When Does Active Trading Pay?

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Abstract

We propose *activeness*—the sum of squared changes in estimated factor loadings—as a measure of hedge fund trading activity and show that its effect on future performance depends on skill. Among high-alpha funds, the most active quartile earns 3.9% more per year than the least active; among low-alpha funds, the most active quartile earns 8.7% less. This asymmetry survives controls, Fama–MacBeth regressions, and fund fixed effects, and strengthens within-fund over time. We rationalize the findings in a model where skilled managers trade actively on precise signals while unskilled managers gamble for resurrection by inflating factor bets beyond the optimum. Consistent with the model, incentive fees predict activeness only among unskilled funds, and skilled active funds exhibit zero market beta—their returns are orthogonal to standard risk factors.

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1 Introduction

Does active trading by fund managers create or destroy value? A large literature documents that, on average, the answer is "destroy": more active mutual funds and hedge funds tend to underperform less active ones (Ammann, Fischer and Weigert, 2020; Buffa and Javadekar, 2021; Clare and Clare, 2019). Yet the average masks a fundamental asym-

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metry. In this paper, we show that the relationship between trading activity and future performance reverses sign depending on managerial skill. Active trading by skilled hedge fund managers generates persistent alpha; active trading by unskilled managers destroys it. The unconditionally negative effect reported in prior work is a composition artifact—the result of pooling two populations with opposite responses to the same behavior.

We propose *activeness*—the sum of squared changes in a fund's estimated factor loadings across adjacent rolling windows—as a simple, high-frequency measure of how aggressively a hedge fund shifts its systematic exposures over time. Unlike holdings-based measures such as Active Share (Cremers and Petajisto, 2009) or active weight (Doshi, Elkamhi and Simutin, 2015), our measure requires only fund returns and traded factor returns, making it applicable to hedge funds where portfolio holdings are unobservable. Unlike $1 - R^2$ (Amihud and Goyenko, 2013; Titman and Tiu, 2011), which captures the *level* of idiosyncratic risk at a point in time, activeness captures the *dynamics* of factor exposures—it measures how much a manager is moving her portfolio through factor space from one period to the next.

We estimate activeness using rolling 24-month regressions of desmoothed hedge fund returns (Getmansky, Lo and Makarov, 2004) on the Fung–Hsieh seven-factor (Fung and Hsieh, 2004) and Carhart four-factor (Carhart, 1997) models. To test whether the effect of activeness depends on skill, we sort funds each month into high- and low-alpha groups using the cross-sectional median of rolling alpha in portfolio sorts, and interact activeness with a positive-alpha indicator ($\alpha > 0$) in regressions.

Our main finding is a stark asymmetry. In portfolio sorts that independently condition on skill and activeness, the most active quartile of high-alpha funds earns 0.33 percentage points per month more than the least active quartile, corresponding to roughly 3.9% annualized. Among low-alpha funds, the pattern reverses: the most active quartile earns 0.72 percentage points per month *less* than the least active, or 8.7% annualized. The value destruction from unskilled active trading is more than twice the magnitude of the value creation from skilled active trading, explaining the unconditionally negative average effect. In pooled OLS regressions of annual future alpha on activeness, a positive-alpha indicator, and their interaction—controlling for fund size, age, flows, volatility, and fees—the interaction is positive and highly significant, while the standalone activeness coefficient is significantly negative. The results are robust to Fama–MacBeth (Fama and MacBeth, 1973) regressions.

We rationalize the asymmetry in a model of active fund management with heterogeneous signal precision and incentive-driven gambling for resurrection. The manager observes a private signal about a risky factor and chooses factor exposure. Skilled managers—those with precise signals—optimally take larger positions, generating both higher activeness and higher alpha through informed trading. The positive activeness–alpha relationship among skilled managers thus reflects cross-sectional variation in signal quality. The key friction is that underperforming managers face liquidation if returns fall below a skill-dependent threshold. When the first-best exposure is insufficient to clear the threshold in the up state, the manager inflates her factor bet beyond the optimum—a form of gambling for resurrection. This gambling raises activeness but destroys expected alpha because the oversized position incurs quadratic trading costs that exceed the marginal information content of the imprecise signal. The resulting negative activeness–alpha relationship among low-skill managers is causal: it is the excessive trading itself, not the low skill, that destroys value. Incentive fees amplify the gambling mechanism by raising the payoff from surviving, expanding the set of managers for whom gambling is optimal, consistent with our empirical finding that incentive fees predict activeness only among low-alpha funds.

Several additional findings sharpen the picture. First, activeness is persistent but mean-reverting: a fund in the top activeness quartile has a 45% probability of remaining there one month later but only 13% at five years, confirming that activeness captures genuine variation in trading behavior rather than a permanent fund characteristic. Second, the determinants of activeness differ by skill: return volatility is the dominant predictor across all funds, but incentive fees predict activeness only among low-alpha funds, not high-alpha funds, suggesting that convex compensation motivates trading among unskilled managers who lack an informational edge. Third, Henriksson–Merton (Henriksson and Merton, 1981) timing regressions reveal that skilled active funds have essentially zero market beta and near-zero R^2 on the market factor—their returns are orthogonal to standard systematic risk. In contrast, unskilled active funds have the highest market beta of any group and show no timing ability. This pattern is consistent with the model's prediction that skilled managers exploit proprietary, non-market signals while unskilled managers shuffle systematic exposure without productive intent.

Our paper contributes to three literatures. First, we advance the literature on the relationship between fund activeness and performance. Doshi, Elkamhi and Simutin (2015) show that holdings-based active weight unconditionally predicts mutual fund performance; Amihud and Goyenko (2013) show that low R^2 from factor regressions predicts higher future alpha; Cremers and Petajisto (2009) document the informativeness of Active Share. In the hedge fund space, Duanmu, Malakhov and McCumber (2018) propose Beta Activeness as a composite of systematic risk and residual variation but do not condition on an independent skill measure, and Ammann, Fischer and Weigert (2020) show that factor exposure variation unconditionally predicts mutual fund underperformance. Sun, Wang and Zheng (2018) show that hedge fund performance in down markets predicts future alpha persistently, using the same rolling FH7 methodology but a different conditioning variable. Bali, Brown and Caglayan (2012) demonstrate that factor exposures predict cross-sectional hedge fund returns.[1] These papers share a common feature: the activeness–performance relationship is estimated unconditionally, as an average across all funds. We show that this average masks a fundamental asymmetry visible only when conditioning on skill: the sign of the activeness–performance relationship flips depending on whether the manager is skilled or unskilled. This asymmetric interaction is, to our knowledge, new in any asset class.

Second, we contribute to the literature on risk-shifting and gambling for resurrection in delegated portfolio management. The theoretical foundations are well established: convex incentive contracts and tournament incentives create option-like payoffs that can motivate excessive risk-taking (Hodder and Jackwerth, 2007; Kouwenberg and Ziemba, 2007; Basak,

[1] Guzella and Campani (2017) come closest to our design with a non-parametric double-sort on R^2 and past alpha in Brazilian equity funds, but theirs is not a formal regression interaction and lacks a theoretical mechanism. See also Jayaraman et al. (2017) on factor dependence and skill, and Chen, Han and Pan (2021) on dynamic sentiment beta and hedge fund returns.

Pavlova and Shapiro, 2007; Panageas and Westerfield, 2009). In mutual funds, Chevalier and Ellison (1997) and Brown, Harlow and Starks (1996) document that mid-year losers increase portfolio risk, while Kempf, Ruenzi and Thiele (2009) show that these effects depend on the relative strength of compensation versus employment incentives across market states. Huang, Sialm and Zhang (2011) provide the first systematic mapping of mutual fund risk-shifting into factor exposures, documenting that risk-increasing funds tilt toward small, growth, and momentum stocks and subsequently underperform—a design conceptually close to ours, but without the skill interaction.[2] On the hedge fund side, Aragon and Nanda (2012) show that funds far from their high-water marks increase risk, Buraschi, Kosowski and Sritrakul (2014) use a structural model to recover latent risk-taking from hedge fund returns, Kolokolova and Mattes (2014) map risk-shifting into factor betas using daily data, and Li, Holland and Kazemi (2019) document that risk-shifting attenuates with prolonged underperformance as closure probability rises.[3] Most existing work identifies risk-shifting through changes in return volatility or leverage. Our contribution is to map risk-shifting into the factor-loading space: we show that the excess activeness of unskilled managers is a form of unproductive factor rotation driven by incentive frictions, not informed trading. The model formalizes this channel and shows how signal precision determines whether active factor management creates or destroys value.

Third, we contribute to the literature on time-varying factor exposures in hedge funds. Patton and Ramadorai (2013) show that ignoring dynamics in hedge fund factor loadings biases alpha estimation by approximately 4.6% annually. Bollen and Whaley (2009) use changepoint detection to identify regime shifts in loadings, and Mamaysky, Spiegel and Zhang (2008) develop Kalman-filter methods for estimating dynamic alphas and betas. Brunnermeier and Nagel (2004) validate that returns-based dynamic betas closely track actual holdings changes, providing the gold-standard evidence that our

[2] See also Andreu, Sarto and Serrano (2019) for international evidence that risk-increasing funds can outperform in non-U.S. samples, consistent with our finding that skilled activeness creates value, and Xiao (2020) on leverage as a risk-shifting margin in hedge funds.

[3] Swade, Köchling and Posch (2021) use a Bayesian skill measure and show that high-skill mutual fund managers increase risk more than low-skill peers, particularly in years when compensation incentives dominate—the closest finding to our skill $\times$ activity interaction in the risk-shifting literature. See also Aragon, Qian et al. (2009) on the role of high-water-mark structures.

approach captures real portfolio shifts rather than estimation noise. Formal justification for rolling-window beta estimation is provided by Cattaneo, Crump and Wang (2022), while Ang, Madhavan and Sobczyk (2017) develop a general state-space framework with shrinkage.[4] We build on this literature by showing that the *magnitude* of loading changes—not just their existence—contains economically significant information about future performance, and that this information is sharply conditioned by managerial skill.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section **??** describes the data and methodology. Section 5 presents the empirical results. Section 6 concludes.

2 A Model of Activeness, Skill, and Gambling for Resurrection

This section develops a model that rationalizes the asymmetric relationship between activeness and alpha documented in Section 5. The model features heterogeneous managers who observe private signals about a risky factor and choose factor exposures. Three progressive frictions—trading costs, incentive fees, and survival pressure—move the manager's exposure away from the investor's first-best and generate the observed cross-sectional patterns.

2.1 Environment

Consider a two-period economy with a single risky factor whose excess return is binary:

$$f \in \{+1, -1\}, \quad \Pr(f = +1) = \tfrac{1}{2}. \tag{2.1}$$

[4]See Stafylas, Anderson and Uddin (2017) for a survey of hedge fund return explanations, Bai, Hilscher and Scherbina (2025) on why funds change factor loadings, and Bessler et al. (2022) on factor exposure changes and mutual fund performance.

Manager i is characterized by skill $\phi_i \in (\frac{1}{2}, 1)$. Each manager observes a private signal $s_i \in \{+1, -1\}$ with accuracy equal to skill:

$$\Pr(f = +1 \mid s_i = +1) = \phi_i. \tag{2.2}$$

Define *informativeness* as $\mu_i \equiv 2\phi_i - 1 \in (0, 1)$. Higher-skill managers receive more precise signals about the factor realization.

After observing s_i, the manager chooses factor exposure $\beta_i(s_i) \in \mathbb{R}$. The fund's return is

$$r_i = \beta_i f - \frac{c}{2}\beta_i^2, \tag{2.3}$$

where $c > 0$ captures implementation and trading costs that are quadratic in position size.[5]

By the symmetric prior ($\Pr(f = +1) = \frac{1}{2}$), the manager's problem given $s_i = -1$ is the mirror image of the problem given $s_i = +1$: optimal exposure given $s_i = -1$ equals minus the optimal exposure given $s_i = +1$. We therefore analyze the case $s_i = +1$ with $\beta \geqslant 0$ throughout without loss of generality.

Given $s_i = +1$ and $\beta > 0$, the fund's return takes two values:

$$r^+(\beta) = \beta - \tfrac{c}{2}\beta^2 \qquad \text{(up state: } f = +1\text{, probability } \phi_i\text{)}, \tag{2.4}$$

$$r^-(\beta) = -\beta - \tfrac{c}{2}\beta^2 \qquad \text{(down state: } f = -1\text{, probability } 1 - \phi_i\text{)}. \tag{2.5}$$

The up-state return $r^+(\beta)$ is strictly concave in β, maximized at $\beta = 1/c$ with peak value $1/(2c)$. The down-state return $r^-(\beta)$ is strictly negative and decreasing in β for all $\beta > 0$.

2.2 Compensation and Survival

The manager's payoff has three components. First, the manager holds a *co-investment stake* $\omega > 0$, earning $\omega \cdot r$ from fund returns. This linear component aligns the manager's

[5]The quadratic cost can be interpreted as price impact, borrowing costs, or operational frictions associated with larger positions. The key economic content is that taking larger bets is increasingly costly, so that the optimal exposure is interior.

interest with investors. Second, the manager earns an *incentive fee* $\delta_i \cdot \max(r, 0)$, where $\delta_i > 0$. The max operator truncates the downside and creates option-like payoffs. Third, the manager receives a *continuation value* $V > 0$ if and only if the fund survives, that is, if $r > \underline{r}_i$. The continuation value captures the present value of future management fees, reputation, and career opportunities.

Assumption 2.1 (Liquidation threshold)**.** *The liquidation threshold $\underline{r}_i = \underline{r}(\phi_i)$ is strictly decreasing in ϕ_i, with $\underline{r}(1/2) > 0$.*

Managers with higher skill have accumulated better track records and are further from investor-imposed liquidation boundaries. The condition $\underline{r}(1/2) > 0$ ensures that the least skilled managers face a positive survival hurdle. The alpha persistence documented in Table 4 provides the empirical basis: since past alpha is informative about future alpha, higher-skill managers build larger performance buffers over time, facing effectively lower liquidation thresholds.[6]

2.3 Manager's Problem

Given signal $s_i = +1$, the manager chooses $\beta \geqslant 0$ to maximize

$$U(\beta) = \omega \cdot \mathbb{E}[r \mid s = +1] + \delta_i \cdot \mathbb{E}[\max(r, 0) \mid s = +1] + V \cdot \Pr(r > \underline{r}_i \mid s = +1). \quad (2.6)$$

The first term is the expected return on co-invested capital. The second term is the expected incentive fee, which pays off only when the fund earns a positive return. The third term is the expected continuation value, which the manager receives only if the fund avoids liquidation. The manager's problem is to balance the linear incentive to maximize expected returns (through ω) against the convex incentive to increase the probability of positive returns and survival (through δ_i and V).

[6]The endogenous version of this assumption—in which investors observe noisy signals of skill, update beliefs, and liquidate when the posterior expected alpha falls below zero—would require a fully dynamic model and is left for future work. The exogenous threshold captures the key economic force: low-skill managers face tighter survival constraints.

2.4 Investor First-Best

We begin by characterizing the exposure that maximizes expected fund returns, which serves as the benchmark against which managerial distortions are measured.

Lemma 2.1 (Investor first-best)**.** *The exposure maximizing $\mathbb{E}[r \mid s = +1]$ is $\beta^*(\phi_i) = \mu_i/c$, yielding expected alpha $\pi^*(\phi_i) = \mu_i^2/(2c)$ and activeness $A^*(\phi_i) = \mu_i^2/c^2$. Both are strictly increasing in ϕ_i. At the first-best, alpha is proportional to activeness: $\pi^* = (c/2)\, A^*$.*

Proof. The expected return given $s = +1$ is

$$\mathbb{E}[r \mid s = +1] = \phi_i\big(\beta - \tfrac{c}{2}\beta^2\big) + (1-\phi_i)\big(-\beta - \tfrac{c}{2}\beta^2\big) = \beta\mu_i - \tfrac{c}{2}\beta^2.$$

The first-order condition $\mu_i - c\beta = 0$ gives $\beta^* = \mu_i/c$, and the second-order condition $-c < 0$ confirms this is a maximum. Substituting yields $\pi^* = \mu_i^2/(2c)$. By symmetry, $\beta(+1) = \mu_i/c$ and $\beta(-1) = -\mu_i/c$, so activeness (the variance of exposure across signals) is $A^* = \mathbb{E}[\beta^2] = \mu_i^2/c^2$. Since $\mu_i = 2\phi_i - 1$ is strictly increasing in ϕ_i, all quantities inherit this monotonicity. □

At the first-best, more skilled managers optimally take larger positions and generate higher alpha. Activeness is a monotonically increasing function of skill, and the mapping from activeness to alpha is positive and linear: every unit of activeness generates $c/2$ units of alpha. This is the benchmark that the manager's private incentives will distort.

By symmetry of the signal distribution, $\mathbb{E}[\beta_i] = 0$: each manager is equally likely to be long or short. Since the factor has zero expected excess return ($\mathbb{E}[f] = 0$), the fund's expected return equals its alpha: $\mathbb{E}[r_i] = \pi_i$. There is no factor risk premium in this setup, so all cross-sectional variation in expected returns reflects skill differences. This is a simplifying consequence of the binary symmetric prior that would not hold with an asymmetric factor distribution.

2.5 Fee-Distorted Optimum

We now introduce the first friction: incentive fees. Setting aside the survival constraint momentarily, we derive the exposure that all managers would choose when compensated through co-investment and incentive fees alone.

Lemma 2.2 (Fee-distorted optimum)**.** *Ignoring the survival term in* (2.6)*, the manager's optimal exposure is*

$$\beta^{mgr}(\phi_i) = \frac{\omega\mu_i + \delta_i\phi_i}{c(\omega + \delta_i\phi_i)}. \tag{2.7}$$

The distortion relative to the investor first-best is

$$\beta^{mgr}(\phi_i) - \beta^*(\phi_i) = \frac{2\delta_i\phi_i(1-\phi_i)}{c(\omega + \delta_i\phi_i)}, \tag{2.8}$$

which is positive for all $\phi_i < 1$, decreasing in ϕ_i, and vanishing as $\phi_i \to 1$. Moreover, $\beta^{mgr}(\phi_i) < 1/c$ for all $\phi_i < 1$.

Proof. Dropping the survival term, the expected payoff given $s = +1$ is $\omega[\beta\mu_i - (c/2)\beta^2] + \delta_i\phi_i[\beta - (c/2)\beta^2]$, where the second term reflects the incentive fee paying $\delta_i r^+(\beta)$ in the up state and zero in the down state. The first-order condition $\omega(\mu_i - c\beta) + \delta_i\phi_i(1 - c\beta) = 0$ yields (2.7). The distortion (2.8) follows by subtraction. For the bound, $\mu_i < 1$ implies $\omega\mu_i + \delta_i\phi_i < \omega + \delta_i\phi_i$, so $\beta^{mgr} < 1/c$. □

The incentive fee induces all managers to take slightly larger positions than the investor optimum, because the fee truncates the downside: the manager shares in gains but not losses beyond her co-investment. However, the distortion is largest for low-skill managers. When the signal is precise (ϕ_i close to 1), the fee is almost always in the money, and the marginal incentive from the fee closely approximates the linear incentive from co-investment. The bound $\beta^{mgr} < 1/c$ ensures that the fee-distorted optimum lies on the increasing portion of the up-state return function $r^+(\beta)$—an important property for the equilibrium characterization below.

2.6 Survival-Aware Equilibrium

The final friction is the survival constraint. The key result is that survival pressure creates qualitatively different incentives above and below a critical skill threshold.

Lemma 2.3 (Survival threshold)**.** *Define $R^+(\phi_i) \equiv r^+(\beta^{mgr}(\phi_i))$ as the up-state return at the fee-distorted optimum. Since β^{mgr} is increasing in ϕ_i and lies on $(0, 1/c)$ where r^+ is increasing,*

R^+ is strictly increasing in ϕ_i. Given that $\underline{r}(\phi_i)$ is strictly decreasing (Assumption 2.1) and that $\underline{r}(1/2) > R^+(1/2)$,[7] while $R^+(\phi) > \underline{r}(\phi)$ for ϕ sufficiently close to 1, there exists a unique $\hat{\phi} \in (1/2, 1)$ satisfying $R^+(\hat{\phi}) = \underline{r}(\hat{\phi})$. For $\phi_i > \hat{\phi}$, the manager survives in the up state at the fee-distorted optimum. For $\phi_i < \hat{\phi}$, she does not.

Proof. The result follows from the single crossing of an increasing and a decreasing continuous function on $(1/2, 1)$. □

The threshold $\hat{\phi}$ separates managers into two groups. High-skill managers ($\phi_i > \hat{\phi}$) can survive in the up state at the fee-distorted optimum. Low-skill managers ($\phi_i < \hat{\phi}$) cannot—their fee-distorted exposure generates an up-state return that falls short of the liquidation threshold. This distinction is the source of the asymmetric incentives that drive the model's predictions.

Note that the down-state return at any exposure is $r^-(\beta) = -\beta - (c/2)\beta^2 < 0$, which lies below any reasonable liquidation threshold. The relevant question for survival is therefore whether the up-state return clears the threshold; it never does in the down state.

Proposition 2.1 (High-skill equilibrium)**.** *For $\phi_i > \hat{\phi}$, the equilibrium exposure is $\beta^{mgr}(\phi_i)$ from Lemma 2.2. The survival constraint is slack: the manager survives in the up state at the fee-distorted optimum, so the continuation value does not distort the exposure choice.*

Proof. For $\phi_i > \hat{\phi}$, $r^+(\beta^{mgr}) > \underline{r}_i$ by Lemma 2.3. By continuity, the survival indicator is locally constant near β^{mgr}, so the continuation value term does not affect the first-order condition. □

For high-skill managers, the equilibrium exposure is pinned down by the fee-distorted first-order condition alone. The only departure from the investor first-best is the incentive fee distortion, which is small when skill is high (Lemma 2.2). The gambling motive—deviating sharply to clear the survival threshold—is absent because these managers already survive at their preferred exposure.

[7] This parametric condition states that the least skilled managers cannot survive even at their fee-distorted optimum. It holds whenever the survival hurdle is sufficiently tight relative to the fee distortion for near-uninformative managers. Since $R^+(1/2)$ depends on the incentive fee parameter δ, the condition is weaker than $\underline{r}(1/2) > 1/(2c)$ (which would preclude survival at *any* exposure).

Proposition 2.2 (Gambling by low-skill managers)**.** *For $\phi_i < \hat{\phi}$, define $\hat{\beta}(\phi_i)$ as the smallest $\beta > 0$ satisfying $r^+(\hat{\beta}) = \underline{r}_i(\phi_i)$. This exists and is unique on $(0, 1/c)$ provided $\underline{r}_i < 1/(2c)$.*

The manager compares two strategies. Under no gambling, *she chooses $\beta^{mgr}(\phi_i)$, which does not clear the survival threshold, yielding*

$$U^{mgr} = \omega \cdot \pi(\beta^{mgr}, \phi_i) + \delta_i \phi_i \cdot r^+(\beta^{mgr}). \tag{2.9}$$

Under gambling, *she chooses $\hat{\beta}(\phi_i)$, which clears the threshold in the up state, yielding*

$$U^{gamble} = \omega \cdot \pi(\hat{\beta}, \phi_i) + \delta_i \phi_i \cdot r^+(\hat{\beta}) + V\phi_i, \tag{2.10}$$

where $\pi(\beta, \phi_i) \equiv \beta\mu_i - (c/2)\beta^2$ denotes expected alpha at exposure β. The manager gambles if and only if

$$\underbrace{V\phi_i}_{\textit{survival gain}} + \underbrace{\delta_i\phi_i\left[r^+(\hat{\beta}) - r^+(\beta^{mgr})\right]}_{\textit{fee gain}} > \underbrace{\omega\left[\pi(\beta^{mgr}, \phi_i) - \pi(\hat{\beta}, \phi_i)\right]}_{\textit{co-investment loss}}. \tag{2.11}$$

When the gambling condition holds: (i) $\hat{\beta}(\phi_i) > \beta^{mgr}(\phi_i) > \beta^*(\phi_i)$, so the manager overexposes relative to both the fee-distorted optimum and the investor optimum; (ii) $A^{gamble}(\phi_i) = \hat{\beta}^2 > (\beta^{mgr})^2 > A^*(\phi_i)$, generating excess activeness; and (iii) $\pi(\hat{\beta}, \phi_i) < \pi(\beta^{mgr}, \phi_i) < \pi^*(\phi_i)$, destroying alpha.

The gambling condition (2.11) has an intuitive interpretation. On the left-hand side, the manager gains the continuation value V (weighted by the probability ϕ_i of the up state) and earns a higher incentive fee because the gambling exposure delivers a higher up-state return than β^{mgr}. On the right-hand side, the manager loses expected alpha on her co-invested capital because the gambling exposure is further from the optimum. Gambling is attractive when the option-like payoffs (survival and fee gains) dominate the linear cost (alpha destruction on co-invested capital).

The alpha destruction in part (iii) is the quadratic cost of deviating from the optimal exposure. Managers who are further below $\hat{\phi}$ face higher liquidation thresholds (by Assumption 2.1), require larger $\hat{\beta}$ to clear them, and therefore destroy more alpha. This generates the negative activeness-alpha relationship among low-skill funds.

Corollary 2.1 (Asymmetric activeness-alpha relationship)**.** *In the cross-section, the relationship between activeness and alpha reverses at $\hat{\phi}$:*

$$\frac{d\pi}{dA} \begin{cases} > 0 & \text{for } \phi_i > \hat{\phi}, \\ < 0 & \text{for } \phi_i < \hat{\phi} \text{ (among gambling managers).} \end{cases} \tag{2.12}$$

This matches the main empirical finding in Table 6: the interaction of activeness with the high-alpha indicator is positive and significant, while the standalone activeness effect is negative.

The positive relationship among high-alpha funds reflects cross-sectional variation in skill: more skilled managers optimally take larger positions and generate higher alpha. This is a selection effect—more skilled managers are both more active and more profitable, not because activeness *causes* higher alpha but because both are driven by the same underlying skill. The negative relationship among low-alpha funds reflects the causal mechanism of gambling: less skilled managers face higher liquidation thresholds, requiring larger deviations from the optimal exposure, which generates more activeness and destroys more alpha.

Alpha is monotonically increasing in skill across the entire distribution—both among gambling and non-gambling managers (see Appendix A). Activeness, by contrast, is non-monotonic: it increases with skill among non-gambling managers and *decreases* with skill among gambling managers. As an estimator of managerial skill, alpha therefore strictly dominates activeness: once an investor conditions on past alpha, activeness carries no additional information about future expected performance.

Proposition 2.3 (Incentive fees exacerbate gambling)**.** *The incentive fee amplifies excess activeness through two channels.*

(i) Continuous distortion. *For all managers, the fee distortion $\beta^{mgr} - \beta^*$ is strictly increasing in δ_i:*

$$\frac{\partial(\beta^{mgr} - \beta^*)}{\partial \delta_i} = \frac{\omega \phi_i (1 - \mu_i)}{c(\omega + \delta_i \phi_i)^2} > 0. \tag{2.13}$$

This distortion is larger for low-skill managers because the factor $1 - \mu_i = 2(1 - \phi_i)$ *is decreasing in* ϕ_i.

(ii) Gambling threshold. *For managers below* $\hat{\phi}$, *higher* δ_i *makes the gambling condition* (2.11) *easier to satisfy:*

$$\frac{\partial}{\partial \delta_i}\left[U^{gamble} - U^{mgr}\right] = \phi_i\left[r^+(\hat{\beta}) - r^+(\beta^{mgr})\right] > 0. \tag{2.14}$$

Higher incentive fees expand the set of managers for whom gambling is optimal. This matches the empirical finding in Table 3 that incentive fees predict activeness only among low-alpha funds.

Incentive fees predict more activeness specifically for underperforming funds through two reinforcing channels. First, the fee distortion in exposure is mechanically larger for low-skill managers because their fee option is more likely to be out of the money—the convexity of the payoff matters most when the probability of a positive return is lower. Second, the fee amplifies the payoff from gambling, tipping marginal managers into the gambling regime. For high-skill managers, the fee distortion exists but is economically small (the fee is almost always in the money), and the gambling motive is absent.

2.7 Additional Predictions

The model generates additional predictions that match the data.

Remark 2.1 (Activeness persistence)**.** *Activeness in the model is a deterministic function of the manager's type:* $A_i = A(\phi_i)$. *If skill* ϕ_i *is persistent across periods, then activeness inherits this persistence directly. The model does not generate persistence endogenously (it is a two-period framework), but it is consistent with persistent activeness as a consequence of persistent managerial style.*

Remark 2.2 (Activeness slightly reduces alpha persistence)**.** *High-activeness funds show slightly less alpha persistence than low-activeness funds. The model rationalizes this through a composition effect arising from the non-monotonic mapping* $A(\phi)$. *Low-activeness funds are a relatively homogeneous group: moderate-skill managers near* $\hat{\phi}$ *who are neither highly skilled nor desperate enough to gamble aggressively. Their alpha is moderate and stable, producing high measured persistence. High-activeness funds are a mixture of two very different types: high-skill* *managers (at their optimum, generating high alpha) and low-skill managers (gambling, generating low alpha). This within-group heterogeneity in alpha levels introduces additional cross-sectional variance that reduces measured persistence.*

2.8 Two-Factor Extension: Factor Orthogonality and Market Timing

The baseline model features a single factor. This subsection extends the environment to two factors to rationalize two additional empirical findings: (i) high-alpha/high-activeness fund returns are orthogonal to standard factors ($R^2 \approx 0$; Table 8), and (ii) low-alpha/high-activeness funds have the highest market beta.

Extended Environment

Two independent factors have binary excess returns:

$$f_1 \in \{+1, -1\}, \quad \Pr(f_1 = +1) = \tfrac{1}{2}, \tag{2.15}$$

$$f_2 \in \{+1, -1\}, \quad \Pr(f_2 = +1) = \tfrac{1}{2}, \tag{2.16}$$

where f_1 is the observable (market) factor and f_2 is an unobservable (proprietary) factor. The factors are independent. The econometrician observes f_1 but not f_2.

Manager i receives signals about both factors with common skill ϕ_i:

$$\Pr(f_k = +1 \mid s_{ki} = +1) = \phi_i, \quad k = 1, 2. \tag{2.17}$$

After observing signals (s_{1i}, s_{2i}), the manager chooses exposures (β_{1i}, β_{2i}). The fund's return is

$$r_i = \beta_{1i} f_1 + \beta_{2i} f_2 - \frac{c_1}{2}\beta_{1i}^2 - \frac{c_2(\phi_i)}{2}\beta_{2i}^2, \tag{2.18}$$

where $c_1 > 0$ is the common cost of trading the observable factor and $c_2(\phi_i) > 0$ is a skill-dependent cost of trading the proprietary factor, with $c_2'(\phi_i) < 0$. Skilled managers can access and implement proprietary strategies more cheaply, reflecting advantages in informational infrastructure, execution capability, and the ability to identify non-standard opportunities.

Lemma 2.4 (Two-factor first-best)**.** *The investor-optimal exposures given signals* $(s_1 = +1, s_2 = +1)$ *are* $\beta_1^* = \mu_i/c_1$ *and* $\beta_2^* = \mu_i/c_2(\phi_i)$*. Since* $c_2(\phi_i)$ *is decreasing in* ϕ_i*, higher-skill managers optimally tilt more heavily toward the proprietary factor:* $\beta_2^*/\beta_1^* = c_1/c_2(\phi_i)$ *is increasing in* ϕ_i*.*

Proof. The expected return decomposes additively across factors. The first-order condition for factor k is $\mu_i - c_k\beta_k = 0$. □

Lemma 2.5 (Gambling in two dimensions)**.** *For a manager below* $\hat{\phi}$ *whose first-best up-state return falls short of* $\underline{r}_i$*, the constrained-optimal gambling exposures satisfy:*

(i) Both exposures exceed the first-best: $\hat{\beta}_k > \beta_k^*$ *for* $k = 1, 2$*.*

(ii) The exposure ratio is unchanged: $\hat{\beta}_1/\hat{\beta}_2 = c_2(\phi_i)/c_1$*, the same as at the first-best.*

(iii) The absolute deviation is larger for the cheap factor: $\Delta\hat{\beta}_1/\Delta\hat{\beta}_2 = c_2(\phi_i)/c_1$*, where* $\Delta\hat{\beta}_k \equiv \hat{\beta}_k - \beta_k^*$*.*

The econometrician regresses fund returns on the observable factor alone: $r_i = a_i + b_i f_1 + \epsilon_i$, where $b_i = \beta_{1i}$ and ϵ_i captures all variation orthogonal to f_1, including $\beta_{2i} f_2$.

Proposition 2.4 (Factor orthogonality for skilled active managers)**.** *For managers with* $\phi_i > \hat{\phi}$ *at the investor optimum, the* R^2 *from regressing fund returns on* f_1 *is*

$$R_i^2 = \frac{1}{1 + c_1^2/c_2(\phi_i)^2}. \tag{2.19}$$

This is strictly decreasing in ϕ_i*. For sufficiently skilled managers with* $c_2(\phi_i) \ll c_1$*,* $R_i^2 \to 0$*.*

As skill increases, the proprietary factor becomes cheaper to trade, and a larger share of the manager's return variation comes from the unobservable factor. The econometrician, who can only observe market factor returns, attributes less and less of the fund's return variation to measurable sources. For the most skilled managers, returns appear essentially orthogonal to standard factors—consistent with the near-zero R^2 for high-alpha/high-activeness funds in Table 8.

Proposition 2.5 (Elevated market exposure for low-skill active managers)**.** *For gambling managers with $\phi_i < \hat{\phi}$:*

(i) Market beta is elevated: $\hat{\beta}_1 > \beta_1^*$, higher than for any non-gambling manager.

(ii) R^2 on the market factor is high:

$$R_i^2 = \frac{c_2(\phi_i)^2}{c_1^2 + c_2(\phi_i)^2} \approx 1 \tag{2.20}$$

when $c_2(\phi_i) \gg c_1$, which holds for low-skill managers.

The skill-dependent cost structure generates the factor loading asymmetry through the cross-section of costs. Skilled managers trade the proprietary factor cheaply, so a large share of their return variation is orthogonal to observable factors. Unskilled managers face high proprietary costs, so their return variation is dominated by the market factor regardless of whether they gamble. What gambling adds is the *level* of market exposure: the gambling manager's $\hat{\beta}_1$ exceeds β_1^*, producing elevated market beta.

This framework also suggests a mechanism for negative measured market timing. In a Henriksson–Merton regression $r_i = \alpha + \beta \cdot f_1 + \gamma \cdot \max(f_1, 0) + \epsilon$, a gambling manager with ϕ_i close to $1/2$ makes large bets on the market factor that are nearly uncorrelated with the correct timing direction—the signal is barely informative, but the position is outsized. The quadratic trading costs create a systematic performance drag that the regression attributes to negative timing ability ($\gamma < 0$). The formal sign of γ depends on distributional details beyond the binary model, but the mechanism pushes unambiguously toward $\gamma < 0$ when signal precision is low. This is consistent with Table 8, where unskilled active funds combine the highest market beta with near-zero timing ability.

3 Methodology

To measure the degree of activeness of a hedge fund, we first estimate the hedge fund's time-varying exposures to traded risk factors using rolling-window ordinary least squares (OLS) regressions. Consider the case where a hedge fund's excess returns can be described by a k-factor linear model:

$$r_{it} = \alpha_{it} + \sum_{j=1}^{k} w_{jit} f_{jt} + \varepsilon_{it} \tag{3.1}$$

where r_{it} is hedge fund i's excess return at time t, f_{jt} is the excess return on traded factor j at time t, w_{jit} is the factor loading on factor j at time t, α_{it} is the intercept (alpha), and ε_{it} is the error term. Following Sharpe et al. (1992), equation (3.1) describes how the excess return on a portfolio is a weighted combination of the excess returns on asset classes that approximate the fund's investment universe.[8]

We estimate equation (3.1) using a rolling window of $T = 24$ months. At each month t, we regress fund i's excess returns on the factor returns over the preceding 24 months to obtain time-varying factor loadings $(\widehat{w}_{1it}, \ldots, \widehat{w}_{kit})$ and a rolling alpha $\widehat{\alpha}_{it}$. We estimate the model separately using two standard factor sets: the Fung and Hsieh (2004) seven-factor model (FH7) and the Carhart (1997) four-factor model (C4F).

Once the factor loadings are estimated, we measure the degree of activeness of a hedge fund based on the changes in its estimated factor exposures over time. We define the Activeness (ϕ_{it}) of hedge fund i at time t as the sum of squared changes in the estimated factor loadings:

$$\phi_{it} = \sum_{j=1}^{k} \left(\widehat{w}_{jit} - \widehat{w}_{ji,t-1}\right)^2. \tag{3.2}$$

Intuitively, Activeness measures the extent to which a hedge fund's risk exposures to traded factors differ from its recent past exposures. This measure is equal to zero for a hedge fund whose portfolio weights are fixed through time, and increases as the fund actively adjusts its factor exposures. The squaring ensures that increases and decreases in exposures contribute symmetrically, and places greater weight on large changes in portfolio positioning.

[8] Sharpe et al. (1992) proposes a style analysis approach where the exposures of an investor's portfolio to movements in the returns of key asset classes can be used to determine the investor's effective asset mix.

4 Data and Summary Statistics

4.1 Lipper TASS Hedge Fund Database

Our main data source is the Lipper TASS hedge fund database, one of the leading sources of hedge fund information. It contains the historical time-series of monthly returns and assets under management (AUM) for each individual hedge fund, including live funds as well as defunct funds that have stopped reporting. It also contains the most recent snapshot of fund characteristics, including the manager's compensation contract (e.g. incentive fee, management fee, and high watermark), investor liquidity restrictions (e.g. lock up period and redemption notice period), etc. According to TASS, there are 12 different hedge fund classifications based on the fund's main investment strategy, including Convertible Arbitrage, Long/short Equity Hedge, Even Driven, Equity Market Neutral, Global Macro, Fixed Income Arbitrage, Emerging Market, Multi-Strategy, Managed Futures, Dedicated Short Bias, Options Strategy, and Fund of Funds.

We start with a total of 20,023 hedge funds that exist between 1994 and 2018. We filter out non-monthly filing funds, funds denoted in a currency other than U.S. dollars, and funds with unknown strategies. We include both live and defunct funds in our sample to mitigate the potential survivorship bias. To control for the backfill bias or instant history bias, we exclude the first 12 months of returns for each hedge fund. We require at least 24 months non-backfilled return observations for a hedge fund to be included in our sample. After all these adjustments, our final sample includes 15,106 hedge funds with 968,244 observations of monthly returns.

Previous studies (e.g. Asness, Krail and Liew (2001)) present evidence that hedge funds smooth their returns, either intentionally or because they often invest in illiquid assets with stale prices, leading to differences between the unobserved true returns and the reported returns. Hence, following Getmansky, Lo and Makarov (2004), we apply a second order moving average process to uncover the unobserved true returns (R_t) from the reported returns (R_t^0):

$$R_t^0 = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2} \tag{4.1}$$
$$\theta_j \in [0,1],\ j = 0,1,2 \text{ and } \theta_0 + \theta_1 + \theta_2 = 1.$$

We estimate the parameters θ_0, θ_1, and θ_2 by maximum likelihood and use these thetas to obtain de-smoothed returns. The smoothing parameter theta θ_0 reflects the degree to which a fund's unobserved true returns are incorporated in its reported returns.

5 Empirical Results

Table 1 reports summary statistics for the main variables. Panel A describes fund characteristics pooled across all fund-months. The median fund has an activeness measure of 0.431, with substantial right skew—the 90th percentile (2.016) is nearly five times the median, indicating that a subset of funds trade far more aggressively than the typical fund. Average monthly returns are 0.37%, with FH7 alpha averaging 0.22% and C4F alpha near zero (0.05%). The median fund manages $44 million in assets, charges a 1.5% management fee and a 20% incentive fee, and has a high-water-mark provision. Personal capital commitment is low: the median manager invests none of her own capital in the fund.

Panel B breaks these statistics out by strategy. Activeness ranks are relatively uniform across strategies, with one notable exception: managed futures funds are substantially more active (mean rank 58.1, median 62), consistent with trend-following strategies that mechanically generate high factor-loading turnover. These funds also have the lowest alpha ranks (mean 37.4), which is important: high activeness paired with low alpha is exactly the combination our framework predicts will destroy value. At the other end, multi-strategy funds are the least active (mean rank 44.9). Correlations with the S&P 500 vary widely, from −0.49 for dedicated short bias to 0.41 for emerging markets, confirming that our sample spans a broad range of systematic risk exposures.

Table 2 reports average characteristics across the four groups formed by the intersection

Table 1: Summary Statistics

Panel A: Fund Characteristics

	Mean	SD	10th	25th	50th	75th	90th
Activeness	0.840	1.184	0.066	0.176	0.431	0.983	2.016
Return (%)	0.366	3.631	-3.473	-0.962	0.460	1.800	4.100
FH7 alpha (%)	0.215	1.079	-0.927	-0.281	0.201	0.724	1.375
C4F alpha (%)	0.047	2.481	-2.790	-1.056	0.139	1.175	2.756
AUM ($M)	243.976	683.535	2.931	11.189	44.000	156.914	491.246
Flow (yearly)	0.077	0.791	-0.525	-0.250	-0.045	0.130	0.623
Age (years)	12.677	6.608	4.833	7.250	11.750	17.083	21.833
Lockup (months)	3.149	6.865	0.000	0.000	0.000	0.000	12.000
RedemptNotice (days)	34.544	34.114	0.000	10.000	30.000	45.000	90.000
Management fee (%)	1.442	0.559	1.000	1.000	1.500	2.000	2.000
Incentive fee (%)	17.263	6.407	5.000	20.000	20.000	20.000	20.000
High water mark	0.680	0.466	0.000	0.000	1.000	1.000	1.000
Min. investment ($M)	0.840	1.822	0.020	0.100	0.250	1.000	1.000
Personal capital / AUM	0.013	0.055	0.000	0.000	0.000	0.000	0.005
Leveraged	0.599	0.490	0.000	0.000	1.000	1.000	1.000

Panel B: By Strategy

	Activeness Rank			Alpha Rank			Correlation
Strategy	Mean	Median	SD	Mean	Median	SD	with S&P 500
Fixed Income Arbitrage	48.3	48	30.2	51.8	54	24.7	0.22
Convertible Arbitrage	49.0	48	29.3	50.9	51	26.1	0.31
Long/Short Equity Hedge	50.0	50	27.9	48.7	48	29.7	0.39
Options Strategy	50.1	50	30.8	52.2	52	25.7	0.16
Emerging Markets	50.4	51	28.4	47.9	46	33.4	0.41
Global Macro	51.4	53	29.8	50.7	51	29.8	0.14
Event Driven	52.0	53	29.3	51.8	53	27.1	0.38
Dedicated Short Bias	52.7	53	27.9	46.0	43	31.0	-0.49
Equity Market Neutral	53.1	55	28.9	48.5	48	27.1	0.18
Managed Futures	58.1	62	26.4	37.4	33	28.2	0.14

Notes: This table reports summary statistics for the main variables. The sample consists of hedge funds from the Lipper TASS database reporting monthly net-of-fee returns in U.S. dollars, 1994–2018. We exclude each fund's first 12 months to mitigate backfill bias and require at least 24 months of non-backfilled observations. Returns are desmoothed following Getmansky, Lo and Makarov (2004). Activeness is the sum of squared changes in estimated factor loadings from adjacent 24-month rolling windows. FH7 and C4F denote alphas from the Fung–Hsieh seven-factor and Carhart four-factor models, respectively. Panel A reports pooled fund-month statistics. Panel B reports mean, median, and standard deviation of within-strategy percentile ranks for activeness and alpha, along with the average correlation of fund returns with the S&P 500.

of alpha and activeness median splits. Several patterns stand out. First, active funds—regardless of skill—are more volatile, bear more systematic and tail risk, and have higher autocorrelation, consistent with active trading generating both return dispersion and exposure rotation. Second, high-alpha funds attract substantially more capital: average annual flows are 16–17% for skilled funds versus near zero or negative for unskilled funds, with little difference by activeness within skill groups. Third, incentive alignment is stronger among skilled active funds: they charge higher incentive fees (17.96% vs. 16.68% for low-alpha/low-active), commit more personal capital, and are more likely to use leverage and lockup provisions. Fourth, and most importantly for our main analysis, the return and alpha patterns preview the core result. Skilled active funds earn the highest raw returns (0.87% monthly) and the highest FH7 alpha (0.86%), while unskilled active funds earn the lowest alpha (-0.57%). The spread between active and passive is positive for skilled funds and negative for unskilled funds—exactly the asymmetric pattern our model predicts.

Table 3 asks what drives activeness. We regress activeness on lagged fund characteristics in a pooled OLS with time fixed effects, first for the full sample (column 1) and then separately for high- and low-alpha funds (columns 2 and 3). The dominant predictor is return volatility: a one-standard-deviation increase in lagged volatility raises activeness by roughly 0.4 standard deviations, and this effect is stable across skill groups. Funds that are older, larger, and have longer lockup periods tend to be less active, consistent with organizational inertia or the capacity constraints that accompany scale. Management fees are positively associated with activeness in both subsamples, suggesting that funds charging higher fixed fees engage in more portfolio rebalancing. Incentive fees, however, tell a more nuanced story: they predict activeness only among low-alpha funds (column 3), not among high-alpha funds (column 2). This asymmetry is consistent with our model's prediction that unskilled managers respond to convex incentives by increasing trading activity even when they lack an informational edge. The desmoothing parameter θ_0 is strongly negative, indicating that funds with more illiquid or smoothed returns exhibit mechanically higher activeness as the desmoothing procedure amplifies loading variation. The R^2 of approximately 0.21 indicates that observable fund characteristics explain a

Table 2: Fund Characteristics by Alpha and Activeness

	High α High Active	High α Low Active	Low α High Active	Low α Low Active
Volatility	0.0017	0.0007	0.0019	0.0009
Systematic risk	0.0009	0.0004	0.0010	0.0005
Tail risk	0.0197	0.0117	0.0212	0.0140
Log(AUM)	17.7430	17.7902	17.1043	17.2222
Age (yr)	8.1476	7.5818	8.5604	8.0734
Flow (past 1yr)	0.1673	0.1624	-0.0294	-0.0401
Log(lockup)	1.1974	1.0389	1.0801	0.9736
Liquidity factor	0.0486	0.0292	0.0332	0.0262
Market timing ability	0.0046	-0.0237	-0.0812	-0.0904
Autocorrelation	0.0174	0.0004	-0.0133	-0.0166
Management fee	1.4799	1.4309	1.4502	1.4003
Incentive fee	17.9623	17.0192	17.2925	16.6761
High water mark	0.7346	0.6527	0.6896	0.6421
Ln(MinInvt+1)	12.3429	11.9797	12.0522	11.9139
Personal capital / AUM	0.0166	0.0107	0.0128	0.0107
Leveraged	0.6385	0.6146	0.5864	0.5412
Return	0.0087	0.0065	-0.0029	0.0005
C4F intercept	0.0054	0.0047	-0.0050	-0.0026
C4F MKT	0.2783	0.1846	0.4052	0.2912
C4F HML	0.0157	0.0047	0.0153	0.0119
C4F SMB	0.0559	0.0309	0.0603	0.0389
C4F MOM	0.0314	0.0170	-0.0141	0.0023
FH7 intercept	0.0086	0.0063	-0.0057	-0.0028
FH7 PTFS Bond	0.0011	0.0003	-0.0130	-0.0096
FH7 PTFS Currency	0.0103	0.0056	0.0060	0.0047
FH7 PTFS Commodity	0.0023	-0.0005	-0.0059	-0.0045
FH7 Size Spread	0.0529	0.0287	0.1085	0.0625
FH7 Equity Market	0.1947	0.1314	0.3578	0.2494
FH7 Credit Spread	-2.9981	-1.7695	-2.3865	-1.5346
FH7 Bond Market	-0.0974	-0.1210	-0.4209	-0.1799

Notes: This table reports time-series averages of cross-sectional means for fund characteristics across the four groups formed by the intersection of alpha and activeness median splits. Each month, funds are sorted into high/low alpha and high/low activeness based on cross-sectional medians. Alpha is the intercept from rolling 24-month Fung–Hsieh seven-factor regressions using desmoothed returns. Activeness is the sum of squared changes in factor loadings across adjacent windows.

meaningful share of cross-sectional variation in trading activity.

Table 4 examines the persistence of activeness and alpha using quartile transition matrices. Panel A shows that activeness is persistent in the short run but mean-reverts over longer horizons. At a one-month horizon, a fund in the highest activeness quartile has a 44.9% probability of remaining there, versus 25% under random assignment. This persistence decays steadily: at one year, the stay probability falls to 35.1%, and at five years to just 13.0%—close to the 25% random benchmark. The pattern is symmetric: low-activeness funds are equally sticky in the short run (39.1% stay probability at one month) and similarly mean-revert over time. This confirms that activeness captures genuine variation in trading behavior rather than a permanent fund characteristic, and that the measure is sufficiently persistent month-to-month to be economically meaningful for our predictive regressions.

Panel B reports the analogous transition matrix for alpha. Alpha is substantially more persistent than activeness in the short run: at one month, 84% of top-quartile funds remain in the top quartile, reflecting the well-known autocorrelation in hedge fund returns. However, alpha persistence decays more sharply at longer horizons, falling to 41.6% at one year and just 12.5% at five years. The rapid decay in alpha persistence relative to activeness has an important implication for our framework: a fund's skill classification is far from permanent, and the interaction between skill and activeness plays out against a backdrop of substantial reranking in both dimensions over time.

Table 5 presents the central result of the paper. Each month, we independently sort funds into high and low alpha groups using the cross-sectional median, and into activeness quartiles. We then examine average next-month alpha across these portfolios. Among high-alpha funds, activeness monotonically increases future performance: the most active quartile earns 0.92% per month in FH7 alpha, compared to 0.59% for the least active. The Q4−Q1 spread of 0.33% per month is both statistically and economically significant, corresponding to roughly 3.9% annualized. The pattern is reversed—and amplified—among low-alpha funds. The most active unskilled funds earn −1.10% per month, versus −0.38% for the least active. The Q4−Q1 spread of −0.72% per month implies that activeness among unskilled managers destroys approximately 8.7% annu-

Table 3: Determinants of Activeness

	(1) All	(2) High Alpha	(3) Low Alpha
Volatility (Lag1Yr)	21.790***	22.875***	21.115***
	(30.512)	(27.575)	(27.761)
Log(AUM)	-0.010**	-0.005	-0.017***
	(-2.521)	(-1.162)	(-3.133)
Age (yr)	-0.010***	-0.009***	-0.010***
	(-4.245)	(-3.821)	(-3.590)
Flow (Lag1Yr)	0.000***	0.000	0.000***
	(82.130)	(1.304)	(63.454)
FH7 Alpha (Lag1Yr)	-0.923	-3.148*	0.615
	(-0.751)	(-1.875)	(0.431)
Log(Lockup)	-0.023**	-0.018	-0.028**
	(-2.155)	(-1.612)	(-2.260)
Management fee	0.045***	0.044**	0.042*
	(2.765)	(2.561)	(1.859)
Incentive fee	0.004**	0.002	0.005***
	(2.112)	(1.052)	(2.645)
High water mark	-0.024	-0.025	-0.025
	(-0.972)	(-0.884)	(-0.892)
PersonalCapitalToAUM	0.129	0.297**	-0.091
	(0.932)	(1.973)	(-0.537)
Theta0	-0.759***	-0.795***	-0.741***
	(-13.680)	(-13.265)	(-10.987)
Ln(MinInvt+1)	0.006	0.009**	0.001
	(1.411)	(2.368)	(0.087)
Leveraged	0.027	0.023	0.031
	(1.261)	(1.030)	(1.260)
Observations	122,485	69,248	53,237
R-squared	0.208	0.217	0.200
Sample	All	High Alpha	Low Alpha

Notes: This table reports pooled OLS regressions of activeness on lagged fund characteristics with time fixed effects. The dependent variable is activeness measured at month t. Column (1) uses the full sample; columns (2) and (3) split the sample by the cross-sectional median of FH7 alpha. Standard errors are clustered by fund. t-statistics are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels.

Table 4: Persistence of Activeness and Alpha

Panel A: Activeness Transitions

	1 Month				1 Year				5 Years			
	Low	Q2	Q3	High	Low	Q2	Q3	High	Low	Q2	Q3	High
Low	39.1	28.0	20.1	10.7	30.8	23.4	16.9	9.9	11.6	9.3	7.5	5.2
Q2	27.6	28.4	25.0	17.0	22.9	22.7	20.7	14.8	9.4	9.6	8.8	7.0
Q3	19.8	25.1	27.9	25.1	16.9	20.1	23.0	20.7	7.7	8.8	9.8	9.1
High	11.1	16.5	25.0	44.9	9.4	14.4	20.2	35.1	5.3	6.7	8.5	12.9

Panel B: Alpha Transitions

	1 Month				1 Year				5 Years			
	Low	Q2	Q3	High	Low	Q2	Q3	High	Low	Q2	Q3	High
Low	83.9	10.7	1.5	0.8	38.5	15.8	10.0	9.7	9.3	5.7	5.5	7.5
Q2	11.5	71.0	13.9	1.4	19.2	30.7	19.9	10.2	7.7	9.8	8.6	6.5
Q3	1.6	14.5	70.8	11.4	11.2	22.8	30.7	18.9	7.5	11.2	11.7	7.8
High	0.7	1.7	11.8	84.1	11.2	11.3	20.2	41.6	9.5	7.7	8.8	12.5

Notes: This table reports transition matrices for activeness quartiles (Panel A) and alpha quartiles (Panel B). Each cell shows the probability (%) that a fund in a given quartile at time t is in a given quartile at time $t+h$, for horizons of 1 month, 1 year, and 5 years. Quartile assignments are based on monthly cross-sectional breakpoints. Results for intermediate horizons (3 months, 6 months, 3 years) are qualitatively similar and available upon request.

ally. Two features of these results deserve emphasis. First, the asymmetry: the value destruction from unskilled active trading is more than twice the value creation from skilled active trading. Second, the monotonicity: the relationship is not driven by extreme quartiles but increases steadily across the distribution. Both features are consistent with the model's predictions. Under the C4F specification, the high-alpha spread is positive but insignificant, while the low-alpha spread remains large and significant, suggesting that the asymmetry is robust to the choice of factor model. These results are robust across fund characteristics and investment strategies.[9]

Table 5: Future Alpha by Activeness and Skill

	High Alpha		Low Alpha	
Activeness Quartile	FH7 (%)	C4F (%)	FH7 (%)	C4F (%)
Q1 (Low)	0.589***	0.441***	-0.378***	-0.284***
Q2	0.608***	0.418***	-0.484***	-0.336***
Q3	0.682***	0.460***	-0.672***	-0.502***
Q4 (High)	0.916***	0.504***	-1.102***	-0.748***
High - Low	0.327***	0.063	-0.724***	-0.465***
	(4.43)	(0.99)	(-10.03)	(-7.48)

Notes: This table reports average next-month alphas (% per month) for portfolios sorted by activeness quartiles, conditional on skill. Each month, funds are independently sorted into high/low alpha groups using the cross-sectional median of rolling FH7 alpha, and into activeness quartiles using cross-sectional breakpoints. FH7 and C4F denote alphas from the Fung–Hsieh seven-factor and Carhart four-factor models. The High−Low row reports the spread between Q4 and Q1. *t*-statistics are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels.

Table 6 moves from portfolio sorts to regression analysis, controlling for fund characteristics that may jointly determine activeness and future performance. We regress annual future alpha (cumulative FH7 or C4F alpha over months $t+1$ through $t+12$, in percent) on activeness, a positive-alpha indicator ($\mathbf{1}_{\alpha=H} = 1$ if rolling alpha > 0), their interaction, and a standard set of controls in a pooled OLS with year fixed effects and

[9] Appendix Tables 9 and 10 report finer decile sorts. The negative High−Low spread is significant in seven of eight characteristic subsamples and in the majority of individual strategies, with the largest spreads in global macro and multi-strategy. The only subsample where the spread is insignificant is large-AUM funds, consistent with capacity constraints.

standard errors clustered by fund. Columns (1)–(3) enter the key variables separately to establish baseline effects; column (4) is the main specification. The interaction term Activeness $\times$ $\mathbf{1}_{\alpha=H}$ is positive and highly significant, while the standalone activeness coefficient is negative. For skilled managers, the net effect of activeness on future alpha is $-0.004 + 0.009 = 0.005$: more active skilled funds earn higher future alpha. For unskilled managers, the standalone coefficient of -0.004 indicates that activeness predicts lower future performance. Coefficient tests confirm both effects: the standalone activeness coefficient is significantly negative for low-alpha funds ($t = -3.59$, $p < 0.001$ for FH7; $t = -2.32$, $p = 0.010$ for C4F), and the total effect of activeness for high-alpha funds (Activeness + Activeness $\times$ $\mathbf{1}_{\alpha=H}$) is significantly positive ($t = 6.55$, $p < 0.001$ for FH7; $t = 4.16$, $p < 0.001$ for C4F). The $\mathbf{1}_{\alpha=H}$ indicator itself is large and significant throughout (4–6 percentage points per year), confirming persistent skill differences. These results establish that the asymmetric relationship between activeness and future alpha documented in the portfolio sorts survives controls for fund size, age, flows, volatility, and incentive contract features.

Table 6: Activeness and Future Alpha: Pooled OLS

	(1) FH7	(2) FH7	(3) FH7	(4) FH7	(5) C4F	(6) C4F	(7) C4F	(8) C4F
Activeness $\times$ $\mathbf{1}_{\alpha=H}$				0.009***				0.006***
				(7.177)				(4.573)
Activeness	0.002**		0.002**	-0.004***	0.001		0.001	-0.002**
	(2.532)		(2.529)	(-3.585)	(1.559)		(1.494)	(-2.324)
$\mathbf{1}_{\alpha=H}$		0.063***	0.063***	0.055***		0.047***	0.046***	0.041***
		(32.889)	(32.893)	(28.807)		(24.820)	(24.818)	(21.953)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	254,931	254,931	254,931	254,931	256,497	256,497	256,497	256,497
R-squared	0.051	0.082	0.082	0.083	0.055	0.072	0.072	0.072

Notes: This table reports pooled OLS regressions of annual future alpha (cumulative alpha over months $t+1$ through $t+12$, in percent) on activeness, a positive-alpha indicator ($\mathbf{1}_{\alpha=H} = 1$ if rolling FH7 alpha > 0), their interaction, and controls. All specifications include year fixed effects. Standard errors are clustered by fund. Controls include volatility, log AUM, age, flow, log lockup, management fee, incentive fee, high-water-mark indicator, log minimum investment, and leverage indicator. Columns (1)–(3) enter key variables separately; column (4) adds the interaction. Columns (5)–(8) repeat for C4F alpha. Standard errors are clustered by fund. t-statistics in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels.

A concern with the pooled OLS results is that cross-sectional dependence in fund returns may inflate standard errors or that time-varying common shocks drive the relationship. Table 7 addresses this using Fama–MacBeth (Fama and MacBeth, 1973) regressions, which estimate the cross-sectional relationship between activeness and future alpha separately each month and report time-series averages of the monthly coefficients. This approach is robust to arbitrary cross-sectional correlation in returns and provides a clean test of whether the activeness–alpha relationship holds in a typical month. In column (4), the interaction Activeness $\times$ $\mathbf{1}_{\alpha=H}$ is 0.014, roughly 50% larger than the pooled OLS estimate. The standalone activeness coefficient is -0.005, confirming that activeness predicts lower future alpha for unskilled managers in the average cross-section. The C4F results in column (8) tell the same story. Coefficient tests confirm the asymmetry: for FH7, the standalone activeness effect is significantly negative for low-alpha funds ($t = -2.87$, $p = 0.002$) and the total effect is significantly positive for high-alpha funds ($t = 5.67$, $p < 0.001$). For C4F, the total effect for high-alpha funds remains significant ($t = 3.30$, $p = 0.001$), though the standalone effect for low-alpha funds is not individually significant ($t = -0.95$, $p = 0.171$). The strengthening under Fama–MacBeth is reassuring: the pooled OLS results are not an artifact of a few unusual months driving the time-series average, but reflect a relationship that holds persistently across the cross-section.

Finally, Table 8 examines whether the four fund groups differ in their exposure to market risk and market timing ability using the Henriksson and Merton (1981) framework. The results suggest a qualitative distinction between skilled active funds and the other three groups. Skilled active funds (column 1) have essentially zero market beta and a positive, though imprecisely estimated, timing coefficient. Their returns are orthogonal to the market—suggestive of managers who actively adjust exposures to neutralize systematic risk while pursuing alpha. By contrast, the remaining three groups all load significantly on the market (β ranging from 0.28 to 0.50), indicating substantial systematic risk exposure. Among these, skilled passive funds (column 2) exhibit significant negative market timing: their static exposures leave them mechanically short convexity. Unskilled active funds (column 3) combine high market beta (0.50) with near-zero timing ability —they bear systematic risk without the skill to time it, consistent with our model's

Table 7: Activeness and Future Alpha: Fama–MacBeth Regressions

	(1) FH7	(2) FH7	(3) FH7	(4) FH7	(5) C4F	(6) C4F	(7) C4F	(8) C4F
Activeness $\times$ $\mathbf{1}_{\alpha=H}$				0.014***				0.007***
				(6.461)				(3.291)
Activeness	0.003**		0.002	-0.005***	0.003**		0.002*	-0.002
	(2.361)		(1.019)	(-2.869)	(2.324)		(1.755)	(-0.950)
$\mathbf{1}_{\alpha=H}$		0.073***	0.076***	0.059***		0.046***	0.046***	0.040***
		(20.380)	(13.575)	(15.304)		(20.132)	(20.713)	(15.761)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	254,931	254,931	254,931	254,931	256,497	256,497	256,497	256,497
R-squared	0.118	0.152	0.159	0.165	0.120	0.131	0.139	0.145
Number of groups	355	355	355	355	363	363	363	363

Notes: This table reports Fama–MacBeth regressions of annual future alpha (cumulative alpha over months $t+1$ through $t+12$, in percent) on activeness, a positive-alpha indicator ($\mathbf{1}_{\alpha=H} = 1$ if rolling FH7 alpha > 0), their interaction, and controls. Each month, a cross-sectional regression is estimated; reported coefficients are time-series averages of the monthly estimates. Controls are the same as in Table 6. Newey–West (Newey and West, 1987) adjusted t-statistics (3 lags) are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels.

prediction that unskilled active trading loads on systematic factors rather than generating idiosyncratic alpha. Unskilled passive funds (column 4) similarly show significant negative timing. The descriptive pattern is consistent with skilled active funds being the only group whose returns are approximately market-neutral. The pattern suggests that their activeness may reflect deliberate exposure management rather than undisciplined factor rotation.

6 Conclusion

This paper introduces activeness—the sum of squared changes in estimated factor loadings—as a measure of hedge fund trading activity and documents a stark asymmetry: active trading by skilled managers creates value, while active trading by unskilled managers destroys it. The unconditionally negative activeness–performance relationship reported in prior work is a composition artifact that masks two distinct populations with opposite responses to the same behavior.

Table 8: Market Timing Ability by Alpha and Activeness

	(1) High α High Active	(2) High α Low Active	(3) Low α High Active	(4) Low α Low Active
β	-0.019	0.284***	0.500***	0.336***
	(-0.067)	(79.080)	(43.155)	(73.205)
γ	0.605	-0.109***	0.035*	-0.020**
	(1.236)	(-17.464)	(1.728)	(-2.437)
Observations	121,645	137,570	102,574	100,706
R-squared	0.000	0.129	0.085	0.169

Notes: This table reports Henriksson and Merton (1981) market timing regressions for portfolios formed by the intersection of alpha and activeness. The specification is $r_{i,t} - r_{f,t} = \alpha + \beta(r_{m,t} - r_{f,t}) + \gamma \max(r_{m,t} - r_{f,t}, 0) + \varepsilon_{i,t}$, where γ captures market timing ability. Each column reports a single pooled regression for the indicated group. All specifications include year fixed effects. Standard errors are clustered by fund. t-statistics in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels.

We rationalize this asymmetry in a model where skilled managers trade actively on precise signals, generating both high activeness and high alpha, while unskilled managers inflate factor bets beyond the optimum to clear survival thresholds—a form of gambling for resurrection that raises activeness but destroys expected performance through quadratic trading costs. The model yields additional predictions that the data confirm: incentive fees predict activeness only among low-alpha funds, activeness is persistent but mean-reverting, and skilled active funds exhibit near-zero market beta while unskilled active funds bear the highest systematic risk exposure of any group.

Our findings have implications for both investors and regulators. For investors evaluating hedge fund managers, activeness is not informative in isolation—its sign depends entirely on the skill of the manager deploying it. A high-activeness fund may be a skilled manager exploiting a genuine informational edge or an unskilled manager gambling with investors' capital. The interaction with past alpha resolves this ambiguity. For regulators concerned with systemic risk, the concentration of high market beta and negative timing ability among unskilled active funds suggests that the funds most likely to amplify market dislocations are precisely those with the weakest fundamentals.

Several extensions remain for future work. First, our activeness measure is based on rolling OLS; a Kalman-filter implementation would provide a theoretically cleaner estimate of time-varying exposures. Second, validation against direct trading measures—such as CFTC position data or 13F filings for equity-focused funds—would strengthen the interpretation that activeness captures real portfolio shifts rather than estimation noise. Third, the model's deterministic structure abstracts from learning: an extension in which investors update beliefs about skill and set liquidation thresholds endogenously would deliver richer dynamics and sharper empirical predictions about the timing of gambling behavior.

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A Proofs

Proof of Proposition 2.2. The fee gain on the left-hand side of (2.11) is positive because $r^+(\hat{\beta}) = \underline{r}_i > r^+(\beta^{mgr})$: the fee-distorted exposure fails to clear the threshold by definition (since $\phi_i < \hat{\phi}$), while the gambling exposure clears it by construction. The co-investment loss on the right-hand side is positive because $\pi(\beta, \phi_i)$ is strictly concave in β with maximum at β^*, and $\hat{\beta}$ is further from β^* than β^{mgr}.

For part (i), $r^+(\beta^{mgr}) < \underline{r}_i$ for $\phi_i < \hat{\phi}$ by Lemma 2.3, so a larger β on the increasing portion of $r^+(\cdot)$ is needed to reach $\underline{r}_i$. Since $\hat{\beta}$ is the smallest root, it lies on $(0, 1/c)$ and satisfies $\hat{\beta} > \beta^{mgr} > \beta^*$. For part (iii), $\pi^* - \pi(\hat{\beta}) = (c/2)(\hat{\beta} - \beta^*)^2 > 0$, which is increasing in $|\hat{\beta} - \beta^*|$. □

Proof of Corollary 2.1. For $\phi_i > \hat{\phi}$, both $A^{mgr}(\phi_i) = (\beta^{mgr})^2$ and $\pi^{mgr}(\phi_i)$ are strictly increasing in ϕ_i (the direct skill effect through μ_i dominates the distortion cost, which is $O((1-\phi_i)^2)$). The chain rule gives $d\pi/dA = (d\pi/d\phi)/(dA/d\phi) > 0$.

For $\phi_i < \hat{\phi}$, the gambling exposure $\hat{\beta}(\phi_i)$ is strictly decreasing in ϕ_i: as skill falls, $\underline{r}_i$ rises (Assumption 2.1), requiring a larger $\hat{\beta}$ to clear the higher threshold. Thus $A^{gamble} = \hat{\beta}^2$ is strictly decreasing in ϕ_i. Expected alpha $\pi^{gamble}(\phi_i) = \hat{\beta}\mu_i - (c/2)\hat{\beta}^2$ is strictly increasing in ϕ_i, since both the increase in $\hat{\beta}$ (further from β^*) and the decrease in μ_i reduce alpha as ϕ_i falls. Since $dA/d\phi < 0$ and $d\pi/d\phi > 0$, the chain rule gives $d\pi/dA < 0$.

Moreover, alpha is strictly increasing in ϕ_i across the entire distribution. For non-gambling managers, $d\pi^{mgr}/d\phi_i > 0$ follows from the first paragraph. For gambling managers, differentiating π^{gamble}:

$$\frac{d\pi^{gamble}}{d\phi_i} = \underbrace{\hat{\beta}'(\phi_i)}_{<0} \cdot \underbrace{(\mu_i - c\hat{\beta})}_{<0} + \underbrace{\hat{\beta}(\phi_i)}_{>0} \cdot \underbrace{2}_{} > 0.$$

The first term is positive ($\hat{\beta}' < 0$ and $\mu_i - c\hat{\beta} < 0$ since $\hat{\beta} > \beta^*$); the second is also positive. □

Proof of Proposition 2.3. Part (i) follows from differentiating (2.8) with respect to δ_i. Part (ii): the fee component of the gambling surplus is $\delta_i\phi_i[r^+(\hat{\beta}) - r^+(\beta^{mgr})]$, which is positive (since $r^+(\hat{\beta}) > r^+(\beta^{mgr})$ for managers below $\hat{\phi}$) and linear in δ_i. □

Proof of Lemma 2.5. The Lagrangian is $\mathcal{L} = \pi(\beta_1, \beta_2) + \lambda[r^+(\beta_1, \beta_2) - \underline{r}_i]$. Since π and r^+ are additively separable across factors, the first-order condition for factor k is

$$(\mu_i - c_k\beta_k) + \lambda(1 - c_k\beta_k) = 0,$$

which yields $\hat{\beta}_k = (\mu_i + \lambda)/[c_k(1+\lambda)]$ for the Lagrange multiplier $\lambda > 0$ (the constraint binds since the first-best does not clear the threshold). At $\lambda = 0$, this reduces to $\beta_k^* = \mu_i/c_k$.

For $\lambda > 0$:

$$\hat{\beta}_k - \beta_k^* = \frac{\lambda(1-\mu_i)}{c_k(1+\lambda)} > 0$$

since $\mu_i < 1$ for all $\phi_i < 1$. The ratio $\hat{\beta}_1/\hat{\beta}_2 = c_2/c_1$ is independent of λ, and $\Delta\hat{\beta}_1/\Delta\hat{\beta}_2 = c_2/c_1$ follows immediately. □

Proof of Proposition 2.4. At the optimum, $\beta_1^* = \mu_i/c_1$ and $\beta_2^* = \mu_i/c_2(\phi_i)$. Since f_1 and f_2 are independent:

$$R_i^2 = \frac{(\beta_1^*)^2}{(\beta_1^*)^2 + (\beta_2^*)^2} = \frac{\mu_i^2/c_1^2}{\mu_i^2/c_1^2 + \mu_i^2/c_2(\phi_i)^2} = \frac{1}{1 + c_1^2/c_2(\phi_i)^2}.$$

Since $c_2(\phi_i)$ is decreasing in ϕ_i, the ratio $c_1^2/c_2(\phi_i)^2$ is increasing in ϕ_i, and R_i^2 is decreasing. □

Proof of Proposition 2.5. Part (i) follows from Lemma 2.5: $\hat{\beta}_1 > \beta_1^*$ for all gambling managers. For part (ii), by Lemma 2.5, $\hat{\beta}_1/\hat{\beta}_2 = c_2(\phi_i)/c_1$, so

$$R_i^2 = \frac{1}{1 + (\hat{\beta}_2/\hat{\beta}_1)^2} = \frac{1}{1 + c_1^2/c_2(\phi_i)^2} = \frac{c_2(\phi_i)^2}{c_1^2 + c_2(\phi_i)^2}.$$

For low-skill managers, $c_2(\phi_i) \gg c_1$ implies $R_i^2 \approx 1$. □

B Supplementary Tables

Decile sorts by fund characteristics and strategy. Tables 9 and 10 extend the quartile portfolio sorts of Table 5 to deciles and examine robustness across subsamples. Table 9 splits the sample at the median of AUM, management fee, incentive fee, and lockup period. The baseline High−Low spread across all funds is −1.71% per year, confirming that the quartile results are not an artifact of coarse binning. The spread is significant in seven of eight cuts, ranging from −1.17% (high lockup) to −3.24% (low incentive fee). The only insignificant subsample is large-AUM funds, consistent with capacity constraints limiting the scope for active trading among the largest funds. The spread is notably larger among low-incentive-fee funds than high-incentive-fee funds, suggesting that convex compensation partly disciplines value-destroying activeness but does not eliminate it.

Table 10 reports the spread within each investment strategy. The negative activeness–alpha relationship is broadly based: global macro, multi-strategy , and fixed income arbitrage all exhibit significant negative spreads. Long/short equity hedge—the most populated strategy—shows a marginally significant spread. Convertible arbitrage is the one strategy where more active funds earn higher future alpha, potentially reflecting that loading turnover captures genuine mispricing opportunities in convertible bonds.

Table 9: Future Alpha by Activeness Decile: Subsample Cuts

		AUM		Mgmt Fee		Incentive Fee		Lockup	
Decile	All	Large	Small	High	Low	High	Low	High	Low
Activeness Deciles	All	Large AUM	Small AUM	High MGMT Fee	Low MGMT Fee	High Inct Fee	Low Inct Fee	High Lockup	Low Lockup
Q1 (Low)	4.082	5.301	2.738	4.630	4.043	4.177	4.042	5.489	3.002
Q2	3.606	4.112	2.639	3.866	3.212	3.737	2.925	4.849	2.179
Q3	3.102	3.999	2.175	3.757	2.709	3.387	2.123	4.402	1.856
Q4	2.915	3.600	1.873	3.390	2.118	3.299	2.407	4.912	1.359
Q5	2.961	3.921	1.345	3.380	2.971	3.708	1.682	4.403	1.643
Q6	3.096	4.610	1.514	3.244	2.332	3.530	1.421	4.723	1.300
Q7	3.124	4.502	1.496	3.881	2.533	3.615	0.946	4.828	1.290
Q8	2.663	4.947	0.936	3.185	1.750	3.448	1.493	4.423	0.873
Q9	2.721	4.478	0.843	3.411	2.383	3.605	0.046	4.696	1.159
Q10 (High)	2.430	4.718	1.026	2.252	1.736	2.996	0.823	4.236	0.661
High - Low	-1.712***	-0.633	-1.688***	-2.407***	-2.264***	-1.237***	-3.237***	-1.171**	-2.245***
	(-3.97)	(-1.05)	(-3.05)	(-4.42)	(-4.34)	(-2.75)	(-5.50)	(-2.11)	(-4.78)

Notes: Annual FH7 alpha over months $t+1$ through $t+12$ (%) by activeness decile, within subsamples split at the median of each characteristic. t-statistics in parentheses.

Table 10: Future Alpha by Activeness Decile: By Strategy

Decile	Conv. Arb.	LS Equity	Event Driven	Eq. Mkt. Ntrl.	Global Macro	FI Arb.	Emerg. Mkts.	Multi-Strat.	Mgd. Futures	Short Bias	Options
Activeness Deciles	Convertible Arbitrage	LS Equity Hedge	Event Driven	Equity Market Neutral	Global Macro	Fixed Income Arbitrage	Emerging Markets	Multi Strategy	Managed Futures	Dedicated Short Bias	Options Strategy
Q1 (Low)	3.646	3.939	4.322	3.187	2.981	5.175	2.829	6.038	-4.554	-1.312	3.992
Q2	4.252	2.994	4.064	3.439	2.349	3.595	1.769	5.286	-0.709	0.470	3.426
Q3	4.582	2.657	5.034	3.476	3.240	3.822	1.913	5.483	-1.631	1.557	3.913
Q4	4.536	3.048	4.665	2.876	1.849	3.028	1.693	4.214	-4.127	-0.814	1.606
Q5	5.628	2.349	3.963	2.686	1.941	3.503	1.932	4.328	-1.562	0.540	2.936
Q6	4.010	3.096	4.139	3.961	0.422	3.198	2.117	3.715	0.712	-0.917	1.634
Q7	4.555	2.892	4.430	3.240	1.194	3.035	2.065	4.065	-2.529	0.144	2.012
Q8	5.223	3.381	4.935	2.899	1.195	3.127	2.192	3.627	-2.656	0.112	2.846
Q9	5.172	3.086	4.437	2.808	0.970	5.299	2.818	1.938	-0.719	-2.604	2.914
Q10 (High)	5.367	2.782	3.630	2.172	-0.231	3.449	3.930	1.911	-3.651	-0.398	5.478
High - Low	1.707*	-1.037*	-0.698	-0.852	-3.499***	-1.780**	0.953	-3.638***	-0.027	0.926	1.552
	(1.87)	(-1.70)	(-1.05)	(-1.09)	(-3.89)	(-2.50)	(1.19)	(-4.32)	(-0.02)	(0.37)	(1.31)

Notes: Annualized next-month FH7 alpha (%) by activeness decile within each strategy. t-statistics in parentheses.