

Why Venture Later? Incentives, Learning, and Industry Allocation in VC Funds*

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Abstract

Why do private equity firms often delay investment in high-uncertainty sectors like deep tech, despite their potential for long-term gains? This paper examines how agency frictions shape cross-industry portfolio allocation decisions. I develop a dynamic model in which limited partners cannot observe fund managers' effort to explore, and fund managers cannot fully monitor entrepreneurs' experimental designs. These two layers of moral hazard reduce incentives for early exploration and distort capital commitments across funds. The model predicts that firms with high opportunity costs are more likely to specialize, while even low-cost firms underinvest in exploration when frictions are severe. Empirical evidence from matched fund-level data confirms these predictions, showing that higher moral hazard is associated with reduced exploratory investment in earlier funds and lower follow-on capital in subsequent funds. Ongoing work uses structural estimation to quantify the welfare implications of these frictions.

Keywords: Exploration–Exploitation Trade-off, Venture Capital, Moral Hazard, Industry Allocation, Dynamic Agency Model

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1 Introduction

Venture capital and private equity firms continually deal with the exploration–exploitation trade-off: should they concentrate investments in well-known sectors where experience yields reliable returns, or venture into high-uncertainty markets (e.g. deep tech) where breakthroughs promise outsized gains? Although some firms specialize narrowly, others diversify across industries (Gompers et al. (2005)), and yet the forces driving these cross-industry allocation choices remain poorly understood, especially when it comes to deep-tech entry. Moreover, agency frictions between limited partners (LPs) and general partners (GPs), as well as between GPs and entrepreneurs, may distort these decisions; but how, and to what extent?

Private equity funds today manage over \$1 trillion globally, and VC in particular has driven breakthroughs in AI, biotechnology, and advanced manufacturing. Still, capital allocation decisions can be distorted by agency frictions, moral hazard and information asymmetries between investors and fund managers. These frictions may inhibit optimal exploration, misallocate resources, and ultimately slow technological progress. Understanding how layered moral hazard shapes cross-industry allocation is therefore crucial for both contract design and efficient capital deployment.

Figure 1 illustrates a pronounced U-shaped cycle: venture portfolios diversified markedly during the mid-2010s, followed by a steady re-specialization in recent years. Rather than returning to their original sectors, firms increasingly concentrate again, often in newly explored areas, after a cycle of broad experimentation. This pattern suggests that early exploration is delayed and conditional, with specialization emerging only after sufficient learning occurs. What contractual or informational barriers induce this delay, and why do firms not explore aggressively from the outset?

This paper addresses these questions by developing a dynamic agency framework in which a private equity firm raises two consecutive funds and chooses how much to invest in its core industry versus a new, unexplored sector. Drawing on Holmstrom and

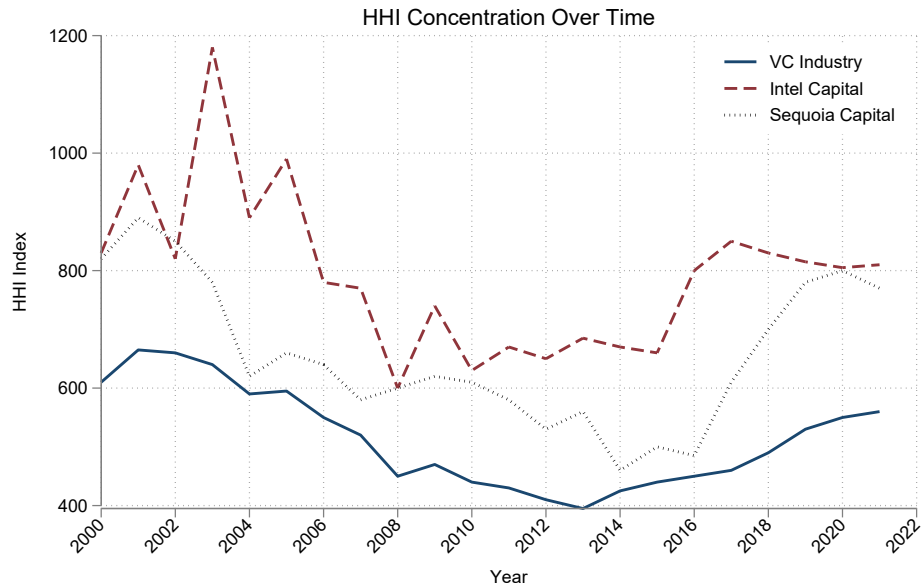


Figure 1. Evolution of industry concentration (HHI) in VC portfolios. After a period of diversification (2008–2015), funds re-specialize post-2015 at both the aggregate level and within firms.

Tirole (1997) analysis of moral hazard and incentive transfers, the first layer of the model captures the conflict between limited partners and general partners: investors must design compensation schemes that elicit effort from fund managers, even though they cannot perfectly observe how that effort is allocated.

At the second layer, the model builds on Holmstrom and Milgrom (1991) multitask principal–agent framework to describe the relationship between GPs and entrepreneurs. Entrepreneurs conduct experiments to test the technical feasibility of deep-tech ventures, but they may bias experimental design (favoring false positives or inconclusive tests) if their private incentives diverge from the fund’s goals.

By nesting these two agency problems, the model highlights how incentive misalignments compound. In the absence of frictions, exploratory investments yield valuable learning that enhances returns in the follow-on fund. But once LP–GP moral hazard reduces the manager’s monitoring effort, and GP–entrepreneur moral hazard distorts experiment quality, the informational benefits of early exploration shrink.

The reduced signal precision forces fund managers to underinvest in new industries,

since a larger exploratory allocation no longer reliably improves future performance. In turn, limited partners respond by trimming their follow-on commitments, creating a cycle of underinvestment and stagnated learning.

Despite the potential for deep-tech breakthroughs, these dual frictions generate a suboptimal equilibrium: few firms explore aggressively in their first fund, and many postpone entry into high-uncertainty sectors until after they have demonstrated success in more familiar markets.

The model endogenizes portfolio choice and LP capital commitments for both funds, identifying two key parameters: the opportunity cost of exploration (the firm’s comparative disadvantage in new ventures) and the learning intensity (the speed at which experience translates into improved screening). In the frictionless first-best benchmark, low opportunity cost and high learning intensity yield interior exploration. However, once moral hazard is introduced, even firms with modest exploration costs optimally underinvest, and LPs allocate less to successor funds as GP shirking intensifies.

Importantly, the framework yields testable predictions: firms facing stronger LP–GP frictions should allocate less to new industries initially, and those with greater GP–entrepreneur misalignment should exhibit lower quality experimentation and weaker follow-on performance.

In the empirical section, I bring these theoretical insights to data assembling a dataset from PitchBook—combining deal-level, exit, and fund-level information for VC firms (1995–2023). I show that roughly one-third of firms that raise a second fund shift their industry focus, consistent with model predictions. Firms facing greater moral hazard (proxied by shorter fundraising intervals and weaker test-reliability measures) allocate significantly less to new industries in Fund 1 and receive smaller follow-on commitments in Fund 2. These results hold under rich controls—including industry, strategy, vintage, and GP fixed effects—and using alternative measures of learning and opportunity cost, thus providing robust evidence that layered agency frictions impede optimal cross-industry

investment.

My analysis builds on the principal-agent framework in the context of financing innovation and venture capital, specifically addressing moral hazard issues related to investor financing decisions. Early contributions to this literature include Aghion and Bolton (1992), Hellmann (1998), Bergemann and Hege (1998), and Cornelli and Yosha (2003), which explore various agency frictions such as the diversion of funds or effort, performance signaling, conflicts surrounding project termination, and entrepreneurs ability to manipulate experiment testing reliability. However, my model introduces a novel moral hazard problem between the LPs and GPs, adding a new layer of difficulty to financing deep tech. The LPs face this moral hazard because GPs may choose to exert effort or shirk in influencing the reliability of experiment testing.

Additionally, my work builds on the empirical literature on agency frictions in financing innovation, particularly in deep tech ventures, as explored by Gompers (1996), Hellmann and Puri (2000). Recent studies highlight the challenges faced by deep tech ventures in securing funding, such as delays in venture capital investment and the difficulty of obtaining funding for high-risk projects (Dalla Fontana and Nanda (2023)). I contribute to this discussion by introducing a two-layered moral hazard in experimental design as a severe friction that can lead to huge decline in allocation capital to deep tech ventures. Furthermore, the empirical results corroborates my model’s findings.

Moreover, existing research on dynamic contracts and innovation incentives highlights key mechanisms for experimentation, persuasion, and contest design. Guo (2016) models a dynamic principal-agent relationship where an agent, with private information about his belief in project success, engages in experimentation under an optimal delegation rule that balances over- and under-experimentation. Ely (2017) studies dynamic persuasion, analyzing how a principal influences an agent’s actions over time through controlled information disclosure. Halac et al. (2017) explores innovation contests with learning, demonstrating how strategic prize-sharing and disclosure policies maximize in-

novation incentives. While these studies offer valuable insights into dynamic incentives, they do not address the endogenous allocation of investment portfolios in deep tech under a two-layered moral hazard structure. My paper extends this literature by introducing a dynamic investment model that endogenizes optimal portfolio allocation in deep tech, considering both GP-LP and GP-entrepreneur moral hazard, thereby capturing the interplay between capital allocation and incentive design in breakthrough innovation.

This paper also extends the exploration-exploitation framework in the context of venture capital investments (e.g., Manso (2011); Sorensen (2008); Bergemann and Hege (2005)), by incorporating agency frictions into the model. Moreover, it enriches the broader literature on agency issues in private equity, building on work by Maurin et al. (2020) and Gryglewicz and Mayer (2023). Lastly, it adds to the dynamic financial contracting literature, showing how exploration-exploitation trade-offs interact with agency problems to shape portfolio allocation decisions (e.g., Bolton and Scharfstein (1990)).

Overall, the results underscore that contractual and monitoring constraints (not merely risk preferences) drive delayed deep-tech entry and subsequent re-specialization. Mitigating dual moral hazard may therefore be key to unlocking earlier, more efficient exploration in high-uncertainty sectors.

The remainder of this extended abstract is organized as follows: Section 2 presents the main theoretical framework and model, including key results (proposition 9, and proposition 10). Section 3 summarizes the findings and key predictions of the model. Section 4 outlines the empirical results.

2 Theoretical Model

In this section, I present and solve the theoretical model that forms the basis of the empirical analysis. Subsection 2.1 introduces the contract between the GP and the entrepreneur, and Subsection 2.2 introduces the contract between LP and GP. Subsection

2.2.1 introduces the key players and outlines the model’s timing structure. Subsection 2.2.2 explores the exploration-exploitation trade-off and the learning dynamics within the framework. In Subsection 2.2.3, I derive the first-best solution, as characterized in Proposition 6, and present the comparative statics in Propositions 7 and 8. Subsection 2.3 examines the dynamic agency problem, assuming that the limited partner holds full bargaining power, with the primary results outlined in Propositions 9 and 10.

2.1 The Contract between the GP and the Entrepreneur

2.1.1 Model Framework

In the realm of Holmstrom and Milgrom (1991), I develop a contracting framework for a venture involving a risk-neutral GP and a risk-neutral entrepreneur, where the conditions for the venture’s success can be evaluated through an experiment.

2.1.2 Timing

The model follows a static structure divided into three distinct periods. In Period 0, the contracting phase takes place, during which the investor presents a take-it-or-leave-it investment offer to the entrepreneur. Period 1 serves as the experimentation phase, where the entrepreneur conducts a test to assess whether the necessary conditions for the venture’s success are met. For clarity, I focus on the venture’s technical feasibility as a key determinant. Finally, Period 2 marks the implementation phase, in which the venture is fully developed, provided that a follow-up investment is made based on the experiment’s outcome. If the investor chooses not to proceed with additional funding, the venture is abandoned, and no further development occurs.

2.1.3 Technology

The venture, denoted by v , has two possible states: success ($v = R$) or failure ($v = 0$). If successful, the venture generates a return R , and if it fails, it has a scrap value of zero.

The cost of fully developing the venture is K , where $K < R$. The ex-ante probability of success in getting the return R through this experiment is p , which is common knowledge. I assume that:

$$pR - K < 0. \quad (1)$$

Since the expected net present value of the venture is negative ex-ante, no investment occurs without additional information. The entrepreneur can acquire such information through experimentation.

2.1.4 Experimentation

The entrepreneur possesses the exclusive ability to conduct an experiment, such as a lab test, to gather information on the feasibility of the technology. The experiment incurs a cost of $C > 0$. The entrepreneur has discretion in designing the experiment, allowing for variations in false negative and false positive outcomes. The test yields one of two possible signals: F if the technology fails the test and P if it passes.

To establish a benchmark, it is useful to define the concept of a perfect experiment. If technical feasibility is both a necessary and sufficient condition for the venture's success, an experiment that provides definitive conclusions regarding feasibility qualifies as a perfect experiment. In this scenario, a successful test result ensures that the venture will succeed, while a failed test result guarantees that the venture will not be viable. Given the prior belief p , the expected payoff from conducting a perfect experiment and proceeding with implementation only if the test yields a positive result is:

$$\text{Expected payoff} = p(R - K) - C. \quad (2)$$

I assume:

$$p(R - K) - C > 0, \quad (3)$$

which implies that experimentation can make the venture a positive-NPV project.

In practice, verifying the existence of a necessary condition can mitigate a venture's risk, but only to a limited extent. To simplify the risk assessment associated with venture development, I characterize the experiment using two parameters: r_1 , which represents the probability that the test fails when the technology is indeed viable and capable of generating R ; and r_2 , which denotes the probability that the test succeeds when the technology is unworkable, meaning the venture would ultimately be unsuccessful.

$$P(s = F|v = R) = r_1, \quad (4)$$

$$P(s = P|v = 0) = r_2, \quad (5)$$

where r_1 represents the rate of false negative test outcomes and r_2 represents rate of false positive test outcomes. The entrepreneur selects (r_1, r_2) within a feasible set $\rho = [\underline{r}_1, \bar{r}_1] \times [\underline{r}_2, \bar{r}_2]$, where $0 \leq \underline{r}_i < \bar{r}_i \leq 1$ for $i = 1, 2$.

Therefore, the expected payoff from experimentation and investment conditional on a pass signal is:

$$\pi_{r_1, r_2} = p(1 - r_1)R - [p(1 - r_1) + (1 - p)r_2]K - C. \quad (6)$$

The net gain from experimentation is:

$$\Delta\pi_{r_1, r_2} = (K - pR - C) + p(1 - r_1)(R - K) - (1 - p)r_2K. \quad (7)$$

Since $R > K$, the net gain is decreasing in both r_1 and r_2 , implying that lower false negative rate and lower false positive rate yield greater value.

This implies that the best experimental design, in terms of maximizing value gained from information discovery, is the one with the lowest false negative rate (the lowest r_1) and the lowest false positive rate (the lowest r_2). Note that a perfect experiment has

$r_1 = 0$ and $r_2 = 0$. Therefore, if available, it yields the highest gain.

The entrepreneur, in effect, faces a multitasking problem in designing the test (how specific and how sensitive to make the test). The contracting problem between the entrepreneur and the GP, therefore, has elements of a multitask Principal-Agent problem (Holmstrom and Milgrom (1991)). A key consideration in multitasking Principal-Agent problems is whether the tasks are complementary or substitutes. For this problem, I consider whether a more specific test is also more sensitive (complementary tasks), or whether greater specificity necessarily means less sensitivity (substitutable tasks). I consider each case in turn.

For simplicity, I assume that each test specification has the same cost $C > 0$. In the case of substitute tasks, I consider the extreme case where false positivity and false negativity are perfect substitutes, meaning that the sum of the parameter values r_1 and r_2 always adds up to a constant κ , so that $r_1 + r_2 = \kappa$, where $0 < \kappa < 2$. The test set in this case of perfect substitutes thus takes the form $\rho = \{(r_1, r_2) \mid r_1 + r_2 = \kappa, \underline{r}_1 \leq r_1 \leq \bar{r}_1, \underline{r}_2 \leq r_2 \leq \bar{r}_2\}$. I assume $\underline{r}_1 + \underline{r}_2 > \kappa$. Hence, the entrepreneur can minimize false negativity by setting $r_1 = \underline{r}_1$ or minimize false positivity (setting $r_2 = \underline{r}_2$), but she cannot do both. At the frontier, a test with less false negatives is inevitably a test with more false positives, and vice versa.

In the case of complementary tasks, I again consider an extreme case where false negativity and false positivity are perfect complements. This means that the test designs are all such that $r_2 = \eta r_1$, where $\eta \in (0, 1)$, so that the test set is given by $\rho = \{(r_1, r_2) \mid r_2 = \eta r_1, \underline{r}_1 \leq r_1 \leq \bar{r}_1, \underline{r}_2 \leq r_2 \leq \bar{r}_2\}$. Thus, in the perfect complements case, any improvements in test design decreases both the false negativity and false positivity of the test.

To illustrate this concept, consider an experimental setup where r_1 and r_2 function as substitutes. Imagine a scenario in which researchers control the temperature of a chemical reaction. At higher temperatures, the reaction is more likely to occur, independent of the

technology's precision. As a result, the experiment yields a high number of true positives but also an increased number of false positives. Conversely, at lower temperatures, both true and false negatives become more frequent. In this setting, r_1 and r_2 move in opposite directions, demonstrating a trade-off between the two.

A setting where false positivity and false negativity are complements could be one where the signal strength of an MRI scanner used for detecting tumors is enhanced. At a higher signal strength, both false positives and false negatives would likely be reduced, meaning that r_1 and r_2 would positively covary with one another.

I am interested in situations where, absent any additional costs or frictions, it is socially desirable to run an experiment to test conditions such that a profitable venture can be realized in case of favorable test results, i.e., $\exists(r_1, r_2) \in \rho$ such that $\pi_{r_1, r_2} > 0$.

Given the above analysis, a necessary condition for this to be the case is that:

$$p(1 - \underline{r}_1)R - [p(1 - \underline{r}_1) + (1 - p)\underline{r}_2]K - C > 0 \quad (8)$$

That is under the most informative test with $r_1 = \underline{r}_1$ and $r_2 = \underline{r}_2$, it must be the case that running the experiment and developing the technology when the experiment is successful yields a positive net present value. I shall assume that condition 8 holds strictly.

The Contracting Problem While the GP is solely concerned with the monetary payoff from their investment, the entrepreneur derives non-pecuniary utility Z from working on the venture. The entrepreneur has no money and requires funding from the investor to carry out the experiment. I also assume that the investor can only use equity shares (in a successful venture) to incentivize the entrepreneur. The outside option for both the investor and the entrepreneur is zero, and there is no discounting.

Suppose that the investor and entrepreneur agree on a contract which commits the investor to pay C for the experiment and the entrepreneur to undertake the experiment. The contract may also specify an ownership stake ω for the GP in the venture if the

venture proceeds, with an ownership stake $(1 - \omega)$ for the entrepreneur. However, the contract cannot specify the design (r_1, r_2) of the experiment because this is not describable.

To satisfy the GP's participation constraint, a necessary condition is:

$$\omega R \geq K$$

With that, the first question I want to answer is: Do the entrepreneur and the investor have conflicting objectives regarding the test design?

For any given $\omega \in [\frac{K}{R}, 1]$, the GP's utility function from an experiment characterized by r_1 and r_2 is given by:

$$U_{GP} = p(1 - r_1)\omega R - [p(1 - r_1) + (1 - p)r_2] K - C \quad (9)$$

Likewise, the entrepreneur's utility function if she pursue the venture with an experiment characterized by r_1 and r_2 is:

$$U_E = p(1 - r_1)(1 - \omega)R + [p(1 - r_1) + (1 - p)r_2] Z \quad (10)$$

2.1.5 Solving the Contracting Problem: Choice of Experiment

What experimental design will the entrepreneur choose under a contract that provides funding C to run an experiment and grants the investor an ownership stake $\omega \in [\frac{K}{R}, 1]$ (a typical structure observed in venture capital financing)? I begin my analysis of the entrepreneur's test design problem by first considering the case of substitute tasks.

Substitute Tasks The entrepreneur's test set is $\rho = \{(r_1, r_2) \mid 0 \leq r_i \leq \kappa, \text{ and } r_1 + r_2 = \kappa\}$, with the condition $\underline{r}_1 + \underline{r}_2 > \kappa$. Suppose again that the venture proceeds if the technology successfully passes the test in the lab.

For any given ω , the entrepreneur's optimal experiment design problem is given by:

$$\max_{r_1} (1 - r_1)p[(1 - \omega)R + Z] + (1 - p)(\min(\kappa - r_1, \bar{r}_2)Z + Z).$$

Since the entrepreneur always minimizes r_1 , I have

$$r_1 = \underline{r}_1, \quad \text{and} \quad r_2 = \min(\kappa - \underline{r}_1, \bar{r}_2),$$

regardless of ω .

Understanding this, the GP will not issue equity shares, implying $\omega = 1$. The GP's utility function, conditional on funding the experiment with r_1 and $r_2 = \kappa - r_1$, is given by:

$$U_I = p(1 - r_1)R - [p(1 - r_1) + (1 - p)\min(\kappa - r_1, \bar{r}_2)]K - C.$$

In this case, the GP's preferred experiment depends on whether the substitution constraint is binding at the corner solution. Specifically,

Proposition 1. *With substitute tasks, for all ω , the entrepreneur's optimal experiment satisfies:*

$$r_1 = \underline{r}_1, \quad r_2 = \min(\kappa - \underline{r}_1, \bar{r}_2),$$

while the GP's preferred experiment varies:

- *If $\kappa - \underline{r}_1 \geq \bar{r}_2$, the GP's preferred experiment aligns with that of the entrepreneur, i.e.,*

$$r_1 = \underline{r}_1, \quad r_2 = \bar{r}_2.$$

- *If $\kappa - \underline{r}_1 < \bar{r}_2$, the GP's preferred experiment differs significantly from that of the entrepreneur, i.e.,*

$$r_1 = \bar{r}_1, \quad r_2 = \kappa - \underline{r}_1.$$

Proof: Basic algebra. ■

Intuition: the GP and the entrepreneur are fundamentally aligned on the parameter choice of false negativity. Low false negativity increases both the likelihood of passing the test and the expected payoff. However, they are misaligned on the parameter choice of false positivity. Lower false positivity enhances the value of the experiment by reducing the false positive rate and decreasing the likelihood of passing the test. Thus, choosing the highest probability of a pass signal is not costly if false positivity is sufficiently low. In such cases, decreasing false negativity does not increase false positivity. However, if decreasing false negativity increases false positivity, the GP would prefer a less false positive test to filter out low-quality projects. The entrepreneur, on the other hand, always prefers a more false positive test.

Complementary Tasks The entrepreneur's test set is now given by

$$\rho = \{(r_1, r_2) \mid r_2 = \underline{r}_2 + \eta r_1, \underline{r}_1 \leq r_1 \leq \bar{r}_1, \underline{r}_2 \leq r_2 \leq \bar{r}_2\},$$

where $\eta \in (0, 1)$. For simplicity, suppose that $\underline{r}_2 + \eta r_1 \leq \bar{r}_2$. If the venture proceeds conditional on the technology successfully passing the test in the lab, the entrepreneur's optimal experiment design problem for a given ω is:

$$\max_{r_1} (1 - r_1)p[(1 - \omega)R + Z] + (1 - p)Z(\underline{r}_2 + \eta r_1) + Z.$$

Differentiating with respect to r_1 , the entrepreneur minimizes r_1 (and therefore also r_2 as much as possible) if and only if

$$p[(1 - \omega)R + Z] - (1 - p)\eta Z > 0. \tag{11}$$

When $(1 - \omega) = 0$, this condition reduces to

$$pZ - (1 - p)\eta Z > 0.$$

If $p < \frac{1}{2}$ and η is close to 1, this condition is violated. Hence, the entrepreneur is willing to choose a minimally false positive (and false negative) test only if she has sufficient skin in the game.

Proposition 2. *When (r_1, r_2) are complementary tasks, the entrepreneur's optimal experiment sets*

$$r_1 = \underline{r}_1, \quad r_2 = \underline{r}_2 + \eta \bar{r}_1$$

if and only if the entrepreneur has sufficient skin in the game such that

$$1 - \omega \geq \frac{Z(\eta(1 - p) - p)}{pR}. \quad (12)$$

Proof: See the discussion above. ■

It follows from condition 12 that the more the entrepreneur values conducting research (i.e., the higher Z), the greater the financial incentives required to design a more conclusive test—where $r_1 = \underline{r}_1$, and $r_2 = \underline{r}_2 + \eta \bar{r}_1$. A more conclusive test may reveal that the technology under study is a dead end, at least for the application of interest to the GP. This could jeopardize future funding opportunities for further research. Consequently, under weak financial incentives, the entrepreneur prefers to design a test that is more likely to pass, ensuring the continuation of their scientific endeavors, rather than risk failure with a more stringent (but more conclusive) test. Similarly, the more conclusive the test (as measured by a lower η), the greater the financial compensation required to incentivize the entrepreneur.

However, financing the experiment may still be profitable for the GP under certain parameter values when the tasks are complementary, whereas such an investment would

not be justified if r_1 and r_2 were substitute tasks.

Proposition 3. *When (r_1, r_2) are complementary tasks, the GP is willing to fund the experiment if*

$$(1 - \underline{r}_1)(pR - (Z - K)(\eta(1 - p) - p)) \geq [(1 - p)\underline{r}_2]K + C. \quad (13)$$

Proof: The investor must provide a share of the final value of the venture $(1 - \omega)$ to the entrepreneur such that condition 12 holds. Assuming this condition is binding and substituting for ω , I obtain condition , ensuring that the GP at least breaks even in expectation. ■

2.1.6 Paying for Failure

One reason for the deep tech market failure is the inadequate financial compensation structure for entrepreneurs. Essentially, entrepreneurs are rewarded when the venture is successfully implemented. They receive a share of the realized value if the technology succeeds, and they face no downside risk, as they do not invest their own money into the venture. Additionally, entrepreneurs gain private benefits from conducting scientific work and are further rewarded with more private benefits if they can continue their research. In contrast, the GP bear all the downside risk and face a potentially risky upside, especially if lab results are inconclusive.

So, how can entrepreneurs be provided with better incentives to design more conclusive experiments? I demonstrate that entrepreneurs can be motivated to choose the most definitive experiment designs (i.e., $r_1 = \underline{r}_1$ and $r_2 = \underline{r}_2$) if they are compensated for a failed test, rather than only for passing a lab test.

Substitute Tasks When tasks are perfect substitutes, it is impossible to incentivize the entrepreneur to minimize both r_1 and r_2 . It is either one or the other. This is the typical multitask moral hazard problem.

As Holmstrom and Milgrom (1991) have argued, providing low-powered incentives to the entrepreneur may be the best solution for the investor. If $\bar{r}_1$ is low enough, it may indeed be more beneficial for the GP not to give any skin in the game to the entrepreneur but instead to reward the entrepreneur for proof of failure. This approach would incentivize the entrepreneur to minimize the false positivity of the experiment (focusing less on improving the rate of true positives).

Proposition 4. *When (r_1, r_2) are substitute tasks, the GP is willing to fund an experiment with a reward for proof of failure $X = Z$ if*

$$p[(1 - \bar{r}_1)(R - K) - \bar{r}_1 Z] - (1 - p)[(1 - \underline{r}_2)Z + \underline{r}_2 K] - C \geq 0 \quad (14)$$

Proof: When condition 14 holds, the VC at least breaks even in expectation when funding a conclusive test that rewards the entrepreneur for proof of failure and minimizes the false positivity of the experiment. ■

Complementary Tasks When tasks are perfect complements, it is possible for the GP to incentivize the entrepreneur to minimize both r_1 and r_2 by either providing sufficient “skin in the game” or by rewarding proof of failure. Providing sufficient skin in the game requires condition 12 to hold, which could be burdensome for the GP. Alternatively, if the VC rewards proof of failure, the entrepreneur has sufficient incentives to minimize both r_1 and r_2 if $X = Z$, which could be cheaper for the GP.

Proposition 5. *When (r_1, r_2) are complementary tasks, it is cheaper to reward the entrepreneur for proof of failure if*

$$p\underline{r}_1 + (1 - p_0)(\underline{r}_2 + \eta\bar{r}_1) \leq \eta(1 - p) - p \quad (15)$$

Proof: When $X = Z$, the entrepreneur is indifferent between any $(r_1, r_2) \in \rho$, as regardless of the outcome of the experiment, the entrepreneur receives Z . If the

technology passes the test, the entrepreneur can continue doing science and receives Z in kind. If the technology fails the test, the entrepreneur is rewarded financially with Z for proof of failure.

When indifferent, the entrepreneur can be assumed to choose the test design that benefits the GP the most. Under this test design, the GP pays the entrepreneur Z with probability $p\underline{r}_1 + (1 - p_0)(\underline{r}_2 + \eta\bar{r}_1)$.

If the GP instead provides skin in the game incentives, the GP must grant the entrepreneur a share of the value of the venture:

$$1 - \omega = \frac{Z(\eta(1 - p) - p)}{pR} \quad (16)$$

which is worth ex-ante:

$$(1 - \omega)pR = pR \left[\frac{Z(\eta(1 - p) - p)}{pR} \right] = Z(\eta(1 - p) - p) \quad (17)$$

It is straightforward to verify that when condition 15 holds, this approach is more expensive for the VC. ■

2.2 The Contract between GP and LP

2.2.1 Players and Timing of Events

Now, I study the exploration versus exploitation trade-off of a private equity firm raising capital for consecutive funds. In the baseline model, a single private equity firm (GP) can raise capital from a single investor (LP). There are two periods. At time $t = 0$, the LP supplies 1 unit of capital to the GP for fund 1 investments. At time $t = 1$, the GP liquidates fund 1 and raises capital for fund 2. At time $t = 2$, fund 2 is liquidated. The timing and sequence of events are as follows:

- At $t = 0$, the GP chooses a fraction α of capital to allocate to a new market

(exploration) and a fraction $1 - \alpha$ to allocate to a known market (exploitation).

- The return from fund 1 is realized, and based on these returns, fund 2 is raised.
- The LP decides the total amount to invest in fund 2, and the GP again allocates capital between exploration and exploitation.

2.2.2 Technology

The private equity firm can allocate each fund's capital to exploit a well-known industry or explore a new market. By investing a fraction of fund 1's capital in the new market, the private equity firm improves its ability to manage investments in this new market, thereby increasing its chance of success in the new market. The probability of success for investment in the new market in fund 2 will be positively related to the fraction of capital invested in the new market in fund 1, representing a form of learning by doing.

For fund 1, I assume that the gross utility per unit of capital invested in new and core follows a distribution with a mean of U_N and U_C respectively; however. The ex-ante probability of success in the new industry through fund 1 is denoted by $\mathbb{E}(P_1) = p$, which is encoded in U_N , and I assume $U_N < U_C$. This assumption specifies that, ex-ante, the expected return for fund 1 in the known industry is greater than the expected return in the exploratory industry. In fund 1, the VC invests a fraction α of the total capital in the new industry. Hence, the total return to fund 1 is given by:

$$\alpha U_N + (1 - \alpha) U_C.$$

I assume that as the VC invests in the new industry through Fund 1, it gains expertise in due diligence within that sector. Consequently, the probability of success in the new industry for Fund 2 depends on the share α of capital invested in the new industry during Fund 1, which I model using a learning function $p_2(\alpha)$.

I assume $p_2(0) = p$, so that if the firm makes no initial investment in the new industry,

the probability of success in Fund 2 remains equal to the ex-ante baseline. To ensure an interior solution in the firm's dynamic optimization problem, I assume that $p_2(\alpha)$ is strictly increasing and concave, with $p'_2(\alpha) > 0$ and $p_2(\alpha) \cdot p'_2(\alpha)$ convex. These properties capture diminishing learning returns and ensure tractability.

Empirically, Figure 2 shows a clear concave relationship between the share of Fund 1 capital invested in the new industry and the subsequent success rate (demeaned by industry-year) for Fund 2 among firms that shifted focus. This pattern supports a learning mechanism with decreasing marginal gains and motivates my choice of the following functional form:

$$p_2(\alpha) = p + (1 - p) (1 - e^{-k\alpha}) , \quad (18)$$

where k captures the intensity of learning. This form has several advantages: it increases smoothly in α , is bounded between p and 1, and maps well onto the empirical evidence on industry-specific performance.

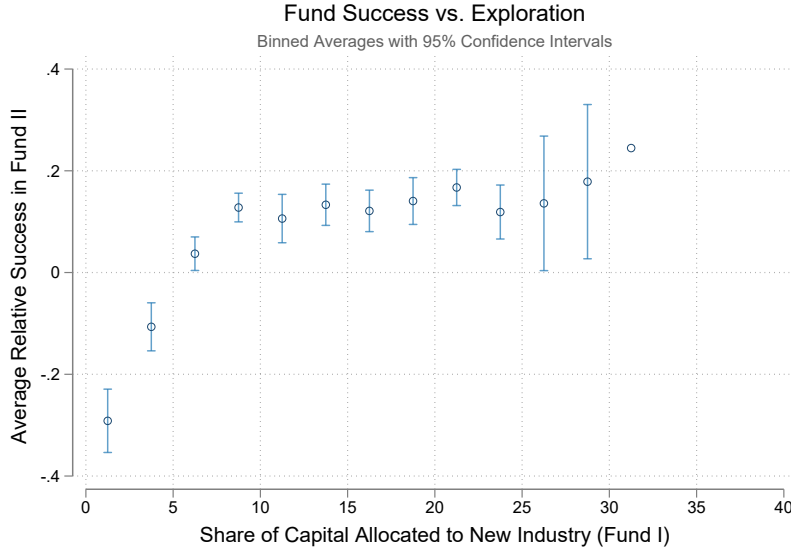


Figure 2. Success rate (demeaned by industry-year) in Fund 2 vs. share invested in the same industry in Fund 1. Data is binned with 95% confidence intervals.

Let the fraction of fund 2's capital allocated to the exploratory industry be θ and the fraction allocated to the known industry be $1 - \theta$. If the limited partner (LP) allocates

one unit of capital to fund 1 at time 0 and I_1 units of capital to fund 2 at time 1, the total expected return on investment across the two funds is given by:

$$(\alpha U_N + (1 - \alpha)U_C) + I_1(\theta U_N^2 + (1 - \theta)U_C). \quad (19)$$

Finally, I assume a convex cost of managing a fund of size I given by $\gamma \frac{I^2}{2}$, where γ is the marginal cost of managing capital.

2.2.3 First Best

From expression 19, fund 2 investment has a bang-bang solution. If $U_N^2 > U_C$, then $\theta = 1$, and if $U_N^2 < U_C$, then $\theta = 0$ and $\alpha = 0$. I focus on the case where $\theta = 1$ and subsequently derive parameter conditions ensuring that $U_N^2 > U_C$. For $\theta = 1$, the social surplus is given by:

$$W(I_S, I_F, \alpha) = (\alpha U_N + (1 - \alpha)U_C) + p[U_N^2 - \gamma \frac{(I_S)^2}{2}] + (1 - p)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \quad (20)$$

Substitute Tasks In the first-best scenario, the social surplus is maximized:

$$\max_{I_S, I_F, \alpha} W(I_S, I_F, \alpha) = (\alpha U_N + (1 - \alpha)U_C) + p[U_N^2 - \gamma \frac{(I_S)^2}{2}] + (1 - p)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \quad (21)$$

where:

$$U_N = p[(1 - r_1)(R - K) - r_1 Z] - (1 - p)[(1 + r_1 - \kappa)Z + (\kappa - r_1)K] - C.$$

$$U_N^2 = p_2(\alpha)[(1 - r_1)(RI_S - KI_S) - r_1 Z] - (1 - p_2(\alpha))[(1 + r_1 - \kappa)Z + (\kappa - r_1)KI_S] - C.$$

Taking the first-order conditions yields:

$$\begin{cases} 1. & pp'_2(\alpha) = \frac{U_C - U_N}{I_S[(1-r_1)(R-K) + (\kappa-r_1)K + (1+\kappa)Z]} \\ 2. & I_S = \frac{p_2(\alpha)[(1-r_1)(R-K) - r_1Z] - (1-p_2(\alpha))[(1+r_1-\kappa)Z + (\kappa-r_1)K]}{\gamma} \\ 3. & I_F = \frac{U_C}{\gamma} \end{cases}$$

The first first-order condition states that the opportunity cost of exploration is equal to the investment in Fund 2 in the event of success, multiplied by the ex-ante probability of success and the incremental probability of success after learning.

The second first-order condition asserts that the investment level in Fund 2 in the event of success is equal to the second-round probability of success in the new industry, multiplied by the return on the fund per unit of capital invested, divided by the marginal cost of managing the fund.

The third first-order condition states that the investment in the second round, in the event of failure in the first-round investment in the new market, equals the ratio of the return on the fund per unit of capital invested to the marginal cost of managing the fund.

Proposition 6. *The first-best problem admits an interior solution (α^{NF}, I_S^{NF}) provided the following condition is satisfied:*

$$\gamma < \frac{p^3 k U_N}{1 - p}$$

Proof: See online appendix. ■

Proposition 7. α^{NF} and I_S^{NF} increase in p and decrease in, U_C , k and γ , and the

comparative statistics are given by:

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{kp(1-p)[(1-r_1)(R-K) - r_1Z] - \sqrt{(kp(1-p)[(1-r_1)(R-K) - r_1Z])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)(R-K) + (\kappa - r_1)K + (1+\kappa)Z]} \right]$$

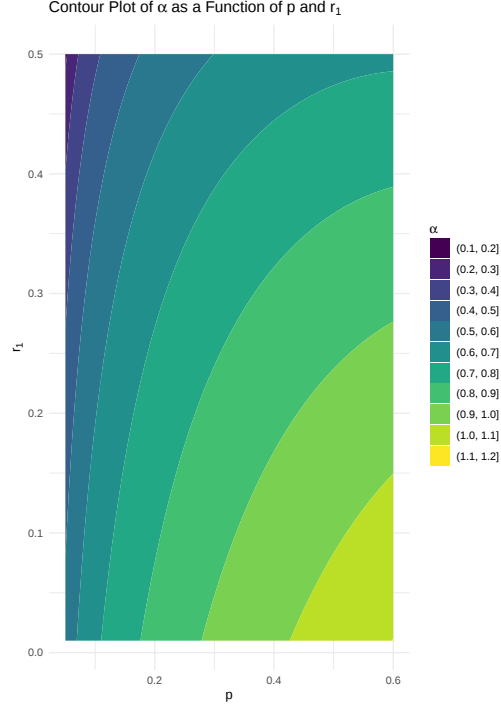
$$I_S^{NF} = \frac{p_2(\alpha^{NF})[(1-r_1)(R-K) - r_1Z] - (1-p_2(\alpha^{NF}))[(1+r_1-\kappa)Z + (\kappa-r_1)K]}{\gamma}$$

Proof: See appendix. ■

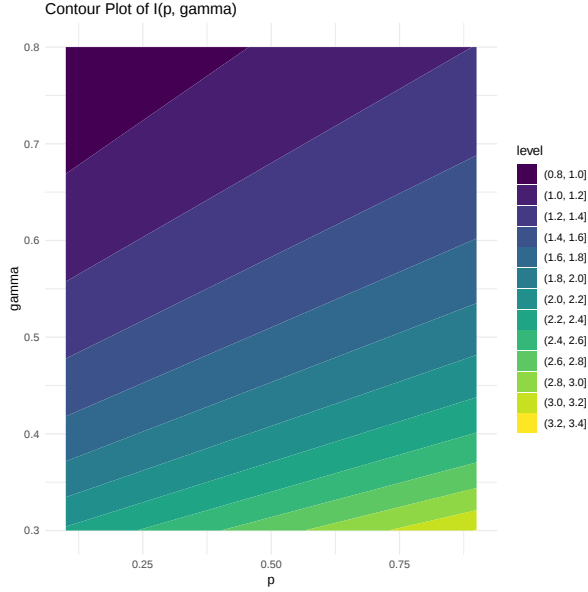
Since p measures how effective the private equity firm is at exploring, a higher p encourages greater exploration in the first period. Similarly, increased exploration in the first period promotes second-period investments because returns in the second period increase with α . The parameter $U_C - U_N$ represents the opportunity cost of exploration. A higher opportunity cost reduces the incentives to explore in the first period, leading to a decrease in α and consequently in I_S through α . The parameter γ denotes the marginal cost of managing a unit of capital. An increase in γ reduces second-period investment (I_S decreases), which in turn lowers the incentive for exploration, thereby decreasing α . The parameter k represents the inverse of learning intensity, meaning that a lower k corresponds to a higher learning rate. A higher learning rate increases the probability of success in the new market through fund 2, thereby enhancing returns. This, in turn, leads to an increase in I_S and consequently in α . At the same time, α decreases in κ , which represents how reliable the tests are in the new industry. These effects are depicted in figure 3.

In the first-best scenario, when there is no agency costs, the first-period return is sacrificed to obtain a high return on the second fund. The total social surplus in the first-best case is:

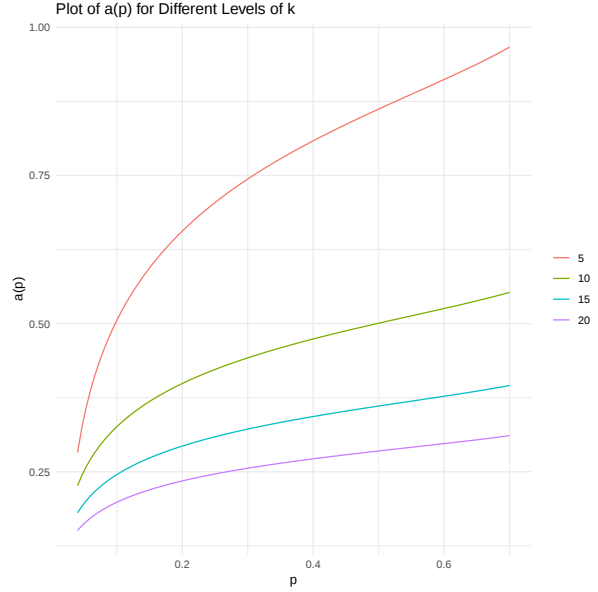
$$W(I_S, I_F, \alpha) = (\alpha U_N + (1-\alpha)U_C) + P_1[U_N^2 - \gamma \frac{(I_S)^2}{2}] + (1-P_1)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \quad (22)$$



(a) α^{NF} with respect to r_1 and p



(b) I_S^{NF} with respect to γ and p



(c) α^{NF} with respect to p for different ks

Figure 3. This figure shows the first-best optimal values of exploration and investment levels with respect to the ex-ante probability of success in exploration, the marginal cost of managing capital, learning intensity, and false negativity rate of test.

Complementary Tasks In the first-best scenario, the social surplus is maximized:

$$\max_{I_S, I_F, \alpha} W(I_S, I_F, \alpha) = (\alpha U_N + (1 - \alpha) U_C) + P_1 [U_N^2 - \gamma \frac{(I_S)^2}{2}] + (1 - P_1) [U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \quad (23)$$

where:

$$U_N = p\omega(1 - r_1)R - [p(1 - r_1) + (1 - p)(r_2 + \eta r_1)] K - C.$$

$$U_N^2 = p_2(\alpha)\omega(1 - r_1)RI_S - [p_2(\alpha)(1 - r_1) + (1 - p_2(\alpha))(r_2 + \eta r_1)] KI_S - C.$$

and, from 12:

$$\omega = 1 - \frac{Z(\eta(1 - p) - p)}{pR}$$

Taking the first-order conditions yields:

$$\begin{cases} 1. & pp'_2(\alpha) = \frac{U_C - U_N}{I_S[(1 - r_1)\omega R + (1 - r_1 + r_2 + \eta r_1)K]} \\ 2. & I_S = \frac{p_2(\alpha)(1 - r_1)\omega R - (1 - p_2(\alpha))(1 - r_1 + r_2 + \eta r_1)K}{\gamma} \\ 3. & I_F = \frac{U_C}{\gamma} \end{cases}$$

The first first-order condition states that the opportunity cost of exploration is equal to the investment in Fund 2 in the event of success, multiplied by the ex-ante probability of success and the incremental probability of success after learning.

The second first-order condition asserts that the investment level in Fund 2 in the event of success is equal to the second-round probability of success in the new industry, multiplied by the return on the fund per unit of capital invested, divided by the marginal cost of managing the fund.

The third first-order condition states that the investment in the second round, in the event of failure in the first-round investment in the new market, equals the ratio of the return on the fund per unit of capital invested to the marginal cost of managing the fund.

Proposition 8. α^{NF} and I_S^{NF} increase in p and decrease in, U_C , k and γ , and the

comparative statistics are given by:

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{kp(1-p)(1-\underline{r}_1)\omega R - \sqrt{(kp(1-p)(1-\underline{r}_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-\underline{r}_1)\omega R + (1-\underline{r}_1 + \underline{r}_2 + \eta \underline{r}_1)K]} \right]$$

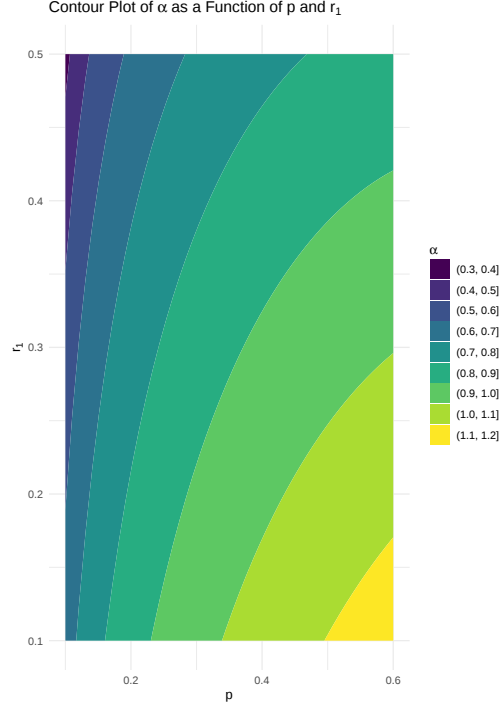
$$I_S^{NF} = \frac{p_2(\alpha^{NF})(1-\underline{r}_1)\omega R - (1-p_2(\alpha^{NF}))(1-\underline{r}_1 + \underline{r}_2 + \eta \underline{r}_1)K}{\gamma}$$

Proof: See appendix. ■

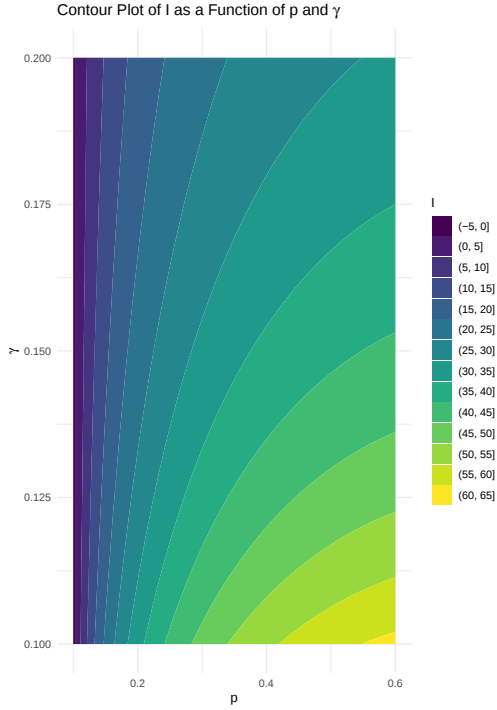
Since p measures how effective the private equity firm is at exploring, a higher p encourages greater exploration in the first period. Similarly, increased exploration in the first period promotes second-period investments because returns in the second period increase with α . The parameter $U_C - U_N$ represents the opportunity cost of exploration. A higher opportunity cost reduces the incentives to explore in the first period, leading to a decrease in α and consequently in I_S through α . The parameter γ denotes the marginal cost of managing a unit of capital. An increase in γ reduces second-period investment (I_S decreases), which in turn lowers the incentive for exploration, thereby decreasing α . The parameter k represents the inverse of learning intensity, meaning that a lower k corresponds to a higher learning rate. A higher learning rate increases the probability of success in the new market through fund 2, thereby enhancing returns. This, in turn, leads to an increase in α and consequently in I_S . At the same time, α decreases in η , which represents how reliable the tests are in the new industry. These effects are depicted in figure 4

In the first-best scenario, when there is no agency costs, the first-period return is sacrificed to obtain a high return on the second fund. The total social surplus in the first-best case is:

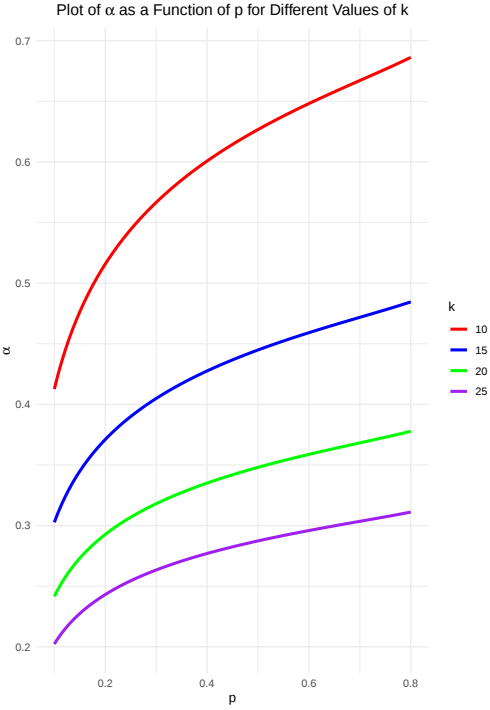
$$W(I_S, I_F, \alpha) = (\alpha U_N + (1-\alpha)U_C) + P_1[U_N^2 - \gamma \frac{(I_S)^2}{2}] + (1-P_1)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \quad (24)$$



(a) α^{NF} with respect to r_1 and p for complementary test



(b) I_S^{NF} with respect to γ and p for complementary test



(c) α^{NF} with respect to p for different k s for complementary test

Figure 4. This figure shows the first-best optimal values of exploration and investment levels with respect to the ex-ante probability of success in exploration, the marginal cost of managing capital, learning intensity, and false negativity rate of test for complementary test case.

2.3 The Problem with Dual-sided Moral Hazard

In this section, I analyze the problem where the limited partners have full bargaining power. A moral hazard issue arises because the general partner may shirk, and this behavior is unobservable to the LP. If the GP exerts effort, they can affect the level of κ in cases where the nature of the experimental test is substitutable, and the level of η when the nature of the experimental test is complementary. In both cases, the GP may reduce κ or η when exerting effort. To fix ideas, one can think of a setting where false positivity and false negativity are substitutes as one where the experimental design involves setting a temperature for experimentation on a particular chemical reaction. It may be that the chemical reaction is more likely to occur at higher temperatures, regardless of the quality of the technology being used. This would imply a high number of true positives but also a high number of false positives at higher temperatures. In such a scenario, the GP might hire a chemist to observe the test and set the ideal temperature for the test, thereby reducing κ and ensuring that the experiment's outcome is more reliable. At the same time, shirking can be interpreted as not hiring the chemist to observe the test and instead enjoying the private benefit of avoiding the cost of observation.

Substitute Tasks In this condition, the GP can exert effort to monitor the experiment. If the GP exerts effort, then $\kappa = \kappa^L$, and if the GP shirks, then $\kappa = \kappa^H$, with $\kappa^L < \kappa^H$. Since $r_1 + r_2 = \kappa$, the experiment yields a lower total value of false negativity and false positivity when the GP exerts effort. Let B denote the private benefit of shirking (interpreted as the costs needed to monitor and ensure that $\kappa = \kappa^L$). In this case, the LP would be better off providing the GP with a transfer of B to incentivize the GP to monitor the experiment, if the condition $(\Delta\kappa)(1-p)K > B$ holds. I assume that this condition is satisfied. To mitigate this issue, the LP must incentivize the GP by offering transfers as a percentage of the returns. Let λ_1 and λ_2^S represent the shares of the return through the first and second periods, respectively, as transfers from the LP to the GP.

The incentive compatibility (IC) conditions are:

$$\begin{cases} 1. & \lambda_2^S U_N^2 \geq B_2 \\ 2. & \lambda_1 U_N \geq B_1 \end{cases}$$

where B denotes the private benefit gained by the GP in the case of shirking, λ_1 denotes the share of payoff allocated to the GP through exploring in period 1, and λ_2^S denotes the share of payoff allocated to the GP through exploring in period 2.

Given that I have assumed the LP has full bargaining power, the LP solves the following optimization problem:

$$\begin{aligned} W(I_S, I_F, \alpha) &= (\alpha U_N + (1 - \alpha) U_C - B_1) \\ &+ p[U_N^2 - \gamma \frac{(I_S)^2}{2} - B_2] \\ &+ (1 - p)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \end{aligned}$$

subject to the following constraints:

$$\begin{cases} 1. & \lambda_2^S U_N^2 \geq B_2 \\ 2. & \lambda_1 U_N \geq B_1 \end{cases}$$

Since the objective function is linear in transfer values, the incentive compatibility constraints binds. Therefore, it holds that:

$$\begin{cases} 1. & \lambda_2^S U_N^2 = B_2 \\ 2. & \lambda_1 U_N = B_1 \end{cases}$$

Plugging in transfers from incentive compatibility constraints into the payoff function gives:

$$\begin{aligned}
W(I_S, I_F, \alpha) &= (\alpha(1 - \frac{B_1}{U_N})U_N + (1 - \alpha)U_C - B_1) \\
&+ p[(1 - \frac{B_2}{U_N^2})U_N^2 - \gamma \frac{(I_S)^2}{2}] \\
&+ (1 - p)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}.
\end{aligned}$$

and the maximization problem becomes:

$$\max_{I_S, I_F, \alpha} W(I_S, I_F, \alpha)$$

The first-order conditions for a maximum (with second-order conditions satisfied) are as follows:

$$\begin{cases}
1. & pp'_2(\alpha) = \frac{U_C - (1 - \frac{B}{U_N})U_N}{(1 - \frac{B}{U_N})I_S[(1 - r_1)(R - K) + (\kappa - r_1)K + (1 + \kappa)Z]} \\
2. & I_S = (1 - \frac{B}{U_N}) \frac{p_2(\alpha)[(1 - r_1)(R - K) - r_1Z] - (1 - p_2(\alpha))[(1 + r_1 - \kappa)Z + (\kappa - r_1)K]}{\gamma} \\
3. & I_F = \frac{U_C}{\gamma}
\end{cases}$$

Proposition 9. α^* and I_S^* both decrease in the severity of moral hazard (B), and the comparative statistics are:

$$\begin{aligned}
\alpha^* &= -\frac{1}{k} \ln \left[\frac{kp\lambda(1 - p)[(1 - r_1)(R - K) - r_1Z] - \sqrt{(kp\lambda(1 - p)[(1 - r_1)(R - K) - r_1Z])^2 - 4\gamma kp(1 - p)^2(U_C - \lambda U_N)}}{2kp\lambda(1 - p)^2[(1 - r_1)(R - K) + (\kappa - r_1)K + (1 + \kappa)Z]} \right] \\
I_S^* &= \frac{p_2(\alpha^*)\lambda[(1 - r_1)(R - K) - r_1Z] - (1 - p_2(\alpha^*))\lambda[(1 + r_1 - \kappa)Z + (\kappa - r_1)K]}{\gamma}
\end{aligned}$$

where $\lambda = (1 - \frac{B}{U_N})$.

Proof: See appendix. ■

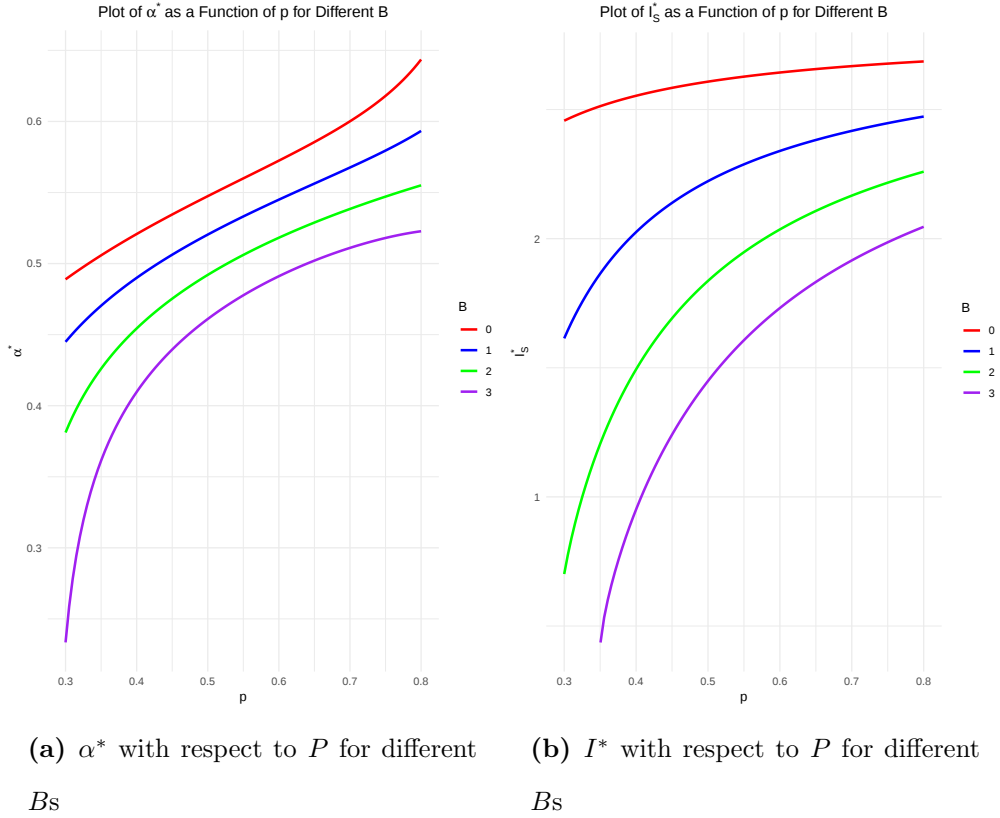


Figure 5. This figure shows the optimal values of exploration with respect to the ex-ante probability of success in exploration, as well as varying levels of moral hazard severity. Moral hazard issues impede exploration and financing innovation of deep tech when tasks are substitute.

The model predicts that PEs experiencing greater moral hazard between GP and LP will allocate a lower portion of their first fund's capital outside their core industry. Additionally, these PEs will have a lower ratio of investment in subsequent funds.

As shown in Figure 6, the effect is economically significant. Figure 6 indicates that a 1% increase in the moral hazard level—measured as a one-percentage-point increase in private benefits relative to the fund's total payoff—reduces exploration by approximately 12.5%.

Complementary Tasks In this condition, the GP can exert effort to monitor the experiment. If the GP exerts effort, then $\eta = \eta^L$, and if the GP shirks, then $\eta = \eta^H$, with $\eta^L < \eta^H$. Since $r_2 = \underline{r}_2 + \eta \bar{r}_1$, the experiment yields a lower false positivity when the GP exerts effort. Let B denote the private benefit of shirking (interpreted as the costs

needed to monitor and ensure that $\eta = \eta^L$). In this case, the LP would be better off providing the GP with a transfer of B to incentivize the GP to monitor the experiment, if the condition $(\Delta\eta)(1-p)r_1K > B$ holds. I assume that this condition is satisfied. To mitigate this issue, the LP must incentivize the GP by offering transfers as a percentage of the returns. Let λ_1 and λ_2^S represent the shares of the return through the first and second periods, respectively, as transfers from the LP to the GP. The incentive compatibility (IC) conditions are:

$$\begin{cases} 1. & \lambda_2^S U_N^2 \geq B_2 \\ 2. & \lambda_1 U_N \geq B_1 \end{cases}$$

where B denotes the private benefit gained by the GP in the case of shirking, λ_1 denotes the share of payoff allocated to the GP through exploring in period 1, and λ_2^S denotes the share of payoff allocated to the GP through exploring in period 2.

Given that I have assumed the LP has full bargaining power, the LP solves the following optimization problem:

$$\begin{aligned} W(I_S, I_F, \alpha) &= (\alpha U_N + (1-\alpha)U_C - B_1) \\ &+ p[U_N^2 - \gamma \frac{(I_S)^2}{2} - B_2] \\ &+ (1-p)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \end{aligned}$$

subject to the following constraints:

$$\begin{cases} 1. & \lambda_2^S U_N^2 \geq B_2 \\ 2. & \lambda_1 U_N \geq B_1 \end{cases}$$

Since the objective function is linear in transfer values, the incentive compatibility

constraints binds. Therefore, it holds that:

$$\begin{cases} 1. & \lambda_2^S U_N^2 = B_2 \\ 2. & \lambda_1 U_N = B_1 \end{cases}$$

Plugging in transfers from incentive compatibility constraints into the payoff function gives:

$$\begin{aligned} W(I_S, I_F, \alpha) &= (\alpha(1 - \frac{B_1}{U_N})U_N + (1 - \alpha)U_C - B_1) \\ &+ p[(1 - \frac{B_2}{U_N^2})U_N^2 - \gamma \frac{(I_S)^2}{2}] \\ &+ (1 - p)[U_C - \gamma \frac{(I_F)^2}{2}] - \frac{\gamma}{2}. \end{aligned}$$

and the maximization problem becomes:

$$\max_{I_S, I_F, \alpha} W(I_S, I_F, \alpha)$$

The first-order conditions for a maximum (with second-order conditions satisfied) are as follows:

$$\begin{cases} 1. & pp'_2(\alpha) = \frac{U_C - (1 - \frac{B}{U_N})U_N}{(1 - \frac{B}{U_N})I_S[(1 - r_1)(R - K) + (\kappa - r_1)K]} \\ 2. & I_S = (1 - \frac{B}{U_N}) \frac{p_2(\alpha)(1 - r_1)(R - K) - (1 - p_2(\alpha))(\kappa - r_1)K}{\gamma} \\ 3. & I_F = \frac{U_C}{\gamma} \end{cases}$$

Proposition 10. α^* and I_S^* both decrease in the severity of moral hazard (B), and the

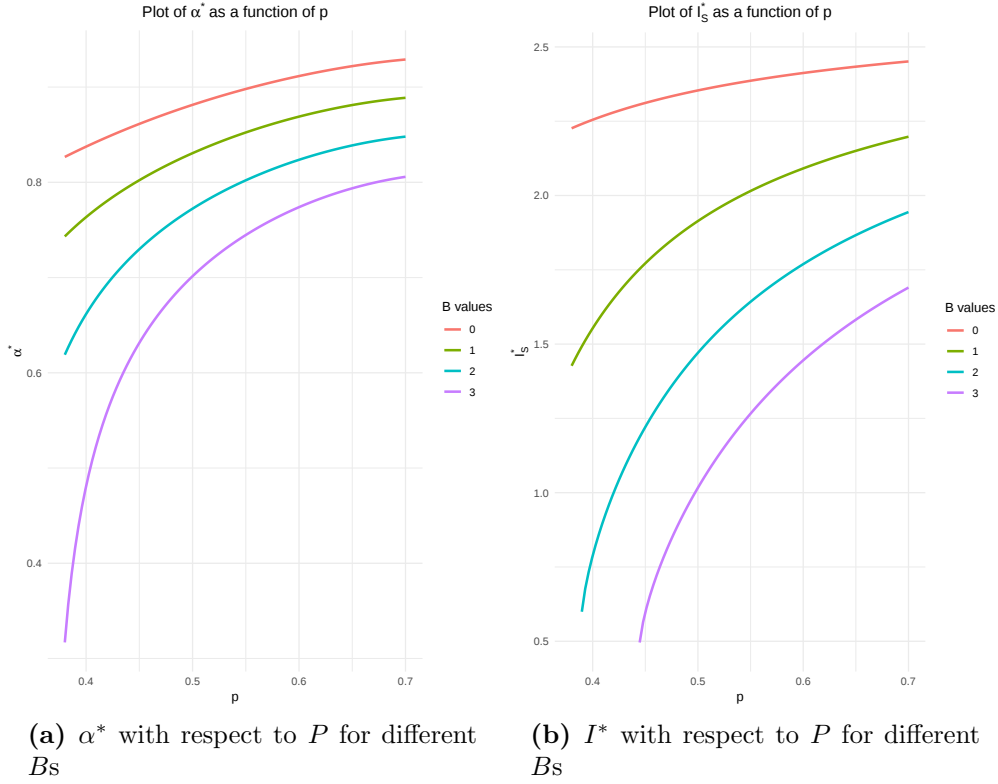


Figure 6. This figure shows the optimal values of exploration with respect to the ex-ante probability of success in exploration, as well as varying levels of moral hazard severity. Moral hazard issues impede exploration and financing innovation of deep tech when tasks are complimentary.

comparative statistics are:

$$\alpha^* = -\frac{1}{k} \ln \left[\frac{kp\lambda(1-p)(1-r_1)\omega R - \sqrt{(kp\lambda(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - \lambda U_N)}}{2kp\lambda(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]} \right]$$

$$I_S^* = \frac{p_2(\alpha^*)(1-r_1)\omega\lambda R - (1-p_2(\alpha^*))(1-r_1+r_2+\eta r_1)\lambda K}{\gamma}$$

where $\lambda = (1 - \frac{B}{U_N})$.

Proof: See appendix. ■

The model predicts that PEs experiencing greater moral hazard between GP and LP will allocate a lower portion of their first fund's capital outside their core industry. Additionally, these PEs will have a lower ratio of investment in subsequent funds.

As shown in Figure 6, the effect is economically significant. Figure 6 indicates that a

1% increase in the moral hazard level—measured as a one-percentage-point increase in private benefits relative to the fund’s total payoff—reduces exploration by approximately 12.5%.

3 Summary of findings and main predictions

This paper develops a dynamic agency model to explore the trade-off between exploration and exploitation for private equity firms managing successive funds. The model examines how a PE firm allocates capital either to a familiar industry within its core expertise (exploitation) or to a new, unfamiliar industry (exploration). Central to the model is the concept of learning-by-doing: by allocating part of its first fund’s capital to new markets, the PE firm gains experience, enhancing its ability to screen, select, and manage investments more effectively. This learning improves the probability of success in the firm’s second fund, enabling better decision-making in subsequent investments. Furthermore, the model accounts for dual-sided moral hazard problem: agency frictions between general partners and limited partners, as well as between GPs and entrepreneurs, which influence the firm’s capital allocation decisions and its ability to fully capitalize on exploration opportunities. The presence of these moral hazard issues reduces the incentives for exploration and limits the firm’s ability to diversify effectively, especially when the firm faces high opportunity costs in exploring new industries.

3.1 Frictionless Benchmark

In the absence of frictions, the cross-industry capital allocation for both funds and the investment levels by limited partners are determined by three key parameters:

- **Opportunity cost of exploration** ($U_C - U_N$): The difference in the likelihood of investment success between the core industry and the new market.

- **Learning intensity** (k): The firm's ability to improve investment management in the new industry through experience.
- **Marginal cost of capital management** (γ): The cost incurred per unit of capital managed.

In the first-best scenario, firms with a very high opportunity cost of exploration ($U_C - U_N$) optimally avoid new industries, allocating all capital to their core sector across both funds. Firms with moderate or low opportunity costs choose to explore in the first fund, with exploration increasing as learning intensity ($1/k$) rises and as γ decline.

3.2 Impact of Moral Hazard

When moral hazard exists between GPs and LPs, agency frictions cause the PE to deviate from the first-best investment decisions, limiting PEs' willingness to invest in new industries where they could gain valuable experience through learning and realize higher future returns. This deviation has a significant welfare impact and can hinder entrepreneurship and innovation in society.

3.3 Empirical Predictions

The model yields testable predictions regarding the cross-sectional behavior of PE firms that explore non-core industries for the first time:

- **Prediction 1:** PE firms facing severe moral hazard allocate a smaller fraction of their first fund's capital outside their core industry.
- **Prediction 2:** In firms with high moral hazard severity, the ratio of fund 2 to fund 1 investment by LPs is lower.

These findings provide insights into the strategic allocation decisions of PE firms and the role of agency frictions in shaping investment behavior.

4 Data and Empirical Results

In this section, I provide tests for the main empirical implications of the model as outlined in section 3.3. Section 4.1, describes the data used. Section 4.2, describes the construction of the empirical proxies for the theoretical variables. Section 4.3.2 presents the main empirical results. The main results and tests for the model are presented in Tables 4 and 5.

4.1 Data Sources

To test the predictions of the model, I use Pitchbook’s venture capital deal, exit, and fund-level data. I begin by aggregating the number of investments per industry classification for each fund using deal-level data. The core industry of a fund is defined as the industry in which it has made the majority of its investments. Table 1 summarizes the distribution of funds across industries.

Next, I integrate exit data at both the investment and fund levels to calculate the number of IPOs and merger exits per fund for each industry. I then match the deal-level data with fund-level data to extract key fund characteristics, including fund sequence, vintage year, size, and fund manager details.

Since the model primarily focuses on first and second funds, I filter the dataset to retain only these fund sequences. To ensure accurate matching, I link second funds within the same strategy to their predecessor using the firm ID and fund name. The final sample consists of funds with vintages between 1995 and 2023. To ensure a representative investment sample, I include only funds with at least five registered investments.

The final dataset comprises 1790 matched first and second funds. The industries covered, along with the number of funds in each, are detailed in Table 1.

Table 1
Number of Funds by Industry in Final Sample

Core Industry	First-time Funds	Second-time Funds
Real Estate	2	4
Business Services	11	7
Raw Materials and Natural Resources	8	7
Energy and Utilities	27	19
Telecom and Media	17	23
Financial and Insurance Services	35	39
Industrials	30	43
Consumer Discretionary	107	124
Healthcare	328	335
Information Technology	1225	1244

4.2 Construction of Key Variables

To empirically test the main predictions of the model, I construct proxies for key variables: $U_C - U_N$, representing the opportunity cost of exploration for Fund 1; B , the severity of moral hazard between GP and LP; I_S , the proportion of invested capital in Fund 2 relative to Fund 1; and k , the learning intensity. This section outlines the methodology used to derive empirical proxies for these variables, ensuring alignment with the theoretical framework.

4.2.1 Opportunity Cost of Exploration

The venture capital literature commonly measures a fund’s success by the number of IPO and merger exits among its portfolio companies (e.g., Sorensen (2008)). To construct a proxy for $U_C - U_N$ (the opportunity cost of exploration), I estimate a fund’s industry-specific exit performance as follows.

For each fund j with vintage year t , I compute the number of investments and successful exits (IPOs or mergers) in each industry i . The successful exit rate for fund j in industry i at time t is given by:

$$s_{ijt} = \frac{S_{ijt}}{N_{ijt}} \quad (25)$$

where S_{ijt} is the number of successful exits (IPOs or mergers) and N_{ijt} is the total number of investments in industry i by fund j .

Since IPO and merger rates vary across industries and business cycles, I normalize the successful exit rate by demeaning it relative to the average exit rate of all funds with vintage year t that invested in the same industry i , defined as:

$$\bar{s}_{it} = \frac{\sum_j s_{ijt}}{\mathcal{N}_t} \quad (26)$$

where $\mathcal{N}_t$ represents the number of funds with vintage year t .

Thus, the industry-specific skill of fund j in industry i is given by:

$$\nu_{ijt} = s_{ijt} - \bar{s}_{it} \quad (27)$$

I define the core industry of a fund j in vintage year t as the industry where the fund made the highest number of investments. The fund's skill in its core industry is denoted as ν_{jt}^C .

To measure the fund's performance outside its core industry, I sum ν_{ijt} across all industries except the core industry:

$$\nu_{jt}^N = \sum_{i \neq C} \nu_{ijt} \quad (28)$$

Finally, I define the opportunity cost of exploration as the difference between the fund's skill in its core industry and its aggregated skill in non-core industries:

$$\zeta_{jT} = \nu_{jt}^C - \nu_{jt}^N \quad (29)$$

This measure captures how much exploration outside the core industry comes at the

expense of leveraging existing expertise.

4.2.2 The Investment Ratio

I compute the investment ratio by dividing the total investment of fund 2 to the total investment of fund 1.

4.2.3 Learning intensity

To estimate the learning intensity parameter k from the data, I first calculate $\nu_{jt_1}^N$, which represents the success rate for Fund 1, and $\nu_{jt_2}^N$, the success rate for Fund 2. I then back out k using the following equation (Equation 18):

$$\nu_{jt_2}^N = \nu_{jt_1}^N + [1 - \nu_{jt_1}^N] (1 - e^{-k\alpha})$$

This equation captures the relationship between the success rates of the two funds and allows for the estimation of k , the learning intensity parameter.

4.2.4 Moral Hazard Proxy

The cost associated with exerting effort to influence parameters such as κ and η (as discussed in Section 2.1.5) is not directly observable in the data. To proxy for the moral hazard problem, I focus on the time difference between the launch of Fund 1 and Fund 2. This time lag can provide the GP with an opportunity to misreport the Net Asset Value (NAV), which is easier to manipulate during this interval.

Typically, when a subsequent fund is launched, the private equity firm has not yet liquidated all the investments from the prior fund. As a result, at the time of raising capital for a follow-up fund, LPs must rely on NAV estimates for the non-exited investments. These NAV estimates may be imprecise or strategically adjusted upwards to improve the chances of securing commitments. The gap between reported NAV and eventual realized values creates a window for GP misreporting, introducing the potential for diversion.

To empirically proxy for this diversion potential (i.e., the empirical counterpart of B), I define a Moral Hazard Dummy that equals 1 if the vintage-year gap between Fund 1 and Fund 2 is less than 3 years, and 0 otherwise. A shorter gap implies that fewer Fund 1 investments have been realized at the time Fund 2 is raised, thereby increasing the scope for NAV manipulation and, hence, moral hazard.

However, the timing of fundraising may itself be endogenous. For instance, more reputable GPs may be able to raise capital faster, and these same reputational factors may also affect investment allocation behavior in Fund 1. To address this potential endogeneity, I implement an instrumental variable approach.

I use the total dollar amount of venture capital raised in the year following Fund 1’s vintage year—denoted by VC Capital_f —as an instrument for the MH Dummy:

$$\text{VC Capital}_f = \text{Total dollar amount of VC funds raised in year } t_f + 1$$

This instrument captures fundraising congestion or LP capital saturation in the period following Fund 1. When the overall VC capital raised in $t_f + 1$ is high, LPs are likely over-committed, reducing the availability of capital for new fund launches in that year. As a result, GPs may face delays in launching Fund 2. This introduces exogenous variation in the time between Fund 1 and Fund 2 fundraising that is not driven by GP-specific traits.

This instrument satisfies the two key IV conditions:

- **Relevance:** High VC capital raised in $t_f + 1$ reduces the probability that a GP can raise Fund 2 quickly. This is testable via the first-stage regression.
- **Exclusion Restriction:** VC capital raised in year $t_f + 1$ should not directly affect the share of Fund 1 investments allocated outside the core industry, which were largely determined in year t_f . It affects exploration only through its influence on the GP’s moral hazard incentives.

4.3 Empirical Results

In this section, I present the summary statistics of my sample and outline tests for the key implications of my model. To assess the model’s predictions, I classify private equity firms into *shifters* and *non-shifters* based on their investment focus across consecutive funds. shifters are firms that shift their core industry focus between their first and second funds. For instance, a firm that primarily invested in the Healthcare sector in its first fund but allocated the majority of its second fund to Information Technology would be considered a shifter.

As highlighted in the theoretical section, the model’s key predictions pertain to cross-sectional variations in investment patterns and industry allocation dynamics within the shifters sample.

4.3.1 Summary Statistic

Table 2 provide summary statistics for the key variables in two groups: the full sample of first-time funds and the subset of first-time funds within the shifters sample. Funds in the shifter sample tend to be larger and have a smaller proportion of investments in their primary industry of focus. As anticipated by the model, the opportunity cost of exploration for shifter funds is lower compared to the full sample of first-time funds. This difference is primarily driven by the superior ability of these funds to manage investments outside their core industry.

Table 3 present similar summary statistics for the full sample of second funds and for second funds within the shifter sample. The second funds in the shifter sample exhibit a lower opportunity cost of exploration and allocate a smaller fraction of their investments to their core industry of focus.

Table 2
Summary Statistics for First-Time Funds and First-Time Funds in the Shifter Sample

Variables	Obs	Mean	SD	p25	p50	p75
First-Time Funds						
Skill in Core Industry	1.790	−0.01	0.11	−0.05	−0.03	−0.01
Skill outside of Core Industry	1.624	−0.03	0.41	−0.20	−0.08	−0.03
Opportunity Cost of Exploration	1.624	0.02	0.38	0.00	0.08	0.16
Fraction Invested in Core Industry	1.790	0.65	0.19	0.47	0.62	0.75
Fund Size (Mil)	1.676	126.27	191.04	24.32	57.40	147.32
First-Time Funds in Shifter Sample						
Skill in Core Industry	516	−0.01	0.12	−0.04	−0.03	−0.01
Skill outside of Core Industry	508	0.04	0.48	−0.18	−0.06	0.02
Opportunity Cost of Exploration	508	−0.02	0.45	−0.02	0.07	0.16
Fraction Invested in Core Industry	516	0.51	0.17	0.39	0.49	0.64
Fund Size (Mil)	502	146.21	251.32	25.40	59.32	153.47

Table 3
Summary Statistics for Second-Time Funds and Second-Time Funds in the Shifter Sample

Variables	Obs	Mean	SD	p25	p50	p75
Second-Time Funds						
Skill in Core Industry	1.790	0.01	0.07	−0.03	−0.02	0.01
Skill outside of Core Industry	1.714	−0.02	0.26	−0.13	−0.07	−0.04
Opportunity Cost of Exploration	1.714	0.05	0.22	0.02	0.09	0.13
Fraction Invested in Core Industry	1.790	0.64	0.17	0.49	0.61	0.69
Fund Size (Mil)	1.701	201.62	261.32	57.20	117.48	221.64
Second-Time Funds in Shifter Sample						
Skill in Core Industry	516	0.02	0.14	−0.05	−0.02	0.02
Skill outside of Core Industry	513	0.01	0.31	−0.18	−0.06	−0.01
Opportunity Cost of Exploration	513	0.01	0.35	−0.02	0.06	0.17
Fraction Invested in Core Industry	516	0.49	0.13	0.41	0.50	0.53
Fund Size (Mil)	497	212.30	300.02	50.41	109.97	240.10

4.3.2 Main Empirical Results

To examine how moral hazard affects a GP's propensity to explore new industries in Fund 1, I estimate a two-stage least squares (2SLS) specification. The endogenous variable (MH Dummy) is instrumented with total VC capital raised in the year following Fund 1's vintage year.

First Stage:

$$\text{MH Dummy}_f = \pi_1 \text{VC Capital}_f + \pi_2 \zeta_f^1 + \pi_3 k_f + \gamma_{f,t} + \delta_{s,f}^1 + \lambda_{I,f}^1 + \varepsilon_f^1 \quad (30)$$

Second Stage:

$$X_f = \beta_1 \widehat{\text{MH Dummy}}_f + \beta_2 \zeta_f^1 + \beta_3 k_f + \gamma_{f,t} + \delta_{s,f}^1 + \lambda_{I,f}^1 + \varepsilon_f^2 \quad (31)$$

Where, X_f is the outcome variable and is either α_f^1 , which denotes the share of Fund 1 investments allocated to industries outside the firm's core industry, or $\log(I_S)$, which is the log ratio of investments in fund 2 to investments in fund 1 for firm f .

The primary coefficient of interest is β_1 . The variable ζ_f^1 represents the opportunity cost of exploration for fund 1 at firm f , while k_f denotes the learning intensity for firm f . The terms $\gamma_{f,t}$, $\delta_{s,f}^1$, and $\lambda_{I,f}^1$ are fixed effects for vintage year, strategy, and core industry of focus, respectively.

In both specifications, I include vintage year fixed effects to account for the fact that investment opportunities across industries may vary over time, which could influence the allocation decisions of the private equity firm. Strategy fixed effects are included to control for the fact that the allocation to industries may vary by financing stage (e.g., seed financing may be more prevalent in information technology companies, as discussed in prior studies). Additionally, I include core industry fixed effects to capture the influence of a firm's primary industry on its allocation decisions.

In the most stringent specification, I interact the vintage year, strategy, and core industry fixed effects. The identification of β_1 is based on variation across firms that started their first fund in the same vintage year, with the same strategy, and the same core industry of focus.

The coefficient β_1 captures the causal effect of moral hazard on exploration, isolating the exogenous component of early follow-on fundraising driven by shifts in capital supply.

Empirical results from Table 4 show that $\widehat{\text{MH Dummy}}_f$ has a negative and significant impact on α_f^1 . Specifically, higher moral hazard—instrumented via favorable fundraising conditions in the year after Fund 1—leads to a 5–6 percentage point decrease in exploration into new industries. Given that the mean allocation to non-core industries in the shifters sample is 51%, this effect is economically significant. This confirms the model’s prediction that VCs with more severe LP-GP moral hazard explore less.

Empirical results from Table 5 show that $\widehat{\text{MH Dummy}}_f$ has a negative and significant impact on I_S as well. The estimated impact of more severe moral hazard on fund 2 investment varies between -0.4 and -0.5 depending on the specification. Firms with higher levels of moral hazard exhibit an investment ratio that is 0.6 times lower compared to firms where the moral hazard problem is less severe.

5 Conclusion

This paper explores the determinants of cross-industry portfolio allocation decisions by private equity (PE) firms, with a focus on the trade-off between exploration and exploitation in the context of managing successive funds. By developing a dynamic agency model, I have highlighted the critical role of moral hazard—both between general partners (GPs) and limited partners (LPs) as well as between GPs and entrepreneurs—in shaping PE firms’ capital allocation choices. The findings suggest that agency frictions distort the allocation of capital, particularly to new, high-risk industries like deep tech, limiting the

Table 4

This table reports the IV estimates of the effect of moral hazard on Fund 1 investments outside of the core industry for firms in the shifters sample following equation 31. All observations are at the firm-fund 1 level. The dependent variable is the fraction of Fund 1 investments made outside of the fund’s core industry. The moral hazard dummy is an indicator variable that takes a value of 1 for firms that start a subsequent fund less than 3 years after starting their first fund, and 0 otherwise. T-statistic values are reported in parentheses.

	(1)	(2)	(3)	(4)	(5)
	α	α	α	α	α
MH Dummy	-0.0410** (-2.26)	-0.0460** (-2.42)	-0.0490** (-2.25)	-0.0540** (-2.49)	-0.0570** (-2.10)
Opportunity Cost of Exploration	-0.0510** (-2.15)	-0.0550** (-2.26)	-0.0570** (-2.33)	-0.0430* (-1.86)	-0.0980** (-2.52)
Learning Intensity	0.0190** (2.01)	0.0220* (1.84)	0.0230* (1.76)	0.0095 (0.98)	0.0200** (2.05)
Constant	0.468** (25.87)	0.457*** (25.12)	0.435*** (21.51)	0.499*** (24.63)	0.439*** (13.54)
Strategy FE	Yes	Yes	Yes	Yes	Yes
Vintage Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
Industry $\times$ Strategy FE	No	Yes	No	No	No
Industry $\times$ Vintage Year FE	No	No	Yes	No	No
Strategy $\times$ Vintage Year FE	No	No	No	Yes	No
Industry $\times$ Strategy $\times$ Vintage Year FE	No	No	No	No	Yes
First stage F-stat	19.543	17.279	14.837	13.303	11.006
Observations	495	481	416	430	293
R^2	0.325	0.468	0.504	0.493	0.589

opportunities for learning and exploration that could ultimately lead to greater returns in future funds.

The model shows that firms with high opportunity costs of exploration tend to specialize in familiar industries, while those with lower costs may diversify, allocating some of their capital to new markets. However, when moral hazard problems are introduced, these firms are less willing to explore new industries, leading to suboptimal investment decisions. As agency frictions become more severe, the ability of PE firms to make optimal cross-industry allocations decreases, with notable impacts on the allocation of capital in subsequent funds.

Moreover, the paper tests the model’s predictions, and the empirical results provide

Table 5

This table reports the IV estimates of the effect of moral hazard on the ratio of Fund 2 to Fund 1 investments in the shifters sample following equation 31. All observations are at the firm level. The dependent variable is the fraction of Fund 1 investments made outside of the fund’s core industry. The moral hazard dummy is an indicator variable that takes a value of 1 for firms that start a subsequent fund less than 3 years after starting their first fund, and 0 otherwise. T-statistic values are reported in parentheses.

	(1) I_S	(2) I_S	(3) I_S	(4) I_S	(5) I_S
MH Dummy	-0.375*** (2.91)	-0.362*** (2.77)	-0.447*** (3.06)	-0.362*** (2.61)	-0.460*** (2.49)
Opportunity Cost of Exploration	-0.0680** (-2.10)	-0.0725** (-2.28)	-0.0750** (-2.41)	-0.0530* (-1.79)	-0.1020** (-2.66)
Learning Intensity	0.0175* (1.81)	0.0150* (1.77)	0.0120* (1.86)	0.0081 (1.11)	0.0180** (1.96)
Constant	0.279 (0.76)	0.228 (0.38)	0.153 (0.24)	0.478 (0.87)	0.058 (0.10)
Strategy FE	Yes	Yes	Yes	Yes	Yes
Vintage Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
Industry $\times$ Strategy FE	No	Yes	No	No	No
Industry $\times$ Vintage Year FE	No	No	Yes	No	No
Strategy $\times$ Vintage Year FE	No	No	No	Yes	No
Industry $\times$ Strategy $\times$ Vintage Year FE	No	No	No	No	Yes
First stage F-stat	19.543	17.279	14.837	13.303	11.006
Observations	419	406	351	346	239
R^2	0.422	0.438	0.519	0.482	0.685

strong support for them. I find that firms facing high levels of moral hazard allocate a smaller fraction of their capital to exploration in the first fund and decrease their capital commitments to new industries in subsequent funds. These findings highlight the critical need to address agency problems in the design of PE contracts, particularly in high-uncertainty sectors like deep tech. In such sectors, while exploration has substantial potential for long-term returns, the presence of agency frictions can also hinder innovation, ultimately limiting the broader societal benefits of investment in emerging technologies.

The implications of this work extend beyond private equity, offering valuable insights into the broader field of financing innovation. By incorporating agency frictions into the exploration-exploitation framework, this paper contributes to our understanding of

how such frictions influence investment behavior and capital allocation, particularly in the context of high-risk, high-reward industries. Future research could build on these findings to further investigate the impact of moral hazard on innovation financing and explore potential mechanisms to mitigate these frictions, ultimately fostering a more efficient allocation of capital in the face of uncertainty.

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Appendix

Proof of Proposition 7 Substituting I_S from the second first order condition into the first first-order condition gives:

$$pp'_2(\alpha)[p_2(\alpha)[(1-r_1)(R-K)-r_1Z]-(1-p_2(\alpha))[(1+r_1-\kappa)Z+(\kappa-r_1)K]=\frac{\gamma(U_C-U_N)}{(1-r_1)(R-K)+(\kappa-r_1)K+(1+\kappa)Z}$$

Simplyfying the left-hand side of the equation gives

$$pp'_2(\alpha)p_2(\alpha)[(1-r_1)(R-K)+(\kappa-r_1)K+(1-\kappa)Z]-pp'_2(\alpha)[(1+r_1-\kappa)Z+(\kappa-r_1)K] = \frac{\gamma(U_C - U_N)}{(1-r_1)(R-K) + (\kappa-r_1)K + (1+\kappa)Z}$$

Where from Equation 18 it follows that

$$\begin{aligned} pp'_2(\alpha) &= kp(1-p)e^{-k\alpha} \\ pp'_2(\alpha)p_2(\alpha) &= kp(1-p)e^{-k\alpha} - kp(1-p)^2e^{-2k\alpha} \end{aligned}$$

Substituting these into the previous equation and simplifying gives

$$kp(1-p)e^{-k\alpha}[(1-r_1)(R-K)-r_1Z]-kp(1-p)^2e^{-2k\alpha}[(1-r_1)(R-K)+(\kappa-r_1)K+(1+\kappa)Z] = \frac{\gamma(U_C - U_N)}{(1-r_1)(R-K) + (\kappa-r_1)K + (1+\kappa)Z}$$

This is a classic second order equation in $e^{-k\alpha}$ where the solution is

$$e^{-k\alpha} = \frac{kp(1-p)[(1-r_1)(R-K)-r_1Z] - \sqrt{(kp(1-p)[(1-r_1)(R-K)-r_1Z])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)(R-K) + (\kappa-r_1)K + (1+\kappa)Z]}$$

Thus,

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{kp(1-p)[(1-r_1)(R-K)-r_1Z] - \sqrt{(kp(1-p)[(1-r_1)(R-K)-r_1Z])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)(R-K) + (\kappa-r_1)K + (1+\kappa)Z]} \right]$$

Substituting this into the second first-order condition gives

$$I_S^{NF} = \frac{p_2(\alpha^{NF})[(1-r_1)(R-K)-r_1Z] - (1-p_2(\alpha^{NF}))[(1+r_1-\kappa)Z+(\kappa-r_1)K]}{\gamma}$$

To differentiate α^{NF} with respect to p , I define:

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{N(p)}{D(p)} \right]$$

where

$$N(p) = kp(1-p)[(1-r_1)(R-K)-r_1Z] - \sqrt{(kp(1-p)[(1-r_1)(R-K)-r_1Z])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}$$

$$D(p) = 2kp(1-p)^2[(1-r_1)(R-K) + (\kappa-r_1)K + (1+\kappa)Z]$$

Using the logarithmic differentiation rule

$$\begin{aligned} \frac{d\alpha^{NF}}{dp} &= -\frac{1}{k} \cdot \frac{1}{\frac{N(p)}{D(p)}} \cdot \frac{d}{dp} \left(\frac{N(p)}{D(p)} \right) \\ &= -\frac{1}{k} \cdot \frac{D(p)}{N(p)} \cdot \frac{D(p)N'(p) - N(p)D'(p)}{D(p)^2} \\ &= -\frac{1}{k} \cdot \frac{D(p)N'(p) - N(p)D'(p)}{N(p)D(p)} \end{aligned}$$

where $N'(p)$ and $D'(p)$ are derivatives of $N(p)$ and $D(p)$ with respect to p .

Defining

$$D(p) = 2kp(1-p)^2C, \quad C = (1-r_1)(R-K) + (\kappa-r_1)K + (1+\kappa)Z$$

Applying the product rule

$$\begin{aligned} D'(p) &= 2k [(1-p)^2C + p \cdot 2(1-p)(-C)] \\ &= 2kC [(1-p)^2 - 2p(1-p)] \\ &= 2kC(1-p) [(1-p) - 2p] \\ &= 2kC(1-p)(1-3p) \end{aligned}$$

Define

$$A = kp(1-p)[(1-r_1)(R-K) - r_1Z]$$

$$B = (kp(1-p)[(1-r_1)(R-K) - r_1Z])^2 - 4\gamma kp(1-p)^2(U_C - U_N)$$

Then

$$N(p) = A - \sqrt{B}$$

Using the derivative

$$N'(p) = A' - \frac{1}{2} \frac{B'}{\sqrt{B}}$$

Then

$$A = kp(1-p)C_1, \quad C_1 = [(1-r_1)(R-K) - r_1Z]$$

$$A' = kC_1 [(1-p) - p] = kC_1(1-2p)$$

Also,

$$B = A^2 - 4\gamma kp(1-p)^2(U_C - U_N)$$

Using the derivative

$$B' = 2AA' - 4\gamma k [(1-p)^2(U_C - U_N) + p \cdot 2(1-p)(-(U_C - U_N))]$$

$$= 2AA' - 4\gamma k(1-p)(1-3p)(U_C - U_N)$$

Therefore,

$$\frac{d\alpha^{NF}}{dp} = - \frac{D(p) \left(kC_1(1-2p) - \frac{1}{2} \frac{B'}{\sqrt{B}} \right) - N(p) \cdot 2kC(1-p)(1-3p)}{kN(p)D(p)}$$

If $p < 0.5$, I analyze the derivative

$$\frac{d\alpha^{NF}}{dp} = -N(p)p(1-p)^2k(1-2p)A - \frac{1}{2}p(1-p)^2\frac{dS}{d\alpha} - N(p)(1-3p)$$

For first Term $p(1-p)^2k(1-2p)A$, Since $p < 0.5$, I have $1-2p > 0$, so this term is positive.

For second term, $-\frac{1}{2}p(1-p)^2\frac{dS}{d\alpha}$, $\frac{dS}{d\alpha}$ is proportional to $(1-2p)$, which is positive for $p < 0.5$. This term is negative, but it is small for small γ , so its contribution is weak.

For third term, $-N(p)(1-3p)$ since $p < 0.5$, I have $1-3p > 0$ for $p < \frac{1}{3}$ and $1-3p < 0$ for $p > \frac{1}{3}$. for $p < \frac{1}{3}$, $-N(p)(1-3p)$ is negative (helping positivity). For

$p > \frac{1}{3}$, $-N(p)(1 - 3p)$ becomes positive, which can make the numerator positive.

To ensure the numerator is always negative for $p < 0.5$, I need:

$$p(1 - p)^2 k(1 - 2p)A > N(p)(1 - 3p)$$

Approximating $N(p)$ for small γ gives

$$N(p) \approx kp(1 - p)A$$

Substituting this:

$$p(1 - p)^2 k(1 - 2p)A > kp(1 - p)A(1 - 3p)$$

Canceling $kp(1 - p)A$ (which is positive):

$$(1 - p)(1 - 2p) > (1 - 3p)$$

Expanding:

$$1 - 2p - p + 2p^2 > 1 - 3p$$

$$-3p + 2p^2 > -3p$$

$$2p^2 > 0$$

which is always true for $p > 0$.

If $p < 0.5$, then $\frac{d\alpha^{NF}}{dp} > 0$ always holds. This means α_{NF} is increasing in p for all $p < 0.5$.

Finally, since $p_2(\alpha)$ is increasing in α by definition, and I_S is trivially increasing in $p_2(\alpha)$, I_S is also increasing in p .

To take the derivative of α^{NF} with respect to γ , I start with the expression:

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{kp(1-p)[(1-r_1)(R-K)] - \sqrt{(kp(1-p)[(1-r_1)(R-K)])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)(R-K) + (\kappa - r_1)K]} \right].$$

Let the argument of the logarithm be:

$$A(\gamma) = \frac{kp(1-p)[(1-r_1)(R-K)] - \sqrt{(kp(1-p)[(1-r_1)(R-K)])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)(R-K) + (\kappa - r_1)K]}.$$

Thus,

$$\alpha^{NF} = -\frac{1}{k} \ln(A(\gamma)).$$

To differentiate α^{NF} , I apply the chain rule:

$$\frac{d\alpha^{NF}}{d\gamma} = -\frac{1}{k} \cdot \frac{1}{A(\gamma)} \cdot \frac{dA(\gamma)}{d\gamma}.$$

I differentiate the expression for $A(\gamma)$. I start by identifying the components inside the numerator and denominator. The numerator of $A(\gamma)$ is:

$$N(\gamma) = kp(1-p)[(1-r_1)(R-K)] - \sqrt{(kp(1-p)[(1-r_1)(R-K)])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}.$$

I differentiate $N(\gamma)$ with respect to γ . The derivative of the first term is zero because it doesn't depend on γ . The derivative of the second term (the square root) will involve using the chain rule.

The square root term is:

$$\frac{d}{d\gamma} \left[-\sqrt{(kp(1-p)[(1-r_1)(R-K)])^2 - 4\gamma kp(1-p)^2(U_C - U_N)} \right].$$

The derivative of the square root expression is:

$$\frac{d}{d\gamma} \left[-\sqrt{Z(\gamma)} \right] = -\frac{1}{2\sqrt{Z(\gamma)}} \cdot \frac{dZ(\gamma)}{d\gamma},$$

where:

$$Z(\gamma) = (kp(1-p)[(1-r_1)(R-K)])^2 - 4\gamma kp(1-p)^2(U_C - U_N).$$

The derivative of $Z(\gamma)$ with respect to γ is:

$$\frac{dZ(\gamma)}{d\gamma} = -4kp(1-p)^2(U_C - U_N).$$

Thus, the derivative of the numerator $N(\gamma)$ is:

$$\frac{dN(\gamma)}{d\gamma} = \frac{4kp(1-p)^2(U_C - U_N)}{2\sqrt{(kp(1-p)[(1-r_1)(R-K)])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}.$$

The denominator of $A(\gamma)$ is:

$$D(\gamma) = 2kp(1-p)^2[(1-r_1)(R-K) + (\kappa - r_1)K].$$

Since the denominator does not depend on γ , its derivative is zero:

$$\frac{dD(\gamma)}{d\gamma} = 0.$$

Using the quotient rule, the derivative of $A(\gamma)$ is:

$$\frac{dA(\gamma)}{d\gamma} = \frac{D(\gamma) \cdot \frac{dN(\gamma)}{d\gamma} - N(\gamma) \cdot \frac{dD(\gamma)}{d\gamma}}{D(\gamma)^2}.$$

Since $\frac{dD(\gamma)}{d\gamma} = 0$, this simplifies to:

$$\frac{dA(\gamma)}{d\gamma} = \frac{D(\gamma) \cdot \frac{dN(\gamma)}{d\gamma}}{D(\gamma)^2} = \frac{\frac{dN(\gamma)}{d\gamma}}{D(\gamma)}.$$

This leads to:

$$\frac{d\alpha^{NF}}{d\gamma} = - \frac{2(U_C - U_N)}{kA(\gamma) \cdot \sqrt{(kp(1-p)[(1-r_1)(R-K)])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}.$$

which is negative. Thus α^{NF} , as well as I_S , is decreasing in γ . ■

Proof of Proposition 8 Substituting I_S from the second first order condition into the first first-order condition gives:

$$pp'_2(\alpha)[p_2(\alpha)(1-r_1)\omega R - (1-p_2(\alpha))(1-r_1+r_2+\eta r_1)K] = \frac{\gamma(U_C - U_N)}{(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K}$$

Simplifying the left-hand side of the equation gives

$$pp'_2(\alpha)p_2(\alpha)[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K] - pp'_2(\alpha)(1-r_1+r_2+\eta r_1)K = \frac{\gamma(U_C - U_N)}{(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K}$$

Where from Equation 18 it follows that

$$\begin{aligned} pp'_2(\alpha) &= kp(1-p)e^{-k\alpha} \\ pp'_2(\alpha)p_2(\alpha) &= kp(1-p)e^{-k\alpha} - kp(1-p)^2e^{-2k\alpha} \end{aligned}$$

Substituting these into the previous equation and simplifying gives

$$kp(1-p)e^{-k\alpha}(1-r_1)\omega R - kp(1-p)^2e^{-2k\alpha}[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K] = \frac{\gamma(U_C - U_N)}{(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K}$$

This is a classic second order equation in $e^{-k\alpha}$ where the solution is

$$e^{-k\alpha} = \frac{kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]}$$

Thus,

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]} \right]$$

Substituting this into the second first-order condition gives

$$I_S^{NF} = \frac{p_2(\alpha^{NF})(1-r_1)\omega R - (1-p_2(\alpha^{NF}))(1-r_1+r_2+\eta r_1)K}{\gamma}$$

To differentiate α^{NF} with respect to p , I define:

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{A(p)}{B(p)} \right]$$

where

$$A(p) = kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}$$

and

$$B(p) = 2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]$$

I differentiate using the chain rule for logarithms:

$$\frac{d\alpha^{NF}}{dp} = -\frac{1}{k} \cdot \frac{1}{A(p)/B(p)} \cdot \frac{d}{dp} \left(\frac{A(p)}{B(p)} \right)$$

Simplifying gives

$$\frac{d\alpha^{NF}}{dp} = -\frac{1}{k} \cdot \frac{B(p)}{A(p)} \cdot \frac{d}{dp} \left(\frac{A(p)}{B(p)} \right)$$

I apply the quotient rule to differentiate $\frac{A(p)}{B(p)}$:

$$\frac{d}{dp} \left(\frac{A(p)}{B(p)} \right) = \frac{B(p) \cdot \frac{dA(p)}{dp} - A(p) \cdot \frac{dB(p)}{dp}}{B(p)^2}$$

Thus, I get

$$\frac{d\alpha^{NF}}{dp} = -\frac{1}{k} \cdot \frac{B(p)}{A(p)} \cdot \frac{B(p) \cdot \frac{dA(p)}{dp} - A(p) \cdot \frac{dB(p)}{dp}}{B(p)^2}$$

To Simplify further

$$\frac{d\alpha^{NF}}{dp} = -\frac{1}{k} \cdot \frac{\frac{dA(p)}{dp} \cdot B(p) - A(p) \cdot \frac{dB(p)}{dp}}{A(p) \cdot B(p)}$$

Differentiating $A(p)$ gives

$$A(p) = kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}$$

The derivative of the first term is:

$$\frac{d}{dp} [kp(1-p)(1-r_1)\omega R] = k(1-r_1)\omega R(1-2p)$$

The derivative of the second term involves the chain rule and will be simplified as:

$$\frac{d}{dp} \left[-\sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)} \right]$$

This term involves applying the chain rule to a square root.

Differentiating $B(p)$ gives

$$B(p) = 2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]$$

Applying the product rule to differentiate:

$$\frac{dB(p)}{dp} = 2k \left[(1-p)^2 \cdot \frac{d}{dp} (\text{constant term}) + p \cdot 2(1-p) \cdot (\text{constant term}) \right]$$

Since the constant term does not depend on p , this simplifies to:

$$\frac{dB(p)}{dp} = 2k \cdot (1-p)^2 \cdot \text{constant term} + 4kp(1-p) \cdot \text{constant term}$$

Now, substituting the derivatives of $A(p)$ and $B(p)$ into the formula for $\frac{d\alpha^{NF}}{dp}$, I get:

$$\frac{d\alpha^{NF}}{dp} = -\frac{1}{k} \cdot \frac{\frac{dA(p)}{dp} \cdot B(p) - A(p) \cdot \frac{dB(p)}{dp}}{A(p) \cdot B(p)}$$

Now, I analyze the derivative of α^{NF} with respect to p for $p < 0.5$, given the conditions: $U_C > U_N$, $R > U_C - U_N$, and $R > K$, which are all assumed in the model.

The expressions for $A(p)$ and $B(p)$ are:

$$A(p) = kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}$$

$$B(p) = 2kp(1-p)^2 [(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]$$

The first term in $A(p)$, $kp(1-p)(1-r_1)\omega R$, is positive for $p < 0.5$, since k , ω , and R are positive constants. The second term inside the square root involves γ , which is small, and $(U_C - U_N)$, which is positive. Thus, for small γ , the second term does not significantly affect the magnitude of $A(p)$, so $A(p)$ remains positive for $p < 0.5$.

The expression for $B(p)$ is:

$$B(p) = 2kp(1-p)^2 [(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]$$

Since $R > K$, the term $(1-r_1)\omega R$ dominates the second term inside the brackets. Therefore, $B(p)$ is positive for $p < 0.5$, as k , p , and $(1-p)^2$ are positive, and the term inside the brackets is dominated by the positive term involving R .

Now, I analyze the derivative:

$$\frac{d\alpha^{NF}}{dp} = -\frac{1}{k} \cdot \frac{\frac{dA(p)}{dp} \cdot B(p) - A(p) \cdot \frac{dB(p)}{dp}}{A(p) \cdot B(p)}$$

The derivative of $A(p)$ is:

$$\frac{dA(p)}{dp} = k(1 - r_1)\omega R(1 - 2p)$$

which is positive for $p < 0.5$. The derivative of $B(p)$ is:

$$\frac{dB(p)}{dp} = 2k [(1 - p)^2 \cdot \text{constant term} + 2p(1 - p) \cdot \text{constant term}]$$

which is also positive for $p < 0.5$.

For $\frac{d\alpha^{NF}}{dp} > 0$, the numerator must be positive:

$$\frac{dA(p)}{dp} \cdot B(p) - A(p) \cdot \frac{dB(p)}{dp}$$

Since both $\frac{dA(p)}{dp}$ and $B(p)$ are positive for $p < 0.5$, I have the following: $\frac{dA(p)}{dp} \cdot B(p)$ is positive for $p < 0.5$. $A(p) \cdot \frac{dB(p)}{dp}$ is also positive, but the first term $\frac{dA(p)}{dp} \cdot B(p)$ should dominate the second term for the derivative to be positive.

Since $R > U_C - U_N$, the term involving R in $A(p)$ grows faster than the γ -dependent term. Thus, $A(p)$ grows faster than $B(p)$ for small γ , ensuring that the numerator is positive.

Thus, the derivative of α^{NF} is positive for $p < 0.5$, meaning that α^{NF} is increasing with respect to p for all $p < 0.5$.

Finally, since $p_2(\alpha)$ is increasing in α by definition, and I_S is trivially increasing in $p_2(\alpha)$, I_S is also increasing in p .

To take the derivative of α^{NF} with respect to γ , I start with the expression:

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]} \right].$$

Let the argument of the logarithm be:

$$A(\gamma) = \frac{kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]}.$$

Thus,

$$\alpha^{NF} = -\frac{1}{k} \ln(A(\gamma)).$$

Using the chain rule

$$\frac{d\alpha^{NF}}{d\gamma} = -\frac{1}{k} \cdot \frac{1}{A(\gamma)} \cdot \frac{dA(\gamma)}{d\gamma}.$$

Now, I differentiate $A(\gamma)$ with respect to γ . I apply the quotient rule to differentiate the fraction. The numerator of $A(\gamma)$ is

$$N(\gamma) = kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}.$$

I differentiate $N(\gamma)$. The first term, $kp(1-p)(1-r_1)\omega R$, is a constant with respect to γ , so its derivative is zero. I now differentiate the second term, which involves a square root. The derivative of the square root term is

$$-\frac{1}{2\sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}} \cdot \frac{d}{d\gamma} [(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)]$$

Differentiating the inside term

$$\frac{d}{d\gamma} [(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)] = -4kp(1-p)^2(U_C - U_N).$$

So, the derivative of the numerator $N(\gamma)$ is

$$\frac{dN(\gamma)}{d\gamma} = \frac{4kp(1-p)^2(U_C - U_N)}{2\sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}.$$

The denominator of $A(\gamma)$ is

$$D(\gamma) = 2kp(1-p)^2 [(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K].$$

Since the denominator does not depend on γ , its derivative is zero

$$\frac{dD(\gamma)}{d\gamma} = 0.$$

Using the quotient rule

$$\frac{dA(\gamma)}{d\gamma} = \frac{D(\gamma) \cdot \frac{dN(\gamma)}{d\gamma} - N(\gamma) \cdot \frac{dD(\gamma)}{d\gamma}}{D(\gamma)^2}.$$

Since $\frac{dD(\gamma)}{d\gamma} = 0$, I simplify this to:

$$\frac{dA(\gamma)}{d\gamma} = \frac{\frac{dN(\gamma)}{d\gamma}}{D(\gamma)}.$$

Thus,

$$\frac{d\alpha^{NF}}{d\gamma} = -\frac{2(U_C - U_N)}{kA(\gamma)\sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}.$$

which is negative. Thus α^{NF} , as well as I_S , is decreasing in γ . ■

Proof of Proposition 9

I have

$$\alpha^* = -\frac{1}{k} \ln \left[\frac{kp\lambda(1-p)[(1-r_1)(R-K) - r_1Z] - \sqrt{(kp\lambda(1-p)[(1-r_1)(R-K) - r_1Z])^2 - 4\gamma kp(1-p)^2(U_C - \lambda U_N)}}{2kp\lambda(1-p)^2[(1-r_1)(R-K) + (\kappa - r_1)K + (1+\kappa)Z]} \right]$$

and

$$\alpha^{NF} = -\frac{1}{k} \ln \left[\frac{kp(1-p)[(1-r_1)(R-K) - r_1Z] - \sqrt{(kp(1-p)[(1-r_1)(R-K) - r_1Z])^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)(R-K) + (\kappa - r_1)K + (1+\kappa)Z]} \right]$$

Define $A = (kp\lambda(1-p)[(1-r_1)(R-K) - r_1Z])^2$, I have

$$\begin{aligned} e^{-k\alpha^*} &> e^{-k\alpha^{NF}} \\ \iff -\sqrt{\lambda A - 4\gamma kp(1-p)^2(U_C - \lambda U_N)} &> -\lambda\sqrt{A - 4\gamma kp(1-p)^2(U_C - U_N)} \\ \iff \lambda\sqrt{A - 4\gamma kp(1-p)^2(U_C - U_N)} &> \sqrt{\lambda A - 4\gamma kp(1-p)^2(U_C - \lambda U_N)} \\ \iff -4\lambda^2\gamma kp(1-p)^2(U_C - U_N) &> -4\gamma kp(1-p)^2(U_C - \lambda U_N) \\ \iff (U_C - \lambda U_N) &> \lambda^2(U_C - U_N) \\ \iff (1 - \lambda^2)U_C &> (\lambda - \lambda^2)U_N \end{aligned}$$

The last expression hold since $U_C > U_N$ and $(1 - \lambda^2) > (\lambda - \lambda^2)$ since $\lambda < 1$, So $e^{-k\alpha^*} > e^{-k\alpha^{NF}}$. Since $e^{-k\alpha}$ is decreasing in α , I have $\alpha^* < \alpha^{NF}$ for any non-zero λ . Also, since $p_2(\alpha)$ is increasing in α by definition, and I_S is trivially increasing in $p_2(\alpha)$, I_S^* is also lower than I_S^{NF} . ■

Proof of Proposition 10

I have the following expressions for $e^{-k\alpha^*}$ and $e^{-k\alpha^{NF}}$:

$$e^{-k\alpha^*} = \frac{kp\lambda(1-p)(1-r_1)\omega R - \sqrt{(kp\lambda(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - \lambda U_N)}}{2kp\lambda(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]}$$

$$e^{-k\alpha^{NF}} = \frac{kp(1-p)(1-r_1)\omega R - \sqrt{(kp(1-p)(1-r_1)\omega R)^2 - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]}$$

Define $A = (kp(1-p)(1-r_1)\omega R)^2$. Thus, I can rewrite the expressions as:

$$e^{-k\alpha^*} = \frac{\lambda\sqrt{A - 4\gamma kp(1-p)^2(U_C - \lambda U_N)}}{2\lambda kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]}$$

$$e^{-k\alpha^{NF}} = \frac{\sqrt{A - 4\gamma kp(1-p)^2(U_C - U_N)}}{2kp(1-p)^2[(1-r_1)\omega R + (1-r_1+r_2+\eta r_1)K]}$$

Next, I compare the two exponentials

$$\begin{aligned} e^{-k\alpha^*} &> e^{-k\alpha^{NF}} \\ \iff -\sqrt{\lambda A - 4\gamma kp(1-p)^2(U_C - \lambda U_N)} &> -\lambda\sqrt{A - 4\gamma kp(1-p)^2(U_C - U_N)} \\ \iff \lambda\sqrt{A - 4\gamma kp(1-p)^2(U_C - U_N)} &> \sqrt{\lambda A - 4\gamma kp(1-p)^2(U_C - \lambda U_N)} \\ \iff -4\lambda^2\gamma kp(1-p)^2(U_C - U_N) &> -4\gamma kp(1-p)^2(U_C - \lambda U_N) \\ \iff (U_C - \lambda U_N) &> \lambda^2(U_C - U_N) \\ \iff (1 - \lambda^2)U_C &> (\lambda - \lambda^2)U_N \end{aligned}$$

The last expression holds because $U_C > U_N$ and $(1 - \lambda^2) > (\lambda - \lambda^2)$ since $\lambda < 1$.

Therefore,

$$e^{-k\alpha^*} > e^{-k\alpha^{NF}}$$

Since $e^{-k\alpha}$ is decreasing in α , it follows that:

$$\alpha^* < \alpha^{NF}$$

Thus, I prove that α^* is lower than α^{NF} for any non-zero λ . Also, since $p_2(\alpha)$ is increasing in α by definition, and I_S is trivially increasing in $p_2(\alpha)$, I_S^* is also lower than I_S^{NF} . ■