What Are Intangibles Worth? Production versus Market Power

Yicheng Liu*

July 28, 2026

Abstract

Intangibles create value for firms by raising productivity and increasing market power. The first channel boosts growth, while the second could weaken competition. This paper quantifies both channels with a dynamic model in which firms invest in intangible capital to raise production and market power. I combine the model with data to decompose intangible value for each U.S. public firm. By 2024, intangibles derive more value from market power than from production, contributing 28% and 21%, respectively, to aggregate market capitalization. Intangible value rises as market-power value grows within firms and firms with greater production-related intangible value gain market share. Over the firm life cycle, it shifts from production toward market power. Incorporating the value of intangible market power restores 60% of the decline in the value premium and explains firms' incentives to invest in intangibles.

*Fisher College of Business, The Ohio State University. Email: liu.9716@osu.edu.

I am deeply indebted to Andrei Gonçalves, Yufeng Wu, and Lu Zhang for their invaluable advice and guidance. For helpful comments, I thank Aditya Chaudhry, Carter Davis, Mingrui Ma, Alessandro Melone, Andrea Ricciardi, Sergey Sarkisyan, Kris Shen, René Stulz, Mike Weisbach, Mike Wittry, Cynthia Yin, and Shaojun Zhang. All errors are my own.

1 Introduction

The rise of intangible capital has transformed both how firms produce and how they compete. Investments in R&D, software, and organizational know-how can raise productivity, but they can also strengthen market power by differentiating products and protecting firms from competition (Romer, 1990; De Ridder, 2024). While a large literature focuses on the productive role of intangibles for firm valuation and investment dynamics (e.g., Peters and Taylor, 2017; Belo et al., 2022), much less is known about their market-power value. Distinguishing these channels is economically important. Productivity gains can promote growth and investment, whereas market power may weaken competition and reduce investment incentives. This paper separates and quantifies these two sources of intangible value.

In this paper, I construct a dynamic firm model in which investment in intangible capital both enhances productivity and generates market power. Combining the estimated model with data on U.S. firms, I decompose the market value for each firm-year into four components, including the value of physical capital, intangible capital in production, intangible capital in market power, and market power unrelated to intangibles. I then examine the evolution of the two economic roles of intangibles and how they affect value and firm decisions.

I begin by constructing a model that links intangible capital endogenously to market power. Three features distinguish it from the standard neoclassical framework. First, intangible capital raises markups, increasing the share of rents in profits and firm value. Second, firms face idiosyncratic markup shocks that capture changes in the competitive environment and other non-intangible forces, such as shifts in consumer preferences or rivals' strategies. Together, these two sources of market power introduce imperfect competition and generate non-constant returns to scale. Third, firms differ in how efficiently they use physical and intangible capital, generating additional variation in firm value.

I estimate the model using the simulated method of moments (SMM), targeting key empirical moments from U.S. firms over 1976–2024. I then combine the model's value function with state variables extracted from the data to recover model-implied value for each firm-

year. The model fits the data well and explains over 80% of the cross-sectional variation in valuation. Next, I apply the Shapley decomposition method (Shapley, 1951) to quantify the marginal contribution of each state variable and disentangle the dual roles of intangibles.

I find that, on average, the value of intangible capital in market power exceeds its value in production, accounting for 28.2% and 16.5% of firm value, respectively. I validate the interpretation of the intangible market-power component using patent events, which provide observable changes in firms' innovative assets. Following a firm's first highly cited patent, intangible-driven market-power value rises, while non-intangible market power remains largely unchanged. As a complementary test, exposure to the China tariff shock affects non-intangible market power but not the intangible-driven component. Together, these results support the model's ability to isolate the value of intangible-driven market power and provide suggestive evidence that non-intangible market power captures the external competition environment.

The value and composition of intangibles vary significantly over time, across the firm life cycle, and in the cross section. Over time, the value share of intangibles in production rises from 12.5% in the first half to 20.6% in the second half of the sample, largely because revenues shift toward more intangible-intensive firms. In contrast, intangible-driven market power remains stable around 28%, suggesting that technological change has increased the productive value of intangibles while their market-power role has remained persistently important over the past five decades. Non-intangible market power also rises, almost entirely because firms with greater market power capture larger revenue shares. This pattern is consistent with weaker competition in the aggregate economy and the growing market shares of firms facing less competition.

As firms mature, the production value of intangibles declines while its market-power contribution rises, suggesting that young firms initially benefit from proprietary technologies in production and increasingly monetize them through market power over time. Because firms usually enter in favorable conditions, the non-intangible market power is initially high

for young firms, while they face more competition and transition to intangible market power as they age. Across firms, intangibles explains 45% of the variation in value. Moreover, as firms increasingly differ in the strength of their market power, the two rent components together account for around 30% of cross-sectional variation by the end of the sample.

Incorporating the market-power role of intangibles is important for understanding both asset prices and firm investment. I first show that accounting for market-power value helps explain the decline in the value premium over the past two decades (Fama and French, 2021). Book-to-market predicts returns by comparing book equity, which primarily reflects capital used in production, with market value, which also includes market-power rents. As firms become increasingly heterogeneous in market power, book-to-market becomes a noisier signal of expected returns. Consistent with this mechanism, removing both intangible and non-intangible market-power value from firm value recovers 60% of the value premium after 2000. In contrast, removing only non-intangible market power leaves the value premium close to zero in recent decades, highlighting the importance of intangible-driven market power.

The value decomposition also allows me to construct a separate q for each economic component and identify the forces underlying investment. Neoclassical investment theory predicts that q summarizes a firm's investment opportunities (Hayashi, 1982), but conventional measures aggregate economically distinct sources of value. I therefore construct four component q's from the value decomposition. Both intangible components strongly predict intangible investment, while market power provides a stronger incentive for intangible investment, with the market-power q carrying a larger coefficient. By contrast, non-intangible market-power q predicts neither physical nor intangible investment strongly, consistent with weaker investment incentives in less competitive environments.

This paper contributes to several literatures that, despite deep theoretical connections, have developed largely in parallel. A central insight from the endogenous growth literature is that intangible capital and market power are inherently linked. Monopoly rents are necessary to finance intangible investments, and intangibles can strengthen market power by

differentiating products and deterring entry (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; De Ridder, 2024). Yet in the finance literature, intangible capital and market power have been studied largely separately. This paper bridges them by structurally modeling the endogenous link between intangible capital and market power.

This paper contributes to the literature on the rise of intangible capital and its impact on firm valuations and investments (e.g., Peters and Taylor, 2017; Kogan et al., 2017; Falato et al., 2022). A closely related paper in this literature is Crouzet and Eberly (2023). They decompose the wedge between average and marginal q into intangibles, exogenous rents, and a nonlinear interaction term, highlighting key aggregate and sectoral patterns. My paper complements their analysis along two dimensions. First, I model the endogenous link between intangible capital and markups, which allows me to understand their joint dynamics and contributions to firm value. Second, I provide firm-level value decompositions that reveal how the value composition varies across firms and identify the driving forces of investments.

This paper also contributes to the investment-based asset pricing literature that studies firm value and asset prices through the lens of firm investment decisions (e.g., Merz and Yashiv, 2007; Liu, Whited and Zhang, 2009). Building on Belo, Xue and Zhang (2013), Belo et al. (2022) decompose firm value into contributions from physical capital, labor, and intangible inputs, including knowledge and brand capital, using a neoclassical investment model. This paper departs from the standard framework by modeling non-constant returns to scale and allowing for market power. The resulting decomposition quantifies the contribution of market power separately from that of production inputs. Additionally, my framework applies SMM estimation at the firm level and recovers firm-specific value decompositions, extending beyond the portfolio-level estimation in Belo et al. (2022).

This paper is also connected to the recent literature on declining competition and rising market power. Several papers document a broad increase in market power over recent decades. For example, De Loecker, Eeckhout and Unger (2020) find that aggregate markups for U.S. public firms rose from 21% to 61% above marginal cost over the past four decades.

Grullon, Larkin and Michaely (2019) similarly show that industries have become more concentrated. Moreover, market power affects firm risk and expected returns (Bustamante and Donangelo, 2017; Corhay, Kung and Schmid, 2020; Dou, Ji and Wu, 2021) and helps explain the growing divergence between small and superstar firms (Corhay, Kung and Schmid, 2025). This paper adds to this literature by looking into the sources of market power and showing that the rise in aggregate market power is driven mainly by non-intangible forces, consistent with declining competition.

The rest of the paper is organized as follows. Section 2 presents the motivating facts and theories on the relation between intangible capital and market power. Section 3 describes a dynamic firm model with intangible capital and endogenous market power. Section 4 outlines the estimation and the decomposition methodology. Section 5 presents key quantitative results on value decomposition. Section 6 shows two applications on the value premium decline and the investment-q relation. Finally, Section 7 concludes.

2 Motivation

The relation between intangible capital and market power is pronounced in the data and is also well-established in economic theory. Section 2.1 presents motivating empirical evidence on the relation between intangibles and markups, including the cross-sectional correlation and the markup response after filing and issuance of influential patents. Section 2.2 discusses the economic mechanism through which intangibles can generate market power.

2.1 Empirical Facts

2.1.1 Data

I use accounting data on publicly traded firms from the Compustat annual file and market equity from the Center for Research in Security Prices (CRSP) monthly file. I keep only firms with a share code (SHRCD) of 10 or 11 and exchange code (EXCHCD) of 1, 2, or 3. I

also exclude firms with non-positive sales, book assets, or firm value and firms with less than $5 million in physical capital (Compustat item PPEGT, deflated in 1990 dollars). I exclude firms from regulated utilities (SIC codes 4900–4999), financial firms (6000–6999), and firms categorized as public service, international affairs, or nonoperating establishments (9000+).

I use the measure of Peters and Taylor (2017), which capitalizes R&D and 30% of SG&A expenses, along with balance-sheet goodwill, to construct the intangible stock. I exclude firms with missing or zero intangible stock. The final sample contains 122,701 firm-year observations from 1976 to 2024, which correspond to 11,272 distinct firms. The sample starts in 1976 because it is the first year when the intangible capital measure is available.

In this section, I measure market power using firm-level markups based on the production function approach, which recovers the markup as the ratio of a variable input's output elasticity to its revenue share.[1] In the main text, I focus on the production function estimates under the Cobb-Douglas production technology with variable inputs and physical and intangible capital as production inputs. This measure is consistent with the model production function in the next section. To further ensure the validity of the markup estimates, I also include five additional measures: two model-free markup proxies and three production-function-based estimates under different assumptions on the production technology. Detailed definitions and results based on these measures are provided in Appendix B.1 and C.

2.1.2 Correlation between Intangibles and Market Power

Figure 1(a) plots the annual cross-sectional correlation between intangible capital share and markup, along with 95% confidence intervals. The correlation is consistently positive and statistically significant throughout the sample period, ranging from approximately 0.2 to 0.4. Moreover, the correlation is largely stable across time and exhibits a modest upward

[1]While this method is widely used and provides markup estimates for nearly all public firms, it raises several concerns such as identification without quantity data and imperfect input proxies (Bond et al., 2021; Demirer, 2022; Raval, 2023; Traina, 2018). Nevertheless, De Ridder, Grassi and Morzenti (2026) argue that, although production-based estimates may be biased in levels, they accurately capture cross-sectional dispersion in markups.

trend, suggesting that the link between intangible intensity and market power has been a persistent phenomenon in the economy. Figure 1(b) further illustrates this relation through binned scatter plots. Both variables are calculated as residuals after purging out industry and year fixed effects to isolate within-industry variation. The relationship is positive and approximately linear, with markups rising from roughly 1.0 at the lowest intangible share bins to close to 2.0 at the highest. Results are largely similar when using alternative markup measures (Figures C1 and C2 in Appendix) and in a regression framework with firm-level controls (Table C2).

2.1.3 Patent Event Studies

Successful patents are an observable outcome of prior intangible investment and provide a direct setting to measure the market power gains from successful innovations. Because the importance of most innovations is difficult to observe directly, I focus on patents ultimately revealed to be influential. Specifically, I classify a patent as important if either its forward citation count or its economic value, as measured by Kogan et al. (2017), lies above the 90th percentile within its Fama-French 48 industry-year. Citations and economic value capture an innovation's technological and economic significance, so these patents are more likely to represent successful innovations that generate substantial producer surplus. I then trace the evolution of markups around the filing and grant dates of these important patents.

Figure 2 reports event study results around the filing (left column) and grant (right column) of each firm's first important patent, under both definitions.[2] I focus on the first important patent because patents create durable market power, and earlier patents would contaminate estimates around subsequent ones. A stacked design in Appendix Figure C3 using all highly-cited or high-value patents delivers similar results but exhibits strong pre-trends, consistent with this contamination.

Markups rise by roughly 3% for highly-cited patents and 8% for high-value patents

[2]Results under alternative markup measures are reported in Figures C4–C7.

around the filing year of influential patents. The larger response for high-value patents reflects the fact that the economic value measure embeds investors' expectations of future profits and market power. Patent filing is the time when a firm has already developed an innovation ultimately revealed to be valuable. As firms commercialize these innovations, product differentiation and other competitive advantages translate into greater pricing power and higher markups. Consistent with this interpretation, the response around the grant date is positive but weaker and preceded by pre-trends, indicating that markups capitalize part of the economic value of successful innovations before formal patent approval.

2.2 Why Intangibles Generate Market Power

The relation between innovation and market power is central to the endogenous-growth literature. Successful innovation generates temporary monopoly rents that allow firms to recover the fixed and sunk costs of creating knowledge (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Klette and Kortum, 2004). In the industrial-organization literature, patents and other barriers to imitation protect these rents, and firms may also strategically invest in innovation to obtain or preserve market leadership (Nordhaus, 1969; Loury, 1979; Dasgupta and Stiglitz, 1980). In these models, intangible inputs are modeled as disembodied ideas rather than as firms' own assets. However, the same logic carries over to dynamic firm models in which innovations are capitalized into an intangible capital stock. Accumulating intangibles both raises productivity and strengthens market power by increasing the profitability and durability of innovation rents.

Intangibles other than innovations and patents can also create market power. Advertising, branding, and customer relationships can reduce consumers' price sensitivity by increasing switching costs or strengthening brand loyalty (Gourio and Rudanko, 2014). Investments in product development and perceived quality can further differentiate a firm's products from its competitors', allowing the firm to charge higher prices (Dixit and Stiglitz, 1977; Doraszelski and Markovich, 2007). Organizational and technological intangibles may also raise fixed

costs or generate scale advantages, discouraging entry and weakening competitive pressure (De Ridder, 2024).

These theoretical mechanisms suggest that accumulated intangible capital can generate firm value through both production and market power. The empirical evidence presented in Section 2.1 is consistent with these mechanisms: firms with greater intangible capital have higher markups, and successful innovations are followed by increases in markups. These findings motivate the model developed in the next section, in which intangible capital simultaneously affects production and market power.

3 Model

Time is discrete, and the horizon is infinite. Firms use physical capital, intangible capital, and costlessly adjustable inputs to produce heterogeneous products. Firms face imperfect competition in the product market and can invest in intangibles to raise their markup. In each period, firms jointly choose the price and flexible inputs to maximize operating profits, subject to the market demand. Taking the operating profits as given, firms then choose physical and intangible investments to maximize the market value of equity.

3.1 Technology

Firm i produces under a Cobb-Douglas technology: $Y_{it} = \mathbf{K}_{it}^{\eta} L_{it}^{1-\eta}$, in which L_{it} is a bundle of flexible inputs, e.g., labor and materials. $\mathbf{K}_{it}$ is a CES capital aggregator that follows

$$\mathbf{K}_{it} = \mathbf{K}(z_{it}^K, z_{it}^N, K_{it}, N_{it}) = \left[\alpha \cdot (\exp(z_{it}^K) K_{it})^{\rho} + (1-\alpha) \cdot (\exp(z_{it}^N) N_{it})^{\rho}\right]^{\frac{1}{\rho}}, \tag{1}$$

in which K_{it} is physical capital and N_{it} is intangible capital. Two capital-specific productivity shocks z_{it}^K and z_{it}^N capture the uncertainty in the productivity of physical and intangible capital. They are firm-specific and thus contribute to cross-sectional heterogeneity in the model. I assume production to exhibit constant returns to scale in capital and the flexi-

ble input. I model intangible capital as a single state variable N, which summarizes the firm's stock of R&D, brand capital, organizational capital, and related intangible assets. This parsimonious representation captures the common role of these assets in affecting both production and market power. Section 3.5 discusses these modeling choices in more detail.

Firms have pricing power in the product market and use the markup as a measure of the degree of market power. Consistent with the industrial organization literature (e.g., Tirole, 1988), markup reflects a firm's ability to extract rents, reduce pass-through, and protect profits from competition. Firm i charges a markup of μ_{it}, defined as price over marginal cost. μ_{it} follows the functional form below:

$$\mu_{it} = \mu(N_{it}, \zeta^{\mu}_{it}) = \exp(\zeta^{\mu}_{it})(1 + \bar{\mu} \cdot N^{\phi}_{it}). \tag{2}$$

The exogenous shock ζ^{μ}_{it} captures the competitive environment the firm faces, and parameters $\bar{\mu}$ and ϕ govern the magnitude of intangible capital's effect on market power.

The markup specification embeds two distinct sources of market power. First, firms endogenously accumulate market power through investment in intangible capital.[3] For example, firms with efficient IT systems and management practices could produce at lower costs but still charge relatively high prices relative to their rivals. Firms with high brand value can reduce customer price sensitivity and sustain higher markups over time. Second, market power may shift exogenously through changes due to non-intangible forces. Such shifts may reflect entry barriers, shifts in antitrust policy, or competitors' innovation that erodes incumbents' rents through creative destruction. I do not allow physical capital to directly affect market power. Section 3.5 discusses this assumption, and Section 4.5 validates the economic meaning of this shock.

[3]Under Bertrand competition, the producer with the highest productivity captures the entire market for a particular good, and the winner charges a markup of $\omega_{it}/(\max_{j \neq i} \omega_{jt})$, where ω represents the quality-adjusted productivity and firm j is the producer with second-highest productivity (Aghion et al., 2023; De Ridder, 2024). I do not take a stand on the competitive behavior and instead model the firm's problem. The specification in Equation (2) assumes that firms' quality-adjusted productivity depends on intangible capital and an exogenous shock.

In each period, the firm chooses its optimal level of flexible input and price to maximize profits, taking demand and the price of the input, P_{Lit}, as given. Let $\Pi_{it} = \Pi(z_{it}^K, z_{it}^N, \zeta_{it}^\mu, K_{it}, N_{it})$ denote the operating profits for firm i at time t (see Appendix A for derivations):

$$\Pi_{it} = \max_{L_{it}, P_{it}} P_{it}Y_{it} - P_{Lit}L_{it} = \begin{cases} \underbrace{\left[1 - \frac{1-\eta}{\mu_{it}}\right]\left(\frac{1-\eta}{\mu_{it}}\right)^{\frac{1-\eta}{\mu_{it}-1+\eta}} \bar{Y}}_{\bar{\Pi}(\mu_{it})} \cdot \mathbf{K}_{it}^{\frac{\eta}{\mu_{it}-1+\eta}} & \text{if } \mu_{it} > 1 \\ \eta(1-\eta)^{\frac{1-\eta}{\eta}} \bar{Y} \cdot \mathbf{K}_{it} & \text{if } \mu_{it} \leq 1, \end{cases} \tag{3}$$

where $\bar{Y}$ is the size of the market and is a constant. When markup is less than or equal to 1, the firm is assumed to operate under perfect competition and take prices as given.

The profit function shows two effects of market power. On the one hand, firms earn more profits with higher price-setting power, as $\bar{\Pi}(\mu_{it})$ is increasing in μ_{it}. On the other hand, market power decreases the incentive to invest through the second term $\mathbf{K}_{it}^{\frac{\eta}{\mu_{it}-1+\eta}}$ as higher μ_{it} decreases the marginal benefit of capital. Moreover, for a given μ_{it}, the profit function is homogeneous of degree $\eta/(\mu_{it} - 1 + \eta)$. When the market structure approaches perfect competition ($\mu_{it} \to 1$), the profit function has constant returns to scale. As firms accumulate market power, rents constitute a larger share of firms' profits and marginal returns to capital fall given that returns to scale $\eta/(\mu_{it} - 1 + \eta)$ is decreasing in μ_{it}. This feature is in line with the recent trend of declining capital share in the economy (Barkai, 2020).

3.2 Investment

Capital equals investment plus beginning-of-period capital net of depreciation:

$$K_{it+1} = (1 - \delta_K)K_{it} + I_{it}^K, \tag{4}$$

$$N_{it+1} = (1 - \delta_N)N_{it} + I_{it}^N, \quad I_{it}^N \geq 0. \tag{5}$$

The depreciation rate δ_N for intangible capital can be interpreted as the rate of creative destruction. New technology renders old ones obsolete, and brand loyalty wears off over time. I do not allow disinvestment in intangible capital. Intangible investments are difficult to resell in secondary markets and are therefore largely sunk and irreversible once incurred.

Investment in both types of capital incurs quadratic costs:

$$\Phi^K(I_{it}^K, K_{it}) = I_{it}^K + \frac{b^K}{2}\left(\frac{I_{it}^K}{K_{it}}\right)^2 K_{it}, \tag{6}$$

$$\Phi^N(I_{it}^N, N_{it}) = I_{it}^N + \frac{b^N}{2}\left(\frac{I_{it}^N}{N_{it}}\right)^2 N_{it}. \tag{7}$$

The quadratic adjustment cost for physical investment is standard in the q-theory literature. The same intuition can generalize to intangible capital. The convex adjustment cost captures diminishing returns to intangible investment as well as constant returns to scale in I^N and N jointly, which is in line with the stylized fact that innovation rate is independent of firm size (Klette and Kortum, 2004).

3.3 Stochastic Processes and the Stochastic Discounting Factor

I jointly model aggregate and idiosyncratic productivity shocks as a single state variable, which implicitly assumes that they have the same persistence. This choice mitigates the curse of dimensionality and ensures that the structural estimation remains computationally feasible. Two capital-specific productivity shocks follow:

$$z_{it+1}^K = x_{t+1}^K + \zeta_{it+1}^K = \rho^K(x_t^K + \zeta_{it}^K) + \varepsilon_{x,t+1}^K + \varepsilon_{\zeta,it+1}^K, \tag{8}$$

$$z_{it+1}^N = x_{t+1}^N + \zeta_{it+1}^N = \rho^N(x_t^N + \zeta_{it}^N) + \varepsilon_{x,t+1}^N + \varepsilon_{\zeta,it+1}^N, \tag{9}$$

where, for $j \in \{K, N\}$, x_{t+1}^j is aggregate productivity, ζ_{it+1}^j is firm-specific productivity, and $\varepsilon_{x,t+1}^j$ and $\varepsilon_{\zeta,it+1}^j$ are i.i.d. aggregate and firm-specific shocks, respectively.

Following Zhang (2005), I specify the stochastic discount factor exogenously as:

$$M_{t+1} = \exp\left(-r_f - \gamma^K \varepsilon^K_{x,t+1} - \gamma^N \varepsilon^N_{x,t+1} - \frac{(\gamma^K)^2(\sigma^K_x)^2}{2} - \frac{(\gamma^N)^2(\sigma^N_x)^2}{2}\right), \tag{10}$$

where r_f, γ^K, γ^N correspond to the interest rate and the prices of risk for physical and intangible productivity shocks. σ^K_x and σ^N_x are the standard deviations of the aggregate productivity shocks for physical and intangible capital, respectively.

The exogenous shock to markup is given by

$$\zeta^\mu_{it+1} = \rho^\mu \zeta^\mu_{it} + \varepsilon_{\mu,it+1}, \tag{11}$$

in which $\varepsilon_{\mu,it+1}$ is independently and identically distributed and uncorrelated across firms.

3.4 Firms' Maximization Problem

The payout of firm i equals

$$D_{it} \equiv \Pi(z^K_{it}, z^N_{it}, \zeta^\mu_{it}, K_{it}, N_{it}) - \Phi^K(I^K_{it}, K_{it}) - \Phi^N(I^N_{it}, N_{it}) - f, \tag{12}$$

where f is the fixed operating cost. Firm i maximizes its cum-dividend market value of equity:

$$V^{\text{cum}}_{it} = \max_{\{I^K_{it+s}, I^N_{it+s}\}_{s=0}^\infty} \mathbb{E}_t\left[\sum_{s=0}^\infty M_{t+s} D_{it+s}\right]. \tag{13}$$

Firms are characterized by the vector of state variables $(z^K_{it}, z^N_{it}, \zeta^\mu_{it}, K_{it}, N_{it})$. Firms' maximization problem can then be written recursively as

$$V^{\text{cum}}(z^K_{it}, z^N_{it}, \zeta^\mu_{it}, K_{it}, N_{it}) = \max_{K_{it+1}, N_{it+1}} D_{it} + \mathbb{E}_t\left[M_{t+1} V^{\text{cum}}(z^K_{it+1}, z^N_{it+1}, \zeta^\mu_{it+1}, K_{it+1}, N_{it+1})\right], \tag{14}$$

subject to capital accumulation equations (4) and (5). Lastly, I rewrite the value function as the ex-dividend value to match the data valuation:

$$V(z_{it}^K, z_{it}^N, \zeta_{it}^\mu, K_{it}, N_{it}) = V^{\text{cum}}(z_{it}^K, z_{it}^N, \zeta_{it}^\mu, K_{it}, N_{it}) - D_{it}. \tag{15}$$

3.5 Discussion of Modeling Assumptions

Constant returns-to-scale. The assumption of constant returns to scale in production is motivated by both econometric identification and consistency with the prior literature. First, recent works argue that returns to scale and markups generally cannot be separately identified (Flynn, Gandhi and Traina, 2019). Second, CRS is a standard assumption in a large body of empirical and theoretical work (e.g., Atkeson and Burstein, 2008; Edmond, Midrigan and Xu, 2023). Flynn, Gandhi and Traina (2019) further document that estimated returns to scale in U.S. firm-level data are centered close to one, supporting CRS as a reasonable empirical approximation.

Intangible capital as a single state variable. While intangible capital encompasses a variety of components, I treat it as a single firm-level input in the model. This modeling choice reflects both data limitations and the conceptual overlap among different types of intangibles in their economic roles. Under the current accounting standards, balance sheet measures do not fully recognize all types of intangible capital. While measures for some specific intangibles might be available, they typically do not cover the whole universe of public firms. On the other hand, both R&D and organizational know-how can enhance product differentiation, lower marginal costs, or build customer loyalty, thereby allowing firms to produce more efficiently and charge markups. Aggregating these components allows for a parsimonious framework without adding multiple state variables while remaining a meaningful representation of the economic role of intangibles.

Markup and physical capital. I do not allow physical capital to directly affect markups. This modeling choice is motivated by the empirical relationship between intangibles and markups, as well as by the economic properties of intangibles, discussed in Section 2. Other factors, including physical capital, may also contribute to market power—for example, through equipment scarcity or entry barriers. These non-intangible sources of market power are captured by the exogenous shock ζ_{it}^{μ}, allowing for a flexible representation of market power variation not directly tied to intangible assets.

4 Estimation and Decomposition

In this section, I first outline the strategy for identifying model parameters in Section 4.1 and show the estimation results in Section 4.2. Then I discuss the decomposition framework and the measurement of state variables in Section 4.3. Finally, Section 4.4 evaluates the model fit and Section 4.5 validates the decomposition.

4.1 Identification

There are 22 parameters in the model. I calibrate four parameters as reported in Panel A of Table 1 and estimate the remaining 18 parameters. Specifically, I set the constant annual risk-free rate r_f to be 4%, matching the data value during the sample period 1976–2024. The size of the product market, $\bar{Y}$, is set to be 20, which normalizes the long-term average level of total capital to be around four. The annual depreciation rate of physical capital, δ^K, is set to be 0.12. The depreciation rate for intangible capital, δ^N, is calibrated to 0.2, and it is close to both the average rate in Peters and Taylor (2017) and the rate calibrated in Falato et al. (2022).

I estimate the remaining 18 parameters using the Simulated Method of Moments (SMM). Identification relies on selecting moments that are informative about underlying structural parameters. To discipline the estimation, I group the parameters into four categories accord-

ing to the margins they primarily govern. The first group concerns the production technology. I use the mean of the profits-to-total-capital ratio to identify the share of capital in production, η. I include the average ratio of intangible capital to total capital to pin down the share of physical capital in production, α. To identify the elasticity of substitution between the two types of capital, ρ, I rely on the standard deviation of the intangible-to-total-capital ratio. When the two capital types are complementary, intangible intensity should remain close to its optimal ratio and exhibit smaller variation in the data. Lastly, I include the average total value-to-total-capital ratio to identify the fixed operating cost f.

The second group includes parameters that govern the investment process of physical and intangible capital. I use the standard deviations of the physical and intangible investment rates to pin down the adjustment cost parameters, $\{b^K, b^N\}$, respectively. For the parameters that govern two productivity processes, I first match the AR(1) coefficient and the standard deviation of profits scaled by physical capital to identify the persistence and volatility of idiosyncratic physical productivity shock, ρ^K and σ_ζ^K. For the intangible-productivity shock, I use AR(1) and standard deviation of profits scaled by intangible capital to identify the persistence and volatility, $\{\rho^N, \sigma_\zeta^N\}$, respectively. These two profitability moments load differently on the two shocks and help disentangle their respective contributions. Since I jointly model the persistence of aggregate and idiosyncratic productivity shocks, I additionally target the AR(1) coefficient of both firm-level and aggregate profits scaled by total capital to discipline the persistence of shocks.

The third group of parameters captures the interaction between intangible capital and markup. I use moments of the ratio between sales and cost of goods sold (Compustat item SALE/COGS) to discipline markup-related parameters. I identify the average effect of intangibles, $\bar{\mu}$, and the curvature of the markup function, ϕ, using the average SALE/COGS and the cross-sectional correlation between intangible capital and SALE/COGS. I then rely on the AR(1) coefficient of SALE/COGS to identify the persistence of exogenous markup shocks, ρ^μ, and the standard deviation of SALE/COGS to pin down their volatility, σ^μ.

The last group governs the stochastic discounting factor and the aggregate productivity. I use the average market return to identify the risk price of physical productivity shocks, γ^K, and the average return spread between high- and low-profitability firms to identify the risk price of intangible productivity shocks, γ^N. I then use the volatility of aggregate physical and intangible investment to identify the standard deviation of two aggregate productivity shocks, $\{\sigma_x^K, \sigma_x^N\}$. I also include the volatility of aggregate profitability to help identify the volatility of these aggregate shocks.

Because the return moments are estimated much less precisely than the quantity moments, I estimate the parameters iteratively in two stages. In the first stage, I use only the quantity moments. In the second stage, holding the first-stage estimates fixed, I use two return moments to identify the prices of risk associated with physical- and intangible-productivity shocks. I alternate between the two stages until the estimates converge.

4.2 Estimation Results

Panel B of Table 1 reports the estimated parameters. Overall, the estimated parameters are economically plausible and broadly consistent with values used in the prior literature. Regarding production technology, physical capital accounts for around 73% of total capital, in line with recent studies such as Crouzet and Eberly (2023). The total capital share is around 0.27, close to that in Bloom (2009). The substitution between physical and intangible capital is estimated to be 0.45, which implies modest substitution between two capital types. The substitutability is consistent with the large dispersion of intangible intensity in the data. In contrast, a typical Cobb-Douglas production function would suggest that a firm cannot operate under low intangible capital.

In the second group, the adjustment cost for physical capital is estimated to be larger than that for intangible capital, which is consistent with technological progress over recent decades as intangible investment has become less costly. In addition, physical productivity shocks are larger and less persistent than intangible productivity shocks. This difference

suggests that heterogeneity in intangible productivity reflects durable differences in firms' know-how and organizational capital, whereas physical productivity is subject to more transitory shocks. The third group characterizes the behavior of markups and the interaction with intangible capital. The estimate of $\phi < 1$ implies diminishing marginal effects of intangible capital on markups. The exogenous markup shock is less persistent and less volatile than the productivity shocks, implying that market power is shaped by more transitory forces and evolves more gradually over time.

Table 2 presents the 21 moment conditions used in the estimation. Panel A reports moments based on firm-level characteristics and Panel B reports the aggregate moments. All simulated moments are reasonably close to their empirical counterparts.

4.3 Shapley Decomposition

Interactions among the state variables make their individual contributions to firm value inherently order-dependent. I therefore employ the Shapley decomposition (Shapley, 1951) to decompose firm value. The method averages each state variable's marginal contribution across all possible orderings, thereby fairly allocating firm value while accounting for interactions among the states.[4]

To fix ideas, consider a simplified example with only two state variables, physical capital K and intangible capital N. The Shapley decomposition of firm value is

$$V(K_{it}, N_{it}) = V(K_{\min}, N_{\min}) + V_{it}^{K} + V_{it}^{N}, \tag{16}$$

where $K_{\min}, N_{\min}$ correspond to the minimum grid values for the two types of capital. There are two orderings in which the two variables can be added: K enters first, followed by N; or N enters first, followed by K. The Shapley values average the contribution of variables over

[4]Shapley decompositions have been widely used in economics and finance, including studies of income inequality (Shorrocks, 1982), worker- and firm-level wage dispersion (Abowd, Kramarz and Margolis, 1999), and heterogeneity in investor expectations (Couts, Gonçalves and Loudis, 2023).

these two orderings:

$$
\begin{aligned}
V_{it}^{K} &= \frac{1}{2}\Big[\underbrace{V(K_{it}, N_{\min}) - V(K_{\min}, N_{\min})}_{K \text{ enters first}} + \underbrace{V(K_{it}, N_{it}) - V(K_{\min}, N_{it})}_{K \text{ enters second}}\Big], \\
V_{it}^{N} &= \frac{1}{2}\Big[\underbrace{V(K_{\min}, N_{it}) - V(K_{\min}, N_{\min})}_{N \text{ enters first}} + \underbrace{V(K_{it}, N_{it}) - V(K_{it}, N_{\min})}_{N \text{ enters second}}\Big].
\end{aligned} \tag{17}
$$

By construction, $V_{it}^{K} + V_{it}^{N} = V(K_{it}, N_{it}) - V(K_{\min}, N_{\min})$, so the decomposition is exact. Crucially, the Shapley approach accounts for complementarities between K and N as any interaction effect is split between the two variables, rather than being attributed entirely to whichever variable is added last.

Formally, I apply this same logic to the full 5-variable value function, and further decompose the contribution of intangible capital into production and markup channels (details in Appendix B.3):

$$
\begin{aligned}
V_{it} &= V(z_{it}^{K}, z_{it}^{N}, \zeta_{it}^{\mu}, K_{it}, N_{it}) \\
&= \underbrace{\bar{V} + V_{it}^{z^K} + V_{it}^{K}}_{\equiv V_{it}^{\text{phy}}} + \underbrace{V_{it}^{z^N} + V_{it}^{N,\text{prod}}}_{\equiv V_{it}^{\text{prod-int}}} + \underbrace{V_{it}^{N,\text{mkp}}}_{\equiv V_{it}^{\text{mkp-int}}} + \underbrace{V_{it}^{\zeta^\mu}}_{\equiv V_{it}^{\text{mkp-non}}},
\end{aligned} \tag{18}
$$

where $\bar{V} \equiv V(z^K = 0, z^N = 0, \zeta^\mu = 0, K = K_{\min}, N = N_{\min})$ is the value function evaluated at constant benchmark values for the five state variables. $\bar{V}$ represents the baseline value of a firm with close-to-zero levels of capital and a zero exogenous shock. Each Shapley value term V_{it}^{comp} measures the marginal contribution of each variable to firm value, averaged over all possible orderings.

$V_{it}^{N,\text{prod}}$ and $V_{it}^{N,\text{mkp}}$ capture the contribution of intangible capital through production and markup, respectively. Because the baseline model includes only one intangible state variable, these channels cannot be separated directly. I therefore solve an auxiliary model with separate intangible variables for production and markup, and use it in a six-variable Shapley decomposition that treats N^{prod} and N^{mkp} as distinct. I use the original value function when the two variables are equal and the auxiliary value function when only one

differs. The two Shapley values sum to the total contribution of intangible capital.

I group the terms in Equation (18) into four economic components: value of physical capital, V_{it}^{phy}, value of intangible capital in production, $V_{it}^{\text{prod-int}}$, value of intangible market power, $V_{it}^{\text{mkp-int}}$, and value of non-intangible market power, $V_{it}^{\text{mkp-non}}$, where the value of capital in production contains both the capital-specific productivity and the capital stock. The value of physical capital, V_{it}^{phy}, contains the constant term $\bar{V}$ as it represents the benchmark value of a firm without intangible capital and market power.

To carry out the decomposition, I need to map all five state variables $(z_{it}^K, z_{it}^N, \zeta_{it}^\mu, K_{it}, N_{it})$ for each firm-year onto the model's state space. Physical and intangible capital are directly observable in the data, and I normalize total capital to match the model distribution:

$$\begin{aligned}
\widehat{K+N}_{it} &= \overline{K+N} + \sigma^{K+N} \cdot z(K_{it} + N_{it}), \\
\hat{K}_{it} &= \widehat{K+N}_{it} \cdot \frac{K_{it}}{K_{it} + N_{it}}, \\
\hat{N}_{it} &= \widehat{K+N}_{it} \cdot \frac{N_{it}}{K_{it} + N_{it}},
\end{aligned} \tag{19}$$

where $z(K_{it}+N_{it})$ is the cross-sectional z-score of firm i's total capital in the data, and $\overline{K+N}$ and σ^{K+N} are the mean and standard deviation of total capital in the model simulation. This transformation removes the empirical time trend in the level while preserving the observed capital composition.

In the model, the markup is $\mu_{it} = (1-\eta) \cdot P_{it}Y_{it}/P_{Lit}L_{it}$ under the assumption of Cobb-Douglas technology between capital and variable inputs. Consistent with the model, I measure the empirical markup as $(1-\eta) \cdot \text{SALE}_{it}/\text{COGS}_{it}$, where SALE and COGS are from Compustat, following De Loecker, Eeckhout and Unger (2020). I then back out the value of the exogenous markup shock $\hat{\zeta}_{it}^\mu$ using the estimated parameters and the functional form of the markup function in Equation (2).

Next, I estimate the two productivity terms z_{it}^K and z_{it}^N by minimizing the following

objective function for each firm-year:

$$\begin{aligned}\min_{z^K,\, z^N} \quad & \left[\frac{V}{K+N}(z^K, z^N, \hat{\zeta}^{\mu}_{it}, \hat{K}_{it}, \hat{N}_{it}) - \frac{V_{it}}{K_{it+1}+N_{it+1}}\right]^2 / \sigma^2_{V/(K+N)} \\ & + \left[\frac{N}{K+N}(z^K, z^N, \hat{\zeta}^{\mu}_{it}, \hat{K}_{it}, \hat{N}_{it}) - \frac{N_{it+1}}{K_{it+1}+N_{it+1}}\right]^2 / \sigma^2_{N/(K+N)},\end{aligned} \tag{20}$$

where $\frac{V}{K+N}(\cdot)$ and $\frac{N}{K+N}(\cdot)$ are the model-implied ratios, and the errors are scaled by their variances $\sigma^2_{V/(K+N)}$ and $\sigma^2_{N/(K+N)}$ in the data.[5] The system has two unknowns and two target moments, so in most firm-years the minimization achieves an exact match, though residuals remain for firms with extreme realizations. By construction, the model therefore largely captures cross-sectional variation in valuations. The goal of the decomposition is to understand the composition of firm value rather than to explain its level.

With the estimated state variables $\hat{\mathbf{s}}_{it} \equiv (\hat{z}^K_{it}, \hat{z}^N_{it}, \hat{\zeta}^{\mu}_{it}, \hat{K}_{it}, \hat{N}_{it})$ for each firm-year, I decompose the firm value as:

$$V_{it} = V^{\text{model}}_{it} + V^{\varepsilon}_{it} \tag{21}$$

$$= V^{\text{phy}}_{it} + V^{\text{prod-int}}_{it} + V^{\text{mkp-int}}_{it} + V^{\text{mkp-non}}_{it} + V^{\varepsilon}_{it}, \tag{22}$$

where

$$\begin{aligned} V^{\text{model}}_{it} &\equiv (K_{it+1} + N_{it+1}) \cdot \frac{V}{K+N}(\hat{\mathbf{s}}_{it}), \\ V^{j}_{it} &\equiv V^{j}(\hat{\mathbf{s}}_{it}) \cdot \frac{K_{it+1} + N_{it+1}}{K(\hat{\mathbf{s}}_{it}) + N(\hat{\mathbf{s}}_{it})}, \\ & \qquad j \in \{\text{phy}, \text{prod-int}, \text{mkp-int}, \text{mkp-non}\}, \end{aligned} \tag{23}$$

and V^{ε}_{it} is actual firm value minus model-implied firm value.

[5] Since z^K and z^N contain both aggregate and firm-specific productivity shocks, I adopt an iterative procedure to make sure the estimates are consistent with their respective distributions. Details can be found in Appendix B.4.

4.4 Model Fit

Figure 3 plots the time series for average productivity estimates z^K and z^N as well as the standard deviation across firms in each year. Both z^K and z^N show fluctuations over the business cycle. Intangible productivity z^N has been steadily increasing over time and exhibits larger dispersion in the cross section, which suggests that firms differ more in the efficiency with which they utilize intangibles.

Panel A of Table 3 provides a summary of all state variables for the whole sample as well as the first and second halves. The intangible productivity z^N has seen a large increase in the later sample. Markup, μ, also increases from 1.19 to 1.42. By construction, average normalized total capital is constant in each cross section, but the composition tilts more towards intangible capital over recent years.

Figure 4(a) plots the cross-sectional R^2 in each year that measures how closely the model-implied firm value V_{it}^{model} matches the data counterpart V_{it}:

$$\hat{R}_t^2 = 1 - \frac{\sum_i \left(V_{it} - V_{it}^{\text{model}}\right)^2}{\sum_i \left(V_{it} - \bar{V}_t\right)^2}, \tag{24}$$

where $\bar{V}_t$ is the cross-sectional mean of V_{it}. Unlike the standard regression R^2, this measure does not allow for a free intercept or slope and therefore penalizes both level and scale deviations between the model and the data. On average, the R^2 is 87%, and it is stable over time and above 80% for most years, except periods with special economic conditions such as the dot-com bubble and the 2008 financial crisis. I also compute R^2 for value scaled by capital, $V_{it}/(K_{it+1} + N_{it+1})$. The average R^2 for the value-to-capital ratio is 78%, and it remains stable over time as shown in Figure 4(b).

Panel B of Table 3 reports the coefficient and adjusted R^2 from the following panel regression:

$$V_{it} = b \cdot V_{it}^{\text{model}} + FE_t + \varepsilon_{it}. \tag{25}$$

The model fits the data reasonably well: in column 1, the regression coefficient b is close to 1

economically (albeit statistically different from 1) and the adjusted R^2 for the entire panel is around 80%. As shown in columns 2 and 3, the fit is roughly similar in the first and second halves of the sample.

Direct evaluation of model fit at the firm level remains relatively uncommon in the literature. A closely related and important benchmark is Belo et al. (2022), who obtain a portfolio-level R^2 of 94% in matching model-implied and observed valuations in the cross section. Building on this line of work, I assess fit at the individual firm level and obtain an R^2 of around 80%. This comparison is inherently demanding, as firm-level analysis preserves idiosyncratic variation that is naturally attenuated through portfolio aggregation.

4.5 Validation

4.5.1 Example Firms

Figure 5 decomposes the average market value of representative firms into four model-implied components, computed at the firm-year level and averaged over time.

Technology and pharmaceutical firms derive substantial value from intangible capital, but through different channels. Amazon and Microsoft benefit from both production and market power, whereas Pfizer and Johnson & Johnson derive more of their intangible value through market power, consistent with the importance of patent protection and pricing power in pharmaceuticals. By contrast, firms such as Costco, 3M, and ExxonMobil derive more value from tangible capital and physical infrastructure.

The decomposition also reveals differences in non-intangible market power. Oracle, for example, has a sizable positive contribution, potentially reflecting network effects, switching costs, or other competitive advantages not captured by its current intangible stock. Costco and ExxonMobil instead exhibit negative contributions from this component.

Finally, Amazon and Microsoft have substantial residual values, indicating that part of their market valuations is not explained by physical capital, intangible capital, or market power. This residual may reflect investor expectations or other forces outside the model.

4.5.2 Value share and firm characteristics

Figure 6(a) plots the relation between the intangible intensity and the total value of intangibles as a percentage of total value across firms. The intangible intensity is measured as the ratio of intangible capital to total capital, and the value of intangibles is the sum of the value of intangible capital in production and intangible-related market power. As expected, firms with higher intangible intensity tend to have a larger share of their value attributed to intangibles. Similarly, Figure 6(b) plots the relation between the markup and the value of market power as a percentage of total value across firms. Firms with higher markups tend to derive more value from market power, and this relation is more concave.

4.5.3 Interpreting the Two Sources of Market-Power Value

The two market-power components capture distinct economic forces. They should respond to different types of shocks and vary with industry dynamics in different ways. In this section, I provide external validation for the decomposition by examining how the two market-power components respond to innovation and competition shocks.

Patents and intangible market power. I first examine changes in the two market-power value shares around the filing and issuance of highly cited patents, using the same event definition and citation threshold as in Section 2.1.3. I use citations rather than the economic value measure of Kogan et al. (2017), because that measure is itself constructed from stock-market reactions, while the outcomes here are also value-based.

Figure 7 shows that the market-power share rises around both filing and issuance, consistent with the markup response in Section 2.1.3. More importantly, the increase is concentrated in intangible-driven market power, while the non-intangible market-power share remains broadly unchanged. This differential response supports the decomposition's attribution of patent-related market power to intangible assets.

Tariff and non-intangible market power. Next, I exploit the granting of Permanent Normal Trade Relations (PNTR) status to China as a plausibly exogenous increase in product-market competition. Before PNTR, China's access to low Normal Trade Relations tariffs required annual renewal, creating uncertainty because nonrenewal would trigger much higher non-NTR tariffs. PNTR permanently removed this uncertainty in 2001 and accelerated Chinese import competition. Following Pierce and Schott (2016), I measure industry exposure using the NTR gap, defined as the difference between non-NTR and NTR tariff rates. I then estimate a difference-in-differences specification that interacts the industry-level NTR gap with a post-PNTR indicator.

Table 4 reports the responses of market-power value shares to the PNTR shock. Panel A presents static difference-in-differences estimates, while Panel B traces the dynamic effects. Consistent with Corsini (2025), greater import competition does not significantly reduce the level of market power. Instead, it slows the growth of the market-power share, with the effect driven entirely by non-intangible market power. Intangible-driven market power remains unchanged, consistent with this component capturing innovation-based rents rather than changes in the competitive environment.

Taken together, the patent and PNTR results provide external validation for the decomposition. Intangible-driven market power responds to innovation, whereas non-intangible market power is more sensitive to shifts in the external competitive environment.[6]

5 An Anatomy of Firm Value

This section presents the main quantitative results on firm value decomposition. Section 5.1 shows how the value of intangibles varies in the time series. Section 5.2 documents the composition shifts over firms' life cycles. Section 5.3 focuses on the cross-sectional variation.

[6]Appendix Table C3 additionally shows that the two market-power components exhibit different cross-sectional relationships with standard product-market characteristics, although these correlations should be interpreted descriptively because market structure and firms' intangible investments are jointly determined.

5.1 Time Variation

Figure 8(a) shows the decomposition for the market, with value components summed across all firms. Figure 8(b) shows each component as a percentage of the total market valuation. Panel A of Table 5 formalizes the results and statistically tests the difference between the early and late periods. First of all, the value of intangible capital increases over the sample period. By 2024, it accounts for around 50% of the total market valuation explained by the model, which amounts to around $20 trillion in dollar terms.

The value of intangible capital in production grows steadily, increasing from 12.5% to 20.6% of total market value. On the other hand, the value of intangible-driven market power remains steady over time. Recent decades have seen substantial growth in technology, pushing up the production value of intangibles, whereas intangibles have long been a source of market power and so their contribution along this channel has remained relatively stable. Moreover, market power tends to be more valuable during economic downturns, because the ability to charge higher prices makes firms less vulnerable to negative shocks.

At the same time, the value attributed to non-intangible market power also grows. During the first half of the sample period, firms operate in a relatively competitive environment, with negative value from non-intangible sources. After 2000, however, firms increasingly earn rents in excess of intangible-driven markups, as reflected by the increasing non-intangible market power component. This rise in non-intangible market power provides suggestive evidence of a decline in competition, consistent with findings in the IO and macroeconomic literature (De Loecker, Eeckhout and Unger, 2020).

Finally, the residual component is generally small and becomes slightly more pronounced during market booms and busts. In the results that follow, I focus on the value components explained by the model and abstract from the residual term, which is not directly linked to the model's state variables and may reflect forces beyond the scope of this analysis.

Variation across time can be attributed to within-firm change or compositional change. For example, the increasing value of market power can either be driven by a within-firm in-

crease (all firms accumulate more market power) or a compositional shift (high-markup firms become larger). Following Paron (2025), I decompose the increase in the value components into these two channels. I use sales, $P_{it}Y_{it}$, to represent the firm-size distribution. Define firm i's sales share as $w_{it} \equiv P_{it}Y_{it}/(P_tY_t)$ and its value-to-sales ratio as $v_{it} \equiv V_{it}/(P_{it}Y_{it})$. The aggregate value-to-sales ratio can then be written as a sales-weighted average of firm-level value-to-sales ratios:

$$\frac{V_t}{P_tY_t} = \sum_i w_{it}v_{it}. \tag{26}$$

The change in the value-to-sales ratio from $t-1$ to t can be decomposed into within-firm and compositional change:

$$\Delta\left(\frac{V_t}{P_tY_t}\right) = \underbrace{\sum_i \bar{w}_{it}\Delta v_{it}}_{\text{Within-firm change}} + \underbrace{\sum_i \Delta w_{it}\bar{v}_{it}}_{\text{Compositional change}}. \tag{27}$$

For either $a = w$ or $a = v$, the overbar denotes the midpoint of the variable across the two periods:

$$\bar{a}_{it} \equiv \frac{1}{2}\left(a_{it} + a_{i,t-1}\right). \tag{28}$$

Intuitively, the within-firm change focuses on changes in firm-level valuations while holding sales weight fixed. The compositional change measures the effect of changing sales shares while holding valuations fixed for each firm. The total value V in Equation (27) can be replaced with each value component to understand their drivers.

Figure 9 plots the cumulative change in the aggregate value-to-sales ratio and decomposes it into within-firm and compositional components. It also shows the same decomposition for the ratio of the value components to sales. Consistent with Paron (2025), Panel (a) shows that the rise in aggregate valuation is driven mainly by compositional change, indicating a

reallocation of revenue toward higher-valuation firms. Panel (b) shows that compositional changes contribute more than within-firm changes to the rise in the value of intangible capital in production, suggesting that firms have broadly increased intangible investment while those that use intangible capital more effectively have become more dominant.

Panels (c) and (d) isolate the contributions of intangible-driven and non-intangible market power, respectively. Both market power components have increased over time relative to sales. While the rise in the value of non-intangible market power largely reflects compositional change, mirroring the pattern observed for total valuation, the increase in the value of intangible-driven market power is instead dominated by within-firm change. This contrast suggests that while many firms expand market power through the accumulation of intangible capital at the firm level, firms with less competition and higher pure rents are capturing a larger market share.

5.2 Firm Life Cycle

Figure 10 traces how the value components evolve over firm age and shows the life cycle pattern in the sources of firm value. Each panel plots the average component share against firm age in years since first appearing in Compustat, with the shaded area indicating 95% confidence intervals. More specifically, I run the following regression and require firms included in the regression to be observed for at least 15 years to ensure a stable composition of firms in the sample:

$$\frac{V_{it}^{\text{comp}}}{V_{it}^{\text{model}}} = \sum_{j=1}^{25} \beta_j \cdot \mathbb{1}\{\text{age}_{it} = j\} + FE_t + FE_c + \varepsilon_{it}. \tag{29}$$

Following the literature on estimating the life cycle effects (e.g., Fienberg and Mason, 1985), I include the year and cohort fixed effects in the regression. Panel (a) shows that the physical capital component remains relatively stable over the firm life cycle, with a slight upward trend after 15 years.

Panel (b) shows a monotonic decline in the production value share of intangible capital of around 4%. One interpretation is that newly public firms enter the sample with relatively fresh and productive intangible stocks, such as recently developed technologies, novel organizational capital, or early-stage intellectual property. These productivity advantages erode over time as competitors imitate or as the intangibles themselves become obsolete. In contrast, Panel (c) documents that the value of intangible-driven market power rises by roughly 10%, with the steepest gains occurring in the first 10 to 15 years. This trajectory is consistent with firms progressively relying on their accumulated intangible capital to build and entrench market power, and the resulting rents become an increasingly important component of firm value.

Panel (d) shows that the non-intangible market power component declines over the firm life cycle. Because this component captures markup variation attributable to competitive conditions rather than the firm's own intangible investments, the decline indicates that older firms tend to operate in increasingly competitive environments. This may reflect the entry of new competitors over time, or the fact that firms tend to enter the market during favorable periods with less competition. This pattern is directly in contrast with the intangible-driven market power component, which rises over the life cycle, suggesting that firms progressively rely more on intangible assets to preserve their market power.

Taken together, the four panels provide a coherent picture of firm value dynamics over the life cycle. Young firms benefit from fresh productive intangibles and operate in less competitive environments. As firms age, a compositional shift occurs within the intangible channel: the direct productivity value of intangibles decays, while firms increasingly convert their intangible stocks into durable market power.

5.3 Cross-sectional Heterogeneity

There is substantial heterogeneity in the composition of firm value across firms and across industries. I first perform a covariance decomposition to understand the contribution of

value components to the cross-sectional variation. To avoid having the decomposition be dominated by large firms, I scale both the actual and model-implied firm value by total capital, $K_{it} + N_{it}$, and let $v_{it} \equiv V_{it}/(K_{it} + N_{it})$ denote the scaled value (and analogously for each component). Equation (18) then implies:

$$\begin{aligned} Cov(v_{it}, v_{it}^{\text{model}}) &= Cov(v_{it}, v_{it}^{\text{phy}} + v_{it}^{\text{prod-int}} + v_{it}^{\text{mkp-int}} + v_{it}^{\text{mkp-non}}) \\ \Rightarrow 1 &= \frac{Cov(v_{it}, v_{it}^{\text{phy}})}{Cov(v_{it}, v_{it}^{\text{model}})} + \frac{Cov(v_{it}, v_{it}^{\text{prod-int}})}{Cov(v_{it}, v_{it}^{\text{model}})} + \frac{Cov(v_{it}, v_{it}^{\text{mkp-int}})}{Cov(v_{it}, v_{it}^{\text{model}})} + \frac{Cov(v_{it}, v_{it}^{\text{mkp-non}})}{Cov(v_{it}, v_{it}^{\text{model}})}. \end{aligned} \tag{30}$$

This equation decomposes the cross-sectional variation of the scaled firm value, v_{it}, that can be explained by the model, v_{it}^{model}. Figure 11 shows the results of the covariance decomposition for each cross section and Panel B of Table 5 reports the estimates. Consistent with the level decomposition, intangible capital plays a larger role in driving firm-level variation while physical capital becomes less important. Moreover, though the level of intangible-driven market-power value is higher than the value of intangibles in production, the contribution of the latter to cross-sectional variation is much larger. This suggests that firms differ more in their production intangibles and their efficiency in utilizing intangibles in production. On the other hand, the total contribution of market power has increased slightly across time, accounting for roughly 29% of the value variation in the past two decades.

There is also substantial heterogeneity in value composition across sectors. Figure 12 reports the level and covariance decomposition for Fama-French 12 industries (excluding utilities and financial firms). The importance of intangible capital varies across sectors. Sectors such as Business Equipment and Healthcare exhibit large contributions from both intangible capital used in production and intangible-driven markup. These industries are built around intellectual capital and differentiated services, and thus firms not only accumulate more intangible assets but also convert them into pricing power. In contrast, traditional industries like Energy and Consumer Durables derive a much larger portion of their value from physical capital, with limited contributions from intangibles or market power.

Moreover, Figure 13(a) shows the relative importance of intangible capital in production and market power across sectors. Sectors such as Business Equipment and Healthcare are located in the upper right corner, indicating that they derive a large portion of their value from both channels. On the other hand, sectors such as Manufacturing are located in the lower area, suggesting that the value of intangibles is mainly from raising production efficiency, while industries such as Telecom and Consumer Nondurables derive a large portion of value from intangible-driven market power.

The size and composition of market power also differ across sectors. Among these sectors, firms in Business Equipment, Healthcare, and Telecom exhibit a large amount of value from market power. Figure 13(b) plots the relative importance of intangible-driven and non-intangible market power across sectors. Sectors such as Healthcare and Business Equipment exhibit a large amount of non-intangible market power, suggesting that consumer tastes or other exogenous shifts allow firms to charge higher prices. On the other hand, sectors including Chemicals and Consumer Durables mostly accumulate market power through intangibles, and they tend to face more intense competition in the market, which limits their ability to charge prices above the level justified by their intangible capital.

6 Economic Implications of the Value Decomposition

Understanding the role of intangibles in production and market power is important for understanding asset prices and corporate decisions. In this section, I focus on two applications using the value decomposition in the previous section. Section 6.1 creates a model-implied value signal and recovers the value premium in recent decades. Section 6.2 constructs Tobin's q using each value component and explores the investment-q relation.

6.1 Resurrecting the Value Premium

A value signal, such as the book-to-market ratio, is designed to capture discount-rate information through the ratio of firms' market value to fundamental value. If a firm is cheaper relative to its fundamental value, it will earn high returns going forward and vice versa. However, the value premium based on the traditional book-to-market ratio has declined over time (Fama and French, 2021) and is close to 0 after 2000. Several papers have argued that book equity no longer captures the fundamental value accurately, so the book-to-market ratio becomes noisy and cannot predict returns. For example, Eisfeldt, Kim and Papanikolaou (2022) and Park (2022) propose to include intangible capital, and Gonçalves and Leonard (2023) construct fundamental equity by calculating the present value of future cash flows under a constant discount rate. However, the book-to-market ratio adjusted for intangibles still shows a visible decline after 2000, and Gonçalves and Leonard (2023) do not provide an economic explanation of the wedge between book equity and fundamental equity.

I propose a different explanation. The book-to-market ratio has become less informative because market value increasingly reflects the value of market power, which is not captured by book equity. To fix ideas, consider a simple Gordon growth model with a constant growth rate and a firm-specific discount rate. Suppose the book equity reflects the value generated by production:

$$BE_i = \frac{CF_i^{\text{prod}}}{\bar{r} - g}, \tag{31}$$

while market equity contains both value from production and market power:

$$ME_i = \frac{CF_i^{\text{prod}} + CF_i^{\text{mkp}}}{r_i - g}. \tag{32}$$

Then the resulting book-to-market ratio is confounded by the share of cash flows attributable to production:

$$BM_i = \frac{BE_i}{ME_i} = \frac{r_i - g}{\bar{r} - g} \cdot \underbrace{\frac{CF_i^{\text{prod}}}{CF_i^{\text{prod}} + CF_i^{\text{mkp}}}}_{\text{Noise term}}. \tag{33}$$

To restore the value signal, I create two model-implied valuation ratios. The first one removes the value of non-intangible market power from market value, and the second one removes the total value of market power:

$$BM_i^{\text{no-mkp-non}} = \frac{BE_i^{\text{total}}}{V_i - V_i^{\text{mkp-non}}}, \tag{34}$$

$$BM_i^{\text{prod}} = \frac{BE_i^{\text{total}}}{V_i - V_i^{\text{mkp-non}} - V_i^{\text{mkp-int}}}, \tag{35}$$

where BE^{total} contains both the book equity from Fama and French (1992) and intangible capital.

Table 6 reports the monthly return spread from portfolios sorts based on the data and model-implied book-to-market ratio. The first and second rows report the value premium using the book-to-market ratio constructed from the data. Sorts based on the traditional book-to-market ratio show a low value premium in both the first and second halves of the sample period, potentially due to the sample restrictions: the sample starts in 1976 and I require all firms to have intangible capital. Adjusting for intangible capital in book equity creates a value premium before 2000 (0.59%, $t = 1.76$) but sees a large decline post-2000 (0.18%, $t = 0.42$).

Next, the model-implied ratio removing non-intangible market power improves the accuracy pre-2000 with a value spread of 0.53% ($t = 2.10$), but the return spread is 0 in the second half. The last measure that removes all market power performs the best. The high-minus-low return is 0.52% ($t = 2.22$) in the first half and 0.35% ($t = 1.65$) in the second half. Relative to the 0.41% decline using the intangible-adjusted book-to-market ratio, the small decline of 0.17% restores around 60% of the decline, and the value premium remains strongly significant over the full sample period (0.43%, $t = 2.70$).

To further establish the economic channel, I take the measure of the fundamental-to-market ratio (FE/ME) from Gonçalves and Leonard (2023) and use my model value shares to understand the wedge between FE/ME and the standard book-to-market ratio

BE^{total}/ME. Specifically, FE is defined as the present value of future cash flows under a common discount rate across firms. Under the Gordon growth example in Equations (31) and (32), FE can be written as:

$$FE_i = \frac{CF_i^{\text{prod}} + CF_i^{\text{mkp}}}{\bar{r} - g}. \tag{36}$$

Therefore, the wedge between these two ratios should be mainly driven by market power terms. I run the following panel regression:

$$\log\left(\frac{FE_{it}}{ME_{it}}\right) = b_1 \log\left(\frac{BE_{it}^{\text{total}}}{ME_{it}}\right) + b_2 \frac{V_{it}^{\text{prod-int}}}{V_{it}^{\text{model}}} + b_3 \frac{V_{it}^{\text{mkp-int}}}{V_{it}^{\text{model}}} + b_4 \frac{V_{it}^{\text{mkp-non}}}{V_{it}^{\text{model}}} + FE_t, \tag{37}$$

The value share of physical capital is omitted as it equals one minus the other three shares.

Table 7 reports the regression results for the full sample and two subsample periods. Overall, the wedge between the fundamental-to-market and book-to-market ratios has highly positive loadings on two market power terms, consistent with book equity not capturing the value of market power. The wedge loads negatively on the share of intangible production, which suggests that the book equity measure overestimates the production value relative to the true fundamental equity. More importantly, the coefficients on both market power terms increase significantly from the first to the second half of the sample. Meanwhile, R^2 is low when only $\log BE^{\text{total}}/ME$ is included in the post-2000 regression. When the three model-implied value shares are included, R^2 is close to the level in the first half of the sample. This pattern suggests that the decline in the value premium is associated with market equity increasingly reflecting the differences in the value of market power across firms.

6.2 The Investment-q Relation

The neoclassical theory of investment predicts that Tobin's q, the ratio of capital's market value to its replacement cost, should fully capture investment opportunities (Hayashi, 1982). As firms increasingly rely on intangible assets, Peters and Taylor (2017) further incorpo-

rate intangible capital into the replacement cost and show that this total q measure better predicts investment. However, a large total q may reflect the market-power rents, which raise the firm value without increasing the marginal return to capital (Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019). More importantly, firms may have very different investment incentives even when they share the same total q, because that market value may arise from fundamentally different economic sources. My decomposition framework provides market-power-adjusted and component-specific q measures and thus helps explain the different incentives for physical and intangible investment separately.

6.2.1 Summary statistics

Panel A of Table 8 reports summary statistics for the empirical and model-implied q measures. Standard q, defined as the market value of the firm relative to physical capital, has a mean of 4.73 and exhibits substantial dispersion and positive skewness. Incorporating intangible capital into the capital stock substantially lowers the level of total q, with the mean falling to 1.42, while maintaining a similar distribution.[7] The next three measures are counterparts to the data total q, where I purge out the value of residuals, non-intangible market power, or the total value of market power. They have similar distributions but different levels. The component q measures show that physical and intangible capital contribute comparably to firm value on average, with mean physical and intangible q of 1.47 and 1.43, respectively. Within intangible capital, the market-power component (0.84) exceeds the production component (0.59), indicating that a substantial share of intangible value reflects firms' ability to generate rents rather than improvements in production efficiency. Overall, the level and distribution of model-implied q measures are reasonable and comparable to the data benchmarks.

Panel B of Table 8 reports the correlation matrix among these q measures. Model total

[7]Note that the definition of total q is slightly different from Peters and Taylor (2017), where they subtract current assets (Compustat item ACT) from firm value. Here I subtract cash (CHE) to be consistent with my definition of firm value throughout the paper. Table C4 reports the regressions in the next two subsections for the exact Peters-Taylor q. The results are similar to using the total q here.

qs covary with the data counterpart with a correlation of around 0.8. At the same time, the decomposed q measures are far from perfectly correlated. In particular, the production and market-power components of intangible q have a correlation of only 0.40, and physical q is only weakly correlated with intangible market-power q (0.08). These moderate correlations suggest that the decomposition isolates economically distinct sources of firm value rather than mechanically partitioning total q.

6.2.2 Horse race

Table 9 reports the horse race of model-implied total qs against the data benchmarks. Panel A regresses physical investment onto q measures and Panel B uses intangible investment as the left-hand variable.[8] The first column reports the investment-q relation using standard Tobin's q. It explains some variation in both investment rates with very small coefficients. Under the neoclassical framework, the inverse q slope measures the adjustment costs. Therefore, the small coefficient implies an unreasonably large adjustment cost. Data total q in the second column predicts investment with a much larger R^2 and a more reasonable slope.

If market-power value and residual valuation components do not generate the same investment incentives as productive capital, removing them from total q should increase the estimated investment-q slope. Columns (3)–(5) of Table 9 examine this prediction. The adjusted model-implied total q measures deliver slightly lower within-R^2 values than the empirical benchmarks, but their coefficients are roughly twice as large. Under the standard quadratic adjustment-cost interpretation, these larger coefficients imply more moderate adjustment costs. The coefficients are smaller for intangible investment than for physical investment, consistent with greater adjustment frictions for intangible capital.

[8]Here I scale each investment rate by its own capital rather than by total capital. This is the main specification for two reasons. First, the model specifies separate adjustment-cost functions for physical and intangible capital, so each first-order condition is expressed in terms of its corresponding investment rate. Second, scaling by total capital mechanically introduces variation in capital composition. For example, even in steady state, a firm with a larger intangible share will have lower physical investment relative to total capital. Because intangible component q covaries with the intangible share, this mechanical relation would bias its estimated coefficient downward for physical investment-q regressions. Nevertheless, Table C5-C7 report the results for $(K+N)$-scaled investment rates.

6.2.3 Component q

Table 10 reports investment-q regressions using the individual value components. Columns (1)–(3) examine physical investment. Physical q alone explains nearly 9% of the within-firm variation in physical investment. When all four components are included in column (2), physical q remains the dominant predictor, while the intangible-production and market-power components have substantially smaller coefficients. Column (3) adds total q to the component regression. The coefficient on non-intangible market-power q becomes significantly negative, suggesting that total q overstates investment incentives when it assigns the same role to valuation arising from productive capital and from non-intangible market power. By contrast, physical q and intangible market-power q remain positively related to physical investment, indicating that total q may underweight these sources of value.

Columns (4)–(6) report the corresponding results for intangible investment. Firms may invest in intangible capital either to improve production efficiency or to build market power. Consistent with this interpretation, both intangible-production q and intangible market-power q strongly predict future intangible investment in columns (4) and (5), with the market-power component carrying the larger coefficient. In column (6), intangible market-power q remains strongly positive even after controlling for total q, whereas intangible-production q becomes negative. This pattern suggests that the market-power channel is an important determinant of intangible investment beyond the information contained in aggregate valuation. Non-intangible market-power q, in contrast, is economically small and statistically insignificant.

To quantify the relative importance of the components, Table 11 decomposes the regression R^2 using Shapley values. Variation in physical investment is explained primarily by physical q, whereas variation in intangible investment is concentrated in the two intangible components, especially intangible market-power q. Non-intangible market-power q contributes little to the explanatory power of either regression.

Taken together, the results show that aggregate q obscures economically distinct invest-

ment incentives. Productive-capital components predict investment in the corresponding capital stock, while intangible-driven market-power value provides an additional and particularly important incentive for intangible investment. Non-intangible market-power value, by contrast, raises firm valuation without generating comparable investment incentives. This distinction helps explain why conventional total q can deliver attenuated investment slopes even when its overall predictive power remains similar.

7 Conclusion

Firms actively acquire market power through investments in marketing and product differentiation. This paper extends the neoclassical framework to incorporate monopolistic competition and endogenous market power. In the model, firms produce heterogeneous products and invest in intangibles to accumulate pricing power. I estimate the model and provide a quantitative framework to decompose firm value into components attributable to physical capital, intangible capital in production, intangible capital in market power, and residual market power. I show that, on average, intangibles account for around 45% of firm value, and 28% of value is driven by intangible-related market power.

Over time, intangible capital and market power account for a growing share of total market valuation, with market power increasingly tilting toward non-intangible sources. The rise in intangible-driven market power is primarily a within-firm shift, while the rise in non-intangible market power is largely driven by reallocation toward high-valuation firms. Over the firm life cycle, young firms enter with productive intangible stocks and favorable competitive conditions, but the direct productivity value of intangibles decays with age while firms increasingly convert their intangible capital into durable market power. Across sectors, intangible-intensive industries derive substantial value from both the production and market-power channels, while traditional industries remain dominated by physical capital.

I show that incorporating the value of intangible market power is important for under-

standing asset prices and incentives for intangible investments. Specifically, the value of market power could contaminate the book-to-market ratio, contributing to the value premium decline in recent decades. Removing the value of market power explains part of the decline. Moreover, I construct a separate q measure for each economic component and show that firms' intangible investment reacts more strongly to market power incentives.

There are several ways to extend the current framework. First, the paper hinges on the measure of intangibles that capitalizes R&D and SG&A expenses. Future work on improving the measure could enhance the model fit. Moreover, measures for different types of intangibles could allow for a better understanding of the specific channels through which intangibles contribute to market power. Second, while the focus of this paper is to understand the composition of firm value and the dual roles of intangibles, the framework is flexible and future research can adopt it to explore other important questions in finance. For example, the model can be used to understand the cross-sectional variation in expected returns, as the value components may have different implications for risk and return.

References

Abowd, J. M., Kramarz, F., & Margolis, D. N. (1999). High Wage Workers and High Wage Firms. *Econometrica*, *67*(2), 251–333.

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J., & Li, H. (2023). A Theory of Falling Growth and Rising Rents. *Review of Economic Studies*, *90*(6), 2675–2702.

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J., & Li, H. (2025). Good Rents versus Bad Rents: R&D Misallocation and Growth.

Aghion, P., & Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica*, *60*(2), 323.

Atkeson, A., & Burstein, A. (2008). Pricing-to-Market, Trade Costs, and International Relative Prices. *American Economic Review*, *98*(5), 1998–2031.

Ayyagari, M., Demirgüç-Kunt, A., & Maksimovic, V. (2024). The Rise of Star Firms: Intangible Capital and Competition (H. Mueller, Ed.). *The Review of Financial Studies*, *37*(3), 882–949.

Baqaee, D. R., Farhi, E., & Sangani, K. (2024). The Darwinian Returns to Scale. *Review of Economic Studies*, *91*(3), 1373–1405.

Barkai, S. (2020). Declining Labor and Capital Shares. *The Journal of Finance*, *75*(5), 2421–2463.

Belo, F., Gala, V. D., Salomao, J., & Vitorino, M. A. (2022). Decomposing firm value. *Journal of Financial Economics*, *143*(2), 619–639.

Belo, F., Xue, C., & Zhang, L. (2013). A Supply Approach to Valuation. *Review of Financial Studies*, *26*(12), 3029–3067.

Bloom, N. (2009). The Impact of Uncertainty Shocks. *Econometrica*, *77*(3), 623–685.

Bond, S., Hashemi, A., Kaplan, G., & Zoch, P. (2021). Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data. *Journal of Monetary Economics*, *121*, 1–14.

Bustamante, M. C., & Donangelo, A. (2017). Product Market Competition and Industry Returns. *The Review of Financial Studies*, *30*(12), 4216–4266.

Corhay, A., Kung, H., & Schmid, L. (2020). Competition, Markups, and Predictable Returns (S. Van Nieuwerburgh, Ed.). *The Review of Financial Studies*, *33*(12), 5906–5939.

Corhay, A., Kung, H., & Schmid, L. (2025). Q: Risk, rents, or growth? *Journal of Financial Economics*, *165*, 103990.

Corsini, A. P. (2025). The Competitive Effects of Imports. *Working paper*.

Couts, S. J., Gonçalves, A., & Loudis, J. (2023). The Subjective Risk and Return Expectations of Institutional Investors. *SSRN Electronic Journal*.

Crouzet, N., & Eberly, J. (2019). Understanding Weak Capital Investment: The Role of Market Concentration and Intangibles. *Proceedings of the 2018 Jackson Hole symposium*, 87–148.

Crouzet, N., & Eberly, J. (2023). Rents and Intangible Capital: A Q+ Framework. *The Journal of Finance*, *78*(4), 1873–1916.

Dasgupta, P., & Stiglitz, J. (1980). Uncertainty, Industrial Structure, and the Speed of R&D. *The Bell Journal of Economics*, *11*(1), 1.

De Loecker, J., Eeckhout, J., & Unger, G. (2020). The Rise of Market Power and the Macroeconomic Implications. *The Quarterly Journal of Economics*, *135*(2), 561–644.

De Ridder, M. (2024). Market Power and Innovation in the Intangible Economy. *American Economic Review*, *114*(1), 199–251.

De Ridder, M., Grassi, B., & Morzenti, G. (2026). The Hitchhiker's Guide to Markup Estimation: Assessing Estimates From Financial Data. *Econometrica*, *94*(1), 137–168.

Demirer, M. (2022). Production Function Estimation with Factor-Augmenting Technology: An Application to Markups. *Working paper*.

Dixit, A. K., & Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, *67*(3), 297–308.

Doraszelski, U., & Jaumandreu, J. (2022). Reexamining the De Loecker & Warzynski (2012) method for estimating markups.

Doraszelski, U., & Markovich, S. (2007). Advertising dynamics and competitive advantage. *The RAND Journal of Economics*, *38*(3), 557–592.

Dou, W. W., Ji, Y., & Wu, W. (2021). Competition, profitability, and discount rates. *Journal of Financial Economics*, *140*(2), 582–620.

Edmond, C., Midrigan, V., & Xu, D. Y. (2023). How Costly Are Markups? *Journal of Political Economy*, *131*(7), 1619–1675.

Eisfeldt, A., Kim, E., & Papanikolaou, D. (2022). Intangible Value. *Critical Finance Review*, *11*(2), 299–332.

Erickson, T., & Whited, T. M. (2002). Two-Step GMM Estimation of the Errors-in-Variables Model Using High-Order Moments. *Econometric Theory*, *18*(3), 776–799.

Falato, A., Kadyrzhanova, D., Sim, J., & Steri, R. (2022). Rising Intangible Capital, Shrinking Debt Capacity, and the U.S. Corporate Savings Glut. *The Journal of Finance*, *77*(5), 2799–2852.

Fama, E. F., & French, K. R. (1992). The Cross-Section of Expected Stock Returns. *The Journal of Finance*, *47*(2), 427–465.

Fama, E. F., & French, K. R. (2021). The Value Premium. *The Review of Asset Pricing Studies*, *11*(1), 105–121.

Fienberg, S. E., & Mason, W. M. (1985). Specification and Implementation of Age, Period and Cohort Models. In W. M. Mason & S. E. Fienberg (Eds.), *Cohort Analysis in Social Research* (pp. 45–88). Springer New York.

Flynn, Z., Gandhi, A., & Traina, J. (2019). Measuring Markups with Production Data.

Gonçalves, A. S., & Leonard, G. (2023). The fundamental-to-market ratio and the value premium decline. *Journal of Financial Economics*, *147*(2), 382–405.

Gourio, F., & Rudanko, L. (2014). Customer Capital. *The Review of Economic Studies*, *81*(3), 1102–1136.

Grossman, G. M., & Helpman, E. (1991). Quality Ladders in the Theory of Growth. *Review of Economic Studies*.

Grullon, G., Larkin, Y., & Michaely, R. (2019). Are US Industries Becoming More Concentrated? *Review of Finance*, *23*(4), 697–743.

Gutiérrez, G., & Philippon, T. (2017). INVESTMENT-LESS GROWTH: AN EMPIRICAL INVESTIGATION.

Hall, R. E. (1988). The Relation between Price and Marginal Cost in U.S. Industry. *Journal of Political Economy*, *96*(5), 921–947.

Hayashi, F. (1982). Tobin's Marginal q and Average q: A Neoclassical Interpretation. *Econometrica*, *50*(1), 213.

Klette, T. J., & Kortum, S. (2004). Innovating Firms and Aggregate Innovation. *Journal of Political Economy*, *112*(5), 986–1018.

Kogan, L., Papanikolaou, D., Seru, A., & Stoffman, N. (2017). Technological Innovation, Resource Allocation, and Growth. *The Quarterly Journal of Economics*, *132*(2), 665–712.

Liu, L. X., Whited, T. M., & Zhang, L. (2009). Investment-Based Expected Stock Returns. *Journal of Political Economy*, *117*(6), 1105–1139.

Loury, G. C. (1979). Market Structure and Innovation. *The Quarterly Journal of Economics*, *93*(3), 395.

Matsuyama, K., & Ushchev, P. (2017). Beyond CES: Three Alternative Classes of Flexible Homothetic Demand Systems. *SSRN Electronic Journal*.

Merz, M., & Yashiv, E. (2007). Labor and the Market Value of the Firm. *American Economic Review*, *97*(4), 1419–1431.

Nordhaus, W. D. (1969). An Economic Theory of Technological Change. *American Economic Review*, *59*(2), 18–28.

Park, H. (2022). An Intangible-Adjusted Book-to-Market Ratio Still Predicts Stock Returns. *Critical Finance Review*, *11*(2), 265–297.

Paron, J. (2025). The Wealth of Stagnation: Falling Growth, Rising Valuations.

Peters, R., & Taylor, L. A. (2017). Intangible capital and the investment-q relation. *Journal of Financial Economics*, *123*(2), 251–272.

Pierce, J. R., & Schott, P. K. (2016). The Surprisingly Swift Decline of US Manufacturing Employment. *American Economic Review*, *106*(7), 1632–1662.

Raval, D. (2023). Testing the Production Approach to Markup Estimation. *Review of Economic Studies*, *90*(5), 2592–2611.

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy*, *98*(5, Part 2), S71–S102.

Shapley, L. (1951). *Notes on the n-person game*. Rand Corporation.

Shorrocks, A. F. (1982). Inequality Decomposition by Factor Components. *Econometrica*, *50*(1), 193.

Sun, L., & Abraham, S. (2021). Estimating dynamic treatment effects in event studies with heterogeneous treatment effects. *Journal of Econometrics*, *225*(2), 175–199.

Tirole, J. (1988). *The theory of industrial organization*. MIT press.

Traina, J. (2018). Is Aggregate Market Power Increasing? Production Trends Using Financial Statements. *SSRN Electronic Journal*.

Zhang, L. (2005). The Value Premium. *The Journal of Finance*, *60*(1), 67–103.

Table 1: Parameter estimates

This table reports the model parameters. Panel A reports the calibrated parameters. Panel B lists the estimated parameters and their standard errors. The model is solved and simulated at annual frequency.

Variable	Notation	Value	SE
Panel A: Calibrated parameters			
Discount rate	r_f	0.04	
Scaling of profits	$\bar{Y}$	20	
Depreciation of physical capital	δ^K	0.12	
Depreciation of intangible capital	δ^N	0.2	
Panel B: Estimated parameters			
Group 1: Production technology			
Share of physical capital	α	0.7297	0.0916
Total share of capital	η	0.2662	0.0048
Substitution between capitals	ρ	0.4515	0.0619
Fixed operating cost	f	0.3766	0.0274
Group 2: Investment and capital-specific productivity			
Adjustment cost of physical capital	b^K	2.9059	0.1957
Adjustment cost of intangible capital	b^N	1.9468	0.3501
Autocorrelation of z_t^K	ρ^K	0.8131	0.0342
Volatility of ζ_t^K shocks	σ_ζ^K	0.6106	0.1521
Autocorrelation of z_t^N	ρ^N	0.9788	0.0126
Volatility of ζ_t^N shocks	σ_ζ^N	0.2227	0.0825
Group 3: Markup-related			
Average effect of intangible capital on markup	$\bar{\mu}$	0.1267	0.0134
Curvature of the markup function	ϕ	0.7576	0.0259
Autocorrelation of ζ_t^μ	ρ^μ	0.7587	0.0151
Volatility of ζ_t^μ shocks	σ^μ	0.1456	0.0036
Group 4: SDF			
Volatility of ε_x^K	σ_x^K	0.0437	0.0153
Volatility of ε_x^N	σ_x^N	0.2344	0.0112
Risk price of ε_x^K	γ^K	19.9670	14.8757
Risk price of ε_x^N	γ^N	16.3640	2.8020

Table 2: Moments

This table reports moments used in the SMM. The model is solved and simulated at an annual frequency. I simulate the model for 100 independent samples, with each sample including 3500 firms over 200 years (the first 150 years form a burn-in period). I report the data moments and the means of the corresponding simulated moments. Definitions of the variables are in Table C1.

Moment	Data	Simulated	SE
Panel A: Firm-level moments			
Average value-to-total-capital ratio	1.3834	1.4425	0.0133
Std. dev. of physical investment	0.1213	0.1213	0.0009
Std. dev. of intangible investment	0.1189	0.1170	0.0010
Average intangible share	0.4839	0.4917	0.0033
Std. dev. of intangible share	0.0852	0.0818	0.0011
Average profits-to-total-capital ratio	0.4284	0.4350	0.0027
AR(1) of profits-to-total-capital ratio	0.6813	0.6424	0.0047
Std. dev. of profits-to-physical-capital ratio	0.6947	0.7376	0.0101
AR(1) of profits-to-physical-capital ratio	0.6935	0.6969	0.0073
Std. dev. of profits-to-intangible-capital ratio	0.9142	0.7794	0.0152
AR(1) of profits-to-intangible-capital ratio	0.6397	0.6260	0.0090
Average SALE/COGS	1.7538	1.7963	0.0097
Std. dev. of SALE/COGS	0.3960	0.3749	0.0082
AR(1) of SALE/COGS	0.6433	0.7532	0.0130
Cross-sectional correlation between SALE/COGS and intangible share	0.2487	0.3431	0.0093
Panel B: Aggregate moments			
Average market excess return	0.0893	0.0753	0.0240
Average HML sorted by profitability	0.0402	0.0415	0.0276
Volatility of aggregate profits-to-total-capital	0.0361	0.0336	0.0030
AR(1) of aggregate profits-to-total-capital	0.8681	0.8320	0.0085
Volatility of aggregate physical investment	0.0218	0.0064	0.0024
Volatility of aggregate intangible investment	0.0152	0.0173	0.0013

Table 3: State variables and model fit

This table reports the estimates of state variables and the fit of the model. The first column reports the entire sample, and the second and third columns show results for the first and second halves of the sample. In Panel A, z^K and z^N are estimated following the procedure outlined in Section B.4, μ is given by $(1-\eta)\cdot$ SALE/COGS, and $\hat{K}$ and $\hat{N}$ are normalized capital. Panel B reports the coefficient and R^2 of the regression $V_{it} = a + b \cdot V_{it}^{\text{model}} + FE_t + \varepsilon_{it}$. Standard errors in parentheses are two-way clustered by firm and year.

	1976–2024	1976–2000	2001–2024
Panel A: Estimates of state variables			
z^K	-0.6891	-0.5874	-0.8198
z^N	0.1373	-0.1335	0.4852
μ	1.2897	1.1916	1.4158
$\hat{K}$	2.2741	2.5108	1.9699
$\hat{N}$	2.1033	1.8672	2.4066
Intangible share	0.4890	0.4352	0.5582
Panel B: Goodness of fit			
b	1.3164	1.2634	1.3188
	(0.1197)	(0.1128)	(0.1253)
Adjusted R^2	0.8042	0.7911	0.8025
No. of observations	122,701	68,997	53,704

Table 4: PNTR exposure and market-power value shares (1991–2007)

This table reports difference-in-differences estimates of the effect of PNTR-driven import competition on the two market-power value shares; Section 4.5.3 describes the setting and the NTR gap measure of industry exposure. The sample is an unbalanced firm-year panel. The Level columns use the value share itself, and the Growth columns use its annual change. Panel A interacts the NTR gap with a post-2000 indicator. Panel B replaces that with separate interactions for the distant pre-period, the short run, and the medium run, omitting 1997–2000 as the reference period. All specifications include firm and year fixed effects, with standard errors clustered by four-digit NAICS industry in parentheses.

	Intangible market-power share		Non-intangible market-power share	
	Level	Growth	Level	Growth
Panel A: Static difference-in-differences				
Gap × Post	0.0304	0.0064	0.0048	-0.0209***
	(0.0219)	(0.0066)	(0.0468)	(0.0063)
Firm FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	52,505	44,245	52,505	44,245
R^2	0.7418	0.2230	0.7841	0.1411
Panel B: Dynamic difference-in-differences				
Gap × Pre (1991–1996)	0.0010	-0.0093	0.0011	-0.0112
	(0.0150)	(0.0072)	(0.0193)	(0.0075)
Gap × Short run (2001–2003)	0.0264	0.0032	0.0165	-0.0270**
	(0.0182)	(0.0090)	(0.0421)	(0.0109)
Gap × Medium run (2004–2007)	0.0355	0.0025	-0.0069	-0.0234***
	(0.0259)	(0.0079)	(0.0462)	(0.0086)
Firm FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	52,505	44,245	52,505	44,245
R^2	0.7418	0.2230	0.7841	0.1412

Table 5: Aggregate decomposition

This table reports the level and covariance decomposition of V into terms attributed to physical capital, intangibles in production, intangible-driven market power, and non-intangible market power. Panel A presents the percentage contribution of each term to total market value. Each term is calculated as $\sum_i V_{it}^{\text{comp}} / \sum_i V_{it}^{\text{model}}$ and then averaged over time. Panel B reports the covariance decomposition following Equation (30) (average across years). The Late − Early row reports the difference between the late and early subperiod averages; standard errors are in parentheses.

		% V^{phy}	% $V^{\text{prod-int}}$	% $V^{\text{mkp-int}}$	% $V^{\text{mkp-non}}$
Panel A: Level decomposition					
Full sample	(1976–2024)	55.11	16.47	28.33	0.10
Early	(1976–2000)	63.74	12.56	27.74	-4.05
Late	(2001–2024)	46.11	20.54	28.93	4.42
Late − Early		-17.64	7.98	1.19	8.47
		(1.15)	(1.12)	(0.75)	(0.80)
Panel B: Covariance decomposition					
Full sample	(1976–2024)	53.04	22.65	13.01	11.30
Early	(1976–2000)	63.72	16.49	10.79	9.00
Late	(2001–2024)	41.91	29.06	15.32	13.71
Late − Early		-21.81	12.57	4.53	4.71
		(1.85)	(1.81)	(0.45)	(0.81)

Table 6: Resurrecting the value premium

This table reports the value premium (value minus growth, in percent per month) for four book-to-market measures over the full sample and the 1976–1999 and 2000–2024 subperiods. Each month firms are sorted into NYSE deciles on the measure; the premium is the value-weighted return spread between the value and growth deciles. The traditional and intangible-adjusted rows use book equity (the latter augmented with estimated intangible capital) in the numerator; the model-implied rows replace market value in the denominator with a model value that strips out non-intangible market power, or all market power. Newey-West t-statistics are in parentheses.

Measure	Full sample	1976–1999	2000–2024
Book-to-market	0.25	0.27	0.24
	(0.84)	(0.83)	(0.48)
Intangible-adjusted	0.37	0.59	0.18
	(1.32)	(1.76)	(0.42)
$BM^{\text{no-mkp-non}}$	0.25	0.53	-0.00
	(1.48)	(2.10)	(-0.01)
BM^{prod}	0.43	0.52	0.35
	(2.70)	(2.22)	(1.65)

Table 7: The fitted value signal, book-to-market, and market-power value shares

This table regresses the log fundamental-to-market ratio (FE/ME) on the log book-to-market ratio and the model-implied value shares of intangible production, intangible markup, and non-intangible markup. All specifications include year fixed effects; standard errors, two-way clustered by firm and year, are in parentheses. The first half is 1976–1999 and the second half is 2000–2024.

	Full sample		1976–1999		2000–2024	
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(BE^{\text{total}}/ME)$	0.1550***	0.1043***	0.2259***	0.1881***	0.0518*	-0.0053
	(0.0199)	(0.0285)	(0.0196)	(0.0367)	(0.0250)	(0.0337)
Intangible (production)		-0.5104***		-0.6621***		-0.3596***
		(0.0490)		(0.0561)		(0.0569)
Intangible (markup)		1.1353***		0.9699***		1.3008***
		(0.1157)		(0.1740)		(0.1421)
Non-intangible markup		0.7411***		0.5815***		0.9348***
		(0.0607)		(0.0814)		(0.0609)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Within R^2	0.0233	0.1215	0.0559	0.1346	0.0022	0.1317
Observations	87,070	87,070	51,908	51,908	35,162	35,162

Table 8: Properties of the q measures

This table reports the properties of the q measures used in the investment regressions: Panel A gives summary statistics and Panel B the pooled correlation matrix. Every measure divides a value by end-of-period capital, where K is the physical-capital stock and N is the intangible-capital stock; V is the sum of market equity and the book value of debt, minus cash and short-term investments. $V^{\text{model}} = V^{\text{phy}} + V^{\text{prod-int}} + V^{\text{mkp-int}} + V^{\text{mkp-non}}$ is the model-implied value, which removes the unexplained residual from market value. Skew is the sample skewness $g_1 = m_3/m_2^{3/2}$, which is zero under symmetry. All q measures are winsorized within each year at the 1st and 99th percentiles, and the sample runs from 1976 to 2024.

Panel A: Summary statistics

Measure	Definition	Mean	SD	Median	Skew
Standard q	V/K	4.73	8.73	1.98	5.46
Total q	$V/(K+N)$	1.42	1.77	0.92	5.36
Model total q	$V^{\text{model}}/(K+N)$	1.34	0.97	1.06	1.68
ex. non-intangible markup	$(V^{\text{model}}-V^{\text{mkp-non}})/(K+N)$	1.29	0.88	0.98	1.54
ex. all markup	$(V^{\text{model}}-V^{\text{mkp-int}}-V^{\text{mkp-non}})/(K+N)$	0.95	0.78	0.70	1.50
Physical q	V^{phy}/K	1.47	1.72	0.87	3.30
Intangible q (total)	$(V^{\text{prod-int}}+V^{\text{mkp-int}})/N$	1.43	1.06	1.13	3.24
Intangible q (production)	$V^{\text{prod-int}}/N$	0.59	0.61	0.37	2.10
Intangible q (markup)	$V^{\text{mkp-int}}/N$	0.84	0.62	0.71	4.19

Panel B: Pooled correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) Standard q	1.00								
(2) Total q	0.74	1.00							
(3) Model total q	0.62	0.82	1.00						
(4) ex. non-intangible markup	0.58	0.82	0.91	1.00					
(5) ex. all markup	0.50	0.79	0.86	0.98	1.00				
(6) Physical q	0.70	0.73	0.76	0.83	0.81	1.00			
(7) Intangible q (total)	0.24	0.45	0.53	0.51	0.55	0.33	1.00		
(8) Intangible q (production)	0.41	0.58	0.63	0.71	0.70	0.48	0.83	1.00	
(9) Intangible q (markup)	0.01	0.18	0.28	0.17	0.23	0.08	0.84	0.40	1.00

Table 9: Investment and q: horse race

This table races the total q measures against each other. Each column is a separate regression of the investment rate on a single q, so the coefficients in a panel are directly comparable across columns. Panel A uses physical investment I^K/K and Panel B intangible investment I^N/N; both dependent variables are measured one year ahead, so the regression is of investment in $t+1$ on q in t. Model total q is the model-implied value over $K+N$; the last two columns strip the non-intangible market-power component $V^{\text{mkp-non}}$, and then both market-power components $V^{\text{mkp-int}}$ and $V^{\text{mkp-non}}$, out of that value. All specifications include firm and year fixed effects, with standard errors two-way clustered by firm and year in parentheses.

	Standard q	Total q	Model total q	Model total q (ex. non-intangible markup)	Model total q (ex. all markup)
	(1)	(2)	(3)	(4)	(5)
Panel A: Physical investment, I^K/K					
q	0.0066***	0.0323***	0.0654***	0.0684***	0.0729***
	(0.0005)	(0.0029)	(0.0034)	(0.0034)	(0.0036)
Observations	107,803	107,803	107,803	107,803	107,803
R^2	0.4463	0.4661	0.4653	0.4639	0.4602
Within R^2	0.0897	0.1223	0.1209	0.1187	0.1126
Panel B: Intangible investment, I^N/N					
q	0.0038***	0.0290***	0.0567***	0.0569***	0.0647***
	(0.0004)	(0.0023)	(0.0030)	(0.0030)	(0.0032)
Observations	107,778	107,778	107,778	107,778	107,778
R^2	0.5824	0.6230	0.6184	0.6132	0.6172
Within R^2	0.0426	0.1357	0.1251	0.1132	0.1223

Table 10: Investment and q: value components

This table decomposes q into its four value components and races them against total q. Columns (1)–(3) explain physical investment I^K/K and columns (4)–(6) intangible investment I^N/N; both rates are led one year. The value components are scaled by the capital stock of the margin being explained: by K in columns (1)–(3) and by N in columns (4)–(6). Column (1) uses only the physical component and column (4) only the two intangible components, which are the components generated by the corresponding capital stock; columns (2) and (5) use all four components; columns (3) and (6) add total q alongside them. All q measures and investment rates are winsorized within each year at the 1st and 99th percentiles. All specifications include firm and year fixed effects, with standard errors two-way clustered by firm and year in parentheses.

	Physical investment, I^K/K			Intangible investment, I^N/N		
	(1)	(2)	(3)	(4)	(5)	(6)
V^{phy}	0.0286***	0.0262***	0.0089***		0.0012**	0.0011**
	(0.0024)	(0.0023)	(0.0022)		(0.0004)	(0.0004)
$V^{\text{prod-int}}$		0.0051***	-0.0022*	0.0257***	0.0276***	-0.0067**
		(0.0011)	(0.0011)	(0.0026)	(0.0027)	(0.0030)
$V^{\text{mkp-int}}$		0.0035*	0.0193***	0.0595***	0.0432***	0.0344***
		(0.0021)	(0.0024)	(0.0061)	(0.0067)	(0.0061)
$V^{\text{mkp-non}}$		0.0041***	-0.0042***		0.0005	0.0013**
		(0.0013)	(0.0012)		(0.0006)	(0.0005)
Total q			0.0262***			0.0269***
			(0.0031)			(0.0023)
Observations	107,803	107,803	107,803	107,778	107,778	107,778
R^2	0.4455	0.4507	0.4771	0.5951	0.5958	0.6342
Within R^2	0.0885	0.0970	0.1403	0.0718	0.0733	0.1615

Table 11: Investment and q: Shapley decomposition of the within-R^2

This table reports the Shapley decomposition of the within-R^2 for the joint four-component regression reported in columns (2) and (5) of Table 10. Each component's share is how much it raises the within-R^2 when added to the regression, averaged over all possible orders in which the four components can be added. The shares therefore do not depend on any particular order and add up to 100. The I^K/K column reports the decomposition for physical investment, and the I^N/N column reports it for intangible investment. Both dependent variables are measured one year ahead, and the components are scaled by K in the first column and by N in the second. Firm and year fixed effects are partialled out before the decomposition.

	Share of within-R^2 (%)	
Value component	I^K/K	I^N/N
V^{phy}	76.23	21.61
$V^{\text{prod-int}}$	13.47	32.28
$V^{\text{mkp-int}}$	9.10	40.52
$V^{\text{mkp-non}}$	1.20	5.59
Total within R^2	0.0970	0.0733
Observations	107,803	107,778

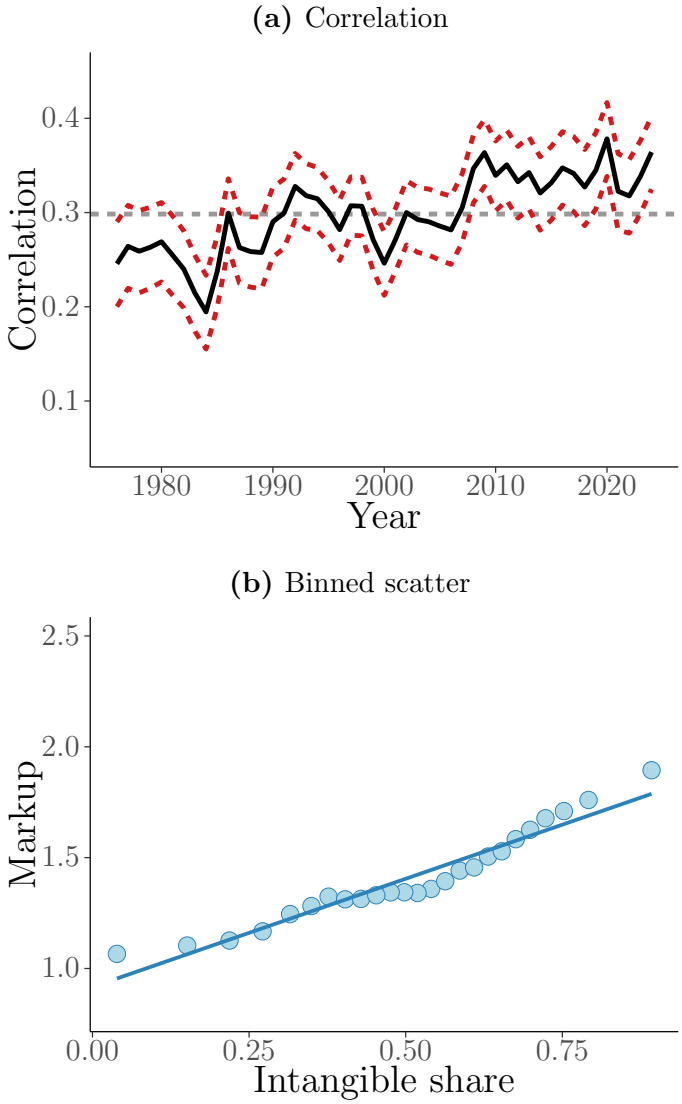


Figure 1: Cross-sectional correlation between intangible share and markups (1976–2024)

This figure plots the relationship between intangible capital share and markups, using the Cobb-Douglas estimate that includes intangible capital as a production input. Panel (a) plots the annual cross-sectional correlation, demeaning both variables within each two-digit industry-year cell, with the 95% confidence interval in red and the sample average in gray. Panel (b) plots a binned scatter: both variables are residualized on two-digit industry and year fixed effects, sorted into 25 equal-sized bins on the residualized intangible share, and plotted as bin means with the sample means added back, with an OLS fitted line. Markups are winsorized within year at the 2.5th and 97.5th percentiles. Intangible share is intangible capital divided by the sum of physical and intangible capital, with intangible capital from Peters and Taylor (2017) and physical capital measured by Compustat item PPEGT.

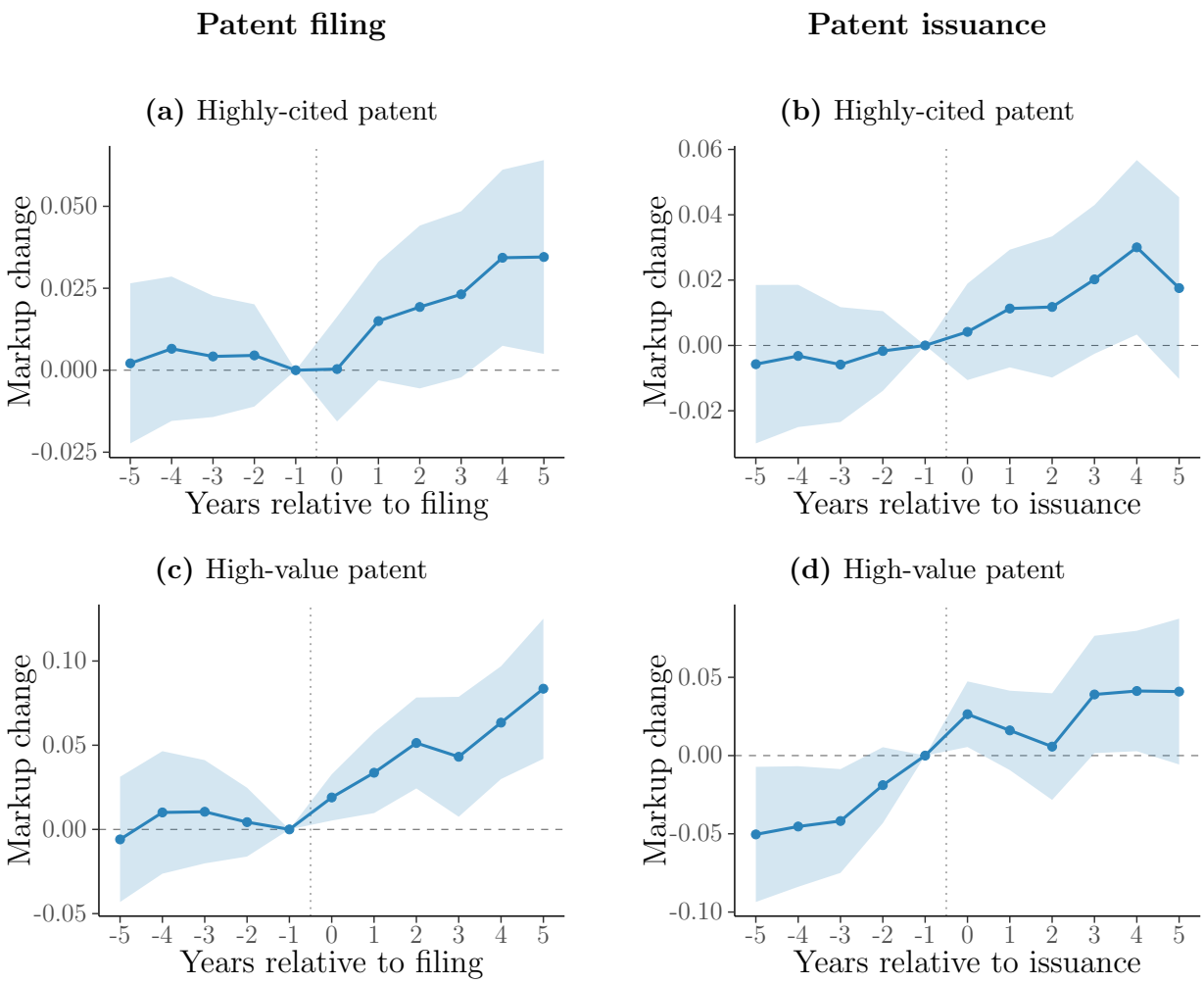


Figure 2: Event studies under Cobb-Douglas with intangibles

This figure plots dynamic treatment effects of important patent filings (left column) and issuances (right column) on markups, using the Cobb-Douglas production-function estimate that includes intangible capital as a production input. Panels (a) and (b) classify a patent as important if its forward-citation count lies in the top 10% within its Fama-French 48 industry and filing or issuance year; Panels (c) and (d) instead classify a patent as important if its stock-market value, as measured by Kogan et al. (2017), lies in the top 10% within the same industry-year cell. A firm is treated in the year of its first important patent, and firms that never have an important patent serve as the never-treated control group. Coefficients are estimated with the Sun and Abraham (2021) interaction-weighted estimator, include firm and year fixed effects, and are normalized relative to the year before the event ($\tau = -1$). The estimation window is $\tau \in [-5, +5]$, and treated firms are required to have markup data in every pre-event year $\tau \in \{-5, \ldots, -1\}$. Shaded areas are 95% confidence intervals, with standard errors clustered by firm.

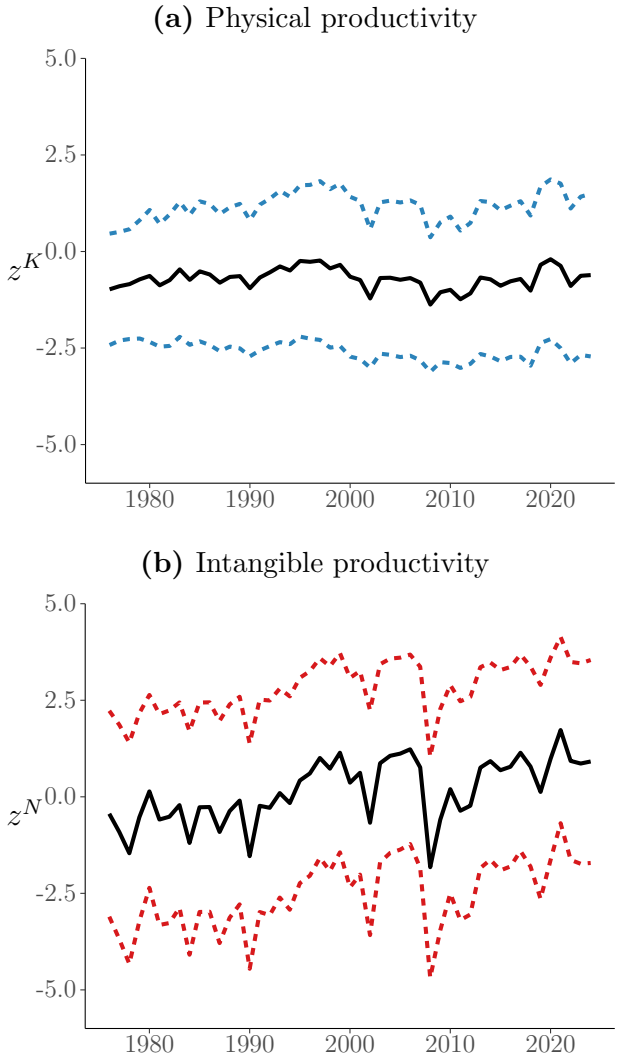
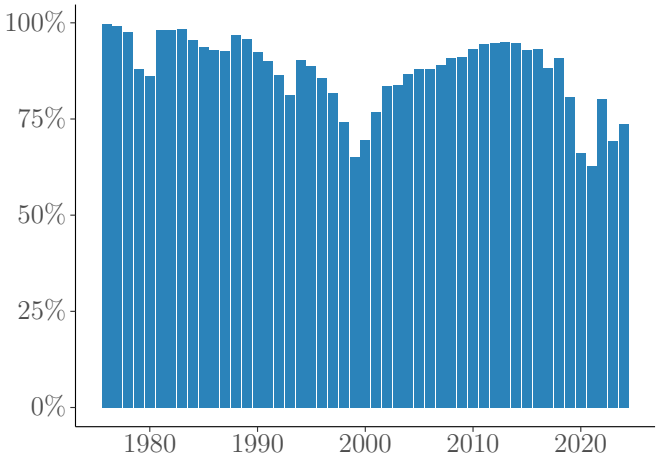


Figure 3: Time series of productivity estimates (1976–2024)

This figure plots the estimated productivity processes, averaged over firms in each year. Panels (a) and (b) show the cross-sectional average productivity for physical capital, $\bar{z}_t^K = x_t^K$, and intangible capital, $\bar{z}_t^N = x_t^N$, respectively. The dashed lines represent $\bar{z}_t^K \pm \sigma_{z^K,t}$ and $\bar{z}_t^N \pm \sigma_{z^N,t}$, where $\sigma_{z,t}$ is the cross-sectional standard deviation, reflecting the dispersion across firms.

(a) R-squared for V



(b) R-squared for $V/(K+N)$

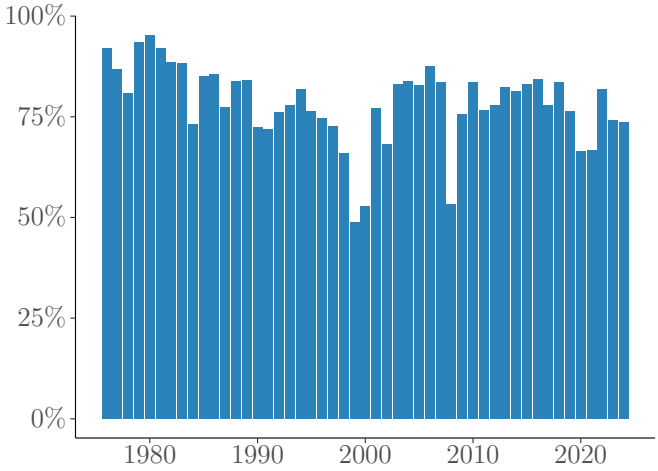


Figure 4: Cross-sectional R-squared over time (1976–2024)

This figure plots the cross-sectional R-squared defined as $\hat{R}^2_t = 1 - \frac{\sum_i \left(V_{it} - V_{it}^{\text{model}}\right)^2}{\sum_i \left(V_{it} - \bar{V}_t\right)^2}$, which restricts both $a = 0$ and $b = 1$. Panel (a) uses firm value V_{it} and panel (b) uses V_{it} scaled by total capital. V_{it}^{model} is the model-implied counterpart and $\bar{V}_t$ is the cross-sectional mean.

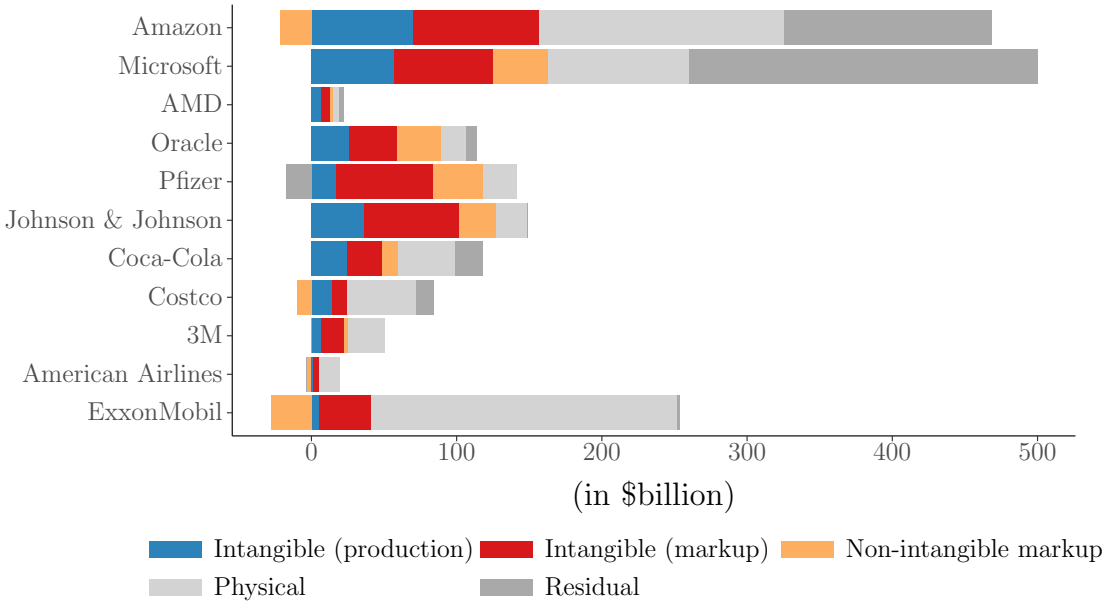


Figure 5: Decomposition for individual firms

This figure plots the value decomposition for a set of example firms based on Equation (18). Each bar represents a firm's average value across sample years.

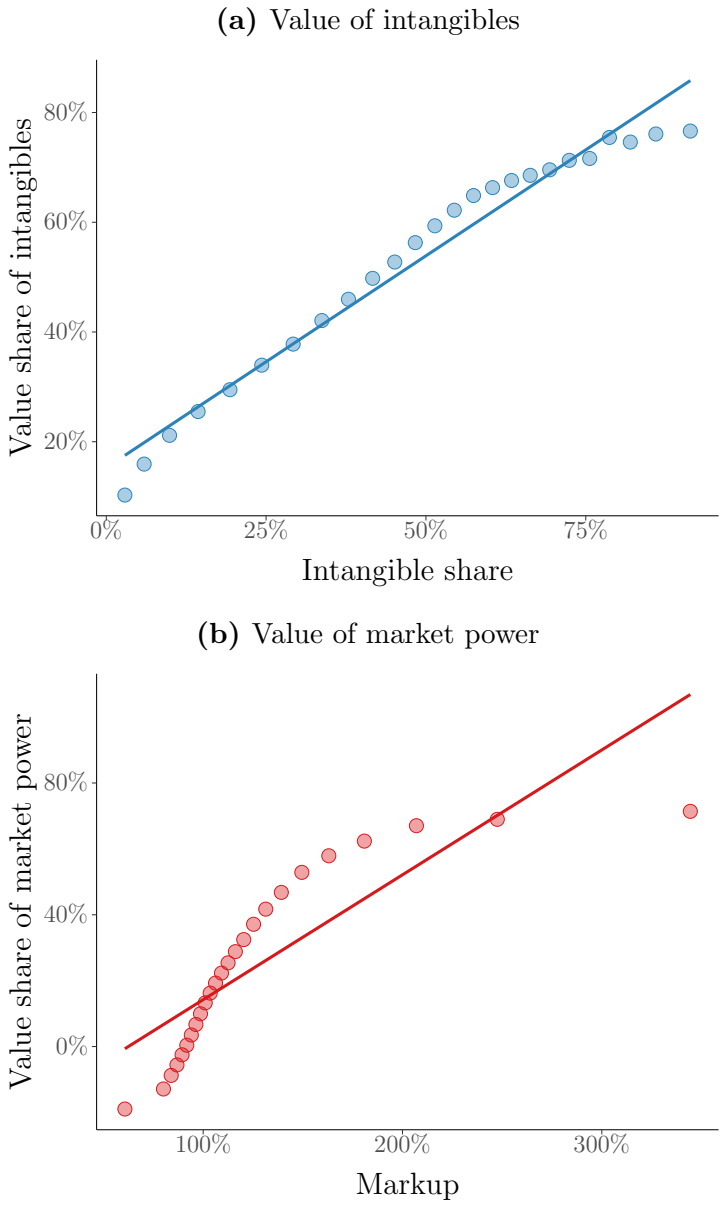


Figure 6: Value shares against intangible share and markup

This figure plots binned scatter plots of model-implied value shares against firm characteristics. Panel (a) plots the share of firm value attributable to intangibles ($V_{it}^{\text{prod-int}} + V_{it}^{\text{mkp-int}}$, scaled by V_{it}) against the firm's intangible share. Panel (b) plots the share of firm value attributable to market power ($V_{it}^{\text{mkp-int}} + V_{it}^{\text{mkp-non}}$, scaled by V_{it}) against the firm's markup. Observations are grouped into equal-sized bins based on the x-axis variable, and bin means are plotted. The dashed line shows the linear fit.

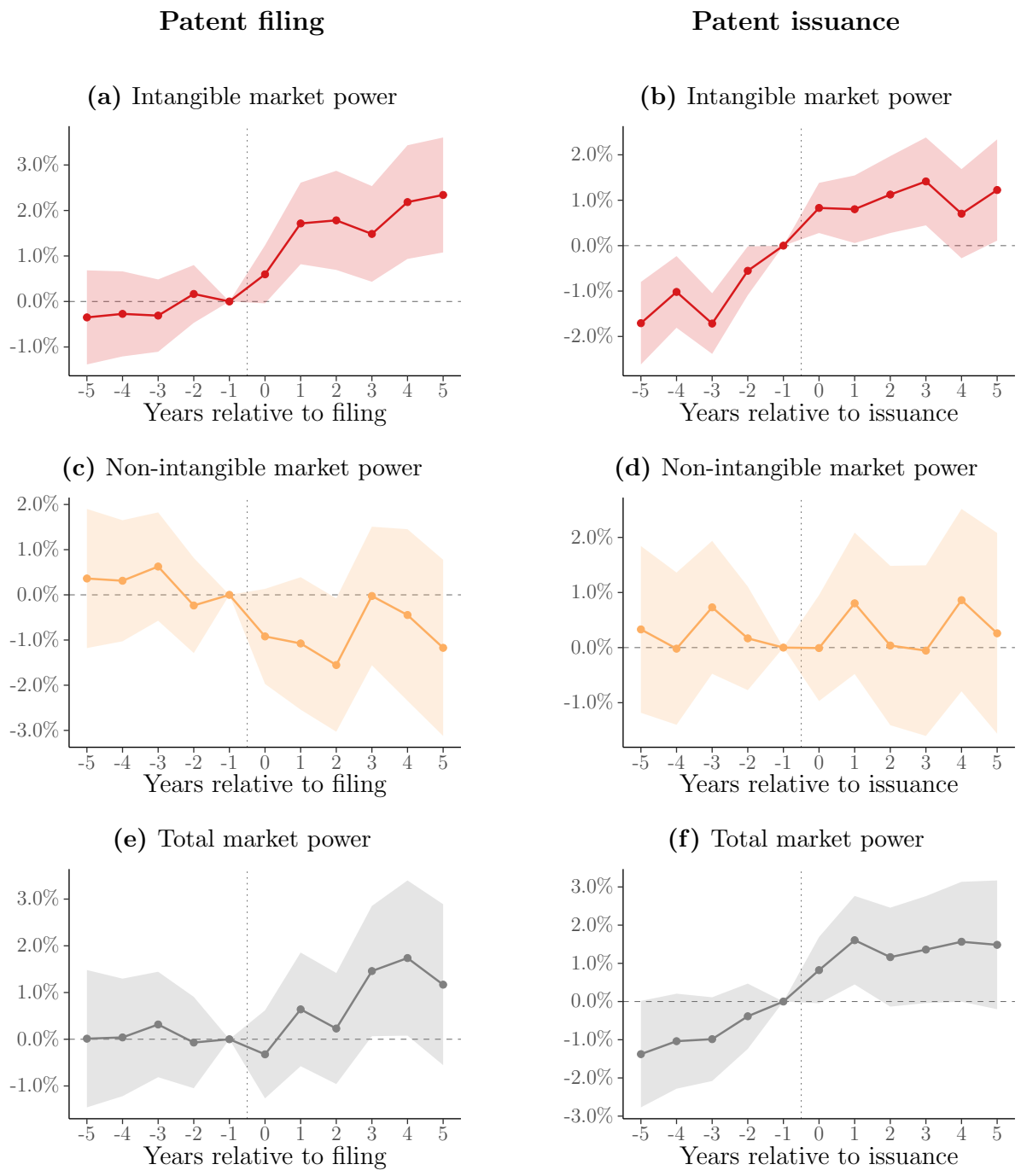


Figure 7: Change in the share of market power around patent filing and issuance

This figure plots dynamic treatment effects of important patent filings (left column) and issuances (right column) on the share of firm value attributable to market power. The outcomes are the intangible-driven market-power value share in Panels (a) and (b), the non-intangible market-power value share in Panels (c) and (d), and the total market-power value share in Panels (e) and (f). Each share is defined as the corresponding component of V^{model} divided by V^{model} and winsorized within each year at the 2.5th and 97.5th percentiles. The event-study design, patent-importance definition, and estimation follow Figure 2. Shaded areas are 95% confidence intervals, with standard errors clustered by firm.

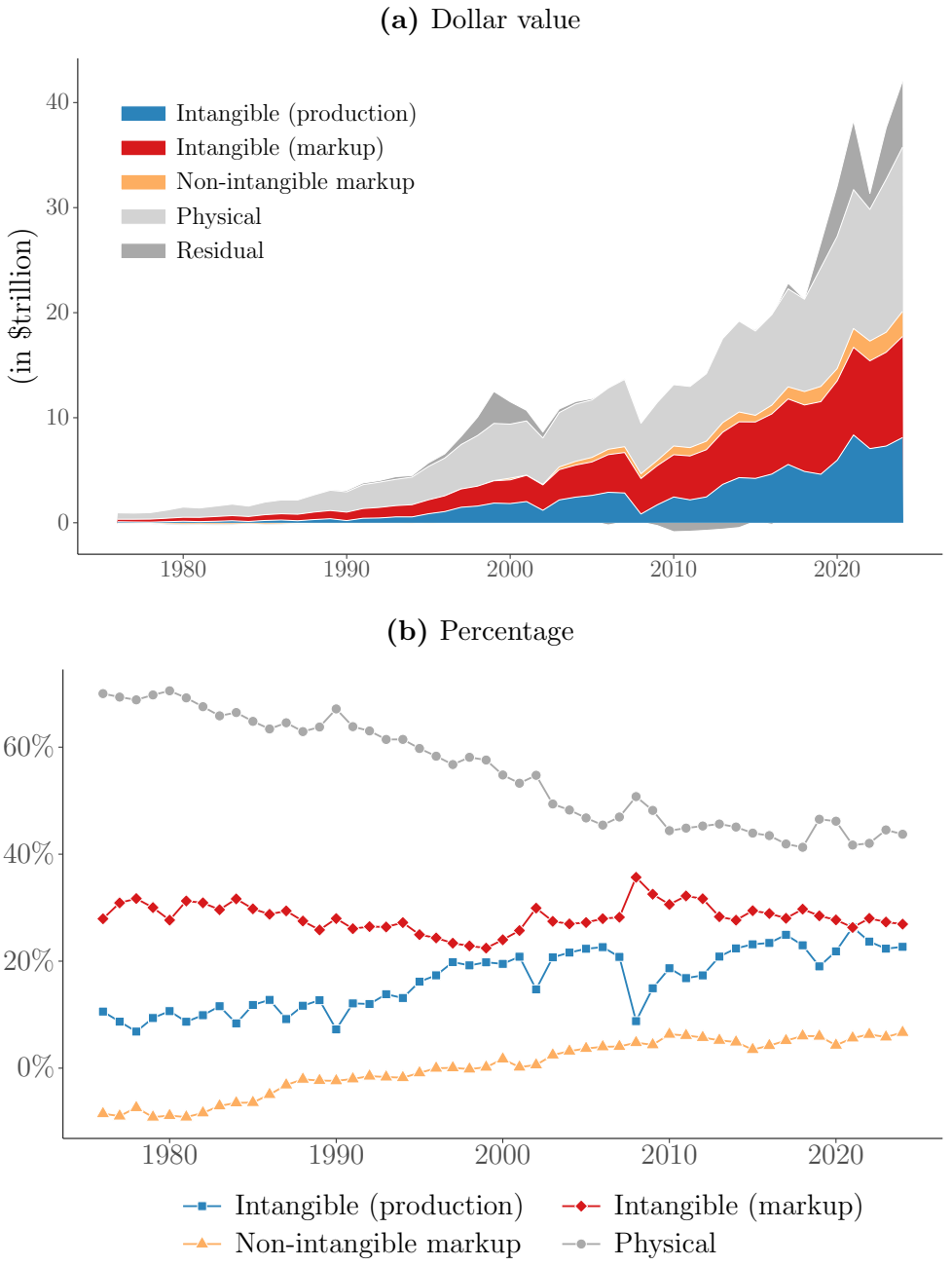


Figure 8: Level decomposition over time (1976–2024)

This figure plots the aggregate value decomposition based on Equation (18). Panel (a) sums each value component across all firms in the cross section for each year and reports the totals in trillions of dollars as a stacked area chart. Panel (b) shows the percentage decomposition, where each component's share is computed as its aggregate dollar value divided by total model-implied firm value.

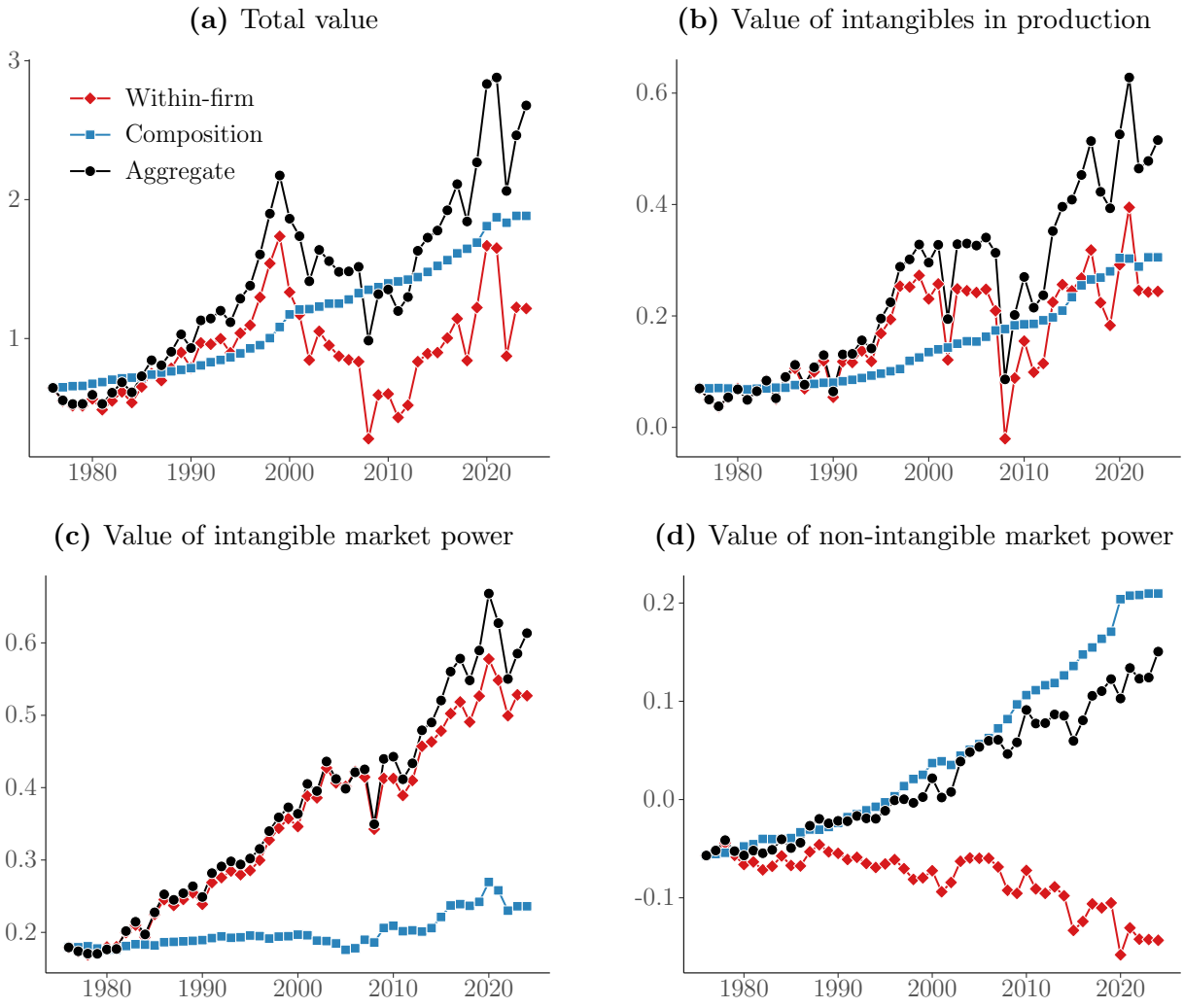


Figure 9: Compositional vs. within-firm change

This figure plots the decomposition of changes in aggregate value into within-firm and compositional components. The decomposition is defined in Equation (27). Panel (a) shows the decomposition of the total value-to-sales ratio. Panel (b) shows the decomposition of the value of intangibles in production scaled by sales. Panels (c) and (d) show the corresponding decompositions of intangible and non-intangible market-power value scaled by sales.

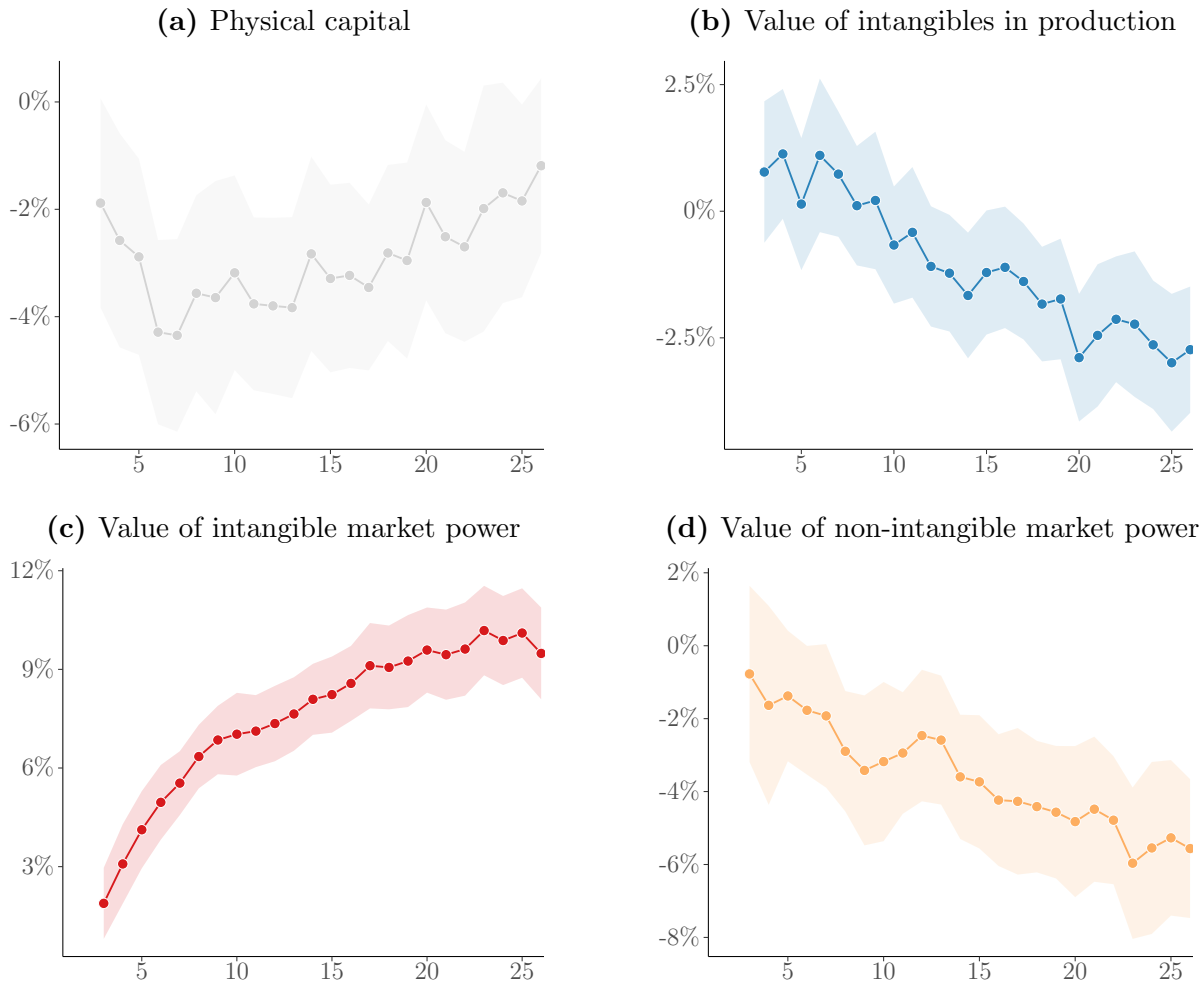


Figure 10: Life cycle changes

This figure plots the evolution of each value component's share over a firm's life cycle. For each component, I regress the ratio of the component to total model-implied value on firm age dummies with year and cohort fixed effects, restricting the sample to firms with at least 15 years of data. The solid line shows the estimated coefficients and the shaded area represents the 95% confidence interval.

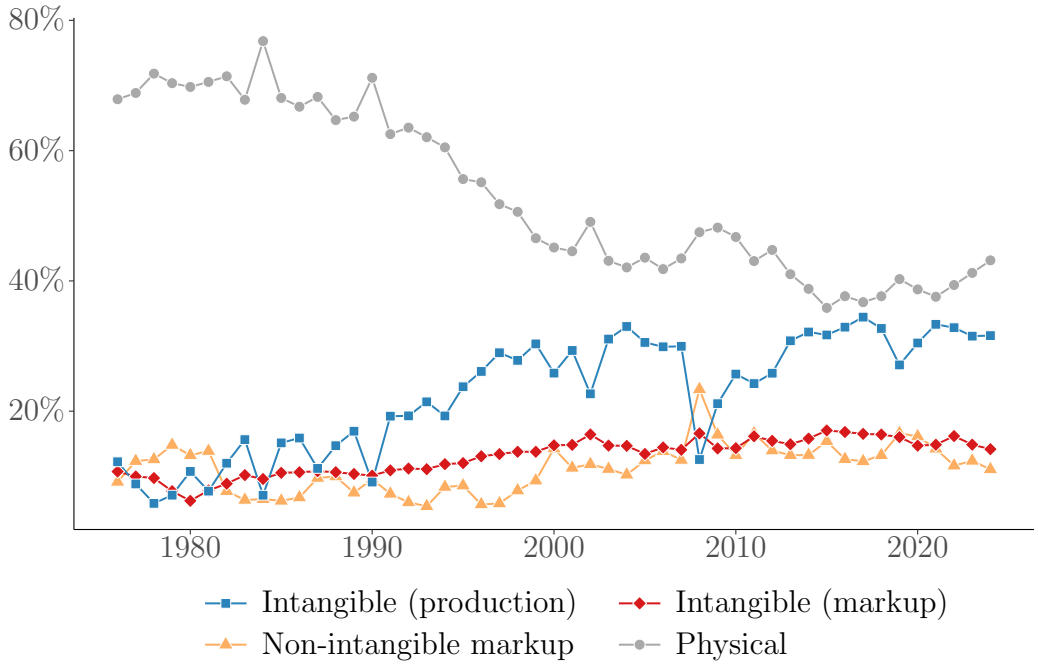
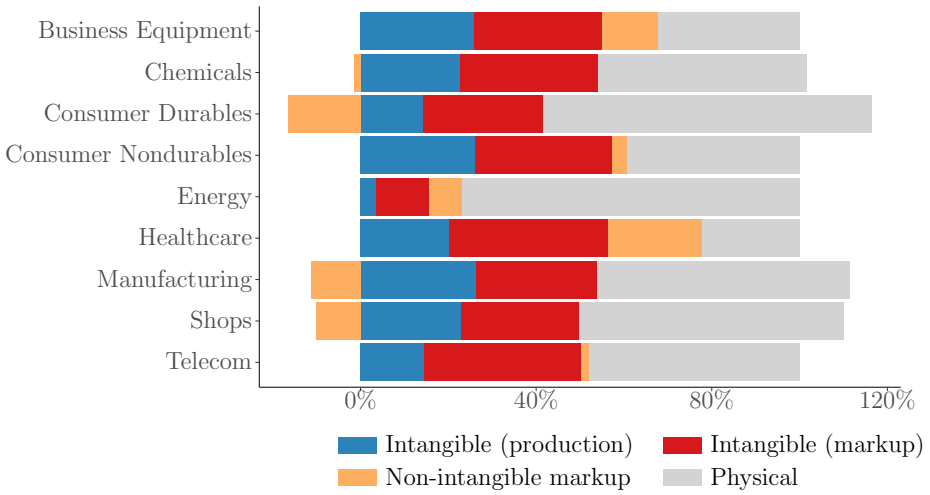
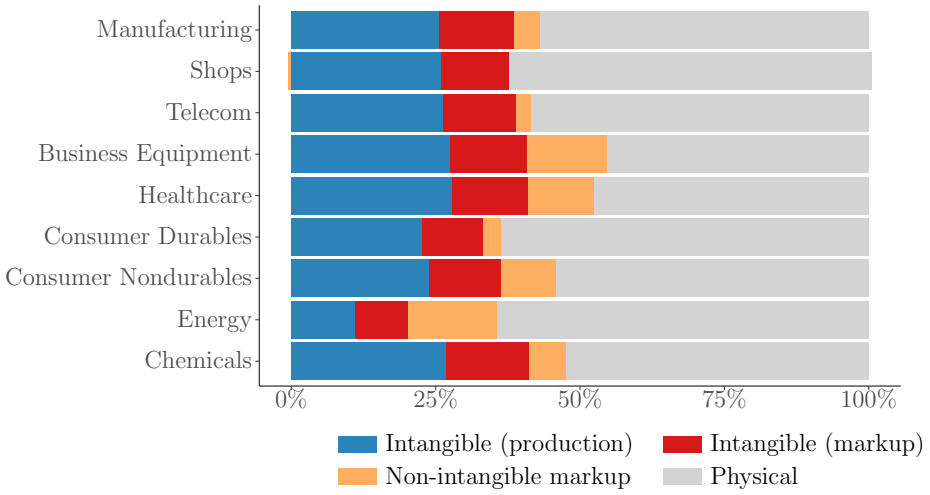


Figure 11: Covariance decomposition over time (1976–2024)

This figure plots the aggregate covariance decomposition for each year based on Equation (30).



(a) Level decomposition

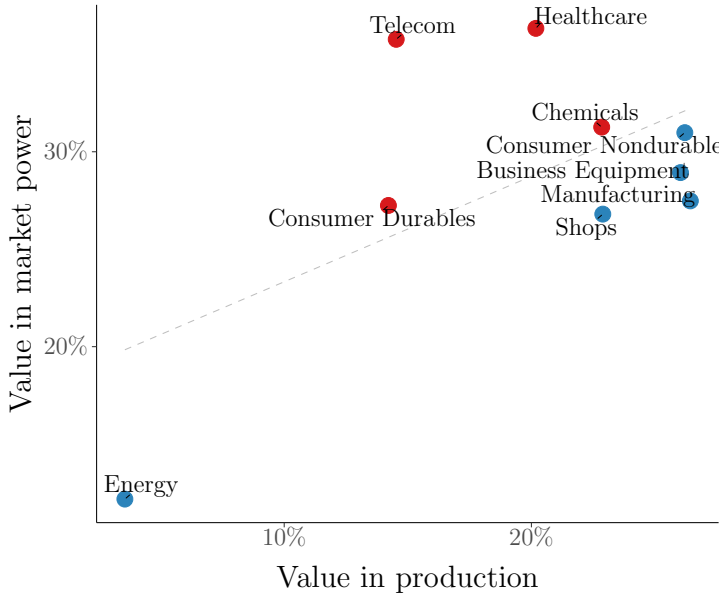


(b) Covariance decomposition

Figure 12: Value decomposition: by sectors

Panel (a) plots the value decomposition for the Fama–French 12-industry classification based on Equation (18), excluding Financials, Utilities, and Other. Each bar decomposes model-implied value into the shares attributable to physical capital, intangibles in production, intangible market power, and non-intangible market power. Panel (b) plots the covariance decomposition for these sectors based on Equation (30). I apply the covariance decomposition for each cross section within a sector and then average across time.

(a) Value of intangibles



(b) Value of market power

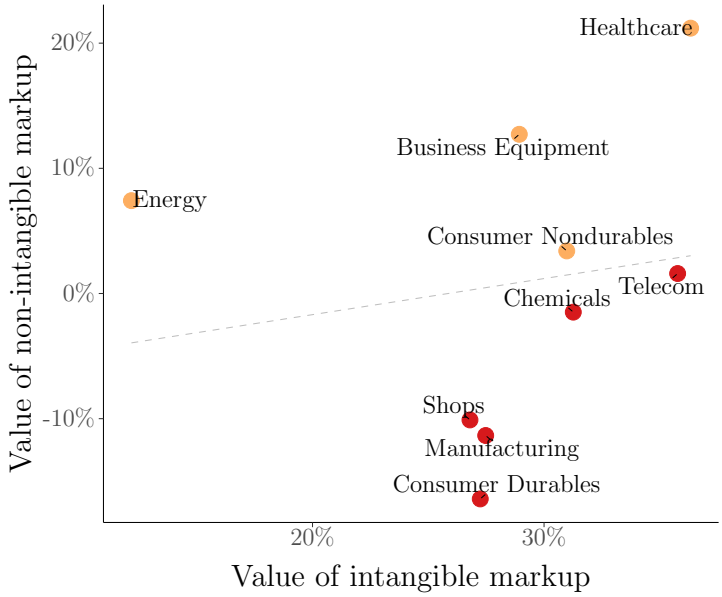


Figure 13: Relation between markup, intangible share, and value

This figure plots sector-level scatter plots using the Fama-French 12 industry classification (excluding Financials, Utilities, and Other). Panel (a) plots the share of firm value attributable to intangible market power against the share from intangibles in production. Panel (b) plots the share from non-intangible market power against the share from intangible market power. Each point represents a sector average, and the dashed line shows the linear fit.

A Derivation of the Operating Profits

In this section, I derive the profit function in Equation (3) based on the consumer demand and production function specified in the model.

A.1 Consumer Demand

Assume each firm operates within its own distinct market segment, producing a single, differentiated variety. The demand function faced by firm i is $Y(P_{it})$, where P_{it} is the price of product i. I parameterize $Y(\cdot)$ as

$$Y(P_{it}) = \left(\frac{P_{it}}{P_t}\right)^{-\frac{\mu_{it}}{\mu_{it}-1}} \frac{\bar{Y}}{P_t}, \quad \mu_{it} > 1 \tag{A.1}$$

where μ_{it} represents the firm's pricing power for product i, $\bar{Y}$ is total market size for this specific product[9], and P_t is an exogenous price index that is absorbed by the productivity shocks (see the next section). Intuitively, when μ_{it} is high, consumers become less sensitive to prices and firms can set higher prices to extract rents. In the next subsection, I show that firms optimally choose markup to be μ_{it} in their profit maximization.

This demand function is consistent with a rational consumer preference system as shown in Matsuyama and Ushchev (2017) and Baqaee, Farhi and Sangani (2024), and it accommodates heterogeneous markups across firms. In this partial equilibrium framework, market power is captured through the shape of the demand function, which governs consumers' price sensitivity. This modeling choice does not imply that consumer preferences are the sole primitive source of market power. In a general equilibrium setting, markups are equilibrium outcomes jointly determined by demand and supply forces: a firm's ability to charge

[9]The total expenditure is set to be a constant and essentially the numeraire in this market. This demand function implicitly assumes that all firms face the same demand curve, and demand variation comes from movement along the curve and variation of demand elasticity (through changing market power), while the total expenditure $\bar{Y}$ is constant across firms. This assumption is not too restrictive since I scale the firm size in data to match the distribution in the model simulation. Therefore, variation in $\bar{Y}$ is scaled down in my estimation.

high prices reflects not only consumers' willingness to pay but also supply-side factors such as product differentiation, proprietary technology, or competitive structure. The demand function in my model should therefore be interpreted as a reduced-form representation of these combined forces.

A.1.1 Special Cases of the Demand Function

The markup is less than or equal to 1. When $\mu_{it} \leq 1$, I assume the firm operates under perfect competition and takes the price P_t as given. This assumption corresponds to the limiting case of the demand function (A.1) as μ_{it} approaches 1 from above.

P_{it} is below the price index P_t. If the firm has the ability to produce with low marginal cost and undercut the price index, I assume the firm captures the entire market and earns total revenue $\bar{Y}$. This case corresponds to a sector monopoly as in De Ridder (2024) and Aghion et al. (2025), where the most efficient producer dominates the entire sector and earns constant revenue. The price this firm can set is constrained by the second-best competitor (potential entrant), and μ_{it} corresponds to the productivity gap between the incumbent and its closest rival. It is consistent with the relation between μ_{it} and intangible capital, as investments in intangibles enhance productivity and widen this competitive advantage.

A.2 Profit Maximization

At each time t, given the demand equation (A.1), firm i solves the following problem:

$$\begin{aligned} \Pi(z_{it}^K, z_{it}^N, \zeta_{it}^\mu, N_{it}, K_{it}) = \max_{P_{it}, L_{it}} \; & P_{it}Y_{it} - P_{Lit}L_{it} \\ \text{subject to} \quad & \mathbf{K}(z_{it}^K, z_{it}^N, K_{it}, N_{it})^\eta L_{it}^{1-\eta} = Y_{it} \end{aligned} \tag{A.2}$$

where P_{Lit} is the price of the costlessly adjustable input and firms take it as given. Let λ_{it} be the Lagrangian multiplier for the production constraint. λ_{it} represents the marginal cost of production (given the capital inputs).

Taking the first-order condition with respect to P_{it} yields the optimal markup, $P_{it}/\lambda_{it} = \mu_{it}$. Substituting P_{it} out using the constraint and rewrite the problem as

$$\max_{L_{it}} \left[\mathbf{K}_{it}^{\eta} L_{it}^{1-\eta}\right]^{\frac{1}{\mu_{it}}} \bar{Y}^{\frac{\mu_{it}-1}{\mu_{it}}} P_t^{\frac{1}{\mu_{it}}} - P_{Lit} L_{it} \tag{A.3}$$

The first-order condition with respect to L_{it} is $P_{Lit} = \frac{1-\eta}{\mu_{it}} \bar{Y}^{\frac{\mu_{it}-1}{\mu_{it}}} P_t^{\frac{1}{\mu_{it}}} Y_{it}^{\frac{1}{\mu_{it}}} / L_{it}$. Solving for L_{it} yields

$$L_{it} = \left(\frac{(1-\eta)\bar{Y}^{\frac{\mu_{it}-1}{\mu_{it}}} P_t^{\frac{1}{\mu_{it}}}}{\mu_{it} P_{Lit}} \right)^{\frac{\mu_{it}}{\mu_{it}-1+\eta}} \mathbf{K}_{it}^{\frac{\eta}{\mu_{it}-1+\eta}} \tag{A.4}$$

Substituting the first-order condition back into Π gives

$$\Pi_{it} = \left(1 - \frac{1-\eta}{\mu_{it}}\right) P_{it} Y_{it} \tag{A.5}$$

Plugging the optimal L_{it} into the profit function,

$$\begin{aligned}
\Pi_{it} &= \left[1 - \frac{1-\eta}{\mu_{it}}\right] \left(\frac{1-\eta}{\mu_{it}}\right)^{\frac{1-\eta}{\mu_{it}-1+\eta}} P_t^{\frac{1}{\mu_{it}-1+\eta}} \bar{Y}^{\frac{\mu_{it}-1}{\mu_{it}-1+\eta}} P_{Lit}^{-\frac{1-\eta}{\mu_{it}-1+\eta}} \mathbf{K}_{it}^{\frac{\eta}{\mu_{it}-1+\eta}} \\
&= \left[1 - \frac{1-\eta}{\mu_{it}}\right] \left(\frac{1-\eta}{\mu_{it}}\right)^{\frac{1-\eta}{\mu_{it}-1+\eta}} \bar{Y} \left(\frac{\mathbf{K}_{it}^{\eta} P_t}{P_{Lit}^{1-\eta} \bar{Y}^{\eta}}\right)^{\frac{1}{\mu_{it}-1+\eta}} \\
&= \left[1 - \frac{1-\eta}{\mu_{it}}\right] \left(\frac{1-\eta}{\mu_{it}}\right)^{\frac{1-\eta}{\mu_{it}-1+\eta}} \bar{Y} \cdot \tilde{\mathbf{K}}_{it}^{\frac{\eta}{\mu_{it}-1+\eta}}
\end{aligned} \tag{A.6}$$

The last equality is due to the fact that the term $\frac{P_t}{P_{Lit}^{1-\eta}\bar{Y}^{\eta}}$ can be absorbed by shocks z_{it}^K and z_{it}^N within the capital aggregator. In the main text, I drop the tilde above $\mathbf{K}$ to be more concise.

The demand function (A.1) is well-defined for $\mu_{it} > 1$. When $\mu_{it} = 1$ with price equal to the marginal cost, the firm operates under perfect competition and is a price taker. In this

case, the firm solves

$$\Pi(z_{it}^K, z_{it}^N, \zeta_{it}^\mu, N_{it}, K_{it}) = \max_{L_{it}} P_t \mathbf{K}_{it}^\eta L_{it}^{1-\eta} - P_{Lit} L_{it} \tag{A.7}$$

The first order condition is given by $P_{Lit} = (1-\eta) P_t \mathbf{K}_{it}^\eta L_{it}^{-\eta}$, which implies $L_{it} = \left(\frac{(1-\eta)P_t}{P_{Lit}}\right)^{\frac{1}{\eta}} \mathbf{K}_{it}$. Therefore, the profit function is

$$\begin{aligned} \Pi_{it} &= P_t \mathbf{K}_{it}^\eta L_{it}^{1-\eta} - (1-\eta) P_t \mathbf{K}_{it}^\eta L_{it}^{1-\eta} \\ &= \eta P_t \mathbf{K}_{it}^\eta L_{it}^{1-\eta} \\ &= \eta (1-\eta)^{\frac{1-\eta}{\eta}} P_t^{\frac{1}{\eta}} P_{Lit}^{-\frac{1-\eta}{\eta}} \mathbf{K}_{it} \end{aligned} \tag{A.8}$$

which is equivalent to Equation (A.6) when setting $\mu_{it} = 1$. Therefore, the profit function (A.6) nests the perfect competition case.

When the firm produces at very low cost, it captures the entire product market niche and the profit is then given by (following Equation (A.5) and the fact that total sales equal the market size)

$$\Pi_{it} = \bar{Y} \cdot \left(1 - \frac{1-\eta}{\mu_{it}}\right) \tag{A.9}$$

B Data, Measurement, and Methodology Details

B.1 Measure of Markups

Estimation of firm-level markups typically relies on the production function approach in the IO literature (e.g., Hall, 1988; De Loecker, Eeckhout and Unger, 2020). This approach exploits the cost-minimization condition that ties marginal cost to input demand. Markups are then estimated as the wedge between the output elasticity of a variable input and that

input's observed revenue share. The measure is given by

$$\mu_{it} = \eta_{it} \frac{\text{SALE}_{it}}{\text{VarCost}_{it}} \tag{B.1}$$

where VarCost_{it} is the cost of a variable input that can be freely adjusted, and η_{it} is the output elasticity of that input.

While this measure provides markup estimates for nearly all public firms in Compustat, it is criticized for relying on strong assumptions: the production function can be identified from revenue-based data when quantity is not observed, and COGS from Compustat fully represents the variable inputs[10]. However, De Ridder, Grassi and Morzenti (2026) argue that even though the level of markup can be biased under the production approach, the dispersion among firms is accurately captured by such estimates, which is the focus of this paper. To further ensure the validity of this measure, I include 6 estimates based on different specifications.

Model-free markup proxies. As shown in De Ridder, Grassi and Morzenti (2026), firm-level variation in markups is primarily driven by the variation in the revenue-to-cost ratio $\text{SALE}_{it}/\text{VarCost}_{it}$. I first document the correlation between intangible capital share and this ratio, which is free of production function estimation. I consider two versions of VarCost_{it}. The first uses COGS following De Loecker, Eeckhout and Unger (2020). The second includes both COGS and some of the SG&A expenses. Traina (2018) suggests that at least some of the selling and management expenses are also variable costs. I follow Ayyagari, Demirgüç-Kunt and Maksimovic (2024) to measure variable costs as:

$$\text{COGS} + \text{XSGA}^* = \text{COGS} + (\text{XSGA} - \text{XRD} - \text{RDIP} - 0.3 \times \text{XSGA}) \tag{B.2}$$

[10]See Traina (2018), Bond et al. (2021), Doraszelski and Jaumandreu (2022), Raval (2023), and Demirer (2022).

where XSGA is Compustat item selling, general, and administrative expenses. XRD and RDIP measure R&D and in-process R&D expenses in Compustat. In other words, this measure of variable costs recognizes R&D and some SG&A expenses as capital investment, while the rest is variable.

Markup based on different production functions. I further estimate markups under four production function specifications, varying along two dimensions: production technology (translog vs. Cobb-Douglas) and whether intangible capital is included as a production input. The translog production function approximates arbitrary production technologies via a second-order Taylor expansion in logarithms. In general, a translog function with n inputs $X_1, X_2, ..., X_n$ can be written as:

$$\log Y = \alpha_0 + \sum_{i=1}^{n} \alpha_i \log X_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \log X_i \log X_j \tag{B.3}$$

Under the translog specification, the output elasticity of the variable input, $\eta_i = \partial \log Y / \partial \log X_i$, is firm-specific and depends on the levels of all production inputs. Cobb-Douglas is a nested special case with $\beta_{ij} = 0$ for all i, j, yielding a constant output elasticity. For each production technology, I consider two input configurations: one with physical capital and the variable input only, and one that additionally includes intangible capital. In all specifications, I use cost of goods sold (COGS) as the variable input expenditure.

B.2 Estimation Details

The SMM estimation minimizes the distance between data moments and simulated moments. The estimation is implemented in the following steps. For an arbitrary set of parameters Θ, I solve the model for the value and policy functions. Then I simulate K panels of N firms $\times$ $T + 150$ years and drop the first 150 years as a burn-in period to remove the impact of initial states. In the simulation, if a firm's value drops to 0, I assume the firm exits and is

immediately replaced by a new firm with the same capital and redrawn shocks.

Next, I compute the model-implied moments in the simulation. Let x_{it} be the actual data vector, $i \in \{1, \ldots, N\}$, $t \in \{1, \ldots, T\}$, and let $y_{itk}(\Theta)$ be the simulated vector from simulation k, $i \in \{1, \ldots, N\}$, $t \in \{1, \ldots, T\}$, and $k \in \{1, \ldots, K\}$. The simulated data vector, $y_{itk}(\Theta)$, depends on a vector of structural parameters, Θ. Define the moment conditions as

$$\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left[h(x_{it}) - \frac{1}{K} \sum_{k=1}^{K} h(y_{itk}(\Theta)) \right] \equiv \Psi^A - \Psi^S(\Theta) \tag{B.4}$$

where $h(y_{itk}(\Theta))$ is a vector of simulated moments and $h(x_{it})$ is the vector of moments from the actual data. $\Psi^A = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} h(x_{it})$, $\Psi^S(\Theta) = \frac{1}{NTK} \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=1}^{K} h(y_{itk}(\Theta))$.

The simulated moments estimator is defined as the solution to the minimization of

$$\hat{\Theta} = \arg\min_{\Theta} \left[\Psi^A - \Psi^S(\Theta) \right]' \hat{W} \left[\Psi^A - \Psi^S(\Theta) \right] \tag{B.5}$$

where $\hat{W}$ is a positive definite matrix that converges in probability to a deterministic positive definite matrix W. It is constructed by calculating the inverse of the variance-covariance matrix of the data moments. I construct the variance-covariance matrix using the influence function method following Erickson and Whited (2002). I demean each variable at the industry-year level and then calculate the influence functions for each moment using the demeaned data. I then covary these influence functions by summing in both the cross-sectional and time series dimensions.

I use the simulated annealing algorithm to solve the minimization problem. The algorithm starts with an initial guess of parameters and iteratively updates the parameters by adding a random perturbation. The new parameters are accepted with a probability that depends on the change in the objective function and a temperature parameter that gradually decreases over iterations. This allows the algorithm to explore the parameter space and avoid local minima. I set the initial temperature to be sufficiently high to allow for exploration and also start with different initial guesses to ensure the robustness of the estimation results.

The simulated moments estimator is asymptotically normal for fixed K. Denote $g(\Theta) \equiv \Psi^A - \Psi^S(\Theta)$, let Θ_0 be the true parameter vector, and define $G \equiv \partial g(\Theta_0)/\partial \Theta'$. The asymptotic distribution of $\hat{\Theta}$ is given by:

$$\sqrt{n}(\hat{\Theta} - \Theta_0) \xrightarrow{d} \mathcal{N}\left(0, \text{avar}(\hat{\Theta})\right), \tag{B.6}$$

where

$$\text{avar}(\hat{\Theta}) = \left(1 + \frac{1}{K}\right)(G'WG)^{-1}G'W\Omega WG(G'WG)^{-1}, \tag{B.7}$$

where Ω is the asymptotic covariance matrix of the moment vector. I estimate this covariance matrix using the influence function of the moment vector, clustered at the firm level, following Erickson and Whited (2002).

B.3 Shapley Decomposition

As described in Section 4.3, separating the production and markup channels of intangible capital requires treating N^{prod} and N^{mkp} as distinct variables. Since the original value function V has only a single state variable N, I solve an auxiliary value function $\tilde{V}$ from a model with the same parameters but two separate intangible capital stocks: N^{prod} enters only the production function and N^{mkp} enters only the markup function. One concern is that in the auxiliary problem, firms need to invest in two copies of intangible capital to achieve the same level of production and market power as in the original problem. To ensure that the two problems have a comparable cost structure, I set the adjustment cost as a function of total investment rates in two intangibles, which is given by $\frac{b^N}{2}\left(\frac{I_{it}^{N,\text{prod}}+I_{it}^{N,\text{mkp}}}{N_{it}^{\text{prod}}+N_{it}^{\text{mkp}}}\right)^2(N_{it}^{\text{prod}} + N_{it}^{\text{mkp}})$, where $I_{it}^{N,\text{prod}}$ and $I_{it}^{N,\text{mkp}}$ are the investments in the two intangible capital stocks.

I then define a composite evaluation rule $\hat{V}$ that governs how coalitions are evaluated

throughout the Shapley decomposition:

$$\hat{V}(\cdot) = \begin{cases} V(\cdot) & \text{if } N^{\text{prod}} = N^{\text{mkp}} \\ \tilde{V}(\cdot) & \text{if } N^{\text{prod}} \neq N^{\text{mkp}} \end{cases} \tag{B.8}$$

That is, when N^{prod} and N^{mkp} are at the same value - both at benchmark or both at their firm-level value - the coalition is evaluated using the original V. The auxiliary $\tilde{V}$ is used only when exactly one of the two has been switched on. This ensures that the decomposition recovers the original V_{it} at the grand coalition while allowing the production and markup channels to be separated.

I then run a Shapley decomposition over six variables $\mathcal{N} = \{z^K, z^N, \zeta^\mu, K, N^{\text{prod}}, N^{\text{mkp}}\}$. For a given variable $j \in \mathcal{N}$ and a subset $S \subseteq \mathcal{N} \setminus \{j\}$, define $\hat{V}(S \cup \{j\})$ as the composite value function evaluated with variable j and every variable in S set to their firm-level values, and all remaining variables set to their benchmark values. Similarly, $\hat{V}(S)$ sets every variable in S to its firm-level value and all other variables (including j) to their benchmark values. The Shapley value of variable j is:

$$V_{it}^j = \sum_{S \subseteq \mathcal{N} \setminus \{j\}} \frac{|S|!\,(5 - |S|)!}{6!} \left[\hat{V}(S \cup \{j\}) - \hat{V}(S)\right] \tag{B.9}$$

where $|S|$ denotes the cardinality of S. The sum ranges over all $2^5 = 32$ subsets of the five variables other than j, weighting each marginal contribution by the Shapley coefficients. The empty coalition corresponds to all variables at their benchmark values, yielding $\hat{V}(\emptyset) = \bar{V}$, and the grand coalition corresponds to all variables at their firm-level values, yielding $\hat{V}(\mathcal{N}) = V_{it}$, since $N^{\text{prod}} = N^{\text{mkp}} = N_{it}$ at the grand coalition.

Intuitively, the Shapley decomposition calculates the average change in firm value when a variable moves from its benchmark to its firm-level value, across all possible orderings in which the six variables are added one by one. I set the benchmark productivity and exogenous markup shocks to zero and the benchmark capital stocks to their minimum grid

values. The decomposition term $V_{it}^{\zeta^\mu}$ captures the marginal contribution of the exogenous markup shock relative to its zero-shock benchmark, averaged across all possible orderings of the other variables. The term $V_{it}^{N,\text{prod}}$ measures the value of adding intangible capital to the production process only, without its effect on markup, since N^{mkp} is treated as a separate variable. Similarly, $V_{it}^{N,\text{mkp}}$ captures the value of intangible capital's contribution to market power, holding the production channel fixed. By construction, $V_{it}^{N,\text{prod}} + V_{it}^{N,\text{mkp}}$ equals the total contribution of intangible capital to firm value.

B.4 Estimation of state variables

This appendix details the iterative procedure for estimating the productivity terms z_{it}^K and z_{it}^N in Section 4.3. Each is the sum of an aggregate and a firm-specific component, $z_{it}^j = x_t^j + \zeta_{it}^j$ for $j \in \{K, N\}$, governed by different distributions (Section 3.3). Inverting the valuation moments in Equation (20) firm-by-firm recovers only the sum z_{it}^j. The aggregate component is the cross-sectional average of the firm-level productivity in each year, $x_t^j = |\mathcal{I}_t|^{-1} \sum_{i \in \mathcal{I}_t} z_{it}^j$, and each firm's idiosyncratic component is the residual $\zeta_{it}^j = z_{it}^j - x_t^j$. I require both components to be consistent with their respective distributions. Since the admissible range for each firm's estimate is centered on the year's aggregate, which is itself the average of those estimates, I impose the two consistency conditions jointly by iteration.

For each capital type j, the aggregate and idiosyncratic components follow AR(1) processes with common persistence ρ^j and innovation standard deviations σ_x^j and σ_ζ^j, so their unconditional standard deviations are $s_x^j = \sigma_x^j / \sqrt{1 - (\rho^j)^2}$ and $s_\zeta^j = \sigma_\zeta^j / \sqrt{1 - (\rho^j)^2}$. I bound the aggregate component by $|x_t^j| \leq 3\, s_x^j$ and the idiosyncratic component by $|\zeta_{it}^j| \leq 3\, s_\zeta^j$, and keep every estimate within the model's productivity grid $[\underline{z}^j, \bar{z}^j]$.

Let $\mathcal{I}_t$ be the set of firms observed in year t. Starting from $x_t^{j,(0)} = 0$ for every year, I iterate two steps until convergence. At iteration m:

Step 1 (Estimation). Taking $x_t^{j,(m-1)}$ as given, for each firm-year I solve Equation (20),

restricting each productivity term to the intersection of its idiosyncratic and grid bounds,

$$z_{it}^{j} \in \left[x_t^{j,(m-1)} - 3\, s_\zeta^j, \;\; x_t^{j,(m-1)} + 3\, s_\zeta^j \right] \cap \left[\underline{z}^j, \; \bar{z}^j \right], \qquad j \in \{K, N\}, \tag{B.10}$$

so a firm may deviate from the year's aggregate only within the support of its firm-specific shock. I obtain a starting value by grid search over the admissible rectangle and refine it with a local (Nelder–Mead) search.

Step 2 (Aggregation). I update each year's aggregate to the cross-sectional average of the firm-level estimates, bounded by $3\, s_x^j$:

$$x_t^{j,(m)} = \frac{1}{|\mathcal{I}_t|} \sum_{i \in \mathcal{I}_t} \hat{z}_{it}^{j}, \quad \text{bounded to } |x_t^{j,(m)}| \le 3\, s_x^j, \qquad j \in \{K, N\}. \tag{B.11}$$

I stop when $\max_t \left(|x_t^{K,(m)} - x_t^{K,(m-1)}| + |x_t^{N,(m)} - x_t^{N,(m-1)}| \right) < 5 \times 10^{-3}$, or when successive changes stagnate. At the fixed point I set $\hat{x}_t^j = x_t^{j,\star}$ and recover each firm's idiosyncratic productivity as $\hat{\zeta}_{it}^j = \hat{z}_{it}^j - \hat{x}_t^j$. This split makes both components consistent with their model-implied distributions.

C Additional Tables and Figures

Table C1: Variable definitions

Variable	Definition
Intangible capital	Accumulated R&D and capitalized SG&A using the perpetual-inventory method, plus balance-sheet goodwill, from Peters and Taylor (2017)
Physical capital	PPEGT
Market equity	Price (CRSP item PRC) × shares outstanding (SHROUT)
Total firm value	Market equity + DLC + DLTT − CHE
Profitability	(REVT − COGS) / (PPEGT + intangible capital)
Physical investment rate	CAPX / PPEGT
Intangible investment rate	(XRD + capitalized share of XSGA) / intangible capital
Intangible share	Intangible capital / (PPEGT + intangible capital)

Table C2: Relation between intangible share and markups

This table reports OLS regressions of markups on the intangible capital share and firm-level controls. The dependent variable in each column is a different markup measure: column (1) uses SALE/COGS; column (2) uses SALE/(COGS + XSGA*); columns (3) and (4) use Cobb-Douglas production function estimates without and with intangible capital as a production input; columns (5) and (6) use translog production function estimates without and with intangible capital. Intangible share is intangible capital divided by the sum of physical and intangible capital. Leverage is total debt divided by total assets. ROA is return on assets. The sample is the model decomposition sample used throughout the paper. All specifications include two-digit industry and year fixed effects. Standard errors in parentheses are two-way clustered by firm and year.

	(1) $\frac{\text{SALE}}{\text{COGS}}$	(2) $\frac{\text{SALE}}{\text{COGS+XSGA}^*}$	(3) CD	(4) CD (intan.)	(5) TL	(6) TL (intan.)
Intan. share	1.1880***	0.2435***	1.0881***	0.9750***	1.0806***	0.9903***
	(0.0730)	(0.0255)	(0.0614)	(0.0543)	(0.0615)	(0.0576)
Log age	-0.1227***	-0.0329***	-0.1107***	-0.0941***	-0.1104***	-0.1085***
	(0.0134)	(0.0047)	(0.0108)	(0.0095)	(0.0109)	(0.0103)
ROA	1.0849***	0.7200***	0.7369***	0.6737***	0.7283***	0.6583***
	(0.1101)	(0.0765)	(0.0923)	(0.0847)	(0.0896)	(0.0839)
Leverage	-0.1295**	0.0083	-0.1364***	-0.1070***	-0.1346***	-0.1449***
	(0.0500)	(0.0266)	(0.0390)	(0.0351)	(0.0385)	(0.0360)
Log assets	0.0047	0.0208***	0.0045	0.0030	-0.0083	-0.0328***
	(0.0063)	(0.0027)	(0.0052)	(0.0046)	(0.0051)	(0.0048)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	122,670	115,089	109,846	109,844	109,846	109,844
Adj. R^2	0.2481	0.3311	0.2357	0.2347	0.2437	0.2156
Within R^2	0.1105	0.2041	0.1219	0.1251	0.1204	0.1226

Table C3: Market-power value shares and industry dynamics

This table reports panel regressions of the intangible and non-intangible market-power value shares on product-market competition measures. Each column is a separate univariate regression of a value share on one competition measure. Panel A is estimated at the industry level: the value shares are aggregated to NAICS-3 industry-year cells (value-weighted, the ratio of aggregate markup value to aggregate firm value in the industry) and regressed on industry-level competition measures with year fixed effects. The measures are the sales Herfindahl (HHI) and the four-firm concentration ratio (CR4), both computed at the NAICS-3 level, and the Census Business Dynamics Statistics establishment net entry rate (entry minus exit). Panel B is estimated at the firm level with NAICS-3 industry-by-year fixed effects. The measures are the Hoberg and Phillips (2016) TNIC Herfindahl (TNIC HHI), TNIC similarity, and product-market fluidity (the change in rivals' product vocabulary relative to the firm's own). All Panel B regressions control for firm size (log sales). Standard errors in parentheses are two-way clustered by industry and year in Panel A and by firm and year in Panel B.

Panel A: Industry-level market structure measures (1976–2024)

	Intangible market-power share			Non-intangible market-power share		
	HHI	CR4	Net entry	HHI	CR4	Net entry
Coefficient	0.0038	-0.0118	-0.0055***	-0.0306	-0.1024*	0.0069**
	(0.0277)	(0.0331)	(0.0016)	(0.0617)	(0.0552)	(0.0028)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4,013	4,013	3,339	4,013	4,013	3,339
R^2	0.0864	0.0869	0.1095	0.0293	0.0449	0.0296
Within R^2	0.0001	0.0006	0.0221	0.0015	0.0176	0.0117

Panel B: Firm-level product market measures (1988–2023)

	Intangible market-power share			Non-intangible market-power share		
	TNIC HHI	Similarity	Fluidity	TNIC HHI	Similarity	Fluidity
Coefficient	0.0207***	0.0004	-0.0014**	-0.0443***	-0.0011*	0.0011
	(0.0061)	(0.0003)	(0.0005)	(0.0101)	(0.0006)	(0.0009)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm size control	Yes	Yes	Yes	Yes	Yes	Yes
Observations	89,879	89,879	87,461	89,879	89,879	87,461
R^2	0.3122	0.3114	0.3127	0.2876	0.2868	0.2867
Within R^2	0.0179	0.0166	0.0178	0.0076	0.0065	0.0061

Table C4: Investment and q: Peters-Taylor q and the value components

This table asks whether Peters-Taylor q still explains investment after controlling for the model's four value components. Columns (1)–(2) explain physical investment I^K/K and columns (3)–(4) intangible investment I^N/N; both dependent variables are measured one year ahead, so the regression is of investment in $t+1$ on q in t. Columns (1) and (3) use Peters-Taylor q on its own, and columns (2) and (4) add the four value components alongside it. The components are scaled by the capital stock of the margin being explained: by K in column (2) and by N in column (4). The sample is restricted to the firm-years for which Peters-Taylor q is available, and is therefore slightly smaller than in the other investment tables. All specifications include firm and year fixed effects, with standard errors two-way clustered by firm and year in parentheses.

	Physical, I^K/K		Intangible, I^N/N	
	(1)	(2)	(3)	(4)
Peters-Taylor q	0.0342***	0.0257***	0.0301***	0.0266***
	(0.0032)	(0.0028)	(0.0024)	(0.0022)
V^{phy}		0.0121***		0.0011**
		(0.0020)		(0.0004)
$V^{\text{prod-int}}$		-0.0012		0.0006
		(0.0011)		(0.0026)
$V^{\text{mkp-int}}$		0.0171***		0.0341***
		(0.0026)		(0.0062)
$V^{\text{mkp-non}}$		-0.0033**		0.0011**
		(0.0013)		(0.0005)
Observations	107,557	107,557	107,532	107,532
R^2	0.4598	0.4742	0.6158	0.6278
Within R^2	0.1118	0.1355	0.1198	0.1473

Table C5: Investment and q: horse race, scaled by total capital

This table repeats Table 9 with every investment rate scaled by total capital $K+N$ rather than by the capital stock of its own margin, which makes the three margins comparable on a common base. Each column is a separate regression of the investment rate on a single q, so the coefficients in a panel are directly comparable across columns. Panel A uses total investment $I/(K+N)$, Panel B uses physical investment $I^K/(K+N)$, and Panel C uses intangible investment $I^N/(K+N)$; all three are led one year, so the regression is of investment in $t+1$ on q in t. Model total q is the model-implied value over $K+N$; the last two columns strip the non-intangible market-power component $V^{\text{mkp-non}}$, and then both market-power components $V^{\text{mkp-int}}$ and $V^{\text{mkp-non}}$, out of that value. All specifications include firm and year fixed effects, with standard errors two-way clustered by firm and year in parentheses.

	Standard q	Total q	Model total q	Model total q (ex. non-intangible markup)	Model total q (ex. all markup)
	(1)	(2)	(3)	(4)	(5)
Panel A: Total investment, $I/(K+N)$					
q	0.0046***	0.0305***	0.0612***	0.0632***	0.0689***
	(0.0004)	(0.0027)	(0.0031)	(0.0031)	(0.0032)
Observations	107,803	107,803	107,803	107,803	107,803
R^2	0.5589	0.6099	0.6072	0.6036	0.6030
Within R^2	0.0735	0.1807	0.1750	0.1675	0.1663
Panel B: Physical investment, $I^K/(K+N)$					
q	0.0018***	0.0147***	0.0330***	0.0337***	0.0378***
	(0.0002)	(0.0018)	(0.0021)	(0.0021)	(0.0022)
Observations	107,803	107,803	107,803	107,803	107,803
R^2	0.5019	0.5348	0.5443	0.5408	0.5433
Within R^2	0.0220	0.0867	0.1053	0.0985	0.1034
Panel C: Intangible investment, $I^N/(K+N)$					
q	0.0028***	0.0152***	0.0269***	0.0281***	0.0296***
	(0.0002)	(0.0008)	(0.0015)	(0.0015)	(0.0017)
Observations	107,803	107,803	107,803	107,803	107,803
R^2	0.7736	0.7915	0.7802	0.7796	0.7771
Within R^2	0.1082	0.1786	0.1344	0.1320	0.1221

Table C6: Investment and q: value components, scaled by total capital

This table repeats Table 10 with every investment rate and every value component scaled by total capital $K+N$ rather than by the capital stock of the margin being explained. Because all four components now share one denominator, the coefficients are comparable across the three margins as well as within them. Columns (1)–(2) explain total investment $I/(K+N)$, columns (3)–(5) physical investment $I^K/(K+N)$ and columns (6)–(8) intangible investment $I^N/(K+N)$; all rates are led one year. Columns (3) and (6) use only the components generated by the corresponding capital stock: the physical component for physical investment and the two intangible components for intangible investment. Columns (1), (4), and (7) use all four components, and columns (2), (5), and (8) add total q alongside them. All specifications include firm and year fixed effects, with standard errors two-way clustered by firm and year in parentheses.

	Total, $I/(K+N)$		Physical, $I^K/(K+N)$			Intangible, $I^N/(K+N)$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
V^{phy}	0.0734***	0.0428***	0.0508***	0.0505***	0.0406***		0.0211***	0.0009
	(0.0030)	(0.0033)	(0.0024)	(0.0023)	(0.0025)		(0.0015)	(0.0015)
$V^{\text{prod-int}}$	0.0426***	0.0013		0.0106***	-0.0028	0.0280***	0.0307***	0.0033
	(0.0060)	(0.0048)		(0.0027)	(0.0025)	(0.0039)	(0.0040)	(0.0032)
$V^{\text{mkp-int}}$	0.0433***	0.0532***		-0.0241***	-0.0209***	0.1199***	0.0703***	0.0768***
	(0.0136)	(0.0127)		(0.0080)	(0.0077)	(0.0089)	(0.0085)	(0.0081)
$V^{\text{mkp-non}}$	0.0510***	0.0330***		0.0360***	0.0302***		0.0134***	0.0015
	(0.0055)	(0.0053)		(0.0036)	(0.0036)		(0.0030)	(0.0028)
Total q		0.0187***			0.0061***			0.0124***
		(0.0016)			(0.0007)			(0.0008)
Observations	107,803	107,803	107,803	107,803	107,803	107,803	107,803	107,803
R^2	0.6100	0.6239	0.5514	0.5561	0.5593	0.7737	0.7830	0.7959
Within R^2	0.1809	0.2101	0.1193	0.1285	0.1348	0.1087	0.1451	0.1960

Table C7: Investment and q: Shapley decomposition of the within-R^2, scaled by total capital

This table repeats Table 11 with every investment rate and value component scaled by total capital $K+N$ rather than by the capital stock of its own margin, and adds the total-investment margin. Each column reports a regression of one one-year-ahead investment rate on the four $(K+N)$-scaled value components with firm and year fixed effects, together with each component's Shapley share of the model's within-R^2. Shares within a column sum to 100.

	Share of within-R^2 (%)		
Value component	$I/(K+N)$	$I^K/(K+N)$	$I^N/(K+N)$
V^{phy}	68.10	85.29	35.15
$V^{\text{prod-int}}$	11.91	3.07	29.36
$V^{\text{mkp-int}}$	14.62	5.23	32.18
$V^{\text{mkp-non}}$	5.37	6.42	3.30
Total within R^2	0.1809	0.1285	0.1451
Observations	107,803	107,803	107,803

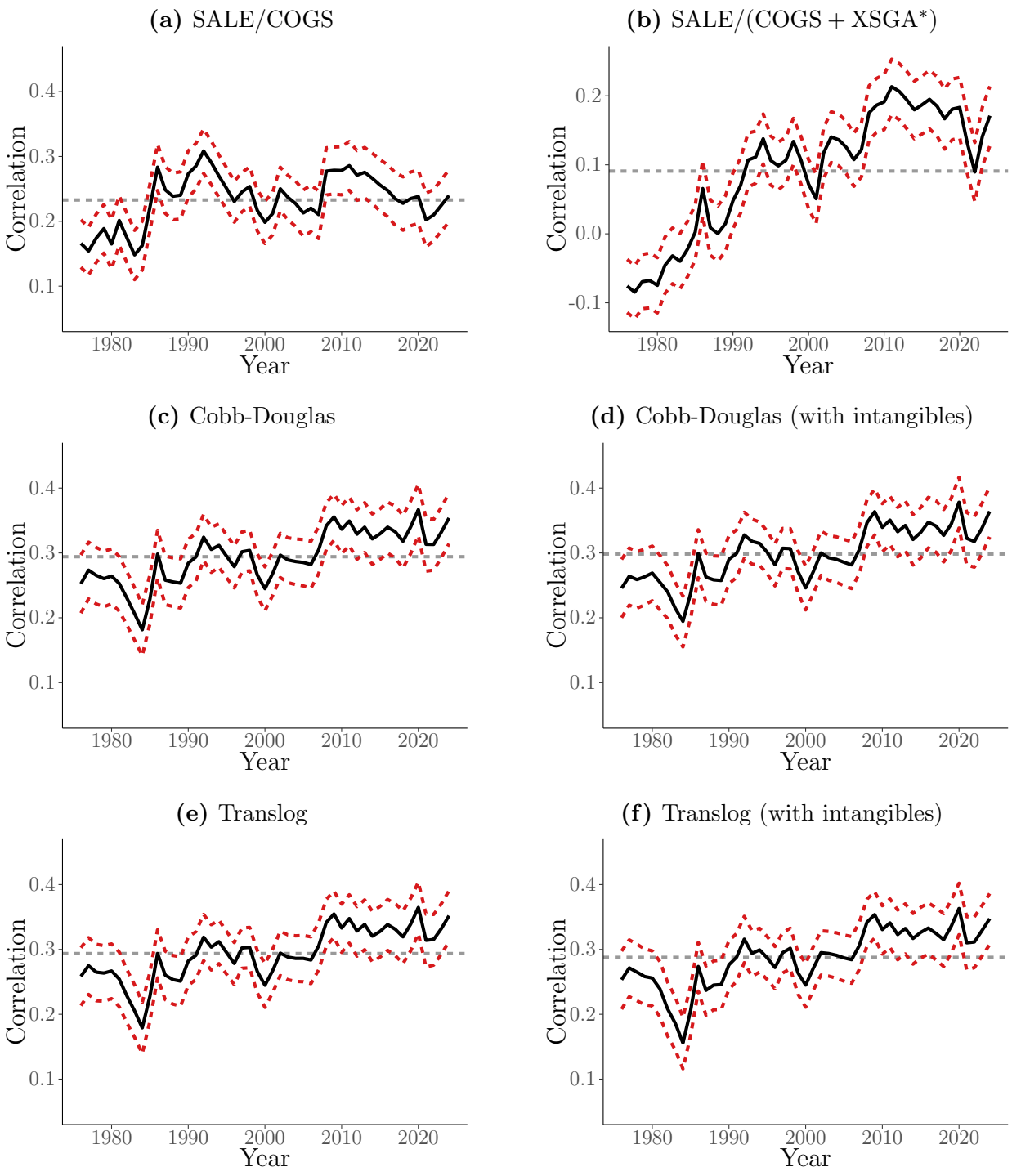


Figure C1: Correlation between intangibles and markup: alternative specifications (1976–2024)

This figure repeats Panel (a) of Figure 1 for all six markup measures. The specification is identical; see the note to Figure 1.

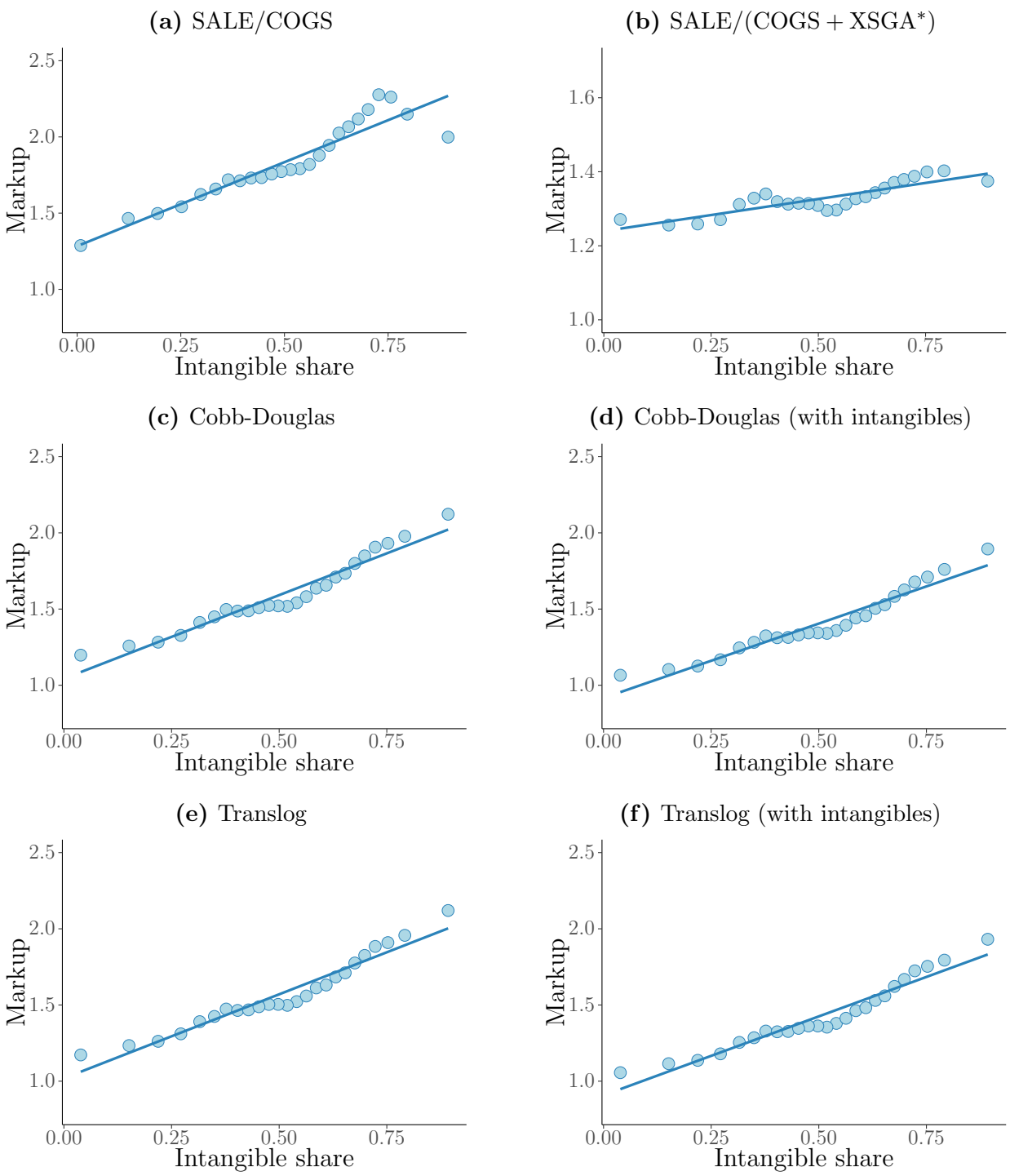


Figure C2: Binned scatter plots of markup against intangible share: alternative specifications (1976–2024)

This figure repeats Panel (b) of Figure 1 for all six markup measures. The specification is identical; see the note to Figure 1.

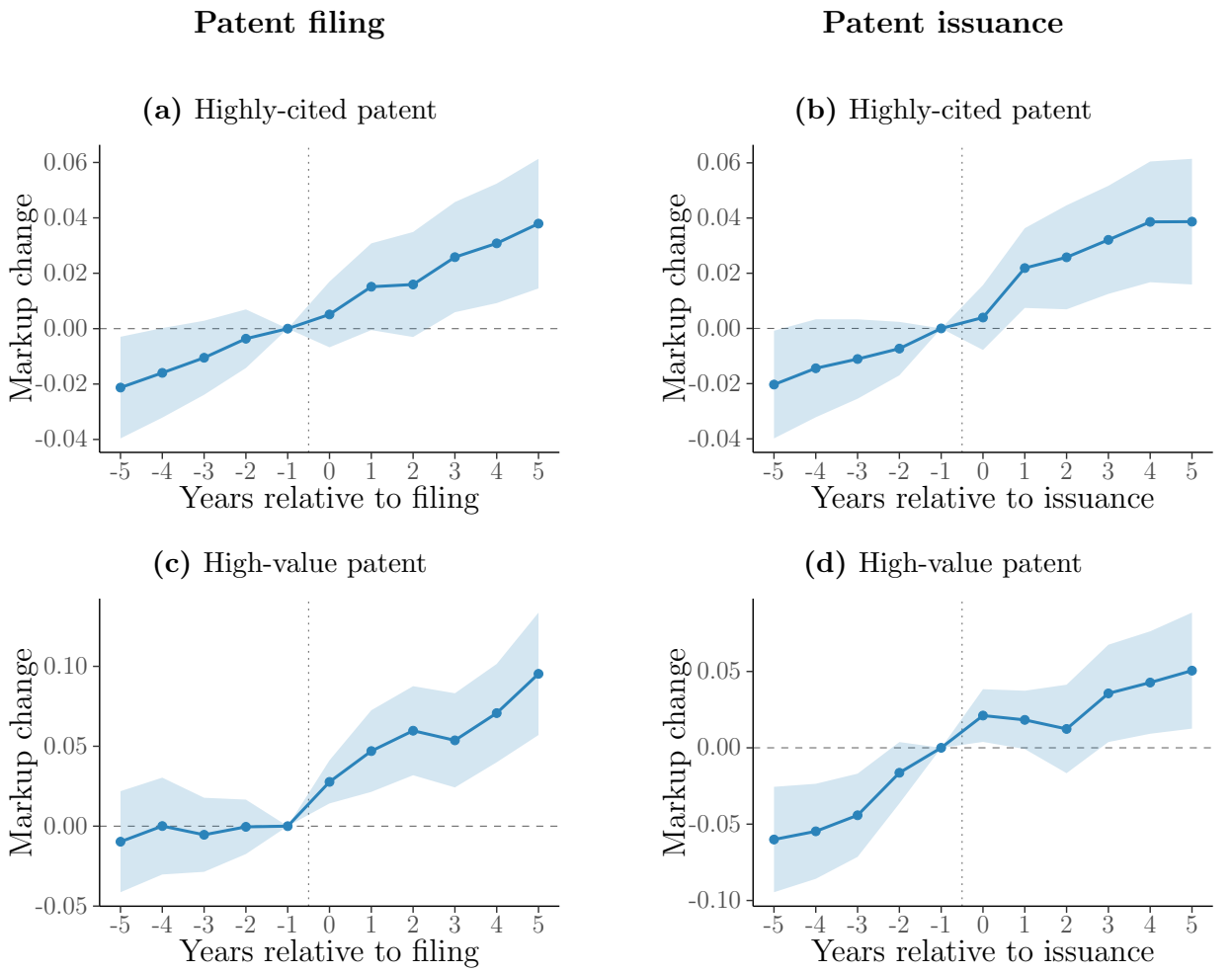


Figure C3: Event studies under Cobb-Douglas with intangibles: stacked difference-in-differences

This figure reproduces Figure 2 using a stacked difference-in-differences design in place of the first-event specification for important patent filings (left column) and issuances (right column), using the "Cobb-Douglas with intangibles" markup measure. Panels (a) and (b) define important patents by forward citations in the top 10% within their Fama-French 48 industry and year; Panels (c) and (d) instead use the Kogan et al. (2017) stock-market value of the patent. Each firm-year with an important patent forms a separate event stack: the treated firm is followed over $\tau \in [-5, +5]$, and controls are firms in the same Fama-French 48 industry with no important patent within that window. Stacks are pooled with event-specific firm and year fixed effects, coefficients are normalized relative to the year before the event ($\tau = -1$), and treated firms are required to have markup data in every pre-event year $\tau \in \{-5, \ldots, -1\}$. Shaded areas are 95% confidence intervals, with standard errors two-way clustered by event and firm.

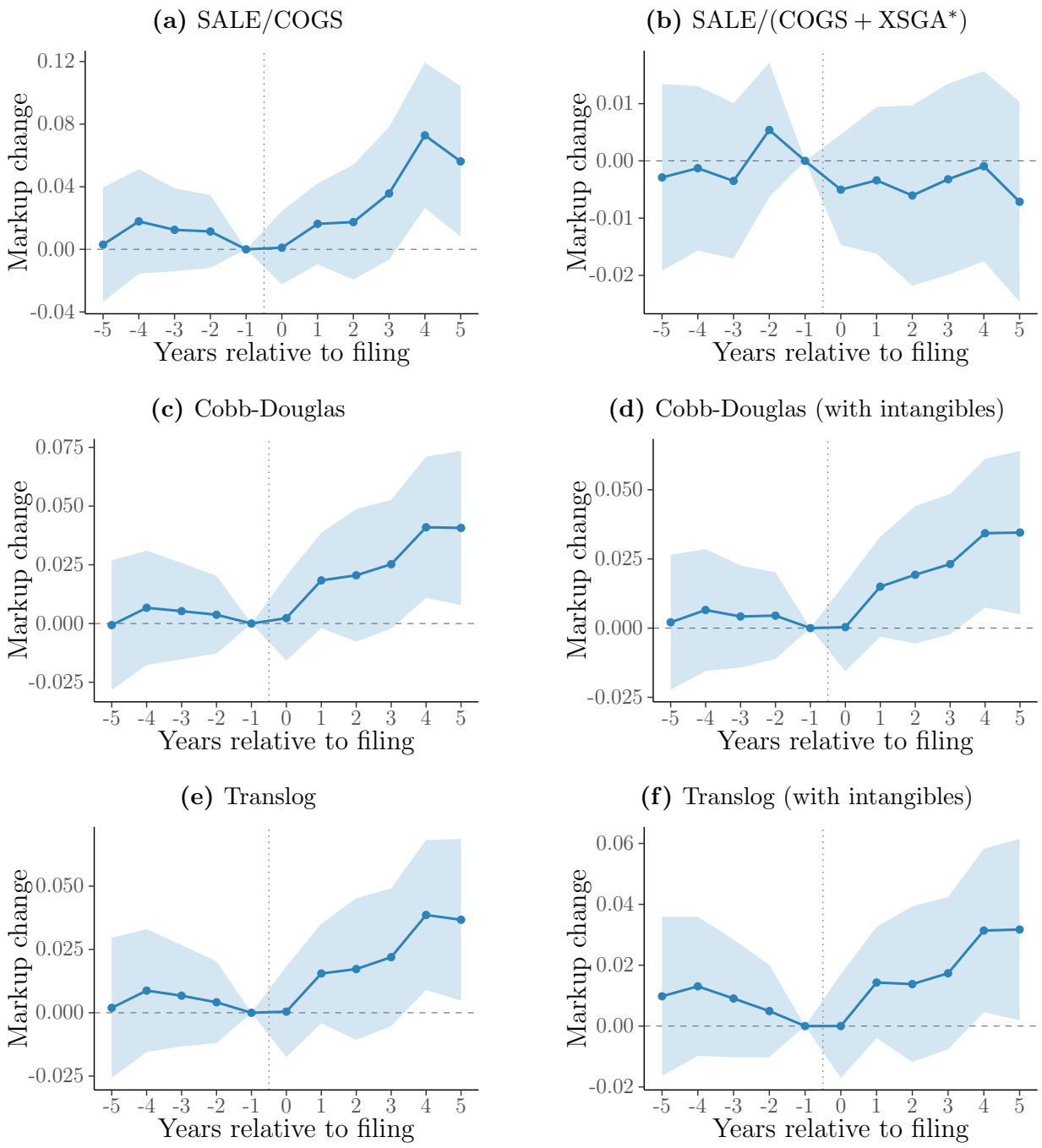


Figure C4: Event study around patent filing: highly-cited patents

This figure repeats the event study of Figure 2 (Panels (a) and (b)) around patent filing, for all six markup measures. The specification is identical; see the note to Figure 2.

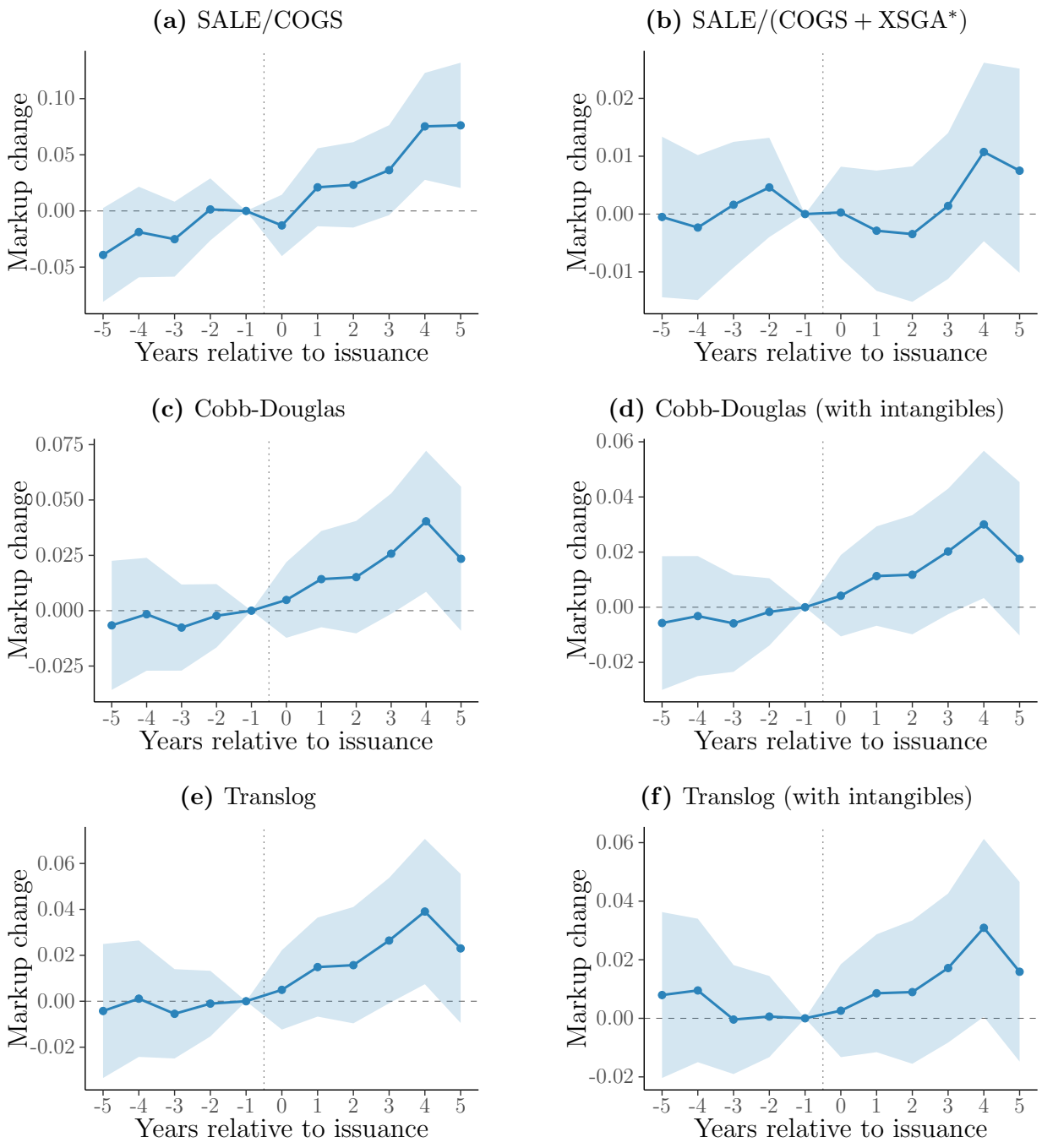


Figure C5: Event study around patent issuance: highly-cited patents

This figure repeats the event study of Figure 2 (Panels (a) and (b)) around patent issuance, for all six markup measures. The specification is identical; see the note to Figure 2.

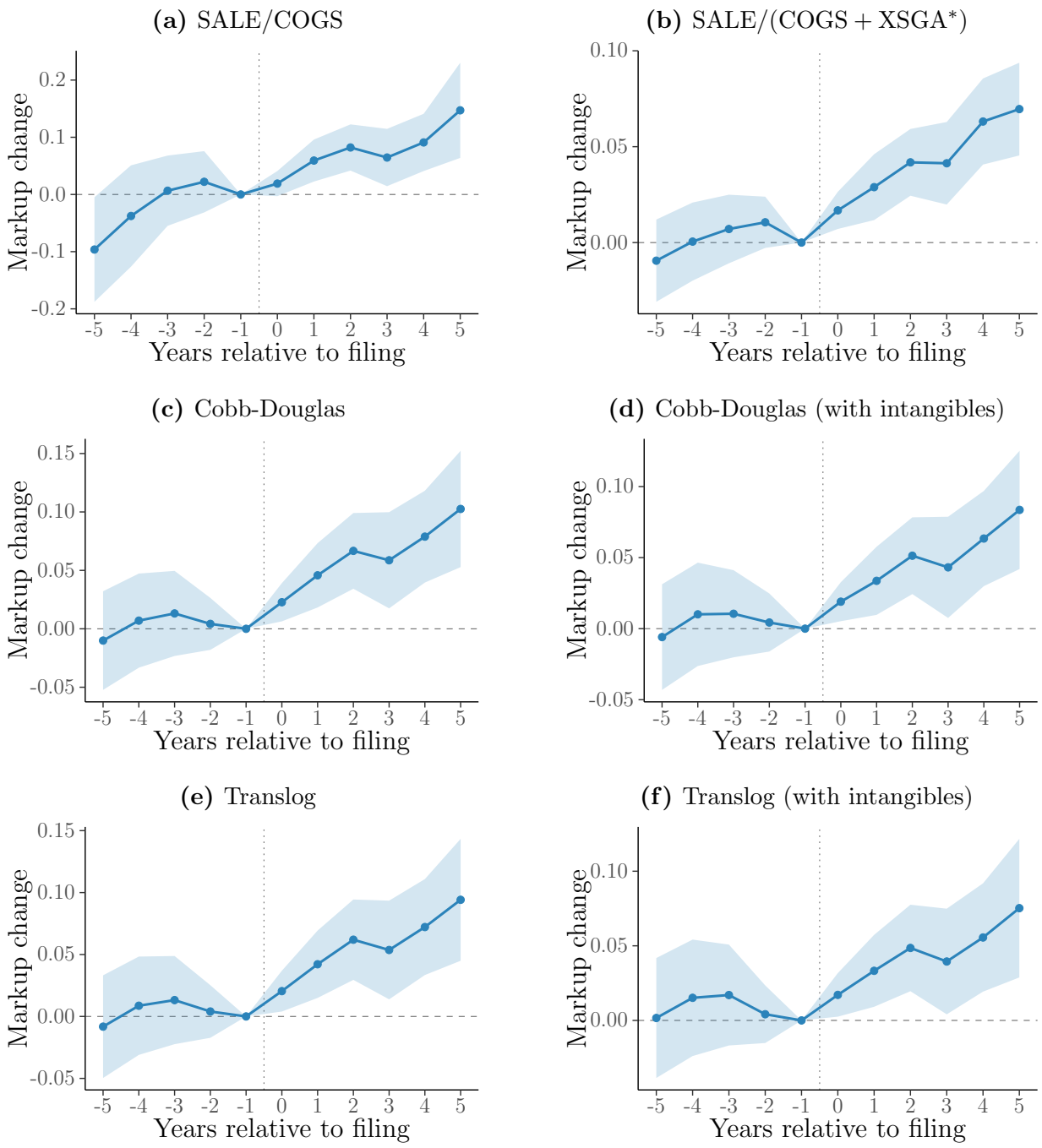


Figure C6: Event study around patent filing: high-value patents

This figure repeats the event study of Figure 2 (Panels (c) and (d)) around patent filing, for all six markup measures, where important patents are defined by the Kogan et al. (2017) stock-market value rather than forward citations. The specification is identical; see the note to Figure 2.

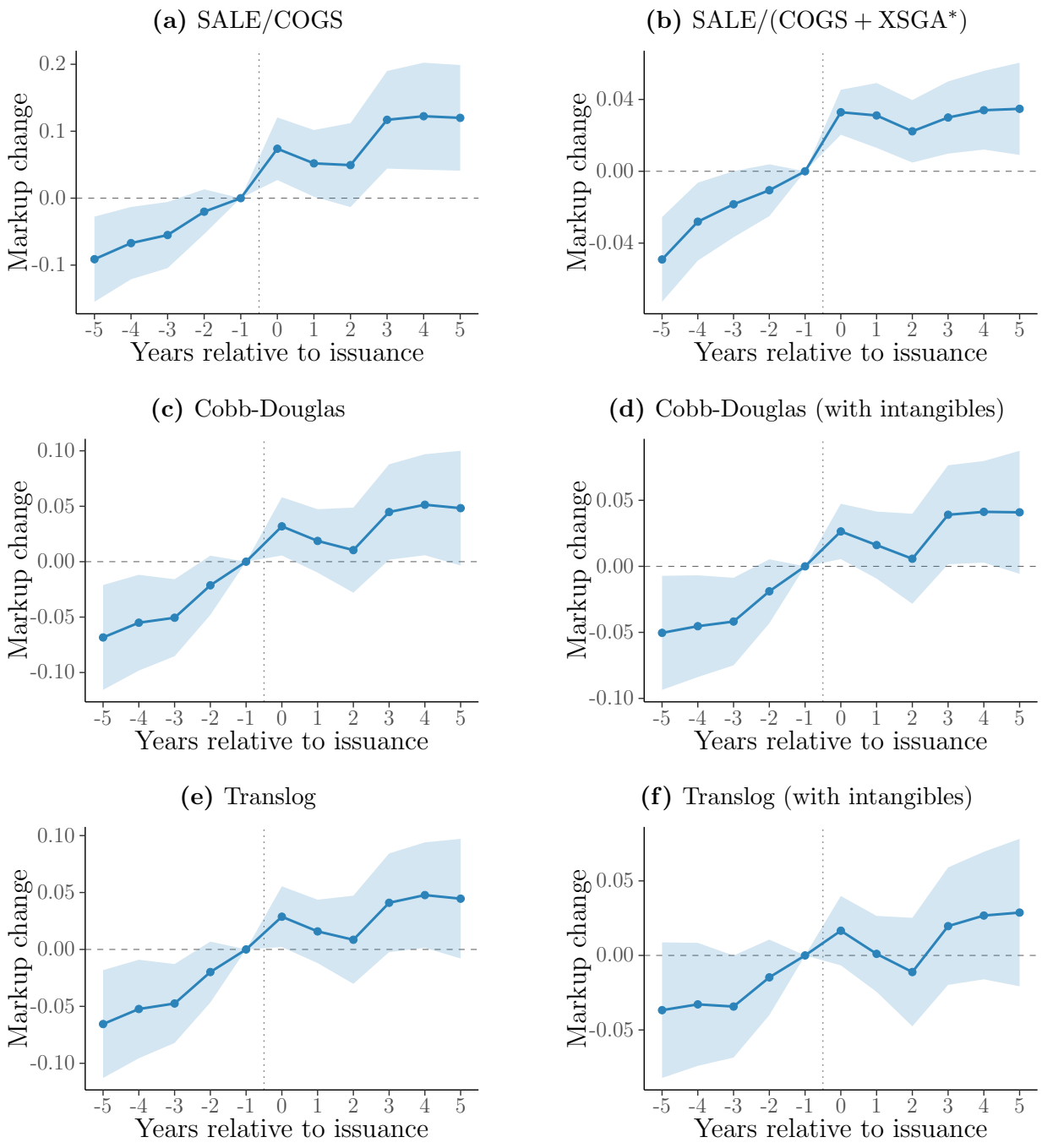


Figure C7: Event study around patent issuance: high-value patents

This figure repeats the event study of Figure 2 (Panels (c) and (d)) around patent issuance, for all six markup measures, where important patents are defined by the Kogan et al. (2017) stock-market value rather than forward citations. The specification is identical; see the note to Figure 2.