

Corporate Governance by Workers^{*}

Anthony Lee Zhang[†], Hongda Zhong[‡]

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Abstract

We show that the departure of informed workers can play a governance role for firms. When workers are compensated partly in equity, they may leave the firm if they observe low managerial effort, reducing the firm's value and the manager's own compensation. Therefore, diluting the manager's equity stake in the firm through worker equity grants can *increase* managers' effort incentives. Worker equity also exerts downward pressure on compensation, since worker exits are more costly to the firm and manager when workers are underpaid. The model provides a new explanation for the prevalence of worker equity compensation and generates several testable predictions regarding worker compensation packages, labor bargaining power, capital structure, and managerial performance.

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[†]University of Chicago, Booth School of Business; anthony.zhang@chicagobooth.edu.

[‡]Corresponding Author. University of Texas at Dallas, CEPR, and FTG; hongda.zhong@utdallas.edu

1 Introduction

Firms employ many individuals, and employees often possess private information about the firm. This paper asks a simple question: can informed rank-and-file workers play a role in corporate governance? More specifically, how are the effort incentives of managers and senior officers shaped by the presence of large numbers of informed workers and their decisions whether to remain employed at the firm? How does the nature of worker compensation influence their governance role? How does the firm’s capital structure interact with the structure of worker compensation?

In this paper, we propose a new governance effect we call the *worker monitoring channel*. Suppose informed rank-and-file workers are paid partially in equity. If the workers observe the manager shirking, they anticipate that their equity grants are worth less, and are more likely to leave the firm. Managers, knowing that workers may leave if they shirk, thus have increased incentives for effort. Thus, counterintuitively, diluting the manager’s equity stake in the firm through worker equity grants can *increase* managers’ effort incentives. Worker monitoring also exerts downward pressure on compensation, because workers who are paid less harm the firm more when they leave, thus providing stronger effort incentives for managers. This paper thus offers a novel explanation for the widespread use of equity compensation for workers, even when workers are too numerous for equity to provide effective direct incentives for effort, and delivers implications for compensation design, capital structure, and managerial performance.

We analyze a simple static moral-hazard model of firm financing, most similar to [Holmstrom and Tirole \(1997\)](#), to which we add a worker component. There is a manager, who produces output by exerting costly private effort; when she is not a full residual claimant on output, she will be unable to commit to exerting the efficient level of effort. There is a competitive representative investor, who has a lower cost of capital than the manager, meaning that it is welfare-improving for the investor to fund the firm’s operations. There is a set of workers, with a random reservation wage, who can produce some exogenous output if they are employed at the firm. We assume the workers can observe the manager’s effort, and condition their decision whether to work at the firm on the manager’s effort.

We first consider a benchmark where the manager and investor hold equity shares in the firm, but workers are paid in fixed wages with no equity component. The manager and investor then face a classic tradeoff: equity granted to investors increases the efficiency of financing, but decreases the manager’s incentives for effort. The worker’s compensation does not interact with this tradeoff: the manager faces a monopsony pricing problem and simply chooses the fixed wage which maximizes surplus extracted from workers, which is equal to

the markdown between worker’s output and their wage multiplied by the probability that workers are retained. Intuitively, under fixed-wage contracts, the manager’s effort does not affect workers’ payoffs and thus the probability of their retention; the manager effort problem is fully separated from the worker wage-setting problem.

The worker monitoring channel emerges when workers are paid with general wage-and-equity contracts. Managerial effort then influences worker retention, causing the problems of manager incentive provision and worker compensation to become intertwined. This mechanism relies on two key conditions. First, workers must care about managerial effort, which implies their compensation must be tied to firm performance. In the model, we restrict attention to wage-and-equity compensation; the performance-sensitive component of worker compensation is naturally equity. It is worth mentioning that a wide range of commonly observed compensation arrangements, such as employee stock options and bonus multipliers based on aggregate performance, can serve a similar purpose. Second, the manager must care about (specifically, must be harmed by) worker departure, so that the desire to retain workers increases the manager’s incentives for effort. This force implies that the level of worker compensation must be lower than the monopsony-optimal wage level in the fixed-wage benchmark: if workers are compensated at the optimal fixed wage, the departure of a marginal worker has no first-order impact on the firm value and the manager’s payoffs. In other words, workers must be “cheap” for the manager to care about retention.

Our model therefore rationalizes the practice of granting unvested equity to a collection of small workers. This practice is difficult to rationalize through the traditional incentive-provision channel: while equity can arguably create incentives for *large* workers, it is ineffective in motivating effort among many *small* workers, since effort incentives are diluted as the number of workers becomes large. In contrast, in our model, the optimal compensation structure for workers is pure equity. This is because the probability of worker retention – which determines the amount of surplus extracted from workers – only depends on total expected compensation, not the wage-equity split; thus, replacing fixed wages by equity strengthens the worker monitoring channel without adversely affecting profit extraction from workers. We want to caution that this result should not be narrowly interpreted as an argument against the existence of fixed wages in practice, but rather as a qualitative explanation for the common practice of linking small workers’ compensation to aggregate firm performance.¹ Finally, it is important that equity is *unvested*, so workers forfeit their equity grants when they leave the firm; such workers then decide to quit by comparing the expected value of their unvested equity to their outside options.

¹Nonzero fixed wages could easily be rationalized through worker risk aversion, for example, which we abstract from for analytical tractability.

The most salient prediction of our model is that worker equity increases managerial effort, despite crowding out managerial equity stake in the firm. Since the manager can always eliminate the worker monitoring channel by offering either zero or monopsony compensation levels, the fact that underpaid workers are employed in equilibrium suggests that the worker monitoring channel always alleviates the underlying moral hazard problem and improves managerial incentives. The simplicity of the model allows us to extend it and study the effect of capital structure. Suppose the firm is financed with defaultable debt and can also default on wages, the fixed wages can be effective to induce worker monitoring if the firm is close to default, because the value of worker equity is insensitive to firm performance when the firm is close to default and equity is “zeroed out”. Hence, the model implies that safer firms tend to use equity and riskier firms tend to use fixed wage to compensate workers. We can also consider bargaining power of workers, such as the existence of a labor union. If workers have high bargaining power such that they are overpaid (relative to the monopsony compensation level), they should be paid in fixed wages because otherwise, managers may deliberately shirk in order to motivate quitting of the overpaid workers.

Other comparative statics of our model are driven by the dual role of workers as both monitors and as profit sources for the firm. As compensation decreases from the monopsony-optimal level, workers are more effective as monitors for the manager, but profit extraction from workers is less efficient. The optimal compensation level trades off these two forces. Workers are more underpaid when output is more sensitive to manager effort, since the role of workers as monitors is more important. Conversely, when profits from workers are comparatively important for the firm, manager incentive provision through workers is more costly, and the optimal contract tends to underpay workers less.

Finally, while we assume workers are identical for simplicity, our results hold in a simple extension to heterogeneous workers, which introduces the possibility of “worker runs” ([Hoffmann and Vladimirov, 2025](#)).

The primary contribution of our paper is to consider the implications of workers’ private information about firms for corporate finance and worker compensation. We contribute mainly to two literatures.

Our contribution to the corporate finance literature is that we analyze the implications of workers’ private information for optimal equity financing. We provide a new answer to the classic question of why workers are often compensated using equity. A classic idea in the literature is that equity is ineffective for inducing effort from regular workers, because equity grants infinitesimal effort incentives when the number of workers is large ([Holmstrom, 1982](#)). The worker monitoring channel we propose does not suffer from this “dilution” problem:

when a large number of workers are paid in equity, it remains true that each individual worker has incentives to quit if the firm is doing sufficiently poorly.

Oyer (2004) argues that worker equity compensation improves worker retention, when wage adjustment is costly and the firm’s equity value is related to the worker’s outside options. Our theory is distinct: rather than insuring workers against outside-option risk, the primary purpose of equity in our setting is to give workers incentive to monitor managers. There are a number of other theories explaining worker equity compensation, including incentive and sorting theories (Oyer and Schaefer, 2005), theories based on worker equity as a defense against hostile takeovers (Pagano and Volpin, 2005), and institutional explanations based on the accounting and tax treatment of employee equity compensation (Blasi, Conte and Kruse, 1996; Hall and Murphy, 2002, 2003; Freeman, Kruse and Blasi, 2010). Our goal is of course to provide a new theoretical channel, not to argue against the relevance of other channels discussed in prior literature.

We also relate to a corporate finance literature on the idea of internal governance of firms. Acharya, Myers and Rajan (2011) studies how subordinate managers contribute to providing incentives for top management, analyzing a model where non-CEO managers’ effort incentives arise from the potential to be promoted to upper management. We instead focus on regular workers, who have no promotion prospects, but may discipline the manager by exiting the firm.

We also contribute to a literature on the determinants of worker wages. In simple industrial-organization models of wage setting, optimal worker pay trades off markdowns with retention, as in classic monopolistic price-setting problems. A classic literature on moral hazard emphasizes that, in their capacity as agents imperfectly observed by firm management, workers should be overpaid somewhat relative to their outside options (Solow, 1979; Shapiro and Stiglitz, 1984; Yellen, 1984). We consider workers in their capacity as *principals*, disciplining the efforts of firm leadership through their credible threats to quit if they observe shirking. In our model, workers are only effective principals if they are undercompensated: the optimal wage-and-equity compensation contract unambiguously lowers worker surplus relative to the optimal fixed wage.

The paper proceeds as follows. We introduce the model in Section 2. We discuss the fixed-wage benchmark in Section 3. Our main results are in Section 4. We explore extensions of the model in Section 5, discuss the model’s assumptions in Section 6, and conclude in Section 7.

2 Setup

There is a single project, which requires a fixed upfront investment of $\bar{I} = 1$. There are three sets of agents: a manager, a unit mass of small homogeneous workers, and competitive investors. There is no discounting and all agents are risk-neutral.

Output. The project’s output is the sum of contributions from the manager and workers. As is standard in the static moral-hazard literature, we assume the manager exerts effort e , at some private convex cost $c(e)$, such that $c''(e)$ is bounded away from 0 for any e .² The manager’s contribution to output is $e + \tilde{\epsilon}$, where $\tilde{\epsilon}$ is some mean-zero shock. The “noise” is included only to justify why we cannot contract directly on managerial effort, and we will largely disregard it going forward, focusing on expected contribution e . We will assume the worker observes e , and can condition her decision to quit on e . The assumption that workers have private information about the firm, while somewhat unusual in the literature on theoretical corporate governance, is supported by a large academic literature and many institutional sources, which we discuss in Subsection 6.2. We assume workers observe effort perfectly for analytical simplicity; allowing noisy signals would not qualitatively change our results.

The unit mass of workers can produce fixed output Δ . Workers have homogeneous random outside option $\omega \sim F(\cdot)$, which can be thought of as a prevailing wage offer from a competing firm, which is not observed by the manager at the time the compensation contract is written. The assumption that workers are identical greatly simplifies the analysis; we show in Subsection 5.4 that all conclusions of the model still hold in an extension where workers are heterogeneous. Under this simplification, only two outcomes are possible: either all workers stay employed, or all workers quit. The firm’s expected output is thus

$$\begin{cases} e + \Delta & \text{if workers stay} \\ e & \text{if workers quit} \end{cases}.$$

We abstract from worker effort in the baseline model; in Subsection 5.3, we show that equity is ineffective at providing effort incentives when the number of workers is large, due to the classic incentive dilution problem (Holmstrom, 1982). We assume e and Δ affect output additively, so there are no complementarities in production: the only interactions between the worker and manager problems will be informational in nature.

Investment. A group of competitive investors can provide funding for the firm. Invest-

²Formally, there exists a constant $\epsilon > 0$ such that $c''(e) > \epsilon$ for all e . For example, any quadratic cost function trivially satisfies this condition.

ment is valuable because investors' cost of funds is 1, and the CEO's cost is some higher number $\theta > 1$.³ Let I be the amount invested by the investors, and $(1 - I)$ by the CEO.⁴ The investors' zero-profit condition implies that I must eventually equal investors' expected payoffs from the firm.

Contracts. We consider a simple set of wage-and-equity contracts. Workers can be paid through a combination of a fixed wage ψ , and an equity share s_W in the firm's residual profits. The manager and investor each receive equity shares s_M, s_I respectively. All parties have limited liability, implying that ψ, s_W, s_I, s_M must all be nonnegative. Moreover, equity shares must add to 1:

$$s_W + s_M + s_I = 1. \quad (1)$$

Throughout the baseline model, we assume the firm never defaults on wage, i.e., $\Delta - \tilde{\epsilon} \geq \psi$ (or equivalently the support of the shock $\tilde{\epsilon}$ is sufficiently small around 0).⁵ In Subsection 5.1, we relax the restriction and analyze the case of defaultable wage together with debt financing. Restricting attention to equity contracts is not without loss of generality, but substantially simplifies our analysis technically. We discuss this simplifying assumption and its relation to the literature in more detail in Subsection 6.1.

A compensation contract is thus described by a triple ψ, s_W, s_M . If workers stay employed, the expected payoffs of each party are:

$$\text{Payoff}_W = \psi + s_W (e + \Delta - \psi) \quad (2)$$

$$\text{Payoff}_M = s_M (e + \Delta - \psi) \quad (3)$$

$$\text{Payoff}_I = s_I (e + \Delta - \psi)$$

where, since all agents are risk-neutral, we suppress the $E[\epsilon] = 0$ terms in payoffs for notational simplicity. Intuitively, the manager and investor each receive their equity shares of residual output $(e + \Delta - \psi)$, and the worker gets her wage and her equity share.

If workers quit, they forfeit both the wage and equity components of their compensation.

³There are several closely related ways to motivate financing in moral hazard models. [Holmstrom and Tirole \(1997\)](#) assume that the firm has fixed initial wealth A , necessitating a fixed amount $I - A$ in external financing. [Jensen and Meckling \(1976\)](#) allow the scale of the firm to vary depending on the amount of external financing. In our model, we fix the size of the firm's investment, and assume there is a constant gap between the costs of inside and outside financing; this setup has economically similar intuitions to these models, and is analytically convenient for our setting. Our model can be thought of as simply fixing the "shadow value of equity", as discussed in [Tirole \(2010, ch. 3.4\)](#).

⁴In the case where $I > 1$, the CEO can save the surplus $(I - 1)$ at the interest rate θ .

⁵Since the firm would never want to pay workers more than their output, $\Delta > \psi$ must hold in equilibrium. Hence, the assumption of risk-free wage is equivalent to assuming sufficiently small shocks $\tilde{\epsilon}$.

The manager and investors share the no-worker firm based on the relative equity shares s_M and s_I , attaining payoffs:

$$Payoff_M = \frac{s_M}{s_M + s_I} e \quad (4)$$

$$Payoff_I = \frac{s_I}{s_M + s_I} e$$

We have implicitly made three important assumptions about the nature of employee equity compensation: the employee is granted a fixed number of equity units in the firm; this grant vests to the employee conditional on employment; and if the employee leaves, the unvested portion of the equity award is cancelled, so the associated dilution of the managers' and investors' equity stakes is never realized. All three assumptions are important for our later analysis, and to our understanding they are standard practice in public firms.⁶ Importantly, some firms set the size of equity grants based on a targeted dollar value *at the grant date*; however, the award must still be expressed as a fixed number of equity units once granted,⁷ so such value-based approaches to equity grant sizing are consistent with our model's assumptions.

The game is divided into five stages as follows.

1. The CEO proposes a contract s_M, s_W, ψ .
2. Investment I is determined through investors' zero-profit condition.
3. The manager chooses effort e .
4. Workers observe their outside option ω and manager effort e , and decide whether to quit.
5. Firm payoffs are realized and the game ends.

We focus on the subgame perfect equilibrium. It is convenient to specify players' objectives backwards. In stage 4, the workers' expected compensation is

$$\Gamma(e, \psi, s_W) \equiv s_W(e + \Delta - \psi) + \psi, \quad (5)$$

and they quit the firm if $\Gamma(e, \psi, s_W)$ is less than their outside option ω . The probability the firm retains these workers is thus $F(\Gamma(e, \psi, s_W))$.

⁶For example, the discussion of share-based payment in the [International Financial Reporting Standards \(IFRS\) 2](#) states: "a grant of shares or share options to an employee is typically conditional on the employee remaining in the entity's employ for a specified period of time."

⁷For example, the [National Association of Stock Plan Professionals](#) writes: "ultimately... grants must be expressed as a number of shares."

In stage 3, fixing the investment I , the manager's effort problem is

$$\mathcal{M} \equiv \max_e \frac{s_M}{s_M + s_I} e (1 - F(\Gamma(e, \psi, s_W))) + s_M (e + \Delta - \psi) F(\Gamma(e, \psi, s_W)) - c(e) - \theta(1 - I) \quad (6)$$

Intuitively, the manager's payoff is either (3) or (4), depending on whether the realization of ω makes workers stay or quit. In addition, the manager pays private effort cost $c(e)$, and cost of capital $\theta(1 - I)$.

In stage 2, expecting manager's equilibrium effort decision $e^*(s_M, s_W, \psi)$, investors break even on their investment:

$$I(s_M, s_I, s_W, \psi) = \underbrace{\frac{s_I}{s_M + s_I} e^* (1 - F(\Gamma(e^*, \psi, s_W)))}_{\text{Workers Quit}} + \underbrace{s_I (e^* + \Delta - \psi) F(\Gamma(e^*, \psi, s_W))}_{\text{Workers Work}}. \quad (7)$$

Since both the investors and the manager are equity holders, expression (7) is simply the manager's payoff gross of effort and financing costs in (6), with s_M replaced by the investor's share s_I .

Finally, in stage 1, the optimal compensation and financing structure $s_M^*, s_I^*, s_W^*, \psi^*$ maximizes the manager's ex-ante payoff $\mathcal{M}$ (defined in (6)), where I is given by (7).

3 Surplus Extraction from Workers and the Fixed-Wage Benchmark

3.1 Surplus Extraction from Workers

Before proceeding, we construct a key quantity which helps us understand the economic mechanisms driving our results.

Definition 1. The expected surplus extracted by the firm from the workers, denoted by $\Pi(\Gamma)$, is a function of workers' expected compensation level Γ :

$$\Pi(\Gamma) \equiv (\Delta - \Gamma) F(\Gamma) \quad (8)$$

In words, the surplus extracted from workers is the markdown $(\Delta - \Gamma)$ between workers' output Δ and their compensation Γ , multiplied by the retention probability $F(\Gamma)$. Just as a monopolist's price-setting decision trades off increased markups against decreased sale quantities, the firm's monopsonistic compensation-setting decision trades off larger markdowns

with lower worker retention probabilities.

We further impose the standard assumption that $\Pi(\cdot)$ is strictly concave:⁸

$$\Pi''(\Gamma) < 0, \quad (9)$$

which ensures a unique compensation level Γ_{FW}^* that maximizes Π , given by the solution to

$$\Pi'(\Gamma_{FW}^*) = 0. \quad (10)$$

The subscript “FW” is short for “fixed-wage.” As will be clear immediately below, when workers are offered a fixed wage without equity ($s_W = 0$), they receive the compensation level Γ_{FW}^* . For future use, the concavity of Π immediately implies:

$$\Gamma < \Gamma_{FW}^* \implies \Pi'(\Gamma) > 0, \quad \Gamma > \Gamma_{FW}^* \implies \Pi'(\Gamma) < 0. \quad (11)$$

Proposition 1. *The manager’s effort problem (6) can be expressed as*

$$\mathcal{M} = \max_e \underbrace{\frac{s_M}{s_M + s_I} e}_{\text{Manager Output}} + \underbrace{\frac{s_M}{s_M + s_I} \Pi(\Gamma(e, \psi, s_W))}_{\text{Surplus Extracted From Workers}} - c(e) - \theta(1 - I). \quad (12)$$

Similarly, investors’ payoff (7) can be expressed as

$$I = \underbrace{\frac{s_I}{s_M + s_I} e^*(s_M, s_W, \psi)}_{\text{Manager Output}} + \underbrace{\frac{s_I}{s_M + s_I} \Pi(\Gamma(e^*(s_M, s_W, \psi), \psi, s_W))}_{\text{Surplus Extracted From Workers}}. \quad (13)$$

Finally, the manager’s contract design problem in stage 1 can be expressed as

$$\mathcal{M} = \max_{s_M, s_I, s_W, \psi} \frac{s_M + \theta s_I}{s_M + s_I} [e^*(s_M, s_W, \psi) + \Pi(\Gamma(e^*(s_M, s_W, \psi), \psi, s_W))] - c(e^*(s_M, s_W, \psi)) - \theta. \quad (14)$$

Proposition 1 provides an intuitive and important view of the firm. If we treat workers as if they are “outsiders” to the firm, the manager-investor firm contains both the stand-alone value of the managerial firm e , and the expected surplus $\Pi(\Gamma(e, \psi, s_W))$ extracted from workers. The manager receives a share $\frac{s_M}{s_M + s_I}$ of this firm as in (12) and the investors receive the remaining $\frac{s_I}{s_M + s_I}$ of this firm as in (13).

⁸Related conditions are commonly imposed for tractability in imperfect-competition models (Bulow and Pfleiderer, 1983; Caplin and Nalebuff, 1991) as well as the related literature on one-dimensional screening problems (Myerson, 1981; Maskin and Riley, 1984).

The comparison between the manager's ex-ante payoff objective (14) and his ex-post effort objective (12) reveals a standard tradeoff between external financing and incentive provision, similar to [Holmstrom and Tirole \(1997\)](#). On the one hand, raising equity from external investors is cheaper due to their lower cost of capital $1 < \theta$. On the other hand, external equity dilutes manager's stake in the firm, reducing effort incentives. Our focus is on how the presence of informed workers can alleviate this friction.

3.2 Fixed-Wage Contracts

We first consider the case in which workers are paid fixed wages, with no equity component: $\Gamma = \psi$ and $s_W = 0$. Relevant variables are subscripted by "FW." This case will be used as a benchmark to compare the general optimal contract to.

Since workers are paid a fixed wage, they quit with probability $F(\psi)$, independent of the manager's effort e . Furthermore, since $s_M + s_I = 1 - s_W = 1$, the manager's effort problem (12) in stage 3 reduces to

$$\max_e s_M [e + \Pi(\psi)] - c(e) - \theta(1 - I), \quad (15)$$

and the FOC for e implies that the unique optimal effort $e_{FW}^*(s_M)$ satisfies

$$c'(e_{FW}^*) = s_M. \quad (16)$$

Using the same simplification, the manager's contracting problem (14) in stage 1 reduces to

$$\max_{s_M, \psi} (s_M + \theta(1 - s_M)) [e_{FW}^*(s_M) + \Pi(\psi)] - c(e_{FW}^*(s_M)) - \theta. \quad (17)$$

This contract design problem is straightforward. It is unambiguously optimal to set wages to maximize $\Pi(\psi)$, which from (10) implies $\psi = \Gamma_{FW}^*$. The choice of s_M navigates a classic tradeoff between financing and incentive provision. Differentiating (17) and setting to 0, we have:

$$\frac{\partial}{\partial s_M} : (\theta - 1) [e_{FW}^*(s_M) + \Pi(\Gamma_{FW}^*)] = [(s_M + \theta(1 - s_M)) - c'(e_{FW}^*(s_M))] \frac{\partial e_{FW}^*}{\partial s_M} \quad (18)$$

When s_M increases, the LHS captures the marginal cost from reduced financing, which is the cost difference $(\theta - 1)$ multiplied by total output. The RHS captures the marginal output gains from increasing the manager's effort incentives. Applying the implicit function theorem

to (16), we have:

$$\frac{\partial e_{FW}^*}{\partial s_M} = \frac{1}{c''(e_{FW}^*(s_M))}$$

Moreover, (16) implies that $c'(e_{FW}^*(s_M)) = s_M$; substituting into (18), we have the FOC:

$$(\theta - 1)[e_{FW}^*(s_M) + \Pi(\Gamma_{FW}^*)] = \frac{\theta(1 - s_M)}{c''(e_{FW}^*(s_M))} \quad (19)$$

Expression (19), alongside $\psi = \Gamma_{FW}^*$, thus characterize the optimal contract without worker equity. Intuitively, when workers are paid a fixed wage, the contract design problem separates cleanly into the problem of optimally extracting surplus from workers through the wage ψ , and optimally trading off manager incentive provision and financing capacity through the equity share s_M .

What is interesting about the general contract, which we analyze next, is that these problems become intertwined: when workers are paid in equity, worker retention affects manager incentives, and the manager internalizes this effect in determining the level of worker compensation.

4 Results

4.1 Worker Retention

When workers hold equity s_W , their expected compensation Γ depends on managerial effort e , as in (5). Worker retention is thus sensitive to managerial effort. From (5), we have that:

$$\frac{\partial \Gamma}{\partial e} = s_W, \quad (20)$$

which in turn implies

$$\frac{d}{de} F(\Gamma(e, \psi, s_W)) = f(\Gamma(e, \psi, s_W)) s_W$$

Increasing managerial effort increases retention, since it increases expected worker compensation $\Gamma(e, \psi, s_W)$. Effort affects retention more when s_W is higher, since workers who are paid more in equity have compensation which is more sensitive to firm output.

4.2 Manager Effort and Worker Monitoring Channel

Differentiating (12) with respect to e , the manager's optimal effort must satisfy:

$$\frac{s_M}{s_M + s_I} \left(1 + \Pi'(\Gamma(e, \psi, s_W)) \frac{\partial \Gamma}{\partial e} \right) - c'(e) = 0. \quad (21)$$

Together with (20), we have the key result of this subsection.⁹

Proposition 2. *Under a contract s_M, s_W, ψ , optimal manager effort $e^*(s_M, s_W, \psi)$ is the unique solution to:*

$$c'(e) = \frac{s_M}{s_M + s_I} (1 + \Pi'(\Gamma(e, \psi, s_W)) s_W). \quad (22)$$

Proposition 2 captures the core economic force in our paper. When workers are compensated using equity, and they observe managers' effort, their exit decisions depend on managers' effort choices. Since workers' exits can also influence firm profits, managers consider the effects of their effort on worker retention, as captured by the $\Pi'(\Gamma(e, \psi, s_W)) s_W$ term in (22). We call this the *worker monitoring channel*.

Two elements are necessary for the worker monitoring channel to increase effort incentives for the managers relative to the benchmark case (16). First, worker compensation must include an equity component $s_W > 0$: only then do workers care about the aggregate performance of the firm, and thus managerial effort. Second, the manager must also care about worker retention, which implies that workers must be under-compensated relative to the monopsony-optimal level: $\Gamma < \Gamma_{FW}^*$, or equivalently $\Pi'(\Gamma) > \Pi'(\Gamma_{FW}^*) = 0$. If workers are paid the monopsony-optimal compensation Γ_{FW}^* , since $\Pi'(\Gamma_{FW}^*) = 0$, workers quitting on the margin have no effect on equity values, because the change in retention rates is perfectly offset by the change in markdowns. If either element vanishes – $s_W = 0$ or $\Gamma = \Gamma_{FW}^*$ – condition (22) reduces to (16), and the presence of workers does not affect managerial effort incentives.

Counterintuitively, granting equity to workers can thus *increase* the manager's incentives for effort, even though the manager's financial equity stake in the firm is reduced. Specifically, holding the manager-investor split constant $k \equiv \frac{s_M}{s_M + s_I}$, an increase in worker equity s_W mechanically reduces the manager stake, $s_M = (1 - s_W)k$. However, condition (22) implies

⁹The second order condition can be easily verified. Using the fact that $\Pi(\cdot)$ is concave and $c(\cdot)$ is convex, we calculate another derivative of (21):

$$\frac{\partial^2}{\partial e^2} : \frac{s_M}{s_M + s_I} \Pi''(\Gamma) s_W^2 - c''(e) < 0$$

implying that (22) is a necessary and sufficient condition for the manager's optimal choice of e .

that managerial effort increases with s_W . Treating workers as “outsiders” of the firm, the potential dilution of managerial stake is accounted for as worker compensation in the $\Pi(\Gamma)$ term. The remaining net effect is that workers retention becomes more sensitive to managerial performance, and hence provides stronger effort incentive for the manager.

Proposition 2 also implies a convenient property about the monopsony optimal compensation level Γ_{FW}^* .

Proposition 3. *Suppose $s_W < 1$, given the ratio between the manager’s and investors’ ownerships $k = \frac{s_M}{s_M + s_I}$, all worker compensation contracts $\{\psi, s_W < 1\}$ that deliver $\Gamma = \Gamma_{FW}^*$ in equilibrium are equivalent in that they generate the same managerial effort e^* and identical payoffs for all players.*

Intuitively, all worker contracts that implement compensation level $\Gamma = \Gamma_{FW}^*$ for the workers remove the governance channel for the manager. Hence, the managerial effort is insensitive to worker contract, which in turn implies that the equilibrium outcome is similarly unaffected. This is a useful benchmark to see how worker equity and underpayment can improve the outcome later.

With these intuitions in mind, we move on to characterize the optimal contracting problem faced by the manager in stage 1.

4.3 Optimal Contract Design

Finally, we consider the manager’s choice of contract s_M, s_I, s_W, ψ in the first stage. We prove a number of claims characterizing the optimal contract, and show that workers must be compensated with equity $s_W^* > 0$ and $\psi^* = 0$. We then provide first-order conditions illustrating the main tradeoff the manager faces when choosing the equity shares for workers and investors.

It is useful to remind ourselves that the manager’s classic moral-hazard problem can be framed as a commitment problem. The comparison between (12) and (14) shows that in stage 1, the manager would like to commit to extracting all surplus from workers, by setting cash compensation Γ_{FW}^* , and also exerting first-best effort, which is characterized by the FOC:

$$c'(e) = \frac{s_M + \theta s_I}{s_M + s_I}$$

However, in stage 3, after the investment is sunk, the manager cannot commit to exerting this effort as the FOC of (12) is

$$c'(e) = \frac{s_M}{s_M + s_I},$$

which means effort will be inefficiently low. In this subsection, we show how the optimal contract, in particular the workers' contract s_W and ψ can be designed to alleviate this commitment problem.

For exposition, we decompose manager's stage 1 payoff (14) into two concave components attributable to manager effort and worker surplus extraction

$$\mathcal{M} = \underbrace{\frac{s_M + \theta s_I}{s_M + s_I} e^*(s_M, s_W, \psi) - c(e^*(s_M, s_W, \psi))}_{\text{Manager Surplus}} + \underbrace{\frac{s_M + \theta s_I}{s_M + s_I} \Pi(\Gamma(e^*(s_M, s_W, \psi), \psi, s_W))}_{\text{Surplus Extracted From Workers}} \quad (23)$$

The goal of contract design is to optimally trade off losses to the two components of (23), which will be graphically illustrated in Figure 1 as we establish the result in several steps.

Claim 1. The optimal financing structure features strictly positive internal and external equity $s_M^*, s_I^* > 0$.

The reason for strictly positive external equity ($s_I^* > 0$) is as follows. Suppose the entire firm is owned by manager (and possibly also partially by the worker), there is no “wedge” between the manager's ex-ante payoff objective (14) and his ex-post effort objective (12). Since the manager's ex-post effort choice is optimized, marginal reduction in effort due to small equity dilution (s_I) also has a zero marginal cost on his ex-ante payoff. In contrast, by raising capital from investors, there is a discrete benefit of $\theta - 1$ per dollar raised. Hence, the optimal financing structure must feature some external equity.

Claim 2. For any $k = \frac{s_M}{s_M + s_I} \in (0, 1)$, there exists a pure equity contract for workers $\tilde{\psi} = 0$ and $\tilde{s}_W < 1$ such that their total compensation $\Gamma = \Gamma_{FW}^*$. Furthermore, holding k and $\psi = 0$ fixed, a small reduction of worker equity $\tilde{s}_W$ to $\tilde{s}_W - \delta$ increases the manager's payoff $\mathcal{M}$.

Proof sketch. The existence of $\tilde{s}_W$ is by construction. To see the second part of the claim, recall we expressed the manager's surplus $\mathcal{M}$ as the sum of manager surplus terms (the blue curve in Figure 1) and surplus extracted from workers Π (the brown curve in Figure 1), both of which are concave. If workers are compensated Γ^* , Π is maximized, but the manager surplus term is not. A small decrease in s_W from this point thus has no first-order effect on Π , since $\Pi'(\Gamma_{FW}^*) = 0$. In other words, marginal worker quitting when they are compensated Γ_{FW}^* has no marginal effect on the firm value. In contrast, a reduction in s_W frees up available equity for the manager, which increases managerial effort, which in turn has a first-order increase on the manager surplus term. The result then follows. The full proof is presented in Appendix A.4. $\square$

This claim establishes a key step in showing that worker monitoring channel is present in

equilibrium. Recall that Proposition 2 establishes the two necessary conditions for workers positive worker equity $s_W > 0$ and undercompensation for workers $\Pi'(\Gamma) > 0$. It is not clear whether these conditions hold in equilibrium. The second part of Claim 2 shows that reducing worker equity from Γ_{FW}^* level is beneficial. Proposition 3 shows that all contracts delivering Γ_{FW}^* are equivalent, which includes the best contract that can be attained by fixed wage $\psi^* = \Gamma_{FW}^*$. Hence, all fixed wage contracts are dominated and the worker monitoring channel must exist in equilibrium. The next proposition is the main result of this subsection that formally summarizes the discussion so far and further establishes equity compensation as the optimal contract for workers.

Proposition 4. *The optimal contract always compensates workers entirely in equity: $\psi^* = 0$ and $s_W^* \in (0, 1)$. Furthermore, workers are undercompensated in that $s_W^* < \tilde{s}_W$ or equivalently $\Gamma^* < \Gamma_{FW}^*$.*

Proof sketch. Intuitively, if worker compensation has a cash component ψ^* , the manager can always marginally swap the cash payment for some additional equity payment s_W^* , while maintain the total compensation to the worker. Such a modification makes the worker's payoff and hence his departure decision more sensitive to the firm's performance, providing a stronger effort incentive for the manager through the worker monitoring channel, i.e., the $\Pi'(\Gamma) s_W$ term in (22). Since the manager ex-ante prefers to commit to a higher effort level e^* , the modified contract therefore improves manager surplus in (23).

There is a secondary “feedback” effect: the higher managerial effort e^* in turn increases firm value and compensation to the worker Γ . Since workers are underpaid relative to the monopsony-optimal level, higher worker compensation additionally increases surplus extraction from workers $\Pi(\Gamma)$ in (23). Both effects work in the same direction and the conclusion follows. The proof in the Appendix shows the details.

□

Proposition 4 reduces the optimal contract problem to the choice of two free variables: $k = \frac{s_M}{s_M + s_I}$, which affects the amount of investment and the manager's baseline level of incentives; and s_W , which affects the tradeoff between worker surplus extraction, and manager incentive provision through worker monitoring.

It is worth noting that for any fixed k , manager effort e cannot be monotonic in s_W over $[0, \tilde{s}_W]$ since worker monitoring is completely ineffective at the two endpoints (recall that $\tilde{s}_W$ is given in Claim 2 and $\Pi'(\Gamma(\tilde{s}_W)) = \Pi'(\Gamma_{FW}^*) = 0$). The two endpoints also yield identical manager effort that coincides with the case where workers are not present. Therefore, worker

equity $s_W \in [0, \tilde{s}_W]$ increases manager's effort incentive rather than diluting it, even though manager's equity s_M is reduced.

We now provide an FOC for the optimal choice of s_W .

Proposition 5. *The optimal choice of s_W , fixing $k = \frac{s_M}{s_M + s_I}$, satisfies:*

$$[(\theta + (1 - \theta)k) - c'(e^*(k, s_W))] \frac{de^*}{ds_W} + (\theta + (1 - \theta)k) \Pi'(\Gamma(e^*(k, s_W), s_W)) \frac{d\Gamma}{ds_W} = 0 \quad (24)$$

Where:

$$\frac{d\Gamma}{ds_W} = e^*(k, s_W) + \Delta + s_W \frac{de^*}{ds_W} \quad (25)$$

$$\frac{de^*}{ds_W} = \frac{k(\Pi'(s_W(e^*(k, s_W) + \Delta)) + s_W(e^*(k, s_W) + \Delta)\Pi''(s_W(e^*(k, s_W) + \Delta)))}{c''(e) - k\Pi''(s_W(e + \Delta))s_W^2} \quad (26)$$

Intuitively, Proposition 5 states that, at the optimal contract, the effect of a small change in s_W on the manager and worker components of surplus should be equal. These terms are the products of “wedge” terms:

$$[(\theta + (1 - \theta)k) - c'(e^*(k, s_W))], \Pi'(\Gamma(e^*(k, s_W), s_W))$$

and “passthrough” terms $\frac{de^*}{ds_W}$ and $\frac{d\Gamma}{ds_W}$, which ultimately each depend on the curvature properties of the cost function $c(\cdot)$ and the profit function $\Pi(\cdot)$.

A simple implication of Proposition 5 is that $\frac{de^*}{ds_W}$ must be negative at the optimal contract (reflected by the downward sloping blue curve at the optimal point s_W^* in Figure 1): if $\frac{de^*}{ds_W} > 0$, then all terms in (24) are positive, and an increase in s_W unambiguously makes the manager better off. In the range where both effort and worker compensation are below their optimal values, (24) can only hold if $\frac{de^*}{ds_W}$ is negative and $\frac{d\Gamma}{ds_W}$ is positive.

Proposition 6. *The optimal choice of $k \equiv \frac{s_M}{s_M + s_I}$ satisfies:*

$$\underbrace{(1 - \theta)[e^*(k, s_W) + \Pi(\Gamma(e^*(k, s_W), s_W))]}_{\text{Financing Benefits}} + \underbrace{[(\theta + (1 - \theta)k) - c'(e^*(k, s_W))]}_{\text{Effort Costs}} \left[\frac{\partial e^*}{\partial k} - \frac{\partial e^*}{\partial s_W} \frac{s_W \frac{\partial e^*}{\partial k}}{e^*(k, s_W) + \Delta + s_W \frac{\partial e^*}{\partial s_W}} \right] = 0 \quad (27)$$

where $\frac{\partial e^*}{\partial s_W}$ is defined in (26), and:

$$\frac{\partial e^*}{\partial k} = \frac{1 + \Pi'(s_W(e + \Delta)) s_W}{c''(e) - k\Pi''(s_W(e + \Delta)) s_W^2} \quad (28)$$

Intuitively, (27) states that the choice of the ratio $k = \frac{s_M}{s_M + s_I}$ – the relative shares of manager versus investor equity – is pinned down by the classic tradeoff between financing and incentive provision, generalizing (18) in the fixed-wage problem in Section 3.2. The first term in (27) reflects the marginal financing gains from increasing k , and the second term reflects the net cost from reducing the manager’s effort, multiplied by the “wedge”

$$(\theta + (1 - \theta)k) - c'(e^*(k, s_W))$$

The effort term is slightly complex, because changing k changes the worker component of profit as well as the manager’s effort; the $\frac{\partial e^*}{\partial s_W}$ term in (27) reflects the shift in worker equity s_W needed to keep worker profit Π constant, which in turn has a feedback effect on the manager’s optimal effort level. Essentially, we derive (27) in Appendix A.7 by differentiating $\mathcal{M}$ with respect to k , and then substituting the s_W FOC from (24) and simplifying, to eliminate the effects of k on the worker profit component of surplus.

4.4 Model Simulations

To illustrate the model’s results, we assume manager effort, and we assume the worker’s outside option is uniformly distributed on $[0, \Delta]$, implying a quadratic profit function. That is,

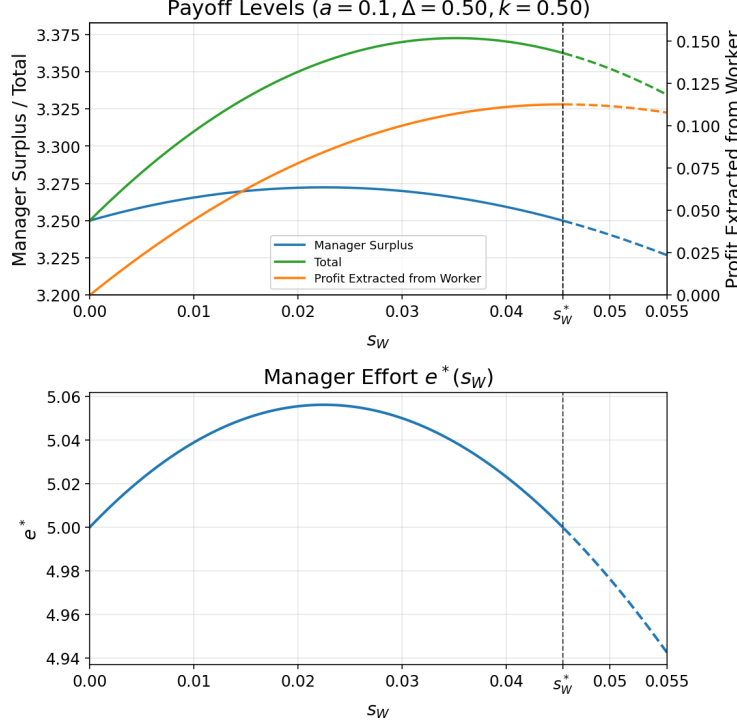
$$c(e) = \frac{ae^2}{2}, \quad \Pi(\Gamma) = (\Delta - \Gamma)F(\Gamma) = \frac{\Gamma(\Delta - \Gamma)}{\Delta} \quad (29)$$

The manager component of surplus is thus more important when a is small and effort costs are less convex, and the worker component is more important when Δ is larger. Throughout the simulations, we maintain $\theta = 1.5$. For any contract s_M, s_W , we then solve for the manager’s optimal effort using the result of Proposition 2, and then use (23) to evaluate the manager effort and worker profit components of surplus.

We illustrate the intuition behind the optimal choice of s_W in Figure 1, where we plot the manager and worker components of surplus in (23), as well as their sum, for a fixed k . There is an interior level of s_W which implements Γ_{FW}^* , and thus maximizes profit from worker (orange). This is never the optimal contract, since worker profit is locally flat in s_W at this peak, whereas the slope of manager effort surplus (blue) is positive. Thus, a small decrease

Figure 1: Manager and Worker Components of Profit

Notes. This figure illustrates how, for fixed k , the choice of worker equity share s_W trades off the manager and worker components of surplus. The blue and orange colored lines show the manager and worker component of surplus in (23), each normalized to have mean-0 over the plotted range. The green line shows their sum.

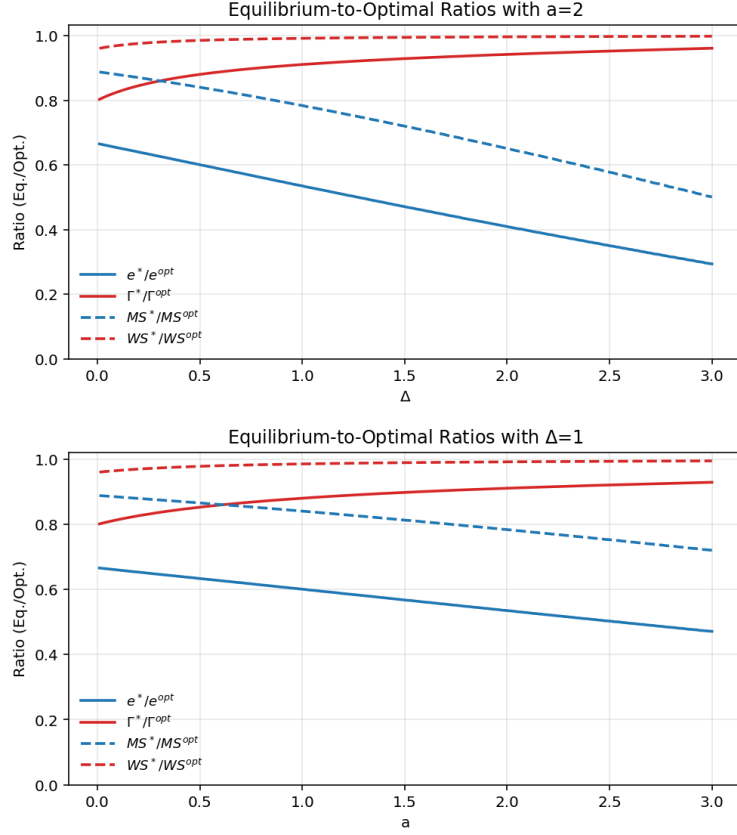


in s_W has a first-order effect on manager surplus, at the cost of only a second-order loss in profit from workers. As s_W decreases further, we reach a local maximum for the blue line; decreasing s_W past this point actually decreases managerial effort, since the reduction in s_W offsets the increase in $\Pi'(\Gamma)$ in the manager's effort FOC in Proposition 2. The optimal s_W thus lies between the peaks of the blue and orange lines.

In Figure 2, we plot the optimal contract – characterized by a k, s_W pair – as we vary a and Δ . We plot the ratios of managerial effort e , worker compensation Γ , manager surplus, and worker profit respectively to their values if manager surplus and worker profit respectively are optimized in isolation. When a decreases and the firm's output is more dependent on manager surplus, workers are more underpaid and worker profit extraction is less efficient, but manager effort and the manager component of surplus increase. Conversely, when Δ increases and worker surplus is more important, the manager component is more distorted and the worker component is less distorted.

Figure 2: Comparative Statics

Notes. This figure illustrates how features of the optimal contract vary as we shift a , the coefficient on managers' costs in (29), and Δ , which parametrizes the worker profit function in (29). In each panel, we show four lines: manager effort divided by its optimal value, fixing the value of k at the optimal contract; worker compensation Γ divided by its fixed-wage optimal value Γ_{FW}^* ; and the ratios of manager and worker surplus, as defined in (23), to their maximized values.



5 Model Extensions

5.1 Defaultable Wage and Risky Debt

So far we have only considered equity financing from external investors. The result is that equity compensation s_W is necessary to create the worker monitoring channel because equity loads worker pay directly on manager's effort and output. Fixed wage in contrast is insensitive to the firm performance. In this subsection, we consider a mix of equity and debt financing, and the possibility that the firm may default on external debt and fixed wage promised to workers. The extension yields some interesting insights. First, the fixed wage can create worker monitoring incentive because the probability of receiving it depends on the firm's performance, which in turn depends on manager's effort. Second, equity is no longer necessarily the optimal compensation contract to the worker.

Suppose the firm can raise debt from investors with a face value of D in addition to issuing s_I shares. Following common practice, in the event of a default, we assume wage ψ is more senior than external debt. Other ingredients are identical to the baseline model in Section 2. Since default is the focus of this subsection, the random shock $\tilde{\epsilon}$ in the production becomes important, and we denote its CDF by $H(\tilde{\epsilon})$. In what follows, we denote $x^+ \equiv \max(x, 0)$.

If the worker stays, the total output is $V_1 \equiv e + \tilde{\epsilon} + \Delta$, and

$$\begin{cases} W(V_1) \equiv \min\{V_1^+, \psi\} & \text{fixed wage payoff} \\ B_1(V_1) \equiv \min\{(V_1 - \psi)^+, D\} & \text{debt payoff} \\ E_1(V_1) \equiv (V_1 - \psi - D)^+ & \text{equity payoff} \end{cases}.$$

If the worker quits, the total output is $V_0 \equiv e + \tilde{\epsilon}$, and

$$\begin{cases} B_0(V_0) = \min\{V_0^+, D\}, & \text{debt payoff} \\ E_0(V_0) = (V_0 - D)^+. & \text{equity payoff} \end{cases}.$$

Expected worker compensation is

$$\Gamma(e) = \mathbb{E}[W(V_1) + s_W E_1(V_1)]. \quad (30)$$

Let $\bar{E}_i(e) \equiv \mathbb{E}[E_i(V_i)]$ denote the expected equity value where the expectation is taken over $\tilde{\epsilon}$. The manager's optimal effort choice is given by

$$(1 - F(\Gamma)) k \bar{E}_0(e) + F(\Gamma) s_M \bar{E}_1(e) - c(e), \quad (31)$$

where $k = \frac{s_M}{s_M + s_I}$. Define

$$\begin{cases} S(e) \equiv \mathbb{E}[V_1^+ - V_0^+] & \text{additional value creation by workers} \\ \Delta B(e) \equiv \mathbb{E}[B_1(V_1)] - \mathbb{E}[B_0(V_0)] & \text{debt-overhang leak to creditors} \end{cases}.$$

Essentially, the value creation captures the additional output from the workers' production. In the baseline model, it is simply Δ . In this subsection, since wage can be defaulted upon, which means that the output net of wage can be negative, the workers' contribution to the "before-wage-claim" firm value $S(e)$ in expectation is smaller than Δ . The presence of worker has another effect due to the debt in place. Some of the workers' value creation goes to the creditors by making their debt safer, and the $\Delta B(e)$ term captures the "debt-overhang leak" to creditors.

Using the accounting identities $V_1^+ = W + B_1 + E_1$, $V_0^+ = B_0 + E_0$, and the fact that $s_M = k(1 - s_W)$, the manager's objective function can be rewritten as

$$k [\bar{E}_0(e) + F(\Gamma) (S(e) - \Delta B(e) - \Gamma(e))] - c(e), \quad (32)$$

which is similar to (50) in our baseline model.¹⁰ Essentially, the workers' value creation net of debt-overhang leakage and the workers' compensation is the net contribution to equity value. The first-order condition of (32) determines the optimal managerial effort:

$$c'(e) = k \bar{E}'_0(e) + k \left(f(\Gamma) \Gamma'(e) [S - \Delta B - \Gamma] + F(\Gamma) [S' - \Gamma' - \Delta B'] \right). \quad (35)$$

where the derivatives are given by the likelihood of outcomes in the relevant regions:

$$\begin{cases} \Gamma'(e) = \Pr(0 < V_1 < \psi) + s_W \Pr(V_1 > \psi + D), \\ \Delta B'(e) = \Pr(\psi < V_1 < \psi + D) - \Pr(0 < V_0 < D), \\ S'(e) = \Pr(-\Delta < V_0 \leq 0). \end{cases}$$

¹⁰To see the derivation, from $s_M = k(1 - s_W)$ expression (31) becomes

$$k \bar{E}_0(e) + k F(\Gamma) [(1 - s_W) \bar{E}_1(e) - \bar{E}_0(e)] - c(e) \quad (33)$$

and from the accounting identities, we have

$$S(e) = W + \Delta B(e) + \bar{E}_1(y) - \bar{E}_0(y).$$

Plugging in the definition of Γ from (30) yields:

$$(1 - s_W) \bar{E}_1(y) - \bar{E}_0(y) = S(y) - \Gamma(y) - \Delta B(y). \quad (34)$$

Substituting into (33) yields (32).

These derivatives capture the additional economic forces and insights. First, the $\Gamma'(e)$ term captures the sensitivity of worker expected compensation to managerial effort as in our baseline model. Interestingly, worker equity is no longer necessary or sufficient. Wage matters for incentives only in the “partially-paid wage” region $0 < V_1 < \psi$, hence fixed wage ψ may create the strongest worker monitoring channel if wage is risky. In contrast, when firm is unlikely to default ($V_1 > \psi + D$), equity provides stronger monitoring channel as we saw in the baseline model. As in standard debt overhang literature, the leakage to creditors hinders managerial effort, which is reflected by the negative coefficients in front of $\Delta B'(e)$ in (35). Finally, $S'(e)$ captures the worker’s gross value added increases managerial effort only insofar as effort makes the worker pivotal for avoiding total bankruptcy. Otherwise, the gross value added is a constant Δ as in our baseline model and does not create managerial effort incentives.

To summarize, the main implication of our analysis is:

Claim 3. When the firm is safe (resp. risky), worker equity s_W (resp. fixed wage ψ) is more likely to provide significant effort incentive for the manager.

5.2 Worker Bargaining Power

This section extends the baseline model to incorporate worker bargaining power. We find that, if workers have high bargaining power and are able to negotiate high total compensation, the worker monitoring channel becomes ineffective, and workers are optimally compensated with fully fixed wages.

To introduce workers’ bargaining power formally, in stage 1 of the game as described in Section 2, we assume that the manager first agrees to an external equity financing term: $k = \frac{s_M}{s_M + s_I}$. Then the workers’ compensation package s_W, ψ is negotiated through generalized Nash bargaining between the CEO and workers; all later game stages are unchanged. The actual game between the manager and external investors that endogenizes k is inconsequential, as the result in this subsection holds for any given k .

At expected compensation Γ , the worker’s expected surplus, over uncertainty in ω , is:¹¹

$$\mathcal{W}(\Gamma) = \int_{\omega=-\infty}^{\Gamma} (\Gamma - \omega) dF(\omega) = \int_{\omega=-\infty}^{\Gamma} F(\omega) d\omega \quad (36)$$

¹¹For the second equality, we have:

$$\int_{-\infty}^{\Gamma} (\Gamma - \omega) dF(\omega) = \underbrace{[(\Gamma - \omega)F(\omega)]_{-\infty}^{\Gamma}}_0 - \int_{-\infty}^{\Gamma} F(\omega)(-d\omega) = \int_{-\infty}^{\Gamma} F(\omega) d\omega$$

The chosen worker contract maximizes the weighted geometric average of manager and worker surplus:

$$\max_{s_W, \psi} (\mathcal{M}(s_W, \psi) - \mathcal{M}_0)^{1-\alpha} (\mathcal{W}(\Gamma(e^*(s_W, \psi), \psi, s_W)))^\alpha \quad (37)$$

where $\mathcal{M}_0$ is the surplus the manager obtains if the worker were to leave entirely, and α parametrizes the worker's bargaining power.¹² As $\alpha \rightarrow 0$, the outcome simply maximizes $\mathcal{M}$, as in the baseline model; as $\alpha \rightarrow 1$, the outcome maximizes worker surplus $\mathcal{W}$ conditional on delivering at least $\mathcal{M}_0$ to the manager.

We solve (37) by characterizing the Pareto frontier of the set of $(\mathcal{M}, \mathcal{W})$ pairs, which we call Φ . It is clear that $\mathcal{W}$ is a strictly increasing function of expected compensation Γ ; thus, any Pareto-optimal $(\mathcal{M}, \mathcal{W})$ must maximize $\mathcal{M}$ conditional on delivering some value of Γ to the worker. Thus, we can thus trace out Φ by solving, for different choices of $\tilde{\Gamma}$:

$$\begin{aligned} \max_{s_W, \psi} \mathcal{M}(s_W, \psi) \\ \text{s.t. } \Gamma(e^*(s_W, \psi), \psi, s_W) = \tilde{\Gamma} \end{aligned} \quad (38)$$

Essentially different levels of workers' bargaining power is reflected by different total worker compensation $\tilde{\Gamma}$ in equilibrium.

Let Γ^* be the Γ value from the manager-optimal contract, characterized in Propositions 5 and 6. Clearly, it is never optimal to have $\Gamma < \Gamma^*$, since this is detrimental to both $\mathcal{M}$ and $\mathcal{W}$. It is clear that manager's outside option $\mathcal{M}_0$ is attained in (38) when $\Gamma = \Delta$, since workers' presence does not affect the equity value of the firm in this case, and the outcome is equivalent to having no workers. Thus, Φ is spanned by the solutions to (38) for values $\Gamma \in [\Gamma^*, \Delta]$.

Claim 4. When $\tilde{\Gamma} < \Gamma_{FW}^*$, the constrained-efficient contract pays the worker only in equity; $\psi = 0$. When $\tilde{\Gamma} > \Gamma_{FW}^*$, the constrained-efficient contract pays the worker only in fixed wages; $s_W = 0$.

Proof. The proof is essentially a repetition of the third and final step in the proof of Proposition 4. When $\tilde{\Gamma} > \Gamma_{FW}^*$, paying a fixed wage $\psi = \tilde{\Gamma}$ maximizes the manager's effort incentive since workers are over paid $\Pi'(\Gamma) < 0$ and manager therefore benefits from their departure. Hence, manager has incentive to shirk and thereby motivating workers to quit. Minimizing worker equity s_W reduces manager's desire to shirk.

When $\tilde{\Gamma} < \Gamma_{FW}^*$, we have shown in the proof of Proposition 4 that replacing fixed pay by

¹²Note that (37) has no $\mathcal{W}_0$ term because we accounted for workers' outside option in the definition of worker surplus in (36).

equity pay is a Pareto improvement for both the manager and workers. Hence, the optimal contract is pure equity in this case. □

Claim 4 shows that, on the Φ -frontier, the efficient contract “jumps” from a full-equity contract to a full fixed-wage contract. We can thus solve (38) for any Γ simply by choosing s_M , varying either s_W or ψ to satisfy the Γ -constraint, depending on whether $\tilde{\Gamma} > \Gamma_{FW}^*$.

Figure 3 illustrates the Φ frontier, calculated by numerically solving (38) under certain parameter settings. The “jump” in Claim 4 induces a kink in the frontier at $\Gamma = \Gamma^*$: when we switch to equity compensation for the worker, worker compensation begins to discipline manager incentives, which causes the rate of substitution between manager and worker surplus to change.

The solution to (37), for some bargaining power α , can be written as:

$$\max_{s_M, s_W, \psi} (1 - \alpha) \log (\mathcal{M}(s_W, \psi) - \mathcal{M}_0) + \alpha \log (\mathcal{W}(\Gamma(e^*(s_W, \psi), \psi, s_W))) \quad (39)$$

Thus, it is the support function of the frontier Φ in the direction $(1 - \alpha, \alpha)$; we illustrate the solution in the case where $\alpha = 0.5$ in Figure 3. As α increases from 0 to 1, the optimal contract moves from the $\mathcal{M}$ -optimal contract to the contract which pays workers $\Gamma = \Delta$ in fixed wages. In the example, the optimal contract is mostly continuous in α ; however, there is a discontinuity induced by the nonconvexity at the kink at Γ^* .

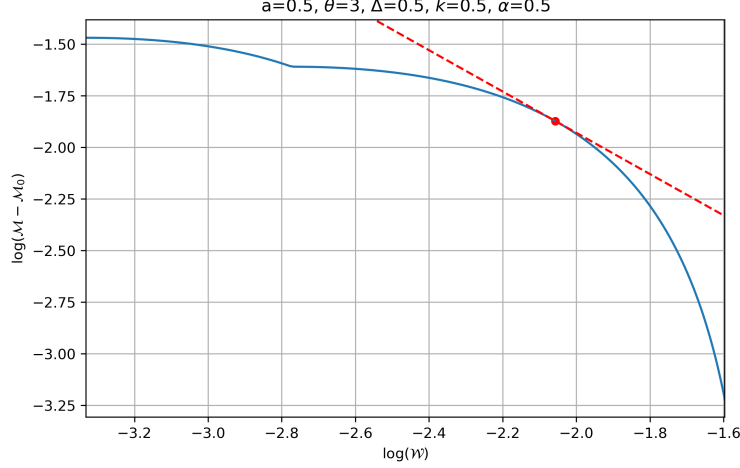
Our model predicts that, when workers have high bargaining power, we should see more fixed-wage contracts and less equity contracts. This is due to Claim 4, which builds on Proposition 2. Worker equity is only effective at increasing managerial incentives when workers are undercompensated; if workers have enough bargaining power to demand $\Gamma > \Gamma^*$, then the optimal compensation contract for the worker is fixed-wage and has no equity.

This prediction is somewhat counterintuitive: we might naively expect that worker equity should be more prevalent when workers have *more* bargaining power, though we are not aware of papers that formally make this prediction.¹³ In our model, this is because equity is purely an incentive device. Our workers are infinitesimal, so equity does not affect workers’ own incentives for effort; worker equity is purely a tool for influencing managers’ effort incentives,

¹³Worker bargaining power might be positively correlated with worker equity compensation in a model where workers contribute a variable amount to surplus, and the firm faces limited-liability constraints, implying that the only way to pay workers large amounts in expected-value terms without violating these constraints is through equity. While this channel may be important in reality, we assume it away: since the worker produces a fixed amount Δ , the firm will never pay the worker more than Δ , implying that any level of worker compensation can be paid entirely in fixed-wages without violating limited liability constraints.

Figure 3: Pareto Frontier of Manager and Worker Surplus

Notes. This figure illustrates the Pareto frontier of manager and worker surplus, which we call Φ in the text. The x and y axes respectively show log worker surplus, $\log(\mathcal{W})$, and log manager surplus, $\log(\mathcal{M} - \mathcal{M}_0)$. The tangency point to the dashed red line illustrates the bargaining outcome – the solution to (39) – when $\alpha = 0.5$.



and it is only effective at this task when workers have low enough bargaining power that their net compensation Γ is lower than Γ^* .

Anecdotally, an observation consistent with our model is that labor unions, which are plausibly associated with higher worker bargaining power, overwhelmingly negotiate the *wages* of employees; it seems less common for unions to negotiate *equity* contracts on behalf of their workers.¹⁴

5.3 Worker Equity and Worker Effort

A key implication of our model is that the equity component in workers' compensation package is crucial for workers to play a governance role. There is a classic explanation for equity compensation in the moral hazard literature: performance-based pay provides effort incentives for players. While this explanation is plausible for large workers, such as C-suite executives, we argue that it is unlikely the explanation for equity compensation for small workers, because of a classic *dilution problem*: with any nontrivially large number of workers, equity has no quantitatively meaningful effects on workers' effort incentives (Holmstrom,

¹⁴Exceptions exist: UAW-represented auto workers have a profit-sharing component of pay, though this is not explicitly an equity stake in the firm. Unions sometimes negotiate for equity when firms are distressed: for example, in the 1994 United Airlines employee buyout, workers accepted equity in exchange for wage concessions, specifically as a means to save the carrier from looming bankruptcy.

1982; Oyer, 2004). In this subsection, we demonstrate this by calculating the optimal contract for effort provision when a manager collaborates with N workers, and show that as $N \rightarrow \infty$, the collective equity compensation for workers tends to zero.

Suppose there are $N + 1$ players: 1 manager, labeled as player 0, and N workers, indexed by $i = 1, 2, \dots, N$. The manager has a technology to produce an average output of e with a private cost of $c(e)$. The N workers can each produce an average output of $\frac{e_i}{N}$ at the private cost of $c_i(e_i) \equiv \frac{c(e_i)}{N}$ for $i \geq 1$. Hence, in terms of production technology, the manager is “equivalent to” the collection of N small players. When $N = 1$, the setting is a standard team production with two members; and when $N \rightarrow \infty$, the setting is a big manager with many infinitesimal workers. The total output (equity) of the firm is

$$e_0 + \frac{\sum_{i=1}^N e_i}{N}$$

Denote by s_i ($i = 0, 1, \dots, N$) the equity share of each player. We are interested in the optimal equity allocation that maximizes the total surplus:

$$\max_{s_i} e_0^* + \frac{\sum_{i=1}^N e_i^*}{N} - c(e_0^*) - \frac{1}{N} \sum_{i=1}^N c(e_i^*). \quad (40)$$

s.t. the incentive compatibility condition of each player

$$e_i^* = \arg \max_{e_i} s_i \left(e_0 + \frac{\sum_{i=1}^N e_i}{N} \right) - c_i(e_i), \quad (41)$$

and the natural restriction that sum of equity shares is 1:

$$\sum_{i=0}^N s_i = 1. \quad (42)$$

While the conclusion holds for more general cost function, for illustration, we specify the cost function as

$$c(e_i) = \frac{1}{2} e_i^2.$$

A simple calculation in the appendix yields the optimal equity allocation.

Claim 5. The optimal equity allocation that maximizes the total surplus is $s_0^* = 1 - \frac{N}{N^2+1}$ and $s_i^* = \frac{1}{N^2+1}$ for all $i \geq 1$. Furthermore, the equity shares that workers collectively receive $\sum_{i=1}^N s_i^* = \frac{N}{N^2+1} \rightarrow 0$, as $N \rightarrow \infty$.

When $N = 1$, the firm is a partnership of two identical players, and hence the optimal

equity share is $s_0^* = s_1^* = \frac{1}{2}$. When the number of workers N becomes large, despite the fact that workers collectively possess the same production technology as the manager, their collective equity share in equilibrium vanishes, and the entire equity optimally belongs to the manager. Intuitively, as individual workers become small, they do not respond to performance pay (equity), because their individual effort has a negligible impact on the outcome. Hence, all incentive pay is allocated to the large player (manager).

Our worker monitoring channel does not suffer from this dilution problem. Technically, worker equity compensation in our setting is constrained not by the dilution of other parties' incentives, but instead by workers' limited liability constraint. It is optimal to pay workers entirely in equity, making worker exit decisions as sensitive to managerial effort as possible, without delivering negative payoffs to the worker in some states of the world. It is thus equivalent whether the representative worker, pushed against her $\psi = 0$ constraint, is thought of as a single worker or an arbitrary number of identical workers: their exit decisions, and thus their effects on managerial incentives, are identical. Interestingly, as we discussed, dilution is not even an issue for *managers*: from (12) and Proposition 2, worker equity can be viewed as having no direct dilution effect on managers' incentives, instead only influencing effort through the profit term $\Pi(\Gamma(e, \psi, s_W))$.

5.4 Robustness: Heterogeneous Workers and Run Incentives

In our baseline model, we have assumed that workers are homogeneous in that they have the same (randomly realized) outside option. The representative worker assumption is a technical simplification, which allows us to analyze interactions between workers' decisions and the manager's decision, while disregarding interactions between workers. We sacrifice some realism here to sharpen the economic insights the stylized model produces.

In this subsection, we modify the model slightly to consider a continuum of ex-post heterogeneous workers with different realizations of reservation payoffs. Other aspects of the model remain the same. We show the additional complication of incentive to run among workers, and illustrate why this complication does not materially change our insight.

Instead of having two cases based on whether workers stay or quit as in Section 2, the expected equity value in the modified model becomes

$$e + (\Delta - \psi) F(\Gamma),$$

where $F(\Gamma)$ is the mass of employed workers from the Law of large numbers, and the expected

compensation to worker becomes

$$\Gamma = \psi + s_W [e + (\Delta - \psi) F(\Gamma)]. \quad (43)$$

Immediately, we can see that there is a fixed point problem in Γ from (43): The more workers quit, the lower the equity value, which in turn motivates workers to quit. As standard in coordination games, there may be multiple solutions to (43), and hence a run incentive among workers. To proceed, one either has to specify a selection rule among the solutions or assume (43) has a unique solution. For simplicity, we take the second approach, and note if the selection rule picks the solution that is continuous in other variables, the logic below remains valid.

Now consider the manager's effort choice problem

$$\max_e s_M [e + (\Delta - \psi) F(\Gamma)] - c(e) - \theta(1 - I). \quad (44)$$

Unlike (1), the total equity that workers hold is $s_W F(\Gamma)$, hence, the share ownership identity becomes

$$s_W F(\Gamma) + s_M + s_I = 1. \quad (45)$$

Using this identity, it is easy to verify as in our main model, that the manager's objective (44) can be rewritten in the same way as (12) with the same first-order condition as (21). Investors' breakeven condition can also be identically expressed as in (13). Therefore, the only difference in the analysis is that unlike (20), where the sensitivity of workers' compensation on manager's effort is simply workers' share s_W , the fixed-point problem in (43) makes $\frac{\partial \Gamma}{\partial e}$ slightly more complex. Take implicit derivative with respect to e in (43), we have

$$\frac{\partial \Gamma}{\partial e} = s_W \left[1 + (\Delta - \psi) f(\Gamma) \frac{\partial \Gamma}{\partial e} \right],$$

or equivalently,

$$\frac{\partial \Gamma}{\partial e} = \frac{s_W}{1 - s_W (\Delta - \psi) f(\Gamma)}. \quad (46)$$

As in the baseline model, the worker monitoring channel relies on the sign of $\Pi'(\Gamma) \frac{\partial \Gamma}{\partial e} > 0$, which based on (46), is equivalent to $\Pi'(\Gamma) > 0$ (the undercompensation of workers) and $s_W > 0$ (worker equity).¹⁵ Therefore, the key insights from our baseline model, i.e.,

¹⁵To see that the denominator in (46) is positive, note that when (43) or equivalently, $\Lambda(\Gamma) \equiv \Gamma - [\psi + s_W [e + (\Delta - \psi) F(\Gamma)]] = 0$ has a unique solution $\Gamma^* > 0$, the derivative $\Lambda'(\Gamma^*) = 1 - s_W (\Delta - \psi) f(\Gamma^*) > 0$ must hold. This is because $\Lambda(0) < 0$ and the uniqueness of solution implies that the function must be increasing at the solution Γ^* .

Propositions 2 and 4, remain valid when workers have different realizations of outside options.

6 Discussion of Model Assumptions

6.1 Technical Structure

Our model takes a textbook tradeoff between manager incentive provision and competitive financing in a risk-neutral setting, and adds a worker component. The manager and investment component is similar to [Tirole \(2010, ch. 3.2\)](#), which is adapted from [Holmstrom and Tirole \(1997\)](#); a difference is that we cannot work in the simple two-outcome, high-low return setting of [Holmstrom and Tirole \(1997\)](#) because the possibility of workers leaving decreases surplus, implying that our contracts must specify payouts for more than two possible output states. We instead assume that managerial output is a random variable whose expectation is increasing in effort, and restrict attention to affine contracts – fixed wages, plus equity shares of output.

This restriction on contracts is technically substantive: there is no reason why linear contracts should be optimal among general contracts in our setting. The upside of this simplification is that we are able to derive technically simple and economically intuitive results. An interesting extension, which we leave to future work, is to characterize the generally optimal contracts in our setting.

The moral hazard literature is vast, and we disregard many other forces analyzed in the literature, such as risk aversion [Holmström \(1979\)](#), dynamics, imperfect observability, and other forces.

6.2 Substantive Assumptions

Worker Knowledge. A core assumption of our model is that employees observe value-relevant information about the firm. This is consistent both with how public companies regulate employee trading, and with empirical evidence that worker-generated signals predict fundamentals and returns.

U.S. public firms must disclose whether they maintain insider-trading policies and procedures and file them with the Form 10-K (or explain non-adoption) under Item 408(b) of Regulation S-K, making such policies near-universal in practice among listed firms. These policies typically impose quarterly trading windows or blackouts (e.g., from late quarter-end until after earnings) and allow ad-hoc “special blackouts” around pending material events.

While directors and officers face stricter rule-based constraints,¹⁶ firm blackout policies routinely extend beyond executives to broad groups of employees likely to encounter material nonpublic information (MNPI).

There are also a number of empirical papers arguing that workers have private information about firms they work at. In firms with employee stock purchase plans (ESPPs), higher aggregate employee purchases predict future stock returns (Babenko and Sen, 2016). Employees’ stock option exercise decisions – distinct from executives’ exercise decisions – also predict returns (Huddart and Lang, 2003).

Employee-generated signals outside trading also forecast fundamentals and returns. Changes in crowdsourced employer reviews by current employees predict one-quarter-ahead earnings surprises and future stock returns (Green et al., 2019); broader measures of employee satisfaction correlate with long-run abnormal returns (Edmans, 2011) and, across countries, with future profitability and earnings surprises (Edmans et al., 2024).

Employee turnover is also negatively associated with future firm performance (Li et al., 2022). In our model, this could be driven both by selection – employees compensated with equity are more likely to leave if the firm is doing poorly – and causal effects – employees leaving further decreases the firm’s output.

Labor Market Power. We assume that firms have market power in labor markets. This is motivated by a large recent literature that finds evidence for quantitatively meaningful labor market markdowns: wages are set well below workers’ marginal revenue product to the firm, consistent with imperfect competition in labor markets (Berger, Herkenhoff and Mongey, 2022; Yeh, Macaluso and Hershbein, 2022; Lamadon, Mogstad and Setzler, 2022; Kroft et al., 2025). Two complementary findings which underpin this interpretation are that firm-specific labor-supply elasticities can be quite low (Dube et al., 2020); and that greater labor-market concentration is associated with lower wages (Azar, Marinescu and Steinbaum, 2022).

¹⁶Under SEC Rule 10b5-1 (as amended in 2022), directors and officers may not trade under a newly adopted or modified 10b5-1 plan until a cooling-off period has elapsed: the later of (i) 90 days after adoption/modification or (ii) two business days after the issuer files its next periodic report (Form 10-Q or 10-K), capped at 120 days. Under Sarbanes-Oxley 306, during a “pension plan blackout period” (when plan participants are temporarily restricted from trading issuer equity in an individual account plan), directors and executive officers are prohibited from trading the issuer’s equity securities outside the plan; issuers must give advance notice and file a Form 8-K announcing the blackout.

7 Conclusion

In this paper, we analyzed how workers' private information influences firm's equity financing. When informed workers are paid partially in equity, they have an incentive to leave the firm if managers shirk. Worker equity compensation thus partially delegates to workers the problem of monitoring the firm: the manager understands that shirking can induce workers to quit, and thus exerts increased effort to influence worker retention. Interestingly, private information also affects the level of worker compensation: the worker monitoring channel only works when workers are undercompensated relative to the monopsony-optimal wage, since only at this point does the manager lose surplus when workers leave. We thus provide a novel explanation for the common practice of compensating workers using equity, and we also introduce a new link between information frictions and the level of worker compensation.

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Appendix

A Proofs

A.1 Proof of Proposition 1:

We have:

$$\begin{aligned} & (s_M + s_I)(e + \Delta - \psi) - e \\ &= (s_M + s_I)(\Delta - \psi) - e(1 - s_M - s_I) \\ &= (1 - s_W)(\Delta - \psi) - es_W \end{aligned}$$

Now, using the definition of $\Gamma(e, \psi, s_W)$ in (5),

$$\begin{aligned} \Delta - \Gamma(e, \psi, s_W) &= \Delta - (\psi + s_W(e + \Delta - \psi)) \\ &= (\Delta - \psi)(1 - s_W) - es_W \end{aligned} \tag{47}$$

implying that:

$$(s_M + s_I)(e + \Delta - \psi) - e = \Delta - \Gamma(e, \psi, s_W). \tag{48}$$

Rearranging (48) slightly, we have:

$$s_M(e + \Delta - \psi) - \frac{s_M}{s_M + s_I}e = \frac{s_M}{s_M + s_I}(\Delta - \Gamma(e, \psi, s_W)) \tag{49}$$

Substituting (49) into (6), we can write the manager's payoff as:

$$\frac{s_M}{s_M + s_I}e + \frac{s_M}{s_M + s_I}F(\Gamma(e, \psi, s_W))(\Delta - \Gamma(e, \psi, s_W)) - c(e) - \theta(1 - I), \tag{50}$$

which becomes (12), using the definition of the extracted surplus $\Pi(\cdot)$ in (8). Similar substitution of (49) into (7) yields (13).

A.2 Proof of Proposition 3

For convenience in this proof, denote

$$k \equiv \frac{s_M}{s_M + s_I}.$$

Suppose a contract induces $\Gamma = \Gamma_{FW}^*$ as the outcome. Since $\Pi'(\Gamma^*) = 0$, the manager's effort level is determined by:

$$c'(e^*) = k(1 + \Pi'(\Gamma^*) s_W) = k, \quad (51)$$

which is independent of the workers' compensation contract $\{\psi, s_W\}$. Since k , e^* , and Γ_{FW}^* are fixed, it is easy to see that both the investors' payoff (13) and the manager's payoff in (23) are independent of the worker's compensation, proving the desired result.

A.3 Proof of Claim 1

If $s_M^* = 0$, condition (22) implies that manager's optimal effort $e^* = 0$. The payoff in (14) is therefore bounded by $\theta \max_{\Gamma} \Pi(\Gamma)$. This is implemented by granting the worker a fixed wage, and is therefore dominated by the optimal contract (19) in this case. Hence, $s_M^* > 0$ and $e^* > 0$.

Define $\kappa \equiv \frac{s_I}{s_M} \in [0, \infty)$. Condition (22) becomes

$$\frac{1}{1 + \kappa} [1 + \Pi'(\Gamma(e^*, \psi, s_W)) s_W] = c'(e^*) \quad (52)$$

and the ex-ante contract design problem (14) can be written as

$$\max_{s_W, \psi, \kappa} \left(\frac{1 - \theta}{1 + \kappa} + \theta \right) [e^* + \Pi(\Gamma(e^*, \psi, s_W))] - c(e^*). \quad (53)$$

We therefore need to show the optimal $\kappa^* > 0$.

Calculate the impact of κ on e^* , and totally differentiate (52) with respect to κ :

$$c''(e^*) \frac{\partial e^*}{\partial \kappa} = -\frac{1}{(1 + \kappa)^2} [1 + \Pi'(\Gamma(e^*, \psi, s_W)) s_W] + \frac{1}{1 + \kappa} \Pi''(\Gamma) s_W^2 \frac{\partial e^*}{\partial \kappa}.$$

Hence,

$$\frac{\partial e^*}{\partial \kappa} = \frac{-\frac{1}{(1 + \kappa)^2} [1 + \Pi'(\Gamma(e^*, \psi, s_W)) s_W]}{c''(e^*) - \frac{1}{1 + \kappa} \Pi''(\Gamma) s_W^2} < 0.$$

At $\kappa = 0$,

$$\left| \frac{\partial e^*}{\partial \kappa} \right|_{\kappa=0} = \frac{1 + \Pi'(\Gamma(e^*, \psi, s_W)) s_W}{c''(e^*) - \Pi''(\Gamma) s_W^2} \leq \frac{1 + \Pi'(\Gamma(e^*, \psi, s_W))}{c''(e^*)},$$

which is bounded since $c''(e)$ is bounded away from 0 for any e .

Next, consider the impact of κ on the manager's design objective around a neighbourhood

of $\kappa = 0$. Totally differentiate (53) with respect to κ :

$$-\frac{1-\theta}{(1+\kappa)^2} [e^* + \Pi(\Gamma(e^*, \psi, s_W))] + \left[\left(\frac{1-\theta}{1+\kappa} + \theta \right) [1 + \Pi'(\Gamma) s_W] - c'(e^*) \right] \frac{\partial e^*}{\partial \kappa}.$$

Using (52) to plug in the expression for $c'(e^*)$, the total derivative becomes

$$-\frac{1-\theta}{(1+\kappa)^2} [e^* + \Pi(\Gamma(e^*, \psi, s_W))] + \frac{\theta\kappa}{1+\kappa} [1 + \Pi'(\Gamma) s_W] \frac{\partial e^*}{\partial \kappa}.$$

Evaluating around $\kappa = 0$, the total derivative is

$$\frac{\theta-1}{(1+\kappa)^2} [e^* + \Pi(\Gamma(e^*, \psi, s_W))] > 0.$$

Hence, a marginal increase in κ over 0 strictly improves the manager's payoff and therefore $\kappa^* > 0$.

A.4 Proof of Claim 2

A.4.1 There exists an equity-only contract $\tilde{s}_W$ which implements Γ_{FW}^*

Define $\tilde{e}$ as the unique solution to (51) in the proof of Proposition 3, which is the optimal managerial effort when workers receive Γ_{FW}^* .

By definition, $\Gamma_{FW}^* \leq \Delta$, otherwise $\Pi(\Gamma_{FW}^*) < \Pi(\Delta) = 0$, which is clearly suboptimal. Therefore, we can construct the workers' equity contract as

$$\tilde{s}_W \equiv \frac{\Gamma_{FW}^*}{\tilde{e} + \Delta} < 1 \tag{54}$$

From (5), since the fix wage is zero, total compensation for workers is

$$\Gamma = \tilde{s}_W (\tilde{e} + \Delta) = \Gamma_{FW}^*.$$

Therefore, the constructed $\tilde{s}_W$ implements Γ_{FW}^* .

A.4.2 A small deviation from $\tilde{s}_W$ to $\tilde{s}_W - \delta$ increases $\tilde{e}$

Consider a marginal reduction of $\tilde{s}_W$ in a full-equity contract. Using the definition of Γ in (5), we can rewrite the manager's effort FOC (22) in this case as:

$$c'(e) = k(1 + \Pi'(s_W(e + \Delta))s_W) \quad (55)$$

We want to show that the solution e as a function of s_W satisfies

$$\frac{\partial e}{\partial s_W} \Big|_{s_W = \tilde{s}_W} < 0.$$

To do this, we apply the implicit function theorem to (55) at the contract $\tilde{s}_W$. Defining:

$$\Lambda \equiv c'(e) - k(1 + \Pi'(s_W(e + \Delta))s_W)$$

$$\frac{\partial \Lambda}{\partial e} = c''(e) - k s_W^2 \Pi''(s_W(e + \Delta))$$

$$\frac{\partial \Lambda}{\partial s_W} = -[k(\Pi'(\tilde{s}_W(\tilde{e} + \Delta)) + \tilde{s}_W(\tilde{e} + \Delta)\Pi''(\tilde{s}_W(\tilde{e} + \Delta)))]$$

where $\tilde{e}$ is defined in (51). Now, we have by assumption:

$$\Pi'(\tilde{s}_W(\tilde{e} + \Delta)) = \Pi'(\Gamma_{FW}^*) = 0$$

Thus, we have:

$$\frac{\partial e}{\partial s_W} \Big|_{s_W = \tilde{s}_W} = \frac{k\tilde{s}_W(\tilde{e} + \Delta)\Pi''(\tilde{s}_W(e + \Delta))}{c''(e) - k\tilde{s}_W^2\Pi''(\tilde{s}_W(e + \Delta))} \quad (56)$$

Now, we assumed Π'' is everywhere negative and c'' is everywhere positive, and k , $\tilde{s}_W$ and $\tilde{e} + \Delta$ are everywhere positive, so we have shown that $\frac{\partial e}{\partial s_W} \Big|_{s_W = \tilde{s}_W} < 0$.

A.4.3 A small deviation from $\tilde{s}_W$ to $\tilde{s}_W - \delta$ increases $\mathcal{M}$

Differentiating (23) with respect to s_W , we have:

$$\frac{d\mathcal{M}}{ds_W} : \left[\frac{s_M + \theta s_I}{s_M + s_I} - c'(\tilde{e}) \right] \frac{\partial e}{\partial s_W} + \frac{s_M + \theta s_I}{s_M + s_I} \Pi'(\Gamma(\tilde{s}_W)) \frac{\partial \Gamma}{\partial s_W} \Big|_{\Gamma = \Gamma_{FW}^*} \quad (57)$$

But, since $\Gamma(\tilde{s}_W) = \Gamma_{FW}^*$ by definition of $\tilde{s}_W$, we have $\Pi'(\Gamma(\tilde{s}_W)) = 0$, hence the second term vanishes.

In contrast, a small downwards deviation from $\tilde{s}_W$ unambiguously increases the manager

surplus component of $\mathcal{M}$, because effort is always too low due to the manager's commitment problem. To see that, substituting the manager's FOC at $\tilde{s}_W$, (51), into (57), and using that $\Pi'(\Gamma(\tilde{s}_W)) = 0$, we have:

$$\frac{d\mathcal{M}}{ds_W} \Big|_{s_W=\tilde{s}_W} = \left[\frac{s_M + \theta s_I}{s_M + s_I} - \frac{s_M}{s_M + s_I} \right] \frac{\partial e}{\partial s_W} \Big|_{s_W=\tilde{s}_W} = \left[\frac{\theta s_I}{s_M + s_I} \right] \frac{\partial e}{\partial s_W} \Big|_{s_W=\tilde{s}_W}$$

Clearly, $\frac{\theta s_I}{s_M + s_I} > 0$. Moreover, we showed in Appendix A.4.2 above that $\frac{\partial e}{\partial s_W} \Big|_{s_W=\tilde{s}_W} < 0$: marginally decreasing s_W increases manager effort. Thus, we have shown that

$$\frac{d\mathcal{M}}{ds_W} \Big|_{s_W=\tilde{s}_W} < 0$$

and that a small deviation from $\tilde{s}_W$ to $\tilde{s}_W - \delta$ increases $\mathcal{M}$.

A.5 Proof of Proposition 4

First, it is clear that the optimal $s_W^* < 1$. Otherwise, $s_W^* = 1$ implies that $s_M^* = s_I^* = 0$, contradicting Claim 1.

Second, we show $s_W^* > 0$. Otherwise, if $s_W^* = 0$, then the worker is on fixed wage contract. The optimal contract in this case is given by $\psi = \Gamma_{FW}^*$. Proposition 3 implies that this contract generates identical outcomes to the pure equity contract $\tilde{s}_W$ in Claim 2, which we showed is suboptimal. Hence, $s_W^* \in (0, 1)$.

Third, we show $\Gamma^* < \Gamma_{FW}^*$. By the same logic as above, all contracts generating $\Gamma^* = \Gamma_{FW}^*$ have identical outcomes. We next show that all contracts generating $\Gamma > \Gamma_{FW}^*$ are strictly dominated by the fixed-wage contract of $\hat{\psi} = \Gamma_{FW}^*$ and $\hat{s}_W = 0$. Consider a contract s_M, s_W, ψ that induces some $\tilde{\Gamma} > \Gamma_{FW}^*$. From (11), we know $\Pi'(\tilde{\Gamma}) < 0$.

From (23), the manager's payoff can be written as:

$$\mathcal{M} = \underbrace{\frac{s_M + \theta s_I}{s_M + s_I} e^* - c(e^*)}_{\text{Manager Surplus}} + \frac{s_M + \theta s_I}{s_M + s_I} \Pi(\Gamma(e^*, \psi, s_W)) \quad (58)$$

where we suppress the arguments in $e^*(s_M, s_W, \psi)$ and $\Gamma(e^*, \psi, s_W)$ for notational simplicity.

From (22) of Proposition 2, the manager's effort $\tilde{e}$ under the contract we are considering satisfies:

$$c'(\tilde{e}) = \frac{s_M}{s_M + s_I} \left(1 + \Pi'(\tilde{\Gamma}) s_W \right) < \frac{s_M}{s_M + s_I} < \frac{s_M + \theta s_I}{s_M + s_I} \quad (59)$$

Now, consider $\hat{\psi} = \Gamma_{FW}^*$. Since $\Pi'(\Gamma_{FW}^*) = 0$, the manager's effort $\hat{e}$ under the new fixed-wage

contract is determined by:

$$c'(\hat{e}) = \frac{s_M}{s_M + s_I} < \frac{s_M + \theta s_I}{s_M + s_I} \quad (60)$$

Comparing (59) and (60), clearly $\tilde{e} < \hat{e}$, and furthermore both are below the value of e which maximizes the manager surplus term in (58), which satisfies:

$$c'(e^*) = \frac{s_M + \theta s_I}{s_M + s_I}$$

Thus, concavity of the manager surplus term implies that the manager surplus term in (58) is higher under $\hat{e}$ than $\tilde{e}$, that is:

$$\frac{s_M + \theta s_I}{s_M + s_I} \hat{e} - c(\hat{e}) > \frac{s_M + \theta s_I}{s_M + s_I} \tilde{e} - c(\tilde{e})$$

By definition, the last term in (58) is also clearly higher under the new contract:

$$\Pi(\Gamma_{FW}^*) > \Pi(\tilde{\Gamma})$$

Thus, both terms of (58) are higher under the new contract, and the manager strictly prefers the new contract. This completes the proof of $\Gamma^* < \Gamma_{FW}^*$ and therefore $\Pi'(\Gamma^*) > 0$.

Finally, we show $\psi^* = 0$. Suppose otherwise that $\psi^* > 0$. We shall construct another contract with $\psi^\dagger = \psi^* - \epsilon$ that generates higher payoff to the manager. In fact, we show that this contract is a Pareto improvement for both the manager and workers. As a notational convention, we denote by variables with $*$ the ones under the conjectured optimal contract and those with $\dagger$ the ones under the newly constructed contract. Define $s_W^\dagger$ such that

$$\Gamma(e^*, \psi^\dagger, s_W^\dagger) = s_W^\dagger(e^* + \Delta - \psi^* + \epsilon) + \psi^* - \epsilon = s_W^*(e^* + \Delta - \psi^*) + \psi^* = \Gamma(e^*, \psi^*, s_W^*).$$

Hence

$$s_W^\dagger = \frac{s_W^*(e^* + \Delta - \psi^*) + \epsilon}{(e^* + \Delta - \psi^* + \epsilon)} = s_W^* + \frac{(1 - s_W^*)\epsilon}{(e^* + \Delta - \psi^* + \epsilon)} > s_W^*.$$

It is possible to choose a small ϵ such that $\psi^\dagger > 0$ and $s_W^\dagger < 1$ hold. Define

$$s_M^\dagger = s_M^* \frac{1 - s_W^\dagger}{1 - s_W^*} \text{ and } s_I^\dagger = s_I^* \frac{1 - s_W^\dagger}{1 - s_W^*}$$

that maintain the equity ratio between the manager and the investor: $s_M^\dagger/s_I^\dagger = s_M^*/s_I^*$. Using

the fact that $\Pi'(\Gamma(e^*, \psi^*, s_W^*)) > 0$, we have

$$\frac{s_M^\dagger}{s_M^\dagger + s_I^\dagger} \left[1 + \Pi'(\Gamma(e^*, \psi^\dagger, s_W^\dagger)) s_W^\dagger \right] = \frac{s_M^*}{s_M^* + s_I^*} \left[1 + \Pi'(\Gamma(e^*, \psi^*, s_W^*)) s_W^* \right] > \frac{s_M^*}{s_M^* + s_I^*} [1 + \Pi'(\Gamma^*) s_W^*] = c'(e^*).$$

Hence, the managerial effort under $\dagger$ contract is higher: $e^\dagger \geq e^*$. From (22) and $\Pi' > 0$, we have

$$\frac{s_M^* + \theta s_I^*}{s_M^* + s_I^*} [1 + \Pi'(\Gamma^*) s_W] > \frac{s_M^*}{s_M^* + s_I^*} [1 + \Pi'(\Gamma^*) s_W] = c'(e^*).$$

Comparing the managerial payoff under the two contracts and using the fact that $e^\dagger > e^*$, we have

$$\frac{s_M^\dagger + \theta s_I^\dagger}{s_M^\dagger + s_I^\dagger} \left[e^\dagger + \Pi(\Gamma(e^\dagger, \psi^\dagger, s_W^\dagger)) \right] - c(e^\dagger) > \frac{s_M^* + \theta s_I^*}{s_M^* + s_I^*} \left[e^* + \Pi(\Gamma(e^*, \psi^\dagger, s_W^\dagger)) \right] - c(e^*) = \frac{s_M^* + \theta s_I^*}{s_M^* + s_I^*} [e^* + \Pi(\Gamma^*)] - c(e^*),$$

where the last equality follows from the construction of the $\dagger$ contract. The contradiction with optimality of the $*$ contract implies that $\psi^* = 0$.

Finally, again since $e^\dagger > e^*$, workers' payoffs also strictly increase under the $\dagger$ contract:

$$\Gamma(e^\dagger, \psi^\dagger, s_W^\dagger) > \Gamma(e^*, \psi^\dagger, s_W^\dagger) = \Gamma(e^*, \psi^*, s_W^*).$$

A.6 Proof of Proposition 5

First, note that, with $k \equiv \frac{s_M}{s_M + s_I}$, we have:

$$\frac{s_M + \theta s_I}{s_M + s_I} = \theta + (1 - \theta) k \quad (61)$$

We differentiate manager surplus $\mathcal{M}$ in (23) with respect to s_W and set to zero, holding the ratio $\frac{s_M}{s_M + s_I}$ fixed:

$$\frac{\partial}{\partial s_W} : [(\theta + (1 - \theta) k) - c'(e^*(k, s_W))] \frac{de^*}{ds_W} + (\theta + (1 - \theta) k) \Pi'(\Gamma(e^*(k, s_W), s_W)) \frac{d\Gamma}{ds_W}$$

where, with slight abuse of notation, we use $\frac{d\Gamma}{ds_W}$ to denote:

$$\frac{d\Gamma}{ds_W} \equiv \frac{d}{ds_W} \Gamma(e^*(k, s_W), s_W)$$

This is (24). Now, using (5), note that for full-equity contracts, $\psi = 0$, we have:

$$\Gamma(e, s_W) = s_W(e + \Delta) \quad (62)$$

which implies:

$$\frac{d}{ds_W} \Gamma = \frac{d}{ds_W} (s_W(e^*(k, s_W) + \Delta)) = e^*(k, s_W) + \Delta + s_W \frac{de^*}{ds_W}$$

This is (25). Finally, we can calculate $\frac{de^*}{ds_W}$ by applying the implicit function theorem to the manager's effort FOC. Using (62), we can write (22) as:

$$\Lambda = c'(e) - \frac{s_M}{s_M + s_I} (1 + \Pi'(s_W(e + \Delta)) s_W) = 0 \quad (63)$$

We then have:

$$\begin{aligned} \frac{\partial \Lambda}{\partial e} &= c''(e) - \frac{s_M}{s_M + s_I} \Pi''(s_W(e + \Delta)) s_W^2 \\ \frac{\partial \Lambda}{\partial s_W} &= -[k(\Pi'(s_W(e + \Delta)) + s_W(e + \Delta) \Pi''(s_W(e + \Delta)))] \end{aligned}$$

From which we get:

$$\frac{de^*}{ds_W} = \frac{k(\Pi'(s_W(e^*(k, s_W) + \Delta)) + s_W(e^*(k, s_W) + \Delta) \Pi''(s_W(e^*(k, s_W) + \Delta)))}{c''(e) - k\Pi''(s_W(e + \Delta)) s_W^2}$$

this is (26).

A.7 Proof of Proposition 6

We differentiate manager surplus $\mathcal{M}$ in (23) with respect to k , to get:

$$\begin{aligned} \frac{\partial \mathcal{M}}{\partial k} = & \underbrace{(1 - \theta) [e^*(k, s_W) + \Pi(\Gamma(e^*(k, s_W), s_W))]}_{\text{Financing}} + \\ & \underbrace{\frac{\partial e^*}{\partial k} [(\theta + (1 - \theta)k) - c'(e^*(k, s_W))]}_{\text{Manager Effort}} + \\ & \underbrace{(\theta + (1 - \theta)k) \Pi'(\Gamma(e^*(k, s_W), s_W)) \frac{\partial \Gamma}{\partial e^*} \frac{\partial e^*}{\partial k}}_{\text{Worker Profit}} \quad (64) \end{aligned}$$

where we used (61) in Appendix A.6 to express $\frac{s_M + \theta s_I}{s_M + s_I}$ in terms of k . Intuitively, (64) has 3 terms: shifting k impacts financing I , manager effort through e^* , and worker profit through Π . We can simplify slightly further by noting that $\frac{\partial \Gamma}{\partial e^*} = s_W$, implying that the worker profit term can be written as:

$$(\theta + (1 - \theta)k) \Pi'(\Gamma(e^*(k, s_W), s_W)) s_W \frac{\partial e^*}{\partial k} \quad (65)$$

We can eliminate the worker profit term in (64), intuitively, by shifting s_W together with k to hold Π fixed. Technically, this involves substituting the s_W FOC, (24) of Proposition (5), into the k FOC in (64). First, we apply the implicit function theorem to the manager's effort FOC, (63) in Appendix A.6, to calculate $\frac{de^*}{dk}$:

$$\frac{\partial \Lambda}{\partial e} = c''(e) - \frac{s_M}{s_M + s_I} \Pi''(s_W(e + \Delta)) s_W^2$$

$$\frac{\partial \Lambda}{\partial k} = 1 + \Pi'(s_W(e + \Delta)) s_W$$

Hence we have:

$$\frac{\partial e^*}{\partial k} = \frac{\frac{\partial \Lambda}{\partial k}}{\frac{\partial \Lambda}{\partial e}} = \frac{1 + \Pi'(s_W(e + \Delta)) s_W}{c''(e) - k \Pi''(s_W(e + \Delta)) s_W^2} \quad (66)$$

This gives (28). Now, note that we can write (24) of Proposition 5 as:

$$\begin{aligned} [(\theta + (1 - \theta)k) - c'(e^*(k, s_W))] \frac{\partial e^*}{\partial s_W} = \\ - (\theta + (1 - \theta)k) \Pi'(\Gamma(e^*(k, s_W), s_W)) \left(e^*(k, s_W) + \Delta + s_W \frac{\partial e^*}{\partial s_W} \right) \end{aligned}$$

We can multiply both sides by:

$$\frac{s_W \frac{\partial e^*}{\partial k}}{e^*(k, s_W) + \Delta + s_W \frac{\partial e^*}{\partial s_W}}$$

To get:

$$\begin{aligned} (\theta + (1 - \theta)k) \Pi'(\Gamma(e^*(k, s_W), s_W)) s_W \frac{\partial e^*}{\partial k} = \\ - [(\theta + (1 - \theta)k) - c'(e^*(k, s_W))] \frac{\partial e^*}{\partial s_W} \frac{s_W \frac{\partial e^*}{\partial k}}{e^*(k, s_W) + \Delta + s_W \frac{\partial e^*}{\partial s_W}} \end{aligned} \quad (67)$$

Now, the LHS of (67) is identical to the worker profit term (65) of the manager's FOC with respect to k . We can thus substitute (67) in to (64), getting:

$$\begin{aligned} \frac{\partial \mathcal{M}}{\partial k} : (1 - \theta) [e^*(k, s_W) + \Pi(\Gamma(e^*(k, s_W), s_W))] + \\ \frac{\partial e^*}{\partial k} [(\theta + (1 - \theta)k) - c'(e^*(k, s_W))] - \\ [(\theta + (1 - \theta)k) - c'(e^*(k, s_W))] \frac{\partial e^*}{\partial s_W} \frac{s_W \frac{\partial e^*}{\partial k}}{e^*(k, s_W) + \Delta + s_W \frac{\partial e^*}{\partial s_W}} \end{aligned}$$

This allows us to group the last two terms, into:

$$\begin{aligned} \frac{\partial \mathcal{M}}{\partial k} : (1 - \theta) [e^*(k, s_W) + \Pi(\Gamma(e^*(k, s_W), s_W))] + \\ [(\theta + (1 - \theta)k) - c'(e^*(k, s_W))] \left[\frac{\partial e^*}{\partial k} - \frac{\partial e^*}{\partial s_W} \frac{s_W \frac{\partial e^*}{\partial k}}{e^*(k, s_W) + \Delta + s_W \frac{\partial e^*}{\partial s_W}} \right] \end{aligned}$$

This gives (27).

A.8 Proof of Claim 5

The optimal effort is given by (41) is straightforwardly calculated from the first order condition

$$e_i^* = s_i.$$

Therefore the design objective (40) becomes

$$\max_{s_i} s_0 + \frac{\sum_{i=1}^N s_i}{N} - \frac{1}{2} (s_0)^2 - \frac{1}{2N} \sum_{i=1}^N s_i^2.$$

subject to (42). Denote by $\frac{\lambda}{N}$ the Lagrangian multiplier on this constraint. First order conditions with respect to s_i are as follows:

$$N - Ns_0^* - \lambda = 0$$

and

$$1 - s_i^* - \lambda = 0.$$

Hence, $s_i^* = 1 - \lambda$ and $s_0^* = 1 - \frac{\lambda}{N}$. Plugging into the constraint (42) yields

$$N - N\lambda + 1 - \frac{\lambda}{N} = 1.$$

Solve for λ :

$$\lambda = \frac{N}{N + \frac{1}{N}}.$$

Hence, $s_0^* = 1 - \frac{1}{N + \frac{1}{N}}$ and $s_i^* = \frac{\frac{1}{N}}{N + \frac{1}{N}}$.