

Anatomy of a VC Boom: A Theory of Skills and Bargaining Power

April 27, 2026

Abstract

This paper provides the first microfounded theory of how bargaining power evolves over the lifecycle of a venture capital boom. Bargaining power acts as the Walrasian price that clears the market for talent when entry is endogenous. The initial phase of a boom features pure price adjustment, with incumbent VCs capturing windfall rents. High prospective returns induce a long-run phase of quantity adjustment, as new VCs and entrepreneurs enter the market. The ultimate long-run reallocation of bargaining power is governed by which party's talent is rendered scarce at the margin—a function of the initial distribution of skills, asymmetric sensitivities to market heat, and a “*rich-get-richer*” dynamic consistent with incumbent VC performance persistence.

JEL: G24, M13, C78

Keywords: Startup, Venture Capital, Bargaining Power, Hot Markets

1 Introduction

The allocation of bargaining power between entrepreneurs and venture capitalists affects nearly every variable observed by empiricists, from financial returns and governance choices, to exit timing and the patience VCs exhibit toward under-performing projects. Yet any attempt to isolate the drivers of this variable faces an identification problem. Specifically, while researchers observe these realized outcomes, they do not observe the underlying supply of talent and demand for capital that jointly produce them. Moreover, this challenge is compounded by multiple, nested layers of unobserved negotiation: the bargain struck between an entrepreneur and a VC’s general partners is distinct from the one between those general partners and their limited partners. Third, some empirical proxies —e.g., returns to limited partners—is realized only with long lags and is confounded by unexpected shocks occurring years after the initial contract is signed. Faced with these layers of unobservability, the empirical literature relies on “theory-lite” narratives of what determines bargaining power. While these stories can provide ex-post rationalization for observed patterns, they lack the formal structure needed to rigorously analyze the underlying economic mechanisms or make new predictions.

This paper provides the first microfounded theory of how bargaining power evolves over the lifecycle of a venture boom. Our approach departs from the existing search literature, which typically relies on a Nash bargaining framework where parties split the surplus according to a fixed rule—an exogenous parameter often denoted as β . Such a “fixed-split” approach is structurally inconsistent with free entry. If β were fixed at a level favorable to one side, the resulting returns would entice a mass of new entrants to that side of the market, while the other side would shrink. To maintain a competitive equilibrium, the division of surplus cannot be a fixed parameter; it must be the price that adjusts to balance the entry incentives of both venture capitalists and entrepreneurs.

We use this market-clearing framework to trace the “anatomy” of a venture boom by endogenizing both the division of surplus and the participation thresholds for each side. A shock to exit values triggers a sequence of adjustments: an initial short-run phase of price adjustments where incumbents capture windfall rents, followed by a long-run phase where new entry expands the pool. Because we endogenize these boundaries, the model demonstrates that a boom does not just change the price of a deal; it changes the composition of the market itself.

The long-run shift in bargaining power is not a uniform response to market heat. Instead, the direction depends on the relationship between active participants and the latent pool of potential entrants. This is determined by whether the latent talent is specialized or democratized relative to the current pool. A democratizing shock, for example, lowers the entry bar for a large reservoir of entrepreneurs with similar business plans—as seen in the dot-com era. In contrast, a specializing shock, such as those in AI or CRISPR, creates demand for a scarce and non-substitutable set of skills. As we demonstrate, these two regimes trigger fundamentally different—and at times opposing—reallocations of bargaining power.

Our model’s key insight is that the ultimate shift in bargaining power is determined not by the average scarcity of talent, but by the local density of skill at the margin. This density determines whether the market must accept a significant drop in quality to attract a wave of entrants of a given magnitude, or can instead draw from a deep pool of close substitutes. It is this marginal elasticity of talent supply that dictates which party’s bargaining power improves as investment opportunities shift. Further, these elasticities need not be fixed as a boom matures and the market dives deeper into both talent pools.

Our approach models a market with free entry for a continuum of heterogeneous agents on both sides. Agents with varying skill levels choose whether to become venture capitalists

or entrepreneurs based on their expected profits. Equilibrium arises when the division of surplus—our measure of bargaining power—adjusts to equate the supply of VCs with the demand from entrepreneurs. In this market-clearing equilibrium, the marginal (i.e., worst active) agent on each side earns zero economic profit, while all inframarginal agents earn rents based on the scarcity and substitutability of their unique skills. This framework allows us to analyze not only how the division of surplus is determined, but also how the composition of active market participants changes over the economic cycle.

More specifically, we model agent quality as coming from a continuous distribution. We refer to an agent as ‘special’ if the probability density is low in the region adjacent to him; that is, there are few agents of comparable quality. One key parameter in our model is v , defined as the exit value of successful projects. As higher exit values directly increase the payoff of an investment, v drives market activity and serves as our measure of market heat.¹ A second key parameter, k , acts as a proxy for bargaining power. This framework allows us to analyze how bargaining power varies with market heat. While the general effect is ambiguous, the final long-run outcome in a hotter market is governed by three key properties of our model:

1. Hot markets favor the party who is more ‘special’ at the margin (i.e., the party whose skills are less easily replaced at the current equilibrium).
2. Hot markets favor the party whose payoffs are less sensitive to it at equilibrium.
3. (“*Rich get richer*”) Hot markets amplify the the existing allocation of rents between VCs and entrepreneurs.

¹The link between high public market valuations, which increase the value of the exit option for entrepreneurs, and booms in business creation is a standard theoretical result.

To understand Property 1, imagine a technological shock that increases exit values. More entrepreneurs are enticed to create startups, which in turn requires more VCs to enter and satisfy this new capital demand. The new allocation of bargaining power depends on the relative elasticity of supply on each side of the market. Suppose, for instance, that there is a large mass of potential entrepreneurs with skills similar to the current marginal agent. In this case, the entrepreneurial supply is highly elastic; a small improvement in their contract terms would induce a large influx of new entrants. By contrast, suppose the supply of VC talent is inelastic, with a steep drop-off in quality below the current margin. To prevent an imbalance where capital demand vastly outstrips supply, the market must adjust: bargaining power endogenously shifts toward the VCs, the relatively scarce and less substitutable factor of production.

To understand Property 2, consider the source of each party’s surplus. The two sides contribute a fundamentally different mix of inputs—from “sweat equity” and human capital to financial and reputational capital—and there is no reason to assume their returns scale in lockstep as exit values rise. Whether due to the relative intensity of these contributions or other underlying asymmetries, one party’s net payoff may be significantly more sensitive to market heat than the other’s. Our framework shows how these sensitivities dictate market outcomes. To maintain equilibrium, the division of surplus must shift to temper the entry incentives of the more sensitive party. Consequently, bargaining power endogenously moves toward the side that is less sensitive to the boom, acting as a necessary stabilizer to keep the supply and demand for talent in balance.

Property 3 captures the interaction between market heat and inframarginal rents. In a standard competitive market, one might expect a boom to attract a wave of new capital that eventually bids away all excess profits. In our framework, however, this competitive erosion is incomplete. While the equilibrium mechanism forces the marginal (worst) VC to

zero profit, it cannot extract the surplus from high-skill incumbents. Because the market must attract even low-skilled entrants to satisfy the rising demand for capital, the “price” of capital remains high enough for these marginal players to break even. This effectively creates a protective floor for the rest of the industry; since elite VCs are significantly more efficient than the marginal entrant, they capture a disproportionate share of the new surplus.

This “rich get richer” dynamic provides a market-based resolution to the long-standing observation of performance persistence in venture capital. Empirical studies show that, unlike in mutual funds, the performance of top-quartile VC funds is highly likely to persist in subsequent funds. The puzzle is why competitive capital inflows do not erode these returns to skill. Our model answers this by highlighting the role of the extensive margin: in a boom, the entry of lower-quality VCs prevents the market-clearing price of capital from tightening enough to extract the surplus generated by elite incumbents. More formally, a hot market steepens the slope of the return-to-skill function—a mechanism we term *skill amplification*. The market cycle thus acts to widen the performance gap between elite and average VCs, yielding the novel prediction that performance persistence should be procyclical—strongest when the market is hot.

The remainder of the paper proceeds as follows. Section 2 develops the long-run model. Section 3 analyzes the short-run dynamics and presents the full anatomy of a venture boom that illustrates our key results. Section 4 provides empirically testable predictions for the two types of market booms: *Democratizing* versus *Specializing shocks*. Section 5 concludes.

1.1 Related Literature

A vast empirical literature documents the cyclical nature of venture capital markets. A central stylized fact is the “*money chasing deals*” phenomenon, where large capital inflows into the industry are associated with higher startup valuations and subsequently lower returns

for VC funds (Gompers and Lerner, 2000). Recent work shows this dynamic persists, with increased competition among VCs leading to more founder-friendly terms, such as lower VC equity stakes and less board control (Dai, Jo, and Ross, 2023). Our paper provides a rigorous microfoundation for these cyclical swings in bargaining power.

While many models of venture capital focus on a single entrepreneur-VC pair, a growing stream of the literature recognizes that financing is the outcome of a matching process with search frictions, building on the canonical frameworks from labor economics (e.g., Pissarides (2000)). For example, Inderst and Müller (2004) layer search over top of a model in which firm value depends upon effort provision by both the entrepreneur and VCs. Agents are matched in a costly search model. Once matched, they engage in Nash bargaining, which requires specifying each party’s reservation utility—what happens if the deal breaks down—as well as a parameter β which captures the notion of bargaining power. Inderst and Müller (2004) employ Walrasian-like reasoning in determining reservation utilities, arguing that a larger number of VCs should improve entrepreneurs’ outside options. The Nash bargaining itself, however, uses an exogenous bargaining power parameter β . The central finding of Inderst and Müller (2004) is that imbalances in this parameter are harmful, as the party with too little power will under-contribute effort, thus destroying firm value.

While the mechanics are similar, our approach differs from the canonical search-and-matching paradigm in its central economic question. That literature’s normative benchmark, established in the classic work of Hosios (1990), is to identify the division of bargaining power that is socially efficient by internalizing search externalities for a *given* pool of market participants. Our paper, by contrast, asks a positive question: what determines the division of surplus in a competitive market with free entry?. In our framework, bargaining power is not a parameter to be optimized for social welfare, but is instead the market-clearing price that governs the extensive margin—who enters the market in the first place.

Our assumptions and conclusions are therefore quite different from [Inderst and Müller \(2004\)](#). In their model, entrepreneurs are identical, as are VCs for most of the analysis. An extension that allows for an exogenous “entry cost” for VCs requires that net profits are driven to zero. Our paper differs on all of these margins. Both entrepreneurs and VCs are heterogeneous, and there is free entry on both sides. In our model, all but the marginal agent on each side earn strictly positive economic profits, as their skills are imperfectly substitutable.

Our approach complements foundational work on sorting in VC markets, such as [Sørensen \(2007\)](#). Sørensen’s two-sided matching model explains the crucial cross-sectional pattern of why high-reputation VCs endogenously match with high-quality entrepreneurs. Our model, by contrast, isolates a different mechanism: how the aggregate level of bargaining power itself acts as a Walrasian price to clear the market, explaining the time-series variation in why the entire market can shift to become more VC- or entrepreneur-friendly.

[Ewens, Gorbenko, and Korteweg \(2022\)](#) provide another recent, important contribution by structurally estimating a dynamic search-and-matching model to quantify the impact of specific contract terms on firm value, while correcting for endogenous selection. A key finding is that while an optimal, value-maximizing contract exists, VCs use their bargaining power to negotiate more investor-friendly terms. While their work provides a detailed micro-level view of how contracts function within a given market state, our model provides the macro-level theory of how the entire market state—the prevailing level of bargaining power—evolves over the economic cycle.

Our model’s equilibrium approach also relates to other large-scale models of the venture ecosystem. [Gornall and Strebulaev \(2020\)](#), for instance, build a large-scale, computational equilibrium model of the entire U.S. venture capital sector to quantify its impact on innova-

tion and the broader economy. While their focus is on quantifying aggregate welfare effects, our tractable analytical model is designed to isolate the specific mechanisms that drive the allocation of bargaining power during market cycles.

This paper builds directly on [Yung \(2017\)](#), which establishes the empirical fact that hot markets lead to wider dispersion in both entrepreneur and VC quality. However, that prior work focused exclusively on these second-moment predictions and, by assuming a uniform talent distribution, could not analyze the division of surplus. The present paper introduces two theoretical innovations: first, we endogenize bargaining power, making the division of surplus the central object of study. Second, by allowing for general talent distributions, we can analyze how scarcity at the margin determines market outcomes. This richer framework allows us to explain the entire dynamic lifecycle of a boom, distinguishing between the short-run price adjustments that generate incumbency rents and the long-run quantity adjustments that ultimately determine how the gains from a boom are divided.

Our work shares a focus on potential conflicts with [Jovanovic and Szentes \(2013\)](#). They argue that VCs, being in short supply and having better outside options, are more impatient than entrepreneurs, and they study the resulting conflicts over liquidation decisions. Contrary to their approach, we allow free entry of VCs rather than assuming they are in short supply. In our model, the ability of each side to enter and exit the market in response to economic conditions is the very mechanism that determines the equilibrium division of surplus.

Our framework also provides a new, dynamic mechanism for the well-documented puzzle of performance persistence in venture capital. The persistence of top-quartile fund performance is a robust stylized fact, firmly established in surveys such as [Kaplan and Sensoy \(2015\)](#) and carefully disentangled from luck by [Korteweg and Sørensen \(2015\)](#). Existing

explanations for this phenomenon are largely micro-level and static, focusing on the informational hold-up power derived from a VC’s network ([Hochberg, Ljungqvist, and Vissing-Jørgensen, 2014](#)) or the reputational feedback loops generated by early career success ([Nanda, Samila, and Sorenson, 2020](#)). Our model complements these views by introducing a distinct, market-wide amplification channel. While mechanisms like informational hold-up are persistent across market states, our “amplification of skill” result (Theorem [2.5.3](#)) shows how market cycles themselves act as an engine that dynamically widens the performance gap between elite and average VCs. This suggests that the strength and statistical observability of performance persistence should be procyclical, a key prediction of our framework.

2 Model

We develop a search model of the venture capital market in which the investment process is characterized by three distinct phases: entry, matching, and screening. The economy consists of two populations of risk-neutral agents: potential entrepreneurs (mass Ω_D) and potential VCs (mass Ω_S). At $T = 1$, agents observe their idiosyncratic endowments. Each entrepreneur (he, denoted by i) receives an idea with a success probability $p_i \in (0, 1)$ drawn from $G(\cdot)$, representing the likelihood that his project is of high quality. Similarly, each VC (she, denoted by j) receives a value $e_j \in (0, 1/2)$ drawn from $F(\cdot)$ corresponding to her screening error rate (described in more detail below) such that lower values represent higher skill. Upon observing these values, agents decide whether or not to pay the required search cost to join the active market. Once in the market, entrepreneurs must be sampled by a VC through a stochastic matching process before their idea can be evaluated and potentially funded.

Endogenous Entry and Stationarity: We model a stationary equilibrium where entry and exit flows are balanced such that the active distributions $G(\cdot)$ and $F(\cdot)$ remain constant. Entry is endogenous: an agent pays the required search cost (C_e for entrepreneurs and C_{VC} for VCs) to join the active searching pool if and only if their expected return from the matching process is positive. Because an agent's expected payoff is strictly increasing in their talent level (p_i for entrepreneurs and $1 - e_j$ for VCs), there exists a marginal (worst) participant on each side of the market whose expected return exactly equals their cost of entry. This defines the participation thresholds p_{min} and e_{max} , such that the set of active entrepreneurs is $[p_{min}, 1]$ and the set of active VCs is $(0, e_{max}]$. Thus, at any moment, the masses of active entrepreneurs (M_E) and VCs (M_{VC}) are determined by those whose talent exceeds these thresholds.

We define market tightness as the ratio of active participants:

$$\theta := \frac{M_E}{M_{VC}} = \frac{\omega \{1 - G(p_{min})\}}{F(e_{max})} \quad (1)$$

where $\omega = \Omega_D/\Omega_S$. We assume $\theta > 1$, reflecting the empirical reality of the venture market where the volume of potential ideas exceeds the capacity of available capital attention.

Table 1: The Search and Screening Timeline

Period	Actions
$T = 1$ (Endowment)	Agents observe their own skill levels (p_i or e_j).
$T = 2$ (Entry)	Agents pay costs C_e or C_{VC} to enter the active searching pool.
$T = 3$ (Sampling)	VCs sample entrepreneurs from the pool at a rate determined by θ .
$T = 4$ (Screening)	The VC screens the project. If $s_{i,j} = L$, the VC continues sifting.
$T = 5$ (Funding)	Upon a High signal ($s_{i,j} = H$), the VC stops searching and invests I .
$T = 6$ (Realization)	Project outcomes are realized and shared according to the stake k .

The Search and Screening Process: Entrepreneurial ideas are perishable; an entrepreneur must be sampled by a VC before their window of opportunity closes. Given the market tightness θ , an entrepreneur is sampled at a Poisson arrival rate $\eta(\theta)$, where $\eta'(\theta) < 0$. This sampling rate acts as a “search tax”: in a crowded market, the probability of being “seen” falls, raising the effective barrier to entry.

Upon a match, the VC utilizes a screening technology to generate a noisy signal $s \in \{H, L\}$. The precision of this signal is governed by the VC’s skill e_j :

$$\alpha_j := \Pr(s_{i,j} = H | w_i = w_B, e_j) = \lambda e_j \quad (2)$$

$$\beta_j := \Pr(s_{i,j} = L | w_i = w_G, e_j) = e_j \quad (3)$$

To maintain tractability and focus on the role of overall skill, we assume $\lambda \in (0, 1)$ is common

across all active searchers, and there is therefore a single measure of skill.

If the signal is low ($s = L$), the VC rejects the entrepreneur and immediately moves to sample the next candidate. This process continues instantaneously until the VC receives a high signal ($s = H$), at which point they commit the investment I . This behavior endogenously generates assortative matching: on average, high-skill VCs populate their portfolios with higher-quality projects, while high-talent entrepreneurs are more likely to clear the sifting process and secure funding.

Payoffs and Surplus Division: The ultimate success of a project is revealed only after investment. A bad project (w_B) fails with certainty, yielding a payoff of zero to both parties. A good project (w_G) succeeds and generates an exit value v . In reality, the actual surplus captured by each participant depends on their specific contributions—such as specialized effort, management expertise, or reputational capital—which need not scale linearly with the firm’s total value. For example, the effort required from an entrepreneur or the distinct services provided by a VC may vary significantly across different scales. For maximum generality, we avoid assuming a specific functional split and instead represent the indirect utility of each party using the generic functions $V^e(w_G, k, v)$ and $V^{VC}(w_G, k, v) - I$, and assume only that these functions satisfy the following standard regularity conditions:

$$\frac{\partial V^{VC}}{\partial v} > 0, \quad \frac{\partial V^e}{\partial v} > 0, \quad \frac{\partial V^{VC}}{\partial k} < 0, \quad \frac{\partial V^e}{\partial k} > 0 \quad (4)$$

In equilibrium, the share k and market tightness θ are jointly determined. We define the conditional means of active entrepreneur and VC qualities in the market as follows:

$$\mathbb{E}(p|p_{min}) \equiv \mathbb{E}(p_i|p_i \geq p_{min}) := \frac{1}{1 - G(p_{min})} \int_{p_{min}}^1 p dG(p),$$

$$\mathbb{E}(e|e_{max}) \equiv \mathbb{E}(e_j|e_j \leq e_{max}) := \frac{1}{F(e_{max})} \int_0^{e_{max}} e dF(e).$$

2.1 Equilibrium and Model Properties

To characterize the market equilibrium, we define a system of four conditions that ensure individual optimality and aggregate consistency. Specifically, we must verify that the marginal entrepreneur and the marginal venture capitalist earn non-negative expected profits so that they have the incentive to pay their entry costs and join the search pool. Furthermore, the market must clear so that the aggregate supply of capital matches the aggregate flow of projects. Finally, we require that venture capitalists actually find it optimal to act upon their private signals; without this incentive-compatibility, the sifting process would not function as a meaningful filter, rendering the other equilibrium conditions invalid.

Specifically, the system must hold for the marginal entrepreneur (those with $p_i = p_{min}$) and the marginal VC (those with $e_j = e_{max}$):

$$\textbf{VC Entry (L):} \quad \Pr(s_{i,j} = H \mid p_i \geq p_{min}, e_j = e_{max}) \cdot \pi_H(e_{max}) = C_{VC}, \quad (5a)$$

$$\textbf{VC IC:} \quad \forall j; \text{ s.t. } e_j \leq e_{max} :: \pi_H(e_j) \geq 0 \quad \text{and} \quad \pi_L(e_j) \leq 0, \quad (5b)$$

$$\textbf{Ent. Entry (J):} \quad \eta(\theta) \cdot \Pr(s_{i,j} = H, w_i = w_G \mid p_i = p_{min}, e_j \leq e_{max}) \cdot V^e = C_e, \quad (5c)$$

$$\textbf{Market Clr (H):} \quad \omega 1 - G(p_{min}) \cdot \Pr(s_{i,j} = H \mid p_i \geq p_{min}, e_j \leq e_{max}) = F(e_{max}). \quad (5d)$$

where:

$$\Pi_H(p_{min}, e_j) := \Pr(w_i = w_G \mid s_{i,j} = H, p_i \geq p_{min}, e_j), \quad (6)$$

$$\Pi_L(p_{min}, e_j) := \Pr(w_i = w_G \mid s_{i,j} = L, p_i \geq p_{min}, e_j), \quad (7)$$

$$\pi_H(e_j) := \Pi_H(p_{min}, e_j) \cdot \{V^{VC} - I\} - \{1 - \Pi_H(p_{min}, e_j)\} \cdot I, \quad (8)$$

$$\pi_L(e_j) := \Pi_L(p_{min}, e_j) \cdot \{V^{VC} - I\} - \{1 - \Pi_L(p_{min}, e_j)\} \cdot I, \quad (9)$$

The condition define the Nash equilibrium of the model by ensuring that the entry

and investment decisions of every agent are individually optimal. In (8), $\pi_H(e_j)$ represents the expected payoff for a VC with skill e_j who funds a project after receiving a high signal ($s = H$). Conversely, $\pi_L(e_j)$ in (9) represents the payoff from funding after a low signal ($s = L$). While the incentive compatibility condition (5b) is a formal requirement for the Nash equilibrium, it is strictly satisfied whenever λ is sufficiently low (will be discussed in detail in Proposition 2.2) and C_{VC} is positive. Because a VC must pay a search cost to enter the market and acquire a signal, the resulting low signal must be “bad enough” to justify rejection; otherwise, the VC would have been better off investing randomly without paying C_{VC} . Consequently, the IC condition is never a binding constraint in our analysis, leaving the entry thresholds p_{min} and e_{max} as the primary determinants of the equilibrium identity.

In (5c), the entrepreneur’s participation is governed by the “search tax” $\eta(\theta)$; a crowded market reduces the probability of being sampled before an idea expires, thereby raising the effective cost of entry. Finally, the market-clearing condition (5d) ensures that the aggregate flow of high signals (the demand for capital) matches the mass of active VCs (the supply of capital). Note that the probability of a high signal in (5d) is averaged over the equilibrium distributions of both searching entrepreneurs and active VCs.

Proposition 2.1 (Entry Decisions in Sequential Equilibrium). *A screening equilibrium (p_{min}, e_{max}, k) is defined by the simultaneous satisfaction of the following three conditions:*

$$J := \eta(\theta) \cdot p_{min} \cdot (1 - \mathbb{E}[e|e_{max}]) \cdot V^e - C_e = 0, \quad (10)$$

$$L := (V^{VC} - I) \cdot \mathbb{E}[p|p_{min}](1 - e_{max}) - I \cdot (1 - \mathbb{E}[p|p_{min}]) \cdot \lambda \cdot e_{max} - C_{VC} = 0, \quad (11)$$

$$H := F(e_{max}) - \omega(1 - G(p_{min})) \cdot \Phi = 0, \quad (12)$$

where Φ is the average probability that a VC observes a “High” signal and funds a project:

$$\Phi := (1 - \mathbb{E}[e|e_{max}]) \cdot \mathbb{E}[p|p_{min}] + \lambda \cdot \mathbb{E}[e|e_{max}](1 - \mathbb{E}[p|p_{min}]). \quad (13)$$

Proof. See Appendix A.

Equation (10) represents the zero-profit condition for the marginal entrepreneur, whose entry depends on the probability of being matched with a VC and successfully passing the screen. Equation (11) represents the VC’s ledger: an investor enters only if the expected return is sufficiently high. Finally, Equation (12) ensures the market clears by balancing the total mass of active VCs against the total mass of fundable projects.

It turns out that for this screening process to function as a meaningful filter, the false-positive rate λ must be sufficiently low. It can be shown via a simple application of Bayes’ Rule that if λ were too high, a “High” signal would actually be more likely to originate from a failure than a success, rendering the screening technology economically worthless. Beyond mere informativeness, this property allows us to rule out a variety of unintuitive edge cases that we consider a pathological boom. If the market’s screening technology is too noisy, an increase in market heat (v) could trigger a perverse incentive: it might attract a wave of “opportunistic” entrepreneurs and low-skill VCs looking to exploit a lenient environment. In such a regime, the resulting flood of false positives could drown out legitimate signals, potentially leaving high-quality incumbents worse off than they were before.

To ensure the model exhibits natural comparative statics and avoids these perverse dynamics, we establish the following existence and uniqueness results under a bounded level of noise.

Proposition 2.2 (Existence and Uniqueness of Equilibrium). *There exists an endogenous threshold λ^* such that, given $\lambda < \lambda^*$ and the following conditions, a unique interior screening equilibrium (p_{min}, e_{max}, k) exists:*

1. **(The Value of Expertise)** *The marginal VC possesses enough skill to justify the search cost C_{VC} , ensuring that screening is a productive filter:*

$$e_{max} \leq \left[1 + \lambda + \frac{C_{VC}}{I(1 - \mathbb{E}[p|p_{min}])} \right]^{-1}. \quad (14)$$

2. **(The Zone of Agreement)** *There exists a range of equity splits $k \in (0, 1)$ where the surplus is large enough to cover the entry costs of both the founder and the VC:*

$$0 \leq (\mathbf{V}^e)^{-1} \left(\frac{C_e}{\eta(1)} \right) < (\mathbf{V}^{VC})^{-1}(C_{VC} + I) \leq 1. \quad (15)$$

Proof. See Appendix B.

The first condition formally captures the “elite” nature of the venture capital industry: if an agent is too unskilled, they can never recoup the overhead of searching for projects. The second condition ensures that the venture is fundamentally viable—that there is a “win-win” split of the pie that covers the entry costs of both parties. Provided these common-sense conditions hold, the market reaches a stable and unique equilibrium.

2.2 Comparative Statics

For the rest of the model, we assume that the conditions stated in Proposition 2.2 hold. To analyze the equilibrium response to market shocks, consider the 3×3 system defined in Proposition 2.1. Totally differentiating the equilibrium system $\{J, L, H\}$ with respect to

market heat v yields:

$$\begin{aligned}
\frac{dJ}{dv} &= \underbrace{\frac{\partial J}{\partial v}}_{\oplus} + \underbrace{\frac{\partial J}{\partial p_{min}}}_{\oplus} \times \frac{dp_{min}}{dv} + \underbrace{\frac{\partial J}{\partial e_{max}}}_{\ominus} \times \frac{de_{max}}{dv} + \underbrace{\frac{\partial J}{\partial k}}_{\oplus} \times \frac{dk}{dv} + \underbrace{\frac{\partial J}{\partial \theta}}_{\ominus} \times \frac{d\theta}{dv} = 0, \\
\frac{dL}{dv} &= \underbrace{\frac{\partial L}{\partial v}}_{\oplus} + \underbrace{\frac{\partial L}{\partial p_{min}}}_{\oplus} \times \frac{dp_{min}}{dv} + \underbrace{\frac{\partial L}{\partial e_{max}}}_{\ominus} \times \frac{de_{max}}{dv} + \underbrace{\frac{\partial L}{\partial k}}_{\ominus} \times \frac{dk}{dv} = 0, \\
\frac{dH}{dv} &= \underbrace{\frac{\partial H}{\partial p_{min}}}_{\oplus} \times \frac{dp_{min}}{dv} + \underbrace{\frac{\partial H}{\partial e_{max}}}_{\oplus} \times \frac{de_{max}}{dv} = 0,
\end{aligned}$$

Our objective is to solve this system for the unknowns $\{\frac{de_{max}}{dv}, \frac{dp_{min}}{dv}, \frac{dk}{dv}\}$ and to identify their signs. Under the assumption $\lambda < \lambda^*(p_{min}, e_{max})$, the model exhibits the following intuitive properties that govern the direction of the total derivatives:

- **Direct Exit Value Effect** ($\frac{\partial J}{\partial v} > 0, \frac{\partial L}{\partial v} > 0$): A rising tide lifts all boats; an increase in exit value v directly improves the expected payoffs for both searching agents.
- **Congestion Relief** ($\frac{\partial J}{\partial p_{min}} > 0, \frac{\partial H}{\partial p_{min}} > 0$): A higher quality threshold p_{min} thins out the pool of searching entrepreneurs. This reduces market tightness, which improves the sampling rate for the remaining founders and eases the search burden on VCs.
- **Selection Benefit** ($\frac{\partial L}{\partial p_{min}} > 0$): High-quality founders are better for investors; a higher-quality entrepreneurial pool increases the expected success rate of the VC's portfolio, directly boosting profits.
- **Noise Penalties** ($\frac{\partial L}{\partial e_{max}} < 0, \frac{\partial J}{\partial e_{max}} < 0$): Incompetence acts as a tax on the market; an increase in VC noise degrades the efficiency of matches and reduces value for both founders and investors.

Solving the 3×3 system via the implicit-function theorem yields that dk/dv is proportional to a weighted combination of direct and indirect effects:

$$f(e_{max}) \cdot \mathcal{W}_{VC} \cdot \left[\underbrace{\frac{\partial L}{\partial v} \cdot \frac{\partial J}{\partial p_{min}}}_{\oplus\oplus \text{ Direct 1}} - \underbrace{\frac{\partial J}{\partial v} \cdot \frac{\partial L}{\partial p_{min}}}_{\oplus\oplus \text{ Indirect 1}} \right] + \omega g(p_{min}) \cdot \mathcal{W}_e \cdot \left[\underbrace{\frac{\partial J}{\partial v} \cdot \frac{\partial L}{\partial e_{max}}}_{\oplus\ominus \text{ Direct 2}} - \underbrace{\frac{\partial J}{\partial e_{max}} \cdot \frac{\partial L}{\partial v}}_{\ominus\oplus \text{ Indirect 2}} \right] \quad (16)$$

where $\mathcal{W}_{VC}$ and $\mathcal{W}_e$ are positive under the assumption $\lambda < \lambda^*$ in Proposition 2.2:

$$\mathcal{W}_{VC} := 1 + \theta \{e_{max} - \mathbb{E}(e|e_{max})\} \left\{ \underbrace{(1 + \lambda)\mathbb{E}(p|p_{min}) - \lambda}_{>0 \text{ under } \lambda < \lambda^*} \right\} > 0,$$

$$\mathcal{W}_e := \mathbb{E}(e|e_{max}) \left\{ \underbrace{(1 + \lambda)\mathbb{E}(p|p_{min}) - \lambda}_{>0 \text{ under } \lambda < \lambda^*} \right\} > 0.$$

The directional movement of dk/dv is governed by the relative magnitudes of these terms. We refer to terms involving a party's own windfall from market heat ($\partial L/\partial v$ or $\partial J/\partial v$) as “direct” effects. For example, Direct 1 captures the VC's direct benefit from a boom, scaled by the entrepreneur's benefit from the resulting congestion relief. Conversely, “indirect” effects capture how the other party's direct windfall pulls the equilibrium terms in the opposite direction through changing entry thresholds.

One can see immediately from (16) that the ultimate sign of dk/dv is endogenously ambiguous. This ambiguity is a central feature of the model: it establishes that a venture boom does not inherently favor one side of the market. Instead, the long-run shift in bargaining power is a function of which specific friction, such as the scarcity of skill at the margin or the sensitivity to noise, is most acute at the current equilibrium.

Theorem 2.3 (Comparative statics related to costs, population, and λ). *The equilibrium $\{p_{min}, e_{max}, k\}$ satisfies the following:*

1. **VC Costs:** $\frac{de_{max}}{dC_{VC}} < 0, \frac{dp_{min}}{dC_{VC}} > 0, \frac{dk}{dC_{VC}} < 0$. *Higher VC costs reduce entry on both sides and shift bargaining power toward VCs.*
2. **Ent. Costs:** $\frac{de_{max}}{dC_e} < 0, \frac{dp_{min}}{dC_e} > 0, \frac{dk}{dC_e} > 0$. *Higher entrepreneur costs reduce participation and shift power toward entrepreneurs.*
3. **Population** ($\frac{de_{max}}{d\omega} > 0, \frac{dp_{min}}{d\omega} > 0, \frac{dk}{d\omega} < 0$): *As the population of potential entrepreneurs expands, market tightness θ rises. The market must attract new VCs from a lower skill distribution ($e_{max} \uparrow$). For these less-efficient participants to cover their entry costs, the entrepreneur's share k must fall, shifting bargaining power toward VCs.*
4. **Type-I Errors:** $\frac{de_{max}}{d\lambda} < 0, \frac{dp_{min}}{d\lambda} > 0, \frac{dk}{d\lambda} < 0$. *An increase in type-I errors reduces NPV, leading to fewer participants on both sides. Bargaining power shifts towards VCs in order to compensate for the cost of funding bad firms.*

Proof. See Appendix C.

The results in Theorem 2.3 serve as a foundational check, confirming that the equilibrium responds intuitively to shifts in entry costs, populations, and error rates. Because these parameters tend to affect the participation incentives of one side of the market in relative isolation, the resulting shifts in bargaining power and participation thresholds follow a straightforward logic. We next turn to our main result, which examines the impact of market heat—a shock to exit values v . This triggers a more complex set of reallocations. Unlike a simple cost shift, an increase in v pulls both parties into the market while simultaneously

degrading the quality of the matches they find, leading to a series of results that require a deeper examination of the marginal densities of talent.

Theorem 2.4 (Comparative statics related to market heat). *An increase in market heat v has the following long-run effects:*

1. **Market Expansion:** *Quality thresholds fall for both parties ($\frac{dp_{min}}{dv} < 0$, $\frac{de_{max}}{dv} > 0$), and market tightness rises ($\frac{d\theta}{dv} > 0$).*
2. **Specialness:** *Provided $e_{max} < \epsilon_{\eta,\theta} + \bar{e}(1 - \epsilon_{\eta,\theta})$ for $\epsilon_{\eta,\theta} := -\frac{d\eta(\theta)/\eta(\theta)}{d\theta/\theta}$ and $\bar{e} \equiv \mathbb{E}(e|e_{max})$, bargaining power favors the party whose talent is more scarce at the margin: $\frac{\partial}{\partial(\frac{\omega g(p_{min})}{f(e_{max})})}(\frac{dk}{dv}) < 0$.*
3. **Payoff Sensitivity:** *Power shifts to the party with lower sensitivity to v : $\frac{\partial}{\partial(\frac{\partial V^e}{\partial v})}(\frac{dk}{dv}) < 0$ and $\frac{\partial}{\partial(\frac{\partial V^{VC}}{\partial v})}(\frac{dk}{dv}) > 0$.*
4. **Rich Get Richer:** *Market heat tends to amplify existing rent allocations: $\frac{\partial}{\partial V^e}(\frac{dk}{dv}) > 0$ and $\frac{\partial}{\partial(V^{VC}-I)}(\frac{dk}{dv}) < 0$.*

Proof. See Appendix D.

Theorem 2.4 formalizes the economic forces that dictate the long-run allocation of bargaining power in a venture capital boom.

The first property in Theorem 2.4.1 describes the basic quantity adjustment of the market. As a boom increases exit values (v), more projects become theoretically profitable, enticing a new wave of entrepreneurs to enter. To clear the market, the number of VCs must

also increase. However, the more subtle result is the rise in market tightness (θ). Because the agents entering the market on both sides are of lower quality than the incumbents (marginal p_i falls and e_j rises), the average project in the pool becomes harder to verify. Consequently, VCs must reject a higher percentage of projects. This creates a sifting clog: even high-quality entrepreneurs find themselves waiting longer to be "seen" as the screening machinery of the market becomes increasingly congested with marginal participants.

The property in Theorem 2.4.2 identifies how the identity of the talent pool determines the division of surplus. Consider a democratizing shock, where the marginal density of entrepreneurs $g(p_{min})$ is large relative to the density of VCs $f(e_{max})$. In this environment, entrepreneurial ideas are a commodity; there is a large supply of potential founders almost as skilled as the current marginal entrepreneur.

In such a market, even a small improvement in exit values triggers a disproportionate flood of new founders. To restore market-clearing equilibrium and prevent infinite congestion, bargaining power must shift away from the founders—the more elastic side of the market—toward the VCs.

However, this effect can perversely reverse if the marginal VC is highly unskilled (e_{max} is very large). Such VCs are unable to distinguish the new influx of talent from the duds. In this case, the congestion tax—the drop in the matching rate $\eta(\theta)$ —harms the VCs more than the larger pool of entrepreneurs benefits them. In such a perverse equilibrium, the VCs would actually lose bargaining power despite their relative scarcity.

We regard the condition on e_{max} as a mild one. In the real world, the setup costs and reputational requirements to become a VC are substantial. Being a VC is an elite professional choice; it is unlikely to be profitable for an agent to enter unless their screening signal is of

relatively high quality. Thus, in any well-functioning market, the scarcity of talent at the margin remains the primary driver of bargaining power.

The property in Theorem 2.4.3 relates to how different agents value the boom itself. An increase in market heat incentivizes entry on both sides, but if one party's value function is particularly sensitive to the change in v , it creates an immediate, disproportionate influx of that agent type. To restore equilibrium, bargaining power must shift toward the less sensitive party. By "taxing" the more sensitive agent through worse contract terms, the market tempers their entry incentive and keeps the pool in balance.

Finally, Theorem 2.4.4 describes how a hot market tends to amplify the existing allocation of rents. This is driven by the marginal agent's zero-profit condition. If VCs already hold significant bargaining power (k is low), contract terms cannot shift too far in the entrepreneurs' favor during a boom without making the marginal VC's entry unprofitable. Because the marginal agent must break even, the inframarginal incumbents—the elite VCs and high-quality founders—capture the lion's share of the new surplus created by the heat.

This analysis characterizes how the market clears in the aggregate. However, to understand the puzzle of performance persistence, we must move beyond aggregate rents and analyze how this market-clearing mechanism affects the cross-sectional returns to skill. We turn to this micro-level analysis in the next subsection.

2.3 The Amplification of Skill and Performance Persistence

Having established how the market-wide division of surplus k depends upon market conditions, we now examine the implications for the performance of individual market participants. For example, we characterize the difference in expected returns between elite (inframarginal) market participants and marginal ones, and how these change over the cycle.

Consider the expected performance of an incumbent VC with a fixed skill level $e < e_{max}$. Her expected payoff is given by:

$$L(e, v) := \Pr(s_{i,j} = H | p_i \geq p_{min}, e_j = e) \cdot \pi_H(e) - C_{VC}, \quad (17)$$

where $\pi_H(\cdot)$ is as defined in (6), the expected payoff of the VC upon funding after receiving a high signal, evaluated at the equilibrium market terms $\{p_{min}, k\}$. Next, rearranging (11), we have

$$V^{VC}(w_G, k, v) - I = \frac{\lambda I \cdot \{1 - \mathbb{E}(p | p_{min})\} \cdot e_{max} + C_{VC}}{\mathbb{E}(p | p_{min}) \{1 - e_{max}\}}.$$

Then, by plugging it into (17), $L(e, v)$ is simplified as follows:

$$\begin{aligned} L(e, v) &\equiv \{V^{VC}(w_G, k, v) - I\} \cdot \mathbb{E}(p | p_{min}) (1 - e) - \lambda I \cdot \{1 - \mathbb{E}(p | p_{min})\} \cdot e - C_{VC} \\ &= \underbrace{\frac{\lambda I \cdot \{1 - \mathbb{E}(p | p_{min})\} + C_{VC}}{1 - e_{max}}}_{:= A(v)} \cdot (e_{max} - e) \end{aligned}, \quad (18)$$

where $A(v)$ strictly decreases in v , deriving from Theorem 2.4.1:

$$A'(v) := \frac{dA}{dv} = \underbrace{\frac{\partial A}{\partial p_{min}}}_{\ominus} \cdot \underbrace{\frac{dp_{min}}{dv}}_{\ominus} + \underbrace{\frac{\partial A}{\partial e_{max}}}_{\oplus} \cdot \underbrace{\frac{de_{max}}{dv}}_{\oplus} > 0.$$

The mean and variance of VC performance in the market are computed as

$$\begin{aligned} \mathbb{E}[L(e, v) | e_{max}] &= \frac{1}{F(e_{max})} \cdot \int_0^{e_{max}} L(e, v) dF(e) \\ &= A(v) \cdot [e_{max} - \mathbb{E}(e | e_{max})] \\ &= L(e = \mathbb{E}(e | e_{max}), v), \end{aligned}$$

$$Var[L(e, v) | e_{max}] = A(v)^2 \cdot Var(e | e_{max}),$$

where the conditional variance of e under $e < e_{max}$ is

$$Var(e|e_{max}) = \mathbb{E}(e^2|e_{max}) - \mathbb{E}(e|e_{max})^2.$$

The comparative statics $\frac{d}{dv}Var(e|e_{max}) > 0$ and $\frac{d}{dv}Var[L(e, v)|e_{max}] > 0$ follow from the properties of truncated variance and $A'(v) > 0$.

By the same approach, we compute an inframarginal entrepreneur's rent as follows. First, define the expected payoff of an entrepreneur with a fixed quality $p > p_{min}$, $J(p, v)$ as

$$J(p, v) := \eta(\theta) \cdot \Pr(s_{i,j} = H, w_i = w_G | p_i = p, e_j \leq e_{max}) \cdot V^e(w_G, k, v) - C_e. \quad (19)$$

Next, rearranging (10), we have

$$V^e(w_G, k, v) = \frac{C_e}{\eta(\theta) \cdot p_{min} \{1 - \mathbb{E}[e|e(p_{min})]\}}.$$

Then, by plugging it into (19), it is simplified as follows:

$$J(p, v) = C_e \cdot \left(\frac{p}{p_{min}} - 1 \right). \quad (20)$$

The mean and variance of entrepreneur performance in the market are computed as

$$\begin{aligned} \mathbb{E}[J(p, v)|p_{min}] &= \frac{1}{1 - G(p_{min})} \cdot \int_{p_{min}}^1 J(p, v) dG(p) \\ &= J(p = \mathbb{E}(p|p_{min}), v), \end{aligned}$$

$$\begin{aligned} Var[J(p, v)|p_{min}] &= \left(\frac{C_e}{p_{min}} \right)^2 \cdot Var(p|p_{min}) \\ &\equiv \left(\frac{C_e}{p_{min}} \right)^2 \{ \mathbb{E}(p^2|p_{min}) - \mathbb{E}(p|p_{min})^2 \}. \end{aligned}$$

Theorem 2.5. (*Procyclical skill amplification and dispersion*) The expected payoff $L(e, v)$ for an incumbent VC with skill $e < e_{max}$ and $J(p, v)$ for an incumbent entrepreneur with skill $p > p_{min}$ exhibit the following properties in the long-run equilibrium:

1. **Invariant Ranking:** Better skills yield higher expected returns.

$$-\frac{dL}{de} = A(v) > 0 \quad \text{and} \quad \frac{dJ}{dp} = \frac{C_e}{p_{min}} > 0.$$

2. **Rising Tide:** All incumbents benefit from a hotter market, regardless of the direction of the shift in k .

$$\frac{dL}{dv} = A'(v)(e_{max} - e) + A(v) \cdot \frac{de_{max}}{dv} > 0 \quad \text{and} \quad \frac{dJ}{dv} = -\frac{dp_{min}}{dv} \cdot \frac{pC_e}{p_{min}^2} > 0.$$

3. **Skill Amplification:** The marginal benefits of market heat are strictly higher for higher-skilled participants (i.e., those with lower e and higher p).

$$-\frac{d^2L}{dedv} = A'(v) > 0 \quad \text{and} \quad \frac{d^2J}{dpdv} = -\frac{dp_{min}}{dv} \cdot \frac{C_e}{p_{min}^2} > 0.$$

4. **Procyclical Dispersion:**

(a) **Skill Dispersion:** Variances of skills among market participants increase.

$$\frac{d}{dv} \text{Var}(e|e_{max}) > 0 \quad \text{and} \quad \frac{d}{dv} \text{Var}(p|p_{min}) > 0.$$

(b) **Performance Dispersion:** The cross-sectional variances of incumbent participant performance increase.

$$\frac{d}{dv} \text{Var}[L(e, v)|e_{max}] > 0 \quad \text{and} \quad \frac{d}{dv} \text{Var}[J(p, v)|p_{min}] > 0.$$

Proof. See Appendix E.

In the following explanations, we focus on cases regarding inframarginal VCs. First, to see why Theorem 2.5 generates performance persistence, consider two incumbent VCs active in a market with a given heat v . Suppose they have skills e_{high} and e_{low} , where $e_{high} < e_{low}$ (recall that lower error rates imply higher skill). Consequently, the high-skill VC earns higher

expected profits than the low-skill VC in the initial state.

Now consider an upward shock to v . By Theorem 2.5.2 (“Rising Tide”), both VCs earn higher profits in the hotter market. However, by Theorem 2.5.3 (“Skill Amplification”), the profit gains accrue disproportionately to the high-skill VC. As the market heats up, the performance spread between the two widens. This mechanism generates positive autocorrelation in returns: funds that outperformed in the cold market are predicted to outperform by an even larger margin in the hot market.

Theorem 2.5.4 (“Procyclical Dispersion”) extends this logic to the aggregate distribution. As formalized in the proof, the variance of returns increases through two reinforcing channels: a *selection effect*, driven by the increase in marginal error rate e_{max} which widens the range of active skills in the market (4a), and a *magnification effect*, as the steeper slope of returns to skill ($A'(v) > 0$) amplifies the economic value of that heterogeneity (4b).

Finally, we clarify the interpretation of VC payoffs in our framework. Our model treats the venture firm as a unitary economic agent, abstracting from the internal contract between General Partners (GPs) and Limited Partners (LPs). While a distinct literature examines how this joint surplus is divided, our results regarding persistence extend to LPs under standard market frictions. Specifically, provided that GPs cannot perfectly and instantaneously extract all scarcity rents via higher fees—due to sticky fee structures or reputational concerns—the ‘amplification of skill’ in Theorem 2.5 is shared between the manager and the investor. Thus, the procyclical widening of the performance gap in our model predicts persistence in net-of-fee returns for LPs, consistent with the empirical evidence.

3 The Anatomy of a Boom

Ideas are fast, but institutions are slow. Economic shocks often arrive with a suddenness that outpaces the infrastructure designed to fund them. When investment opportunities improve ($v \uparrow$), the existing supply of venture capital firms might initially struggle to accommodate a surge of new entrepreneurial ideas. This creates a dynamic tension: how does a market clear when the demand for capital explodes, but the supply remains temporarily under-prepared for the boom? To address this, we decompose the market’s response into two distinct phases: a short-run period of institutional adjustment and a long-run period of competitive entry.

The central assumption of our short-run analysis is that the supply of venture capital is temporarily fixed, or “sticky.” The economic justification for this stickiness is the significant lead time required to institutionalize new capital. While a founder can launch a seed startup business pitch relatively quickly, raising a new venture capital firm is a multi-year endeavor. Prospective general partners must build a track record, cultivate relationships with limited partners, and secure capital commitments before they can begin the deployment cycle. This structural lag mismatch is a well-documented feature of the venture market (e.g., [Gompers and Lerner \(2004\)](#), [Kaplan and Schoar \(2005\)](#)).

We begin by characterizing the equilibrium dynamics during this phase where capital supply is fixed and the division of surplus k must carry the full burden of market clearing.

3.1 Short Run Equilibrium with Sticky VC Supply

Suppose that the market is initially endowed with a market heat represented by $v > 0$, with equilibrium values denoted by $\{p_{min}, e_{max}, k\}$ and $\theta = \frac{\omega \cdot g(p_{min})}{f(e_{max})}$. Then, assume that market heat increases to $v^* > v$ while the market faces a sticky VC supply in the short run, fixing the marginal VC entry to the previous equilibrium’s level, $\bar{e} \equiv e_{max}$.

When $\lambda < \lambda^*(p_{min}, e_{max})$ holds, we can represent $p_{min} \equiv p(e_{max})$ in equilibrium, where $p(\cdot)$ is strictly decreasing². Also, denote the new short-run market tightness by $\bar{\theta} = \frac{\omega \cdot g(p(\bar{e}))}{f(\bar{e})}$. Then, the entry condition of the marginal entrepreneur can be written as in (10):

$$\eta(\theta) \cdot p(\bar{e}) \cdot \{1 - \mathbb{E}(e|\bar{e})\} \cdot V^e(w_G, \bar{k}, v^*) - C_e = 0,$$

where $\bar{k}$ represents the new equilibrium level of k in the short run. By plugging $p_{min} = p(\bar{e})$ and rearranging the equation above, we have

$$V^e(w_G, \bar{k}, v^*) = \hat{J}(\bar{e}) := \frac{C_e}{\eta(\bar{\theta}) \cdot p(\bar{e}) \cdot \{1 - \mathbb{E}(e|\bar{e})\}} = V^e(w_G, k, v),$$

where $\hat{J}(\cdot)$ is a strictly increasing function, and the last equality follows from (10) in the previous equilibrium before a change in v .

Using the implicit function theorem, we verify the following property:

Proposition 3.1. *In the short run with a fixed VC supply,*

$$\frac{dk}{dv} = -\frac{\partial V^e / \partial v}{\partial V^e / \partial k} < 0 \quad \text{and} \quad L(\bar{e}, v^*) > L(e_{max}, v) = 0.$$

The first property of Proposition 3.1, $\frac{dk}{dv} = -\frac{\partial V^e / \partial v}{\partial V^e / \partial k} < 0$, delivers the central result of the short-run analysis: an increase in market heat (v) unambiguously shifts bargaining power to VCs ($dk/dv < 0$). This stands in sharp contrast to the long-run equilibrium, where the effect is ambiguous. The intuition for this stark result stems from the market's adjustment mechanism when one side is constrained. In the long run, a hotter market leads to quantity adjustment, as quality thresholds for entry fall and more deals are done. In the

²For further details, see *Uniqueness of the screening equilibrium* in Appendix B.

short run, however, this channel is shut down. Because the number of VCs is fixed, the number of fundable projects is also fixed, meaning the market can only respond through price adjustment. VCs effectively become the fixed factor of production. With rising project values increasing the demand for a fixed supply of capital, competition forces entrepreneurs to accept progressively more VC-friendly terms (k falls), allowing incumbent VCs to capture the entire windfall from the market boom as an “incumbency rent.”

The second property, $L(\bar{e}, v^*) > L(e_{max}, v) = 0$, implies that during this short-run equilibrium, VCs benefit doubly: both because of improved investment opportunities, and because of shifting bargaining power. Thus, even marginal VCs in the previous and the new short-run equilibrium enjoy strictly positive rents *ex ante*.

3.2 Transition to the long-run equilibrium

When the temporarily sticky VC supply is resolved, the market shifts to a new equilibrium, denoted by $\{p_{min}^* = p(e_{max}^*), e_{max}^*, k^*\}$ and $\theta^* = \frac{\omega \cdot g(p_{min}^*)}{f(e_{max}^*)}$. By Theorem 2.4.1, we know $p_{min}^* < p_{min} = p(\bar{e})$ and $e_{max}^* > e_{max} = \bar{e}$, reflecting the expansion of the talent pool in both sides of the market. Combined with the monotonicity of value functions (or equivalently, the *ex-post payoffs* when $w = w_G$ is realized after funding) in v and k , we have the following properties:

Proposition 3.2. *The high rents earned during the short-run phase induce new VC entry, resolving the supply constraint. This transition from the hot short-run equilibrium $(\bar{k}, v^*)$ to the new hot long-run equilibrium (k^*, v^*) has the following effects:*

1. *New VC entry intensifies competition for deals, shifting bargaining power back toward entrepreneurs: $k^* > \bar{k}$.*

2. *The new competition erodes the incumbency rents captured by VCs during the short-run phase, while entrepreneurs are better off relative to the short run:*

$$V^{VC}(w_G, \bar{k}, v^*) > V^{VC}(w_G, k^*, v^*) > V^{VC}(w_G, k, v),$$

$$V^e(w_G, k^*, v^*) > V^e(w_G, \bar{k}, v^*) = V^e(w_G, k, v).$$

Despite the erosion of temporary super-profits, the market boom is welfare-improving for all inframarginal participants. In the new long-run equilibrium, both parties are strictly better off than in the original cold market.

3. *The market becomes more congested in the long run:*

$$\theta^* > \bar{\theta} = \theta \quad \text{and} \quad \eta(\theta^*) < \eta(\bar{\theta}) = \eta(\theta).$$

Proof. See Appendix [F](#).

In principle, the supply of venture capital can adjust on either the intensive margin (incumbents raising larger funds) or the extensive margin (new firm creation). This paper focuses on the extensive margin, isolating our central mechanism: the deterioration of marginal talent (e_{max}) as the market expands. While the intensive margin is also operative during booms, it generates similar quality-degrading effects via attention dilution, as partners must spread their limited bandwidth across larger portfolios. Thus, whether through new, lower-skilled entrants or overstretched incumbents, the aggregate economic outcome is the same: a decline in the average quality of VC services. Our framework captures this first-order effect in the most tractable way.

4 Empirical Predictions

Our model provides a framework for understanding how market heat (v) translates into observable outcomes. A central insight of our analysis is that not all booms are created equal. The ultimate allocation of bargaining power depends critically on the nature of the technological shock—specifically, whether it democratizes entrepreneurship or favors specialized, scarce talent. We distinguish between these two regimes to generate a set of testable predictions for empirical researchers.

Before detailing specific predictions, we note a bridge between our abstraction and empirical reality. Strictly speaking, our model derives the equilibrium division of surplus, k . In practice, this abstract bargaining power manifests through a complex bundle of observable contract terms and investor behaviors. More broadly, we interpret k as proxying for a summary statistic for the entire deal structure. For example, when our model predicts a shift in bargaining power to VCs, we expect to see this reflected in lower pre-money valuations, stricter protective provisions (such as full-ratchet anti-dilution or participating liquidation preferences), and more aggressive governance. Conversely, a shift toward entrepreneurs should result in “cleaner” term sheets and looser oversight. This mapping allows us to extrapolate from our formal results on surplus division to predictions about contract design and investor patience.

The Nature of the Shock and Contract Terms. We first consider the evolution of contract terms. In a “Democratizing shock” (e.g., the dot-com boom), barriers to entry fall, creating a deep pool of substitute entrepreneurs (high marginal density $g(p)$). Our model predicts that in this regime, the supply of entrepreneurial talent is highly elastic. Consequently, despite rising valuations (v), bargaining power shifts toward VCs, the relatively scarce bottleneck. Empirically, this implies that while post-money valuations may rise

with the tide, pre-money valuations (a proxy for the founder’s distinct contribution) will lag, effectively diluting the founder’s ownership share. Protective terms (such as liquidation preferences and board control) should also tighten. Conversely, in a “Specializing shock” (e.g., Generative AI or biotech), talent is scarce and inelastic (low $g(p)$). Here, the low density of marginal entrepreneurs implies that the average entrant is of high quality, leading to a higher overall probability of successful exits. VCs must compete for this limited pool of experts, driving up pre-money valuations and leading to founder-friendly terms (e.g., dual-class stock, “clean” term sheets) that mirror the high exit expectations. These predictions coincide with Property 1, the argument on the relative specialness of talents, formally stated in Theorem 2.4.2.

Outcome Dispersion and Performance Persistence. The market cycle also shapes the second moment of outcomes. As derived in Theorem 2.5.4, hot markets strictly increase dispersion. For VCs, the entry of lower-skilled (“tourist”) capital increases the cross-sectional variance of fund returns. Ideally, empirical work would also examine the dispersion of entrepreneur payoffs. Just as the boom brings in marginal VCs, it attracts marginal business ideas (low p). In a Democratizing shock, this extensive margin creates a fat left tail of failed startups. Furthermore, our model predicts that *performance persistence* for VCs is procyclical. The influx of unskilled capital prevents the market-clearing price from tightening enough to extract the full surplus from high-skill incumbents, strengthening the autocorrelation of top-tier fund returns during booms.

Scarcity, Patience, and Staging. Finally, we consider implications for governance and deal staging. The equilibrium bargaining power (k) reflects the opportunity cost of the scarce party. In a Democratizing shock where VCs are the bottleneck, their time is the constraining resource. We therefore expect VCs to exhibit *impatience* and impose tighter staging (more frequent milestones, shorter runways), allowing them to abandon underperforming projects

quickly and reallocate resources. In contrast, in a Specializing shock where talent is the bottleneck, VCs must exhibit *patience* to retain scarce human capital. This predicts looser staging (longer runways) and implies that the startup failure rate may not rise as sharply as in a democratizing boom, as investors are willing to nurture marginal projects for longer. In addition, a smaller influx of lesser talent during booms, due to the specialized nature of required talent (low $g(p_{min})$), may also contribute to significantly lower failure probabilities than under a Democratizing shock.

Table 2 summarizes these predictions, mapping the type of shock to the expected directional change in observable market moments relative to a baseline cold market.

Table 2: Predicted Shifts in Observable Moments during Market Booms

Outcome Variable	Democratizing Shock (High $g(p)$, Elastic Talent)	Specializing Shock (Low $g(p)$, Inelastic Talent)
<i>Panel A: Contract Terms (Deal-Level)</i>		
Pre-Money / Post-Money Ratio	↓ (Higher Dilution)	↑ (Lower Dilution)
Investor Protections (e.g., Liq. Prefs)	↑ (Stricter)	↓ (Looser)
<i>Panel B: Outcomes & Performance (Fund & Founder Level)</i>		
Startup Failure Rate	↑↑	Ambiguous
VC Fund Return Variance	↑↑	↑
Entrepreneur Payoff Variance	↑↑	↑
VC Performance Persistence	↑ (Stronger)	↑ (Stronger)
<i>Panel C: Governance & Duration</i>		
Time to Liquidation (for Failures)	↓ (Impatient Capital)	↑ (Patient Capital)
Staging Intensity (Frequency of Rounds)	↑ (Tighter Control)	↓ (Looser Control)
Matching (VC-Founder Sorting)	↓ (Weaker)	↑ (Stronger)

5 Conclusion

This paper provides the first microfounded theory of how bargaining power evolves over the lifecycle of a venture capital boom. By endogenizing bargaining power as the Walrasian price that clears the market for talent, our framework reveals a distinct, two-phase “anatomy of a boom.” Following a positive shock, the market’s initial response is a short-run phase of pure price adjustment. Constrained by a fixed supply of incumbent VCs, the market clears by shifting bargaining power unambiguously in their favor, allowing them to capture windfall “incumbency rents.” These high returns, in turn, induce a long-run phase of quantity adjustment. As new, lower-quality entrants flood the market, the supply constraint is relaxed, and competition for deals intensifies, eroding the incumbent VCs’ super-normal profits and allowing entrepreneurs to finally capture a share of the boom’s surplus. The ultimate long-run division of bargaining power is nuanced, depending on the nature of the technological shock itself, which determines the marginal elasticity of talent supply on each side of the market.

This framework not only provides a rigorous explanation for the venture cycle but also offers new insights into other key empirical features of the market. For instance, the model provides a new, dynamic mechanism for the well-documented puzzle of performance persistence. A “rich get richer” property of the equilibrium shows how market booms act as a powerful amplifier of the returns to skill for elite VCs. Furthermore, the cyclical sorting of talent in our model offers a natural explanation for the procyclicality of innovation types, where cold markets dominated by elite agents are more conducive to radical innovation, while hot markets with lower-quality entrants favor more incremental projects. By providing a unified theory of these interconnected phenomena, our model offers a new lens through which to understand the complex dynamics of skill-intensive markets.

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A Proof of Proposition 2.1

Proof. We compute the conditional probability $\Pi_H(p_{min}, e_j)$ in (6) as follows:

$$\Pi_H(p_{min}, e_j) = \frac{\Pr(w_i = w_G, s_{i,j} = H | p_i \geq p_{min}, e_j)}{\Pr(s_{i,j} = H | p_i \geq p_{min}, e_j)},$$

where

$$\begin{aligned} \Pr(w_i = w_G, s_{i,j} = H | p_i \geq p_{min}, e_j) &= \Pr(s_{i,j} = H | w_i = w_G, p_i \geq p_{min}, e_j) \\ &\quad \times \Pr(w_i = w_G | p_i \geq p_{min}, e_j) \\ &= \Pr(s_{i,j} = H | w_i = w_G, e_j) \times \Pr(w_i = w_G | p_i \geq p_{min}) \\ &= (1 - e_j) \times \mathbb{E}[\Pr(w_i = w_G | p_i) | p_i \geq p_{min}] \\ &= (1 - e_j) \mathbb{E}(p | p_{min}), \end{aligned}$$

$$\begin{aligned} \Pr(w_i = w_B, s_{i,j} = H | p_i \geq p_{min}, e_j) &= \Pr(s_{i,j} = H | w_i = w_B, p_i \geq p_{min}, e_j) \\ &\quad \times \Pr(w_i = w_B | p_i \geq p_{min}, e_j) \\ &= \Pr(s_{i,j} = H | w_i = w_B, e_j) \times \Pr(w_i = w_B | p_i \geq p_{min}) \\ &= \lambda e_j \times \mathbb{E}[\Pr(w_i = w_B | p_i) | p_i \geq p_{min}] \\ &= \lambda e_j \{1 - \mathbb{E}(p | p_{min})\}, \end{aligned}$$

$$\begin{aligned} \Pr(s_{i,j} = H | p_i \geq p_{min}, e_j) &= \Pr(w_i = w_G, s_{i,j} = H | p_i \geq p_{min}, e_j) \\ &\quad + \Pr(w_i = w_B, s_{i,j} = H | p_i \geq p_{min}, e_j) \\ &= (1 - e_j) \mathbb{E}(p | p_{min}) + \lambda e_j \{1 - \mathbb{E}(p | p_{min})\}, \end{aligned}$$

$$\Rightarrow \Pi_H(p_{min}, e_j) = \frac{(1 - e_j) \mathbb{E}(p | p_{min})}{(1 - e_j) \mathbb{E}(p | p_{min}) + \lambda e_j \{1 - \mathbb{E}(p | p_{min})\}}.$$

Likewise, we compute $\Pi_L(p_{min}, e_j)$ in (7) as follows:

$$\begin{aligned}\Pi_L(p_{min}, e_j) &= \frac{\Pr(w_i = w_G, s_{i,j} = L | p_i \geq p_{min}, e_j)}{\Pr(s_{i,j} = L | p_i \geq p_{min}, e_j)} \\ &= \frac{e_j \cdot \mathbb{E}(p | p_{min})}{e_j \cdot \mathbb{E}(p | p_{min}) + (1 - \lambda e_j) \{1 - \mathbb{E}(p | p_{min})\}}.\end{aligned}$$

Then, the expected payoffs of a VC with $e_j \leq e_{max}$, (5a), is computed as follows:

$$\begin{aligned}\pi_H(e_j) &= \frac{(1 - e_j) \mathbb{E}(p | p_{min})}{(1 - e_j) \mathbb{E}(p | p_{min}) + \lambda e_j \{1 - \mathbb{E}(p | p_{min})\}} \cdot \{V^{VC}(w_G, k, v) - I\} \\ &\quad - \frac{\lambda e_j \{1 - \mathbb{E}(p_i | p_{min})\}}{(1 - e_j) \mathbb{E}(p | p_{min}) + \lambda e_j \{1 - \mathbb{E}(p | p_{min})\}} \cdot I, \\ \pi_L(e_j) &= \frac{e_j \cdot \mathbb{E}(p | p_{min})}{e_j \cdot \mathbb{E}(p | p_{min}) + (1 - \lambda e_j) \{1 - \mathbb{E}(p | p_{min})\}} \cdot \{V^{VC}(w_G, k, v) - I\} \\ &\quad - \frac{(1 - \lambda e_j) \{1 - \mathbb{E}(p | p_{min})\}}{e_j \cdot \mathbb{E}(p | p_{min}) + (1 - \lambda e_j) \{1 - \mathbb{E}(p | p_{min})\}} \cdot I,\end{aligned}$$

and the ex-ante rent of the corresponding VC upon entry, (5b), is computed as

$$\begin{aligned}\Pr(s_{i,j} = H | p_i \geq p_{min}, e_j) \cdot \pi_H(e_j) - C_{VC} &= \{V^{VC}(w_G, k, v) - I\} (1 - e_j) \mathbb{E}(p | p_{min}) \\ &\quad - \lambda e_j \{1 - \mathbb{E}(p | p_{min})\} I - C_{VC}.\end{aligned}$$

Since the RHS of the equation above (henceforth, denote by $RHS(e_j)$) strictly decreases in e_j , we find that $RHS(e_j) > 0$ necessarily holds if $RHS(e_{max}) = 0$. Therefore, we derive the VC entry condition (11).

The second incentive-compatibility condition in (5b), $\pi_L(e_j) \leq 0$, is equivalent to

$$e_j \cdot \mathbb{E}(p_i | p_{min}) \cdot \{V^{VC}(w_G, k, v) - I\} - (1 - \lambda e_j) \{1 - \mathbb{E}(p | p_{min})\} \cdot I \leq 0,$$

where its LHS (henceforth, denote by $LHS(e_j)$) strictly increases in e_j . Hence, we verify that $LHS(e_j) < 0$ necessarily holds if $LHS(e_{max}) = 0$. Therefore, we derive the second incentive-compatibility condition in (5b).

For an entrepreneur with $p_i \geq p_{min}$, we have

$$\begin{aligned} \Pr(w_i = w_G, s_{i,j} = H | p_i, e_j \leq e_{max}) &= \Pr(s_{i,j} = H | w_i = w_G, p_i, e_j \leq e_{max}) \\ &\quad \times \Pr(w_i = w_G | p_i, e_j \leq e_{max}) \\ &= \Pr(s_{i,j} = H | w_i = w_G, e_j \leq e_{max}) \times \Pr(w_i = w_G | p_i) \\ &= \mathbb{E}[\Pr(s_{i,j} = H | w_i = w_G, e_j) | e_j \leq e_{max}] \times \Pr(w_i = w_G | p_i) \\ &= \{1 - \mathbb{E}(e | e_{max})\} \cdot p_i. \end{aligned}$$

Hence, the corresponding entrepreneur's entry condition, (5c), follows as

$$\eta(\theta) \cdot \{1 - \mathbb{E}(e | e_{max})\} \cdot p_i \cdot V^e(w_G, k, v) - C_e \geq 0,$$

where the LHS strictly increases in p_i . Thus, we verify that $LHS(p_i) > 0$ necessarily holds if $LHS(p_{min}) = 0$. Therefore, we derive the entrepreneur entry condition (10).

To simplify the market-clearing condition in (5d), we compute Φ as follows and derive (12):

$$\begin{aligned}
\Pr(s_{i,j} = H | p_i \geq p_{min}, e_j \leq e_{max}) &= \Pr(w_i = w_G, s_{i,j} = H | p_i \geq p_{min}, e_j \leq e_{max}) \\
&\quad + \Pr(w_i = w_B, s_{i,j} = H | p_i \geq p_{min}, e_j \leq e_{max}) \\
&= \Pr(s_{i,j} = H | w_i = w_G, e_j \leq e_{max}) \times \Pr(w_i = w_G | p_i \geq p_{min}) \\
&\quad + \Pr(s_{i,j} = H | w_i = w_B, e_j \leq e_{max}) \times \Pr(w_i = w_B | p_i \geq p_{min}) \\
&= \{1 - \mathbb{E}(e | e_{max})\} \mathbb{E}(p | p_{min}) + \lambda \mathbb{E}(e | e_{max}) \{1 - \mathbb{E}(p | p_{min})\} \equiv \Phi.
\end{aligned}$$

□

B Proof of Proposition 2.2

Proof. Assume that λ is low enough to be bounded by an endogenous threshold

$$\lambda < \lambda^*(p_{min}, e_{max}) := \min \left\{ \frac{\mathbb{E}(p | p_{min})}{1 - \mathbb{E}(p | p_{min})}, \frac{1 - \mathbb{E}(e | e_{max})}{\mathbb{E}(e | e_{max})} \right\}.$$

1. (*Proposition 2.2.1*) As we rearrange (11), it follows that

$$V^{VC}(w_G, k, v) - I = \frac{\lambda I \cdot \{1 - \mathbb{E}(p | p_{min})\} \cdot e_{max} + C_{VC}}{\mathbb{E}(p | p_{min}) (1 - e_{max})}.$$

Then, as we plug the equation above into the second inequality in (5b) and rearrange it, we have

$$e_{max} \cdot C_{VC} + \lambda I \cdot e_{max}^2 \{1 - \mathbb{E}(p | p_{min})\} \leq I \cdot (1 - e_{max}) (1 - \lambda e_{max}) \{1 - \mathbb{E}(p | p_{min})\},$$

which is further rearranged as

$$[C_{VC} + (1 + \lambda) I \cdot \{1 - \mathbb{E}(p | p_{min})\}] \cdot e_{max} \leq I \cdot \{1 - \mathbb{E}(p | p_{min})\}.$$

By simplifying the equation above, we derive (14).

2. (*Proposition 2.2.2*) We can rearrange (10) and (11) as follows:

$$\mathbf{V}^e(k) = J(p_{min}, e_{max}) := \frac{C_e}{\eta(\theta) \cdot p_{min} \{1 - \mathbb{E}[e|e_{max}]\}},$$

$$\mathbf{V}^{VC}(k) - I = L(p_{min}, e_{max}) := \frac{\lambda I \cdot \{1 - \mathbb{E}(p|p_{min})\} \cdot e_{max} + C_{VC}}{\mathbb{E}(p|p_{min}) \{1 - e_{max}\}},$$

subject to the market clearing condition (12).

In this proof, assume that both $\mathbf{V}^e(k)$ and $\mathbf{V}^{VC}(k)$ can increase boundlessly in k if it is unbounded: i.e., $\mathbf{V}^e(k) \xrightarrow{k \rightarrow \infty} \infty$ and $\mathbf{V}^{VC}(k) \xrightarrow{k \rightarrow -\infty} \infty$. First, when $p_{min} \rightarrow 1$, $e_{max} \rightarrow$ and $\theta \rightarrow 1$ should hold to meet (12). In this case, $J(p_{min}, e_{max}) \rightarrow \frac{C_e}{\eta(1)}$ and $L(p_{min}, e_{max}) \rightarrow C_{VC}$ hold. Second, when $p_{min} \rightarrow 0$, provided that there exists $e^* \in (0, \frac{1}{2})$ holding $F(e^*) = \omega \lambda \mathbb{E}(e|e_{max} = e^*)$, $e_{max} \rightarrow e^*$ should hold to meet (12). In this case, both $J(p_{min}, e_{max}) \rightarrow \infty$ and $L(p_{min}, e_{max}) \rightarrow \infty$ hold.

Since $\mathbf{V}^e(k)$ and $\mathbf{V}^{VC}(k)$ are strictly monotone in k , provided that (12) can be represented in the form of $e_{max} = e(p_{min})$ for some function $e(\cdot)$, we can rearrange the system $\{(10), (11), (12)\}$ in the following form:

$$k = k_e(p_{min}) := (\mathbf{V}^e)^{-1}[J(p_{min}, e_{max} = e(p_{min}))],$$

$$k = k_{VC}(p_{min}) := (\mathbf{V}^{VC})^{-1}[L(p_{min}, e_{max} = e(p_{min})) + I],$$

where the equilibrium is determined at $k_{eq}(p_{min}) \equiv k_e(p_{min}) - k_{VC}(p_{min}) = 0$

We know $k_{eq}(p_{min}) \xrightarrow{p_{min} \rightarrow 0} \infty$ and $k_{eq}(p_{min}) \xrightarrow{p_{min} \rightarrow 1} (\mathbf{V}^e)^{-1}\left(\frac{C_e}{\eta(1)}\right) - (\mathbf{V}^{VC})^{-1}(C_{VC} + I)$. Provided that (15) holds, we have $(\mathbf{V}^e)^{-1}\left(\frac{C_e}{\eta(1)}\right) - (\mathbf{V}^{VC})^{-1}(C_{VC} + I) < 0$. Since

$k_{eq}(p_{min})$ is a continuous function, these conditions imply that there exists $p_{min}^* \in (0, 1)$ for which $k_{eq}(p_{min}^*) = 0$. In addition, (15) guarantees $k_e(p_{min}^*) = k_{VC}(p_{min}^*) \in (0, 1)$.

3. (Uniqueness of the screening equilibrium) Using the implicit function theorem, we derive the relationship between p_{min} and e_{max} as follows:

$$e'(p_{min}) \equiv \frac{de_{max}}{dp_{min}} = -\frac{\partial H / \partial p_{min}}{\partial H / \partial e_{max}},$$

where

$$\begin{aligned} \frac{\partial H}{\partial p_{min}} &= \omega \cdot g(p_{min}) \cdot \Phi - \omega \cdot \{1 - G(p_{min})\} \cdot \frac{\partial \Phi}{\partial p_{min}} \\ &= \omega \cdot g(p_{min}) \cdot [p_{min} \{1 - \mathbb{E}[e|e_{max}]\} + \lambda(1 - p_{min}) \mathbb{E}(e|e_{max})] > 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial H}{\partial e_{max}} &= f(e_{max}) - \omega \cdot \{1 - G(p_{min})\} \cdot \frac{\partial \Phi}{\partial e_{max}} \\ &= f(e_{max}) \cdot [1 - \theta \{\lambda - (1 + \lambda) \mathbb{E}(p|p_{min})\} \{e_{max} - \mathbb{E}(e|e_{max})\}]. \end{aligned}$$

Given $\lambda < \lambda^*(p_{min}, e_{max})$, as $\lambda - (1 + \lambda) \mathbb{E}(p|p_{min}) < 0$ holds, we have $\frac{\partial H}{\partial e_{max}} > 0$.

Hence, it follows that

$$e'(p_{min}) = -\frac{\omega \cdot g(p_{min})}{f(e_{max})} \cdot \frac{p_{min} \{1 - \mathbb{E}[e|e_{max}]\} + \lambda(1 - p_{min}) \mathbb{E}(e|e_{max}) = \oplus}{1 - \theta \underbrace{\left\{ \lambda - (1 + \lambda) \mathbb{E}(p|p_{min}) \right\}}_{<0} \{e_{max} - \mathbb{E}(e|e_{max})\} = \oplus} < 0. \quad (21)$$

Then, the system is simplified as follows:

$$J^* \equiv \mathbf{V}^e(k) - \left(\tilde{J}(p_{min}) := \frac{C_e}{\eta(\theta) \cdot p_{min} \{1 - \mathbb{E}[e|e(p_{min})]\}} \right) = 0, \quad (22)$$

$$L^* \equiv \mathbf{V}^{VC}(k) - I - \left(\tilde{L}(p_{min}) := \frac{\lambda I \cdot \{1 - \mathbb{E}(p|p_{min})\} \cdot e_{max} + C_{VC}}{\mathbb{E}(p|p_{min}) \{1 - e_{max}\}} \right) = 0, \quad (23)$$

where $e_{max} = e(p_{min})$ follows from rearranging (12).

As we rearrange the market-clearing condition (12) as $\theta \left(:= \frac{\omega\{1-G(p_{min})\}}{F(e_{max})} \right) = \Phi^{-1}$, it follows that

$$\begin{aligned} \frac{d\Phi}{dp_{min}} &= \frac{\partial\Phi}{\partial p_{min}} + \frac{\partial\Phi}{\partial e_{max}} \cdot e'(p_{min}) \\ &= \underbrace{\left\{ 1 - (1 + \lambda) \mathbb{E}(e|e_{max}) \right\}}_{>0} \cdot \underbrace{\frac{d}{dp_{min}} \mathbb{E}(p|p_{min})}_{>0} \\ &\quad + \underbrace{\left\{ \lambda - (1 + \lambda) \mathbb{E}(p|p_{min}) \right\}}_{<0} \cdot \underbrace{\frac{d}{de_{max}} \mathbb{E}[e|e_{max}]}_{>0} \cdot \underbrace{e'(p_{min})}_{<0} > 0, \end{aligned} \quad (24)$$

Then, we verify that $\tilde{J}(p_{min})$ and $\tilde{L}(p_{min})$ strictly decrease in p_{min} , as follows:

$$\begin{aligned} \frac{d\tilde{J}(p_{min})}{dp_{min}} &= - \frac{\tilde{J}(p_{min})^2 (= V^e(w_G, k, v))^2}{C_e} \cdot (*)' < 0, \\ (*)' &:= \frac{d}{dp_{min}} [\eta(\theta) \cdot p_{min} \{1 - \mathbb{E}[e|e(p_{min})]\}], \\ &= \underbrace{\eta'(\theta)}_{<0} \cdot \underbrace{\frac{d\theta}{dp_{min}}}_{<0} \cdot p_{min} \{1 - \mathbb{E}[e|e(p_{min})]\} + \eta(\theta) \{1 - \mathbb{E}[e|e(p_{min})]\} \\ &\quad - \eta(\theta) \cdot p_{min} \cdot \underbrace{\frac{d}{de_{max}} \mathbb{E}[e|e_{max}]}_{>0} \cdot \underbrace{e'(p_{min})}_{<0} > 0, \\ \frac{d\tilde{L}}{dp_{min}} &= - \frac{\tilde{L}(p_{min})^2 \left(= \{V^{VC}(w_G, k, v) - I\}^2 \right)}{C_{VC} + \lambda I \{1 - \mathbb{E}(p|p_{min})\} e_{max}} \cdot (**) < 0, \end{aligned} \quad (25)$$

$$\begin{aligned}
(**)' &:= \left\{ V^{VC}(w_G, k, v) - I \right\} \left\{ (1 - e_{max}) \cdot \underbrace{\frac{d}{dp_{min}} \mathbb{E}(p|p_{min})}_{>0} - \mathbb{E}(p|p_{min}) \cdot \underbrace{e'(p_{min})}_{<0} \right\} \\
&+ \lambda I \left[e_{max} \cdot \underbrace{\frac{d}{dp_{min}} \mathbb{E}(p|p_{min})}_{>0} - \{1 - \mathbb{E}(p|p_{min})\} \cdot \underbrace{e'(p_{min})}_{<0} \right] > 0.
\end{aligned} \tag{26}$$

Then, we can summarize the system $\{(10), (11), (12)\}$ as follows:

$$k_e(p_{min}) := (\mathbf{V}^e)^{-1} \left[\tilde{J}(p_{min}) \right],$$

$$k_{VC}(p_{min}) := (\mathbf{V}^{VC})^{-1} \left[\tilde{L}(p_{min}) + I \right],$$

where $k_e(p_{min})$ strictly decreases in p_{min} , while $k_{VC}(p_{min})$ strictly increases in p_{min} .

Therefore, provided that (15) holds, the system has a unique solution within $(0, 1)^3$.

□

C Proof of Theorem 2.3

Proof. Consider a parameter $x \in \{v, C_e, C_{VC}, I, \omega, \lambda\}$. Then, by solving the following system of two unknowns, $\left\{ \frac{dp_{min}}{dx}, \frac{dk}{dx} \right\}$:

$$\frac{dJ^*}{dx} = \frac{\partial J^*}{\partial x} + \frac{dJ^*}{dk} \cdot \frac{dk}{dx} + \frac{dJ^*}{dp_{min}} \cdot \frac{dp_{min}}{dx},$$

$$\frac{dL^*}{dx} = \frac{\partial L^*}{\partial x} + \frac{dL^*}{dx} \cdot \frac{dk}{dx} + \frac{dL^*}{dp_{min}} \cdot \frac{dp_{min}}{dx},$$

we have the solution as follows:

$$\frac{dp_{min}}{dx} = \frac{\frac{\partial J^*}{\partial x} \cdot \frac{\partial L^*}{\partial k} - \frac{\partial J^*}{\partial k} \cdot \frac{\partial L^*}{\partial x}}{\{V^{VC}(w_G, k, v) - I\} \cdot \frac{\partial V^e}{\partial k} \cdot (**) - V^e(w_G, k, v) \cdot \frac{\partial V^{VC}}{\partial k} \cdot (*) = \oplus}, \quad (27)$$

$$\frac{dk}{dx} = \frac{V^e(w_G, k, v) \cdot \frac{\partial L^*}{\partial x} \cdot (*) - \{V^{VC}(w_G, k, v) - I\} \cdot \frac{\partial J^*}{\partial x} \cdot (**)}{\{V^{VC}(w_G, k, v) - I\} \cdot \frac{\partial V^e}{\partial k} \cdot (**) - V^e(w_G, k, v) \cdot \frac{\partial V^{VC}}{\partial k} \cdot (*) = \oplus}, \quad (28)$$

where

$$(*) := \frac{\tilde{J}(p_{min}) (= V^e(w_G, k, v))}{C_e} \cdot (*)' > 0,$$

$$(**) := \frac{\tilde{L}(p_{min}) (= V^{VC}(w_G, k, v) - I)}{C_{VC} + \lambda I \{1 - \mathbb{E}(p|p_{min})\} e_{max}} \cdot (**) > 0,$$

and $(*)' > 0$ and $(**) > 0$ are computed in (25) and (26), respectively.

Since $e'(p_{min}) < 0$ and $\frac{d\theta}{dp_{min}} < 0$ by Proposition 2.2 (see the proof of uniqueness in Appendix B), the signs of $\frac{de_{max}}{dx}$ and $\frac{d\theta}{dx}$ are always opposite to that of $\frac{dp_{min}}{dx}$:

$$\frac{de_{max}}{dx} = \frac{dp_{min}}{dx} \cdot \underbrace{e'(p_{min})}_{<0} \quad \text{and} \quad \frac{d\theta}{dx} = \frac{dp_{min}}{dx} \cdot \underbrace{\frac{d\theta}{dp_{min}}}_{<0}. \quad (29)$$

□

D Proof of Theorem 2.4

Proof. 1. (Theorem 2.4.1) Computing the numerator of (27) for $x = v$, we have

$$\frac{\partial V^e}{\partial v} \cdot \frac{\partial V^{VC}}{\partial k} - \frac{\partial V^e}{\partial k} \cdot \frac{\partial V^{VC}}{\partial v} = \oplus \cdot \ominus - \oplus \cdot \oplus < 0,$$

thus giving us $\frac{dp_{min}}{dx} < 0$. Then, by (29), $\frac{de_{max}}{dx} > 0$ and $\frac{d\theta}{dx} > 0$ hold.

2. (*Theorem 2.4.2*) Computing the numerator of (28) for $x = v$, we have

$$\frac{dk}{dv} \equiv \frac{\Lambda_1}{\Lambda_2} := \frac{V^e \cdot \frac{\partial V^{VC}}{\partial v} \cdot (*) - \{V^{VC} - I\} \cdot \frac{\partial V^e}{\partial v} \cdot (**)}{\{V^{VC} - I\} \cdot \frac{\partial V^e}{\partial k} \cdot (**) - V^e \cdot \frac{\partial V^{VC}}{\partial k} \cdot (*)}, \quad (30)$$

where the sign of Λ_1 is ambiguous, while $\Lambda_2 > 0$.

For the rest of the proof, denote $y \equiv \frac{\omega \cdot g(p_{min})}{f(e_{max})}$ and $z \equiv e'(p_{min})$. By (21), we know

$\frac{\partial z}{\partial y} = -\frac{z}{y} < 0$. Then, it follows that

$$\begin{aligned} \frac{\partial}{\partial y} \left(\frac{dk}{dv} \right) &= \frac{\partial z}{\partial y} \cdot \frac{\partial}{\partial z} \left(\frac{dk}{dv} \right) \\ &= \frac{\partial z}{\partial y} \cdot \frac{1}{\Lambda_2} \cdot \left[\begin{array}{c} V^e \cdot \left(\frac{\partial V^{VC}}{\partial v} + \frac{dk}{dv} \cdot \frac{\partial V^{VC}}{\partial k} \right) \cdot \frac{\partial (*)}{\partial z} \\ - \{V^{VC} - I\} \cdot \left(\frac{\partial V^e}{\partial v} + \frac{dk}{dv} \cdot \frac{\partial V^e}{\partial k} \right) \cdot \frac{\partial (**)}{\partial z} \end{array} \right] \\ &\equiv \frac{\partial z}{\partial y} \cdot \frac{1}{\Lambda_2} \cdot \left[\begin{array}{c} V^e \cdot \underbrace{\frac{dV^{VC}}{dv}}_{>0} \cdot \frac{\partial (*)}{\partial z} - \{V^{VC} - I\} \cdot \underbrace{\frac{dV^e}{dv}}_{>0} \cdot \frac{\partial (**)}{\partial z} \end{array} \right], \end{aligned}$$

where, in equilibrium,

$$\begin{aligned} \frac{dV^e}{dv} &= \frac{d\tilde{J}}{dv} = \underbrace{\frac{dp_{min}}{dv}}_{<0} \cdot \underbrace{\frac{d\tilde{J}}{dp_{min}}}_{<0} > 0, \\ \frac{dV^{VC}}{dv} &= \frac{d\tilde{L}}{dv} = \underbrace{\frac{dp_{min}}{dv}}_{<0} \cdot \underbrace{\frac{d\tilde{L}}{dp_{min}}}_{<0} > 0. \end{aligned}$$

As we compute the partials with respect to $z \equiv e'(p_{min})$, we have

$$\frac{\partial (**)}{\partial z} = \frac{V^{VC}(w_G, k, v) - I}{C_{VC} + \lambda I \{1 - \mathbb{E}(p|p_{min})\} e_{max}} \cdot \frac{\partial (**)'}{\partial z} < 0,$$

which follows from

$$\frac{\partial (**)'}{\partial z} = -(V^{VC} - I) \cdot \mathbb{E}(p|p_{min}) - \lambda I \{1 - \mathbb{E}(p|p_{min})\} < 0,$$

$$\frac{\partial (*)}{\partial z} = -\frac{V^e}{C_e} \cdot p_{min} \cdot \frac{f(e_{max})}{F(e_{max})} \cdot [\theta \cdot \eta'(\theta) \cdot \{1 - \mathbb{E}(e|e_{max})\} + \eta(\theta) \cdot \{e_{max} - \mathbb{E}(e|e_{max})\}].$$

Define $\epsilon_{\eta,\theta}$ as the sensitivity of the sampling rate to a change in the market tightness.

Then, if the following condition holds in equilibrium:

$$\epsilon_{\eta,\theta} \left(:= -\frac{d\eta(\theta)/\eta(\theta)}{d\theta/\theta} \right) > 1 - \frac{1 - e_{max}}{1 - \mathbb{E}(e|e_{max})} \left(= 1 - \frac{\Pr(s = H|w_G, e_{max})}{\mathbb{E}[\Pr(s = H|w_G, e)|e \leq e_{max}]} \right),$$

we have $\frac{\partial(*)}{\partial z} > 0$, thereby having

$$\frac{\partial}{\partial y} \left(\frac{dk}{dv} \right) = \underbrace{\frac{\partial z}{\partial y}}_{<0} \cdot \frac{1}{\Lambda_2} \cdot \left[V^e \cdot \underbrace{\frac{dV^{VC}}{dv}}_{>0} \cdot \underbrace{\frac{\partial(*)}{\partial z}}_{>0} - \{V^{VC} - I\} \cdot \underbrace{\frac{dV^e}{dv}}_{>0} \cdot \underbrace{\frac{\partial(**)}{\partial z}}_{<0} \right] < 0.$$

3. (*Theorems 2.4.3 and 2.4.4*) The rest of the of partials are computed as follows:

$$\frac{\partial}{\partial \left(\frac{\partial V^e}{\partial v} \right)} \left(\frac{dk}{dv} \right) = -\frac{\{V^{VC} - I\} \cdot (**)}{\Lambda_2} < 0,$$

$$\frac{\partial}{\partial \left(\frac{\partial V^{VC}}{\partial v} \right)} \left(\frac{dk}{dv} \right) = \frac{V^e \cdot (*)}{\Lambda_2} > 0,$$

$$\frac{\partial}{\partial V^e} \left(\frac{dk}{dv} \right) = \frac{(*)}{\Lambda_2} \cdot \left(\frac{\partial V^{VC}}{\partial v} + \frac{dk}{dv} \cdot \frac{\partial V^{VC}}{\partial k} \right) = \frac{(*)}{\Lambda_2} \cdot \frac{dV^{VC}}{dv} > 0,$$

$$\frac{\partial}{\partial (V^{VC} - I)} \left(\frac{dk}{dv} \right) = -\frac{(**)}{\Lambda_2} \cdot \left(\frac{\partial V^e}{\partial v} + \frac{dk}{dv} \cdot \frac{\partial V^e}{\partial k} \right) = -\frac{(**)}{\Lambda_2} \cdot \frac{dV^e}{dv} < 0.$$

□

E Proof of Theorem 2.5

Proof. As we differentiate $L(e, v)$ in (18) with respect to e and v , respectively, we have

$$\frac{dL}{de} = -\frac{1}{1 - e_{max}} \{I[1 - \mathbb{E}(p|p_{min})] + C_{VC}\} < 0,$$

$$\frac{dL}{dv} = \frac{1 - e}{(1 - e_{max})^2} \cdot \{I[1 - \mathbb{E}(p|p_{min})] + C_{VC}\} \cdot \underbrace{\frac{de_{max}}{dv}}_{\oplus} - \underbrace{\frac{d}{dp_{min}} \mathbb{E}(p|p_{min})}_{\oplus} \cdot \frac{e_{max} - e}{1 - e_{max}} \cdot \underbrace{\frac{dp_{min}}{dv}}_{\ominus} > 0,$$

verifying the first two partials. Hence, the cross derivative follows as

$$\frac{d^2 L}{dvde} = -\frac{1}{(1 - e_{max})^2} \cdot \{I[1 - \mathbb{E}(p|p_{min})] + C_{VC}\} \cdot \underbrace{\frac{de_{max}}{dv}}_{\oplus} + \underbrace{\frac{d}{dp_{min}} \mathbb{E}(p|p_{min})}_{\oplus} \cdot \frac{1}{1 - e_{max}} \cdot \underbrace{\frac{dp_{min}}{dv}}_{\ominus} < 0,$$

verifying the third partial.

The derivative of $Var[L(e, v)|e_{max}]$ with respect to v is initially computed as

$$\begin{aligned} \frac{d}{dv} Var[L(e, v)|e_{max}] &= \frac{d}{dv} [A(v)^2 \cdot Var(e|e_{max})] \\ &= 2A(v) \cdot \underbrace{A'(v)}_{>0} \cdot Var(e|e_{max}) + A(v)^2 \cdot \frac{d}{dv} Var[L(e, v)|e_{max}]. \end{aligned}$$

The derivative of $Var(e|e_{max})$ with respect to v is initially computed as

$$\frac{d}{dv} Var(e|e_{max}) = \underbrace{\frac{de_{max}}{dv}}_{>0} \cdot \frac{d}{de_{max}} Var(e|e_{max}).$$

The derivative of $Var(e|e_{max})$ with respect to e_{max} is computed as

$$\frac{d}{de_{max}} Var(e|e_{max}) = \frac{f(e_{max})}{F(e_{max})} \cdot [\{e_{max} - \mathbb{E}(e|e_{max})\}^2 - Var(e|e_{max})] > 0.$$

Hence, we have $Var[L(e, v)|e_{max}]$ and $Var(e|e_{max})$ strictly increasing in v .

Similarly, as we differentiate $J(p, v)$ in (20) and its conditional variance, it follows that

$$\frac{dJ}{dp} = \frac{C_e}{p_{min}} > 0 \quad \text{and} \quad \frac{dJ}{dv} = \frac{dp_{min}}{dv} \cdot \frac{dJ}{dp_{min}} = - \underbrace{\frac{dp_{min}}{dv}}_{<0} \cdot \frac{pC_e}{p_{min}^2} > 0,$$

$$\frac{d^2 J}{dp dv} = - \frac{C_e}{p_{min}^2} \cdot \frac{dp_{min}}{dv} > 0,$$

$$\frac{d}{dp_{min}} Var(p|p_{min}) = \frac{g(p_{min})}{1 - G(p_{min})} \cdot [Var(p|p_{min}) - \{p_{min} - \mathbb{E}(p|p_{min})\}^2] < 0,$$

$$\frac{d}{dp_{min}} Var(J(p, v)|p_{min}) = - \frac{2C_e^2}{p_{min}^3} \cdot Var(p|p_{min}) + \frac{C_e^2}{p_{min}^2} \cdot \frac{d}{dp_{min}} Var(p|p_{min}) < 0,$$

$$\Rightarrow \frac{d}{dv} Var(p|p_{min}) = \frac{dp_{min}}{dv} \cdot \frac{d}{dp_{min}} Var(p|p_{min}) > 0 \quad \text{and} \quad \frac{d}{dv} Var(J(p, v)|p_{min}) > 0.$$

□

F Proof of Proposition 3.2

Proof. The verification follows directly from $e_{max}^* > \bar{e} = e_{max}$, which is a result of Theorem 2.4.1. As we rearrange (10) and (11) under the new long-run equilibrium, we have

$$V^e(w_G, k^*, v^*) = \widehat{J}(e_{max}^*) \equiv \frac{C_e}{\eta(\theta^*) \cdot p(e_{max}^*) \cdot \{1 - \mathbb{E}(e|e_{max}^*)\}},$$

$$V^{VC}(w_G, k^*, v^*) - I = \widehat{L}(e_{max}^*) \equiv \frac{\lambda I \{1 - \mathbb{E}(p|p(e_{max}^*))\} e_{max}^* + C_{VC}}{\mathbb{E}(p|p(e_{max}^*)) (1 - e_{max}^*)},$$

where $\widehat{J}(\cdot)$ and $\widehat{L}(\cdot)$ are strictly increasing functions in e_{max} by (25), (26), and $e'(p_{min}) < 0$. Hence, $e_{max}^* > e_{max}$ implies that

$$V^e(w_G, k^*, v^*) = \widehat{J}(e_{max}^*) > \widehat{J}(\bar{e}) = V^e(w_G, \bar{k}, v^*) = V^e(w_G, k, v),$$

$$V^{VC}(w_G, k^*, v^*) = \widehat{L}(e_{max}^*) > \widehat{L}(\bar{e}) = V^{VC}(w_G, k, v),$$

verifying the second inequality in Proposition 3.2.2.

Since $\partial V^e / \partial k > 0$, $V^e(w_G, k^*, v^*) > V^e(w_G, \bar{k}, v^*)$ implies $k^* > \bar{k}$, verifying Proposition 3.2.1. In consequence, since $\partial V^{VC} / \partial k < 0$, $k^* > \bar{k}$ implies $V^{VC}(w_G, k^*, v^*) < V^{VC}(w_G, \bar{k}, v^*)$, verifying the first inequality in Proposition 3.2.2.

The inequalities in Proposition 3.2.3 follow directly from (24).

□