Regulation and Intermediation in Over-the-counter Markets*

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Abstract

This paper investigates how banking regulation affects the trading behavior of bank-affiliated dealers in over-the-counter (OTC) financial markets. To micro-found the regulatory costs of holding inventories of OTC-traded assets, I develop a search model of an inter-dealer OTC market in which risk-averse dealers, facing idiosyncratic uncertainty in their endowment income and OTC asset returns, choose portfolios of risk-free and risky OTC assets subject to a regulatory balance sheet constraint. Acquiring additional inventories tightens regulatory requirements: dealers close to their constraint purchase assets only at sufficiently discounted prices to remain in compliance and avoid the penalty of lower utility from a violation. I solve the model numerically and show that inventory costs are state-dependent: tighter constraints lower the marginal value of OTC assets and increase trading costs. I examine stricter regulation by increasing the risk weight on risky OTC assets from 50% to 150%: stricter regulation raises holding costs and average spreads.

Keywords: Over-the-counter markets, regulation, inventory management
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1 Introduction

In the last decade since the great financial crisis, a variety of different reforms have been initiated to improve resilience and limit risk in the financial sector. International standards such as Basel II.5 and Basel III, as well as the Dodd-Frank Act in the U.S., have all updated or created new rules and laws that hope to avoid excessive risk-taking by financial institutions. However, an unintended consequence of these new regulations has been their effect on the trading behavior of dealers in over-the-counter (OTC) markets. Many dealers in OTC markets are *bank-affiliated*[1], that is, they operate as a subsidiary of a bank holding company (BHC) (Cetorelli and Stern, 2015)[2]. The largest dealers in OTC markets are bank-affiliated, and they comprise a large part of the market: about 50% of transaction volume in U.S. corporate bond markets in 2020 (Anderson et al., 2023; The Federal Reserve Board, 2023). What is key is that recent regulations aimed at improving resilience in the financial sector are applied to a BHC's consolidated balance sheet, which includes all subsidiary dealer trading assets (Adrian et al., 2018; Cimon and Garriott, 2019). That is, inventories of OTC-traded assets held by bank-affiliated dealers for trading purposes are explicitly included in their parent BHC's consolidated balance sheet, and so are also included in the calculation of key regulatory ratios. When making trading decisions, a bank-affiliated dealer anticipates the impact on their parent BHC's regulatory position.

Among the regulations most often cited as having the largest impact on bank-affiliated dealers' trading behavior, this paper focuses on the impact of Basel-style risk-weighted asset capital requirements[3]. While Basel II (2008) and Basel II.5 (2011) refined the notion of risk-weighted assets to better reflect the market risks that assets held for trading face, Basel III (expected implementation 2025) has updated these market risk weights. When risk weights increase, stocks of those asset classes affected are weighted more heavily in the calculation of risk-weighted assets: total risk-weighted assets are higher for the same amount of assets held, resulting a higher capital requirement. Compared to Basel II.5, these new market risk weights are estimated (as of 2019) to increase market risk capital requirements by 22% on average (BIS, 2019). This affects bank-affiliated dealers because many of the affected

[1] I use the term *bank-affiliated* to refer to dealers that operate as a subsidiary of a bank holding company (BHC). This is not to be confused with the term "broker-dealers", usually used to refer to the largest investment banks prior to the 2008 financial crisis. While Bear Stearns, Lehman Brothers, and Merrill Lynch were either taken over or declared bankruptcy, Goldman Sachs and Morgan Stanley were converted into BHCs post-crisis (Adrian and Shin, 2010).

[2] A BHC may have many subsidiaries, including but not limited to: commercial banking, specialty lending, asset management, underwriting, and, of course, dealer trading desks for multiple OTC markets (Cetorelli and Stern, 2015).

[3] Other notable important regulations being the supplementary leverage ratio (SLR) and the liquidity coverage ratio (LCR).

asset classes trade OTC. In an informal survey of practitioners by the Committee on the Global Financial System (CGFS), approximately 50% of survey respondents cited that these expected revisions of the market risk framework will result in "at least a moderate decline" in market making activity for those asset classes affected (CGFS, 2014).

Since these tighter regulations have taken effect, a large body of empirical evidence shows several clear changes in OTC markets: dealers now hold fewer trading assets, bid-ask spreads have widened, and standard liquidity metrics have deteriorated (Adrian et al., 2017; Duffie, 2023; Breckenfelder and Ivashina, 2021; Cimon and Garriott, 2019; Dick-Nielsen and Rossi, 2019; Cohen et al., 2024; Bao et al., 2018; Bessembinder et al., 2018; Choi and Huh, 2017). But, precisely, through what mechanism do such regulations shape dealer decision-making, producing these outcomes? Theoretical investigation of the impact of this type of regulation on OTC markets is just beginning, and has thus far focused on the exploration of dealer behavior subject to a reduced-form regulatory cost of inventories; for example, by either imposing a regulatory cap on OTC inventory stock, as in Duffie et al. (2023), or placing a holding cost per unit of inventory, as in Cohen et al. (2024). In contrast, in this paper I aim to micro-found the regulatory balance sheet costs of OTC assets: dealers choose a portfolio of assets, including risky OTC traded assets, under a balance-sheet level regulatory constraint. OTC inventories are costly to hold due to the *potential threat* of penalties (in the form of lower utility) faced if the regulatory constraint is violated.

I develop a search model of an inter-dealer OTC market in which bank-affiliated dealers, referred to interchangeably in this text as *agents*, manage their portfolio of risk-free liquid assets and risky OTC-traded assets subject to a balance sheet regulatory constraint. As in Gârleanu (2009), agents are risk averse and face idiosyncratic uncertainty regarding their endowment income and their private valuation of the OTC asset's return. Furthermore, these two processes can be correlated. The agent's risky endowment income in this model represents the earnings allocated to a dealer's trading desk by the parent BHC from other lines of business, which are therefore independent of dealer decisions. In the same vein, the OTC asset also provides some outside unmodeled income called the OTC asset "dividend". This return includes not only any *direct* income (interest payments, true dividends), but also provides an agent *indirect* income from holding the OTC asset. Since this is a model of an inter-dealer market, for a bank-affiliated dealer this indirect income also includes any intermediation profits from the retail side of the market.

The risk-free asset exists in perfectly elastic supply: agents can save or borrow any amount at an exogenous risk-free rate, and have constant access to this outside market. The risky OTC asset is traded in a decentralized market characterized by random pairwise meetings: when agents meet, they engage in proportional (Kalai) bargaining. Agents can hold strictly positive units of the risky OTC asset. I micro-found regulatory inventory costs through a balance sheet constraint. All agents are subject to a risk-based capital requirement inspired by Basel II and Basel III in that they must maintain a certain level of common equity tier 1 capital (equity) relative to the value of the risk-weighted assets (the risk-weighted sum of the market value of their risk-free and risky OTC assets) on their balance sheet in a proportion set by a policymaker.[4] An increase in the strictness of the regulation increases the minimum level of equity an agent must hold, all else equal. I further assume that there is perfect enforcement of the regulatory capital requirement: if an agent's equity falls such that it is below the regulatory minimum, they must immediately take action to remedy it.

These two last features are at the heart of this model. First, as a consequence of the risk-weighted asset ratio, the more risky OTC assets a dealer has the higher the amount of regulatory capital they must hold. In this way, the regulatory constraint is endogenous since it depends on a dealer's stock of risky OTC assets, which follows from dealer trading decisions. The second is a consequence of the perfect enforcement assumption: in between trading opportunities in the OTC market, if a dealer finds themselves in violation of the constraint the only tool at their disposal is to cut their consumption to boost their savings in the risk-free asset in order to increase their equity to the regulatory minimum. Since this leads to lower utility, violating the regulatory constraint is costly for agents. As a result of these two features, when an agent buys additional units of the risky OTC asset, they also consider the impact on their regulatory position. If a dealer is close to their constraint, they only buy at sufficiently low prices to remain in compliance and avoid the lower utility penalty of violation. Search frictions are crucial to this mechanism, as they prevent agents from being able to re-balance their portfolio continuously.

I solve the steady state numerically using an iterative procedure to solve a high-dimensional fixed-point problem in the agent value function, trading rules, and distributions. To do so, I modify the finite difference algorithm developed by Achdou et al. (2022) to account for what is essentially a state-based boundary condition. I solve the model for a lower risk weight (50%) to establish baseline results. I find that the presence of a risk-based capital requirement causes the marginal value of an additional unit of the risky OTC asset to collapse when

[4] In this model, I focus exclusively on how regulation affects market making through inventory balance sheet costs, abstracting from funding costs which have been shown to be an additional important factor in determining how dealers intermediate. Usually, dealers finance their positions via short-term funding on the repo market - tighter regulation reduces a dealer's ability to get funding (Cimon and Garriott, 2019; Adrian et al., 2018). I also abstract away from competition from non bank-affiliated dealers and focus solely on the choices of bank affiliated dealers. The rise of non-bank dealers to fill the immediacy gap left in recent years is thus an observed trend this model will be unable to explain. Lastly, I abstract away from the choice of risky-principal vs. agency trades on the part of dealers, considering only risk-principal trades (Saar et al., 2019; Cimon and Garriott, 2019; An and Zheng, 2023).

an agent's state is near the regulatory bound; far from the bound, marginal values level off as the risk of violating the constraint falls. Thus, the balance sheet costs of risky OTC assets are heterogeneous and state-based, depending crucially on dealers' distance-to-constraint. These lower marginal values near the bound result in these dealers demanding lower prices on asset purchases, leading to higher trading costs for their counter-party. As a consequence, I find that spreads widen significantly near the regulatory threshold.

I use this model to study the impact of stricter regulation by increasing the risk weight on risky OTC assets from 50% to 150%. The stricter regulation increases the set of agent states that are constrained, making the risky OTC asset less attractive to agents near the regulatory bound. The marginal value of an additional unit of the risky OTC asset falls sharply near the new regulatory bound, with the steepest decline for agents that hold more inventory. Consequently, I find that average spreads increase by 10 bps. I also find that market participation tilts toward agents that have high levels of equity, and low-equity traders withdraw from the market. Despite a smaller pool of traders, turnover increases as the remaining better-capitalized agents comprise a higher fraction of meetings. This slightly increases turnover in comparison to the low-regulatory weight environment.

This paper is organized as follows. Section 2 reviews the related literature. Section 3 outlines the quantitative model, and Section 4 discusses the method used to solve the model and the parameterization. Section 5 presents the baseline model outcomes, and section 6 explores the counterfactual when risk weights increase due to stricter regulation. Section 7 concludes.

2 Related Literature

Many search-theoretic approaches to studying OTC markets follow the seminal works of Duffie et al. (2005) and Lagos and Rocheteau (2009), in which dealers can intermediate trades in an illiquid asset for customers. However, in this setting, the study of dealer inventory management is limited: dealers do not have any private valuation of the OTC traded asset, are risk-neutral, and have access to a centralized inter-dealer market. Consequently, dealers do not hold steady state inventories and engage only in matchmaking activities for customers. Weill (2007); Rocheteau and Weill (2011); Fleskes (2024) extend this framework to study the out of steady state dynamics of dealer inventory, identifying under what conditions dealers use their inventory capacity temporarily to smooth imbalances in customer trading needs due to unexpected liquidity crises. In Cohen et al. (2024), the authors impose an asset-in-advance constraint: dealers must hold assets to fulfill customer orders, and so have incentive to hold inventories in the steady state. The regulatory cost of inventories here is modeled as a per-unit cost to holding inventories.

Dealer inventories play a more natural role when markets are fully decentralized.[5] Hugonnier et al. (2020) allow for fully decentralized two-tier markets for trade in an OTC asset. Risk-neutral dealers and customers are heterogeneous in their private valuations of the asset and engage in bilateral trade with binary asset holdings. A key result of their paper is that, in the steady state, only some dealers with middle-of-the-road private valuations are willing to provide intermediation services and that intermediation by dealers is characterized by intermediation chains. Yang and Zeng (2021) use a similar two-tier market framework in which risk-neutral dealers can now hold multiple units of the asset. By relaxing the binary asset holding assumption, the authors show that there are multiple equilibria in which dealers coordinate their liquidity provision depending on asset fundamentals.

In the above models, dealers have a special exogenous ability to intermediate in a retail market. Other works in this literature focus instead on one-tier fully decentralized markets to study endogenous intermediation: which agents decide to become intermediaries. Rather than designating a special set of agents who exogenously have the ability to intermediate, heterogeneous agents all can trade with each other. These markets are characterized by a core-periphery structure and intermediation chains: at the core of the trading network are agents who engage heavily in intermediation, and at the periphery are those who behave more as value investors. The agents that arise as intermediaries tend to be those that have a type that allows them to extract better terms of trade: they have faster meeting technologies, better information, or more bargaining power such as in Jarosch et al. (2016); Donaldson et al. (2018); Üslü (2019a); Bethune et al. (2022); Hugonnier et al. (2021); Farboodi et al. (2023). In contrast to the above works, I find that it is certain agent states rather than fixed agent types which drive the heterogeneity in trading behavior. In my model, agents are all identical in that they face the same risky income processes. However, realizations for each agent generate heterogeneity in the first dimension, equity.

In the vast majority of studies of OTC markets in this literature, agents are risk neutral. Duffie et al. (2007) and Gârleanu (2009) are the foundational works that depart from this assumption, relying on CARA utility preferences to microfound quadratic utility over multiple units of the OTC asset.[6] Kargar et al. (2023) is the exception, explicitly considering risk

[5]Another parallel literature that studies dealer inventory management is based on a class of search models in the tradition of Rubinstein and Wolinsky (1987), such as in Johri and Leach (2002); Shevchenko (2004); Masters (2007); Watanabe (2010); Wright and Wong (2014); Nosal et al. (2015, 2019); Watanabe (2020); Gu et al. (2024); Gong and Wright (2024). These types of inventory models differ from the previously discussed works in that they primarily study three-sided goods markets: there are usually distinct sellers, middlemen, and buyers who all occupy different roles in the economy and face different frictional markets. The middleman's inventory management problem is therefore summarized by their pricing strategy on both sides of the market, wholesale and retail. Carrasco and Smith (2017); Carrasco and Harrison (2023) focus on the pricing strategies of a dealer liquidating their assets.

[6]The same method is used to micro-found the reduced form utility specifications in Vayanos and Weill

averse customers and aggregate risk. However, the focus of their work is on the pricing of the OTC asset and the portfolio choice problem of customers; dealers act as simple risk-neutral profit maximizing matchmakers. In contrast, in this paper I investigate the portfolio problem of risk-averse dealers who are subject to a regulatory balance sheet constraint. Liquidity in the inter-dealer market therefore a consequence of the constrained portfolio decisions of dealers.

There is also a literature on inventory management by dealers in OTC markets in the finance literature based on the seminal works of Amihud and Mendelson (1980) and Ho and Stoll (1981, 1983).[7]. Works of this tradition primarily focus on the determination of bid-ask spreads set by a (usually) monopolistic risk neutral dealer, where investors arrive to the OTC market with fixed trading demands. Duffie (2023) extends Amihud and Mendelson (1980) by imposing an maximum level of inventory that the dealer can hold: the higher the balance sheet costs of inventories, the lower the inventory cap. As the level of inventories approaches the regulatory implied limit, bid-ask spreads widen to account for the increasing marginal cost of dealer balance sheet space. Similarly, Wang and Zhong (2022) study the impact of the Basel III risk weight changes by varying the maximum level of inventories a dealer can hold: they find that the increased capital requirements result in an increase in order rejection rates by dealers. Adrian et al. (2020) study the impact of increased overnight inventory costs and show that dealers adjust their pricing strategies as the end of the trading day approaches to offload inventories and avoid them.

In contrast to this style of model, in this work all agents engage in bilateral trade; distributions and agent demands are equilibrium objects. Furthermore, inventory costs are derived via a balance sheet constraint rather than through an inventory cap. This paper is the first, to the best of my knowledge, to study dealer inventory management from this joint portfolio choice and search perspective in a general equilibrium framework.

3 A portfolio choice model with an illiquid asset

Time is continuous and infinite. Dealers, referred to interchangeably in this paper as *agents*, are infinitely lived and discount time at a constant rate $r \in (0, 1)$. There is a numeraire good called consumption c_t, a risk-free asset ω_t in perfectly elastic supply, and a risky over-the-counter (OTC) traded asset n_t in some positive supply in shares $S > 0$. The risk-free asset can be invested in an outside account that pays the discount rate and can be costlessly converted into the numeraire. Alternatively, agents can also borrow in the risk-free asset at

(2008); Praz (2014); Üslü (2019b).

[7]Both this literature and the middlemen literature a la Rubinstein and Wolinsky (1987) have origins in Garman (1976).

the discount rate: ω is the stock of *net* risk-free assets an agent holds. The risky OTC asset earns a risky dividend D_t, paid out in units of the risk-free asset. All agents earn income from a risky endowment η_t also paid in units of the risk-free asset. Agents can hold multiple divisible units of the risk-free asset ω and the risky OTC asset $n \geq 0$.

I frame the above as the problem of a trading desk manager operating as a subsidiary of a larger institution (i.e., a bank holding company). The risk-free asset is interpreted as cash, or another equally liquid and safe asset like reserves, which serves as the unit of account. The manager uses their cash holdings to buy and sell the OTC asset, and all income (from dividends or endowments) is realized in this unit of account. The outside account paying the discount rate r is interpreted as a reserve account; alternatively, they can borrow from an outside market at the same rate. The manager controls their trading strategies and portfolio decisions: their holdings of cash (the risk-free asset) and OTC asset inventories (the risky OTC asset). However, they do not control the activities of the rest of the institution, for example, the other BHC subsidiaries (e.g., commercial banking, investment banking, etc.). The endowment is interpreted as any outside income the trading desk may periodically receive from the institution's other lines of business, or alternatively as a cost if the trading desk has to transfer cash to other subsidiaries at the discretion of institution's higher management.

All agents have constant absolute risk aversion (CARA) utility over consumption of the numeraire good with risk aversion parameter κ: $u(c) = -e^{-\kappa c}$. As such, consumption of the numeraire good can be negative. Consumption can be thought of as profits remitted from the trading desk to the larger institution, with negative consumption indicating losses taken on the trading desk at that time. Agents have idiosyncratic uncertainty over their endowment income and their private valuation of the OTC traded asset's dividend. Let $Z_t = [Z_{\eta,t}, Z_{D,t}]'$ be a two-dimensional standard Brownian motion for $t \geq 0$ defined on a filtered probability space $(\Omega, F, \mathbb{P})$ where $Z_{\eta,t}$ and $Z_{D,t}$ are independent. Following Gârleanu (2009); Duffie et al. (2007); Praz (2014), the stochastic cumulative dividend process per unit of risky OTC asset n follows for constants m_D and σ_D:

$$dD_t = m_D dt + \sigma_D dZ_{D,t}. \tag{1}$$

The stochastic endowment process follows for constants m_η and σ_η:

$$d\eta_t^i = m_\eta dt + \sigma_\eta Z_{\eta,t}^i. \tag{2}$$

$Z_{\eta,t}^i$ is defined as:

$$Z_{\eta,t}^i = \rho_t^i dZ_{D,t} + \sqrt{1 - (\rho_t^i)^2} dZ_{\eta,t}. \tag{3}$$

An agent's endowment process is stochastic with drift m_η and with an element of uncertainty $\sigma_\eta dZ^i_{\eta,t}$. An agent's type i determines how correlated their endowment is with their private valuation of the OTC asset's dividend: ρ^i_t is the instantaneous correlation for an agent of type i between the OTC asset dividend and the endowment of agent i. If an agent's OTC asset dividend income is negatively correlated with their endowment income, the OTC asset dividend acts as a hedge. In this setting, all agents have the same fixed correlation $\rho^i = \rho\ \forall i$. I assume that all agents have the same type, and so liquidity preferences are fixed and constant $\rho^i = \rho$.

Each agent records on the asset side of their balance sheet their net holdings of the risk-free asset ω at a market price normalized to 1, and the OTC asset n is marked to market at price $\tilde{p}$, to be defined more precisely shortly. An agent's equity E is defined as the sum of the market value of their net assets:

$$E = \omega + \tilde{p}n.$$

There is a unit measure of a continuum of agents, where $F(E, n)$ denotes the distribution of agents over equity E and risky OTC asset holdings n.

Trade in the risky OTC asset is fully bilateral. Agents receive a trading opportunity according to a Poisson process with intensity $\lambda > 0$. Upon receiving a trade opportunity, an agent draws a counter-party from the distribution $F(E, n)$. Once a trading partner is drawn, agents engage in proportional (Kalai) bargaining over the terms of trade of the OTC asset, where agents have equal bargaining power. The market value of the OTC asset, $\tilde{p}$, is defined as the *average* price of the risky asset based on current trades in the OTC market[8]. It is also assumed that when trading the risky OTC asset, agents cannot consume the numeraire good.

Each agent is also subject to a risk-based capital requirement in the spirit of Basel II and Basel III, which stipulates that they must maintain a level of tier 1 capital relative to their risk-weighted assets of at least $\Lambda\%$. Typically the policy ratio is $\Lambda = 4.5\%$ unless the BHC parent is a globally systemically important bank (G-SIB); depending on their systemic importance, G-SIBs are subject to an additional capital requirement of 1% to 3.5% (BIS, 2019). In this model, common equity tier 1 capital is simply agent equity, E, and risk-weighted assets are calculated as the sum of the market value of dealer risk-free and risky OTC asset holdings, where risk-free assets have a 0% risk weight and risky OTC asset

[8]The average price is used to value agent asset holdings as it best reflects current accounting standards in the United States. According to Financial Accounting Standards Board (FASB) regulation[9], assets for which there are active (even if decentralized) markets are consider *level 1* assets and therefore quoted prices for similar assets at a given date must be used to value an institution's illiquid assets.

inventories have some positive risk weight set by policy, $\nu\%$. The risk-weight $\nu\%$ typically ranges from 0% to 250% in increments of 50%, with cash and U.S. treasuries using a risk weight of 0%. The risk-based capital requirement in this model is in practice a risk-weighted assets ratio:

$$\frac{E}{0 \times \omega + \nu \times \tilde{p}n} \geq \Lambda. \tag{4}$$

Define the minimum level of equity required by this regulation for an agent with a stock of risky OTC assets n to be:

$$\bar{E}(n) = \Lambda \nu \tilde{p} n. \tag{5}$$

Due to the stochastic nature of agent income, it is possible that agents experience negative income shocks severe enough to cause their equity to fall to or below the regulatory bound, $E \leq \bar{E}(n)$. I assume that there is *perfect enforcement* of the capital requirement. In such a case, there is no fine, fee, or other cost for the violation of the constraint; rather, the agent is required to immediate action to remedy their regulatory position. That is, dealers must adjust their consumption policy and trading behavior such that their equity is expected to quickly recover to comply with the capital requirement.[10]

Agent problem Agents are born with an initial level of the risk-free asset, $\omega_0 > 0$. Agents receive income from their endowment $d\eta_t$, their holdings of the risk-free asset $r\omega_t dt$, and their holdings of the risky OTC asset $n_t dD_t$. Agents consume the numeraire good $c_t dt$, and can trade a_t units at price P_t determined by bargaining when the trading opportunity arrives. An agent's equity therefore evolves according to:

$$\begin{aligned} dE_t =& d\eta_t - c_t dt + rE_t dt + n_t \left(dD_t + d\tilde{p}_t - r\tilde{p}_t dt \right) \\ &+ (\tilde{p}_t - P_t) dn_t. \end{aligned} \tag{6}$$

An agent's OTC asset holdings evolve according to:

$$dn_t = 1\{\text{OTC trade opportunity}\} a_t. \tag{7}$$

Higher endowment income, $d\eta_t$, and higher direct income from the risk-free and risk assets, $rE_t dt$ and $n_t dD_t$, increase an agent's equity. When an agent consumes, $c_t dt$, equity falls. $d\tilde{p}_t$ captures the change in equity per unit of risky OTC asset due to changes in the market price: as the asset appreciates in value, equity rises, The term $r\tilde{p}_t dt$ captures the per-unit opportunity cost of holding a risky OTC asset, the value $\tilde{p}_t$ of which otherwise could be earning the discount rate r. Finally, when an agent trades, they balance their

[10]Equivalently, one could impose a fine for violation sufficiently high that agents optimally choose such behavior.

effective trade price, P, relative to the market price, $\tilde{p}$: the term $\tilde{p}_t - P_t$ is called *slippage* by practitioners and captures the net gain or loss to an agent's equity when trading the illiquid asset. In centralized markets, slippage is always zero as the trade price coincides with the market price.

At time t, an agent of state (E, n) maximizes their expected discounted value of consumption subject to equations (6), (7), as well as the regulatory constraint (4):

$$W_t(E,n) = \sup_{\{c(s)\}_{s=t}^{\infty}} E_t\left[\int_t^{\infty} -e^{r(s-t)}e^{-\kappa c(s)}ds \mid E_t = E, n_t = n\right]. \tag{8}$$

Trade the OTC asset When agents meet in the OTC market, they engage in proportional bargaining, also referred to as Kalai bargaining, with equal bargaining power over the terms of trade (a, P) with trade size a. When $a > 0$, this indicates a "home" agent (E, n) buy and an "away" agent (E', n') sell. The home agent maximizes their post-trade surplus with respect to (a, P),

$$\max_{P\geq 0, a}\left[W(E + (\tilde{p} - P)a, n + a) - W(E, n)\right], \tag{9}$$

subject to the following constraints:

$$\begin{gathered}
W(E + (\tilde{p} - P)a, n + a) - W(E, n) = W(E' - (\tilde{p} - P)a, n' - a) - W(E', n'), \\
-n \leq a \leq n', \\
E + (\tilde{p} - P)a \geq \Lambda\nu\tilde{p} \times (n + a), \\
E' - (\tilde{p} - P)a \geq \Lambda\nu\tilde{p} \times (n' - a).
\end{gathered}$$

The first is the surplus sharing Kalai constraint, which stipulates that the post-trade surplus of each agent must be equal when there is equal bargaining power. The second equation reflects feasibility: agents can only trade assets they have. Since agents cannot consume simultaneous to trading, the last two equations ensure that post-trade both agents remain in compliance with the regulatory constraint given by (4). Note that agents do not necessarily always trade, as $a = 0$ is in the set of possible outcomes.

In a similar environment without an occasionally binding regulatory capital requirement, as in Duffie et al. (2007); Gârleanu (2009); Praz (2014); Üslü (2019b), CARA utility implies that an agent's valuation of assets is independent of her equity position. In these frameworks, one can effectively replicate the risk-return trade-off faced by risk-averse agents in the model outlined here using risk-neutral agents with a specific reduced-form quadratic utility over OTC assets. However, when an occasionally binding constraint is introduced, this equityindependence feature of CARA utility no longer holds: agents must now also must track their equity position and proximity to the regulatory constraint, and take their position into account when trading in the OTC asset.

3.1 Stationary equilibrium

I consider a steady state in which the market price $\tilde{p}$ is constant ($d\tilde{p} = 0$).

3.1.1 Agent value function and optimal consumption policy

Assuming appropriate differentiability and using Ito's Lemma, the agent's Hamilton-Jacobi-Bellman (HJB) equation is:

$$
\begin{aligned}
rW(E,n) = \max_c \{ & u(c) + \partial_E W(E,n)\left(m_\eta + rE - c + n\left(m_D - r\tilde{p}\right)\right) \\
& + \frac{1}{2}\partial_E^2 W(E,n)\left(\sigma_\eta^2 + \sigma_D^2 n^2 + 2n\rho\sigma_\eta\sigma_D\right) \\
& + \lambda \int_{E',n'} \left[W(E + (\tilde{p} - P^*)\, a^*, n + a^*) - W(E,n)\right]^+ dF(E',n')
\end{aligned}
\tag{10}
$$

The first term is the flow utility that agents earn from consuming the numeraire good c; the second and third terms capture risk-aversion due to the stochastic nature of the endowment and the OTC asset dividend (drift and volatility, respectively); the final two terms capture the expected change in value due to trade where (a^*, P^*) are determined by the bargaining process outlined in the previous section.

Now consider the agent's optimal consumption policy. The agent maximizes their consumption c given their current level of equity E and a fixed stock of OTC risk assets n. As stated previously, the agent must maintain a high enough level of equity E relative to their risk weighted assets, as defined by (4). Optimal agent consumption when equity is above the regulatory bound, $E > \bar{E}(n)$, satisfies:

$$
u'(c^*(E,n)) = \partial_E W(E,n). \tag{11}
$$

Note that an agent may receive a negative endowment or dividend realization large enough that the agent's equity falls *below* the regulatory bound. This is possible because these two sources of income are modeled directly as a diffusion processes: they are random variables, and so agents do not know with certainty what their realization will be when making consumption decisions. Since there is perfect enforcement of the capital requirement, the agent's only recourse in the short run is to sharply reduce consumption and boost savings, ensuring that her equity, at least in *expectation*, quickly recovers to comply with the capital requirement. In the medium term agents may adjust their holdings of risky OTC assets as trading opportunities become available. However, the perfect enforcement assumption requires that, before those trading opportunities arrive, the agent must adjust their consumption/savings behavior to try to regain compliance. An agent's consumption policy therefore follows:

$$c^*(E,n) = \begin{cases} u'^{-1}\left(\partial_E W(E,n)\right) & E > \bar{E}(n), \\ m_\eta + rE + n\left(m_D + d\tilde{p} - r\tilde{p}\right) - \left(\bar{E}(n) - E\right) & E \leq \bar{E}(n). \end{cases} \tag{12}$$

When equity is above the regulatory bound for a given stock of risky OTC assets, $E > \bar{E}(n)$, consumption is at the unconstrained optimum. When equity is below the regulatory bound, $E \leq \bar{E}(n)$, the agent consumes such that the *drift* in her equity is large enough to recover the regulatory minimum. That is, a dealer consumes in such a way that they expect their equity to rise to the regulatory minimum. The term $\bar{E}(n) - E$ captures the penalty to agent consumption based on the distance from the bound: the lower your equity, the more you have to cut your consumption to regain the regulatory minimum. In this way, being in violation of the regulatory constraint is costly for an agent: lower consumption results in lower utility.

A state-based boundary condition I implement this consumption policy through a state-based boundary condition following Achdou et al. (2022). Achdou et al. (2022) handle borrowing constraints in continuous time by implementing what is essentially a constraint on the shape of the value function, which results in agents consuming in such a way that the borrowing constraint is never violated. I implement the above consumption policy in a similar manner by placing a condition on the shape of the value function that ensures consumption and savings choices *in expectation* keep equity above the regulatory minimum. The state-based boundary condition in this framework is $\forall n$:

$$\partial_E W(E,n) \geq u'\left(m_\eta + rE + n\left(m_D + d\tilde{p} - r\tilde{p}\right) - \left(\bar{E}(n) - E\right)\right),\ \forall E \leq \bar{E}(n) \tag{13}$$

Under this condition, optimal consumption is always satisfied by (11). In other words, the boundary condition requires that the marginal value of equity below the regulatory bound is sufficiently high such that it is optimal for agents to make severe cuts to their consumption. This boundary condition thus enforces the regulatory constraint indirectly by encoding the implicit cost of violating the regulatory constraint into the shape of the value function.

There are two key differences between the implementation of the state-based boundary condition in this framework and that of Achdou et al. (2022). First, in theory, agents can potentially realize a negative income shock such that the regulatory constraint is violated despite their efforts to remain in compliance. This is because endowment $d\eta_t$ and dividend dD_t incomes are random variables and are not known ex-ante, in contrast to Achdou et al. (2022)

in which income at each instant is known (though it may evolve over time via a diffusion or jump process). In practice, however, the discretization of the value function required by the numerical solution renders the effective probability of realizing a negative income shock that pushes agents into violation is zero. As a consequence, in the computed equilibrium solution the regulatory constraint is violated due to income shocks with probability zero.

Second, the state-based boundary constraint in this framework is *endogenous*: dealers can adjust their stock of risky OTC assets n, and thereby adjust the regulatory minimum level of equity they must hold. In comparison to Achdou et al. (2022), it is not sufficient to only restrict the boundary point $E = \bar{E}(n)$. To ensure that dealers find trades that place them in violation of the regulatory constraint undesirable, I must also impose off-equilibrium-path boundary conditions for all $E \leq \bar{E}(n)$.

3.1.2 Equilibrium distribution of agent types

Define the probability that, post-trade, an agent of state (E', n') becomes state (E, n) to be $\alpha_{E,n,E',n'}$. The steady state distribution of agents $F(E, n)$ with probability density function $f(E, n)$ satisfies the following Kolmogorov forward (Fokker-Planck) equation:

$$\begin{aligned}
0 = &- \partial_E \left\{ (m_\eta - c^*(E,n) + rE + n(m_D - r\tilde{p}))\, f(E,n) \right\} \\
&+ \frac{1}{2}\partial_E^2 \left\{ \left(\sigma_\eta^2 + \sigma_D^2 n^2 + 2n\rho\sigma_\eta\sigma_D\right) f(E,n) \right\} \\
&+ \lambda \left(\int_{E',n'} \alpha_{E,n,E',n'} f(E',n') d(E',n') \right) \\
&- \lambda f(E,n) \left(\int_{E',n'} \alpha_{E',n',E,n} d(E',n') \right).
\end{aligned} \tag{14}$$

The first two terms capture the changes in the distribution due to the stochastic diffusion processes on the endowment and the OTC asset dividend. The last two lines show the inflows and outflows of agents due to trade in the OTC asset.

3.1.3 The market price of the OTC asset

The market price of the OTC asset $\tilde{p}$ is the average of all the realized trade prices in the economy. Denote the probability that two agents meet and trade, conditional on receiving a trade opportunity and trading a non-zero amount of the asset, to be $P_{trade}(E, n, E', n')$. The weighted market price $\tilde{p}$ is therefore:

$$\tilde{p} = \int\int \left(P_{trade}(E,n,E',n') \times P^*(E,n,E',n')\right) dF(E',n') dF(E,n). \tag{15}$$

3.1.4 Market clearing

Finally, market clearing requires that all asset shares $S > 0$ be held by agents. With a unit measure of agents, it must be that the steady state weighted average of asset holdings of all agents equals the supply in shares:

$$\int n dF(E, n) = S. \tag{16}$$

3.1.5 Stationary equilibrium definition

A stationary equilibrium is a value function $W(E, n)$, consumption policy $c^*(E, n)$, a distribution $f(E, n)$, a set of trading rules (a^*, P^*) for the OTC asset, and a market price $\tilde{p}$ such that, taking the model primitives $\{r, \lambda, m_\eta, \sigma_\eta, m_D, \sigma_D, \rho, \Lambda, \nu\}$ as given, the following are satisfied:

1. Value function $W(E, n)$ satisfies (10) and (13);
2. Consumption policy $c^*(E, n)$ satisfies (11);
3. Trading rules (a^*, P^*) satisfy the bargaining protocol defined in (9);
4. Distribution $f(E, n)$ satisfies (14);
5. The market price $\tilde{p}$ is defined as in (15) and market clearing holds as in (16).

3.1.6 Discussion

The key mechanism in this model is as follows: the presence of the risk-based capital requirement distorts the marginal valuation of risky and illiquid OTC assets in the region close to the regulatory constraint by penalizing consumption if the regulatory constraint is violated. Thus, the true "balance sheet cost" of holding OTC inventories is, in fact, the off-equilibrium-path *threat* of lower utility if they violate the regulatory constraint.

The fact that the risky asset is traded in a market characterized by search frictions is critical to this dynamic. In a model with centralized trading, agents with CARA utility would simply hold a fixed level of assets either at their preferred level, or up to the regulatory limit. Agents could continuously re-balance their level of assets freely, and as such never face the possibility of violating the regulatory capital requirement. Unless agents face frictions in buying and selling the OTC asset, the threat of violating the constraint holds no power.

The regulatory cost to holding inventories as modeled here is a purely implicit one and differs depending on the agent's distance to the constraint. As the regulatory bound is approached, the threat of lower utility looms more menacingly and agents adjust their consumption and trading behavior. When an agent has plenty of slack in their regulatory ratio, the risk of violation is minimized. This is in stark contrast to models in which balance sheet costs are modeled as per-unit costs to OTC inventories: all agents change their trading behavior in the same manner. This work emphasizes how this behavior changes depending on the distance-to-constraint, and that cost changes over time as agents realize different incomes.

4 Solution Method & Parameterization

The steady-state problem amounts to a high-dimensional fixed point problem in the value function $W(E,n)$ and the distribution $F(E,n)$ of agents as discussed in Hugonnier et al. (2020). The algorithm I have developed to solve the model numerically is available in Appendix (A). I discretize the state space for the liquid risk-free assets of the agent to be $E_i \in [0,10]$ with $I = 100$ points and the risky OTC assets to be $n_j \in \{0,1,2,3,4,5\}$. Since an agent's stock of risky OTC assets n is meant to represent different inventory levels, I also modify the regulatory constraint to include a regulatory scaling parameter $\phi \geq 1$:

$$E > \bar{E}(n) = \Lambda\nu \underbrace{\phi} n$$

By appropriately choosing this scaling parameter, I can ensure that when asset levels are at their highest level (n_H), this truly means that agents have a very large stock of risky OTC assets relative to their equity. This ensures that at these higher inventory levels, agents have a significantly lower risk-weighted assets ratio.

I use the parameterization as in Tables 1 and 2. A unit of time is one year. The following parameters are set directly: r, S, κ, λ, and ϕ. The discount rate (the return on the risk-free asset) is set to $r = 5\%$. The asset supply in shares is set to $S = 1$, and the CARA risk aversion parameter is set to $\kappa = 0.4$. The Poisson intensity parameter on agent meetings in the OTC market is set to $\lambda = 30$, indicating that agents receive a trading opportunity once every 8.4 days on average. Furthermore, I set the regulatory scaling parameter ϕ to 10. This indicates that when agents hold the highest level of risky OTC assets ($n = n_H = 5$), the highest possible risk-weighted asset ratio is 18.50%; this is relative to when agents hold the lowest level of positive risky OTC assets ($n = 1$), at which the highest possible risk weighted asset ratio is 92.50%.[11]

The parameters on the stochastic process $m_\eta, \sigma_\eta, m_D, \sigma_D$ and the correlation parameter ρ are directly estimated using quarterly balance sheet level data from the FR-Y9C filings of bank holding companies (BHCs). The data and the maximum likelihood procedure used are

[11] The parameters λ and ϕ are set directly for this draft, but a calibration is in progress targeting annual turnover and the distribution of risk-weighted asset ratios observed in the data, respectively.

Variable	Parameter	Value	Variable	Parameter	Value
Discount rate	r	5%	Asset supply (in shares)	S	1
Income process (drift)	m_η	0.0736	Meeting intensity	λ_d	30
Income process (vol.)	σ_η	0.1366	Correlation	ρ_m	0.0013
Dividend process (drift)	m_d	0.1265	Risk aversion	κ	0.4
Dividend process (vol.)	σ_d	0.3445	Regulatory Scaling	ϕ	10

Table 1: Baseline model parameters

Variable	Parameter	Value
OTC asset risk weight	ν	50%, 150%
Risk weighted assets ratio	Λ	4.5%

Table 2: Baseline policy parameters

outlined in Appendix (B), as well as the standard errors of the estimates.

5 Baseline model outcomes

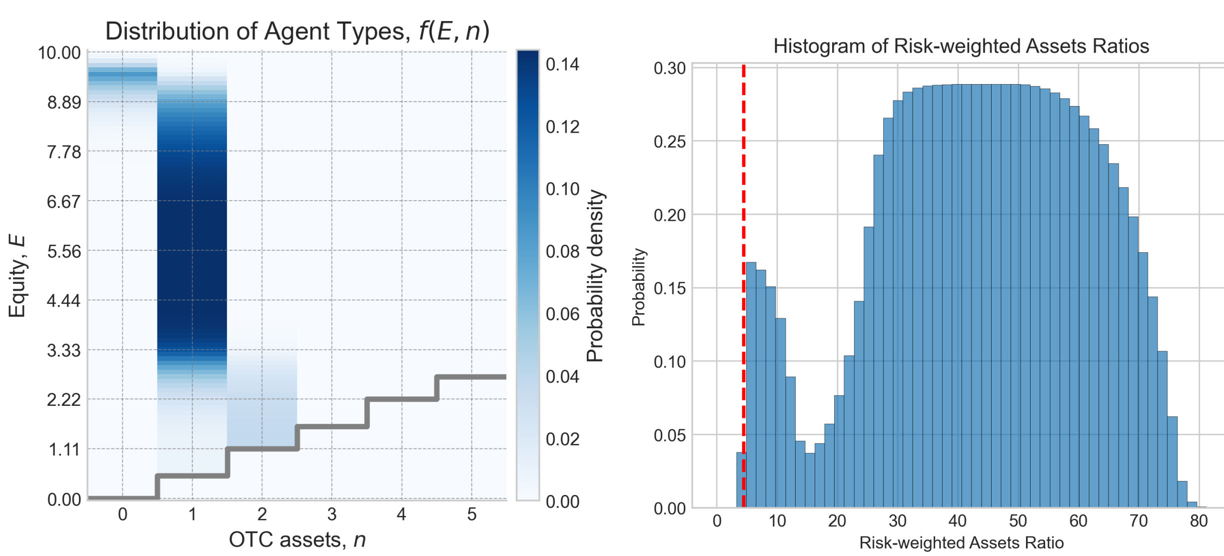
First, consider the baseline model to be the case with the lower risk weight on the risky OTC asset, $\nu = 50\%$. The policy parameter is constant and set to $\Lambda = 4.5\%$. The baseline model statistics are available in Table 3, column 1. As shown, the equilibrium market price is $\tilde{p} = 2.4604$. Figure 1a presents a heatmap of the distribution of agent types, where the gray step function represents the minimum level of equity $\bar{E}(n)$ agents must have for each level of risky OTC asset n. Recall that agents are periodically receiving income shocks and trading opportunities, and so move around the state space. Figure 1a shows that agents spend the most time holding $n = 1$ units of the risky OTC asset. Agents rarely ever hold more than $n = 2$ assets.

On average, agents have a risk-weighted asset ratio of 43.76%, much higher than the 4.5% required by policy. Figure 1b presents a histogram of the risk-weighted asset ratios of all agents and shows that the distribution is bimodal: most agents have sufficiently high risk-weighted assets clustered around the mean. These agents are primarily those who hold one risky OTC asset, $n = 1$. However, there is another concentration of agents just above the regulatory limit of 4.5%. These agents are primarily those who have two assets, $n = 2$.

The effect of the regulatory constraint The presence of the regulatory constraint distorts agent valuations of the risky OTC asset as they approach the regulatory bound. Figure 2 presents the marginal value of an additional unit of the asset, $W(E, n+1) - W(E, n)$, as a function of the distance from regulatory equity minimum, $E - \bar{E}(n)$. The graph shows that as agent equity approaches the bound, they increasingly dislike the risky OTC asset. This

	(1)	(2)
	$\nu = 50\%$	$\nu = 150\%$
Equilibrium market price ($\tilde{p}$)	2.4629	2.4604
Equity		
mean	5.68	6.89
std. dev	2.19	1.66
25th	3.93	5.65
50th	5.75	6.86
75th	7.47	8.18
Risk-weighted Asset Ratio		
mean	43.76%	17.20%
std. dev	17.83	5.34
25th	31.17%	15.05%
50th	44.30%	18.06%
75th	58.25%	21.07%
Meetings that result in a trade (%)	1.24%	1.71%
Annual Turnover (%)	18.62%	25.66%
Buy side agents	99.98%	97.51%
Sell side agents	93.85%	89.39%
Buy & Sell side agents	93.85%	86.97 %
Average Spread (bps)	46.98	59.88
Average Return, buy (bps)	37.25	42.36
Average Return, sale (bps)	12.84	22.95

Table 3: Model statistics



(a) Distribution of agent types $f(E, n)$

(b) Histogram of risk-weighted asset ratios

Figure 1: Distribution of agent types & histogram of risk-weighted asset ratios

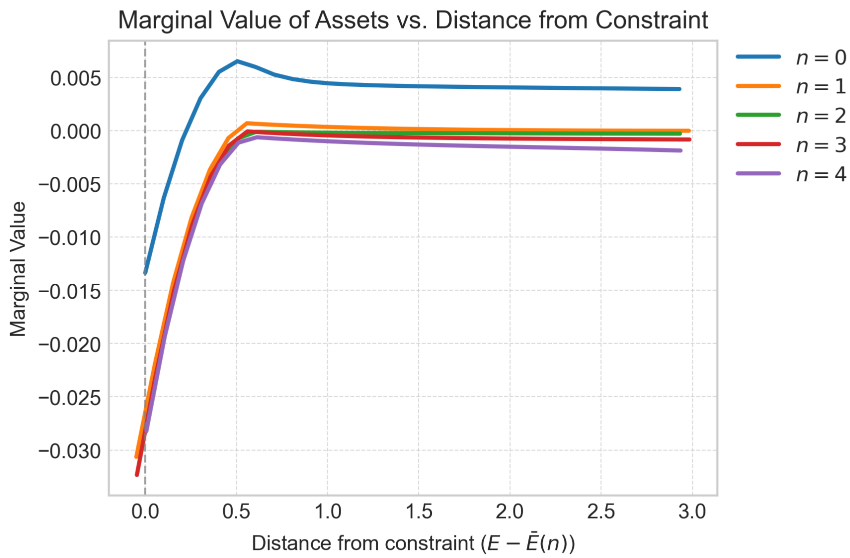
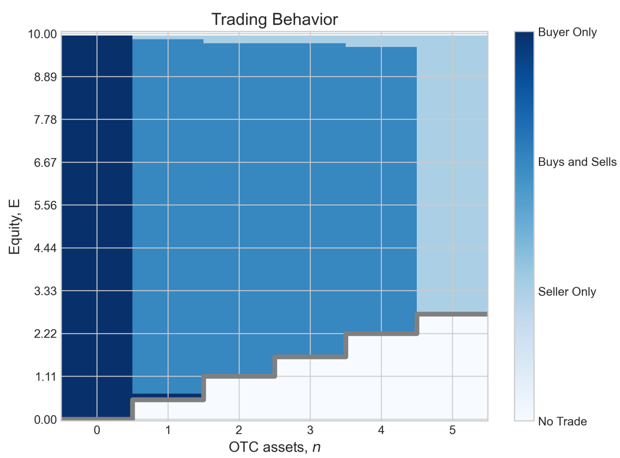
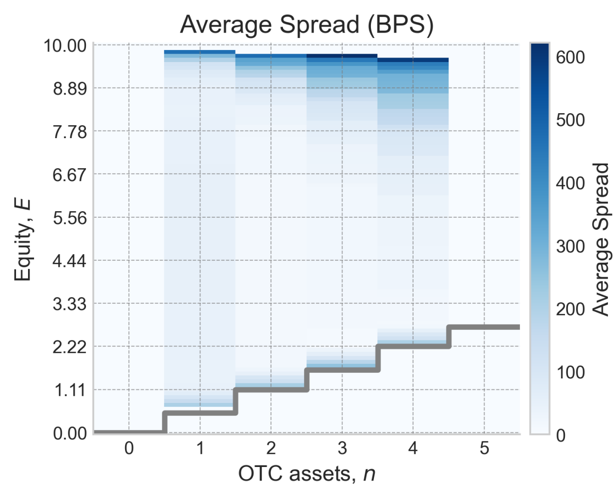


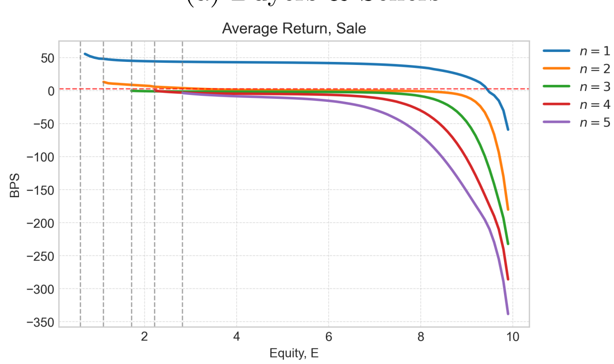
Figure 2: Marginal value of an additional unit of the risky OTC asset



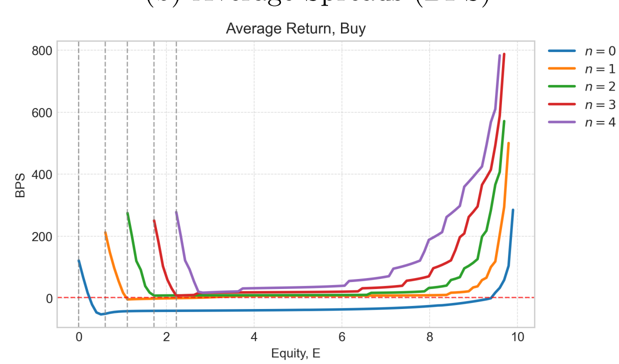
(a) Buyers & Sellers



(b) Average Spreads (BPS)



(c) Average Return, Sale (BPS)



(d) Average Return, Buy (BPS)

Figure 3: Trading Behavior

parameterization suggests that the relative change in the marginal value is the same for all agents, regardless of their risky OTC asst inventories n, but that their inventories determine the level. As agents move away from the bound, their marginal values begin to level off as the risk of falling into the constrained equity region becomes very low.

Trade in the OTC market As shown in Table 3, only about 1.24% of the matches result in trade, and the annual turnover in the market is 18.62%, below the 40%-60% usually observed corporate bond markets. An agent of type (E, n) is classified as a buyer if they face a non-zero probability of buying the risky OTC asset and as a seller if they face a non-zero probability of selling. Figure 3a displays the classification indicator across agent types (E, n). Most agents are willing to buy or sell, 99.98% and 93.85%, respectively. The fraction of agents facing both potential purchases and sales is also 93.85%. As Figure 3a demonstrates, for this parameterization most agents are willing to buy and sell and are strict buyers or sellers mechanically due to the no-short selling constraint ($n \geq 0$) and the finiteness of the grid ($n \leq n_H$).

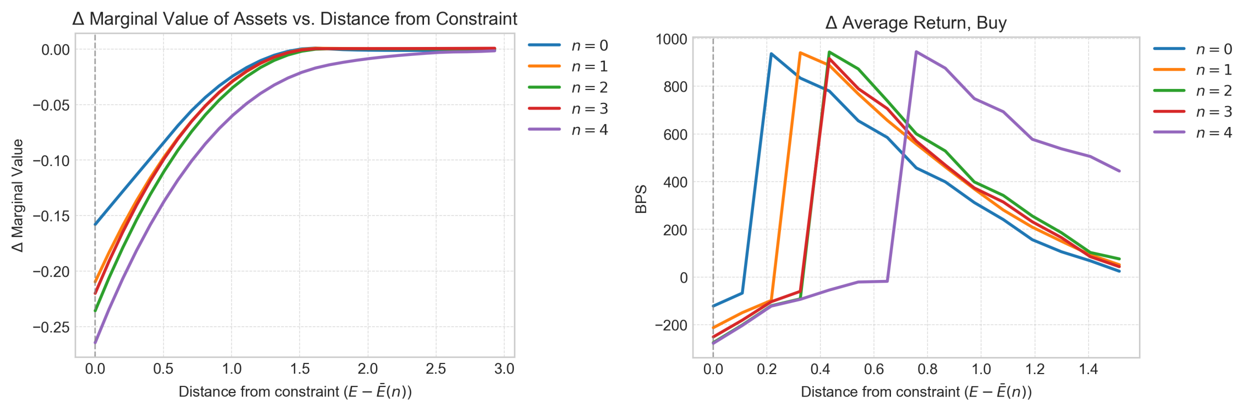
Figure 3b presents a heatmap of the average spreads faced by each agent in basis points, and Figures 3c and 3d the average returns on purchases and sales of the risky OTC asset over the market price. Average spreads in basis points are calculated as follows:

$$Spread(E, n) = \frac{E[P_{sell}|E, n] - E[P_{buy}|E, n]}{\tilde{p}} \times 10,000.$$

The one-sided returns in basis points for purchases and sales are calculated as:

$$Return_{sell}(E, n) = \frac{E[P_{sell}|E, n] - \tilde{p}}{\tilde{p}} \times 10,000;$$
$$Return_{buy}(E, n) = \frac{\tilde{p} - E[P_{buy}|E, n]}{\tilde{p}} \times 10,000.$$

On average, agents earn a spread of 46.98 bps, with purchases earning higher returns (37.25 bps) than sales (12.84 bps). As shown in Figure 3b, spreads mechanically increase as agents approach the upper bound of the equity grid. However, spreads also increase as agents approach the regulatory bound on equity ($\bar{E}(n)$, denoted by the gray step function). The effect the regulatory capital requirement is visible here: as agents approach the regulatory bound, they demand to be compensated when buying assets for the increased risk of violating the constraint that they bear. Figures 3c and 3d show that the return on the sale of the risky OTC asset over the market price is close to zero for most states. However, agents that hold one asset require higher compensation when they sell the asset: this reflects the positive marginal value observed by $n = 1$ agents shown in Figure 2. Since they enjoy holding one unit of the risky OTC asset, they require a strictly positive return on their sale to compensate



(a) Change in the marginal value of an additional unit of the risky OTC asset

(b) Change in the average return on purchases of the OTC assets

Figure 4: Change in marginal valuations and trading costs following an increase in the risk weight ν%

for the loss of the asset.

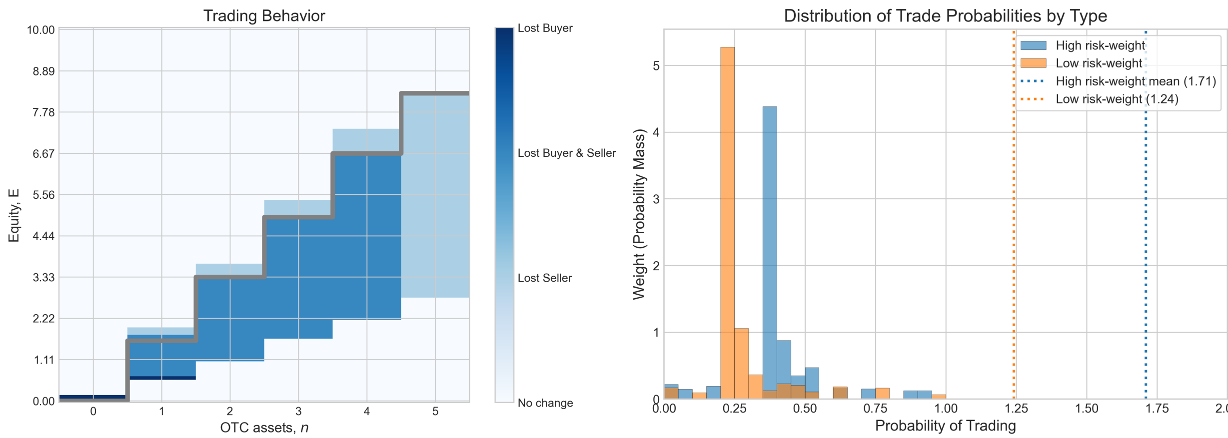
The regulatory capital requirement affects the willingness of agents to buy more than their willingness to sell. As shown in Figure 3d, the required return for agent buys quickly rise to upwards of 200 bps as they approach the regulatory bound. These agents will indeed buy the asset, but only if they are able to increase their equity such that they move *away* from the regulatory bound. However, this is a very, very expensive trade for the counterparty. Thus, the presence of the regulatory constraint can result in an increase in trading costs, depending on the agent's distance to the constraint.[12]

6 Regulation and OTC trade

Now consider the counterfactual of stricter regulation in which the risk weight on risky OTC assets ν increases from 50% to 150%. The second column of Table 3 displays the corresponding model statistics for this new steady state. Following an increase in the risk weight, there is:

1. An increase in average regulatory tightness and average equity;
2. An increase in trading costs via higher spreads;
3. A decline in agents willing to simultaneously buy and sell, and a decline in the percentage of agents willing to trade;

[12]I omit a discussion regarding the distortion of the spreads and returns as agents approach the top of the equity grid, as this is a mechanical result of the finite grid used and not due to the risk-return trade-off that agents make in this model.



(a) Change in Buyers & Sellers (b) Histogram of trade probabilities (%)

Figure 5: Change in trading behavior following an increase in the risk weight $\nu\%$

4. An increase in the fraction of meetings that result in a trade, and consequently an increase in turnover in the OTC market.

How does a change in the risk weight $\nu\%$ affect equilibrium outcomes? First, an increase in the risk weight, all else equal, increases the minimum level of equity agents must hold, with a larger change for higher levels of OTC assets (high n). This mechanically increases agent equity (rising from 5.68 to 6.89 units on average), and increases average regulatory tightness - risk-weighted asset ratios fall from 43.7% to 17.20% on average.

Agents are now more likely to get close to the regulatory minimum of equity capital. Additionally, since there is now a larger constrained equity space, agents now face the risk of even lower penalties for violating the constraint. This results in a decline in the marginal value of the risky OTC asset. Figure 4a presents the change in the marginal value if one additional unit of the risky OTC asset as a function of the distance from the constraint. That is, I am comparing directly the marginal valuations in both regulatory cases ($\nu = 50\%, \nu = 150\%$) in the region close the respective constraint. As is shown, the marginal value of risky OTC assets falls; this decline is greater the more assets one has.

This change in the compliant state-space and the marginal valuation of risky OTC assets has several implications for trade in the OTC asset. As Table 3 column 2 shows, the percentage of buy-side agents falls from 99.98% to 97.51%; sell-side agents from 93.85% to 89.39%; and agents who both buy and sell from 93.85% to 86.97%. Figure 5a shows the change in trading behavior across agent types (E, n). The gray step function represents the new higher regulatory minimum equity $\bar{E}(n)$. As is shown, in this new steady state agents that have valid equity but are close to the regulatory bound are no longer willing to buy the asset; only sell[13].

Additionally, agents close to the regulatory bound charge higher prices. Figure 4b shows that the change in the average return on a purchase of an asset is significantly higher as the agent approaches their regulatory bounds. These costs are sufficiently high that, once one gets extremely close to the bound, they are no longer willing to buy the asset for any terms of trade offered by agents in the economy. On average, spreads increase by 10 bps, from 49.90 bps to 59.88 bps (Table 3).

Finally, for this parameterization this regulatory change has the somewhat counterintuitive outcome that turnover and the percentage of meetings that result in a trade *increase*. This is due to a combination of changes in the distribution of agents. First, relative to the lower risk-weight economy, agents with lower levels of equity are less likely to trade, and agents with higher levels of equity are more likely to trade. This change is in junction with a shift in the distribution of equity: now, on average, agents are also larger in terms of their equity. Therefore, on average, we see a slight rise in the fraction of meetings that result in a trade. This is demonstrated by Figure 5b, which plots a histogram of trading probabilities. As is shown, there is a shift post-regulatory change towards agents that have higher trade probabilities. In summary, fewer agents are trading, but those that are trading trade more often, resulting in higher turnover.

7 Conclusion

In this paper, I ask how recent regulatory changes, such as the Basel III market risk framework, have affected dealer market making behavior in OTC markets. To answer this question, I develop a model of an interdealer OTC market in which agents make a portfolio choice regarding their holdings of risk-free liquid assets and risky OTC-traded assets subject to a risk-based capital requirement. This occasionally binding financial constraint alters the marginal valuation of the risky OTC traded asset near the regulatory bound on equity through the threat of reduced utility when equity drops below the required threshold.

I solve the model numerically using an original iterative algorithm based on the finite difference methods presented in Achdou et al. (2022). I show that agents change their trading behavior in the region close to the regulatory bound, charging higher transaction costs to buy the asset to avoid violating the constraint. I show that following an increase in the risk weight on the risky OTC asset, the distortions introduced by this constraint cause these trading costs to increase.

The next steps of the project are to calibrate the parameter governing the rate of meet-

[13] If trade sizes were not fixed at 1, it is likely that these agents would buy increasingly small increments of the asset for very high prices.

ings λ, and the regulatory scaling parameter, ϕ, using turnover and the distribution of risk-weighted asset ratios, respectively. Additionally, I am working on a set of motivational regressions using transaction level bond data from TRACE. Finally, I am working on extending the model to include a retail market in addition to the inter-dealer market analyzed here.

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Appendix

A Stationary equilibrium solution method

The steady state problem amounts to a high-dimensional fixed point problem in the value function $W(E, n)$ and distribution $f(E, n)$ as discussed in Hugonnier et al. (2020). At its core, there are two main sub-problems. The first involves solving for the intensities with which agents change their type in the E and n dimensions due to optimal trade outcomes as a function of the value function and distribution. This step is solved by simply finding the grid point that best satisfies the Kalai bargaining problems for each possible trade, if indeed one exists.[14]

The second involves solving for the agent value function (taking as given switching intensities due to trade), and then solving for the implied agent distribution. I solve for the value function using finite-difference methods as outlined in Achdou et al. (2022) with some modifications. Due to the regulatory constraint, the boundary of equity grid is staggered: agents with more OTC asset holdings must hold higher equity. The modifications can be found in Appendix (A.1). I solve the steady state using the algorithm defined in Algorithm 1.

A.1 State-based boundary conditions and finite-difference methods

In this section I outline the finite-difference algorithm based on Achdou et al. (2022) modified to handle the state-based boundary condition as it appears in my model. Let there be I liquid asset grid points E_i with step size ΔE in the domain $[E_L, E_H]$ and J OTC asset holding points n_j with step size $\Delta n = 1$ in the domain $[0, n_H]$. Let $T_{ij,i'j'}$ be the trade outcome probabilities due to trade in the OTC market: $T_{ij,i'j'}$ represents the probability of switching from type ij to type $i'j'$ in the defined grid. Note that $\sum_{i',j'} T_{ij,i'j'} = 1$. Define the following:

$$s_{ij} = m_\eta - c_{ij} + rE_i + n_j(m_D - r\tilde{p})$$
$$\sigma_j^2 = \sigma_\eta^2 + \sigma_D^2 n_j^2 + 2n_j\rho^m\sigma_\eta\sigma_D$$

Note the relevant forward and backward differences are:

$$\partial_{E,F} w_{ij} = \frac{w_{i+1,j} - w_{ij}}{\Delta E}$$

[14] This is a bottleneck; with 100 liquid asset grid points and up to 5 units of the OTC asset held, obtaining the optimal trade outcomes for all agents takes about 39 seconds on a Windows laptop with a 12th Gen Intel Core i7 processor using Python 3.11 when using parallelization.

Algorithm 1: Steady State Equilibrium

Data : Initial guess $\tilde{p}$; tolerances $\mathrm{tol}_1, \mathrm{tol}_2, \mathrm{tol}_3$; weights w_1, w_2, w_3; max iterations K, I.
Result: Converged market price $\tilde{p}$, distribution f, and value functions W.

begin

Start: Define a agent type grid given $\tilde{p}$. Choose an initial arbitrary guess of trade switching intensities $\bar{T}$. Solve for $W^{(0)}$ and distribution $f^{(0)}$ using the modified finite-difference method defined in appendix (A.1).

for $k \leftarrow 1$ **to** K **do**

```
/* Inner loop: Value Function */
```

$W_0^{(k)} \leftarrow W^{(k)}$ **for** $i \leftarrow 1$ **to** I **do**

Compute optimal trade intensities T_i using $W_{i-1}^{(k)}$. Obtain new value function $W_i^{(k)}$. $c_1 \leftarrow \max\{|W_i^{(k)} - W_{i-1}^{(k)}|\}$ **if** $c_1 < \mathrm{tol}_1$ **then**

$W^{(k)} \leftarrow W_i^{(k)}$; **break**

else

$W_i^{(k)} \leftarrow w_1 W_i^{(k)} + (1 - w_1) W_{i-1}^{(k)}$

```
/* Outer loop: Distribution */
```

Solve for $f^{(k)}$ given $W^{(k)}$. $c_2 \leftarrow \max\{|f^{(k)} - f^{(k-1)}|\}$ **if** $c_2 < \mathrm{tol}_2$ **then**

break

else

$f^{(k)} \leftarrow w_2 f^{(k)} + (1 - w_2) f^{(k-1)}$

```
/* Market price convergence */
```

Compute $\tilde{p}_K$ from $\{W^{(K)}, f^{(K)}\}$. $c_3 \leftarrow |\tilde{p}_K - \tilde{p}|$ **if** $c_3 < \mathrm{tol}_3$ **then**

stop

else

$\tilde{p} \leftarrow w_3 \tilde{p}_K + (1 - w_3) \tilde{p}$ **go back to Start**

$$\partial_{E,B} w_{ij} = \frac{w_{ij} - w_{i-1,j}}{\Delta E}$$

$$\partial_E^2 w_{ij} = \frac{w_{i,j+1} - 2w_{ij} + w_{i,j-1}}{(\Delta E)^2}$$

The discretized Dealer HJB is w_{ij}, given a guess w_{ij}^n the next guess w_{ij}^{n+1} satisfies:

$$\begin{aligned}\frac{w_{ij}^{n+1} - w_{ij}^n}{\Delta} + r w_{ij}^{n+1} =& u(c_{ij}^n) + \partial_E w_{ij}^{n+1} s_{ij}^n + \frac{1}{2}\partial_E^2 w_{ij}^{n+1}\sigma_j^2 \\ &+ \lambda \sum_{i',j'} T_{ij,i'j'}\left(w_{i'j'}^{n+1} - w_{ij}^{n+1}\right)\end{aligned}$$

The implicit guess using the upwind scheme is a system of $I \times J$ equations:

$$\begin{aligned}\frac{w_{ij}^{n+1} - w_{ij}^n}{\Delta} + r w_{ij}^{n+1} =& u(c_{ij}^n) + \partial_{E,F} w_{ij}^{n+1}\left[s_{ij,F}^n\right]^+ + \partial_{E,B} w_{ij}^{n+1}\left[s_{ij,B}^n\right]^- + \frac{1}{2}\partial_E^2 w_{ij}^{n+1}\sigma_j^2 \\ &+ \lambda \sum_{i',j'} T_{ij,i'j'}\left(w_{i'j'}^{n+1} - w_{ij}^{n+1}\right)\end{aligned} \tag{17}$$

Note that optimal agent consumption is always:

$$c_{ij}^n = (u')^{-1}(\partial_E w_{i,j}^n)$$

Where the marginal value of equity is:

$$\begin{aligned}\partial_E w_{i,j} =& \partial_E w_{ij,F} 1\{s_{ij,F} > 0\} + \partial_E w_{ij,B} 1\{s_{ij,B} < 0\} \\ &+ u'(m_\eta + rEi + n_j\left(m_D - r\tilde{p}\right)) 1\{s_{ij,F} \leq 0 \leq s_{ij,B}\}\end{aligned}$$

Writing equation (17) in matrix notation and using the backwards and forward difference equations, I obtain the following:

$$\begin{aligned}\frac{w_{ij}^{n+1} - w_{ij}^n}{\Delta} + r w_{ij}^{n+1} =& u(c_{ij}^n) + \frac{w_{i+1,j}^{n+1} - w_{i,j}^{n+1}}{\Delta E}\left[s_{ij,F}^n\right]^+ + \frac{w_{i,j}^{n+1} - w_{i-1,j}^{n+1}}{\Delta E}\left[s_{ij,B}^n\right]^- \\ &+ \frac{w_{i+1,j}^{n+1} - 2w_{ij}^{n+1} + w_{i-1,j}^{n+1}}{(\Delta E)^2}\frac{\sigma_j^2}{2} \\ &+ \lambda \sum_{i',j'} T_{ij,i'j'}\left(w_{i'j'}^{n+1} - w_{ij}^{n+1}\right)\end{aligned}$$

Which can be written for generic grid points ij as:

$$\begin{aligned}\frac{w_{ij}^{n+1}-w_{ij}^{n}}{\Delta}+rw_{ij}^{n+1}=&u(c_{ij}^{n})+z_{ij}^{n}w_{i+1,j}^{n+1}+y_{ij}^{n}w_{i,j}^{n+1}+x_{ij}^{n}w_{i-1,j}^{n+1}\\&+\lambda\sum_{i',j'}T_{ij,i'j'}w_{i'j'}^{n+1}\end{aligned}\tag{18}$$

Where:

$$z_{ij}^{n}=\left(\frac{1}{\Delta E}\left[s_{ij,F}^{n}\right]^{+}+\frac{1}{2}\sigma_j^2\frac{1}{(\Delta E)^2}\right)$$

$$y_{ij}^{n}=\left(\frac{1}{\Delta E}\left[s_{ij,B}^{n}\right]^{-}-\frac{1}{\Delta E}\left[s_{ij,F}^{n}\right]^{+}-\sigma_j^2\frac{1}{(\Delta E)^2}-\lambda\right)$$

$$x_{ij}^{n}=\left(\frac{1}{2}\sigma_j^2\frac{1}{(\Delta E)^2}-\frac{1}{\Delta E}\left[s_{ij,B}^{n}\right]^{-}\right)$$

Up until now this method is consistent with Achdou et al. (2022). The modification lies in how I handle the boundary condition. As in Achdou et al. (2022), implementing the boundary condition requires a modification to the backwards difference of the value function at the boundary point. The difference here is now the boundary point is no longer the bottom of the grid only, and I have multiple points which need to be constrained for all levels of equity that violate the constraint: $E \leq \bar{E}(n_j)$ for each n_j. Therefore for intermediate OTC asset holdings $n_j \leq n_H$ there is a region of the grid where consumption is constrained.

I impose the boundary condition as follows. For all i, j such that $E_i < E(n_j)$, the boundary condition needs to be satisfied for the backward and forward difference. Identify the boundary point of risk-free assets, or the minimum level of risk-free assets for each j that satisfies the constraint, as $\bar{E}(n_{j^*})$ and the grid point as i^*j^*. The backward and forward differences for all $\hat{i} \in \{i < i^*\}$ must satisfy the boundary condition (13):

$$\partial_{E,B}w_{\hat{i}j^*}=u'(m_\eta+rE_{\hat{i}}+n_{j^*}(m_D-r\tilde{p})-(\bar{E}(n_{j^*})-E_{\hat{i}}))$$

$$\partial_{E,F}w_{\hat{i}j^*}=u'(m_\eta+rE_{\hat{i}}+n_{j^*}(m_D-r\tilde{p})-(\bar{E}(n_{j^*})-E_{\hat{i}}))$$

Now, consider the point at which the regulatory constraint first binds, i^*j^*. The backward difference at point i^*j^* is:

$$\partial_{E,B}w_{i^*j^*}=\frac{w_{i^*j^*}-w_{i^*-1,j^*}}{\Delta E}=u'(m_\eta+r\bar{E}(n_{j^*})+n_{j^*}(m_D-r\tilde{p}))$$

Therefore the value function at the point i^*-1, j^* in the risk-free asset grid is by definition:

$$w_{i^*-1,j^*} = w_{i^*j^*} - \Delta E u'(m_\eta + r\bar{E}(n_{j^*}) + n_{j^*}(m_D - r\tilde{p}))$$

Similarly, the value function for any point below the constraint point is:

$$w_{\hat{i}-1,j^*} = w_{\hat{i}j^*} - \Delta E u'(m_\eta + rE_{\hat{i}} + n_{j^*}(m_D - r\tilde{p}) - (\bar{E}(n_{j^*}) - E_{\hat{i}}))$$

Therefore equation (18) at the boundary point i^*, j^* is:

$$\begin{aligned}\frac{w^{n+1}_{i^*j^*} - w^n_{i^*j^*}}{\Delta} + rw^{n+1}_{i^*j^*} =& u(c^n_{i^*j^*}) - x^n_{i^*j^*}\Delta E u'(m_\eta + rE_{i^*}(n_{j^*}) + n_{j^*}(m_D - r\tilde{p})) \\ &+ z^n_{i^*j^*} w^{n+1}_{i^*+1,j^*} + \left(x^n_{i^*j^*} + y^n_{i^*j^*}\right) w^{n+1}_{i^*j^*} \\ &+ \lambda \sum_{i',j'} T_{ij,i'j'} w^{n+1}_{i'j'}\end{aligned}$$

And for any point below the constraint point, $\hat{i} \in \{i < i^*\}$, is:

$$\begin{aligned}\frac{w^{n+1}_{\hat{i}j^*} - w^n_{\hat{i}j^*}}{\Delta} + rw^{n+1}_{\hat{i}j^*} =& u(c^n_{\hat{i}j^*}) - x^n_{\hat{i}j^*}\Delta E u'(m_\eta + rE_{\hat{i}} + n_{j^*}(m_D - r\tilde{p}) - (E_{i^*}(n_{j^*}) - E_{\hat{i}})) \\ &+ z^n_{\hat{i}j^*} w^{n+1}_{\hat{i}+1j^*} + \left(x^n_{\hat{i}j^*} + y^n_{\hat{i}j^*}\right) w^{n+1}_{\hat{i}j^*} \\ &+ \lambda \sum_{i',j'} T_{ij,i'j'} w^{n+1}_{i'j'}\end{aligned}$$

The upper bound of the liquid asset grid is handled in the standard manner as in Achdou et al. (2022) by requiring that the marginal value of wealth at the upper most liquid asset grid point I for all OTC asset holdings n_j be zero:

$$\partial_E w(E_I, n_j) = 0 \implies w_{Ij} = w_{I+1,j}$$

And so equation ((18)) at the upper boundary point Ij is:

$$\frac{1}{\Delta} w^{n+1}_{Ij} - \frac{1}{\Delta} w^n_{Ij} + rw^{n+1}_{Ij} = u(c^n_{Ij}) + x^n_{Ij} w^{n+1}_{I-1,j} + \left(y^n_{Ij} + z^n_{Ij}\right) w^{n+1}_{I,j}$$

Note that such boundary constraints are not necessary on the OTC asset grid j as the constraints on the bargaining process prevent agents from trading in violation of feasibility or regulation.

Equation (18) can therefore be written for all valid points on the i, j grid as:

$$\begin{aligned}\frac{w_{ij}^{n+1} - w_{ij}^{n}}{\Delta} + rw_{ij}^{n+1} =& u(c_{ij}^{n}) + z_{ij}^{n} w_{i+1,j}^{n+1} + y_{ij}^{n} w_{i,j}^{n+1} + x_{ij}^{n} w_{i-1,j}^{n+1} \\ &+ \lambda \sum_{i',j'} T_{ij,i'j'} w_{i'j'}^{n+1}\end{aligned} \tag{19}$$

Where:

$$z_{ij}^{n*} = \begin{cases} z_{ij}^{n} & \text{if } i < I \\ 0 & \text{if } i = I \end{cases}$$

$$y_{ij}^{n*} = \begin{cases} y_{ij}^{n} & \text{if } i < I \text{and } i > i_j^* \\ y_{ij}^{n} + x_{ij}^{n} & \text{if } i < I \text{and } i \leq i_j^* \\ y_{ij}^{n} + z_{ij}^{n} & \text{if } i = I \end{cases}$$

$$x_{ij}^{n*} = \begin{cases} x_{ij}^{n} & \text{if } i > i_j^* \\ 0 & \text{if } i \leq i_j^* \end{cases}$$

Define A^n to be a sparse matrix with $I \times J$ columns and $I \times J$ rows with y_{ij}^{n*} on the center diagonal, x_{ij}^{n*} on the lower diagonal, and z_{ij}^{n*} on the upper diagonal. Define $u_{ij}^n = u(c_{ij}^n)$. Define w^n as a vector of length $I \times J$, with entries $(w_{11}, ..., w_{I1}, w_{12}..., w_{I2}, w_{1J}, ..., w_{IJ})$. Define b^n to be a vector of length $I \times J$ with entries:

$$b_{ij}^{n} = \begin{cases} u_{ij}^{n} - x_{ij}^{n} \Delta \omega u'(m_\eta + rE_i + n_j(m_D - r\tilde{p}) - (\bar{E}(n_{j^*}) - E_i)) & \text{if } i \leq i_j^* \\ u_{ij}^{n} & \text{otherwise} \end{cases}$$

Define T to be a matrix with $I \times J$ columns and $I \times J$ rows populated with the trading probabilities $T_{ij,i'j'}$. Therefore the difference equation can be written in matrix form as:

$$\underbrace{\frac{1}{\Delta}\left(w^{n+1} - w^{n}\right) + rw^{n+1}}_{\text{dim}=(I\times J)\times 1} = \underbrace{u^n}_{\text{dim}=(I\times J)\times 1} + \underbrace{A^n + \lambda T}_{\text{dim}=(I\times J)\times(I\times J)} \times \underbrace{w^{n+1}}_{\text{dim}=(I\times J)\times 1}$$

Which can be solved following the standard iterative procedure outlined in Achdou et al. (2022).

B Estimation of stochastic processes

I use quarterly balance sheet level data from the FR-Y9C filings of bank holding companies (BHCs) to estimate via maximum likelihood the constants in the stochastic processes in equations (1) and (2), m_η, σ_η, m_D and σ_D. FR-Y9C filings are publicly available financial statements that financial institutions are required to prepare and submit for regulatory purposes. I filter the dataset to only include domestic U.S. based bank holding companies with strictly positive levels of assets: the data spans 76 quarters from Q1 2006 to Q4 2024, and includes 1580 distinct BHC's. Figure (6) displays the count of BHC's in the sample over time.

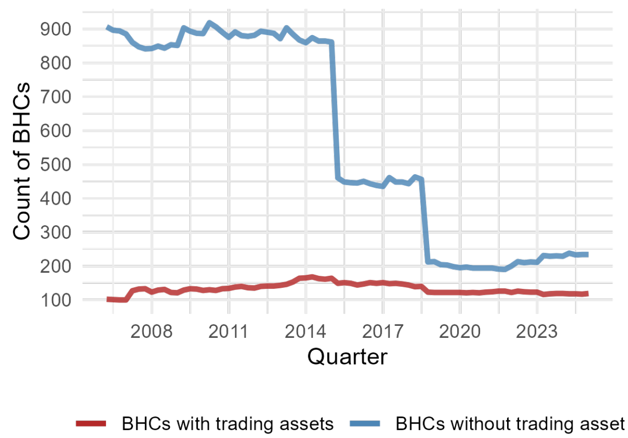

Figure 6: Number of bank holding companies (BHCs) in the sample over time.

I define an agent's endowment income in the data, $\hat{\eta}$, to be a BHC's recorded consolidated net income less their income from trading assets, which is defined as the sum of total interest on their stock of trading assets and trading revenues. I define an agent's dividend income per unit of risky OTC asset in the data, $\hat{D}$, to be the sum of a BHC's total consolidated income from trading assets divided by their stock of trading assets.

Since equity in my model spans $[0, 10]$, I need to scale $\hat{\eta}$ appropriately before estimation of the endowment process. I first scale the line item for total equity, $\hat{E}$ such that the 95th percentile value is equal to 10, and obtain the scaling factor. I then scale $\hat{\eta}$ using this scaling factor, to obtain $\hat{\eta}^*$. Since $\hat{D}$ is a per unit return, it does not need to be scaled. Table 4 reports the summary statistics for the scaled variables of interest.

I now derive the log-likelihood function for the endowment process (the procedure is exactly the same for the dividend process). Since the unit of time is one year and data is observed quarterly, the time step is $\Delta t = 0.25$. Equation (2) can be discretized as:

$$\Delta\eta_{it} = \eta_{i,t+\Delta t} - \eta_{it} \approx m_\eta \Delta t + \sigma_\eta \sqrt{\Delta t}\epsilon_{it}$$

Where $\epsilon_{it} \sim N(0, 1)$. Therefore, for a given BHC i at time t, the change in their endow-

	All BHCs							BHCs with Trading Assets						
	Mean	*Std. Dev.*	*25th*	*50th*	*75th*	*90th*	*Max*	*Mean*	*Std. Dev.*	*25th*	*50th*	*75th*	*90th*	*Max*
Equity, $\hat{E}^*$	4.211	27.773	0.106	0.212	0.745	3.196	561.044	20.013	61.230	0.654	2.096	9.585	36.970	561.044
Endowment income, $\hat{\eta}^*$	0.071	1.384	0.003	0.009	0.031	0.143	39.100	0.238	3.081	0.009	0.064	0.273	1.188	39.100
Δ Endowment income, $\Delta\hat{\eta}^*$	0.000	0.966	–0.003	0.003	0.010	0.049	36.206	–0.003	2.145	–0.015	0.015	0.089	0.492	36.206
Dividend income, $\hat{D}$	–	–	–	–	–	–	–	0.526	12.190	0.008	0.037	0.103	0.320	672.800
Δ Dividend income, $\Delta\hat{D}$	–	–	–	–	–	–	–	0.105	11.928	–0.016	0.007	0.034	0.119	594.666
Total assets* (log)	1.283	1.569	0.175	0.810	1.975	3.330	8.829	3.246	1.966	1.921	3.014	4.490	5.862	8.829
Return on Assets (%)	5.750	721.460	0.200	0.430	0.760	1.110	118286.150	0.496	0.931	0.224	0.460	0.496	1.090	19.368
No. Subsidiaries	1.436	1.770	1.000	1.000	1.000	2.000	57.000	1.780	2.470	1.000	2.000	2.000	6.000	40.000
Age (years)	2.460	3.010	1.000	2.000	3.000	6.000	33.000	2.420	2.620	1.000	2.000	3.000	6.000	26.000

Table 4: Summary Statistics

This table presents the summary statistics for the variables and controls used for all BHCs in the sample over all quarters observed. Note that a * indicates that the variable is scaled such that the 95% percentile of BHC total equity is equal to $E_H = 10$, the maximum level of equity in the model's grid.

	(1) Endowment $d\eta_t$	(2) Dividend dD_t	(3) Correlation ρ
$\hat{m}_\eta$	0.0736	–	–
	(0.0017)		
$\hat{\sigma}_\eta$	0.1366	–	–
	(0.0004)		
$\hat{m}_D$	–	0.1265	–
		(0.0103)	
$\hat{\sigma}_D$	–	0.3443	–
		(0.0024)	
$\hat{\beta}$	–	–	0.0030
			(0.0010)
$\hat{\rho}$	–	–	0.0014
			(0.0023)
N	49,214	9,523	9,523
LL	-62,383.57	-3,241.77	–
r	–	–	0.098

Table 5: Stochastic process MLE estimates

Column (1) presents the MLE estimates of the parameters (m_η, σ_η) for the process defined by equation (2). Column (2) presents the MLE estimates of the parameters (m_D, σ_D) for the process defined by equation (1) using the log-likelihood defined in equation (20). Column (3) presents the estimate of β from equation (21), and the implied correlation ρ that follows by definition of equation (3).

ment income is:

$$\Delta\eta_{it} \sim N(m_\eta \Delta t, \sigma_\eta^2 \Delta t)$$

The log-likelihood function for N_t BHCs in each time t is therefore:

$$\ell(m_\eta, \sigma_\eta) = \sum_{t=1}^{T} \sum_{i=1}^{N_t} \left[-\frac{1}{2} \ln\left(2\pi\sigma_\eta^2 \Delta t\right) - \frac{(\Delta\eta_{it} - m_\eta \Delta t)^2}{2\sigma_\eta^2 \Delta t} \right] \tag{20}$$

I minimize the above log-likelihood using the observed scaled net income $\hat{\eta}^*_{it}$ from the FR-Y9C filings for each BHC. The log-likelihood function for the dividend process is of a similar form, for which the observed return on trading assets $\hat{D}_{it}$ is used. I additionally use for both BHC level controls: log of total assets (scaled), return on assets, age, the number of subsidiary companies, and a first quarter dummy variable to control for seasonality. Table 4 reports the summary statistics for these control variables. All continuous variables are winsorized at the 3% level. The MLE estimates are presented in Table 5.

Given the above MLE estimates, I can then obtain an estimate for the correlation parameter, ρ. By definition of the stochastic process given in equations (1), (2), and (3):

$$\underbrace{\frac{cov(d\eta_t, dD_t)}{var(d\eta_t)}}_{\beta} = \frac{\sigma_\eta \sigma_D \rho}{\sigma_\eta^2} = \frac{\sigma_D}{\sigma_\eta}\rho$$

Where the term $\frac{cov(d\eta_t, dD_t)}{var(d\eta_t)}$ is in fact the OLS coefficient β in the following regression:

$$\Delta D_t = \alpha + \beta \Delta\eta_t + \epsilon_t \tag{21}$$

I run the above OLS regression using $\hat{\eta}^*_{it}$, $\hat{D}_{it}$, as well as the same BHC level controls as in the MLE estimation, to obtain an estimate of $\hat{\beta}$, and use the previously obtained estimates of $\hat{\sigma_\eta}$ and $\hat{\sigma_D}$. The estimate of the correlation coefficient is therefore:

$$\hat{\rho} = \hat{\beta}\frac{\hat{\sigma}_\eta}{\hat{\sigma}_D}.$$