Green Mandate in Two-Sided Markets*

Briana Chang and Harrison Hong

December 2022

Abstract

We model the welfare effects of a green mandate in two-sided markets. Examples include banks and workers conditioning lending or employment on firms cutting emissions, respectively. We identify three differences when compared to the first-best carbon-emissions tax. First, despite complementarities, productive firms need not hire productive agents due to abatement costs. Second, the welfare-maximizing mandate requires firms abate for others, which might be infeasible. Third, agents without a mandate earn more and productive firms do better than under an emissions tax. Calibrating to lending and labor markets, a mandate approximates first best only when firms and agents are relatively homogeneous.

*We thank seminar participants at the University of Zurich for helpful comments.

1 Introduction

We analyze the welfare consequences of a green mandate in two-sided markets. Two prominent markets we have in mind are bank loans and workers. Nearly 40% of global banking assets have committed to aligning their lending portfolios with net-zero emissions by 2050 (United Nations Net-Zero Banking Alliance (2022)). Human resource surveys, such as those by job-search platform Glassdoor, indicate that workers are demanding that firms meet carbon emissions targets as a precondition for employment.

We are particularly interested in comparing outcomes achieved with an optimal green mandate to the planner's first-best solution. To do this, we introduce a carbon-emissions mandate into an integrated assessment model (Nordhaus 1992). Firm output is determined each period by combining firm productivity and agent productivity. Carbon emissions, which increase with firm output, accumulate over time, damage the economy and reduce social welfare. Firms under-spend on costly cleanup or abatement due to externalities.

However, a fraction of agents are green and will only work with firms that meet the mandate, thereby incentivizing firms to abate in order to attract them. Firms and agents otherwise match in a standard frictionless and competitive market over time. Risk-neutral firms consume profits and risk-neutral agents consume fees or earnings each period, respectively.

We identify three differences in outcomes under a green mandate compared to the planner's first-best solution. The first difference is misallocation. In the first-best solution, the planner optimally chooses the sorting to maximize output and also sets the emissions tax. Sorting between firms and agents is based on productivity — more productive agents work for more productive firms due to a complementarity in the production function. Firms face an emissions tax that is equal to the social cost of carbon.

Under a green mandate, sorting is determined in a decentralized equilibrium, as opposed to being chosen by the planner. With a mandate, agents with less productivity but who do not adhere to a mandate (i.e. non-green agents) can be as attractive as more productive agents who adhere to a mandate (i.e. green agents). The reason is the abatement costs incurred in hiring a green agent. Sorting can be summarized by an adjusted-talent index of agent productivity, whereby agent productivity is discounted by a factor equal to the per unit cost of carbon cleanup times the transformation of output to emissions. There is positive matching or sorting of more productive firms with agents with more adjusted-productivity.

The second difference is that the mandate that maximizes social welfare requires firms to clean up for others. The extent to which this is feasible depends on underlying distributions of firm productivity, and the joint distribution of agent productivity and green preferences. Only certain firms – the ones hiring green agents– will abate carbon emissions. Suppose that net-zero emissions in the aggregate (i.e. complete cleanup each period of total emissions) is socially optimal, then these green firms must clean up more for others, necessarily implying a net-negative mandate. Facing a very net-negative mandate, firms might then choose to shut down since they have to pay a high cleaning cost. When this happens, the mandate then distorts aggregate production and generates unemployment.

The third difference has to do with distributional consequences. Holding fixed agent productivity, firms hiring green agents (i.e. green firms) pay lower wages than firms with non-green agents (i.e. brown firms). This result is natural since there are abatememt costs incurred in order to hire green agents. We show that the equilibrium fee or earnings of an agent is comprised of not just this difference in abatement costs to hiring different types of agents but also bidding by productive firms that benefit the most by hiring them, i.e a competition cost. As a result, productive firms do better under a green mandate regime than an optimal emissions tax regime, i.e. their tax burden is lower under a mandate regime than an emission-tax regime.

The size of these differences in general depends on the distributions of firm productivity and worker talent, the cost of abatement, the fraction of green agents and whether productive agents are more likely to be activists. The more green agents the smaller the differences. These differences are non-monotonic in the correlation of agent productivity and agent type (i.e. green or non-green). If these two agent attributes are uncorrelated, the most productive firms will hire non-green agents and be brown, while the least productive firms will hire green agents and be green. Suppose instead that productive agents are all green, i.e. perfect correlation of types. Then the most productive firms will now be more likely to be green and the least productive firms more likely to be brown.

In our quantitative analysis, we apply our model to the US labor market and the bank loan market. We solve for the optimal green mandate subject to a no-shut down constraint and compare it with the first-best solution. We find that a mandate approximates the first best only when firms and agents are relatively homogeneous. The reason is that the differences articulated above are exacerbated by heterogeneity of firms and agents.

Corporations face pressure from a range of stakeholders to address the global warming externality. Research has mainly focused on consumer and investors in centralized markets (see Hong and Shore

2022 for a review). Generally, this research finds optimistic results when it comes to the ability of consumers (Besley and Ghatak 2007) and investors in centralized markets (Heinkel, Kraus, and Zechner 2001, Hong, Wang, and Yang 2021, Broccardo, Hart, Zingales, et al. 2022). Our paper highlights the limits of green mandates in two-sided markets. The importance of heterogeneity of agents in limiting the effectiveness of green mandates is echoed in Magill, Quinzii, and Rochet 2015.

2 Model

Production. Time is continuous. There is a continuum of heterogeneous firms and agents. We assume that firm and agent characteristics affecting production can be summed up by one number, which we refer them as their productivity, denoted by k and s, respectively. Let $A_t f(k, s)$ denote the flow production within the pair (k, s), where A_t represents the aggregate productivity.

Assumption 1. *The production function is continuous and multiplicatively separable $f(k, s) = a(k)b(s)$, where $a'(k) > 0$ and $b'(s) > 0$.*

For example, in the labor market, it can be nested as k as the firm size and $a(k) = k^\theta$ and $b(s) = s$. In the market for banks, one can then interpret $a(k)$ as the firm's productivity and $b(s)$ as the loan size provided by bank s. The distribution of firm is denoted by $G_f(k)$ with support $[k_L, k_H]$, and the distribution of agents is denoted by $G_w(s)$ with support $[s_L, s_H]$. As we explain later in Section 6, our model, which features one-to-one matching, can be applied to environments where a firm hires multiple workers or a bank lends to multiple firms by reinterpreting $G_w(s)$ and $G_f(k)$. All agents and capital owners (firms) are risk-neutral and consume their profits each period, respectively.

Aggregate output each period is given by

$$Y_t = A_t F_t, \tag{1}$$

where F_t is endogenous and depends on the matches between firms and agents in the economy. The process that changes the aggregate productivity is Poisson with an arrival rate μ. When there is a change, the new value of A' is drawn from the fixed distribution $H(A)$, with support $[A_L, A_H]$ and a mean $\bar{A}$. For simplicity, we assume that there are no frictions regarding the matching decisions, and we normalize agents' outside option to zero.

Carbon emissions and abatement. Following integrated assessment models (Nordhaus 1992, Jensen and Traeger 2014), we assume that production results in emission and all firms have access to an abatement technology. Production at time t leads to firm emissions $\sigma A_t f(k,s)$ and aggregate emission of $\sigma(A_t F_t)$. In the meantime, all firms can remove m_t of emissions at a linear cost c. Let E_{t-} denote the accumulated stock of emissions *before* period t. The aggregate level of emissions at period t is then given by $E_{t-} + \sigma A_t F_t - M_t$, where M_t represents the abatement by all firms at period t.

We assume that the accumulated emissions is depreciated at the rate δ. Hence, the law of motion for carbon emissions is given by

$$dE_t = ((\sigma A_t F_t - M_t) - \delta E_{t-})\, dt, \tag{2}$$

where the first two terms represent the newly added emissions net of abatement at period t.

Damages from carbon emissions. The damages of emissions to the economy at each point in time is strictly convex and increasing in the level of aggregate emission. It is modeled as a flow cost,

$$\frac{1}{1+\chi}\left(E_{t-} + \sigma A_t F_t - M_t\right)^{\chi+1}, \tag{3}$$

with $\chi > 0$.

We assume that the damage function is strictly convex ($\chi > 0$) to ensure interior solutions. Moreover, recent integrated assessment models emphasize that convex damage functions are more in line with climate science due to concerns about climate tipping points (Cai and Lontzek 2019, Bretschger and Pattakou 2019, Lemoine and Traeger 2014).

Assumption 2. *(Abatement is Socially Optimal)*

$$(\sigma A_L f(s_L, k_L))^{\chi} > c.$$

Assumption 2 means that the cleanup cost c is low enough so that zero abatement m is never socially optimal. Specifically, this condition states that the cleanup cost c is lower than the damage of emissions for the least productive pair of firm and agent (denoted by k_L and s_L) even when the accumulated stock is zero $E_{t-} = 0$. Hence, for any other pair and/or for any $E_{t-} > 0$, it is socially optimal to have positive abatement.

Emissions mandate. The mandate, denoted by ζ_t, is modeled as a constraint on a firm's emissions minus its abatement m, which yields

$$\sigma A_t f(k, s) - m \leq \zeta_t. \tag{4}$$

That is, ζ_t represents the carbon emissions tolerance, where the lower the ζ_t means a lower tolerance or a tighter standard. In the special case where $\zeta = 0$, the firm is cleaning up just its own emissions. In general, it can be negative, which would require the firm to clean up for others.

Green agents. We model some agents are only willing to match with firms that satisfy the emissions mandates, which we refer them as green agents. For example, one can interpret these agents are banks (workers) that only lend to (work for) firms that satisfy the carbon emissions mandate. Equation (4) thus serves as an additional constraint for firms that hire green agents. Such a constraint would not exist if a firm hires agents that do not care.

Agents' types are thus two-dimensional $(s, \theta) \subseteq [s_L, s_H] \times \{0, 1\}$, where $\theta = 1$ denotes green agents and zero otherwise. Let $g_w(s, \theta)$ denote the joint probability density, and the measure of green agents is given by $\lambda \equiv \int_S g_w(s, 1) ds$.

3 Decentralized Equilibrium with Green Mandate

Firms optimization problem. We now analyze the equilibrium outcome given any mandate. (In Section 4, we then consider the optimal mandate.) Given the mandate ζ_t, the firm chooses the type of worker (s, θ) to hire (if any) and the amount of removal m to maximize the present value of firm profits discounted at the risk-free rate r.

Let $w_t(s, \theta)$ denotes the fee for agent (s, θ), which can be interpreted as interest payments for banks or wages for workers. The Hamilton-Jacobi-Bellman (HJB) equation of the firm can be expressed as

$$\begin{aligned} rJ_t(k, A_t, \zeta_t) = & \max_{\{m_t \geq 0, (s,\theta)\}} A_t f(k, s) - w_t(s, \theta) - cm_t \\ & + \mu \left(\int J_t(k, A_t, \zeta_t) dH(A') - J(k, A_t, \zeta_t) \right) + J_\zeta d\zeta_t \end{aligned} \tag{5}$$

subject to Equation (4) iff $\theta = 1$ (i.e. the firm hires a green agent).

The HJB Equation (5) has the following terms. The first three terms on the right-hand side are the flow revenues net of fees and the cost of cleanup. The fourth term is the probability of a change in aggregate productivity times the expected change in the value function J depending on the draw of the productivity distribution A_t. The fifth term is the change in the value function with changes in the mandate ζ_t.

Profit-maximizing firms do not internalize the cost of emissions and thus the damage of emissions do not directly enter their objective function. The cost of emissions and the accumulated stock of emissions will affect the firm's problem only through the mandate ζ_t. It is also clear from Equation 5 that firms will not have incentives to abate if the firm does not hire a green agent.

3.1 Sorting with Mandates in Competitive Equilibrium

Definition 1. Given (A_t, ζ_t), a competitive equilibrium consists of a fee function $w_t(s, \theta)$, the assignment $\kappa_t(s, \theta)$, and cleanup $m_t(k)$ at period t such that (1) matching is stable, (2) $m_t(k)$ solves Equation (5) given the optimal match; and (3) the agent market clears.

Observe that, taking the mandate ζ_t and equilibrium fee function $w_t(s, \theta)$ as given, firms' hiring and abatement decisions are effectively static. This occurs for two reasons that need to be highlighted. First, matching between firms and agents is assumed to be frictionless. Hence, firms and agents can change their matches at any point of time. Second, the mandate ζ_t imposes a constraint only on time t emission but is not history dependent.

We thus solve the sorting and abatement problem for each time t and for given any (ζ_t, A_t). The *flow* surplus between firm k and agent type (s, θ) can be expressed as

$$\Omega_t(k, (s, \theta)) = A_t f(k, s) - C^\theta \max\{0, (\sigma A_t f(k, s) - \zeta_t)\}, \tag{6}$$

where

$$C^\theta = \begin{cases} c & \text{if } \theta = 1 \\ 0 & \text{otherwise} \end{cases}. \tag{7}$$

To simplify our analysis, we consider the case where the mandate is binding in the sense that every firm needs to do *some* cleanup in order to satisfy the mandate. In fact, as we will focus on the environment where net-zero is socially optimal, this condition is naturally satisfied.

Assumption 3. *(Binding Mandate)*

$$\sigma A_t f(k,s) > \zeta_t \ \forall k, s.$$

Under Assumption 3, firms that hire green agents must then choose abatement (a positive value of m) so that the constraint Equation (4) is binding $\forall(k,s)$. Thus, the flow surplus in Equation 6 can be further reduced to

$$\Omega_t(k,(s,\theta)) = (1 - C^\theta \sigma) A_t f(k,s) + C^\theta \zeta_t. \tag{8}$$

One-dimensional sorting by pseudo-index z.

Lemma 1. *Under Assumption 3, the sorting outcome can be summarized by an one-dimensional index $z(s,\theta) = (1 - C^\theta \sigma) b(s)$, where firm with higher k is matched with an agent with a higher index $z(s,\theta)$.*

To see this, observe that

$$\frac{\partial \Omega_t(k,(s,\theta))}{\partial k} = \left\{(1 - C^\theta \sigma) b(s)\right\} A_t a'(k)$$

only depends on (s,θ) through the one-dimensional index $z(s,\theta)$. That is, the marginal gain of matching to a more productive firm is the same for two agents have the same index $z(s,\theta)$. Hence, these two agents must have the sorting outcome. Moreover, since $a'(k) > 0$, this also means that $\frac{\partial \Omega_t(k,z(s,\theta))}{\partial k \partial z} > 0$. In other words, the sorting can thus be understood as larger firm are matched with agents with higher index z.[1]

Intuitively, since hiring the green agents requires cleaning up and thus becomes more costly, the index summarizes the ranking of agents' added value. All else equal, the green agent now receives a discount of $(1 - c\sigma)$ relative to the agent's productivity; and thus the green agent's ranking is the same as a lower skilled non-green agent $b(s') < b(s)$ where $b(s') = (1 - c\sigma)\, b(s)$.

Assumption 4. *(Production is Socially Optimal)*

$$(1 - c\sigma) > 0$$

Assumption 4 means that, taking into account the emission and the abatement costs, the produc-

[1] According to Chiappori (2016), this is the case is referred as "pseudo-index" model.

tion is nevertheless socially optimal even for the least productive pair. As explained later from the planner's problem, Assumption 4 implies that the first-best solution is to maximize production and to clean up accordingly without shutting down the firms.

Very negative ζ_t and potential for firm shut down. It is important to note that, despite that $1 - c\sigma > 0$, the surplus can be negative with a very negative ζ_t, as

$$\Omega_t(k, (s, 1)) = (1 - c\sigma) A_t f(k, s) + c\zeta_t. \tag{9}$$

Recall from our discussion of Euation 4 that $\zeta_t = 0$ represents the mandate that requires firms to clean up their own emission. A very negative ζ_t, however, can arise when green firms are required to clean up for other brown firms. As a result, when facing a very tight mandate (when ζ_t is very negative), firms may optimally choose to shut down. Moreover, according to Equation 9, given any ζ_t, firms that produce less are more likely to shut down.[2]

Distribution of pseudo-index z. Given that the surplus can become negative for activists, let $\hat{s}_t$ denote the cutoff type of green agentssuch that $\Omega_t(k, (\hat{s}_t, 1)) = 0$. Given any $\hat{s}_t$, the distribution of z depends on the joint distribution $g_w(s, \theta)$, which is given by

$$G_w(z|\hat{s}_t) \equiv \int_{\hat{s}_t}^{b^{-1}\left(\frac{z}{1-c\sigma}\right)} g_w(\tilde{s}, 1) d\tilde{s} + \int_{s_L}^{b^{-1}(z)} g_w(\tilde{s}, 0) d\tilde{s}. \tag{10}$$

From Equation 10, the first term is the mass of green agents such that $(1 - c\sigma) b(s) \leq z$ conditional on being employed $s \geq \hat{s}_t$, and the second term is the mass of non-green agents such that $b(s) \leq z$. Note that, because of some green agents may not work, the measure of total agents that are employed is given by $G_w(s_H|\hat{s}_t) = 1 - \int_{s_L}^{\hat{s}_t} g_w(\tilde{s}, 1) d\tilde{s} \in [(1 - \lambda), 1]$. For simplicity, we focus on the case where the lowest value of z in the market is the least skill green agents $(1 - c\sigma)\, \hat{s}_t$, as we show that it is also the relevant case under the optimal mandate.[3]

[2] A positive ζ_t, on the other hand, means that firms can produce positive net emission, which automatically guaranteed positive surplus under Assumption 4.

[3] More generally, the lowest value of z can also be the least skilled non-green agent, which happens when there are lots of green workers that are out of market and thus $(1 - c\sigma)\hat{s}_t \geq s_L$.

3.2 Equilibrium Assignment, Fees, and Green vs. Brown Firms

Proposition 1. *Given any ζ_t, the equilibrium is characterized by the cutoff type $(\hat{s}_t, \hat{k}_t)$, where agents (firms) are out of the market iff $s \leq \hat{s}_t$ ($k < \hat{k}_t$). For any agent (s, θ), the assignment function is given by $\kappa(s, \theta) = \kappa^*(z(s, \theta))$, where $\kappa^*(z(s, \theta))$ solves (1) $G_w(s_H|\hat{s}_t) - G_w(z|\hat{s}_t) = 1 - G_f(\kappa_t^*(\tilde{z}))$ for any $z \geq z_{L,t}$ with $\hat{k}_t = \kappa^*(z_{L,t})$,and (2) for any $\hat{s}_t \in (s_L, s_H)$, $\Omega_t\left(\kappa^*\left((1 - c\sigma)\hat{s}_t\right), \hat{s}_t\right) = 0$. If the firm k is matched with a green agent with skill s,*

$$m^*(k) = \sigma A_t f(k, s) - \zeta_t, \tag{11}$$

and zero otherwise. The fee for a green agent is given by

$$w_t(s, \theta) = W_t(z(s, \theta)) + C^\theta \zeta_t \tag{12}$$

where

$$W_t(z) = \int_{z_{L,t}}^{z} A_t a\left(\kappa^*(\tilde{z})\right) d\tilde{z} + W_t\left(z_{L,t}\right) \tag{13}$$

and $W_t(z_{L,t}) = -c\zeta_t$.

Since firms only abate carbon emissions when they end up hiring green agents, the sorting outcome thus determines firms' abatement. That is, if a firm hires a green agent in equilibrium, it chooses the abatement so that the mandate is satisfied, which gives Equation 11 and zero otherwise. We thus refer the firms that (do not) hire green agents and thus engage emissions abatement as the green (brown) firms.

The assignment function $\kappa^*(z)$ determines the firm type for the worker with index z. A firm with size k can thus either be matched with a non-green agent or a green agent but with a relatively high skill, as long as both have the same index z. The wage equation further implies that firm k is indeed indifferent between these two options. This is because that, for any $b(s) = (1 - c\sigma)b(s') = z$, they result in the same flow payoff to the firm, which yields

$$A_t f(k, s) - w_t(s, 0) = (1 - c\sigma)A_t f(k, s') + c\zeta_t - w_t(s', 1),$$

where we use the fact that $w_t(s', 1) = W_t(z) + c\zeta_t$ according to Equation 12. Note that $W_t(z)$ given in Equation (13) is simply the equilibrium utility for an agent with index one-dimensional index z, as in Tervio (2008).

The second term in Equation (12) is an adjustment for the abatement cost. Only for the special case with $\zeta_t = 0$, two agents with the same index z generates the same matching surplus value. More generally, the level of the mandate affects firms' abatement costs and hence the surplus and thus fees differ for green and non-green agents conditional on index z.

Note that, since we normalize agent's outside option to be zero, the utility for the lowest type $W_t(z_{L,t})$ is pinned down so that his utility is zero. Specifically, since the lowest type is a green agent, then $w(\hat{s}_t, 1) = W_t(z_{L,t}) + c\zeta_t = 0$.

The effect of mandate on aggregate abatement. Given ζ_t, the aggregate level of abatement is essentially the abatement by all firms that hire green agents, which can be expressed as

$$M_t(\zeta_t) = \int_{\hat{s}_t(\zeta_t)}^{s_H} \{\sigma A_t f(\kappa^* ((1 - c\sigma)s), s) - \zeta_t\} g_w(s, 1) ds. \tag{14}$$

This highlights two effects of the mandate. First of all, it directly affects the level of abatement of green firms. Second, it affects the surplus function and may result in shut down, affecting the cutoff type $\hat{s}_t(\zeta_t)$. Recall that the sorting outcome only depends on the underlying distribution $G_w(z|\hat{s}_t)$. Hence, if the mandate does not affect the cutoff type $\hat{s}_t(\zeta_t)$, a lower ζ_t only increases the aggregate abatement but does not affect the sorting outcome.

Earnings premium for non-green agents. Since green agents require firms to clean up, they would thus recieve a lower fee relative to non-green agent counterpart in equilbiurm. Formally, the earnings premium for non-green agents with skill s yields

$$EP_t(s) \equiv w_t(s, 0) - w_t(s, 1) = W_t(z(s, 0)) - W_t(z(s, 1)) - c\zeta_t. \tag{15}$$

Similarly, the premium is affected by the mandate ζ_t through two channels. Fixing $\hat{s}_t$, a lower ζ_t leads to more abatement and thus higher premium. When the mandate also affects the cutoff type, it results in different $G_w(z|\hat{s}_t)$ and thus $W_t(z)$.

Equation 15 highlights that the earnings premium is determined by the difference in compensation of two different z-index, which can be expressed as

$$
\begin{aligned}
w(s,0)-w(s,1) &= A_t \int_{(1-c\sigma)b(s)}^{b(s)} a\left(\kappa^*(z)\right) d\tilde{z} - c\zeta_t \qquad (16)\\
&= A_t \underbrace{\left\{ \int_{(1-c\sigma)b(s)}^{b(s)} \left\{ a\left(\kappa^*(z)\right) - a\left(\kappa^*\left((1-c\sigma)b(s)\right)\right)\right\} d\tilde{z} \right\}}_{competition cost} + \underbrace{c\left(\sigma A_t f\left(\kappa^*\left((1-c\sigma)b(s)\right), s\right) - \zeta_t\right)}_{cleaning cost}.
\end{aligned}
$$

The earning premium in our model can thus be decomposed by two terms. The first term captures the fact that type-s green worker is now hired by the smaller firm, relative to type-s non-green worker, which is the loss of having a lower ranking. This term is zero if and only if firms are homogeneous, and increases in firm dispersion. The second term represents the cleaning cost for the employees that hires type-s activist, who needs to clean up $\{\sigma A_t f(\kappa^*((1-c\sigma)s, s) - \zeta_t\}$ in order to satisfy the mandate. This expression further highlights that, without misallocation (i.e., when firms are all homogeneous), the wage gap collapses to the standard cleaning cost.

Lemma 2. *Given ζ_t, $EP_t(s)$ increases with s and λ $\forall t$.*

Lemma 2 highlights that the premium is larger for more productive agents. This is because more productive agents are employed at more productive firms, which thus predicts a higher $\int_{(1-c\sigma)b(s)}^{b(s)} \kappa^*(\tilde{z})d\tilde{z}$ in Equation 16. This result holds for any correlation between skill and green preference. In other words, this result does not rely on the fact that higher skilled agents are more likely to be green.

The effect of λ, the total measure of green agents is also driven by the competition effect. Intuitively, when there are less green agents (lower λ), non-green agents become more scare and attractive, and thus enjoy higher rents. Formally, one can show that $\kappa^*(z)$ is weakly higher for a higher λ, and thus increases the premium.

4 First-Best Solution or Socially Optimal Mandate

A mandate can be understood as a special policy instrument that the planner can use given the existence of green agents. We are interested in how the planner can use a mandate to achieve the socially optimal abatement. To do so, in Section 4.1, we begin by establishing the first-best benchmark, where we analyze the planner's problem and show that this first-best solution can be implemented through an emissions tax. In Section 4.2, we then study the optimal mandate and compare it to the first-best benchmark in Section 5.

4.1 First-Best Emissions Tax

The planner's problem The planner chooses (1) the assignment function $\kappa_t(s)$, which determines the total production in this economy, denoted by $F_t\{\kappa_t(s)\}$, and (2) the aggregate emission removal, denoted by M_t, to maximize social welfare:

$$\max_{\{\kappa_t(s), M_t\}} \mathbb{E} \int_0^{\infty} e^{-rt} \left\{ A_t F_t(\kappa_t(s)) - \frac{d}{1+\chi} \left(E_{t-} + \sigma A_t F_t(\kappa_t(s)) - M_t\right)^{\chi+1} - cM_t \right\} dt. \tag{17}$$

Since the planner can change the sorting at each point of time (i.e., no reallocation or search frictions), the assignment problem is again effectively static and it only affects the production F_t at period t. However, unlike the firms in competitive markets that take the policy at period t as given, the planner's abatement problem is dynamic as the continuation value is affected by the accumulated emissions. The HJB equation thus yields

$$\begin{aligned} rV^{FB}(A_t, E_{t-}) = \max_{\{\kappa_t(s), M_t\}} & A_t F_t\{\kappa_t(s)\} - \frac{1}{1+\chi} \left(E_t + \sigma A_t F_t\{\kappa_t(s)\} - M_t\right)^{\chi+1} - cM_t \\ & + \mu \left\{ \int V^{FB}(A', E_{t-}) dG(A') - V(A_t, E_{t-}) \right\} + \frac{\partial V^{FB}(A_t, E_{t-})}{\partial E} dE_t \end{aligned} \tag{18}$$

The first three terms on the right-hand size are the total production, net of total damages and cost of abatement. The fourth term is the probability of a change in aggregate productivity times the expected change in the value function depending on the draw of the productivity distribution A_t. The fifth term is the change in the value function with changes in the accumulated emission stock E_t.

Lemma 3. *Under Assumption 4,* $\kappa^{FB}(s) = \arg\max_{\kappa(s)} \int f(\kappa(s), s) dG_w(s)$*.*

$$V^{FB}(A_t, E_{t-}) = \left(\frac{(1 - c\sigma) F\{\kappa^{FB}(s)\}}{r} \right) \left(\frac{rA_t + \mu \bar{A}}{r + \mu} \right) + \gamma_E E_{t-} + v_0,$$

where $\gamma_E = -\left(\frac{c}{1+r+\delta}\right)$*. The first-best allocation and welfare can be achieved with an emissions tax in competitive markets, where*

$$T_t(e) = \left\{ \frac{d}{1+\chi} (e + E_t)^{\chi+1} - \gamma_E e \right\} + \tau(E_t),$$

where $E_t = E_{t-} + \hat{e}_t$ *is the current stock of emission, and* $\tau(E_t) = \left(\frac{\chi}{1+\chi}\right)(\gamma_E + c)^{\frac{\chi+1}{\chi}} - (c + \gamma_E) E_t$.

Recall that Assumption 4, $(1 - c\sigma) > 0$, implies that it is optimal to maximize the production, taking into account the abatement costs. Hence, the optimal sorting must simply maximize total production at each point of time, obtaining the first-best production, which yields $F^{FB} \equiv \max_{\kappa(s)} \int f(\kappa(s), s) dG_w(s)$.

Given the production F^{FB}, the optimal mandate is then simply chosen to generate the optimal cleaning up M_t, so that the marginal benefit of decreasing emission stock must equal to the cost of abatement. That is, the FOC is satisfied

$$d\left(E_{t-} + \sigma A_t F^{FB} - M_t\right)^{\chi} - \frac{\partial V^{FB}(A_t, E_{t-})}{\partial E} = c, \tag{19}$$

where RHS is the marginal benefit of decreasing emission stock that includes the flow costs as well as the cost of increasing the stock next period.

To implement this, the tax function must internalize the social cost of emissions. The first term in the tax is the flow damage cost and the second term represents the cost of increasing future emission stock, using the fact that $\frac{\partial V^{FB}(A_t, E_{t-})}{\partial E} = \gamma_E$.

The flow payoff to the firm, denoted by $J_t^{Tax}(k)$, under the emissions tax can be rewritten as

$$J_t^{Tax}(k) = \max_s A_t f(k, s) - C_t(f(k, s)) - w_t(s),$$

where $C_t(f) \equiv \min_m T_t(\sigma f - m) + cm$ represents the effective cost of production, taking into account the abatement costs and tax payment. In this implementation, the constant term $\tau(E_t)$ is chosen so that firms' problem under the taxation schedule can be expressed as $C_t(f) = c\sigma A f$ and thus

$$J_t^{Tax}(k) = \max_s (1 - c\sigma) A_t f(k, s) - w(s).$$

In other words, taxation effectively generates a discount factor $(1 - c\sigma)$ in productivity for all pairs. Notice that while such a discount will change firm's profits and agent's earnings, it will *not* affect the sorting in decentralized market; hence, the first-best allocation is thus guaranteed.

Notice that, since we focus on heterogeneous agents in our framework, we take the productivity distribution as given (the capital stock and skill) as given and shut down the intensive margin for simplicity. If one adds back such a choice, then the taxation will induce distortion on the intensive margin (such as the capital and labor investment), consistent with standard model with homogeneous agents with tax distortion.

4.2 Socially Optimal Green Mandate

We now analyze the optimal green mandate. There are two key differences between the mandate vs. first-best taxation. First, given any mandate ζ_t, the sorting is determined in a decentralized equilibrium, and the corresponding aggregate abatement is then given by Equation 14. That is, the planner is no longer able to choose the sorting as in the first-best benchmark.

Second, only certain firms – the ones hiring green agents– will abate carbon emissions. Suppose that net-zero is socially optimal, these green firms must clean up more for others, necessarily implying a negative mandate $\zeta_t < 0$. Facing a very negative mandate, firms might then choose to shut down since they have to pay a high cleaning cost. When this happens, the mandate then distorts aggregate production and generates unemployment.

We now solve for the optimal mandate subject to a no-shut-down constraint. That is, we restrict ourselves to the mandate ζ_t that ensures positive surplus for all pairs. According to Equation 9, since the least productive pairs are more likely to shut down given by ζ_t, the no-shut down constraint can thus be further reduced to having non-negative surplus for the least productive pair, which yields

$$\Omega_t(\kappa(s_L, 1), (s_L, 1)) = A_t z(s_L, 1) a(\kappa(s_L, 1)) + c\zeta_t \geq 0. \tag{20}$$

That is, taking into account the cleaning costs, the surplus of hiring the least skill green agent must be positive. In the i.i.d case, $\kappa(s_L, 1) = k_L$ will then be the smallest firm. Note that, this condition necessarily implies that all pairs that hire green agents have positive surplus, and, as a result, all firms and agents must also receive non-negative payoff in equilibrium.

Let $F_t(\zeta_t)$ denote the total production in decentralized markets under mandate ζ_t. The planner chooses the mandate to maximize the aggregate payoff, taking into account the welfare cost of emissions and abatement costs. The HJB equation is given by

$$\begin{aligned}
rV^M(A_t, E_{t-}) = \max_{\zeta_t} A_t F_t(\zeta_t) - \frac{d}{1+\chi}\left(E_{t-} + \sigma A_t F_t(\zeta_t) - M_t(\zeta_t)\right)^{\chi+1} - cM_t(\zeta_t) \\
+ \mu\left\{\int V^M(A', E_{t-})dH(A') - V^M(A_t, E_{t-})\right\} \\
+ \frac{\partial V^M(A_t, E_{t-})}{\partial E}\left\{(\sigma A_t F_t(\zeta_t) - M_t(\zeta_t)) - \delta E_{t-}\right\},
\end{aligned} \tag{21}$$

subject to Equation 20.

Since the no-shut-down constraint guarantees that $\hat{s}_t = s_L$, the level of abatement itself will not affect the sorting and thus the level of output. In other words, $F_t(\zeta_t) = F^* \ \forall \zeta_t$. Hence, given the production F^*, the optimal mandate is thus given by either the interior solution or the corner solution when Equation 20 is binding. Specifically, the interior solution must imply that the mandate induces the abatement such that the FOC is satisfied, which yields

$$d\left(E_{t-} + \sigma A_t F^* - M_t(\zeta_t)\right)^{\chi} - \frac{\partial V^M(E_{t-})}{\partial E} = c, \tag{22}$$

where F^* represents the matching under mandate.

One can show that in the unconstrained regime, the interior optimal mandate, denoted by ζ_t^I, is thus given by:

$$\zeta_t^I = \frac{1}{\lambda}\left\{\left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}} - E_{t-} - \sigma A_t\left(F^* - F^g\right)\right\}, \tag{23}$$

where $F^g \equiv \int_{s_L}^{s_H} f(\kappa^*((1-c\sigma)s), s) g_w(s,1) ds$ represents the production generated by all firms who hire activists. The higher emission by the brown firms, captured by the term $\sigma A_t(F^* - F^g)$, implies more negative mandates.

Specifically, suppose that we are at the state in which net-zero is the optimal (i.e., $E_{t-} = E_{ss} = (\frac{c+\gamma_E}{d})^{\frac{1}{\chi}}$), the green firms must clean up for others, and thus $\zeta_t < 0 \ \forall \lambda < 1$. Only in the extreme case where all agents are green agents $\lambda = 1$, we thus have $F^* = F^g$ and $\zeta_t = 0$. Whenever the constraint is binding, we thus have

$$\bar{\zeta}_t = \frac{-(1-c\sigma)A_t f(\kappa^*(s_L, 1), s_L)}{c}.$$

5 Differences in Outcomes Between Socially Optimal Mandate and First Best

Relative to the first-best benchmark, the mandate has three sets of differences. First, it generates misallocation in decentralized equilibrium. Second, the no-shut-down constraint limits how much green firms can clean up for brown firms. Third, there are distributional consequences. To our earlier point regarding non-green agents' earning premium, we show that moe productive firms do relatively better under a green mandate than under a carbon-emissions tax.

5.1 Misallocation

Proposition 4 below highlights the skill misallocation generated by the optimal employee mandate, as low skilled non-green agents become relatively valuable, and thus $F^* \leq F^{FB}$.

Lemma 4. *Compared to the first-best allocation, green agents (non-green agents) work for smaller (larger) firms* $\kappa_t(s,1) \leq \kappa^{FB}(s)$ $\left(\kappa_t(s,0) \geq \kappa^{FB}(s)\right)$ *and* $F^* \leq F^{FB}$*. The distortion in the optimal employee mandate equilibrium, denoted by* $\frac{F^*}{F^{FB}}$*, increases with* c*,* σ*, and heterogeneity of agents and firms, and non-monotonic in correlation between talent and activism.*

Correlation between workers' skills and preference. To understand the effects of the correlation between skill and green preference, let $\lambda(s) \equiv \frac{g_w(s,1)}{g_w(s,1)+g_w(s,0)}$ represent the measure of green agents conditional on skill s. In the special case that there is no correlation between talent and activism, we thus have $\lambda(s) = \lambda \ \forall s$.

To illustrate the effects, it is useful to consider three special cases: no correlation $\lambda^0(s) = \lambda \ \forall s$, the extreme positive, and the extreme negative correlation case, denoted by $\lambda^+(s)$ and $\lambda^-(s)$, respectively.

$$\lambda^+(s) = \begin{cases} 1 & \text{if } s \geq s^+ \equiv G_w^{-1}(1-\lambda) \\ 0 & \text{otherwise} \end{cases} . \ \lambda^{-1}(s) = \begin{cases} 0 & \text{if } s \geq s^- \equiv G_w^{-1}(\lambda) \\ 1 & \text{otherwise} \end{cases} .$$

That is, $\lambda^+(s)$ ($\lambda^-(s)$) means that agents with relatively high skill $s \geq s^+$ ($s \geq s^-$) are all green (non-green) agents, where s^+ and s^- are pinned down so that the total measure of green agents is λ.

Figure 1 shows that when skills and preference are uncorrelated, the modified-talent z distribution of non-green agents (red line) lies above that of the green agents. Hence, non-green agents are more

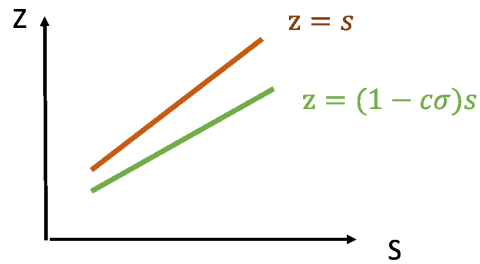

Figure 1: Uncorrelated skill and green preference.

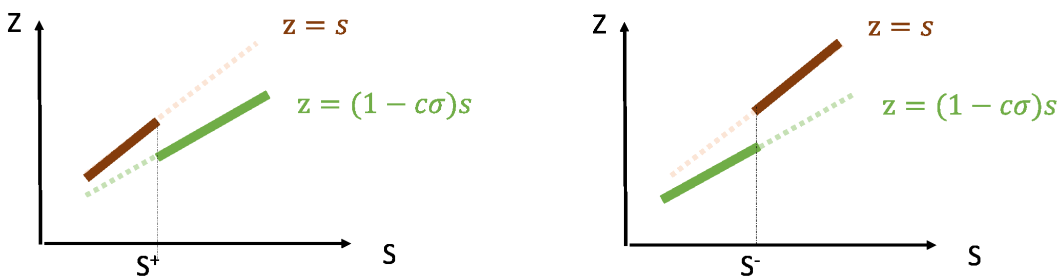

Figure 2: Correlated skills and green preference. The first subfigure represents the extreme positive correlation and the second subfigure represents extreme negative correlation. The width of the green and brown line represent the size of green agents and non-green agents given the skill s.

attractive to productive firms and will be employed at and offered higher wages by productive firms. Less productive firms hire green agents and do perform the abatement for society.

Figure 2 below illustrative the effect of distribution on z when talent and activism are correlated. We use the width of the green and brown line represent the size of green agents and non-green agents given the skill s.

Recall that $\kappa^*(z)$ is increasing in z. Hence, the negative-correlation case predicts that the most productive firm must hire non-green agents. On the other hand, in the case with positive correlation, agents with highest z also have higher skills and thus are matched with the he most productive firm.

This example also illustrates why the size of distortion is generally non-monotonic in correlation. Note that the distortion arises whenever the index z and s are not perfectly correlated. In the case with the positive correlation, the top skill agents are always agents with top index. However, moving from the positive correlation to zero correlation, some of these high-skill agents now have lower ranking z and thus be outcompeted by some agents with lower skill but non-activists, which thus increases the distortion. Similarly, the distortion also increases when moving from the negative correlation to zero correlation. In fact, the negative correlation case is the special case where z and s is perfectly correlated (i.e., higher s must have higher z), which thus means no distortion.

5.2 Binding No-Shut-Down Constraint

Recall that the no shut down constraint represents that there is a limit of how much green firms are willing to abate for brown firms. Observe from Equation 23, one can also see that the no-shut down constraint is likely to bind when emission stock is high (E_{t-}), lower measure of green agents, and when brown firms produce more emissions.

Importantly, which firms become brown or green depends on the equilibrium sorting. Intuitively, if green firms happen to be larger firms, then the constraint is unlikely to bind as they only need to abate for firms that have lower emission. Through this channel, the correlation between talent and activism is thus crucial for the aggregate abatement.

Lemma 5. *No shut down constraint is more likely to bind with lower λ, higher emission stocks E_{t-}, lower correlation between between talent and activism.*

5.3 Distributional Effects

We now analyze how firms' profits compares in the optimal employee mandate equilibrium and the first best. As in the case with taxation, firms' profits are lower; however, the mandates affect firms differently due to the sorting. To be precise, we thus compare firm profits under the optimal mandates vs. emission tax, which are denoted as $J_t^M(k)$ and $J_t^{Tax}(k)$, respectively.

To allow for arbitrary correlation between talents and activists, we assume that $\lambda(s) = \lambda + \chi(s) \in [0, 1]$, where $\int \chi(s) dG_w(s) = 0$ and thus $\int \lambda(s) dG_w(s) = \lambda$.

Lemma 6. *$J_t^M(k)$ decreases with λ and $J_t^M(k) - J_t^{Tax}(k) \geq 0$, where equality holds when $\lambda(s) = 1$ $\forall s$. Moreover, $J_t^M(k) - J_t^{Tax}(k)$ increases with firm size k.*

The idea behind the proof uses the fact the outcome of taxation can is equivalent to the special case where all agents are green ($\lambda = 1$), where all firms are effectively face the same discount factor $(1 - c\sigma)$ in all pairs. For any $\lambda < 1$, it can then be understood as if a better distribution of types z, as some agents do not care about the mandate. As a result of that, firms now can hire agents with a weakly higher index z and thus earn higher profits.

Moreover, larger firms benefit most for the improvement of distribution on the top, they thus benefit relatively more as well. In other words, compared to taxation, the effective costs for larger firms are now lower. Since a lower λ means more non-activities, which again improves the agents' effective distributions further. Hence, for the same reason, it increases firm profits.

In the taxation benchmark, firms have lower marginal profits and agents thus also have lower wages. On the other hand, under the mandate, non-green agents enjoy higher wages as they're more valuable to firms. Hence, a higher measure of green agents means non-green agents becomes more scarce, which thus increases their fees $w_t(s,0)$.

Lemma 7. *$w_t(s,0)$ increases with λ, and $w_t(s,0) - w_t^{Tax}(s) \geq 0$ where equality holds when $\lambda = 1$.*

6 Quantitative Analysis

We have shown that the optimal mandate can deviate from the first-best and the amount of the deviation crucially depends on the underlying distribution of firm and agent productivity. In this section, we now provide the quantitative analysis for two specific applications: the labor market and the bank-loan market.

6.1 Calibrating First-Best Emissions Abatement

For both applications, we will assume that $\delta \to 0$ and that the current emission is at the the steady state level, i.e. $E_{ss} = E_0$. As such, abatement has to be such that "net-zero" is indeed optimal, i.e. there are no further changes to the carbon stock in the atmosphere, which can only happen when all further flow of emissions is abated.

The current emission stock is $E_0 = 2100$ (gigaton). Moreover, since the optimal abatement is net-zero $M_t = \sigma A_t F_t$, the ratio of abatement to output is then given by $\frac{cM_t}{A_t F_t} = c\sigma$, which is set to 4%, following estimates from the literature. Using that the emission rate is around 100 ton of carbon emission per 1 million of revenue ($\sigma = 1 \cdot 10^{-4}$), the cleaning cost is thus around 400 dollar cost per ton. The interest rate is set to be 6%.

The effectiveness of the mandates, however, depends on the underlying distributions of firms and agents. We thus now proceed to calibrate these parameters for these two applications.

6.2 Application: Labor Market with Climate Activists

For the labor market, we follow the literature and assume that $a(k) = k^\theta$ and $b(s) = s$, where k represents the firm size and θ represents the impact of the size. While our baseline model is about one-to-one matching, it can be reinterpreted as firm k have multiple positions $\ell(k)$ and the matching is between positions and workers, under the assumption that the production function is additively separable across types and within types (see shown in Branikas et al. 2022). Moreover, assuming that the distribution of firms k is Pareto with index α with density $g_f(k)$ and $\ell(k) = \ell_0 \left(\frac{k}{k_L}\right)^m$, the measure of positions provided by firms that is smaller than k is then given by $G(k) = \int \ell(k)g_f(k)dk$, which is a Pareto distribution with index $\gamma = \alpha - m$.

Note that a firm must satisfy either the mandate or not. Hence, when firms have multiple positions, we interpret that a firm k can either hire green or brown workers for all his positions. Thus, the trade-off of hiring green vs. brown workers derived in our baseline model remain intact.

Our calibration consists of parameters obtained externally from various sources and parameters calibrated internally. The fraction of workers who adhere to a green mandate $\lambda = 0.2$ is obtained externally based on surveys from Glassdoor. Other than λ, our other parameters are from Branikas et al. 2022. The Pareto index of the firm productivity distribution γ is 1.1. The complementarity parameter θ is 0.149. The heterogeneity of worker talent is given by $\beta = -0.13$. The support of k (firm productivity or size) is given by $[100, 2.5^{*}10^6]$

The distribution of workers is given by $G_w(s) = 1 - \left(\frac{s_H - s_L}{s_H - s}\right)^{-\frac{1}{\beta}}$, where $\beta = -0.13$. We assume constant TFP $A_t = A$, which can be interpreted as the long term (detrend) average productivity. For the benchmark, we further assume that there is no correlation between worker talent and green preferences.[4]

The remaining parameters are thus measure of activists (λ), TFP (A), and the support of agents' skills (s_L, s_H), which are calibrated to match the following four moments: (1) the revenue for the smallest, median, and the largest firm, which is given by 1, 50, and 572,000 (millions), respectively; (2) the average wage gap which we take be 5%. Table 1 below summarizes the parameter values.

Table below summarizes the misallocation and the aggregate abatement relative the first-best benchmark as well as the wage-gap where the average wage gap is defined as the cross sectional average of $\frac{w_t(s,0) - w_t(s,1)}{w_t(s,0)}$. Our result shows that that, quantitatively, the distortion of the mandate is relatively small in terms of the misallocation.

To highlight the effect of mandate, we further compare these result with two counterfactual environments. First, we consider the case where firms are homogeneous $k[j] = \bar{k}\ \forall j$, and thus there

[4] Empirical findings in the literature point to more productive firms being more green (Hong, Kubik, and Scheinkman 2012, Servaes and Tamayo 2013, Albuquerque, Koskinen, and Zhang 2019). Hence, it would be appear that we are in the non-negative region. We take the i.i.d. as a lower bound on the effectiveness of these employee mandates.

Parameter	value	source
λ	0.2	external
γ	1.1	external
θ	0.149	external
β	-0.13	external
(k_L, k_H)	$(10^{-4}, 2.5)$ (million)	external
(s_L, s_H)	$(0.0897, 1.136 * 10^4)$	internal
A	$7.163 * 10^5$	internal

Table 1: Parameter Values

	Benchmark	Homogeneous firms	Homogeneous firms and workers
$\left(F^{FB} - F^*\right)/F^{FB}$ (%)	$3.418 \cdot 10^{-4}$	0	0
$(M^*)/M^{FB}(\%)$	19.98	20.14	100
Binding NSD Constraint	Yes	Yes	No
Average Wage Gap (%)	5.1624	3.4634	100

Table 2: Outcomes of the Optimal Green Mandate

is no mis-allocation effect. The no-shut down constraint is nevertheless binding in this case, which highlights the role of heterogeneous workers.

Now, consider the benchmark where both firms and workers are homogeneous, all pairs have the same production AF. One can see that the constraint is not binding iff

$$AF + c\left(\frac{-\sigma AF}{\lambda}\right) > 0,$$

where $\frac{\sigma AF^*}{\lambda}$ represents the aggregate emission divided by the share of green firms. That is, as long as λ is large enough, then these green firms will be able to implement the first-best solution. This condition holds in our calibration as $\lambda = 0.2$ and $c\sigma = 4\%$, which explains that one would be able to implement the first-best when both firms and workers are homogeneous.

Note that in this special case, since we assume that green workers receive zero wages (as we assume their outside option is zero), we thus have $w(s, 1) = 0$ and $w(s, 0) = c\zeta_t$, thus explaining why the wage gap is 100%. That is, brown workers receive premiums as they save the cleaning costs. All firms are thus indifferent between paying high wages to brown workers or implementing the cleaning up.

6.3 Application: Bank Loan Market with Green Mandate

In the content of banks, it would be more natural to interpret that one bank can lend to multiple firms. Hence, in this case, we assume that each bank s has $m(s)$ measure of managers and each manager can lend to a firm with a loan size $b(s)$. That is, the matching is between each manager and firm. Similar as before, as long as the payoff of each loan is independent, each manager then make their loan decision separately. Hence, all characterization remains the same, where the only difference is the underlying distribution of s is now given by $G_w(s) = \int_{s_L}^{s} m(\tilde{s}) f(\tilde{s}) d\tilde{s}$,where $f(s)$ is the measure of bank s.

In this application, since a bank can do normal loans vs. green loan, one can then interpret λ as the share of bank's portfolio that can only lend to firms that satisfy the mandates. The fees in this framework $w(s, \theta)$ thus represent the total repayment for the loan with size $b(s)$, where $\theta = 1$ if it's a green loan.

TO BE COMPLETED

7 Conclusion

We model the welfare effects of a green mandate in two-sided markets. Examples we have in mind include markets for bank loans and workers. Compared to the first-best carbon-emissions tax, we identify three differences that arise in the green mandate setting. The first difference is that despite complementarities, productive firms need not hire productive agents due to abatement costs. The second difference is that the welfare-maximizing mandate requires firms abate for others, which might be infeasible. The third difference is distributional in nature — agents without a mandate earn more and productive firms do better than under an emissions tax. We calibrate our model to markets for bank loans and labor markets. A green mandate approximates first best only when firms and agents are relatively homogeneous.

References

Albuquerque, Rui, Yrjö Koskinen, and Chendi Zhang (2019). "Corporate social responsibility and firm risk: Theory and empirical evidence". In: *Management Science* 65.10, pp. 4451–4469.

Besley, Timothy and Maitreesh Ghatak (2007). "Retailing public goods: The economics of corporate social responsibility". In: *Journal of public Economics* 91.9, pp. 1645–1663.

Branikas, Ioannis et al. (2022). *Non-Pecuniary Preferences of Talented Employees*. Tech. rep. National Bureau of Economic Research.

Bretschger, Lucas and Aimilia Pattakou (2019). "As bad as it gets: how climate damage functions affect growth and the social cost of carbon". In: *Environmental and resource economics* 72.1, pp. 5–26.

Broccardo, Eleonora, Oliver Hart, Luigi Zingales, et al. (2022). "Exit versus voice". In: *Journal of Political Economy* 130.12, pp. 3101–3145.

Cai, Yongyang and Thomas S Lontzek (2019). "The social cost of carbon with economic and climate risks". In: *Journal of Political Economy* 127.6, pp. 2684–2734.

Heinkel, Robert, Alan Kraus, and Josef Zechner (2001). "The effect of green investment on corporate behavior". In: *Journal of financial and quantitative analysis* 36.4, pp. 431–449.

Hong, Harrison, Jeffrey D Kubik, and Jose A Scheinkman (2012). *Financial constraints on corporate goodness*. Tech. rep. National Bureau of Economic Research.

Hong, Harrison and Edward P Shore (2022). *Corporate Social Responsibility*. Tech. rep. National Bureau of Economic Research.

Hong, Harrison, Neng Wang, and Jinqiang Yang (2021). *Welfare consequences of sustainable finance*. Tech. rep. National Bureau of Economic Research.

Jensen, Svenn and Christian P Traeger (2014). "Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings". In: *European Economic Review* 69, pp. 104–125.

Lemoine, Derek and Christian Traeger (2014). "Watch your step: optimal policy in a tipping climate". In: *American Economic Journal: Economic Policy* 6.1, pp. 137–66.

Magill, Michael, Martine Quinzii, and Jean-Charles Rochet (2015). "A theory of the stakeholder corporation". In: *Econometrica* 83.5, pp. 1685–1725.

Nordhaus, William D (1992). "An optimal transition path for controlling greenhouse gases". In: *Science* 258.5086, pp. 1315–1319.

Servaes, Henri and Ane Tamayo (2013). “The impact of corporate social responsibility on firm value: The role of customer awareness”. In: *Management science* 59.5, pp. 1045–1061.

Tervio, Marko (2008). “The difference that CEOs make: An assignment model approach”. In: *American Economic Review* 98.3, pp. 642–68.

A Appendix

A.1 Omitted Proofs

A.1.1 Proof for Lemma 2

Proof. For any $s' > s$,we have

$$EP(s',t) - EP(s,t) = \int_{(1-c\sigma)s'}^{s'} a(\kappa^*(z))dz - \int_{(1-c\sigma)s}^{s} a(\kappa^*(z))dz > 0,$$

as $Q(s) \equiv \int_{(1-c\sigma)s}^{s} a(\kappa^*(z))dz$ and $Q'(s) \equiv a(\kappa^*(s)) - a(\kappa^*((1-c\sigma)s))(1-c\sigma) > 0$ since $\kappa^*(z)$ increases in z. □

A.1.2 Derivation for the un-constrained Value function

Define $\hat{V}(A_t, E_{t-}|\tilde{F})$ the value function given any sorting that gives the production $\tilde{F}$ when the no-shut down constraint is not binding.

$$\begin{aligned} r\hat{V}(A_t, E_{t-}|\tilde{F}) = \max_{M_t} A_t F - \frac{d}{1+\chi}\left(E_{t-} + \left(\sigma A_t \tilde{F} - M_t\right)\right)^{\chi+1} - cM_t \\ + \mu\left\{\int V(A', E_{t-}|\tilde{F})dG(A') - V(A_t, E_{t-}|\tilde{F})\right\} + \frac{\partial \hat{V}(A_t, E_{t-}|\tilde{F})}{\partial E_t} dE_t, \end{aligned} \tag{24}$$

which can be understood as choosing the optimal clean-up given any F. The lemma below first provides the analytical solution for $\hat{V}(A_t, E_{t-}|\tilde{F})$.

Lemma 8. *Given any* $\tilde{F}$,

$$\hat{V}(A_t, E_t|\tilde{F}) = \left(\frac{(1-c\sigma)\tilde{F}}{r}\right)\left(\frac{rA_t + \mu\bar{A}}{r+\mu}\right) + \gamma_E E_t + v_0. \tag{25}$$

where $\gamma_E \equiv -\left(\frac{c}{1+r+\delta}\right)$ *and* $v_0 = \frac{d}{r}\left(\frac{\chi}{1+\chi}\right)\left(\frac{c}{d}\left(\frac{r+\delta}{1+r+\delta}\right)\right)^{\frac{\chi+1}{\chi}}$. *The socially optimal cleanup is given by*

$$M^*(A_t, E_t|\tilde{F}) = \left(\sigma A_t \tilde{F} + E_{t-}\right) - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}}. \tag{26}$$

Proof. We guess and verify that $\hat{V}(A_t, E_t|\tilde{F}) = \gamma_E E_t + \gamma_A A + \tilde{v}_0$, and thus

$$M^*(A_t, E_t|\tilde{F}) = \left(\sigma A_t \tilde{F} + E_t\right) - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}}.$$

Plugging into Equation 24, we thus have

$$r\left(\gamma_E E_t + \gamma_A A_t + \tilde{v}_0\right) = A_t\tilde{F} - \frac{d}{1+\chi}\left(\frac{\gamma_E + c}{d}\right)^{\frac{\chi+1}{\chi}} - c\left\{\left(\sigma A_t\tilde{F} + E_t\right) - \left(\frac{\gamma_E + c}{d}\right)^{\frac{1}{\chi}}\right\} + \gamma_E\left\{\left(\frac{\gamma_E + c}{d}\right)^{\frac{1}{\chi}} - E_t(1+\delta)\right\} +$$

and thus

$$r\gamma_E = -\left\{c + \gamma_E(1+\delta)\right\} \Rightarrow \gamma_E = \frac{-c}{1+r+\delta}$$

and

$$r\gamma_A = (1-c\sigma)\tilde{F} - \mu\gamma_A \Rightarrow \gamma_A = \frac{(1-c\sigma)\tilde{F}}{r+\mu}$$

and hence

$$\begin{aligned}
r\tilde{v}_0 &= -\frac{d}{1+\chi}\left(\frac{\gamma_E + c}{d}\right)^{\frac{\chi+1}{\chi}} + c\left(\frac{\gamma_E + c}{d}\right)^{\frac{1}{\chi}} + \gamma_E\left(\frac{\gamma_E + c}{d}\right)^{\frac{1}{\chi}} + \mu\gamma_A\bar{A} \\
&= \left\{-\frac{d}{1+\chi}\left(\frac{\gamma_E + c}{d}\right) + c + \gamma_E\right\}\left(\frac{\gamma_E + c}{d}\right)^{\frac{1}{\chi}} + \mu\gamma_A\bar{A} \\
&= (c+\gamma_E)\left(\frac{\chi}{1+\chi}\right)\left(\frac{\gamma_E + c}{d}\right)^{\frac{1}{\chi}} + \mu\gamma_A\bar{A} \\
&= d\left(\frac{\chi}{1+\chi}\right)\left(\frac{\gamma_E + c}{d}\right)^{\frac{1+\chi}{\chi}} + \mu\gamma_A\bar{A}
\end{aligned}$$

We thus have

$$\hat{V}(A_t, E_{t-}|\tilde{F}) = \left(\frac{(1-c\sigma)\tilde{F}}{r}\right)\left(\frac{rA_t + \mu\bar{A}}{r+\mu}\right) + \gamma_E E_t + \frac{d}{r}\left(\frac{\chi}{1+\chi}\right)\left(\frac{c}{d}\left(\frac{r+\delta}{1+r+\delta}\right)\right)^{\frac{\chi+1}{\chi}}$$

Since ζ_t only affects the total cleanup, the optimal mandate when the no-shut down constraint is not binding thus solves

$$\begin{aligned}
M(\zeta_t) &= \int_{s_L}^{s_H}\left\{A_t f(\kappa^*\left((1-c\sigma)s\right), s) - \zeta_t\right\} g_w(s,1)ds = F^g - \lambda\zeta_t, \\
&= M^*(A_t, E_{t-}|F^*) = (\sigma A_t F^* + E_{t-}) - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}},
\end{aligned}$$

which gives Equation 23. □

A.1.3 Proof for Lemma 3

Proof. We first show that, $F = F^{FB}$, under the first-best allocation. Suppose (m, F) is such that $F < F^{FB}$, by increasing the production to F^{FB} while increasing the removal by $\sigma A\left(F^{FB} - F\right)$ so that $\sigma AF - m = \sigma AF^{FB} - \left(m + \sigma A\left(F^{FB} - F\right)\right)$. Hence, the gain by doing so is positive when $1 - c\sigma > 0$, as

$$\left\{A\left(F^{FB} - F\right) - c\sigma\left(A\left(F^{FB} - F\right)\right)\right\} = (1 - c\sigma)A\left(F^{FB} - F\right) \geq 0.$$

Hence, we have $V^{FB}(A, E) = \hat{V}(A_t, E_t | F^{FB})$.

Now, we prove that $T(e)$ can implement the first-best. Firm's optimization under $T(e)$ yields,

$$J_t(k) = \max_{s,m} A_t f(k, s) - T_t\left(\sigma A_t f(k, s) - m\right) - w_t(s) - cm.$$

Note that, given (s, θ), the FOC of clean-up for firm k yields

$$T'(\sigma A_t f(k, s) - m) = d\left((\sigma A_t f(k, s) - m) + (E_{t-} + \hat{e}_t)\right)^{\chi} + \left(\frac{c}{1 + r + \delta}\right) = c,$$

and thus

$$m(k, s) = \sigma A_t f(k, s) + (E_{t-} + \hat{e}_t) - \left(\frac{c}{d}\left(\frac{r + \delta}{1 + r + \delta}\right)\right)^{\frac{1}{\chi}}. \tag{27}$$

The we have

$$J_t(k) = \max_{s} A_t f(k, s) - C_t(f(k, s)) - w_t(s),$$

where $E_t = E_{t-} + \hat{e}_t$ represents the stock at period t given the new emission $\hat{e}_t$,

$$T_t(e) = \left\{\frac{d}{1 + \chi}\left(e + E_t\right)^{\chi + 1} - \gamma_E e\right\} + \tau(E_t),$$

$$
\begin{aligned}
C_t(f) &\equiv \min{}_m T_t(f-m) + cm \\
&= \frac{d}{1+\chi}\left(\frac{\gamma_E + c}{d}\right)^{\frac{\chi+1}{\chi}} - \gamma_E\left\{\left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}} - E_t\right\} + \tau(E_t) - c\left\{\sigma A_t f(k,s) + E_t - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}}\right\} \\
&= c\sigma A f(k, s(k)) + (c+\gamma_E)(E_t) - d\left(\frac{\chi}{1+\chi}\right)\left(\frac{\gamma_E + c}{d}\right)^{\frac{\chi+1}{\chi}} + \tau_t(E_t) \\
&= c\sigma A f(k, s(k)),
\end{aligned}
$$

using the fact that $\tau_t(E_t) = d\left(\frac{\chi}{1+\chi}\right)(\gamma_E + c)^{\frac{\chi+1}{\chi}} - (c+\gamma_E)\,E_t$. Hence, given any s, the problem can be rewritten as

$$
J(k) = \max_s (1 - c\sigma) A_t f(k,s) - w(s),
$$

hence emission tax lower the marginal value of production but no distortion on F. All workers and firms' profit then decreases by the factor $(1 - c\sigma)$ equally and wages solve

$$
\{1 - c\sigma\}\, A f_s(k,s) - w'(s) = 0.
$$

□

A.1.4 Proof for Proposition 6

Proof. Given that $\frac{dJ^M{}_t(k)}{dk} = A_t z^*(k)$ and $\frac{dJ_t^{Tas}(k)}{dk} = A_t(1-c\sigma)s^*(k)$,we thus have

$$
D(k) \equiv J_t^M(k) - J_t^{Tax}(k) = A_t \int_{k_L}^{k} \left(z^*(\tilde{k}) - (1-c\sigma)s^*(\tilde{k})\right) d\tilde{k} \geq 0,
$$

where inequality uses the fact that $z^*(\tilde{k}) - s^*(\tilde{k}) \geq 0$. This is because that $(1-c\sigma)s^*(k)$ is equivalent to the case with $\lambda = 1$. Hence for any $\lambda < 1$, all firms must now hire agents with higher index z. Moreover, for the same reason, $D'(k) = z^*(k) - (1-c\sigma)s^*(k) \geq 0$. □