Demand-driven Corporate Leverage Choice: Implications for Intermediary Regulation*

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Abstract

With home bias, the participation of domestic households in the equity market is a key determinant of the amount of equity domestic firms can issue. I explore theoretically and empirically the consequences of constraints on households' equity investment for regulated intermediaries and the corporate capital structure. In cross-country comparison, I document a negative relationship between the equity share in households' financial assets and corporate leverage, as well as financial sector leverage. I develop a general equilibrium model consistent with these observations: firms optimally choose leverage, trading off interest tax deduction, and costly default, while only a subset of households can participate in equity markets directly with the remainder saving via regulated intermediaries. I explore the effects of capital requirement regulation (including Basel-style risk-weighting of corporate equity) on corporate leverage, output, return inequality, and financial stability. In the first-best equilibrium the planner invests in equity on behalf of the households, and in the second-best equilibrium financial intermediaries hold more corporate equity than in the market solution. Capital regulation that imposes a high risk weight on equity pushes the economy even further into equity scarcity, relative to the laissez-faire solution. Therefore, this paper implies that regulators might want to be more lenient toward financial intermediaries that hold corporate equity as assets.

Keywords: Corporate Leverage Choice, Equity Market Participation, Financial Stability, Financial Regulation

JEL codes: G11, G21, G28, G32, G51

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1 Introduction

High levels of corporate debt are of concern to many policymakers who fear amplification of negative shocks, costly firm defaults, as well as diminished corporate investment incentives. Nevertheless, our understanding of firms' leverage choices remains limited. An influential strand of the literature, following Leland (1994), models leverage choice as a trade-off between the tax advantage of interest rate deductibility of debt and the possibility of costly default. There exist rich theoretical models of leverage choice in this vein (most recently DeMarzo and He 2021, a model of dynamic leverage choice without commitment), in which investors are cast as deep-pocketed price takers. Aiming to understand the bigger picture, there exist general equilibrium macro models, in which firms optimally choose leverage, such as Covas and Den Haan (2011), Jermann and Quadrini (2012), and Begenau and Salomao (2019), who build frameworks to understand the dynamics of debt and equity issuance of firms over the business cycles. In these models, corporate debt and equity are again held and priced by households who act as perfect arbitrageurs between these asset classes.

With my research, I want to take seriously that, in reality, neither households nor the financial intermediaries who invest on their behalf, are perfect arbitrageurs. There are multiple frictions preventing debt and equity from being priced according to a no-arbitrage benchmark. I will focus on two key frictions: 1. a large share of households does not participate directly (or via unregulated intermediaries such as mutual funds) in equity markets, especially in European economies (as displayed in Figure 1), but save via bank deposits or life insurance; 2. Banks and insurers do not hold a large share of corporate equity as assets, and regulation strongly disincentivizes such investment. Consequentially, in most economies, most equity is held directly or via lightly regulated intermediaries by a minority of high-income households.

These two frictions suggest that the partial equilibrium models omit an important consideration: prices for financing sources reflect scarcity, and thus if only a small share of aggregate savings can be channeled into firms as equity, equity prices will be low and firms will prefer to issue debt to finance their operations, especially if there exists a tax advantage of corporate debt. This effect is especially pronounced in economies with low equity participation. European economies have lower equity participation rates than the United States and on average higher levels of corporate debt. In my theoretical analysis, the tax advantage distorts the financing choices of firms towards debt, which is exacerbated by a lack of equity investment by households. Government intervention, either directly or via financial regulation, can decrease this distortion.

The European Commission's 2015 Green Paper [1], charting out the path towards the "Capital Markets Union", reflects the policy relevance of this channel at length: there is a perceived lack of equity-financing options for mid-sized firms, a bank-dependence of firms, as well as a lack of equity market participation of households, which are all described as interrelated parts of the malaise of low investment and high corporate leverage. In the United States, a large share of equity investments are

[1] https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex%3A52015DC0063

conducted via capital-based, government-subsidized retirement schemes, which do not exist in most European countries. The policy discussion on such capital-based retirement schemes to complement or replace pay-as-you-go schemes is an ongoing topic of public attention in large European economies but faces strong political opposition. Beck et al. (2023) show that corporate leverage in the Eurozone, as opposed to the United States, has decreased since 2008, which cannot be explained by credit supply frictions. During the same period, the share of savings via equity assets such as pension and investment funds by Eurozone households has increased, as displayed in Figure 2.

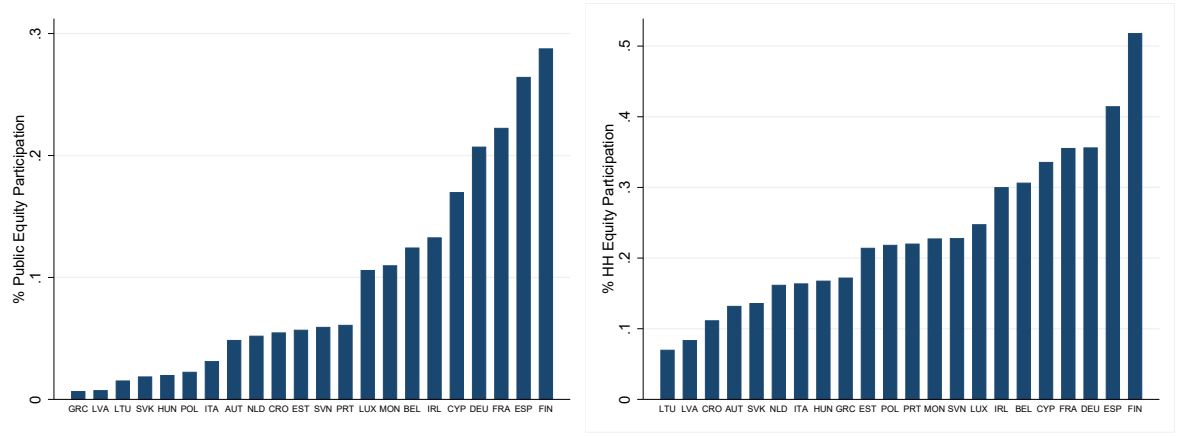


Figure 1: The extensive margin of household equity participation in the Eurozone. Data from the ECB Household Finance and Consumption Survey (2017). Left Panel: Participation in Public Equity. Right panel: Participation in All Equity (including public equity, equity mutual funds, and active and passive private business shares).

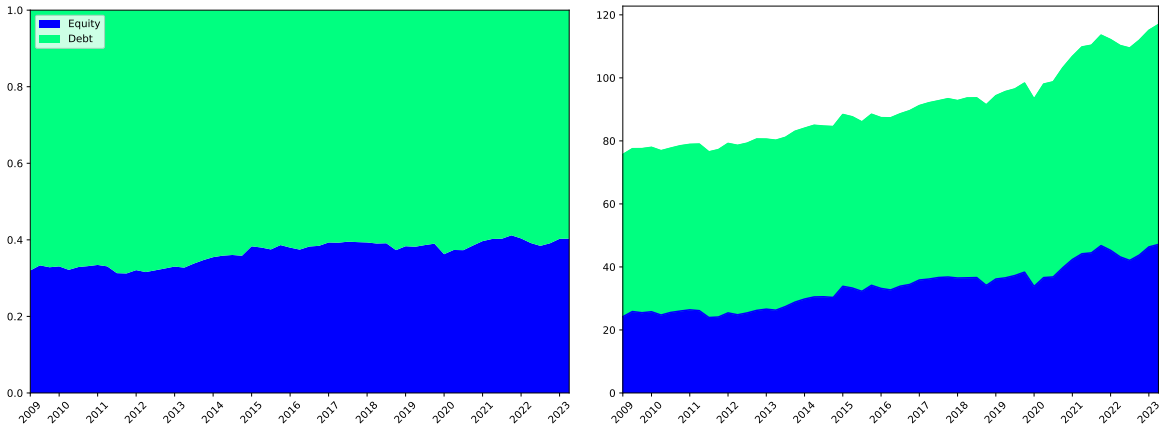


Figure 2: Financial assets of Eurozone households, split by debt and equity. Data from the Distributed Wealth Accounts (DWA), compiled by the ECB. Left panel: Shares of debt and equity assets in total financial assets. Right panel: Financial assets per Eurozone household in €1000.

In this paper, I derive the implications of limited investment into corporate equity for the macroeconomy and financial stability. I show that economies with low corporate equity investment by households feature higher leverage in their corporate and financial sectors. I demonstrate that there is a hidden cost to restricting banks' investment into corporate equity: if households are not investing directly in corporate equity and intermediaries are not allowed to do so, then firms will tilt their financ-

ing structure toward debt, which will lead to less investment, more defaults, and fewer savings. The restriction of intermediary investment into corporate equity has a little-considered side effect on financial stability: high regulatory risk weights on corporate equity make households divert intermediary capital by directly investing in debt instead. The lack of capital and the risky corporate sector increase the risk of the intermediaries' loan portfolios and decrease the intermediary capital, thus raising both leverage and asset risk as a consequence. Lastly, capital regulation increases return inequality, which has been identified as an important source of wealth inequality in Xavier (2021): bank asset returns are lower due to regulatory restriction, and thus deposit rates fall, while equity investors receive high returns.

I contribute to the literature by analyzing the effects of limited equity market investment in a general equilibrium OLG model to understand its effect on the aggregate economy. In this model, two types of households offer their labor inelastically and receive the same wage when they are young: only *investors* can invest in capital markets directly, while *savers* can only save via regulated intermediaries. These regulated intermediaries finance themselves with deposits and equity capital, which are respectively provided by *savers* and *investors*. Pass-through of banks' asset returns to deposits is imperfect and thus owning the banks yields a benefit to *investors*: they can leverage up cheaply with *savers'* deposits. However, operating the bank is costly: there is a cost of intermediation, which is proportional to the asset size of the bank.

On their asset side, banks hold corporate debt and equity, while being subject to regulation, which takes the form of a Basel-type risk-weighted assets capital regulation constraint, in which corporate equity is weighted more strongly than corporate debt.

In a competitive corporate sector, many identical firms produce using capital and labor. Capital can be raised both in the form of debt and equity, which are perfect substitutes in production. However, debt financing is beneficial for the firm due to a tax advantage, as interest rate payments are deductible from the firm's corporate tax bill. Debt can also be costly as firms' equity holders are subject to limited liability and thus will choose to default if the firm is at a loss after debt repayment. Default is more likely for high-leverage firms and is priced in and thus in equilibrium there is an optimal internal solution for the leverage choice of the corporate sector.

I characterize the planner solution and market equilibrium of this model. Without any constraint, the bank equalizes the risk-adjusted prices of debt and equity, and firms choose their leverage trading off tax advantage and costly default. As bank returns are high, *investors* save a large share of their wage via bank equity and thus banks are well capitalized. Regulatory constraints decrease the investment of banks into corporate equity, upon which there are two cases: either *investors* households can replace the missing equity investment of the bank, in which case the allocation will still be efficient, OR there will be a positive equity premium in equilibrium, due to equity being inefficiently scarce, and as a consequence *investors* will save exclusively via corporate equity. If there is an equity premium, *investors* will divert equity capital away from the bank, thereby increasing bank leverage. Firms choose higher leverage if equity is scarce and expensive and thus aggregate investment, wages, and output

are lower, while corporate defaults are more frequent. Finally, when equity investment by banks is restricted, return inequality between *savers* and *investors* increases, as the return on bank assets (the source of deposit rates) is lower.

I further characterize the optimal regulation if banks are subject to potential insolvency when facing systemic shocks. In the first-best solution, in which the planner can allocate household savings to firms without restrictions, the firms would be fully equity financed as the planner does not take into account the tax advantage that drives firms towards higher leverage. In the second-best setup, in which the planner can choose the asset allocation of the bank, the planner would make the bank invest in both equity and debt to minimize the default probability of the bank. I show that both bank competition and deposit insurance drive the optimal asset allocation of banks towards debt, which, in this benchmark economy, means further away from the first-best. Basel-type capital requirement constraints provide precise instruments to choose the asset allocation of banks, but in this economy, the optimal risk weights incentivize banks to hold more corporate equity. Overly restricting equity investment comes at the detriment of output, investment, and financial stability (the primary goal of the regulation). This result poses the question of why intermediaries' holdings of corporate equity are made so costly by the current regulation, especially for economies in which households invest little directly into corporate equity. It can be optimal to loosen equity regulation specifically while tightening capital requirements specifically, which connects to the work of Elenev, Landvoigt, and Van Nieuwerburgh (2021), who quantitatively estimate optimal regulatory capital requirements in the US, but do not consider regulation of bank investment into corporate equity in their analysis. In my model, both firm and bank defaults are connected, as in Mendicino et al. (2020), however, I show how bank equity investment regulation can increase the default risk of the corporate sector and thus make a "twin default" of both banks and firms more likely.

My modeling approach and focus are most closely related to Diamond (2020) and Scharfstein (2018). Diamond (2020) studies the role of households' preferences for safe assets for the organization of financial markets. In his work the low participation of households in equity markets is a consequence of this preference for safe deposits: households hold safe assets and to a limited extent equity, while banks hold debt and provide safe deposits. The driving force behind this segmentation is that banks face an agency cost for holding risky assets and that equity is assumed to be more sensitive to aggregate risk. Compared to Diamond (2020), I differ in two key points: I am not assuming that debt and equity have inherently different systemic risk exposure. Instead, I explain the distribution of different asset types across sectors through the regulation of intermediaries, which allows me to focus on the question of optimal regulation of intermediary investment into corporate equity. As in my model, the financing choice of firms is endogenously determined according to the established theories of leverage choice, equity is endogenously riskier than debt, as it is the residual claim on the firm's profits. Due to Diamond (2020)'s assumption of equities' larger systemic risk, banks in his model naturally choose a corporate debt portfolio. However, as I detail in Section 2, this is not universally true, e.g. in the past, in large European economies in which regulation was permissive, banks did hold considerable equity

stakes in domestic firms, thus casting doubt on the idea that banks naturally assume the role of solely acting as loan providers. Therefore, the two papers differ in their core assumptions, as well as in the stylized facts that they set out to explain.

Scharfstein (2018) analyzes the role of pension systems in the size and efficiency of national financial markets, specifically comparing pay-as-you-go systems with market-based alternatives. He finds that market-based systems enable larger and more efficient financial markets, while pay-as-you-go systems increase housing investment and bank reliance on firm financing. Relative to Scharfstein (2018), I focus not only on market- vs. bank-based financing but specifically on the leverage choice of firms and on the regulation of intermediaries, rather than different pension systems. Notwithstanding this difference in focus, I find that an introduction of an equity-based pension system should be a crucial policy change for economies with low equity participation.

Melcangi and Sterk (2021) find in a new Keynesian framework that changes in household equity investment are important to account for the effect of transmission of monetary policy on output. However, their channel only transmits via fund flows to fully equity-financed firms, while in this paper the role of equity investment for corporate leverage plays a central role. Firms do not only invest less due to drying-up equity flows, but they cannot replace these missing equity flows with bank investment, as firm leverage would become unsustainable. Doerr, Drechsel, and Lee (2021) find that, at the state level of the US, an increase in the share of the top earners leads to a decrease in investment channeled to small firms. As it is assumed that top-earner equity investment is channeled mostly to out-of-state firms, their channel operates via the reduction of regional bank deposits (as top-earners rely less on bank deposits). In contrast to this, my paper sets out to understand the role of household investment for both equity and debt financing and focuses on the national level, at which home bias in equity investment is well-established. Other authors have studied what equity investment of households implies for optimal taxation choices, specifically corporate taxation as in González, Montecino, and Stiglitz (2025) in which households' savings in equity shares implies that optimal corporate taxes are a function of market power wealth. In my model, the level of corporate tax rate increases the intensity of the distortion caused by low equity investment by households.

Several papers have examined a distinct aspect of a bank's equity investment, namely whether banks holding minority equity stakes in firms alleviates agency problems (by aligning lender and owner incentives). The seminal theoretical contribution was Mahrt-Smith (2006), while Beschwitz and Foos (2016) and Fecht et al. (2021) analyze the problem empirically and find small, or even detrimental effects on solving the hold-up problem. In my work, I focus on another aspect of banks' equity provision to firms, namely the effects of such equity investments in an economy where equity is scarce. The paper proceeds as follows: in Section 2, I describe the institutional context of households' participation in equity markets and intermediary regulation, Section 3 documents stylized facts about the relationship of household equity investment and aggregate leverage, Section 4 lays out the general equilibrium model, Section 5 extends the model to banks subject to systemic risk and discusses optimal leverage and equity regulation, Section 7 concludes.

2 Institutional Setting: Households' Equity Market Participation, Intermediary Regulation, and Leverage

In the United States, equity participation of households is high, hovering around the 50% mark. Indeed, if we study the US Financial accounts data, we find that US households hold a considerable share of their financial assets either directly in corporate equity or via mutual, pension, and exchange-traded funds. Figure 12 shows this in the exact financial asset breakdown of US households. As seen in Figure 13, households also constitute the primary owners of US equity, either directly or via funds. Banks, on the other hand, hold a very small share of total US corporate equity. Figures 14-16 denote the holdings of such funds, which consist of equity and debt holdings, with equity accounting for a large share of all assets. In European countries however, equity participation is distinctively lower as can be seen from the OECD financial household accounts: in the UK, only 10% of households' assets are categorized as "Shares and other Equity", in Germany and the Netherlands 12%, in Italy and France around 24%. Only the Scandinavian countries come close to the US equity share, with around 35% (vs. 40% for the US). Instead, for low-equity European nations, bank deposits, and life insurance are the predominant savings vehicles, e.g. for Germany 40% of financial assets are in deposits and currency, and 15% in life insurance contracts. Corporate leverage, on the other hand, is higher in Europe than in the US: according to BIS Statistics total credit to non-financial corporations is 112%, as a share of GDP, in the Eurozone, relative to 82% in the US in 2022Q2. The difference becomes even more pronounced if we only consider bank credit, with 50% (US) vs. 92% (Eurozone).

If equity is scarce and households are constrained in their investment opportunities, why do intermediaries not invest on their behalf? In Europe, where household participation is less than in the United States, large banks used to hold considerable equity positions in firms, a phenomenon that was dubbed "Deutschland AG" in the case of Germany, with the largest banks holding considerable equity stakes in the largest industry firms. One example is Deutsche Bank, which in 1989 held considerable shares in many of Germany's large industrial firms, including a 28 % share in Daimler-Benz.[2] Beschwitz and Foos (2016) show that German banks made use of the capital gains tax breaks associated with the German Tax Reduction Act of 2000 to liquidate these equity positions.

Similarly, the French privatizations of industrial firms and banks in the late 1980s left large banks and insurers with considerable stakes in these firms. As Hancké (2001) details, the financial conglomerates around Union des Assurances de Paris - Banque Nationale de Paris and Assurances Générales de France-PARIBAS, held stakes in firms like Air France, Elf, Saint-Gobain, Péchiney, Aérospatiale, Usinor-Sacilor, Rhône-Poulenc, and Total. The historical fact that regulated intermediaries, banks, and insurers, in large European economies have historically held considerable equity shares, casts doubt on the hypothesis that the business model of banks would naturally tend towards the exclusive provision of debt. In contrast, banking regulation in the United States has been disincentivizing bank investment into equity for a long time, beginning with the Glass-Steagall Act and subsequently

[2]For a detailed description of the ownership structure of the *Deutschland AG*, see Beyer (2002).

confirmed in regulation since then, outright banning most of such investment activity. In more recent times, the framework of the Basel reforms (BIS 2016) views banks primarily as the providers of loans to households and firms.

According to the Basel Accords, corporate equity investments receive a high risk weight of 250% in the standardized calculation method for risk-weighted assets. The rationale of the framers of Basel is that a social planner would not want banks to invest in risky investments, especially those for which other investor types are ubiquitous, to not pose a risk to the stability of the banking system. Indeed, equity is more volatile than debt, both in theory (as most junior claim on the firm) and empirically (according to well-documented excess volatility in equity prices). Interestingly, the US policy discussion in the 1980s focussed on whether the US should move closer to a bank-based system, as France or Germany (see Langohr and Santomero 1985). After the early 80's recession, equity financing might have seemed destabilizing, especially relative to the established, relationship-based bank finance, that small and medium enterprises in Europe enjoyed. Instead of the US moving more towards bank finance, however, European banks moved towards the US system of banks as pure loan providers, although without having as many equity investor households as the US. A question remains whether the restrictive treatment of banks' corporate equity investments indeed enhances financial stability and if equity investors are ubiquitous, as is the underlying regulatory model. My paper explores this question intending to contribute to the discussion on banking regulation.

3 The Empirical Relationship of Household Equity Holdings and Corporate Leverage

The share of households' investments into equity, as well as the corporate leverage, displays significant cross-country differences, as displayed in Figure 3. In the left panel, the share of equity (including private equity) in the aggregate household financial portfolio is displayed, as reported in the national accounts collected by the OECD. There are substantial differences, with German, Japanese, and Dutch households holding only around 10% of their financial assets as equity. The aggregate leverage ratio, measured as (Debt - Liquid Assets)/Equity varies between 0.2 and 0.5.

My working hypothesis is that in countries in which equity constitutes a large share of the household portfolios, corporate leverage should be low, as firms will find it easy to raise equity at a lower price. Indeed, plotting the share of household equity holdings against corporate leverage in Figure 4, confirms that the statistical relationship is indeed significantly negative. It is noticeable that economies for which aggregate leverage is low, despite having a low equity participation are very open economies: the UK, Ireland, Netherlands, and Germany. However, as seen in the right panel, even within this group the relationship is negative over time.

One could rightfully be concerned that this relationship is driven more by country fixed effects driven by institutional factors, as discussed by the literature following Porta et al. (1998). However, if

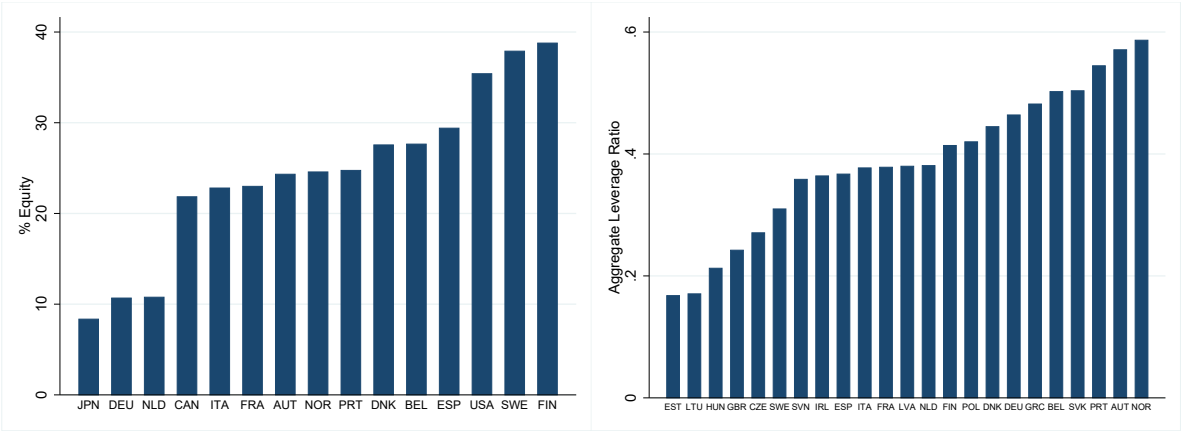


Figure 3: Left Panel: Share of Equity in the Aggregate Households' Financial Assets across OECD countries. Right panel: Aggregate Book Leverage Ratio of Non-financial Companies in the Eurozone. Data: National Financial Accounts.

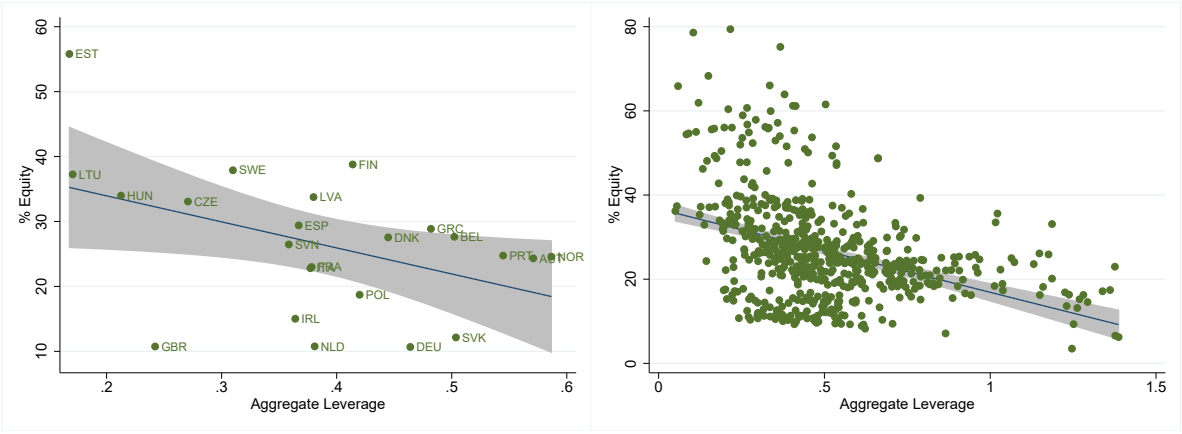


Figure 4: Share of Equity in the Aggregate Households' Financial Assets vs Corporate Book Leverage. Left Panel: 2019, Right Panel: 1970-2021. Data: National Financial Accounts.

we employ a fixed effect regression of corporate leverage on the equity share in households' financial portfolios, we find that even with country-time level variation, the relationship is negative. The regression results for

$$\text{Corporate Leverage}_{c,t} = \alpha + \beta \% \text{ Equity Households}_{c,t} + \eta_t + \phi_c + \varepsilon_{c,t},$$

are reported in Table 1. Economically, the size of the effect is significant: for a one percentage point lower equity share in the household financial portfolio, the aggregate corporate leverage is approximately one percentage point higher in the econometric model with all fixed effects included.

Not only does non-financial companies leverage seem to be related to equity participation, but financial leverage as well. Figure 3 shows the relationship of financial sector book leverage and household equity. Here as well a significantly negative statistical relationship emerges: a one percentage point lower equity participation by households implies a around ten percentage points higher financial sector book leverage.

VARIABLES	(1)	(2)	(3)	(4)
	Corporate Leverage			
% HH Equity	-0.758***	-0.672***	-1.385***	-1.088***
	(0.220)	(0.220)	(0.399)	(0.335)
Observations	613	613	613	613
R-squared	0.150	0.293	0.552	0.681
Time FE	No	Yes	No	Yes
Country FE	No	No	Yes	Yes

Table 1: Fixed-effect regression of corporate leverage on the household equity share in financial assets. Standard errors are double-clustered at the time and country level. *** p<0.01, ** p<0.05, * p<0.1

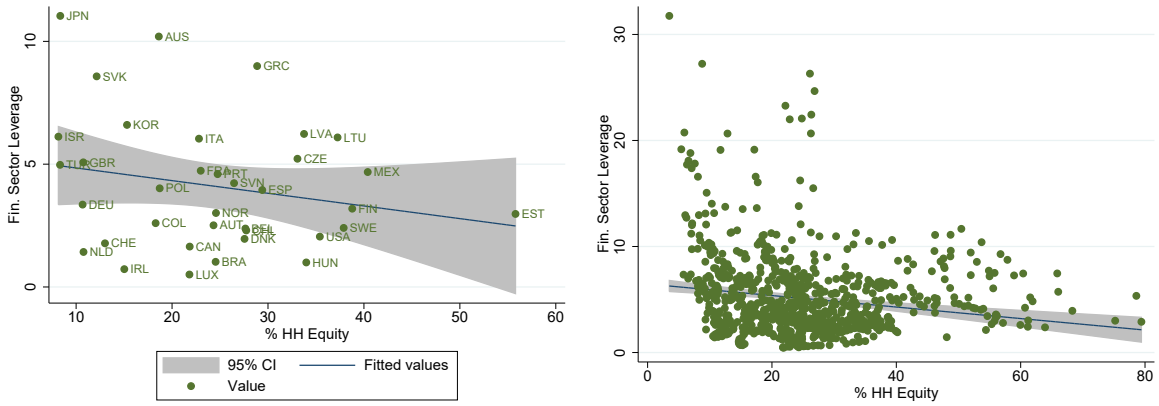


Figure 5: Share of Equity in the Aggregate Households' Financial Assets vs Financial Sector Book Leverage. Left Panel: 2019, Right Panel: 1970-2021. Data: National Financial Accounts.

Demand Side vs. Institutional Features

To further lend credibility to the suggestion that equity participation by households cannot only be explained by institutional features, Figures 17 to 19 in the appendix describe the relationship of various measures of institutional features of the national financial market with equity investments by households. We would expect that in countries with high equity premia, relative to safe bonds, households should be especially eager to invest, but the relationship displayed in Figure 17 shows that in fact, the relationship of average equity premia and equity participation is negative. This suggests rather that equity premia might be high because domestic households don't invest in equity as much. A literature on institutional determinants of financial markets, following Porta et al. (1998), suggests that institutional factors, such as the quality of the domestic accounting standards, determine the share of inside equity (as shown by Peter (2021) for European economies) and corporate leverage. Indeed, as Figure 18 shows, the index of the strength of accounting standards computed in Porta et al. (1998) is negatively correlated with aggregate corporate book leverage but is unrelated to the share of equity holdings of households. Likewise, Figure 19 reports that there is no significant relationship between the inside equity share, as computed in Peter (2021) and the equity share in households portfolios. Finally, the share of national firm equity that is listed is a proxy for the accessibility of the domestic financial

market: if households want to invest, but are somehow barred from doing so, we should expect that in economies where more firms list their equity, the share of household equity holdings should be higher. However, on the contrary, the statistical relationship between households' equity investment share and the share of listed equity in all equity is negative as well, as reported in Figure 20.

In conclusion, if we want to make sense of these observed correlations, we would like to have a model in which households' participation in the equity market is driven by factors outside of the institutional features of the market, so households need to inherently choose not to participate driven by preferences for safe assets or high mental costs of participation. Such a model should match the relationship between households' equity holdings and firm leverage, as well as the leverage of the financial sector. It should explain why banks invest little into equity and why some households hold significant equity, while others forgo equity market participation entirely. Section 4 lays out exactly such a model.

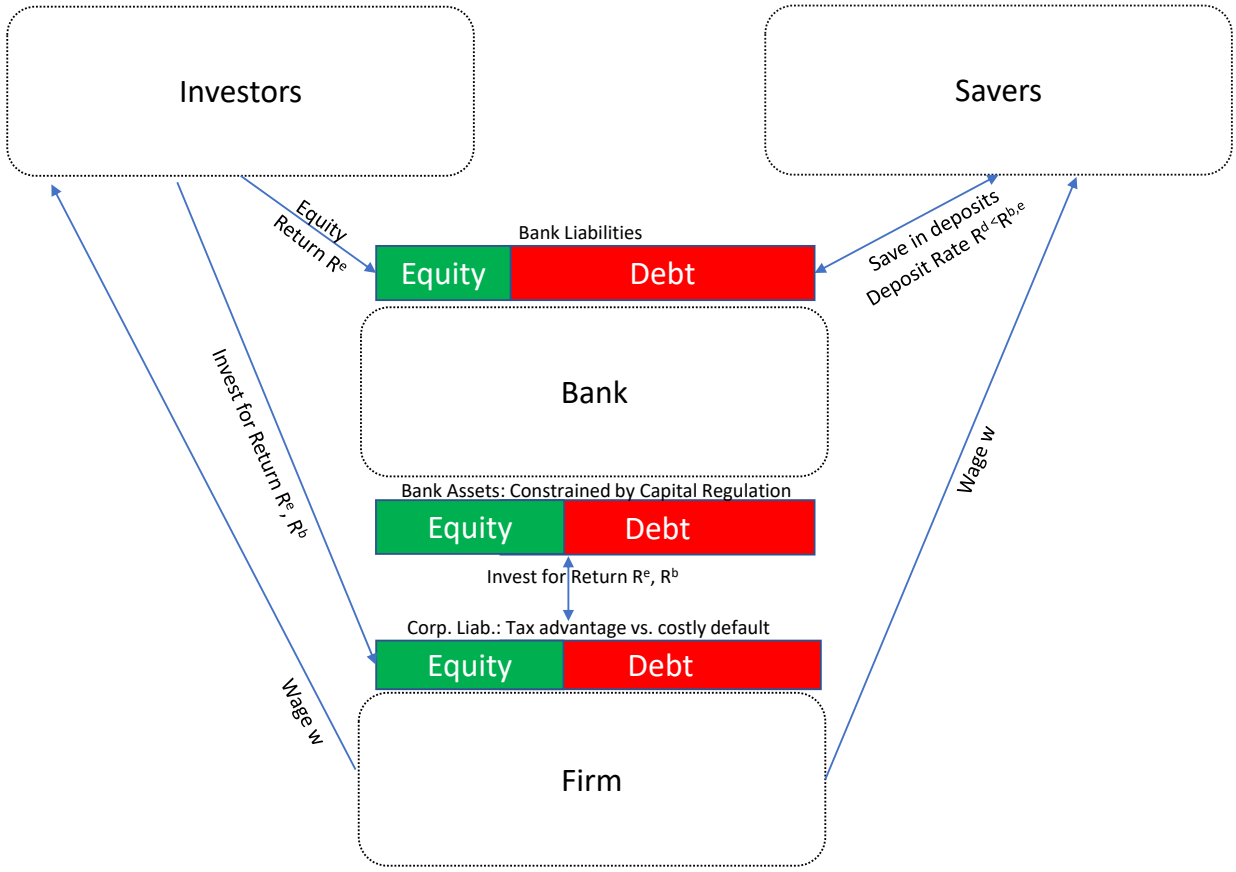
4 A General Equilibrium Model of Leverage Choice

In this section, I present an overlapping-generations model that allows me to analyze the role of household equity investment and financial intermediation for the macroeconomy. A subset of households do not participate in financial markets but save through bank deposits, while financial intermediaries face Basel-style capital regulation constraints with risk weights differing by asset class (equity vs debt). In the Basel regulation, banks' corporate equity investments are assigned a higher risk weight relative to corporate debt in the calculation of risk-weighted assets. The share of bank equity relative to these risk-weighted assets must be below a regulatory threshold value. Firms produce with capital and labor (which young households offer inelastically), according to a Cobb-Douglas production schedule. Capital can be procured from intermediaries and unconstrained households in the form of both equity or debt. Firms aim to choose leverage optimally in a trade-off between the tax advantage of corporate debt and costly firm default. Banks hold a portfolio of corporate assets, through which they are subjected to systemic shocks and can potentially default. Figure 6 provides a high-level overview of the sectors in this general equilibrium model. The model is similar to Diamond (2020) and Scharfstein (2018) in spirit, however, I do not assume that corporate equity is inherently systemically riskier than corporate debt, which allows me to understand the role of the investment choices of households and banks. I take it as a given that a subset of households cannot access financial markets and let firms optimally choose leverage according to the trade-off theory of capital structure. In equilibrium, equity will be endogenously more volatile than the debt due to it being the residual claim to firms' profits.

4.1 Households

The model is a two-period OLG model, in which I will analyze a static equilibrium. There are two types of households: savers (a share η of all households) and investors (a share $1-\eta$ of all households). Both household types offer their labor inelastically to the corporate sector and receive a wage w.

Figure 6: Schematic overview of the general equilibrium model set-up.



The households simply maximize their old-age consumption and are risk-neutral, thus they save their young-age wage through whichever asset yields them the highest returns. The key difference between the two household types is that savers do not have access to financial markets and can choose between a storage asset with a net return of 0 and depositing their wages into a regulated intermediary (which I will refer to as a bank going forward), which grants them a gross return of R^d on their deposits. In contrast, investors can access financial markets and can choose between five assets: corporate bonds with expected return R^b, corporate equity with expected return R^e, bank equity with expected return $R^{e,i}$, bank deposits, and storage.

Problem of the Saver

$$\max E(C),$$

with

$$C = R^d q + s^s$$

and

$$w = q + s^s.$$

Here, q are the deposits and s^s is investment into storage. The saver household will invest all their wage into deposits as long as $R^d > 1$.

Problem of the Investor

$$\max E(C),$$

with

$$C = R^e e^x + R^{e,i} e^i + R^b b^x + s^i,$$

and

$$w = e^x + e^i + b^x + s^i$$

Here, R^e and $R^{e,i}$ are the return on equity on corporate and financial intermediary equity, respectively. e^x and e^i is the equity investment into the corporate sector and the financial intermediary. b^x is investment into corporate bonds with return R^b and s^i is investment into storage.

Again, here the straightforward solution is that the investors save via the assets with the higher return.

4.2 Firms

Firms produce using a Cobb-Douglas production technology with capital and labor as inputs and are subject to a stochastic productivity shock z. Capital can be raised via two types of contracts: equity or debt. The firm is owned by its shareholders and maximizes its expected profit to be paid out to its owners. It raises debt and equity in t and pays workers to produce in $t + 1$. However, the firm can choose whether to repay debt and issue the residual earnings to equity holders, or whether not to repay debt, default, and accept a gross return of zero for equity holders. This is a limited liability assumption: losses cannot be imposed on the equity holders. Regarding the cost of default, I make the harshest possible assumption that in the case of default, the rate of recovery is zero. There is a linear tax of τ on profits, which is in lump-sum distributed to the old generation. Interest payments and wages can be deducted from these profits, implying a tax advantage of debt. The tax proceeds are redistributed to the old generations as lump-sum payments. Debt and equity are equivalent in production, but the two types of capital differ in their implication for firm default. Debt is attractive to the firm, as it creates a tax shield, but a high level of debt comes with a higher probability of default. Equity does not come with a tax advantage, but it lowers the probability of default.

Problem of the firm

I assume that the firm takes market prices of capital and labor as given, but it takes into account how its capital mix affects the risk premium included in the price of capital. This is consistent with the firm being atomistic, as even then its capital structure has an effect on the interest rate it has to pay on its debt. I derive in Appendix B.1 that the formal problem of the firm can be written as:

$$\max_{e,b,l}[1-F(\bar{z})][(1-\tau)[E(z|z>\bar{z})(e+b)^{\alpha}l^{1-\alpha}-wl]-(1-\tau)R^{b}b-[1-F(\bar{z})]\tau b-R^{e}e.$$

Here, e, b, l are the firm's respective equity/debt/labor choices. R^e is the gross expected return on equity, and R^b is the gross expected interest rate on debt, of which the net interest rate $R^b - 1$ is tax deductible. $\bar{z}$ is the critical value for the productivity shock at which the equity holders will choose default. Note that conditional on not defaulting, the ex-post returns on debt and equity would be $\frac{R^b}{1-F(\bar{z})}$ and

$$\bar{R}^{e}=(1-\tau)(z(e+b)^{\alpha}l^{1-\alpha}-wl]-(1-\tau)\frac{R^{b}}{1-F(\bar{z})}b-\tau b,$$

respectively.

The labor choice is made after z has been realized, thus the wage is given as

$$w=z(1-\alpha)(e+b)^{\alpha}l^{-\alpha}.$$

Using this optimal w, $l = 1$, and the fact that the distribution of z is independent of the choice of debt and equity, we can formulate the firm problem as

$$\max_{e,b}[1-F(\bar{z})](1-\tau)\alpha E[z|z>\bar{z}](e+b)^{\alpha}-(1-\tau)R^{b}b-[1-F(\bar{z})]\tau b-R^{e}e.$$

The default threshold $\bar{z}$ is defined as the value for z that yields exactly zero to equity holders after debt repayment (the cost of equity is sunk and thus not considered in this decision):

$$(1-\tau)\alpha\bar{z}(e+b)^{\alpha}-(1-\tau)\frac{R^{b}b}{1-F(\bar{z})}-\tau b=0.$$

Therefore, the threshold for $\bar{z}$ can be written as:

$$\bar{z}=\frac{(1-\tau)\frac{R^{b}}{1-F(\bar{z})}+\tau}{(1-\tau)\alpha(e+b)^{\alpha}}b$$

In the theory appendix, in Section B.2, I derive the FOCs of the firms' problem, which are:

$$[1-F(\bar{z})](1-\tau)\alpha^{2}E[z|z>\bar{z}](e+b)^{\alpha-1}=R^{e}+f(\bar{z})\frac{d\bar{z}}{de}\left[(1-\tau)\frac{R^{b}}{1-F(\bar{z})}+\tau\right]b, \tag{1}$$

$$[1-F(\bar{z})]\left[(1-\tau)\alpha^{2}E[z|z>\bar{z}](e+b)^{\alpha-1}-\tau\right]=(1-\tau)R^{b}\left[1+f(\bar{z})\frac{\frac{d\bar{z}}{db}}{1-F(\bar{z})}b\right]. \tag{2}$$

To understand these FOCs, the firm issues both debt and equity until the benefit of additional

output equals the cost, which is the interest rate, in case of debt diminished by the marginal benefit of the tax advantage, and an additional term which denotes the effect on the default probability. This additional default risk effect lowers the cost of equity ($d\bar{z}/de < 0$), while increasing the cost of debt ($d\bar{z}/db > 0$).

If R^b and R^e are taken as given, the optimal choice of debt and equity is unique. Intuitively, if leverage is determined, then total capital $e + b$ is chosen according to the downward-sloping Cobb-Douglas capital schedule, setting the marginal unit of capital equal to the weighted average cost of capital. If total capital $e + b$ is held constant, the leverage choice can be solved as a function of b via:

$$R^e = [1 - F(\bar{z})]\tau + (1 - \tau)R^b \left[1 + f(\bar{z})\frac{\frac{d\bar{z}}{db}}{1 - F(\bar{z})}b\right], \tag{3}$$

where RHS strictly increases in b and thus leads to a unique leverage choice. The demand in the financial market is determined by the choices of *investors* and financial intermediaries.

4.3 Financial Intermediary

The bank finances itself through equity capital e^i ("bank equity" to distinguish it from corporate equity) and deposits q. It invests in firm equity and firm debt to maximize the expected profit distributed amongst its equity holders (who are investor households). In this Section, to serve the exposition, I assume that the bank is fully diversified in the corporate sector and thus cannot default, i.e. there are no systemic shocks to the bank balance sheet. Section 5 discusses the case of a risky bank, "twin defaults", and optimal regulation.

The problem of the bank is:

$$\max_{e^y, b^y} R^e e^y + R^b b^y - R^d q - R^e e^i - \Gamma(e^i + q)$$

subject to

$$q + e^i = e^y + b^y.$$

The saver households have no access to financial markets other than via the bank and thus will not get to take advantage of the returns that their deposits create in the form of bank assets. Specifically, I model their return R^d as a Nash-bargaining outcome between the bank and the depositors, taking into account their relative outside options. Accordingly,

$$R^d = \lambda \left[\frac{e^y}{e^y + b^y}R^e + \frac{b^y}{e^y + b^y}R^b\right] + 1 - \lambda.$$

The parameter λ governs the pass-through from bank returns to depositors. As pass-through is imperfect, investing via the bank is advantageous for investor households, as they can finance their investments with cheap deposits, however, it comes at the cost of operating the bank. I model

such cost as a variable cost Γ that increases with the asset size of the bank. Banks are furthermore subject to Basel-style capital regulation. To exemplify how regulatory constraints affect the equilibrium outcome, I showcase in the following the banks's optimal capital allocation for three types of regulatory constraints: 1. no constraints; 2. banks cannot invest into equity; 3. a Basel-type constraint, in which regulatory leverage, defined as the ratio of bank equity to risk-weighted assets, must exceed a certain threshold. Note that in this example z is i.i.d. across firms, an assumption that we will weaken in Section 5 to take the risk of bank default and optimal regulation seriously.

4.3.1 No Capital Regulation

In this case, the bank will invest freely into whichever asset yields the highest return. As we have shown the corporate sector will in equilibrium not be financed solely by either debt or equity, it must be that in this case R^b=R^e.

Then the bank's profit collapses to:

$$\cancel{(R^b - R^e)}b^y + (R^e - R^d - \Gamma)q - \Gamma e^i,$$

which (using $R^d = \lambda R^e + 1 - \lambda$) implies that equilibrium bank equity is given by:

$$e^i = \frac{R^e - R^d - \Gamma}{\Gamma} q = \frac{(1-\lambda)(R^e - 1) - \Gamma}{\Gamma} q$$

The investor households provide capital to the bank until the benefit (profiting from cheap deposits, due to imperfect pass-through) equals the cost of intermediation.

4.3.2 No Equity Investment allowed

This setting corresponds to the US regulation, in which banks cannot invest directly into speculative assets (such as corporate equity) at all (or only in very restrictive settings, such as venture capital or private equity under strict guidelines). In this case, the profit of the bank simplifies to:

$$(R^b - R^e - \Gamma)e^i + (R^b - \Gamma - R^d)q,$$

as $b^y = q + e^i$. Using $R^d = \lambda R^b + 1 - \lambda$, we get:

$$(R^b - R^e - \Gamma)e^i + ((1-\lambda)(R^b - 1) - \Gamma)q$$

As long as this profit is positive, investor households will continue to invest into bank equity. Thus, in equilibrium

$$e^i = \frac{(1-\lambda)(R^b - 1) - \Gamma}{R^e + \Gamma - R^b} q.$$

4.3.3 With a Risk-weighted Asset Constraint

Lastly, I consider a constraint in which e^i as a share of risk-weighted bank assets, has to reach a certain threshold. Such a constraint will be of the form

$$e^i > \chi^e e^y + \chi^b b^y,$$

with $0 < \chi^b < \chi^e < 1$. The bank is allowed to invest into both asset types, however investment into equity increasing regulatory assets more than investment into debt. If the constraint does not bind, the solution is as in Subsection 4.3.1. If the constraint binds however, the investment choice of the bank is tilted towards equity as now for every unit of equity the bank does not forgo one, but $\frac{\chi^e}{\chi^b}$, units of debt investment.

Using the binding constraint in the bank's profit, it can be written as:

$$((1-\chi^e)(R^e - R^d) - \Gamma)e^y + (R^b - R^d - \Gamma - \chi^b(R^e - R^d))b^y.$$

This suggests that investor households will invest in both debt and equity via the bank up to the point where:

$$(1-\chi^e)(R^e - R^d) - \Gamma = R^b - R^d - \Gamma - \chi^b(R^e - R^d) = 0.$$

Here, a higher χ^e reduces the incentive to invest in equity, accounting for the downside of equity if the bank's leverage constraint is binding.

We can summarize this condition as:

$$R^e - R^d = \frac{\Gamma}{1-\chi^e},$$

and

$$R^e - R^b = (\chi^e - \chi^b)(R^e - R^d),$$

which, combined, yield this equilibrium relationship between the price for debt and the price for equity:

$$R^e - R^b = \frac{\chi^e - \chi^b}{1-\chi^e}\Gamma > 0.$$

An equity premium is introduced, due to the relative scarcity of equity. Equity will be used less in equilibrium but is more valuable, due to its effect on default.

4.4 Equilibrium

I will look at an equilibrium in period t. w has been determined by capital allocation decisions in $t-1$. An equilibrium is defined by R^e and R^b, such that market clearing for debt and equity is achieved.

Specifically, we are looking for a R^e and R^b, such that:

$$e = e^x + e^y \tag{4}$$

$$b = b^x + b^y \tag{5}$$

$$e + b = w \tag{6}$$

$$(1-\eta)w = e^x + b^x + e^i \tag{7}$$

$$\eta w = q \tag{8}$$

Dynamically, the law of motion of capital governs the economy:

$$w_t = [1 - F(\bar{z})]E[z|z > \bar{z}](1-\alpha)(e_t + b_t)^\alpha = e_{t+1} + b_{t+1}.$$

Note that this is a law of motion of capital that is common to the neoclassic growth model, with the addition of the probability of destruction of capital due to default and the split of capital into debt and equity. With the resource constraint of the economy $w = e + b$, we have already pre-determined the aggregate capital. Now we can use all other equations to understand the leverage choice of the firm. From the bank's problem, we find the equilibrium relationship between the returns on debt and equity R^e and R^b. Then from Condition 3, we find the optimal mix of debt and equity.

4.4.1 Equilibrium without Regulation

In the equilibrium without regulation, we find that $R^e = R^b$ and that *investor* households split their investment between corporate equity and debt, as well as bank equity. What does that imply for the leverage choice and default probability of the firm? Condition (3) tells us that

$$R^e = [1 - F(\bar{z})]\tau + (1-\tau)R^b \left[1 + f(\bar{z})\frac{\frac{d\bar{z}}{db}}{1 - F(\bar{z})}b\right].$$

Now we can use $R^e = R^b$, to find the equilibrium capital mix, which will depend on the exact distribution of $f(z)$:

$$R^e \left[1 - (1-\tau)\left[1 + f(\bar{z})\frac{\frac{d\bar{z}}{db}}{1 - F(\bar{z})}b\right]\right] = [1 - F(\bar{z})]\tau.$$

Any distribution with an increasing hazard ratio

$$\frac{f(z)}{1 - F(z)},$$

will lead to a unique equilibrium leverage choice. One simple example is the exponential distribution with its constant hazard ratio. In this simplified case, the condition becomes:

$$R^e \left[1 - (1-\tau)\left[1 + \frac{d\bar{z}}{db} b\right]\right] = [1 - F(\bar{z})]\tau$$

LHS increases in b while RHS decreases, yielding a unique equilibrium b. With this equilibrium b, we can now solve for the interest rate! For this we use condition (14):

$$[1 - F(\bar{z})][(1-\tau)\alpha^2 E[z|z > \bar{z}](e+b)^{\alpha-1} - \tau] = (1-\tau)R^b \left[1 + f(\bar{z})\frac{\frac{d\bar{z}}{db}}{1 - F(\bar{z})} b\right].$$

With $e + b = w$ and b determined in equilibrium, this condition becomes (again simplifying for the exponential distribution):

$$[1 - F(\bar{z})][(1-\tau)\alpha^2 E[z|z > \bar{z}]w^{\alpha-1} - \tau] = (1-\tau)R^b \left[1 + \frac{d\bar{z}}{db} b\right].$$

Note that $\frac{\partial \bar{z}}{\partial R^b} > 0$, RHS increases in R^e, while LHS decreases for small values of τ (which is a requirement for a unique equilibrium).

4.4.2 Equilibrium when the bank cannot invest into equity

In this equilibrium, we do not get a relationship between the interest rates on equity and debt from the bank problem. However, there are two cases here: either banks' investment into debt is already "excessive", such that it distorts the optimal allocation, as seen in the equilibrium without regulation. In this case, $R^e > R^b$ and the investor household splits its wage between bank equity and corporate equity, while the bank invests only in debt. The alternative case would be that despite the bank investing all funds into debt, it is still possible to achieve the leverage choice of the equilibrium without regulation. In this case, the households split their investment between corporate equity, corporate debt, and bank equity and $R^e = R^b$. Investment of the household is given by

$$e^i = \frac{(1-\lambda)(R^b - 1) - \Gamma}{R^e + \Gamma - R^b} q,$$

implying that the larger the differential of $R^e - R^b$, the less the household will allocate to bank equity, thereby increasing bank leverage.

Corporate leverage increases in the case in which $R^e > R^b$, according to Condition 3:

$$R^e = [1 - F(\bar{z})]\tau + (1-\tau)R^b \left[1 + f(\bar{z})\frac{\frac{d\bar{z}}{db}}{1 - F(\bar{z})} b\right].$$

The larger R^e, while holding R^b constant, the larger b must be to offset the increase. Therefore the firm chooses higher leverage.

In summary: if equity participation is low, prohibiting bank equity investment, increases both bank and corporate leverage, as equity is scarce. Furthermore, in such an equilibrium, return inequality is high. Previously, saver households received $\lambda R^e + 1 - \lambda$ and investor households received R^e. However,

now investor households receive $R^e > R^b$ and saver households receive $\lambda R^b + 1 - \lambda$.

4.4.3 Equilibrium with a Basel-type Risk-weighted Asset Capital Regulation Constraint

This case is now simply a slightly more general case of Section 4.4.2. Again, if the regulatory constraint binds, it must be true either that $R^e > R^b$ or that the bank only invests in debt and chooses $e^y = 0$, which would imply that we are in a similar case to when the bank is barred from investing into corporate equity. The interesting case is again $R^e > R^b$, in which the bank would want to invest more into corporate equity, but is disincentivized by the regulatory constraint. In this case, again, corporate leverage is increasing in regulatory tightness via Equation (3), bank leverage is higher as bank returns are lower because the bank invests excessively (relative to the unregulated equilibrium) into lower-return debt. As bank returns are lower, once again, return inequality rises. As debt is "excessive" relative to what we would see in the absence of regulation, defaults are higher, which contracts the economy via lower wages to the young generation and lower returns on savings for the old generation.

5 Modeling Bank Risk

In the previous section, where the aim was to display the workings of the model, it is not surprising to find that bank regulation is purely detrimental to aggregate outcomes. There is no downside for banks to hold equity and as equity de-risk firms, banks should hold more equity as they do under a Basel-type capital constraint. However, I purposefully left aside the topic of bank risk in Section 4, as well as the question of optimal regulation, and restricted the discussion to bank and firm leverage to show in a simplified setting the effects of regulation on the general equilibrium allocation of corporate debt and equity. In Section 5, I will solve the same model, but with banks that are subject to systemic risk and potential insolvency. This implies that the increase in bank leverage due to equity regulation has implications for financial stability and thereby welfare. I further explore the role of bank competition and deposit insurance, as both of these elements interact with risk-taking by banks and thus affect optimal regulatory choices.

5.1 Model Set-Up with Systemic Bank Risk

In Section 4, I assumed that firms' productivity shocks were idiosyncratic and thus the banks were able to hold a diversified portfolio that would deliver a certain return. This ruled out bank default. To introduce systemic risk, I now assume that the firm productivity shock z is

$$z = i + a,$$

where i is an idiosyncratic firm-specific component, distributed according to $f_i(i)$ and a is an aggregate component, drawn from distribution $f_(a)$. a maps to the systemic risk that banks are exposed to.

Each bank now ex-ante optimizes, similar as in Section 4.3,

$$\max_{e^y, b^y} [1 - F_a(\bar{a})] \left[R^e e^y + R^b b^y - R^d q - \Gamma(e^i + q)\right] - R^e e^i$$

subject to

$$q + e^i = e^y + b^y.$$

Here $\bar{a}$ is a threshold value for the realization of the systemic productivity shock a: for $a < \bar{a}$, the bank will default, and as before I assume that there are no recoverable assets after default.

If we denote the *realized* (as opposed to *expected* R^e, R^b) gross returns on debt and equity as $\bar{R}^e(a)$ and $\bar{R}^b(a)$, then the bank will default if

$$\bar{R}^e(a)e^y + \bar{R}^b(a)b^y - R^d q - \Gamma(e^i + q) < 0,$$

ergo if the ex-post payout on deposits and intermediation cost is lower than the realized value of the bank's investments.

$$\bar{R}^e(a)e^y + \bar{R}^b(a)b^y - R^d q - \Gamma(e^i + q) < 0,$$

implicitly pins down $\bar{a}$ for a given b^y, e^y, and e^i.

In Appendix B.3, I show that

$$\bar{R}^b(a) = \frac{R^b}{1 - F_z(\bar{z})} \left[1 - F_i(\bar{z} - a)\right],$$

and

$$\bar{R}^b(a) = \frac{1}{e} \left[1 - F_i(\bar{z} - a)\right] \left((1 - \tau)\alpha \left(a + E[i | i > \bar{z} - a]\right)(e + b)^\alpha - (1 - \tau)\bar{R}^b(a) - \tau b\right).$$

Note that the returns of equity and debt increase in a, however featuring different functional forms. $\bar{R}^b(a)$ increases quickly for low a, but then levels off as $F_i(\bar{z} - a) \to 0$. The return on equity is lower for low values of a but increases convexly in a. Figure 8 below showcases this relationship. I will now derive the portfolio allocation of the bank, both with and without capital regulation, and also explore the role of deposit insurance.

5.1.1 No capital regulation

With competitive banks, investors invest into the banking sector as long as there is positive profit. Thus, in equilibrium,

$$[1 - F_a(\bar{a})] \left[R^e e^y + R^b b^y - R^d q - \Gamma(e^i + q)\right] - R^e e^i = 0.$$

In Appendix B.4, I show that this boils down to

$$\left(R^e - \Gamma - \frac{R^e}{1 - F_a(\bar{a})}\right) e^y + \left(R^b - \Gamma - \frac{R^e}{1 - F_a(\bar{a})}\right) b^y - \left(1 + \Gamma - \frac{R^e}{1 - F_a(\bar{a})}\right) q = 0.$$

If we ignored the effect of the asset allocation between e^y and b^y on bank default risk (treated $\bar{a}$ as constant), we would find again that

$$R^e = R^b.$$

However, taking into account that $\bar{a}$ decreases in b^y around $R^e = R^b$ (as the tax advantage of corporate debt makes debt too cheap on the margin in equilibrium, as shown below in Section 5.3), it follows that the bank is willing to pay a premium for corporate equity to reduce risk, thus

$$R^b = R^e - \frac{R^e}{(1 - F_a(\bar{a}))^2}\left[e^y \frac{dF_a(\bar{a})}{de^y} - b^y \frac{dF_a(\bar{a})}{db^y}\right] + \frac{R^e q}{(1 - F_a(\bar{a}))^2}\left[\frac{dF_a(\bar{a})}{de^y} - \frac{dF_a(\bar{a})}{db^y}\right]$$

.

Here, as $\frac{dF_a(\bar{a})}{de^y} < 0$, $\frac{dF_a(\bar{a})}{db^y} > 0$, and $e^y + b^y > q$, it follows that

$$R^b > R^e.$$

Banks are willing to take on corporate equity despite a lower expected return for corporate equity to de-risk their portfolio.

Without deposit insurance Without deposit insurance, the depositors take into account the risk of bank default and thus demand the deposit rate $R^d = \frac{1}{1 - F_a(\bar{a})}$. In this case, the result changes slightly to

$$R^b = R^e + \frac{R^e}{(1 - F_a(\bar{a}))^2}\left[e^y \frac{dF_a(\bar{a})}{de^y} - b^y \frac{dF_a(\bar{a})}{db^y}\right] - \frac{(R^e + 1)q}{(1 - F_a(\bar{a}))^2}\left[\frac{dF_a(\bar{a})}{de^y} - \frac{dF_a(\bar{a})}{db^y}\right]$$

.

If bank depositors take into account the potential insolvency of the bank, the attractiveness of equity to de-risk increases even more and the premium on debt increases further.

5.2 Capital Regulation

Now I, analogously to Section 4.4.3 impose a regulatory constraint with Basel-style capital weights:

$$e^i > \chi^e e^y + \chi^b b^y,$$

with $0 < \chi^b < \chi^e < 1$.

In Appendix B.5, I derive that, in this case, the problem of the bank becomes

$$\left(R^e - \frac{\chi^e R^e}{1-F_a(\bar{a})} - \chi^e \Gamma\right) e^y + \left(R^b - \frac{\chi^b R^e}{1-F_a(\bar{a})} - \chi^b \Gamma\right) b^y - (1+\Gamma)q.$$

Here, we find the following relationship between R^e and R^b:

$$R^b = R^e - (\chi^e - \chi^b)\left(\frac{R^e}{1-F_a(\bar{a})} + \Gamma\right) - \left(\chi^e e^y \frac{dF_a(\bar{a})}{de^y} - \chi^b b^y \frac{dF_a(\bar{a})}{db^y}\right) \frac{R^e}{(1-F_a(\bar{a}))^2}$$

There are two effects: the second term on the RHS is the effect of the leverage constraint, which makes holding equity less attractive. The third term on the RHS makes holding equity more attractive, as it de-risks the bank. If the capital weight on equity is high enough, $\chi^e >> \chi^b$, we have that

$$R^b > R^e.$$

In other words: corporate equity has such a high risk weight that despite its benefit the bank reduces its holdings relative to the market solution.

Without deposit insurance, there emerges a fourth term:

$$R^b = R^e - (\chi^e - \chi^b)\left(\frac{R^e}{1-F_a(\bar{a})} + \Gamma\right) - \left(\chi^e e^y \frac{dF_a(\bar{a})}{de^y} - \chi^b b^y \frac{dF_a(\bar{a})}{db^y}\right) \frac{R^e}{(1-F_a(\bar{a}))^2} - \left(\frac{dF_a(\bar{a})}{de^y}\frac{dF_a(\bar{a})}{db^y}\right) \frac{R^e}{(1-F_a(\bar{a}))^2}$$

which pushes the bank towards more corporate equity holdings. Thus, deposit insurance decreases the cost of holding debt for the bank and thus increases bank default risk.

5.3 Optimal Bank Regulation

5.3.1 First Best

I will look at the case in which the social planner has a utilitarian social welfare function. This implies that the planner ignores the tax advantage of corporate debt and thus there they optimize steady-state aggregate consumption according to a Cobb-Douglas return to capital R. Accordingly, their objective function is:

$$\max_{e,b} P\left(\text{Profit} \geq 0\right) \times \left[R - \Gamma\right] w$$

$$\text{Profit} = (R^{*e} - \lambda R^e)e + (R^{*b} - \lambda R^b)b - \left[1 - \lambda + \Gamma\right] w,$$

where "Profit" denotes the profit of the bank. With this utilitarian planner, the optimal allocation maximizes total household consumption. Let's look at a first-best planner who is able to freely allocate resources from households to the firm. Their power includes circumventing financial intermediation, in the sense that they can allocate saver household deposits directly to the firm. The first-best trivially

would be to aggregate all household endowments and to provide them to the firm in the form of equity for production. In this case bank profit is zero, Profit $= 0$ and therefore $P\left(\text{Profit} \geq 0\right) = 1$. This is optimal, as it minimizes bank default risk, thus maximizing expected output, which does not directly depend on the capital structure of the firm. Firm investment is no longer distorted by the corporate tax advantage of debt and costly corporate and intermediary defaults are ruled out. It is important to note that this result is driven by the fact that in this economy, without doing justice to the whole extent of literature on corporate leverage choice, 100% equity financing is the socially desirable benchmark, as the tax advantage of corporate debt is, in the aggregate, a distortion that the social planner does not need to take into account in the first-best equilibrium case.

Proposition 1 *With a utilitarian planner that maximizes steady-state consumption and can allocate household savings without restrictions to assets, the first-best allocation is $e = w$ (fully equity-financed firms).*

5.3.2 Second Best

Let the second-best equilibrium feature the optimal asset allocation, in which the planner must respect the restriction of *saver* households to invest via the financial intermediary (so they cannot be made to invest directly through financial markets). The planner in this case chooses the asset composition of savings of and financial intermediaries. Now, the planner needs to take into account that a 100 % equity portfolio might be sub-optimal due to a high probability of default for the financial intermediary, which strategically defaults without recovery if it encounters profit values below 0. There now is a trade-off for the planner: minimizing corporate default risk by choosing more corporate equity investment of the intermediary into corporate equity which as the residual claim is more volatile, or reducing bank default risk, by choosing more intermediary investment into corporate debt. The key difference to the first best is that it exposes the economy to costly intermediary defaults.

Suppose the economy is in the first best of fully equity financed firms and the productivity shock z is the sum of two components: a, which is an aggregate productivity shock, and i, which is an idiosyncratic, firm specific productivity shock.

$$z = a + i$$

Given this structure for the productivity shock, the planner optimizes:

$$\max_{e^y, b^y} P\left(\text{Profit} \geq 0\right) \times \left[R - \Gamma\right] w$$

$$\text{Profit} = (R^{*e} - \lambda R^e)e + (R^{*b} - \lambda R^b)b - \left[1 - \lambda + \Gamma\right] w$$

For steady-state considerations, the planner only cares about the default probability of the bank, but now 100 % equity does not minimize $P\left(\text{Profit} \geq 0\right)$.

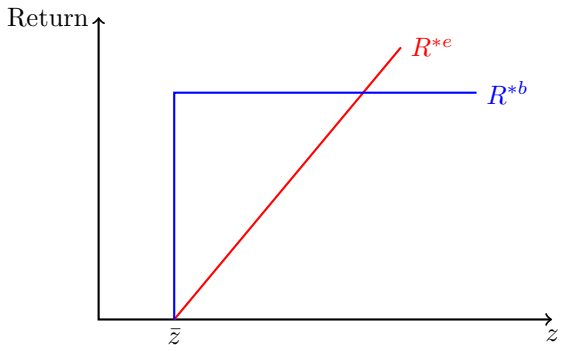


Figure 7: Payout schedules of the firms' liabilities as a function of realized productivity z.

We can now understand the second-best optimization problem if we consider the following payoff schedules for the two distinct corporate assets. Essentially, the planner is solving a Value at Risk problem: they want to choose debt and equity to minimize the probability that a shock a realizes that would lead to a default of the bank. Figure 7 shows the payout schedule of the firms' liabilities as a function of productivity. Debt is preferable to hold when facing high realizations of z, but $\bar{z}$ increases in b. Figure 8 shows the payout schedules of the diversified portfolio of firm liabilities held by the banks as a function of the aggregate shock a, while the idiosyncratic shock i washes out. In the left panel, we see an asset allocation with little debt and a lot of equity. Clearly, the planner will add more corporate debt to the banks' asset side to decrease the probability of asset return realizing below the VaR line. On the right-hand side, the probability of default is minimized by employing a mix of debt and equity. Only for very low realizations of a the bank will default. Thus, we can conclude that in the second-best solution, the planner will want the bank to hold corporate debt.

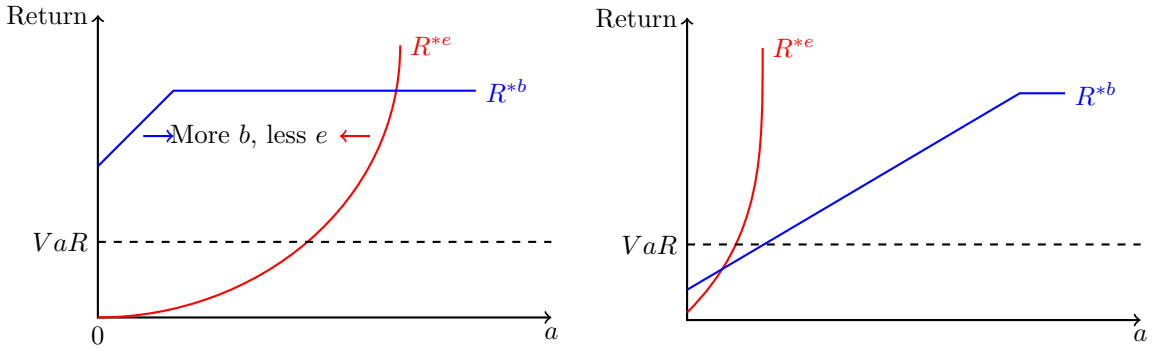


Figure 8: Payout schedules of the diversified portfolio of firms liabilities that the bank holds as assets as a function of the aggregate component of productivity a. On the left-hand side additional equity decreases bank default risk, on the right-hand side the curves have shifted for bank default risk to be minimized.

Therefore, even if intermediaries hold a balanced portfolio of debt and equity, they are exposed to systemic risk through the aggregate productivity shock a. If the economy is in the first-best, the intermediary holds 100 % corporate equity on its asset side and thus there is a positive default probability from an adverse equity shock. By changing one unit of intermediary assets from equity to debt, the planner reduces expected firm output but reduces the default probability of the bank. For

low values of debt, expected firm output does not decrease a lot, as the default probability remains low, and thus in the second best, there is an interior solution, which features enough debt for the banks to be stable, and enough equity for the firms to be stable. The optimal tax-saving strategy by the firm plays no role in these considerations.

Proposition 2 *A second-best planner who can only choose the asset allocation of the financial intermediaries optimally will choose a mix of corporate equity and debt on its asset side. The first-best would not be optimal if savings are channeled through the bank.*

5.3.3 Comparison with the market equilibrium

To understand the role of regulation, I will now compare the optimal allocation with the market allocation. Here, I will consider several different cases: 1. banks with deposit insurance vs. banks without deposit insurance, and 2. large banks with market power vs. competitive banks. We will see that both of these elements shift the solution towards more debt. What all cases have in common is, that now the tax advantage of corporate debt induces a friction that the planner previously did not take into account because they don't share the incentives of the firm. If the tax rate is large enough, the tax advantage friction creates price pressure to push the market equilibrium towards a higher debt share than in the second-best. The banker cares about the profit of the bank and thus solves:

$$\max_{e^y, b^y} P\left(\text{Profit} \geq 0\right) \times \text{Profit}$$

$$\text{Profit} = (R^{*e} - \lambda R^e)e^y + (R^{*b} - \lambda R^b)b^y - \left[1 - \lambda + \Gamma\right] q$$

If corporate debt is privately scarce in the second-best equilibrium, which will be the case if the tax advantage is strong, then the profit-motive will increase corporate debt holdings. This effect is even stronger if the banker does not internalize their effect on corporate default risk by demanding corporate debt. Anything that decreases the extent to which the banker internalizes this risk pushes the bank's portfolio allocation towards more debt. This is true for both having smaller banks (which means that they don't take into account their own impact on corporate default risk) and for having deposit insurance (which weakens market discipline and thus induces bank risk-taking).

To see this, recall that in the second-best solution, the planner ignored the tax advantage of corporate debt, and only aimed to maximize expected consumption. In the market equilibrium, however, the tax advantage for corporate debt allows the firm to issue it at a lower cost and thus it will provide a higher return to debt holders. Therefore, it is only natural that the market equilibrium features a higher corporate leverage, compared to the second-best solution. This result only becomes apparent in general equilibrium, since if we look at the individual bank while holding the default probability of the corporate sector constant, it would be optimal for this bank to hold more debt, which is less volatile than equity. Figure 9 shows the hierarchy of the different solutions: the more debt, the further away from the first-best solution the economy is. The market solution induces a higher debt financ-

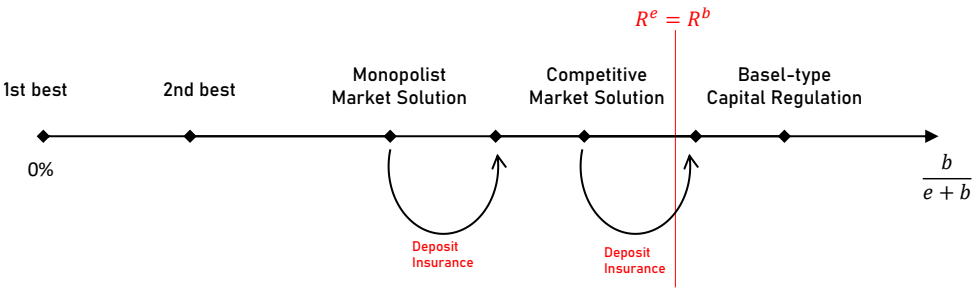


Figure 9: The hierarchy of solutions in terms of welfare and debt.
Stronger tax distortion shifts the vertical red line to the right.

ing share relative to the second-best equilibrium. Figure 9 also shows that regulation that makes it harder for banks to hold corporate equity pushes the equilibrium even further away from the first-best solution, the details of such equilibrium were described in the previous Section 4.3. In that scenario, firm leverage is high and equity is scarce, as we have seen seems to be the case in many European economies.

5.3.4 Deposit insurance and Market Power

With deposit insurance, financial intermediaries do not take into account their own default probability in their allocation decision and only aim to maximize expected return (this is driven by the fact that the equity holders of the banks are risk neutral). That incentivizes them to take on more risk to gain a higher return on their asset side. Without deposit insurance, banks take into account their own default risk because their cost of capital now includes an idiosyncratic risk premium that will increase the default risk of the bank. In this case, the deposit rate would have the following form:

$$R^d = \frac{1}{1 - G(\bar{a})} \left[\lambda \left[\frac{e^y}{e^y + b^y} R^e + \frac{b^y}{e^y + b^y} R^b \right] + 1 - \lambda \right],$$

where G is the cumulative distribution function of a, and $\bar{a}$ is the threshold below which intermediaries would default. As banks now internalize their own default risk, they are being pushed to lower their cost of capital by investing in safer assets. But which are those assets? This depends on the strength of the tax advantage of corporate debt, which shifts the vertical red line in Figure 9 to the right. On the right of this line, banks will increase equity holdings if they can, on the left they will increase debt holdings. Suppose there is only one large bank, which thus internalizes the general equilibrium effect it has on corporate default risk and therefore will increase equity holdings to decrease its own default risk. However, this only happens in a corner solution in which the investor households only hold debt, otherwise, the investor households will simply offset the additional demand by the bank and leave overall corporate default risk unchanged. Thus, with enough investor households, a

large bank will move towards debt as well as conditional on more equity not affecting corporate default probability, it is decreasing the overall volatility of its portfolio by taking on more debt. Similarly, if we have a competitive banking sector, with many competitive banks, these banks will individually want to shift towards debt, which is individually safer on the micro level, but increases the default risk of the whole banking sector on the macro level.

5.3.5 Implementation of the Optimal Regulation

The optimal regulation would bring the economy into the second best allocation by increasing the equity holding of banks, but if investor households can offset the effect on corporate default risk, this move would be useless. If it is possible, by letting banks invest freely into equity while restraining their investment into debt, investor households are being pushed into the corner of only supplying debt. This would be the optimal regulation, which will be achievable if there are only a few investor households. If however there are more investor households the optimal regulatory policy is to incentivize banks to invest more in debt, by restricting investment into corporate equity and thereby shifting banks towards more debt, which decreases bank default risk, while keeping corporate default risk constant. This can be done up to a point where investor households only hold equity. Interestingly, the relationship of bank safety with equity is non-linear: bank safety is globally maximized for high levels of corporate equity in the bank's assets, but around the market equilibrium, it is optimal to increase debt up to the local maximum of 100 % debt holdings for investor households.

In the current regulatory regime, as detailed in Section 2, banks are strongly disincentivized to hold corporate equity on their balance sheets. In the model this pushes investor households towards holding mostly equity, and little corporate debt, the stylized fact that Diamond (2020) sets out to explain. In economies in which there are few investor households, and equity is consequently scarce, banks might be too constrained in their equity holdings and it is even locally optimal to loosen regulation on equity holdings to return to the local maximum. This would work especially well in the realistic case with deposit insurance, as equity, if scarce, provides a higher return in equilibrium and these banks would be happy to invest in the more profitable equity investment. For competitive banks without deposit insurance, this incentive is weakened, because they see equity investment as individually more risky. The theory appendix Section B.6 details further regulatory trade-offs.

6 Quantitative example: United States vs. Germany

To understand the impact of low equity investment by households in conjunction with intermediary restrictions to invest into equity assets, we look at a quantified version of the model that maps features of the United States economy relative to European economies (here values for Germany are used for comparison). In this numerical solution of the model, I compare two economies with equal productivity that matches an average income of around 50.000 Dollar, standard parameters for the default recovery rate, the labor share, the pass-through of bank profits to depositors, as well as the cost of

intermediation. They differ in the share of *investor*-type households (United States: 0.35, Germany: 0.11) and the corporate tax rates (United States: 0.25, Germany 0.3), which determines the severity of the tax shield and therefore debt bias. The parameter values can be found in Table 2.

Parameter	US	DE
α	0.4	0.4
η	0.35	0.11
$f(z)$	uniform$(0,\ 8 \times 10^4)$	$uniform(0,\ 8 \times 10^4)$
τ	0.25	0.3
λ	0.5	0.5
ϕ	0.4	0.4
Γ	0.2	0.2

Table 2: Parameters for the United States and Germany

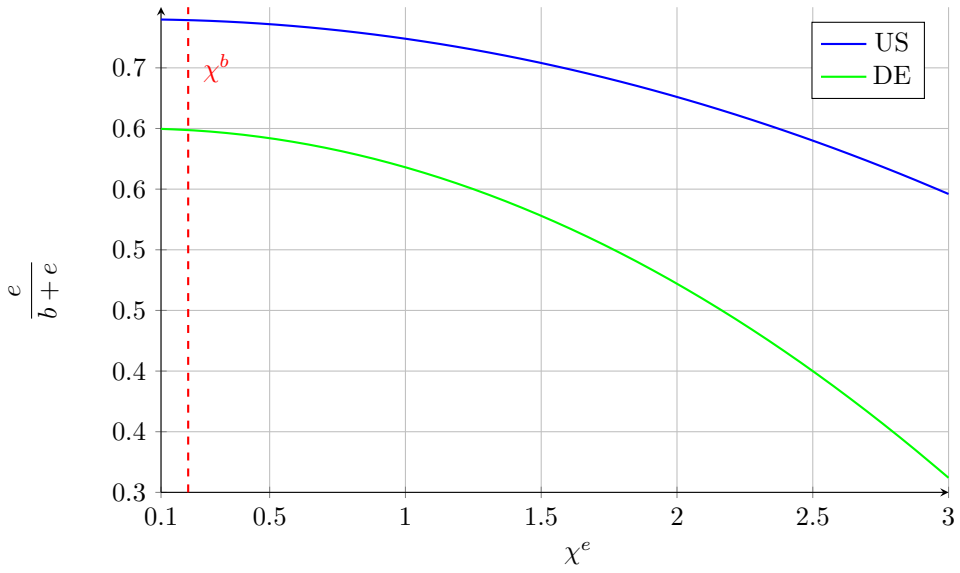


Figure 10: Aggregate corporate equity share in stylized German and U.S. economies as a funciton of χ^e.

Figures 10 and 11 showcase the results: at the current value of χ^e, which is 2.5, the simulated "German" economy is smaller and firms finance themselves with a lower share of corporate equity, which implies a higher share of firm defaults. However, reducing the χ^e parameter benefits the economy with a higher share of *saver*-type households. This is what we saw analytically before an economy with less equity investing households profits more from relaxing restrictions for regulated intermediaries to invest in equity assets. The convergence is not perfect however, as the simulated "United States" economy still has features that allow it to funnel more equity financing to its firms: a lower corporate tax rate, combined with a structurally higher share of household equity investment.

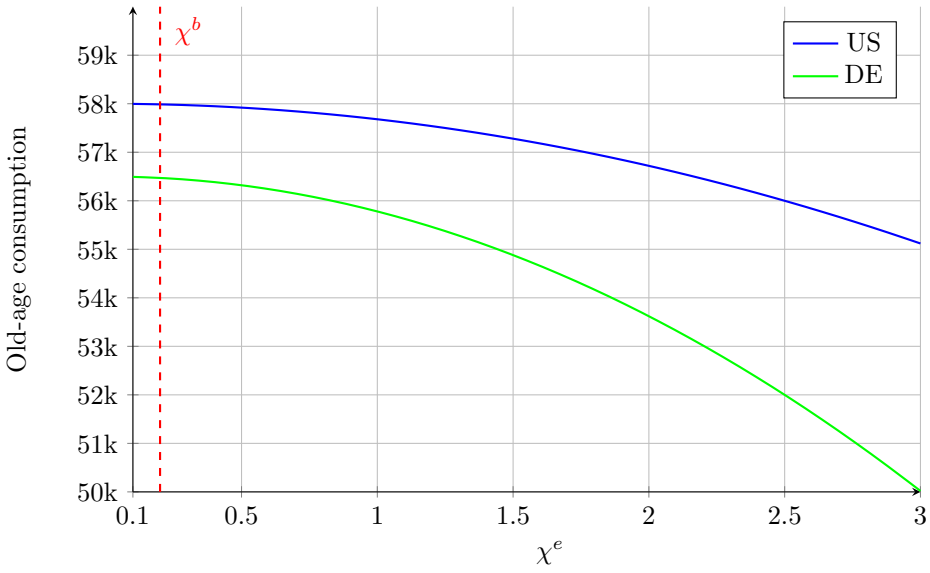


Figure 11: Aggregate corporate equity share in stylized German and U.S. economies as a funciton of χ^e.

7 Conclusion

In this paper, I analyzed the relationship between equity market investment by households and the optimal leverage choice by firms. This relationship has profound implications for capital regulation of banks and insurers. I find that low equity investment by households implies lower firm investment and output as it leads to elevated corporate leverage and default rates. Furthermore, it might exacerbate return inequality between households. Intermediaries could invest on behalf of those households that prefer saving through deposits. However, they face two disincentives: equity is risky and might lead to intermediary default and capital requirement regulation that specifically increases the cost of intermediaries' investment in corporate equity. By investing little in corporate equity, intermediaries create an externality through which their loan portfolio becomes more risky. Therefore, it might be optimal that regulators lower the capital regulation risk weight that is applied to corporate equity. This discussion is especially relevant for bank-dependent European economies, in which leverage is high, and equity participation is low. Peter (2021) points out that equity issuance frictions have a strong impact in Europe, which might further exacerbate the frictions introduced by the low participation of households in equity markets. My analysis provides a strong rationale for schemes that facilitate households providing equity capital directly to firms, or public investment schemes that invest in corporate equity on the households' behalf. Capital-based retirement systems, through which the government invests pension contributions in equity, might be welfare-enhancing, not only through the stabilization of pension schemes and by enabling constrained households to participate in financial markets but also through generating equity capital for firms and thereby increasing resilience in both the corporate and intermediary sectors.

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Appendix: Figures

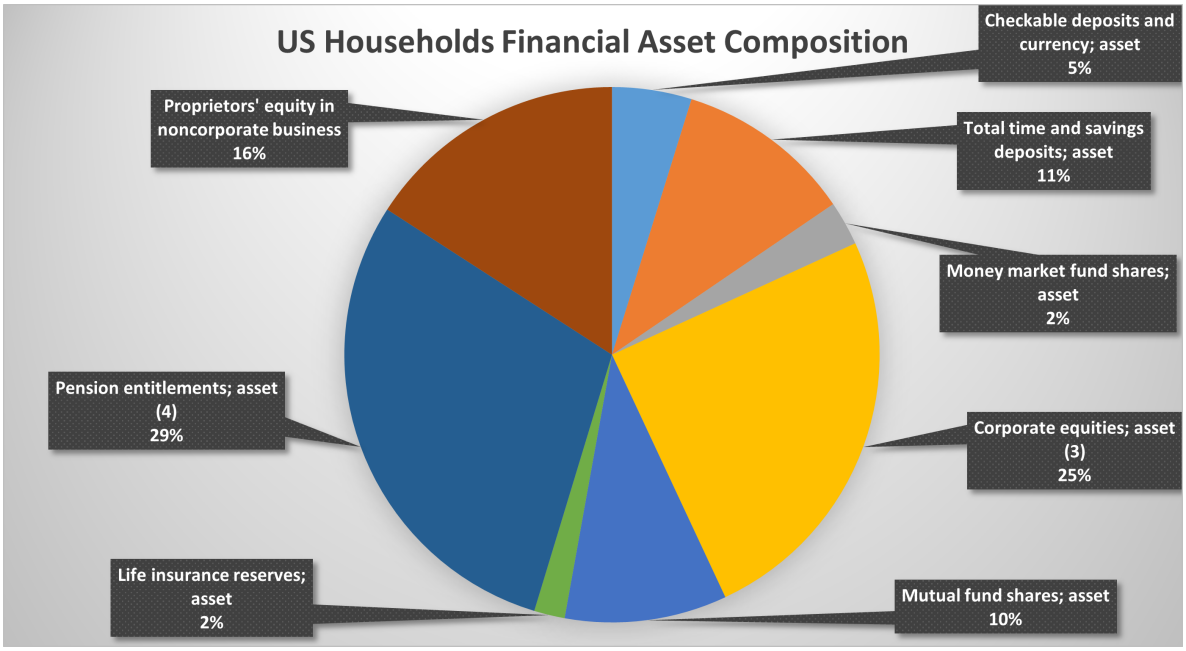

Figure 12: US Households financial asset breakdown 2020. Source: US Financial accounts.

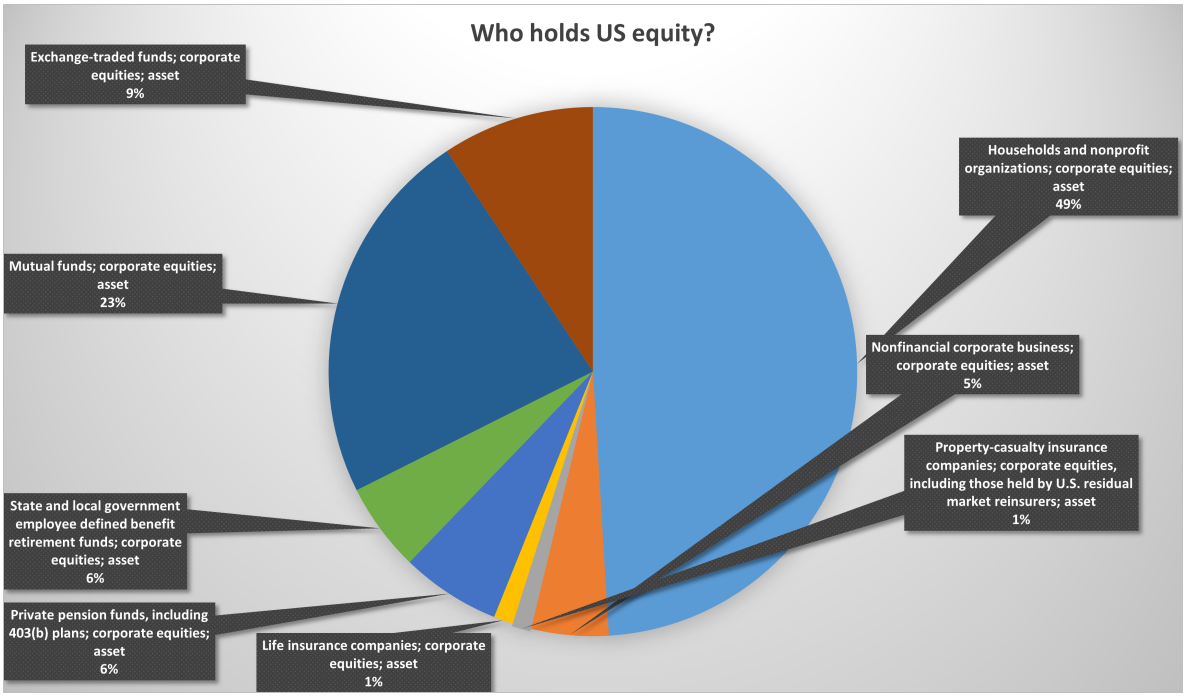

Figure 13: US Corporate Equity Ownership. Source: US Financial accounts.

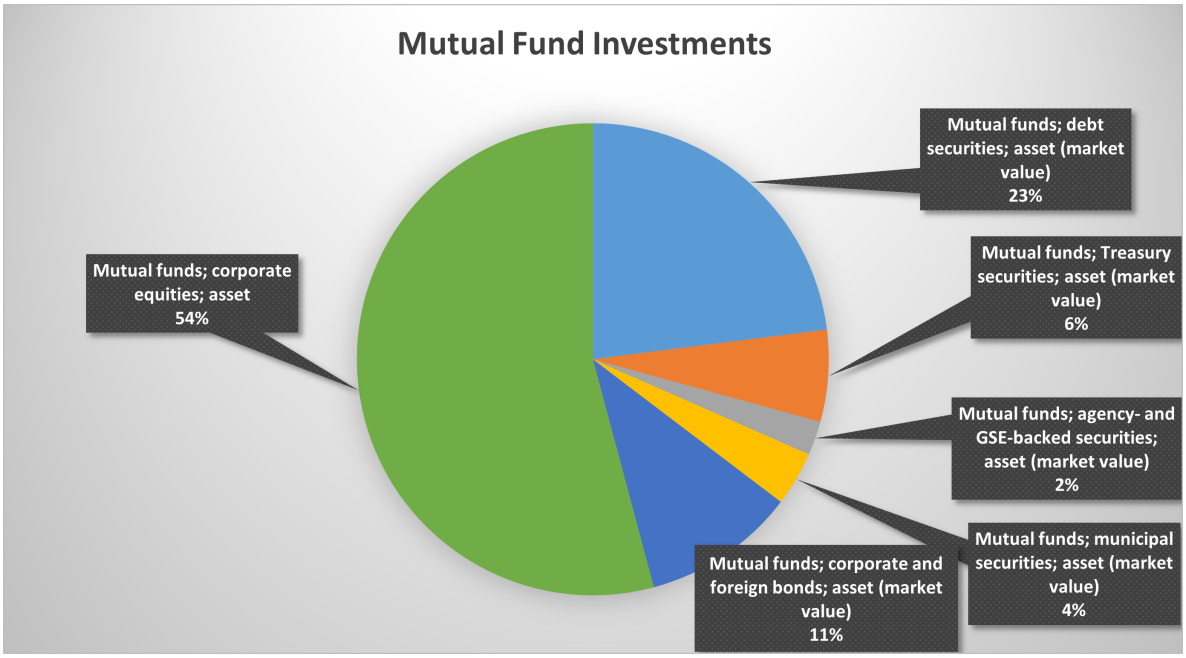

Figure 14: US Mutual Funds Asset Holdings. Source: US Financial accounts.

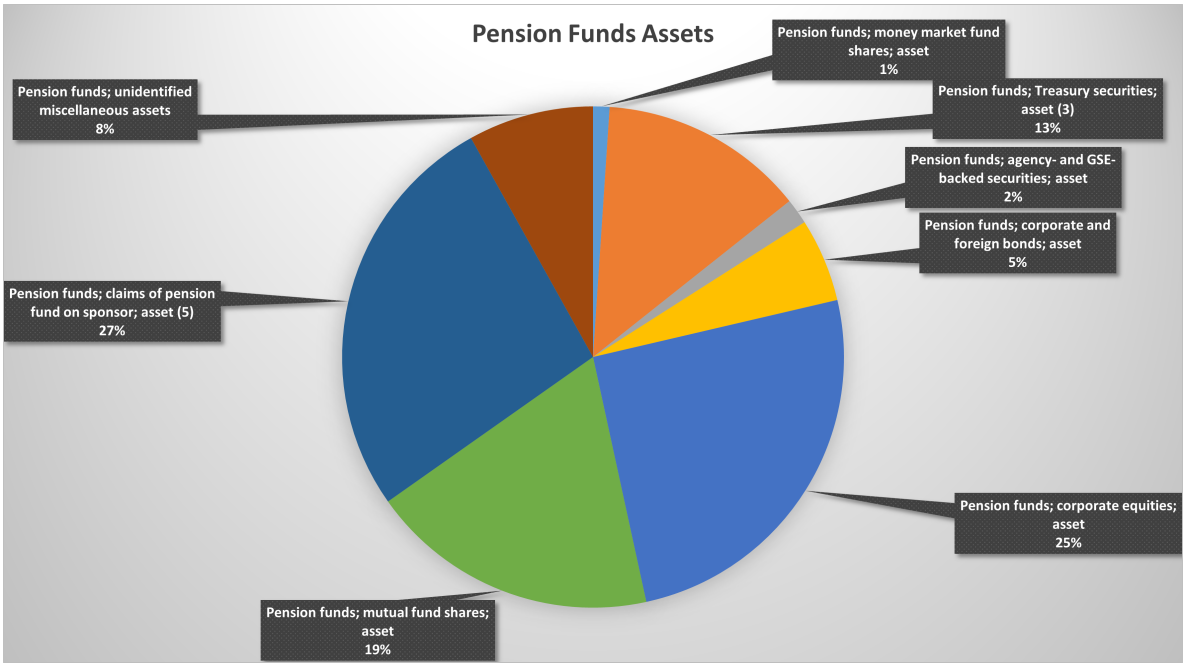

Figure 15: US Pension Funds Asset Holdings. Source: US Financial accounts.

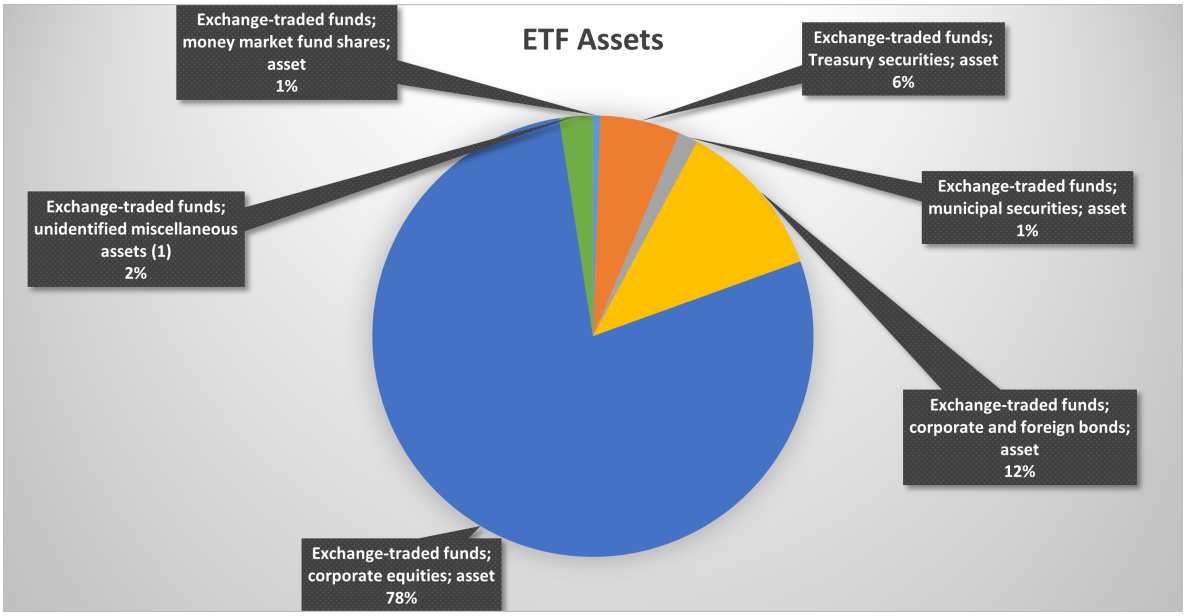

Figure 16: US Exchange Traded Funds Asset Holdings. Source: US Financial accounts.

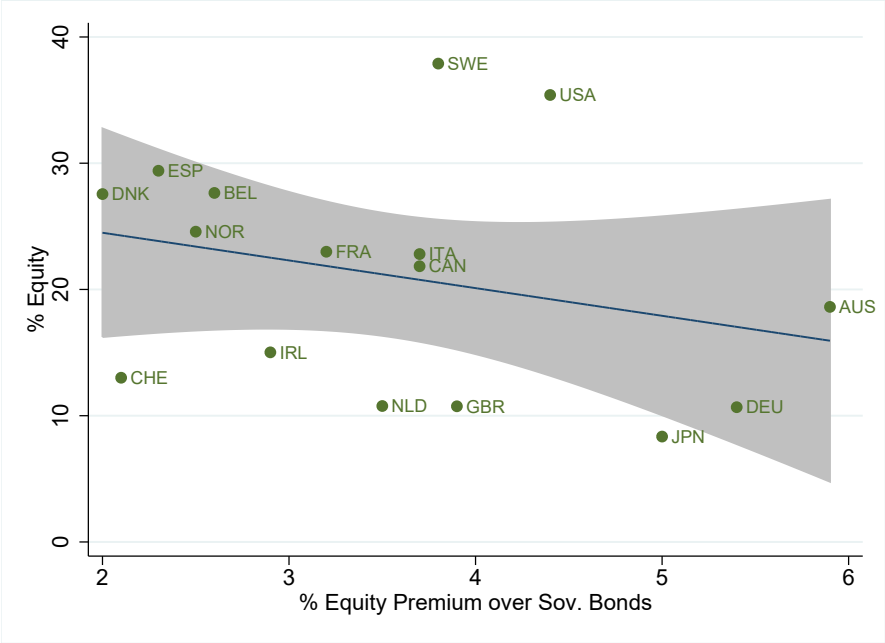

Figure 17: The relationship of the equity premium over sovereign bonds from 1900 to 2010, following Dimson, Marsh, and Staunton (1998), and the share of households' financial investments into equity (Source: National Financial accounts).

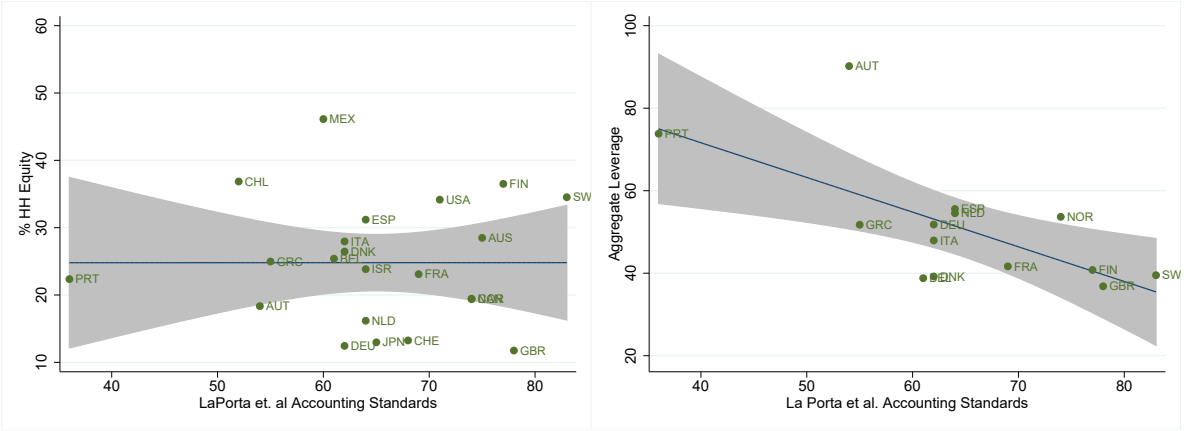

Figure 18: Left Panel: Scatter plot of the accounting standards measure, computed in Porta et al. (1998), and the share of households' financial investments into equity (Source: National Financial accounts). Right panel: Scatter plot of the accounting standards measure, computed in Porta et al. (1998), and aggregate corporate book leverage (Source: National Financial accounts). As the Porta et al. (1998) measure was calculated in 1998, I use households' equity holdings and corporate leverage from 2000 to 2005.

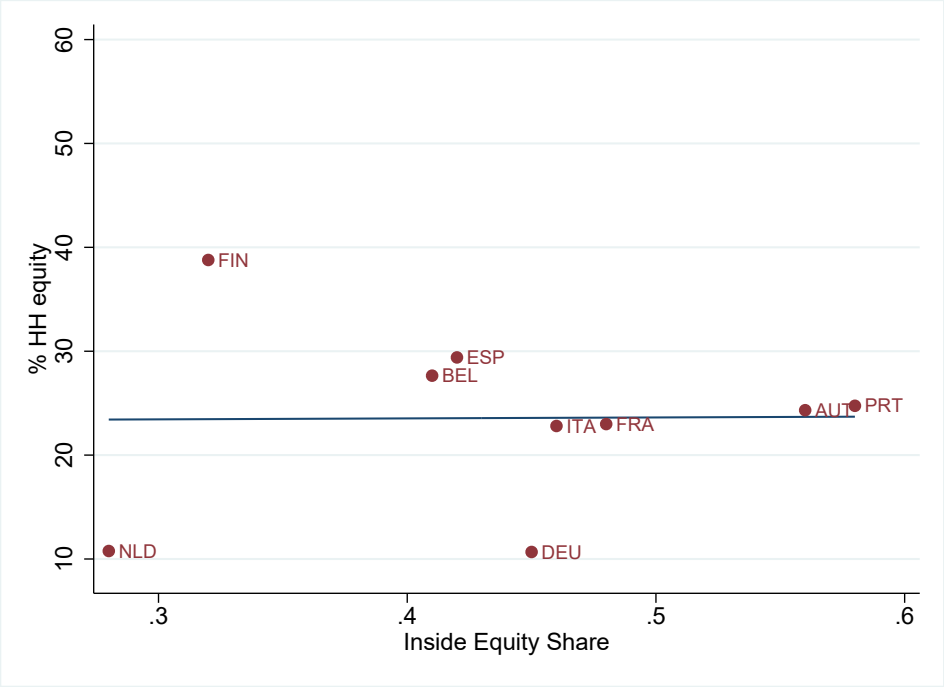

Figure 19: Scatter Plot of the insider share of in domestic equity finance, as computed by Peter (2021) and the share of equity in households' financial portfolios. Data: Peter (2021) and National Financial Accounts.

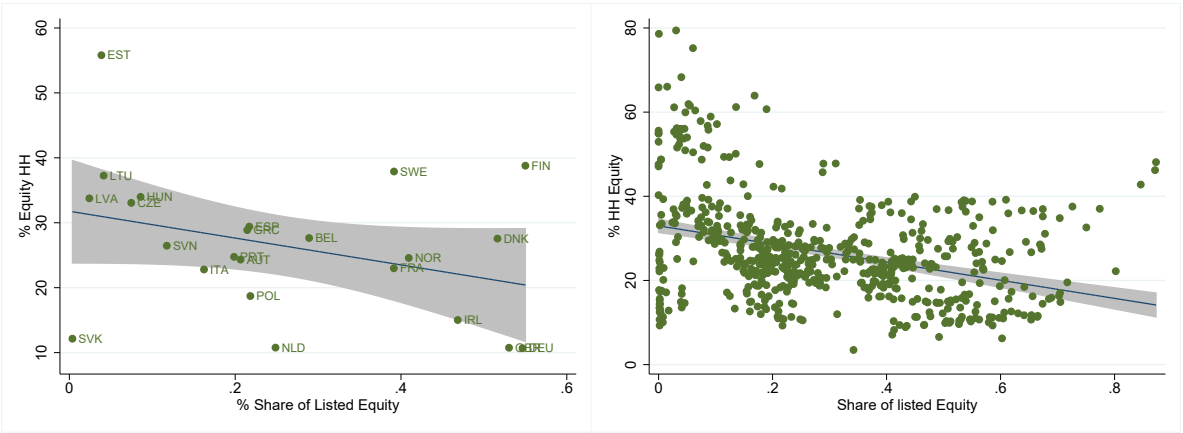

Figure 20: Scatter plots of the share of listed equity in all domestic firm equity (as opposed to unlisted and other equity) and the share of equity in households' financial portfolios. Left panel: 2019. Right panel: 1970-2021. Data: National Financial Accounts.

Appendix: Derivations

B.1 Firm Problem

We want to write down the expected value of the firm's profit while taking into account the tax advantage of corporate debt. Note that the tax advantage is only applied to the interest payment on corporate debt, not the principal. I denote the contractual interest on corporate debt as $\bar{r}^b$ and the expected gross interest rate as R^b.

Both principal b, as well as interest $\bar{r}^b b$ are only repaid if the firm does not default. Thus the expected repaid principal is $[1 - F(\bar{z})]b$, while $\bar{r}^b b$ is deducted from the taxable income of the firm. Therefore the firm problem can be written as

$$\max_{e,b,l}[1 - F(\bar{z})](1 - \tau)[E(z)(e + b)^{\alpha} l^{1-\alpha} - wl - \bar{r}^b b] - [1 - F(\bar{z})]b - R^e e.$$

The relationship between the expected gross interest rate and the realized gross interest rate is

$$R^b = (1 - F(\bar{z}))\bar{R}^b = (1 - F(\bar{z}))(1 + \bar{r}^b)$$
$$\iff \bar{r}^b = \frac{R^b}{1 - F(\bar{z})} - 1$$

If we plug this into the firm's problem we get:

$$\max_{e,b,l} E([1 - F(\bar{z})][(1 - \tau)[z(e + b)^{\alpha} l^{1-\alpha} - wl] - [1 - F(\bar{z})]b - (1 - \tau)R^b b + (1 - \tau)[1 - F(\bar{z})]b - R^e e,$$

or

$$\max_{e,b,l} E([1 - F(\bar{z})][(1 - \tau)[z(e + b)^{\alpha} l^{1-\alpha} - wl] - (1 - \tau)R^b b - [1 - F(\bar{z})]\tau b - R^e e.$$

In this way, the firm problem can be written as a function of the expected interest rates R^e and R^b, taking into account that only the interest rate, not the principal, is tax deductible.

B.2 Firm first order conditions

To find the FOCs, note that

$$[1 - F(\bar{z})]E[z|z > \bar{z}] = \int_{\bar{z}}^{\infty} f(x)x dx,$$

and thus

$$\frac{d[1 - F(\bar{z})]E[z|z > \bar{z}]}{dx} = -\bar{z} f(\bar{z}) \frac{d\bar{z}}{dx}.$$

The first order conditions of the firm are accordingly:

$$\frac{\partial}{\partial e} = -\bar{z}f(\bar{z})\frac{d\bar{z}}{de}(1-\tau)\alpha(e+b)^{\alpha} + [1-F(\bar{z})](1-\tau)\alpha^2 E[z|z>\bar{z}](e+b)^{\alpha-1} - R^e = 0, \tag{9}$$

$$\frac{\partial}{\partial b} = f(\bar{z})\frac{d\bar{z}}{db}[\tau b - \bar{z}(1-\tau)\alpha(e+b)^{\alpha}] + [1-F(\bar{z})][(1-\tau)\alpha^2 E[z|z>\bar{z}](e+b)^{\alpha-1} - \tau] - (1-\tau)R^b = 0. \tag{10}$$

If we use this explicit form of $\bar{z}$ we can simplify these FOCs to:

$$\frac{\partial}{\partial e} = -f(\bar{z})\frac{d\bar{z}}{de}\left[(1-\tau)\frac{R^b}{1-F(\bar{z})} + \tau\right] b + [1-F(\bar{z})](1-\tau)\alpha^2 E[z|z>\bar{z}](e+b)^{\alpha-1} - R^e = 0 \tag{11}$$

$$\frac{\partial}{\partial b} = -f(\bar{z})\frac{d\bar{z}}{db}(1-\tau)\frac{R^b b}{1-F(\bar{z})} + [1-F(\bar{z})][(1-\tau)\alpha^2 E[z|z>\bar{z}](e+b)^{\alpha-1} - \tau] - (1-\tau)R^b = 0 \tag{12}$$

or:

$$[1-F(\bar{z})](1-\tau)\alpha^2 E[z|z>\bar{z}](e+b)^{\alpha-1} = R^e + f(\bar{z})\frac{d\bar{z}}{de}\left[(1-\tau)\frac{R^b}{1-F(\bar{z})} + \tau\right] b, \tag{13}$$

$$[1-F(\bar{z})]\left[(1-\tau)\alpha^2 E[z|z>\bar{z}](e+b)^{\alpha-1} - \tau\right] = (1-\tau)R^b\left[1 + \frac{f(\bar{z})\frac{d\bar{z}}{db}}{1-F(\bar{z})}b\right]. \tag{14}$$

B.3 Derivation of $\bar{R}^b(a)$ and $\bar{R}^e(a)$

The objective function of the bank problem in expectation is:

$$[1-F_a(\bar{a})]\left[R^e e^y + R^b b^y - R^d q - \Gamma(e^i+q)\right] - R^e e^i.$$

Note that the bank's owners are risk-neutral and thus demand to be compensated in their lending to a firm. That means that ex-ante the contracted gross interest rate is

$$\frac{R^b}{1-F_z(\bar{z})},$$

to receive R^b in expectation. The bank holds a portfolio of all firms in the economy. A fraction of these firms, specifically those for which $i+a<\bar{z}$, will default. Thus the bank only receives their loan payment from a measure $1-F_i(\bar{z}-a)$ of all firms. As a function of the systemic shock a, this yields a realized return to the bank debt portfolio of:

$$\bar{R}^b(a) = \frac{R^b}{1-F_z(\bar{z})}\left[1-F_i(\bar{z}-a)\right].$$

The corporate equity portfolio returns zero for all firms that default on their debt, due to the assumption of no recovery. For the remaining firms, it returns the residual of what is left after the firm pays out its creditors, which derives from the problem of the firm, as in Section 4.2.

$$\bar{R}^b(a) = \frac{1}{e}\left[1 - F_i(\bar{z} - a)\right]\left((1-\tau)\alpha\left(a + E[i|i > \bar{z} - a]\right)(e+b)^{\alpha} - (1-\tau)\bar{R}^b(a) - \tau b\right).$$

$a + E[i|i > \bar{z} - a]$ is the average productivity shock for an equity portfolio containing equity of all firms, after the aggregate shock a has been drawn.

B.4 Derivation of the Bank Problem without Capital Regulation

We start with the bank problem:

$$[1 - F_a(\bar{a})]\left[R^e e^y + R^b b^y - R^d q - \Gamma(e^i + q)\right] - R^e e^i = 0,$$

and plug in $e^i = e^y + b^y - q$.

$$[1 - F_a(\bar{a})]\left[R^e e^y + R^b b^y - R^d q - \Gamma(e^y + b^y)\right] - R^e(e^y + b^y - q) = 0$$

$$\iff \left[R^e e^y + R^b b^y - R^d q - \Gamma(e^y + b^y)\right] - \frac{R^e(e^y + b^y - q)}{1 - F_a(\bar{a})} = 0$$

Collecting terms, we find:

$$\left(R^e - \Gamma - \frac{R^e}{1 - F_a(\bar{a})}\right)e^y + \left(R^b - \Gamma - \frac{R^e}{1 - F_a(\bar{a})}\right)b^y - \left(1 + \Gamma - \frac{R^e}{1 - F_a(\bar{a})}\right)q = 0.$$

Without deposit insurance, the initial problem is:

$$[1 - F_a(\bar{a})]\left[R^e e^y + R^b b^y - \frac{R^d}{1 - F_a(\bar{a})}q - \Gamma(e^i + q)\right] - R^e e^i = 0,$$

and the remaining steps of the derivation are the same as above.

B.5 Derivation of the Bank Problem with Capital Regulation

Similarly to B.4, we start with the bank problem

$$[1 - F_a(\bar{a})]\left[R^e e^y + R^b b^y - R^d q - \Gamma(e^i + q)\right] - R^e e^i = 0,$$

but now the regulatory constraint binds

$$e^i = \chi^e e^y + \chi^b b^y.$$

Therefore, the problem becomes:

$$[1 - F_a(\bar{a})] \left[R^e e^y + R^b b^y - R^d q - \Gamma(\chi^e e^y + \chi^b b^y + q)\right] - R^e(\chi^e e^y + \chi^b b^y) = 0,$$

$$\iff \left[R^e e^y + R^b b^y - R^d q - \Gamma(\chi^e e^y + \chi^b b^y + q)\right] - \frac{R^e(\chi^e e^y + \chi^b b^y)}{1 - F_a(\bar{a})} = 0.$$

Collecting terms, we find

$$\left(R^e - \frac{\chi^e R^e}{1 - F_a(\bar{a})} - \chi^e \Gamma\right) e^y + \left(R^b - \frac{\chi^b R^e}{1 - F_a(\bar{a})} - \chi^b \Gamma\right) b^y - (1 + \Gamma) q.$$

Again, for the version without deposit insurance, the initial problem is:

$$[1 - F_a(\bar{a})] \left[R^e e^y + R^b b^y - \frac{R^d}{1 - F_a(\bar{a})} q - \Gamma(e^i + q)\right] - R^e e^i = 0,$$

and the remaining steps of the derivation are the same as above.

B.6 Regulatory trade-offs

B.6.1 Stability/Regulation Trade-off

Two forces characterize this trade-off: the financial intermediary does not internalize the cost of its potential default. On the other hand, a leverage constraint might decrease optimal investment if chosen too tightly. As an example, assume $\chi^e = \chi^b$, such that there is no leverage penalty for the bank to invest in corporate equity. In this case, $R^e = R^b$ and the capital requirement constraint becomes $e^i > \chi^e(e^y + b^y)$. The bank does not internalize that a. high debt b^y makes firms more likely to default; and that b. high equity e^y is more volatile than debt, making firms more likely to default. Thus the bank invests on the margin too much into debt and equity, and, relative to its investment into debt, too much into equity. If the bank would internalize both effects, it would instead solve the following problem:

$$\max_{e^y, b^y, q} [1 - F(\tilde{z})](R^e e^y + R^b b^y) + F(\tilde{z})(\xi(R^e e^y + R^b b^y)) - R^d q - R^e e^i - \Gamma(e^i + q)$$

It is now no longer obvious that the bank wants to accept all deposits, as deposits are costly, if not invested into assets. Thus the bank would reduce leverage below an optimal threshold, which can be induced by the capital requirements constraint. However, this threshold leads to some funds

not being used efficiently, as not all funds of saver households are channeled into productive capital. The optimal leverage constraint trades off these two inefficiencies against one another.

B.6.2 Corporate Equity Investment Trade-off

Even with the optimal leverage constraint the instruments of the regulator remain too blunt. In other words, financial intermediaries choose the riskier investment in the same regulatory category, similar to the findings of Becker and Ivashina (2015) for insurance. Despite the overall restriction of bank investment and leverage, the bank still invests too much into corporate equity relative to the case where it internalizes the risk of doing so. Thus, it is efficient to regulate the equity investment of banks with a separate instrument, in this case, a risk weight on equity that is larger than that of debt $\chi^e > \chi^b$. Thus, equity is treated as a separate regulatory category. This will reduce the banks' investment into corporate equity but will increase the risk of corporate default if investor households are too few to replace the missing equity investment. Thus, we learn a very important lesson: equity investment by intermediaries is optimally tighter regulated in economies in which households participate abundantly in equity markets. To link this to the real world: this constitutes an explanation to why the US (where equity market participation is high) regulates equity investment by intermediaries more tightly than Europe (where equity market participation is low). We learn that there is an optimal equity regulation, which trades off financial stability (preventing corporate defaults and bank defaults) with maximizing output. Note that if the risk weight on equity is loosened relative to debt, it is optimal that investment is more restricted overall.