Subjective Beliefs and the Q-Theory of Investments[1]

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Preliminary – Please Do Not Circulate

Abstract

I study the role of managers' and investors' subjective beliefs for corporate investments and valuation. I augment a q-model allowing *jointly* for managers and investors to have non-rational expectations. The model highlights the importance of small distortions in expectations for investment decisions and valuation. The disagreement between these two economic agents impacts Investment-Q regressions and Investment-CashFlow sensitivity, even without other frictions. I construct a novel dataset on managers' and investors' sales forecasts. I test the model predictions and estimate the model to quantify the role of subjective beliefs. I find that managers are optimistic and overinvest, while investors are pessimistic and underprice firms. Therefore, the documented gap between investments and valuation widens further. This requires higher implied markups to reconcile this evidence. These results highlight the importance of departing from rational expectations to evaluate the role of economic rents for corporate policies.

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1 Introduction

Recent research highlights a disconnection between corporate investments relative to valuation (Gutiérrez and Philippon, 2017). This is in stark contrast with standard q-theory (Tobin, 1969 Hayashi, 1982), which would predict higher investments, ceteris paribus. After accounting for intangible capital in production, we attribute the "missing investments" to economic rents (Peters and Taylor, 2017, Crouzet and Eberly, 2023). Due to data limitations, this literature largely neglects the role of subjective beliefs of the economic agents, which is a key in financial economics. Managers' and investors' expectations about the future affect capital budgeting decisions and the stock price (Greenwood and Shleifer, 2014, Malmendier and Tate, 2015). What is the role of managers' and investors' subjective beliefs in the disconnection between investments and valuation?

In neoclassical models, the gap between investments and valuation can be cast as the growth of the gap between Tobin's Average Q and Marginal Q. We directly observe the Average Q in the data, while Marginal Q is the unobservable shadow value of capital. The theoretical literature on the divergence of Average Q and Marginal Q has rapidly expanded and found many possible theoretical explanations. Recent studies point the finger toward the rise in intangible capital (Peters and Taylor, 2017) and economic rents (Crouzet and Eberly, 2023, Corhay et al., 2025) to explain the disconnection observed in the data. Over the last several decades, the rise of intangible capital has been a well-documented trend (see for example Corrado et al., 2009). This will mechanically imply that the common measure of

Tobin's Q is too high since the denominator will underestimate the real productive capital (Gutiérrez and Philippon, 2017 Crouzet and Eberly, 2023 Peters and Taylor, 2017). Rents act differently for the divergence between Average Q and Marginal Q. Market power and decreasing returns to scale (i.e. rents and quasi-rents) decrease the incentive to invest, since the marginal benefit to investing is low (Barkai, 2020, Abel and Eberly, 2004, Gutiérrez and Philippon, 2017, Cooper and Ejarque, 2003). The expectations of economic agents can play a very important role in this discussion as they are embedded in the stock price and in investment decisions. Optimistic investors and pessimistic managers could generate low investments and high valuations even in the complete absence of economic rents. Alternatively, higher market power could hide behind an optimistic manager and pessimistic investors, as the gap between investments and valuation would mechanically be lower. By jointly considering both managerial and investor expectations, I disentangle the effects of rents from belief-driven distortions.

To guide my work, I start with a theory of the firm that invests to accumulate capital to generate cashflows (Hayashi, 1982). I augment the model with economic rents, which capture market power from the final goods (Lindenberg and Ross, 1981, Cooper and Ejarque, 2003). This makes the average value of the firm (Tobin's Average Q) and its marginal value (Tobin's Marginal Q) diverge. In addition to market power (rent), I also allow managers' and investors' expectations to be non-rational (Greenwood and Hanson, 2015, Greenwood and Shleifer, 2014). The managers will make the investment decision, and they might have expectations about future fundamentals (future demand, productivity...) which differ from the true process observed by the econometrician. More precisely, if the manager has higher expectations about the future than the outcome will display optimism (pessimism if lower). Therefore, the managers' perceived value of investments will be higher than the true value. He will overinvest and destroy firm value compared to the first-best, which is not biased. In a similar spirit, investors can also display irrational expectations. Investors can think the future is brighter than it is for their firm, and they will therefore overprice the firm's stock price. The theory builds on standard neoclassical investment theory in the spirit of Hayashi, 1982 and Abel and Eberly, 1994a. The model displays interesting findings. First, the *disagreement* between investors and managers can create a gap between investments and valuation due to their different beliefs. Second, the model highlights that even small distortions in expectations matter both for investments and valuations. This is due to the nonlinear dynamics of the state variable. Third, I simulate different versions of the model (Moyen, 2005). On these different panels, I run Investment-Q regressions and I include cashflows in the regression. I show that the disagreement between investors and managers can break the implementation of Q-theory and cashflows start becoming significant absent other frictions such as market power, financing constraints, agency conflicts... I will test these predictions in the data. The model speaks to the literature on the empirical implementation of Q-theory with investment-Q regressions and to the cashflow sensitivity (Kaplan and Zingales, 1997, Fazzari et al., 1988 Alti, 2003). It is worth noting that I am potentially assuming departure from rational expectations, then I will confirm this when I bring the model to the data.

I collect a novel dataset on investors' and managers' sales forecasts to test the prediction of the model and to run a structural estimation exercise to quantify the magnitudes of these frictions. I collect from IBES and Factset annual sales forecasts about sales from the analysts' consensus. This will be my proxy as investors' expectations (Gennaioli et al., 2015, Dessaint et al., 2024a, Bordalo et al., 2017). From the same datasets, I also collect managerial guidance about future sales (Ma et al., 2024, C. A. Otto, 2014). One potential issue with guidance is manipulation. Managers will voluntarily disclose future sales, and this will be an average of what they really think and what they want the market to believe. I mitigate this by taking only short-term forecasts released at most one year before the end of the fiscal year. The intuition is that managers might fabricate long-term firm prospects but they can't depart from what they truly believe in the short-run as it will have consequences. To keep the two expectations comparable, I also apply the same filters on the horizon to the analysts consensus data. Moreover, Ma et al., 2024 show that this dataset has similar properties to the survey data from the Bank of Italy. I mitigate the other potential problem of sparsity of the data by using both IBES and Factset, which leads to have longer time-series. I merge this dataset with Compustat Annual to obtain the accounting variables. Moreover, I use the definition of capital and investment that takes into account investment in intangible capital (as in Falato et al., 2023, Einsfeldt and Papanikolaou, 2013, Peters and Taylor, 2017...). The final dataset consists of 7,505 firm-year observations from 2002. I perform a series of analyses and studies on this dataset. First, I compute the difference between investors' and managers' sales forecasts over time. I find that this value decreased over time, as we can see in Fig. 1.

[Insert Figure 1 about here]

This result is surprising and has important implications. Since companies' valuations go up over time and investments go down, one would expect the difference in investors' and managerial expectations to increase over time. Therefore, this is the first signal that the true gap between valuation and investments is bigger than previously established. Second, I assess the forecast accuracy of these forecast data and I show that this dataset has strong informational content. Third, since the proposed model deviates from rational expectations, I check the persistence of forecast errors. I show that forecast errors are predictable and confirm this hypothesis. Third, I show the strong empirical content of investors' sales forecast and managers' sales forecast for corporate investments and for Tobin's Q. Lastly, I test the model predictions about I-Q regression and the sensitivity of cashflows. As the simulation exercises suggest, I document a stronger performance of I-Q regressions for firms where investors and managers do not disagree on future outcomes. Moreover, cash flow is not significant in this regression. On the other hand, for firms with a high level of disagreement in beliefs, the implementation of Q-theory works less well and cash flows is significant in this regression.

In this study, I also run a quantitative exercise to quantify the role of beliefs and rents on investments and valuation. I build an estimation procedure that allows to structurally estimate the model. I start writing the dynamics of the key state variable, the stock of productive capital. Then, I build an iterative procedure that relies on the GMM (Hansen, 1982) and that allows me to estimate the deep parameters of the model. It is worth highlighting that the procedure displays very strong small-sample properties and model fit. This allows me to estimate the model on smaller groups of firms as in Frésard et al., 2023 Gryglewicz et al., 2021). Moreover, the procedure also ensures global identification of the parameters. The estimated parameters highlight that managers overestimate future sales. Therefore, they overinvest and destroy firm value. On the other hand, investors are pessimistic and underprice the firm. The average parameter governing rent points to a significant degree of market power. At the industry level, the pessimism result is driven by the tech and healthcare industries. While in Manufacturing and Consumer Goods investors are right on average. The parameter governing managerial beliefs displays more heterogeneity at the industry level. Managers in manufacturing and Technology are optimistic and overinvest, while in Consumer Goods and Pharma they are pessimistic and leave money on the table by underinvesting. Turning now to the parameter governing rents, manufacturing and tech exhibit the highest level of rents. Afterwards, I perform a series of counterfactual exercises by shutting down the preferred mechanism and see what investments and valuations look like. First, I shut down the role of managerial beliefs (optimism) and find that the investment rate goes from 0.2 to 0.18. Firm value increases by 1% when investments do not depart from the optimal level. Next, I examine investor beliefs and shut down this channel. Investors display pessimism and underprice firm, without this bias the observed firm value would be 1.33% higher. Finally, I investigate the impact of economic rents by shutting down the rent mechanism. Without the presence of rents, the model-implied investment rate jumps substantially from 0.2 to 0.24. Finally, I augment the model and directly link the rent parameter to firm markups. The average markup implied by the model is 1.35 and it is much higher than previously estimated in this sample (for example in Crouzet and Eberly, 2023). This result highlights the importance of including expectations to carefully evaluate the role of rents in the economy.

1.1 Related Literature

This paper belongs to the extensive literature on Q-theory and on its implementation. The empirical performance of Q-theory has been disappointingly low. Some papers tried to improve the measurement of Q such as Abel and Blanchard, 1986 without relying on stock market prices, Philippon, 2009 uses on bond prices, Gala et al., 2023 use state variables and a series of papers that try to correct for measurement errors (Erickson and Whited, 2012 Erickson et al., 2014). Other papers look at the relationship between Investments and Q with a focus on intangible capital. The pioneering work of Peters and Taylor, 2017 corrects the denominator of Tobin's Q to account for intangible capital and shows that I-Q regressions work better. Other examples that consider Intangible Capital are Einsfeldt and Papanikolaou, 2013; Almeida and Campello, 2007; Chen et al., 2006; Gourio and Rudanko, 2014; In addition, Lindenberg and Ross, 1981, Cooper and Ejarque, 2003 and Abel and Eberly, 1994b show that market power could impact the effectiveness of I-Q regressions. The intuition is that companies have built a moat around their castle that allows them to earn high profits, even without heavily investing. Peters and Taylor, 2017 and Crouzet and Eberly, 2023 are the most similar papers to this work, but they differ substantially. I explain the empirical results in Peters and Taylor, 2017, including intangibles in Q decreases the GAP between Average Q and Marginal Q. Hence, Investment-Q regressions work better. More recently, Gormsen and Huber, 2025 claim that the gap between investments and Q lies in their different discount rates (denominator). This paper studies expectations on cashflows (numerator) and finds the opposite since, if anything, the gap increases after considering beliefs. Moreover, this is the first paper that considers *jointly* the beliefs of both investors and managers.

I also contribute to the blossoming literature on expectations and behavioural corporate finance. First, several papers studied firm expectations about macroeconomic variables (e.g., Coibion et al., 2018 for inflation, Tanaka et al., 2020 for GDP growth, Ben-David et al., 2013 for aggregate stock market returns, Guiso and Parigi, 1999 for uncertainty), I study firms' own sales forecasts and how they affect their decisions. For example, Malmendier and Tate, 2015 reviews this literature. More recently, Ma et al., 2024 studies the roles of distortion in expectations for misallocation and Barrero, 2022 studies the micro and macro implications role of different biases. The most similar papers are Cummins et al., 2006, Gennaioli et al., 2015, and Gormsen and Huber, 2025. The first two consider only the expectations of one economic agent at the time, while in my setup what really matters is the disagreement. While in Gormsen and Huber, 2025 they study the discount rate (denominator), while I focus on the cashflow expectations (numerator); moreover, my results are very different and shed light on the role of rents. In addition, I also run a fully fledged quantitative exercise by structurally estimating the model. Other relevant papers are for example Stein, 1996, Graham et al., 2013, Graham et al., 2015, Ben-David et al., 2013, Bordalo et al., 2017,Bordalo et al., 2018.[1]

Third, this work is relevant to the literature on the implications of rising rents. Loecker et al., 2020 shows that market power increased over time. Other examples that study the increase in rents and document the dramatic implications for the economy are Autor et al., 2020, Gutiérrez and Philippon, 2017, and Barkai, 2020. I show that the expectations of economic agents have implications for our assessment of rents.

The rest of the paper is organized as follows. Section 2 introduces the theoretical setup. Section 3 describes the data construction. Section 4 highlights the stylized facts. Section 5 describes the estimation procedure. Section 6 discusses the results. Section 7 concludes.

[1]Recently, the literature on asset pricing has rapidly started to include subjective beliefs Greenwood and Hanson, 2015, Greenwood and Shleifer, 2014, Bouchaud et al., 2019, De La O and Myers, 2021, De La O and Myers, 2024...

2 A Model for Corporate Investments

Managers are risk-neutral, discount cash flows at a rate $r > 0$, and make decisions in the best interests of their shareholders. Time is continuous, the horizon is infinite, and I operate in the usual filtered probability space.

2.1 Fundamental Equations

Consider a firm that uses capital K_t, to generate sales Y_t at time t according to the production function:

$$dY_t = dA_t K_t^{\gamma} \tag{1}$$

where the process dA_t capture the productivity or future demand and γ which governs rents as in Hennessy et al., 2007, Cooper and Ejarque, 2003, Abel and Eberly, 2004, Falato et al., 2023. The process dA_t evolves according to the following arithmetic Brownian motion:

$$dA_t = \overline{A}dt + \sigma_a dW_t^a \tag{2}$$

where $\sigma_a > 0$. I allow for managerial beliefs about dA_t to differ, in particular, they potentially believe:

$$dA_t^{Manager} = \underbrace{(1+\beta)\overline{A}}_{\tilde{A}^{Manager}} dt + \sigma_a dW_t^a \tag{3}$$

and I also allow for investors' beliefs about dA_t to differ in the following way:

$$dA_t^{Investor} = \underbrace{(1+\alpha)\overline{A}}_{\tilde{A}^{Investor}} dt + \sigma_a dW_t^a \tag{4}$$

where α and β capture the distortion. In particular, positive values of these constants imply optimism while negative values imply pessimism in their beliefs [2].

The total capital stock K_t accumulation evolves according to the following stochastic process:

$$dK_t = (\ell_{k,t} - \delta_k K_t)dt \tag{5}$$

where $\ell_{k,t} > 0$ is the long run investment and δ_k is the depreciation rate of capital. Total capital includes both physical and intangible capital as in Peters and Taylor, 2017 Einsfeldt and Papanikolaou, 2013 Falato et al., 2023. Investing is increasingly costly and the firm has the following quadratic cost function:

$$C(\ell_{k,t}) = \frac{\lambda_k}{2}\left(\frac{\ell_{k,t}}{K_t}\right)^2 K_t^{\gamma} \tag{6}$$

with cost parameter $\lambda_k > 0$. [3] As in standard q-models (e.g. Hayashi, 1982) the cost function is homogeneous of degree one in K_t^{γ} and depends on the investment rate. The model solution derived below is robust to alternative specification of the cost function.

2.2 Optimal Investment Policy - First best ($\beta = 0$)

The firm chooses investments to maximize the expected discounted earnings, which gives the firm value.

The firm's objective function is then:

$$V(K_0) = \max_{\ell_{k,t}} \mathbf{E}\Big\{ \int_0^{\infty} e^{-rt}(\overline{A}K_t^{\gamma} - C(\ell_{k,t})dt)\Big\} \tag{7}$$

[2] This specification could be extended to capture overconfidence $dA_t = (1+\beta)\overline{A}dt + (1+\epsilon)\sigma_a dW_t^a$. In this case $\epsilon > 0$ captures underconfidence and $\epsilon < 0$ captures overconfidence. It is trivial to see that this would not affect investments.

[3] This parameter governing the cost function captures the cost to invest and can be seen as a reduced form way to capture frictions.

where $\mathbf{E}$ is the conditional expectations operator given the initial value of total capital. The firm value then satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rV(K_t) = \max_{\ell_{k,t}} \left\{ \overline{A}K_t^{\gamma} - \left[\frac{\lambda_k}{2} \left(\frac{\ell_{k,t}}{K_t} \right)^2 K_t^{\gamma} \right] + V_k(\ell_{k,t} - \delta_k K_t) \right\} \tag{8}$$

where V_k is the first order derivative of the value function $V(K_t)$ with respect to K_t, and V_{kk} is the second order derivative of the value function $V(K_t)$ with respect to K_t. In economic terms, the above equates the returns of investing in the firm (l.h.s.) and the expected change in value due to the maximization problem subject to the investment choice (r.h.s). Indeed, if we look more carefully now at the right-hand side, the first term represents the operating profits, the second term the adjustment costs, and the third term takes into account the change in firm value (V_k) due to the investment decision affecting the change in the stock of capital ($\ell_{k,t} - \delta_k K_t$), and the last terms considers the effect of volatility of the capital evolution process. The investment policy is retrieved by solving $V_k = C_i$, where C_i is the first derivative of the adjustment cost function with respect to the investments. As in any Q-model, the firm is going to set the optimal investment rate such that the marginal benefits (Marginal Q) equals the marginal cost. To solve this model, I guess and verify that the firm value is linear in K_t^{γ}:

$$V(K_t) = qK_t^{\gamma} \tag{9}$$

where the parameter q is function of the model's deep parameters $\overline{A}, \gamma$ and λ_k. This makes Marginal Q and Average Q endogenous in my model.

The first order condition (FOC) for $\ell_{k,t}$ leads to:

$$V_k = \lambda_k \ell_{k,t} K_t^{\gamma-2} \tag{10}$$

$$\ell_{k,t}^{*} = \frac{V_k}{\lambda_k K_t^{\gamma-2}} \tag{11}$$

It is worth noticing that the second-order condition $-\lambda_k K_t^{\gamma-2} < 0$ always holds.

To proceed further with the calculations, I need to compute the partial derivatives of the value function with respect to capital.

$$V_k = q\gamma K_t^{\gamma-1} \tag{12}$$

$$V_{kk} = q\gamma(\gamma-1)K_t^{\gamma-2} \tag{13}$$

The optimal investment $\ell_{k,t}^*$ becomes:

$$\ell_{k,t}^* = \frac{1}{\lambda_k K_t^{\gamma-2}}(q\gamma K_t^{\gamma-1}) = \left(\frac{q\gamma}{\lambda_k}\right)K_t = i^* K_t \tag{14}$$

The steady-state optimal investment is increasing in q and γ, but decreasing in λ_k, as the economic rationale would suggest. The completed solution for the model is available in Appendix A.

I will now describe how Average Q and Marginal Q behave in this simple model. Recall that: $V(K_t) = qK_t^{\gamma}$.

With perfect competition (PC) and constant returns to scale (CRS) then $\gamma = 1$, the production function is linear homogenous of degree one. As in Hayashi, 1982, Average Q equals Marginal Q. It follows that:

$$Marginal\ Q = \frac{\partial V(K_t)}{\partial K_t} = q \tag{15}$$

and

$$Average\ Q = \frac{V(K_t)}{K_t} = q \tag{16}$$

On the other hand, when we depart from the classical hypotheses the curvature of the profit function γ is not unity since companies have rents. It follows that when $\gamma \neq 1$:

$$Marginal\ Q = V_k = q\gamma K_t^{\gamma-1} \tag{17}$$

$$Average\ Q = \frac{V(K_t)}{K_t} = qK_t^{\gamma-1} \tag{18}$$

so Average Q is higher than Marginal Q if and only if $\gamma < 1$. I can already see that higher rents (i.e. low γ) push down Marginal Q and so it depresses the optimal investment rate. As we can see in the Figure A.4 above when rents are high (γ is low), the optimal investment rate decided by the company is low. This is effects of market power and decreasing returns to scale on corporate investments. Moreover, it creates a gap between Average Q and Marginal Q ("investment gap"). The companies will rely on Marginal Q to decided the investment rate but Average Q will be different. From the data, we can observe only Average Q and the Figure above clearly shows why this might mislead our analysis. Interestingly, the difference between Average Q and Marginal Q increases with rents. In the limiting case of linear homogenous of degree of one production function ($\gamma = 1$), Average Q equals Marginal Q as standard in this literature.

2.3 Optimal Investment Policy - Biased Manager ($\beta \neq 0$)

In this section I will show the implication for corporate investments in the case where the manager is potentially biased. She chooses investments to maximize the expected discounted earnings, which gives the firm value. I will define with $\tilde{\ell}_{k,t}$ and $\tilde{V}$ the investment rate and firm value according to the managerial beliefs.

$$\tilde{V}(K_0) = \max_{\tilde{\ell}_{k,t}} \mathbf{E}\Big\{ \int_0^\infty e^{-rt}(\overline{A}(1+\beta)K_t^\gamma - C(\tilde{\ell}_{k,t})dt) \Big\} \tag{19}$$

where $\mathbf{E}$ is the conditional expectations operator given the initial value of total capital. As in the previous case, the firm value then satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rV(K_t) = \max_{\ell_{k,t}} \Big\{ (1+\beta)\overline{A} K_t^{\gamma} - \Big[\frac{\lambda_k}{2} \left(\frac{\tilde{\ell}_{k,t}}{K_t} \right)^2 K_t^{\gamma} \Big] + V_k(\tilde{\ell}_{k,t} - \delta_k K_t) \Big\} \tag{20}$$

which gives us the investment rate in this case:

$$\tilde{\ell}^*_{k,t} = \frac{1}{\lambda_k K_t^{\gamma-2}} (q\gamma K_t^{\gamma-1}) = \left(\frac{\tilde{q}\gamma}{\lambda_k} \right) K_t = i^* K_t \tag{21}$$

It is straightforward to see that a pessimistic manager ($\beta < 0$) will destroy value by under-investing. In particular, the marginal Q perceived by the manager is higher than the true one. Therefore, she will underinvest and destroy value. A similar argument can be made with an optimistic manager who will overinvest. This is the economic mechanism at play.

2.4 Firm Value - Biased Investor ($\alpha \neq 0$)

The firm value of the firm in this case with the biased manager is just the discounted value of realized earnings when the investment rate is $\tilde{i}$.

$$V(K_0)^{Firm} = \mathbf{E}\Big\{ \int_0^{\infty} e^{-rt} (\overline{A}\, K_t^{\gamma} - C(i_t)) dt \Big| i_t = \tilde{i} \} \tag{22}$$

This would be the value that investors would assign to the firms with correct subjective beliefs. I also allow for investors' beliefs about fundamentals to be biased. Therefore, the firm value according to them is:

$$V(K_0)^{Investors} = \mathbf{E}\Big\{ \int_0^{\infty} e^{-rt} ((1+\alpha)\overline{A}\, K_t^{\gamma} - C(i_t)) dt \Big| i_t = \tilde{i} \} \tag{23}$$

The mathematical calculation of firm value can be found in appendix. It is interesting to note that an optimistic manager ($\alpha > 0$) will overstate the value of the company. On the other hand, a pessimistic manager will understate the value of the firm.

The shareholders' problem implies that they know that the managers are biased but they still keep them. In Appendix A I study the contracting problem in this case. I obtain the optimal contract in this case and study the impact for corporate policies. The model highlights that it can be optimal for shareholders to have non-rational manager as they can exploit them by offering a different compensation scheme. A fully fledged [4]

2.5 Model Comparative stats and Economic Mechanisms

I will illustrate and explain the economic mechanisms that the model mimics. I will calibrate the following parameters: $r = 0.1$, $\overline{A} = 0.3$, $\lambda_k = 15$, and $\delta_k = 0.12$.

[Insert Figure A.4 about here]

In the right panel, I plot the investment rate and in the right panel, l I plot the AverageQ and MarginalQ for different values of rents (γ). Perfect competition and constant return to scale arise when $\gamma = 1$, otherwise, economic rents are present. If we consider the first-best case ($\alpha = \beta = 0$), the investment rate decreases with rents. Monopoly forces, for example, decrease the incentive to invest and the gap between AverageQ and MarginalQ increases. As in the standard case, in my model AverageQ and MarginalQ coincide when $\gamma = 1$.

I allow for the manager to be pessimistic ($\beta = -0.3$) and the investor to be optimistic ($\alpha = 0.2$). For illustration purposes, I focus only on this case. In this case, as we can see from the right panel, the manager is going to believe that the firm is less valuable. He

[4] Modelling labour market side of this problem will give interesting general equilibrium analysis, but it is outside the scope of this paper.

will perceive both AverageQ and MarginalQ to be lower than the real one, and therefore, he will underinvest. This is equalent to having a manager that perceives the projects to be less valuable (lower NPV) and underinvests in the capital budgeting decision due to his pessimism. A pessimistic manager will underinvest ($i < i^{FB}$) and will destroy value by leaving money on the table. Similarly, an optimistic manager ($\beta > 0$) will overinvest ($i > i^{FB}$), and destroy shareholder value. We can see the green line is the firm-value when the manager is biased, which is always under the case in the first-best. It's interesting to note that higher rents mitigate managerial beliefs when departing from first best.

Afterwards, I allow also investors to be biased about firm fundamental.

I argue that we observe in the data the dotted black line on the right panel, which is the investment rate decided by the firm managers. Moreover, we also observe the pink line, which is the firm average value (Tobin's Q) according to investors. It's interesting to focus on the case when $\gamma = 1$. I show that even small belief distortions create an important gap between the investment rate and Tobin's Q even in the absence of rents. This has strong policy implications as the current models would confuse belief distortion with rents. It is worth observing that this generalized model boils down to the standard models when $\alpha = \beta = 0$.

2.6 Model Predictions for I-Q relation

In this section, I simulate the model with a different set of parameters. I then perform the classical investment-Q regressions and get new insights on the fit and on the sensitivity of cashflows.

I follow Moyen, 2005 and simulated a panel of 2,000 firms for 10 years. I simulate different models to study the relationship between I-Q and the I-CF sensitivity changes. To mimic the empirical exercise usually carried out by the researcher, I run these panel regressions with firm and time fixed effects. The common parameters for all the models are: $r = 0.1$, $\overline{A} = 0.3$, $\lambda_k = 15$, and $\delta_k = 0.12$. Model 1 ($\gamma = 1$, $\alpha = \beta = 0$) is the workhorse Q-model without friction with the standard assumption. Model 2 ($\gamma = 0.7$, $\alpha = \beta = 0$) includes rents in the standard model. These could come from market power or decreasing returns to scale. Therefore, AverageQ and MarginalQ will not be the same anymore. I now include subjective beliefs. Model 3 ($\gamma = 1$, $\alpha = 0.2$,$\beta = -0.2$) represents a Q-model without rents where the manager is pessimistic and the investors are optimistic. We can see the results of the simulations in the Table 1 below.

[Insert Table 1 about here]

The results from Model 1 show that when regressing investments on Q in the absence of rents, we have a very good fit. The estimated β is $1/\lambda_k$ as the theory predicts. Moreover, cash flow is not significant statistically or economically. The results from Model 2 suggest that the inclusion of rents affects these empirical applications. In particular, the first column shows that investments becomes less responsive to Q as the gap between AverageQ and MarginalQ increases. Moreover, the β will be slightly downward biased and not perfectly reflect the theoretical prediction. In the second column, we see that cashflow becomes significant as in Cooper and Ejarque, 2003, for example. This is because cash flow will be informative of the effects of economic rents on the relation between investments and Q. Model 3 shows that, even in the absence of rents, the relationship between investment and valuation can be compromised. In this case, it is worth noticing that the implied cost parameter is much large. This is because the linear model will try to explain low investments with a high Q. Therefore, it will have to imply a large cost parameter to explain this disconnection. Moreover, when the manager is optimistic and the investor is pessimistic, cashflow will enter this regression with significance. This is because cashflow will contain information about the true process. Moreover, as we would expect investments becomes less sensitive to Q.

In this section, I show that there are other factors that can undermine the relationship between investments and Q. In particular, subjective beliefs of the economic agents and their disagreement play a key role. This is true even in the absence of the common ones in the literature, such as monopoly power, decreasing returns, financial constraints (Fazzari et al., 1988, Kaplan and Zingales, 1997, Alti, 2003, Cooper and Ejarque, 2003, Moyen, 2005...). Moreover, the estimated beta on Q will no longer represent the real sensitivity of investment to investment opportunities; cashflow could be significant, and the sign will depend on the disagreement.

3 Data and Measurements

I use firm accounting variables from WRDS Compustat Annual dataset from 2002 to 2021 . All the data are deflated and expressed in US dollars 2000, with the GDP deflator by the St.Louis Fed. All the variables are winsorized at the 1th and 99th percentile. I exclude financial services firms (SIC codes 6000 to 6999), Utilities (SIC codes 4900 to 4999), Regulated (SIC 8000 to 9999) in any given quarter. I drop firms with missing values or values below 5mln of total assets or sales, or with PPE less than 5mln, as standard in the literature.
I define total capital as in Peters and Taylor, 2017. Total capital is the sum of Physical capital and Intangible capital. The latter is defined as the sum of Organizational capital and Knowledge capital. The stock of Organizational capital and Knowledge capital is computed with the perpetual inventory method on SGA and R&D with a depreciation rate of 30% and 15% respectively. Similarly, intangible investments are defined as R&D plus 30% SGA. I define operating profits as ebitda plus intangible investments. We can see below the description for variable construction and the summary statistics.
A key part of this work revolves around estimating the behavioural biases. I will use the Analyst forecasts consensus to proxy for investors' expectations (as Gennaioli et al., 2015, Dessaint et al., 2024a, Dessaint et al., 2024b...), and I will use managerial guidance to proxy for managerial expectations (as in C. Otto, 2014). The variable that I will focus on is sales. There are mainly two problems when using these data (Ma et al., 2024): manipulation and

panel size, particularly for managerial guidance. I rely on Factset and IBES to extend the coverage and manually collect these data. Moreover, I restrict the disclosure to be only in the short term (12 months before the fiscal year). The reasoning is that if managers manipulate (or are unaware of) sales, they will do it more in the long-term. In the short-term, they cannot go too far from what they truly believe. Moreover, managers and analysts report forecasts for various items. After a careful analysis, I conclude that sales is the best one for this study. I provide a detailed explanation of the data cleaning and collection process in Appendix B.

The final sample includes 7,505 firms-year observations about firms from 2002 to 2021.

[Insert Table 2 about here]

The table above describes the summary statistics of the final sample. The sample is slightly tilted towards firms with high valuation and larger ones. This is not very surprising as one might expect due to the coverage of analysts and due to managerial criteria. Nevertheless, in Appendix B, I compared the properties of my sample with the full Compustat and the distributions are still comparable and therefore, my sample is a good representation of US publicly listed firms.

4 Stylized Facts

This section will run the first empirical tests. I start by showing the evolution of the different in sales forecast between investors and managers over time. Afterwards, I study whether the analysts' and managers' forecasts predict actual sales. Then, I study the persistence of forecast errors. This is a key test to verify the departure from the rational expectations setup. Lastly, I assess the relevance of expectations for firm investments policies and for the I-Q relation.

4.1 Difference in sales forecast over time

In the figure below I report the average yearly scaled difference in sales forecasts between investors and managers.

[Insert Figure 1 about here]

Since in the data valuations go up, while investments go down, one might expect this difference in beliefs to increase over time. Quite surprisingly, I find the opposite. This means that investors are mis-pricing downwards the stock and managers are over-investing, therefore the gap between investments and Q is higher than previously measured.

4.2 Accuracy of Forecasts

I check that managerial and investors forecasts are accurate. To verify the informational content I run the following baseline regression:

$$\frac{Sales_{i,t}}{Ktot_{i,t}} = \alpha + \beta F_{i,t-1}\left[\frac{Sales_{i,t}}{Ktot_{i,t}}\right] + \epsilon_{i,t} \tag{24}$$

where $F_{i,t-1}$ represents the forecasted value of sales for firm i at time t-1. I include in the regressions fixed effects and I cluster standard errors by firm and year.

[Insert Table 3 about here.]

The table above shows that both analysts' and managers' forecasts of sales relative to capital are accurate predictors of realized outcomes. In all specifications, the estimated coefficients are highly significant and close to one when the forecasts are included separately, indicating that forecasted values are strongly aligned with actual performance. The inclusion of firm fixed effects slightly reduces the coefficient magnitudes but they remain statistically significant and economically meaningful.

The last two columns present a specification in which both analyst and managerial forecasts are included simultaneously. In this joint regression, both sets of forecasts continue to be statistically significant, indicating that each source contains incremental information about future firm performance that is not subsumed by the other.

Overall, these findings confirm the strong empirical content of both analysts' and managers' expectations. These results validate the use of forecast-based belief measures in the model and support the premise that heterogeneous beliefs across agents are grounded in observable forecasting behavior.

4.3 Persistence of Forecasts Error

The model proposed deviates from rational expectations. Under the rational expectations hypothesis, forecast errors are not predictable using variables in the economic agents' information set (such as past forecasts). I document in the table below that forecast errors are persistent.

I define forecast error as:

$$\widehat{FE}_{it} = \frac{F_{i,t-1}[Sales_{i,t}] - Sales_{i,t}}{Sales_{i,t}} \tag{25}$$

The results are robust to different specifications. I test the persistence of forecast error with the following empirical setup:

$$\widehat{FE}_{it} = \alpha + \delta_t + \kappa \widehat{FE}_{i,t-1} + \epsilon_{it}, \tag{26}$$

where δ_t denotes year fixed effects and α is a constant. I cluster standard errors at the firm and year level.

[Insert Table 4 about here.]

The table above confirms that forecast errors are persistent and predictable over time. The estimated coefficients on lagged forecast errors are positive and statistically significant for both analysts and managers, with values of 0.35 and 0.24, respectively. This implies that agents who overestimated (or underestimated) sales in the past are likely to make similar errors in subsequent periods. . This persistence indicates that economic agents do not fully correct their beliefs based on past mistakes, leading to systematic deviations from realized outcomes.

These findings provide strong empirical support for the modeling approach taken in this paper, which relaxes the assumption of rational expectations. The presence of predictable and persistent forecast errors is consistent with the idea that agents form biased or sticky beliefs about future outcomes—an assumption central to the theoretical framework developed here.

4.4 Corporate Investments, Tobin's and Expectations

An important relation in my model is the one between managerial expectations and corporate investments. I will test it with the following linear model:

$$i_{i,t} = \alpha_i + \zeta_t + \beta F_{i,t-1}^{Managers}\left[\frac{Sales_{i,t}}{Ktot_{i,t}}\right] + controls_{i,t-1} + \epsilon_{i,t} \tag{27}$$

The estimated coefficients confirm the hypothesis about the tight relationship between investments and expectations. I report the results in the table below.

Another very important assumption is that investors' beliefs affect the valuation of the firm that we observe. I will test it with the following linear model:

$$q_{i,t} = \alpha_i + \zeta_t + \beta F_{i,t-1}^{Investors}\left[\frac{Sales_{i,t}}{Ktot_{i,t}}\right] + controls_{i,t-1} + \epsilon_{i,t} \tag{28}$$

The estimated coefficients confirm the hypothesis about the tight relationship between firm valuation and expectations. I report the results in the table below.

[Insert Table 5 about here.]

Column (1) in Table 5 shows that the coefficient on the lagged managerial forecast of the sales-to-capital ratio is 0.10 and highly significant. Column (2) reports a coefficient of 1.271 on analysts' forecasts in the Tobin's Q regression, indicating that investor beliefs strongly correlate with market valuation. The high R^2 values (0.71 and 0.65) and large sample sizes confirm that expectations—both managerial and investors' are key drivers of corporate investment and valuation.

4.5 Disagreement and the I-Q relation

The model predicts that Q-theory works better when investors and managers do not disagree, absent other frictions. I construct a simple measure of their disagreement by taking the difference of the absolute values of their expectations.

$$D = |Manager\{F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]\} - Analyst\{F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]\}| \tag{29}$$

I divide the dataset in two, differentiating by high or low disagreement. I split only in two groups due to the small sample size. I run the classical I-Q regression with firm and time fixed effects and cluster standard errors by firm. As the model simulation predicts, in case of low disagreement Q has a better fit. Moreover, in the cluster of firms with low disagreement, cashflow is not significant. This result is confirmed also when taking into account the correction for measurement error (Erickson and Whited, 2000 Erickson and Whited, 2012).

[Insert Table 6 about here.]

In the low-disagreement subsample (column 1), the coefficient on lagged Tobin's Q is 0.021 and highly significant, while cashflow is statistically insignificant, consistent with a strong role for Q-theory when managers and investors agree. In contrast, the high-disagreement subsample (column 2) yields a slightly lower but still significant Q coefficient of 0.016, and cashflow becomes significant as predicted. The higher R^2 (0.83 vs. 0.79) in the low-disagreement group further confirms that Q-theory fits better under aligned expectations.

5 Estimation

In this section, I outline the procedure for estimating the model. First, in Section 5.1 I derive the discrete-time moment conditions that link observable investment and the evolution of the stock of productive capital. Next, in Section 5.2 I discuss identification and inference, showing that the chosen moments uniquely pin down each parameter and detailing the GMM inference procedure. Finally, in Section 5.3 I address the granularity of my group-level estimates.

5.1 Dynamics of Capital

The only state variable in the model is K_t, the stock of capital. Therefore, I obtain the law of motion of capital at the equilibrium. Recall that the law of motion of capital is:

$$dK_t = (\ell_{k,t} - \delta_k K_t)dt \tag{30}$$

By substituting into the process above the optimal investment $\ell^*_{k,t} = \left(\frac{q\gamma}{\lambda_k}\right)K_t = i^* K_t$, I obtain the controlled capital equation. The dynamics of capital along the steady-state path follows:

$$\frac{dK_t}{K_t} = \left(i^* - \delta_k\right)dt \tag{31}$$

With a simple application of the ITO lemma and after discretising exactly the process describing the evolution of capital, I link the empirical moments with the implied ones by the model to retrieve the model's deep parameters.

5.2 Identification and inference

In this section, an estimation procedure to obtain the model's deep parameter. I estimate the model using the GMM (Generalized Method of Moments). I show the chosen empirical moments and their distributional counterparty. Then, I provide the theoretical results to illustrate the global identification properties of my estimators.[5]

The key idea is to match a moment that we can measure from the data with one implied by the model. In this specific case, due to the nonlinearities implied by the analytical expression of q, I provide theoretical arguments for the uniqueness of the parameters (identification). The parameters that will be estimated are γ, δ_k, β, α, $\overline{A}$ and λ_k. Then, I retrieve λ_k from the model implied relation. I illustrate below the moments that I use in the estimation.

The first moment is simply the investment rate, as a ratio between total investments and the stock of capital.

$$m_1 \equiv E\left[\frac{I_t}{K_t}\right] \tag{32}$$

The next moment allows me to identify δ_k. I rely on moments of capital growth.

$$m_2 \equiv E\left[\frac{K_{t+1}}{K_t}\right] \tag{33}$$

The third moment is the growth rate of sales, which will allow me to get the parameter governing rents:

$$m_3 \equiv E\left[\frac{Y_{t+1}}{Y_t}\right] \tag{34}$$

The next moment will allow me to obtain $\overline{A}$, I use the ratio between sales and the stock of capital:

$$m_4 \equiv E\left[\frac{Y_t}{K_t^{\gamma}}\right] \tag{35}$$

[5] In an unreported section, I analyse also the small sample properties. This includes the bias of the estimated parameters to the noise-to-signal ratio and the statistical power.

The next moment is going to help me identify the parameter governing the managerial bias (β).

$$m_5 \equiv E\left[\frac{\tilde{Y}_t^{Manager} - Y_t}{K_t^{\gamma}}\right] \tag{36}$$

The next moment is going to help me identify the parameter governing the investors' bias (α). It is worth highlighting that I define with $\tilde{Y}$ the forecast of the economic agent.

$$m_6 \equiv E\left[\frac{\tilde{Y}_t^{Investor} - Y_t}{K_t^{\gamma}}\right] \tag{37}$$

Moreover, I calibrate the discount rate $r = 0.1$. In addition, the model also implies some additional conditions for the existence of parameters and for the value function to be well-behaved, which could affect the discounting factor. To keep the exposition clear, I have omitted all the analytical calculations required for the steps above, but I invite the interested reader to go to Appendix D. I want to highlight that this procedure uniquely and globally identifies all the deep parameters of the model from a theoretical standpoint and in simulations. In alternative exercises, I also ask the model to match the firm value, and the results are not affected by this choice as the model already matches the levels of Q. I decide to keep this proposed estimation procedure to maintain the global identification properties of my estimates.

5.3 Granularity and Grouping Mechanisms

In an ideal world, I would run the estimation procedure for each firm, but this is not possible with the estimation proposed. I estimate the model described in Section 2 for groups of firms for a determined time period.

I run some analysis to select the group size in order to ensure robust small-sample properties and econometric rigour. In particular, a sample size of 100 firms gives statistical power. Indeed, expectations data do not display long time series therefore I need to enlarge the group dimension.

This implies I assume that all the firms belonging to a group have the same deep parameters. The benefit of this approach is that it will allow me to get more cross-sectional heterogeneity in the parameters and investment policies than having one model for the universe of firms.

The grouping mechanism will follow three criteria: industry, managerial forecast error, and analyst forecast error, similarly as in Frésard et al., 2023 Gryglewicz et al., 2021. The first criterion for the group is the industry, I consider Fama French 5 industries since companies belonging to the same industries will be exposed to similar shocks (i.e. labour market shortages, technological disruptions...). The second criterion is the forecast error of analysts, as these firms will have investors with similar beliefs (α). The last criterion that I use is the forecast error of managers, as these firms will have investors with similar beliefs (β). The final number of groups is 15 for the 1,798 firms.

6 Results

6.1 Estimates

The table below reports the updated GMM estimates of the model's structural parameters, along with their corresponding standard deviations and standard errors. Specifically, we estimate the depreciation rate of capital (δ_k), the rents/curvature parameter (γ), the parameter ($\overline{A}$), the belief parameters (β and α), and the investment cost parameter (λ_k).

[Insert Table 7 about here.]

As shown in the first panel, the mean estimated depreciation rate (δ_k) is around 0.1 with relatively small dispersion. The estimated mean for γ is 0.76, hinting at a non-trivial degree of rents or curvature in the production function. We also observe that the parameter $\overline{A}$ exhibits a mean of about 1.15, with moderate variability across the sample groups.

Turning to the belief parameters, the mean of β is approximately 0.017 which indicates that on average manager are optimistic and overinvest. On the other hand, α is slightly negative on average (-0.01), suggesting that on average investors underprice firms. Both parameters display a modest degree of dispersion. Finally, λ_k, which governs investment costs, has a mean of 2.17. Given the absence of other frictions in my model, this parameter would be a reduced-form way to think of financing frictions. In the second panel, I report the mean and dispersion of the standard errors, which are relatively small overall, reinforcing the small sample properties of these estimates.

Overall, these results are consistent the presence of economic rents, and disagreement in beliefs between investors and managers, all of which play a significant role in shaping firms' investment decisions and value.

6.2 Model Fit

In this section, I assess the fit of the model. I compare the moments used in the estimation process (from the data) against the ones that my estimates imply. This is to verify that the model is fitting the data appropirately and that it is a useful tool to make quantitative statements when running counterfactual exercises. I report this in the Table 8 below.

[Insert Table 8 about here]

Overall, the model does a reasonably good job of matching key empirical moments, as indicated by the tight connection between the observed and model-implied values in Table 8. The first moment, m_1, which represents the average investment rate, is replicated almost perfectly, suggesting that the model reliably reproduces the overall level of investment seen in the data. Moments m_2 and m_3 also lie fairly close to their data counterparts, demonstrating that the model adequately captures important aspects of capital growth and sales dynamics.

Turning to m_4, the model-implied mean slightly exceeds the observed mean. Although this difference is somewhat larger than that for the preceding moments, it remains within a reasonable range. The fifth and sixth moments, m_5 and m_6, are relatively small in absolute terms, and the model correctly captures both the negative sign in m_6 (Investor FE) and the near-zero value of m_5 (Manager FE), with only moderate differences in magnitude.

It is important to note that the moments reported in Table 8 are average values computed within each group. Additionally, t-statistics comparing the model-implied and the observed (data) moments have been calculated, and the results indicate that the differences between them are not statistically significant. This further supports the conclusion that the model captures the underlying data patterns quite well. Moreover, the model produces values of Q close to the data. The implied firm value is 1.8 against the 1.88 in the considered sample.

In sum, the comparison of observed and model-implied moments confirms that the model is a useful tool to make quantitative statements about firm behaviours.

6.3 Evolution of α, β, and γ Across Industries

In this section, I will discuss the heterogeneity of the parameter governing rents and the beliefs.

The figure 3 shows that the investor bias parameter, α is negative andross all the industries. The highlight is that investors in the Tech and Pharma industry tend to be quite pessimistic, while those one in manufacturing are, on average correct. This result highlights that if anything, Tobin's Q should be higher and that potentially the gap between investments and valuation is even higher than previously assessed. In the model I depart from rational expectations and the economic agents have full information. An alternative way of seeing this is that the price in the market is less informative in Tech and Pharma, which is reasonable since these industries are more volatile and harder to value due to the higher stock of intangible capital.

[Insert Figure 3 about here]

The figure 4 illustrates the managerial bias parameter, β, across all industries. I find that manager in manufacturing and tech are optimistic and overinvest. In the other industries instead, they tend to be pessimistic and underinvest. Therefore, the investment rate in Manufacturing an Tech is even higher that it should be therefore this could drive downward out assessment of rents and hide them. On the other industries instead we might end up confusing underinvestment due to pessimisitc with market power. The heterogeneity across industries in β across sectors underscores the importance of considering industry-specific estimates.

[Insert Figure 4 about here]

The figure below presents the heterogeneity ih the rents parameter, γ, across industries. It's interesting to observe that in all industries $\gamma \neq 1$. This means that these firms do not operate under the standard assumption of the neoclassical model in Hayashi, 1982. The estimates for γ reveal significant cross-industry variation. In particular, the manufacturing and tech exhibit the highest level of rents (low γ). Quite surprisingly, the pharma industry is the one displaying the lowest level of rents. These results are important as they consider the beliefs of economic agents when evaluating the role of rents for corporate policies.

[Insert Figure 5 about here]

In summary, the updated evidence from the figures highlights that there exists substantial heterogeneity in investor beliefs, managerial beliefs, and rents.

6.4 Counterfactual Analysis

I perform a series of counterfactual experiments by shutting down selected mechanisms, I can examine how managerial biases, investor biases, and economic rents affect investment behaviour and firm value.

First, I consider the role of managerial belief, which is captured by the positive parameter β. The estimates suggest that on average managers are optimistic and overinvest. Therefore, they waste shareholder money with this suboptimal behaviour. I put $\beta = 0$ and compute the optimal policy and firm value. I find that the investment rates declines from 0.2 to 0.18. Moreover, firms value increases by 1% without managerial biases.

Next, I examine investor pessimism, as represented by the negative parameter α. The counterfactual exercise obtained putting $\alpha = 0$ implies that investors underprice firms by around 1.33%.

Finally, I investigate the impact of economic rents by shutting down the rent mechanism ($\gamma = 1$). Without the presence of rents, the model-implied investment rate jumps substantially from 0.2 to 0.24.

6.5 Rents Implications

Consider a firm that uses capital K_t to produce nonstorable output Y_t at time t according to the production function:

$$Q_t = Z_t K_t \tag{38}$$

where the process Z_t describes the stochastic evolution of productivity. The inverse demand curve for the firm's output is:

$$P_t = h_t Q_t^{-\frac{1}{\epsilon}} = h_t Q_t^{\frac{1-\mu}{\mu}} \tag{39}$$

where the process h_t describes the demand locator, and ϵ is the price elasticity of demand is $\epsilon > 1$. I link the elasticity ϵ with the markup μ defining the inverse of the elasticity as $(P - MC)/P$ and the markup as P/MC where P is the price and MC is the marginal cost. This leads to the classical relationship between elasticity and markup: $\mu = \frac{1}{1-\frac{1}{\epsilon}}$. In the limiting case with perfect competition, $\epsilon = \infty$ and $\mu = 1$. Assuming that firm's revenue Y_t is price times quantities $Y_t = P_t Q_t$, this leads over the increment dt to Eq.1 (as in Crouzet and Eberly, 2023 He et al agency Q) where:

$$\gamma = \frac{1}{\mu} \tag{40}$$

Therefore, the parameter γ representing rents will identify markups in my setup. Table 9 reports the value at the industry level. [6].

[Insert Table 9 about here.]

The average markup implied by my model is 1.35. Manufacturing and Tech lead with a markup of 1.62 and 1.38, respectively, reflecting substantial market power. Consumer Goods registers the lowest markup at 1.21, consistent with more competitive retail markets, and Healthcare shows the smallest markup of 1.08. These values is higher than previously estimated in a similar setup, for example Crouzet and Eberly, 2023 finds a lower markup of 1.24 in a similar sample period. This value implied by my model is mechanically much higher than it would be without including subjective beliefs and highlights the importance of including the expectations of economic agents to do a proper assessment of rents.

7 Conclusion

In this paper, I study the role of investors' and managers' beliefs for investments and valuation. I show that the disagreement between these two economic agents affects the implementation of Q-theory and cashflow sensitivity. Moreover, I highlight that after considering subjective beliefs, the documented gap between investments and valuation widens further. Therefore, the role of economic rents in this discussion and the implied markups are higher than previously established. This is a first step to bring the rapidly expanding literature on expectations of economic agents into corporate finance and opens up future research.

[6]This is due to the standard assumption of constant returns to scale which is justified by the complexity in obtaining data about quantities

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Figure 1: Difference in Sales Forecasts. I plot the yearly average different in sales forecast between investors and manager scaled by total capital.

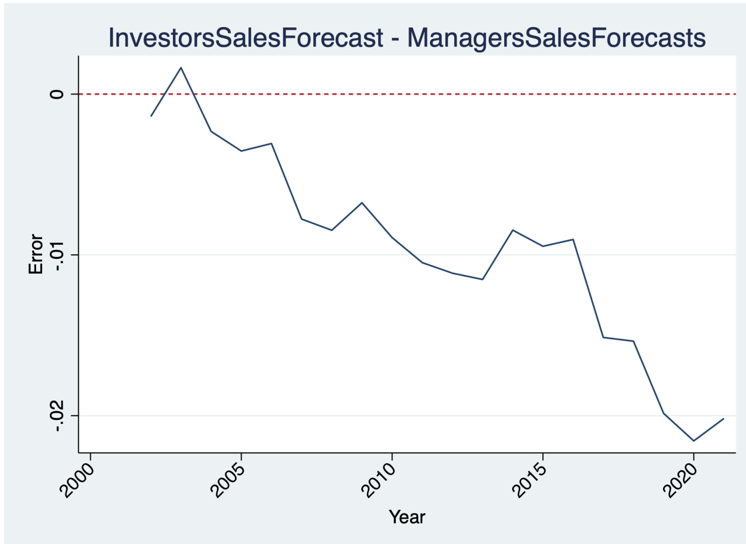


Figure 2: Model Comparative Statics. The figure on the left plots the optimal investment rate for different rent values. The solid black line is the first best, while the dotted black line is the one with the biased manager. The figure on the right shows the model-implied Average Q and Marginal Q for different rent values and in different cases. The solid blue and red line represent Average Q and Marginal Q when both the manager and the investors are rational. The dotted blue and red line are perceived Average Q and Marginal Q for the manager and the green and pink lines are the true and investors perceived Average Q when the manager is biased and invests sub-optimally.

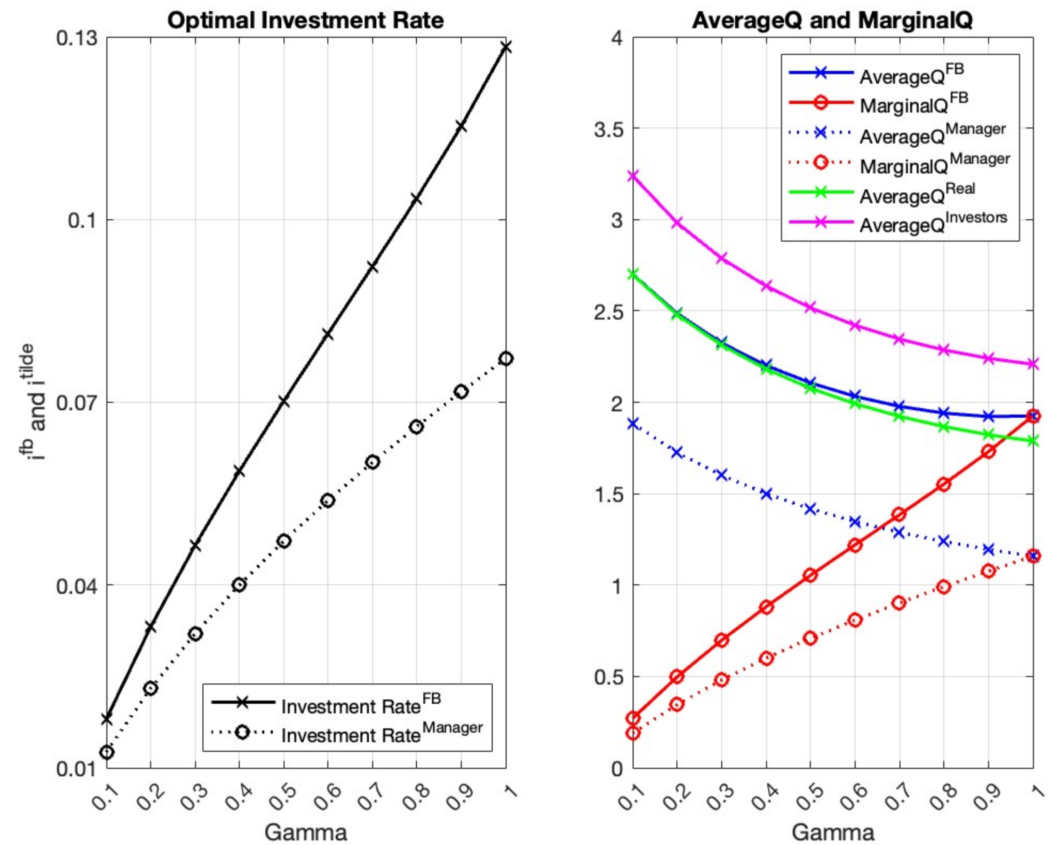


Figure 3: Estimated investor bias across industries. This figure shows the average estimated parameter α across the FF5 industries.

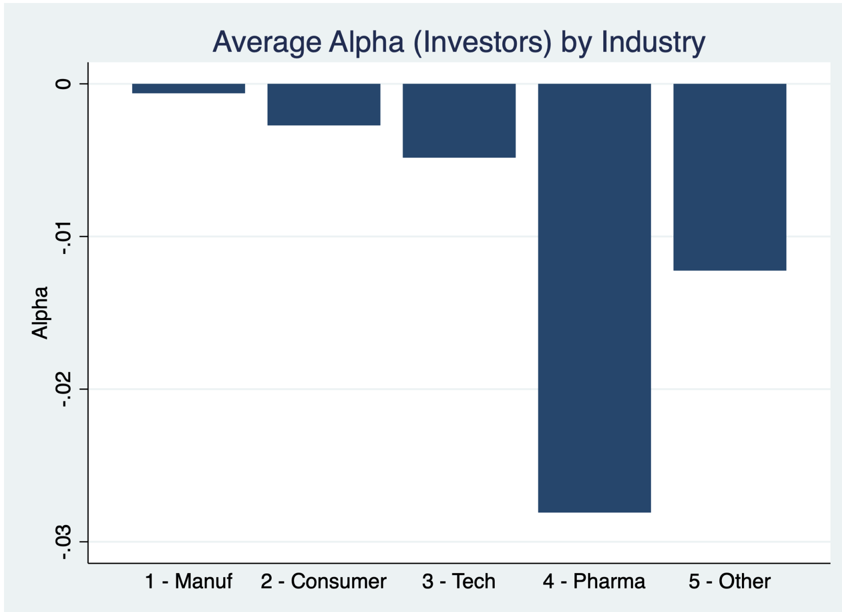


Figure 4: Estimated managerial bias across industries. This figure shows the average estimated parameter β across the FF5 industries.

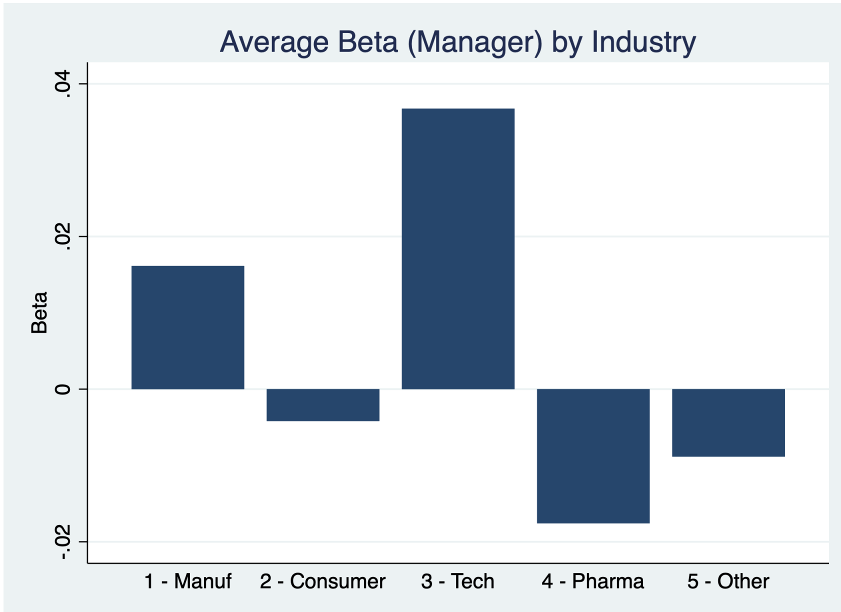


Figure 5: Estimated rents parameter across industries. This figure shows the average estimated parameter γ across the FF5 industries.

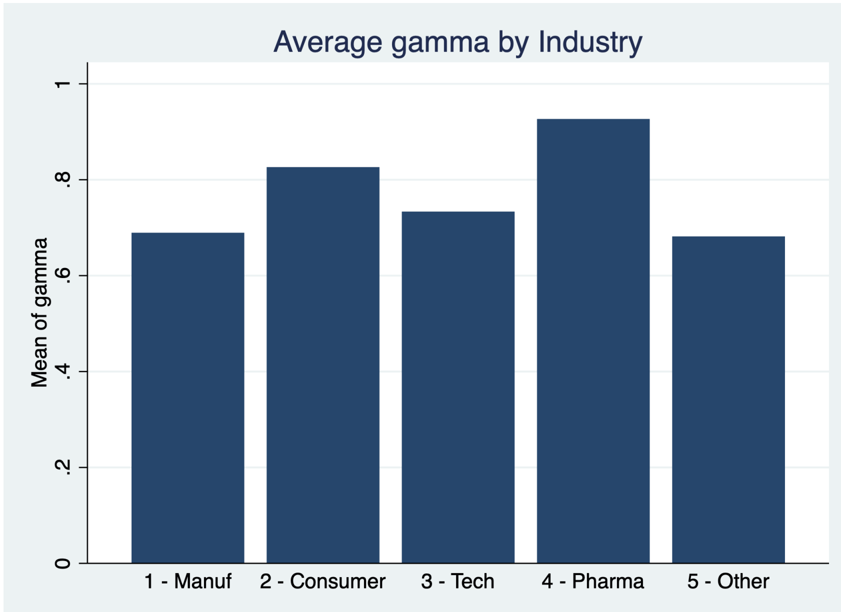


Table 1: The table presents estimates from a panel regression on data simulated from the model in the three different cases. I simulate 2,000 firms for 10 years. Model 1: $\gamma = 1$, $\alpha = \beta = 0$*, Model 2:* $\gamma = 0.7$, $\alpha = \beta = 0$*, and Model 3:* $\gamma = 1$, $\alpha = 0.2$,$\beta = -0.2$*. The regressions have the investment rate as a dependent variable and Q and cash flow as explanatory variables. ***, **, and * indicate significance at the 1, 5, and 10% levels, respectively.*

	Model 1		Model 2		Model 3	
Q	0.064***	0.064***	0.060***	0.027***	0.035***	0.017***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.00)	(0.000)
CF		0.001		0.112***		0.076***
		(0.001)		(0.002)		(0.001)

Table 2: Summary Statistics. This table presents the descriptive statistics of the main variables used in the analysis. The descriptive statistics are: Number of observations (N); mean; standard deviation; and the percentiles p25, p50, and 075. The sample covers the period 2002 to 2021.

	N	Mean	Std	p25	p50	p75
i	7.505	0.2	0.14	0.1	0.17	0.26
q_{tot}	7.505	1.88	2.18	0.62	1.15	2.22
$Size$	7.505	6.27	1,43	5,2	6,18	7.25
$Cashflow$	7.505	0.22	0.18	0.12	0.19	0.29
$Sales$	7.505	1.1	0.92	0.56	0.86	1.32
$AnalystSalesForecasts$	7.505	1.08	0.88	0.56	0.85	1.3
$ManagerSalesForecasts$	7.505	1.09	0.89	0.56	0.85	1.3

Table 3: Sales Forecasts predicting Realized Sales.
This table shows the result of the regression: $\frac{Sales_{i,t}}{Ktot_{i,t}} = \alpha + \beta F_{i,t-1}\left[\frac{Sales_{i,t}}{Ktot_{i,t}}\right] + \epsilon_{i,t}$.
****, **, and * indicate significance at the 1, 5, and 10% levels, respectively.*

	$Sales_{i,t}/Ktot_{i,t-1}$					
Analysts $F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]$	1.03***	1.05***			0.72***	0.63***
	(0.00)	(0.00)			(0.12)	(0.17)
Manager $F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]$			1.00***	0.98***	0.31**	0.42**
			(0.00)	(0.01)	(0.12)	(0.18)
Firm FE	N	Y	N	Y	N	Y
Year FE	Y	Y	Y	Y	Y	Y
$Adj.R^2$	0.98	0.99	0.97	0.98	0.99	0.99
N	7.505	7.505	7,505	7.505	7,505	7.505

Table 4: Persistence of the Forecast Errors.
This table shows the result of the regression: $\widehat{FE}_{it} = \alpha + \delta_t + \kappa\widehat{FE}_{i,t-1} + \epsilon_{it}$,*.*
****, **, and * indicate significance at the 1, 5, and 10% levels, respectively.*

	Analyst$\widehat{FE}_{it}$	Manager$\widehat{FE}_{it}$
Analyst$\widehat{FE}_{i,t-1}$	0.35***	
	(0.04)	
Manager$\widehat{FE}_{i,t-1}$		0.24***
		(0.04)
TimeFE	Y	Y
$Adj.R^2$	0.13	0.07
N	5,120	5,120

Table 5: Managerial Expectations and Investments. This table shows the results when I test the empirical content of the sales forecasts for investments and Q.
****, **, and * indicate significance at the 1, 5, and 10% levels, respectively.*

	i	q
Manager $F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]$	0.10***	
	(0.00)	
Analyst $F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]$		1.271***
		(0.21)
TimeFE	Y	Y
FirmFE	Y	Y
$Adj.R^2$	0.71	0.65
N	7,038	6,749

Table 6: Empirical test of the Investment–Q and Investment–Cashflow relations.
$D = |Manager\{F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]\} - Analyst\{F_{i,t-1}[Sales_{i,t}/Ktot_{i,t-1}]\}|$
****, **, and * indicate significance at the 1, 5, and 10% levels, respectively.*

	i	
	Low D	High D
$Q_{i,t-1}$	0.021***	0.016***
	(0.00)	(0.00)
$CF_{i,t-1}$	0.03	0.09***
	(0.02)	(0.03)
Firm FE	Y	Y
Time FE	Y	Y
$Adj.R^2$	0.83	0.79
N	3,117	3,062

Table 7: Estimation results for the groups. This tables illustrates the results from the estimation procedure. The first panel shows the estimated parameter and the standard deviation of the point estimates. The second panel shows the standard errors and the standard deviation of the estimated standard errors.

	δ_k	γ	$\overline{A}$	β	α	λ_k
Estimator						
Mean	0.1	0.76	1.15	0.02	-0.01	2.17
StDev	0.03	0.15	0.36	0.11	0.05	0.79
Standard Error						
Mean	0.024	0.08	0.04	0.01	0.001	0.02
StDev	0.01	0.03	0.018	0.05	0.005	0.01

Table 8: Comparison of Observed vs. Model-Implied Means for Six Key Moments. The moments reported here are the averages computed within groups.

Moment	Target (Data)	Model Implied
m_1 Investment Rate	0.203	0.203
m_2 Growth of K	1.191	1.171
m_3 Sales Growth	1.164	1.142
m_4 Sales	1.6	1.72
m_5 Manager FE	0.001	0.001
m_6 Investor FE	-0.024	-0.031

Table 9: Implied-Markups. This table shows the implied markup μ by the estimated γ.

Implied Markups	Full Sample	Manufacturing	Consumer Goods	Tech	Healthcare	Other
μ	1.35	1.62	1.21	1.38	1.08	1.47

Appendices

A Theoretical Analysis

A.1 Model Solution

I solve this model by guessing the function formal of the value function and then I verify that it satisfies the HJB. I guess that $V(K_t) = qK_t^\gamma$. I substitute also the partial derivatives V_k and V_{kk}, and the optimal policy at the equilibrium ℓ^* in the HJB below.

Now, I plug the above and, once last time, also the optimal policy $\ell_{k,t}^*$ in the HJB (8).

$$rV(K_t) = \max_{\ell_{k,t}} \Big\{ \overline{A}K_t^\gamma - C(\ell_{k,t}^*) + V_k(\ell_{k,t} - \delta_k K_t) \Big\} \tag{41}$$

A first useful step is substituting the optimal policy $\ell_{k,t}^*$ at the equilibrium onto the cost function:

$$C(\ell_{k,t}^*) = \frac{\lambda_k}{2}\Big(\frac{\ell_{k,t}}{K_t}\Big)^2 K_t^\gamma = \frac{\lambda_k}{2}\Big(\frac{q\gamma K_t}{K_t\lambda_k}\Big)^2 K_t^\gamma = \Big(\frac{q^2\gamma^2}{2\lambda_k}\Big)K_t^\gamma \tag{42}$$

After plugging everything and proceeding with the calculations the term K_t^γ cancels out, and we obtain:

$$q^2\Big(\frac{\gamma^2}{2\lambda_k}\Big) + q\Big[-\gamma\delta_k - r\Big] + \overline{A} = 0 \tag{43}$$

Where economic rationale provides the usual solution for q:

$$q = \frac{[\delta_k\gamma + r] - \sqrt{[-\delta_k\gamma - r]^2 - 2\overline{A}\left(\dfrac{\gamma^2}{\lambda_k}\right)}}{\left(\frac{\gamma^2}{\lambda_k}\right)} \tag{44}$$

when the below conditions hold:

$$\Big[-\delta_k\gamma - r\Big]^2 - 2\overline{A}\left(\frac{\gamma^2}{\lambda_k}\right) \geq 0 \tag{45}$$

It is interesting to notice that v, which directly impacts the optimal investment rate, is increasing in the exogenous productivity term $\overline{A}$.

A.2 Firm Value

I will provide in this appendix more details about the c the firm value according to the manager, investor, the real value when the investment rate in not the first-best and in the first-best case. I compute the value of the firm solving the following:

$$V(K_0)^{Firm} = \mathbf{E}\Big\{\int_0^\infty e^{-rt}(\overline{A}\,K_t^\gamma - C(i_t))dt\Big|i_t = \tilde{i}\} \tag{46}$$

$$V(K_0)^{Investors} = \mathbf{E}\Big\{\int_0^\infty e^{-rt}((1+\alpha)\overline{A}\,K_t^\gamma - C(i_t))dt\Big|i_t = \tilde{i}\} \tag{47}$$

$$V(K_0)^{Manager} = \mathbf{E}\Big\{\int_0^\infty e^{-rt}((1+\beta)\overline{A}\,K_t^\gamma - C(i_t))dt\Big|i_t = \tilde{i}\} \tag{48}$$

$$V(K_0)^{FB} = \mathbf{E}\Big\{\int_0^\infty e^{-rt}(\overline{A}\,K_t^\gamma - C(i_t))dt\Big|i_t = i^{FB}\} \tag{49}$$

In particular, it is important to verify that the usual transversality condition holds and then proceed with the calculations. The calculations are very similar for all the case. For example, the solution for the value from the investor point of view is:

$$V(K_0)^{Investors} = \frac{\overline{A}(1+\alpha) - \frac{\lambda_k}{2}(\tilde{i})^2}{r - \gamma(\tilde{i} - \delta_k)} \tag{50}$$

A.3 Dynamic Contracting Model Extension

In this section, I augment the model in the main section to show the dynamic moral hazard problem between the principal and the agent when they disagree on fundamentals. For clarity of exposition and without loss of generality in this section only the agent will be biased. I will retrieve the properties of the optimal contract with different beliefs and show how this affects corporate policies.

A.3.1 Investment Environment

- A firm generates operating cash flow according to $K_t dA_t$, where dA_t is the productivity shock:

$$dA_t = a_t \mu dt + \sigma dZ_t \tag{51}$$

- The manager beliefs that dA_t evolves according to:

$$dA_t = (1+\alpha) a_t \mu dt + \sigma dZ_t \tag{52}$$

- K_t is the level of capital stock. Capital is determined by investment I_t and depreciation δ:

$$dK_t = (I_t - \delta K_t) dt \tag{53}$$

- Investment entails adjustment costs, homogeneous of degree one in I and K. Define $i = I/K$. Then total investment cost is:

$$c(i)K = \left(\frac{1}{2}\lambda_k i^2\right) K \tag{54}$$

  For simplicity of exposition, I do not consider the linear term.

- Instantaneous cash flows are dY_t:

$$\begin{aligned} dY_t &= K_t\left(dA_t - c(i)dt\right) \\ &= a_t \mu K_t dt + \sigma K_t dZ_t - c(i)K_t dt \end{aligned} \tag{55}$$

A.3.2 First Best - Neoclassical benchmark with shareholder deciding

$$q^{FB} = \theta i^{FB} \tag{56}$$

$$i^{FB} = r + \delta - \sqrt{(r+\delta)^2 - \frac{2\mu}{\theta}} \tag{57}$$

From the investor point of view:

$$p^{FB}(w) = P^{FB}(K, W)/K = q^{FB} - w. \tag{58}$$

A.3.3 Incentive Compatibility and the Dynamics of W

- Using the martingale representation theorem, the dynamics of the agent's perceived continuation utility are:

$$d\tilde{W}_t = \gamma \tilde{W}_t dt - dC_t + \beta_t K_t (dA_t - (1+\alpha)\mu dt) \tag{59}$$

- As before, ICC implies:

$$\beta_t = \frac{\lambda}{1+\alpha} \tag{60}$$

  therefore, the higher the level of optimism of the manager, the cheaper it will be incentive full effort $a_t = 1$.

- The investor's value is a function $P(K, W)$. Since everything is proportional to capital, define $w = W/K$:

- If $dC_t = 0$, the dynamics of w_t are:

$$dw_t = (\gamma - (i_t - \delta))w_t dt + \beta_t \sigma dZ_t \tag{61}$$

- Then $P(K, W) = KP(1, W/K) = Kp(w)$.

A.3.4 Investor's Value

- The investor's (scaled) value satisfies the HJB equation on $w \in [0, w^1]$:

$$rp(w) = \sup_{i,\beta \geq \lambda/(1+\alpha)} \left\{ \mu - c(i) + (i - \delta)p(w) + (\gamma - i + \delta)wp'(w) + \frac{\sigma^2 \beta^2}{2} p''(w) \right\} \tag{62}$$

- Subject to:

$$p(0) = l \qquad \text{(Liquidation at } w = 0) \tag{63}$$

$$p'(w^1) = -1 \qquad \text{(Compensation at } w = w^1) \tag{64}$$

$$p''(w^1) = 0 \qquad \text{(Smooth pasting at } w = w^1) \tag{65}$$

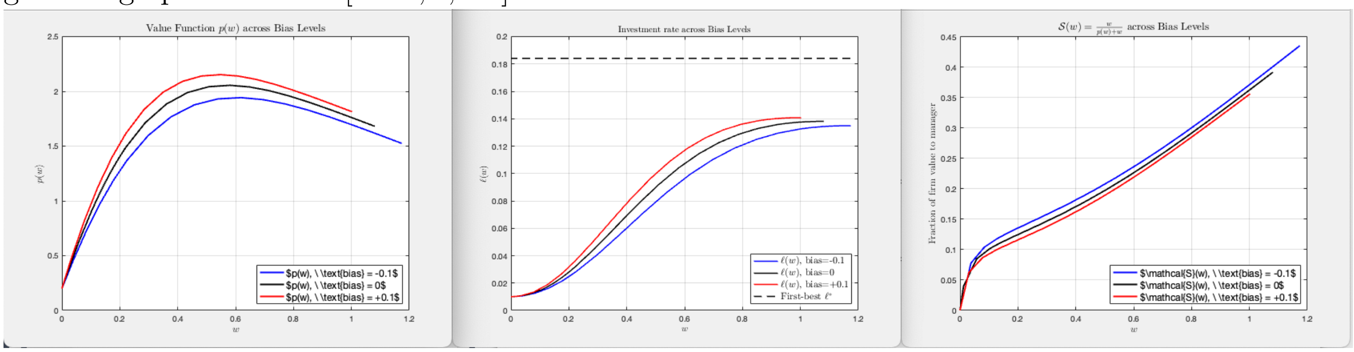
- Concavity of $p(w)$ implies optimal $\beta_t = \lambda/(1 + \alpha)$.

- i solves:

$$c'(i(w)) = p(w) - wp'(w) \tag{66}$$

A.4 Numerical Solution

I show below the model comparative stats for the values: $r = 0.08$, $\gamma = 0.16$, $\sigma_x = 0.25$, $\delta = 0.175$, $\lambda_s = 1$, $\lambda_l = 20$, $L = 0.2$, and $\alpha = 0.6$ for different values of the parameter governing optimism $\alpha = [-0.1, 0, 0.1]$.

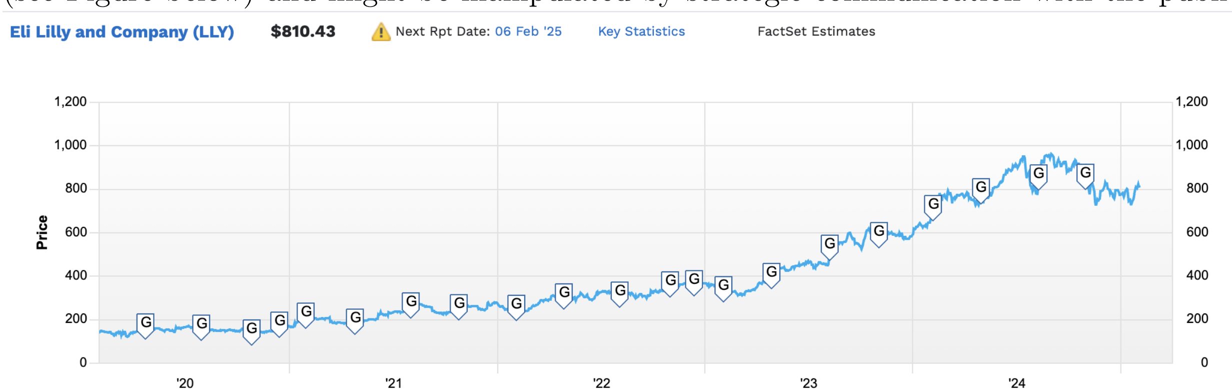


The more optimistic the manager is, the cheaper it will be to incentives and the lower the firm value that he will receive. In this case, therefore the payout threshold is lower and therefore after a sequence of positive shocks cash payments will happen earlier. Managerial optimism will increase firm value to the shareholder and make overinvestment optimal.

B Dataset construction

B.1 Forecasts Data

As explained in the main body of the paper, a key aspect of bringing the model to the data revolves around having accurate data about managerial and investors' expectations.
For the analysts I take the consensus from IBES and Factset. I consider only forecasts done in the short-term (12 months before the fiscal year). In there are more than one, I have the average. In a similar spirit, for managerial forecasts, I consider only short-term guidance. This is particularly important in this case. Indeed, managerial guidances are voluntarily disclosed (see Figure below) and might be manipulated by strategic communication with the public.



These values are expected to be between what the manager believes and what they want the public to believe. Nevertheless, in the short-term these concerns are mitigated as they will face immediate consequences. I want to highlight that I complement the common dataset used IBES with Factset to increase the panel size.
The two most commonly forecasted variables are Sales and EPS, with sales having the better coverage in my setup. Moreover, the choice of using sales is also motivated by not having to adjust the denominator of EPS, which could be problematic.

B.2 Data Cleaning

In this section, I will explain how I constructed the variables in the summary statistics. The variable cashflow is the sum of income before extraordinary items (ib), depreciation expenses (dp) and intangible investments(xrd+ 0.3*sga). The stock of physical capital is just PPEGT. The stock of intangible capital. In the paper, I will use "total capital", which is the sum of physical and intangible capital. The numerator of the definition of Tobin's Q is computed as usual ((prcc * csho) + (dlc+dltt) - act), while the denominator is Total capital for Total Q. I will rely on the data provided by Peters and Taylor, 2017 on WRDS. Lastly, size is the natural logarithm of total assets (atq).

B.3 Sample Representative:

I download and compute the summary stats for the full compustat and report the results in the table below. The goal of this part is to show that the sample that I use in the main paper how from the universe of Compustat. This is to show that the sample used is a good representation of US publicly listed firms, with a tilt towards better-covered larger firms.

Variable	Obs	Mean	Std. Dev.	P25	P50	P75
i	164,759	0.21	0.20	0.09	0.15	0.26
q_{tot}	164,759	1.27	1.80	0.37	0.72	1.38
size	164,759	5.85	1.97	4.36	5.67	7.18
cf	164,759	0.23	0.25	0.10	0.19	0.31
sales	164,759	1.33	1.35	0.48	0.95	1.69

C Supporting material for the empirical analysis

C.1 Persistence of Forecast Errors

In the main text I document that both analysts' and managers' sales forecast errors—defined as the deviation between forecasted and realized sales scaled by total capital—exhibit significant persistence over time. As a robustness check, I re-estimate exactly the same persistence regression but scale each forecast error by the firm's physical capital and obtain similar results as we can see below.

	log Analyst$\widehat{FE}_{it}$	log Manager$\widehat{FE}_{it}$
log Analyst$\widehat{FE}_{i,t-1}$	0.356***	
	(0.045)	
Manager$\widehat{FE}_{i,t-1}$		0.242***
		(0.039)
TimeFE	Y	Y
$Adj.R^2$	0.14	0.06
N	5,119	5,119

C.2 Investment-Q and Investment-CF alternative test

As a further robustness check, I check the results on the main body on the relationship between Investments-Q and Investments-CF. I confirm the results when using the definition of capital that includes only physical capital and only physical investment (capx). Therefore, I recompute the main variables and run the regressions. I report the results in the table below. The results are also robust when taking into account for the measurement error correction.

	i^{phy}	
	Low D	High D
$TQ^{Phy}_{i,t-1}$	0.0034***	0.0027***
	(0.000)	(0.000)
$CF_{i,t-1}$	-0.000	0.028***
	(0.04)	(0.007)
Firm FE	Y	Y
Time FE	Y	Y
$Adj.R^2$	0.53	0.45

D Analytical details around the econometric procedure

I will show how to obtain all the parameters from each moment. I developed this iterative procedure that ensures global identification of the deep parameters and a good match of the empirical moments.

1. From the first moment m_1 we will get i^*.
2. I get the deprecation rate of capital as $\delta_k = i - ln(m_2)$.
3. Using the growth of capital (m_3), I get γ
4. I will use the scaled Sales (m_4) to retrieve $\overline{A}$ from the econometrician point of view
5. I will use the managerial forecast error (m_5) to retrieve the parameter α
6. I will use the managerial forecast error (m_6) to retrieve the parameter β
7. Then I use moment implied condition to get λ_k

In addition to these, the estimatino procedure will ensure the correct sign of the parameter, the existence of the roots and the transversality condition.