What is a Central Bank Worth?

Preliminary and Incomplete

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Abstract

About 5.5% of GDP in modern times, much less than prior estimates for the present value of seigniorage. I adapt the methodology of Jiang, Lustig, Van Nieuwerburgh & Xiaolan (2024) to develop an asset pricing model that prices the Bank of England (BoE), excluding quantitative easing. I leverage the fact that the BoE was publicly traded until 1946, using the observed price to assess the accuracy of the model. The model's relatively modest errors stem from the strong mean reversion embedded in the VAR that forecasts the BoE's dividends. Having a reliable estimate for the value of the central bank since the end of WWII, I assess its contribution to the government's fiscal capacity. On average the central bank has accounted for 8.5% of the market value of gross government debt. This share increases with inflation, peaking at 21.3% in 1975.

Note for AFA Poster session: Ongoing work extends the benchmark exercise by pricing the payoffs from QE and applying the framework to more than 40 central banks.

Keywords: Central Bank Valuation, Seigniorage, Monetary History, Bank of England, Fiscal Capacity.

JEL Classifications: E5, E58, E6, E63, G12, N10, N13, N14.

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1 Introduction

Central banks are losing money as a result of quantitative easing, leading to the suspension of remittances to the government and, in some cases, to the recapitalization of the central bank. At the same time, the decline of cash could erode central banks' seigniorage revenue (Bell et al. 2023), while the rise of private digital money could reallocate it to the private sector (Brunnermeier & Niepelt 2019, BIS 2023). Against this backdrop, politicians have proposed abolishing interest on reserves (Financial Times 2025),[1] while legal and accounting academics have proposed reclassifying banknotes and reserves as "social equity" to lower the measure of public sector net debt (Kumhof et al. 2020, Bossone & Costa 2021). In advanced economies central bank distributions to the treasury hover around 0.40% of GDP per year (Bell et al. 2023). This paper attempts to estimate their present value and assess how much central bank profits contribute to the government's fiscal capacity.

To value the central bank, I adapt the methodology developed by Jiang, Lustig, Van Nieuwerburgh & Xiaolan (2024) to value government debt. Focusing on the post-WWII period, Jiang et al. (2024) price the U.S. government debt based on its expected primary surpluses. They find that the model-implied price is drastically different from the one observed in the market, and as a result conclude that U.S. Treasuries may be overpriced. In a companion paper they take a centuries-long view, studying the fiscal histories of the Netherlands, the U.K., and the U.S. (Chen et al. 2025); they find that when a country is the dominant safe asset supplier, it can issue more debt than is justified by its expected primary surpluses.

Rather than comparing model-implied valuations against the observed debt prices to study the "exorbitant privilege," I leverage the fact that in the past many central banks were publicly traded. Focusing on the Bank of England, which was publicly traded until 1946, I use the historically observed price of the bank to assess the accuracy of the asset pricing model, allowing me to estimate what the central bank is worth today. I do not test a present value identity and in turn I do not speak to the literature on government debt valuation puzzles, bubble terms or incomplete markets (Brunnermeier et al. 2024, Collin-Dufresne et al. 2024).

To be concrete about the goal of the paper, consider the consolidated government budget constraint of Sargent & Wallace (1981):[2]

$$G_t - T_t \;=\; \frac{M_t - M_{t-1}}{P_t} \;+\; B_t^{\text{cons}} \;-\; (1 + r_{t-1})\, B_{t-1}^{\text{cons}}.$$

Where G_t and T_t are spending and taxes, M_t is the money supply, B_t is government debt, P_t is the price level, and r_t is the real interest rate. Assuming a transversality condition, one can show that the present value of consolidated government debt is:

$$B_0^{\text{cons}} = \sum_{j=1}^{\infty} \left(\prod_{k=0}^{j-1} \frac{1}{1 + r_k} \right) (T_j - G_j + \frac{M_j - M_{j-1}}{P_j}).$$

[1] See Cochrane (2025) for a critique. Central banks bought long-term, low-yielding, assets and financed them with short-term interest-bearing reserves. As policy rates rose in 2022 central banks had to pay higher interests on these reserves, while their income from the long-term assets remained unchanged.

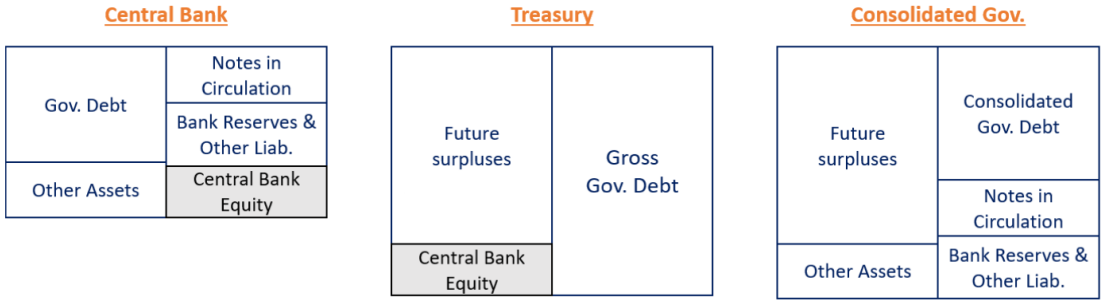
[2] See Appendix B for the brief derivation.

Where the real change in money balances ($\frac{M_j - M_{j-1}}{P_j}$) is, for Sargent & Wallace (1981), seigniorage revenue.[3] One can however consider a budget constraint which does not consolidate the central bank and in which government debt is financed with taxes and the profits from the central bank (Π_t^{CB}). The value of the gross government debt then follows:

$$B_0^{\text{gross}} = \sum_{j=1}^{\infty} \left(\prod_{k=0}^{j-1} \frac{1}{1+r_k} \right) (T_j - G_j + \Pi_j^{CB})$$

Figure 1 makes this point graphically. Although they do not disentangle the central bank's profits from the rest of government revenues, the asset pricing model of Jiang et al. (2024) values gross government debt (B_0^{gross}). The goal then is to decompose the value of the gross government debt and estimate how much of it can be attributed to the value of the central bank.

Figure 1 Illustrative consolidation of the government



Note: The treasury issues debt against its future primary surpluses and the central bank's profits. The central bank issues currency and takes in commercial bank reserves, and holds government debt and other assets.

The methodology involves two steps: (1) forecasting the payoff stream associated with the central bank and (2) discounting it. On the payoff stream, I assume that the central bank should be priced based on its dividends. Keeping in mind Shiller's (1981) excess volatility puzzle this may seem heroic, but it is defensible for two reasons. First, during the publicly traded period the only information available to shareholders were dividend payouts and the weekly balance sheet (Clapham 1946, Horsefield 1953). Second, during the nationalized period, the BoE is peculiar in that it has distinctively little discretion about remittances to the treasury; this renders the problem of central bank dividends being highly artificial (Schwarz et al. 2015, Goncharov & Novotny-Farkas 2022) much less worrisome. From bondholders' perspective, the relevant payoff is the stream of dividends to the treasury: the flows that reach the national budget. I only price the direct payoff streams associated with the bank, I do not price any other benefits like reductions in treasury yields resulting from monetary policy, the effects of such falls in yields on government revenue and spending, or broader benefits to the macroeconomy. I do not price the payoffs from unconventional monetary policy, hoping to tackle its pricing in future work.

To generate the forecast, I implement a vector autoregression model for the state variables as in Campbell (1996) which includes the growth rate and level of the BoE's dividends-to-GDP and assets-to-GDP, along with

[3] In contrast, today central banks typically describe seigniorage as the income (net of production costs) earned on assets funded through currency issuance (Bell et al. 2023, BoE 2025*a*). This is closer to the "stock" or "opportunity cost" definition of seigniorage (Phelps 1973, Klein & Neumann 1990). Reich (2017) overviews the history of the different definitions. Jiang et al. (2024) use the term to describe convenience yields on the government debt.

the same macroeconomic and financial variables as the VAR in Jiang et al. (2024). I introduce a structural break to distinguish two periods: 1845–1945, when the Bank of England held the monopoly right to issue money, conducted monetary policy, and was privately owned, and 1946–2024, when the bank was nationalized.

To discount the cash flows I implement the same two approaches as Jiang et al. (2024). First, I value the bank through Campbell-Shiller decompositions, assuming that the risk premium of the central bank is equal to that of a GDP claim, and that the risk premium of a GDP claim is equal to that of unlevered market equity (Abel 1999, Campbell 2003). By the cointegration of central bank dividends and GDP, in the long run the central bank inherits GDP's long run risk (Bansal & Yaron 2004, Alvarez & Jermann 2005, Hansen & Scheinkman 2009). Second, I take a stance on the stochastic discount factor. I closely follow the specification and estimation of Jiang et al. (2024): a no-arbitrage SDF whose log is linear in the innovations of the VAR's state variables, and whose market prices of risks are estimated to match bond and stock prices.[4] For the nationalized period this entails the fundamental assumption that the SDF that prices assets in the market is the same as that of the government; d'Avernas et al. (2024) make a similar assumption when studying the fiscal costs of QE.

Under the benchmark model based on Campbell-Shiller decompositions, I obtain that, in the 1946-2024 period, the Bank of England has been worth an average of 5.5% of GDP. This is much lower than prior estimates proposed by the literature, most notably with Del Negro & Sims (2015) estimating it to be 94% of GDP, and quantifies concerns about central banks losing seigniorage revenue due to the decline of cash or the rise of private digital money. With the fully-fledged asset pricing model and its SDF, I obtain that during the same period the bank was worth an average of 43.1% of GDP.

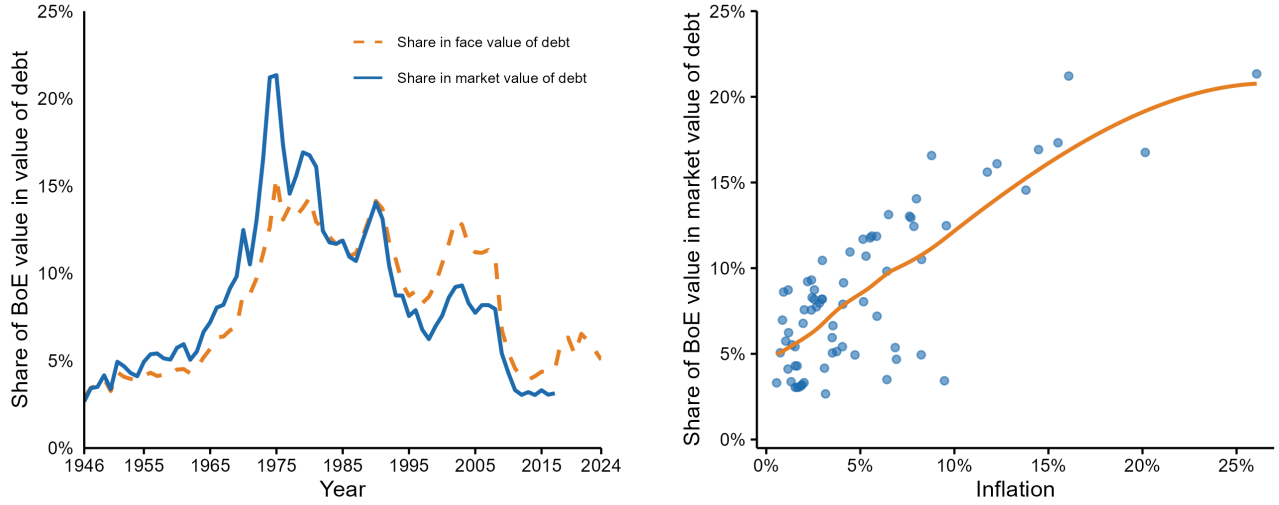
Figure 2 is the main result of the paper, plotting the time series of the share of the market value of gross government debt attributable to the value of the central bank, along with this share's relation with inflation. It is based on the benchmark valuation exercise, and provides an answer to Reis' (2019) question: can the central bank alleviate fiscal burdens? Between 1946-2024, the BoE on average accounted for 8.5% of the market value of government debt. This share increases with inflation, peaking at 21.3% in 1975.

Related Literature: This paper seeks to bring the methods of the literature taking an asset pricing perspective to study fiscal capacity (Jiang et al. 2023) to the literature underlining the importance of separating the budget constraints of the central bank and the treasury.

Although my focus is on central bank profitability, the closest related literature is the work trying to measure and value seigniorage (understood as the real change in money balances). Although a long literature going back to Cagan (1956) has tried to estimate the seigniorage "Laffer curve" (Easterly et al. 1995, Fischer et al. 2002, Benati et al. 2021), few contributions have tried to arrive at an estimate of its present value. In a Ramsey economy, Del Negro & Sims (2015) estimate the present value of seigniorage to be 94% of GDP; Reis (2015*a*) however showed how their estimate is sensitive to the calibration and the functional form assumptions about the money demand function, and that with the same model one could arrive at an estimate of 2-12% of GDP. Hilscher et al.

[4] A no-arbitrage model for the SDF as in Duffie & Kan (1996), Dai & Singleton (2000) Ang & Piazzesi (2003). Lustig, Hanno and Van Nieuwerburgh, Stijn and Verdelhan, Adrien (2013) and Gupta & Van Nieuwerburgh (2021) respectively use a similar combination with a VAR to study a claim to aggregate consumption and the performance of private equity funds.

Figure 2 Share of the value of the BoE in the value of U.K. government debt



Note: The left panel reports the BoE's share in the face value and market value of government debt over time. The right panel plots the BoE's share in the market value of government debt against inflation, along with a non-parametric locally weighted regression fit. The value is based on Campbell-Shiller decompositions and leverages a constant risk premium on the central bank of 3.77%. Chen et al. (2025) find no government debt valuation puzzle in this period of U.K. history. I lack data on the market value of government debt in 2018-2024.

(2015)[5] estimate the market value of the Federal Reserve to be 18-22% of GDP by forecasting seigniorage revenues as a function of inflation, and building the SDF through option contracts on inflation, as in Hilscher et al. (2022). In back of the envelope exercises, Fry (1992) and Buiter & Rahbari (2013) respectively estimate the present value of seigniorage to be 38% and 20% of GDP.

Several papers have taken a stress testing approach to study central bank solvency. Greenlaw et al. (2013), Cavallo et al. (2019) and Christensen et al. (2015) presented statistical projections for the Fed's balance sheet, with the first two of these correctly anticipating a stop to remittances. Using a dynamic term structure model to estimate the interest-rate risk of the Fed, Carpenter et al. (2015) deemed large losses unlikely. More recently d'Avernas et al. (2024) take a public finance approach, quantifying QE's fiscal costs as the increase in expected deadweight losses from future taxation, finding them small relative to standard estimates of output gains. Fujiki & Tomura (2017) and Debrun et al. (2021) present projections for the Bank of Japan and the Eurosystem. Reis (2015*b*) studies different definitions of central bank insolvency, arguing the most important is whether the present value of its dividends to the treasury is non-negative. Hall & Reis (2015) focus on different dividend distribution policies and whether they can avoid an explosive path for bank reserves.

Although this paper is mainly a measurement exercise, it hopes to contribute to the section of the literature focusing on whether profitability concerns can hamper monetary policy. Several contributions have shown that without fiscal support solvency concerns can threaten a central bank's control of inflation: Sims (2005), Del Negro & Sims (2015) and Park (2015) do so with the fiscal theory of the price level, Berriel & Bhattarai (2009) with a New Keynesian model, and Bassetto & Messer (2013), Benigno (2020), and Benigno & Nisticò (2020) with cash-in-advance and endowment monetary economies. Jeanne & Svensson (2007), Berriel et al. (2022) and Bhattarai et al. (2023) leverage balance sheet concerns to generate a signaling channel of monetary policy.

[5]This is an unpublished manuscript, the results are presented in Reis (2015*b*) and Reis' (2015*a*) comment to Del Negro & Sims (2015).

Understandably, there have been few attempts to investigate this question empirically. Most recently and comprehensively, Goncharov et al. (2023) report that central banks are discontinuously more likely to report slightly positive profits than slightly negative losses, and that this propensity correlates with lower interest rates and higher inflation. Adler et al. (2016) document that weakly capitalized central banks tend to have larger (negative) policy rate deviations from the "optimal" Taylor-rule implied rates. Ize (2007), Klüh & Stella (2008), Perera et al. (2013), and Pinter (2018) find similar (descriptive) results, though Benecká et al. (2012), Hampl & Havranek (2020) and Bell et al. (2024) question the robustness of the relationship. Archer & Moser-Boehm (2013), Bunea et al. (2016), and Long & Fisher (2024) provide overviews of central banks' profit distribution, recapitalization, and loss coverage rules, as well as of their accounting policies. Wessels & Broeders (2022) provides an overview of different measures of a central bank's capital adequacy.

Lastly, this paper contributes to the quantitative history of central banking using financial statement data. Ferguson et al. (2025) compiled a four-century long dataset to argue that balance sheet expansions were initially associated with government finance in geopolitical emergencies, but over time liquidity provision became their key driver. With a monthly dataset starting in the late 19th century, Bazot et al. (2024) show that central banks can partially offset international interest rate shocks through balance sheet expansions. Humann et al. (2025) document how the profitability of major central banks increased during the Great Inflation of the 1970s and 1980s. Abildgren (2017), Manas (2019), Martín-Aceña et al. (2020), Anson & Capie (2022), and Grodecka-Messi et al. (2025) respectively study the historical profitability of the central banks of Denmark, France, Spain, the UK, and Sweden, with the last finding that exogenous shocks to the Swedish central bank's profitability resulted in more accommodative monetary policy and higher inflation.

2 History and Data

I use annual data from 1845 to 2024, starting with the first full year after the Bank Charter Act of 1844, which granted the Bank of England the monopoly right over banknote issuance.[6] The BoE conducted lender of last resort operations from the beginning of the sample (Bagehot 1873, Anson et al. 2017), and was the government's banker since its establishment in 1694. I leverage three main datasets produced by the Bank of England or its staff: "A millennium of Macroeconomic Data for the UK" (Thomas & Dimsdale 2017), "Annual data on the Bank of England's balance sheet" (BoE 2020), and Anson & Capie (2022)'s data on the profitability of the BoE. These are complemented by data from the Office of National Statistics, Global Financial Data,[7] Ellison & Scott (2020), Müller et al. (2025), and Bordo et al. (2025). To update the balance sheet and profitability data to 2024 I relied on the BoE's published annual accounts. Details on data construction are in Appendix A.

[6] Technically, the 1844 Bank Charter Act did not immediately eliminate the notes issued by other banks. Instead, it froze their issuance at existing levels and barred new entrants. This led to a gradual phasing out of private note issuance in England and Wales, culminating in 1921 when the last country bank lost its note-issuing rights. When any of these banks gave up their right to issue notes, voluntarily or through merger, the Bank of England could be authorized to assume up to two-thirds of the lapsed issue (BoE 1969).

[7] I am thankful to Princeton University's Bendheim Center for Finance for providing me access to this resource.

2.1 Defining the Payoff Streams: The BoE's Segmented Structure

To pin down the payoffs being priced, it is helpful to examine three institutional features that make the BoE a highly segmented central bank: the separation between the Banking and Issue Departments, the Asset Purchase Facility, and the Exchange Equalisation Account.

The Banking and Issue Departments:

The 1844 Bank Charter Act split the Bank and its accounts into a Banking Department an Issue Department. The historical details are below, but the key point is that in modern times the profits from the note issuing department, what the BoE calls seigniorage (BoE 2025*a*), constitute the bulk of remittances. These are transferred directly to the government, so the bank has no discretion over their magnitude. In the first regime (1845-1945) the total dividends to private shareholders are priced, only a small share of which are seigniorage related. In the second (1946-2024) the profits from both departments are priced together.

The original intention of the separation was to earmark the profits from banknote issuance, with the Issue Department's liabilities consisting entirely of banknotes.[8] The privately-owned BoE had to pay a largely fixed annual fee to the treasury in exchange for the monopoly right over money issuance, and was allowed to retain any excess profits of the department (Anson & Capie 2022). In the period until 1928 the department's profits net of fees amounted to 7.5% of post-tax profits on average. Following the Currency and Banknotes Act of 1928 all Issue Department profits were directly remitted to the treasury; this continued after the 1946 nationalization. In recent years, especially since the start of QE, the department has stopped holding traditional assets and instead only holds a deposit at the banking department, which pays the policy rate (the "Bank rate") (BoE 2010, 2025*a*).

The rest of the central bank's liabilities sit with the Banking Department, including commercial bank reserves. Reserves did not pay interest until 2006 (BoE 1910, Bailey 2024), though in modern times other (for example public) deposits did. Its profits include for example those from open market or liquidity operations. Since the nationalization its dividends have been a small share of remittances, and since 1984 it distributes half of net income. 2019 saw the first recapitalization of the bank since 1816, and since 2021 the Banking department has distributed no dividends.[9] Figure 3 illustrates the relationship between the two departments and their consolidation. Appendix A presents the evolution of assets disaggregated by department.

The Asset Purchase Facility:

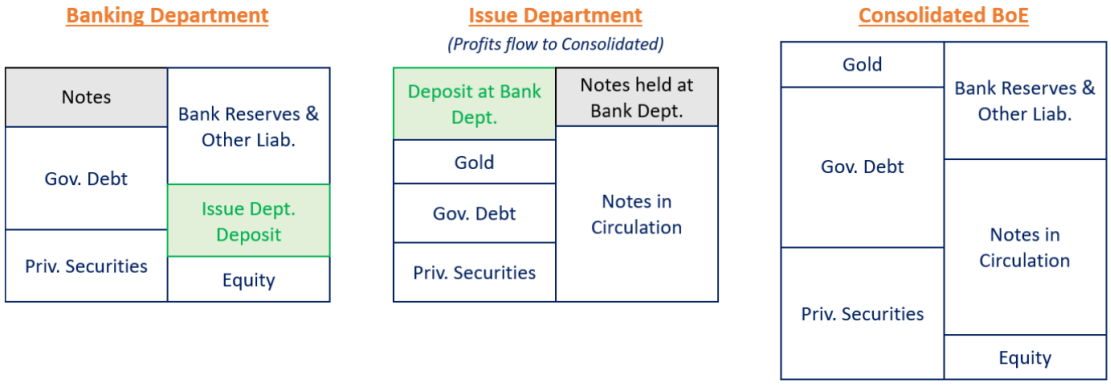
The BoE has a unique indemnified structure to implement QE: the treasury receives any surpluses directly but must compensate any losses.[10] The BoE owns a subsidiary, the Asset Purchase Facility (APF), which takes out a loan from the BoE to purchase assets; it pays the floating Bank Rate on the loan. In its accounts the BoE only

[8] Under the gold standard part of the issue was backed by private and government debt, so issuance generated some profits. There was a limit on how much money could be backed by assets other than gold. For details on this "fiduciary issue" see BoE (1969).

[9] This is because of a 2018 agreement whereby no dividends are distributed if the bank's loss-absorbing capital is under a certain threshold. The recapitalization was largely technical, amounting to 0.05% of GDP (Carney 2018, BoE & HM Treasury 2018, 2025).

[10] According to the BoE, this was meant to avoid any suspicion that monetary policy decisions "might be taken with a mind to their financial implications for the bank, rather than purely in pursuit of the monetary policy objectives"(BoE 2021). Yet, the BoE must obtain the treasury's permission before increasing the maximum size of the APF's assets (House of Lords 2021). Bell et al. (2023) discusses the tradeoffs around such indemnity agreements in monetary policy.

Figure 3 Illustrative consolidation of Bank of England departments

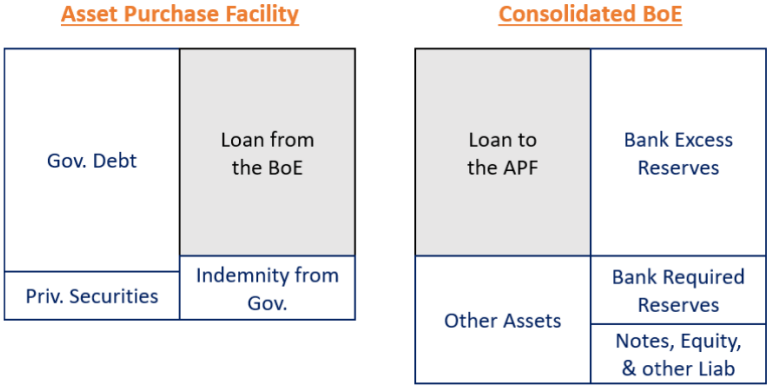


Note: The Issue Department's only liabilities are currency, which it issues to hold government debt, private assets, and a deposit with the Banking Department. The Banking department holds all other liabilities; its note (cash) holdings are not counted into Notes in Circulation. In the private period the Issue Department's profits net of fees flow to the shareholders. In the nationalized period they flow directly to the government.

displays the loan to the APF at amortized cost, which it finances by issuing bank reserves through the Banking Department. Both the loan and the reserves pay the policy rate, so the structure does not have any net interest income impact on the BoE. Indeed, the BoE does not consolidate the APF in its accounts as "it has no economic interest in its activities" (BoE 2009). Figure 4 illustrates the relationship.

Essentially, any losses from the difference between the APF's income and what it must pay on the loan from the APF are covered by the treasury, so that in the fiscal year ending in February 2024 the treasury paid £44.6bn into the APF. [11] In this paper I focus on pricing conventional monetary policy. Including QE in the valuation of the central bank is left for future work.

Figure 4 Illustrative relationship of the Bank of England and the Asset Purchase Facility



Note: The APF borrows from the BoE at the floating policy rate, while the BoE finances the loan by issuing reserves which also pay the policy rate. As a result, the structure results in zero net interest income to the BoE. The government receives any profits from the APF and must compensate any losses. The figure is slightly outdated as in 2025 the BoE abolished its reserve requirement.

The Exchange Equalisation Account:

A year after it left the gold standard, in 1932 the U.K. created the Exchange Equalisation Account (EEA) to hold the country's foreign-currency and gold reserves and conduct exchange rate policy. It is managed by

[11] It is worth noting that the BoE's QE program has a higher average maturity of government bond holdings than other major programs (Bank of Japan 2023); only 0.9% of its holdings are in corporate bonds throughout the period.

the BoE, but all of its surpluses and losses go to the treasury. Because the EEA is an exchange-rate (and not a monetary-policy) instrument, I exclude its payoffs from the asset-pricing model (acknowledging that under the gold standard the line is blurred). Accordingly, the 0.44% of GDP (£3.3 bn) loss incurred during the 1992 failed intervention to keep the U.K. in the European Exchange Rate Mechanism (HM Treasury 1997) is not included.

2.2 Balance sheet and profitability evolution

This section provides an overview of the evolution of the Bank of England's balance sheet and profitability, displayed in Figure 5 (details on the series, along with additional assumptions required to construct them, are in included in Appendix A). To summarize, each regime prices the following series, following the names in Figure 5:

1. **1845-1945**: Banking Dept. Private Dividends (blue) + Issue Dept. Private Dividends (orange, zero from 1929).
2. **1946-2024**: Banking Dept. Government Dividends (purple) + Issue Dept. Government Dividends (green). I do not include ordinary taxes paid by the BoE (gray), the payoffs from QE, nor the 2019 recapitalization of the bank.

In its private era the shareholders elected the Governor and the Court of Directors. Before WWI the policy rate was typically set at the Court's weekly meetings, with the goal of maintaining the gold parity. As the bank put it "the Government has no voice in the management of the Bank" and exercised no direct supervision (BoE 1910). Overall, the BoE was a relatively profitable central bank. Its branch managers had instructions to "get business at fair rates if you can, but get business" (Faber 1896)[12] while, as Keynes (1914) put it, the bank held "one of the smallest gold reserves in Europe, while building up on the basis of it the largest volume of business." During WWI monetary policy was de-facto decided by the government, and during the inter-war period decision making power gradually shifted from the bank to the treasury (Goodhart 2018).

There is no single turning point at which the bank began prioritizing "policy over profit", with historians proposing dates ranging from 1890 to the 1920s (Dowd 1991, Goodhart 2018, Anson & Capie 2022). However there is a clear break in the profitability series during the interwar period. While during the 19th century the bank typically paid out the entirety of profits, its payout ratio started to decline at the turn of the century. Beginning in WWI its retained earnings grew dramatically, while the bank was also extraordinarily taxed.[13] With the decoupling of profits and dividends in the interwar years, some have argued that the shareholders in practice became bondholders (Capie et al. 1994). Note that, with private shareholders not having any knowledge of pre-tax profits, these retained earnings largely went into an "off-the-books" reserve (only displayed in the balance sheet panel of Figure 5).

[12]Commercial "country" banks often complained about unfair competition by the BoE. As Faber (1896) put it: "We do not complain about fair competition but [the BoE's rate undercutting] is fostered by free money costing the lender nothing at all. How can we country bankers who pay well for our deposits meet such competition as this?"

[13]In the context of studying the credibility provided by the gold standard, Bordo & Kydland (1995) argue that temporarily suspending it during WWI allowed governments to raise seigniorage revenue. However due to the fixed fee agreement the seigniorage revenue could only be captured by raising taxes. Figure 11 in Appendix A plots the evolution of the payout ratio during the private period.

I highlight a few final points relevant for understanding the balance sheet and profit series. The balance sheet expansion of 1929 was not due to a response to the Great Depression, but rather due to the delayed consolidation of the "Bradbury notes" into the BoE's balance sheet. This paper money was issued by the treasury during WWI and was legal tender. Similarly, the expansion of the balance sheet excluding the APF loan in 2018 was due to the transfer of the loans from the Term Funding Scheme (a funding-for-lending operation similar to the ECB's TLTROs) from the balance sheet of the APF to that of the bank, and so was also not policy driven.[14] The profits of the issue department are low (zero in 1938-39) during the Great Depression because briefly (starting in 1929) it implemented mark-to-market accounting (Anson & Capie 2024). Almost without interruption, the BoE held a fixed policy rate of 2% from 1932 to 1952 (BoE 1952). Lastly, the spike in profitability during the Great Inflation is consistent with Humann et al. (2025): early-QE profit levels can be achieved with a much smaller balance sheet when yields are high and interest-bearing liabilities are limited.

3 Forecasting the Cashflows: the VAR

To forecast the payoffs of the central bank, I assume that the $N \times 1$ vector of state variables z_t follows a Gaussian first-order VAR:

$$z_t = \mathbf{\Psi} z_{t-1} + \mathbf{u}_t = \mathbf{\Psi} z_{t-1} + \Sigma^{\frac{1}{2}} \boldsymbol{\varepsilon}_t$$

Where $\mathbf{\Psi}$ is the $N \times N$ slope matrix and $\mathbf{u}_t \sim$ i.i.d. $\mathcal{N}(0, \Sigma)$ are the homoskedastic innovations. $\Sigma^{\frac{1}{2}}$ is the Cholesky decomposition of the covariance matrix ($\Sigma = \Sigma^{\frac{1}{2}}(\Sigma^{\frac{1}{2}})'$), such that $\boldsymbol{\varepsilon}_t \sim$ i.i.d. $\mathcal{N}(0, I)$. In this way shocks to each state variable $\mathbf{u}_t$ are linear combinations of its own structural shocks $\boldsymbol{\epsilon}_t$ and the structural shocks to the state variables that precede it in the VAR. The VAR is estimated equation-by-equation on the demeaned state variables, and the estimates for the slope matrix and Cholesky decomposition are applied to the three level variables, whose equations are not estimated. The state variables and their means for each regime are described in Table 1. Throughout the paper the dollar sign superscript indicates a rate is nominal.

This approach takes the variables relating to the central bank as given, and assumes the central bank commits to a dividend policy that is affine in the state vector. The state vector includes a rich set of macroeconomic and financial variables (the same as in Jiang et al. (2024)) which help predict the BoE's dividend growth: inflation, GDP growth, the term structure, the stock market's price-dividend ratio[15] and dividend growth, and the BoE's assets. BoE assets exclude the loan to the APF during the period of QE. Before 1915 the short rate is a 3-month yield.[16] The long rate is the consol yield in the first regime, and the 10-year government bond yield in the second. By placing BoE assets before its dividends in the VAR I allow dividends to respond contemporaneously to shocks to assets and to the rest of the macroeconomic variables.

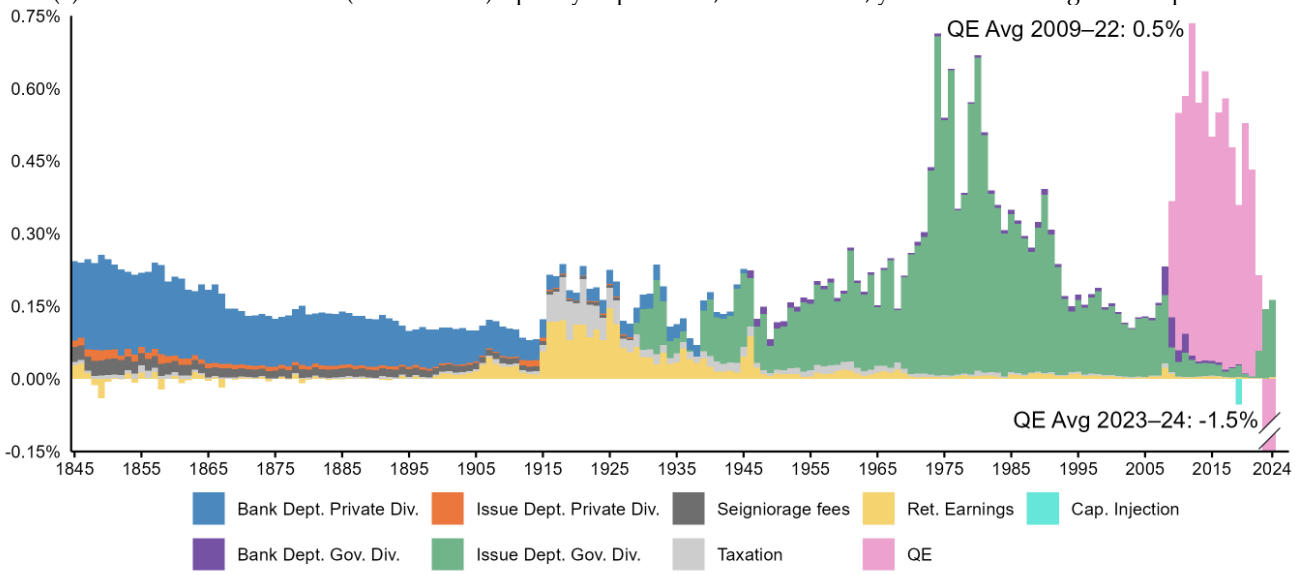
[14] The transfer formally occurred in January 2019, but appears in 2018 due to the interpolation of the balance sheet to transform it from fiscal to calendar years.

[15] One may worry that early in the first regime the Bank of England constituted a large share of the British market capitalization. Although according to some estimates in 1849 the BoE represented 23% of the total market cap of all companies traded in the UK, by 1929 this share declined to 0.8%. Appendix A presents a plot of this evolution.

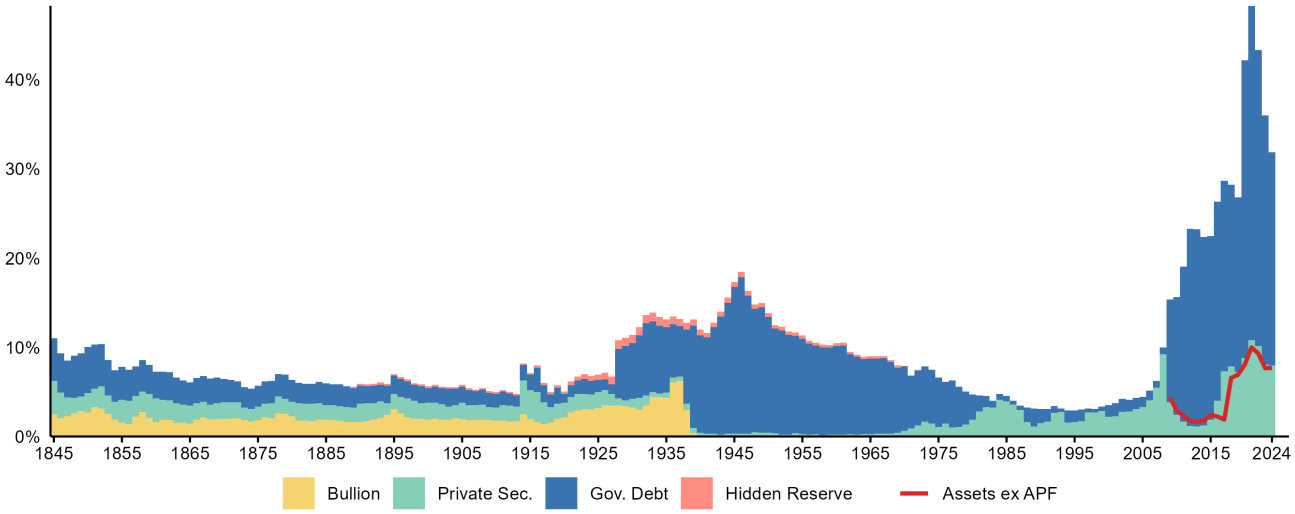
[16] From 1845-1899 it is the 3-month yield on commercial paper, while in 1900-1915 it is the 3-month yield on government bonds. There is no available data on 1-year yields for this period.

Figure 5 The Bank of England's Balance sheet and profitability evolution.

(a) Net income distribution (as % of GDP): split by department, includes APF, y-axis is cut during the loss period



(b) Assets (as % of GDP): consolidates asset distribution of the APF



(c) Liabilities (as % of GDP)

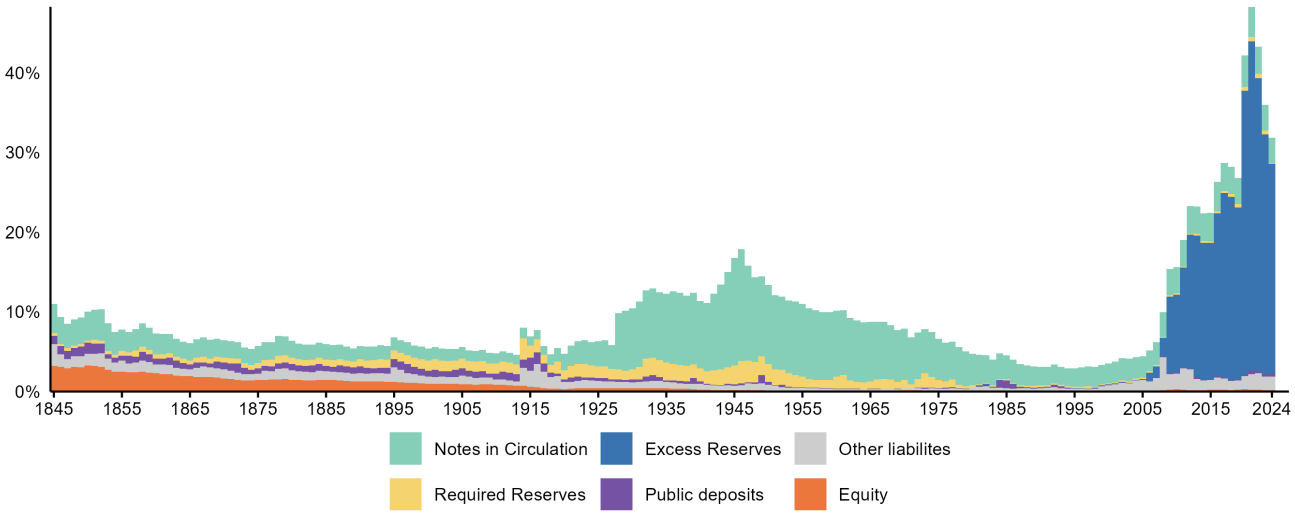


Table 1 VAR state variables

Index	Variable	Mean	Description	Sample Mean 1845–1945	Sample Mean 1946–2024
1	π_t	π_0	Log Inflation	1.05%	4.87%
2	$y_t^{\$}(1)$	$y_0^{\$}(1)$	Log 1-Year Nominal Yield	3.17%	5.20%
3	$\text{yspr}_t^{\$}$	$\text{yspr}_0^{\$}$	Log Long Rate Minus Log 1-Year Yield Spread	0.07%	0.80%
4	x_t	x_0	Log Real GDP Growth	1.83%	2.26%
5	$\Delta \log d_t^M$	μ_d	Log Stock Dividend-to-GDP Growth	0.93%	-0.22%
6	$\log d_t^M$	$\log d_0^M$	Log Stock Dividend-to-GDP Level	2.25%	3.66%
7	pd_t^M	pd_0^M	Log Stock Price-to-Dividend Ratio	21.99	24.34
8	$\Delta \log a_t$	μ_a	Log BoE Asset-to-GDP growth	0.51%	-0.99%
9	$\log a_t$	$\log a_0$	Log BoE Asset-to-GDP Level	7.54%	6.66%
10	$\Delta \log v_t$	μ_v	Log BoE Dividend-to-GDP Growth	-3.00%	-0.01%
11	$\log v_t$	$\log v_0$	Log BoE Dividend-to-GDP Level	0.09%	0.20%

Note: For the level variables the mean of the exponentiated variable is reported, only for illustration.

I estimate the VAR separately across the two regimes for both institutional and macroeconomic reasons. Firstly, the BoE was privately owned before 1946 and nationalized thereafter, carrying distinct average dividend-to-GDP ratios. Secondly, it is defensible that the economy was fundamentally different across the two periods: most of the first regime operated under the gold standard, with lower and more stable inflation, more drastic yield curve inversions, no procyclical behavior of short rates, and with the degree of financialization of the economy entailing lower stock market dividends to GDP. Chen et al. (2025) implement the same break, arguing that the data-generating process differs across periods when the U.K. possessed the exorbitant privilege. Appendix C.2 presents Chow tests supporting the existence of a break and the estimates for the slope and innovation matrices.

Cochrane (2020)'s discussion of Jiang et al. (2024) stresses that the asset to be priced should enter the VAR and that the SDF should load on it (in their case, debt-to-GDP). One could analogously include the BoE's price in the state vector, but I purposefully do not: the goal is not to test a present-value identity, and including the price in the VAR would bring a mechanical fit to the identity. Instead, I use the observed pre-1946 price only to assess the model and then value the bank in the post-1946 regime, where no market price exists.

With the inclusion of the level variables, the system becomes a vector error correction model, such that the associated growth variables exhibit mean reversion to their long term relationships to GDP. As a result, it is required that the BoE's log dividends be cointegrated with log GDP under a $(1, -1)$ cointegration vector. Though this is an assumption, Appendix C.1 presents Johansen tests which support the existence of a cointegrating relationship. With cointegration implying predictability (Engle & Granger 1987), in the long run the central bank's risk premium will be driven by that on GDP, which will be central in valuation.

Similarly to Jiang et al. (2024), there is an issue with the in-sample trend of central bank dividends to GDP, despite the existence of a cointegrating relationship. As a result (and similar to their detrending of spending to GDP) I detrend the BoE's dividends in both regimes (although it is only material in the first). In this way, I

reconstruct the log BoE dividend to GDP level that enters the VAR as follows:

$$\log v_t = \log v_1 + \sum_{k=2}^{t} (\Delta \log v_k - \hat{\mu}_v)$$

The in-sample trend in BoE dividends during the first regime is large and is largely driven by the interwar years, when historians have argued the BoE's shareholders effectively became bondholders (Capie et al. 1994). Excluding the interwar period, the estimated in-sample trend is -1.72%. This has a first-order impact on the VAR's forecasting accuracy. A remedy could be to implement a further structural break in means (and not slope matrices), demeaning the dividend level with period-specific averages (1845–1914 and 1915–1945) and detrending in the first subperiod. Analogously to debt-to-GDP ratios not being stationary (Campbell et al. 2025), BoE assets are not stationary.[17] To avoid biased estimates in the VAR I also detrend them. Importantly, when pricing the BoE I evaluate the state vector at the actual values of dividends and assets, not the detrended ones.

As emphasized by Cochrane (2020), the stock market dividends to GDP ratio has a clear trend; Augmented Dickey–Fuller tests do not reject a unit root in the ratio.[18] For consistency with Jiang et al. (2024), the stock market dividends-to-GDP ratio is not detrended. Lastly, concerns about a non-stationary stock market price–dividend ratio in the U.S., and the challenges it brings for the estimation of valuation models, are well documented (Lettau & Van Nieuwerburgh 2008, Atkeson et al. 2025). For the U.K., ADF tests reported in Appendix C.1 reject a unit root for the variable in both regimes.

3.1 VAR dynamics and implied forecasts

Figure 7 presents the impulse responses to GDP and short rate shocks implied by each regime's VAR. Both models exhibit the mean reversion in BoE assets and dividends motivated by their cointegration with GDP. Figure 6 displays, for each regime, the model's forecasting accuracy at different horizons.

1845-1945 Regime: In the first regime, GDP shocks have little effect on BoE assets or dividends. At the same time, increases to the short rate have a modest impact on BoE dividends, while they lead to temporary GDP contractions. The log BoE dividend equation has a near–unit-root coefficient on $\log v_{t-1}$ while in the innovation matrix most variation in $\Delta \log v_t$ is absorbed by its own shock; the BoE asset equation behaves similarly (and more strongly). In large part, I interpret the unresponsiveness of BoE dividends to the interwar period, which saw large shocks to the economy while profits became decoupled with dividends (historians have argued that during the period BoE shareholders in effect became bondholders (Capie et al. 1994)). As a result, the VAR is unable to capture the countercyclical nature of BoE dividends in the 19th century (Anson et al. 2017, Anson & Capie 2022). The evolution of assets might also be contaminated by the consolidation of the Bradbury notes in 1928 (discussed in Section 2.2), where log assets grew by 50% while the British economy had not yet experienced

[17] Appendix C.1 reports Augmented Dickey–Fuller tests not being able to reject a unit root.

[18] Johansen tests indicate cointegration between log dividends and log GDP; taken together, this suggests the cointegrating vector may differ from $(1, -1)$. Also note that, in the second regime, the short rate cannot be rejected to have a unit root, in line with Campbell & Shiller (1987, 1991).

the full effects of the Great Depression.

The forecasts exhibit a strong mean reversion. The long run forecasts (at 5 and 10 years) are low early in the sample and high late in the sample. This occurs because of the error correction dynamics of the model (when the dividend level is far from its cointegrating mean forecasts pull it back towards it) coupled with the fact that the forecasts are built by evaluating the state vector at the actual data. Early in the sample dividends to GDP are well above the mean, and late in the sample they are well below it. This is the main driver of the asset pricing model's errors very early and very late in the sample.

1946-2024 Regime: In the second regime the VAR exhibits rich dynamics for central bank dividends. While negative shocks to GDP lead to dividend declines, increases in the short rate lead to large increases in dividends. The model also has the counterintuitive behavior that increases in the short rate are associated with increases in BoE assets. This may be due to contamination from the consolidation of the Term Funding Scheme into the BoE's balance sheet in 2018 (originally in the APF), which more than doubled its total assets in a year where rates tripled (although remaining below 1%). The forecasts are more successful in this regime. Absent a strong in-sample trend in dividends to GDP, long-horizon projections reflect macro drivers rather than the mechanical mean reversion of the first regime. Note the spike in dividends at the end of the sample is, together with the rate increases, due to a low starting level in 2022.

4 Valuation

The present discounted value of the central bank relative to GDP is:

$$\frac{PV_t^{CB}}{Y_t} = \frac{V_t}{Y_t} \exp(pd_t^V) \tag{4.1}$$

Where Y_t is GDP, V_t is the BoE's dividend and pd_t^V is the price-dividend ratio of central bank equity. To obtain the log price-dividend ratio two approaches are possible. Valuing the bank through Campbell & Shiller (1988) linearizations of the BoE's returns, which requires taking a stance on the relevant discount rate, or developing a fully fledged model for the SDF which delivers the price-dividend ratio. Both rely on the assumption that the risk premium on a GDP claim is equal to the risk premium on unlevered equity.

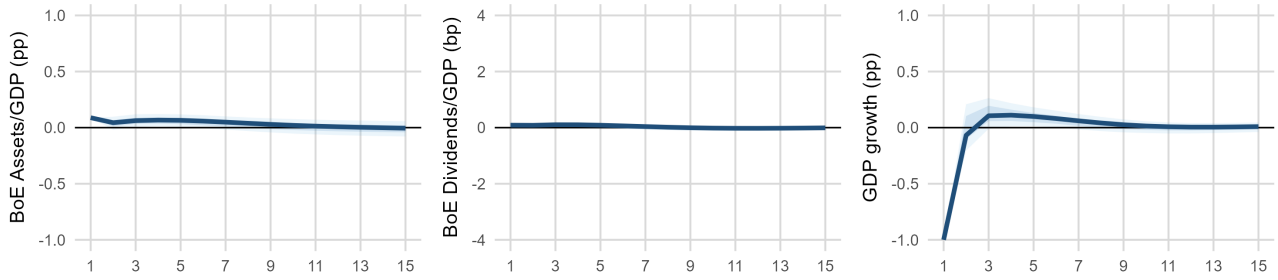
4.1 Valuation based on Campbell-Shiller decompositions

By log-linearizing the returns on BoE equity and iterating forward, I obtain the standard Campbell & Shiller (1988) decomposition of the log price-dividend ratio of the central bank:

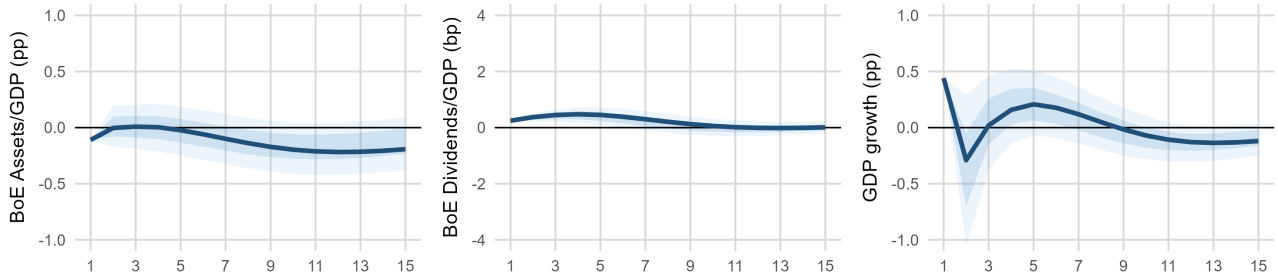
$$pd_t^V = \frac{\kappa_0^V}{1-\kappa_1^V} + \underbrace{\mathbb{E}_t\left[\sum_{j=1}^{\infty}(\kappa_1^V)^{j-1}\,\Delta\log V_{t+j}\right]}_{\substack{\text{Cashflow term}\\ CF_t^V}} - \underbrace{\mathbb{E}_t\left[\sum_{j=1}^{\infty}(\kappa_1^V)^{j-1}\,r_{t+j}^V\right]}_{\substack{\text{Discount rate term}\\ DR_t^V}}.$$

Figure 6 VAR-Implied Impulse Responses

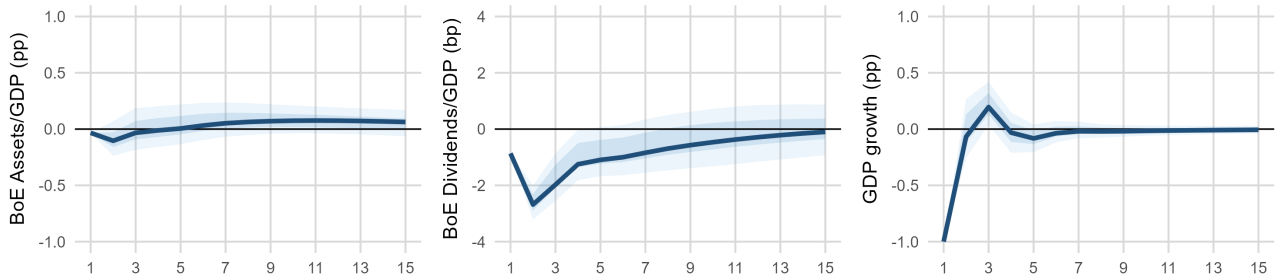
Panel A: -1% Shock to GDP Growth in 1845-1945 Regime



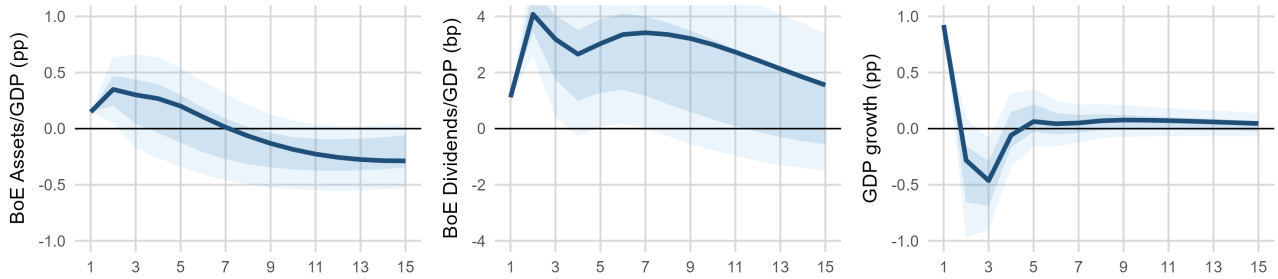
Panel B: +100bps Shock to the Short Rate in 1845-1945 Regime



Panel C: -1% Shock to GDP Growth in 1946-2024 Regime



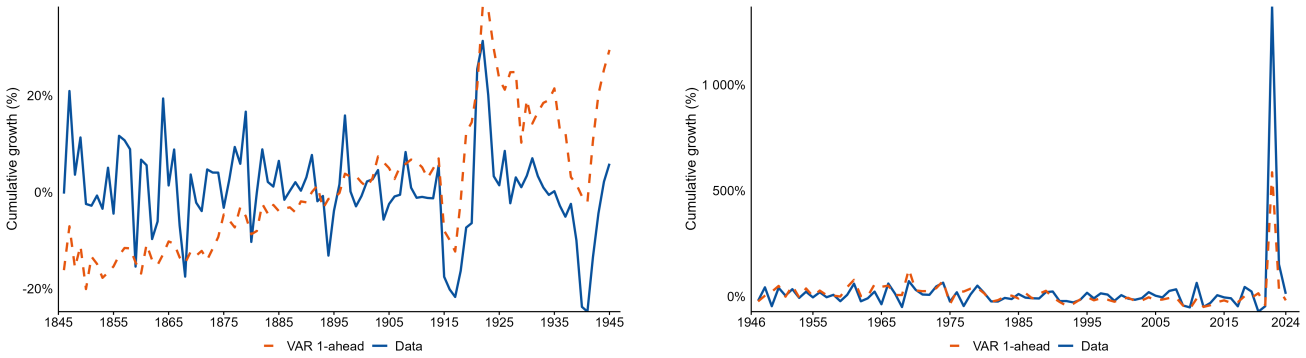
Panel D: +100bps Shock to the Short Rate in 1946-2024 Regime



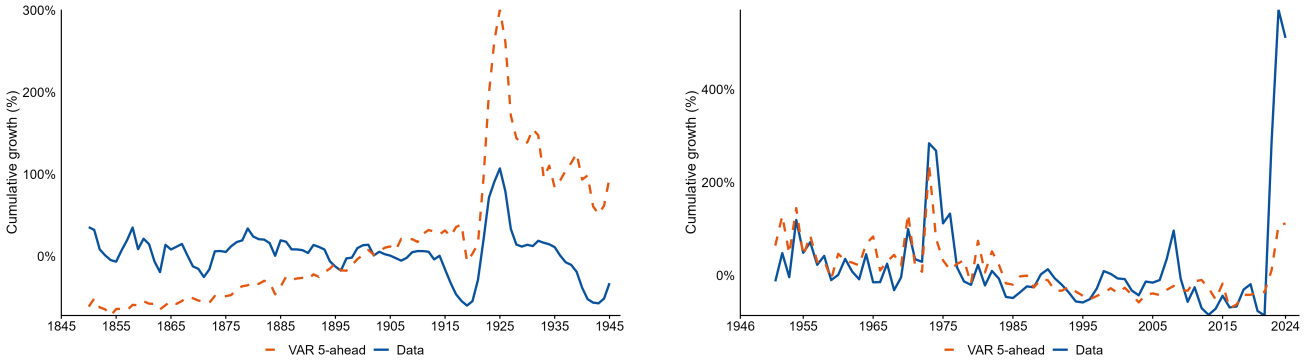
Note: The blue lines depict the impulse responses implied by the estimated VAR. Responses of BoE assets to GDP and GDP growth are shown in percentage points, while those of BoE dividends to GDP are in basis points. I plot the one- and two- standard deviation confidence intervals obtained by resampling residuals and adding them to the data to simulate new samples, re-estimating the VAR, and computing the associated impulse responses.

Figure 7 VAR Forecast Evaluation.

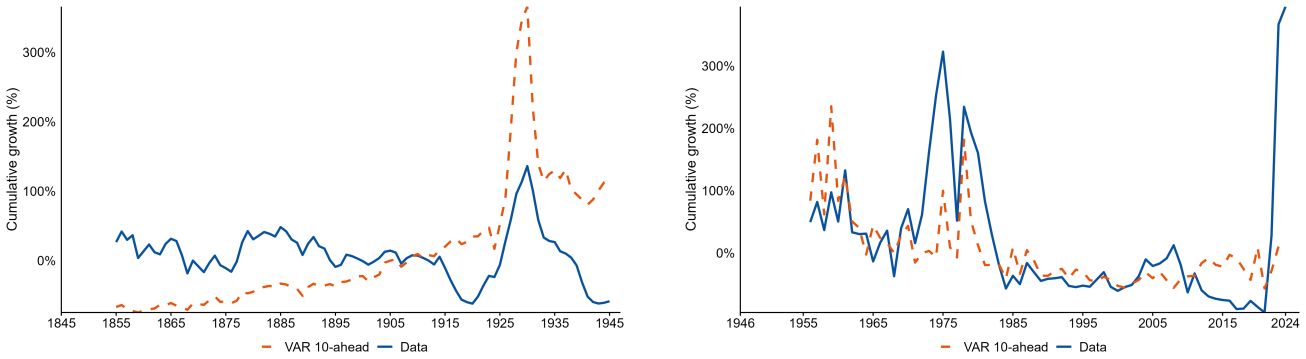
Panel A: 1-year forecast of Log BoE Dividends-to-GDP growth



Panel B: Cumulative 5-year forecast of Log BoE Dividends-to-GDP growth



Panel C: Cumulative 10-year forecast of Log BoE Dividends-to-GDP growth



Note: I plot the actual log BoE dividend growth rates over 1-year, 5-year, and 10-year rolling windows in blue. The value at each year represents the k-year growth rates that end at that year. I also plot these rates as forecasted by the VAR in orange, conditioning on the raw (not detrended) state variables k years in the past.

Where κ_0^V and κ_1^V are linearization constants which depend on the mean of the log price-dividend ratio pd_0^V, and r_{t+j}^V is the expected log return on the BoE's dividend claim. The full derivation is in Appendix D. To make progress, one can denote rp_t^V as the risk premium on the BoE dividend claim relative to the yield of a long-term bond; with this, the expected log returns on central bank equity follow:

$$\mathbb{E}_t[r_{t+1}^V] = y_t^{\$}(1) + yspr_t^{\$} + rp_t^V.$$

The short-term rate and the term spread are part of the VAR's state vector, and with the VAR one can build out the expectation terms. This yields the following expression for the log price-dividend ratio:

$$pd_t^V = pd_0^V + \Big(\underbrace{\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\right)' \mathbf{\Psi}}_{\substack{\text{From cashflow} \\ \text{term}}} + \underbrace{\left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)'}_{\substack{\text{From discount rate} \\ \text{term}}} \Big) \underbrace{(I - \kappa_1^V \mathbf{\Psi})^{-1}}_{\substack{\text{From geometric} \\ \text{series}}} z_t$$

Where $\mathbf{e}_{variable}$ are (column) selector vectors which extract state variables, and the mean log price-dividend ratio pd_0^V solves the following system of equations:

$$\begin{aligned} pd_0^V &= \frac{\kappa_0^V + x_0 + \pi_0 + \mu_v - y_0^{\$}(1) - yspr_0^{\$} - rp_0^V}{1 - \kappa_1^S} \\ \kappa_1^V &= \frac{\exp(pd_0^V)}{1 + \exp(pd_0^V)} \\ \kappa_0^V &= \log(1 + \exp(pd_0^V)) - \kappa_1^V pd_0^V \end{aligned}$$

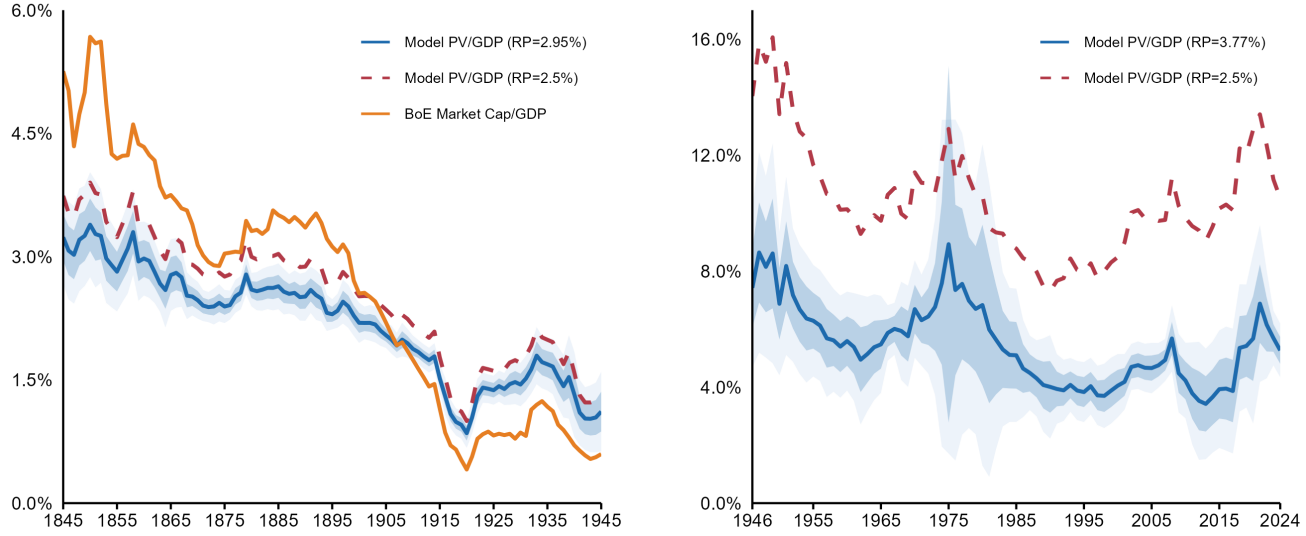
The only parameter that is left to be pinned down to price the central bank is its risk premium rp_0^V. By the assumption of a $(-1, 1)$ cointegrating vector for BoE dividends and GDP, μ_v is set to zero. Further and by the same assumption, in the long run the central bank will inherit the risk premium of a GDP claim. This entails disregarding the short run properties of the risk premium on the central bank. To make progress I adopt the convention that the risk premium on a claim to GDP is equal to the risk unlevered risk premium of stock market equity (Abel 1999, Campbell 2003), such that:

$$rp_0^V = rp_0^Y = \frac{1}{T} \sum_{t=1}^{T} (1 - l_t)\, rp_t^M$$

Where l_t is the corporate debt-to-asset ratio in year t, and rp_0^Y is the constant GDP risk premium. The risk premium on market equity rp_t^M is derived in Appendix D.2, and is given by a standard linearization of the market's log return. In the first regime, this entails a central bank risk premium of 2.95%, and of 3.77% in the second. Figure 8 reports the results. The valuation is split into panels by regime and changes the y-axis in each panel. For comparability across regimes I also report results under an assumed 2.5% risk premium on GDP.[19] In the first regime it displays the observed market value of the BoE relative to GDP.

[19] Jiang et al. (2024) consider both 2.5% and 3%. Assuming a zero risk premium on the claim on central bank dividends fails to deliver a price similar to observed during the publicly traded period by an order of magnitude. Figure 13 in Appendix D.1 presents a unified plot.

Figure 8 Present value of the Bank of England under a constant risk premium (% of GDP)



Note: The average values of the central bank under constant risk premia are 2.2% and 5.5% of GDP, respectively for the first and second regime (2.5% and 10.5% under an assumed risk premium of 2.5%). Confidence bands are obtained by resampling the residuals from the VAR to simulate 50,000 pseudo-samples, re-estimating the VAR, and computing one and two standard deviations. Draws are accepted only if the VAR is stable and the path for the market equity risk-premium is non-negative (this last condition is not imposed in the years where realized equity risk premia were negative).

To a first order of magnitude, the model is decently able to match the observed value of the BoE in the first regime. The average model-implied value under the regime's risk premium is 2.2% of GDP, while the average observed market capitalization to GDP is 2.6%. The average observed price-dividend ratio is 30.9, while the model-implied average ratio ($\exp(pd_0^V)$) is 29.6. The average error is -0.42% of GDP (on average the model under-prices), while in absolute value the average error is 0.77% of GDP (this is 37.11% as a percent of the observed price). The root mean squared error is 0.91%. The model's early undershooting and later overshooting of the observed price is, in large part, due to the strong mean reversion embedded in the VAR. As shown in Figure 6, in the mid 19th the VAR forecasts sharp declines in dividends, while it forecasts dramatic increases in the interwar period (both of which are inaccurate).

The decent performance of the model in the first regime indicates it is suitable to price the central bank in the second, especially because the second regime does not have the in-sample trend issue which is driving the model's inaccuracy in the first. It also supports the assumption that the risk premium on the central bank is equal to that on unlevered equity. The average value of the central bank is estimated at 5.5% of GDP, with an average price-dividend ratio of 37.3. With the second regime's risk premium at 3.77%, the model-implied value under an assumed risk premium of 2.5% is twice as high, at 10.5% of GDP. Nevertheless, these estimates are far from those previously put forward by the literature, with Del Negro & Sims (2015) and Hilscher et al. (2015) estimating the present value of seigniorage to be 94% and 18-22% of GDP, respectively, and in back of the envelope exercises Fry (1992) and Buiter & Rahbari (2013) estimating to it to be 38% and 20% of GDP, respectively.

By focusing on the long run relationship to GDP, the exercise disregards the cyclical properties of the risk premium on the central bank. Section 4.2, which builds the full asset pricing model, finds that in the first

regime the BoE is a safe asset. In the second regime it has business cycle risk, such that at short horizons the risk premium on the central bank is higher than that on GDP, though in the long run is has a risk premium below that on GDP. The second effect dominates, increasing the value of the bank. As a result, in the second regime the benchmark result can be seen as a lower bound on the value of the BoE.

Appendix D.1 presents the relative importance of the decompositions' terms over time. Appendix D.2 presents valuation estimates through the same Campbell-Shiller decomposition but with time-varying risk premia. It makes the same assumption that the risk premium of GDP is that of unlevered market equity, and leverages the yearly returns of the market to estimate the time series of the risk premium of GDP. It arrives (by construction) as at the same average value of the central bank, though delivering very different dynamics. It tracks the observed prices less well, especially in the years leading up to WWII when equity risk premia were negative. In an effort to price the BoE only with information available to agents at each point time, Appendix D.3 presents valuation estimates based on a rolling-window VAR. In this exercise I re-estimate the VAR every year using only data up to that year, recomputing the respective means and constant GDP risk premium. This exercise is also worse able to match the observed market price during the first regime.

4.2 Full asset pricing model

Departing from the assumption of constant risk premia on the central bank rigorously requires building a dynamic asset pricing model. I replicate the model of Jiang et al. (2024), which leverages the VAR together with a no-arbitrage model for the SDF as in Duffie & Kan (1996), Dai & Singleton (2000) and Ang & Piazzesi (2003). First I postulate a nominal SDF $M_t^{\$} = \exp(m_t^{\$})$ which in logs is linear in the VAR's structural innovations:

$$m_{t+1}^{\$} = -y_t^{\$}(1) - \tfrac{1}{2}\,\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1}.$$

Where $\boldsymbol{\Lambda}_t$ is the vector the market prices of risk, which follows:

$$\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 z_t.$$

The market prices of risk are estimated by stipulating asset pricing models for different assets in the economy, and trying to match their model-implied returns to the data. Assets are modeled through the recursions typical in the affine term structure literature. Appendix E details the model and the estimation procedure. Overall, it entails a high-dimensional optimization of a scalar objective function. This approach has the drawback that the estimation is computationally intensive, which makes obtaining confidence intervals on the value infeasible.

I estimate a separate SDF for each regime. Once estimated it loads on inflation, the term spread, GDP growth, and the stock market dividend growth and price-dividend ratio. The model is able to match the real and nominal yield curves and stock price-dividend ratios. It also results in an SDF that in general has enough permanent risk to generate an equity risk premium (Alvarez & Jermann 2005). However, the model fails at matching the time series of equity returns.

The price-dividend ratio used to evaluate equation 4.1 is built by adding up BoE dividend strips of horizons up to infinity (4,000 in practice). The log price-dividend ratio of a dividend strip of horizon h is proven to follow:

$$pd_t^V(h) \;=\; A^V(h) + \mathbf{B}^V(h)' z_t.$$

Where $A^V(h)$ and $\mathbf{B}^V(h)$ depend recursively on the analogous terms of the prior horizon ($A^V(h-1)$ and $\mathbf{B}^V(h-1)$). Adding the dividend strips yields the ultimate price-dividend ratio used in pricing:

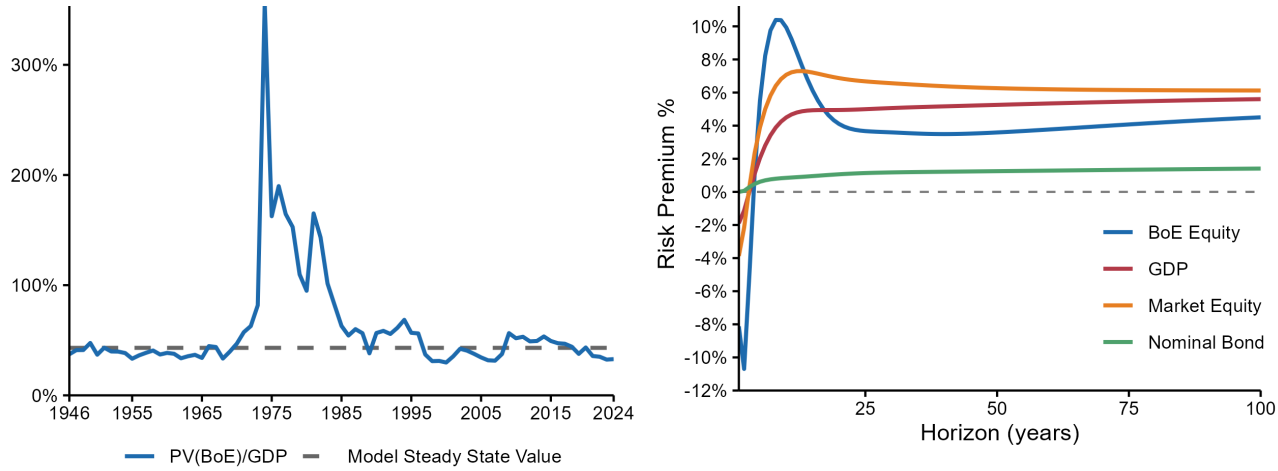
$$\exp(pd_t^V) = \sum_{h=1}^{4{,}000} \exp\big(A^V(h+1) + \mathbf{B}^V(h+1)' z_t\big).$$

Though not strictly necessary for pricing, for reference the model also delivers an expression for the (time-varying) risk premium on the aggregate claim to BoE dividends. It requires the assumption that the log price-dividend on the aggregate claim is also recursive, following: $pd_t^V \;=\; pd_0^V + \big(\bar{\mathbf{B}}^V\big)' z_t$.

The model fails at matching the observed market price of the BoE in the first regime, delivering an average value of the central bank of 111.2% of GDP. I attribute the model's failure to two aspects. First, with BoE dividends being largely unaffected by macroeconomic structural shocks in the VAR, the model sees the BoE as a safe asset and awards it a negative risk premium. Second, the model is estimated based on a small amount of data. The results are reported in Appendix E.2.

In the post-1946 regime, the model delivers a more reasonable estimate, valuing the bank at an average of 43.1% of GDP. BoE dividends have a -0.18 correlation with the nominal SDF. The implied term structure of risk-premia is hump-shaped: at medium horizons the BoE carries a high premium because of its pro-cyclical profits (as the VAR's impulse responses in Figure 6 showed, profits fall when GDP falls). At long horizons the BoE premium converges toward, but remains below, the risk premium on the GDP claim. As a result, the estimates in Section 4.1 provide a lower bound for the value of the central bank. During the stagflationary recessions of the 1970s and 1980s, when profits become countercyclical, the valuation spikes.

Figure 9 Present value and risk premia of the Bank of England under the full model for the SDF



Note: The left panel plots the model-implied present value of BoE dividends. The right panel the term structure of the model-implied risk premia for bonds, market equity, GDP and the BoE. The average model-implied value is 43.1% of GDP.

5 Conclusion

The question of whether profitability concerns can hamper monetary policy was neatly summarized by the former Deputy Governor of the Swedish Riksbank: "The fact that capital is shrinking or even becoming negative need not be a problem for a central bank. [...] However, problems may arise if the central bank loses its long-term earnings capacity" (Flodén 2022). This paper arrived at an estimate of this long term earnings capacity for the Bank of England, finding that it is substantially smaller than the literature's previously reported estimates of the present value of seigniorage. It also measured the extent to which this earnings capacity contributes to the government's fiscal capacity. Although limited (at most contributing 21.3% for the market value of gross government debt), this backing can almost triple with inflation.

Repeating this exercise across developed economies could allow for richer analyses of these two dimensions. Building on Goncharov et al. (2023), one could quantify how policy-rate deviations from Taylor-rules affect central banks' value. Separately, extending the framework could help develop a deeper account of the relationship between the central bank's fiscal support to the government and inflation.

Appendix A: Data sources

I leverage three main datasets produced by the Bank of England or its staff: "A millennium of Macroeconomic Data for the UK" (Thomas & Dimsdale 2017), "Annual data on the Bank of England's balance sheet" (BoE 2020), and Anson & Capie (2022)'s data[20] on the profitability of the BoE. These are complemented by data from the Office of National Statistics, Global Financial Data, Müller et al. (2025) (Global Macro data), and Bordo et al. (2025). To update the data to 2024 I also rely on published annual accounts. The bank's annual accounts follow a fiscal year running from February to February, so I interpolate the balance sheet and profit data to obtain calendar-year series. Below I detail how each series is constructed:

A.1 Macroeconomic data:

- **Nominal GDP:** from the Millenium dataset, at market prices and geographically-consistent borders (that is excluding Ireland before 1922). See column D of sheet A9. Complemented for 2017-2024 with Global Macro Data.
- **Real GDP:** from the Millennium dataset, at market prices and geographically-consistent borders. See column B of sheet A1. Complemented for 2017-2024 with Global Macro Data.
- **Inflation:** from the Millennium dataset, the change in the deflator at market prices. See column AI of sheet A1. Complemented for 2017-2024 with Global Macro Data.
- **Government Debt:** for the period 1870-2024 I use Bordo et al. (2025)'s measure of General Government Gross Debt. They present several improvements on the Millennium dataset, most importantly their inclusion of local government debt. Complemented for 1845-1870 with the Millennium dataset' Public Sector Debt, see column BU of sheet A1. Both are calendar year-based.
- **Market Value Government Debt:** Computing it requires an important assumption. Only marketable debt has a market value, but for a large part of the 20th century total and marketable debt differed drastically due to the international loans received from the U.S. during WWI and WWII (in 1946 marketable debt accounted for less than half of total debt). Assuming marketable debt was representative of non-marketable debt, I multiply the gross government debt series of Bordo et al. (2025) by the market-to-par ratio reported by Ellison & Scott (2020).
- **Short rate:** for the period 1970-2024 it is the yield on 1-year treasury bills from the Bank of England's yield dataset BoE (2025*b*). For the period 1915-1969 it is the 1-year yield from the interpolated yield curve of Ellison & Scott (2020). For the period 1900-1915 it is the yield on 3-month treasury bills from Global Financial Data (ITGBR3D). Lastly, for the period 1845-1899, when the government did not issue short term

[20]The BoE did not begin publishing complete annual accounts until 1971. They dataset is based on internal documents from the BoE's archive.

debt, it is the Prime Commercial Bill/Paper Rate from the Millennium dataset; see Column F of sheet A31. All are zero-coupon yields and year averages.

- **Long rate:** in the first regime it is the yield on consols from the Millennium dataset. Their series is corrected for Goschen's conversion issues (Klovland (1994)); see column AV from sheet A1. In the second regime, for the period 1970-2024 it is the yield on 10-year treasuries from the Bank of England's yield dataset BoE (2025*b*). For the period 1945-1969 it is the 10-year yield from the interpolated yield curve of Ellison & Scott (2020). All are zero-coupon yields and year averages.
- **Dividends to GDP:** The entire sample is built with Global Financial Data, multiplying the dividend yield (DFTASD) by the ratio of market capitalization to GDP of UK-based companies (SCGBRM) and taking the average across months.
- **Stock Price-Dividend ratio:** the inverse of the dividend yield from Global Financial Data (DFTASD). It is a yearly average.

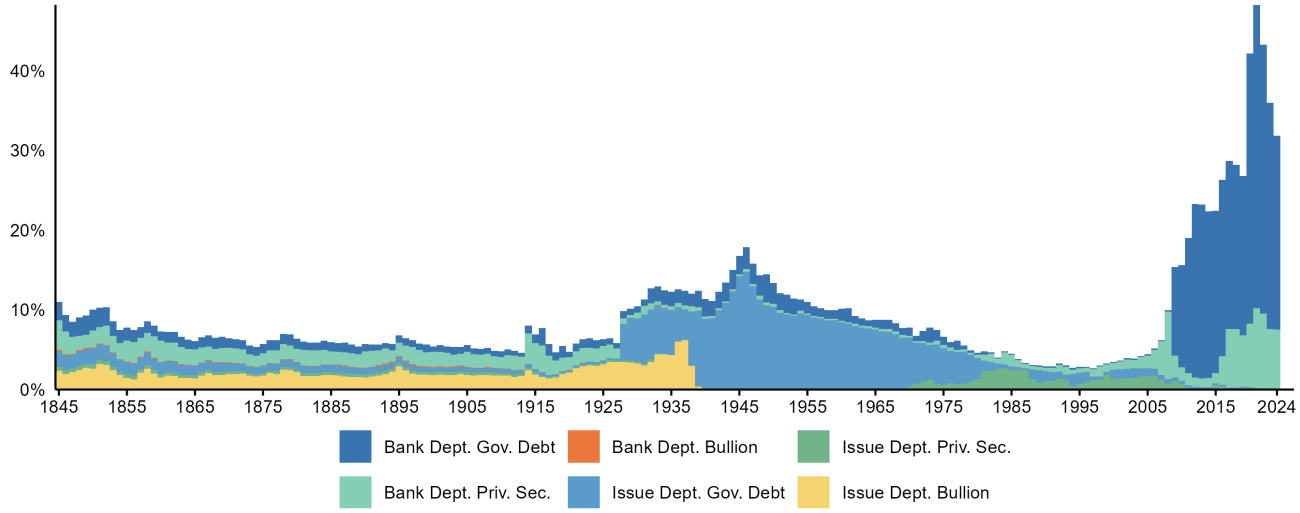
A.2 Balance and profitability sheet data:

Balance sheet data: To produce Figure 5, I made the following modifications to clean the BoE's 2020 balance sheet dataset:

- Despite sitting in the Banking department's liabilities, 7-day bills (small and only present early in the sample) are consolidated into Notes in Circulation, as in Dornbusch & Frenkel (1984). These "post bills" were short-maturity IOU, typically payable seven days after sight, used for safe interbank payments through the mail.
- In the fiscal years of 1999-2000 the balance sheet more than doubled due to the accounting of the EU's TARGET II settlement system, through which central banks accumulated claims on each other. In subsequent years these claims were netted into a single position with the ECB. This netting policy has been back-casted, canceling out the temporary expansion.
- All bank reserves are treated as required until the BoE's balance sheet database begins reporting a breakdown between the Cash Deposit Ratio deposits and other operational balances (in 1982). Note in 2025 the BoE abolished its reserve requirement. During the Great Inflation the BoE implemented what, simplifying, amounted to an increase in the reserve requirement. Through this Supplementary Special Deposit Scheme ("The Corset") the BoE set an allowed growth path for banks' interest-bearing liabilities. Any excess over this growth path had to be matched by (non-interest-bearing) supplementary deposits at the BoE (BoE 1982).
- The "Equity" category contains paid-in capital, "The Rest" account (most similar to retained earnings in modern times), and all other reserves. The off-balance sheet hidden reserve is not included (it is also excluded from the total asset series).

Figure 10 displays the breakdown of assets across departments. The banknotes held at the Banking Department and the Issue Department's deposit at the Banking Department are not included. In the period 1845-1945 cash constituted an average of 27% of the Banking department's assets. The Issue department's deposit only began in 1993, representing an average of 50% of assets for the 1993-2024 period. In the 21st century, with the majority of the Issue department's assets consisting in this deposit, its asset disappear. Its private securities can include reverse repurchase agreements with the private sector.

Figure 10 The BoE's Assets by department



Profitability data: I implemented several cleaning adjustments to the profitability series of Anson & Capie (2022). The main assumption, only relevant for producing Figure 5, is splitting the observed dividends across departments before 1929 (when all Issue Department profits started going to the government). I applied the consolidated payout ratio to both departments post-tax profits. Additional cleaning steps are detailed below, which mostly reconcile the original data with contemporary accounting rules.

- Until 1971 all writedowns, writeoffs and provisions were implemented after bottom line profits. In the 1844–1890 period there is no breakdown of writedowns, writeoffs and provisions (the 'contra' accounts recorded in Anson & Capie (2022)'s dataset). In some years however there is a difference between post-dividend and post-tax profits and the change in "The Rest" account (the historical analog for the retained earnings account). I assume that all differences are due to writedowns not recorded in the statements, and subtract them from profits. With some exceptions in years where prior work has discussed there being large writeoffs (Anson et al. 2017), this rarely changes pre-tax profits by more than 1%.

- In the 1890–1970 period the breakdown of writedowns, writeoffs and provisions is provided and all are subtracted from profits. In the 1971–2024 period these are charged directly to profits, so no adjustment is necessary except for 1972-1973 where the release of provisions were still added to equity.

- Anson & Capie (2022) report that in the 1934-1947 period £38.159 million was allocated into suspense and reserve accounts earmarked for the BoE staff's pensions. This amount accounts for all of the net income

retained during the period. Although inconsistent with modern accounting, I did not subtract these from profits due to the lack of an annual breakdown.

- The data on the seigniorage fees paid to the government is from Anson & Capie (2024).
- It is worth noting that the profits of the Issue Department were not reported in the 1913-1929 fiscal years. As a result Anson & Capie (2022) estimate them based on the reported total income of the Issue Department net of the note production costs.
- The profit series only includes operating net income. Other comprehensive income is only reported in modern times and is rarely material.

In Figure 11 I report the evolution of the BoE's payout ratio during the private period. During the nationalized period the measure is less informative because the majority of profits are remitted directly to the government through the Issue Department. Figure 12 shows the proportion of the total market capitalization of all companies traded in British stock markets accounted for by the Bank of England based on two data sources.

Figure 11 The BoE's payout ratio (dividends over post-tax profits)

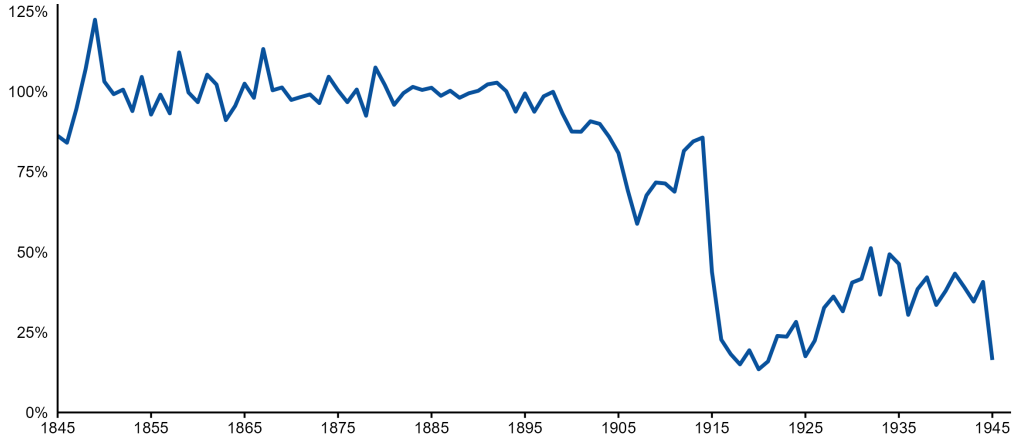
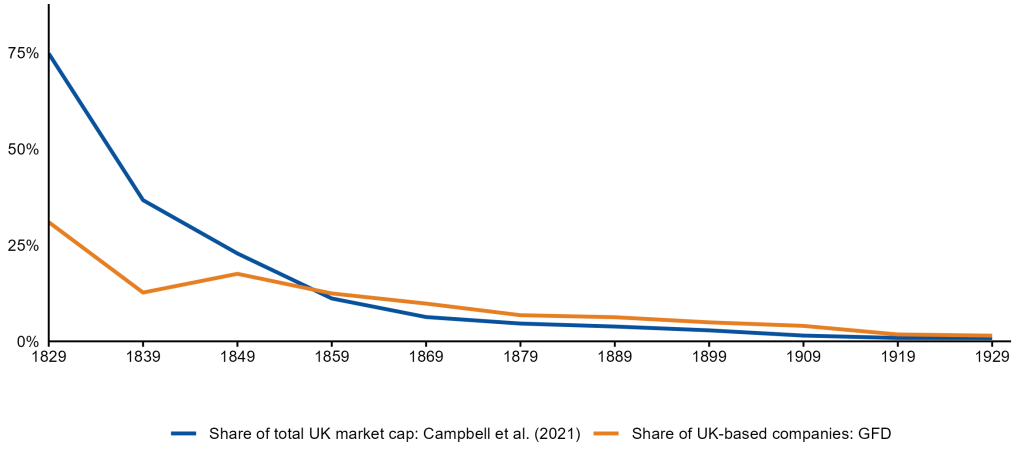


Figure 12 BoE's share of the total market capitalization of stocks listed in the UK



Asset Purchase Facility data and payoff smoothing: The data on the APF is from the ONS. I use the interest income (MDD6) and interest expense (MDD7) on an accrual basis, along with the cash transfers (MT6A for the profits and MF7A for the losses). The payoffs from QE reported in Figure 5 back-cast the cash transfer policy between the APF and the treasury, first implemented in 2013, to the beginning of the program in 2009. According to the policy (Busetto et al. 2022), cash transfers are determined by:

$$\text{Transfer}_{\text{from APF to Gov}} = \text{Int. Income} - \text{Int. Expense} - \text{Valuation and Redemption Loss}$$

With a rise in interest expenses the transfers can become negative, representing transfers from the treasury to the APF. The ONS's (2019) dataset on the APF's cash flows show no redemptions until 2013, so the backcasting simply entails that the cash transfer to the treasury if the net interest income. With this adjustment, the large dividend of 2013 (meant to compensate for the transfer-less initial years) is smoothed out.

The BoE's liquidity provision program during the financial crisis, the Special Liquidity Scheme, was also an indemnified operation. It surpluses are recorded as other comprehensive income, and resulting in a single large dividend of £2.3bn in 2012. I smoothed out this dividend, including in the Banking department's dividend the annual "post-tax surplus from indemnified operations" that is recorded in the accounts.

A.3 Additional data required in the estimation of the SDF:

- **Yield data:** The yield data for the additional tenors required to estimate the SDF is from the Bank of England's yield curve dataset (BoE 2025*b*). It starts in 1970, and I take its spot curve of zero-coupon yields. It also provides yields from inflation-indexed gilts. It is complemented with the dataset of Ellison & Scott (2020), which starts in 1915 and fits a cubic spline to the zero coupon yield curve, subject to a penalty that imposes additional stiffness at the long end. During the older periods there were not that many issuances across the yield curve (see Figure 3 of their paper), so to not rely too heavily on their estimated yield curve I only keep their 1-year (which when available tracks the 1-year yield reported by Global Financial Data), 2-year and and 5-year. In 1900-1915, when no 1-year yield is available, I take the 3-month yield on government bonds, and for 1845-1899 I take the 3-month yield on commercial paper (as discussed in the main data appendix). I assume both are 1-year yields in the model. In all cases I take the yearly average across months.

- **Returns on dividend futures:** The average returns on dividend futures are available for the period 2005–2014 from van Binsbergen & Koijen (2017).

- **Leverage:** To construct the mean of the corporate leverage ratio for each regime break, I built the annual leverage series as follows, integrating and interpolating several sources, always using data on non-financial corporations.

  - **1880–1899:** I combine Jordà et al. (2017)'s measure of Loans to Business with the market cap of

British companies traded in the UK from Global Financial Data (SCGBRM), measuring leverage as JST Loans/(JST Loans + Market Cap).

- **1900–1948:** From Millenium's sheet "A62. Financial claims 1900-48" (Roe 1971), measuring leverage as (Debentures + Bank advances)/(Debentures + Bank advances + Shares). This dataset is only available for 1900, 1910, 1920, 1927, 1937, and 1948. All years in between are interpolated.
- **1957–1966:** From Millennium's sheet "A64a. Sector balance sheets 57+", excluding cooperative society deposits and trade credit from debt, and measuring leverage as (Debt)/(Debt + Equity + Net worth residual).
- **1979–1986:** From Millennium's sheet "A64b. Sector Balance Sheets" (Sbano 2008), measuring leverage as (Loans + Securities other than shares)/(Loans + Securities other than shares + Shares and other equities).
- **1987–2024:** From the ONS' UK Economic Accounts, computing leverage as (Debt securities (AF.2) + Loans (AF.4)/(Debt securities + Loans + Equity and investment fund shares/units (AF.5)).
- **Gap in first regime:** For the 1845–1879 period I impute the mean leverage of the first regime when the data is available (27.5%).
- **Gaps in second regime:** For the periods 1949–1956 and 1967–1978 I interpolate between the last and next available observation.

Appendix B: Sargent-Wallace derivation

Disregarding population and its growth, the one-period constraint of the consolidated government (in units of consumption good after dividing by the price level P_t) is:

$$G_t - T_t \;=\; \frac{M_t - M_{t-1}}{P_t} \;+\; B_t^{\text{cons}} \;-\; (1 + r_{t-1})\, B_{t-1}^{\text{cons}}.$$

Where G_t and T_t are government spending and taxes, M_t are money balances, B_t^{cons} is the outstanding government debt (net of consolidation), r_t is the real interest rate (such that $1 + r_t = \frac{1+i_t}{1+\pi_{t+1}}$). After rearranging it becomes:

$$B_{t-1}^{\text{cons}} \;=\; \frac{1}{1 + r_{t-1}} \left[T_t - G_t + \frac{M_t - M_{t-1}}{P_t} + B_t^{\text{cons}} \right].$$

Setting $t = 0$ and iterating forward:

$$\begin{aligned} B_0^{\text{cons}} &= \frac{1}{1 + r_0} \left[T_1 - G_1 + \frac{M_1 - M_0}{P_1} + B_1 \right] \\ &= \sum_{j=1}^{T} \left(\prod_{k=0}^{j-1} \frac{1}{1 + r_k} \right) (T_j - G_j + \frac{M_j - M_{j-1}}{P_j}) + \left(\prod_{k=0}^{T-1} \frac{1}{1 + r_k} \right) B_T^{\text{cons}} \end{aligned}$$

Assuming the transversality condition that $\lim_{T\to\infty}\Big(\prod_{k=0}^{T-1}\frac{1}{1+r_k}\Big)B_T^{\text{cons}}=0$ yields:

$$B_0^{\text{cons}}=\sum_{j=1}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{1+r_k}\right)(T_j-G_j+\frac{M_j-M_{j-1}}{P_j})$$

Appendix C: Details on the VAR

C.1 Stationarity and Cointegration tests

I test whether the price–dividend ratio exhibits a unit root using Augmented Dickey–Fuller tests with a drift (mean-stationary alternative) and a Bayesian Information Criterion lag selection. In the first regime, the test rejects the unit root at a 5% level, with a test statistic of -4.56 and a critical value -2.88. In the second regime the unit root is rejected as well, with a test statistic of -2.95 and a critical value of -2.89.

With the same test, I am not able to reject that the ratio of stock market dividends-to-GDP exhibits a unit root in neither regime, with a test statistic of -1.93 and a critical value of -2.88 in the first, and a test statistic of -1.58 and a critical value of -2.89 in the second. I am also not able to reject that BoE assets-to-GDP have a unit root, with a test statistic of -0.564 and a critical value of -2.88 in the first, and a test statistic of -2.17 and a critical value of -2.89 in the second. Lastly, in the first regime ADF tests can reject a unit root in the short rate, with a test statistic of -3.44 and a critical value of -2.88. In the second regime a unit root in the short rate cannot be rejected, with a test statistic of -2.04 and a critical value of -2.89.

I perform Johansen cointegration tests for BoE dividends and GDP by first estimating the vector error correction model:

$$\Delta w_t \;=\; A\big(B'w_{t-1}+c\big)\;+\;D\,\Delta w_{t-1}\;+\;\varepsilon_t, \qquad \text{where}\quad w_t\;=\;\begin{pmatrix}\log V_t\\ \log \text{GDP}_t\end{pmatrix}$$

Such that there is no a linear trend and there is a constant in cointegration. I run trace tests at a 5% confidence level. In the first regime the trace test does not reject the null of cointegration rank 1 between BoE dividends and GDP (with a test statistic of 6.51 and a critical value of 9.24), but rejects the null of cointegration rank 0 (with a test statistic of 22.4 and a critical value of 19.96). In the second regime the test does not reject the null of cointegration rank 1 (with a test statistic of 5.47 and a critical value of 9.24), but rejects the null of cointegration rank 0 (with a test statistic of 38.9 and a critical value of 19.96).

I also run similar tests to study cointegration between stock market dividends and GDP. In the first regime the trace test does not reject the null of cointegration rank 1 between stock market dividends and GDP (with a test statistic of 9.67 and a critical value of 9.24), but rejects the null of cointegration rank 0 (with a test statistic of 22.2 and a critical value of 19.96). In the second regime the test does not reject the null of cointegration rank 1 (with a test statistic of 2.44 and a critical value of 9.24), but rejects the null of cointegration rank 0 (with a test

statistic of 30.91 and a critical value of 19.96). The non-rejection of a unit root in stock market dividends to GDP alongside Johansen evidence of cointegration indicates that the cointegrating vector may differ from $(1, -1)$.

C.2 VAR estimates and Chow test

Table 2 reports the first regime's VAR estimates for the slope matrix and Cholesky decomposition. Table 3 does so for the second regime. The zero columns for the structural shocks for stock market dividends-to-GDP ($\log d_t^M$), BoE assets-to-GDP ($\log a_t$) and BoE dividends-to-GDP ($\log v_t$) reflect the fact that innovations to these variables reside in their growth equation, leaving the level deterministic. Both VARs are stable, with the maximum eigenvalue in absolute value being 0.98 in the first regime and 0.91 in the second.

In the first regime 1845–1945 the VAR shows strong persistence in inflation, reflecting the gold standard. In both regimes central bank dividends exhibit a high volatility, especially in the second (for reference, Jiang et al. (2024) report volatiles of taxes and spending to GDP of 2.92 and 4.13). In both regimes BoE assets show a strong persistence. The large reported variance for stock market dividend growth reflects the ten-fold dividend growth of 1945.

Table 4 reports Chow tests for a structural break at 1946. At the 5% significance level, I reject the null of the lack of a break for inflation, the short rate, the stock market price-dividend ratio, BoE dividend growth (and marginally for stock market dividends to GDP, with $p = 0.0545$), as well as for the system as a whole. I do not reject it for the term spread, real GDP growth, and BoE asset growth.

Table 2 1845-1945 VAR estimates

Panel A: slope matrix $\mathbf{\Psi}$

	π_{t-1}	$y^{\$}_{t-1}(1)$	$\text{yspr}^{\$}_{t-1}$	x_{t-1}	$\Delta \log d^M_{t-1}$	$\log d^M_{t-1}$	pd^M_{t-1}	$\Delta \log a_{t-1}$	$\log a_{t-1}$	$\Delta \log v_{t-1}$	$\log v_{t-1}$
π_t	**0.68**	-0.48	0.12	**0.45**	**0.05**	**-0.03**	-0.04	**0.13**	0.01	**-0.15**	0.03
$y^{\$}_t(1)$	0.04	**1.01**	**0.65**	**0.08**	**0.01**	**-0.02**	0.00	0.00	-0.01	-0.01	-0.00
$\text{yspr}^{\$}_t$	-0.01	-0.06	0.36	-0.06	**-0.01**	**0.01**	-0.00	0.00	0.01	0.02	0.01
x_t	-0.05	-0.84	-0.34	0.18	0.02	-0.01	**-0.05**	0.01	**0.03**	0.02	**0.03**
$\Delta \log d^M_t$	-0.48	2.42	5.60	*-1.31*	**-0.36**	-0.11	-0.22	-0.32	0.09	0.42	0.13
$\log d^M_t$	-0.48	2.42	5.60	*-1.31*	**-0.36**	0.89	-0.22	-0.32	0.09	0.42	0.13
pd^M_t	**-0.74**	-1.07	-3.27	**-0.78**	-0.06	**0.11**	**0.79**	-0.08	0.00	-0.17	-0.02
$\Delta \log a_t$	**-0.87**	**5.26**	3.83	0.10	-0.10	**0.12**	0.09	*-0.21*	**-0.09**	-0.06	-0.07
$\log a_t$	**-0.87**	**5.26**	3.83	0.10	-0.10	**0.12**	0.09	*-0.21*	**0.91**	-0.06	-0.07
$\Delta \log v_t$	**-1.03**	**2.73**	1.01	-0.39	-0.03	0.03	**0.29**	**-0.18**	**-0.09**	0.13	**-0.17**
$\log v_t$	**-1.03**	**2.73**	1.01	-0.39	-0.03	0.03	**0.29**	**-0.18**	**-0.09**	0.13	**0.83**

Panel B: $100 \times \mathbf{\Sigma}^{1/2}$

	$\boldsymbol{\varepsilon}^{\pi}_t$	$\boldsymbol{\varepsilon}^{y^{\$}(1)}_t$	$\boldsymbol{\varepsilon}^{yspr^{\$}}_t$	$\boldsymbol{\varepsilon}^{x}_t$	$\boldsymbol{\varepsilon}^{\Delta \log d^M}_t$	$\boldsymbol{\varepsilon}^{d^M}_t$	$\boldsymbol{\varepsilon}^{pd^M}_t$	$\boldsymbol{\varepsilon}^{\Delta \log a}_t$	$\boldsymbol{\varepsilon}^{\log a}_t$	$\boldsymbol{\varepsilon}^{\Delta \log v}_t$	$\boldsymbol{\varepsilon}^{\log v}_t$
π_t	3.21	0	0	0	0	0	0	0	0	0	0
$y^{\$}_t(1)$	0.33	0.83	0	0	0	0	0	0	0	0	0
$\text{yspr}^{\$}_t$	-0.27	-0.81	0.13	0	0	0	0	0	0	0	0
x_t	-0.15	0.37	-0.42	2.65	0	0	0	0	0	0	0
$\Delta \log d^M_t$	-2.72	-3.76	3.47	-1.20	16.18	0	0	0	0	0	0
$\log d^M_t$	-2.72	-3.76	3.47	-1.20	16.18	0	0	0	0	0	0
pd^M_t	0.21	-1.61	-3.63	0.82	-1.17	0	6.74	0	0	0	0
$\Delta \log a_t$	-3.70	-1.22	0.66	-3.09	1.25	0	0.92	8.27	0	0	0
$\log a_t$	-3.70	-1.22	0.66	-3.09	1.25	0	0.92	8.27	0	0	0
$\Delta \log v_t$	-2.12	2.19	-0.08	-2.52	1.53	0	-1.83	-0.48	0	4.26	0
$\log v_t$	-2.12	2.19	-0.08	-2.52	1.53	0	-1.83	-0.48	0	4.26	0

Note: Panel A reports my estimate of the VAR slope matrix for the 1845-1945 regime. Numbers in bold have t-statistics greater than 1.96 in absolute value. Numbers in italics have t-statistics between 1.645 and 1.96. Panel B reports my estimate of the VAR innovation matrix.

Table 3 1946-2024 VAR estimates

Panel A: slope matrix $\mathbf{\Psi}$

	π_{t-1}	$y^{\$}_{t-1}(1)$	$\text{yspr}^{\$}_{t-1}$	x_{t-1}	$\Delta \log d^M_{t-1}$	$\log d^M_{t-1}$	pd^M_{t-1}	$\Delta \log a_{t-1}$	$\log a_{t-1}$	$\Delta \log v_{t-1}$	$\log v_{t-1}$
π_t	**0.50**	0.30	0.50	**0.25**	0.01	*-0.03*	*-0.03*	-0.01	0.01	0.01	-0.01
$y^{\$}_t(1)$	**0.11**	**1.02**	*0.27*	**0.26**	0.00	0.00	*0.01*	-0.01	*0.01*	-0.00	-0.00
$\text{yspr}^{\$}_t$	-0.01	-0.05	**0.66**	**-0.10**	-0.01	-0.00	-0.00	0.00	0.00	-0.00	0.00
x_t	-0.05	0.04	0.48	0.11	0.00	0.00	0.02	-0.00	0.01	**-0.02**	0.00
$\Delta \log d^M_t$	**-1.20**	-1.01	-1.61	**1.15**	-0.09	**-0.25**	*-0.12*	-0.05	-0.04	0.00	0.00
$\log d^M_t$	**-1.20**	-1.01	-1.61	**1.15**	-0.09	**0.75**	*-0.12*	-0.05	-0.04	0.00	0.00
pd^M_t	**-2.68**	1.31	0.54	**-4.11**	-0.10	-0.07	**0.59**	0.03	-0.07	*0.09*	-0.02
$\Delta \log a_t$	*1.78*	-0.77	*-5.44*	1.63	-0.15	0.06	-0.00	0.02	**-0.15**	-0.05	-0.07
$\log a_t$	*1.78*	-0.77	*-5.44*	1.63	-0.15	0.06	-0.00	0.02	**0.85**	-0.05	-0.07
$\Delta \log v_t$	**3.60**	**10.67**	*8.64*	**11.80**	-0.36	0.31	0.29	-0.19	**0.48**	-0.09	**-0.41**
$\log v_t$	**3.60**	**10.67**	*8.64*	**11.80**	-0.36	0.31	0.29	-0.19	**0.48**	-0.09	**0.59**

Panel B: $100 \times \mathbf{\Sigma}^{1/2}$

	ε^{π}_t	$\varepsilon^{y^{\$}(1)}_t$	$\varepsilon^{yspr^{\$}}_t$	ε^{x}_t	$\varepsilon^{\Delta \log d^M}_t$	$\varepsilon^{d^M}_t$	$\varepsilon^{pd^M}_t$	$\varepsilon^{\Delta \log a}_t$	$\varepsilon^{\log a}_t$	$\varepsilon^{\Delta \log v}_t$	$\varepsilon^{\log v}_t$
π_t	2.25	0	0	0	0	0	0	0	0	0	0
$y^{\$}_t(1)$	0.23	0.88	0	0	0	0	0	0	0	0	0
$\text{yspr}^{\$}_t$	0.01	-0.51	0.44	0	0	0	0	0	0	0	0
x_t	-0.85	0.82	-0.11	2.01	0	0	0	0	0	0	0
$\Delta \log d^M_t$	-1.81	-0.09	-0.61	-1.65	9.90	0	0	0	0	0	0
$\log d^M_t$	-1.81	-0.09	-0.61	-1.65	9.90	0	0	0	0	0	0
pd^M_t	-3.12	-4.28	-5.92	1.63	-1.28	0	14.76	0	0	0	0
$\Delta \log a_t$	0.00	1.94	-1.01	1.01	-1.23	0	-4.06	17.42	0	0	0
$\log a_t$	0.00	1.94	-1.01	1.01	-1.23	0	-4.06	17.42	0	0	0
$\Delta \log v_t$	2.28	4.82	2.09	8.90	4.73	0	-4.13	-2.27	0	27.83	0
$\log v_t$	2.28	4.82	2.09	8.90	4.73	0	-4.13	-2.27	0	27.83	0

Note: Panel A reports my estimate of the VAR slope matrix for the 1946-2024 regime. Numbers in bold have t-statistics greater than 1.96 in absolute value. Numbers in italics have t-statistics between 1.645 and 1.96. Panel B reports my estimate of the VAR innovation matrix.

Table 4 Chow tests for structural break at 1946

Variable	F-stat	p-value
π_t	2.67	0.00
$y^{\$}_t(1)$	2.25	0.01
$\text{yspr}^{\$}_t$	1.33	0.21
x_t	1.07	0.39
$\Delta \log d^M_t$	1.79	0.05
pd^M_t	2.30	0.01
$\Delta \log a_t$	1.36	0.19
$\Delta \log v_t$	10.49	0.00
System (all growth equations)	5.47	0.00

Appendix D: Valuation based on Campbell-Shiller decompositions

Let P_t^V be the ex-dividend price at the end of period t of a perpetual claim on the central bank's dividends V_t. Defining the log price–dividend ratio on this claim as $pd_t^V \equiv \log P_t^V - \log V_t$, the log return follows:

$$\begin{aligned} r_{t+1}^V &= \log\Big(\frac{P_{t+1}^V + V_{t+1}}{P_t^V}\Big) \\ &= \log\Big(1 + \exp(pd_{t+1}^V)\Big) + \log\Big(\frac{V_{t+1}}{V_t}\Big) - pd_t^V \end{aligned}$$

Taylor expanding the first term to first order around the unconditional mean pd_0^V gives:

$$r_{t+1}^V \approx \kappa_0^V + \kappa_1^V\, pd_{t+1}^V + \Delta\log V_{t+1} - pd_t^V$$

With

$$\kappa_1^V = \frac{\exp(pd_0^V)}{1+\exp(pd_0^V)}, \qquad \kappa_0^V = \log(1+\exp(pd_0^V)) - \kappa_1^V\, pd_0^V$$

Putting pd_t^V on the left-hand side:

$$pd_t^V \approx \kappa_0^V + \kappa_1^V\, pd_{t+1}^V + \Delta\log V_{t+1} - r_{t+1}^V$$

Iterating forward, taking expectations, and taking the limit to infinity along with imposing the no-bubble condition that $\lim_{k\to\infty}(\kappa_1^V)^k pd_{t+k}^V = 0$ yields the familiar Campbell-Shiller expression:

$$\begin{aligned} pd_t^V &\approx \kappa_0^V \sum_{j=0}^{k-1}(\kappa_1^V)^j + \sum_{j=1}^{k}(\kappa_1^V)^{j-1}\,\Delta\log V_{t+j} - \sum_{j=1}^{k}(\kappa_1^V)^{j-1}\, r_{t+j}^V + (\kappa_1^V)^k pd_{t+k}^V \\ &= \frac{\kappa_0^V}{1-\kappa_1^V} + \underbrace{\sum_{j=1}^{\infty}(\kappa_1^V)^{j-1}\,\mathbb{E}_t[\Delta\log V_{t+j}]}_{\substack{\text{cash-flow term} \\ CF_t^V}} - \underbrace{\sum_{j=1}^{\infty}(\kappa_1^V)^{j-1}\,\mathbb{E}_t[r_{t+j}^V]}_{\substack{\text{discount-rate term} \\ DR_t^V}} . \end{aligned} \tag{D1.1}$$

D.1 Constant risk premia:

Assuming constant risk premia one can make further progress. Denoting the risk premium on the claim to the central bank's dividends as rp_t^V, one can define the claim's expected return as:

$$\mathbb{E}_t[r_{t+1}^V] = y_t(1) + yspr_t + rp_t^V \tag{D1.2}$$

Since both the 1-year yield and the spread are part of the VAR, we can leverage the slope matrix to build out the expectation of the return, as $\mathbb{E}_t[z_{t+j}] = \mathbf{\Psi}^j z_t$. Assuming constant risk premia, denoted by rp_0^V, and denoting the column selector vectors for the short rate and the yield spread as $\mathbf{e}_{yn}$ and $\mathbf{e}_{yspr}$ the expected return follows:

$$\mathbb{E}_t\left[r^V_{t+j}\right] = y^{\$}_0(1) + yspr^{\$}_0 + rp^V_0 + \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)' \mathbf{\Psi}^{j-1} z_t$$

Plugging this into the expression for the discount rate term (DR^V_t):

$$DR^V_t = \mathbb{E}_t\Big[\sum_{j=1}^{\infty} (\kappa^V_1)^{j-1} r^V_{t+j}\Big] = \frac{y^{\$}_0(1) + yspr^{\$}_0 + rp^V_0}{1-\kappa^V_1} \; + \; \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)' (I - \kappa^V_1 \mathbf{\Psi})^{-1} z_t$$

To make progress on the cash flow term (CF^V_t), first note that $\Delta \log V_{t+1} = \Delta \log v_{t+1} + x_{t+1} + \pi_{t+1}$, where all of the variables on the right-hand side are in the state vector. As a result:

$$\mathbb{E}_t[\Delta \log V_{t+j}] = \mu_v + x_o + \pi_0 + \left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\right)' \mathbf{\Psi}^j z_t$$

So that the cash flow term becomes:

$$CF^V_t = \sum_{j=1}^{\infty} (\kappa^V_1)^{j-1} \mathbb{E}_t\Big[\Delta \log V_{t+j}\Big] = \frac{x_0 + \pi_0 + \mu_v}{1-\kappa^V_1} \; + \; \left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\right)' \mathbf{\Psi} (I - \kappa^V_1 \mathbf{\Psi})^{-1} z_t$$

Where $\mathbf{e}_{\Delta v}$ selects the central bank dividends-to-GDP growth rate in the state vector. Putting the cash flow and discount rate terms together yields the following expression for the price-dividend ratio on the central bank dividend claim:

$$\begin{aligned} pd^V_t &= \frac{\kappa^V_0 + x_0 + \pi_0 + \mu_v - y^{\$}_0(1) - yspr^{\$}_0 - rp^V_0}{1-\kappa^V_1} + \left(\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\right)' \mathbf{\Psi} - \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)'\right)(I - \kappa^V_1 \mathbf{\Psi})^{-1} z_t \\ &= pd^V_0 + \left(\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\right)' \mathbf{\Psi} - \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)'\right)(I - \kappa^V_1 \mathbf{\Psi})^{-1} z_t. \end{aligned} \tag{D1.3}$$

Grouping the constant terms as pd^V_0, we know it will solve:

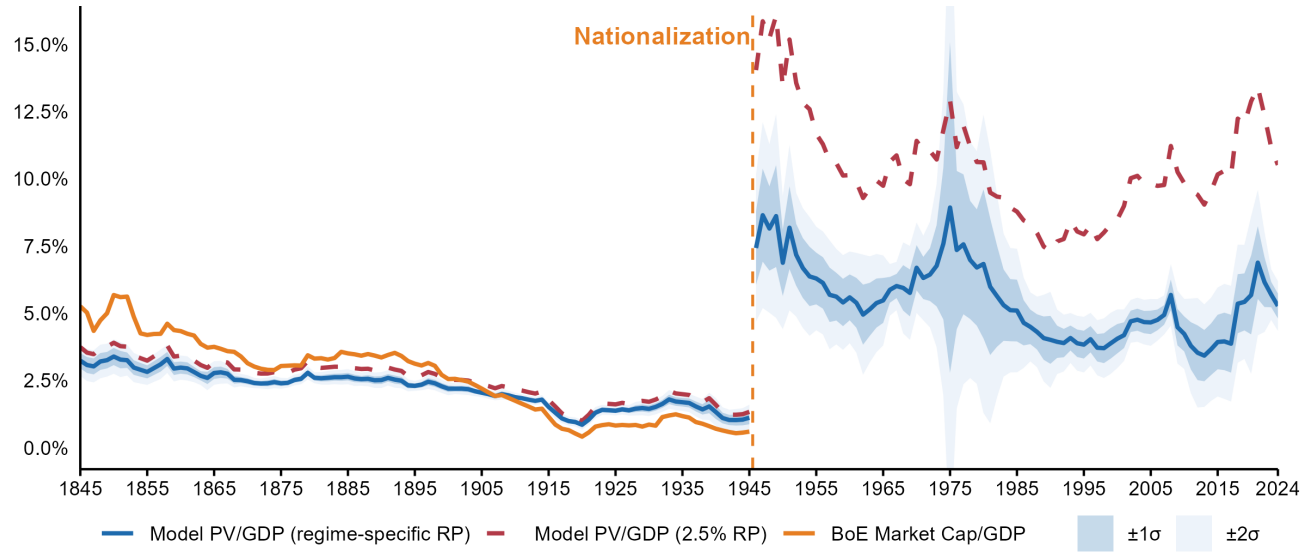
$$\begin{aligned} pd^V_0 &= \frac{\kappa^V_0 + x_0 + \pi_0 + \mu_v - y^{\$}_0(1) - yspr^{\$}_0 - rp^V_0}{1-\kappa^V_1} \\ \kappa^V_1 &= \frac{\exp(pd^V_0)}{1+\exp(pd^V_0)} \\ \kappa^V_0 &= \log(1+\exp(pd^V_0)) - \kappa^V_1 pd^V_0 \end{aligned} \tag{D1.4}$$

Equation (D1.3) delivers the log price-dividend ratio used to price the central bank in Section 4.1. Under the baseline estimation, the risk premium on the central bank is set to that on GDP. In turn, the risk premium on GDP is determined by the unlevered risk premium on equity.

$$\begin{aligned} rp^V_t &= rp^Y_t = (1-l_t) rp^M_t \\ rp^V_0 &= rp^Y_0 = \frac{1}{T}\sum_{t=1}^{T} (1-l_t)\, rp^M_t \end{aligned}$$

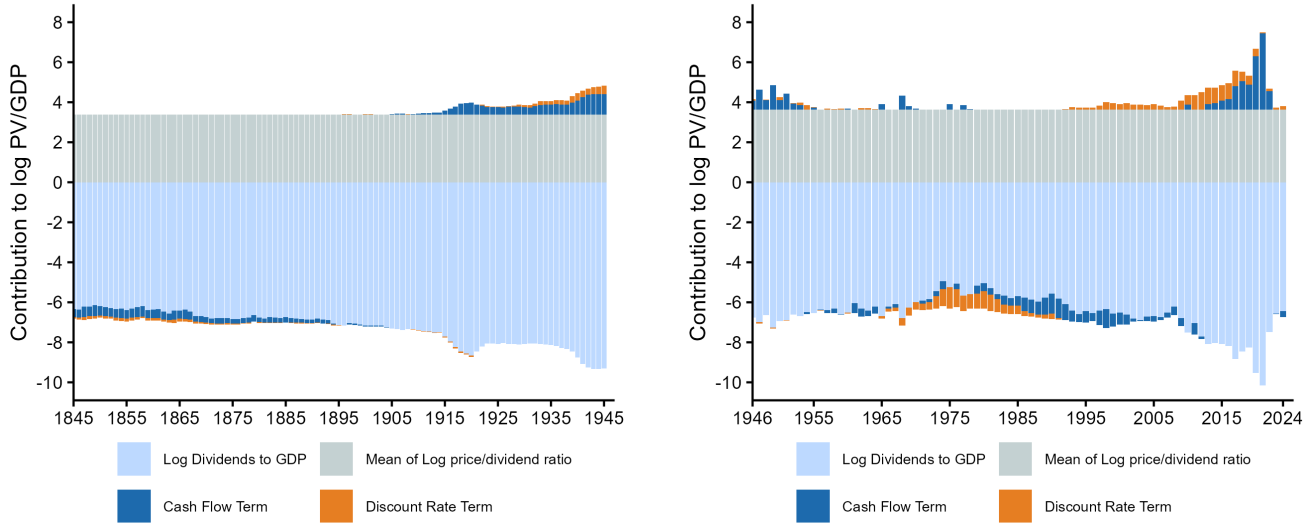
The derivation of the market risk premium rp_t^M is detailed in the following subsection. By the cointegrating relationship of central bank dividends and GDP, the mean growth of dividends to GDP is set to zero ($\mu_v = 0$). Section 4.1 presented the results separated by regime. For illustration Figure 8 presents the entire time series. Taking the log of the valuation expression, $\log(\frac{PV_t^V}{Y_t}) = \log(\frac{V_t}{Y_t}) + pd_0^V + CF_t^V - DR_t^V$, Figure 14 presents the contribution of each of the terms from the Campbell-Shiller decomposition to the final valuation displayed in Figure 8.

Figure 13 Present value of the Bank of England under a constant risk premium (% of GDP)



Note: The average values of the central bank under constant risk premia are 2.2% and 5.5% of GDP, respectively for the first and second regime (2.5% and 10.5% under an assumed risk premium of 2.5%). Confidence bands are obtained by resampling the residuals from the VAR to simulate 50,000 pseudo-samples and re-estimating the VAR.

Figure 14 Terms of the Campbell-Shiller decomposition under constant risk premia



Note: Taking logs of the valuation equation, $\log(PV_t^{CB}/Y_t) = \log(V_t/Y_t) + pd_t^V$, the figure shows annual contributions to $\log(PV_t^{CB}/Y_t)$ from: (i) $\log(V_t/Y_t)$, (ii) the mean of the log price-dividend ratio pd_0^V, (iii) the Campbell–Shiller cashflow and discount rate terms.

D.2 Time varying risk premia:

To obtain time-varying risk premia on the central bank, I also apply the convention that a claim to GDP is equivalent to an unlevered claim to equity (Abel 1999, Campbell 2003), but do so in the time series. First, I keep (D1.2) as the expression for the risk premium on the central bank, so that rp_t^V is constant into the future but time varying for each t. Building out the expectation terms from equation (D1.2) as before delivers:

$$\begin{aligned} pd_t^V &= \frac{\kappa_0^V}{1-\kappa_1^V} + \sum_{j=1}^{\infty}(\kappa_1^V)^{j-1}\,\mathbb{E}_t\big[\Delta \log V_{t+j}\big] - \sum_{j=1}^{\infty}(\kappa_1^V)^{j-1}\,\mathbb{E}_t\big[r_{t+j}^V\big] \\ &= \frac{\kappa_0^V + x_0 + \pi_0 + \mu_v}{1-\kappa_1^V} + \big(\big(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\big)'\boldsymbol{\Psi} - \big(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\big)'\big)(I - \kappa_1^V\boldsymbol{\Psi})^{-1}\,z_t - \frac{y_0^{\$}(1) + yspr_0^{\$}}{1-\kappa_1^V} - \frac{rp_t^V}{1-\kappa_1^V} \end{aligned}$$

Grouping terms into the expression for pd_0^V from equation (D1.4) yields:

$$pd_t^V = pd_0^V + \big(\big(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\big)'\boldsymbol{\Psi} - \big(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\big)'\big)(I - \kappa_1^V\boldsymbol{\Psi})^{-1}\,z_t - \frac{rp_t^V - rp_0^V}{1-\kappa_1^V}$$

Assuming that the risk premium on the central bank is equal to that on a claim to GDP, and that the risk premium on a claim to GDP is equal to that on unlevered equity entails:

$$\begin{aligned} rp_t^V &= rp_t^Y = (1-l_t)rp_t^M \\ rp_0^V &= rp_0^Y = \frac{1}{T}\sum_{t=1}^{T}(1-l_t)\,rp_t^M \end{aligned}$$

where l_t is the corporate leverage of the economy (corporate debt over assets) and l_0 is its mean. With this, the log price-dividend ratio on the central bank under time-varying risk premia is:

$$pd_t^V = pd_0^V + \big(\big(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta v}\big)'\boldsymbol{\Psi} - \big(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\big)'\big)(I - \kappa_1^V\boldsymbol{\Psi})^{-1}\,z_t - \frac{(1-l_t)rp_t^M - rp_0^Y}{1-\kappa_1^V}$$

To obtain the market's risk premium (rp_t^M and rp_0^M), consider the standard Campbell & Shiller (1988) linearization of the market's log return.

$$r_{t+1}^M = \log\Big(\frac{P_{t+1}^M + D_{t+1}}{P_t^M}\Big)$$

Such that the log price-dividend ratio of the market follows:

$$pd_t^M = \frac{\kappa_0^M}{1-\kappa_1^M} + \underbrace{\sum_{j=1}^{\infty}(\kappa_1^M)^{j-1}\,\mathbb{E}_t\big[\Delta \log D_{t+j}\big]}_{\substack{\text{cash-flow term} \\ CF_t^M}} - \underbrace{\sum_{j=1}^{\infty}(\kappa_1^M)^{j-1}\,\mathbb{E}_t\big[r_{t+j}^M\big]}_{\substack{\text{discount-rate term} \\ DR_t^M}}.$$

Where

$$\begin{aligned}\kappa_1^M &= \frac{\exp(pd_0^M)}{1+\exp(pd_0^M)},\\ \kappa_0^M &= \log(1+\exp(pd_0^M)) - \kappa_1^M pd_0^M,\end{aligned}$$

and, unlike with the claim to central bank dividends, pd_0^M comes from the data. Building the expectation terms with the VAR, and assuming the risk premium on the market is given by $\mathbb{E}_t\left[r_{t+1}^M\right] = y_t(1) + yspr_t + rp_t^M$, I obtain:

$$pd_t^M = \frac{\kappa_0^M + x_0 + \pi_0 + \mu_d}{1-\kappa_1^M} + \left(\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta d}\right)'\mathbf{\Psi} - \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)'\right)\left(I - \kappa_1^M\mathbf{\Psi}\right)^{-1} z_t - \frac{y_0^\$(1) + yspr_0^\$}{1-\kappa_1^M} - \frac{rp_t^M}{1-\kappa_1^M}$$

Evaluating this expression at the mean of the state vector (zero) delivers the mean log price-dividend ratio of the market:

$$pd_0^M = \frac{\kappa_0^M + x_0 + \pi_0 + \mu_d}{1-\kappa_1^M} - \frac{y_0^\$(1) + yspr_0^\$}{1-\kappa_1^M} - \frac{rp_0^M}{1-\kappa_1^M}$$

Plugging this into the expression for pd_t^M yields:

$$pd_t^M = pd_0^M + \left(\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta d}\right)'\mathbf{\Psi} - \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)'\right)\left(I - \kappa_1^M\mathbf{\Psi}\right)^{-1} z_t - \frac{rp_t^M - rp_0^M}{1-\kappa_1^M}$$

Solving out for the deviation of the market's risk premium from its mean, and leveraring the fact that the log price-dividend ratio of the market is a part of the state vector yields:

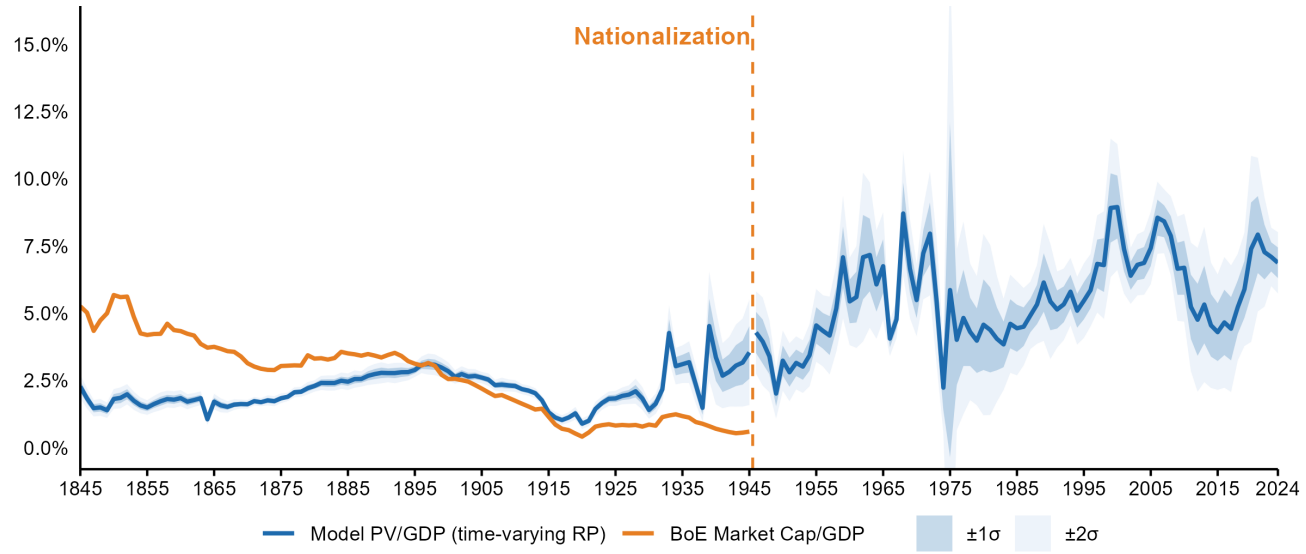
$$\begin{aligned}rp_t^M &= rp_0^M + (1-\kappa_1^M)\left[pd_0^M - pd_t^M + \left(\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta d}\right)'\mathbf{\Psi} - \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)'\right)\left(I - \kappa_1^M\mathbf{\Psi}\right)^{-1} z_t\right]\\ &= rp_0^M + (1-\kappa_1^M)\left[-\mathbf{e}_{pd}' z_t + \left(\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{e}_{\Delta d}\right)'\mathbf{\Psi} - \left(\mathbf{e}_{yn} + \mathbf{e}_{yspr}\right)'\right)\left(I - \kappa_1^M\mathbf{\Psi}\right)^{-1} z_t\right]\end{aligned}$$

Note that the mean equity risk premium rp_0^M can be obtained by solving out for it in the expression for pd_0^M:

$$rp_0^M = \kappa_0^M + x_0 + \pi_0 + \mu_d - y_0^\$(1) - yspr_0^\$ - (1-\kappa_1^M)\, pd_0^M$$

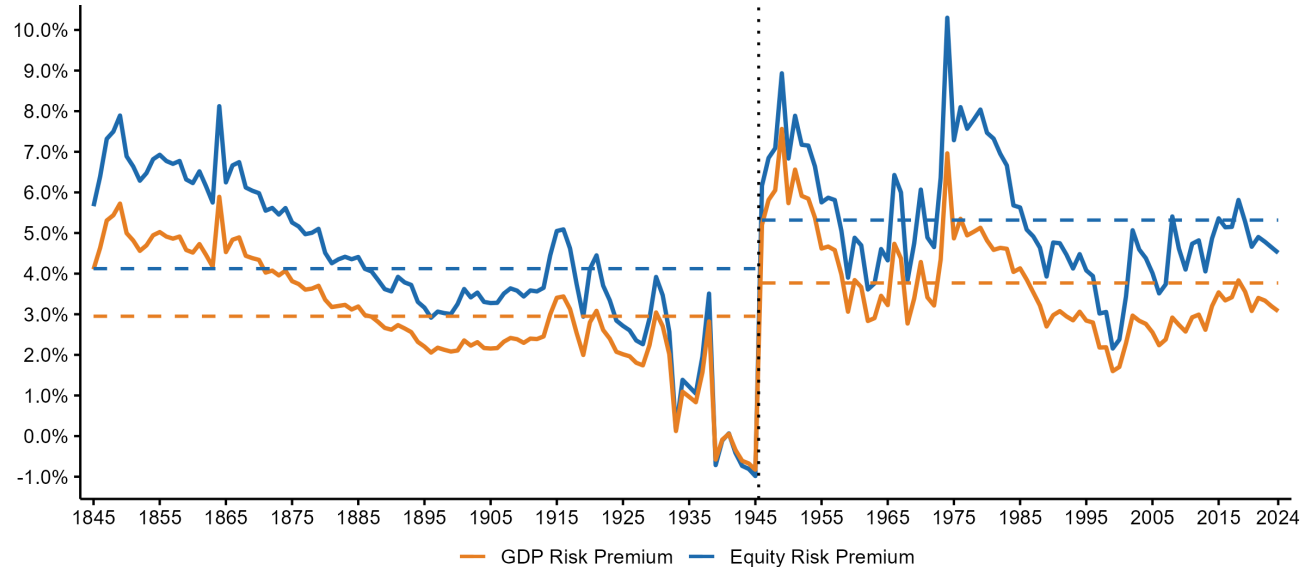
Equation (D1.3) delivers the log price-dividend ratio used to price the central bank in Section 4.1. By the cointegrating relationship of central bank dividends and GDP, the mean growth of dividends to GDP is set to zero ($\mu_v = 0$). Figure 15 presents the results. Figure 16 reports the risk premia used in the estimation, while Figure 17 reports the gap between the results using constant and time varying risk premia.

Figure 15 Present value of the Bank of England under time-varying risk premia (% of GDP)



Note: The average values of the central bank under time-varying risk premia are 2.2% and 5.6% of GDP, respectively for the first and second regime. Confidence bands are obtained by resampling the residuals from the VAR to simulate 50,000 pseudo-samples and re-estimating the VAR. Draws are accepted only if the VAR is stable and the market equity risk-premium path is non-negative (this last condition is not imposed in the years where realized equity risk premia were negative). The time series of leverage is not resampled.

Figure 16 Risk premia on the central bank (on GDP) and on market equity

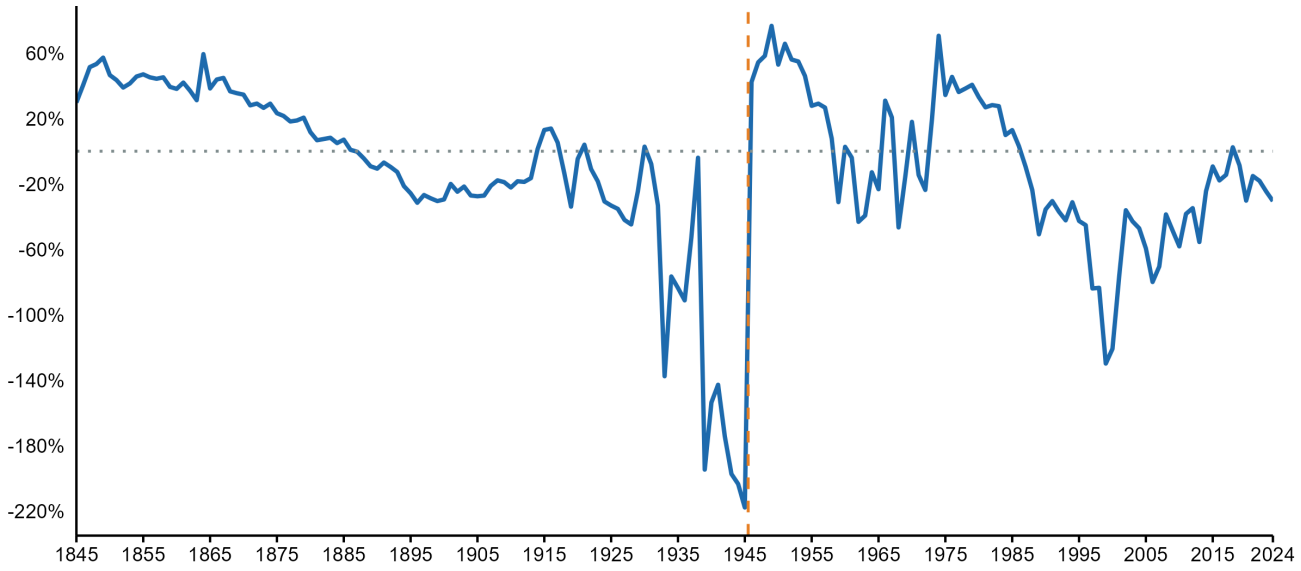


Note: Risk premia used in the Campbell-Shiller estimation with time-varying risk premia. The average risk premia on GDP are 2.95% and 3.77% respectively for each regime.

D.3 Rolling window VAR:

In an effort to price the BoE only with information available to agents at each point time, I re-estimate the VAR every year using only data up to that year, and compute the respective means and constant GDP risk premium. This procedure inevitably leads to a loss of observations due to the degrees-of-freedom constraint, while small samples also result in unstable estimates for the slope matrix. In the first regime, 20% of years

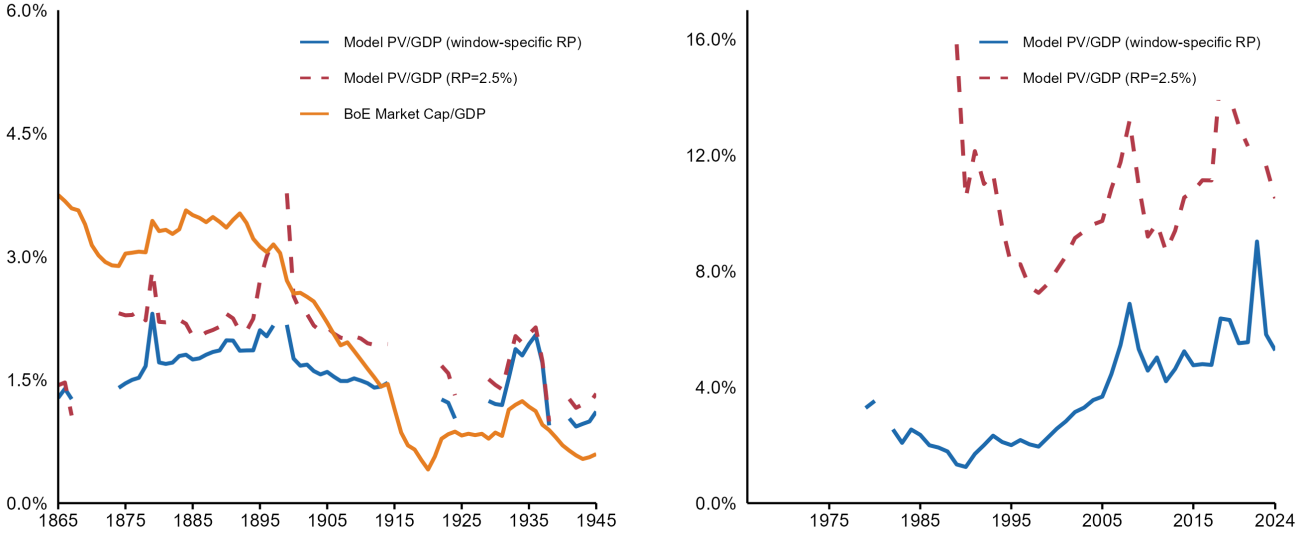
Figure 17 Percentage difference in value under constant and time-varying risk premia



Note: The gap is computed as $(PV_t^{V,\text{constant}} - PV_t^{V,\text{time-varying}})/PV_t^{V,\text{constant}}$.

cannot be estimated and the VAR is unstable in 19% of them; in the second regime, 25% of years are lost and instability affects 18%. Figure 18 reports the results.

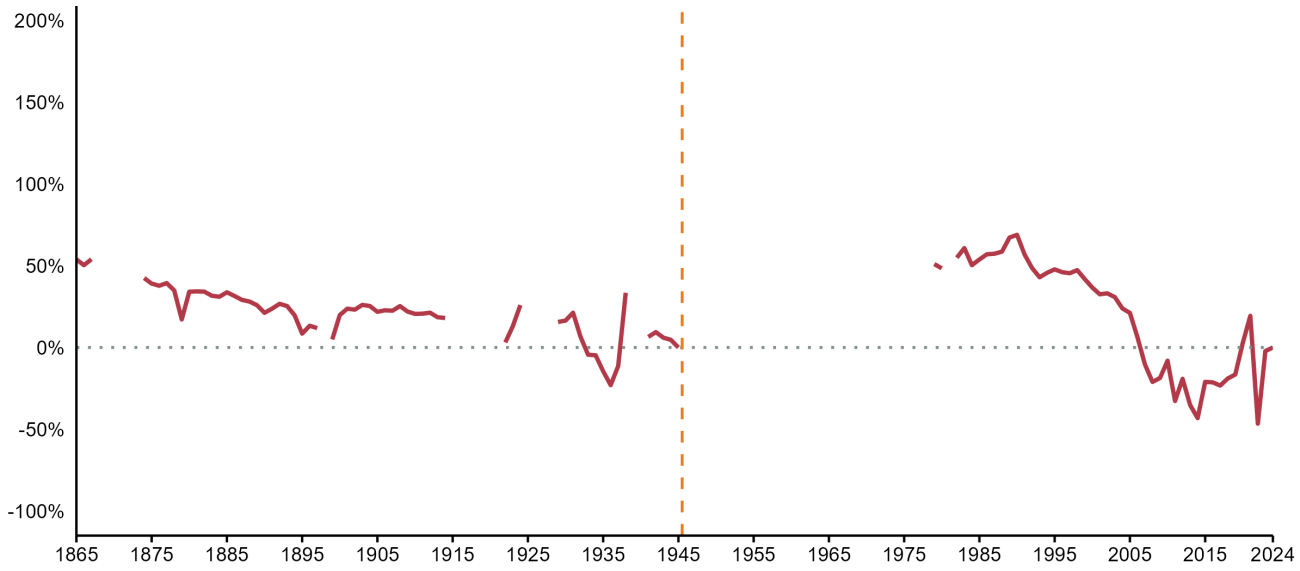
Figure 18 Present value of the Bank of England under a rolling window VAR and constant risk premia



Note: The average values of the central bank under the rolling window VAR are 1.6% and 4.0% of GDP, respectively for the first and second regime (2.0% and 10.7% under the assumed risk premium of 2.5%). Years were the estimated VAR was not stable are not displayed.

In the first regime, this exercise results in an average constant risk premium of 3.33%, an average (average) price-dividend ratio on the central bank of 29.6 and an average value of 1.6% of GDP. The lower valuations early in the sample are largely due to the fact that short rates and stock market returns (which determine the GDP risk premium) were comparatively high. In the second regime, it delivers an average constant risk premium of 3.7%, an average (average) price-dividend ratio on the central bank of 30.3, and an average value of 3.4% of GDP.

Figure 19 Percentage difference in value under benchmark and rolling window VARs (constant risk premia)



Note: The gap is computed as $(PV_t^{V,\text{constant}} - PV_t^{V,\text{rolling VAR, constant}})/PV_t^{V,\text{constant}}$.

In the second regime, it also delivers lower valuations due to the high rates resulting from the Great Inflation taking up a larger fraction of the data.

Appendix E: Details on the full asset pricing model

This Appendix details the procedure to estimate the stochastic discount factor. It details the no-arbitrage model for the SDF similar to those in Duffie & Kan (1996), Dai & Singleton (2000) and Ang & Piazzesi (2003). The appendix closely follows Appendix D and E of Jiang et al. (2024), with a few modifications due to my setting.

To begin, I postulate a log SDF of the following form:

$$m_{t+1}^{\$} = -y_t^{\$}(1) - \tfrac{1}{2}\,\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1}.$$

where the SDF is $M_t^{\$}$, $m_t^{\$} = \exp(M_t^{\$})$, and $\boldsymbol{\Lambda}_t$ are the market prices of risk such that

$$\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 z_t.$$

$\boldsymbol{\Lambda}_0$ and $\boldsymbol{\Lambda}_1$ are, respectively, a vector and a square matrix with dimensions consistent with the state vector. The estimation of the SDF consists on estimating the market price of risk parameters, so that the model-implied real and nominal yield curves and stock prices match their empirical counterparts, also generating an equity risk premium. Equities in the SDF model will be particularly valuable for learning about the price of permanent growth risk. The model's implications for the term structure of risk seek to adequately price the short- and long-run risks of the central bank's dividends.

The objective function is detailed in Section E.1, the results in Section **??**. The models for nominal zero-coupon bonds, real zero-coupon bonds, the dividend claim on aggregate equity, dividend strips, a GDP strip, and the strip on central bank dividends, which are used to build the objective function, are outlined in Section E.3, and in almost all cases rely on affine recursions.

I estimate a separate SDF for each of the two regimes. The estimation proceeds in three steps. In each step, I minimize a scalar objective that aggregates penalty terms for the moment-matching targets and the no-arbitrage and regularity conditions listed below. The resulting unconstrained problem is solved with a standard Nelder–Mead algorithm. To mitigate sensitivity to local minima, I run an outer loop that restarts the solver from the previous iteration, which helps the solver escape shallow local minima.

1. In the first step, I shut down time variation in the risk prices by setting all entries in the $\mathbf{\Lambda}_1$ matrix to zero. Given the small dimension of the Λ_0 vector, this first step is fast.

2. In the second step, I introduce time-varying risk prices by allowing nonzero values in the $\mathbf{\Lambda}_1$ matrix. I hold fixed the unconditional risk prices for inflation, the term spread, and GDP growth (the first, third, and fourth elements of $\mathbf{\Lambda}_0$), and solve for the remaining prices of risk. The moments used are adjusted slightly. As the $\mathbf{\Lambda}_1$ matrix is two-dimensional, this step is slower. I iterate over step 2 until the value function converges (30 times). This step serves to provide a good starting point for the following.

3. In the third term, all risk price parameters are re-estimated as free variables, iterating on the solution until the value function converges (30 times).

Once estimated, the priced shocks are inflation, the term spread, GDP growth, stock market dividend growth, and the price-dividend ratio. I make a few brief points on the data. For the 1845-1915 period the 1-year yield is taken from the 3-month yields of commercial paper and (from 1900) government bonds. Chen et al. (2025) also make this assumption. I also assume that consols (the U.K.'s perpetual government bonds) have a maturity of 30 years. In the empirical counterpart of stock returns, the in-sample mean of stock market dividend-to-GDP growth is used; everywhere else in the model, and motivated by the cointegration of stock market dividends and GDP, the mean growth of dividends-to-GDP (μ_d) is set to zero. This is how Jiang et al. (2024) proceed.

E.1 Target moments

The main set of moments are the following. They are the only moments used in the first estimation step:

1. *No arbitrage restriction on the long maturity bond:* Given that the observed yield spread of long-maturity bond is one of the elements of the state vector, I ensure that the model-implied yield on the long-term bond (of maturity τ) matches the data. In the first regime this is the consol yield, which is treated as a 30-year bond. In the second it is the 10-year yield. Equation (E3.2) provides the relevant expression for the yield in the

model. In the first step with constant prices of risk, I only consider the unconditional moment:

$$-\frac{A^{\$}(\tau)}{\tau} = y_0^{\$}(\tau)$$

2. *Nominal and real yield curve:* I match nominal and real yields for a set of tenors. In the first break I target the 1, 2, 5 and 30-year maturities; I target less maturities so as to not over-rely on the estimated yield curve of Ellison & Scott (2020). In the second regime I target the nominal yields of maturities $1, 2, 5, 10, 20$ and 30, and the real yields (from the U.K.'s inflation-indexed gilts) of maturities $5, 7, 10, 20$ and 30. Equations (E3.2) and (E3.3) provide the relevant expressions for the model's yields.

3. *No arbitrage restriction on stock market:* I ensure that the model-implied stock returns match the VAR-implied stock returns, given that the price–dividend ratio and the dividend growth rate of the aggregate stock market are in the state vector. In this first step, I only consider the unconditional moment, matching the model-implied unconditional equity risk premium to its empirical counterpart. Equation (E3.4) provides the relevant unconditional no arbitrage restriction.

4. *Equity price–dividend ratio:* I match the time series of the stock market's price–dividend ratio in model and data. Equation (E3.6) provides the restriction.

5. *Equity term structure:* To match the equity term structure Jiang et al. (2024) match the price-dividend ratios of 1 and 2-year dividend strips as well as the unconditional risk premium on the dividend futures averaged across the first 7 annual strips. However, there is no readily available data for U.K. dividend-strip price–dividend ratios; only average dividend-futures returns for 2005–2014 are available from van Binsbergen & Koijen (2017). Future work could match the equity term structure using the methodology of Giglio et al. (2024), which requires only cross-sectional returns and dividend yields. The restriction is detailed in Equation (E3.7). Due to a lack of data, this condition is omitted in the first structural break.

6. *GDP risk premium:* Given the cointegrating relationship between central bank dividends and GDP, the price of risk for the GDP shocks is the key variable which will drive the risk premia of the claim to central bank's dividends. I assume that the claim on GDP claim is equivalent to an unlevered equity claim. I compute the unlevered corporate excess return by multiplying the equity excess return with one minus the aggregate corporate leverage ratio. In my samples, the average leverage ratio (i.e., debt/total asset) are 27.5% and 29.5% respectively. I match the GDP strip's risk premium to the equity strip's risk premium times one minus the leverage ratio for strips of horizons 1–10, 20, 30, . . . , 90 years. Equation (E3.8) details the relevant condition.

7. *Good deal bound:* I compute the realized nominal log SDF ($m^{\$}$) implied by the model and, in the spirit of Cochrane & Saa-Requejo (2000), impose a penalty for its standard deviation being in excess of 2.

8. *Regularity conditions:* For tenors 100, 200, 400, 700, 1000, 2000, 3000, 4000 years, I add penalties to (i) ensure the GDP strip's nominal discount rate (its unconditional expected return) stays above the average nom-

inal GDP growth, (ii) ensure the unconditional nominal–real spread stays above average inflation across maturities, (iii) ensure the unconditional bond risk premium stays below the GDP risk premium, (iv) ensure bond return volatility remains below 30%, and (v) ensure the unconditional bond risk premium stays below 2%. Equations (E3.9) through (E3.13) detail the relevant conditions.

The second estimation step includes all of the above moments, while the following are added:

9. *No arbitrage restrictions on the long-maturity bond:* I now target the time-series of the long-term bond yield, in this ensuring that the state-dependent elements of equation (E3.2) are equal:

$$-\frac{\mathbf{B}^{\$}(\tau)}{\tau} \;=\; \mathbf{e}_{yn} + \mathbf{e}_{yspr}.$$

   Together with equation displayed in the first condition, this entails ensuring that Equation (E3.2) holds in the time series. That is I aim to match the entire time series of nominal and real bond yields of the various maturities, rather than just the mean yields.

10. *No arbitrage restriction on the stock market:* Similarly, I ensure that the time series of stock returns from the model match the VAR-implied stock returns. Equation E3.4) provides the restriction.

11. I impose two restrictions on the eigenvalues of the risk-neutral companion matrix $\mathbf{\Psi} - \Sigma\mathbf{\Lambda}_1$. First, I ensure the eigenvalues remain below 0.995 to ensure all valuation ratios are well-defined in the model. Second, I ensure the imaginary parts of the eigenvalues remain below 0.5, which removes oscillatory behavior in the term structure of risk premia that is not supported by the data.

In the third step, I add a final condition:

12. *Regularity on the central bank dividend claim:* I ensure that the price-dividend ratio of the claim to central bank dividends is well behaved (remains below $1,000$). Equation (E3.14) details the relevant condition.

Table 5 summarizes the use and sizes of the penalties associated with each condition.

Table 5 Penalties used in estimation.

#	Condition	Penalty	Steps active	Times activated
1	Unconditional no-arbitrage restriction on the long maturity bond	1×10^6	1, 2, 3	1
2.a	Nominal yield curve	1×10^6	1, 2, 3	1 (Avg. over mat.)
2.b	Real yield curve	1×10^5	1, 2, 3*	1 (Avg. over mat.)
3	Unconditional no-arbitrage restriction on stock market	1×10^4	1, 2, 3	1
4	Equity price–dividend ratio	2×10^4	1, 2, 3	1
5	Equity term structure	5×10^4	1, 2, 3**	1
6	GDP risk premium	1×10^5	1, 2, 3	18 (horizons)
7	Volatility of the SDF (Good-deal bound)	2×10^5	1, 2, 3	1
8	Regularity conditions	1×10^5	1, 2, 3	5×8 (horizons)
9	Conditional no-arbitrage restriction on the long maturity bond	5×10^6	2, 3	T periods
10	Conditional no-arbitrage restriction on stock market	1×10^4	2, 3	T periods
11.a	Risk neutral companion matrix eigenvalues (modulus)	1×10^{10}	2, 3	1
11.b	Risk neutral companion matrix imaginary part of eigenvalues	1×10^7	2, 3	1
12	Central bank price–dividend ratio regularity	1×10^{10}	3	1

* Inactive in the first regime; based on 1985–2024 data.
** Inactive in the first regime; based on 2003–2014 data.

E.2 Results

1845-1945 Regime:

The estimated constant and time-varying market prices of risk for the first regime are:

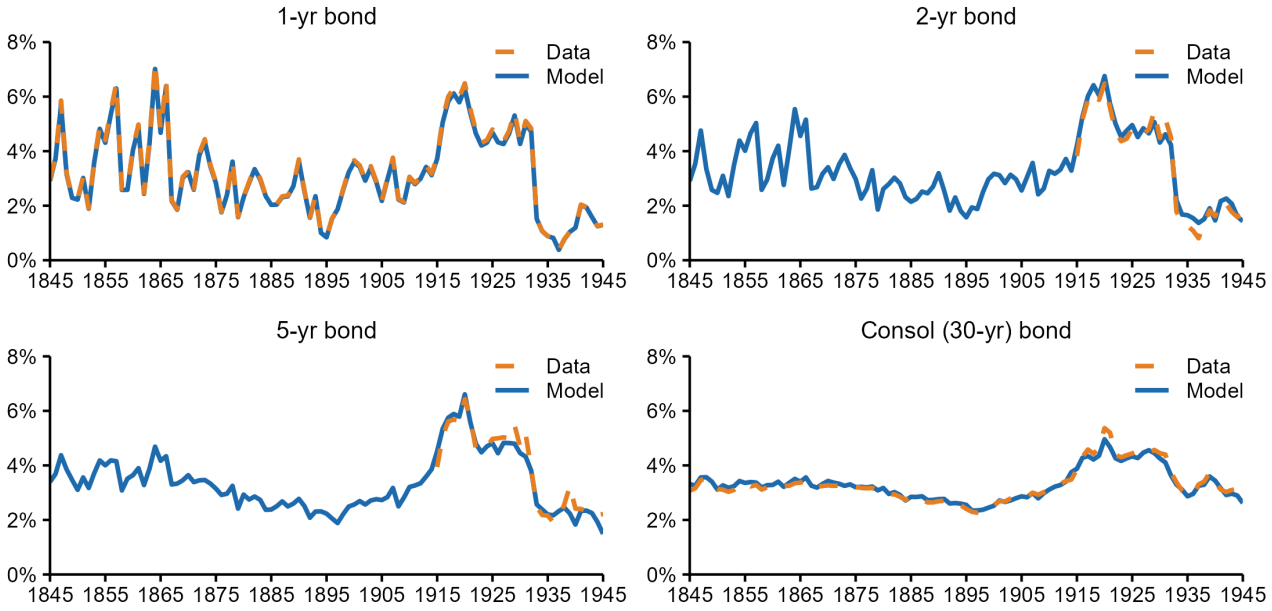
$$\mathbf{\Lambda}_0^{1845\text{–}1945} = \begin{bmatrix} -0.6 & 0.0 & 0.0^+ & 0.7 & 0.0^+ & 0.0 & -1.1 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}'$$

$$\mathbf{\Lambda}_1^{1845\text{–}1945} = \begin{bmatrix} 9.8 & -52.9 & -100.4 & 8.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 4.8 & 48.3 & 88.7 & 1.2 & 1.7 & -1.8 & 0.2 & -0.3 & -0.2 & 1.2 & 0.5 \\ 0.5 & -95.9 & -76.0 & -0.9 & 0.5 & -1.8 & -0.9 & 1.6 & -2.3 & -1.7 & -0.1 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -10.9 & 26.4 & 75.4 & -17.3 & -2.0 & 0.2 & -2.3 & -2.9 & 0.3 & 4.2 & -0.4 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Figures 20 through 23 report the model's results. It is able to match the term structure of nominal yields (there is no real yield data to target), where in the 19th century the yield curve was largely flat. In line with the dynamics of the gold standard, where recessions were typically deflationary, the model generates a negative inflation risk premium. However, the model delivers a lower expected inflation when the U.K. left the gold standard. Although the model is decently able to match the stock market price-dividend ratio, it fails to deliver the equity premia of the mid 19th century.

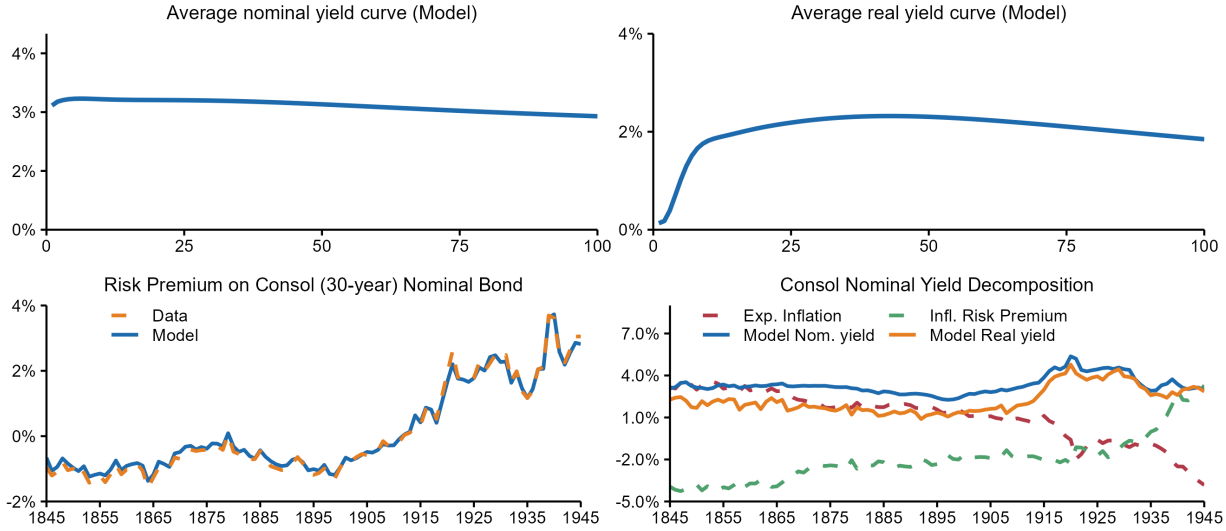
Most importantly, the model overvalues the Bank of England by two orders of magnitude (with an average value of 111.2% of GDP). With price–dividend ratios above 100, the BoE is priced as a completely safe asset, consistent with the VAR's dynamics where its dividends are nearly orthogonal to macroeconomic shocks. Overall, BoE dividends have a 0.01 correlation with the SDF. Consequently, the model implies a large negative risk premium for the BoE, and its term premium converges only very slowly to the GDP risk premium (taking centuries to do so).

Figure 20 1845-1945: Term Structure model and empirical counterparts



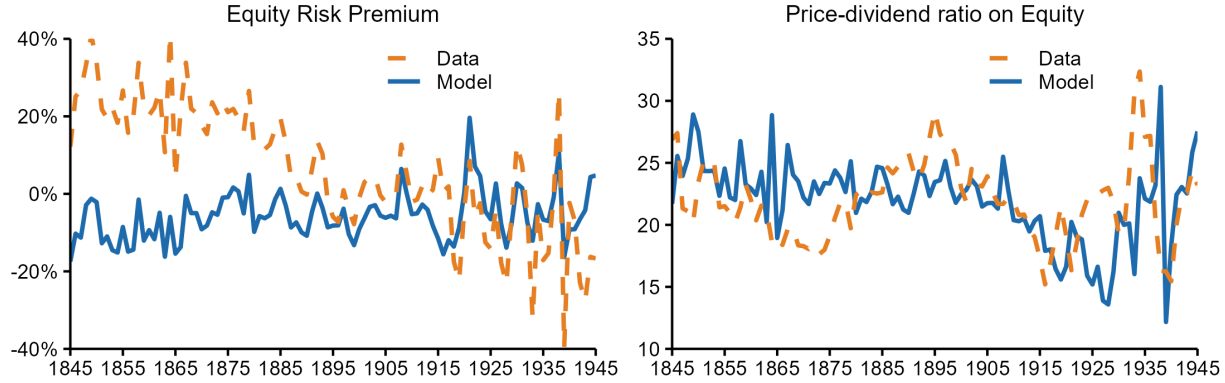
Note: The figure plots the observed and model-implied yields for 1-, 2-, 5-years bonds, as well as the consol (treated as a 30-year) yield. Yields are year averages. Before 1900 the 3-month yield on commercial paper is assumed to be the 1-year yield. For 1900-1915 the 3-month government yield is assumed to be the 1-year. The medium term maturities are those interpolated by Ellison & Scott (2020).

Figure 21 1845-1945: Long-term Yields and Bond Risk Premia



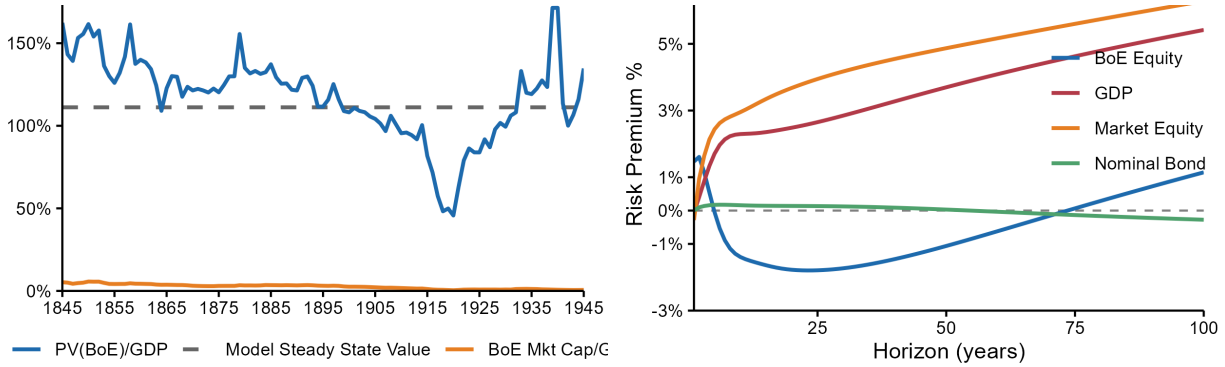
Note: The top panel plots the average nominal and real yields for maturities 1–100 years. The bottom panel plots the nominal consol risk premium (consol yield minus the expected 1-year yield over the next 30 years), along with the consol yield decomposed into the real yield, 30-year expected inflation, and the 30-year inflation risk premium.

Figure 22 1845-1945: Equity Risk Premium and Price-Dividend Ratio



Note: The figure plots the observed and model-implied equity risk premium on the overall stock market in the left panel and the price-dividend ratio in the right panel.

Figure 23 1845-1945: BoE Valuation and Risk Premia



Note: The left panel plots the model-implied present value of BoE dividends along with the observed market capitalization. The right panel plots the term structure of the model-implied risk premia for bonds, market equity, GDP and the BoE

1946-2024 Regime:

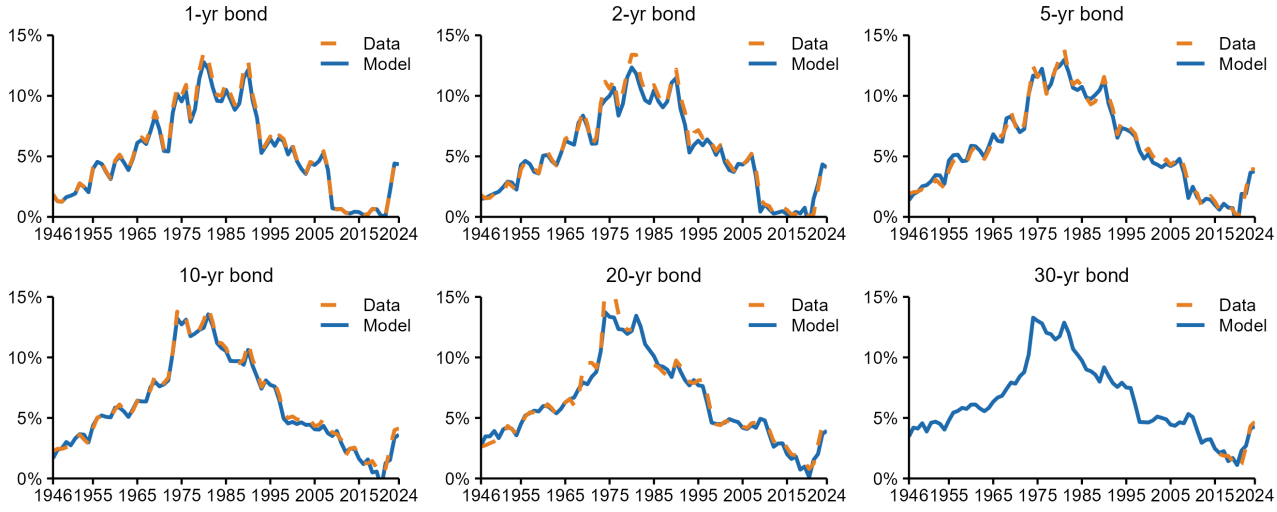
The estimated constant and time-varying market prices of risk for the second regime are:

$$\boldsymbol{\Lambda}_0^{1946\text{–}2024} = \begin{bmatrix} -0.6 & 0.0 & 0.0^{-} & -0.5 & -0.3 & 0.0 & -1.5 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}'$$

$$\boldsymbol{\Lambda}_1^{1946\text{–}2024} = \begin{bmatrix} 4.0 & -2.4 & 11.5 & 1.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 7.3 & -0.4 & -25.5 & 3.5 & -0.3 & 0.8 & 0.8 & 1.2 & 0.9 & 0.2 & 0.3 \\ 14.1 & -27.7 & -48.8 & -1.2 & -1.0 & -0.9 & 3.7 & -2.8 & 1.7 & 0.1 & 0.0^{-} \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -21.0 & 0.7 & -3.1 & -15.4 & -2.5 & -1.7 & -2.0 & 1.9 & -0.6 & 1.0 & -0.3 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

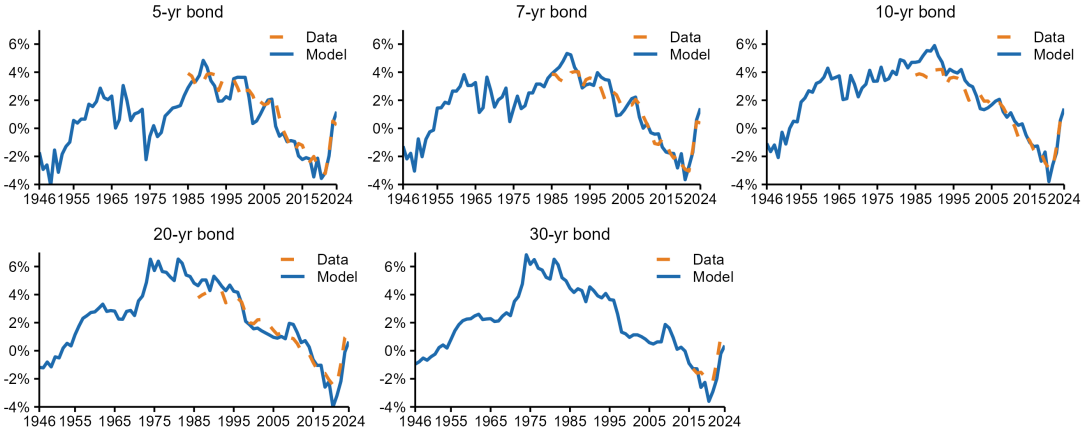
Figures 24 through 27 report the model's results. It is able to match the nominal and real term structure. However, it is unable to generate an equity risk premium (and in fact does the opposite). Given the link between the premium on a GDP claim and that of equity, this could have a first order impact on the delivered value for the BoE.

Figure 24 1946-2024: Term Structure model and empirical counterparts



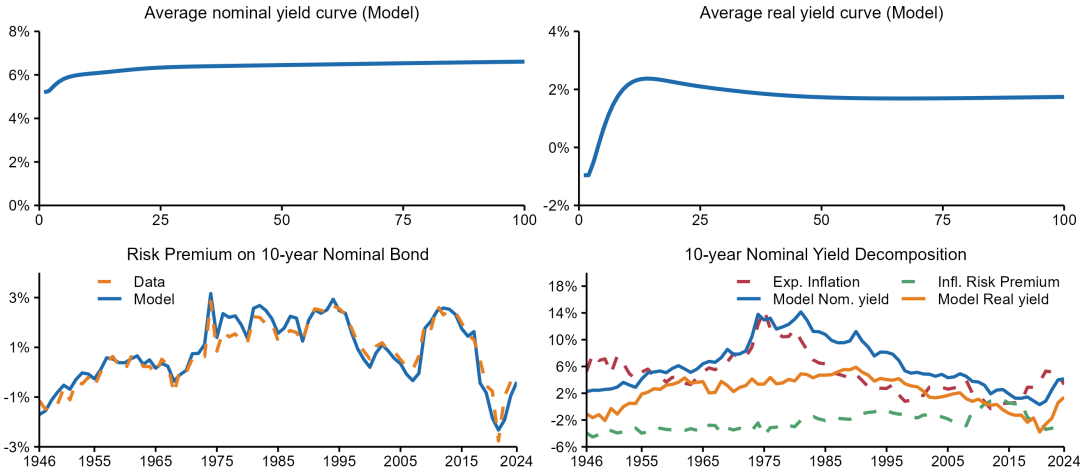
Note: The figure plots the observed and model-implied 1-, 2-, 5-, 10-, 20-, and 30-year nominal Gilt yields. Yields are year averages.

Figure 25 1946-2024: Real term Structure model and empirical counterparts



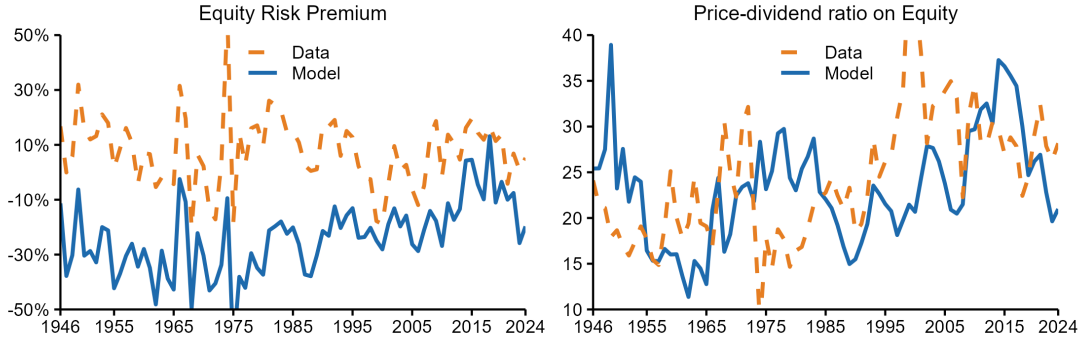
Note: The figure plots the observed and model-implied 5-, 7-, 10-, 20-, and 30-year nominal Gilt yields. Yields are year averages.

Figure 26 1946-2024: Long-term Yields and Bond Risk Premia



Note: The top panel plots the average nominal and real yields for maturities 1–100 years. The bottom panel plots the nominal 10-year bond risk premium (10-year yield minus the expected 1-year yield over the next 10 years), along with the nominal yield decomposed into the real yield, 10-year expected inflation, and the 10-year inflation risk premium.

Figure 27 1946-2024: Equity Risk Premium and Price-Dividend Ratio



Note: The figure plots the observed and model-implied equity risk premium on the overall stock market in the left panel and the price-dividend ratio in the right panel.

E.3 Asset pricing model

For each asset category, below I detail the asset pricing model that delivers the equilibrium asset prices which are matched to the data to estimate the market prices of risk.

1. The Risk-Free Rate

The real short yield $y_t(1)$, or risk–free rate, satisfies the Euler equation $\mathbb{E}_t[\exp\{m_{t+1} + y_t(1)\}] = 1$. The real log SDF follows

$$m_{t+1} = m^{\$}_{t+1} + \pi_{t+1} = -y^{\$}_t(1) - \tfrac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + \pi_0 + (\mathbf{e}_\pi)'\boldsymbol{\Psi} z_t + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1}$$

One can solve the Euler equation to obtain the result that the real short yield is the nominal short yield minus expected inflation minus a Jensen adjustment plus the inflation risk premium:

$$\begin{aligned} y_t(1) &= y^{\$}_t(1) - \mathbb{E}_t[\pi_{t+1}] - \tfrac{1}{2}(\mathbf{e}_\pi)'\boldsymbol{\Sigma}\,\mathbf{e}_\pi + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_t \\ &= y^{\$}_0(1) - \pi_0 - \tfrac{1}{2}(\mathbf{e}_\pi)'\Sigma\mathbf{e}_\pi + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_0 + \big[(\mathbf{e}_{yn})' - (\mathbf{e}_\pi)'\boldsymbol{\Psi} + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1\big] z_t \\ &= y_0(1) + \big[(\mathbf{e}_{yn})' - (\mathbf{e}_\pi)'\boldsymbol{\Psi} + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1\big] z_t\,. \end{aligned} \tag{E3.1}$$

Where in the last step I defined $y_0(1)$ by grouping the first four terms.

Proof: Taking the expression for the real SDF, we can group terms that do and do not depend on the structural shocks $\boldsymbol{\varepsilon}_{t+1}$ such that:

$$\begin{aligned} A^{rf} &= -y^{\$}_t(1) - \tfrac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t + \pi_0 + \mathbf{e}_\pi'\boldsymbol{\Psi} z_t, \\ (\boldsymbol{B}^{rf})'\boldsymbol{\varepsilon}_{t+1} &= -\boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + \mathbf{e}_\pi'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1} \end{aligned}$$

The Euler equation simplifies by the log-normality of the structural shocks:

$$\begin{aligned} \mathbb{E}_t\Big[\exp\big(A^{rf} + (\boldsymbol{B}^{rf})'\boldsymbol{\varepsilon}_{t+1}\big)\Big] &= 1 \\ \exp(A^{rf})\exp\big(\tfrac{1}{2}(\boldsymbol{B}^{rf})'(\boldsymbol{B}^{rf})\big) &= 1 \end{aligned}$$

Taking logs, noting that $(\boldsymbol{B}^{rf})'(\boldsymbol{B}^{rf}) = \boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - 2\mathbf{e}_\pi'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_t + \mathbf{e}_\pi'\boldsymbol{\Sigma}\mathbf{e}_\pi'$, and plugging A^{rf} back in delivers the result:

$$\begin{aligned} y_t(1) &= y^{\$}_t(1) - \pi_0 - (\mathbf{e}_\pi)'\boldsymbol{\Psi} z_t - \tfrac{1}{2}\mathbf{e}_\pi'\boldsymbol{\Sigma}\mathbf{e}_\pi + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\Lambda_t \\ &= y^{\$}_0(1) - \pi_0 - \tfrac{1}{2}(\mathbf{e}_\pi)'\Sigma\mathbf{e}_\pi + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\Lambda_0 + \big[(\mathbf{e}_{yn})' - (\mathbf{e}_\pi)'\boldsymbol{\Psi} + (\mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1\big] z_t\,. \end{aligned}$$

With this, not that the real short rate will be the negative expected log SDF minus a Jensen term. Starting

from the Euler equation:

$$\begin{aligned}
&\mathbb{E}\big[\exp\big(m_{t+1}+y_t\big)\big]=1\\
&\exp(y_t)\,\exp\Big(\mathbb{E}[m_{t+1}]+\tfrac{1}{2}\operatorname{Var}(m_{t+1})\Big)=1\\
&y_t=-\mathbb{E}[m_{t+1}]-\tfrac{1}{2}\operatorname{Var}(m_{t+1}).
\end{aligned}$$

2. Nominal and Real Term Structure

The nominal yields of zero-coupon bonds of horizon h are affine in the state vector:

$$y_t^{\$}(h) \;=\; -\frac{A^{\$}(h)}{h} \;-\; \frac{\mathbf{B}^{\$}(h)'}{h}\,z_t, \tag{E3.2}$$

where the coefficients $A^{\$}(h)$ and $\mathbf{B}^{\$}(h)$ satisfy the recursions

$$A^{\$}(h{+}1) \;=\; -\,y_0^{\$}(1) \;+\; A^{\$}(h) \;+\; \tfrac{1}{2}\,\mathbf{B}^{\$}(h)'\boldsymbol{\Sigma}\,\mathbf{B}^{\$}(h) \;-\; \mathbf{B}^{\$}(h)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_0,$$

$$\mathbf{B}^{\$}(h{+}1)' \;=\; \mathbf{B}^{\$}(h)'\boldsymbol{\Psi} \;-\; (\mathbf{e}_{yn})' \;-\; \mathbf{B}^{\$}(h)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1,$$

initialized at $A^{\$}(0)=0$ and $\mathbf{B}^{\$}(0)=0$.

Similarly, real yields on zero-coupon bonds of horizon h (denoted $y_t(h)$ without the \$ superscript) are affine with coefficients that follow similar recursions and the same initializations:

$$y_t(h) \;=\; -\frac{A(h)}{h} \;-\; \frac{\mathbf{B}(h)'}{h}\,z_t, \tag{E3.3}$$

$$A(h{+}1) = -\,y_0(1) + A(h) + \tfrac{1}{2}\,\mathbf{B}(h)'\boldsymbol{\Sigma}\,\mathbf{B}(h) - \mathbf{B}(h)'\boldsymbol{\Sigma}^{\frac{1}{2}}\big(\boldsymbol{\Lambda}_0 - \boldsymbol{\Sigma}^{\frac{1}{2}}\mathbf{e}_{\pi}\big),$$

$$\mathbf{B}(h{+}1)' = -\,\mathbf{e}_{yn}' + \big(\mathbf{e}_{\pi} + \mathbf{B}(h)\big)'\big(\boldsymbol{\Psi} - \boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1\big).$$

With $h=1$, the real recursion delivers the expression for the risk-free rate in (E3.1). These expressions for the nominal and real yields are the ones used to match the term structure (conditions 1, 2 and 9) in Appendix E.1.

Proof. I include only the proof for the nominal yields (that for the real yields is similar). I conjecture that the $t{+}1$ price of an h-horizon zero-coupon bond is exponentially affine in the state vector:

$$\log\big(P_{t+1}^{\$}(h)\big) \;=\; A^{\$}(h) + \mathbf{B}^{\$}(h)'z_{t+1},$$

and solve for $A^{\$}(h{+}1)$ and $\mathbf{B}^{\$}(h{+}1)$ by verifying this conjecture using the Euler equation:

$$\begin{aligned} P_t^{\$}(h{+}1) &= \mathbb{E}_t\Big[\exp\big\{m_{t+1}^{\$} + \log\big(P_{t+1}^{\$}(h)\big)\big\}\Big] \\ &= \mathbb{E}_t\Big[\exp\big\{-y_t^{\$}(1) - \tfrac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + A^{\$}(h) + \mathbf{B}^{\$}(h)'z_{t+1}\big\}\Big] \\ &= \exp\big\{-y_0^{\$}(1) - (\mathbf{e}_{yn})'z_t - \tfrac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t + A^{\$}(h) + \mathbf{B}^{\$}(h)'\boldsymbol{\Psi}z_t\big\} \times \mathbb{E}_t\Big[\exp\big\{-\boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + \mathbf{B}^{\$}(h)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1}\big\}\Big]. \end{aligned}$$

Using the log-normality of $\boldsymbol{\varepsilon}_{t+1}$ and substituting the expression for $\boldsymbol{\Lambda}_t$, one obtains

$$P_t^{\$}(h{+}1) = \exp\Big\{-y_0^{\$}(1) - (\mathbf{e}_{yn})'z_t + A^{\$}(h) + \mathbf{B}^{\$}(h)'\boldsymbol{\Psi}z_t + \tfrac{1}{2}\mathbf{B}^{\$}(h)'\boldsymbol{\Sigma}\,\mathbf{B}^{\$}(h) - \mathbf{B}^{\$}(h)'\boldsymbol{\Sigma}^{\frac{1}{2}}\big(\boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 z_t\big)\Big\}.$$

Taking logs and collecting terms yields a linear representation for $\log P_t^{\$}(h{+}1)$:

$$\log\big(P_t^{\$}(h{+}1)\big) \;=\; A^{\$}(h{+}1) + \mathbf{B}^{\$}(h{+}1)'z_t,$$

where $A^{\$}(h{+}1)$ and $\mathbf{B}^{\$}(h{+}1)$ satisfy the recursions above. The relationship between log bond prices and bond yields is $-\log\big(P_t^{\$}(h)\big)/h = y_t^{\$}(h)$.

Although unused in the estimation of the market prices of risk, it is helpful to consider the nominal bond risk premium. To do so define the one-period return on the nominal zero-coupon bond as

$$r_{t+1}^{\$}(h) \;=\; \log\big(P_{t+1}^{\$}(h)\big) \;-\; \log\big(P_t^{\$}(h{+}1)\big).$$

The nominal bond risk premium on a bond of maturity h is the expected excess return corrected for a Jensen term, and equals the negative conditional covariance between that bond return and the nominal SDF:

$$\mathbb{E}_t\Big[r_{t+1}^{\$}(h) - y_t^{\$}(1) + \tfrac{1}{2}\operatorname{Var}_t\big(r_{t+1}^{\$}(h)\big)\Big] \;=\; -\operatorname{Cov}_t\big(m_{t+1}^{\$},\, r_{t+1}^{\$}(h)\big) \;=\; \mathbf{B}^{\$}(h)'\,\boldsymbol{\Sigma}^{\frac{1}{2}}\,\boldsymbol{\Lambda}_t.$$

3. Aggregate stock market

To arrive at a no-arbitrage condition on stock returns, first define the nominal return on the aggregate stock market as:

$$R_{t+1}^{M} \;=\; \frac{P_{t+1}^{M} + D_{t+1}^{M}}{P_t^{M}},$$

where P_t^M is the ex–dividend price on the equity market. The standard log–linearization delivers

$$\begin{aligned} r_{t+1}^{M} \;&=\; \kappa_0^{M} \;+\; \kappa_1^{M}\, pd_{t+1}^{M} \;-\; pd_t^{M} \;+\; \log\big(\frac{D_{t+1}^{M}}{D_t^{M}}\big) \\ r_{t+1}^{M} \;&=\; \kappa_0^{M} \;+\; \kappa_1^{M}\, pd_{t+1}^{M} \;-\; pd_t^{M} \;+\; \Delta d_{t+1} \;+\; x_{t+1} + \pi_{t+1} \end{aligned}$$

In the second step I decompose the log growth of dividends into the growth in the dividends-to-GDP ratio, GDP

growth, and inflation, which are all (in logs) a part of the state vector. The linearization constants κ_0^M and κ_1^M follow

$$\kappa_1^M \;=\; \frac{e^{\,pd_0^M}}{e^{\,pd_0^M}+1} \;<\; 1, \qquad \kappa_0^M \;=\; \log\!\big(e^{\,pd_0^M}+1\big) \;-\; \frac{e^{\,pd_0^M}}{e^{\,pd_0^M}+1}\, pd_0^M.$$

Where pd_0^M is mean log price-dividend ratio of the market. Since the price-dividend ratio on the stock market is also in the state vector, we can plug in the selector vectors $\boldsymbol{e}_{pd}$, $\boldsymbol{e}_{\Delta d}$, and $\boldsymbol{e}_x$ (which extract the log pd ratio, the log dividend/GDP ratio growth rate, and the real GDP growth rate) so the log returns become:

$$\begin{aligned} r_{t+1}^M \;&=\; \kappa_0^M \;+\; \kappa_1^M \,\Big[pd_0^M + \mathbf{e}_{pd}'(\boldsymbol{\Psi} z_t + \boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon_{t+1}})\Big] \;-\; pd_0^M \;-\; \mathbf{e}_{pd}' z_t \\ &\qquad +\; \mu_d \;+\; x_0 \;+\; \pi_0 \;+\; (\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'(\boldsymbol{\Psi} z_t + \boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon_{t+1}}) \\ r_{t+1}^M \;&=\; r_0^M \;+\; \Big[(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Psi} - \mathbf{e}_{pd}'\Big] z_t \;+\; (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\,\boldsymbol{\Sigma}^{\frac{1}{2}}\,\boldsymbol{\varepsilon_{t+1}}. \end{aligned}$$

Where in the second algebra step I group the mean terms into the mean return

$$r_0^M = \kappa_0^M - (1 \;-\; \kappa_1^M) pd_0^M + x_0 + \pi_0 + \mu_d = \mathbb{E}[r_{t+1}^M],$$

and group the terms that depend on the state vector or on the structural shocks. With this expression for the log returns, we can begin with the Euler equation (based on the nominal SDF) on the aggregate stock market, and again use the log normality of the structural shocks:

$$\begin{aligned} 1 &= \mathbb{E}_t\left[M_{t+1}^{\$}\,\frac{P_{t+1}^M + D_{t+1}^M}{P_t^M}\right] = \mathbb{E}_t\Big[\exp\{\, m_{t+1}^{\$} + r_{t+1}^M \,\}\Big] \\ &= \mathbb{E}_t\Big[\exp\{\, -y_t^{\$}(1) - \tfrac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + r_0^M + (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)' z_{t+1} - \mathbf{e}_{pd}' z_t \,\}\Big] \\ &= \exp\Big\{ -y_0^{\$}(1) - \tfrac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t + r_0^M + \big[(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Psi} - \mathbf{e}_{pd}' - \mathbf{e}_{yn}'\big] z_t \Big\} \\ &\quad \times \mathbb{E}_t\Big[\exp\{ -\boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1}\}\Big] \\ &= \exp\Big\{ r_0^M - y_0^{\$}(1) + \big[(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Psi} - \mathbf{e}_{pd}' - \mathbf{e}_{yn}'\big] z_t \Big\} \\ &\quad \times \exp\Big\{ \tfrac{1}{2}(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Sigma}\,(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi) - (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_t \Big\}. \end{aligned}$$

Taking logs:

$$\begin{aligned} &r_0^M - y_0^{\$}(1) + \big[(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Psi} - \mathbf{e}_{pd}' - \mathbf{e}_{yn}'\big] z_t \\ &\qquad + \underbrace{\tfrac{1}{2}(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Sigma}\,(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)}_{\text{Jensen term}} = (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_t \\ &\mathbb{E}_t[r_{t+1}^{m\$}] - y_t^{\$}(1) + \tfrac{1}{2}\operatorname{Var}_t(r_{t+1}^{m\$}) = -\operatorname{Cov}_t(m_{t+1}^{\$},\, r_{t+1}^{m\$}). \end{aligned}$$

The left-hand side is the expected excess return on the stock market, corrected for a Jensen term, while the right-hand side is the negative of the conditional covariance between the nominal log stock return and the

nominal log SDF. The left-hand side describes the expected excess return of equity from the VAR's dynamics, and the right-hand side that from the asset pricing model (since it requires knowledge of the market prices of risk).

To arrive at the expressions included in the optimization steps, I plug in the linear expression for $\mathbf{\Lambda}_t = \mathbf{\Lambda}_0 + \mathbf{\Lambda}_1 z_t$ and split the expression into the constant and time varying terms. The constant part will follow:

$$r_0^M - y_0^{\$}(1) + \tfrac{1}{2}(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)' \mathbf{\Sigma} (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi) = (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)' \mathbf{\Sigma}^{\frac{1}{2}} \mathbf{\Lambda}_0$$

To pin down the constant market price of risk on dividends-to-GDP growth, I extract the fifth element of the $\mathbf{\Lambda}_0$ vector (the variable is in the fifth position). That is, I solve for $\mathbf{e}_5' \mathbf{\Lambda}_0$, defining $\mathbf{e}_5$ as the selector vector such that $\mathbf{\Lambda}_0 = (\mathbf{e}_5 \mathbf{e}_5') \mathbf{\Lambda}_0 + (I - \mathbf{e}_5 \mathbf{e}_5') \mathbf{\Lambda}_0$. Consolidating variable selectors for illustration ($K = \kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi$):

$$\mathbf{e}_5' \mathbf{\Lambda}_0 = \frac{r_0^M - y_0^{\$}(1) + \frac{1}{2}(K)' \mathbf{\Sigma} \ (K) \ - \ (K)' \mathbf{\Sigma}^{\frac{1}{2}} (\mathbf{I} - \mathbf{e}_5 \mathbf{e}_5') \mathbf{\Lambda}_0}{(K)' \mathbf{\Sigma}^{\frac{1}{2}} \mathbf{e}_5}. \tag{E3.4}$$

(E3.4) is the equation that targets the unconditional stock return in all steps of the estimation of the model. Proceeding similarly, the state-dependent market prices or risk must satisfy:

$$\left[(\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)' \mathbf{\Psi} - \mathbf{e}_{pd}' - \mathbf{e}_{yn}'\right] z_t = (\kappa_1^M \mathbf{e}_{pd} + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi)' \mathbf{\Sigma}^{\frac{1}{2}} \mathbf{\Lambda}_1 z_t$$

I define $\mathbf{\Lambda}_1 \mathbf{e}_i = (\mathbf{e}_5 \mathbf{e}_5') \mathbf{\Lambda}_1 \mathbf{e}_i + (I - \mathbf{e}_5 \mathbf{e}_5') \mathbf{\Lambda}_1 \mathbf{e}_i$ to select, within the dividend-to-GDP growth row of $\mathbf{\Lambda}_1$, the column corresponding to variable i. For each variable, it must hold that:

$$\frac{\left[(K)' \mathbf{\Psi} - \mathbf{e}_{pd}' - \mathbf{e}_{yn}'\right] \mathbf{e}_i - (K)' \mathbf{\Sigma}^{\frac{1}{2}} (\mathbf{I} - \mathbf{e}_5 \mathbf{e}_5') \mathbf{\Lambda}_1 \mathbf{e}_i}{(K)' \mathbf{\Sigma}^{\frac{1}{2}} \mathbf{e}_5} \mathbf{e}_5 = \mathbf{e}_5 \mathbf{e}_5' \mathbf{\Lambda}_1 \mathbf{e}_i \tag{E3.5}$$

For each of the variables in the state vector, (E3.5) is the equation that targets the conditional stock returns in the second and third steps of the estimation.

4. Dividend Strips

The log price–dividend ratios on dividend strips are affine in the state vector.

$$pd_t^M(h) \ = \ A^M(h) + \mathbf{B}^M(h)' z_t,$$

where the coefficients $A^M(h)$ and $\mathbf{B}^M(h)$ follow the recursions:

$$
\begin{aligned}
A^M(h+1) &= A^M(h) + x_0 + \mu_d - y_0^{\$}(1) + \pi_0 \\
&\quad + \tfrac{1}{2}\left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}\left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right) \\
&\quad - \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_0, \\
\mathbf{B}^M(h+1)' &= \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Psi} - \mathbf{e}_{yn}' - \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1,
\end{aligned}
$$

initialized at $A_0^M = 0$ and $\mathbf{B}_0^M = 0$.

Proof: I conjecture the affine structure and solve for $\mathbf{A}^M(h+1)$ and $\mathbf{B}^M(h+1)$ by it using the nominal Euler equation (so that gross dividend growth is nominal):

$$
\begin{aligned}
\exp\left(pd_t^M(h{+}1)\right) &= \mathbb{E}_t\left[M_{t+1}^{\$}\,\exp\left(pd_{t+1}^M(h)\right)\frac{D_{t+1}^M}{D_t^M}\right] \\
&= \mathbb{E}_t\left[\exp\left\{m_{t+1}^{\$} + \pi_{t+1} + x_{t+1} + \Delta d_{t+1} + pd_{t+1}^M(h)\right\}\right] \\
&= \mathbb{E}_t\Big[\exp\Big\{-y_{t,1}^{\$} - \tfrac{1}{2}\,\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + \pi_0 + \mathbf{e}_\pi' z_t + x_0 + \mu_d \\
&\qquad\qquad + (\mathbf{e}_{\Delta d} + \mathbf{e}_x')z_{t+1} + A^M(h) + \mathbf{B}^M(h)'z_{t+1}\Big\}\Big] \\
&= \exp\Big\{-y_0^{\$}(1) - \mathbf{e}_{yn}'z_t - \tfrac{1}{2}\,\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t + \pi_0 + \mathbf{e}_\pi'\boldsymbol{\Psi}z_t + x_0 + \mu_d \\
&\qquad\qquad + (\mathbf{e}_{\Delta d} + \mathbf{e}_x)'\boldsymbol{\Psi}z_t + A^M(h) + \mathbf{B}^M(h)'\boldsymbol{\Psi}z_t\Big\} \\
&\quad \times \mathbb{E}_t\left[\exp\left\{-\boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1}\right\}\right].
\end{aligned}
$$

Using the log-normality of ε_{t+1} and plugging in the expression for $\boldsymbol{\Lambda}_t$ delivers:

$$
\begin{aligned}
pd_t^M(h{+}1) &= -y_0^{\$}(1) + \pi_0 + x_0 + \mu_d + A^M(h) \\
&\quad + \left[\left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Psi} - (\mathbf{e}_{yn})'\right]z_t \\
&\quad + \tfrac{1}{2}\left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}\left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right) \\
&\quad - \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\left(\boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 z_t\right).
\end{aligned}
$$

Taking logs and collecting terms yields the log-linear expression for $pd_t^M(h{+}1)$ along with its recursive terms:

$$
pd_t^M(h{+}1) \;=\; A^M(h{+}1) + \mathbf{B}^M(h{+}1)'z_t,
$$

where

$$
\begin{aligned}
A^M(h{+}1) &= A^M(h) + x_0 + \mu_d - y^{\$}_0(1) + \pi_0 \\
&\quad + \tfrac{1}{2}\left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}\left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right) \\
&\quad - \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_0, \\
\mathbf{B}^M(h{+}1)' &= \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Psi} - (\mathbf{e}_{yn})' \\
&\quad - \left(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1.
\end{aligned}
$$

For reference, I define the dividend strip risk premium as

$$
\begin{aligned}
\mathbb{E}_t\big[r^M_{t+1}(h)\big] \;-\; y^{\$}_{t,1} \;+\; \tfrac{1}{2}\,\mathrm{Var}_t\big[r^M_{t+1}(h)\big] &= -\,\mathrm{Cov}_t\big(m^{\$}_{t+1},\, r^M_{t+1}(h)\big) \\
&= \big(\mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\big)'\,\boldsymbol{\Sigma}^{\frac{1}{2}}\,\boldsymbol{\Lambda}_t.
\end{aligned}
$$

Adding the exponent of the log price-dividend ratios for all of the strip horizons $(4,000)$ will yield the price-dividend ratio of equity, so that

$$
\exp(pd^M_t) = \sum_{h}^{4{,}000} \exp(pd^M_t(h)). \tag{E3.6}
$$

The third condition of the model consists on penalizing deviations of the market price-dividend ratio from the exponent of its conditional empirical counterpart ($pd^M_t = pd^M_0 + \mathbf{e}'_{pd} z_t$).

Dividend Futures: Having a model for the price-dividend ratios on dividend strips we can turn to the returns on their futures, which are used in estimation. Because a dividend strip pays only at $t+h$, by no-arbitrage the dividend futures price equals the strip's spot value financed at the nominal risk-free rate. As a result, the price of a dividend futures contract that delivers one year's worth of nominal dividends at time $t+h$, divided by the current dividend, is

$$
\frac{F^M_t(h)}{D^M_t} = \exp\big(pd^M_t(h) + h\, y^{\$}_t(h)\big),
$$

where $pd^M_t(h)$ is the log spot price–dividend ratio on the dividend strip of maturity h. Plugging in the affine expressions for the strip price–dividend ratio and the nominal bond price yields:

$$
\frac{F^M_t(h)}{D^M_t} = \exp\Big(A^M(h) - A^{\$}(h) + \big(\mathbf{B}^M(h) - \mathbf{B}^{\$}(h)\big)' z_t\Big).
$$

The one–period return on the dividend future of maturity h is

$$
R^{\mathrm{Fut},M}_{t+1}(h) \;\equiv\; \frac{F^M_{t+1}(h-1)}{F^M_t(h)} - 1 \;=\; \frac{F^M_{t+1}(h-1)/D^M_{t+1}}{F^M_t(h)/D^M_t}\,\frac{D^M_{t+1}}{D^M_t} \;-\; 1.
$$

Plugging in the affine expressions yields:

$$R^{\text{Fut},M}_{t+1}(h) = \exp\big[A^M(h-1) - A^\$(h-1) - A^M(h) + A^\$(h) + x_0 + \mu_d + \pi_0 \\ + \big(\mathbf{B}^M(h-1) - \mathbf{B}^\$(h-1) + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi\big)' z_{t+1} - \big(\mathbf{B}^M(h) - \mathbf{B}^\$(h)\big)' z_t\big]. \tag{E3.7}$$

With this expression, the estimation of the model proceeds as follows. van Binsbergen & Koijen (2017) report the mean monthly realized returns for the 2003–2014 period of dividend futures of maturities 1 through 7 separately. I annualize these and take their mean. I then compute the realized returns implied by the model (equation (E3.6)) for the same maturities over the same time period, and compute their global mean. I then penalize squared deviations of the model-implied global mean from the empirical global mean. This is exactly the procedure in Jiang et al. (2024). Note the data is not available in the first regime, so this condition is omitted.

Note that after rearranging and taking logs and expectations I obtain the the expected log return on dividend futures, which is already a risk premium since the price of the dividend future includes the risk-free carry-cost:

$$\mathbb{E}_t\Big[\log\big(1 + R^{\text{Fut},M}_{t+1}(h)\big)\Big] = A^M(h-1) - A^\$(h-1) - A^M(h) + A^\$(h) + x_0 + \mu_d + \pi_0 \\ + \Big[\big(\mathbf{B}^M(h-1) - \mathbf{B}^\$(h-1) + \mathbf{e}_{\Delta d} + \mathbf{e}_x + \mathbf{e}_\pi\big)'\boldsymbol{\Psi} - \big(\mathbf{B}^M(h) - \mathbf{B}^\$(h)\big)'\Big] z_t.$$

Given that the state variable z_t is mean-zero, the right-hand-side in the first row of the above expression denotes the unconditional dividend futures risk premium.

5. GDP Strips

The log price–dividend ratios on GDP strips are affine in the state vector.

$$pd^x_t(h) \;=\; A^x(h) + \mathbf{B}^x(h)' z_t,$$

where the coefficients $A^x(h)$ and $\mathbf{B}^x(h)$ follow the recursions:

$$\begin{aligned} A^x(h+1) &= A^x(h) + x_0 + \mu_d - y^\$_0(1) + \pi_0 \\ &\quad + \tfrac{1}{2}\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^x(h)\right)'\boldsymbol{\Sigma}\left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^x(h)\right) \\ &\quad - \left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^x(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_0, \\ \mathbf{B}^x(h+1)' &= \left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^x(h)\right)'\boldsymbol{\Psi} - \mathbf{e}'_{yn} - \left(\mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^x(h)\right)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\Lambda}_1, \end{aligned}$$

initialized at $A^x_0 = 0$ and $\mathbf{B}^x_0 = 0$. The proof is analogous to that on dividend strips so it is omitted. Motivated by the assumption that a claim to GDP is an unlevered equity claim, the third condition in the estimation consists on ensuring the unconditional risk premium on GDP matches that on equity adjusted for firms' debt over assets

ratio l:

$$\underbrace{\left[-\frac{A^x(h)}{h} - y_0^{\$}(1) + (x_0 + \pi_0)\right]}_{\text{GDP strip RP at horizon } h} = (1-l)\ \underbrace{\left[-\frac{A^x(h)}{h} - y_0^{\$}(1) + (\mu_m + x_0 + \pi_0)\right]}_{\text{Equity strip RP at horizon } h}. \tag{E3.8}$$

This is ensured at strip horizons 1–10, 20, 30, . . . , 90 years. The weight on each horizon scales down linearly.

Proof: To show that equation (E3.8) represents the risk premium on GDP (the proof for market dividend strips is similar), let X_t denote the one-period payout on a GDP strip (that is, nominal GDP). The affine log price–dividend ratio of the h-horizon strip proved above is:

$$\log\frac{P_t^X(h)}{X_t} \;=\; A^x(h) + \mathbf{B}^x(h)'z_t \quad\Longrightarrow\quad P_t^X(h) \;=\; X_t\,\exp\big(A^x(h) + \mathbf{B}^x(h)'z_t\big).$$

The h-period hold-to-maturity log return is

$$r_{t,t+h}^X = \log\frac{P_{t+h}^X(0)}{P_t^X(h)} = \log X_{t+h} - \log X_t \;-\; A^x(h) \;-\; \mathbf{B}^x(h)'z_t,$$

since by initialization $A^x(0) = 0$ and $\mathbf{B}^x(0) = 0$. Taking unconditional expectations and using $\mathbb{E}[z_t] = 0$,

$$\mathbb{E}\big[r_{t,t+h}^X\big] = \mathbb{E}\big[\log X_{t+h} - \log X_t^X\big] - A^x(h).$$

Now note that the average log growth of nominal GDP is given by the means of the variables of the state vector. Multiplying by the number of periods yields the result:

$$\begin{aligned}\mathbb{E}\big[\log X_{t+h} - \log X_t\big] &= h(x_0 + \pi_0)\\ \frac{1}{h}\,\mathbb{E}\big[r_{t,t+h}^X\big] \;&=\; x_0 + \pi_0 \;-\; \frac{A^x(h)}{h}.\end{aligned}$$

The derivation is similar for the risk premium on equity (based on the dividend strips) so is omitted (just noting that the average growth rate of nominal dividends is $x_0 + \pi_0 + \mu_d$).

6. Regularity conditions:

Similar expressions for risk premia can also be derived for the zero-coupon bonds. For tenors 100, 200, 400, 700, 1000, 2000, 3000, 4000 years, penalties are added to:

1. Ensure the GDP strip's nominal discount rate (its risk premium plus the short rate) stays above the average nominal GDP growth. This guarantees the present value of the GDP claim converges. That is:

$$-\frac{A^{\$}(h)}{h} \;+\; \Big(-\frac{A^x(h)}{h} - y_0^{\$}(1) + x_0 + \pi_0\Big) \;\geq\; x_0 + \pi_0 \tag{E3.9}$$

2. Ensure the average nominal–real spread across maturities stays above average inflation. This helps pin

down the Fisher relation in the long run.

$$-\frac{A^{\$}(h)}{h} \;-\; \Big(-\frac{A(h)}{h}\Big) \;\geq\; \pi_0 \tag{E3.10}$$

3. Ensure the unconditional bond risk premium stays below the GDP risk premium.

$$\Big(-\frac{A^{\$}(h)}{h} - y_0^{\$}(1)\Big) \;\leq\; \Big(-\frac{A^{x}(h)}{h} - y_0^{\$}(1) + + x_0 + \pi_0\Big) \tag{E3.11}$$

4. Ensure the bond return (that is, its change in log prices) volatility remains below 30%

$$r_{t+1}^{\text{bond}}(h) \;\approx\; \big[A^{\$}(h-1) + \mathbf{B}^{\$}(h-1)' z_{t+1}\big] \;-\; \big[A^{\$}(h) + \mathbf{B}^{\$}(h)' z_t\big], \qquad \text{Std}\left(r_{t+1}^{\text{bond}}(h)\right) \;\leq\; 0.30 \tag{E3.12}$$

5. Ensure the unconditional bond risk premium stays below 2%. This prevents the model from delivering large term premia at long horizons. Note $y_0^{\$} = A^{\$}(1)$

$$-\frac{A^{\$}(h)}{h} + A^{\$}(1) \;\leq\; 0.02 \tag{E3.13}$$

7. The Central Bank

I model the claim on central bank dividends like I do equity, aggregating dividend strips whose price-dividend ratio (for a strip of horizon h) is affine in the state vector:

$$pd_t^V(h) \;=\; A^V(h) + \mathbf{B}^V(h)' z_t,$$

where the coefficients $A^V(h)$ and $\mathbf{B}^V(h)$ follow the recursions:

$$\begin{aligned}
A^V(h+1) &= A^V(h) + x_0 + \mu_v - y_0^{\$}(1) + \pi_0 \\
&\quad + \tfrac{1}{2}\left(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right)' \boldsymbol{\Sigma} \left(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)\right) \\
&\quad - \left(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h)\right)' \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\Lambda}_0, \\
\mathbf{B}^V(h+1)' &= \left(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h)\right)' \boldsymbol{\Psi} - \mathbf{e}_{yn}' - \left(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h)\right)' \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\Lambda}_1,
\end{aligned}$$

initialized at $A_0^V = 0$ and $\mathbf{B}_0^V = 0$.

Proof: The proof is identical to that for dividend strips. First conjecture the log price–dividend ratios on central bank dividend strips are affine in the state vector:

$$pd_t^V(h) = A^V(h) + \mathbf{B}^V(h)' z_t.$$

Note that nominal central bank dividend growth equals

$$\Delta \log V_{t+1} = \Delta \log v_{t+1} + x_{t+1} + \pi_{t+1} = x_0 + \pi_0 + \mu_v + (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi)' z_{t+1}.$$

Applying this to the Euler equation:

$$\begin{aligned}
\exp\big(pd_t^V(h+1)\big) &= \mathbb{E}_t\Big[M_{t+1}\, \exp\big(pd_{t+1}^V(h)\big)\, \frac{V_{t+1}}{V_t}\Big] \\
&= \mathbb{E}_t\Big[\exp\Big\{m_{t+1}^{\$} + \Delta \log v_{t+1} + x_{t+1} + \pi_{t+1} + pd_{t+1}^V(h)\Big\}\Big] \\
&= \exp\Big\{-y_0^{\$}(1) - (\mathbf{e}_{yn})' z_t - \tfrac{1}{2}\,\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t + \mu_v + x_0 + \pi_0 \\
&\qquad + (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Psi}\, z_t + A^V(h)\Big\} \\
&\quad \times \mathbb{E}_t\Big[\exp\Big\{-\boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\varepsilon}_{t+1}\Big\}\Big].
\end{aligned}$$

Using the log-normality of $\boldsymbol{\varepsilon}_{t+1}$, substitute in the affine expression for $\boldsymbol{\Lambda}_t$ to get

$$\begin{aligned}
\exp\big(pd_t^V(h+1)\big) = \exp\Big\{ &- y_0^{\$}(1) + \mu_v + x_0 + \pi_0 + \big[(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Psi} - (\mathbf{e}_{yn})'\big] z_t \\
&+ A^V(h) + \tfrac{1}{2}\,(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Sigma}\, (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h)) \\
&- (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Sigma}^{\frac{1}{2}}\, (\boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 z_t)\Big\}.
\end{aligned}$$

Taking logs and collecting terms yields the recursive terms:

$$\begin{aligned}
A^V(h+1) &= A^V(h) + x_0 + \mu_v - y_0^{\$}(1) + \pi_0 \\
&\quad + \tfrac{1}{2}\,(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h))' \boldsymbol{\Sigma}\, (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^M(h)) \\
&\quad - (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\Lambda}_0, \\
\mathbf{B}^V(h+1)' &= (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Psi} - \mathbf{e}_{yn}' - (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \mathbf{B}^V(h))' \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\Lambda}_1,
\end{aligned}$$

Aggregating the price-dividend ratios of the strips up to infinity (in practice a 4,000 horizon) yields the (ex-dividend) price–dividend ratio of the central bank dividend claim:

$$\exp(pd_t^V) = \sum_{h=1}^{4{,}000} \exp\big(A^V(h+1) + \mathbf{B}^V(h+1)' z_t\big). \tag{E3.14}$$

Equation (E3.14) presents condition 13 of estimation of the model. Crucially, having estimated the model and running the recursion with the solutions for $\boldsymbol{\Lambda}_0$ and $\boldsymbol{\Lambda}_1 z_t$, this price-dividend ratio prices the central bank with equation 4.1.

Derivation of the risk premium on the Central Bank. To obtain the risk premium on the entire claim to central bank

dividends, define the nominal holding–period return on the claim as

$$R^V_{t+1} \equiv \frac{V_{t+1} + P^V_{t+1}}{P^V_t}.$$

Log–linearizing around $z_t = 0$:

$$r^V_{t+1} = \kappa^V_0 + \Delta \log V_{t+1} + \kappa^V_1 \, pd^V_{t+1} - pd^V_t.$$

where $pd^V_t = \log P^V_t - \log V_t$, and the linearization constants κ^V_0 and κ^V_1 are defined as

$$\kappa^V_1 = \frac{e^{pd^V_0}}{e^{pd^V_0} + 1} < 1 \qquad \text{and} \qquad \kappa^V_0 = \log\big(e^{pd^V_0} + 1\big) - \frac{e^{pd^V_0}}{e^{pd^V_0} + 1} \, pd^V_0.$$

pd^V_0 can be obtained from evaluating equation (E3.14), the conditional value of the price-dividend ratio of the central bank, at $z_t = 0$. Plugging in the dynamics of the VAR yields:

$$r^V_{t+1} \;=\; \kappa^V_0 + \mu_v + x_0 + \pi_0 + \big(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi\big)' z_{t+1} + \kappa^V_1 pd^V_{t+1} - pd^V_t.$$

To make progress towards the central bank's risk premium, I assume that its price-dividend ratio is affine in the state vector, such that:

$$pd^V_t \;=\; pd^V_0 + \big(\,\bar{\mathbf{B}}^V\,\big)' z_t.$$

With this, grouping constant terms into r^V_0 (the unconditional expected log return on the central bank) yields:

$$r^V_{t+1} \;=\; r^V_0 + \big(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa^V_1 \, \bar{\mathbf{B}}^V\big)' z_{t+1} - \big(\,\bar{\mathbf{B}}^V\,\big)' z_t.$$

Where

$$r^V_0 \;=\; x_0 + \pi_0 + \kappa^V_0 - pd^V_0\,(1 - \kappa^V_1).$$

Taking this expression to the Euler equation:

$$\begin{aligned}
1 &= \mathbb{E}_t\big[\exp\big(m^{\$}_{t+1} + r^V_{t+1}\big)\big] \\
&= \exp\Big\{ - y^{\$}_0(1) - (\mathbf{e}_{yn})' z_t - \tfrac{1}{2}\,\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t + r^V_0 + \big[\big(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa^V_1 \, \bar{\mathbf{B}}^V\big)'\boldsymbol{\Psi} - \big(\bar{\mathbf{B}}^V\big)'\big] z_t \Big\} \\
&\qquad \times \mathbb{E}_t\Big[\exp\Big\{ - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} + \big(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa^V_1 \, \bar{\mathbf{B}}^V\big)'\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1} \Big\}\Big].
\end{aligned}$$

Using the log-normality of ε_{t+1} and substituting for the affine expression for $\mathbf{\Lambda}_t$ yields

$$\begin{aligned}1 = \exp\Big\{ & r_0^V - y_0^{\$}(1) + \big[\,(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa_1^V\,\bar{\mathbf{B}}^V)'\mathbf{\Psi} - (\bar{\mathbf{B}}^V)' - (\mathbf{e}_{yn})'\big] z_t \\ & + \underbrace{\tfrac{1}{2}\,(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa_1^V\,\bar{\mathbf{B}}^V)'\mathbf{\Sigma}\,(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa_1^V\,\bar{\mathbf{B}}^V)}_{\text{Jensen}} \\ & - (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa_1^V\,\bar{\mathbf{B}}^V)'\mathbf{\Sigma}^{\frac{1}{2}}(\mathbf{\Lambda}_0 + \mathbf{\Lambda}_1 z_t)\Big\}.\end{aligned}$$

Taking logs, the equation above can be split into conditional and unconditional parts. The left-hand side of this equation is the unconditional expected excess log return with the Jensen adjustment. The right-hand side is the unconditional covariance of the log SDF with the log return. With this, the unconditional risk premium on the central bank is given by:

$$r_0^V - y_0^{\$}(1) + \text{Jensen} = (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa_1^V\,\bar{\mathbf{B}}^V)'\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{\Lambda}_0. \tag{E3.15}$$

Conditionally the following must hold.

$$(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa_1^V\,\bar{\mathbf{B}}^V)'\mathbf{\Psi} - (\bar{\mathbf{B}}^V)' - (\mathbf{e}_{yn})' = (\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi + \kappa_1^V\,\bar{\mathbf{B}}^V)'\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{\Lambda}_1.$$

This describes the time-varying component of the central bank's risk premium. For reference, given $\mathbf{\Lambda}_1$, we can solve for the vector $\bar{\mathbf{B}}^V$ which establishes the recursion of the log price-dividend ratios on the aggregate claim to central bank dividends.

$$\bar{\mathbf{B}}^V = \left(I - \kappa_1^V(\mathbf{\Psi} - \mathbf{\Sigma}^{\frac{1}{2}}\mathbf{\Lambda}_1)'\right)^{-1}\Big[(\mathbf{\Psi} - \mathbf{\Sigma}^{\frac{1}{2}}\mathbf{\Lambda}_1)'(\mathbf{e}_{\Delta v} + \mathbf{e}_x + \mathbf{e}_\pi) - \mathbf{e}_{yn}\Big].$$

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