

Learning about Noise, Information Disclosure, and Market Stability

Hao Yu*

University of Macau

Minxing Zhu[†]

University of Macau

July 1, 2025

Abstract

How do rational investors' learning about noise trading and firms' information disclosure jointly affect financial market stability? We examine this question in a model in which a part of a risky asset's payoff is publicly disclosed and rational investors can learn about noise traders' demand for the asset. We demonstrate that reducing information disclosure can eliminate the multiple self-fulfilling equilibria caused by learning about noise. In an extended model with endogenous information disclosure, we show that the equilibrium expected asset price is increasing and discontinuous in the intensity of rational investors' learning about noise. Our results predict that (i) more information disclosure can exacerbate the market fragility caused by high-frequency trading, and (ii) market crashes can occur during the process of regulating high-frequency trading. We also discuss the empirical relevance of our results.

Keywords: Multiple equilibria; Information disclosure; Learning; Market stability; High-frequency trading

JEL Classification: D82, D83, G12, G14

*Email: josephhao.yu@connect.um.edu.mo

[†]Email: minxing.zhu@connect.um.edu.mo

1 Introduction

Information disclosure has long been considered a way to increase market efficiency, reduce financial fragility, and increase investors' welfare. Policies have also been proposed to improve the quality of disclosure (e.g., the Sarbanes Oxley Act and the Dodd Frank Act). Nowadays, with the development of information technology, many investors use trading strategies based on information about others' order flows (e.g., high frequency trading) when trading in financial markets. However, such trading strategies have been blamed for creating market instability. For example, the "flash crash" occurred on May 6, 2010, during which major US stock indices collapsed rapidly, was thought to be caused by high frequency trading algorithms.¹ How do information disclosure and high frequency trading jointly affect financial markets? Can information disclosure decrease the probability of market crashes caused by high frequency trading? This paper attempts to address these questions with a simple trading model.

Specifically, we develop a noisy rational expectations equilibrium model with a competitive financial market, in which investors can trade a risky asset whose payoff consists of two parts, as in Goldstein and Yang (2015). Investors can acquire private information about the first part of the payoff, and observe public information about the second part of the payoff. As in Ganguli and Yang (2009) and Farboodi and Veldkamp (2020), the rational investors can also acquire private information about noise traders' demand for the risky asset.

The first main result derived from our model is that, if the rational investors learn about the noise traders' demand and the public information is sufficiently precise (imprecise),

¹See https://en.wikipedia.org/wiki/2010_flash_crash.

then the financial market has multiple equilibria (a unique equilibrium). Notice that the existence of multiple equilibria corresponds to market fragility (Benhabib et al. (2019); Goldstein et al. (2024)) and excess volatility (Ganguli and Yang (2009)), as the asset price can change without any changes in asset fundamentals, the noise of public signal, and the noise traders' demand. Moreover, the rational investors' trading based on information about the noise traders' demand can be interpreted as "high-frequency" trading in reality, because both trading strategies profit from others' trading patterns (Farboodi and Veldkamp (2020)). Therefore, our result implies that, when high-frequency trading exists, instead of stabilizing the market, information disclosure can increase market fragility, consistent with the empirical evidence demonstrating that more information disclosure is associated with higher market volatility (Leuz and Verrecchia (2000); Bushee and Noe (2000); Magnan and Xu (2008)).

The existence of multiple equilibrium prices is due to the self-fulfilling nature of financial market equilibria when the rational investors can learn about noise traders' demand. When the public information is precise and the investors believe that the asset price is very informative about the asset fundamentals, they are very confident in their predictions on the asset payoff. In this case, the rational investors trade aggressively against the noise traders (i.e., respond aggressively to changes in noise trading information), resulting in a high realization of price informativeness. Similarly, when the public information is precise but the investors believe that the asset price is uninformative about the asset fundamentals, they are less confident in their predictions on the asset payoff. Therefore, the rational investors do not trade aggressively against noise traders and thus the noise trading significantly affect the asset price, leading to a low realized price informativeness. In summary, if the public information is precise, the equilibrium price informativeness self-fulfills the

rational investors' initial belief about the price informativeness, which results in multiple self-fulfilling equilibria.

In contrast, when the public information is imprecise, even if the rational investors believe that the asset price is informative, they are still reluctant to trade aggressively against noise traders. The reason is that the asset price and the public information are informative about different dimensions of the asset payoff, and abundant information about one dimension does not eliminate the uncertainty of the total asset payoff. As a result, no matter the rational investors initially believe that the asset price informativeness is high or low, their trading behavior always leads to a low realized price informativeness, and thus the financial market equilibrium is unique.

To further explore the joint effect of learning about noise and information disclosure on the financial market, we endogenize the precision of the public signal by assuming that a firm manager chooses this precision to maximize the unconditional expected asset price, subject to a convex cost of disclosure. The second main result of our paper is about the impacts of the investors' learning about noise trading on the public information disclosure. We find that when the precision of investors' private signal about noise trading is low (high), the precision of public signal is increasing (decreasing) in the precision of noise trading signals.

Intuitively, since the objective of information disclosure is to increase the asset price, the firm manager will disclose more (less) if the marginal impact of disclosure on the price is higher (lower). The investors' learning about noise trading has two opposite effects on the impacts of disclosure on asset price. On one hand, an increase in the precision of noise trading information reduces the risk perceived by the investors, which makes them respond more aggressively to the disclosure, thereby increasing the marginal ef-

fect of disclosure on the asset price. On the other hand, an increase in the precision of noise trading information allows the investors to better filter out the noise contained in the asset price and extract more information about asset fundamentals from the asset price, which increases (reduces) their reliance on the price signal (public signal), decreasing the marginal effect of disclosure on their trading behavior and thus the asset price. These two opposite effects result in a non-monotonic impact of learning about noise trading on information disclosure.

More interestingly, the public signal precision is discontinuous in the noise trading signal precision: as the noise trading signal precision increases, the public signal precision can suddenly jump from a low value to a high value. The reason is that the firm manager's objective function is non-concave in the public signal precision. Specifically, the objective function has two local maximum points. If the noise trading signal precision is less (greater) than a certain threshold, then the local maximum point on the left (right) is the global maximum point. The discontinuity of information disclosure also leads to the discontinuities of price informativeness, liquidity, expected price level, and price volatility in investors' learning about noise trading. Notably, as the precision of noise trading signals decreases, the asset price can experience a sudden drop. Since investors' trading based on information about noise traders' demand resembles high-frequency and quantitative trading in reality, our result suggests that regulations on such trading strategies may result in market crashes.

Related literature. Our paper is broadly related to the literature on learning and information aggregation in financial market (e.g., Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982)). As is concluded in a recent review by Goldstein et al. (2024), investors' learning behavior can lead to multiple equilibria and thus the extreme fragility of

financial markets. Benhabib et al. (2019) demonstrate that when the aggregate output is at an intermediate level, the information productions in the financial and real markets are strategic complementary, resulting in multiple equilibria in both markets. Ganguli and Yang (2009) show that investors' learning about noise trading or random supply of assets can generate multiple equilibria in financial markets. Farboodi and Veldkamp (2020) extend the framework in Ganguli and Yang (2009) to an information choice setting and show that rational investors tend to learn more about noise traders' demand as financial data technologies develop. In a similar setting, Marmora and Rytchkov (2018) show that learning about noise increases price informativeness. Our paper contributes to this strand of literature by identifying a condition under which the market fragility caused by learning about noise can be eliminated. We also demonstrates the impact of noise trading information on voluntary information disclosure, which is new to the literature.

Our paper is also related to the literature on the effects and causes of information disclosure in financial markets. People generally believe that information disclosure is welfare-increasing for investors, as it increases firms' transparency, market efficiency, and liquidity. However, recent research has revealed some negative impacts of information disclosure. Goldstein and Yang (2019) demonstrate that disclosing information that firms know little about to the financial markets can decrease real efficiency. Goldstein and Yang (2017) show that information disclosure lowers investors' incentives to produce information, decreasing stock price informativeness. Christensen and Qin (2014) find that information disclosure can be welfare-decreasing when investors have heterogeneous prior beliefs. We contribute to this strand of literature by revealing the relationship between information disclosure and equilibrium multiplicity, showing that information disclosure can reduce market stability.

The rest of this paper is organized as follows. Section 2 presents and solves the model. Section 3 analyzes the impact of disclosure on equilibrium multiplicity and discusses the empirical relevance. Section 4 studies how investors' learning about noise trading affects market equilibrium outcomes, taking the disclosure as exogenously given. Section 5 endogenizes the disclosure and investigates how it is affected by noise trading information. Section 6 concludes. The proofs of propositions are provided in Appendix A.

2 Model

In this section, we develop a rational expectations equilibrium model in which a risky asset whose payoff consists of two parts is traded in a financial market. Investors can acquire private information about the first part of the payoff, while information about the second part of the payoff is publicly disclosed. We demonstrate that the disclosure is crucial in determining the existence of multiple equilibria when the investors have information about liquidity trading.

2.1 Financial assets and public information

In the financial market, there is a risky asset (e.g., a stock) whose payoff is $\tilde{v} = \tilde{v}_1 + \tilde{v}_2$, where $\tilde{v}_1 \sim N(\bar{v}_1, \tau_1^{-1})$ and $\tilde{v}_2 \sim N(\bar{v}_2, \tau_2^{-1})$. A public signal about $\tilde{v}_2$, $\tilde{s} = \tilde{v}_2 + \tilde{e}$, where $\tilde{e} \sim N(0, \tau_s^{-1})$, is disclosed to the financial market. The risky asset has a unit net supply and an endogenous price $\tilde{p}$. There is also a risk-free asset whose net return is normalized to $r_f = 0$.

2.2 Investors and private information

We consider two kinds of financial traders. First, there are noise traders who trade for liquidity reasons. Their demand for the risky asset is $\tilde{n} \sim N(0, \tau_n^{-1})$. Second, there is a continuum of informed investors. Each investor $i \in [0, 1]$ can observe two private signals. The first signal is about the first part of the asset payoff $\tilde{v}_1$, $\tilde{x}_i = \tilde{v}_1 + \tilde{\varepsilon}_i$, where $\tilde{\varepsilon}_i \sim N(0, \tau_x^{-1})$. The second signal is about the noise traders' demand $\tilde{n}$, $\tilde{z}_i = \tilde{n} + \tilde{\eta}_i$, where $\tilde{\eta}_i \sim N(0, \tau_z^{-1})$. We assume that $(\tilde{v}_1, \tilde{v}_2, \tilde{n}, \tilde{e}, \{\tilde{\varepsilon}_i\}_i, \{\tilde{\eta}_i\}_i)$ are mutually independent. Each informed investor has CARA utility with absolute risk-aversion coefficient $\gamma > 0$. Therefore, investor i 's problem when choosing the risky asset holding D_i is

$$\max_{D_i} \mathbb{E} [-\exp(-\gamma D_i(\tilde{v} - \tilde{p})) | \mathcal{F}_i], \quad (1)$$

where $\mathcal{F}_i = \{\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{p}\}$ is the investor's information set.

2.3 Timeline

In our model, the sequence of events is as follows. At $t = 0$, the precision τ_s of the public signal $\tilde{s}$ is determined. At $t = 1$, after observing their private signals and the public signal, the investors trade in the financial market, and the risky asset price $\tilde{p}$ is realized. At $t = 2$, the asset payoff $\tilde{v}$ is realized.

2.4 Definition of financial market equilibrium

Definition 1 (Financial market equilibrium). *Given the public signal precision τ_s , a financial market equilibrium consists of the investors' optimal risky asset holdings $\{D_i(\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{p})\}_{i \in [0,1]}$*

and the equilibrium asset price $\tilde{p}$, such that (i) each $D_i(\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{p})$ solves problem (1), and (ii) the financial market clears, i.e.,

$$\int_0^1 D_i(\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{p}) di + \tilde{n} = 1. \quad (2)$$

2.5 Equilibrium characterization

We use the standard “conjecture and verify” method to solve for the equilibrium. First, we conjecture that the equilibrium asset price has the form

$$\tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{s} + \alpha_3 \tilde{n}, \quad (3)$$

where $\alpha_0, \alpha_1, \alpha_2$ and α_3 are endogenous deterministic coefficients. Each investor i can transform the equilibrium price into a signal about the first part of the asset payoff,

$$\tilde{s}_{pi} = \frac{\tilde{p} - \alpha_0 - \alpha_2 \tilde{s} - \alpha_3 \mathbb{E}[\tilde{n}|\tilde{z}_i]}{\alpha_1} = \tilde{v}_1 + \frac{\alpha_3}{\alpha_1}(\tilde{n} - \mathbb{E}[\tilde{n}|\tilde{z}_i]), \quad (4)$$

whose precision is $\tau_p = (\text{Var}[\frac{\alpha_3}{\alpha_1}(\tilde{n} - \mathbb{E}[\tilde{n}|\tilde{z}_i])|\tilde{z}_i])^{-1} = (\frac{\alpha_1}{\alpha_3})^2(\tau_n + \tau_z)$. Solving problem (1), we know that an investor’s optimal risky asset holding can be expressed as

$$\tilde{D}_i = D_i(\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{p}) = \frac{\mathbb{E}[\tilde{v}|\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{s}_{pi}] - \tilde{p}}{\gamma \text{Var}[\tilde{v}|\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{s}_{pi}]}. \quad (5)$$

Substituting the optimal asset holdings into the market clearing condition Eq. (2) and rearranging terms, we can derive the implied price function,

$$\tilde{p} = \int_0^1 \mathbb{E}[\tilde{v}|\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{s}_{pi}] di - \gamma(1 - \tilde{n}) \left[\int_0^1 (\text{Var}[\tilde{v}|\tilde{x}_i, \tilde{z}_i, \tilde{s}, \tilde{s}_{pi}])^{-1} di \right]^{-1}. \quad (6)$$

Comparing Eq. (3) and Eq. (6), we have the following proposition that characterizes the financial market equilibrium.

Proposition 1 (Equilibrium asset price). *The equilibrium asset price can be expressed as $\tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{s} + \alpha_3 \tilde{n}$. The price coefficients $(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$ satisfy the following system of equations,*

$$\begin{aligned}\alpha_0 &= \frac{\tau_1 \bar{v}_1 - \gamma}{\tau_1 + \tau_x + \tau_p} + \frac{\tau_2 \bar{v}_2 - \gamma}{\tau_2 + \tau_s} \\ \alpha_1 &= \frac{\tau_x + \tau_p}{\tau_1 + \tau_x + \tau_p}, \\ \alpha_2 &= \frac{\tau_s}{\tau_2 + \tau_s}, \\ \alpha_3 &= \frac{\tau_p \tau_n \frac{\alpha_3}{\alpha_1}}{(\tau_1 + \tau_x + \tau_p)(\tau_n + \tau_z)} + \frac{\gamma}{\tau_1 + \tau_x + \tau_p} + \frac{\gamma}{\tau_2 + \tau_s},\end{aligned}\tag{7}$$

where $\tau_p = (\frac{\alpha_1}{\alpha_3})^2(\tau_n + \tau_z)$. Let $\xi \equiv \frac{\alpha_1}{\alpha_3}$, then ξ satisfies $g(\xi) = 0$, where

$$g(\xi) = k_3 \xi^3 - k_2 \xi^2 + k_1 \xi - k_0,\tag{8}$$

and $k_3 = \gamma \frac{\tau_n + \tau_z}{\tau_2 + \tau_s}$, $k_2 = \tau_z$, $k_1 = \gamma \left(1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s}\right)$, and $k_0 = \tau_x$. Moreover, the solutions to the equation $g(\xi) = 0$ are positive.

Proof. See Appendix A. □

As is standard in the rational expectations equilibrium literature (e.g., Farboodi and Veldkamp (2020)), $\xi \equiv \frac{\alpha_1}{\alpha_3}$ measures the informativeness of the asset price. Proposition (1) shows that an equilibrium price informativeness $\xi > 0$ satisfies Eq. (8). If there is a unique solution to Eq. (8), then the financial market equilibrium is unique.

Proposition 2 (Equilibrium uniqueness when $\tau_z = 0$). *If $\tau_z = 0$, then the equation $g(\xi) = 0$ has a unique solution for all $\tau_s \in [0, +\infty) \cup \{+\infty\}$. In other words, when informed investors have*

no information about liquidity demand, the financial market equilibrium is unique, regardless of the level of information disclosure.

Proof. See Appendix A. □

Proposition (2) shows that when the investors have no information about the liquidity demand (i.e., $\tau_z = 0$), the financial market equilibrium is unique, and the uniqueness is not affected by public information disclosure. However, with the development of financial data technology and quantitative trading, investors tend to acquire more information about liquidity demand (Farboodi and Veldkamp (2020)). The information about liquidity demand can result in multiple equilibria in a financial market (Ganguli and Yang (2009)). In the next section, we investigate how the disclosure of information about the firm's fundamentals affects the equilibrium multiplicity in the financial market, under the condition that investors can acquire information about liquidity demand (i.e., $\tau_z > 0$).

3 Information disclosure and equilibrium multiplicity

In this section, we investigate how the precision τ_s of the public signal about the second part of the asset payoff $\tilde{s} = \tilde{v}_2 + \tilde{e}$ affects the equilibrium multiplicity (i.e., the number of solutions to the equation $g(\xi) = 0$), under the condition that the precision τ_z of investors' private signals about the liquidity demand $\tilde{z}_i = \tilde{n} + \tilde{\eta}_i$ is greater than zero. We also discuss the empirical relevance of our theoretical results.

3.1 Disclosure and multiplicity

Proposition 3 (Disclosure and equilibrium multiplicity). *Assume that $\tau_z > 0$ and $\gamma^2 > 4\tau_z\tau_x$. If $\tau_s < \tau_s^* \equiv \tau_z^{-1}\sqrt{3\gamma^2(\tau_n + \tau_z)(\tau_1 + \tau_x)} - \tau_2$, then the financial market equilibrium*

is unique.² In contrast, if $\tau_s \rightarrow +\infty$, the financial market exhibits multiple equilibria (i.e., the equation $g(\xi) = 0$ has multiple solutions). In other words, multiple equilibria exist (a unique equilibrium exists) if the public signal is sufficiently precise (imprecise).

Proof. See Appendix A. □

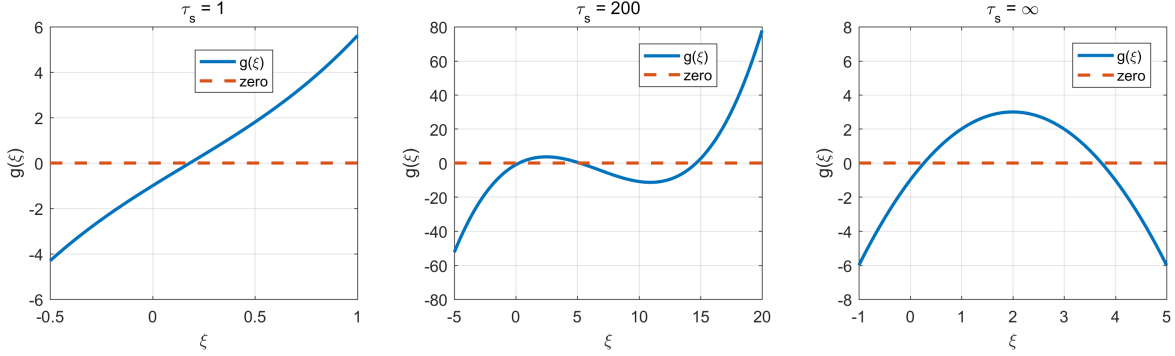


Figure 1: Information disclosure and equilibrium multiplicity. This figure plots the function $g(\xi)$ defined in Proposition (1) under different intensities of information disclosure τ_s . Parameter values: $\gamma = 4$, $\tau_x = 1$, $\tau_n = 1.5$, $\tau_z = 1$, $\tau_1 = 1$, and $\tau_2 = 1$.

Proposition 3 shows that when the investors have information about liquidity demand, the information disclosure about the firm's fundamentals can lead to multiple equilibria in the financial market. Figure 1 demonstrates the result intuitively: When $\tau_s = 1$, the function $g(\xi) = 0$ has a unique real root, indicating a unique financial market equilibrium. In contrast, when $\tau_s = 200$ or $\tau_s \rightarrow +\infty$, the function $g(\xi) = 0$ has more than one real root, indicating multiple equilibria.

To understand the effect of information disclosure on equilibrium multiplicity, follow-

²Notice that the condition $\tau_2 \leq \tau_z^{-1} \sqrt{3\gamma^2(\tau_n + \tau_z)(\tau_1 + \tau_x)}$ is required to make $\{\tau_s \in [0, +\infty) | \tau_s < \tau_s^*\} \neq \emptyset$.

ing Ganguli and Yang (2009), we investigate the investors' aggregate demand,

$$\tilde{D} = \int_0^1 \tilde{D}_i di = H_0 + H_x \tilde{v}_1 + H_s \tilde{s} - H_z \tilde{n} - H_p \tilde{p}, \quad (9)$$

where $(H_0, H_x, H_s, H_z, H_p)$ are positive and deterministic, and

$$H_z = H_z(\xi, \tau_s) = \frac{(\tau_2 + \tau_s)\tau_z \xi}{\gamma[\tau_1 + \tau_x + (\tau_n + \tau_z)\xi^2 + \tau_2 + \tau_s]} > 0. \quad (10)$$

Equations (9) and (10) demonstrate that the informed investors trade against liquidity demand (i.e., $H_z > 0$) when they have information about the liquidity demand (i.e., $\tau_z > 0$). When H_z is high (low), the investors trade against the liquidity demand aggressively (not aggressively), resulting in a low (high) impact of the liquidity demand on the asset price and thus a high (low) price informativeness. Notice that H_z is determined by both the investors' belief about the price informativeness ξ and the information disclosure τ_s .

Proposition 4 (The impact of ξ on H_z). *The derivative of $H_z(\xi, \tau_s)$ with respect to ξ is*

$$\frac{\partial H_z}{\partial \xi} = \frac{\tau_z(\tau_2 + \tau_s)[\tau_1 + \tau_x + \tau_2 + \tau_s - (\tau_n + \tau_z)\xi^2]}{\gamma[\tau_1 + \tau_x + (\tau_n + \tau_z)\xi^2 + \tau_2 + \tau_s]^2}. \quad (11)$$

Notably, we have $\frac{\partial H_z}{\partial \xi} > 0 \iff \xi^2 < (\tau_1 + \tau_x + \tau_2 + \tau_s)/(\tau_n + \tau_z)$.

Proof. The proof is by direct calculation. □

Proposition 4 suggests that when the public signal precision τ_s is sufficiently high, the aggressiveness of trading against liquidity demand H_z is increasing in the investors' belief about the price informativeness ξ . Therefore, if the investors believe that the price informativeness is high (low), then their trading will lead to a high (low) price informativeness. In other words, when the information disclosure is high, the equilibrium price

informativeness self-fulfills the investors' belief about the price informativeness, leading to multiple equilibria.

In contrast, when the public signal precision τ_s is low, the aggressiveness of trading against liquidity demand H_z tends to decrease in the investors' belief about the price informativeness ξ . In this case, the believe of a high (low) ξ will lead to a low (high) H_z . Therefore, although the investors believe that the price informativeness is high (low), their trading will result in a low (high) price informativeness. In other words, a low information disclosure prevents the equilibrium price informativeness from fulfilling (or strengthening) the investors' belief, resulting in a unique equilibrium.

3.2 Empirical relevance

Equilibrium multiplicity in theoretical models corresponds to excess volatility and crash risks (e.g., Ganguli and Yang (2009)). Therefore, Proposition 3 suggests that information disclosure may increase excess volatility, which destabilizes financial markets. This result seems to be contrary to what is commonly believed, but indeed there is empirical evidence showing that more precise information disclosure increases (or does not decrease) volatility. For example, Leuz and Verrecchia (2000) demonstrate that, after switching from the German to an international reporting standard, which increases the levels of disclosure, the stock price volatility of German firms did not decrease. Bushee and Noe (2000) find that an increase in disclosure ranking can attract trading based on short-term strategies and increase stock return volatility. Magnan and Xu (2008) show that excess stock return volatility is increasing in the disclosure of uncertain information. Our result in Proposition 3 is consistent with and provide theoretical explanations for these empirical observations.

4 Learning about noise and financial market

In this section, we investigate the impacts of the precision of noise trading information on equilibrium asset price informativeness, liquidity, expected price level, and price volatility, taking the precision of public information as exogenously given.³ Our theoretical results in this section suggest that investors' learning about noise trading tends to improve market quality, consistent with empirical observations showing that high-frequency tradings benefit financial markets (e.g., Hasbrouck and Saar (2013)).

4.1 Price informativeness

Proposition 5 (Learning about noise and price informativeness). *If the public signal precision $\tau_s \in (0, \tau_s^*)$ is exogenously given, then the asset price informativeness is increasing in the precision of noise trading information, i.e., $\frac{\partial \xi}{\partial \tau_z} > 0$.*

Proof. See Appendix A. □

Proposition 5 shows that investors' learning about noise traders' demand increases asset price informativeness. When the investors have more precise signals about noise trading, they can better filter out the noise contained in the asset price, which allows them to extract more information about the asset fundamentals. With increased amounts of fundamental information, the risk-averse investors perceive lower risks and thus trade more aggressively on their private information, increasing the price informativeness. This result is consistent with the empirical observation that high-frequency trading adds to

³In this section and Section 5, if there exist multiple equilibria in the financial market, we only focus on the equilibrium with the smallest price informativeness (i.e., the smallest real root of the equation $g(\xi) = 0$), as in Farboodi and Veldkamp (2020).

price discovery (e.g., Brogaard et al. (2010); Hasbrouck and Saar (2013)).

4.2 Liquidity

Proposition 6 (Learning about noise and liquidity). *Define*

$$\bar{\tau}_z \equiv \max \left\{ \frac{\gamma^2(\tau_1 + \tau_x)(\tau_2 + \tau_s + \tau_1 + \tau_x)^2}{\tau_x^2(\tau_2 + \tau_s)^2} - \tau_n, \frac{\tau_x \tau_n}{\tau_1 + \tau_2 + \tau_s} \right\}. \quad (12)$$

If $\tau_z > \bar{\tau}_z$, then the asset liquidity is increasing in the precision of noise trading information, i.e., $\frac{\partial(1/\alpha_3)}{\partial \tau_z} > 0$.

Proof. See Appendix A. □

Proposition 6 shows that the investors' learning about noise trading can increase the asset's liquidity (i.e., reduce the price impact of noise traders). Note that the investors can profit from trading against noise traders, because the noise traders make the asset price deviate from its fundamental value. A more precise signal about noise trading enables an investor to trade against noise traders more aggressively, reducing the noise traders' price impact and improving the liquidity. This result is consistent with the empirical evidence demonstrating that high-frequency trading benefits financial markets by providing liquidity (e.g., Brogaard et al. (2018); Jarnecic and Snape (2014)).

4.3 Asset price

Proposition 7 (Learning about noise and expected asset price). *The expected asset price increases in the precision of noise trading information, i.e., $\frac{\partial \mathbb{E}[\bar{p}]}{\partial \tau_z} > 0$.*

Proof. See Appendix A. □

Proposition 7 shows that investors' learning about noise trading tends to increase the asset price. This result can be understood from two perspectives. First, investors' learning about noise trading allows them to extract more fundamental information from the asset price and increases the price informativeness (see Proposition 5), which reduces the risks faced by the investors, decreasing the risk premium. Second, investors' learning about noise trading increases the liquidity (see Proposition 6), decreasing the liquidity premium. This result also predicts that high-frequency trading tends to reduce the cost of capital, which can be empirically tested.

4.4 Price volatility

By the equilibrium price function Eq. (3), the volatility of the asset price can be expressed as

$$\text{Var}[\tilde{p}] = \underbrace{\alpha_1^2 \text{Var}[\tilde{v}_1] + \alpha_2^2 \text{Var}[\tilde{s}]}_{\text{Information-driven}} + \underbrace{\alpha_3^2 \text{Var}[\tilde{n}]}_{\text{Liquidity-driven}}.$$

As in Ozsoylev and Walden (2011), the price volatility can also be decomposed as the “information-driven” volatility and the “liquidity-driven” volatility. The following proposition shows that investors' learning about noise trading has different impacts on different components of price volatility.

Proposition 8 (Learning about noise and different components of volatility). *The information-driven price volatility is increasing in the precision of noise trading signal, i.e., $\frac{\partial(\text{Var}[\alpha_1 \tilde{v}_1] + \text{Var}[\alpha_2 \tilde{s}])}{\partial \tau_z} > 0$. Moreover, if $\tau_z > \bar{\tau}_z$, where $\bar{\tau}_z$ is given by Eq. (12), then the liquidity-driven price volatility is decreasing in the precision of noise trading signal, i.e., $\frac{\partial \text{Var}[\alpha_3 \tilde{n}]}{\partial \tau_z} < 0$.*

Proof. See Appendix A. □

Consistent with the results demonstrating that noise trading information increases price informativeness (see Proposition 5) and liquidity (see Proposition 6), Proposition 8 shows that investors' learning about noise trading increases information-driven price volatility and decreases liquidity-driven price volatility. Regarding the impact of the precision of noise trading information on the total price volatility, it can be shown that

$$\frac{\partial \text{Var}[\tilde{p}]}{\partial \tau_z} < 0 \iff \text{Var}[\tilde{n}] > -\frac{\alpha_1}{\alpha_3} \frac{\partial \alpha_1 / \partial \tau_z}{\partial \alpha_3 / \partial \tau_z} \text{Var}[\tilde{v}_1],$$

suggesting that the investors' learning about noise trading tends to decrease price volatility when noise trading prevails in the market. This result is consistent with the empirical observation that high-frequency trading dampens volatility (e.g., Brogaard et al. (2010)).

5 Learning about noise and information disclosure

In this section, we endogenize the public signal precision τ_s , assuming that the precision is optimally chosen by a manager. We then investigate how the investors' learning about the noise trading affects the information disclosure and the financial market.

5.1 Optimal information disclosure

Assume that at $t = 0$, a manager chooses the public signal precision to maximize the expected asset price net of the cost of disclosure, i.e.,

$$\max_{\tau_s \in [0, +\infty)} \mathbb{E}[\tilde{p}] - C(\tau_s), \tag{13}$$

where the cost of disclosure has the form $C(\tau_s) = \frac{c}{2}(\tau_s)^2$, and $c > 0$. To better understand the firm's incentive to disclose information, we first investigate how the information disclosure τ_s affects the expected asset price $\mathbb{E}[\tilde{p}]$.

Proposition 9 (Information disclosure and price informativeness). *The equilibrium price informativeness ξ increases in the public signal precision τ_s , i.e., $\frac{\partial \xi}{\partial \tau_s} > 0$, where ξ is determined by Eq. (8).*

Proof. See Appendix A. □

Proposition 9 shows that the disclosure of information about the second part of the asset payoff $\tilde{v}_2$ increases the amount of information about the first part of the asset payoff $\tilde{v}_1$ revealed by the asset price. The reason is that more information about $\tilde{v}_2$ reduces the risk of the total asset payoff $\tilde{v} = \tilde{v}_1 + \tilde{v}_2$ perceived by the risk-averse investors, making them trade more aggressively and incorporate more private information about $\tilde{v}_1$ into the asset price.

Proposition 10 (Information disclosure and expected asset price). *The expected asset price increases in the public signal precision, i.e., $\frac{\partial \mathbb{E}[\tilde{p}]}{\partial \tau_s} > 0$.*

Proof. The result is obvious by Proposition 9 and the following Eq. (15). □

Proposition 10 shows that the information disclosure increases the expected asset price, indicating that the firm manager has an incentive to disclose. To understand this result, notice that the expected asset price can be expressed as

$$\mathbb{E}[\tilde{p}] = \bar{v}_1 + \bar{v}_2 - \frac{\gamma}{\tau_1 + \tau_x + \xi^2(\tau_n + \tau_z)} - \frac{\gamma}{\tau_2 + \tau_s}, \quad (14)$$

so the derivative of $\mathbb{E}[\tilde{p}]$ with respect to τ_s is

$$\frac{\partial \mathbb{E}[\tilde{p}]}{\partial \tau_s} = \underbrace{\frac{2\gamma\xi(\tau_n + \tau_z)}{[\tau_1 + \tau_x + \xi^2(\tau_n + \tau_z)]^2} \frac{\partial \xi}{\partial \tau_s}}_{\text{Indirect Effect}} + \underbrace{\frac{\gamma}{(\tau_2 + \tau_s)^2}}_{\text{Direct Effect}}. \quad (15)$$

Eq. (15) shows that the information disclosure has two effects on the expected asset price, one of which is a direct effect. When the information disclosure τ_s increases, the investors face a lower risk of the second part of the asset payoff $\tilde{v}_2$, so the discount with respect to the risk of the second part of the payoff (i.e., $\frac{\gamma}{\tau_2 + \tau_s}$) is smaller. The other effect is an indirect effect. Recall that more disclosure of information about $\tilde{v}_2$ also reduces the uncertainty of $\tilde{v}_1$, because the asset price can provide more information about $\tilde{v}_1$ (see Proposition 9). Therefore, the disclosure of information about $\tilde{v}_2$ also reduces the discount with respect to the risk of $\tilde{v}_1$ (i.e., $\frac{\gamma}{\tau_1 + \tau_x + \xi^2(\tau_n + \tau_z)}$). As a result, the expected asset price is increasing in the amount of disclosed information about the second part of asset payoff.

5.2 The impacts of learning about noise on optimal disclosure

In this subsection, we investigate the impact of the noise trading signal precision τ_z on the optimal public signal precision τ_s . Due to the complexity of the expected price $\mathbb{E}[\tilde{p}]$ as a function of τ_s , we numerically solve for the optimal disclosure problem (13) under different values of τ_z .

Discontinuity. The left panel of Figure 2 shows that the investors' learning about noise trading has a non-trivial impact on the optimal disclosure of fundamental information. First, the optimal public signal precision τ_s is not continuous in the precision of noise trading signal τ_z . In our numerical example, when τ_z increases from 12.75 to 12.76, the optimal τ_s jumps from 2.86 to 4.05. The reason for this discontinuity is that the objective

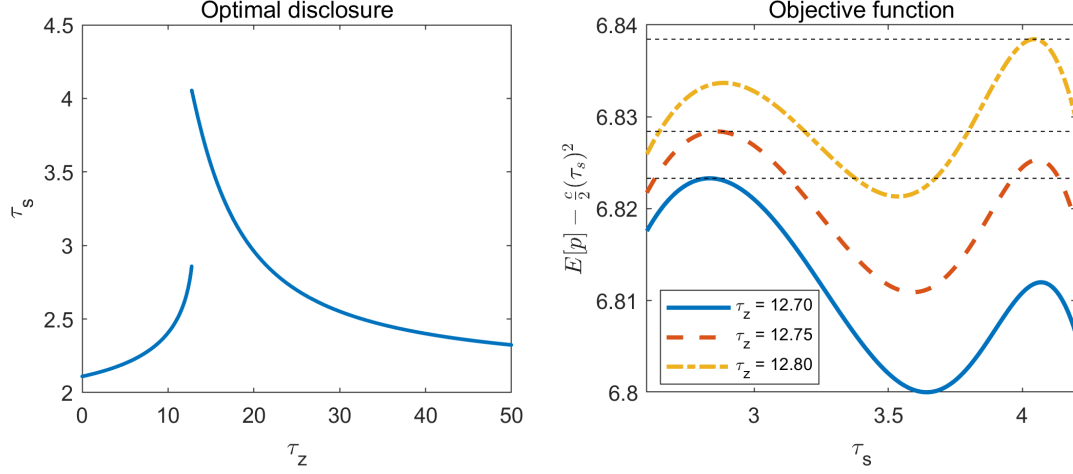


Figure 2: Optimal information disclosure. The left panel plots the optimal information disclosure τ_s as a function of the precision τ_z of private signals about noise trading. The right panel plots the objective functions $\mathbb{E}[p] - C(\tau_s)$ when τ_z takes different values. Parameter values: $\gamma = 4$, $\tau_x = 1$, $\tau_n = 1.5$, $\tau_z = 1$, $\tau_1 = 1$, $\tau_2 = 1$, $\bar{v}_1 = 5$, $\bar{v}_2 = 5$, and $c = 0.2$.

function $\mathbb{E}[p] - C(\tau_s)$ is not concave in τ_s . The right panel of Figure 2 shows that the objective function can have two local maxima. Which local maximum is the global maximum depends on the precision of noise trading signal τ_z . In the example shown in the right panel of Figure 2, when $\tau_z = 12.70$ or 12.75 , the left local maximum is larger than the right local maximum. In contrast, when $\tau_z = 12.80$, the left local maximum is lower than the right local maximum. Therefore, a very small increase in the investors' learning about noise trading can lead to a surge in the disclosure of information about fundamentals.

Non-monotonicity. Second, the optimal public signal precision τ_s is non-monotonic in the precision of noise trading signal τ_z . Recall that τ_s has an indirect effect on the expected asset price $\mathbb{E}[\tilde{p}]$ through the price signal precision $\tau_p = \xi^2(\tau_n + \tau_z)$ (see Eq. (15)). One can also decompose the indirect effect of τ_s as follows,

$$\frac{\partial[-\gamma(\tau_1 + \tau_x + \tau_p)^{-1}]}{\partial \tau_s} = \frac{\gamma}{(\tau_1 + \tau_x + \tau_p)^2} \frac{\partial \tau_p}{\partial \xi} \frac{\partial \xi}{\partial \tau_s}. \quad (16)$$

When τ_z is low, a marginal increase in τ_z increases the sensitivity of τ_p to ξ (i.e., $\frac{\partial \tau_p}{\partial \xi}$), which magnifies the positive impact of τ_s on ξ , increasing the indirect effect of τ_s and thus the marginal effect of τ_s on $\mathbb{E}[\tilde{p}]$. As a result, the optimal level of τ_s increases in τ_z when τ_z is low. In contrast, when τ_z is high, a further increase in τ_z increases τ_p and decreases the term $\frac{\gamma}{(\tau_1 + \tau_x + \tau_p)^2}$ significantly, mitigating the marginal effect of τ_s on $\mathbb{E}[\tilde{p}]$. Consequently, the optimal level of τ_s decreases in τ_z when τ_z is high.

5.3 The impacts of learning about noise on financial market

In this section, we investigate the impacts of investors' learning about noise trading on equilibrium market variables. In Figure 3, we plot the equilibrium price informativeness $\xi \equiv \frac{\alpha_1}{\alpha_3}$, liquidity $\frac{1}{\alpha_3}$, expected price $\mathbb{E}[\tilde{p}]$, and price volatility $\text{Var}[\tilde{p}]$ as functions of the precision of signals about noise trading τ_z , when the public signal precision τ_s is optimally determined by problem (13).

The upper left panel of Figure 2 shows that the equilibrium price informativeness is increasing in the precision of noise trading signals τ_z . The reason is that when the investors have more precise information about noise trading, they can better filter out the noise contained in the asset price, extracting more information about the fundamentals from the price. With a lower uncertainty about the fundamentals, the informed investors trade more aggressively, injecting more information about the fundamentals into the asset price. Moreover, the increased aggressiveness of informed trading against noise trading decreases the price impact of the noisy order flows and thus increases the liquidity (see the upper right panel of Figure 2).

The lower left panel of Figure 2 shows that the expected asset price $\mathbb{E}[\tilde{p}]$ is increasing in the precision of noise trading signals τ_z . As has been explained, a higher τ_z reduces

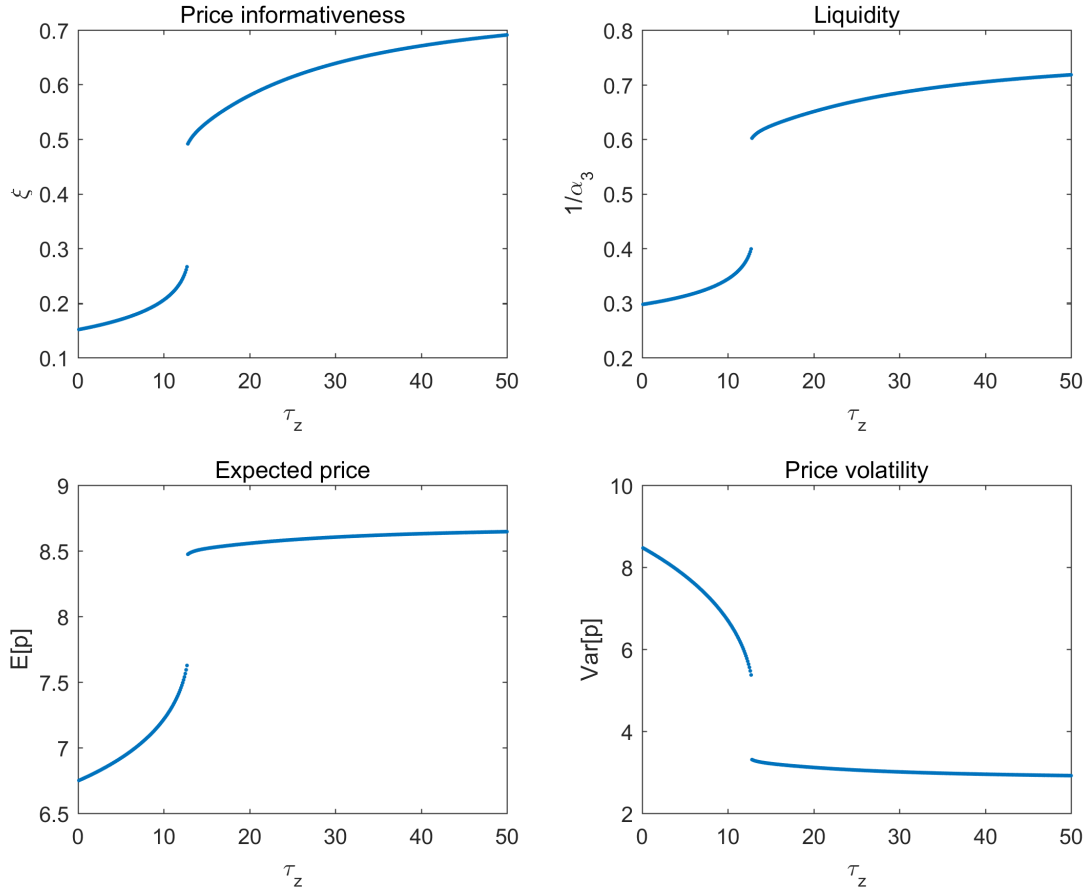


Figure 3: Equilibrium market outcomes. This figure plots equilibrium price informativeness $\xi \equiv \frac{\alpha_1}{\alpha_3}$, liquidity $\frac{1}{\alpha_3}$, expected price $\mathbb{E}[\tilde{p}]$, and price volatility $\text{Var}[\tilde{p}]$ as functions of the precision of signals about noise trading τ_z , when information disclosure τ_s is endogenous. Parameter values: $\gamma = 4$, $\tau_x = 1$, $\tau_n = 1.5$, $\tau_z = 1$, $\tau_1 = 1$, $\tau_2 = 1$, $\bar{v}_1 = 5$, $\bar{v}_2 = 5$, and $c = 0.2$.

the investors' perceived risk of the asset payoff. Therefore, the investors require a lower discount with respect to risk when buying the asset, increasing the expected asset price. Equivalently, the expected return, $\mathbb{E}[\tilde{v} - \tilde{p}]$, is decreasing in τ_z . Moreover, the lower right panel of Figure 2 shows that the price volatility is decreasing in τ_z . The reason is that a higher τ_z reduces the price impact of noise traders, decreasing the noise-driven volatility. Notice that, except for the discontinuity, the impacts of learning about noise trading on equilibrium market variables with endogenous disclosure are consistent with those with exogenous disclosure (see Section 4).

5.4 Discussion: High-frequency trading, disclosure, and instability

As suggested by Farboodi and Veldkamp (2020), investors' trading based on information about noise traders' order flow in the model resembles high-frequency trading in reality. To understand this relationship, we can consider the "front running", which is a kind of high-frequency trading strategy.⁴ For example, a trader submit a market order to a broker to buy some shares of a stock. Upon observing the trader's order flow, the broker who engages in "front running" quickly buys the stock using its own account. After that, the broker places the trader's order, pushing up the stock price, and then sells the stocks she just bought at this increased price. Clearly, in the "front running", the broker profits from her private information about the trader's order flow.

In economic theories, market fragility is often represented by the existence of multiple equilibria (e.g., Diamond and Dybvig (1983); Benhabib et al. (2019); Goldstein et al. (2024)), because equilibrium variables can change (e.g., prices can plunge) without any changes in economic fundamentals. Therefore, Propositions 2 and 3 suggest that high-

⁴See https://en.wikipedia.org/wiki/Front_running.

frequency trading is a source of financial market fragility. This result is consistent with that in Ganguli and Yang (2009). Proposition 3 further suggests that information disclosure, which is commonly thought of as a method to stabilize financial markets, can strengthen, rather than mitigate, the instability caused by high-frequency trading.

Does restricting high-frequency trading stabilize financial markets? While we can be sure that a market without high-frequency trading is more stable than a market with it, as suggested by Propositions 2 and 3, our results in Figure 3 show a market crash can occur as the intensity of high-frequency trading gradually decreases from a high level to a low level. The market crash is caused by a large (i.e., discontinuous) decrease in public (voluntary) information disclosure in response to a small decrease in high-frequency trading. Our results remind policy makers of the potential negative spillover effects of their regulations on high-frequency trading.

6 Conclusion

In this paper, we analyze a model in which a part of an asset's payoff is publicly disclosed and rational investors can learn about noise traders' demand for the asset. We show that reducing information disclosure can eliminate multiple equilibria caused by rational investors' learning about noise traders' demand. We also find that the optimal information disclosure and thus the equilibrium asset price, volatility, liquidity, and informativeness are discontinuous in the precision of rational investors' information about noise trading. Our theoretical results suggest that (i) less information disclosure mitigates the market fragility caused by high-frequency trading, and (ii) restrictions on high-frequency trading can also result in market crashes.

Appendix

A Proofs of propositions

A.1 Proof of Proposition 1

Proof. Using Bayes' rule, the implied price function Eq. (6) can be expressed as

$$\tilde{p} = \int_0^1 \left(\frac{\tau_1 \bar{v}_1 + \tau_x \tilde{x}_i + \tau_p \tilde{s}_{pi}}{\tau_1 + \tau_x + \tau_p} + \frac{\tau_2 \bar{v}_2 + \tau_s \tilde{s}}{\tau_2 + \tau_s} \right) di - \gamma (1 - \tilde{n}) \left(\frac{1}{\tau_1 + \tau_x + \tau_p} + \frac{1}{\tau_2 + \tau_s} \right),$$

where $\tilde{s}_{pi} = \tilde{v}_1 + \xi^{-1}(\tilde{n} - \mathbb{E}[\tilde{n}|\tilde{z}_i])$ and $\xi = \frac{\alpha_1}{\alpha_3}$. By the law of large numbers, we have $\int_0^1 \tilde{x}_i di = \tilde{v}_1$ and $\int_0^1 \tilde{z}_i di = \tilde{n}$. Therefore, the average price signal can be expressed as

$$\int_0^1 \tilde{s}_{pi} di = \tilde{v}_1 + \xi^{-1} \left(\tilde{n} - \frac{\tau_z}{\tau_n + \tau_z} \int_0^1 \tilde{z}_i di \right) = \tilde{v}_1 + \frac{\xi^{-1} \tau_n}{\tau_n + \tau_z} \tilde{n}.$$

Calculating the integration in the implied price function and rearranging terms, we have

$$\begin{aligned} \tilde{p} = & \left(\frac{\tau_1 \bar{v}_1 - \gamma}{\tau_1 + \tau_x + \tau_p} + \frac{\tau_2 \bar{v}_2 - \gamma}{\tau_2 + \tau_s} \right) + \frac{\tau_x + \tau_p}{\tau_1 + \tau_x + \tau_p} \tilde{v}_1 + \frac{\tau_s}{\tau_2 + \tau_s} \tilde{s} \\ & + \left[\frac{\tau_p \tau_n \frac{\alpha_3}{\alpha_1}}{(\tau_1 + \tau_x + \tau_p)(\tau_n + \tau_z)} + \frac{\gamma}{\tau_1 + \tau_x + \tau_p} + \frac{\gamma}{\tau_2 + \tau_s} \right] \tilde{n}, \end{aligned}$$

which yields Eq. (7). From Eq. (7) it is obvious that $\xi \equiv \frac{\alpha_1}{\alpha_3}$ is determined by the following equation,

$$\xi = \frac{\frac{\tau_x + \tau_p}{\tau_1 + \tau_x + \tau_p}}{\frac{\tau_p \tau_n \xi^{-1}}{(\tau_1 + \tau_x + \tau_p)(\tau_n + \tau_z)} + \frac{\gamma}{\tau_1 + \tau_x + \tau_p} + \frac{\gamma}{\tau_2 + \tau_s}}, \quad (17)$$

or equivalently,

$$g(\xi) = \gamma \frac{\tau_n + \tau_z}{\tau_2 + \tau_s} \xi^3 - \tau_z \xi^2 + \gamma \left(1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s} \right) \xi - \tau_x = 0,$$

Now we prove that if $\tau_2 + \tau_s < \infty$, then there exists at least one positive solution to the equation $g(\xi) = 0$. Notice that $\lim_{\xi \rightarrow -\infty} g(\xi) = -\infty$ and $\lim_{\xi \rightarrow +\infty} g(\xi) = +\infty$. Therefore, by the Intermediate Value Theorem, the equation $g(\xi) = 0$ has at least one real root. We can calculate that $g'(\xi) = 3k_3\xi^2 - 2k_2\xi + k_1$. Since $k_j > 0, j = 0, 1, 2, 3$, we know that $g'(\xi) > 0, \forall \xi \leq 0$. Moreover, we have $g(0) = -k_0 < 0$, so $g(\xi) < 0, \forall \xi \leq 0$. Therefore, all the real solutions to the equation $g(\xi) = 0$ are positive. $\square$

A.2 Proof of Proposition 2

Proof. If $\tau_z = 0$, then $k_2 = 0$, so $g(\xi) = k_3\xi^3 + k_1\xi - k_0$. In this case, it is obvious that $g'(\xi) > 0, \forall \xi \in \mathbb{R}$. Therefore, the solution to $g(\xi) = 0$ is unique. $\square$

A.3 Proof of Proposition 3

Proof. The first order derivative of $g(\xi)$ is $g'(\xi) = 3k_3\xi^2 - 2k_2\xi + k_1$. We can calculate that

$$\Delta \equiv 4k_2^2 - 12k_1k_3 < 0 \iff \tau_z^2 < 3\gamma^2 \left(\frac{\tau_n + \tau_z}{\tau_2 + \tau_s} \right) \left(1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s} \right).$$

Therefore, a sufficient condition for $\Delta < 0$ is

$$\tau_z^2 < 3\gamma^2 \left(\frac{\tau_n + \tau_z}{\tau_2 + \tau_s} \right) \left(1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s} \right) - 3\gamma^2 \left(\frac{\tau_n + \tau_z}{\tau_2 + \tau_s} \right) = \frac{3\gamma^2(\tau_n + \tau_z)(\tau_1 + \tau_x)}{(\tau_2 + \tau_s)^2}, \quad (18)$$

or equivalently,

$$\tau_s < \tau_s^* \equiv \sqrt{\frac{3\gamma^2(\tau_n + \tau_z)(\tau_1 + \tau_x)}{\tau_z^2}} - \tau_2. \quad (19)$$

Now we prove that the financial market equilibrium is unique when $\tau_s < \tau_s^*$. If $\tau_s < \tau_s^*$, then $\Delta = 4k_2^2 - 12k_1k_3 < 0$. Therefore, we have $g'(\xi) > 0, \forall \xi$. Moreover, it's obvious that $\lim_{\xi \rightarrow -\infty} g(\xi) = -\infty$ and $\lim_{\xi \rightarrow +\infty} g(\xi) = +\infty$, so the equation $g(\xi) = 0$ has a unique real root.

Now we prove that the financial market exhibits multiple equilibria when $\tau_s \rightarrow +\infty$. Notice that $\lim_{\tau_s \rightarrow +\infty} k_3 = 0$ and $\lim_{\tau_s \rightarrow +\infty} k_1 = \gamma$. Therefore, when $\tau_s \rightarrow +\infty$, we have $g(\xi) = -\tau_z\xi^2 + \gamma\xi - \tau_x$. Solving the quadratic equation $g(\xi) = 0$ yields two solutions,

$$\xi^+ = \frac{\gamma + \sqrt{\gamma^2 - 4\tau_z\tau_x}}{2\tau_z}, \quad \xi^- = \frac{\gamma - \sqrt{\gamma^2 - 4\tau_z\tau_x}}{2\tau_z}. \quad (20)$$

This result is similar to those in Ganguli and Yang (2009) and Farboodi and Veldkamp (2020). □

A.4 Proof of Proposition 5

Proof. A transformation of Eq. (17) yields $\xi = h(\xi, \tau_z)$, where

$$h(\xi, \tau_z) = \frac{\tau_x + \tau_z\xi^2}{\gamma[1 + (\tau_2 + \tau_s)^{-1}(\tau_1 + \tau_x + (\tau_n + \tau_z)\xi^2)]}.$$

Taking derivative of $h(\xi, \tau_z)$ with respect to τ_z , it can be shown that

$$\frac{\partial h(\xi, \tau_z)}{\partial \tau_z} = \frac{\xi^2[1 + (\tau_2 + \tau_s)^{-1}(\tau_1 + \tau_n\xi^2)]}{\gamma[1 + (\tau_2 + \tau_s)^{-1}(\tau_1 + \tau_x + (\tau_n + \tau_z)\xi^2)]^2} > 0, \quad \forall \xi > 0.$$

One can also observe that $h(0, \tau_z) > 0$. Recall from Proposition 2 that the equation of ξ , $\xi = h(\xi, \tau_z)$, has a unique real root. Let $\tau_z^L > 0$, $\Delta\tau_z > 0$, and $\tau_z^H = \tau_z^L + \Delta\tau_z$. Let $\xi(\tau_z^H)$ be the solution to $\xi = h(\xi, \tau_z^H)$, i.e., $\xi(\tau_z^H) = h(\xi(\tau_z^H), \tau_z^H)$. Recall that $\frac{\partial h(\xi, \tau_z)}{\partial \tau_z} > 0$ and $\tau_z^L < \tau_z^H$, so $h(\xi(\tau_z^H), \tau_z^L) < h(\xi(\tau_z^H), \tau_z^H) = \xi(\tau_z^H)$, or equivalently, $h(\xi(\tau_z^H), \tau_z^L) - \xi(\tau_z^H) < 0$. Also recall that $h(0, \tau_z^L) - 0 > 0$. Therefore, by the intermediate value theorem, there exists a $\xi(\tau_z^L) \in (0, \xi(\tau_z^H))$, such that $h(\xi(\tau_z^L), \tau_z) - \xi(\tau_z^L) > 0$, or equivalently, $\xi(\tau_z^L) = h(\xi(\tau_z^L), \tau_z)$. Now we have proved that $\xi(\tau_z^L) < \xi(\tau_z^L + \Delta\tau_z)$, $\forall \tau_z^L, \Delta\tau_z > 0$. Therefore, we have

$$\xi'(\tau_z) = \lim_{\Delta\tau_z \rightarrow 0} \frac{\xi(\tau_z + \Delta\tau_z) - \xi(\tau_z)}{\Delta\tau_z} > 0,$$

which completes the proof. $\square$

A.5 Proof of Proposition 6

Proof. From Eq. (7), we know that the price impact of noise traders can be expressed as

$$\alpha_3 = \frac{\xi\tau_n}{\tau_1 + \tau_x + \xi^2(\tau_n + \tau_z)} + \frac{\gamma}{\tau_1 + \tau_x + \xi^2(\tau_n + \tau_z)} + \frac{\gamma}{\tau_2 + \tau_s}, \quad (21)$$

where $\xi = \frac{\alpha_1}{\alpha_3}$. By Proposition 5, we know that $\frac{\partial \xi}{\partial \tau_z} > 0$. Therefore, it can be observed that the second term on the right-hand side of Eq. (21) is decreasing in τ_z . Notice that the first term on the right-hand side of Eq. (21) by $\tau_n/f(\xi)$, where $f(\xi) = \frac{\tau_1 + \tau_x}{\xi} + (\tau_n + \tau_z)\xi$. It is easy to show that $f'(\xi) > 0 \iff \xi^2 > \frac{\tau_1 + \tau_x}{\tau_n + \tau_z}$.

Denote $t = \xi^2$. Therefore, Eq. (17) is equivalent to $\sqrt{t} = g(t)$, where

$$g(t) \equiv \frac{\tau_x + \tau_z t}{\gamma \left[1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s} + \left(\frac{\tau_n + \tau_z}{\tau_2 + \tau_s} \right) t \right]}.$$

Taking derivative with respect to t , it can be shown that

$$g'(t) = \frac{\tau_z \left(1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s}\right) - \tau_x \left(\frac{\tau_n + \tau_z}{\tau_2 + \tau_s}\right)}{\gamma \left[1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s} + \left(\frac{\tau_n + \tau_z}{\tau_2 + \tau_s}\right) t\right]^2}.$$

Therefore, we have

$$g'(t) > 0 \iff \tau_z > \frac{\tau_x \tau_n}{\tau_1 + \tau_2 + \tau_s}.$$

Assume that $\tau_z > \frac{\tau_x \tau_n}{\tau_1 + \tau_2 + \tau_s}$, so that $g'(t) > 0$. Since $t = \xi^2$, we know that $h'(\xi) > 0$, where

$$h(\xi) = \frac{\tau_x + \tau_z \xi^2}{\gamma [1 + (\tau_2 + \tau_s)^{-1} (\tau_1 + \tau_x + (\tau_n + \tau_z) \xi^2)]}.$$

Therefore, we have

$$\xi = h(\xi) > h(0) = \frac{\tau_x}{\gamma \left(1 + \frac{\tau_1 + \tau_x}{\tau_2 + \tau_s}\right)},$$

so a sufficient condition for $\xi^2 > \frac{\tau_1 + \tau_x}{\tau_n + \tau_z}$ is $[h(0)]^2 > \frac{\tau_1 + \tau_x}{\tau_n + \tau_z}$, or equivalently,

$$\tau_z > \frac{\gamma^2 (\tau_1 + \tau_x) (\tau_2 + \tau_s + \tau_1 + \tau_x)^2}{\tau_x^2 (\tau_2 + \tau_s)^2} - \tau_n.$$

Now we prove the statement in Proposition 6. When $\tau_z > \bar{\tau}_z$, where $\bar{\tau}_z$ is defined in Eq. (12), we have $\xi^2 > \frac{\tau_1 + \tau_x}{\tau_n + \tau_z}$, so $f'(\xi) > 0$. Therefore, together with the result that $\frac{\partial \xi}{\partial \tau_z} > 0$ (see Proposition 5), it can be shown that $f'(\xi) \frac{\partial \xi}{\partial \tau_z} > 0$, which implies that $\tau_n / f(\xi)$ is decreasing in τ_z . Therefore, α_3 is decreasing in τ_z , or equivalently, $1/\alpha_3$ is increasing in τ_z , which completes the proof. $\square$

A.6 Proof of Proposition 7

Proof. By Eq. (7), the unconditional expected asset price can be expressed as $\mathbb{E}[\tilde{p}] = \bar{v}_1 + \bar{v}_2 - \frac{\gamma}{\tau_1 + \tau_x + \xi^2(\tau_n + \tau_z)} - \frac{\gamma}{\tau_2 + \tau_s}$. By Proposition 5, we have $\frac{\partial \xi}{\partial \tau_z} > 0$. It immediately follows that $\frac{\partial \mathbb{E}[\tilde{p}]}{\partial \tau_z} > 0$. $\square$

A.7 Proof of Proposition 8

Proof. From Eq. (7), it can be shown that $\frac{\partial \alpha_1}{\partial \tau_z} = \frac{\tau_1}{(\tau_1 + \tau_x + \tau_p)^2} \frac{\partial \tau_p}{\partial \tau_z}$. Recall that $\tau_p = \xi^2(\tau_n + \tau_z)$. Moreover, Proposition 5 have shown that $\frac{\partial \xi}{\partial \tau_z} > 0$. Therefore, it is obvious that $\frac{\partial \tau_p}{\partial \tau_z} > 0$ and thus $\frac{\partial \alpha_1}{\partial \tau_z} > 0$. It immediately follows that $\frac{\partial \text{Var}[\alpha_1 \tilde{v}_1]}{\partial \tau_z} > 0$. Notice that $\alpha_2 = \frac{\tau_s}{\tau_2 + \tau_s}$ is not affected by τ_z , so $\frac{\partial (\text{Var}[\alpha_1 \tilde{v}_1] + \text{Var}[\alpha_2 \tilde{s}])}{\partial \tau_z} > 0$. From Proposition 6 we know that when $\tau_z > \bar{\tau}_z$, $\frac{\partial \alpha_3}{\partial \tau_z} < 0$, so $\frac{\partial \text{Var}[\alpha_3 \tilde{n}]}{\partial \tau_z} < 0$. $\square$

A.8 Proof of Proposition 9

Proof. This proof is similar to that of Proposition 5. A transformation of Eq. (17) yields $\xi = h(\xi, \tau_s)$, where

$$h(\xi, \tau_s) = \frac{\tau_x + \tau_z \xi^2}{\gamma[1 + (\tau_2 + \tau_s)^{-1}(\tau_1 + \tau_x + (\tau_n + \tau_z)\xi^2)]}.$$

One can observe that $\frac{\partial h(\xi, \tau_s)}{\partial \tau_s} > 0$, $\forall \xi$, and $h(0, \tau_s) > 0$. Recall from the proof of Proposition 1 that the equation of ξ , $\xi = h(\xi, \tau_s)$, has at least one real root. In what follows we only focus on the smallest solution to the equation $\xi = h(\xi, \tau_s)$. Let $\tau_s^L > 0$, $\Delta \tau_s > 0$, and $\tau_s^H = \tau_s^L + \Delta \tau_s$. Let $\xi(\tau_s^H)$ be the solution to $\xi = h(\xi, \tau_s^H)$, i.e., $\xi(\tau_s^H) = h(\xi(\tau_s^H), \tau_s^H)$. Recall that $\frac{\partial h(\xi, \tau_s)}{\partial \tau_s} > 0$ and $\tau_s^L < \tau_s^H$, so $h(\xi(\tau_s^H), \tau_s^L) < h(\xi(\tau_s^H), \tau_s^H) = \xi(\tau_s^H)$, or equivalently,

$h(\xi(\tau_s^H), \tau_s^L) - \xi(\tau_s^H) < 0$. Also recall that $h(0, \tau_s^L) - 0 > 0$. Therefore, by the intermediate value theorem, there exists a $\xi(\tau_s^L) \in (0, \xi(\tau_s^H))$, such that $h(\xi(\tau_s^L), \tau_s) - \xi(\tau_s^L) > 0$, or equivalently, $\xi(\tau_s^L) = h(\xi(\tau_s^L), \tau_s)$. Now we have proved that $\xi(\tau_s^L) < \xi(\tau_s^L + \Delta\tau_s)$, $\forall \tau_s^L, \Delta\tau_s > 0$. Therefore, we have

$$\xi'(\tau_s) = \lim_{\Delta\tau_s \rightarrow 0} \frac{\xi(\tau_s + \Delta\tau_s) - \xi(\tau_s)}{\Delta\tau_s} > 0,$$

which completes the proof. □

References

- Benhabib, J., Liu, X., Wang, P., 2019. Financial markets, the real economy, and self-fulfilling uncertainties. *The Journal of Finance* 74, 1503–1557.
- Brogaard, J., Carrion, A., Moyaert, T., Riordan, R., Shkilko, A., Sokolov, K., 2018. High frequency trading and extreme price movements. *Journal of Financial Economics* 128, 253–265.
- Brogaard, J., et al., 2010. High frequency trading and its impact on market quality. Northwestern University Kellogg School of Management Working Paper 66.
- Bushee, B.J., Noe, C.F., 2000. Corporate disclosure practices, institutional investors, and stock return volatility. *Journal of accounting research* , 171–202.
- Christensen, P.O., Qin, Z., 2014. Information and heterogeneous beliefs: Cost of capital, trading volume, and investor welfare. *The Accounting Review* 89, 209–242.
- Diamond, D.W., Dybvig, P.H., 1983. Bank runs, deposit insurance, and liquidity. *Journal of political economy* 91, 401–419.

- Farboodi, M., Veldkamp, L., 2020. Long-run growth of financial data technology. *American Economic Review* 110, 2485–2523.
- Ganguli, J.V., Yang, L., 2009. Complementarities, multiplicity, and supply information. *Journal of the European Economic Association* 7, 90–115.
- Goldstein, I., Huang, C., Yang, L., 2024. Fragility of financial markets. Available at SSRN .
- Goldstein, I., Yang, L., 2015. Information diversity and complementarities in trading and information acquisition. *The Journal of Finance* 70, 1723–1765.
- Goldstein, I., Yang, L., 2017. Information disclosure in financial markets. *Annual Review of Financial Economics* 9, 101–125.
- Goldstein, I., Yang, L., 2019. Good disclosure, bad disclosure. *Journal of Financial Economics* 131, 118–138.
- Grossman, S.J., Stiglitz, J.E., 1980. On the impossibility of informationally efficient markets. *The American economic review* 70, 393–408.
- Hasbrouck, J., Saar, G., 2013. Low-latency trading. *Journal of Financial Markets* 16, 646–679.
- Hellwig, M.F., 1980. On the aggregation of information in competitive markets. *Journal of economic theory* 22, 477–498.
- Jarnecic, E., Snape, M., 2014. The provision of liquidity by high-frequency participants. *Financial Review* 49, 371–394.
- Leuz, C., Verrecchia, R.E., 2000. The economic consequences of increased disclosure. *Journal of accounting research* , 91–124.

- Magnan, M., Xu, B., 2008. Information uncertainty, corporate disclosure and stock return volatility. *Corporate Disclosure and Stock Return Volatility* (January 4, 2008) .
- Marmora, P., Rytchkov, O., 2018. Learning about noise. *Journal of Banking & Finance* 89, 209–224.
- Ozsoylev, H.N., Walden, J., 2011. Asset pricing in large information networks. *Journal of Economic Theory* 146, 2252–2280.
- Verrecchia, R.E., 1982. Information acquisition in a noisy rational expectations economy. *Econometrica: Journal of the Econometric Society* , 1415–1430.