The Misuse of Knowledge*

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July 30, 2026

Abstract

Small information imperfections can fundamentally distort financial market outcomes. Building on psychological foundations, I develop a behavioral model in which investors universally believe in the Efficient Market Hypothesis, termed the "Believer Expectations Equilibrium" (BEE). The model provides a unified explanation for three central asset pricing anomalies: inelastic asset demand, excess price volatility, and the idiosyncratic volatility (IVOL) anomaly (i.e., stocks with higher idiosyncratic volatility earn lower subsequent returns). It further generates novel predictions, including a U-shaped relationship between price volatility and market risk aversion, and the emergence of noise-driven instability when market risk aversion is low, which amplifies noise trader risk to arbitrarily large levels and provides an explanation for tremendous market fluctuations in the absence of fundamental shocks. These outcomes under BEE arise from a distorted relative scale between two forces: the "market learning echo" and the "learning-absent market confidence." More generally, the relative strength of these two components critically determines the stability of any market in which agents learn from prices. Finally, the model delivers policy implications regarding optimal market transparency in the presence of mislearning.

Keywords: Behavioral, EMH, Imperfect information, Inelastic demands, Price volatility, Learning, Noise trader risk, Market stability

*I am extremely grateful to Francesca Bastianello, Kent Daniel and Paul Fontanier for their invaluable guidance. I am also grateful to Xuelin Li, Yueran Ma, Harry Mamaysky, Simon Oh, Liyan Yang, Anthony Lee Zhang and participants at Chicago Booth RP Workshop for their helpful discussions and suggestions. All errors are my own.

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"It simply wasn't true that a world with almost perfect information was very similar to one in which there was perfect information."

— Joseph Stiglitz

1 Introduction

A central function of prices is to aggregate and convey dispersed information. Hayek (1945) argues that even when no individual sees the full picture, the economy can function effectively so long as prices accurately transmit the information held by each person. Building on this insight, a large literature (Diamond and Verrecchia, 1981; Grossman, 1976; Grossman and Stiglitz, 1980; Hellwig, 1980) formalizes the informational role of prices through the Rational Expectations Equilibrium (REE), in which dispersed information is incorporated into prices through trading, and agents can correctly infer the market's aggregated information from prices provided they know the underlying structure of the market (e.g., others' risk aversion or the fraction of informed traders). This immediately raises a natural question: what happens when such structural information is imperfect, as is likely to be the case in reality?

Under the Grossman-Stiglitz framework (Grossman and Stiglitz, 1980), I show that even slight imperfections in the market's structural information can endogenously generate mislearning from prices, arising from the misuse of well-established knowledge. Specifically, when investors do not know the fraction of informed traders associated with a given piece of information, psychological foundations suggest that they may instead blindly believe in the Efficient Market Hypothesis (EMH) (Fama, 1970). As a result, they construct an incorrect picture of the market through a simple thought experiment and therefore mislearn from prices.

To clarify this idea, I first explain why investors generally cannot know the exact fraction of informed traders associated with a given piece of information when fields of expertise are exogenous, dynamic, and heterogeneous across investors. In the existing literature, "informed traders" are typically defined as those who receive signals about asset fundamentals, and their fraction is typically modeled as common knowledge. Even if the market does not immediately observe their fraction, this structural feature could, in principle, be inferred through repeated rounds of trading if the market structure were sufficiently stable.

This theoretical assumption, however, is unlikely to hold in reality. First, receiving a piece of information does not necessarily imply understanding it. If the information

lies outside an investor's field of expertise, she may observe it without being able to interpret its implications. For example, an investor with graduate training in economics may understand macroeconomic theory well enough to interpret FOMC announcements and their implications for asset values. By contrast, without comparable human capital in political science, the same investor may be much less able to assess how the outbreak of a war in the Middle East affects global stock markets. Receiving information therefore does not imply genuinely understanding it.

To capture this distinction, I refer to investors who understand a given piece of information as "investigators," corresponding to informed traders in the existing literature, because they use their expertise to investigate and interpret the signal. By contrast, I refer to investors who do not understand that information as "imputers," corresponding to uninformed traders in the existing literature, because they can only infer the information from prices.

Importantly, these roles are information-specific. The same investor may be an investigator for one type of information and an imputer for another, depending on her field of expertise, which is shaped by early-life human capital investment. Since information from many different domains continuously arrives in financial markets, and since the market's human capital composition is itself exogenous, dynamic, and hidden, the fraction of investigators for any particular piece of information is unlikely to be transparent to investors and may remain difficult to infer even through repeated trading.

With this imperfection in the market's structural information, imputers who seek to learn from prices must infer the fraction of investigators and fill this informational gap whenever new fundamental information arrives. This impulse to close the gap reflects the psychological concept of "need for cognitive closure" (Kruglanski and Webster, 1994; Kruglanski and Webster, 1996), which describes individuals' desire for definite knowledge on a given issue and their aversion to ambiguity and confusion. According to this theory, when individuals face uncertainty that cannot be easily resolved through direct observation, they tend to rely on the most accessible knowledge and theory that can provide cognitive closure. In financial markets, the EMH is a natural candidate for this role given its prominence as a foundational idea in modern finance. In this paper, I focus on the semi-strong form of the EMH, which states:

"Current prices 'fully reflect' all obviously publicly available information."

Relying on this statement of the EMH, investors can easily infer through a simple thought experiment that all the investors, whether or not they include themselves, must be investigators. The economic intuition is straightforward. If not all investors are

investigators, then imputers, as a group with non-zero mass, would gradually resolve their uncertainty about the news through two channels over time. First, they would repeatedly learn the news from prices. Second, more imputers would become investigators as they gradually come to understand the news. As a result, the news would remain correlated with asset returns after its arrival, which contradicts the statement of the EMH. Moreover, because noise traders are present, this correlation would not be fully eliminated by arbitrageurs (De Long et al., 1990; Shleifer and Vishny, 1997). By contrast, perceiving all other investors as investigators yields a market picture that is consistent with the EMH. I refer to this mode of reasoning, namely believing in the EMH and inferring that all others are investigators, as "believer thinking." Importantly, once investors adopt believer thinking, it is difficult for them to depart from it. One reason is that they are reluctant to undo the cognitive closure they have achieved. In addition, "theory-induced blindness," which Kahneman (2011) describes as the tendency for people to become blind to evidence that contradicts a theory once they accept it, further reinforces believer thinking.[1]

The reader may question that it seems unrealistic for imputers to perceive all other investors as investigators who can understand the news, while only they themselves cannot. There are three responses to this concern. First, "need for cognitive closure" and "theory-induced blindness" make investors unlikely to think carefully about this possibility, especially when the cost of giving up cognitive closure is high or when they firmly believe in the EMH. Second, this pattern is consistent with the psychological notion of "pluralistic ignorance" (Prentice and Miller, 1993), under which each individual believes that they alone think differently from everyone else. Third, I show that even if investors actively and heuristically choose which way of thinking to adopt, without the influence of these exogenous psychological mechanisms, all investors still adhere to believer thinking as long as the model parameters satisfy a particular condition.

I refer to the market equilibrium in which investors universally believe in the EMH and adopt believer thinking as the "Believer Expectations Equilibrium," or BEE. The benchmark for comparison is the Rational Expectations Equilibrium, or REE, in which the fraction of investigators is common knowledge among investors. In our framework, all investors other than noise traders are Bayesian. Investigators trade on the information they directly investigate and understand, so believer thinking does not distort their beliefs about the asset. [2] By contrast, imputers trade on the information they infer from prices based on their perceived market picture.

[1]Note that "theory-induced blindness" alone can generate believer thinking. In that case, narratives about the EMH and finance education become the origin of believer thinking.

[2]Because noise traders are present, investigators expect to be rewarded for their investigation relative to imputers and therefore have an incentive to investigate.

Under BEE, the market overreacts to news and underreacts to stale information. Asset demand becomes more inelastic, and asset prices become more volatile in response to both information and noise trading flows. Cross-sectional heterogeneity in the degree of believer thinking further gives rise to the idiosyncratic volatility (IVOL) anomaly (Ang et al., 2006; Ang et al., 2009), reflected in a negative relation between idiosyncratic volatility and subsequent returns across assets. BEE therefore provides a unified explanation for three central asset pricing anomalies: inelastic asset demand, excess price volatility, and the IVOL anomaly. In the extension with multiple signals, which better reflects the richness of real-world information environments demonstrated by Hayek (1945), asset price volatility displays a non-monotonic U-shaped relation with market risk aversion. Moreover, price volatility can become arbitrarily large when market risk aversion is sufficiently low. In this region, a "noise-driven instability" emerges, as traders respond aggressively to noise rather than to fundamental information.

To investigate the robustness of BEE, I show that it can arise endogenously and sustain itself when investors heuristically choose their way of thinking. In particular, no investor has an incentive to deviate from believer thinking to partially rational expectations [3] as long as the model parameters satisfy a specific condition. This result implies that believer thinking can survive through a fully endogenous heuristic channel, even in the absence of the psychological foundations discussed above.

I then analyze the mechanism behind the outcomes of BEE. Specifically, believer thinking affects belief formation through two channels, which I call the "confidence effect" and the "mapping effect." The confidence effect arises because imputers overestimate the informational efficiency of prices, which makes them overconfident in the information they infer from prices. The mapping effect arises because imputers infer the information magnitude using an incorrectly perceived market structure. Together, these two effects amplify the market's "learning echo," which I define as the risk-aversion-scaled demand response to price generated by the component of asset demand that depends on imputed signals. I also define the "learning-absent market confidence" as the average confidence of the market when traders do not learn from prices. I show that, in a Grossman-Stiglitz economy, the relative magnitude of the market learning echo and the learning-absent market confidence determines both the elasticity of asset demand and market stability. When the market learning echo is large relative to the learning-absent market confidence, asset demand becomes more inelastic and can even become positively sloped

[3] Partially rational expectations refers to a way of thinking in which investors correctly identify the fractions of investigators and imputers in the market but fail to recognize the mislearning of other investors.

in price. Under BEE, this amplification of the market learning echo generates inelastic asset demand, which in turn produces overreaction to information and excess volatility.

When mislearning is present, market transparency can harm the informational efficiency of asset prices. The reason is that mislearning amplifies noise, which weakens the price's informational role. Using BEE as an example, I show that for a policymaker who faces no cost of intervention, the optimal level of market transparency can take only one of two values: no transparency, under which no one is informed of the news' existence, or complete transparency, under which everyone is informed of the news' existence. No transparency is optimal when the news is difficult to understand, as reflected by the fraction of investigators falling below a threshold. Complete transparency is optimal when the news is easy to understand, as reflected by the fraction of investigators exceeding the same threshold.

By proposing the BEE model, this paper shows how slight imperfections in market structure can endogenously generate mislearning and examines the consequences that follow. BEE provides a unified explanation for multiple asset pricing anomalies. A further contribution of the paper is its prediction of a U-shaped relation between price volatility and market risk aversion, as well as the possibility of a "noise-driven instability." This prediction offers a new explanation for extreme market fluctuations, especially when no obvious fundamental driver is present. More broadly, highlighting the importance of the relative magnitude of the market's "learning echo" and its "learning-absent market confidence" deepens our understanding of market stability and yields insights for policy design.

1.1 Literature Review

This paper mainly contributes to three strands of literature. The first is the behavioral asset pricing literature that explains asset pricing anomalies through investor behavior. This literature can be broadly divided into three categories: biased beliefs, non-standard preferences, and cognitive constraints. Within the biased-belief category, extrapolation theories propose that investors irrationally project past returns into the future (Barberis, 2018; Barberis et al., 2015; Glaeser and Nathanson, 2017; Hong and Stein, 1999; Pan et al., 2021). Another prominent biased-belief mechanism is overconfidence, which suggests that investors are excessively confident in their own skills and information, thereby generating belief disagreement and asset pricing anomalies (Bastianello and Fontanier, 2025; Daniel et al., 1998; Eyster et al., 2019; Scheinkman and Xiong, 2003). The literature on non-standard preferences mainly applies prospect theory (Kahneman and Tversky, 1979)

to financial markets (Barberis et al., 2001; Pagel, 2016). The literature on cognitive constraints instead draws on neurobiological insights to model investor behavior (Benartzi and Thaler, 2001; Bordalo et al., 2013; Bordalo et al., 2019). This paper contributes to the biased-belief strand. Relative to the existing literature, it studies a distinct heuristic, namely believing in the EMH grounded in psychological foundations, and shows that this mechanism can provide a unified explanation for multiple central asset pricing anomalies.

The second related literature studies noise trader risk and limits of arbitrage. De Long et al. (1990) develop this idea and show that noise trader risk can keep prices from returning to fundamental value by discouraging informed traders from fully trading on their information. Shleifer and Vishny (1997) further show that when institutional investors care about short-term performance in order to attract capital, noise trader risk can limit their willingness to trade against mispricing. In BEE, the price response to noise trading is amplified and can become extremely large when market risk aversion is sufficiently low. Believer thinking therefore amplifies noise trader risk, further discourages arbitrage, and prevents prices from returning to fundamental value even when the scale of noise trading itself is modest. Another closely related paper is Mendel and Shleifer (2012), who study the "chasing noise" behavior of uninformed traders within a rational framework and show how learning from prices amplifies the effects of noise trading on the market. Relative to their framework, BEE introduces mislearning from prices and allows for interaction among Bayesian traders with different information sets. Moreover, under BEE, noise trader risk can be amplified without bound in absolute terms, rather than in the relative sense discussed by Mendel and Shleifer (2012). Relatedly, Mondria et al. (2022) study costly interpretation of prices, where investors inject sentiment and processing noise when reading the price signal, generating endogenous noise trading, price momentum, and excess volatility. Their bias adds noise to a correctly understood price mapping and yields underreaction. By contrast, believer thinking is a systematic misperception of the mapping itself. More broadly, this paper connects the two pillars of behavioral finance: the psychology of investor beliefs and the limits of arbitrage (Barberis and Thaler, 2003). Believer thinking provides a psychological foundation for systematic mispricing and a mechanism through which noise trader risk can be amplified to a large extent, thereby strongly limiting the scope for arbitrage to correct such mispricing.

This paper also contributes to the fast-growing literature on heuristic behavior. This literature originates with Tversky and Kahneman (1974), who identify representativeness, availability, and anchoring as systematic cognitive shortcuts that depart from normative Bayesian reasoning. Subsequent work (Kahneman, 2003; Kahneman, 2011) further develops the dual-process theory of cognition, distinguishing between System 1, which is

fast, automatic, and intuitive, and System 2, which is slow, deliberate, and analytical. In economics, heuristic reasoning is closely related to the foundational concept of bounded rationality proposed by Simon (1955), who argues that agents satisfice rather than optimize because of cognitive limitations. More recent work formalizes heuristic behavior within economic models. Gabaix (2014) develops a model of sparsity-based bounded rationality in which agents simplify their representation of the world by ignoring less salient variables. Gennaioli and Shleifer (2010) model representativeness as a distortion in probability assessment driven by salience, while Bordalo et al. (2012) formalize salience-based choice. A growing recent literature also studies the heuristic role of memory in belief formation and economic decision-making (Bordalo et al., 2020; Bordalo et al., 2023; Bordalo et al., 2025; Jiang et al., 2025). This paper formalizes a novel heuristic in financial markets, which I call believer thinking, that arises from belief in established theory and knowledge. Within the dual-process framework of Kahneman (2011), this heuristic operates as a System 1 premise, that is, an intuitive and unexamined default assumption about market efficiency, on which agents then perform a System 2 thought experiment to infer the fraction of investigators and carry out Bayesian updating. Because this deliberate reasoning is internally rigorous conditional on the premise, the heuristic becomes self-masking. The sophistication of the System 2 computation discourages agents from questioning the underlying System 1 foundation, which is consistent with Kahneman's observation that System 2 often endorses rather than scrutinizes the frames supplied by System 1.

The closest work to this paper is Bastianello and Fontanier (2025), who also study mislearning from prices in a Grossman-Stiglitz economy and likewise obtain over-reaction, inelastic and even upward-sloping demand, and feedback-driven amplification between prices and beliefs. Two differences distinguish the present paper. First, the bias differs. Their "partial equilibrium thinking" trader correctly knows the fraction of informed traders but neglects that other uninformed traders also learn from prices. Imputer in this paper instead believes in the EMH and therefore infers that all others are investigators. Whereas they treat a misperceived fraction of informed traders as a parameterized example, believer thinking derives this misperception endogenously, grounding it in psychological foundations and the thought experiment of Section 3. Second, and more importantly, the environment differs. Their baseline features fully revealing prices, whereas I place noise traders and partial revelation at the center. This is what allows believer thinking to generate results with no analogue in their framework: the IVOL anomaly and the noise-driven instability.

The rest of the paper proceeds as follows. Section 2 presents the model and derives the

BEE and its benchmark, the REE. Section 3 presents the thought experiment through which investors with believer thinking infer the fraction of investigators. Section 4 analyzes the market outcomes under BEE. Section 5 studies the extension with multiple signals. Section 6 examines the mechanism. Section 7 discusses the impacts of market transparency and the corresponding policy implications. Section 8 concludes.

2 The Model

2.1 Model Layout

In this section, I present the model and state the key assumptions. I begin with the basic environment and then define the BEE and the REE, which serves as the benchmark.

Consider a two-period framework with periods 1 and 2. The market contains one risk-free asset with perfectly elastic supply and one risky asset with constant supply $X > 0$. Both assets are traded in period 1 and pay terminal payoffs in period 2. The price and terminal payoff of the risk-free asset are normalized to 1. The terminal payoff of the risky asset is v, which is not perfectly observed by the market in period 1.

The market consists of a continuum of Bayesian traders of mass 1 and noise traders. Bayesian traders share a common prior belief about v, given by

$$v \sim \mathcal{N}(\mu, \tau^{-1}), \tag{1}$$

where τ denotes the precision of the prior belief. Bayesian traders have exponential utility, or equivalently mean-variance CARA utility over total period 2 payoffs, with homogeneous absolute risk aversion γ. In period 1, they trade conditional on their available information to maximize expected utility. Therefore, the representative Bayesian trader i solves the following optimization problem:

$$\max_{q_i} \left(\mathbb{E}[v \mid \mathcal{I}_i] - P\right) q_i - \frac{1}{2}\gamma \mathbb{V}[v \mid \mathcal{I}_i] q_i^2, \tag{2}$$

where q_i denotes the quantity submitted by trader i, and $\mathcal{I}_i$ denotes her period 1 information set about v. In period 1, the market observes a public signal s about v such that:

$$s \sim \mathcal{N}(v, \tau_\varepsilon^{-1}), \tag{3}$$

where τ_ε is the objective precision of s. This signal partitions Bayesian traders into two groups. A fraction ϕ of Bayesian traders can investigate and correctly interpret s. I

refer to these traders as "investigators." The remaining fraction $1-\phi$ cannot directly understand s and can only infer it from the equilibrium price. I refer to these traders as "imputers." In period 1, noise traders submit an exogenous trading quantity u given by

$$u \sim \mathcal{N}(0, \sigma_u^2), \tag{4}$$

where σ_u^2 denotes the variance of the noise trading quantity.

The market-clearing condition (MCC) for the risky asset equates aggregate demand with supply and is given by

$$\phi q_{IN} + (1-\phi) q_{IM} + u = X, \tag{5}$$

where q_{IN} denotes the trading quantity of a representative investigator, obtained by solving optimization problem (2) conditional on the investigators' information set $\mathcal{I}_{IN}$, and q_{IM} denotes the optimal trading quantity of a representative imputer, obtained by solving the same problem conditional on the imputers' information set $\mathcal{I}_{IM}$.

I now define the Believer Expectations Equilibrium (BEE) and the benchmark model within the framework above. Under BEE, all Bayesian traders in the market believe that all other Bayesian traders can correctly understand the public signal s and map it into the terminal payoff of the risky asset, regardless of whether they themselves can do so. For investigators, this believer thinking has no effect on their beliefs, since they can directly understand s and use it to update their beliefs about v. Their information set in optimization problem (2) is therefore always given by: [4]

$$\mathcal{I}_{IN} = \{(\mu, \tau), (s, \tau_\varepsilon)\}. \tag{6}$$

For imputers, by contrast, believer thinking generates mislearning. Since they perceive all other Bayesian traders as understanding s, their perceived market-clearing condition (MCC) is given by

$$q_{IN} + u = X, \tag{7}$$

which means that imputers perceive the entire market as consisting of investigators, while treating themselves as atomistic traders with no price impact. Under believer thinking, they therefore infer the imputed signal $\tilde{s}^{BEE}$ from equation (7), rather than from the

[4]Note that I represent the information set solely by the prior and signal realizations together with their perceived precisions. This is because both μ and s are normally distributed, so their realizations and precisions summarize all payoff-relevant information and are sufficient for Bayesian traders to form their posterior beliefs about the asset's terminal payoff.

objectively correct MCC in equation (5). As a result, imputers mislearn under BEE, which gives rise to the following biased information set:

$$\mathcal{I}_{IM}^{BEE} = \left\{ (\mu, \tau), \left(\tilde{s}^{BEE}, \tau_{\tilde{s}^{BEE}} \right) \right\}, \tag{8}$$

where $\tau_{\tilde{s}^{BEE}}$ denotes the precision that imputers assign to $\tilde{s}^{BEE}$ that they learn from prices. To assess the effects of believer thinking, I use the Rational Expectations Equilibrium, or REE, as the benchmark. Under REE, all Bayesian traders correctly perceive the market structure and the market-clearing condition. The REE has the same information set for investigators as in equation (6). However, because imputers infer s from the true MCC in equation (5), rather than from equation (7) as under BEE, their information set under REE is given by

$$\mathcal{I}_{IM}^{REE} = \left\{ (\mu, \tau), \left(\tilde{s}^{REE}, \tau_{\tilde{s}^{REE}} \right) \right\}, \tag{9}$$

where $\tilde{s}^{REE}$ and $\tau_{\tilde{s}^{REE}}$ denote respectively the imputed signal under REE and its perceived precision. Based on the framework above, I define market equilibrium as follows.

Definition 1. *A market equilibrium is a collection of beliefs, equivalently information sets, and trading quantities for investigators and imputers, together with the equilibrium asset price,* $\{\mathcal{I}_{IN}, \mathcal{I}_{IM}, q_{IN}, q_{IM}, P\}$*, such that*

(a) Bayesian traders choose their trading quantities by solving problem (2)*.*

(b) The risky asset market clears according to equation (5)*.*

(c) Investigators' beliefs are consistent with the signal s *they directly observe, and imputers' beliefs are consistent with their perceived market structure and the observed price.*

Based on this definition of market equilibrium, I solve for the BEE and REE in the next section.

2.2 Model Solving

2.2.1 Believer Expectations Equilibrium

I solve for the BEE in three steps, each corresponding directly to one of the three components in Definition 1.

Step 1. Derive the optimal trading quantities submitted by investigators and imputers by solving optimization problem (2) under their respective, as yet undetermined, beliefs.

Given the information sets defined above, Bayesian traders form beliefs as follows:

$$\begin{cases} \mathbb{E}[v \mid \mathcal{I}_{IN}^{BEE}] = \frac{\tau\mu + \tau_\varepsilon s}{\tau + \tau_\varepsilon}; \\ \mathbb{V}[v \mid \mathcal{I}_{IN}^{BEE}] = \frac{1}{\tau + \tau_\varepsilon}. \end{cases} \tag{10}$$

$$\begin{cases} \mathbb{E}[v \mid \mathcal{I}_{IM}^{BEE}] = \frac{\tau\mu + \tau_{\tilde{s}BEE}\tilde{s}^{BEE}}{\tau + \tau_{\tilde{s}BEE}}; \\ \mathbb{V}[v \mid \mathcal{I}_{IM}^{BEE}] = \frac{1}{\tau + \tau_{\tilde{s}BEE}}. \end{cases} \tag{11}$$

The first-order condition of problem (2) yields the optimal trading quantity as:

$$q_{type}^{BEE} = \frac{\mathbb{E}[v \mid \mathcal{I}_{type}^{BEE}] - P}{\gamma \mathbb{V}[v \mid \mathcal{I}_{type}^{BEE}]}, \tag{12}$$

where $type \in \{IN, IM\}$ indexes the trader type. Equation (12) can then be rewritten for investigators and imputers as:

$$q_{IN}^{BEE} = \tau \frac{\mu - P}{\gamma} + \tau_\varepsilon \frac{s - P}{\gamma}, \tag{13}$$

and

$$q_{IM}^{BEE} = \tau \frac{\mu - P}{\gamma} + \tau_{\tilde{s}BEE} \frac{\tilde{s}^{BEE} - P}{\gamma}. \tag{14}$$

Step 2. Impose market clearing in the risky asset market to derive the actual market-clearing condition (MCC).

To clear the risky asset market, total demand must equal total supply, as stated in equation (5). Combining equations (5), (13), and (14), the actual MCC is given by:

$$\phi \left(\tau \frac{\mu - P}{\gamma} + \tau_\varepsilon \frac{s - P}{\gamma} \right) + (1 - \phi) \left(\tau \frac{\mu - P}{\gamma} + \tau_{\tilde{s}BEE} \frac{\tilde{s}^{BEE} - P}{\gamma} \right) + u = X. \tag{15}$$

Step 3. Obtain $\tilde{s}^{BEE}$ and $\tau_{\tilde{s}BEE}$ as functions of the price P from the MCC perceived by imputers, and substitute them into equation (15) to solve for the equilibrium.

Combining equations (7) and (13), the MCC perceived by imputers is given by

$$\tau \frac{\mu - P}{\gamma} + \tau_\varepsilon \frac{s - P}{\gamma} + u = X. \tag{16}$$

Therefore, the mapping from price to the perceived signal inferred by imputers is given by:

$$\tilde{s}^{BEE} = \frac{1}{\tau_\varepsilon} \left[(\tau + \tau_\varepsilon) P - \tau\mu + \gamma X \right] \sim \mathcal{N}\left(v, \tau_{\tilde{s}BEE}^{-1} \right), \tag{17}$$

in which

$$\tau_{\tilde{s}^{BEE}}^{-1} = \tau_{\varepsilon}^{-1} + \frac{\gamma^2}{\tau_{\varepsilon}^2}\sigma_u^2. \tag{18}$$

Substituting equations (17) and (18) into the actual MCC in equation (15) yields the BEE. The equilibrium asset price under BEE is given by:

$$\begin{aligned} P^{BEE} = \frac{1}{(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\tau + (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\phi\tau_{\varepsilon}} \big[&(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\tau\mu \\ &+ (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\phi\tau_{\varepsilon}s - (\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\gamma X + (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\gamma u\big]. \end{aligned} \tag{19}$$

The trading quantity submitted by the representative investigator is given by:

$$\begin{aligned} q_{IN}^{BEE} = \frac{1}{(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\tau + (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\phi\tau_{\varepsilon}} \big[&-(1-\phi)\gamma\sigma_u^2\tau_{\varepsilon}\tau\mu \\ &+(1-\phi)\gamma\sigma_u^2\tau_{\varepsilon}\tau s + (\tau + \tau_{\varepsilon})(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)X - (\tau + \tau_{\varepsilon})(\tau_{\varepsilon} + \gamma^2\sigma_u^2)u\big]. \end{aligned} \tag{20}$$

The trading quantity submitted by the representative imputer is given by:

$$\begin{aligned} q_{IM}^{BEE} = \frac{1}{(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\tau + (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\phi\tau_{\varepsilon}} \big[&\phi\gamma\sigma_u^2\tau\tau_{\varepsilon}\mu \\ &- \phi\gamma\sigma_u^2\tau\tau_{\varepsilon}s + \left[\tau(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2) + \phi\tau_{\varepsilon}^2\right]X - \gamma^2\sigma_u^2\tau u\big]. \end{aligned} \tag{21}$$

The signal learned by the representative imputer is given by:

$$\begin{aligned} \tilde{s}^{BEE} = \frac{\tau + \tau_{\varepsilon}}{\tau_{\varepsilon}\left[(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\tau + (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\phi\tau_{\varepsilon}\right]} \big[&(\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\tau\mu + (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\phi\tau_{\varepsilon}s \\ &- (\phi\tau_{\varepsilon} + \gamma^2\sigma_u^2)\gamma X + (\tau_{\varepsilon} + \gamma^2\sigma_u^2)\gamma u\big] - \frac{\tau}{\tau_{\varepsilon}}\mu + \frac{\gamma X}{\tau_{\varepsilon}}. \end{aligned} \tag{22}$$

Expressions (19) through (22), together with the actual signal observed by investigators and the homogeneous prior beliefs, characterize the Believer Expectations Equilibrium (BEE).

2.2.2 Rational Expectations Equilibrium

To solve for the Rational Expectations Equilibrium (REE), I follow the same first two steps as above and modify only Step 3. In this step, I first conjecture the functional form of the equilibrium price and then determine its coefficients by matching them to the equilibrium price implied by the actual MCC in equation (5). I save the detailed derivation for the appendix. The REE is characterized as follows. The equilibrium price

of the risky asset is given by:

$$P^{REE} = \frac{1}{(\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2)\tau + (\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\phi\tau_\varepsilon} \left[(\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2)\tau\mu \right. \\ \left. + (\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\phi\tau_\varepsilon s - (\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2)\gamma X + (\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\gamma u\right]. \tag{23}$$

The trading quantity submitted by the representative investigator is given by:

$$q_{IN}^{REE} = \frac{1}{(\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2)\tau + (\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\phi\tau_\varepsilon} \left[-(1-\phi)\gamma\sigma_u^2\tau\tau_\varepsilon\mu \right. \\ \left. +(1-\phi)\gamma\sigma_u^2\tau\tau_\varepsilon s + (\tau+\tau_\varepsilon)(\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2)X - (\tau+\tau_\varepsilon)(\phi\tau_\varepsilon + \gamma^2\sigma_u^2)u\right]. \tag{24}$$

The trading quantity submitted by the representative imputer is given by:

$$q_{IM}^{REE} = \frac{1}{(\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2)\tau + (\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\phi\tau_\varepsilon} \left\{\phi\gamma\sigma_u^2\tau\tau_\varepsilon\mu \right. \\ \left. - \phi\gamma\sigma_u^2\tau\tau_\varepsilon s + \left[\tau\left(\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2\right) + \phi^2\tau_\varepsilon^2\right]X - \gamma^2\sigma_u^2\tau u\right\}. \tag{25}$$

The signal learned by the representative imputer is given by:

$$\tilde{s}^{REE} = s + \frac{\gamma}{\phi\tau_\varepsilon}u. \tag{26}$$

The precision of the signal $\tilde{s}^{REE}$ perceived by imputers satisfies:

$$\tau_{\tilde{s}^{REE}}^{-1} = \tau_\varepsilon^{-1} + \frac{\gamma^2}{\phi^2\tau_\varepsilon^2}\sigma_u^2. \tag{27}$$

3 Believer Thinking: A Thought Experiment

In this section, I show why investors who believe in the Efficient Market Hypothesis (EMH) can infer, through a simple thought experiment, that all other traders are investigators. Specifically, investors compare two candidate market pictures, one implied by believer thinking and the other by rational expectations, and ask which one is consistent with the central statement of the EMH that publicly available information is already "fully reflected" in prices and therefore should not predict subsequent returns. To capture this dynamic implication, I extend the static model above to a multi-period setting. The multi-period model shows that if imputers with non-zero mass remain in the market after information arrives, as in the market picture implied by rational expectations, then that information predicts future returns. By contrast, if no imputers with non-zero mass re-

main after the information arrives, as in the market picture implied by believer thinking, this predictability disappears and the EMH holds. [5] This is precisely the thought experiment through which investors infer the fraction of investigators and believer thinking emerges.

3.1 Multi-Period Model Layout

Without loss of generality, I extend the two-period static model to a four-period model with periods 0 through 3. I retain the core structure described above. The market contains a continuum of Bayesian traders of mass 1 with homogeneous absolute risk aversion γ, who trade a risky asset with constant supply X and unobservable terminal payoff v, as well as a normalized risk-free asset with perfectly elastic supply. Both assets are traded from period 0 to period 2. In period 3, the terminal payoff is realized and consumption takes place. Bayesian traders have exponential utility over terminal wealth in period 3, and in each trading period they choose their positions to maximize terminal utility (Bastianello and Fontanier, 2026; Daniel et al., 2023; Hong and Stein, 1999). Specifically, for each $t \in \{0, 1, 2\}$, a representative Bayesian trader with information set $\mathcal{I}_t$ solves the following equivalent mean-variance optimization problem over terminal wealth:

$$\max_{q_t} \left(\mathbb{E}[v \mid \mathcal{I}_t] - P_t\right) q_t - \frac{1}{2}\gamma \mathbb{V}[v \mid \mathcal{I}_t] q_t^2. \tag{28}$$

In period 0, all Bayesian traders share the common prior belief μ about the risky asset's terminal payoff v, as given in equation (1), and no news arrives. In period 1, the public signal s about v arrives, as defined in equation (3). Upon observing this signal, a fraction $\phi_1 \in [0, 1]$ of Bayesian traders become investigators and can immediately understand the news. The remaining Bayesian traders accounting for a fraction $1 - \phi_1$ are imputers, who do not understand the news and can only infer the signal s from prices. In period 2, some of the period 1 imputers gradually come to understand the news, so the fraction of investigators rises to $\phi_2 \in [\phi_1, 1]$. The remaining fraction $1 - \phi_2$ continues to learn from prices for one more round. In period 3, the asset pays its terminal payoff v. In each trading period, noise traders submit an exogenous order $u_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_u^2)$. In period 0, the market-clearing condition is therefore given by:

$$q_0 + u_0 = X. \tag{29}$$

[5] Another market picture that can generate the same empirical pattern consistent with the EMH is one in which no investor, in any period, becomes an investigator of the information. In this case, return predictability does not arise because the information is never incorporated into prices. I do not consider this market environment here because it is clearly inconsistent with the economic meaning of the EMH.

In periods 1 and 2, the market-clearing condition is given by:

$$\phi_t q_{IN,t} + (1-\phi_t) q_{IM,t} + u_t = X. \tag{30}$$

Imputers have no delay in learning. In each period, both asset demand and the equilibrium price already incorporate the information that imputers infer from prices.

In periods 1 and 2, the information set of investigators is given by $\mathcal{I}_{IN,t} = \{(\mu, \tau), (s, \tau_\varepsilon)\}$. By contrast, the information set of imputers is given by $\mathcal{I}_{IM,t} = \{(\mu, \tau), (\tilde{s}_t, \tau_{\tilde{s}_t})\}$. The imputers' information set in period 2 therefore differs from that in period 1. This difference arises because imputers who do not become investigators in period 2, and hence do not obtain the full-information set $\mathcal{I}_{IN,t}$, continue to learn from prices for one more round. They use the additional information revealed by the period 2 price to further update their beliefs about the terminal payoff v. When all Bayesian traders have rational expectations, imputers in both periods 1 and 2 learn from the market-clearing condition in equation (30).

3.2 The Thought Experiment

As a thought experiment, traders compare the price dynamics implied by the two candidate market pictures, one under believer thinking and the other under rational expectations. To illustrate this comparison, I first present the equilibrium price in each period of the multi-period model. The detailed derivations are provided in the appendix.

$$P_0 = \mu - \frac{\gamma}{\tau} X + \frac{\gamma}{\tau} u_0, \tag{31}$$

$$\begin{aligned} P_1 = \frac{1}{(\phi_1^2\tau_\varepsilon + \gamma^2\sigma_u^2)\tau + (\phi_1\tau_\varepsilon + \gamma^2\sigma_u^2)\phi_1\tau_\varepsilon} \big[&(\phi_1^2\tau_\varepsilon + \gamma^2\sigma_u^2)\tau\mu \\ &+ (\phi_1\tau_\varepsilon + \gamma^2\sigma_u^2)\phi_1\tau_\varepsilon s - (\phi_1^2\tau_\varepsilon + \gamma^2\sigma_u^2)\gamma X + (\phi_1\tau_\varepsilon + \gamma^2\sigma_u^2)\gamma u_1\big], \end{aligned} \tag{32}$$

and

$$\begin{aligned} P_2 = \frac{1}{(\tau+\tau_\varepsilon)(\phi_1^2+\phi_2^2)\tau_\varepsilon + (\tau+\phi_2\tau_\varepsilon)\gamma^2\sigma_u^2} \big\{ &\left[(\phi_1^2+\phi_2^2)\tau_\varepsilon + \gamma^2\sigma_u^2\right]\tau\mu \\ &+ \left[(\phi_1^2+\phi_2^2)\tau_\varepsilon + \phi_2\gamma^2\sigma_u^2\right]\tau_\varepsilon s - \left[(\phi_1^2+\phi_2^2)\tau_\varepsilon + \gamma^2\sigma_u^2\right]\gamma X \\ &+ (1-\phi_2)\phi_1\tau_\varepsilon\gamma u_1 + \left[(\phi_1^2+\phi_2)\tau_\varepsilon + \gamma^2\sigma_u^2\right]\gamma u_2 \big\}. \end{aligned} \tag{33}$$

The information s arrives in period 1. Under the EMH, the price P_1 should already fully reflect s, so s should not predict subsequent returns. Therefore, for the EMH to hold, the covariance between s and $R_2 = P_2 - P_1$, which is the return from period 1 to period 2, must be zero. Under believer thinking, all traders immediately understand the news

upon its arrival, implying $\phi_1^{BT} = \phi_2^{BT} = 1$. By contrast, under rational expectations, heterogeneity in expertise, attention, and time constraints implies that not all traders immediately understand the news, so the more realistic market picture satisfies $0 < \phi_1^{RE} < \phi_2^{RE} < 1$. Substituting these two sets of values for $\{\phi_1, \phi_2\}$ into the price equations and computing the covariance between s and R_2 in each case leads to the following lemma.

Lemma 1. *The covariance between s and $R_2 = P_2 - P_1$ is zero when $\{\phi_1, \phi_2\} = \{\phi_1^{BT}, \phi_2^{BT}\}$, and positive when $\{\phi_1, \phi_2\} = \{\phi_1^{RE}, \phi_2^{RE}\}$.*

$$Cov^{BT}(s, R_2) = 0, \quad Cov^{RE}(s, R_2) > 0. \tag{34}$$

Proof. Without specific statement, all proofs are saved for appendix.

Lemma 1 shows that information s is fully reflected in price at the moment it becomes publicly available, namely in period 1, only when all Bayesian traders immediately understand the signal upon its arrival. This is exactly the market picture implied by believer thinking, and it is consistent with the EMH. By contrast, the more realistic market picture characterized by $0 < \phi_1 < \phi_2 < 1$ contradicts the EMH, because the information s is not fully incorporated into price when it first becomes public. Instead, it enters asset prices gradually over time. In this setting, two forces jointly determine the relation between information s and the subsequent return R_2. To see this, we can express the prices in periods 1 and 2 as:

$$P_1 = A_1 + B_1\mu + C_1 s + D_1 u_1, \tag{35}$$

and

$$P_2 = A_2 + B_2\mu + C_2 s + D_2 u_1 + E_2 u_2, \tag{36}$$

where C_1 and C_2 represent the extent to which the signal s is incorporated into the asset price. We can further express the return R_2 as:

$$R_2 = P_2 - P_1 = (A_2 - A_1) + (B_2 - B_1)\mu + (C_2 - C_1)s + (D_2 - D_1)u_1 + E_2 u_2. \tag{37}$$

Clearly, whether the covariance between s and R_2 is zero is equivalent to whether $C_1 = C_2$. C_1 and C_2 are determined by the market-clearing conditions in their respective periods. Since u_1 and u_2 are i.i.d., only two factors can alter the market-clearing condition from period 1 to period 2: (1) some fraction $(\phi_2 - \phi_1)$ of imputers in period 1 convert into investigators in period 2 due to a one-period delayed understanding of the news; and (2) Bayesian traders who remain imputers continue learning s from price in period 2, which

enhances their confidence in the precision of the inferred signal. In the market picture delivered by rational expectations (i.e., $0 < \phi_1 < \phi_2 < 1$), both factors are present, which result in $C_2 > C_1$ and consequently a positive $Cov(s, R_2)$. Conversely, in the market picture delivered by believer thinking, both factors are absent because all traders are investigators from period 1 onward. Hence, the market-clearing conditions in periods 1 and 2 differ only by the noise trading term. In this scenario, $C_1 = C_2$, which implies $Cov(s, R_2) = 0$.

The reader may question why in this thought experiment arbitrageurs are absent. In reality, arbitrageurs may exploit the nonzero covariance $Cov(s, R_2)$, driving it towards zero or at least reducing it to a level where it is no longer profitable. We can employ the limits of arbitrage (De Long et al., 1990; Shleifer and Vishny, 1997) to argue this point. As long as no risk-neutral arbitrageur facing no institutional friction exists, the $Cov(s, R_2)$ can never be arbitraged away. Consequently, investors with relatively lower risk aversion or fewer institutional constraints would always find the information s profitable after it becomes publicly available. Hence, asset prices can never fully reflect s as stated by the EMH unless ϕ_1 reaches 1 (i.e., all Bayesian traders immediately understand the news upon its arrival).

Thus, through this thought experiment, traders who believe in the EMH infer that all other traders are investigators and therefore adopt believer thinking. In the following sections, I return to the static model introduced in Section 2. I do not use the multi-period model for the main analysis because the dynamics of ϕ over time are difficult to pin down, and because in real markets new information may arrive frequently, which would greatly increase the complexity of the analysis without being central to the paper's main focus. Moreover, the static model captures the key short-run and long-run market outcomes in a clearer and more tractable way.

4 Market Outcomes of BEE

In this section, I analyze the market outcomes of BEE and relate them to prominent asset pricing anomalies.

4.1 Price Overreaction to News

I first examine how asset prices respond to news under BEE. Here, news refers to the objective signal, which is s in the model. Taking derivatives of equations (19) and (23) with respect to the true signal s yields the following proposition.

Proposition 1. *Relative to REE, asset prices under BEE overreact to the true news s.*

$$\frac{\partial P^{BEE}}{\partial s} > \frac{\partial P^{REE}}{\partial s} > 0. \tag{38}$$

Proposition 1 shows that asset prices under BEE are more sensitive to news s than under REE. This prediction of price overreaction is consistent with empirical evidence on long-run return reversals (Barberis, 2018).

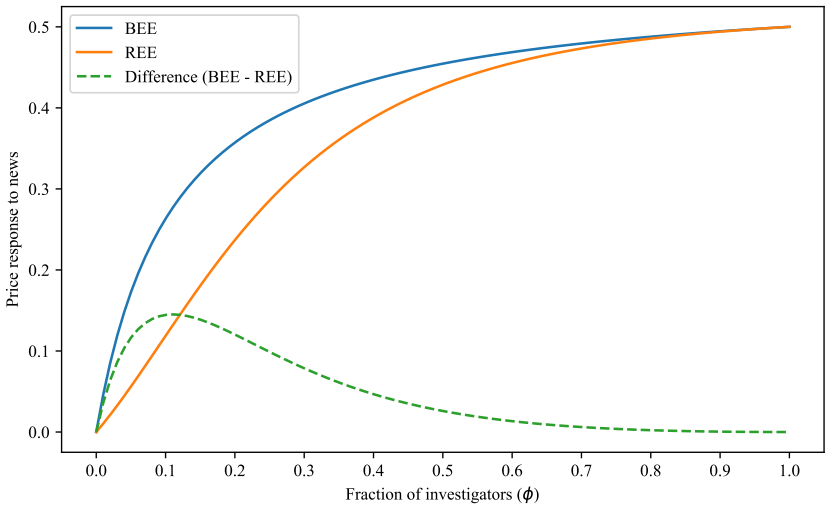


Figure 1: Price Response to News - Fraction of Investigators. This figure plots the respective values of price response to news under BEE and REE and the difference in responses. Parameters in the model take values as $\tau = 1$, $\tau_\varepsilon = 1$, $\sigma_u = 1$, $\gamma = 0.5$.

Proposition 1 directly implies the existence of financial bubbles. In particular, when the fraction of investigators ϕ is small, the overreaction of price to the signal s, measured by the gap between the price response under BEE and that under REE, becomes significantly large relative to the REE benchmark, as illustrated in Figure 1. In this case, the market can exhibit substantial overpricing, potentially exceeding 100%. Historically, major technological revolutions have often been accompanied by large bubbles and subsequent crashes, including the bubble associated with the Second Industrial Revolution in the 19th century, the technology-stock bubble of the late 1990s, and the bubble driven by financial innovation that culminated in the 2008 financial crisis. Proposition 1 suggests an additional force behind such episodes. News about revolutionary technologies is often difficult for most investors to interpret accurately, which implies a low fraction of investigators. When investors apply believer thinking and become overly optimistic about the

informational efficiency of prices, this informational difficulty can generate substantial overpricing.

The other informational component in belief formation upon the arrival of news s is the prior belief μ. While s captures new information, μ reflects stale information. In our model, the following lemma characterizes the relationship between price responses to s and μ.

Lemma 2. *Under both BEE and REE, the asset price responses to news s and to the prior belief μ sum to one. Specifically, for $cond \in \{BEE, REE\}$:*

$$\frac{\partial P^{cond}}{\partial \mu} + \frac{\partial P^{cond}}{\partial s} = 1. \tag{39}$$

Lemma 2 immediately implies the following corollary.

Corollary 1. *Relative to REE, asset prices under BEE underreact to the prior belief μ.*

$$0 < \frac{\partial P^{BEE}}{\partial \mu} < \frac{\partial P^{REE}}{\partial \mu}. \tag{40}$$

Corollary 1 shows that the price overreaction to news under BEE comes at the expense of the prior belief, or stale information. Under BEE, the roles of news and prior belief are distorted in the determination of equilibrium prices. In this sense, believer thinking reinforces the effects of bounded memory (Afrouzi et al., 2023; Jiang et al., 2025; Malmendier and Nagel, 2011) by further increasing the overweighting of the most recent information.

4.2 Inelastic Asset Demands

The inelasticity of asset demand is a long-standing puzzle in finance (Shleifer, 1986). Recently, this question has received renewed attention in the literature, driven by the increasing availability of holdings and beliefs data, as well as advances in estimation methodology (Bastianello, 2026; Gabaix and Koijen, 2021; Koijen and Yogo, 2019). In this section, I show that believer thinking can be one force that generates inelastic markets. To study this mechanism, I define the total demand for the risky asset as follows (He et al., 2025).

Definition 2. *The total market demand for the risky asset is the sum of demands by all Bayesian traders.*

$$D_{TOT}(P) = \phi q_{IN}(P) + (1-\phi) q_{IM}(P). \tag{41}$$

That is, the analysis focuses only on the asset demand of Bayesian traders, given that noise trading is perfectly inelastic and exogenous in the model. By Definition 2, the slope of the total demand curve is a weighted average of the slopes of the investigator's and imputer's demand curves. Lemma 3 identifies the group generating the inelasticity under BEE.

Lemma 3. *Under BEE, the demand curve of the representative imputer is steeper than under REE, implying more inelastic asset demand.*

$$\frac{\partial q_{IM}^{REE}(P)}{\partial P} < \frac{\partial q_{IM}^{BEE}(P)}{\partial P} < 0. \tag{42}$$

Asset price P affects imputers' demand through two opposing channels, reflecting the scarcity role and the information role of price. The scarcity role is unchanged while the information role, which operates through learning from prices, is amplified under BEE as imputers overestimate the informational efficiency of prices, which is shown in the mechanism analysis below. As a result, imputers' demand becomes more inelastic. By Definition 2, Lemma 3 immediately yields the following proposition regarding the aggregate demand elasticity.

Proposition 2. *Under BEE, total asset demand is more inelastic than under REE.*

$$\frac{\partial D_{TOT}^{REE}(P)}{\partial P} < \frac{\partial D_{TOT}^{BEE}(P)}{\partial P} < 0. \tag{43}$$

4.3 Excess Price Volatility

The discussion of excess price volatility goes back at least to Shiller (1981). Excess volatility refers to the fact that asset prices fluctuate much more than fundamentals, or the news about those fundamentals, would seem to justify. Propositions 1 and 2 already suggest the intuition. For a given change in news s or in net supply $X - u$, prices move more under BEE than under REE. This naturally points to higher price volatility under BEE. In this section, I formalize this implication and show that BEE generates excess price volatility. Before turning to the main result, I first clarify several notions. For ease of decomposition, I define price volatility as the variance of prices throughout the paper. Under this Grossman-Stiglitz framework, it is useful to decompose price volatility by its source, as follows.

Definition 3. *The total volatility of price, denoted by $PriceVol$, can be decomposed into a component driven by information, denoted by $PriceVol_{info}$, and a component driven*

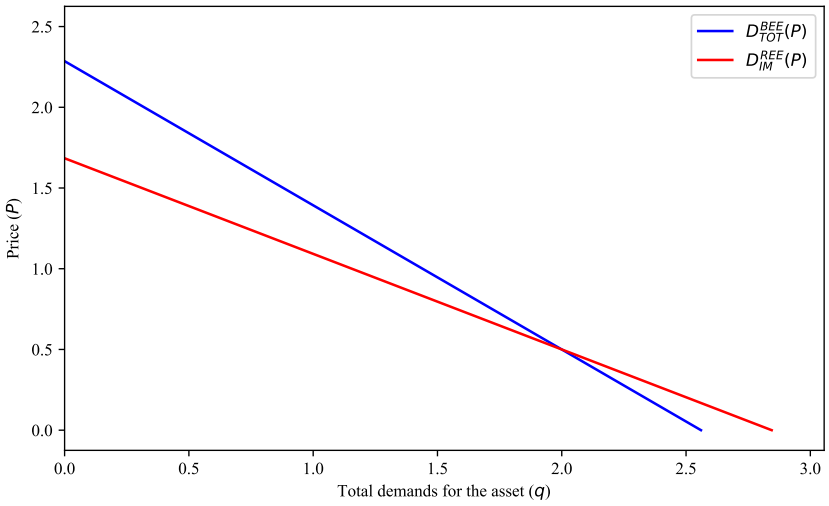


Figure 2: Total Demand Curves under BEE and REE. This figure plots the demand curves (trading quantity to price) under BEE and REE respectively. Parameters in the model take values as $\phi = 0.2$, $\tau = 1$, $\tau_\varepsilon = 1$, $\sigma_u = 1$, $\gamma = 0.5$, $\mu = 1$, $X = 1$, $s = 3$.

by noise trading, denoted by $PriceVol_{noise}$*. Total price volatility is the sum of these two components.*

$$PriceVol = PriceVol_{info} + PriceVol_{noise}. \tag{44}$$

The logic behind this decomposition is straightforward. In equilibrium, the asset price can be written as:

$$P = \beta_0 + \beta_1 \mu + \beta_2 s + \beta_3 u. \tag{45}$$

Conditional on the prior belief μ, which is determined ex ante, the asset price P is normally distributed as:

$$P \sim \mathcal{N}(\beta_0 + \beta_1 \mu + \beta_2 v, \beta_2^2 \tau_\varepsilon^{-1} + \beta_3^2 \sigma_u^2). \tag{46}$$

Therefore, price volatility can be written as:

$$PriceVol = \underbrace{\beta_2^2 \tau_\varepsilon^{-1}}_{PriceVol_{info}} + \underbrace{\beta_3^2 \sigma_u^2}_{PriceVol_{noise}}. \tag{47}$$

In equation (47), the term $\beta_2^2 \tau_\varepsilon^{-1}$ captures the component of price volatility driven by information, while $\beta_3^2 \sigma_u^2$ captures the component driven by noise trading. Note that β_2 and β_3 are exactly the partial derivatives of price with respect to the signal s and the noise trading quantity u. Thus, information-driven price volatility arises from the randomness

of s, while noise-driven price volatility arises from the randomness of noise trading u.

By Definition 3, we obtain the following proposition.

Proposition 3. *Asset prices under BEE exhibit greater volatility than under REE, driven by both components of price volatility.*

$$PriceVol^{BEE} > PriceVol^{REE}, \tag{48}$$

and

$$PriceVol^{BEE}_{info} > PriceVol^{REE}_{info}, \quad PriceVol^{BEE}_{noise} > PriceVol^{REE}_{noise}. \tag{49}$$

Proposition 3 shows that asset prices under BEE are more volatile than under REE, consistent with the excess volatility anomaly. When investors overestimate the market's informational efficiency, prices react more sharply to both randomly arriving news and noise trading. To quantify this effect, I define the relative excess price volatility under BEE as follows.

Definition 4. *Relative excess price volatility under BEE is defined as the ratio of price volatility under BEE to price volatility under REE.*

$$PriceVol^{RelExc} = \frac{PriceVol^{BEE}}{PriceVol^{REE}}. \tag{50}$$

By this definition, relative excess volatility measures the extent to which believer thinking amplifies price volatility relative to the REE benchmark. [6] The relative excess price volatility under BEE is given by:

$$PriceVol^{RelExc} = \left[\frac{(\phi^2\tau_\varepsilon + \gamma^2\sigma_u^2)\tau + (\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\phi\tau_\varepsilon}{(\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\tau + (\tau_\varepsilon + \gamma^2\sigma_u^2)\phi\tau_\varepsilon}\right]^2 \left(\frac{\tau_\varepsilon + \gamma^2\sigma_u^2}{\phi\tau_\varepsilon + \gamma^2\sigma_u^2}\right)^2, \tag{51}$$

from which it follows that:

$$\frac{\partial PriceVol^{RelExc}}{\partial\phi} < 0. \tag{52}$$

Equation (51) shows that relative excess volatility under BEE is decreasing in the fraction of investigators among Bayesian traders, ϕ. As ϕ falls, relative excess volatility rises at an increasing rate, as illustrated by the convex curves in Figure 3. The maximum level

[6] Note that this definition of relative excess volatility captures excess price volatility relative to a rational benchmark, whereas the notion of excess volatility in Shiller (1981) refers to the component of price volatility that cannot be justified by variation in the rationally expected present value of future fundamentals. The two notions are therefore related but not equivalent.

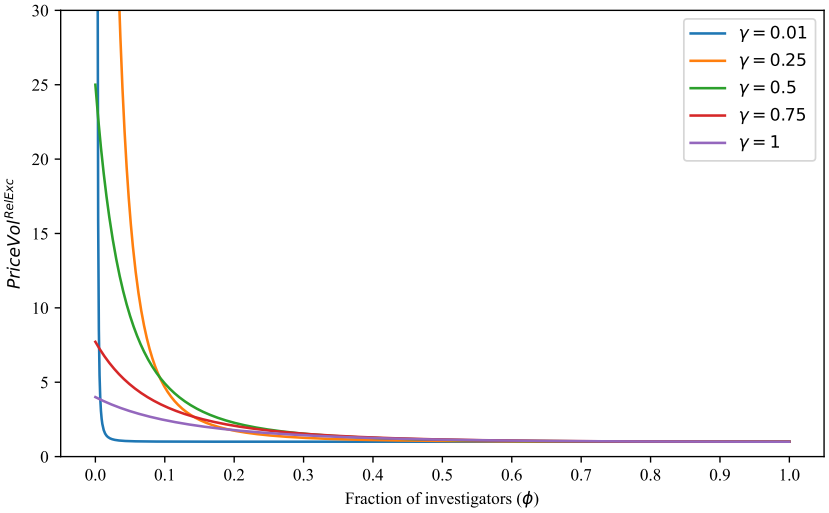


Figure 3: Relative Excess Volatility of Price - Fraction of Investigators. This figure plots the relative excess volatility of price varying with the fraction of investigators among population of Bayesian traders ϕ when γ takes 0.01, 0.25, 0.5, 0.75 and 1. Other parameters in the model take values as $\tau = 1$, $\tau_\varepsilon = 1$, $\sigma_u = 1$

of relative excess price volatility under BEE is attained as the fraction of investigators converges to zero, which is given by:

$$\overline{PriceVol}^{RelExc} = \lim_{\phi \to 0} PriceVol^{RelExc}(\phi) = \left(\frac{\tau_\varepsilon + \gamma^2 \sigma_u^2}{\gamma^2 \sigma_u^2} \right)^2. \tag{53}$$

In this limit, no information is incorporated into prices because no investor truly understands the news. Under REE, price learners therefore trade only on their priors and learn nothing from prices. Price volatility consequently arises solely from noise trading u. Under BEE, by contrast, traders continue to confidently learn from prices and misinterpret noise-driven price movements as informative signals. As a result, noise-driven volatility is amplified. Equation (53) shows that when $\gamma^2 \sigma_u^2$ becomes sufficiently small relative to τ_ε, this upper bound can become very large and even diverge to infinity as $\gamma^2 \sigma_u^2 \to 0$. Therefore, the BEE model predicts that when the market becomes less risk-averse, as reflected by a smaller γ, asset prices exhibit greater volatility. This mechanism is especially relevant during financial bubbles, which are periods of unusually low market risk aversion (Shiller, 2015), and it is consistent with the high volatility of asset prices in such episodes (Barberis, 2018; Cochrane, 2002). I show below that this link between price volatility and market risk aversion continues to hold under weaker conditions, without requiring the fraction of investigators ϕ to converge to zero.

4.4 The IVOL Anomaly

The idiosyncratic volatility (IVOL) anomaly documented by Ang et al. (2006); Ang et al. (2009) refers to the finding that stocks with higher idiosyncratic volatility earn lower subsequent returns. This pattern implies a negative premium on idiosyncratic risk and therefore conflicts with classic asset pricing theory.[7] While some candidate explanations for this puzzle have been proposed, a substantial proportion of it remains unexplained (Hou and Loh, 2016). In this section, I show that the BEE model can account for the IVOL anomaly. Specifically, when stocks are subject to heterogeneous degrees of believer thinking, a negative cross-sectional relation between idiosyncratic volatility and subsequent returns arises naturally.

To show this, I extend the baseline model to a cross-section of stocks and introduce a parameter $\theta_j \in [0, 1]$ to capture the degree of believer thinking associated with stock j. Specifically, among the $(1 - \phi_j)$ imputers for stock j, a fraction θ_j adopts believer thinking, which corresponds to a share $\theta_j(1 - \phi_j)$ of all Bayesian traders in stock j. The remaining fraction $(1 - \theta_j)$ of imputers, corresponding to a share $(1 - \theta_j)(1 - \phi_j)$ of all Bayesian traders in stock j, adopts "partially rational expectations." By "partially rational expectations," I mean a situation in which a trader correctly infers the fraction of investigators but believes that other traders also do not apply believer thinking. In other words, the trader fails to recognize others' mistakes and assumes that their beliefs are correct.[8] For stock j, idiosyncratic risk comes from two sources: noise trader risk, captured by u_j, and firm-specific information, captured by part of s_j. Without loss of generality, I assume that u_j and s_j are each i.i.d. across stocks, so that they represent stock-specific risk, while all other parameters are homogeneous across stocks except for θ_j. Under these assumptions, the model generates cross-sectional price behavior consistent with the IVOL anomaly.

Proposition 4. *Ceteris paribus, when stocks differ in the degree of believer thinking among their traders, assets with higher idiosyncratic risk earn lower average returns. This negative relation is reflected by:*

$$\frac{\partial \mathbb{E}\left[R_j\right]}{\partial \theta_j} < 0, \ \frac{\partial PriceVol_{noise,j}}{\partial \theta_j} > 0, \ \frac{\partial PriceVol_{info,j}}{\partial \theta_j} > 0. \tag{54}$$

[7] In classic asset pricing theory, idiosyncratic risk should generate zero risk premium when diversification is costless, and a positive risk premium when diversification is costly.

[8] If all traders were to adopt partially rational expectations simultaneously, these rational expectations would no longer be "partial," and the market would coincide with the Rational Expectations Equilibrium (REE).

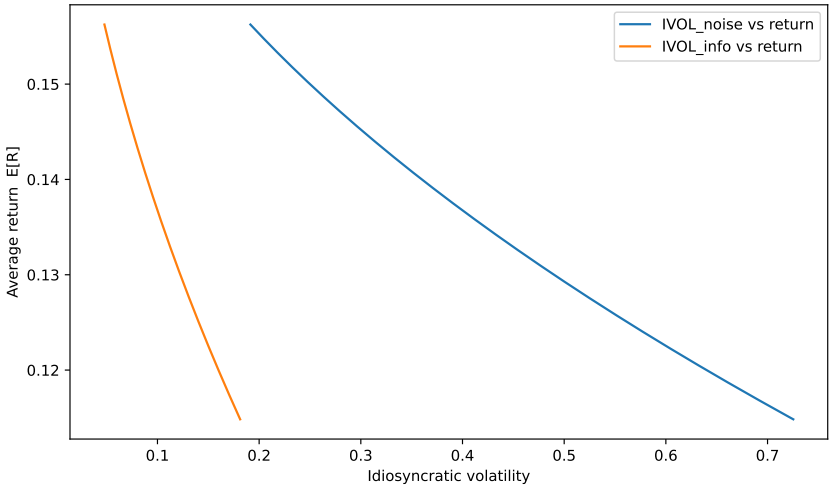


Figure 4: Average Return - Idiosyncratic Volatility. This figure plots the cross-sectional relationship between the average subsequent return and the respective idiosyncratic volatilities driven by noise traders and information. Parameters in the model take values as $\phi = 0.1$, $\tau = 1$, $\tau_\varepsilon = 1$, $\sigma_u = 1$, $\gamma = 0.2$, $v = 1$, $\mu = 1$, $X = 1$, $\theta \in [0, 1]$.

Therefore, BEE provides a new explanation for the IVOL anomaly. When a stock is associated with a higher degree of believer thinking, it exhibits both greater idiosyncratic risk and lower subsequent returns. Moreover, the empirical finding that the IVOL anomaly is stronger among stocks traded more heavily by retail investors (Han and Kumar, 2013) is consistent with the idea that retail investors are more likely to adopt believer thinking because of their underconfidence in their own skills.

4.5 Endogenous Sustainability of BEE

A standard criticism of behavioral economics is that traders with biased beliefs, if they exist, should eventually be dominated by rational traders and driven out of the market, in a process akin to natural selection (Darwin, 1859). If this argument is correct, only unbiased beliefs, and the investors who hold them, should survive in the long run.

In this section, I show that under BEE, imputers who adopt believer thinking can in fact earn higher average returns than those who do not. If imputers choose their thinking way based on historical experience measured by average returns rather than ob-

jective welfare,[9] then the Believer Expectations Equilibrium (BEE) can sustain itself even without the psychological foundations discussed above, as long as a particular parameter condition holds.

To show this, I retain the basic model setting from Section 2, except that all traders now actively choose their way of thinking. Consider a marginal imputer who chooses between two ways of thinking based on experience: believer thinking and partially rational expectations. She adopts believer thinking if her learning objective, namely return, exceeds that under partially rational expectations on average. Otherwise, she adopts partially rational expectations.

Assume that the market is currently at the BEE. I then ask whether this outcome is also an equilibrium in the choice of thinking ways by examining whether a marginal imputer has an incentive to deviate from believer thinking. She chooses between believer thinking and partially rational expectations based on historical experience, adopting the one that yields higher realized returns on average. Since the market is currently at the BEE, the equilibrium price is given by equation (19). If the marginal imputer adopts believer thinking, her demand function is given by:

$$q_{IM}^{BT}(P) = \frac{\gamma^2\sigma_u^2}{\tau_\varepsilon + \gamma^2\sigma_u^2}\tau\frac{\mu - P}{\gamma} + \frac{\tau_\varepsilon}{\tau_\varepsilon + \gamma^2\sigma_u^2}X. \tag{55}$$

If the marginal imputer adopts partially rational expectations, she perceives the market-clearing condition as given by equation (5) and, by definition, believes that all other imputers also use rational expectations. In this case, she uses the same mapping from prices to signals as imputers under REE, even though the actual market is governed by BEE. Her demand function is therefore given by:

$$q_{IM}^{PR}(P) = \frac{\gamma^2\sigma_u^2}{\phi\tau_\varepsilon + \gamma^2\sigma_u^2}\tau\frac{\mu - P}{\gamma} + \frac{\phi\tau_\varepsilon}{\phi\tau_\varepsilon + \gamma^2\sigma_u^2}X. \tag{56}$$

Since the asset's terminal payoff is v, the return on one share of the asset is always $v - P$. Therefore, at the BEE, the difference between the expected return under believer thinking

[9] There are straightforward reasons why imputers might learn from experience based on returns rather than risk-adjusted welfare. First, assessing historical welfare requires agents to explicitly or implicitly calculate both the first moment, average returns, and the second moment, return variance, which involves greater cognitive and memory costs than evaluating historical returns alone. Second, traders' risk aversion may vary over time, which makes learning from historical objective welfare more complicated. For evidence on time-varying risk aversion, see Campbell and Cochrane (1999), Guiso et al. (2018), and Shum and Xin (2022).

and that under partially rational expectations is given by:

$$\Delta R = \mathbb{E}[(v - P^{BEE})q_{IM}^{BT}(P^{BEE})] - \mathbb{E}[(v - P^{BEE})q_{IM}^{PR}(P^{BEE})]. \tag{57}$$

When $\Delta R > 0$, a marginal trader who learns from historical returns has no incentive to deviate from believer thinking and remains a "loyal" believer in the EMH. In this case, BEE is stable and constitutes an equilibrium in ways of thinking. By contrast, if $\Delta R < 0$, the marginal trader has an incentive to deviate from believer thinking to partially rational expectations, and other marginal traders will then follow. In that case, BEE is not an equilibrium in ways of thinking and cannot sustain itself. The following corollary provides the parameter condition under which BEE is an equilibrium in ways of thinking.

Corollary 2. *The Believer Expectations Equilibrium constitutes an equilibrium in ways of thinking, that is,* $\Delta R > 0$*, when:*

$$(\phi\tau_\varepsilon + \gamma^2\sigma_u^2)\phi X^2 > \left[(1 - \phi + \phi^2)\tau_\varepsilon + \gamma^2\sigma_u^2\right]\sigma_u^2. \tag{58}$$

Inequality (58) characterizes the parameter condition required for BEE to sustain itself. In particular, the condition is more likely to hold when the asset's supply X is sufficiently large or when noise trading volatility σ_u is sufficiently small. Note that Corollary 2 identifies the condition under which BEE is fully self-sustaining. However, failure of inequality (58) does not invalidate the foundation of BEE. Even when BEE does not constitute an equilibrium in ways of thinking, investors may still adopt believer thinking because of the psychological foundations discussed above.

5 BEE with Multiple Signals

In this section, I introduce an important extension of the BEE model in which multiple signals coexist and no trader fully understands all available information. This environment better captures the complexity of real-world markets. Each trader is familiar with some fields and can therefore interpret the news coming from them, while remaining unfamiliar with other fields and unable to interpret the corresponding information. Moreover, the set of fields a trader understands differs across traders. This extension is in the spirit of Hayek (1945), where no market participant is required to be fully informed about all available information and the prices can aggregate dispersed information as long as traders act on their own information. As this section shows, however, when believer thinking is present in this richer setting, the informational role of prices can break down

completely, causing the market to become highly unstable.

5.1 Multiple-Signal Model Layout and Solving

In the extended model, the risky and risk-free assets deliver the same payoffs as in the baseline framework. Without loss of generality, I introduce two pieces of news, represented by the signals s_1 and s_2 about the risky asset's terminal payoff v. The signals are distributed as:

$$s_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(v, \tau_\varepsilon^{-1}), \tag{59}$$

where $i \in \{1, 2\}$. The two signals represent information from different fields, for example, macroeconomic news and industry-specific news, and are assumed to have the same precision. The results in this section do not rely on this equal-precision assumption.

For the continuum of Bayesian traders of measure 1, a fraction ϕ, which I refer to as group 1, are investigators for s_1 and imputers for s_2. The remaining fraction $(1 - \phi)$, which I refer to as group 2, are investigators for s_2 and imputers for s_1. Bayesian traders therefore complement one another in their understanding of information. For technical simplicity, I exclude traders who understand neither signal. This restriction is also consistent with the assumption that only traders who "know something" actively participate in the market. At the same time, the economy contains no traders who "know everything" about both signals. This assumption is consistent with the empirical observation that no identifiable group of traders systematically outperforms the market (Jensen, 1968). All other features of the market, including the distribution of noise trading u, are the same as in the single-signal model.

Adopting believer thinking, each trader incorrectly assumes that all other Bayesian traders in the market are investigators for both signals s_1 and s_2. They therefore perceive the market-clearing condition as:

$$\tau \frac{\mu - P}{\gamma} + \tau_\varepsilon \frac{s_1 - P}{\gamma} + \tau_\varepsilon \frac{s_2 - P}{\gamma} + u = X. \tag{60}$$

Traders then learn from prices according to this perceived market-clearing condition and choose their optimal trading quantities based on the corresponding imputed signal. As a result, the actual market-clearing condition is given by:

$$\begin{aligned} &\phi \left(\tau \frac{\mu - P}{\gamma} + \tau_\varepsilon \frac{s_1 - P}{\gamma} + \tau_{\tilde{s}_2^{BEE}} \frac{\tilde{s}_2^{BEE} - P}{\gamma} \right) \\ &+ (1 - \phi) \left(\tau \frac{\mu - P}{\gamma} + \tau_{\tilde{s}_1^{BEE}} \frac{\tilde{s}_1^{BEE} - P}{\gamma} + \tau_\varepsilon \frac{s_2 - P}{\gamma} \right) + u = X. \end{aligned} \tag{61}$$

I save the detailed derivation for the appendix. The market equilibrium under BEE with two signals is solved as follows. The equilibrium price of the risky asset is given by:

$$P^{BEE} = \frac{1}{\tau + \tau_\varepsilon} \left[\tau\mu + \phi\tau_\varepsilon s_1 + (1-\phi)\tau_\varepsilon s_2 - \gamma X\right] + \frac{1}{\tau + \tau_\varepsilon}\left(\gamma + \frac{\tau_\varepsilon}{\gamma\sigma_u^2}\right) u. \tag{62}$$

The trading quantity submitted by group 1 Bayesian traders is given by:

$$q_1^{BEE} = \frac{\gamma\sigma_u^2}{\tau_\varepsilon + \gamma^2\sigma_u^2}(1-\phi)\tau_\varepsilon(s_1 - s_2) + X - u. \tag{63}$$

The trading quantity submitted by group 2 Bayesian traders is given by:

$$q_2^{BEE} = \frac{\gamma\sigma_u^2}{\tau_\varepsilon + \gamma^2\sigma_u^2}\phi\tau_\varepsilon(s_2 - s_1) + X - u. \tag{64}$$

The signal imputed by group 1 Bayesian traders is given by:

$$\tilde{s}_2^{BEE} = \frac{1}{\tau + \tau_\varepsilon}\left\{\tau\mu + (\tau + 2\tau_\varepsilon)\left[\phi s_1 + (1-\phi)s_2\right] - \gamma X\right\} - s_1 + \frac{\tau + 2\tau_\varepsilon}{\tau + \tau_\varepsilon}\left(\frac{\gamma}{\tau_\varepsilon} + \frac{1}{\gamma\sigma_u^2}\right) u. \tag{65}$$

The signal imputed by group 2 Bayesian traders is given by:

$$\tilde{s}_1^{BEE} = \frac{1}{\tau + \tau_\varepsilon}\left\{\tau\mu + (\tau + 2\tau_\varepsilon)\left[\phi s_1 + (1-\phi)s_2\right] - \gamma X\right\} - s_2 + \frac{\tau + 2\tau_\varepsilon}{\tau + \tau_\varepsilon}\left(\frac{\gamma}{\tau_\varepsilon} + \frac{1}{\gamma\sigma_u^2}\right) u. \tag{66}$$

The precisions that the two groups of Bayesian traders assign to their imputed signals are identically given by:

$$\tau_{\tilde{s}_1^{BEE}}^{-1} = \tau_{\tilde{s}_2^{BEE}}^{-1} = \tau_\varepsilon^{-1} + \frac{\gamma^2}{\tau_\varepsilon^2}\sigma_u^2. \tag{67}$$

5.2 Market Outcomes of the Extended BEE

In this section, I discuss market outcomes under the extended BEE with multiple signals, focusing primarily on price behavior.

The equilibrium price of the risky asset is given by equation (62). An immediate implication is that the price response to each signal is proportional to the fraction of Bayesian traders who investigate that signal. In the present setting, the price responses to s_1 and s_2 are in the ratio $\phi/(1-\phi)$. This pattern has a clear economic intuition. Investigators are the traders who incorporate the information they understand into prices through their trading. The more traders understand a given signal, the more strongly that signal is reflected in the equilibrium price. I then turn to the equilibrium trading

quantities. Beyond the common component $X-u$, each group's trading quantity contains a term proportional to the gap between the signal they investigate and the signal they learn from prices, scaled by the size of the other group. These terms always have opposite signs across the two groups and therefore offset each other in aggregation, reflecting disagreement among traders. The group that represents a smaller share of the market trades more aggressively on the signal gap, implying that they are more exposed to mistaken trading generated by believer thinking.

I now focus on how prices respond to noise trading under different levels of risk aversion and derive the main predictions of this section. According to the equilibrium price formula (62), the price response to noise trading u is given by:

$$\frac{\partial P^{BEE}}{\partial u} = \frac{1}{\tau + \tau_\varepsilon}\left(\gamma + \frac{\tau_\varepsilon}{\gamma \sigma_u^2}\right), \tag{68}$$

which implies that the component of price volatility driven by noise trading is:

$$PriceVol^{BEE}_{noise} = \left[\frac{1}{\tau + \tau_\varepsilon}\left(\gamma + \frac{\tau_\varepsilon}{\gamma \sigma_u^2}\right)\right]^2 \sigma_u^2. \tag{69}$$

The novel predictions of the extended BEE model can therefore be summarized as follows.

Proposition 5. *In the presence of multiple signals, and in the absence of a visible fraction of traders who understand all signals, the asset price response to noise trading and the resulting noise-driven price volatility under BEE are both U-shaped in traders' risk aversion γ. Specifically, as γ increases from values close to zero, both the price response and noise-driven volatility initially decrease and then increase. The critical point at which the relationship changes direction is given by:*

$$\gamma^* = \frac{\sqrt{\tau_\varepsilon}}{\sigma_u}. \tag{70}$$

Moreover, both the price response to noise trading and the noise-driven price volatility diverge to positive infinity as γ approaches zero from above. [10]

$$\lim_{\gamma \to 0^+} \frac{\partial P^{BEE}}{\partial u} = +\infty, \quad \lim_{\gamma \to 0^+} PriceVol^{BEE}_{noise} = +\infty. \tag{71}$$

[10] One point worth noting is that the price response to u is not continuous at risk neutrality, $\gamma = 0$. When market risk aversion is exactly zero, the equilibrium must instead be solved by equating the asset price to the expected payoff perceived by traders. This condition yields an arbitrarily determined price, that is, infinitely many equilibria. I do not pursue this knife-edge case further, since homogeneous risk neutrality in the market is unlikely to be empirically relevant.

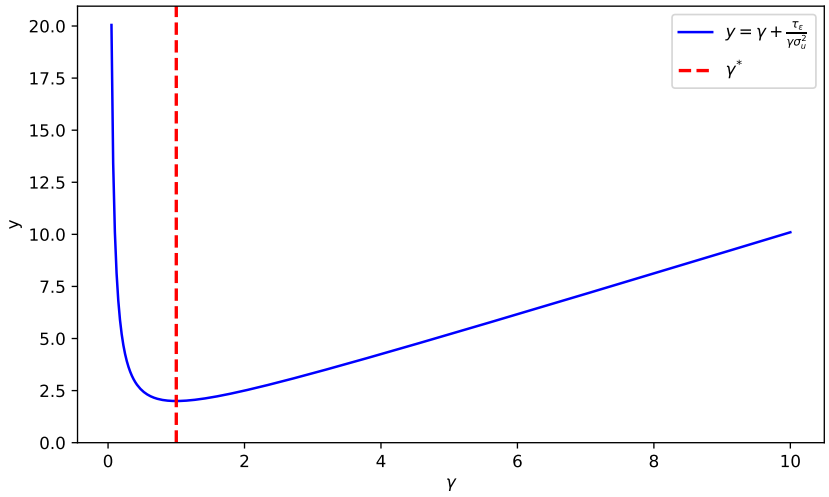


Figure 5: Price Volatility Proxy - Market Risk Aversion. This figure plots the varying component of price response to noise trading versus the homogeneous market risk aversion γ. Other parameters in the model take values as $\tau_\varepsilon = 1$, $\sigma_u = 1$

Figure 5 illustrates the U-shaped relationship between noise-driven price volatility and traders' risk aversion γ in the presence of multiple signals. When γ exceeds the critical value γ^*, the slope of price volatility with respect to γ is positive. When γ falls below γ^*, this slope becomes negative, which reflects the non-monotonic relationship between price volatility and γ.

The extended BEE with multiple signals describes a market in which believer traders mistakenly think that they are trading on information investigated by others through learning from prices, whereas the actual picture of the market is closer to "I trade on you, and you trade on me," with the noise u serving as the original trigger. When traders' risk aversion is sufficiently low and falls below the threshold γ^*, the market enters a highly unstable state in which prices react sharply to noise trading u and display extreme volatility. As risk aversion declines further, the price response to noise and the resulting noise-driven volatility can even diverge to infinity. I refer to this mechanism as "noise-driven instability," which is a regime in which mislearning amplifies the price impact of noise trading without bound, so that price movements are driven not by fundamental information about the asset, but entirely by noise in the market. Because the amplification is unbounded, such a regime can provide one candidate explanation for the dramatic booms and collapses observed throughout financial history in the absence of clear fundamental shocks, such as Tulip Mania in the Netherlands and the Black Monday Crash of 1987. This price behavior shows that believer thinking can act as a multiplier of the noise trader risk formalized by De Long et al. (1990) and Shleifer and Vishny (1997). When

traders adopting believer thinking mistakenly and aggressively trade on u, the impact of u can be amplified without bound. In this sense, Bayesian traders themselves effectively become noise traders (Mendel and Shleifer, 2012), and the total noise trader risk in the market becomes much larger than its original exogenous source u.

6 Mechanism

In this section, I present the mechanism underlying the market outcomes of BEE.

6.1 Belief Distortion: Confidence Effect and Mapping Effect

Believer thinking is, at its core, a misperception of the fraction of investigators, ϕ. For imputers, information and noise trading shocks affect market outcomes through the following chain: shock $\rightarrow$ price $\rightarrow$ belief of imputers $\rightarrow$ demands of imputers $\rightarrow$ price. As a key step in this chain, I begin by analyzing the belief formation of imputers. To update their beliefs about the asset's payoff, Bayesian imputers need two objects: the signal inferred from price and the perceived precision of that imputed signal. Believer thinking endogenously distorts both components of belief formation. I refer to the bias in perceived precision as the "confidence effect," and the bias in the inferred signal itself as the "mapping effect." The confidence effect tends to make imputers overreact to information, whereas the mapping effect tends to make them underreact to information. To see this, recall that imputers learn from prices using their perceived market-clearing condition:

$$\tilde{\phi}\left(\tau\frac{\mu-P}{\gamma}+\tau_{\varepsilon}\frac{s-P}{\gamma}\right)+(1-\tilde{\phi})\left(\tau\frac{\mu-P}{\gamma}+\tau_{\tilde{s}}\frac{\tilde{s}(P)-P}{\gamma}\right)+u=X, \tag{72}$$

where $\tilde{\phi}\in\{\phi,1\}$ is the fraction of investigators perceived by imputers under REE and BEE, respectively. The function $\tilde{s}(P)$ denotes the learning rule that imputers believe other imputers use, which is identical to their own. [11] This perceived market-clearing condition therefore determines imputers' learning behavior. Since u is unobservable to imputers, it follows immediately that they perceive the imputed signal satisfies:

$$\tilde{s}(P)=s+\frac{\gamma}{\tilde{\phi}\tau_{\varepsilon}}u. \tag{73}$$

[11] Other forms of behavioral biases could lead investors to perceive the learning rule used by others as different from the one they use themselves (Bastianello and Fontanier, 2025). Under both REE and BEE, these two learning rules are identical.

That is, imputers perceive $\tilde{s}(P)$ as an unbiased and noisy signal of the original signal s, where the (additional) noise term is $\frac{\gamma}{\tilde{\phi}\tau_\varepsilon}u$. Because u and s are independent, once the imputed signal $\tilde{s}(P)$ is interpreted as a signal about the asset's terminal payoff v, the precision that imputers assign to it is given by:

$$\tau_{\tilde{s}}^{-1} = \underbrace{\tau_\varepsilon^{-1}}_{\text{original noise of } s} + \underbrace{\frac{\gamma^2}{\tilde{\phi}^2\tau_\varepsilon^2}\sigma_u^2}_{\text{learning noise}} . \tag{74}$$

Equation (74) shows that the perceived precision of the imputed signal increases with the perceived fraction of investigators $\tilde{\phi}$, as a higher $\tilde{\phi}$ reduces the perceived learning noise. The intuition is straightforward. Imputers understand that investigators are the traders who incorporate the information into prices. For a given level of noise in the original signal, a larger perceived population of investigators implies that the information is more strongly incorporated into and reflected in prices. This channel gives rise to the confidence effect under BEE, as formalized in the following lemma.

Lemma 4. *Since BEE induces an upward bias in the perceived fraction of investigators, imputers perceive prices to be more informationally efficient and therefore assign greater precision to the signals they learn from prices under BEE than under REE.*

$$\tau_{\tilde{s}BEE} > \tau_{\tilde{s}REE}. \tag{75}$$

Therefore, when imputers with believer thinking perceive the entire market, except themselves, as investigators, they become overconfident in trading on the information they infer from prices. The other source of learning bias operates through the mapping effect. Taking the derivative of the imputed signal (72) with respect to price yields:

$$\frac{\delta\tilde{s}(P)}{\delta P} = \frac{\tau + \tilde{\phi}\tau_\varepsilon + (1-\tilde{\phi})\tau_{\tilde{s}}}{\tilde{\phi}\tau_\varepsilon + (1-\tilde{\phi})\tau_{\tilde{s}}}, \tag{76}$$

in which I use the operator "δ" rather than "∂" or "d" to emphasize that it represents the learning mapping used by imputers. As a measure of learning aggressiveness, the response of the imputed signal to price is given by the ratio of imputers' perceived aggregate market confidence to their perceived aggregate confidence in the signal. Equation (76) implies that, for a given change in price, the imputed signal under BEE is less responsive than under REE. This is the mapping effect, which is summarized in the following lemma.

Lemma 5. *Since BEE raises the perceived aggregate market confidence in the original*

signal, the signal imputed by imputers is less responsive to price than under REE.

$$0 < \frac{\delta \tilde{s}^{BEE}(P)}{\delta P} < \frac{\delta \tilde{s}^{REE}(P)}{\delta P}. \tag{77}$$

Another important property of the learning mapping used by imputers, which can also be observed directly from equation (76), is summarized in the following lemma.

Lemma 6. *As long as the market has prior beliefs, such that $\tau > 0$, we always have:*

$$\frac{\delta \tilde{s}(P)}{\delta P} > 1. \tag{78}$$

This property holds under any information structure, that is, for any collection of priors and signals held by different groups of Bayesian traders. It follows from the fact that imputers always recognize that, when traders are Bayesian, the aggregate confidence of the market exceeds the aggregate confidence in the signal to be inferred from prices.

The confidence effect and the mapping effect push belief formation in opposite directions, yet both arise endogenously from the same mislearning from prices. The mapping effect operates through the information that imputers extract from prices. By making the inferred signal less responsive to price movements, it tends to generate underreaction of beliefs. The confidence effect instead operates through the weight that imputers place on the inferred signal. By making the imputed signal appear more precise, it tends to generate overreaction of beliefs.

These two effects can be seen more clearly through the two-step belief-formation framework formalized by Bastianello and Fontanier (2025). In the first step, imputers map the observed asset price into an inferred signal $\tilde{s}$. In the second step, they combine this imputed signal with their prior belief μ, assigning weights according to the perceived precisions of the prior and the signal. While Bastianello and Fontanier (2025) refer to distortions in the first step as "mislearning from prices" and distortions in the second step as "mislearning from fundamentals," mislearning from prices in the present partially revealing framework endogenously distorts both steps. Believer thinking changes the price-to-signal mapping $\tilde{s}(P)$ in the first step, while simultaneously distorting the perceived precision $\tau_{\tilde{s}}$ and hence the Bayesian weight assigned to the imputed signal in the second step. Thus, the mapping and weighting distortions are not imposed as separate biases, rather, they are jointly generated by the same underlying mislearning from prices.

Under this two-step framework, imputers' beliefs, including both their expectations and perceived variances, can be represented by a modified version of equation (11), as

follows:

$$\begin{cases} \mathbb{E}[v \mid (\tilde{s}, \tau_{\tilde{s}})] = \frac{\tau\mu + \tau_{\tilde{s}}\tilde{s}}{\tau + \tau_{\tilde{s}}}; \\ \mathbb{V}[v \mid (\tilde{s}, \tau_{\tilde{s}})] = \frac{1}{\tau + \tau_{\tilde{s}}}. \end{cases} \tag{79}$$

The updated expectation is given by:

$$\mathbb{E}[v \mid (\tilde{s}, \tau_{\tilde{s}})] = \frac{\tau}{\tau + \tau_{\tilde{s}}}\mu + \frac{\tau_{\tilde{s}}}{\tau + \tau_{\tilde{s}}}\tilde{s}. \tag{80}$$

Equation (80) clearly illustrates the role of both the mapping effect and the confidence effect in expectation formation. If only the mapping effect is present while the confidence effect is shut down, such that $\tau_{\tilde{s}} = \tau_{\tilde{s}^{REE}}$, then a smaller $\frac{\delta\tilde{s}(P)}{\delta P}$ implies that imputers learn more conservatively from prices. Their beliefs therefore respond less to price movements, which makes their demands more elastic. [12] By contrast, if only the confidence effect is present while the mapping effect is shut down, such that $\frac{\delta\tilde{s}(P)}{\delta P} = \frac{\delta\tilde{s}^{REE}(P)}{\delta P}$, then the higher perceived precision $\tau_{\tilde{s}}$ leads imputers to place greater weight on $\tilde{s}$. This makes beliefs respond more strongly to price movements and, in turn, makes demands more inelastic.

The discussion above focuses on expectations about v. A key feature of BEE is that imputers' beliefs are distorted not only in expectation but also in precision. As equation (79) shows, the posterior variance perceived under BEE is lower than that under REE. This downward bias reflects overconfidence in updated beliefs. Imputers perceive their posterior beliefs to be more precise than they actually are because believer thinking leads them to overestimate the informational efficiency of prices. This is a parallel manifestation of the confidence effect in the other component of beliefs.

6.2 Asset Demands and the "Learning Echo"

To illustrate how the biased precision perceived by believer imputers distorts the market, I now move from beliefs to asset demand. The demand of the representative imputer is given by:

$$q_{IM}(P) = \frac{\mathbb{E}[v \mid (\tilde{s}, \tau_{\tilde{s}})] - P}{\gamma\mathbb{V}[v \mid (\tilde{s}, \tau_{\tilde{s}})]}. \tag{81}$$

Since $\mathbb{V}[v \mid (\tilde{s}, \tau_{\tilde{s}})]$ appears in the denominator of equation (81), a downward bias in this term directly amplifies imputers' asset demand, holding all other components fixed. This reflects imputers' overconfidence in their expectations about v. Substituting the expressions for each component into equation (81) and rewriting $\tilde{s}$ as the learning mapping

[12] Note that the informational role of prices generates a force opposite to the scarcity role, and therefore a weaker informational role pushes demand toward greater elasticity.

$\tilde{s}(P)$ used by imputers yields the following explicit demand function of imputers:

$$q_{IM}(P) = \underbrace{\frac{1}{\gamma}\tau(\mu - P)}_{\text{prior demand}} + \underbrace{\frac{1}{\gamma}\tau_{\tilde{s}}\left[\tilde{s}(P) - P\right]}_{\text{updated demand}}. \tag{82}$$

Equation (82) shows that the CARA demand of imputers can be linearly decomposed into components incorporating each piece of information in their beliefs, including the prior and the signal learned from prices. The same decomposition can also be applied to investigators' demand. [13] In this decomposition, I refer to the first component, which incorporates the prior belief, as "prior demand," and the second component, which incorporates the imputed signal, as "updated demand." The updated demand arises entirely from belief updating and would disappear in the absence of updating. While the prior demand reflects only the scarcity role of prices, the updated demand of imputers, which incorporates the signal learned from prices, reflects both the scarcity role and the informational role of prices. The informational role dominates the scarcity role and gives the updated demand a positive slope with respect to price, as implied by Lemma 6.

I refer to $\tau_{\tilde{s}}\left[\frac{\delta\tilde{s}(P)}{\delta P} - 1\right]$ as the "learning echo," namely, the derivative of the updated demand incorporating the imputed signal with respect to price, scaled by risk aversion γ. The learning echo is therefore jointly determined by the perceived precision of the imputed signal and the sensitivity of the imputed signal to price, that is, learning aggressiveness. The confidence effect and the mapping effect discussed earlier therefore jointly shape the distortions in the learning echo.

The learning echo plays a central role in determining market elasticity and price fluctuations. If the learning echo becomes sufficiently large, market demand for the risky asset can become perfectly inelastic or even positively sloped with respect to price. To show this, I write the total demand for the risky asset, as defined in Definition 2, as follows:

$$D_{TOT}(P, s) = \frac{1}{\gamma}\tau(\mu - P) + \phi\frac{1}{\gamma}\tau_{\varepsilon}(s - P) + (1 - \phi)\frac{1}{\gamma}\tau_{\tilde{s}}\left[\tilde{s}(P) - P\right]. \tag{84}$$

[13] The same decomposition can be applied to investigators' demand:

$$q_{IN}(P) = \underbrace{\frac{1}{\gamma}\tau(\mu - P)}_{\text{prior demand}} + \underbrace{\frac{1}{\gamma}\tau_{\varepsilon}(s - P)}_{\text{updated demand}}. \tag{83}$$

Note that while the updated demand of imputers reflects the informational role of prices, both components of investigators' demand reflect only the scarcity role of prices, because investigators do not learn from prices.

Unlike in Definition 2, I now treat the signal s explicitly as a variable of the demand function $D_{TOT}(\cdot)$, because the analysis below studies how changes in s affect prices. The slope of the total demand function with respect to price is therefore given by:

$$\frac{\partial D_{TOT}(P,s)}{\partial P} = -\frac{1}{\gamma}\left\{ \underbrace{\tau + \phi\tau_{\varepsilon}}_{\substack{\text{learning-absent}\\ \text{market confidence}}} \quad \underbrace{-(1-\phi)\tau_{\tilde{s}}\left[\frac{\delta\tilde{s}(P)}{\delta P} - 1\right]}_{\text{market learning echo}} \right\}. \tag{85}$$

Equation (85) immediately implies that the asset demand curve is downward sloping if and only if:

$$(1-\phi)\tau_{\tilde{s}}\left[\frac{\delta\tilde{s}(P)}{\delta P} - 1\right] < \tau + \phi\tau_{\varepsilon}. \tag{86}$$

Here, the right-hand side is the learning-absent market confidence, that is, the market's average confidence when imputers do not learn from prices. It is obtained from the derivative of the market-mass-weighted sum of investigators' demand and imputers' prior demand with respect to price, scaled by risk aversion γ. The left-hand side is the market learning echo, that is, the learning echo of imputers multiplied by their market mass. Inequality (86) therefore compares the market-wide scarcity role and informational role of prices and gives the upper bound on the informational role of prices, as captured by the market learning echo, required for total demand to remain downward sloping.

Within this region, a larger market learning echo makes asset demand more inelastic, holding the other parameters fixed. When the two sides of inequality (86) are equal, market demand becomes perfectly inelastic. If the inequality is reversed, market demand for the risky asset becomes upward sloping in price. In this extreme case, a higher price leads the market, on average, to infer an even higher payoff, which induces additional buying and pushes the price up further. Therefore, any force that raises the market learning echo or lowers the learning-absent market confidence can make asset demand more inelastic or even upward sloping. These forces include a larger fraction of imputers, more aggressive learning from prices, a higher perceived precision of the imputed signal, lower precision of prior beliefs, and lower objective precision of the signal.

After highlighting the importance of the relative magnitude of the market learning echo and the learning-absent market confidence in determining market demand elasticity, I now return to the specific analysis of BEE. Comparing the learning echo under BEE with that under the REE benchmark yields the following lemma.

Lemma 7. *The learning echo of the representative imputer, and therefore the market*

learning echo, is larger under BEE than under REE.

$$0 < (1-\phi)\tau_{\tilde{s}^{REE}} \left[\frac{\delta \tilde{s}^{REE}(P)}{\delta P} - 1 \right] < (1-\phi)\tau_{\tilde{s}^{BEE}} \left[\frac{\delta \tilde{s}^{BEE}(P)}{\delta P} - 1 \right]. \tag{87}$$

Since the fractions of investigators and imputers, the confidence in prior beliefs, and the objective precision of the signal are all the same under BEE and REE, the learning-absent market confidence is identical across the two equilibria. Hence, a larger learning echo under BEE implies that asset demand is more inelastic under BEE than under REE, as stated in Proposition 2.

6.3 The Macro Market

I now turn to macro-level market outcomes. The subjective beliefs and asset demands analyzed above enter the equilibrium through the trading behavior of Bayesian traders, while market outcomes are ultimately determined by the actual market-clearing condition (MCC):

$$D_{TOT}(P, s) + u = X. \tag{88}$$

That is, total asset demand, including demand from Bayesian traders and noise traders, must equal the asset's constant supply X. Taking the total differential of equation (88) yields:

$$\frac{\partial D_{TOT}(P,s)}{\partial P}\, dP + \frac{\partial D_{TOT}(P,s)}{\partial s}\, ds + du = 0. \tag{89}$$

This equation describes the relationship among the information s, the noise trading quantity u, and the equilibrium asset price P. The asset price enters this relationship through both its informational role (i.e., a higher price indicates a higher terminal payoff) and its scarcity role (i.e., a higher price raises the cost of investment). The information s affects this relationship by changing investigators' beliefs about v. The noise trading quantity u affects it by directly changing total demand, or equivalently the asset's net supply. In equation (89), the asset price P enters with multiplier $\frac{\partial D_{TOT}(P,s)}{\partial P}$, the information s enters with multiplier $\frac{\partial D_{TOT}(P,s)}{\partial s}$, and the noise trading quantity u enters with multiplier 1. These multipliers can be interpreted as the marginal effects of the corresponding variables in equation (89). When s and u are treated as exogenous shocks to the market, their changes are multiplied by these marginal effects before being injected into the market. To maintain the MCC, the asset price must adjust so as to offset these shocks. Equation (89)

therefore directly implies:

$$\frac{\partial P}{\partial s} = -\frac{\partial D_{TOT}(P,s)}{\partial s} \Big/ \frac{\partial D_{TOT}(P,s)}{\partial P}, \tag{90}$$

and

$$\frac{\partial P}{\partial u} = -1 \Big/ \frac{\partial D_{TOT}(P,s)}{\partial P}. \tag{91}$$

The equations above describe the price responses to exogenous shocks to information s and noise trading u, respectively. For a given level of $\frac{\partial D_{TOT}(P,s)}{\partial s}$, which is identical under BEE and REE, these price responses are fully determined by the measure of market demand elasticity, $\frac{\partial D_{TOT}(P,s)}{\partial P}$. Specifically, the price response to s is given by the negative inverse of $\frac{\partial D_{TOT}(P,s)}{\partial P}$, scaled by $\frac{\partial D_{TOT}(P,s)}{\partial s}$, while the price response to u is simply given by the negative inverse of $\frac{\partial D_{TOT}(P,s)}{\partial P}$. [14] When $\frac{\partial D_{TOT}(P,s)}{\partial P} < 0$, so that demand is downward sloping, greater demand inelasticity implies stronger price responses to both information and noise trading, thereby generating excess price volatility. Moreover, when $\frac{\partial D_{TOT}(P,s)}{\partial P} > 0$, the signs of the price responses to both information and noise trading are reversed. In this case, good news or greater noise trading flow lowers the asset price, while bad news or lower noise trading flow raises it.

The mechanism through which prices overreact to news and exhibit excess volatility in the presence of believer thinking is therefore clear and is summarized in the following proposition.

Proposition 6. *Under the single-signal BEE, believer thinking induces two distortions in imputers' beliefs and demands. It makes imputers overconfident about the precision of the information they infer from prices, while also making them learn more conservatively from prices. Although the mapping effect associated with the latter force dampens learning, the confidence effect associated with the former dominates. As a result, believer thinking increases the learning echo and thereby makes market demand more inelastic. Relative to REE, the direct demand response to the signal and the multiplier on noise trading in the market-clearing condition remain unchanged under BEE. Prices must therefore respond more strongly to both information and noise trading flows in order to offset their respective impacts on the market-clearing condition. This generates both overreaction to news and excess price volatility.*

[14] This is analogous to the identification of demand elasticity using the inverse of the price multiplier in the recent literature on inelastic markets (Bastianello, 2026; Gabaix and Koijen, 2021; Hartzmark and Solomon, 2025; Pavlova and Sikorskaya, 2023). The difference lies in measurement: the quantities studied in these papers are relative, following the textbook definition of demand elasticity, whereas the quantities in the present model are absolute.

The origin of inelastic demand and its link to price volatility has recently received substantial attention (Bastianello, 2026; Gabaix and Koijen, 2021; He et al., 2025). Proposition 6 identifies believer thinking and mislearning from prices as a potential mechanism through which inelastic demand and the resulting excess price volatility can arise. Importantly, believer thinking does not conflict with other sources of market inelasticity that amplify price volatility, such as the mandate-based mechanism proposed by Gabaix and Koijen (2021). Instead, these mechanisms can interact and operate jointly. In our model, the noise-trading quantity u represents a component of perfectly inelastic demand. Other sources of demand inelasticity can directly affect, or even approximately generate, u, while believer thinking amplifies their effects on asset prices by making the demand of price learners more inelastic.

6.4 The Multiple-Signal Condition

The analysis above clarifies the mechanism that generates market outcomes under BEE with a single signal. I now extend this mechanism analysis to the multiple-signal case. The focus is on the learning echo in asset demand and on how it gives rise to extremely high price volatility as the market approaches risk neutrality, as discussed above.

Under BEE with multiple signals, the perceived precision of the imputed signals is given by equation (67), which, as in the single-signal case, is determined by the perceived fraction of traders investigating the relevant signal. I now examine the response of each imputed signal to price under BEE. Taking the total differential of equation (60) yields:

$$\frac{\delta \tilde{s}_1(P, s_2)}{\delta P} = \frac{\delta \tilde{s}_2(P, s_1)}{\delta P} = \frac{\tau + 2\tau_\varepsilon}{\tau_\varepsilon}. \tag{92}$$

The signal investigated by each group enters that group's learning function as an additional variable, reflecting the fact that imputers form their imputed signals using both the asset price and the signal they directly investigate. The total demand of Bayesian traders, as a function of the asset price and the two signals, can therefore be written as the sum of demand components incorporating each piece of information in their beliefs:

$$\begin{aligned} D_{TOT}(P, s_1, s_2) =& \frac{1}{\gamma}\tau(\mu - P) + \phi\frac{1}{\gamma}\left\{\tau_\varepsilon(s_1 - P) + \tau_{\tilde{s}_2}\left[\tilde{s}_2(P, s_1) - P\right]\right\} \\ &+ (1-\phi)\frac{1}{\gamma}\left\{\tau_\varepsilon(s_2 - P) + \tau_{\tilde{s}_1}\left[\tilde{s}_1(P, s_2) - P\right]\right\}. \end{aligned} \tag{93}$$

By taking the partial derivative of equation (93) with respect to price, we obtain the

slope of the demand function as:

$$\frac{\partial D_{TOT}(P,s_1,s_2)}{\partial P} = -\frac{1}{\gamma}\left\{\underbrace{\tau+\tau_\varepsilon}_{\substack{\text{learning-absent}\\ \text{market confidence}}} - \underbrace{\left(\phi\tau_{\tilde{s}_2}\left[\frac{\delta\tilde{s}_2(P,s_1)}{\delta P}-1\right]+(1-\phi)\tau_{\tilde{s}_1}\left[\frac{\delta\tilde{s}_1(P,s_2)}{\delta P}-1\right]\right)}_{\text{market learning echo}}\right\}. \tag{94}$$

Under the multiple-signal case, the market learning echo consists of two components, each corresponding to the updated demand associated with a particular imputed signal for the group of traders that learns it. The condition required for asset demand to remain downward sloping is unchanged: the market learning echo must be smaller than the learning-absent market confidence, which equals $\tau+\tau_\varepsilon$ in the two-signal case. Combining (67) and (92), the market learning echo under BEE with two signals is given by:

$$LearningEcho^{BEE} = \phi\tau_{\tilde{s}_2}\left[\frac{\delta\tilde{s}_2(P,s_1)}{\delta P}-1\right]+(1-\phi)\tau_{\tilde{s}_1}\left[\frac{\delta\tilde{s}_1(P,s_2)}{\delta P}-1\right] = \frac{\tau_\varepsilon}{\tau_\varepsilon+\gamma^2\sigma_u^2}(\tau+\tau_\varepsilon), \tag{95}$$

which yields the slope of the total demand function as:

$$\frac{\partial D^{BEE}_{TOT}(P,s_1,s_2)}{\partial P} = -\frac{1}{\gamma}\frac{\gamma^2\sigma_u^2}{\tau_\varepsilon+\gamma^2\sigma_u^2}(\tau+\tau_\varepsilon). \tag{96}$$

Notice that $\lim_{\gamma\to 0^+} LearningEcho^{BEE} = \tau+\tau_\varepsilon$. Therefore, as the risk aversion of Bayesian traders approaches zero from the positive side, the market learning echo converges to the learning-absent market confidence. It follows that $\lim_{\gamma\to 0^+}\frac{\partial D^{BEE}_{TOT}(P,s_1,s_2)}{\partial P} = 0$, which means the total demand becomes perfectly inelastic in this limit. The relationship among the variables required by the MCC is then given by:

$$\frac{\partial D_{TOT}(P,s_1,s_2)}{\partial P}dP + \frac{\partial D_{TOT}(P,s_1,s_2)}{\partial s_1}ds_1 + \frac{\partial D_{TOT}(P,s_1,s_2)}{\partial s_2}ds_2 + du = 0, \tag{97}$$

which yields the price response to noise trading flow u as:

$$\frac{\partial P}{\partial u} = -1\Big/\frac{\partial D_{TOT}(P,s_1,s_2)}{\partial P}. \tag{98}$$

Thus, as shown above, asset demand approaches perfect inelasticity as the market approaches risk neutrality. This, in turn, generates an arbitrarily large price response to noise trading u and therefore arbitrarily high price volatility. The fundamental force behind this prediction is that the market learning echo converges to the learning-absent

market confidence. The U-shaped relationship between price volatility and market risk aversion is also captured jointly by equations (96) and (98).

Why, then, does perfectly inelastic demand not also generate an arbitrarily large price response to signals in the model? To answer this question, I examine the response of asset demand to signal s_1. By symmetry, the same analysis applies to s_2. As equation (93) shows, a change in s_1 affects not only the updated demand component directly incorporating s_1 for group 1, but also the updated demand component incorporating the imputed signal $\tilde{s}_2$, because s_1 enters the learning function used by group 1. Differentiating equation (93) with respect to s_1 gives:

$$\frac{\partial D_{TOT}(P, s_1, s_2)}{\partial s_1} = \phi \frac{1}{\gamma} \left[\tau_\varepsilon + \tau_{\tilde{s}_2} \frac{\delta \tilde{s}_2(P, s_1)}{\delta s_1} \right], \tag{99}$$

where the response of the imputed signal $\tilde{s}_2$ to the investigated signal s_1 for group 1, derived from equation (60), is given by:

$$\frac{\delta \tilde{s}_2(P, s_1)}{\delta s_1} = -1. \tag{100}$$

Equation (100) shows that, for Bayesian traders, the investigated signal is netted out from the imputed signal one-for-one. This is a consequence of believer thinking together with the assumption that the two signals have the same precision τ_ε. It then follows that the response of total demand to s_1 under BEE is:

$$\frac{\partial D^{BEE}_{TOT}(P, s_1, s_2)}{\partial s_1} = \frac{1}{\gamma} \frac{\gamma^2 \sigma_u^2}{\tau_\varepsilon + \gamma^2 \sigma_u^2} \phi \tau_\varepsilon. \tag{101}$$

Therefore, as risk aversion approaches zero, $\frac{\partial D^{BEE}_{TOT}(P, s_1, s_2)}{\partial s_1}$ converges to zero at the same rate as $\frac{\partial D^{BEE}_{TOT}(P, s_1, s_2)}{\partial P}$, because the two derivatives share the same component governing their convergence. Specifically,

$$\frac{\partial P^{BEE}}{\partial s_1} = - \frac{\partial D^{BEE}_{TOT}(P, s_1, s_2)}{\partial s_1} \Big/ \frac{\partial D^{BEE}_{TOT}(P, s_1, s_2)}{\partial P} = \frac{\phi \tau_\varepsilon}{\tau + \tau_\varepsilon}, \tag{102}$$

which is independent of risk aversion γ.

7 Market Transparency and Informational Efficiency

The BEE model delivers a simple but important insight: greater market transparency does not necessarily improve informational efficiency when behavioral distortions such as believer thinking are present. The reason is that becoming aware of the existence of specific information does not guarantee that the information will be correctly understood. When traders mislearn from prices, this misunderstanding can distort market outcomes and amplify noise in prices. In this section, I formalize this insight within the BEE framework and derive the corresponding policy implications.

7.1 Market Transparency

I continue with the baseline model from Section 2 and introduce a new parameter $\kappa \in [0, 1]$ for the specific signal s. This parameter denotes the fraction of the continuum of Bayesian traders who are aware of the existence of s, and therefore measures market transparency. In practice, κ may be shaped by regulatory enforcement, disclosure requirements, media coverage, or other channels of information dissemination. Without loss of generality, I assume that κ is the same for investigators and imputers. That is, the fraction κ of investigators who become aware of s is equal to the fraction of imputers who do. Traders who are aware of the signal's existence behave as investigators and imputers do in the baseline BEE model. Investigators trade on the prior and the investigated signal s, while imputers trade on the prior and the signal $\tilde{s}$ inferred from prices. With believer thinking, these imputers continue to perceive all other traders as being aware of the existence of s and as acting as investigators, which they infer through the same thought experiment described in Section 3. By contrast, traders who are unaware of the signal's existence trade only on the prior μ. To derive policy implications, I assume that a policymaker actively chooses the market transparency parameter κ to maximize informational efficiency in asset prices.

7.2 Informational Efficiency

In this section, I formalize the unconditional informational efficiency of prices as the objective in the policymaker's optimization problem. Here, "unconditional" means that the market's prior belief μ is treated as a random variable. This setting is particularly suitable for the analysis of transparency and optimal policy because the transparency parameter κ is much more persistent than the market belief μ. In particular, κ is determined by policy choices and other relatively stable factors, whereas μ evolves over

time as new information arrives. Motivated by this distinction, I formalize informational efficiency through the following definition.

Definition 5. *The informational efficiency of price is the share of price volatility explained by the existing information μ and s.*

$$Eff = \frac{1}{\mathbb{V}[P]} \left\{ \frac{Cov(P,\mu)^2}{\mathbb{V}[\mu]} + \frac{Cov(P,s)^2}{\mathbb{V}[s]} \right\} = Corr(P,\mu)^2 + Corr(P,s)^2. \tag{103}$$

This definition of informational efficiency is consistent with the notion of price informativeness defined by Brogaard et al. (2022), which captures the maximal extent to which price fluctuations can be explained by the existing information μ and s, without observing market noise u. In the absence of noise, that is, when $\mathbb{V}[u] = 0$, informational efficiency reaches its maximum value of 1. In this case, all price fluctuations are attributable to information.

7.3 Optimal Market Transparency

In the presence of believer thinking, which serves here as an example of behavioral bias, greater market transparency therefore has two opposite effects on the informational efficiency of prices. On the one hand, higher transparency allows more investigators to incorporate the signal s into prices by making them aware of its existence. On the other hand, greater transparency also causes more imputers to engage in mislearning from prices, which amplifies market noise u and thereby reduces informational efficiency. Let $Eff(\kappa)$ denote the informational efficiency of price as a function of market transparency κ under BEE. The policymaker's objective is to choose the optimal transparency level, denoted by $\kappa^* \in [0,1]$, so as to maximize $Eff(\kappa)$. Formally, κ^* solves:

$$\max_{\kappa \in [0,1]} Eff(\kappa). \tag{104}$$

The following proposition characterizes the market transparency level that maximizes the informational efficiency of prices.

Proposition 7. *Under BEE, the market transparency level that maximizes the informational efficiency of prices satisfies $\kappa^* \in \{0,1\}$, corresponding to "no transparency" and "complete transparency" respectively. In particular, the optimal transparency level is*

determined by the fraction of investigators ϕ and is given by:

$$\kappa^* = \begin{cases} 0, \ \textit{if } \phi < \phi^*; \\ \{0,1\}, \ \textit{if } \phi = \phi^*; \\ 1, \ \textit{if } \phi > \phi^*. \end{cases} \tag{105}$$

The threshold $\phi^* \in [0,1]$ is given by:

$$\phi^* = \frac{\sqrt{\tau^2\gamma^4\sigma_u^4 + \tau(\tau_\varepsilon + 2\gamma^2\sigma_u^2)\left[(\tau_\varepsilon + \gamma^2\sigma_u^2)^2 + \tau_\varepsilon\tau\right]} - \tau\gamma^2\sigma_u^2}{(\tau_\varepsilon + \gamma^2\sigma_u^2)^2 + \tau_\varepsilon\tau}. \tag{106}$$

Proposition 7 provides an important policy implication of BEE. When believer thinking is prevalent in the market, the optimal choice between low (zero) transparency and high (complete) transparency for a given piece of news s depends on ϕ, the fraction of investigators for that news. If the news is easy to understand, making $\phi > \phi^*$, the optimal policy is complete transparency, which ensures that everyone becomes aware of the news' existence. By contrast, if the news is difficult to understand, making $\phi < \phi^*$, the optimal policy is zero transparency, which keeps the market unaware of the news' existence.[15]

8 Conclusions

This paper develops a behavioral asset pricing framework, the Believer Expectations Equilibrium (BEE), in which investors universally believe in the Efficient Market Hypothesis and, through a simple thought experiment grounded in well-documented psychological tendencies, infer that every other trader must understand any given piece of news. The belief is internally consistent with the EMH, yet it leads imputers to systematically mislearn from prices. Slight imperfections in investors' knowledge of market structure can therefore endogenously produce mislearning.

BEE offers a unified account of three central asset pricing anomalies: inelastic demand, excess volatility, and the IVOL anomaly. And it further yields two novel predictions: a U-shaped relationship between price volatility and market risk aversion, and the possibility of "noise-driven instability," whereby arbitrarily small fluctuations in noise trading can generate arbitrarily large price movements. Believer thinking can therefore amplify noise trader risk without bound and help explain dramatic market episodes that lack any clear fundamental driver. The underlying mechanism rests on a single comparison whose relevance extends well beyond BEE: the comparison between the market average learning

[15] With a cost of intervention, as in a more realistic setting, interior solutions can naturally emerge.

echo and the learning-absent market confidence. Believer thinking affects this comparison through two channels. The confidence effect causes imputers to overestimate the precision of signals learned from prices. The mapping effect dampens how aggressively they translate prices into the magnitude of signals. The confidence effect dominates, so the learning echo increases and moves closer to the learning-absent market confidence, making demand more inelastic. As a result, prices respond more strongly to both information and noise. More generally, in any market where learning from prices is present, the comparison between the market average learning echo and the learning-absent market confidence governs demand elasticity and market stability.

Moreover, BEE has two broader implications. First, it can be self-sustaining: when investors evaluate ways of thinking by realized returns, believer thinking survives endogenously under specific parameter conditions, even in the absence of its psychological foundations. Second, it yields a clear policy lesson: when mislearning is present, the optimal degree of market transparency depends on whether the news is hard or easy to understand. More broadly, the informational role of prices is more fragile than standard models imply. Belief in market efficiency, when held without an accurate picture of who actually understands what, can itself become a source of inefficiency. The misuse of knowledge does not merely distort individual beliefs. It can reshape equilibrium outcomes and undermine the very efficiency of the price system.

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