

The Value of Compliance Carbon Offsets in Cap-and-Trade Markets*

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Abstract

We study compliance carbon offsets in cap-and-trade markets. We develop a model predicting that firms sort into offset use by carbon productivity: highly productive firms rely solely on allowances, while less productive firms use offsets because they trade at a discount but carry invalidation risk. Using administrative data from California's Cap-and-Trade program, we find strong empirical support for these predictions. Structural estimation suggests that roughly one-third of compliance offsets fail to reduce emissions, generating about 2 percent uncounted emissions. Nevertheless, offsets can increase efficiency relative to an equivalent tightening of the emissions cap under certain conditions.

JEL Classification: G30; G33; G38; O44; Q50.

Keywords: carbon offsets, cap-and-trade, biodiversity, invalidation risks, compliance offsets, carbon regulation, climate finance

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An offset is a carbon credit generated by projects that reduce or remove carbon dioxide from the atmosphere. Many offsets are nature-based, such as investments in projects in both developed and developing countries that reduce emissions through protecting forests or reforestation. The market for carbon offsets has grown enormously in recent years: from negligible amounts in the early 2000s, the cumulative issuance of global offset credits surpassed 6 Gt CO₂e in 2024 ([World Bank¹](#)), equivalent to roughly 15% of global emissions in that year. While they remain predominantly a voluntary instrument, carbon offsets are increasingly being integrated into compliance-based carbon pricing schemes such as carbon taxes or cap-and-trade programs,² and they are a central topic in international climate negotiations, including at COP30.³

Practitioners, regulators, and academics are divided on the benefits of compliance offsets. Proponents praise carbon offsets as a cost-effective tool to reduce emissions, facilitate equitable cross-country transfers, and support biodiversity conservation ([Chausson et al., 2020](#)), whereas critics argue that they deliver fewer emission reductions than claimed ([Probst et al., 2024](#)). We contribute to this debate by developing and estimating a model to study the implementation of carbon offsets in California Emissions Trading Scheme (ETS), a cap-and-trade policy which caps total emissions and allows companies to trade permits. ETS schemes of this type cover about 19% of global GHG emissions ([ICAP, 2025](#)) and often incorporate offsets to reduce the number of permits that regulated firms must surrender. In particular, in California, regulated entities are allowed to use offsets to cover 8% of their total emissions over the period we study (2013-2020).

Our analysis yields four main results. First, our model predicts and the data confirms that firms sort into offset use based on their carbon productivity: highly productive firms rely exclusively on carbon allowances, while lower productivity firms use offsets up to the regulatory offset usage limit. Second, we structurally estimate offset quality and the extent to which poor-quality offsets generate uncounted emissions. At a social cost of carbon of \$190 per metric ton of CO₂ emissions equivalent (ton of CO₂e)—the value determined by the Biden administration—we find that about 33% of offsets fail to deliver genuine emission reductions leading to an “invisible” increase in uncounted emissions of 2%

¹The figure refers to cumulative issuance of carbon credits across independent carbon crediting mechanisms, historically used by voluntary buyers, as well as government or international mechanisms used for compliance with carbon-mitigation regulation.

²According to the [World Bank State and Trends of Carbon Pricing](#), in 2024 roughly 75% of retired carbon credits were for voluntary purposes (representing the majority of the market), while the remaining 25% were for compliance purposes, compared with a 90:10 voluntary-to-compliance ratio in 2023. Details are reported in Table A.1 of the Online Appendix.

³See, for example, [Thomson Reuters, 2025](#).

above the state cap in each compliance period. At a social cost of carbon of \$51 per ton—the value used by the Obama administration—we estimate that virtually all offsets are poor quality. Third, we identify a tradeoff: Even if offsets are imperfect, they can, under certain conditions, lead to more output than an equivalent relaxation of the emissions cap. For example, at a social cost of carbon of \$190 per ton each offset generates an extra \$10 dollars in output relative to raising the emissions cap. At a social cost of carbon of \$51 per ton, however, it is better to forego using offsets altogether. Finally, we characterize offset quality when regulators seek to incentivize production by small firms—often a policy priority—using carbon discounts. We show that this implies a starker tradeoff, as carbon discounts for small firms can lead to worse offset quality and more uncounted emissions.

Our simple theoretical model is designed to capture the key features of California’s compliance offset market. Firms produce output which generates emissions and they are heterogeneously carbon productive, i.e., in the amount of output generated per unit of emissions. Emissions are regulated in a cap-and-trade market, in which each firm must surrender carbon allowances and offsets against its generated emissions. Offsets are supplied by a representative producer facing convex costs of offset production; these offsets are imperfect, as some fraction of them are low-quality and do not remove carbon.

We assume an exogenous carbon cap and focus on how offsets are regulated. The two key regulatory features that we model are the main tools used in California’s compliance offset market in practice, namely, usage limits and invalidation risk. The regulator chooses the frequency with which they invalidate offsets as well as the offset usage limit to balance economic output against the carbon emissions which are generated during production. The regulator also faces a cost to invalidate low-quality offsets to capture monitoring and information-gathering costs. Once the regulator sets offset regulation, firms purchase carbon allowances and offsets. If a firm’s offsets are invalidated, they are withdrawn from the firm. After offset invalidation occurs, firms produce under the constraint that they have enough carbon allowances or offsets to back their production.

In equilibrium, firm offset usage depends on their carbon productivity. High productivity firms exclusively use carbon allowances and do not rely on offsets, medium productivity firms use as many offsets as possible given the offset usage limit, while the lowest productivity firms forego production altogether. Offsets trade at a discount relative to carbon allowances because they face the risk of invali-

dation. For firms that are highly productive, the discount to buying offsets is not worth the possible lost production if offsets are invalidated. Firms that are less productive prefer using cheaper offsets because their lost production is not as costly: such firms will purchase the maximum amount of offsets they can up to the offset usage limit. For the lowest productivity firms, the cost of carbon compliance is too high relative to the benefits of production.

The regulator’s decision is determined by the endogenous response of firms to offset regulation. If the regulator relaxes offset regulation by increasing the usage limit or lowering the invalidation probability, it increases output because more firms produce. However, the marginal firms that produce when regulation is relaxed are less carbon productive—generating more emissions per unit of additional output. In the edge case in which invalidation is prohibitively costly, the offset usage limit is the main regulatory tool in the offset market. This case is empirically relevant, as the data show that compliance offsets face negligible invalidation risk. Overall, the optimal regulatory policy prescribes tighter offset limits when offset quality worsens, when the regulator suffers a greater disutility from emissions, or when the emissions cap is already generous which implies that most highly productive firms are not constrained in their production by carbon policy.

To bring this framework to the data, we turn to California’s Cap-and-Trade Program, which provides a rich empirical setting to examine firms’ offset use and the regulator’s policy choices. Specifically, we compile detailed information on regulated entities in California’s Cap-and-Trade Program across four compliance periods (2013-2023). We match compliance data—including total emissions, allowances, and offsets surrendered—to firm and establishment characteristics using the National Establishment Time Series (NETS), supplemented with address information obtained via Freedom of Information Act (FOIA) requests.

We utilize offset price data from Ccarbon, exploiting the price difference between CC0-8 offsets (with an 8-year invalidation risk) and CC0-3 offsets (with a 3-year invalidation risk) to infer the market-implied probability of offset invalidation. To do so, we build on the model of [Litterman and Iben \(1991\)](#), and rely on a no-arbitrage assumption to extract the probability of offset invalidation. The invalidation probability based on this approach is 0.46%, in line with the very few instances of offset invalidation that we observe over the sample period. The final dataset integrates firm emissions, allowances, offset use, sales, and invalidation risk, providing the necessary inputs for our structural estimation exercise.

As a preliminary step, we show simple descriptive statistics that strongly support the model’s key prediction of firm sorting in offset use. Firms with higher carbon productivity—measured as sales per unit of pollution—are significantly less likely to rely on offsets, while less productive firms are more inclined to use them. Graphical evidence shows a clear negative relationship between offset ratios and sales-over-emissions measures, and this pattern holds across both firm-level and establishment-level data. Regression analysis confirms that the relationship is robust: more productive and larger entities consistently exhibit lower offset usage. In contrast, offset adoption is concentrated among low-productivity firms. We also show that offset usage is bimodal: low-productivity firms use the maximum amount of permissible offsets and tend to be at the regulatory limit, whereas more productive firms tend not to use offsets at all.

We estimate the structural model on the data to quantify the underlying parameters that govern offset quality, firms’ offset usage and the regulator’s policy trade-offs, using data from the first three compliance periods of California’s Cap-and-Trade Program (2013-2020), during which the offset-usage limit was fixed at 8%. Our estimation targets offset usage, offset prices, surrendered allowances (emissions), and the probability of invalidation to recover structural parameters including the cost of producing offsets and offset quality. The model is tractable enough to allow for estimation to be performed analytically meaning that we can clearly demonstrate which moments identify which parameters.

The results indicate that offsets are very cheap to produce, but that their quality is imperfect. If we assume the regulator uses a social cost of carbon of \$190 per ton, our estimates suggest that only about 67% of offsets deliver genuine emission reductions, with the remaining 33% not delivering any emission reductions. This share of bad offsets implies the presence of uncounted emissions above the cap, averaging around 2% of total emissions per compliance period. We take this exercise further by estimating offset quality and uncounted emissions for a range of social costs of carbon. Lower social costs of carbon imply worse offset quality and more uncounted emissions. If the regulator uses a social cost of \$51 per ton, our estimates suggest that almost all offsets are of low quality, leading to uncounted emissions that are roughly 8% over the cap per compliance period.

These results are particularly striking in light of the institutional environment in which we estimate the model. California has developed substantial infrastructure to ensure the environmental integrity of compliance offsets. Each offset project undergoes independent verification by accredited third-party

auditors and must comply with detailed regulatory protocols before offset credits are issued. Projects remain subject to ongoing monitoring and audits after the issuance of offset credits. Given this rigorous oversight system, our estimates suggesting that a high fraction of offsets are of poor quality should be interpreted as a lower bound on the fraction of poor-quality offsets that may exist in other regions with less developed monitoring and enforcement systems.

We use structurally estimated offset quality to plot an output emissions frontier which helps us to quantify the tradeoff faced by the regulator when choosing the offset usage limit. For any given value of the social cost of carbon, the frontier shows the percentage increase in emissions relative to the percentage increase in output as the offset usage limit varies. The frontier shows that aggregate emissions intensity increases as the regulator increases the offset usage limit, because firms with worse emission intensity start producing when the limit is relaxed. In particular, at a social cost of carbon of \$190 per ton, our estimates suggest that the use of compliance offsets in California increased emissions by roughly 9% for every 1% increase in output. Lower social costs of carbon imply worse offset quality, giving rise to a steeper output emissions frontier.

We use the structural estimates to evaluate two counterfactuals. First, we consider what output would be if instead of having offsets, the regulator increases the carbon cap by the amount of uncounted emissions. As long as at least some offsets are good quality, adjusting the carbon cap produces less output than using offsets. Intuitively, good quality offsets do remove carbon, allowing for some emission-free output gain. We estimate that at a social cost of \$190 per ton, each offset generates an extra \$10 dollars in output relative to raising the carbon cap by the amount of uncounted emissions. If the costs of establishing and running the offset market exceed this amount, the regulator is better off not allowing offsets and raising the carbon cap instead. This result is sensitive to parameter values: at a social cost of \$51 per ton, it is better to raise the carbon cap rather than using offsets.

Second, we characterize implied offset quality when the regulator values production by small firms more than that of large firms. Specifically, we consider cases in which the average social cost of carbon is \$190 per ton, but small firms benefit from a discount on the social cost of carbon relative to large firms. A higher small firm discount corresponds to a lower marginal social cost of carbon, which in turn implies worse offset quality and more uncounted emissions. For example, if regulators apply a social cost of carbon for small firms that is 40% that of large firms, our analysis implies that only 40% of

offsets are good quality—27% less than offset quality when small and large firm production is valued equally.

Related Literature: We contribute to the growing literature on the environmental integrity of carbon offsets. An extensive interdisciplinary body of work has assessed whether offset programs achieve genuine reductions in greenhouse gas emissions. For offsets to provide real climate benefits, they must satisfy established criteria (see, for e.g., [Gillenwater, 2012](#); [Broekhoff et al., 2019](#); [Guérin et al., 2021](#)), such as additionality (the project would not proceed absent revenues from carbon allowances), conservative quantification (ensuring that emission reductions are not overstated), permanence (guaranteeing that reductions are not reversed, for example by wildfires), and the absence of double counting (ensuring that a given reduction is credited only once). A recent meta-analysis in *Nature Communications* synthesizing more than 60 empirical evaluations of over 2,300 projects concludes that fewer than 16% of issued allowances correspond to verifiable reductions in emissions ([Probst et al., 2024](#)). Other papers study whether offsets also increase biodiversity while reducing carbon ([Zhou and Almond, 2024](#)), or whether offset subsidy goes to projects that would have been financed without the subsidy ([Calel et al., 2025](#)).

In contrast, our study employs a revealed-preference approach to infer the prevalence of low-quality offsets.⁴ We exploit the institutional feature that regulators impose binding limits on offset usage within cap-and-trade systems, and we interpret these limits as a mechanism to restrict the entry of offsets that do not satisfy integrity criteria. Although methodologically distinct from project-level audits, our estimates are consistent with [Badgley et al. \(2022\)](#), who document that roughly 30% of allowances in California’s Cap-and-Trade program are systematically over-credited.

A growing body of research in climate finance and economics examines how financial and governance frictions shape firm responses to pollution regulation, analyzing liability rules ([Akey and Appel, 2021](#); [Ohlrogge, 2023](#); [Bellon, 2021](#); [Chen, 2025](#)), disclosure mandates ([Inderst and Opp, 2025](#); [Gupta and Starmans, 2023](#)), carbon taxes ([Döttling and Rola-Janicka, 2023](#); [Besley and Persson, 2023](#); [Ramadorai and Zeni, 2024](#)), cap-and-trade systems ([Biais and Landier, 2022](#); [Ivanov, Kruttli, and Watugala, 2024](#)), ownership structure ([Duchin, Gao, and Xu, 2025](#); [Bellon, 2025](#)), and the use of offsets. Related to our

⁴[Shapiro and Walker \(2025\)](#) also rely on a revealed-preference approach in the context of offsets. They show that prices in U.S. air pollution offset markets reveal marginal abatement costs and imply that regulation is often too lenient relative to efficiency.

study, [Kim, Li, and Wu \(2024\)](#) show that voluntary offsets are concentrated in low-emission industries. In contrast, we focus on compliance offsets and demonstrate, both theoretically and empirically, that they are disproportionately used by less carbon-efficient firms, underscoring the distinction between voluntary and compliance offsetting. Our framework further highlights the policy trade-offs embedded in the design of cap-and-trade systems and the role of offsets within them. In so doing, we complement other finance papers that analyze cap-and-trade design from different perspectives, such as how carbon price volatility deters investment in green technologies ([Fuchs, Stroebe, and Terstegge, 2024](#)), the presence of waterbed effects ([Akey et al., 2024](#); [Perino, Ritz, and van Benthem, 2025](#)), distributional concerns ([Fowlie, Holland, and Mansur, 2012](#)), and the interaction between financial frictions, investment, and output reallocation ([Bartram, Hou, and Kim, 2022](#); [Colmer et al., 2025](#)).

1 Model

In this section, we introduce and analyze a model of offset markets. Section [1.1](#) presents the model. Section [1.2](#) analyzes the model equilibrium and presents our key results.

1.1 Model Description

We consider an economy with a continuum of producers indexed by $i \in [0, 1]$. There are three time periods in the model, $t = 0, 1$, and 2 . In period $t = 0$, a regulator sets regulatory standards for the offset market, namely, the offset usage limit and invalidation probability, and firms make decisions regarding the purchase of carbon allowances and offsets. In period $t = 1$, offset invalidation may occur. In period $t = 2$, firms produce.

Each firm i chooses the amount of capital to employ in production, $k_i \leq \bar{k}$, where $\bar{k}$ represents the maximum capital available to any firm. Firms differ in their productivity, denoted by A_i , and indexed such that firm $i = 0$ has the lowest productivity and firm $i = 1$ has the highest productivity. Specifically, we assume that firm productivity is drawn from a general distribution with cumulative distribution function $F(\cdot)$ and full support on an interval $[A_{min}, A_{max}]$ with $A_{min} > 0$. Firms are ordered such that $A_i = F^{-1}(i)$ for $i \in [0, 1]$, which implies that the fraction of firms with productivity below any level $A \in [A_{min}, A_{max}]$ is given by $F(A)$. Firm i produces output according to the linear production function $y_i = A_i k_i$. It generates emissions $e_i = \eta k_i$ proportional to capital use, where $\eta > 0$ is the emission

intensity of the production technology. We define firm i 's carbon productivity—the amount of economic output generated per unit of carbon emissions—as $\xi_i = \frac{A_i}{\eta}$.⁵

Firms in the economy are subject to a cap-and-trade system with an aggregate emissions cap $C < \int_0^1 \eta \bar{k} di$. This assumption implies that the aggregate emissions cap is not so large that all firms in the economy can produce using their maximum capital. Each firm i is allocated carbon allowances c_i , with the aggregate allocation satisfying $\int_0^1 c_i di = C$. These allowances can be traded among firms at $t = 0$ at an endogenous market price p_c . In the majority of our analysis, which focuses on the tradeoffs confronting the use of offset regulation instruments, we take the emissions cap as given. In Section 3.2, we estimate the tradeoffs associated with increasing the aggregate emissions cap versus allowing more offset usage.

In addition to allowances, firms can purchase carbon offsets at $t = 0$ at an endogenous market price p_o to comply with emission requirements. Offsets are produced by a representative offset producer who faces convex offset production costs $\frac{\phi}{2} Q_o^2$, where $Q_o \geq 0$ is the amount of carbon offsets produced and $\phi > 0$.

Our model includes two key regulatory features governing compliance offsets that feature in the California compliance offset market, namely, usage limits and invalidation risk. First, each firm can use offsets to cover at most a fraction $\alpha \in [0, 1)$ of their total emissions. Second, offsets purchased by firms carry a risk of invalidation. Specifically, we assume that with probability $\rho \in [0, 1]$ an offset is invalidated at $t = 1$ after purchase but before firm production. We assume that when invalidation occurs, an affected firm loses all of its offsets.

For simplicity, we assume that (i) allowances and offsets are traded before invalidation and production and that (ii) a firm loses all of its offsets if invalidation occurs. These are simple ways of capturing the real cost in terms of lost production coming from offset invalidation that firms cannot hedge.⁶

As such, if a firm has an offset invalidated, it can no longer generate the emissions it meant to offset. Alternatively, we can make the assumption that the firm has to pay an extra cost to produce and generate

⁵For simplicity, we assume that the emissions efficiency parameter η is constant across firms. In principle, firms could also differ in η , reflecting heterogeneity in emissions technologies. Allowing for this would require specifying assumptions on the joint distribution of productivity (A_i) and emissions efficiency (η_i). The key theoretical results would remain unchanged under such an extension, but given data limitations—specifically, the lack of reliable firm-level measures of emissions technology—we maintain the simplifying assumption that η is constant.

⁶If firms knew with certainty that a fraction of their offsets would be invalidated, they could simply purchase additional offsets to hedge this risk so that they would always produce with all of their capital.

emissions upon invalidation, for example a cost of non-compliance. Alternative assumptions could be time to build or to alter production because of assets in place, or a scramble for permits if invalidation generates an aggregate shock. The key assumption we require is that having an offset invalidated is costly for a firm in proportion to how much production it was hoping to use the offset towards.

At $t = 0$, the regulator chooses the offset usage limit, α , and the invalidation probability, ρ . Invalidation occurs at $t = 1$, after firms have purchased offsets.⁷ We assume that with probability δ a sold offset is of poor quality and does not offset any pollution. The probability δ is best interpreted as the risk-neutral regulator's perceived *expected* offset quality.⁸ The regulator faces an increasing and convex cost of invalidating offsets $\kappa(Q_o\rho)$ with $\kappa'(Q_o\rho) > 0$ and $\kappa''(Q_o\rho) > 0$, where Q_o is the offsets purchased by firms and $Q_o\rho$ is the expected number of offsets invalidated. If the regulator chooses $\rho = \delta$, all poor quality offsets are invalidated and all the remaining valid offsets remove emissions. If $\rho < \delta$, the quantity of bad offsets not invalidated are $Q_o(\delta - \rho)$.⁹ Note that ρ is the unconditional invalidation probability. Specifically, we assume that if the offset is good quality (with probability $1 - \delta$), the conditional probability of invalidation is zero, whereas if the offset is poor quality (with probability δ), the conditional probability of invalidation is $\frac{\rho}{\delta}$. This implies that, after invalidation, the remaining bad offsets in the economy are $Q_o(\delta - \rho)$.

The regulator trades off an increase in production with an increase in emissions produced from offsets. Specifically, we assume that the regulator suffers disutility which is an increasing function of the expected emissions from bad offsets $\lambda(Q_o(\delta - \rho))$ with $\lambda'(Q_o(\delta - \rho)) > 0$.¹⁰ In particular, at $t = 0$ the

⁷Our key result on firm sorting obtains if we alternatively assume that the regulator does not commit to an invalidation probability at $t = 0$ and instead chooses it at $t = 1$. In addition, because our estimated invalidation probability is close to 0, our quantitative results do not depend on the timing at which the regulator chooses the invalidation probability.

⁸If instead the regulator is risk-averse and uncertain about offset quality, our estimate of offset quality can be interpreted as an upper-bound on the regulator's perceived expected offset quality.

⁹We assume there are no false positives for simplicity. The results extend naturally if we allow for imperfect invalidation. If instead good offsets are invalidated with probability $\pi > 0$, bad offsets remaining become $Q_o[\delta - \rho + (1 - \delta)\pi]$. The regulator's problem can be rewritten with an augmented cost function $\tilde{\kappa}(Q_o\rho)$ (explicitly written out in footnote 11) which captures both direct invalidation costs and additional uncounted emissions from false positives and is increasing and convex. We can also allow π to be an increasing function of ρ , capturing that more aggressive invalidation leads to more false positives, which further increases the effective cost of invalidation. Thus, false positives simply increase the effective cost of invalidation without altering the model's structure.

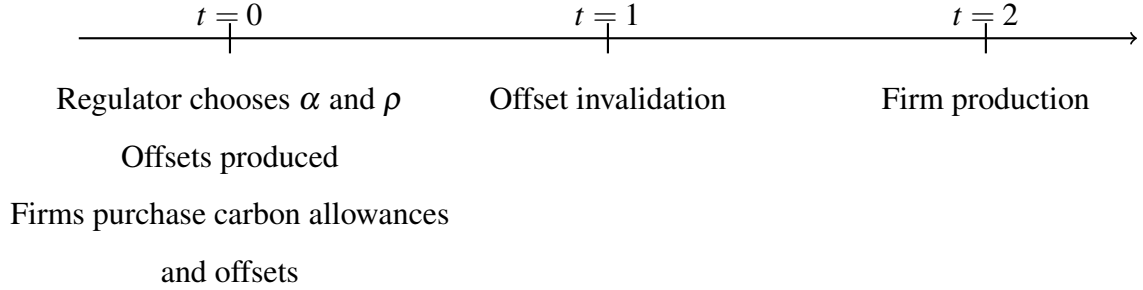
¹⁰Our results are qualitatively similar if instead we assume that the regulator suffers disutility from total emissions (i.e., inclusive of C). Since in practice, the choice of C should already incorporate the regulator's disutility from emissions at the prevailing social cost of carbon, we write down the problem to focus on incremental emissions coming from offsets over and above the cap C .

regulator chooses α and ρ to solve

$$\max_{0 \leq \alpha < 1, 0 \leq \rho \leq \delta} Y(\alpha, \rho) - \lambda(Q_o(\alpha, \rho)(\delta - \rho)) - \kappa(Q_o(\alpha, \rho)\rho), \quad (1)$$

where Y is expected aggregate production.¹¹

The sequence of events is summarized in the timeline below:



1.2 Equilibrium Analysis

We solve for regulation and firm decisions and carbon and offset market clearing using backward induction. In period $t = 2$, after offsets are invalidated, firms choose their production levels given their available compliance instruments (carbon allowances and valid offsets). In period $t = 0$, prior to invalidation, firms decide on the optimal mix of carbon allowances and offsets. Understanding firm decisions, the regulator chooses the offset usage limit and invalidation probability.

1.2.1 Firm Production Decisions Post-Invalidation

Firm i chooses its production level to maximize output, subject to its compliance constraints. Let I_i be an indicator variable defined as

$$I_i = \begin{cases} 0, & \text{if firm } i\text{'s offsets are invalidated,} \\ 1, & \text{otherwise.} \end{cases}$$

¹¹If we allow for false positive, the augmented cost function can be defined as $\tilde{\kappa}(Q_o\rho) \equiv \kappa(Q_o\rho) + (\lambda(Q_o(\delta - \rho + (1 - \delta)\pi)) - \lambda(Q_o(\delta - \rho)))$. Since λ is increasing, $\tilde{\kappa}(Q_o\rho) > \kappa(Q_o\rho)$ for $\pi > 0$, and $\tilde{\kappa}$ remains increasing and convex.

Specifically, firm i chooses the optimal amount of capital to produce with, k_i , to maximize its output given its stock of carbon allowances q_i^c and valid offsets $q_i^o I_i$ subject to its capital and regulatory constraints.

$$\begin{aligned} \max_{k_i} \quad & A_i k_i, \\ \text{s.t.} \quad & k_i \leq \bar{k}, \\ & e_i = \eta k_i \leq q_i^c + \min\{q_i^o I_i, \alpha e_i\}. \end{aligned}$$

The first constraint is the firm's capital constraint: it can use at most $\bar{k}$ units of capital for production. The second constraint is the regulatory constraint: a firm must use either a carbon allowance or a valid offset for every unit of emissions it generates. The minimum condition ensures that the offset usage limit is satisfied; the firm can use valid offsets towards at most α fraction of its emissions. Note that $q_i^o \leq \alpha e_i$ because a firm will never purchase offsets at $t = 0$ if it cannot use them towards production in the future.

Given these constraints, the production decision of firm i is characterized by

$$y_i = A_i k_i = \begin{cases} A_i \bar{k} = \xi_i \eta \bar{k}, & \text{if } q_i^c + q_i^o I_i \geq \eta \bar{k}, \\ \xi_i (q_i^c + q_i^o I_i), & \text{if } q_i^c + q_i^o I_i < \eta \bar{k}, \end{cases}$$

with corresponding emissions given by

$$e_i = \begin{cases} \eta \bar{k}, & \text{if } q_i^c + q_i^o I_i \geq \eta \bar{k}, \\ q_i^c + q_i^o I_i, & \text{if } q_i^c + q_i^o I_i < \eta \bar{k}. \end{cases}$$

A firm employs its full capital $\bar{k}$ in production whenever its available allowances and valid offsets suffice to cover the corresponding emissions. If the firm does not have enough allowances and valid offsets to cover the emissions generated from producing using its entire capital stock, it will produce with as much capital as possible before hitting its regulatory constraint. In this case, for a given amount of carbon allowances and valid offsets, a firm with higher carbon productivity ξ_i , will be able to produce using more capital, as each unit of capital for such a firm generates lower emissions.

1.2.2 Firm Carbon Allowance and Offset Purchase Decisions Pre-Invalidation

At $t = 0$, before offset invalidation, firm i chooses the optimal mix of carbon allowances and offsets to purchase. Specifically, the firm acquires q_i^c units of carbon allowances and q_i^o units of offsets. Firm i receives a free allocation c_i of allowances, so the net amount of carbon allowances purchased is $q_i^c - c_i$. When making their decisions, firms account for the fact that each offset is subject to invalidation with probability ρ , as well as the regulatory limits on the use of offsets to cover at most an α fraction of total emissions.

Firm i chooses (q_i^c, q_i^o) to maximize its expected profits by solving

$$\begin{aligned} \max_{q_i^c \geq 0, q_i^o \geq 0} \quad & \xi_i \left[q_i^c + (1 - \rho) q_i^o \right] - p_c (q_i^c - c_i) - p_o q_i^o, \\ \text{s.t.} \quad & q_i^c + q_i^o \leq \eta \bar{k}, \\ & q_i^o \leq \alpha (q_i^c + q_i^o). \end{aligned}$$

where p_c is the market price of carbon allowances and p_o is the price of offsets. The objective function represents the benefit of holding compliance instruments: allowances provide the full production benefit ξ_i , while offsets deliver only a fraction $(1 - \rho)$ of that benefit due to the risk of invalidation.

The first constraint expresses that any instruments purchased beyond those required to cover the emissions associated with full-capital production ($\eta \bar{k}$) have no use in increasing output. Even though the firm is technically allowed to purchase more allowances and offsets than would be needed for $\eta \bar{k}$ emissions in total, it will never choose to do so because the purchase incurs a cost but the marginal benefit is zero. The second constraint enforces the α offset usage limit. Even though the firm can technically purchase more offsets, it will never do so since such a purchase incurs a cost without providing production benefits. Thus, these constraints are a convenient way to write down the effective decision problem of the firm.

A carbon allowance gives the firm a benefit of ξ_i units of production at a cost of p_c , whereas an offset gives the firm an expected benefit of $(1 - \rho) \xi_i$ units of production at a cost of p_o . A firm i is indifferent

between purchasing carbon allowances and offsets if and only if

$$\underbrace{\xi_i - p_c}_{\text{Expected profit from purchasing carbon allowance}} = \underbrace{(1 - \rho) \xi_i - p_o}_{\text{Expected profit from purchasing carbon offset}}.$$

We define $\bar{\xi}$ as the threshold carbon productivity of the most productive firm that purchases carbon offsets. This is given by

$$\bar{\xi} = \frac{p_c - p_o}{\rho}. \quad (2)$$

Note that for $\bar{\xi} > 0$, we require $p_c > p_o$. As we show below when we analyze carbon allowance and offset market clearing, this is always satisfied for $\rho > 0$. Because offsets may be invalidated, they trade at a discount relative to carbon allowances. For less emission intensive firms with $\xi_i > \bar{\xi}$, the cost of lost production is strictly greater than the discount from buying offsets. These firms, therefore, will purchase only carbon allowances.

Conversely, more emission intensive firms with $\xi_i < \bar{\xi}$ find offsets relatively more attractive than carbon allowances because the cost of lost production is not that high relative to the discount from buying offsets. These firms strictly prefer to purchase cheaper offsets rather than carbon allowances but they will nonetheless purchase additional allowances because they are bound by the offset usage limit. These firms will be willing to buy carbon allowances to be able to use their subsidized offsets if:

$$\underbrace{(1 - \rho) \xi_i + \rho(1 - \alpha) \xi_i}_{\text{Expected output of firm per unit of emissions at offset usage constraint}} \geq \underbrace{(1 - \alpha) p_c + \alpha p_o}_{\text{Cost of carbon allowance and offset purchase}}$$

The left hand side of the above equation shows the expected output per unit of emissions for a firm which purchases the maximum amount of offsets it can feasibly use towards production. With probability $1 - \rho$ its offsets will not be invalidated and it can produce using its full capital, and with probability ρ its offsets will be invalidated and it can only produce using only $1 - \alpha$ fraction of its capital. The right hand side of the equation is the cost of the carbon allowance and offset purchase.

We define $\underline{\xi}$ as the threshold carbon productivity of the least productive firm that purchases carbon offsets. This is given by:

$$\underline{\xi}_i = \frac{(1-\alpha)p_c + \alpha p_o}{1-\alpha\rho}. \quad (3)$$

Firms with $\xi_i < \underline{\xi}$ do not find it beneficial to purchase offsets or carbon allowances.

The carbon allowance and offset purchase of firm i is therefore given by:

$$q_i^o = \begin{cases} \alpha\eta\bar{k} & \text{if } \xi_i \in [\underline{\xi}, \bar{\xi}] \\ 0 & \text{otherwise} \end{cases}, \quad q_i^c = \begin{cases} (1-\alpha)\eta\bar{k} & \text{if } \xi_i \in [\underline{\xi}, \bar{\xi}] \\ \eta\bar{k} & \text{if } \xi_i > \bar{\xi} \\ 0 & \text{otherwise} \end{cases}.$$

Proposition 1 (Firm Sorting). *A firm's decision about whether to purchase carbon allowances and/or offsets is determined by its carbon productivity. Specifically, firms with $\xi_i > \bar{\xi} = \frac{p_c - p_o}{\rho}$ exclusively purchase carbon allowances, firms with $\xi_i < \underline{\xi} = \frac{(1-\alpha)p_c + \alpha p_o}{1-\alpha\rho}$ purchase no carbon allowances or offsets, and firms with intermediate productivity, $\xi_i \in [\underline{\xi}, \bar{\xi}]$, purchase both carbon allowances and offsets and are at the offset usage constraint.*

This analysis demonstrates that only firms with intermediate carbon productivity will purchase offsets. The most productive firms with low emission intensity find it too costly to use offsets because of the offset invalidation risk. The least productive firms with high emission intensity find it too costly to use a combination of carbon allowances and offsets because of the offset usage constraint.

1.2.3 Offset Producer Problem

The representative offset producer's problem is to choose Q_o to maximize its profit from selling offsets at the market price p_o , net of production costs:

$$\max_{Q_o \geq 0} p_o Q_o - \frac{\phi}{2} Q_o^2.$$

Taking the first-order condition, the offset producer's optimal supply of offsets is:

$$Q_o = \frac{p_o}{\phi}. \quad (4)$$

The quantity of offsets supplied by the producer increases with the offset price and decreases with the cost of producing offsets.

1.2.4 Equilibrium for a given α and ρ

We characterize the market equilibrium for a given ρ and α . In Section 1.2.5, we consider the regulator's choice of α and ρ . Equilibrium requires that both the carbon allowance market and the offset market clear. We define two threshold indices: $i_{\underline{\xi}}$ is the firm with $\xi_i = \underline{\xi}$; and $i_{\bar{\xi}}$ is the firm with $\xi_i = \bar{\xi}$.

Market clearing in the carbon allowance market requires that the total demand for carbon allowances across firms equals the aggregate cap C . Firms with emission intensity between $\underline{\xi}$ and $\bar{\xi}$ use offsets towards α fraction of their emissions and carbon allowances for the remaining $1 - \alpha$ fraction of their emissions. Firms with emission intensity greater than $\bar{\xi}$ use only carbon allowances. All firms with $\xi_i \geq \underline{\xi}$ use all their capital for production, while firms with $\xi_i < \underline{\xi}$ use none of their capital for production. Hence:

$$C = \int_{i_{\underline{\xi}}}^{i_{\bar{\xi}}} (1 - \alpha) \eta \bar{k} di + \int_{i_{\bar{\xi}}}^1 \eta \bar{k} di. \quad (5)$$

Market clearing in the offset market requires that the total demand for offsets across firms equals offset supply. Firms with emission intensity between $\underline{\xi}$ and $\bar{\xi}$ use valid offsets towards α fraction of their emissions. No other firms purchase offsets. Hence:

$$\frac{p_o}{\phi} = \int_{i_{\underline{\xi}}}^{i_{\bar{\xi}}} \alpha \eta \bar{k} di. \quad (6)$$

The market clearing conditions (5) and (6), along with the conditions that determine the carbon productivity thresholds governing firm offset and carbon allowance purchase decisions (2) and (3), jointly determine the equilibrium prices p_c and p_o and the allocation of carbon allowances and offsets across firms, $\underline{\xi}$ and $\bar{\xi}$.

In what follows, define $G(\cdot) = F(\eta \cdot)$ as the cumulative distribution function of the carbon productivity ξ_i , with full support on the interval $[\frac{1}{\eta} A_{\min}, \frac{1}{\eta} A_{\max}]$. We assume that G is strictly increasing and continuous on $[\frac{A_{\min}}{\eta}, \frac{A_{\max}}{\eta}]$, with density g that is continuous and satisfies $0 < \underline{g} \leq g(\xi) \leq \bar{g} < \infty$ on any compact interval containing the equilibrium cutoffs. Consequently G^{-1} exists, is continuous, and is C^1 with $(G^{-1})'(\cdot) = 1/g(G^{-1}(\cdot))$ on that set. We make additional restrictions, stated in the proof of

Proposition 2, that $A_{\min}$ is sufficiently low and $A_{\max}$ is sufficiently high to guarantee interior solutions for $i_{\underline{\xi}}$ and $i_{\bar{\xi}}$. Then we can establish the following proposition.

Proposition 2 (Existence and uniqueness of equilibrium). *There exists a unique equilibrium characterized by the quadruple $\{i_{\underline{\xi}}, i_{\bar{\xi}}, p_c, p_o\}$ where $0 \leq i_{\underline{\xi}} < i_{\bar{\xi}} \leq 1$ and $p_o \leq p_c$ satisfy the indifference conditions (2)–(3) and the market-clearing conditions (5)–(6). Let $E \equiv \eta \bar{k}$ and define*

$$x \equiv \alpha(i_{\bar{\xi}} - i_{\underline{\xi}}) = \frac{Q_o}{E} = \frac{p_o}{\phi E}.$$

Then, the unique equilibrium is pinned down by the fixed point $x = h(x)$ with

$$h(x) = \frac{1}{\phi E} \left((1 - \alpha \rho) G^{-1} \left(1 - \frac{C}{E} - x \right) - (1 - \alpha) \rho G^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) \right),$$

with feasible domain $x \in [0, \min \{1 - \frac{C}{E}, \frac{\alpha}{1 - \alpha} \frac{C}{E}\}]$. In the unique equilibrium, cutoff types are given by

$$i_{\underline{\xi}} = 1 - \frac{C}{\eta \bar{k}} - x, \quad i_{\bar{\xi}} = 1 - \frac{C}{\eta \bar{k}} + \left(\frac{1}{\alpha} - 1 \right) x,$$

and offset and carbon allowance prices are given by,

$$p_o = \phi \eta \bar{k} x, \quad p_c = \rho G^{-1}(i_{\bar{\xi}}) + \phi \eta \bar{k} x.$$

The above proposition characterizes carbon allowance and offset usage and prices in equilibrium. In the proposition, x captures the share of total emissions in the economy that are covered by offsets. A higher x implies more offsets are used in equilibrium. The threshold $i_{\underline{\xi}}$ decreases in x , while the threshold $i_{\bar{\xi}}$ increases in x . The equilibrium price of offsets increases in x because the offset producer requires higher prices to supply more offsets. The equilibrium price of offsets also increases in the cost of producing offsets.

Comparative Statics. To further understand how offset and carbon allowance usage is affected by regulation, we investigate how firm demand for the two instruments changes as the regulation governing the use of offsets changes. We first consider a change in the offset usage limit.

COROLLARY 1. *An increase in the offset usage limit, α , holding ρ fixed, increases the total offset usage, Q_o , increases the offset price, p_o , and decreases the threshold lowest productivity firm that uses offsets, $\underline{\xi}$. In particular,*

$$\frac{\partial \underline{\xi}}{\partial \alpha} < 0, \quad \frac{\partial p_o}{\partial \alpha} > 0, \quad \frac{\partial Q_o}{\partial \alpha} > 0. \quad (7)$$

Furthermore, if the invalidation rate $\rho < \underline{\rho}$, where $\underline{\rho}$ is defined in the Appendix, an increase in the offset usage limit, α , also decreases the price wedge between carbon allowances and offsets, $p_c - p_o$, and decreases the threshold highest productivity firm that uses offsets, $\bar{\xi}$. In particular,

$$\frac{\partial (p_c - p_o)}{\partial \alpha} < 0, \quad \frac{\partial \bar{\xi}}{\partial \alpha} < 0, \quad \frac{d}{d\alpha} (i_{\bar{\xi}} - i_{\underline{\xi}}) < 0. \quad (8)$$

An increase in the offset usage limit increases the demand for offsets, which increases the relative offset price and equilibrium offset usage. For the lowest productivity firm that is willing to buy offsets, there are two effects. First, the increase in the relative offset price makes offsets less attractive as it reduces the relative discount to buying offsets. Second, the increase in the offset usage limit allows the firm to use more subsidized offsets, making offsets relatively more attractive. Intuitively, low productivity firms like using cheap offsets but because of offset usage constraints, they are forced to also use more expensive carbon allowances. An increase in α increases how much they can benefit from cheaper offsets. This latter effect always dominates, causing the lowest productivity firm which uses offsets to shift to the left of the firm productivity distribution. The highest productivity firm that is willing to buy offsets, $\bar{\xi}$, is determined by the relative discount to buying offsets. If the discount decreases because of the offset price increase, the highest productivity firm which uses offsets also shifts to the left of the firm productivity distribution.

Looking at offset usage, there are two effects. The amount of offsets used per firm conditional on that firm using offsets—the intensive margin effect—increases because of the laxer offset usage constraint. However, the total number of firms that use offsets—the extensive margin effect—decreases when α increases if the price wedge between offsets and carbon allowances decreases. The intensive margin effect always dominates the extensive margin effect, causing the aggregate usage of offsets to increase when the offset usage limit increases. Figure 1 illustrates how the distribution of firms using offsets changes with α . An increase in α causes more emission intensive firms to purchase offsets and shifts the distribution of firms using offsets to the left.

Next, we consider how a change in the invalidation probability, ρ , affects the equilibrium.

COROLLARY 2. *An increase in the invalidation probability, ρ , narrows the distribution of firms using offsets by increasing the threshold lowest productivity firm that uses offsets, $\underline{\xi}$, and decreasing the threshold highest productivity firm that uses offsets, $\bar{\xi}$, decreases total offset usage, Q_o , and decreases the offset price, p_o . In particular,*

$$\frac{\partial \underline{\xi}}{\partial \rho} > 0, \quad \frac{\partial \bar{\xi}}{\partial \rho} < 0, \quad \frac{\partial Q_o}{\partial \rho} < 0, \quad \frac{\partial p_o}{\partial \rho} < 0.$$

Furthermore, for $\rho < \underline{\rho}$, where $\underline{\rho}$ is defined in the Appendix, an increase in the invalidation probability ρ increases the price wedge between carbon allowance and offset prices,

$$\frac{\partial (p_c - p_o)}{\partial \rho} > 0. \tag{9}$$

An increase in the invalidation probability makes offsets less attractive. This decreases the productivity of the highest productivity firm that is willing to buy offsets, $\bar{\xi}$ and increases the productivity of the lowest productivity firm that is willing to buy offsets, $\underline{\xi}$, narrowing the productivity distribution of firms using offsets. Overall, the decrease in the demand for offsets, lowers the price of offsets relative to carbon allowances. Looking at the aggregate usage of offsets by firms, there is only an extensive margin effect. The total number of firms that use offsets increases when ρ decreases. Figure 2 illustrates how the distribution of firms using offsets changes with ρ , showing that an increase in ρ narrows the distribution of firms using offsets.

1.2.5 The Regulator

We now turn to the regulator's problem. We cannot obtain a general closed form solution for the regulator's optimal choice of ρ and α . However, we can derive a closed form solution for α if we assume that invalidation is prohibitively costly for the regulator, i.e., $\kappa(Q_o \rho) \rightarrow \infty \forall \rho > 0$. Since the model is analytically tractable under this assumption, we characterize the regulator's optimal choice of α in this special case.

We also assume that the regulator faces a linear disutility from emissions generated by offsets, i.e. $\lambda(x) = \lambda x$. This assumption allows us to interpret λ as the social cost of carbon in our estimation. Then, we can establish the following proposition.

Proposition 3. *Assume that $\kappa(Q_o\rho) \rightarrow \infty \forall \rho > 0$, $\lambda(x) = \lambda x$, $\phi < \bar{\phi}$ (defined in the proof), $\lambda\delta \in (\xi_{min}, \xi_{max})$ and $C < \eta\bar{k}(1 - G(\lambda\delta))$. Then, the regulator will choose $\rho = 0$ and will choose*

$$\alpha = \frac{1 - \frac{C}{\eta\bar{k}} - G(\lambda\delta)}{1 - G(\lambda\delta)}. \quad (10)$$

In this special case, the regulator will not invalidate any offsets as the cost of doing so is prohibitively high. The regulator chooses a lower α as offset quality deteriorates (an increase in δ) and the regulator suffers a greater disutility from emissions (an increase in λ). The regulator also chooses a lower offset usage limit the higher the carbon cap C . This is because a higher carbon cap implies that an increase in α will allow relatively less productive firms to start producing.

In the subsequent sections, we present empirical results that support our key predictions about which firms use carbon allowances versus offsets. We also perform an estimation in Section 3 using the closed form solution for α we obtain in Proposition 3. We also estimate offset quality numerically without making this simplifying assumption.

2 Background, Data, and Empirical Analysis

This section introduces the institutional features of California's Cap-and-Trade Program, details our data sources, outlines the construction of our variables, and presents summary statistics and empirical tests that validate the key prediction of firm sorting in the model and provide the moments for the structural estimation.

2.1 Background on California Cap-and-Trade

California's Cap-and-Trade Program is a market-based mechanism designed and administered by the California Air Resources Board (CARB) to reduce greenhouse gas (GHG) emissions across the state.¹² Adopted in 2011 and launched on January 1, 2013, when the first emissions cap took effect, the program was initially authorized through 2020, later extended to 2030, and has now been reauthorized to operate through 2045 in line with California's long-term decarbonization goals. It functions under an annually declining statewide emissions cap, which ensures that permissible emissions across covered sectors fall steadily over time and remain consistent with the state's broader climate targets.

Under California's Cap-and-Trade regulation, each regulated entity has an emissions obligation.¹³ This obligation is to surrender allowances or eligible carbon offsets to cover the aggregated emissions from all facilities under its responsibility. Entities may meet this obligation by purchasing allowances through state-run quarterly auctions in the primary market or from other parties in the secondary market, and by purchasing eligible carbon offsets from CARB-approved developers.

CARB sets several multi-year compliance periods.¹⁴ Within each period, regulated entities must meet both annual and full-period surrender obligations: annual compliance instruments (allowances and offsets) equal to 30% of the prior year's verified emissions are due by November 1st of the following year, while the remaining obligation must be surrendered by November 1st of the multi-year period's conclusion. For example, for the 2021-2023 compliance period, surrender deadlines are Nov 1, 2022 (30% of the 2021 emissions), Nov 1, 2023 (30% of the 2022 emissions), and Nov 1, 2024, for the remaining emissions (remaining balance of 2021 and 2022 emissions, plus 100% of 2023 emissions).¹⁵ CARB also regulates the fraction of compliance offset credits entities can use to cover their overall compliance obligation. Before 2021, regulated entities could not use offsets for more than 8% of their compliance obligation. The limit was reduced to 4% for 2021-2025. The limit will increase to 6% for 2026-2030. In addition, beginning in 2021, at least half of all compliance offsets must originate from projects that directly benefit California—DEBS (Direct Environmental Benefits).

¹²Title 17 of the California Code of Regulations, §95800-96023. More information on California's Cap-and-Trade Program is available [here](#).

¹³California Code of Regulations, Title 17, §95812.

¹⁴The compliance periods have been: 2013-2014, 2015-2017, 2018-2020, 2021-2023, and 2024-2026.

¹⁵The Cap-and-Trade Program permits intertemporal banking of compliance instruments. Allowances and offsets may be banked indefinitely for use in future compliance periods and do not expire. Allowances are subject only to entity-level holding limits designed to mitigate market power. There is no borrowing against future vintages. See California Code of Regulations, Title 17, §95920-95922.

Compliance offsets must satisfy a set of CARB-certified criteria to ensure that credited emission reductions are "real, additional, quantifiable, permanent, verifiable, and enforceable."¹⁶ Eligible offset project types include forestry (representing roughly 80% of the total issuances), livestock methane capture, mine methane capture, and Ozone-depleting substance (ODS) destruction, such as the destruction of refrigerants with high global warming potential.¹⁷ These offsets represent verified emission reductions from sources that are not directly covered by compliance obligations under California's Cap-and-Trade Program, to avoid double-counting and ensure additionality.

Each offset project is independently verified by accredited third-party auditors, must comply with detailed regulatory protocols, and undergoes CARB review before credits are issued. Once issued, offsets remain subject to continuous monitoring, audits, and a formal investigation process that leads to invalidation only when credible evidence of material non-compliance exists. Despite the rigorous process leading to their issuance, CARB-certified offsets can still be invalidated by CARB within eight years after their issuance. The invalidation window can be reduced to three years after issuance if a second verification is submitted.¹⁸ Common reasons for offset invalidation include:

- **Material Misstatement:** if the offset overstates the amount of GHG reductions or GHG removal enhancements by more than 5 percent. The overstated amount will be subject to invalidation.
- **Regulatory Non-Compliance:** if the offset project activity and implementation are not in accordance with all local, regional, state, and national environmental and safety regulations for which the CARB-certified offset credit was issued.
- **Double Crediting:** if the offset credits have been claimed in another voluntary or regulatory program.

If an offset is invalidated, even if it has been previously surrendered, the regulated entity must replace it with either a valid offset or emission allowances. This creates invalidation risk which contributes to heterogeneity in offset prices. Offsets offering a three-year invalidation window typically command higher prices than standard offsets with an eight-year invalidation window. As discussed later, we exploit this

¹⁶Source: https://ww2.arb.ca.gov/sites/default/files/2021-02/ct_reg_unofficial.pdf.

¹⁷This information is available on the [CARB Program Data Dashboard](#), Accessed Nov 6, 2025.

¹⁸This second regulatory verification must be completed within three years of CARB-certified offset issuance. See California Code of Regulations, Title 17, § 95985.

price difference to infer the probability of offset invalidation.¹⁹ Offset invalidation events under the CARB offset program are rare. Since 2013, CARB has invalidated only a very small fraction of issued offset credits. Specifically, 0.9% (0.05%) of offset projects (credited offsets) have been invalidated.²⁰

Wrongful invalidation of offsets under California’s Cap-and-Trade Program is highly unlikely because CARB enforces a stringent, multi-layered system of oversight. The burden of proof rests with CARB, and the process incorporates multiple safeguards to prevent errors. Reflecting the robustness and credibility of this system, there has never been a successful or even initiated legal challenge to an offset invalidation decision since the program’s inception.

2.2 Datasets

We collect detailed administrative information on the list of regulated entities from the Cap-and-Trade Program Data Dashboard.²¹ We extract the compliance information of each entity for the 2013-2014, 2015-2017, and 2018-2020 compliance period. We omit the 2021-2023 compliance period due to substantial changes in the quantity and types of offsets entities are allowed to surrender. The dataset includes emissions obligations, as well as total surrendered allowances and compliance offsets, for each compliance period. Our sample comprises 383 unique regulated entities across these three periods, which we refer to as the compliance-level sample.

When a regulated entity operates one or more covered facilities, we observe the physical address of each facility. We use the Freedom of Information Act to obtain entity addresses when this information is missing from public records. When the emission source is not tied to a specific generating facility we instead observe the firm’s headquarters address. Note that this headquarters address can be located outside of California: for example, some regulated entities, such as transportation fuel suppliers, have compliance obligations that are not linked to a particular facility within California.

We then match the names and addresses of the facilities of regulated entities in the compliance-level sample to the names and addresses of establishments in the National Establishment Time Series (NETS) dataset. A direct match between NETS establishments and CARB facilities does not necessarily hold.

¹⁹Additionally, sellers can provide invalidation insurance within the contract, covering the offset for different periods within either the three-year or eight-year windows. However, since this insurance is provided by a private party, the offset buyer is exposed to counterparty risk.

²⁰The full list of invalidation events is available [here](#), accessed November 6, 2025.

²¹The data are publicly available on the [CARB Program Data Dashboard](#), Accessed Nov 6, 2025.

The reason is that the NETS establishment and CARB facilities definitions are different: NETS defines an establishment as a specific business location tied to an economic activity, whereas CARB defines a covered facility by its emission sources. As a result, one CARB facility can match to multiple NETS establishments or vice-versa because CARB regulates legal or operational units responsible for emissions—such as distinct processes—rather than the single physical establishment captured in NETS.

Given these data limitations, we take two nested approaches. The first approach is to only keep regulated entities whose facilities can be matched to one or multiple NETS establishments. We call this sample the establishment-level sample, which comprises 154 unique regulated entities. The second approach matches regulated entities directly to the firm headquarters. We call this sample the firm-level sample. The firm-level sample is larger and comprises 345 unique entities. As illustrated in Figure 3, the establishment-level sample is nested in the firm-level sample, and the firm-level sample is a subsample of the compliance-level sample. Note that our unit of analysis is always at the regulated-entity level, but we use the terminology compliance-level (all entities in the CARB database), establishment-level (CARB entities whose facilities are matched to NETS establishments) and firm-level (CARB entities whose firm headquarters is used to determine all NETS establishments linked to the firm) for ease of exposition. The matching procedure is described in detail in Section D of the Online Appendix.

We then extract relevant information by combining NETS accounting and financial data with regulatory information from CARB. Specifically, we focus on two statistics: sales, and pollution intensity, measured as sales per unit of pollution. Sales are obtained from NETS while pollution is obtained from CARB. We compute these statistics for both the firm-level and establishment-level samples. In the establishment-level sample, we aggregate sales from the matched CARB facilities to obtain the sales for each regulated entity in a given year. This approach plausibly reflects the economic activity of the entity attributable to activities based in California. In the firm-level sample, we aggregate sales across all establishments in the United States belonging to the firm linked to each regulated entity in a given year, thereby capturing the overall economic activity of the firm. This can be useful to capture the economic activity of firms who generate revenues from business in California and are therefore regulated under CARB without having any facilities located there. For example, this may be the case for transportation firms that are headquartered outside of California.

We primarily focus on the establishment-level sample in our analysis as it focuses directly on the regulated operating facilities, providing a precise link to the regulation. However, the firm-level sample is also useful to study since it covers a larger universe of entities regulated under CARB than the establishment-level sample. However, it also includes activity from unregulated sources. Given these trade-offs, we present results using both samples.

We use the Summary of Market Transfers Report data to obtain annual offset and allowance prices.²² For every year, the report provides a summary of the transfers of allowances and offsets between entities registered in the Compliance Instrument Tracking System Service (CITSS) and subject to the California or the Quebec Cap-and-Trade program. While all the other data we use are measured only for California and CARB-regulated entities, the price data are averages of all transactions across Quebec and California.²³ From the report, we collect the number of transactions between entities registered in CITSS, as well as an annual weighted average price in USD per 1 ton of CO₂ equivalent. We then convert these annual prices into compliance-period prices by taking a quantity-weighted average of the annual prices within each compliance period.

These price data are representative of secondary market transactions for allowances and offsets, but they do not distinguish offset prices by invalidation risk. We therefore supplement these data with prices from the vendor cCarbon, which provides monthly average transaction prices for offsets aggregated across multiple brokers. Using these data, we recover offset prices by invalidation category. Our analysis focuses on CCO-8 offsets, which expose buyers to an eight-year invalidation window, and CCO-3 offsets, which expose buyers to a three-year invalidation window. These two offset types allow us to construct a market-level measure of invalidation risk following the procedure described in Section 3 and Table E of the Online Appendix.

2.3 Summary Statistics

Table 1 presents summary statistics for the key variables used in the analysis, for the three levels of matched data: firm, establishment, and compliance. These variables represent measures of economic

²²The data are publicly available on the [Summary of Market Transfers Report](#), Accessed Nov 6, 2025.

²³California's Cap-and-Trade Program has been linked with Quebec's Cap-and-Trade program since 2014 under the Western Climate Initiative, forming a single integrated carbon market. Allowances and eligible offsets are mutually fungible across jurisdictions, with transactions recorded in the CITSS. Consequently, reported secondary market prices reflect aggregated activity from both California and Quebec rather than jurisdiction-specific markets.

activity, captured by sales, at either the firm or establishment level, as well as carbon productivity measured by sales per unit of total surrendered compliance instruments. Additionally, we define the offset ratio as the share of an entity's compliance obligation met with carbon offsets. For all three samples, the temporal dimension corresponds to the compliance period, and the unit of observation is the regulated entity.²⁴

Panel A of Table 1 shows the sample that is matched at the firm-level. Firms are large on average, with total firm-level sales per compliance period averaging approximately \$29.3 billion, with wide variation (standard deviation of \$98.43 billion). The ratio of firms' sales to pollution has a median of just over \$3,000 per metric ton of CO₂e. Figure A.1 plots the distribution of the log of this ratio by compliance period.

Panel B of Table 1 reports descriptive statistics for the sample of entities that can be matched at the establishment level. This sample contains only the facilities that are directly matched to NETS establishments. Establishment-level sales over pollution has an average of \$5,833 per metric ton of CO₂e. Similarly, Figure A.2 plots the distribution of the log of this ratio by compliance period.

Finally, panel C of Table 1 summarizes the data from all 383 compliance entities. On average, a regulated entity surrenders 4 million allowances, corresponding to 4 million metric tons of CO₂e. Roughly 45% of entities use offsets (dummy mean = 0.447), but the offset ratio, i.e., the share of emissions obligations satisfied by offsets, is low on average (mean = 0.03 and median = 0). In line with our theoretical assumptions, Figure 4 shows that firms either use offset directly at the 8% compliance limit or don't use offset at all. The corresponding distribution of offset usage is highly dispersed, with a mean of over 300,000 offsets surrendered, corresponding to 300,000 metric tons of CO₂e.

Finally, Figure 5 shows monthly prices of CCO-3 and CCO-8 offsets in California's Cap-and-Trade market from 2013 to 2021 as provided by the vendor cCarbon. CCO-3 offsets trade at a modest but persistent premium relative to CCO-8 offsets, consistent with their lower exposure to invalidation risk.

²⁴To match the frequency of the compliance emissions data, annual sales are aggregated at the compliance period level. The first compliance period spans two years, while the second and third compliance periods each span three years. Additional details on all variable definitions are provided in Section E of the Online Appendix.

2.4 Empirical Tests

Before describing our structural estimation approach, we qualitatively test Proposition 1 of the model, which states that firms with higher (carbon) productivity ξ , measured in the data as sales per unit of emissions obligations, are less likely to use offsets.

We begin with graphical evidence. Figure 6 plots the binscatter of offset usage on our proxy for carbon productivity. Panel A and B focus on the establishment-level sample, while Panel C and D focus on the firm-level sample; Panel B and D control for a compliance period fixed effect. Overall, in line with the model's predictions, we observe a negative relationship between offset ratios and firms' carbon productivity, as captured by sales over pollution.

Figure 7 plots the histogram of firms' and establishments' sales over pollution, for two different groups: first, the group of entities that never use offsets, and second, the group of entities that surrender some offsets. Overall, the graphical evidence confirms the pattern shown in the binscatter: firms that are more carbon productive rely less on offsets.

A Kolmogorov–Smirnov (KS) test supports the graphical evidence that firms using offsets have lower carbon productivity than firms that do not use offsets. The two-sided KS test rejects the null hypothesis that the two distributions are equal ($p < 0.001$). The one-sided KS test rejects the null hypothesis that the CDF of sales-over-emissions for firms not using offsets lies above that of firms using offsets ($p < 0.001$). Conversely, the test fails to reject the null hypothesis that the CDF of sales-over-emissions for firms using offsets lies above that of firms not using offsets ($p = 1.000$). These results indicate that the distribution of carbon productivity for firms not using offsets first-order stochastically dominates that of firms using offsets.

Table 2 explores the relationship between a firm's or establishment's productivity and the share of offsets surrendered. Columns (1), (2), and (3) use establishment-level data, while columns (4), (5), and (6) use firm-level data. Columns (2) and (5) include a compliance period fixed effect. Columns (4) and (6) add controls for the size of the entity: employment, number of establishments, and their interaction. They also include several fixed effects interacted with a period fixed effect. These fixed effects control for time-invariant differences across compliance entities that are public firms, located in the same state, owned by government entities, or operating in the same industry. Because these characteristics may

themselves capture some dimension of carbon productivity, these fixed effects may act as contaminated controls, in the sense that they may absorb part of the variation of interest. That said, across all model specifications, higher carbon productivity at the establishment or firm-level is significantly negatively associated with the offsets ratio. The coefficient on sales over pollution is consistently negative and highly statistically significant ($p < 0.01$). The economic magnitudes are meaningful. For instance, a 10% increase in sales-over-emissions is associated with a reduction in the offset ratio of about 1.1% relative to its mean. The inclusion of period fixed and other controls effects slightly improves the explanatory power and reinforces the robustness of these patterns.

Table A.2 of the Online Appendix repeats the baseline regression using an alternative method of aggregating sales at the compliance period level.²⁵ The results are very similar to those in the baseline analysis. Overall, the empirical exercise supports the hypothesis that more carbon-efficient firms are less reliant on offsets for compliance purposes.

3 Estimation

We estimate the model at the compliance-period level using data from the first three California Cap-and-Trade compliance periods (2013-2020), during which the offset-usage limit was fixed at 8%. All parameters are obtained analytically by matching theoretical moments to their sample counterparts.

For the estimation, we fit a lognormal productivity distribution to the establishment-level distribution of sales per unit of pollution (Figure 7), obtaining a mean log productivity of 4.83 and a standard deviation of 3.27. We further assume a linear disutility from emissions, $\lambda(x) = \lambda x$, and set $\lambda = \$190$ per metric ton of CO₂e, in line with recent recommendations by the Environmental Protection Agency (EPA).²⁶

Targeted Moments: We target six moments—the regulatory offset usage limit, the implied probability of invalidation, the average prices of carbon allowances and carbon offsets (from transactions between CITSS registered entities), the total quantity of retired carbon offsets, and the total surrendered allowances. The first column in Table 3 summarizes all targeted moments.

²⁵For 3.9% of our sample, sales for every year of the compliance period are not observed. Rather than treating these missing observations as zeros, we replace them with the average sales per observed year for each entity. This approach allows us to impute missing sales, rather than implicitly assuming that an entity with unobserved sales generates no revenue during that year.

²⁶The report is available [here](#).

For the shorter 2013–2014 compliance period, we multiply the total quantity of retired carbon offsets, and the total surrendered allowances by 3/2 to ensure that all compliance period observations are on a common three-year scale. The average implied invalidation probability is inferred from the monthly price spread between CCO-8 and CCO-3 offsets reported in Figure 5, building on the framework of Litterman and Iben (1991). We assume that the risk-neutral probability of offset invalidation in any given year is q . Under the risk-neutral measure, all assets earn the same expected return, implying $\frac{(1-q)^8}{p_{\text{CCO-8}}} = \frac{(1-q)^3}{p_{\text{CCO-3}}}$. CCO-8 offsets are subject to an eight-year invalidation window during which the buyer (i.e., the regulated entity) bears the invalidation risk. In contrast, for CCO-3 offsets, this risk drops to zero after three years. We use this relationship to back out the implied invalidation probability ρ , setting our measure of invalidation risk to $\rho = q$.²⁷

Figure 8 shows time-series plots of the main targeted moments, broken down by compliance period. Panel A of Figure 8 displays the average prices of compliance allowances and carbon offsets for each period, weighted by the volume of instruments traded during the year. Panel B of Figure 8 shows the average implied probability of invalidation, derived from the price spread between CCO-8 and CCO-3 offsets. Panel C of Figure 8 shows the average number of compliance instruments and offset credits surrendered by regulated entities in each compliance period.

Untargeted Moments: We also use total sales and productivity thresholds as untargeted moments to validate the estimation, reported in the second column of Table 3. The log threshold productivities, $\log(\underline{\xi}) = 2.78$ and $\log(\bar{\xi}) = 6.81$, corresponding to the 25th and 75th percentiles of the empirical productivity distribution, identify the lowest- and highest-productivity regulated entities that use offsets, respectively.

Total sales at the establishment-level are not observed for all regulated entities. As a result, we construct a variable of imputed sales as follows. First, when establishment-level sales are observed, we use them directly but only consider regulated entities with sales-to-emissions ratio equal to or greater than the estimated $\underline{\xi}$. This because, in the model, regulated entities with carbon productivity below $\underline{\xi}$ do not produce. Second, when establishment-level sales are missing, but firm-level sales are available, we impute establishment sales using the median establishment-to-firm sales share estimated from the (truncated)

²⁷Our model assumes risk-neutral firms and regulators, implying that the physical and risk-neutral probability measures coincide. In practice, if market participants are risk-averse and invalidation risk is not diversifiable, the market-implied q would exceed the physical invalidation probability.

establishment-level sample ($0.09335 \times$ firm sales). Third, if both establishment-level and firm-level sales are missing, we impute sales by using the average ratio of (truncated) establishment-level sales to surrendered compliance instruments observed in the establishment-level sample ($327.6703 \times$ surrendered instruments).²⁸

Estimation Procedure: Below, we detail each step of the estimation procedure. The first column of Table 4 reports the estimated parameters.

Equations 2 and 3 give us the threshold carbon productivities, $\bar{\xi}$ and $\underline{\xi}$, as a function of four of our target moments—the carbon allowance price, the carbon offset price, the offset usage limit and the invalidation probability. Using the implied $\bar{\xi}$ and $\underline{\xi}$, we can calculate the implied $\bar{i}$ and $\underline{i}$.

Next, the offset producer’s first order condition (equation 4) gives us the parameter ϕ , which governs the price of producing offsets as a function of two of the target moments—the average number of total offsets retired per compliance period and the price of carbon offsets. We estimate ϕ as being close to 0. This implies that offsets are very cheap to produce as evidenced by the high supply of offsets available at relatively cheap prices.

We then use the offset market clearing (equation 6) to calculate the implied $\eta\bar{k}$. We truncate the productivity distribution at $\xi_{\max} = 2424$ to satisfy market clearing for the carbon allowance market (equation 5) to exactly match our target moment of total surrendered emissions. We then use our estimates to calculate the model-implied aggregate regulated entity sales.

Interpreting invalidation probability: Given the very low invalidation probability, there are two possible explanations. Either (i) offsets are of very high quality which implies that δ is close to 0, or (ii) the invalidation cost κ is prohibitively high. In case (i), the regulator wants to choose an α close to 1 and allow for as many offsets as possible as offsets increase output without generating additional emissions. Given the low observed α in practice, we conclude that we must have case (ii) — the invalidation cost is so high that the regulator does not invalidate bad offsets.

From Proposition 3, we have a closed form solution for α for the case when invalidation is prohibitively costly for the regulator. Using equation 10, we can calculate the implied $\lambda\delta$ at an offset usage limit of

²⁸Once again, to place all observations on a common three-year scale, consistent with the scaling applied to surrendered offsets and allowances, total imputed sales for the 2013–2014 period are multiplied by 3/2.

8%. We can interpret λ as the social cost of carbon to obtain an estimate of offset quality, δ . Table 4 reports the estimated parameters at a social cost of carbon of \$190 per metric ton of CO₂e.²⁹

We note that because the system is just identified, the estimation procedure is equivalent to GMM with an identity weighting matrix. We use six moments (offset usage limit, the invalidation probability, the prices of carbon allowances and carbon offsets, the quantity of retired carbon offsets and surrendered allowances/emissions) to recover six parameters ($\bar{\xi}, \underline{\xi}, \phi, \bar{k}\eta, \lambda\delta$, and ξ_{max}).

Bootstrap procedure: We estimate the standard errors of the structural parameters using a bootstrap procedure designed to preserve the structure of the original dataset, which consists of cross-sectional observations across three compliance periods. In each bootstrap replication ($N = 10,000$), we draw with replacement from the cross-sectional data within each compliance period separately. Specifically, for each compliance period, we generate a resampled dataset by sampling with replacement from that period’s original cross-section. We then concatenate the resampled data from all three periods to reconstruct a full dataset with the same compliance-period structure as the original. In parallel, for each replication, we draw allowance and offset prices from the annual average prices of the corresponding compliance period (from transactions between CITSS registered entities). We feed the bootstrapped panel-level data and resampled prices into the structural estimation routine, obtaining one set of estimated parameters per replication. Repeating this process 10,000 times yields a distribution of parameter estimates, from which we compute standard errors as the standard deviation across replications. In Table 4 we report the means, standard errors, and confidence intervals calculated using the bootstrap procedure.

3.1 Offset Quality, Uncounted Emissions, and the Output Emissions Frontier

We can interpret λ as the social cost of carbon. Estimates of the social cost of carbon vary substantially. The Obama administration used a value of \$51 per metric ton of CO₂e, while the Biden administration

²⁹For robustness, we also perform a numerical estimation to determine offset quality without imposing the simplifying assumption in the final analytical step that $\rho = 0$. Specifically, we implement a GMM procedure to estimate κ and δ by matching the model-implied optimal policy moments (α^*, ρ^*) to their data counterparts (α, ρ) . We fix the input parameters estimated analytically $(\phi, k\eta, \xi_{max})$, use an identity weighting matrix, and assume a quadratic invalidation cost $\kappa(Q_o\rho)^2$. The resulting estimate of offset quality is similar: $\hat{\delta} \approx 0.27$, implying that about 27% of offsets are of low quality. At these estimates, the regulator chooses $\alpha^* \approx 8\%$ and a low invalidation probability $\rho^* \approx 0.07\%$. The estimated κ is 0.59, and the minimized GMM criterion (“ J -statistic”) is 2.83×10^{-5} . This numerical estimation also facilitates counterfactual exercises with respect to ρ , which we plan to explore in future work.

used a value of \$190.³⁰ The California Air Resources Board states that it relies on EPA values for the social cost of carbon of between \$145 and \$403 depending on the discount rate used.³¹

Figure 9 plots the implied fraction of good offsets, $1 - \delta$, as a function of the social cost of carbon, λ . A social cost of \$190 per ton, implies that 67% (33%) of offsets are of good (bad) quality. A social cost of \$51 per ton implies that all offsets are bad quality. Figure 9 also plots the implied fraction of uncounted emissions produced by bad offsets as a function of the social cost of carbon. A social cost of \$190 (\$51) per ton implies that over our sample period, each compliance period had 2% (8%) of uncounted emissions above the cap set by the state. Assuming no change in the social cost of carbon, firm productivity, or the carbon cap, the reduction of the offset limit to 4% implies that offset quality has fallen such that 41% of all offsets are of bad quality.

Based on our estimated offset quality, we can quantify the tradeoff faced by the regulator when choosing the offset usage limit. The first plot of Figure 10 plots the output emissions frontier, i.e., the percentage increase in emissions relative to the percentage increase in output given our estimated implied offset quality for two different values of the social cost of carbon, $\lambda = \$51$ per ton and $\lambda = \$190$ per ton. The output emissions frontier is traced by varying α for a given social cost of carbon, and a black dashed line shows the 45-degree line. The frontiers are above the 45-degree line, demonstrating that for both values of the implied offset quality, the percentage increase in emissions at a positive α is higher than the percentage increase in output. In other words, aggregate emissions intensity increases as we increase α . This occurs because firms with worse emission intensity start producing when α increases, resulting in a move to the right of the frontier as the trade-off between emissions and output worsens.

The lower the social cost of carbon, the steeper the output emissions frontier, because lower social costs of carbon imply worse offset quality. The second plot of Figure 10 shows the tradeoff between emissions and output fixing α at 8% as a function of the social cost of carbon. A social cost of carbon of \$190 implies that emissions increase by roughly a factor of 9 compared to the increase in output when α is 8% relative to a benchmark of $\alpha = 0\%$.

³⁰The report is available [here](#).

³¹The information is available [here](#).

3.2 Counterfactuals

We consider two counterfactuals using the estimated model. First, we consider what output would be if instead of having offsets, the regulator increases the carbon cap by the amount of uncounted emissions. Figure 11 plots the additional output per offset relative to simply raising the carbon cap as a function of the social cost of carbon. In general, as long as at least some offsets are good quality (i.e., $\delta < 1$) adjusting the carbon cap produces less output than using offsets. This occurs because good quality offsets actually remove carbon allowing for some output gain without increasing emissions. At a social cost of \$190 per ton, each offset generates an extra \$10 in output relative to raising the carbon cap. If the costs of establishing and running the offset market exceed this amount, the regulator is better off not allowing offsets and raising the carbon cap instead. As offset quality worsens, i.e. for lower values of the social cost of carbon, the additional output generated per offset decreases. At a social cost of \$51 per ton, it is better to raise the carbon cap rather than using offsets.

Second, we also consider implied offset quality if the regulator places a higher weight on production by small firms. In this case, the net social cost of carbon that the regulator would apply to small firms should be smaller than the one they would apply to large firms. The regulator will use the marginal social cost of carbon when making policy decisions. Because offset usage allows less productive firms, which are typically small firms, to start producing, the regulator will therefore use the social cost of carbon for small firms when choosing α and ρ . If the average social cost of carbon is \$190 per ton, we can calculate the difference in the social cost of carbon for small and large firms as we vary the marginal social cost of carbon (i.e. the social cost of carbon for small firms).

In particular, we define "small firms" as those whose productivity is below the median of productive firms and "large firms" as those whose productivity is above the median. We define λ_S as the social cost of carbon for small firms and λ_H as the social cost of carbon for large firms. Then, if the average social cost of carbon is \$190 per ton we have

$$190(E_S + E_L) = \lambda_S E_S + \lambda_H E_L,$$

where E_S and E_L are emissions by all small firms and all large firms respectively.

The first plot in Figure 12 shows the implied social cost of carbon for large firms as a function of the social cost of carbon for small firms. When λ_S is \$190 per ton, λ_H is also \$190 per ton and the regulator values production by small and large firms equally. As λ_S decreases below \$190 per ton, λ_H increases. The second plot in Figure 12 plots the implied discount a small firm gets relative to a large firm on the social cost of carbon. A higher small firm discount corresponds to a lower marginal social cost of carbon, which in turn implies worse offset quality and more uncounted emissions. This analysis helps to provide an estimate of offset quality for any given carbon discount applied to small firms. For example, if regulators are likely to use a social cost of carbon for small firms which is about 40% that of what they apply to large firms, then the implied λ_S is 100 (Figure 12). Based on our estimates, this would imply that only 40% of offsets are good quality (Figure 9)—27% less than the offset quality we estimate under the assumption that the regulator applies the same social cost of carbon to small and large firms.

4 Conclusion

Our paper highlights the nuanced role of carbon offsets within cap-and-trade systems. By combining theory and evidence from California’s program, we demonstrate that offset usage is not uniform but instead reflects systematic sorting by firms: highly productive firms avoid offsets due to the risks of invalidation, while less productive firms rely on offsets up to the regulatory limit. This sorting mechanism underscores that offsets are not a neutral compliance tool but one that shapes which firms expand production and how the burden of emissions reduction is distributed across the economy.

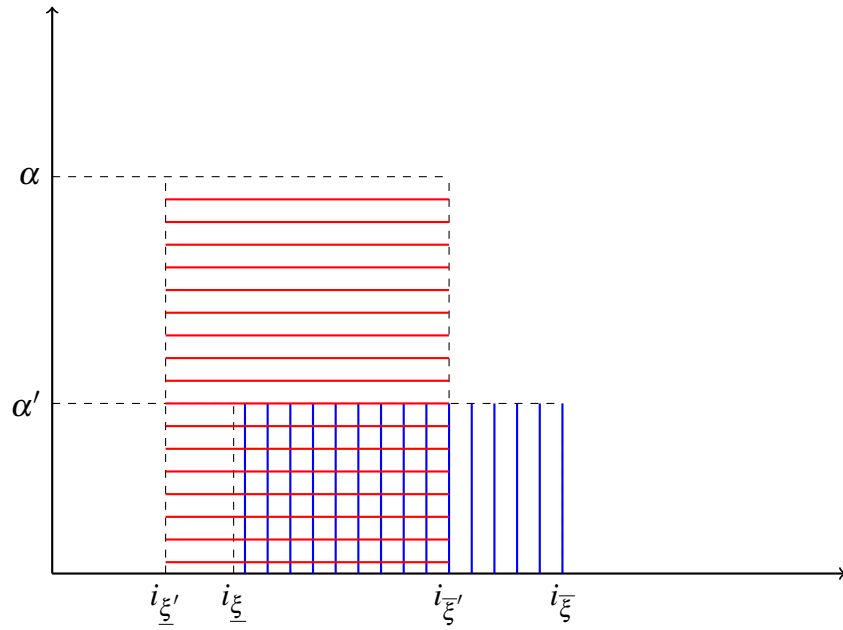
Our structural estimates reveal both the promise and the shortcomings of offsets. At a social cost of carbon of \$190, while roughly 67% of offsets deliver genuine reductions, the remaining 33% contribute to uncounted emissions, thus undermining the integrity of the cap. At the same time, we show that even imperfect offsets can enable more output than a simple relaxation of the emissions cap, because good offsets allow some production to take place without increasing emissions. These findings stress the regulator’s dilemma: invalidation is prohibitively costly, yet failing to regulate offset quality generates additional emissions above the cap. Overall, optimal policy in this growing market will need to carefully balance offset limits, the social cost of carbon, and the realities of enforcement.

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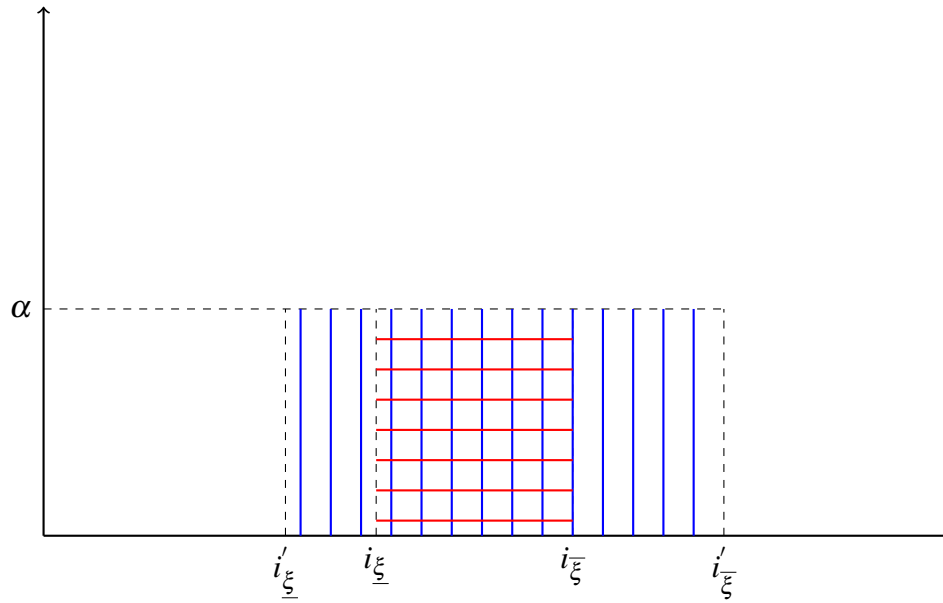
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Figure 1: Offset Usage at Different Regulatory Limits



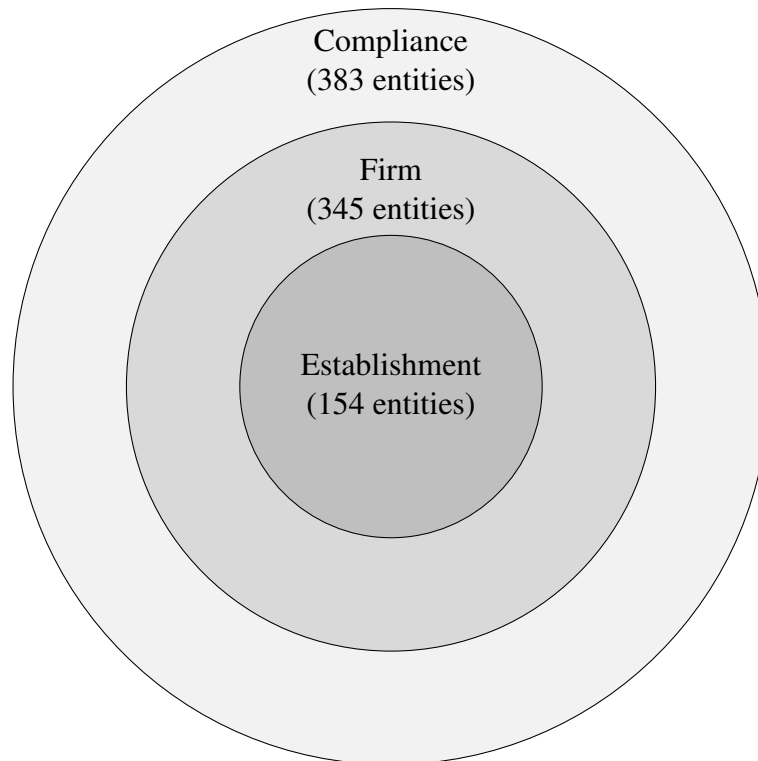
In the plot above, the red horizontal lines illustrate offset usage for a higher α , while the blue vertical lines represent offset usage for a lower α . As α increases, the measure of firms using offsets decreases, representing an extensive margin effect. Simultaneously, the offset usage per firm, conditional on the firm using offsets, increases, capturing an intensive margin effect. The latter effect always dominates the former effect, causing total offset usage to increase with α .

Figure 2: illustration



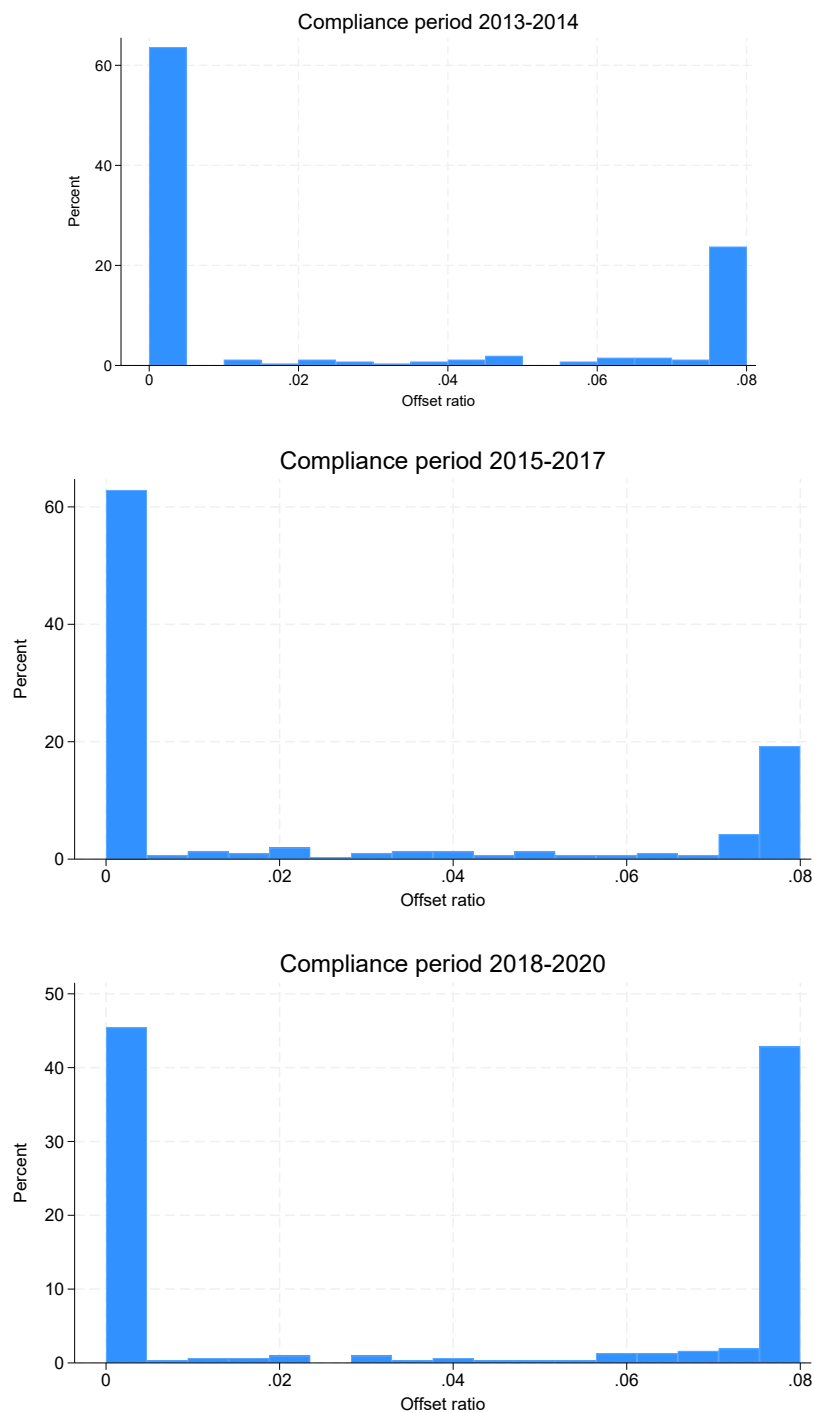
In the graph, the blue vertical lines represent offset usage for a lower invalidation probability, ρ , while the red horizontal lines illustrate offset usage for a higher invalidation probability. As ρ increases, only the extensive margin effect operates: the measure of firms using offsets decreases. Since the intensive margin effect is absent, total offset usage decreases solely due to the extensive margin effect.

Figure 3: Samples used



The figure displays a Venn diagram illustrating the three nested samples used in the empirical analysis. The compliance-level sample includes all compliance entities from the 2013-2014, 2015-2017, and 2018-2020 compliance periods and contains 383 unique entity IDs. The firm-level sample consists of the subset of regulated entities successfully matched at the firm level across the same compliance periods, totaling 345 unique entity IDs. The establishment-level sample represents the subset of establishments matched at the entity level to regulated entities in the cap-and-trade and includes 154 unique entities.

Figure 4: Histograms: offset ratios



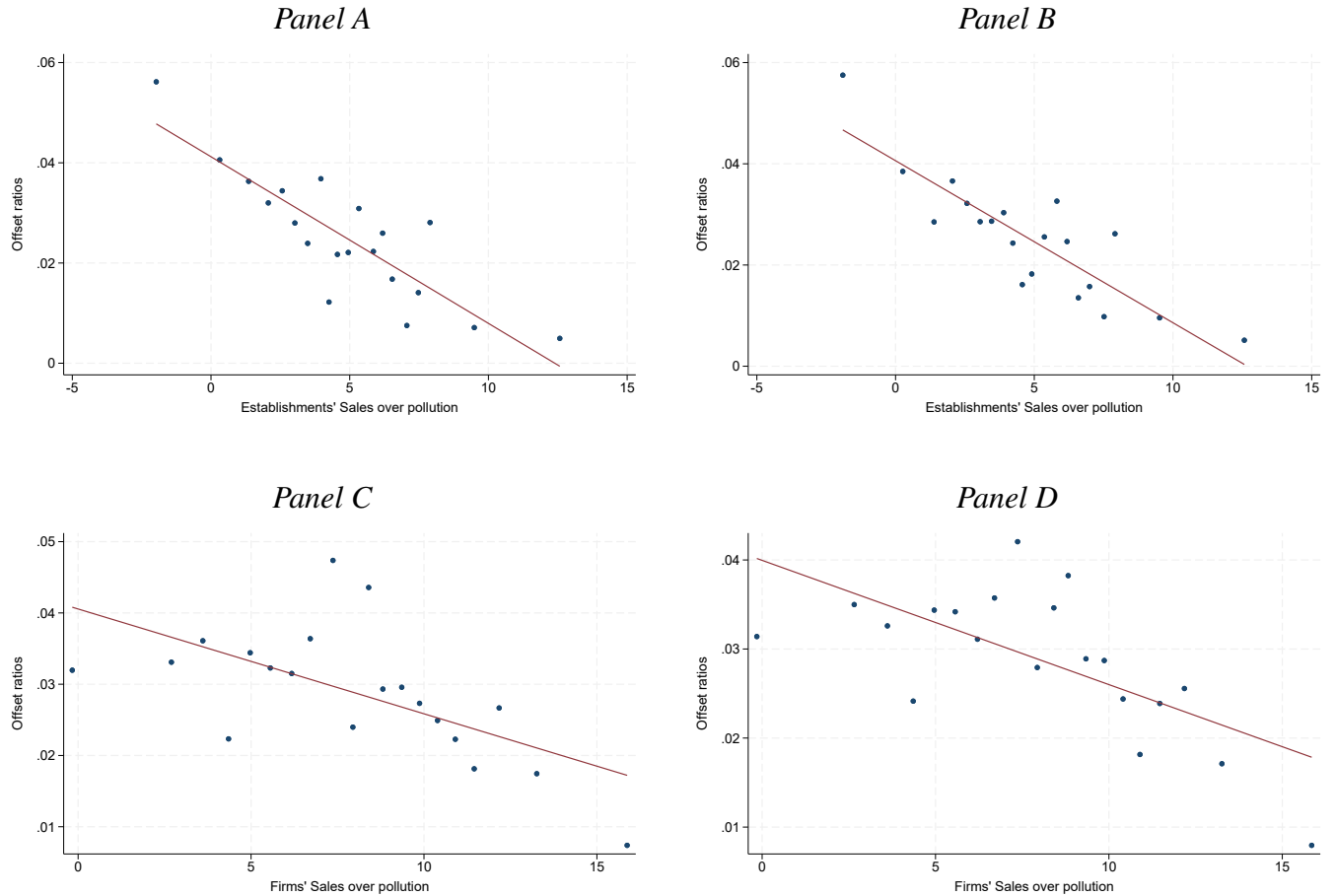
This graph plots the distribution of offset ratios for each compliance period, that is, the fraction of offset used in the instruments surrendered. Each distribution corresponds to one of the compliance periods used in the empirical analysis. Source: CARB's Cap-and-Trade Program Data Dashboard.

Figure 5: Offset prices over time



This figure plots monthly average transaction prices for CCO-3 and CCO-8 offsets in the California Cap-and-Trade market from 2013 to 2021 as provided by the vendor cCarbon. CCO-3 offsets (blue) consistently trade at a small premium relative to CCO-8 offsets (red). Both series exhibit strong co-movement. The persistent premium of CCO-3 over CCO-8 provides visual evidence that invalidation risk is priced in offset markets. Prices are expressed in US dollars.

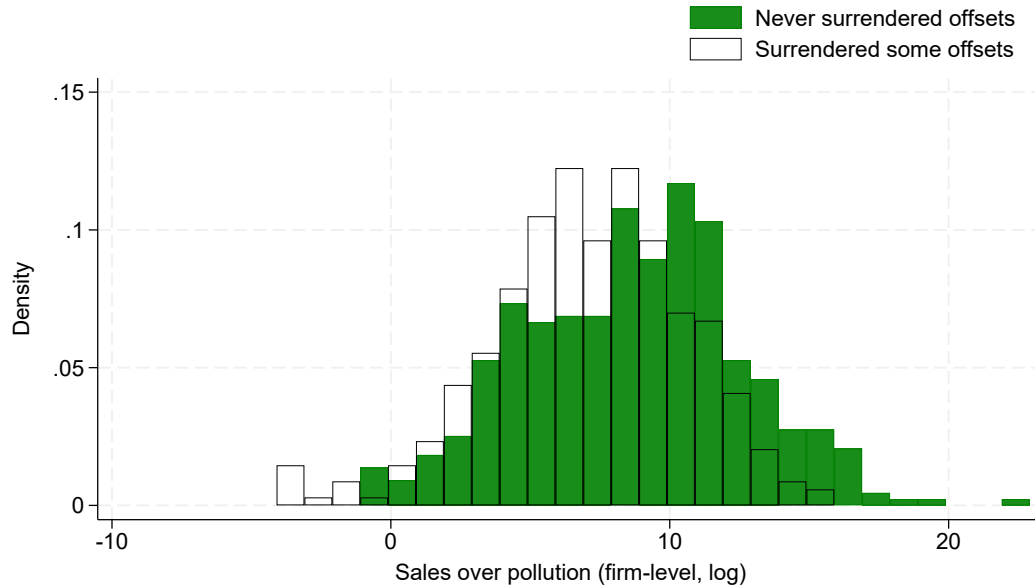
Figure 6: Productivity distribution and offset usage (intensive margins)



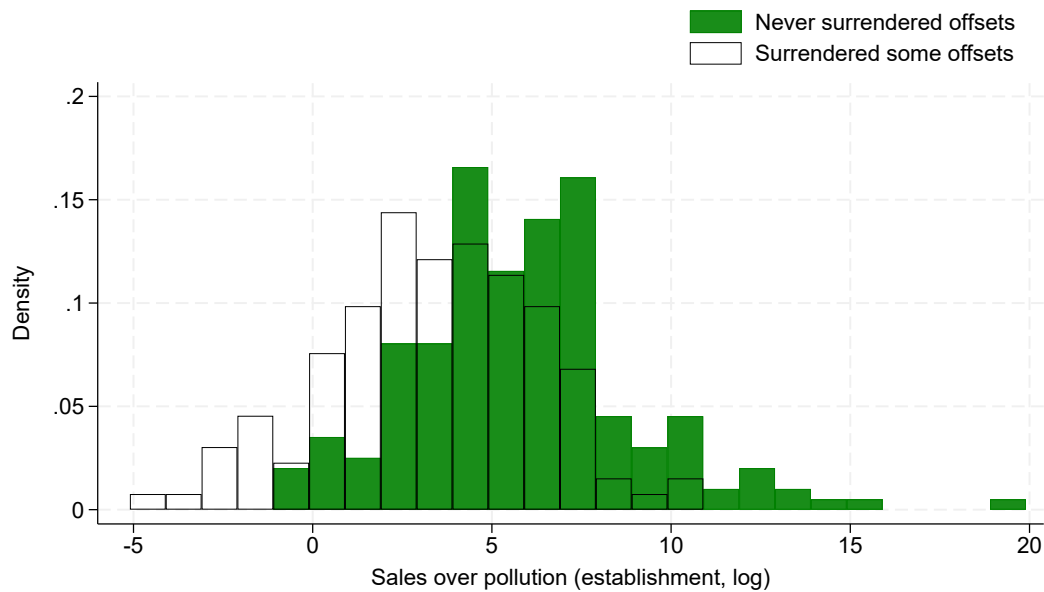
These graphs show the offset usage as a function of carbon productivity. Specifically, Panel A shows the binscatter between the Offset ratio and Sales (at the establishment level) over pollution, while Panel B shows the same relationship, but controls for a period fixed effect. Similarly, Panel C shows the binscatter between the Offset ratio and Sales (at the firm level) over pollution, while Panel D adds a period fixed effect. The Offset ratio is defined as the total number of offsets surrendered by an entity during a compliance period over the total number of instruments surrendered. Section E of the Online Appendix describes in details how the variables are constructed. Source: CARB's Cap-and-Trade Program Data Dashboard, NETs, cCarbon, and authors' calculations.

Figure 7: Productivity distribution and offset usage (extensive margins)

Panel A

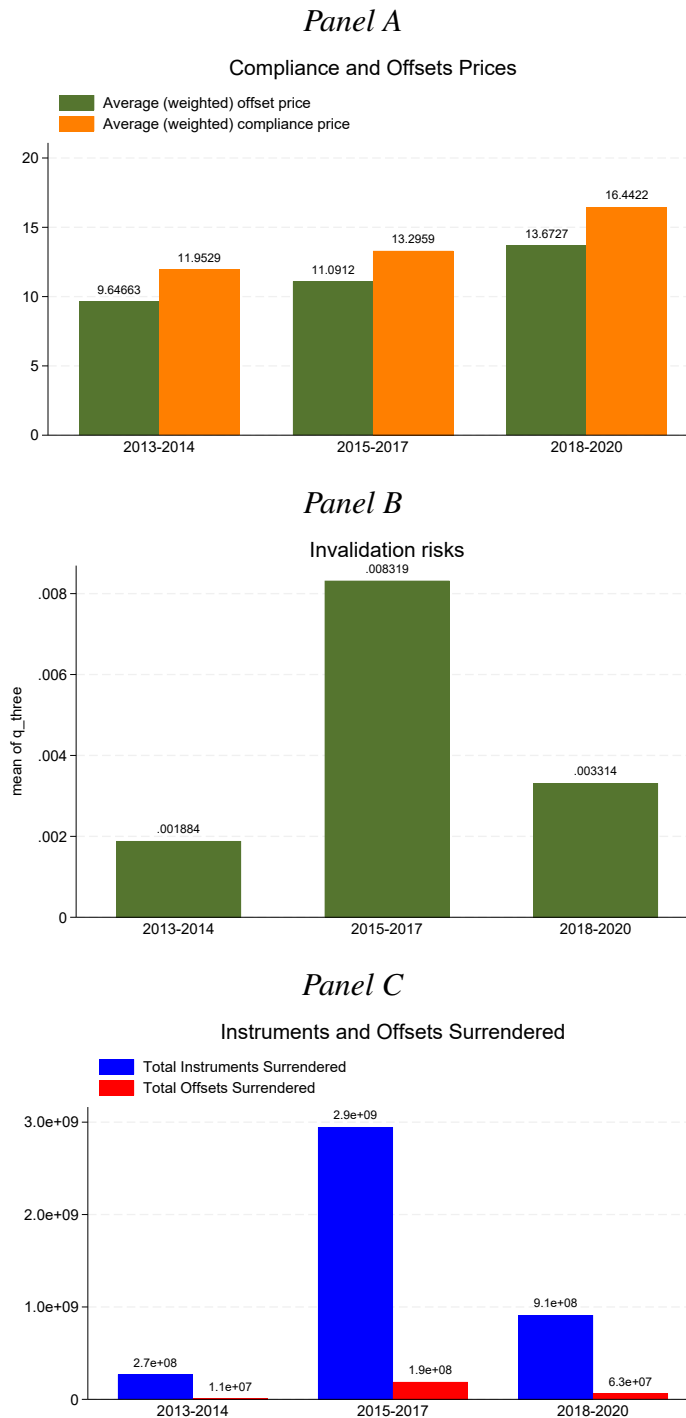


Panel B



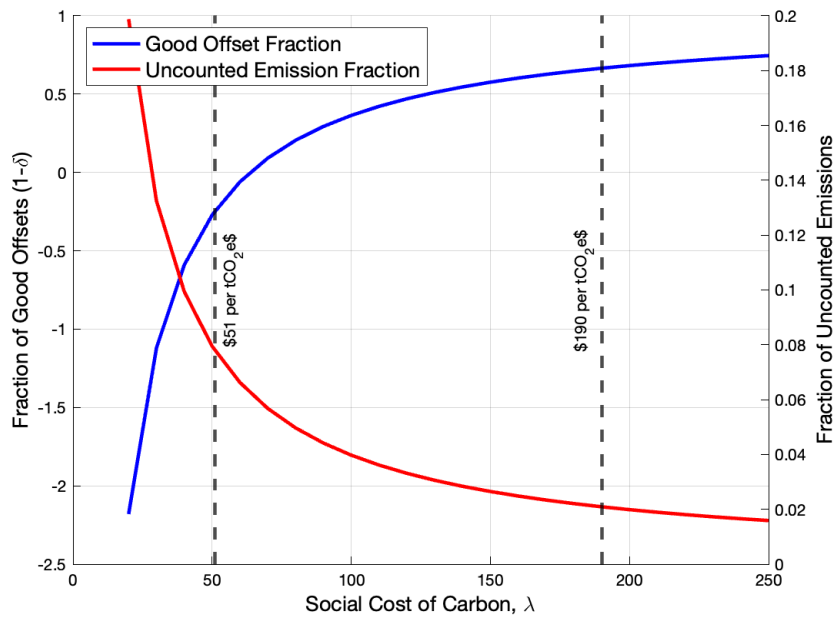
These graphs show the distribution of Sales over pollution, measured both at the establishment and firm level, grouped by offsets usage. Section E of the Online Appendix described in detail how the variables are constructed. Source: CARB's Cap-and-Trade Program Data Dashboard, NETs, Ccarbon, and authors' calculations.

Figure 8: Main Moments per compliance period



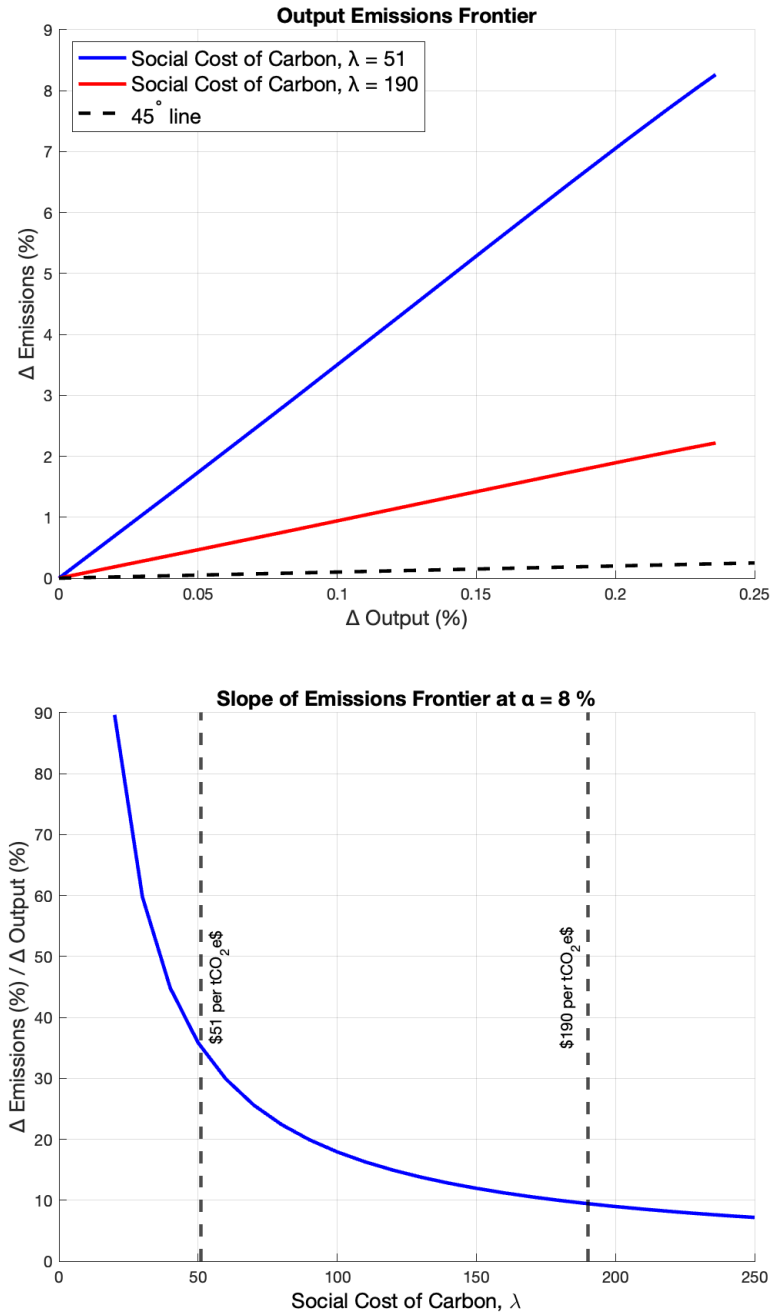
The graph plots the main moments used in the estimation for each compliance period. Panel A plots the average price of compliance allowance and carbon offset for each compliance period (rounded to 4 digits after the decimal point). The prices are expressed in USD per metric tons of CO₂e as provided by the Summary of Market Transfers Report and weighted by the amount of compliance instrument traded during the year. Panel B shows the average probability of invalidation as implied by the price differences between CCO-8 and CCO-3 monthly prices reported in Figure 5. Panel C shows the average number of compliance instruments and offset credits that are surrendered by the firms, on average, and per compliance period.

Figure 9: Offset Quality, Uncounted Emissions and the Social Cost of Carbon



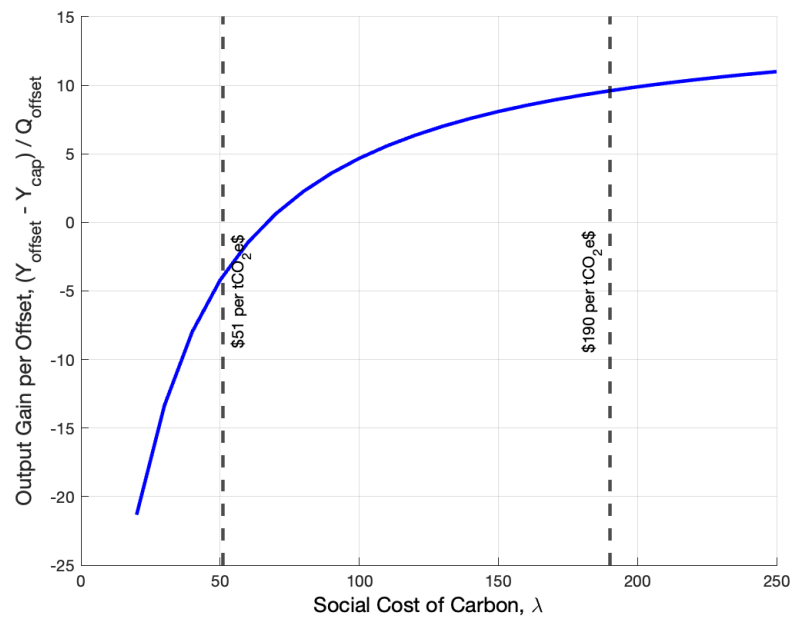
The plots above shows the estimated fraction of good offsets ($1 - \delta$) and fraction of uncounted emissions as a function of the social cost of carbon.

Figure 10: Output Emissions Frontier



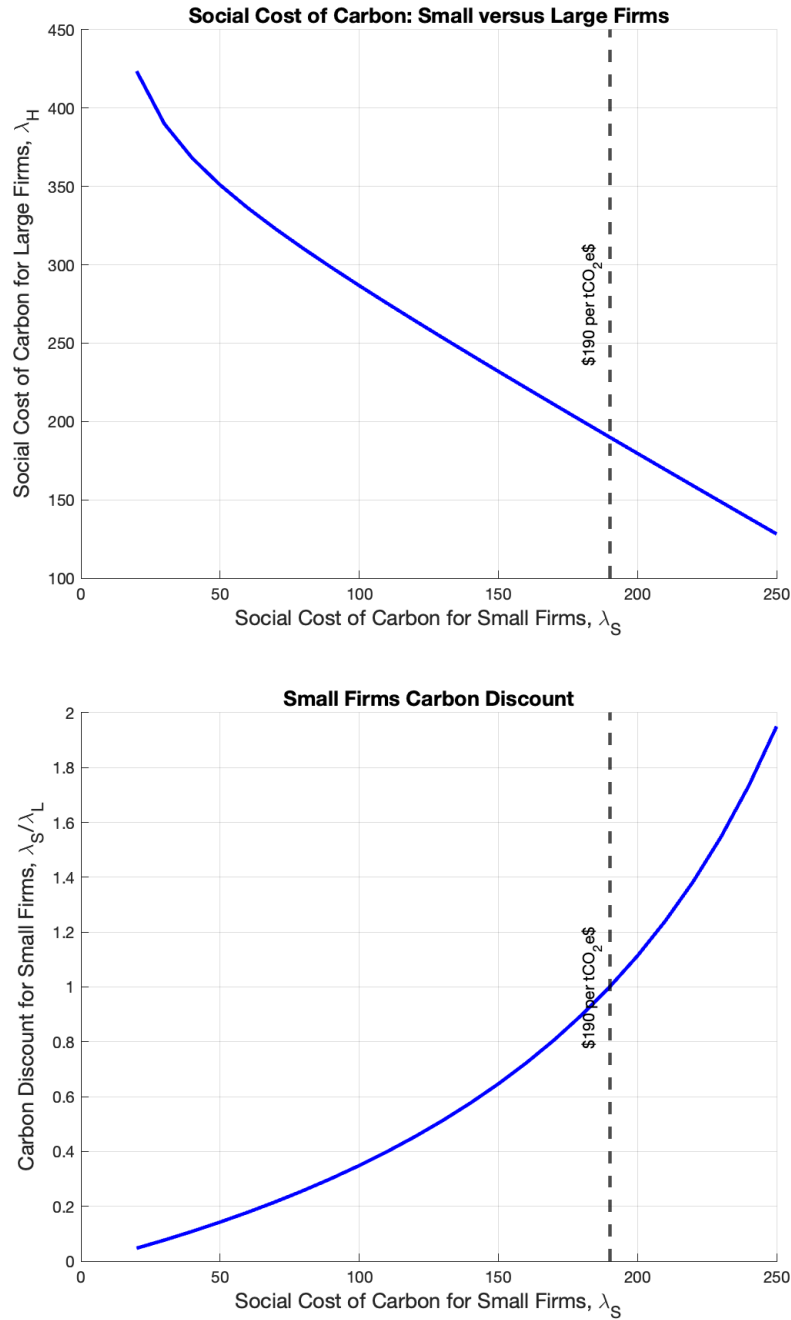
The upper plot shows the output emission frontier by varying α for different values of the social cost of carbon. The bottom plot shows the percentage increase in emissions relative to the percentage increase in output fixing $\alpha = 8\%$ against the social cost of carbon.

Figure 11: Carbon Cap Versus Offsets



The above plot shows the additional output that can be gained per offset relative to simply raising the carbon cap by the amount of uncounted emissions.

Figure 12: Social Cost of Carbon: Small versus Large Firms



The first plot above shows the estimated social cost of carbon for large firms assuming an average cost of carbon of \$190 per metric ton of CO₂e. The x-axis is the social cost of carbon for small firms which are the marginal firms that produce. The second plot plots the implied carbon discount for small firms as a function of the social cost of carbon for small firms.

Table 1: Descriptive statistics

	mean	sd	p25	p50	p75
Panel A: Firm-level					
Firm's Sales	29,368.901	98,425.167	48.566	1,087.678	11,331.747
Firms' Sales over pollution	253,039.092	938,399.841	182.014	3,676.500	46,176.336
<i>Unique Entities</i>	345				
Panel B: Establishment-level					
Establishment's Sales	226.644	566.195	5.966	28.788	219.706
Establishments' Sales over pollution	5,833.778	22,014.189	16.042	116.520	909.162
<i>Unique Entities</i>	154				
Panel C: Compliance-level					
1(Offset Surrendered)	0.447	0.497	0	0	1
Offset Ratio	0.030	0.036	0	0	0.08
Total Offsets Surrendered	297,579.151	4,310,265.655	0.000	0.000	22,924.000
Total Instruments Surrendered	4,679,603.617	67,037,585.573	68,664.000	181,422.000	896,783.000
<i>Unique Entities</i>	383				

This Table reports descriptive statistics for our main sample. The unit of analysis is at the entity level, and the time dimension corresponds to the compliance period. Sales, at both the firm and establishment levels, are reported in millions of USD. The variable Sales over Pollution represents a firm's or establishment's sales in USD per metric ton of CO₂e. 1(Offset Surrendered) is a dummy variable equal to one if the regulated entity surrenders at least one offset. The variables Total Number of Offsets Surrendered and Total Instruments Surrendered are measured in metric tons of CO₂e. Section E of the Online Appendix provides a detailed description of variable construction.

Table 2: Offset ratio and firm's productivity

	Dependent variable: Offset ratio					
	(Establishment-Level)			(Firm-level)		
Log(Sales over pollution)	-0.00333*** (0.000507)	-0.00320*** (0.000505)	-0.00319*** (0.000547)			
Log(Sales over pollution)				-0.00147*** (0.000375)	-0.00140*** (0.000372)	-0.00313*** (0.000567)
Observations	331	331	331	779	779	695
R-squared	0.10	0.15	0.16	0.024	0.071	0.35
Period FE		X	X		X	X
Controls			X			X
Period FE $\times$ Public FE			X			X
Period FE $\times$ State FE			X			X
Period FE $\times$ Gov FE			X			X
Period FE $\times$ Industry FE			X			X
Period FE $\times$ State FE			X			X

This Table explores the relationship between carbon productivity and the reliance on carbon offsets. The dependent variable is the Offset ratio. Columns (1), (2), and (3) report establishment-level regressions, while Columns (4), (5) and (6) report firm-level regressions. In Columns (1), (2), and (3), sales are defined at the establishment level, focusing on establishments that are directly matched to a regulated entity. In Columns (4), (5), and (6), sales are defined at the firm-level, and include all sales that belong to a firm that has some regulated establishments. The variable Controls includes proxies for the size of the entity—employment, number of establishments, and their interaction. Public FE, State FE, Government FE, and Industry FE control for time-invariant differences across compliance entities that are public firms, located in the same state, owned by government entities, or operating in the same industry. The Period FE is defined at the compliance level. The offset ratio is the total number of offsets used for compliance, divided by the total emissions obligation (winsorized at 2%). Section E of the Online Appendix describes in detail how the variables are constructed. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 3: Summary of Estimation Inputs

Panel A: Empirical Moments	Targeted	Untargeted
1. Average (per period) total offsets used (tCO ₂ e)	89,318,401	—
2. Average (per period) total emissions (tCO ₂ e)	1,420,597,530	—
3. Average (per period) aggregate sales (billion USD)	—	673
4. Average price of carbon allowance (USD/tCO ₂ e)	14.40	—
5. Average price of carbon offset (USD/tCO ₂ e)	11.94	—
6. Average invalidation probability (%)	0.46	—
7. Offset usage limit (%)	8.00	—
8. Log sales over pollution (establishment-level) of highest productivity firm using offsets	—	6.81
9. Log sales over pollution (establishment-level) of lowest productivity firm using offsets	—	2.78
Panel B: Additional Inputs	Value	
1. Average (establishment-level) log Sales over Emissions	4.83	
2. Std. dev. (establishment-level) log Sales over Emissions	3.27	
3. Social Cost of Carbon (USD/tCO ₂ e)	190.00	

The Table reports summary statistics for the sample of firms used in the estimation procedure. The column Targeted moments lists the moments that are exactly matched by the model, while the column Untargeted moments contains additional moments that are not used in the estimation. The lowest and highest productivity firm using offsets is based on the 25% and 75% percentiles. The full distribution is depicted in Figure 7.

Table 4: Estimated Parameters and Confidence Intervals

Parameter	Value	Mean	t-stat	95% C.I.
Log threshold productivity $\ln(\bar{\xi})$	6.26	6.26***	5.14	[5.78 ; 6.56]
Log threshold productivity $\ln(\underline{\xi})$	2.62	2.62***	37.78	[2.57 ; 2.66]
Indifferent firm $\bar{i}$	0.67	0.67***	20.80	[0.60 ; 0.72]
Indifferent firm $\underline{i}$	0.25	0.25***	12.98	[0.21 ; 0.29]
Production cost ϕ	1.3×10^{-7}	1.6×10^{-7} **	2.07	$[6.2 \times 10^{-8} ; 3.3 \times 10^{-7}]$
Total emissions capacity $\bar{k}\eta$ (million tCO ₂ e)	2,654	2,670**	2.03	[1,025 ; 5,712]
Aggregate output (billion USD)	628	656*	1.94	[220 ; 1,480]
Offset quality δ	0.33	0.37**	2.16	[0.14 ; 0.79]

The Table reports the mean of the estimated parameters, associated t-statistics, and 95% confidence intervals computed on the bootstrap distribution. The column Value shows the parameter estimates from the single estimation in which the targeted moments are exactly matched to their counterparts in the data.

Appendix

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A Additional Tables

Table A.1: Implemented Cap-and-Trade ETS and Cap-and-Trade-Adjacent Systems

Instrument name	Jurisdiction	Year implemented	Compliance Offsets
<i>Cap-and-Trade ETS</i>			
Beijing pilot ETS	Beijing	2013	Yes
California CaT	California	2013	Yes
China national ETS	China	2021	Yes
Chongqing pilot ETS	Chongqing	2014	Yes
EU ETS	EU	2005	No
Fujian pilot ETS	Fujian	2016	Yes
Guangdong pilot ETS	Guangdong	2013	Yes
Hubei pilot ETS	Hubei	2014	Yes
Kazakhstan ETS	Kazakhstan	2013	Yes
Korea ETS	Korea, Rep.	2015	Yes
Massachusetts ETS ^a	Massachusetts	2018	Yes
Mexico ETS	Mexico	2020	No
Montenegro ETS	Montenegro	2020	No
New Zealand ETS	New Zealand	2008	No
Oregon ETS	Oregon	2022	Yes
Quebec CaT	Quebec	2013	Yes
Saitama ETS	Saitama	2011	Yes
Shanghai pilot ETS	Shanghai	2013	Yes
Shenzhen pilot ETS	Shenzhen	2013	Yes
Switzerland ETS	Switzerland	2008	No
Tianjin pilot ETS	Tianjin	2013	Yes
Tokyo CaT	Tokyo	2010	Yes
UK ETS	United Kingdom	2021	No
Washington CCA	Washington	2023	Yes
RGGI	RGGI	2009	Yes
<i>Cap-and-Trade-Adjacent</i>			
Alberta TIER OBPS	Alberta	2020	Yes
Australia Safeguard Mechanism BCS	Australia	2016	Yes
BC OBPS	British Columbia	2024	Yes
Canada federal OBPS	Canada	2019	Yes
New Brunswick OBPS	New Brunswick	2021	No
Newfoundland and Labrador PSS EPS	Newfoundland and Labrador	2019	No
Nova Scotia OBPS ^b	Nova Scotia	2023	Yes
Ontario EPS	Ontario	2022	No

Compliance offset eligibility is based on the World Bank Carbon Pricing Dashboard - Crediting Mechanisms Dataset (accessed November 2025). Each ETS or adjacent system is matched to mechanisms listed under “Compliance CPIs accepting credits,” and only credits eligible for compliance are included. Eligibility years were verified using ICAP and regulator documentation. ^aMassachusetts participates in RGGI. ^bNova Scotia operated a cap-and-trade program (2019-2022) before transitioning to an OBPS in 2023. OBPS = Output-Based Pricing System; EPS = Emissions Performance Standards; BCS = Baseline-and-Credit System; CaT = Cap and Trade.

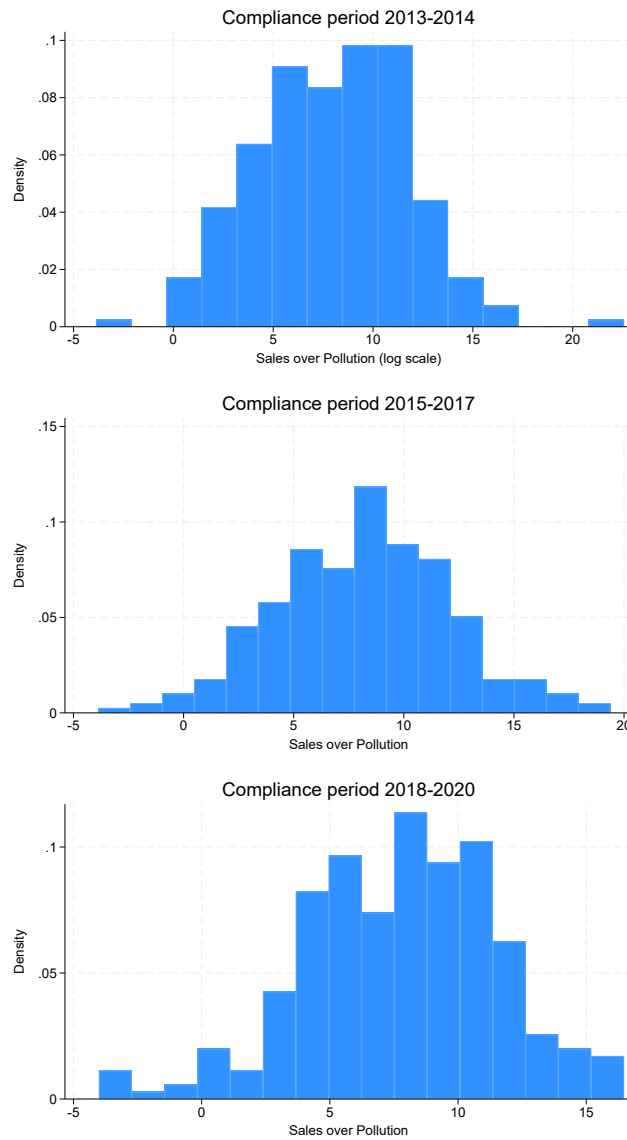
Table A.2: Offset ratio and firm's productivity

	Dependent variable: Offset ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Sales over pollution)	-0.00335*** (0.000514)	-0.00333*** (0.000511)	-0.00339*** (0.000550)			
Log(Sales over pollution)				-0.00145*** (0.000379)	-0.00144*** (0.000376)	-0.00243*** (0.000517)
Observations	331	331	331	779	779	695
R-squared	0.098	0.15	0.15	0.023	0.072	0.32
Period FE		X	X		X	X
Controls			X			X
Period FE $\times$ Public FE			X			X
Period FE $\times$ State FE			X			X
Period FE $\times$ Gov FE			X			X
Period FE $\times$ Industry FE			X			X
Period FE $\times$ State FE			X			X

This Table explores the relationship between carbon productivity and the reliance on carbon offsets. The dependent variable is the Offset ratio. Columns (1), (2), and (3) report establishment-level regressions, while Columns (4), (5), and (6) report firm-level regressions. In Columns (1), (2), and (3), the variable sales is defined at the establishment level, focusing on establishments that are directly matched to a regulated entity. In Columns (4), (5), and (6), sales are redefined at the firm-level, and include all sales that belong to the firm that has some regulated establishments. The variable Controls includes proxies for the size of the entity—employment, number of establishments, and their interaction. Public FE, State FE, Government FE, and Industry FE control for time-invariant differences across compliance entities that are public firms, located in the same state, owned by government entities, or operating in the same industry. The Period FE is defined at the compliance level. The offset ratio is the total number of offsets used for compliance, divided by the total compliance burden. Section E of the Online Appendix describes in detail how the variables are constructed. The sample is constructed by replacing missing sales with the average sales per observed year for each entity, because for 3.9% of our sample, sales for every year of the compliance period are not observed. This approach allows us to impute missing sales, rather than implicitly assuming that an entity with unobserved sales generates no revenue during that year. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

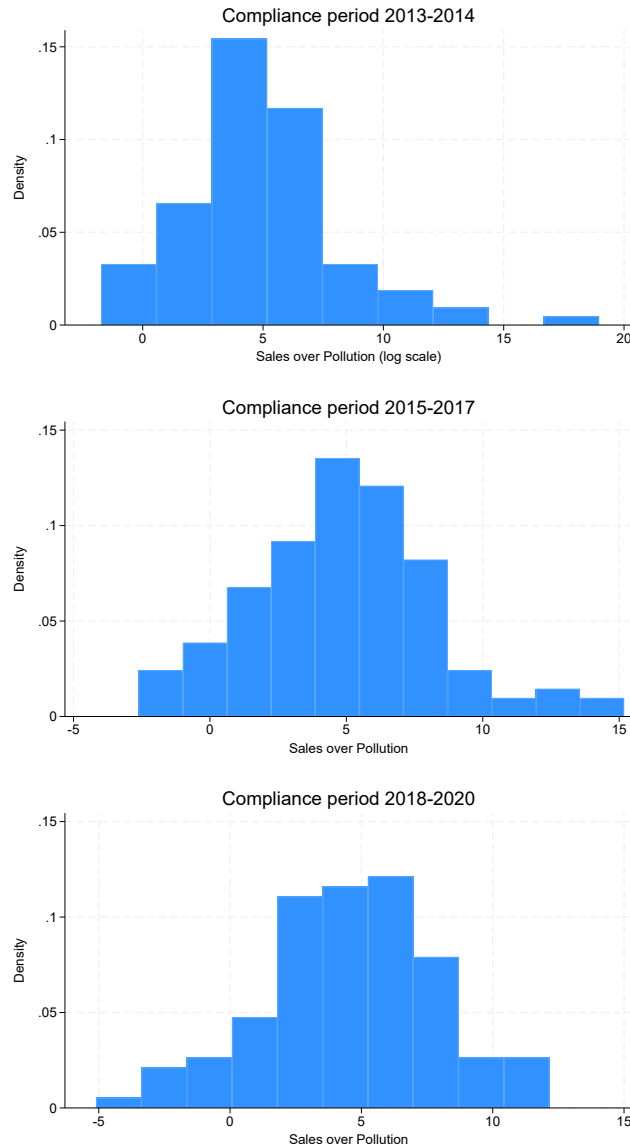
B Additional Figures

Figure A.1: Histograms: sales over pollution (firm-level, log scale)



This graph plots the distribution of total sales at the firm level over pollution regulated under the Californian Cap-and-Trade system, for different compliance periods. Source: CARB's Cap-and-Trade Program Data Dashboard, NETs, and authors' calculations.

Figure A.2: Histograms: sales over pollution (establishment's level, log scale)



This graph plots the distribution of the total sales, defined at the establishment regulated under California's Cap-and-Trade Program, over the pollution regulated in the California's Cap-and-Trade Program, for different compliance periods. Source: CARB's Cap-and-Trade Program Data Dashboard, NETs, and authors' calculations.

C Model Appendix

Proof of Proposition 1. This proof follows directly from arguments in the text. $\square$

Proof of Proposition 2. Recall the market clearing conditions

$$C = \int_{i_{\underline{\xi}}}^1 \eta \bar{k} di - \alpha \int_{i_{\underline{\xi}}}^{i_{\bar{\xi}}} \eta \bar{k} di \quad \text{and} \quad Q_o = \alpha \int_{i_{\underline{\xi}}}^{i_{\bar{\xi}}} \eta \bar{k} di, \quad (11)$$

where C is the exogenous supply of carbon allowances and $Q_o = \frac{p_o}{\phi}$ is the equilibrium supply of carbon offsets maximizing the offset producer problem in (4). Equations (11) can be simplified as

$$\frac{C}{\eta \bar{k}} = 1 - G(\underline{\xi}) - \alpha [G(\bar{\xi}) - G(\underline{\xi})], \quad \frac{p_o}{\phi \eta \bar{k}} = \alpha [G(\bar{\xi}) - G(\underline{\xi})]. \quad (12)$$

Denote $x = \alpha [G(\bar{\xi}) - G(\underline{\xi})]$, and obtaining $p_o = (1 - \alpha \rho) \underline{\xi} - (1 - \alpha) \rho \bar{\xi}$ from (3) and (2), we have

$$\begin{cases} G(\underline{\xi}) = i_{\underline{\xi}} &= 1 - \frac{C}{\eta \bar{k}} - x \\ G(\bar{\xi}) = i_{\bar{\xi}} &= 1 - \frac{C}{\eta \bar{k}} + \left(\frac{1}{\alpha} - 1\right)x \\ x &= \frac{1}{\phi \eta \bar{k}} [(1 - \alpha \rho) \underline{\xi} - (1 - \alpha) \rho \bar{\xi}] = \frac{1}{\phi \eta \bar{k}} [(1 - \alpha \rho) G^{-1}(i_{\underline{\xi}}) - (1 - \alpha) \rho G^{-1}(i_{\bar{\xi}})] \end{cases} \quad (13)$$

Thus the equilibrium is uniquely characterized by the system of three equations (13) and three unknown parameters $\{x, \underline{\xi}, \bar{\xi}\}$.

Denote for convenience $E = \eta \bar{k}$. Define the function $h(x)$ as

$$h(x) := \frac{1}{\phi E} \left((1 - \alpha \rho) G^{-1} \left(1 - \frac{C}{E} - x \right) - (1 - \alpha) \rho G^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) \right). \quad (14)$$

Define $f(x) = x - h(x)$. Then we want to find the conditions under which the solution to $f(x) = 0$ exists and is unique. Imposing that $0 \leq i_{\underline{\xi}} \leq i_{\bar{\xi}} \leq 1$ implies that x lies in the interval $(0, \min \{1 - \frac{C}{E}, \frac{\alpha}{1 - \alpha} \frac{C}{E}\})$, with $C < E$ and $\alpha \in (0, 1)$.

Uniqueness: Note that since G is continuous and increasing, so is G^{-1} . The first term in (14) is therefore decreasing in x , whereas the second term is increasing in x because $\alpha \in (0, 1)$. Given that the second term enters negatively, the function $h(x)$ decreases in x . This implies that $f(x)$ increases monotonically in x and thus if there is a solution, it is necessarily unique.

Existence: It is sufficient to verify that $f(0) \leq 0 \leq f(x_{\max})$ for $x_{\max} = \min\{1 - \frac{C}{E}, \frac{\alpha}{1-\alpha} \frac{C}{E}\}$. Consider first $x = 0$. We have $h(0) = \frac{1}{\phi E} \left((1 - \alpha\rho - \rho + \rho\alpha) G^{-1} \left(1 - \frac{C}{E} \right) \right) \geq 0$ for $\rho \in [0, \delta]$ because $\delta \leq 1$. Thus $f(0) = 0 - h(0) \leq 0$ with equality if and only if $\rho = 1$.

Consider then $x_{\max} = 1 - \frac{C}{E}$. We have

$$h(x_{\max}) = \frac{1}{\phi E} \left((1 - \alpha\rho) \xi_{\min} - (1 - \alpha) \rho \bar{\xi} \right) \quad (15)$$

with $\xi_{\min} = \frac{A_{\min}}{\eta}$. If $A_{\min} = 0$, then $h(x_{\max}) = -\frac{1}{\phi E} (1 - \alpha) \rho \bar{\xi} \leq 0$ and $f(x_{\max}) = 1 - \frac{C}{E} + \frac{1}{\phi E} (1 - \alpha) \rho \bar{\xi} > 0$. More generally, it suffices that

$$\xi_{\min} < \frac{(1 - \alpha)\rho}{1 - \alpha\rho} G^{-1} \left(\frac{1 - \frac{C}{E}}{\alpha} \right).$$

Equivalently, $A_{\min} < \eta \frac{(1 - \alpha)\rho}{1 - \alpha\rho} G^{-1} \left((1 - \frac{C}{E}) / \alpha \right)$.

Consider finally the case $x_{\max} = \frac{\alpha}{1 - \alpha} \frac{C}{E}$.

$$h(x_{\max}) = \frac{1}{\phi E} \left((1 - \alpha\rho) \underline{\xi} - (1 - \alpha) \rho \xi_{\max} \right) \quad (16)$$

with $\xi_{\max} = \frac{A_{\max}}{\eta}$. If $A_{\max} = +\infty$, then $h(x_{\max}) = -\infty < 0$ and $f(x_{\max}) = +\infty > 0$. More generally, it suffices that

$$\xi_{\max} > \frac{1 - \alpha\rho}{(1 - \alpha)\rho} G^{-1} \left(1 - \frac{C}{E(1 - \alpha)} \right).$$

Equivalently, $A_{\max} > \eta \frac{1 - \alpha\rho}{(1 - \alpha)\rho} G^{-1} \left(1 - \frac{C}{E(1 - \alpha)} \right)$.

As such, under the assumptions that $A_{\min} < \eta \frac{(1 - \alpha)\rho}{1 - \alpha\rho} G^{-1} \left((1 - \frac{C}{E}) / \alpha \right)$ and $A_{\max} > \eta \frac{1 - \alpha\rho}{(1 - \alpha)\rho} G^{-1} \left(1 - \frac{C}{E(1 - \alpha)} \right)$, there exists a unique solution $x \in [0, \min\{1 - \frac{C}{E}, \frac{\alpha}{1 - \alpha} \frac{C}{E}\}]$.

Endpoint $\alpha = 0$. When $\alpha = 0$ we have $x = \alpha [G(\bar{\xi}) - G(\underline{\xi})] = 0$ and $p_o = \phi E x = 0$. Carbon allowance market clearing gives $G(\underline{\xi}) = 1 - \frac{C}{E}$, hence

$$\underline{\xi} = G^{-1} \left(1 - \frac{C}{E} \right), \quad \bar{\xi} = \underline{\xi}, \quad p_c = \underline{\xi}, \quad p_o = 0.$$

Thus the boundary equilibrium is unique at $\alpha = 0$.

□

Proof of Corollary 1. We aim to prove that, for the model parameters values specified as in Proposition 2,

$$\frac{\partial \xi}{\partial \alpha} < 0, \quad \frac{\partial p_o}{\partial \alpha} > 0, \text{ and } \frac{\partial Q_o}{\partial \alpha} > 0, \quad (17)$$

and we want to find the parameters conditions under which

$$\frac{\partial(p_c - p_o)}{\partial \alpha} < 0, \text{ and } \frac{\partial \bar{\xi}}{\partial \alpha} < 0. \quad (18)$$

Consider first $\frac{\partial \xi}{\partial \alpha} < 0$. Note that since $i = G(\xi_i)$ with G continuous and monotonically increasing, this is the same as proving that $\frac{\partial i_{\xi}}{\partial \alpha} < 0$. Differentiating the first equation in (13) w.r.t α , we have that

$$\frac{\partial i_{\xi}}{\partial \alpha} = -\frac{\partial x}{\partial \alpha} \quad (19)$$

We have to verify that $\frac{\partial x}{\partial \alpha} > 0$ where x is the solution to the equation $f(x) = 0$ with $f(x) = x - h(x)$ and $h(x)$ given by (14). We can write an equivalent equation that gives us the unique fixed-point x as

$$f(x, \alpha) := \phi E x - \left[(1 - \alpha \rho) G^{-1} \left(1 - \frac{C}{E} - x \right) - (1 - \alpha) \rho G^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) \right]. \quad (20)$$

By the Implicit Function Theorem,

$$\frac{\partial x}{\partial \alpha} = -\frac{\frac{\partial f(x, \alpha)}{\partial \alpha}}{\frac{\partial f(x, \alpha)}{\partial x}}. \quad (21)$$

Computing the derivative, we obtain

$$\frac{\partial x}{\partial \alpha} = -\frac{\rho G^{-1} \left(1 - \frac{C}{E} - x \right) - \rho G^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) - (1 - \alpha) \rho G'^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) \frac{x}{\alpha^2}}{\phi E + (1 - \alpha \rho) G'^{-1} \left(1 - \frac{C}{E} - x \right) + (1 - \alpha) \rho G'^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) \left(\frac{1}{\alpha} - 1 \right)} \quad (22)$$

$$= \rho \frac{G^{-1} \left(i_{\bar{\xi}} \right) - G^{-1} \left(i_{\underline{\xi}} \right) + (1 - \alpha) G'^{-1} \left(i_{\bar{\xi}} \right) \frac{x}{\alpha^2}}{\phi E + (1 - \alpha \rho) G'^{-1} \left(i_{\underline{\xi}} \right) + (1 - \alpha) \rho G'^{-1} \left(i_{\bar{\xi}} \right) \left(\frac{1}{\alpha} - 1 \right)}. \quad (23)$$

given that $G'^{-1}(x) > 0$, $i_{\underline{\xi}} \leq i_{\bar{\xi}}$, and $\alpha \in (0, 1)$, $\frac{\partial x}{\partial \alpha}$ is positive.

It immediately follows that $\frac{\partial p_o}{\partial \alpha} > 0$ because $p_o = \phi E x$ and that $\frac{\partial Q_o}{\partial \alpha} > 0$ because $Q_o = \frac{p_o}{\phi} = x E$.

Next, we want to establish conditions for when $\frac{\partial \bar{\xi}}{\partial \alpha} < 0$. Differentiating the second equation in (13) w.r.t α , we have that

$$\frac{\partial i_{\bar{\xi}}}{\partial \alpha} = \left(\frac{1}{\alpha} - 1 \right) \frac{\partial x}{\partial \alpha} - \frac{x}{\alpha^2} \quad (24)$$

For $\frac{\partial \bar{\xi}}{\partial \alpha} < 0$, we require that

$$\frac{\partial x}{\partial \alpha} < \frac{1 - \alpha}{\alpha} x. \quad (25)$$

To obtain a sufficient parameter restriction for (25), we use the Mean Value Theorem,

$$G^{-1}(i_{\bar{\xi}}) - G^{-1}(i_{\underline{\xi}}) = G'^{-1}(i^*)(i_{\bar{\xi}} - i_{\underline{\xi}}) = \frac{i_{\bar{\xi}} - i_{\underline{\xi}}}{g(\xi^*)} = \frac{x}{\alpha} \frac{1}{g(\xi^*)} \leq \frac{x}{\alpha \underline{g}},$$

where g is the density of G and $g(\xi) \geq \underline{g} > 0$ for any $\xi \in [\underline{\xi}, \bar{\xi}]$. Also, $G'^{-1}(i_{\bar{\xi}}) = 1/g(\bar{\xi}) \leq 1/\underline{g}$.

Therefore, from (23), we get

$$\frac{\partial x}{\partial \alpha} \leq \rho \frac{\frac{x}{\alpha \underline{g}} + \frac{1 - \alpha}{\alpha^2} \frac{x}{\underline{g}}}{\phi E} = \frac{\rho}{\phi E} \frac{x}{\alpha^2 \underline{g}}. \quad (26)$$

A sufficient condition for (25) is then

$$\frac{\rho}{\phi E} \cdot \frac{1}{\alpha^2 \underline{g}} < \frac{1 - \alpha}{\alpha} \iff \rho < \phi E \underline{g} \alpha (1 - \alpha) \quad (27)$$

Since $\alpha(1 - \alpha) \leq \frac{1}{4}$, a global sufficient condition that is independent of α is

$$\rho < \frac{1}{4} \phi E \underline{g} = \rho^* \quad (28)$$

If the inequality in (28) is satisfied, then $\frac{\partial \bar{\xi}}{\partial \alpha} < 0$. This also immediately implies that $\frac{\partial(p_c - p_o)}{\partial \alpha} < 0$ given that $p_c - p_o = \rho \bar{\xi}$.

Finally we have $i_{\bar{\xi}} - i_{\underline{\xi}} = \frac{x}{\alpha}$.

$$\frac{d}{d\alpha}(i_{\bar{\xi}} - i_{\underline{\xi}}) = \frac{1}{\alpha} \frac{\partial x}{\partial \alpha} - \frac{x}{\alpha^2}.$$

For $\frac{d}{d\alpha}(i_{\bar{\xi}} - i_{\underline{\xi}})$ to be negative, we require that

$$\frac{\partial x}{\partial \alpha} < \frac{x}{\alpha}.$$

Because $\frac{\partial x}{\partial \alpha} < \frac{1 - \alpha}{\alpha} x < \frac{x}{\alpha}$, $\frac{d}{d\alpha}(i_{\bar{\xi}} - i_{\underline{\xi}}) < 0$ when $\rho < \rho^*$. □

Proof of Corollary 2. We aim to prove that, for the model parameters values specified as in Proposition 2,

$$\frac{\partial \xi}{\partial \rho} > 0, \quad \frac{\partial \bar{\xi}}{\partial \rho} < 0, \quad \frac{\partial Q_o}{\partial \rho} < 0, \quad \frac{\partial p_o}{\partial \rho} < 0, \quad (29)$$

and we want to find the parameters conditions under which,

$$\frac{\partial(p_c - p_o)}{\partial \rho} > 0. \quad (30)$$

Consider first $\frac{\partial \xi}{\partial \rho} > 0$. Note that since $i = G(\xi_i)$ with G continuous and monotonically increasing, this is the same as proving that $\frac{\partial i_{\xi}}{\partial \rho} > 0$. Differentiating the first equation in (13) w.r.t ρ , we have that

$$\frac{\partial i_{\xi}}{\partial \rho} = -\frac{\partial x}{\partial \rho} \quad (31)$$

We have to verify that $\frac{\partial x}{\partial \rho} < 0$ where x is the solution to the equation $f(x) = 0$ with $f(x) = x - h(x)$ and $h(x)$ given by (14). We can write an equivalent equation that gives us the unique fixed-point x as

$$f(x, \rho) := \phi E x - \left[(1 - \alpha \rho) G^{-1} \left(1 - \frac{C}{E} - x \right) - (1 - \alpha) \rho G^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) \right] \quad (32)$$

By the Implicit Function Theorem,

$$\frac{\partial x}{\partial \rho} = -\frac{\frac{\partial f(x, \rho)}{\partial \rho}}{\frac{\partial f(x, \rho)}{\partial x}}. \quad (33)$$

Computing the derivative, we obtain

$$\frac{\partial x}{\partial \rho} = -\frac{[\alpha G^{-1}(1 - \frac{C}{E} - x) + (1 - \alpha) G^{-1}(1 - \frac{C}{E} + (\frac{1}{\alpha} - 1)x)]}{\phi E + (1 - \alpha \rho) G'^{-1}(1 - \frac{C}{E} - x) + (1 - \alpha) \rho G'^{-1}(1 - \frac{C}{E} + (\frac{1}{\alpha} - 1)x) (\frac{1}{\alpha} - 1)}, \quad (34)$$

which is always negative since both the numerator and denominator in (34) are strictly positive for $\alpha \in (0, 1)$. It immediately follows that $\frac{\partial p_o}{\partial \rho} < 0$ because $p_o = \phi E x$, that $\frac{\partial Q_o}{\partial \rho} < 0$ because $Q_o = \frac{p_o}{\phi} = xE$, and that $\frac{\partial \bar{\xi}}{\partial \rho} < 0$ because $\frac{\partial \bar{\xi}}{\partial \rho} \propto \frac{\partial i_{\bar{\xi}}}{\partial \rho} = (\frac{1}{\alpha} - 1) \frac{\partial x}{\partial \rho} < 0$, where the last identity is obtained by differentiating the second equation in (13) w.r.t ρ .

It remains then to check the conditions under which $\frac{\partial(p_c - p_o)}{\partial \rho} > 0$. Since $p_c - p_o = \rho \bar{\xi}$, we require

$$\bar{\xi} + \rho \frac{\partial \bar{\xi}}{\partial \rho} > 0 \implies -\frac{\bar{\xi}}{\rho} < \frac{\partial \bar{\xi}}{\partial \rho}. \quad (35)$$

We have

$$\frac{\partial \bar{\xi}}{\partial \rho} = G'^{-1} \left(1 - \frac{C}{E} + \left(\frac{1}{\alpha} - 1 \right) x \right) \left(\frac{1}{\alpha} - 1 \right) \frac{\partial x}{\partial \rho} \quad (36)$$

$$= \frac{1}{g(\bar{\xi})} \left(\frac{1}{\alpha} - 1 \right) \frac{\partial x}{\partial \rho} \quad (37)$$

$$= -\frac{1}{g(\bar{\xi})} \left(\frac{1}{\alpha} - 1 \right) \left(\frac{\alpha \bar{\xi} + (1 - \alpha) \bar{\xi}}{\phi E + (1 - \alpha \rho) g(\bar{\xi})^{-1} + (1 - \alpha) \rho g(\bar{\xi})^{-1} \left(\frac{1}{\alpha} - 1 \right)} \right) \quad (38)$$

$$> -\frac{1}{g(\bar{\xi})} \frac{(1 - \alpha)}{\alpha} \frac{\bar{\xi}}{\phi E}, \quad (39)$$

where the last inequality uses the fact that $\bar{\xi} < \bar{\xi}$. Denoting $\underline{g}$ as before such that $g(\bar{\xi}) \geq \underline{g} > 0$ for any $\bar{\xi} \in [\bar{\xi}, \bar{\xi}]$, a sufficient condition for (35) to be verified is that $\rho < \bar{\rho}$. The condition for (30) to be verified is then that

$$\rho < \frac{\alpha}{1 - \alpha} \underline{g} \phi E = \underline{\rho}, \quad (40)$$

meaning that the price wedge increases in the invalidation probability as long as the probability of invalidation is sufficiently small.

□

Proof of Proposition 3. Assume that $\kappa(Q_o \rho) \rightarrow \infty \forall \rho > 0$. Then, the regulator optimally sets $\rho = 0$ because any $\rho > 0$ yields infinite cost. When $\rho = 0$, offsets are perfect substitutes for allowances and the no-arbitrage condition yields $p_c = p_o$, and consequently the high-indifference type $i_{\bar{\xi}} \equiv 1$.

The maximum demand for offsets for a given α and C is $\frac{\alpha}{1 - \alpha} C$. This is also the offset usage that maximizes production for a given α . This implies that all firms, if indifferent between using offsets and carbon allowances, use both instruments. Assuming that ϕ is sufficiently small (exact condition specified below) that offset production meets demand, market clearing in the combined offset and carbon allowances gives us

$$(1 - i_{\bar{\xi}}) \eta \bar{k} = C + \frac{\alpha}{1 - \alpha} C \implies i_{\bar{\xi}} = 1 - \frac{C}{(1 - \alpha) \eta \bar{k}}. \quad (41)$$

Assuming $\lambda(x) = \lambda x$, the regulator problem can be expressed as

$$\max_{0 \leq \alpha \leq 1} \int_{i_{\underline{\xi}}}^1 \xi_i \eta \bar{k} di - \lambda \delta \alpha (1 - i_{\underline{\xi}}) \eta \bar{k} \quad (42)$$

The first order condition is

$$-\underline{\xi} \frac{\partial i_{\underline{\xi}}}{\partial \alpha} - (1 - i_{\underline{\xi}}) \lambda \delta + \alpha \frac{\partial i_{\underline{\xi}}}{\partial \alpha} \lambda \delta = 0. \quad (43)$$

recalling that $i_{\underline{\xi}} = 1 - \frac{C}{\eta \bar{k} (1 - \alpha)}$, this gives

$$\underline{\xi} \frac{C}{\eta \bar{k}} \frac{1}{(1 - \alpha)^2} - \frac{C}{\eta \bar{k}} \frac{1}{(1 - \alpha)} \lambda \delta - \alpha \lambda \delta \frac{C}{\eta \bar{k}} \frac{1}{(1 - \alpha)^2} = 0, \quad (44)$$

which can be written as

$$\frac{1}{(1 - \alpha)^2} (\underline{\xi} - \lambda \delta) = 0 \quad (45)$$

For the above to give an interior solution for α , we require that $\underline{\xi} = \lambda \delta$. Therefore, α solves

$$G^{-1} \left(1 - \frac{C}{\eta \bar{k} (1 - \alpha)} \right) = \lambda \delta.$$

Rearranging the above, we get

$$\alpha = \frac{1 - \frac{C}{\eta \bar{k}} - G(\lambda \delta)}{1 - G(\lambda \delta)}. \quad (46)$$

To guarantee the existence of an interior solution, we require that $\lambda \delta \in (\xi_{\min}, \xi_{\max})$ and that $C < \eta \bar{k} (1 - G(\lambda \delta))$.

From the threshold lowest productivity firm's indifference condition (Equation 3) we have

$$p_c = \lambda \delta. \quad (47)$$

For the offset producer to be willing to supply $\frac{\alpha}{1 - \alpha} C$ offsets, we require that $\phi \leq \frac{\lambda \delta}{\frac{\alpha}{1 - \alpha} C} = \bar{\phi}$.

□

D Matching Procedure

We describe below the matching procedure between the CARB dataset and the NETS files. We start by pre-processing the data (step 0), and then implement a multi-step nested matching procedure (step 1 to 3).

Step 0. We start by pre-processing the data. From the California Cap-and-Trade program data, we append each regulated entity's name with the address of its establishment or, when unavailable, its headquarters. We geocode all addresses using the Google API. We standardize and clean the legal names of all entities in both the NETS data and the California Cap-and-Trade program data. Finally, we remove empty and duplicate records and retain only observations with valid latitude and longitude.

Step 1. Using the `geonear` package, we identify all NETS establishments located within a specified radius of each CARB entity. We retain every NETS establishment located within eight miles of a California-regulated entity or its headquarters. We next apply fuzzy-name matching to this radius-based subset. All candidate NETS-CARB pairs are then ranked using address similarity and name-matching scores. We manually review these pairs and confirm the matches.

Step 2. We remove from the CARB database all entities matched in Step 1 and run a new fuzzy-matching procedure based on both names and addresses. We manually review the resulting candidate pairs and retain those that represent high-quality matches.

Step 3. After removing all entities matched in Steps 1 and 2, we search for perfect name or perfect address matches. For these remaining potential links, we conduct online research to identify alternative names or alternative addresses. We retain pairs where an alternative name matches the NETS entity and the recorded address matches the CARB entity, as well as pairs where the recorded name matches and the alternative address (identified through external research) aligns.

Sample construction We use the headquarters identifier of each matched NETS establishment to retrieve firm-level information. We construct the establishment-level sample by identifying the set of NETS establishments whose name and address match those reported by CARB for a given regulated entity.

E Variables construction

The table below describes how the main variables of the empirical section are constructed.

Variables	Sample	Description
Total Offsets Surrendered	Compliance-level	We construct the variable directly from the CARB individual compliance reports by importing the official compliance spreadsheets for each period (2013-2014, 2015-2017, 2018-2020). This data is public. Unit: metric tons of CO ₂ e.
Total Instruments Surrendered	Compliance-level	We construct the variable directly from the CARB individual compliance reports by importing the official compliance spreadsheets for each period (2013-2014, 2015-2017, 2018-2020). This data is public. Unit: metric tons of CO ₂ e.
Offset Ratio	Compliance-level	The share of compliance met with offsets by dividing total offsets surrendered by total instruments surrendered. We winsorize this ratio at the 2 percent tails to reduce the influence of extreme values. Unit: metric tons of CO ₂ e.
Sales (firm-level)	Firm-level	The total sales at the firm level where at least one NETS establishment is matched to a given CARB entity within that compliance period. This variable is constructed by extracting establishment-level sales from NETS for the years 2013 through 2020. All establishments linked to the same NETS headquarters identifier (hqduns) are then aggregated by summing sales across establishments for each year. After this aggregation, the NETS headquarters data are merged with the CARB-NETS linkage file so that each CARB-regulated entity is connected to the corresponding NETS headquarters. We then sum this variable over a compliance period to obtain the total sales made during that compliance period. Unit: USD million.
Sales (establishment-level)	Establishment-level	The total annual sales from NETS for the establishments that can be directly matched to a CARB regulated entity. We then sum this variable over a compliance period to obtain the total sales made during that compliance period. Unit: USD million.
Establishments' Sales over pollution (establishment-level)	Establishment-level	We construct a carbon productivity measure by dividing establishment-level sales by total compliance instruments surrendered and winsorizing the resulting ratio at the two percent tails to limit the influence of extreme outliers. Unit: USD per metric tons of CO ₂ e.

log(Establishments' Sales over pollution (establishment-level))	Establishment-level	We construct a carbon productivity measure by dividing establishment-level sales by total compliance instruments surrendered. We then take the logarithm of this variable to obtain a more stable and normally distributed outcome. Unit: log USD per metric tons of CO ₂ e.
Firm' Sales over pollution (firm-level)	Firm-level	We construct a carbon productivity measure by dividing Firm-level sales by total compliance instruments surrendered and winsorizing the resulting ratio at the two percent tails to limit the influence of extreme outliers. Unit: USD per metric tons of CO ₂ e.
log(Firm' Sales over pollution (firm-level))	Firm-level	We construct a carbon productivity measure by dividing Firm-level sales by total compliance instruments surrendered. We then take the logarithm of this variable to obtain a more stable and normally distributed outcome. Unit: log USD per metric tons of CO ₂ e.
Invalidation Risk	Constant	We build on the model of Litterman and Iben (1991) to estimate the invalidation probability. We assume the risk-neutral probability of offset invalidation in any given year is q. Then, because in the risk-neutral world the expected return of all assets should be the same, we have: $\frac{(1 - q)^8}{p_{CCO-8}} = \frac{(1 - q)^3}{p_{CCO-3}},$ where $CCO - 8$s are subject to an 8-year invalidation window following a single verification, during which the buyer (a regulated entity) bears the invalidation risk. In contrast, for $CCO - 3$s, this risk shifts back to the original project issuer after three years. We use this approach to construct our measure of invalidation risk. Unit: probability.
