Model-Based Cross-Section Factors in Corporate Bonds

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Abstract

Using a structural model of default with stochastic interest rates and liquidity shocks, we construct three common risk factors – credit, duration, and liquidity – from the cross-section of corporate bond returns. This theory-based three-factor model can explain the risk premiums of portfolios within the credit market and beyond, including U.S. equity and emerging sovereign bonds. Our estimation is unique in that the three latent factors are extracted not from bond characteristics but the model-implied factor loadings, offering a clean identification of the risk of credit, duration, and liquidity. As such, our model-based factors can be used to gauge and differentiate their relative importance and identify distinct episodes of flight-to-quality, flight-to-cash, and flight-to-liquidity. Moreover, our duration factor is used to decompose U.S. Treasury bond returns into duration risk and safety premium, differentiating flight-to-safety from flight-to-cash, while our liquidity factor sheds light on the post-regulation capacity of dealers' balance sheet and its asset-pricing implication. Finally, using our credit factor to predict the next-month liquidity factor, we document the prevalence of credit-driven liquidity risk in the corporate bond market.

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1 Introduction

The U.S. corporate bond market, with an amount outstanding \$11.38 trillion by 2025Q2, is the most important debt financing channel for large U.S. corporations and an essential investment vehicle for global financial institutions and investors. Sitting at the intersection of finance, macro, and intermediation, it also contains rich information on duration, credit, and liquidity risks. Specifically, as a fixed-income asset, its pricing is linked directly to the time-varying interest rates; as a defaultable security, it reflects the overall health and survival of Corporate America; and as a predominantly dealer-intermediated security, its liquidity condition reflects the balance-sheet capacity of financial institutions.

In addition to its considerable size, the U.S. corporate bond market also offers a large cross-section of bonds with varying credit quality, maturity, and liquidity. Focusing on this rich and vast cross-section of corporate bond returns, our main objective in this paper is to extract the common risk factors of duration, credit, and liquidity from it. Central to our empirical approach is a structural model of default with stochastic interest-rate risk and Poisson-driven liquidity shock. Associated with the three shocks in the model are three risk factors – credit, liquidity, and duration, which we infer using bond-level factor loadings implied by the model.

Unlike the existing cross-sectional literature in equity and bond, which uses stock or bond characteristics to form empirically-motivated factors, our empirical approach is unique in that our risk factors emerge from an arbitrage-free asset-pricing model. Specifically, they are extracted from the cross-section of corporate bond returns using model-implied factor loadings. Intuitively, bonds of varying maturity can be informative about the pricing of the duration risk factor, while bonds of different credit/liquidity qualities can inform us about the credit/liquidity risk factors. This is exactly the essence of our estimation strategy, except that, in our model-based approach, such cross-sectional variations in duration, credit quality, and liquidity are captured by our model-implied factor loadings. Accordingly, our extracted factors offer a clean identification of credit, liquidity, and duration.

Since 2008, the corporate bond market has experienced significant shocks in credit, liquidity, and duration (e.g., 2008 financial crisis, post-2008 financial regulations, Covid-19, post-Covid inflation surge and the ensuing rate hikes). Against this backdrop, our model-implied factors can be used to gauge and differentiate the relative importance of credit, liquidity, and duration over the varying degrees of financial and macro uncertainties and intermediary constraints.[1] The extreme outcomes of our model-implied factors can further

[1]For example, by separately identifying the credit and liquidity risk factors, we can disentangle liquidity from credit – an issue of great importance during the 2008 financial crisis. Examining the interactions of

help us identify episodes of flight-to-cash (i.e., dash-for-cash) when our duration factor suddenly drops. Likewise, episodes of flight-to-quality and flight-to-liquidity can be separately identified using extreme values of our credit and liquidity factors.[2] Importantly, the clean identification of our model-based factors allows us to separately locate such important and influential episodes in history. A better understanding of their nature and origin can help us gauge the effectiveness of the policies and regulations enacted at the time.

Model-Based Extraction of Common Risk Factors – Different from the equity market, the corporate bond market enjoys a close connection between market pricing and the fundamental risk factors, as captured by the long line of structural models of default pioneered by Merton (1974). In our model, assuming the existence of a risk-neutral measure and let P_t^i be the time-t price of asset i,

$$\frac{dP_t^i}{P_t^i} - r_t\,dt = b_i\,dF_t^Q + dB_t^i\,,\ i = 1, 2, \ldots, N\,, \tag{1}$$

where the latent risk factor, dF_t^Q, is zero mean under the risk-neutral measure Q and non-zero under the physical measure, with the factor risk premium λ_t driving the difference:

$$dF_t^Q = \lambda_t\,dt + dF_t\,. \tag{2}$$

Representing unpriced idiosyncratic risk, dB_t^i is zero mean under both measures. Given the time-t market-observed returns across i and their respective model-implied factor loadings b_i, we can back out the time-t risk factors dF_t^Q.

Central to our factor extraction are the model-implied factor loading b_i, which we derive from structural models of default. Taking the classic default model of Merton (1974) as an example, the time-t bond value is $P_t = Ke^{-rT}N(d_2) + V_tN(-d_1)$, where V_t is the time-t value of asset. The corresponding model-implied loading on the asset value is therefore $b_i^V = \frac{\partial \ln P_t}{\partial \ln V_t}$. We further generalize this framework by introducing stochastic interest rate and a system-wide liquidity shock. Overall, our three-factor model of credit, duration, and liquidity is

the risk factors, we can further identify trends of comovement between liquidity and credit and analyze the impact of duration risk on credit and liquidity risks.

[2]Prominent examples of flight-to-cash include June 20, 2013, the day after Bernanke's taper speech; March 12, 17, and 19, 2020, events of dash for cash amid Covid-19; and June 13, 2022, when the CPI announcement for May indicates accelerating inflation. Episodes of flight-to-quality captured by our credit factor include September 15, 2008 (Lehman bankruptcy), the peak of the 2008 financial crisis (early October 2008), March 9, 2020 (Covid-19), and June 13, 2022, (CPI announcement indicating accelerating inflation). Flight-to-liquidity days captured by our liquidity factor include August 8, 2011 (the black Monday due to U.S. debt ceiling crisis), October 15, 2014 (U.S. Treasury flash crash), March 23, 2020 (announcement of Fed SMCCF to provide liquidity to corporate bonds), and March 13, 2023 (SVB bankruptcy).

estimated using the corresponding factor loadings – the model-implied factor loading for the credit risk, b_i^V, is closely associated with the distance-to-default measure; the factor loading for the interest-rate risk r_t can be derived by $b_i^r = \frac{\partial \ln P_t}{\partial r_t}$, which is essentially the duration of the bond. We further add a liquidity risk factor using the bond-level liquidity measure of Bao, Pan, and Wang (2011) to proxy for the bond-level exposure to the systematic liquidity risk.

Leveraging the large cross-section of bond returns, we extract the time-t latent factors using the model-based factor loadings. Specifically, we run the cross-sectional regression of excess bond returns on their respective factor loadings to obtain the time-t slope coefficients, which, according to equation (1), are in fact the time-t latent risk factors of credit, duration, and liquidity. Stacking the time-t latent factors over time, we obtain the time-series of credit, duration, and liquidity factors. While this empirical approach of running cross-sectional regression to extract the latent factors is similar to that of Kelly, Pruitt, and Su (2019), Fama and French (2020) and Chen et al. (2024), there is an important and conceptual difference between the two. In particular, instead of using characteristics, we use model-implied factor loadings to extract the latent factors and, more importantly, the pricing relation in in equation (1) is dictated by an arbitrage-free asset pricing model. This model-based factor approach differentiates our paper importantly from the existing cross-sectional studies on corporate bonds with empirically motivated factors (e.g., Fama and French (1993), Bai, Bali, and Wen (2019), and Dickerson, Mueller, and Robotti (2023)) and a long-run consumption growth factor (Elkamhi, Jo, and Nozawa (2024)), as well as from recent work employing a duration-based valuation framework to resolve the CAPM's pricing failure (Binsbergen, Nozawa, and Schwert (2025)).

A Three-Factor Model of Credit, Liquidity, and Duration – We first examine whether our model-based three factors carry significant risk premia. Our analysis reveals that both the credit and liquidity factors carry economically and statistically significant risk premia, with credit being particularly pronounced during the post-financial-crisis period. The credit factor averages 24 bps (t-stat = 1.90) from 2005 to 2023, increasing substantially to 39 bps (t-stat = 2.86) in the post-crisis era. Meanwhile, the liquidity factor maintains a consistent premium of 4 bps (t-stat = 4.29) from 2005 to 2023 and 5 bps (t-stat = 4.45) after the crisis, reflecting the persistent pricing of liquidity risk. Although the duration factor does not exhibit statistically significant risk premium, it demonstrates a strong correlation with Treasury returns, suggesting its primary importance lies in capturing systematic interest rate exposure.

Building on these baseline results, we investigate how the relative importance of credit, liquidity, and duration risks has evolved over time. Employing principal component analysis

(PCA) and Shapley value decompositions, we document significant shifts in the dominant source of risk across different market regimes. Credit risk emerges as the primary driver during the 2008 financial crisis and the 2016 junk bond closure, explaining up to 95% the variation in the first principal component. Liquidity risk gains prominence after 2017, coinciding with regulatory changes that increased dealers' balance sheet costs, while duration risk takes the lead during the 2022 interest rate hike and the 2012 European debt crisis. These patterns highlight the shifting importance of credit, liquidity, and duration risks over time.

To further understand the macro drivers of our model-based bond factors, we examine their time-series relationships with established uncertainty measures. The credit factor demonstrates particularly strong sensitivity to the VIX index, alongside financial uncertainty measure from Jurado, Ludvigson, and Ng (2015) and the MOVE index, with the VIX emerging as the strongest driver. The liquidity factor shows negative correlations with financial uncertainty index, indicating that liquidity conditions are closely tied to the financial sector's health and the balance-sheet costs faced by intermediaries. The duration factor shows its strongest association with the MOVE index. These intuitive alignments confirm that each factor captures distinct dimensions of market risk that correspond to their economically relevant uncertainty measures.

The Empirical Performance of the Three-Factor Model – We demonstrate our framework's broad applicability to asset pricing by first evaluating its performance across major asset classes. Our three-factor model substantially reduces significant excess returns across asset classes: investment-grade corporate bonds decline from 26 bps (t-stat = 1.95) to 6 bps (t-stat = 0.77), high-yield bonds from 49 bps (t-stat = 2.47) to 16 bps (t-stat = 1.17), and the aggregate stock market from 76 bps (t-stat = 2.55) to 35 bps (t-stat = 1.62). The model also reduces foreign corporate bond returns from 34 bps (t-stat = 2.69) to 7 bps (t-stat = 0.81) and emerging market sovereign bond from 36 bps (t-stat = 2.16) to 8 bps (t-stat = 0.55). Credit exposure drives most of this reduction, with particularly strong loadings for equities and high-yield bonds. Duration and liquidity factors provide complementary explanatory power, especially for investment-grade and foreign corporate bonds. Treasury returns exhibit the expected flight-to-safety pattern, with positive duration loadings and negative credit loadings.

Extending to cross-sectional asset pricing, we examine 20 portfolios sorted by bond characteristics including CAPM beta, duration, rating, and illiquidity. Monthly excess returns range from 13 to 46 bps, nearly all statistically significant. After controlling for our three factors, 19 portfolios show insignificant alphas.[3] The pattern of factor loadings reveals economically intuitive relationships: credit risk dominates for lower-rated and higher-beta bonds, duration risk is primary for higher-quality, longer-duration bonds, and both credit and liquidity factors gain importance for less liquid bond portfolios.

Economic Insights from the Three Factors of Duration, Credit, and Liquidity – Our estimation is unique in that the three latent factors are extracted not from bond characteristics but the model-implied factor loadings, offering a clean identification of the risk of credit, duration, and liquidity. Applying our model-based factors more broadly, 1) we use the duration factor to decompose U.S. Treasury bond returns into duration risk and safety premium, differentiating flight-to-safety from flight-to-cash; 2) we use the credit and liquidity factors to decompose the credit spreads and shed light on the varying components that give rise to the observed correlation between credit spreads and interest rates; 3) we use the credit and liquidity shocks to examine the presence of credit-driven liquidity risk and the liquidity-driven credit risk; and finally 4) we use our liquidity factor to shed light on the post-regulation capacity of dealers' balance sheet and its asset-pricing implication.

First, different from the U.S. Treasury, which, as the premier safe asset, is driven both by flight-to-safety and time-varying interest rates, our duration factor extracted from the corporate bond market offers a pure risk factor on the time-varying interest rates. Decomposing the U.S. Treasury returns into two distinct components: pure duration risk and safety premium, we find a strong negative correlation between returns to risky assets (e.g., aggregate stock returns, high-yield corporate bond returns, and investment-grade corporate bond returns) and the safety premium component of U.S. Treasury returns. By contrast, the pure duration factor returns are positively correlated with the returns to the risky assets, indicating that higher interest rates (i.e., negative duration) actually negatively affects the pricing of the risky assets.

Focusing on the safety of the U.S. Treasury, we further document a strong comovement between our measure of U.S. Treasury safety and that of Jiang, Krishnamurthy, and Lustig (2021) – the comovement between these two safety measures intensifies markedly during months identified as acute flight-to-safety episodes. Opposite to U.S. Treasury safety are moments when U.S. Treasury temporarily loses its safety status. While such moments are often characterized by dash-to-cash, our framework can further differentiate dash-to-cash and flight-away-from-safety. We find that across all historical events, March 2020 is unique in that

[3] For the remaining high-rated portfolio, our three factors explain half of the excess return. Essentially, these are the firms with highest credit quality, whose returns are not fully driven by credit, duration and liquidity risks. This is consistent with the credit spread puzzle in the literature, a model incorporating crash risk may help explain the remaining one-third of the return.

it was the only time when dash-to-cash and flight-away-from-safety occurred simultaneously. In other words, both the duration and safety components of U.S. Treasury turned negative, driven most likely by massive selling pressure on U.S. Treasury from foreign central banks and other liquidity-constrained U.S. Treasury investors. By contrast, most dash-to-cash episodes are often accompanied by flight-to-safety (e.g., the 2022 inflation surge). In other words, while the duration component of U.S. Treasury turns negative, its safety premium component remains salient.

Second, we re-examine the well-documented negative correlation between credit spreads and interest rates documented by Longstaff and Schwartz (1995) through the lens of our three-factor decomposition. By separating credit spreads into credit and liquidity components, we find that the returns to the credit factor are in fact positively correlated with our duration factor, while the liquidity factor are negatively correlated with duration. In other words, rising interest rates are accompanied with deteriorating credit qualities and improving liquidity conditions. It is the liquidity component in the credit spreads that gives rise to the conventional negative correlation between credit spreads and interest rates.

Third, our investigation of the lead-lag relationships between credit and liquidity factors indicates that it is the credit risk that leads to subsequent liquidity risk, while liquidity-driven default is mostly a 2008 phenomenon. Specifically, we find that credit factor strongly predicts the next-month liquidity factor, with a predictive R^2 of 25%. This credit-driven liquidity effect remains statistically significant throughout our sample period, particularly strengthening after the 2008 financial crisis. In sharp contrast, the evidence of liquidity-driven credit risk is considerably weaker and confined primarily to the 2008 global financial crisis. The full-sample regression of liquidity factor predicting the next-month credit factor yields an R^2 of only 1%. Our findings provide robust empirical support for the theoretical mechanisms proposed by He and Xiong (2012) and He and Milbradt (2014), demonstrating that both directions of influence exist but with markedly different strengths and persistence.

Finally, we establish a direct link between our liquidity factor and dealers' post-crisis balance-sheet capacity, offering insights into how regulatory changes have transformed liquidity risks. We document a significant structural break around 2017, coinciding with the full implementation of post-crisis regulations under Dodd–Frank and Basel III. During the post-2017 period, increased volatility in our liquidity factor becomes systematically correlated with reduced dealer net positions in corporate bonds, whereas this relationship remains statistically insignificant in the preceding years. This finding aligns with Goldberg and Nozawa (2021), who identify dealer inventory capacity as a key driver of market liquidity and expected returns from 2002 to 2016. We further show that after 2017, the association between dealers' net position and liquidity factor volatility tightens substantially. As rising inventory costs compel dealers to maintain smaller balance sheets, liquidity provision becomes more constrained, resulting in heightened liquidity volatility that our factor effectively captures.

Related Literature – Our paper contributes to the literature on structural models of default, a field developed by the seminal work of Merton (1974), followed by important contributions from Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), He and Xiong (2012) and many others. These models have been pivotal in understanding the dynamics of credit risk and its connection with interest rate risk or liquidity risk. Our study aims to provide a unified framework to jointly analyze these three types of risk. Moreover, the existing literature has primarily focused on the models' implication on the credit spread puzzle, including Huang and Huang (2012), Chen, Collin-Dufresne, and Goldstein (2008), Almeida and Philippon (2007), Bhamra, Kuehn, and Strebulaev (2009), Chen (2010), and Huang, Nozawa, and Shi (2025). However, the implications of structural models on bond returns have been less explored, highlighting a gap that our research aims to address. By integrating liquidity and interest rate risks into the structural models, our study seeks to uncover the model-implied cross-sectional factors from corporate bond returns and further disentangle the credit, liquidity, and duration risks over time, thereby providing new insights into this classical literature.

Our paper is also related to the growing literature on identifying a common corporate bond factor structure. This research stream begins with Fama and French (1993), who identify two key bond-market factors: the term factor and the default factor, both of which are crucial in pricing bond returns. More recently, Bai, Bali, and Wen (2019) propose a four-factor model, which was later retracted due to lead/lag errors identified by Dickerson, Mueller, and Robotti (2023). After correcting these statistical issues, they demonstrated that the bond Capital Asset Pricing Model (CAPM) is not dominated by either traded or non-traded factor models in the existing literature. Related work also examines single-factor specifications, such as a long-run consumption growth factor (Elkamhi, Jo, and Nozawa (2024)) and a duration-based valuation framework aimed at resolving the CAPM's pricing failure (Binsbergen, Nozawa, and Schwert (2025)). Most existing studies adopt statistical methodologies. Kelly, Palhares, and Pruitt (2023) use instrumented principal components analysis (IPCA) to propose a conditional factor model for corporate bond returns. Dickerson, Julliard, and Mueller (2023) explore the corporate bond factor zoo and find that most tradable factors are unlikely sources of priced risk, except for post-earnings announcement drift in corporate bonds. Dang, Hollstein, and Prokopczuk (2023) identify carry, duration, equity momentum, and the term structure as the most important risk factors in corporate bond markets. Dick-Nielsen et al. (2024) document replication failures in corporate bond factors and find that several equity signals can predict bond returns under their robust factor construction. Chen et al. (2024) study the common risk factors in the returns on stocks, bonds, and options, using the Regressed-PCA approach proposed by Chen, Roussanov, and Wang (2025). In global corporate bonds, Bekaert, Santis, and Mondino (2024) identify a robust three-factor model incorporating market, maturity, and liquidity, while Deng, Hou, and Shi (2024) emphasize a two-factor framework based on market and a composite equity factor. In contrast, our study adopts a structural default model-based approach, providing a theoretical foundation for some factors identified by existing statistical methods and offering valuable economic insights into several well-established findings documented in the literature.

Finally, our paper also enriches the literature on the safety premium or convenience yield of U.S. Treasury – a concept of central importance in prior work (e.g., Krishnamurthy and Vissing-Jorgensen (2012); Du, Im, and Schreger (2018); Liao (2020); Jiang, Krishnamurthy, and Lustig (2021); Jiang, Krishnamurthy, and Lustig (2023); Jiang et al. (2024)). While existing proxies such as the Treasury basis of Jiang, Krishnamurthy, and Lustig (2021) are derived from currency-hedged sovereign yield differentials, our approach extracts a duration-based safety premium directly from the cross-section of U.S. corporate and Treasury bonds, which helps mitigate potential clientele effects across different sovereign markets. We validate our new measure by showing strong positive comovement with the Treasury basis, with the correlation significantly strengthening during flight-to-safety episodes. More importantly, our framework decomposes U.S. Treasury returns into two distinct components: pure duration risk and safety premium. This decomposition enables novel insights, including differentiating between dash-for-cash and flight-to-safety (and flight-away-from-safety) episodes, with particular focus on the unprecedented Treasury market dislocations during the COVID-19 crisis. We also show that the well-documented negative correlation between credit spreads and Treasury yields (Longstaff and Schwartz (1995)) is driven primarily by the safety premium channel.

The rest of our paper is organized as follows. Section 2 presents a structural model of default with stochastic interest rates and liquidity jump shocks. Section 3 describes the data and constructs our model-based three factors, then examines their risk premia, and analyzes the macroeconomic drivers of these factors. Section 4 shows the empirical performance of our three-factor framework, including its ability to explain risk premia in both aggregate markets and cross-sectional portfolios. Section 5 demonstrates new insights from our three-factor framework into four established findings: decomposing Treasury returns into duration risk and safety premium, analyzing credit-spread – Treasury-yield correlations, examining credit-liquidity lead-lag relationships, and linking liquidity to dealers' post-regulation balance-sheet capacity. Section 6 concludes. Further details are provided in the Appendix.

2 The Model

Our model framework builds on a sequence of work by Merton (1974) and Longstaff and Schwartz (1995). We first consider a structural model of default (Merton (1974)) augmented with stochastic interest rate (Longstaff and Schwartz (1995)). Later we will introduce a jump component capturing the liquidity shock (Pan (2002), Bao, Pan, and Wang (2011), He and Xiong (2012)).

2.1 Default Risk and Interest Rate Risk

Under the risk-neutral measure, the aggregate output V_t follows a geometric Brownian motion

$$\frac{dV_t}{V_t} = r_t dt + \sigma_V dB_t^V \,, \tag{3}$$

where r is the short-term risk-free rate. Following Longstaff and Schwartz (1995), we assume the dynamics of r is drawn from Vasicek (1977), given by

$$dr_t = (\alpha - \zeta r_t)dt + \eta dB_t^r \,, \tag{4}$$

The firm i's asset value V_t^i follows a geometric Brownian motion

$$\frac{dV_t^i}{V_t^i} = r_t dt + \beta_i \sigma_V dB_t^V + \sigma_i dB_t^{V,i} \,, \tag{5}$$

where $B_t^{V,i}$ is an independent standard Brownian motion that generates idiosyncratic shocks specific to the firm i. β_i is the sensitivity of the firm-level systematic volatility to variation in the aggregate volatility. We can further simplify the equation (5) to

$$\frac{dV_t^i}{V_t^i} = r_t dt + \sigma dB_t \,, \tag{6}$$

where $\sigma = \sqrt{\beta_i^2 \sigma_V^2 + \sigma_i^2}$ and B_t is a standard Brownian motion.

For any security with payoff at time T contingent on the values of V and r, the price $H(V, r, T)$ follows the partial differential equation,

$$\frac{\sigma^2}{2} V^2 H_{VV} + \frac{\eta^2}{2} H_{rr} + \rho\sigma\eta V H_{Vr} + rVH_V + (\alpha - \zeta r)H_r - rH = H_T, \tag{7}$$

where ρ is the instantaneous correlation between dB_t^V and dB_t^r.

For riskfree bond, following Vasicek (1977), the value of a riskless discount bond $R(r_t, t, T)$

is given by

$$R(r_t, t, T) = e^{A(t,T)+B(t,T)r_t}, \tag{8}$$

where

$$\begin{aligned} A(t,T) &= \left(\frac{\eta^2}{2\zeta^2} - \frac{\alpha}{\zeta}\right)(T-t) + \left(\frac{\eta^2}{\zeta^3} - \frac{\alpha}{\zeta^2}\right)(e^{-\zeta(T-t)} - 1) - \frac{\eta^2}{4\zeta^3}(e^{-2\zeta(T-t)} - 1), \\ B(t,T) &= \frac{1}{\zeta}(e^{-\zeta(T-t)} - 1). \end{aligned}$$

For risky bond, following Merton (1974), the payoff for the debt holder is $\min(V_T, K)$. Therefore, the debt value issued by firm i can be solved by

$$P_t^i = \mathbb{E}_t^Q \left[e^{-\int_t^T r_s\, ds} \cdot \left(K - (K - V_T^i) \cdot \mathbb{I}_{\{V_T^i \le K\}} \right) \right] \tag{9}$$

Following Longstaff and Schwartz (1995), the bond price P_t^i can be solved as follows,

Proposition 1 *The value of a risky bond under Merton with stochastic interest rate is,*

$$P_t^i = KR(r_t, t, T)N(d_2) + V_t^i R(r_t, t, T) e^{M(r_t,t,T)+\frac{1}{2}S^2(t,T)} N(-d_1), \tag{10}$$

where

$$d_1 = \frac{\ln \frac{V_t^i}{K} + M(r_t, t, T) + S^2(t,T)}{S(t,T)}, \quad d_2 = \frac{\ln \frac{V_t^i}{K} + M(r_t, t, T)}{S(t,T)},$$

and

$$\begin{aligned} M(r_t, t, T) &= \left(\frac{\alpha}{\zeta} - \frac{\eta^2}{\zeta^2} - \frac{1}{2}\sigma^2 - \frac{\rho\sigma\eta}{\zeta}\right)(T-t) + \left(\frac{\eta^2}{2\zeta^3} + \frac{\rho\sigma\eta}{\zeta^2}\right) e^{-\zeta T}(e^{\zeta(T-t)} - 1) \\ &+ \left(\frac{r}{\zeta} - \frac{\alpha}{\zeta^2} + \frac{\eta^2}{\zeta^3} - \frac{\eta^2}{2\zeta^3}e^{-\zeta T}\right)(1 - e^{-\zeta(T-t)}), \end{aligned}$$

$$S^2(t,T) = \left(\frac{\rho\sigma\eta}{\zeta} + \frac{\eta^2}{\zeta^2} + \sigma^2\right)(T-t) - \left(\frac{\rho\sigma\eta}{\zeta^2} + \frac{2\eta^2}{\zeta^3}\right)(1 - e^{-\zeta(T-t)}) + \frac{\eta^2}{2\zeta^3}(1 - e^{-2\zeta(T-t)}).$$

In the special case when the stochastic interest rate process degenerates to a constant r, we have

$$R(r,t,T) = e^{-r(T-t)}, \quad M(r,t,T) = (r - \frac{1}{2}\sigma^2)(T-t), \quad S(t,T) = \sigma\sqrt{T-t},$$

Hence, the bond value becomes

$$P_t^i = Ke^{-r(T-t)}N(d_2) + V_t^i N(-d_1).$$

where $d_1 = \frac{\ln \frac{V_t^i}{K} + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$, which reduce to the standard result in Merton's model.

For model-implied factor loadings in the bond market, we can derive the proposition,

Proposition 2 *The instantaneous return of bond i in excess of the riskfree rate follows*

$$\frac{dP_t^i}{P_t^i} - r_t dt = b_i^V\, dF_t^V + b_i^r\, dF_t^r,$$

where $dF_t^V = \sigma_V dB_t^V$, $dF_t^r = \eta dB_t^r$ and

$$\begin{aligned} b_i^V &= \frac{\partial \ln P_t^i}{\partial \ln V_t} = \beta_i \frac{\partial \ln P_t^i}{\partial \ln V_t^i} = \beta_i \frac{V_t^i}{P_t^i} R(r_t, t, T) e^{M(r_t,t,T)+\frac{1}{2}S^2(t,T)} N(-d_1), \\ b_i^r &= \frac{\partial \ln P_t^i}{\partial r_t} = \frac{e^{-\zeta(T-t)} - 1}{\zeta}\left[1 - \frac{\frac{f(d_2)K}{S(t,T)} + V_t^i e^{M(r_t,t,T)+\frac{1}{2}S^2(t,T)}[N(-d_1) - \frac{f(d_1)}{S(t,T)}]}{KN(d_2) + V_t^i e^{M(r_t,t,T)+\frac{1}{2}S^2(t,T)} N(-d_1)}\right]. \end{aligned}$$

We assume that, in the short run, a 1% change in V leads to a β_i% change in the firm's asset value V_i, consistent with the instantaneous correlation structure of their returns.

2.2 Adding Liquidity Shock

Following the idea in He and Xiong (2012), we further introduce the liquidity shock, which arrives according to a Poisson process N_t with intensity λ. Upon the arrival of the liquidity shock, the bond investor has to sell the bond at a fractional cost of κ. To simplify the analysis, we model the liquidity shock as an instantaneous price drop of magnitude κ, which can be measured using the effective bid-ask spread from Bao, Pan, and Wang (2011). Therefore, for bond i at time t with jump component, we have

$$\frac{dP_t^i}{P_t^i} - r_t dt = \frac{\partial \ln P_t^i}{\partial \ln V_t}\sigma_V dB_t^V + \frac{\partial \ln P_t^i}{\partial r_t}\eta dB_t^r + (e^{-\kappa_i} - 1)(dN_t - \lambda dt),$$

Hence, Proposition 2 can be further expanded to incorporate liquidity shocks,

$$\frac{dP_t^i}{P_t^i} - r_t dt = b_i^V\, dF_t^V + b_i^r\, dF_t^r + b_i^N\, dF_t^N, \tag{11}$$

where $b_i^N = e^{-\kappa_i} - 1$ and $dF_t^N = dN_t - \lambda dt$.

3 Estimating the Three-Factor Model

In this section, we present our model-based approach to extract three common risk factors – Credit, Liquidity and Duration – from cross-sectional corporate bond returns. As shown in (9), the bond i's price in our model P_t^i is a function of three state variables, where V_t is the systematic component of the firm's asset value, r_t is the stochastic interest rate, and N_t is a poisson process capturing the liquidity shocks. Associated with the three state variables of the model are the three risk factors: credit, duration, and liquidity risks. To extract these factors from cross-sectional regression, following equation (11), we have

$$\frac{dP_t^i}{P_t^i} - r_t dt = b_i^V \, dF_t^V + b_i^r \, dF_t^r + b_i^N \, dF_t^N + dB_t^i \,,$$

where dF_t^V, dF_t^r, and dF_t^N are the latent risk factors associated with the three state variables, whose means are zero under the risk-neutral measure Q and non-zero under the physical measure, with the factor risk premium driving the difference. Representing idiosyncratic risk, dB_t^i is zero mean under both the risk-neutral and the physical measure. From proposition 2 and equation (11), for bond i, the model-implied factor loadings are $b_i^V = \partial \ln P_t^i / \partial \ln V_t$ for credit, $b_i^r = \partial \ln P_t^i / \partial r_t$ for duration, and $b_i^N = e^{-\kappa_i} - 1$ for liquidity.[4] Then we take advantage of the availability of a vast cross-section of bond returns and use the cross-sectional information to extract the time-t latent factors.

3.1 Data Description

To estimate the model and the resultant factor loading, we focus on the bond issuers with equity listed in NYSE, Nasdaq and AMEX, which are larger and more important to the economy and whose financial statements and equity market information help us measure their credit quality.

We obtain our data on bond pricing from TRACE, bond returns from WRDS, bond characteristics information from Mergent FISD, equity pricing and treasury pricing from CRSP and issuer-level balance-sheet information from Compustat. For the purpose of studying credit pricing, we construct a sample of bonds with actively traded stocks and corporate bonds as follows. We include fixed-rate dollar-denominated bonds issued by non-financial firms and exclude bonds with remaining maturity less than six months, bonds with issuance size of less than 100 thousand dollars, bonds whose issuer has less than 10 trading days in

[4] In the empirical estimation, we actually use $-\partial \ln P_t^i / \partial r_t$ as the factor loading for duration risk, which measures the sensitivity of interest rate exposure, similar to the concept of modified duration. Likewise, we use $-(e^{-\kappa_i} - 1)$ as the factor loading for liquidity risk.

the equity market during a quarter or has missing financial statements during a quarter. To calibrate liquidity parameter at the bond-month level, we use the effective bid-ask spread derived from the gamma measure proposed by Bao, Pan, and Wang (2011). Therefore, we also require bonds to be actively traded, with a minimum of 11 trading days per month to facilitate the accurate calculation of the illiquidity measure. Our data sample spans a period from January 2005 to December 2023. The rationale for this time frame is that prior to 2005, public transactions were not fully disclosed on TRACE, which could potentially introduce data inconsistencies. To prevent potential data errors or outliers from driving our results, we also take the conservative treatment by winsorizing the credit spreads at lower 0.5% and upper 99.5% of the sample.

Our bond-level data are summarized in Table I, consisting of monthly bond prices, bond characteristics, and bond trading information. For pricing data, we directly use the month-end return and yield to maturity data from WRDS Bond Returns database. Following the convention, we use the Fed Constant Maturity Bonds as the reference curve and calculate the credit spread as the difference between the corporate bond yield and treasury yield of the same maturity. BondBeta is the bond beta estimated from a CAPM model in the bond market over a rolling window of past 24 months. Maturity is the bond's time to maturity in years. Rating is a numerical translation of Moody's rating: 1=Aaa and 21=C. Age is the time since issuance in years. Coupon is the bond's coupon payment in percent. Amount is the bond's amount outstanding in millions of dollars. Gamma is the illiquidity measure. Callable is 1 for bonds issued with callable options.

For issuer-level equity information, we report the total market value of its equity in logarithm under EquitySize. EquityBeta is the equity beta estimated from a CAPM model in the stock market over a rolling window of past 60 months. BM is the book value of the equity to market capitalization. We further report three important inputs used to measure the credit quality of a firm. We use daily stock returns over a rolling window of past 3 months to measure equity volatility. To calculate firm leverage, we collect information on the firm's short- and long-term debt and its book value of asset from the quarterly financial statements. Leverage is calculated as the ratio of the total short- and long-term debt to the total asset value. The quarterly asset growth is computed as the growth rate of the asset value averaged over the past three years. Using these bond and equity information, we can estimate the model-based factor loadings for credit (b_i^V), liquidity (b_i^N), and duration (b_i^r).

Overall, our sample encompasses 14,761 bonds issued by 1,480 firms over the period from 2005 to 2023. The bonds in our sample exhibit a mean monthly return of 27 basis points (bps) and a credit spread of 128 bps. In terms of credit quality, the median rating stands at 8, which corresponds to Moody's Baa1 rating, indicating a relatively low credit risk profile.

The median time to maturity is approximately 6 years, while the average age of the bonds is around 4 years. Regarding bond size and liquidity, the bonds in our sample are notably larger and more liquid than average. The median amount outstanding is a substantial USD 500 million, and the gamma measure, which reflects illiquidity, is 0.17, indicating a relatively high liquidity. A significant proportion of the bonds in our sample are issued with embedded call options. Shifting focus to equity market variables, the average issuer in our sample is relatively large, with a median equity size of USD 24.5 billion. This suggests that our sample is skewed towards larger corporations. The median BM is 0.4 and the median equity volatility is 0.25. The median leverage ratio is 0.37 and the median asset growth rate is 0.05. In terms of estimated factor loadings, given that the majority of the bonds in our sample are issued by high quality firms characterized by large size, low leverage, and low equity volatility, the corresponding distance-to-default measures are quite high. Under the assumption of a normal distribution, approximately 60% of the credit factor loadings in our sample are close to zero. The average liquidity factor loading is 0.77 and the average duration factor loading is 4.80.

3.2 Factor Estimation

To calibrate the model at the firm level, we need to first estimate the firm-level asset value and asset volatility. Following Moody's KMV (Kealhofer and Kurbat (2001)), we estimate the firm's asset value V^i and its corresponding asset volatility σ by solving the following non-linear equations simultaneously,

$$E_t^i = V_t^i R(r_t, t, T) e^{M(r_t,t,T)+\frac{1}{2}S^2(t,T)} N(d_1) - KR(r_t, t, T)N(d_2) \text{ and } \sigma_E = \frac{V_t^i}{E_t^i}\frac{\partial E_t^i}{\partial V_t^i}\sigma\,, \quad (12)$$

where E_t^i is the value of the firm's equity, V_t^i is the ssset value of the firm, r_t is the risk-free rate, σ_E is equity volatility, σ is its asset volatility, and K is its default boundary, and T is the time horizon of interest.

To estimate monthly factors, we use the bond monthly return as the dependent variable in equation (2). For calculating the factor loadings on the right-hand side at month t, we adhere to equation (1) and utilize the most recent available information up to month t. Specifically, for balance-sheet variables, we consistently use the data from two quarters prior to month t. This includes the default boundary K, which is calculated as the sum of the firm's current debt and its long-term debt. Regarding the firm's equity value, we refer to the information from month $t-1$, calculated by multiplying the month-end stock price by the number of common equity shares outstanding. For the equity volatility σ_E, we compute it using daily equity returns spanning from month $t-3$ to month $t-1$. We require that the issuer has a minimum of 10 trading days within this three-month window to ensure sufficient data for accurate calculation. The debt maturity (T) is computed as the weighted average of preset maturities across all interest-bearing debt categories. Specifically, we assign maturities of 0.5, 1.5, 2.5, 3.5 and 7 years to debt categories dlc, dd2, dd3, dd4 and dd5, respectively, and weight each by its share of total interest-bearing debt. This consolidated measure captures the issuer's effective maturity and feeds directly into default-risk assessment.

For bond-specific variables, we use the bond's modified duration as the time horizon for discounting.[5] Additionally, we proxy the transaction cost κ with the bond's effective bid-ask spread, which is calculated from the gamma measure at month $t-1$. For interest rate parameters, we calibrate the values $\beta = 0.01$, $\alpha = 0.0005$, $\eta = 0.03$ and $\rho = -0.25$ based on interest rate data from 2005 to 2023. The risk-free rate is obtained from French's website.

With these inputs, we are able to estimate the asset value V and asset volatility σ_A using equations (12). Subsequently, we compute the factor loadings for month t by applying equation (1). This methodological framework can also be extended to estimate daily factors. We use the most recent available information from the previous month or two quarters prior to day t to calculate the factor loadings. Then, we employ bond daily returns as the dependent variable in equation (2) to extract the daily factor returns. This approach ensures consistency and accuracy in our factor estimation across different time frequencies.

Table II presents the risk premia of our three model-based factors at the monthly frequency. The results indicate that both credit and liquidity factors carry significant risk premia, particularly after the global financial crisis. Specifically, the credit factor exhibits an average premium of 24 bps (t-stat = 1.90) from 2005 to 2023, which turned negative during the 2008 financial crisis. When focusing on the post-financial-crisis period from 2009 to 2023, the credit premium rises to 39 bps (t-stat = 2.86). The liquidity premium averages 4 bps (t-stat = 4.29) in the full sample and increases to 5 bps (t-stat = 4.45) in the post-crisis sample. In contrast, we observe no significant premium for the duration factor, with an average of 1 bp (t-stat = 0.83). However, over time, we find interesting dynamics, particularly its strong correlation with Treasury returns. For daily factors, from 2005 to 2023, the risk premia are 0.82 bps (t-stat = 1.32) for credit, 0.34 bps (t-stat = 2.39) for liquidity, and 0.12 bps (t-stat = 1.53) for duration. Focusing on the sample period from 2009 to 2023, these premia increase to 1.76 bps (t-stat = 2.89) for credit, 0.49 bps (t-stat = 3.20) for liquidity,

[5]Our empirical estimation is conducted at the bond level to ensure sufficient cross-sectional variation for measuring the duration factor. When a firm has multiple bonds outstanding, the estimation jointly employs (i) the firm-level weighted-average maturity described above for credit-risk components and (ii) each bond's own modified duration for discount-rate components. This approach reflects the economic distinction that default risk is borne by the firm, whereas interest-rate risk is bond-specific.

and 0.16 bps (t-stat = 1.90) for duration.

As illustrated in Figure 1, panel (a) plot the cumulative returns of the three monthly factors for credit, liquidity, and duration. Panel (b) display the cumulative returns for the daily factors. Overall, we can observe a high correlation between monthly and daily cumulative returns for each of the three factors. By running cross-sectional regression as specified in equation (2), our credit factor captures the average return, with greater weight assigned to issuers exhibiting higher credit risk. Likewise, the liquidity factor captures the average return, with more weight attributed to bonds with higher liquidity risk. For the duration factor, it captures the average return weighted more heavily by bonds with longer duration. Therefore, we expect the three factors to have positive returns in normal market conditions. However, during market turmoil, as investors may sell high credit risk bonds and seek quality in bonds with low credit risk (flight-to-quality), the credit factor should experience negative returns in these periods. This intuition aligns with our empirical findings. Indeed, while the credit factor carries positive returns for most of the time, it becomes significantly negative during the 2008 financial crisis, the 2016 junk bond closure, and the 2020 Covid-19 pandemic. For the liquidity factor, the negative periods include 2016 and 2020. The duration factor is less volatile than the credit and liquidity factors, with the general increasing trend stopping after the 2020 pandemic and switching to a decreasing trend after the 2022 interest rate hike, similar to the pattern observed for US Treasury bonds.

Figure 2 shows the normalized daily returns of credit factor, liquidity factor, and duration factor. Focusing first on the extreme negative outcomes of daily factors, we are able to identify episodes of flight-to-cash or dash-for-cash when our duration factor suddenly drops. Prominent examples of flight-to-cash include June 20, 2013, the day after Bernanke's taper speech; March 12, 17, and 19, 2020, events of dash for cash amid Covid-19; and June 13, 2022, when the CPI announcement for May indicates accelerating inflation. Likewise, episodes of flight-to-quality and flight-to-liquidity can be separately identified using extreme values of our credit and liquidity factors. For example, episodes of flight-to-quality captured by our credit factor include September 15, 2008 (Lehman bankruptcy), the peak of the 2008 financial crisis (early October 2008), March 9, 2020 (Covid-19), and June 13, 2022, (CPI announcement indicating accelerating inflation). Flight-to-liquidity days captured by our liquidity factor include August 8, 2011 (the black Monday due to U.S. debt ceiling crisis), October 15, 2014 (U.S. Treasury flash crash), March 23, 2020 (announcement of Fed SMCCF to provide liquidity to corporate bonds), and March 13, 2023 (SVB bankruptcy). Importantly, the clean identification of our model-based factors allows us to separately locate such important and influential episodes in history. A better understanding of their nature and origin can help us gauge the effectiveness of the policies and regulations enacted at the time.

To better understand the relative importance of credit, liquidity, and duration risks over time, we conduct a Principal Component Analysis (PCA) on the daily returns of our three factors, credit, liquidity and duration, over a rolling window of past three months. Panel (a) of Figure 3 shows the R^2 contribution of the first three principal components (PCs) in explaining the variation in these factors. We find that the first PC accounts for approximately 60% of the total variation, while the second and third PCs account for 25% and 15%, respectively. This indicates that the first PC dominates the common movements in our factors. To further examine the contributions of each factor to the first PC, Panel (b) plots the Shapely R^2 decomposition of PC1 to overcome the overlapping information contained in three factors. Our results reveals that different factors plays key roles in explaining the first PC over time. For example, our credit factor (blue bar) can explain up to 90% of the first PC during the 2008 financial crisis and the 2016 junk bank closure, highlighting credit risk as the primary driver in these periods. Liquidity risk (red bar) emerges as the dominant factor post-2017, coinciding with regulatory changes that increased dealers' balance sheet costs. Duration risk (orange bar) takes the lead during specific events such as the 2022 interest rate hike and the 2012 European debt crisis. Overall, our analysis underscores the shifting importance of credit, liquidity, and duration risks over time.

3.3 The Macro Drivers of Three Factors

Next we delve into the macroeconomic drivers of our model-based bond factors. Specifically, we examine the time-series relationships between our three factors – credit, liquidity, and duration – and key macroeconomic and market variables. For macroeconomic uncertainty, we utilize the macro uncertainty index, financial uncertainty index, and real uncertainty index developed by Jurado, Ludvigson, and Ng (2015). These indices provide a comprehensive measure of the overall economic and financial uncertainty in the market. For market-based variables, we employ the VIX index from the CBOE to gauge stock market uncertainty, the MOVE index to assess Treasury market uncertainty, and the noise measure from Hu, Pan, and Wang (2013) to measure Treasury market liquidity. As shown in Table III, we conduct individual and joint regressions to determine which uncertainty measures have the strongest explanatory power for each factor. The regression results are presented in Panel A for the credit factor, Panel B for the liquidity factor, and Panel C for the duration factor.

Our findings reveal that the credit factor is negatively correlated with changes in the VIX index, financial uncertainty index, and MOVE index, with the VIX emerging as the strongest indicator. This suggests that credit risk is highly sensitive to overall market volatility and economic uncertainty. The liquidity factor, on the other hand, is negatively correlated with changes in the financial uncertainty index, indicating that liquidity conditions are closely tied to the financial sector's health and the balance-sheet costs faced by intermediaries. Macro uncertainty also negatively affects liquidity, though to a lesser extent. The duration factor is most strongly and consistently associated with the MOVE index, which specifically measures interest rate uncertainty – an intuitive alignment given duration's sensitivity to rate movements. The VIX also shows a significant negative relationship with duration. Overall, these results demonstrate that our three factors capture distinct dimensions of market risk that align logically with established uncertainty measures.

4 The Three-Factor Model: Empirical Performance

In this section, we first examine whether our model-based three risk factors can explain aggregate market risk premia across major asset classes, including corporate bonds, government bonds, and equities. We then extend our analysis to cross-sectional asset pricing tests within credit market. Specifically, we construct and test portfolios sorted by various bond characteristics (such as duration, credit rating, and liquidity) to evaluate the model's performance in explaining cross-sectional return variations.

4.1 Explaining Aggregate Market Returns Across Asset Classes

To evaluate the performance of our three factors, we first run the following monthly regression to see if the three factors can explain the risk premia across major and important markets.

$$R_t^M - R_t^f = \alpha + \beta^{Credit} F_t^V + \beta^{Liquidity} F_t^N + \beta^{Duration} F_t^r + \epsilon_t \,, \tag{13}$$

where R_t^M is the market return for bond, stock, or treasury. F_t^V, F_t^N, and F_t^r are the three factors extracted from cross-sectional corporate bond returns in Section 3. Table IV presents the regression results. Panel A shows the mean excess returns across five market indices in US, including CorpBond (Aggregate Corporate Bond Market Index), HY (Bloomberg Barclays High-Yield Index), IG (Bloomberg Barclays Investment-Grade Index), Stock (Fama-French MktRf Portfolio), and Treasury (Bloomberg Barclays Aggregate Treasury Index), as well as three market indices for Foreign entities, including CorpBond (aggregate corporate dollar bond return by foreign issuers), SovereignEM (Bloomberg Emerging Market Sovereign Index), and SovereignEU (Bloomberg EU Sovereign Index). We observe significant risk premia in both the corporate bond and equity markets: 30 bps (t-stat = 2.53) for US corporate bond, 49 bps (t-stat = 2.47) for HY, 26 bps (t-stat = 1.95) for IG, 76 bps (t-stat = 2.55) for Stock, 34 bps (t-stat = 2.69) for foreign corporate bond, and 36 bps (t-stat = 2.16) for emerging market sovereign bond. For the Treasury portfolio, the average excess return is not significant after accounting for the risk-free rate, with a mean of 13 bps (t-stat = 1.45) for US and 13 bps (t-stat = 1.22) for EU.

Panel B reports the regression results controlling only for the credit factor. This specification reveals that credit exposure plays a dominant role in explaining risk premia across most asset classes. For HY, IG, stocks, and emerging market sovereign bonds, the risk premia become largely insignificant after accounting for credit risk, with alphas reducing to 19 bps (t-stat = 1.43), 14 bps (t-stat = 1.13), 34 bps (t-stat = 1.63), and 18 bps (t-stat = 1.11), respectively. The credit factor exhibits economically and statistically strong loadings across all markets, particularly for equities (coefficient = 1.77, t-stat = 14.85) and high-yield bonds (coefficient = 1.25, t-stat = 10.60). For Treasuries, the credit factor shows a negative relationship (coefficient = -0.13, t-stat = -2.05), consistent with flight-to-safety dynamics.

Panel C presents the full three-factor model, incorporating duration, credit, and liquidity factors. While credit remains the most important explanatory variable, adding duration and liquidity factors further enhances the model's explanatory power, particularly for investment-grade bonds and foreign corporate bonds where these additional factors remain significant. For instance, the investment-grade bond portfolio shows strong positive loadings on both duration (coefficient = 7.44, t-stat = 14.23) and liquidity (coefficient = 1.69, t-stat = 2.12) factors. The three-factor model also produces higher R^2 values across most asset classes compared to the single-factor credit model, indicating a more complete characterization of risk exposures. For Treasury returns, the three-factor specification yields a significantly positive alpha of 13 bps (t-stat = 2.12), which rolling window regressions reveal to be more pronounced during post-crisis periods (2008 − 2012 and 2020 − 2022). Overall, the results demonstrate that while the credit factor is the primary driver of risk premia across most corporate bond and equity markets, the inclusion of duration and liquidity factors provides additional explanatory power, particularly for IG.

4.2 Explaining Cross-Sectional Returns Within Credit Market

To further demonstrate the importance of our three factors, we examine their explanatory power for the risk premia across a comprehensive set of test portfolios within the credit market. We construct four categories of quintile portfolios sorted by bond characteristics: bond CAPM beta, bond duration, bond rating, and illiquidity, resulting in 20 portfolios. As shown in Table V, the average monthly excess returns for these portfolios range from 13 to

46 bps, with nearly all being statistically significant. After controlling for our three-factor bond model, the alphas become insignificant for 19 out of the 20 portfolios, underscoring the model‘s strong explanatory power.

The pattern of factor loadings reveals important insights into which risks drive returns for different types of bonds. In Panel A (bond beta sorted), the credit factor's loadings rise from 0.18 (t-stat = 3.22) in the lowest-beta quintile to 0.73 (t-stat = 4.57) in the highest-beta quintile. This indicates that bonds with higher systematic market risk are also more exposed to credit risk. In contrast, Panel B (duration sorted) shows an inverse relationship: as portfolio duration increases, the credit factor's loading declines, while the duration factor's loading rises substantially (from 1.09 to 13.41). This pattern suggests that longer-duration bonds tend to be issued by higher-quality firms for which interest rate risk, rather than credit risk, is the primary return driver. In Panel C (rating sorted), for the lowest-rated (G5) portfolios, credit loadings reach 1.22 (t-stat = 8.40), while for the highest-rated portfolios, the credit loading is insignificant and even negative. Conversely, duration loadings are strongest for high-quality bonds. This aligns with the intuition that credit risk is prominent for lower-quality issuers. Panel D (illiquidity sorted) shows that both credit and liquidity factors gain importance for less liquid bonds, with liquidity loadings surging from 0.16 to 4.63 across the illiquidity spectrum.

For the remaining portfolio whose alpha remains significant – the highest-rated quintile in Panel C – our three factors reduce the mean excess return from 22 bps (t-stat = 1.88) to 11 bps (t-stat = 2.39). This portfolio consists of firms with the highest credit quality, whose returns may not be fully captured by credit, duration, and liquidity risks alone.[6] Overall, our three-factor model successfully explains the majority of returns across the 20 test portfolios. The results demonstrate that while credit risk is the dominant factor for lower-quality and higher-beta bonds, duration and liquidity risks provide important additional explanatory power, particularly for higher-quality bonds.

5 The Three-Factor Model: Fresh Insights

Our three-factor framework, which extracts duration, credit, and liquidity components from corporate bond returns, yields new insights into several well-established findings documented in the literature. Specifically, we provide four distinct applications: First, in Section 5.1, we use the duration factor to decompose U.S. Treasury returns into pure duration risk and safety

[6]This is consistent with the credit spread puzzle in the literature; a model incorporating crash risk may help explain the remaining portion of the return.

premium components, differentiating flight-to-safety from flight-to-cash. Second, Section 5.2 employs the credit and liquidity factors to decompose credit spreads and sheds light on the varying components that give rise to the observed correlation between credit spreads and Treasury yields. Third, Section 5.3 utilizes the credit and liquidity shocks to investigate both credit-driven liquidity and liquidity-driven credit effects. Finally, Section 5.4 uses our liquidity factor to elucidate the post-regulation capacity of dealers' balance sheet. The following subsections develop each of these applications in detail.

5.1 UST = Duration Risk + Safety Premium

Our model-implied cross-sectional factors offer a novel framework to decompose U.S. Treasury (UST) returns into two distinct components: a *pure duration risk* component and a *safety premium* component. This decomposition sheds new light on the economic forces driving the UST convenience yield – a concept of central importance in the literature (e.g., Krishnamurthy and Vissing-Jorgensen (2012); Du, Im, and Schreger (2018); Liao (2020); Jiang, Krishnamurthy, and Lustig (2021); Jiang, Krishnamurthy, and Lustig (2023); Jiang et al. (2024)). We first construct a duration-based measure of the safety premium. Second, we validate this measure against established convenience yield proxies, demonstrating its alignment with existing literature. Third, we utilize this decomposition to differentiate between dash-for-cash and flight-to-safety (flight-away-from-safety) episodes, with particular focus on the unprecedented Treasury market dynamics during the COVID-19 crisis.

5.1.1 Safety Premium = DurationUST - DurationCorp

We begin by extracting the duration factor from the cross-section of investment-grade U.S. corporate bonds (controlling for credit and liquidity effects), denoted as *DurationCorp*. This factor captures the compensation investors demand for bearing interest rate risk, and we interpret it as a measure of pure duration risk. Separately, we extract a duration factor from the U.S. Treasury market, denoted as $DurationUST$.[7] Because Treasury securities are considered virtually default-free and highly liquid, their duration factor should reflect not only interest-rate exposure but also the convenience yield (or safety premium) unique to government bonds. The difference between the two thus isolates the excess return attributable to the safety and liquidity benefits of Treasuries, providing a direct, return-based measure of the convenience yield, which denotes as $SafetyPremium$.

[7] Our *DurationUST* factor exhibits a correlation of 94% with the aggregate U.S. Treasury return, and 96% with the change in the 10-year U.S. Treasury yield, validating its effectiveness in capturing Treasury market dynamics.

This decomposition allows us to examine how each component contributes to returns across different asset classes. We run predictive regressions of monthly excess returns for (1) the aggregate equity market, (2) investment-grade (IG) corporate bonds, (3) high-yield (HY) corporate bonds, and (4) U.S. Treasuries on the two estimated factors: $DurationCorp$ and $SafetyPremium$. Table VI summarizes the results.

Consistent with intuition, $DurationCorp$ loads positively and significantly on equity excess returns, indicating that higher duration risk coincide with higher equity premia. In contrast, $SafetyPremium$ exhibits a strong negative relationship with equity returns, reflecting pronounced flight-to-safety episodes: when equities decline, the safety premium embedded in Treasuries rises. A similar, though more nuanced, pattern emerges in corporate bond markets. For HY bonds, the coefficient on $DurationCorp$ is positive and significant, while the coefficient on $SafetyPremium$ is negative and highly significant – mirroring the equity-market result and underscoring the flight-to-safety nature of HY returns. For IG bonds, the pure duration component remains positive, but the safety premium component is less negative, consistent with IG bonds' intermediate position between equities/Treasuries in the safety hierarchy. Finally, for U.S. Treasury returns themselves, both factors enter positively, as expected: Treasury returns are driven both by compensation for duration risk and by variation in the convenience yield.

5.1.2 Connection to Convenience Yield

To validate our safety premium measure against established proxies, we compare it with the Treasury basis of Jiang, Krishnamurthy, and Lustig (2021), a yield-based convenience yield measure derived from currency-hedged sovereign yield differentials. However, differences in the investor bases for U.S. and foreign sovereign bonds may introduce clientele effects that also influence measured convenience yields. Our proposed approach above can better account for such effects. Since our measure is expressed in returns and the Treasury basis in yields, we plot changes in the Treasury basis against our duration-based return differential to ensure comparability. As shown in Panel A of Figure 4, the two series exhibit a strong positive comovement. For instance, in September 2008, the collapse of Lehman Brothers triggered a flight to safety, causing a sharp increase in the Treasury basis. Consistently, our duration-based measure also yield a high return that month, indicating that investors strongly preferred long-duration Treasuries over comparable investment-grade corporate bonds, even after adjusting for credit and liquidity risks. Similar co-movements are evident during other stress episodes, such as when Citigroup stood on the brink of collapse in November 2008 and during the COVID-19 outbreak in March 2020, supporting the idea that both metrics capture the convenience yield embedded in U.S. Treasuries.

We further hypothesize that the relationship between the two measures may be stronger during periods of market stress. To test this, we use the BondSafety measure from Hu, Jin, and Pan (2025) to identify months with pronounced flight-to-safety episodes – characterized by sharp equity declines and simultaneous Treasury rallies. As shown in Panel B, the positive association between the two convenience yield measures is indeed significantly stronger in these BondSafety months, when investor demand for safe assets is most acute. In summary, our duration-based measure of the U.S. Treasury convenience yield provides a useful and intuitive alternative to existing proxies, capturing similar economic intuition through a different methodological lens. Moreover, our measure offers distinct advantages by potentially mitigating investor clientele effects inherent in cross-country yield comparisons and providing a direct reading of the convenience yield as a risk premium in return space, thereby complementing the yield-level measure of Jiang, Krishnamurthy, and Lustig (2021).

5.1.3 Flight-to-Safety vs Dash-to-Cash

Figure 5 provides a visual decomposition of the dynamic interactions between our two key risk measures: the pure duration risk component ($DurationCorp$) on the x-axis and the safety premium component ($SafetyPremium$) on the y-axis. By examining the joint distribution of these two factors, we can differentiate between various market stress regimes, with particular emphasis on the distinctive patterns observed during the COVID-19 crisis of March 2020.

The red data points in the figure represent the critical March 2020 period, which offers a unique laboratory for studying extreme market behavior. When $DurationCorp$ becomes increasingly negative (left side of the plot), this indicates a "dash for cash" or "flight to cash" episode – a phenomenon captured by the blue arrow direction in the figure. In such episodes, investors rapidly reduce duration exposure. The most severe instances occur when observations fall beyond the 2.5σ or 4σ thresholds in the negative x-direction, representing extreme dash-for-cash events. Notable examples include 2020-03-09, 2020-03-12, 2020-03-18 during the COVID crisis, as well as 2008-10-10 during the global financial crisis and 2022-06-23 during the aggressive monetary tightening cycle.

Our framework's distinctive contribution lies in its ability to further decompose these dash-for-cash episodes based on the behavior of the safety premium component (y-axis). Specifically, we can differentiate between two distinct patterns within the dash-for-cash regime: flight to safety (green arrow direction) and flight away from safety (red arrow direction). Flight to safety episodes occur when investors increase their demand for Treasury safety, resulting in positive $SafetyPremium$. The most pronounced flight-to-safety events in our sample include several COVID-19 dates: 2020-03-20, 2020-03-09, and 2020-03-16. Additionally, we observe similar patterns during the global financial crisis (2008-11-20 and 2008-09-15) and during the 2022 monetary tightening period (2022-06-13 and 2022-09-13). Flight away from safety episodes occur when investors move away from Treasury securities, driving down safety premiums. Notable examples include 2020-03-17 and 2020-03-10 during the COVID crisis, as well as specific days during the financial crisis when equity market experienced sharp rebounds (2008-11-13). More recently, 2022-11-10 and 2023-11-14 represents another flight-away-from-safety episode, triggered by unexpectedly low CPI data that reduced expectations of further monetary tightening.

What makes the flight-away-from-safety phenomenon particularly noteworthy is its conditional nature. While flight away from safety can occur with positive $DurationCorp$ (as in 2022-11-10), the most economically meaningful instances occur when combined with negative duration factors – that is, within a dash-for-cash regime. Strikingly, within our entire sample, the only days exhibiting both dash-for-cash and flight-away-from-safety simultaneously were during the COVID crisis: specifically, 2020-03-17 and 2020-03-10. This concentration in the bottom-left quadrant (red shaded region) underscores the unprecedented nature of the March 2020 Treasury market dislocation. The economic mechanisms behind these dates are illuminating. On 2020-03-10, the 10-year U.S. Treasury yield surged by approximately 20 bps, and on 2020-03-17, it jumped by nearly 30 bps – extraordinary moves for the typically stable Treasury market. These episodes reflected a liquidity crisis where even traditional safe havens were temporarily abandoned as investors scrambled for cash, highlighting the distinctive characteristics of the COVID-induced market stress compared to other crises.

In contrast, the combination of dash-for-cash and flight-to-safety (green region) appears more frequently and across diverse market stress episodes, including not only the COVID crisis but also the global financial crisis and the 2022 monetary tightening cycle. This broader distribution suggests that while flight-to-safety represents a more conventional response to market stress, flight-away-from-safety represents a more extreme, liquidity-driven phenomenon largely confined to the most severe market dislocations.

5.2 Credit Spread = Credit + Liquidity

An important stylized fact in fixed income markets is the strong negative correlation between credit spreads and Treasury yields, documented by Longstaff and Schwartz (1995). Our factor framework provides new insight into this relationship by decomposing both sides: credit spreads into credit and liquidity components, and Treasury returns into pure duration risk and safety premium components. This 2×2 decomposition reveals that the aggregate negative correlation masks important opposing forces.

Panel E and F of Table VI report regression results among the four components. Three key patterns emerge. First, the credit factor is positively correlated with the pure duration factor ($DurationCorp$). Second, the credit factor exhibits a strong negative correlation with the safety premium ($SafetyPremium$). During flight-to-safety episodes, credit deteriorates while the safety premium of Treasuries rises. Third, the liquidity factor is negatively correlated with both the pure duration factor and the safety premium. Illiquidity tends to coincide with a preference for short-duration assets (negative duration factor) and a higher safety premium.

These findings clarify the economic forces behind the aggregate negative credit-spread–Treasury-yield correlation. Our decomposition indicates that the overall correlation is driven primarily by the safety premium component, which is negatively related to both the credit factor and the liquidity factor. When focusing on the effects of pure duration risk on credit and liquidity factors, we find a positive association between credit and pure duration risk. Moreover, the liquidity component itself is negatively correlated with pure duration risk, tilting the total relationship toward negativity.

Figure 6 further illustrates these dynamics using daily correlation within each year. Panel (a) shows the positive co-movement between the credit factor and the pure duration factor, consistent with the notion that flights-to-cash often accompany credit deterioration. Panel (b) shows that liquidity deteriorates when duration demand rises (positive duration factor) and when the safety premium increases, reflecting simultaneous flights-to-duration and flights-to-liquidity. Panel (c) confirms that the combined credit-and-liquidity measure – weighted as in Table IV – exhibits an overall negative correlation with the duration factor, especially before 2022. To understand the positive correlation between credit and duration factors, note that a negative duration factor typically signals flights-to-cash, as observed after the 2022 interest rate hike, when bond investors tend to flight to quality because issuers with high credit quality are less affected by rate increases. Consequently, our credit factor turns negative, aligning with its positive correlation with the duration factor. The negative correlation between liquidity and duration factors can be driven by the fact that flights-to-duration (positive duration factor) and flights-to-liquidity (negative liquidity factor) can occur simultaneously. These findings aligns with the regression results in Panel E and F of Table VI. Panel (d) of Figure 6 shows that the correlation between credit and liquidity risks has increased since 2016.

To summarize, our decomposition therefore offers a nuanced reinterpretation of the mechanism behind the credit-spread–Treasury-yield result. While Longstaff and Schwartz (1995) attribute the negative correlation to an interest-rate-induced change in asset drift, our results emphasize the role of the safety premium and liquidity dynamics. The negative association is not primarily credit-driven but is instead liquidity-driven and amplified by flight-to-safety forces. These insights, enabled by the separate identification of credit, liquidity, and safety components, extend the existing literature and provide a more granular understanding of how credit and Treasury markets interact.

5.3 Credit-Driven Liquidity vs Liquidity-Driven Credit

In this subsection, we further examine the lead-lag relationships between credit and liquidity factors using predictive regressions. Figure 7 presents rolling window estimates of two complementary predictive specifications: Panel (a) examines whether current credit predict future liquidity, while Panel (b) investigates the reverse direction – whether current liquidity predicts future credit.

The results in Panel (a) demonstrate a strong and persistent credit-driven liquidity effect. The month t credit factor significantly predicts the liquidity factor in month $t+1$, with a full-sample coefficient of 0.05 (t-stat $= 8.56$) and an impressive predictive R^2 of 25%. The rolling window analysis reveals that this predictive relationship has remained statistically significant throughout most of our sample period, particularly strengthening after the 2008 financial crisis. Economically, this pattern suggests that credit deterioration – manifesting through heightened default concerns – systematically leads to subsequent liquidity worsening.

In contrast, Panel (b) reveals a much weaker liquidity-driven credit effect. The full-sample regression yields a coefficient of 1.18 but with a borderline t-statistic of 1.70 and a negligible R^2 of only 1%. The rolling window analysis provides further nuance: while the relationship is generally insignificant throughout most of the sample, two distinct periods exhibit significant predictive power. First, immediately following the 2008 financial crisis, liquidity risk significantly predicted future credit risk, consistent with the "liquidity-driven default" mechanism theorized by He and Xiong (2012). Second, during the 2022 monetary tightening cycle, another episode of significant liquidity-driven credit prediction emerged.

Our empirical evidence establishes a clear asymmetry in the credit-liquidity relationship: credit-driven liquidity effects are strong, persistent, and economically significant, while liquidity-driven credit effects are relatively weak, episodic, and largely confined to 2008 global financial crisis. Our findings provide robust empirical support for the theoretical mechanisms proposed by He and Xiong (2012) and He and Milbradt (2014), demonstrating that both directions of influence exist but with markedly different strengths and persistence. Moreover, our rolling window approach reveals that the credit-driven liquidity effect has become increasingly prominent in the post-financial crisis era, potentially reflecting structural changes in market making and dealer intermediation.

5.4 Liquidity Factor and Dealers' Balance-Sheet Capacity

Panel (a) of Figure 8 plots the Exponentially Weighted Moving Average (EWMA) volatility of the daily returns of our three factors over a three-month rolling window, with a decay parameter of 0.98. Using volatility as a proxy for risk, we observe that credit risk (red line) is particularly pronounced during several financial upheavals: the 2008 global financial crisis, the 2011 European debt crisis, the 2016 junk bond closure, the 2020 Covid-19 pandemic, and the 2022 interest rate hike. Regarding duration risk (green line), the most notable peaks occur during the 2022 interest rate hike, the 2020 pandemic, and the 2008 financial crisis, which is consistent with our intuitions. Liquidity volatility (blue line) reaches its peak during the 2020 Covid-19 pandemic, a period marked by a dash for cash.

More importantly, liquidity volatility has grown substantially since 2017, coinciding with the full implementation of stringent regulations under the Dodd-Frank Act and Basel III. To investigate the link between liquidity and dealer activity, Panel (b) of Figure 8 plots the EWMA volatility of our liquidity factor against primary dealers' net positions in investment-grade and high-yield corporate bonds, using data from the Federal Reserve Bank of New York (2013 – 2023).[8] The scatter plot reveals a significant negative relationship between liquidity factor volatility and dealer net positions from 2017 to 2023, while the relationship remains statistically insignificant from 2014 to 2017. This pattern is consistent with the post-crisis regulatory environment: as rising inventory costs compel primary dealers to reduce their net positions, the market's liquidity tends to become more volatile, which is precisely captured by the increase in our liquidity factor's volatility.

6 Conclusion

In this paper, we extract three model-based risk factors – credit, duration, and liquidity – from the cross-section of U.S. corporate bond returns. Our approach differs from characteristic-based factor construction by relying on loadings implied by a structural, arbitrage-free pricing model. This design yields cleanly identified factors that separately capture systematic credit, interest-rate, and liquidity risks.

We show that these factors carry economically meaningful risk premia, exhibit intuitive

[8]We omit pre-2013 data as the Fed did not separately report net positions for investment-grade and high-yield corporate bonds during that period, instead reporting only aggregate net positions that included other bond types.

macroeconomic drivers, and shift in relative importance across different market regimes. Specifically, credit and liquidity risks command significant premiums, while duration primarily captures systematic interest-rate exposure. Their evolution is closely tied to recognizable economic forces: credit risk is most sensitive to overall market volatility, liquidity risk to financial-sector stress, and duration risk to interest-rate uncertainty. These linkages, together with time-varying dominance among the three factors – credit during the 2008 crisis, liquidity after post-crisis regulations, and duration during monetary-tightening episodes – confirm that our framework captures the shifting nature of fundamental risks in financial markets.

More importantly, the framework delivers fresh insights into several classic questions in fixed income and asset pricing. First, decomposing Treasury returns into pure duration risk and safety premium allows us to distinguish dash-for-cash, flight-to-safety, and flight-away-from-safety episodes, revealing the unique nature of Treasury dislocations during March 2020. Second, the same decomposition clarifies that the well-known negative correlation between credit spreads and Treasury yields operates primarily through the safety premium channel. Third, we document a strong and persistent credit-driven liquidity effect, while liquidity-driven credit effects appear episodic and weaker. Fourth, our liquidity factor captures the post-2017 tightening link between dealer balance-sheet capacity and market liquidity, highlighting how post-crisis regulation has reshaped liquidity provision.

Collectively, these results demonstrate that a model-based three-factor framework not only explains risk premia across corporate bond, equity, and sovereign markets, but also provides a unified lens to analyze dynamic interactions between credit, duration, liquidity, and safety. Our work thus offers researchers and practitioners a theoretically grounded toolset for dissecting key risks in fixed income markets and beyond.

References

Almeida, H. and T. Philippon (2007). The risk-adjusted cost of financial distress. *The Journal of Finance 62*(6), 2557–2586.

Bai, J., T. G. Bali, and Q. Wen (2019). Retracted: Common risk factors in the cross-section of corporate bond returns. *Journal of Financial Economics 131*(3), 619–642.

Bao, J., J. Pan, and J. Wang (2011). The illiquidity of corporate bonds. *Journal of Finance 66*(3), 911–946.

Bekaert, G., R. A. D. Santis, and T. Mondino (2024). The global cross-section of corporate bonds: Market, maturity and liquidity. *Working paper*.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2009, 10). The levered equity risk premium and credit spreads: A unified framework. *The Review of Financial Studies 23*(2), 645–703.

Binsbergen, J. H. v., Y. Nozawa, and M. Schwert (2025, 01). Duration-based valuation of corporate bonds. *The Review of Financial Studies 38*(1), 158–191.

Black, F. and J. C. Cox (1976). Valuing corporate securities: Some effects of bond indenture provisions. *Journal of Finance 31*(2), 351–367.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *Journal of Finance 65*(6), 2171–2212.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein (2008, 08). On the relation between the credit spread puzzle and the equity premium puzzle. *The Review of Financial Studies 22*(9), 3367–3409.

Chen, Q., N. Roussanov, and X. Wang (2025). Semiparametric conditional factor models in asset pricing. *Working paper*.

Chen, Z., N. Roussanov, X. Wang, and D. Zou (2024). Common risk factors in the returns on stocks, bonds (and options), redux. *Working paper*.

Dang, T. D., F. Hollstein, and M. Prokopczuk (2023, 02). Which factors for corporate bond returns? *The Review of Asset Pricing Studies 13*(4), 615–652.

Deng, J., K. Hou, and Z. Shi (2024). What drives global corporate bond returns? *Working paper*.

Dick-Nielsen, J., P. Feldhütter, L. H. Pedersen, and C. Stolborg (2024). Corporate bond factors: Replication failures and a new framework. *Working paper*.

Dickerson, A., C. Julliard, and P. Mueller (2023). The corporate bond factor zoo. *Working paper*.

Dickerson, A., P. Mueller, and C. Robotti (2023). Priced risk in corporate bonds. *Journal of Financial Economics 150*(2), 103707.

Du, W., J. Im, and J. Schreger (2018). The u.s. treasury premium. *Journal of International Economics 112*, 167–181.

Elkamhi, R., C. Jo, and Y. Nozawa (2024). A one-factor model of corporate bond premia. *Management Science 70*(3), 1875–1900.

Fama, E. and K. French (2020). Comparing cross-section and time-series factor models. *The Review of Financial Studies 33*, 1891–1926.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Goldberg, J. and Y. Nozawa (2021). Liquidity supply in the corporate bond market. *The Journal of Finance 76*(2), 755–796.

He, Z. and K. Milbradt (2014). Endogenous liquidity and defaultable bonds. *Econometrica 82*(4), 1443–1508.

He, Z. and W. Xiong (2012). Rollover risk and credit risk. *Journal of Finance 67*(2), 391–429.

Hu, G. X., Z. Jin, and J. Pan (2025). The stock-bond correlation: A tale of two days in the u.s. treasury market. *Working paper*.

Hu, G. X., J. Pan, and J. Wang (2013). Noise as information for illiquidity. *The Journal of Finance 68*(6), 2341–2382.

Huang, J.-Z. and M. Huang (2012, 11). How Much of the Corporate-Treasury Yield Spread Is Due to Credit Risk? *The Review of Asset Pricing Studies 2*(2), 153–202.

Huang, J.-Z., Y. Nozawa, and Z. Shi (2025). The global credit spread puzzle. *The Journal of Finance 80*(1), 101–162.

Jiang, Z., A. Krishnamurthy, and H. Lustig (2021). Foreign safe asset demand and the dollar exchange rate. *The Journal of Finance 76*(3), 1049–1089.

Jiang, Z., A. Krishnamurthy, and H. Lustig (2023, 11). Dollar safety and the global financial cycle. *The Review of Economic Studies 91*(5), 2878–2915.

Jiang, Z., H. Lustig, S. V. Nieuwerburgh, and M. Z. Xiaolan (2024). Bond convenience yields in the eurozone currency union. *Working paper*.

Jurado, K., S. C. Ludvigson, and S. Ng (2015, March). Measuring uncertainty. *American Economic Review 105*(3), 1177–1216.

Kelly, B., D. Palhares, and S. Pruitt (2023). Modeling corporate bond returns. *The Journal of Finance 78*(4), 1967–2008.

Kelly, B., S. Pruitt, and Y. Su (2019). Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics 134*, 501–524.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy 120*(2), 233–267.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance 49*(4), 1213–1252.

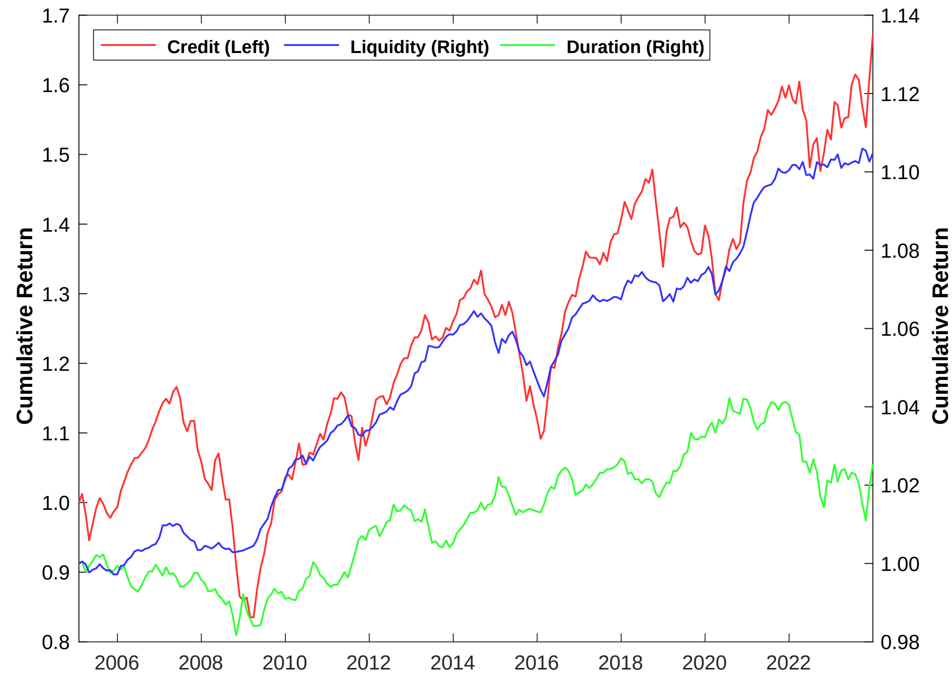
Leland, H. E. and K. B. Toft (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance 51*(3), 987–1019.

Liao, G. Y. (2020). Credit migration and covered interest rate parity. *Journal of Financial Economics 138*(2), 504–525.

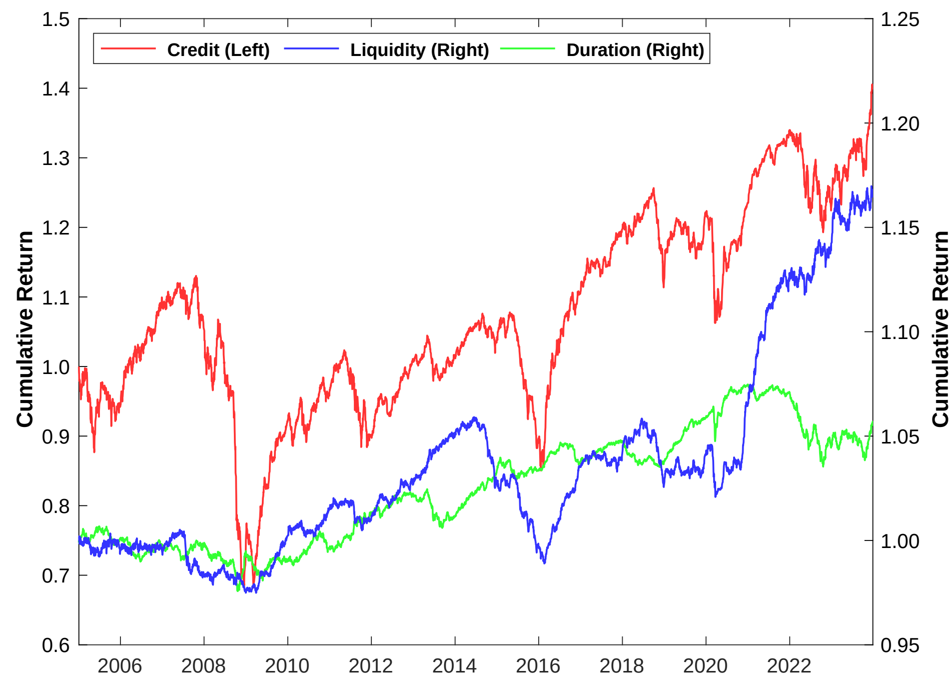
Longstaff, F. A. and E. S. Schwartz (1995). A simple approach to valuing risky fixed and floating rate debt. *Journal of Finance 50*(3), 789–819.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance 29*(2), 449–470.

Pan, J. (2002). The jump-risk premia implicit in options: evidence from an integrated time-series study. *Journal of Financial Economics 63*(1), 3–50.



(a) Monthly Cumulative Return



(b) Daily Cumulative Return

Figure 1. This figure plots the cumulative return of our three factors – credit, liquidity, and duration – at a monthly frequency in Panel (a) and a daily frequency in Panel (b).

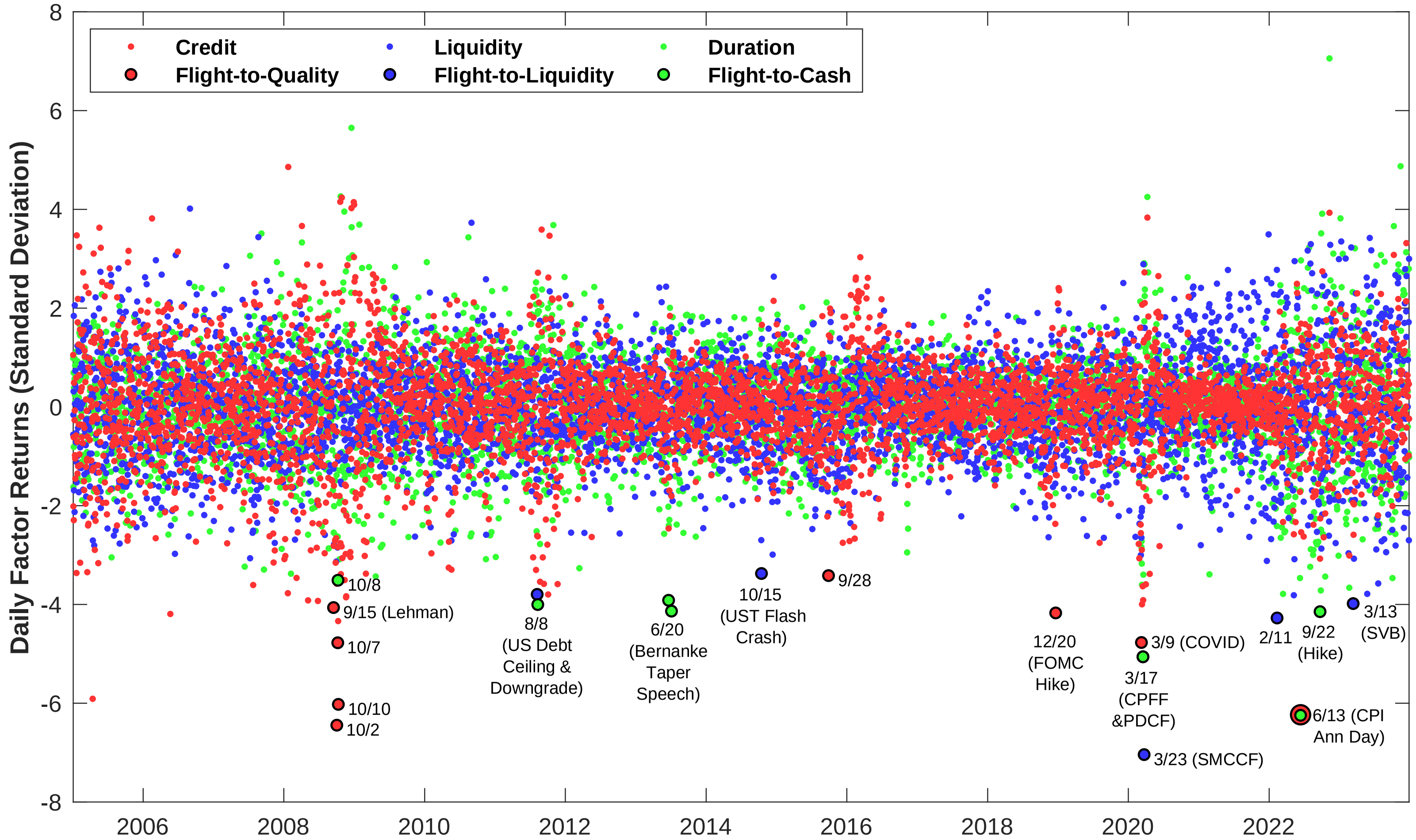
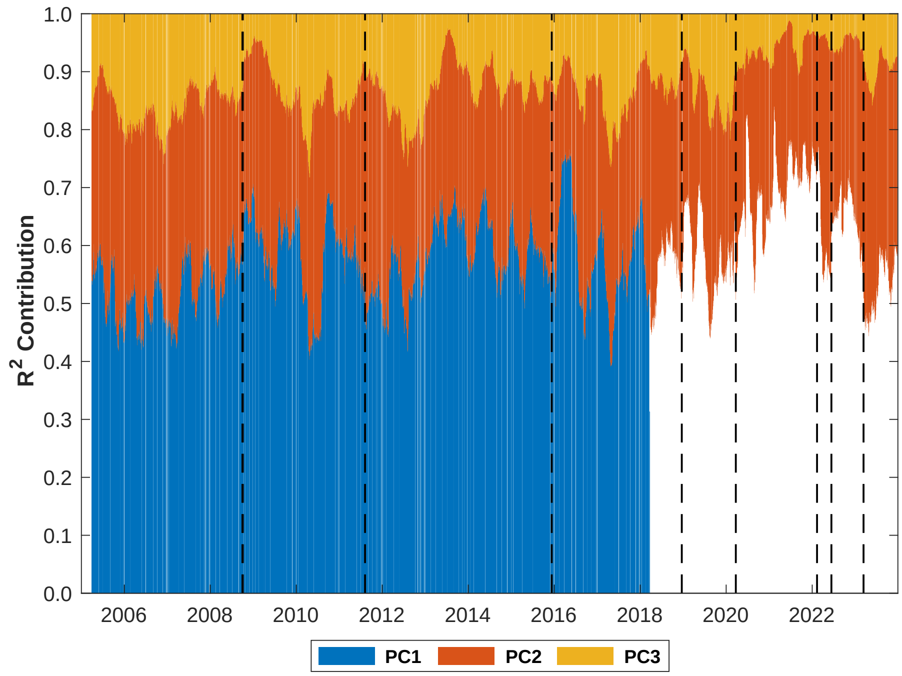
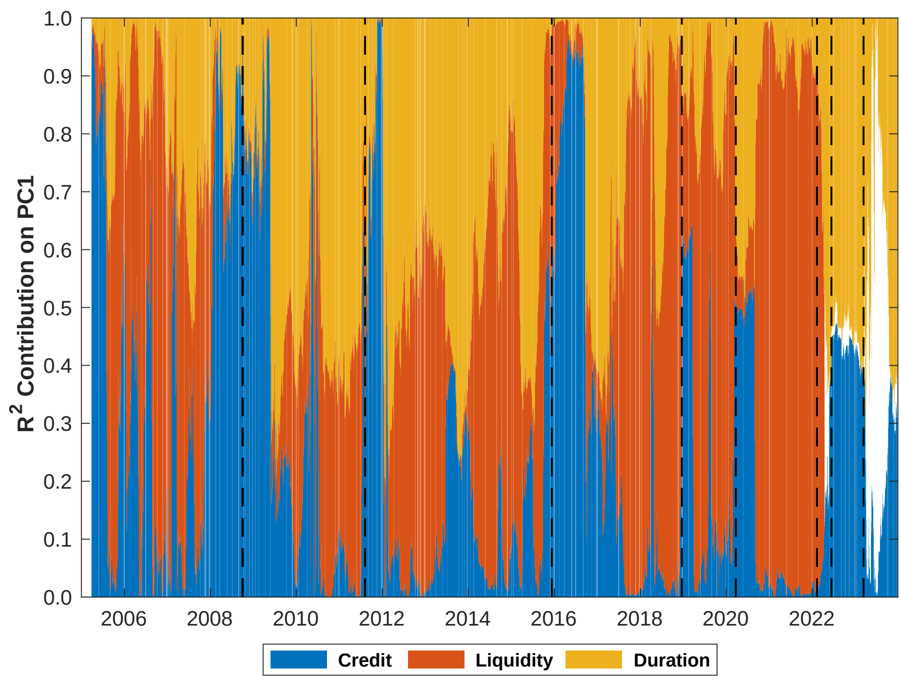


Figure 2. Factor Normalized Returns. This figure plots the normalized daily returns of our three factors: credit, liquidity and duration.

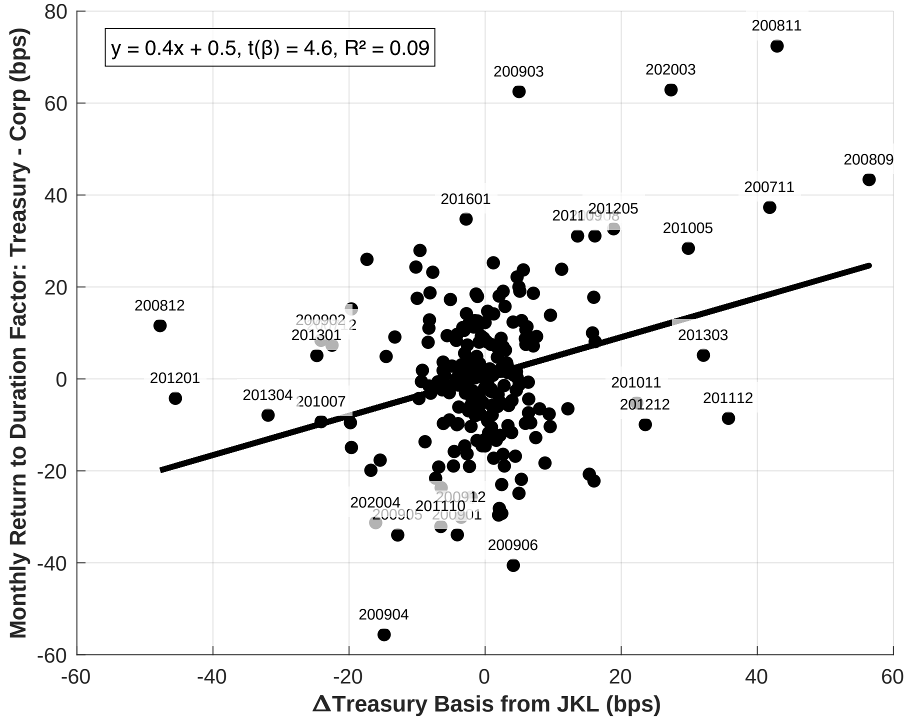


(a) R^2 **Contribution for PCA**

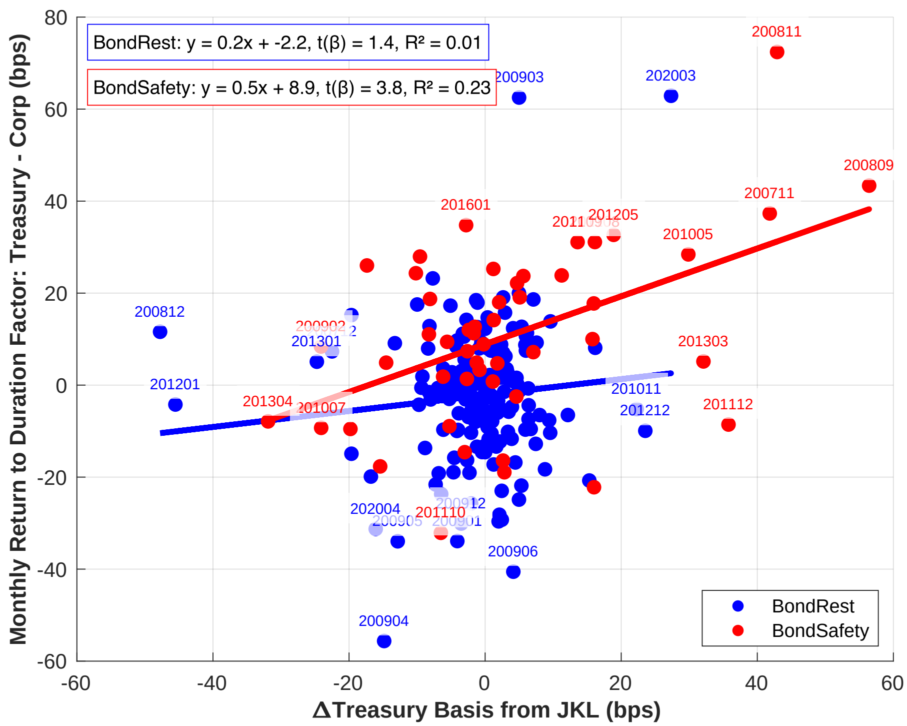


(b) R^2 **Contribution for PC1**

Figure 3. This figure plots the R^2 Contribution from PCA analysis based on our three factors daily returns, credit, liquidity and duration, over a rolling window of past three months. Panel (a) shows the R^2 contribution of each three PC in explaining the three factor returns. Panel (b) shows the Shapely R^2 decomposition of PC1 with respect to three factors.



(a) Treasury Basis Comparison



(b) Treasury Basis Comparison by Group

Figure 4. This figure plots the change in the Treasury basis from Jiang, Krishnamurthy, and Lustig (2021) against our safety premium measure, defined as the monthly return difference in duration factors estimated from the U.S. Treasury and corporate-bond markets, respectively. Panel A uses the full sample; Panel B uses the subsample. BondSafety denotes months with pronounced flight-to-safety episodes – sharp equity declines coupled with 10-year Treasury rallies (See Hu, Jin, and Pan (2025)).

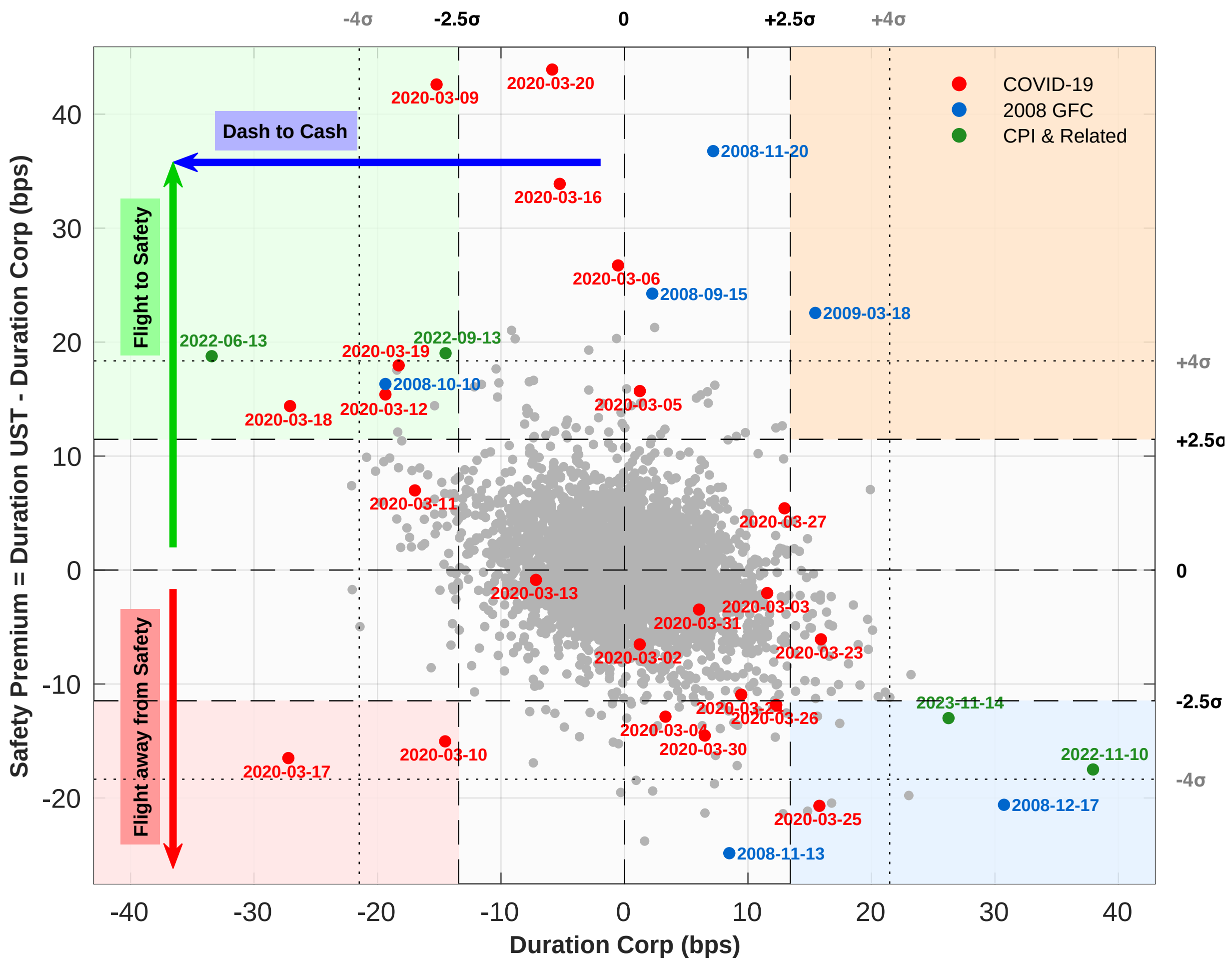
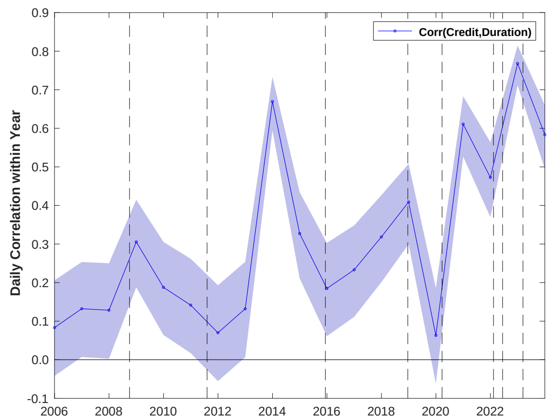
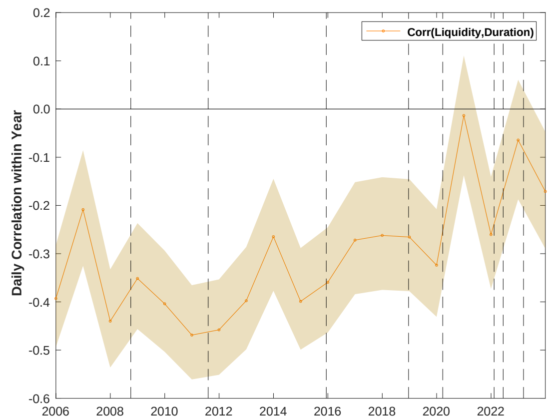


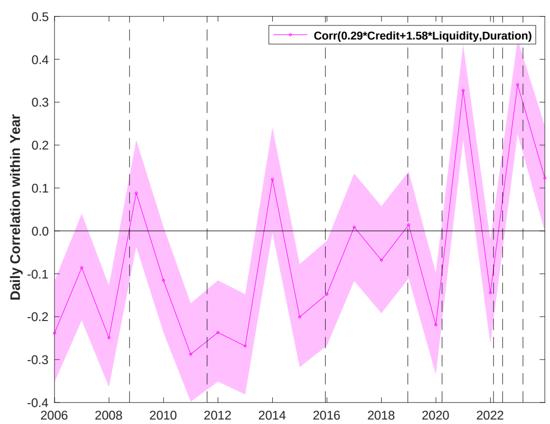
Figure 5. Flight-to-Safety vs Dash-to-Cash. This figure plots the daily duration risk factor ($DurationCorp$) against the safety premium component ($SafetyPremium$) based on our decomposition of U.S. Treasury returns. The red points highlight March 2020 during the COVID-19 crisis, blue points mark key dates from the 2008 financial crisis, green points indicate CPI announcement days and related events, and gray points represent all other trading days. The colored regions delineate different market regimes: dash-for-cash (blue arrow), flight-to-safety (green arrow), and flight-away-from-safety (red arrow). Reference lines at $\pm 2.5\sigma$ and $\pm 4\sigma$ demarcate extreme observations in both dimensions.



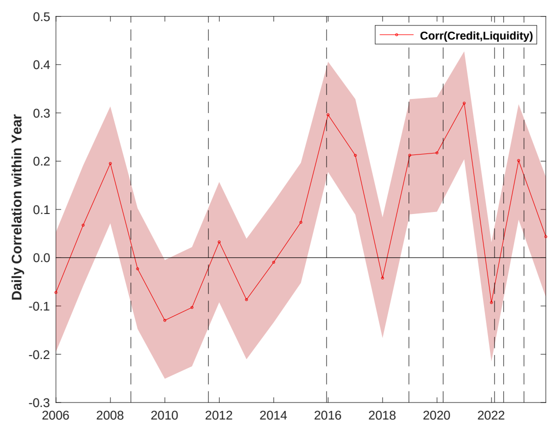
(a) Corr(Credit, Duration)



(b) Corr(Liquidity, Duration)

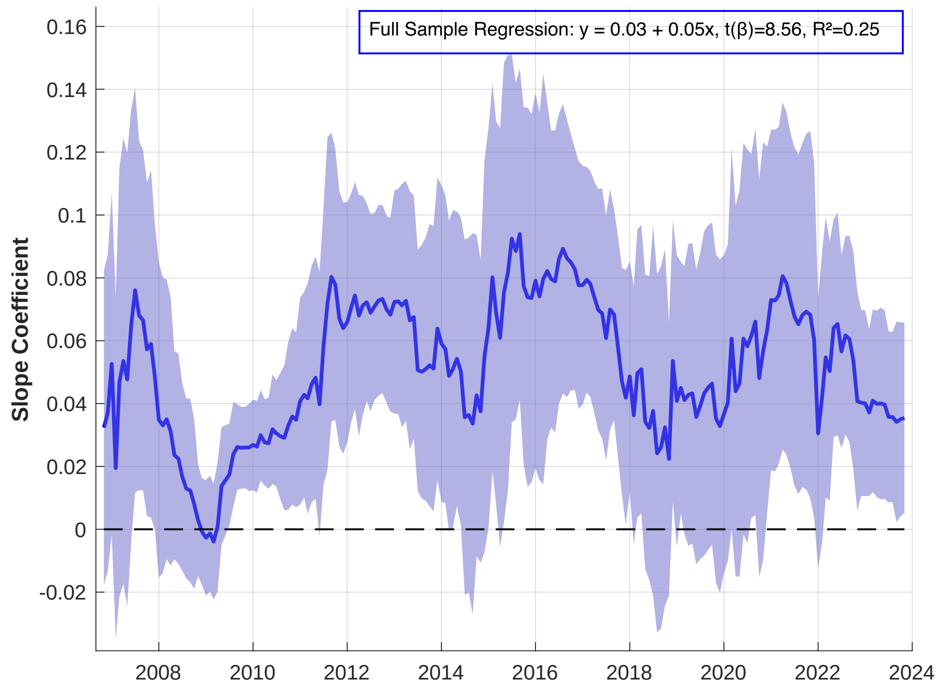


(c) Corr(Credit+Liquidity, Duration)

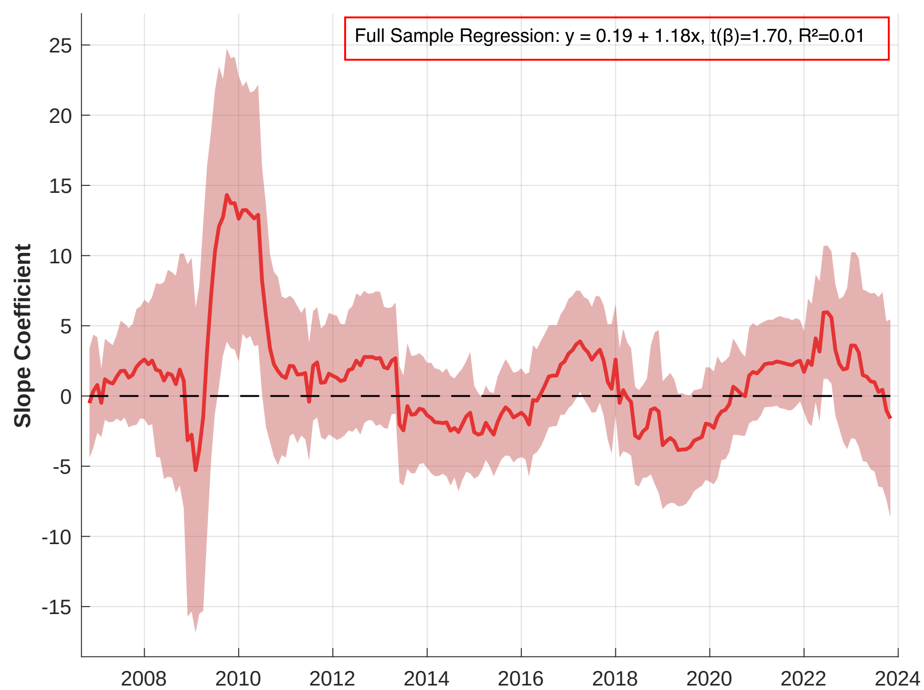


(d) Corr(Credit, Liquidity)

Figure 6. This figure plots the time-varying annual correlations among credit risk, liquidity risk, and duration risk. Panel (a) shows the correlation between credit and duration factors; Panel (b) displays the correlation between liquidity and duration factors; Panel (c) illustrates the correlation between duration and a combined credit-liquidity measure, where the weights are estimated from regressing corporate bond returns on the three factors; Panel (d) exhibits the correlation between credit and liquidity factors directly. All correlations are computed using daily factor returns within each calendar year. The shaded bands represent 95% confidence intervals.

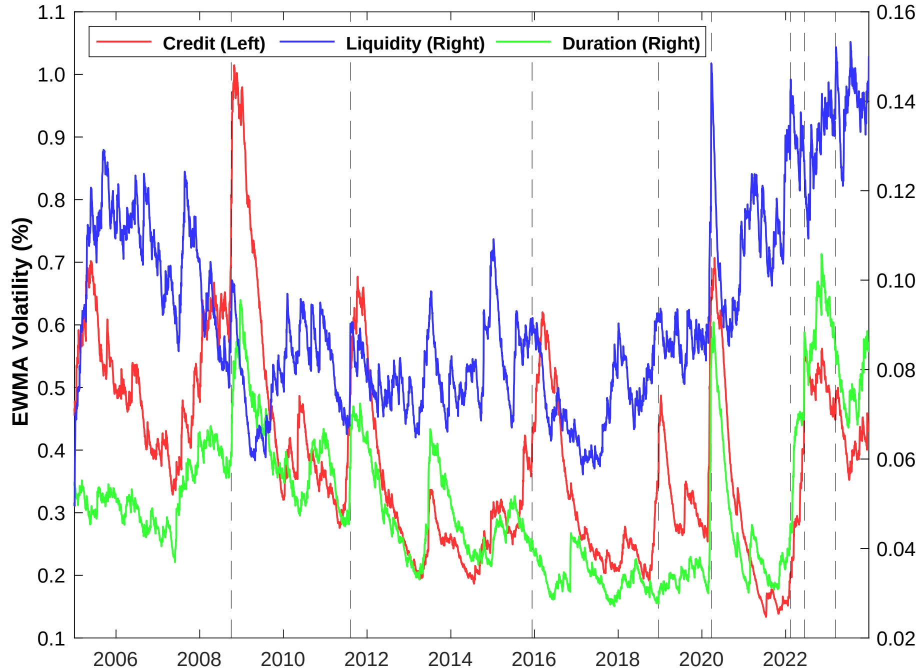


(a) Regress $Liquidity_{t+1}$ **on** $Credit_t$

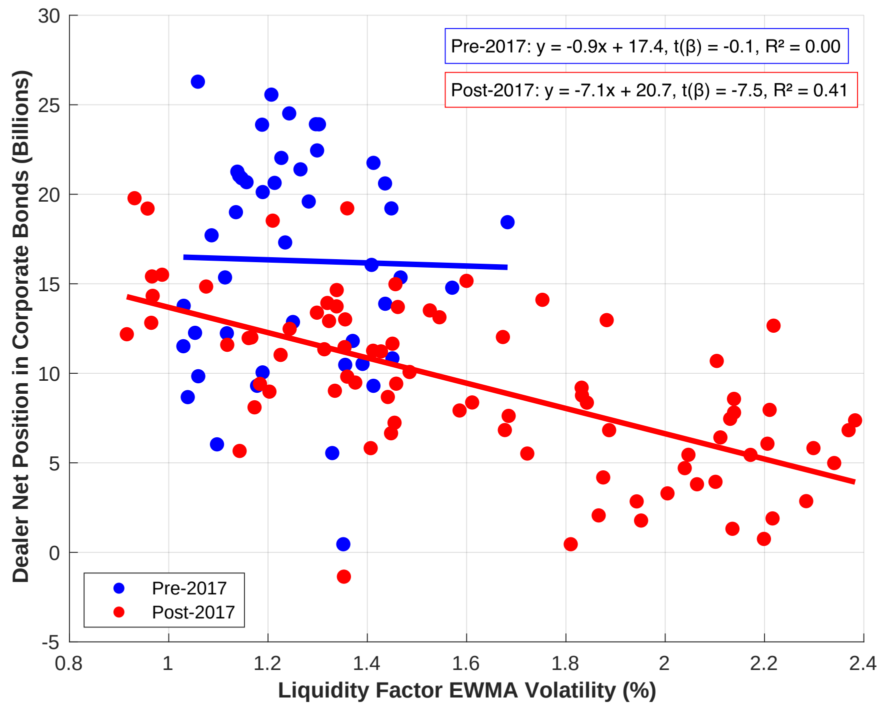


(b) Regress $Credit_{t+1}$ **on** $Liquidity_t$

Figure 7. This figure plots time-varying lead-lag relationships between credit and liquidity risk factors. Panel (a) displays rolling window estimates from regressing the liquidity factor at month $t+1$ on the credit factor at month t. Panel (b) shows the reverse predictive relationship. Each regression is estimated using a 24-month rolling window ending at time t, with the resulting slope coefficient plotted at the end date of each estimation window. The shaded areas represent 95% confidence intervals. Full-sample regression results are reported in the text boxes.



(a) **Time-Series of Factor Return Volatility**



(b) **Liquidity Factor Volatility and Dealer Net Position**

Figure 8. Panel (a) plots the EWMA volatility of our three factors, credit, liquidity and duration, with decay parameter 0.98. Panel (b) shows a scatter plot of the EWMA volatility of the liquidity factor against primary dealers' net positions in investment-grade and high-yield corporate bonds, estimated using data from the Federal Reserve Bank of New York's website.

Table I

Summary Statistics

This table reports summary statistics for our sample of bonds. The sample period is from January 2005 to December 2023. NumBonds is the number of bonds. BondRet is the month-end bond return from WRDS Bond Returns database. CreditSpread is the difference in yield between corporate bonds and Treasury bonds of the same maturity. BondBeta is the bond beta estimated from a CAPM model in the bond market over a rolling window of past 24 months. Maturity is the bond's time to maturity in years. Rating is a numerical translation of Moody's rating: 1=Aaa and 21=C. Age is the time since issuance in years. Coupon is the bond's coupon payment in percent. Amount is the bond's amount outstanding in millions of dollars. Gamma is the illiquidity measure from Bao, Pan, and Wang (2011). Callable is 1 for bonds issued with callable options. NumIssuers is the number of issuers. EquitySize is the log of the firm's equity size. EquityBeta is the equity beta estimated from a CAPM model in the stock market over a rolling window of past 60 months. BM is the book value of the equity to market capitalization. EquityVolatility is the annualized equity volatility. Leverage is the ratio of total short- and long-term debt to the total asset value. AssetGrowth is the average growth rate of the asset value in the past three years. b_i^V, b_i^r, and b_i^N are the bond's factor loading associated with credit, duration, and liquidity factors, respectively.

Variable	Mean	StdDev	Min	P25	Median	P75	Max
	Bond-Level Variables						
NumBonds	14,761						
BondRet (%)	0.37	2.68	-9.25	-0.53	0.27	1.31	10.23
CreditSpread (%)	1.92	2.13	-0.09	0.75	1.28	2.22	14.11
BondBeta	1.12	0.87	-0.44	0.50	0.95	1.57	5.53
Maturity (yr)	9.47	8.53	0.50	3.26	6.13	14.05	29.99
Rating	8.59	3.33	1.00	6.00	8.00	10.00	17.00
Age (yr)	5.67	4.73	1.00	2.00	4.00	7.00	24.00
Coupon (%)	5.13	1.90	1.20	3.65	5.10	6.50	9.75
Amount	659	520	1.80	300	500	800	2850
Gamma (bps)	0.83	2.20	-1.09	0.03	0.17	0.65	16.76
Call	0.88	0.32	0.00	1.00	1.00	1.00	1.00
	Equity-Level Variables						
NumIssuers	1,480						
EquitySize (log)	23.79	1.62	18.46	22.87	23.92	24.85	27.84
EquityBeta	1.02	0.62	-0.08	0.56	0.95	1.34	3.50
BM	0.46	0.40	-0.78	0.22	0.41	0.64	2.23
EquityVolatility	0.30	0.19	0.09	0.18	0.25	0.35	1.29
Leverage	0.39	0.17	0.07	0.27	0.37	0.48	0.97
AssetGrowth	0.07	0.11	-0.17	0.01	0.05	0.10	0.53
	Estimated Factor Loadings						
b_i^V	0.10	0.29	0.00	0.00	0.00	0.02	2.09
b_i^r	5.87	4.30	-1.94	2.64	4.80	7.79	16.70
b_i^N	1.19	1.35	0.00	0.31	0.77	1.57	8.11

Table II

Factor Returns

This table reports the average returns and CAPM alphas for monthly factors and daily factors. Reported in square brackets are *t*-statistics. *, **, and *** indicate significant at the 10%, 5%, and 1% level, respectively.

	Average Factor Returns and Alphas							
	Monthly (%)				Daily (bps)			
	2005-2023		2009-2023		2005-2023		2009-2023	
	Mean	Alpha	Mean	Alpha	Mean	Alpha	Mean	Alpha
Credit	0.24*	0.06	0.39***	0.13	0.82	-0.61	1.76***	-0.29
	[1.90]	[0.61]	[2.86]	[1.21]	[1.32]	[-1.07]	[2.89]	[-0.55]
Liquidity	0.04***	0.04***	0.05***	0.05***	0.34**	0.32**	0.49***	0.45***
	[4.29]	[3.98]	[4.45]	[3.98]	[2.39]	[2.23]	[3.20]	[2.88]
Duration	0.01	-0.02*	0.02	-0.02**	0.12	-0.18***	0.16*	-0.26***
	[0.83]	[-1.78]	[1.08]	[-2.20]	[1.53]	[-3.17]	[1.90]	[-4.36]

Table III

Macro Drivers of Three Factors (Num of STDs)

This table reports the result of time-series regressions of factor returns on a set of macro and market uncertainty variables, including the macro uncertainty index, financial uncertainty index, and real uncertainty index developed by Jurado, Ludvigson, and Ng (2015), the VIX index from the CBOE, the MOVE index, and the noise measure from Hu, Pan, and Wang (2013). Reported in square brackets are *t*-statistics. *, **, and *** indicate significant at the 10%, 5%, and 1% level, respectively.

	Credit			Liquidity			Duration		
Fin Uncertainty	-0.35***		-0.18**	-0.25***		-0.25***	-0.17**		-0.05
	[-4.14]		[-2.19]	[-3.31]		[-3.05]	[-2.02]		[-0.63]
Macro Uncertainty	-0.17		-0.10	-0.34**		-0.35**	-0.01		0.05
	[-1.06]		[-0.71]	[-2.10]		[-2.11]	[-0.08]		[0.30]
Real Uncertainty	0.02		-0.03	0.19		0.20	-0.00		-0.05
	[0.16]		[-0.19]	[1.26]		[1.37]	[-0.03]		[-0.35]
VIX		-0.42***	-0.32***		-0.15	-0.02		-0.18**	-0.16*
		[-5.60]	[-4.19]		[-1.62]	[-0.25]		[-2.13]	[-1.85]
MOVE		-0.11*	-0.10*		0.05	0.07		-0.19**	-0.19**
		[-1.74]	[-1.73]		[0.56]	[0.97]		[-2.35]	[-2.32]
Noise		-0.15**	-0.08		-0.11	0.00		-0.04	-0.04
		[-2.31]	[-1.24]		[-1.49]	[0.03]		[-0.53]	[-0.44]
Constant	-0.00	0.00	0.00	0.00	0.01	0.00	-0.00	-0.00	-0.00
	[-0.00]	[0.08]	[0.07]	[0.00]	[0.08]	[0.08]	[-0.00]	[-0.07]	[-0.07]
Obs	228	227	227	228	227	227	228	227	227
R^2	0.20	0.28	0.33	0.15	0.03	0.14	0.02	0.10	0.09

Table IV

Explaining the Market Excess Returns

This table reports the result of monthly panel regressions of market-level excess returns on our three model-based factors: credit, liquidity and duration. We consider five market indices in US, including CorpBond (Aggregate Corporate Bond Market Index), HY (Bloomberg Barclays High-Yield Index), IG (Bloomberg Barclays Investment-Grade Index), Stock (Fama-French MktRf Portfolio), and Treasury (Bloomberg Barclays Aggregate Treasury Index), as well as three market indices for Foreign countries, including CorpBond (aggregate corporate dollar bond index by foreign issuers), SovereignEM (Bloomberg Emerging Market Sovereign Index), and SovereignEU (Bloomberg EU Sovereign Index). Panel A reports the mean excess returns. Panel B reports the regression results after controlling for credit factor. Panel C reports the regression results after controlling for our three factors. Reported in square brackets are *t*-statistics. *, **, and *** indicate significant at the 10%, 5%, and 1% level, respectively.

	Panel A: Mean Excess Return							
	US					Foreign		
	CorpBond	HY	IG	Stock	Treasury	CorpBond	SovereignEM	SovereignEU
Mean	0.30**	0.49**	0.26*	0.76**	0.13	0.34***	0.36**	0.13
	[2.53]	[2.47]	[1.95]	[2.55]	[1.45]	[2.69]	[2.16]	[1.22]
	Panel B: Credit Factor Model							
α	0.16	0.19	0.14	0.34	0.16*	0.19*	0.18	0.04
	[1.49]	[1.43]	[1.13]	[1.63]	[1.72]	[1.74]	[1.11]	[0.39]
Credit	0.59***	1.25***	0.47***	1.77***	-0.13**	0.64***	0.78***	0.36***
	[8.20]	[10.60]	[5.25]	[14.85]	[-2.05]	[7.72]	[5.58]	[4.13]
Obs	228	228	228	228	228	228	228	228
R^2	0.34	0.63	0.20	0.55	0.03	0.41	0.34	0.18
	Panel C: Three Factors Model							
α	0.09	0.16	0.06	0.35	0.13**	0.07	0.08	-0.02
	[1.20]	[1.17]	[0.77]	[1.62]	[2.12]	[0.81]	[0.55]	[-0.25]
Duration	6.20***	1.86	7.44***	0.88	5.49***	5.67***	5.76***	4.39***
	[10.37]	[1.52]	[14.23]	[0.64]	[16.36]	[10.68]	[6.52]	[7.33]
Credit	0.29***	1.16***	0.12*	1.75***	-0.36***	0.32***	0.47***	0.13*
	[4.48]	[7.06]	[1.95]	[11.39]	[-9.36]	[4.94]	[3.43]	[1.66]
Liquidity	1.58**	0.62	1.69**	-0.37	0.53	2.78***	2.32*	1.48*
	[2.07]	[0.39]	[2.12]	[-0.19]	[0.90]	[3.06]	[1.82]	[1.69]
Obs	228	228	228	228	228	228	228	228
R^2	0.76	0.64	0.77	0.55	0.71	0.73	0.53	0.48

Table V

Explaining the Bond Market Testing Portfolios

This table reports the result of monthly panel regressions of bond market testing portfolios on our three model-based factors: credit, liquidity and duration. We construct four categories of quintile portfolios based on bond CAPM beta (Panel A), bond duration (Panel B), bond rating (Panel C), and illiquidity (Panel D). Reported in square brackets are t-statistics. *, **, and *** indicate significant at the 10%, 5%, and 1% level, respectively.

	Panel A: BondBeta Sorted Portfolio					Panel B: Duration Sorted Portfolio				
	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
	Raw Excess Monthly Return					Raw Excess Monthly Return				
Mean	0.13**	0.20**	0.28**	0.33**	0.46**	0.17***	0.25**	0.30**	0.37**	0.34
	[2.01]	[2.35]	[2.26]	[2.03]	[2.11]	[2.66]	[2.37]	[2.19]	[2.25]	[1.59]
	Three Factors Model					Three Factors Model				
α	0.01	0.06	0.09	0.06	0.11	0.05	0.05	0.08	0.09	0.07
	[0.17]	[0.85]	[1.02]	[0.63]	[1.09]	[0.90]	[0.62]	[0.75]	[0.88]	[0.97]
Duration	1.95***	3.75***	5.98***	8.40***	9.77***	1.09***	2.62***	4.43***	8.45***	13.41***
	[4.43]	[7.10]	[7.65]	[10.56]	[8.47]	[2.80]	[4.65]	[5.92]	[11.30]	[17.17]
Credit	0.18***	0.13**	0.20**	0.32***	0.73***	0.24***	0.44***	0.50***	0.30***	0.19***
	[3.22]	[2.05]	[2.44]	[3.94]	[4.57]	[4.49]	[5.45]	[4.90]	[3.95]	[2.74]
Liquidity	1.20**	1.42**	1.58*	1.98*	1.47	1.04*	1.42*	1.05	2.29**	1.50*
	[1.98]	[2.05]	[1.78]	[1.86]	[1.03]	[1.88]	[1.79]	[1.07]	[2.14]	[1.86]
R^2	0.48	0.57	0.65	0.77	0.80	0.46	0.60	0.62	0.77	0.92
	Panel C: Rating Sorted Portfolio					Panel D: Illiquidity Sorted Portfolio				
	Raw Excess Monthly Return					Raw Excess Monthly Return				
Mean	0.22*	0.25*	0.27**	0.34**	0.39*	0.25*	0.23**	0.27**	0.36**	0.39**
	[1.88]	[1.95]	[1.98]	[2.19]	[1.92]	[1.81]	[2.47]	[2.24]	[2.31]	[2.06]
	Three Factors Model					Three Factors Model				
α	0.11**	0.08	0.05	0.05	0.02	0.07	0.10	0.09	0.09	-0.05
	[2.39]	[1.26]	[0.66]	[0.44]	[0.12]	[0.73]	[1.64]	[1.19]	[1.11]	[-0.47]
Duration	7.62***	7.67***	7.48***	5.26***	1.85*	5.31***	4.57***	6.07***	7.91***	8.43***
	[15.45]	[14.43]	[12.41]	[5.46]	[1.92]	[6.11]	[9.97]	[10.88]	[11.57]	[10.88]
Credit	-0.06	0.06	0.22***	0.55***	1.22***	0.42***	0.16***	0.23***	0.35***	0.58***
	[-1.32]	[1.18]	[3.13]	[4.48]	[8.40]	[3.98]	[2.62]	[3.75]	[4.85]	[6.91]
Liquidity	0.70	1.22*	1.77**	2.10	1.32	0.16	0.76	1.13	1.86**	4.63***
	[1.44]	[1.86]	[2.16]	[1.62]	[0.99]	[0.15]	[1.32]	[1.55]	[2.11]	[3.84]
R^2	0.85	0.82	0.79	0.65	0.71	0.64	0.68	0.76	0.80	0.81

Table VI

Decompose UST Returns

This table reports time-series regressions of monthly excess returns for four asset classes on the two components of Treasury duration factor ($DurationUST$): the pure duration risk factor ($DurationCorp$) and the safety premium ($SafetyPremium$). Panels A through D present results for (1) aggregate equity market, (2) U.S. Treasuries, (3) investment-grade (IG) corporate bonds, and (4) high-yield (HY) corporate bonds. Panels E and F examine the relationships with credit and liquidity factors, respectively. Reported in square brackets are t-statistics. *, **, and *** indicate significant at the 10%, 5%, and 1% level, respectively.

	Panel A: Stock Excess Return				Panel B: Treasury Excess Return			
DurationUST	-0.02				0.05***			
	[-1.28]				[32.28]			
DurationCorp		0.06***		0.03**		0.04***		0.05***
		[3.84]		[2.57]		[13.67]		[28.61]
SafetyPremium			-0.17***	-0.16***			0.04***	0.05***
			[-6.59]	[-6.26]			[6.45]	[21.15]
Constant	0.80***	0.68**	0.85***	0.79***	0.04	0.07	0.11	0.04
	[2.65]	[2.36]	[3.57]	[3.33]	[1.13]	[1.14]	[1.39]	[1.14]
Obs	228	228	228	228	228	228	228	228
R^2	0.01	0.09	0.36	0.39	0.89	0.50	0.19	0.89
	Panel C: IG Excess Return				Panel D: HY Excess Return			
DurationUST	0.05***				-0.01			
	[7.59]				[-0.67]			
DurationCorp		0.07***		0.07***		0.05***		0.03***
		[18.47]		[18.77]		[4.38]		[3.53]
SafetyPremium			-0.04***	-0.02**			-0.12***	-0.11***
			[-3.05]	[-2.00]			[-7.30]	[-6.53]
Constant	0.17	0.15**	0.28**	0.17**	0.51**	0.42**	0.55***	0.50***
	[1.58]	[2.24]	[2.27]	[2.60]	[2.53]	[2.24]	[3.88]	[3.63]
Obs	228	228	228	228	228	228	228	228
R^2	0.33	0.74	0.11	0.76	0.01	0.16	0.46	0.52
	Panel E: Credit Factor				Panel F: Liquidity Factor			
DurationUST	-0.01*				-0.25***			
	[-1.69]				[-5.86]			
DurationCorp		0.03***		0.02***		-0.16***		-0.22***
		[5.01]		[3.95]		[-3.62]		[-4.75]
SafetyPremium			-0.08***	-0.08***			-0.27***	-0.34***
			[-11.51]	[-11.20]			[-4.17]	[-4.49]
Constant	0.26**	0.20*	0.28***	0.25***	4.80***	4.55***	4.46***	4.79***
	[2.08]	[1.69]	[3.15]	[2.97]	[5.24]	[4.62]	[4.62]	[5.27]
Obs	228	228	228	228	228	228	228	228
R^2	0.02	0.12	0.50	0.53	0.17	0.06	0.08	0.18

Internet Appendix for
"Model-Based Credit, Liquidity, and Duration Factors in Cross-Sectional Corporate Bond Returns"

ZHE GENG and JUN PAN*

This Internet Appendix provides additional proofs, tables and figures supporting the main text.

I Technical Proofs

Proof of Proposition 1: From equation (9), we have,

$$\begin{aligned} P_t^i &= \mathbb{E}_t^Q\left[e^{-\int_t^T r_s ds}\left(K-(K-V_T^i)\mathbb{I}_{V_T^i\le K}\right)\right] \\ &= KR(r_t,t,T) - K\mathbb{E}_t^Q\left[e^{-\int_t^T r_s ds}\mathbb{I}_{V_T^i\le K}\right] + \mathbb{E}_t^Q\left[e^{-\int_t^T r_s ds}V_T^i\mathbb{I}_{V_T^i\le K}\right]. \end{aligned}$$

We first solve for the second term on the right-hand side $K\mathbb{E}_t^Q\left[e^{-\int_t^T r_s ds}\mathbb{I}_{V_T^i\le K}\right]$. Following Longstaff and Schwartz (1995), let $H(V_t^i,r,t,T)$ denote the price of a risky discount bond with maturity date T and payoff function $\mathbb{I}_{V_T^i\le K}$. Let $H(V_t^i,r,t,T)=R(r_t,t,T)Q(V_t^i,r,t,T)$. Substituting into equation (7) and denoting X as the ratio V_t^i/K, we have

$$\frac{\sigma^2}{2}X^2QXX + \frac{\eta^2}{2}Q_{rr} + \rho\sigma\eta XQ_{Xr} + \left[r+\rho\sigma\eta B(t,T)\right]XQ_X + \left[\alpha-\zeta r+\eta^2B(t,T)\right]Q_r = -Q_t,$$

subject to the condition $Q(X,r,T,T)=\mathbb{I}_{V_T^i\le K}$. $Q(V_t^i,r,t,T)$ is the probability that $\ln X_T$ is less than zero. Following the proof in the appendix of Longstaff and Schwartz (1995), $\ln X_T$ is normally distributed with mean $\ln X_t + M(r_t,t,T)$ and standard deviation $S(t,T)$, where $M(r_t,t,T)$ and $S(t,T)$ are given in Proposition 1. Therefore, $K\mathbb{E}_t^Q\left[e^{-\int_t^T r_s ds}\mathbb{I}_{V_T^i\le K}\right] = KR(r_t,t,T)N(-d_2)$, where $d_2=\frac{\ln\frac{V_t^i}{K}+M(r_t,t,T)}{S(t,T)}$ and $d_1=\frac{\ln\frac{V_t^i}{K}+M(r_t,t,T)+S^2(t,T)}{S(t,T)}$.

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Similarly, we can solve for the third term $\mathbb{E}_t^Q\left[e^{-\int_t^T r_s ds} V_T^i \mathbb{I}_{V_T^i \leq K}\right]$. Hence,

$$P_t^i = KR(r_t, t, T)N(d_2) + V_t^i R(r_t, t, T)e^{M(r_t,t,T)+\frac{1}{2}S^2(t,T)}N(-d_1).$$

Proof of Proposition 2: Denote the time-t bond i price $P_t^i = f(X_t, t)$, where $X_t = (\ln V_t, r_t)$ and

$$\begin{aligned} d\ln V_t &= (r_t - \frac{1}{2}\sigma_V^2)dt + \sigma_V dB_t^V, \\ dr_t &= (\alpha - \zeta r_t)dt + \eta dB_t^r, \end{aligned}$$

Applying Ito's Lemma on P_t^i, we have

$$dP_t^i = \left(\frac{\partial P_t^i}{\partial t} + \sum_{j=1}^d \mu_X^j \frac{\partial P_t^i}{\partial X_j} + \frac{1}{2}\sum_{j,k=1}^d \rho_{jk}\sigma_X^j\sigma_X^k \frac{\partial^2 P_t^i}{\partial X_j \partial X_k}\right)dt + \sum_{j=1}^d \frac{\partial P_t^i}{\partial X_j}\sigma_X^j dB_t^j. \quad \text{(IA.1)}$$

where $d = 2$, $\mu_X = (r_t - \frac{1}{2}\sigma_V^2, \alpha - \zeta r_t)'$, $\sigma_X = (\sigma_V, \eta)'$, $dB_t = (dB_t^V, dB_t^r)'$ and ρ is the 2×2 correlation matrix between the two brownian shocks. From Feynman-Kac Theorem,

$$\frac{\partial P_t^i}{\partial t} + \sum_{j=1}^d \mu_X^j \frac{\partial P_t^i}{\partial X_j} + \frac{1}{2}\sum_{j,k=1}^d \rho_{jk}\sigma_X^j\sigma_X^k \frac{\partial^2 P_t^i}{\partial X_j \partial X_k} - r_t P_t^i = 0. \quad \text{(IA.2)}$$

Substituting to equation (IA.1),

$$dP_t^i = r_t P_t^i dt + \sum_{j=1}^d \frac{\partial P_t^i}{\partial X_j}\sigma_X^j dB_t^j.$$

Hence,

$$\begin{aligned}
\frac{dP_t^i}{P_t^i} - r_t dt &= \sum_{j=1}^{d} \frac{1}{P_t^i} \frac{\partial P_t^i}{\partial X_j} \sigma_X^j dB_t^j, \\
&= \sum_{j=1}^{d} \frac{\partial \ln P_t^i}{\partial X_j} \sigma_X^j dB_t^j, \\
&= \frac{\partial \ln P_t^i}{\partial \ln V_t} \sigma_V dB_t^V + \frac{\partial \ln P_t^i}{\partial r_t} \eta dB_t^r.
\end{aligned}$$