] f(q|e),$$

so, using complementary slackness, the Kuhn-Tucker conditions are

$$u'(w_I(q)) \begin{cases} \geq \lambda_{IR}^I & \text{if } w_I(q) = q \\ = \lambda_{IR}^I & \text{if } w_I(q) \in (0, q) \\ \leq \lambda_{IR}^I & \text{if } w_I(q) = 0 \end{cases} \quad (10)$$

Suppose there exists some $q > 0$ such that $w_I(q) = 0$. By the Kuhn-Tucker conditions, we must have $u'(0) \leq \lambda_{IR}^I$, which, because u' is strictly decreasing, implies $u'(w) < \lambda_{IR}^I$ for all $w > 0$. This is only possible if $w_I(q) = 0$ for all q , which does not satisfy IR. Therefore, the solution entails $w_I(q) > 0$ for all $q > 0$.

Next, suppose there exists some subinterval (q_1, q_2) of $[0, \bar{q}]$ such that $w_I(q) \in (0, q)$, so that $u'(w_I(q)) = \lambda_{IR}^I$ by the KT conditions. Since u' is strictly decreasing, by the KT conditions this is only possible if $w_I(q) = q$ for $q \in [0, q_1]$.

Finally, suppose $w_I(q^*) = q^*$ for some q^* . Since u' is strictly decreasing, it follows that $u'(w_I(q)) > u'^*$ for all $w_I(q) \leq q \leq q^*$. By the Kuhn-Tucker conditions, we must have $w_I(q) = q$ for all $q < q^*$.

In sum, the optimal contract is given by:

$$w_I(q) = \min\{q, q_I^*\},$$

which satisfies monotonicity, verifying that the omitted constraint doesn't bind. Moreover, q_I^* is uniquely determined by IR.

In the last part of the proof, we verify that λ_{IR}^I is bounded. Recall that from (9), we have:

$$\lambda_{IC}^I = \lambda_{IR}^I \frac{N_I}{D_I}. \quad (11)$$

where

$$0 < N_I \equiv \int [q - w_I(q)] f_e(q|e_I) dq,$$

and

$$D_I \equiv - \left[\int u(w_I(q)) f_{ee}(q|e_I) dq - C'''(e_I) \right] \rightarrow - \left[\int u(q) f_{ee}(q|e^*) dq - C'''^* \right] \equiv D^* > 0$$

by our FOA assumption at e^* , and where we used the fact that IR and LL imply $w_I^* \rightarrow q$ and $e_I^* \rightarrow e$. Therefore, for I small enough, $D_I \geq \frac{D^*}{2} > 0$ (where D^* is not a function of I).

As calculated above, the pointwise FOC w.r.t. $w_I(\cdot)$ is:

$$u'(w_I(q)) f(q|e_I) - \lambda_{IR}^I f(q|e_I) + \lambda_{IC}^I u'(w_I(q)) f_e(q|e_I) = 0.$$

Substitute (11):

$$u'(w_I(q)) f(q|e_I) + \lambda_{IR}^I \left[\frac{N_I}{D_I} u'(w_I(q)) f_e(q|e_I) - f(q|e_I) \right] = 0. \quad (12)$$

The sign of this derivative pins down whether the solution entails $w_I(q)$ is at the boundary at 0, on the interior, or at the boundary at q .

By the Implicit Function Theorem, for each $\delta > 0$, there exists $\varepsilon > 0$ such that for $I \in (0, \varepsilon)$, we have $e_I \in [e^* - \delta, e^* + \delta]$. That is, for all such I , e_I belongs to the compact interval $[e^* - \delta, e^* + \delta]$. Since output is also in a compact interval and f_e is a continuous function (of both arguments), it attains a finite maximum M in the domain $[0, \bar{q}] \times [e^* - \delta, e^* + \delta]$. Hence,

$$0 \leq N_I = \int_0^{\bar{q}} [q - w_I(q)] f_e(q|e_I) dq \leq M \int_0^{\bar{q}} [q - w_I(q)] dq.$$

since $q \geq w_I(q) \forall q$ by LL. By assumption $f(q|e) \geq \delta > 0$ for all q, e . Therefore,

$$\int_0^{\bar{q}} [q - w_I(q)] dq \leq \int_0^{\bar{q}} [q - w_I(q)] \frac{f(q|e_I)}{\delta} dq = \frac{1}{\delta} \underbrace{\int_0^{\bar{q}} [q - w_I(q)] f(q|e_I) dq}_{=I} = \frac{I}{\delta},$$

where the first inequality used $\frac{f(q|e)}{\delta} \geq 1$ for all q, e and the second one used IR. Thus,

$$0 \leq N_I \leq \frac{M}{\delta} I \rightarrow 0$$

as $I \rightarrow 0$. Since $D_I \geq \kappa > 0$ for I small enough,

$$0 \leq \frac{N_I}{D_I} \leq \frac{M}{\kappa \delta} I \rightarrow 0$$

as $I \rightarrow 0$.

For the remaining terms, note that the concavity of u gives $u'(w_I(q)) \leq u'(0)$ (uniform in q) and recall the uniform bound: $f_e(q|e) \leq M$ in the domain $[0, \bar{q}] \times [e^* - \delta, e^* + \delta]$ (and e_I belongs to this interval if $I < \varepsilon$). Then:

$$\underbrace{\frac{N_I}{D_I}}_{\rightarrow 0} \times \underbrace{u'(w_I(q))}_{\leq u'(0)} \underbrace{f_e(q|e_I)}_{\leq M} \rightarrow 0.$$

Using these limits on expression (12), the LHS is:

$$\underbrace{u'(w_I(q))}_{\leq u'(0)} \underbrace{f(q|e_I)}_{\rightarrow f(q|e^*)} + \lambda_{IR}^I \left[\underbrace{\frac{N_I}{D_I} u'(w_I(q)) f_e(q|e_I)}_{\rightarrow 0} - \underbrace{f(q|e_I)}_{\rightarrow f(q|e^*)} \right]. \quad (13)$$

Suppose $\lim_{I \searrow 0} \lambda_{IR}^I = +\infty$. Since $f(q|e) \geq \delta$ for all q, e , we have

$$\lambda_{IR}^I f(q|e^*) \geq \lambda_{IR}^I \delta \rightarrow +\infty,$$

and because the other terms are all bounded, we can find $I > 0$ (close enough to zero) such that the expression in equation (13) is negative for all q . But then pointwise optimization would give $w_I(q) = 0$ for all q , violating IR.

Suppose instead $\lim_{I \searrow 0} \lambda_{IR}^I = -\infty$, so that $\lambda_{IR}^I < 0$ for I small enough. Again, since $f(q|e) \geq \delta$ for all q, e , we must have

$$\lambda_{IR}^I f(q|e^*) \leq \lambda_{IR}^I \delta \rightarrow -\infty,$$

and because the other terms are all bounded, we can find $I > 0$ (close enough to zero) such that the expression in equation (13) is positive for all q . But then pointwise optimization would give $w_I(q) = q$ for all q , again violating IR (since $I > 0$). Hence λ_{IR}^I must remain bounded. ■

Proof of Proposition 2:

Note that (Mon) and (LL) hold if and only if (Mon) and $w(0) = 0$ hold. To see this, first note that evaluating (LL) at $q = 0$, one finds that $w(0) = 0$. Moreover, any w that starts at zero and has slope between 0 and 1 cannot fall below zero and cannot exceed q .

Therefore, the program can be written as:

$$\max_{w(\cdot), x(\cdot), e} \int_0^{\bar{q}} [q - w(q)] f(q|e) dq \quad (14)$$

$$\text{s.t.} \quad \int_0^{\bar{q}} u(w(q)) f_e(q|e) dq = C'(e) \quad (15)$$

$$0 \leq \dot{w}(q) \leq 1 \quad \forall q \quad (16)$$

$$w(0) = 0 \quad (17)$$

It is helpful to introduce the likelihood ratio $L(q|e) \equiv \frac{f_e(q|e)}{f(q|e)}$, so we can write the program as:

$$\max_{w(\cdot), x(\cdot), e} \int_0^{\bar{q}} [q - w(q)] f(q|e) dq \quad (18)$$

$$\text{s.t.} \quad \int_0^{\bar{q}} u(w(q)) L(q|e) f(q|e) dq = C'(e) \quad (19)$$

$$0 \leq x(q) \leq 1 \quad \forall q \quad (20)$$

$$\dot{w}(q) = x(q), \quad (21)$$

$$w(0) = 0. \quad (22)$$

Set up the Hamiltonian:

$$H = [q - w(q) + \lambda u(w(q)) L(q|e)] f(q|e) + \mu(q) x(q).$$

- Pontryagin condition:

$$x(q) = \begin{cases} 0 & \text{if } \mu(q) < 0 \\ 1 & \text{if } \mu(q) > 0 \end{cases}.$$

- ODE of co-state:

$$\dot{\mu}(q) = [1 - \lambda u'(w(q)) L(q|e)] f(q|e).$$

- Transversality: $\mu(\bar{q}) = 0$.

- FOC with respect to effort:

$$\int_0^{\bar{q}} [q - w(q)] f_e(q|e) dq + \lambda \underbrace{\left[\int_0^{\bar{q}} u(w(q)) f_{ee}(q|e) dq - C'''(e) \right]}_{\leq 0 \text{ by FOA}} = 0. \quad (23)$$

The proof of our main result will use the fact that the Lagrange multiplier associated with IC is non-negative:

Claim 1 $\lambda \geq 0$.

Proof. Use the definition of L to write:

$$\int_0^{\bar{q}} [q - w(q)] f_e(q|e) dq = \int_0^{\bar{q}} [q - w(q)] L(q|e) f(q|e) dq.$$

Since $h(q) \equiv q - w(q)$ and $L(q|e)$ are both increasing functions of q , it follows by Chebyshev inequality for monotone functions that

$$\int_0^{\bar{q}} [q - w(q)] L(q|e) f(q|e) dq \geq \int_0^{\bar{q}} [q - w(q)] f(q|e) dq \cdot \underbrace{\int_0^{\bar{q}} L(q|e) f(q|e) dq}_0 = 0,$$

where the equality at the end follows from the fact that L has mean zero:

$$\int_0^{\bar{q}} L(q|e) f(q|e) dq = \int_0^{\bar{q}} f_e(q|e) dq = \frac{d}{de} \underbrace{\left[\int_0^{\bar{q}} f(q|e) dq \right]}_1 = 0.$$

Substituting in (23), establishes that $\lambda \geq 0$. ■

We now prove the Proposition. By Pontryagin's condition and $w(0) = 0$, it suffices to show that $\mu(0) < 0$. We now show that, in fact, $\mu(0) \leq -1$.

Integrate ODE and use transversality:

$$\underbrace{\mu(\bar{q})}_0 - \mu(0) = \int_0^{\bar{q}} [1 - \lambda u'(w(q)) L(q|e)] f(q|e) dq$$

$$\therefore \mu(0) = -1 + \lambda \int_0^{\bar{q}} u'(w(q))L(q|e)f(q|e)dq. \quad (24)$$

Let $g(q) \equiv u'(w(q))$ and note that g is a (weakly) decreasing function. As shown before, $L(q|e)$ is an increasing function of q with mean zero. By Chebyshev inequality for anti-monotone functions, we have

$$\int_0^{\bar{q}} g(q)L(q|e)f(q|e)dq \leq \int_0^{\bar{q}} g(q)dq \cdot \underbrace{\int_0^{\bar{q}} L(q|e)f(q|e)dq}_0 = 0.$$

Substituting back in (24), gives

$$\mu(0) = -1 + \lambda \int_0^{\bar{q}} u'(w(q))L(q|e)f(q|e)dq \leq -1.$$

■

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