The Term Structure of Interest Rates with Dealer Market Power in Repo Market

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ABSTRACT

Dealers finance most of their Treasury holdings through repo borrowing, yet term-structure models treat them as price takers despite their market power over lenders. I embed this market power into a preferred-habitat model. Dealers strategically restrict carry trades and earn a funding rent proportional to their market power and the policy rate. In their first-order condition, this rent acts as a marginal shadow funding cost. Because it moves with the policy rate, market power makes carry positions more sensitive to monetary shocks, increasing bond-price sensitivity to interest-rate risk and return volatility. Higher volatility raises the compensation dealers require for absorbing quantity shocks, which increases their price impact and thereby raises volatility again—an equilibrium feedback I call the volatility spiral. The mechanism lowers Treasury valuations and strengthens the transmission of both forward guidance and quantitative easing. Forward-guidance amplification is largest for short bonds and fades with maturity, whereas quantitative-easing amplification is smallest for short bonds and grows with maturity. Calibrated to U.S. Treasury and repo market data, the model implies that dealer market power accounts for $2.6 trillion, about 9%, of the market value of outstanding Treasury debt, and amplifies forward guidance by 1.8 to 2.0 times and quantitative easing by 3.9 to 6.0 times relative to a competitive repo market. Thus, reforms to repo-market structure can alter government financing costs and monetary-policy transmission well beyond the repo market itself.

Keywords: Monetary Policy, Market Power, Bond Risk Premia, Term Structure of Interest Rates

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I. Introduction

Dealers finance the large majority of their Treasury holdings through repurchase agreements (Fleming, Nguyen, and Rosenberg, 2008), borrowing overnight against Treasury collateral from cash-rich lenders such as money market funds. Repo market intermediation is also concentrated among a small number of large dealers, a feature that has drawn sustained regulatory attention. In December 2023, the Securities and Exchange Commission mandated central clearing for most Treasury repo and cash transactions, with the stated aim of making Treasury markets more competitive. Yet the asset-pricing consequences of repo-market structure are largely unknown. If dealers possess market power over their funding, how does that market power affect Treasury valuations, the term structure of risk premia, and monetary-policy transmission? I answer these questions by embedding dealer market power in repo markets into a preferred-habitat model of the term structure.

The leading preferred-habitat theory of the term structure (e.g., Vayanos and Vila, 2021) explains bond prices and monetary policy transmission through dealers who carry trade between short-term funding markets and long-term bonds, providing a natural framework for these questions. It assumes dealers take funding costs as given, an assumption that holds only under perfect competition. Financial intermediaries, however, wield substantial market power in funding markets and strategically internalize how their own borrowing moves rates (Drechsler, Savov, and Schnabl, 2017; Huber, 2023).

I develop a preferred-habitat model with monopolistic competition among dealers in repo markets, which endogenizes an upward-sloping supply curve for repo financing. The model adds a single new object to the standard framework—dealers' market power—while preserving its tractable affine term structure. Dealers internalize that expanding repo borrowing raises their funding rate and therefore strategically restrict their carry positions, earning a rent proportional to the policy rate, in line with Drechsler et al. (2017).

This rent enters the dealer's first-order condition as the marginal shadow funding cost of expanding a repo-financed position. Because it is proportional to the policy rate, a one-unit change in the policy rate changes this marginal shadow funding cost by more than one unit. Market power therefore has two distinct effects on carry trades: it reduces positions in levels, but makes those positions more sensitive to movements in the policy rate. In response to a given monetary shock, dealers rebalance more aggressively than competitive dealers do, increasing the bond-price loading on the short

rate and hence bond return volatility.

This initial increase in volatility starts a separate equilibrium feedback. In the first round, a more volatile bond requires greater compensation when dealers absorb a quantity shock, so the same shock has a larger price impact, increasing price sensitivity to quantity shock. That larger price sensitivity itself adds to return volatility, which further increases the compensation required for quantity risk and magnifies price sensitivity to quantity shock again. The feedback continues until the equilibrium fixed point is reached. I call this amplification mechanism the "*volatility spiral*." Market power starts the mechanism and the volatility spiral amplifies it. It carries three consequences for the term structure and monetary-policy transmission.

First, market power increases bond risk premia at every maturity, and increasingly so at longer maturities. The expected excess bond return includes both the rent dealers extract from their market power and the amplified compensation for duration risk. I use the Herfindahl-Hirschman Index (HHI) of repo borrowing from money market funds as a time-varying proxy for repo-market concentration. I document that HHI correlates with the 10-year term premium from Adrian, Crump, and Moench (2013) at 0.72 and predicts a 3.2 percentage point higher one-year excess return on 10-year bonds per standard deviation increase in concentration.

Second, with market power, the term premium responds more strongly to interest rate shocks. When the short rate falls, the yield curve steepens and long-term bond prices rise. Preferred-habitat investors demand fewer long-maturity bonds. Dealers must absorb this additional supply, increasing their exposure to interest rate risk and therefore requiring higher risk premia. The risk premium's response to changes in dealers' holdings is determined by the quantity of risk each unit of bond carries, which is amplified by market power through the volatility spiral. Therefore, dealer market power amplifies the sensitivity of bond returns to the yield curve slope: a given change in slope now maps into a much larger change in the compensation dealers require, so a steeper curve predicts a larger excess return under market power than under competition (Fama and Bliss, 1987). I document that moving up one HHI decile is associated with a 0.3 to 1 increase in the Fama–Bliss coefficient, depending on maturity.

Third, dealer market power amplifies the transmission of both forward guidance and quantitative easing, but with sharply different maturity profiles. Two forces shape these profiles. Through the rent-extraction channel, market power makes dealers' marginal shadow funding cost respond more

than one-for-one to the policy rate, directly increasing bond-price sensitivity to interest-rate shocks. The volatility spiral adds a term-premium channel that grows with maturity because market power amplifies the quantity of risk carried by each bond. For forward guidance, the term-premium response works against the direct effect of policy on forward rates, generating the underreaction as in the standard preferred-habitat model. Because this offset grows with maturity and is itself amplified by market power, the overall amplification declines along the yield curve. It is therefore strongest at short maturities, consistent with the high- versus low-concentration comparison in the data. For quantitative easing, the term-premium channel instead reinforces the direct effect of the quantity shock. Market power's amplification of quantity-shock pass-through therefore grows with maturity and is largest at long maturities.

I quantify consequences of dealer market power for the Treasury market by comparing the calibrated model against the competitive counterfactual at the same common structural parameters. Because dealer market power raises yields at every maturity relative to a competitive counterfactual, it lowers the market value of outstanding Treasury debt and raises the government's cost of borrowing. Discounting the current maturity structure of outstanding U.S. marketable Treasury debt against the model's yield curve, this effect is worth roughly $2.6 trillion, about 9% of the market value of outstanding debt. Moreover, dealer market power also strengthens the transmission of monetary policy. The same 100 basis point forward-guidance move shifts the ten-year yield by roughly 1.9× more under dealer market power than under competition, and the response to quantitative easing is 3.9–6.0× larger, growing monotonically with maturity. A more competitive repo market would therefore lower the government's cost of borrowing, at the cost of weaker monetary policy transmission.

This paper extends four strands of literature. First, this paper contributes to the preferred-habitat framework for the term structure of interest rates. The canonical model of Vayanos and Vila (2021) offers a tractable equilibrium in which risk-averse arbitrageurs intermediate between segmented markets, with rich implications for how shocks propagate through the yield curve. This framework has traveled well beyond Treasury markets, into mortgage-backed securities (e.g., Hanson, 2014; Malkhozov, Mueller, Vedolin, and Venter, 2016), foreign exchange (Gourinchas, Ray, and Vayanos, 2025), corporate bonds (Cavaleri, 2023), and interest rate swaps (Khetan, Li, Neamțu, and Sen, 2023). He, Nagel, and Song (2022) explicitly model the role of repo market in this framework and apply it to Treasury prices during the COVID-19 crisis. However, none of these studies incorporate dealer market power in

funding markets. I add this empirically well-documented feature to the preferred-habitat framework and trace its implications for bond pricing and monetary policy transmission, while preserving the affine structure of bond prices that makes the framework tractable, so the mechanism should extend readily wherever the framework is already applied.

Second, this paper contributes to the literature on imperfect competition in financial intermediation. The deposit channel of Drechsler et al. (2017) shows that intermediary market power shapes monetary policy transmission. When policy rates rise, banks with deposit-market power pass through the policy shock incompletely to deposit rates, widening deposit spreads, inducing deposit outflows, and contracting lending. This insight has spurred a growing literature on banking market power and monetary policy (e.g., Xiao, 2020; Wang, Whited, Wu, and Xiao, 2022). This literature, however, focuses primarily on deposit and lending markets and does not develop the implications of intermediary market power for the term structure of interest rates. Huber (2023) documents substantial dealer market power in repo markets, arising from money market funds' aversion to concentrating their cash allocations among a small number of dealers. Because repo markets finance most of dealers' Treasury positions (Fleming et al., 2008), a natural parallel to the deposit channel emerges: dealers intermediate between money market funds and bond markets through repo financing, much as banks intermediate between depositors and borrowers. I bring this market-power mechanism into a term structure model and show that it initiates an equilibrium feedback that amplifies bond-return volatility.

More broadly, this volatility spiral belongs to a class of financial amplification mechanisms in which intermediary balance sheets, asset prices, and endogenous risk mutually reinforce one another (Brunnermeier and Pedersen, 2009; Brunnermeier and Sannikov, 2014). In Brunnermeier and Pedersen (2009), deteriorating market liquidity feeds back through tighter margins and intermediary losses; in Brunnermeier and Sannikov (2014), declining asset prices erode intermediary net worth, which further depresses asset prices. The feedback in this paper instead operates through the equilibrium compensation for quantity risk: dealer market power initially raises bond exposure to short-rate shocks, and the resulting volatility increases the compensation required for absorbing quantity shocks, further increasing price impact and volatility.

Third, this paper speaks to the oversensitivity of long-term rates to short-rate changes, a well-documented puzzle (e.g., Hanson and Stein, 2015; Hanson, Lucca, and Wright, 2021). A growing literature explains this within the preferred-habitat framework by assuming short-rate shocks positively

correlate with supply shocks from preferred-habitat investors (e.g., Hanson, 2014; Malkhozov et al., 2016; Hanson et al., 2021; Carboni and Ellison, 2022). As Greenwood, Hanson, and Vayanos (2023b) note, this mechanism is equivalent to assuming that preferred-habitat investors have downward-sloping demand curves with respect to yields, $\alpha(\tau) < 0$ in my model, which conflicts with the positive relationship between bond risk premia and yield curve slope (Fama and Bliss, 1987) that requires $\alpha(\tau) > 0$. My model bridges this gap: dealer market power generates yield oversensitivity while preserving $\alpha(\tau) > 0$, since the rent extraction channel amplifies dealers' response to short-rate shocks without disturbing the positive relation between term premia and yield curve slope. Both forward-rate sensitivity and Fama-Bliss coefficients rise with dealer market power.

Fourth, this paper connects to the literature on quantity shocks and term premia, including the effects of Treasury issuance and central bank asset purchases on the yield curve (e.g., Greenwood and Vayanos, 2014a; Greenwood et al., 2023b; Greenwood, Hanson, Stein, and Sunderam, 2023a), typically studied under price-taking dealers. I show that the market power channel amplifies quantity-shock pass-through, and that this amplification grows with maturity. At the paper's calibration, this channel alone makes a quantitative easing program 3.9–6.0 times more effective in a concentrated repo market than in a competitive one, so dealer market power is a first-order determinant of how effectively balance-sheet policy transmits to long-term yields.

II. Model

The model has three ingredients: dealers with market power in the repo market, a short-rate shock and a quantity shock, and a repo spread arising from a funding-cost regulation on the money market lender. This section characterizes the model's equilibrium in full generality, with the regulation-cost parameter ψ unrestricted. Section III then sets $\psi = 0$ for tractability and specializes the resulting equilibrium channel by channel: Section III.A sets the demand-shock volatility to zero as well, to obtain closed-form results for the short-rate channel, while Section III.B restores the demand shock and asks how market power shapes the pass-through of quantity shocks to the yield curve. Section V restores $\psi \neq 0$ and calibrates the combined model to the data.

A. Setup

Money market. A repurchase agreement (repo) is a collateralized short-term loan where one party temporarily sells Treasury securities to another with an agreement to repurchase them at a specified price, with the price difference determining the repo rate. Treasury dealers are the main cash borrowers, using repo financing to fund their bond inventories, while money market funds (MMFs) are the dominant cash lenders. Huber (2023) documents that MMFs exhibit *concentration aversion* when allocating cash across dealers: MMFs simultaneously accept different repo rates from different dealers, with dealer identity driving persistent cross-sectional rate dispersion. This concentration aversion gives dealers market power. The Federal Reserve's Overnight Reverse Repurchase Agreement (ON-RRP) facility serves as a crucial outside option for MMFs, effectively setting a floor on repo rates.

I formalize this market power as follows. There are overlapping generations of lenders and repo-financing arbitrageurs (*dealers*), each living for an infinitesimal period dt. At each instant t, a representative money market fund manager with cash Q chooses the investment $D_{i,t}\,dt$ placed with each dealer $i \in [0,1]$, taking dealers' repo rates $\{r_{i,t}\}$ as given, and places the remainder with the outside option at rate r_t. Following Huber (2023) and Drechsler et al. (2017), the manager's concentration aversion is captured by a constant elasticity of substitution (CES) aggregator over dealers. In addition, the manager's aggregate repo exposure is subject to a regulation cost, e.g., a balance-sheet or leverage-ratio cost, that rises with the scale of that exposure. The manager's problem is

$$\max_{\{D_{i,t}\}} (Q - D_t) r_t + \left(\int_0^1 (D_{i,t} r_{i,t})^{1-\frac{1}{\eta}}\, di \right)^{\frac{1}{1-\frac{1}{\eta}}} - \frac{\psi}{2Q} D_t^2, \qquad D_t \equiv \int_0^1 D_{i,t}\, di, \tag{1}$$

where $\psi \geq 0$ is the regulation-cost parameter and $\eta \in (1,\infty)$ is the elasticity of substitution across dealers: as $\eta \to \infty$ investments become perfectly substitutable and the repo market is perfectly competitive.

I solve the manager's problem in two steps. First, given the aggregate investment D_t, the manager allocates optimally across dealers; the regulation cost depends only on the aggregate D_t, so it does not affect this first-stage allocation.

Proposition 1 (**Optimal Allocation Across Dealers**): *Given the total investment D_t and private repo rates $\{r_{i,t}\}$, the optimal cash investment to dealer i is given by:*

$$D_{i,t} = \left(\frac{r_{i,t}}{r_{d,t}} \right)^{\eta - 1} D_t, \tag{2}$$

where $r_{d,t}$ is the aggregate private repo rate that is given by CES average of the individual repo rates $\{r_{i,t}\}$:

$$r_{d,t} \equiv \left(\int_0^1 r_{i,t}^{\eta-1} \, di \right)^{\frac{1}{\eta-1}}. \tag{3}$$

The elasticity of repo funding supply to repo rate $r_{i,t}$ is hence given by

$$\nu_i \equiv \frac{\partial \ln D_{i,t}}{\partial \ln r_{i,t}} = \eta - 1 \equiv \frac{1}{\mathscr{M}}. \tag{4}$$

The proof is in Appendix A.1. I let $\mathscr{M} \equiv 1/(\eta - 1)$ measure dealers' market power: a higher $\mathscr{M}$ means lenders are less sensitive to rate competition among dealers, so $\mathscr{M} = 0$ corresponds to price-taking dealers. Throughout the theoretical results below, I maintain $\mathscr{M} \in (0, \bar{\mathscr{M}})$, a single threshold characterized in Appendix A.11 and invoked by several propositions.[1]

Second, the value of allocating D_t optimally across dealers is $D_t r_{d,t}$, independent of η, as in the frictionless CES aggregator, so the manager's problem in equation (1) reduces to a choice of the aggregate investment alone:

$$\max_{D_t} \; D_t r_{d,t} - D_t r_t - \frac{\psi}{2Q} D_t^2. \tag{5}$$

The first-order condition gives the equilibrium repo spread endogenously:

$$r_{d,t} = r_t + \psi \frac{D_t}{Q}. \tag{6}$$

When $\psi = 0$, equation (6) reduces to $r_{d,t} = r_t$: the manager is willing to invest any amount of cash as long as dealers offer the aggregate repo rate equal to the outside option. For $\psi > 0$, the repo rate rises with the manager's aggregate exposure D_t, generating a repo spread that comoves with dealers' bond holdings. To close the money market, the outside option return r_t follows an exogenous process:

$$dr_t = \kappa_r(\bar{r} - r_t)\, dt + \sigma_r \, dB_{r,t}, \tag{7}$$

where $\kappa_r > 0$ is the speed of mean reversion, $\bar{r}$ is the long-run mean, $\sigma_r > 0$ is the volatility, and $\{B_{r,t} :$

[1] At this paper's calibration, $\bar{\mathscr{M}} \approx 6.4$, verified numerically in Appendix A.11.1, more than six times the value $\mathscr{M} = 1.0$ that Huber (2023) estimates and that this paper uses throughout its quantitative results (Section V.B). Appendix A.11.2 shows $\bar{\mathscr{M}}$ stays above three times the calibrated value under one-at-a-time perturbations of the underlying primitives.

$0 \leq t < \infty\}$ is a standard Brownian motion.

Bond market. The bond market follows a standard two-demand-factor version of the preferred-habitat model in Vayanos and Vila (2021). At each time t, there is a continuum of zero-coupon, default-free bonds with face value one that mature at time $t+\tau$ where $\tau \in (0, T]$. I denote the time-t price of the bond that matures at $t+\tau$ by $P_t^{(\tau)}$. The yield of the bond with maturity τ at time t is

$$y_t^{(\tau)} \equiv -\frac{\log\left(P_t^{(\tau)}\right)}{\tau}, \tag{8}$$

and the forward rate between maturities $\tau - \Delta\tau$ and τ at time t is

$$f_t^{(\tau-\Delta\tau,\tau)} \equiv \frac{\log P_t^{(\tau-\Delta\tau)} - \log P_t^{(\tau)}}{\Delta\tau}. \tag{9}$$

Following Vayanos and Vila (2021), I consider an exogenous demand curve from preferred-habitat investors for bonds with different maturities. The dollar value of bond holdings by preferred-habitat investors with maturity τ at time t is denoted by $H_t^{(\tau)}$, given by

$$H_t^{(\tau)} = -\alpha(\tau)\log P_t^{(\tau)} - \beta_t^{(\tau)}, \tag{10}$$

where $\alpha(\tau) > 0$ is a maturity-dependent constant capturing the price elasticity of habitat investors for maturity τ, and $\beta_t^{(\tau)}$ is a time-varying demand intercept:

$$\beta_t^{(\tau)} = \theta_0(\tau) + \theta(\tau)\beta_t, \tag{11}$$

where $\theta_0(\tau)$ is a constant baseline demand and $\theta(\tau)$ is the maturity-specific loading on the aggregate quantity shock β_t. The shock β_t captures time-varying demand pressure from preferred-habitat investors, e.g., insurance companies, pension funds, central banks, and follows an Ornstein-Uhlenbeck process:

$$d\beta_t = -\kappa_\beta \beta_t\, dt + \sigma_\beta\, dB_{\beta,t}, \tag{12}$$

where $\kappa_\beta > 0$ is the speed of mean reversion, $\sigma_\beta > 0$ is the volatility, and $B_{\beta,t}$ is a standard Brownian motion independent of $B_{r,t}$. A positive β_t corresponds to an exogenous supply shock, reduced habitat demand, while negative β_t represents a demand shock.

Dealers' problem. The dealer sector consists of a continuum of dealers indexed by $i \in [0,1]$. Following Vayanos and Vila (2021) and He et al. (2022), there are overlapping generations of dealers living over infinitesimal periods. The generation born at time t engages in repo-financed carry trades from t to $t+dt$. Specifically, they borrow short-term funds through repos at rate $r_{i,t}$ and invest in longer-term bonds, consume at $t+dt$, and then die. Therefore, they maximize a mean–variance objective over instantaneous changes in wealth:

$$\max_{\{X_{i,t}^{(\tau)}\}_{\tau\in[0,T]}} \mathbb{E}_t\left[dW_{i,t}\right] - \frac{\gamma}{2}\mathrm{Var}_t\left[dW_{i,t}\right] \tag{13}$$

where the wealth change over the period $[t, t+dt]$ is

$$dW_{i,t} = \int_0^T X_{i,t}^{(\tau)}\left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_{i,t}dt\right)d\tau \tag{14}$$

where $X_{i,t}^{(\tau)}$ is the dollar amount borrowed through repos at rate $r_{i,t}$ to invest in bonds with maturity τ. Notably, the dealer's repo rate $r_{i,t}$ endogenously depends on her bond holdings through an upward-sloping supply curve of repo funding derived from the lender's optimal allocation in Proposition 1.

B. Equilibrium

I focus on the symmetric monopolistic competition equilibrium in which individual dealers employ the same strategy. In aggregate, the dealer sector has $X_t^{(\tau)} = X_{i,t}^{(\tau)}$ amount of direct Treasury holdings with maturity τ and finances the total bond holding $X_t = \int_0^T X_t^{(\tau)} d\tau$ by borrowing $D_t = D_{i,t}$ from the money market lender at the repo rate $r_{d,t} = r_{i,t}$.

Definition 1 (**Symmetric Monopolistic Competition Equilibrium**): *A symmetric monopolistic competition equilibrium is an allocation* $\{D_t, X_t^{(\tau)}, \{D_{i,t}, X_{i,t}^{(\tau)}\}_{i\in[0,1]}\}_{\tau\in[0,T]}$ *and prices* $\{P_t^{(\tau)}, r_{d,t}, \{r_{i,t}\}_{i\in[0,1]}\}_{\tau\in[0,T]}$ *such that the following conditions hold:*

1. *The allocation and repo rate setting are symmetric across dealers:* $D_{i,t} = D_t$, $X_{i,t}^{(\tau)} = X_t^{(\tau)}$, *and* $r_{i,t} = r_{d,t}$ *for all* $i \in [0,1]$.

2. *Given the prices and bond holdings of all other dealers, for each dealer* $i \in [0,1]$*, the bond holding* $X_{i,t}^{(\tau)} = X_t^{(\tau)}$ *solves the dealer's problem in equation* (13).
3. *The aggregate repo rate* $r_{d,t}$ *satisfies the repo-spread equation* (6).
4. *The repo market clears:* $D_t = \int_0^T X_t^{(\tau)} d\tau$.
5. *The bond market clears:* $X_t^{(\tau)} + H_t^{(\tau)} = 0$.

Under this equilibrium definition, I can solve for the equilibrium bond prices $\{P_t^{(\tau)}\}_{\tau=0}^T$. My key innovation is to incorporate dealers' market power in setting repo borrowing rates through the lender's optimal allocation in Proposition 1. Different from the standard preferred-habitat model in Vayanos and Vila (2021) where dealers take funding rates as given, here the funding cost for dealer i endogenously depends on her bond holding $X_{i,t} = \int_0^T X_{i,t}^{(\tau)} d\tau$. To solve this equilibrium, I first characterize the supply curve of repo funding for an individual dealer i. Substituting the symmetric equilibrium condition $r_{i,t} = r_{d,t}$ for other dealers, $D_t = X_t$, and the repo market clearing condition $D_{i,t} = X_{i,t}$ into equation (2), I obtain the funding cost for dealer i as a function of her bond holding:

$$r_{i,t} = r_{d,t} \left(\frac{X_{i,t}}{X_t} \right)^{\mathscr{M}}, \tag{15}$$

where $r_{d,t}$ and X_t are taken as given by dealer i, as she is too small to affect the aggregate dealer bond holding and repo rate. This implies that the elasticity of dealer i's repo rate with respect to her bond holding is

$$\frac{\partial \ln r_{i,t}}{\partial \ln X_{i,t}} = \mathscr{M}. \tag{16}$$

I conjecture that equilibrium yields are affine in the two risk factors r_t and β_t. There exist three functions $A_r(\tau)$, $A_\beta(\tau)$, and $C(\tau)$ depending only on τ such that the time-t price of the bond with maturity τ is

$$P_t^{(\tau)} = e^{-[A_r(\tau) r_t + A_\beta(\tau)\beta_t + C(\tau)]}. \tag{17}$$

Ito's lemma implies

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} = \mu_t^{(\tau)} dt - A_r(\tau)\sigma_r dB_{r,t} - A_\beta(\tau)\sigma_\beta dB_{\beta,t} \tag{18}$$

where $\mu_t^{(\tau)}$ is the expected return of bonds with maturity τ at time t, given by

$$\mu_t^{(\tau)} \equiv A_r'(\tau) r_t + A_\beta'(\tau)\beta_t + A_r(\tau)\kappa_r(r_t - \bar{r}) + A_\beta(\tau)\kappa_\beta\beta_t + \frac{1}{2}A_r(\tau)^2\sigma_r^2 + \frac{1}{2}A_\beta(\tau)^2\sigma_\beta^2 + C'(\tau). \tag{19}$$

Dealer i's problem can be rewritten as

$$\max_{\{X_{i,t}^{(\tau)}\}_{\tau\in[0,T]}} \int_0^T X_{i,t}^{(\tau)}\left(\mu_t^{(\tau)} - r_{i,t}\right)d\tau - \frac{\gamma}{2}\left[\sigma_r^2\left(\int_0^T X_{i,t}^{(\tau)}A_r(\tau)d\tau\right)^2 + \sigma_\beta^2\left(\int_0^T X_{i,t}^{(\tau)}A_\beta(\tau)d\tau\right)^2\right] \tag{20}$$

where dealer i's repo rate $r_{i,t}$ is endogenously determined by her bond holding through the funding supply curve in equation (15). The first-order condition is

$$\mu_t^{(\tau)} - r_{i,t} = \gamma\sigma_r^2 A_r(\tau)\int_0^T X_{i,t}^{(u)}A_r(u)du + \gamma\sigma_\beta^2 A_\beta(\tau)\int_0^T X_{i,t}^{(u)}A_\beta(u)du + \frac{\partial r_{i,t}}{\partial X_{i,t}^{(\tau)}}\int_0^T X_{i,t}^{(u)}du. \tag{21}$$

The right-hand side has three components: risk compensation for interest rate risk, risk compensation for quantity shock risk, and the market power rent. Using equation (15), the rent term is

$$\frac{\partial r_{i,t}}{\partial X_{i,t}^{(\tau)}}\int_0^T X_{i,t}^{(u)}du = \frac{\partial r_{i,t}}{\partial X_{i,t}}X_{i,t} = \mathscr{M} r_{i,t}, \tag{22}$$

proportional to $\mathscr{M}$ and $r_{i,t}$ and vanishes when $\mathscr{M} = 0$. Applying symmetric equilibrium conditions $X_{i,t}^{(\tau)} = X_t^{(\tau)}$ and $r_{i,t} = r_{d,t}$ to equation (21) yields the following.

Lemma 1 (**Dealer's First-Order Condition in Symmetric Equilibrium**): *The dealer's first-order condition in symmetric equilibrium is*

$$\mu_t^{(\tau)} - r_{d,t} = -A_r(\tau)\lambda_{r,t} - A_\beta(\tau)\lambda_{\beta,t} + \mathscr{M} r_{d,t}, \tag{23}$$

where $\lambda_{r,t}$ and $\lambda_{\beta,t}$ are the endogenous prices of interest rate and quantity shock risk:

$$\lambda_{r,t} \equiv -\gamma\sigma_r^2\int_0^T X_t^{(u)}A_r(u)du, \qquad \lambda_{\beta,t} \equiv -\gamma\sigma_\beta^2\int_0^T X_t^{(u)}A_\beta(u)du, \tag{24}$$

and $\mathscr{M}$ measures dealers' market power.

The coefficients $-A_r(\tau)$ and $-A_\beta(\tau)$ are the factor loadings of the τ-maturity bond on interest rate risk and quantity-shock risk, respectively. The first two right-hand-side terms are the corresponding

risk compensations, endogenously determined by aggregate dealer holdings. The third term, $\mathscr{M} r_{d,t}$, is the market-power rent: dealers internalize that expanding positions raises their funding rate and, by strategically restricting carry trades, capture a rent proportional to market power $\mathscr{M}$ and the funding rate $r_{d,t}$, consistent with the deposit channel of Drechsler et al. (2017).

Equation (23) isolates the new object introduced by dealer market power. The rent $\mathscr{M} r_{d,t}$ enters the first-order condition as the marginal shadow funding cost of expanding a repo-financed position. Section III.A.2 traces its direct effect on the bond-price loading on the short rate; Section III.B.2 shows how that initial amplification feeds back through bond volatility into the loading on the quantity shock.

Bond market clearing requires dealers' holdings equal the negative of habitat investors' demand:

$$X_t^{(\tau)} = \theta_0(\tau) + \theta(\tau)\beta_t - \alpha(\tau)\left[A_r(\tau) r_t + A_\beta(\tau)\beta_t + C(\tau)\right]. \tag{25}$$

Aggregating equation (25) across maturities and substituting into the repo-spread equation (6) shows that the equilibrium repo rate is itself affine in (r_t, β_t):

$$r_{d,t} = d_0 + d_r\, r_t + d_\beta\, \beta_t, \tag{26}$$

with

$$\begin{aligned} d_r &\equiv 1 - \psi_Q \int_0^T \alpha(\tau) A_r(\tau)\, d\tau, \qquad d_\beta \equiv \psi_Q \left(\int_0^T \theta(\tau)\, d\tau - \int_0^T \alpha(\tau) A_\beta(\tau)\, d\tau \right), \\ d_0 &\equiv \psi_Q \left(\int_0^T \theta_0(\tau)\, d\tau - \int_0^T \alpha(\tau) C(\tau)\, d\tau \right), \qquad \psi_Q \equiv \psi / Q. \end{aligned} \tag{27}$$

Substituting equations (19), (25), and (26) into Lemma 1 gives an equation affine in both r_t and β_t. Equating coefficients yields the following ODE system.

Proposition 2 (**Equilibrium Yield Curve With Repo Spread**): *The functions $A_r(\tau)$, $A_\beta(\tau)$, and $C(\tau)$ satisfy the system of ODEs:*

$$A_r(\tau)(\kappa_r + \Omega_{rr}) + A_r'(\tau) + A_\beta(\tau)\Omega_{\beta r} = (1 + \mathscr{M})\, d_r, \tag{28}$$

$$A_\beta(\tau)(\kappa_\beta - \Omega_{\beta\beta}) + A_\beta'(\tau) - A_r(\tau)\Omega_{r\beta} = (1 + \mathscr{M})\, d_\beta, \tag{29}$$

$$C'(\tau) + \tfrac{1}{2}\sigma_r^2 A_r(\tau)^2 + \tfrac{1}{2}\sigma_\beta^2 A_\beta(\tau)^2 - A_r(\tau)\kappa_r \bar{r} - A_r(\tau)\Omega_r - A_\beta(\tau)\Omega_\beta = (1 + \mathscr{M})\, d_0, \tag{30}$$

where

$$\Omega_{fr} \equiv \gamma\sigma_f^2 \int_0^T \alpha(u) A_f(u) A_r(u)\, du, \quad f \in \{r, \beta\}, \tag{31}$$

$$\Omega_{f\beta} \equiv \gamma\sigma_f^2 \int_0^T A_f(u)\big(\theta(u) - \alpha(u) A_\beta(u)\big)\, du, \quad f \in \{r, \beta\}, \tag{32}$$

$$\Omega_f \equiv \gamma\sigma_f^2 \int_0^T \big(\theta_0(u) - \alpha(u) C(u)\big) A_f(u)\, du, \quad f \in \{r, \beta\}, \tag{33}$$

d_r, d_β, d_0 *are given in equation* (27)*, and boundary conditions are* $A_r(0) = 0$*,* $A_\beta(0) = 0$*, and* $C(0) = 0$*. When* $\psi = 0$*,* $d_r = 1$ *and* $d_\beta = d_0 = 0$*, recovering* $r_{d,t} = r_t$ *and the tractable special case of Proposition* 3 *in Section III exactly.*

The system is a fixed point in the nine scalars $\{\Omega_{rr}, \Omega_{\beta r}, \Omega_{r\beta}, \Omega_{\beta\beta}, \Omega_r, \Omega_\beta, d_r, d_\beta, d_0\}$, all themselves integrals of A_r, A_β, C: the six risk-compensation integrals, plus three scalars characterizing the affine repo rate. Despite depending on the entire yield curve through D_t, the equilibrium repo rate $r_{d,t}$ remains affine in (r_t, β_t), so the repo-spread regulation cost does not break the affine structure of the model; it only enlarges the fixed point from six scalars to nine. Section III sets $\psi = 0$ for tractability and characterizes each risk factor's pass-through channel in closed form; Section V restores $\psi > 0$ and solves the full nine-scalar fixed point numerically to take the model to the data.

III. Theoretical Results

For tractability, I set the regulation-cost parameter $\psi = 0$ for the remainder of this section; the section V restores $\psi > 0$ and shows that all qualitative results in this section remain valid. By equation (6) this means $r_{d,t} = r_t$ identically, so $d_r = 1$ and $d_\beta = d_0 = 0$ in equation (26), and Lemma 1's first-order condition specializes to

$$\mu_t^{(\tau)} - r_t = -A_r(\tau)\lambda_{r,t} - A_\beta(\tau)\lambda_{\beta,t} + \mathscr{M} r_t. \tag{34}$$

Substituting this simplification into Proposition 2 of Section II gives the following.

Proposition 3 (**Equilibrium Yield Curve**): *When* $\psi = 0$*, the functions* $A_r(\tau)$*,* $A_\beta(\tau)$*, and* $C(\tau)$ *satisfy the*

system of ODEs:

$$A_r(\tau)(\kappa_r + \Omega_{rr}) + A'_r(\tau) + A_\beta(\tau)\Omega_{\beta r} = (1 + \mathscr{M}), \tag{35}$$

$$A_\beta(\tau)(\kappa_\beta - \Omega_{\beta\beta}) + A'_\beta(\tau) - A_r(\tau)\Omega_{r\beta} = 0, \tag{36}$$

$$C'(\tau) + \tfrac{1}{2}\sigma_r^2 A_r(\tau)^2 + \tfrac{1}{2}\sigma_\beta^2 A_\beta(\tau)^2 - A_r(\tau)\kappa_r\bar{r} - A_r(\tau)\Omega_r - A_\beta(\tau)\Omega_\beta = 0, \tag{37}$$

with $\Omega_{rr}, \Omega_{\beta r}, \Omega_{r\beta}, \Omega_{\beta\beta}, \Omega_r, \Omega_\beta$ *given in equation* (33) *and boundary conditions* $A_r(0) = 0$, $A_\beta(0) = 0$, *and* $C(0) = 0$.

The system is a fixed point in the six scalars $\{\Omega_{rr}, \Omega_{\beta r}, \Omega_{r\beta}, \Omega_{\beta\beta}, \Omega_r, \Omega_\beta\}$, all themselves integrals of A_r, A_β, C. In the $\sigma_\beta = 0$ special case, this reduces to the scalar fixed point for κ_r^* solved in closed form in Section III.A; in the general $\sigma_\beta > 0$ case it is solved numerically by the dedicated closed-form-affine solver described in Section III.B and Section V.B's Implementation paragraph. Section V restores $\psi > 0$ and calibrates the combined model to the data.

A. Short-Rate Risk

This section specializes the general model to the case without demand-shock risk as well ($\sigma_\beta = 0$). This special case admits closed-form results that make the market power mechanism transparent.

A.1. Equilibrium

To obtain closed-form results and isolate the market power mechanism, I set $\sigma_\beta = 0$, so that $\beta_t = 0$ for all t. The bond price then depends only on the short rate:

$$P_t^{(\tau)} = e^{-A_r(\tau)r_t - C(\tau)}. \tag{38}$$

Setting $\sigma_\beta = 0$ (so $A_\beta(\tau) \equiv 0$) in Proposition 3 above reduces the ODE system to two equations that admit the following analytical solution.

Proposition 4 (Equilibrium Yield Curve Without Demand Risk): *When $\sigma_\beta = 0$, we have:*

$$A_r(\tau) = \frac{(1+\mathscr{M})(1-e^{-\kappa_r^*\tau})}{\kappa_r^*}, \tag{39}$$

$$C(\tau) = \kappa_r^*\bar{r}^* \int_0^\tau A_r(u)\,du - \frac{\sigma_r^2}{2}\int_0^\tau A_r(u)^2\,du, \tag{40}$$

where the risk-neutral mean reversion κ_r^* and long-run mean $\bar{r}^*$ solve the fixed-point equations:

$$\kappa_r^* = \kappa_r + \gamma\sigma_r^2(1+\mathscr{M})^2 \int_0^T \alpha(\tau)\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right)^2 d\tau, \tag{41}$$

$$\begin{aligned}\bar{r}^* = \bar{r} + \gamma\sigma_r^2\Bigg(&(1+\mathscr{M})\int_0^\infty \left[\theta_0(\tau) - (1+\mathscr{M})\bar{r}\alpha(\tau)\tau\right]\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\,d\tau \\ &+\frac{\sigma_r^2}{2}(1+\mathscr{M})^3\int_0^\infty \alpha(\tau)\left[\int_0^\tau\left(\frac{1-e^{-\kappa_r^*u}}{\kappa_r^*}\right)^2 du\right]\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\,d\tau\Bigg) \\ \Bigg/\Bigg(\kappa_r^*\Bigg[1+\gamma\sigma_r^2(1+\mathscr{M})^2&\int_0^\infty \alpha(\tau)\left[\int_0^\tau \frac{1-e^{-\kappa_r^*u}}{\kappa_r^*}\,du\right]\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\,d\tau\Bigg]\Bigg).\end{aligned} \tag{42}$$

The proof is in Appendix A.2. The results in this subsection are analytical consequences of Proposition 4. I discuss the core mechanism in Section III.A.2, examine implications for bond risk premia in Section III.A.3, and analyze monetary policy transmission in Section III.A.4; Section V.C illustrates each result numerically at the paper's calibration.

A.2. Rent Extraction and Dealers' Carry Trade

Market power affects the yield curve through dealers' strategic rent extraction. Equation (34) shows that dealers' equilibrium condition equates the marginal revenue from carry trading, $\mu_t^{(\tau)} - r_t$, to the marginal cost of holding bonds: duration risk compensation $-A_r(\tau)\lambda_{r,t}$ plus the rent term $\mathscr{M} r_t$.

The rent term $\mathscr{M} r_t$ captures a distinctive feature of imperfect competition: dealers internalize that expanding their positions raises their borrowing rate and therefore strategically limit their bond holdings. Under perfect competition, $\mathscr{M} = 0$, dealers take the repo rate as given and the rent vanishes. With market power, $\mathscr{M} > 0$, the rent enters their first-order condition as the marginal shadow funding cost of expanding a repo-financed position. Because it is proportional to r_t, a one-unit movement in the policy rate changes this marginal shadow funding cost by $(1+\mathscr{M})$. Thus, market power reduces carry positions in levels but makes those positions more sensitive to policy-rate movements. Dealers consequently rebalance more in response to a given monetary shock, amplifying the bond-price response

and return volatility. The following proposition formalizes the price-sensitivity result.

Proposition 5 (**Market Power and Bond Price Sensitivity**): *For every $\tau > 0$, the bond price's sensitivity to the short rate, $A_r(\tau; \mathscr{M})$, is strictly increasing in market power: $\partial A_r(\tau; \mathscr{M})/\partial \mathscr{M} > 0$.*

The proof is in Appendix A.3. This amplified price sensitivity gives rise to higher bond return volatility, which in turn raises both the level of risk premia and their responsiveness to the position adjustments dealers make in response to interest rate shocks.

A.3. Bond Risk Premia

Lemma 2 (**Steady-State Risk Premium**): *At steady state ($r_t = \bar{r}$), the risk premium is*

$$\mu_t^{(\tau)} - \bar{r} = (1 + \mathscr{M})\left(1 - e^{-\kappa_r^* \tau}\right)(\bar{r}^* - \bar{r}) + \mathscr{M}\bar{r}. \tag{43}$$

The proof is in Appendix A.4. Equation (43) decomposes the risk premium into two terms. The rent term $\mathscr{M}\bar{r}$ is the marginal cost a dealer incurs by expanding its repo position: flat across maturities, since it depends only on the policy rate and market power. The duration risk compensation term $(1 + \mathscr{M})(1 - e^{-\kappa_r^* \tau})(\bar{r}^* - \bar{r})$ compensates dealers for bearing interest rate risk, grows with maturity since $1 - e^{-\kappa_r^* \tau}$ is increasing in τ, and its dependence on $\mathscr{M}$ works through two opposing forces. The term $(1 + \mathscr{M})$ in equation (43) captures the volatility effect: a monopolistically competitive dealer's marginal shadow funding cost moves by $(1 + \mathscr{M})$ for a one-unit change in r_t, which amplifies the bond price's sensitivity to interest rate shocks, formalized in Proposition 5, so τ-maturity Treasury returns become more volatile, an effect that is always positive and grows with τ since longer-maturity bonds carry proportionally more of this amplified interest rate risk. The position-adjustment effect instead operates through κ_r^*, which appears inside $e^{-\kappa_r^* \tau}$ and inside $\bar{r}^*$. Greater market power raises dealers' funding cost, incentivizes dealers to take less carry-trade positions, which lowers bond prices. Since $\alpha(\tau) > 0$, preferred-habitat investors respond to lower bond prices by raising their demand for bonds, and dealers accommodate that demand by further reducing their own bond holdings. Smaller positions mean less exposure to interest rate risk, which lowers the price of interest rate risk $\lambda_{r,t}$ dealers must be compensated for. This effect on risk premium is always negative and scales in strength with $\alpha(\tau)$.

At $\alpha(\tau) \equiv 0$, dealers' holdings are fixed, so the quantity effect vanishes, $\kappa_r^* = \kappa_r$ is independent of

$\mathscr{M}$, and only the volatility effect operates. The effect of market power collapses to

$$\frac{\partial(\mu_t^{(\tau)} - \bar{r})}{\partial \mathscr{M}} = 2(1+\mathscr{M})\left(\frac{1-e^{-\kappa_r \tau}}{\kappa_r}\right)\Omega_r^0 + \bar{r} > 0, \tag{44}$$

where $\Omega_r^0 \equiv \gamma\sigma_r^2 \int_0^T \theta_0(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r}\,d\tau > 0$. The constant $\bar{r}$ is the rent term's contribution and the remaining, strictly increasing term is the volatility effect's, so the marginal effect is positive and strictly increasing in τ. As $\alpha(\tau)$ rises from zero, the quantity effect activates and strengthens, dampening the duration risk compensation term $-A_r(\tau)\lambda_{r,t}$ in equation (34) and working against the volatility effect; once $\alpha(\tau)$ is large enough, it overturns the sign or the maturity profile of the marginal effect. The following proposition formalizes this intuition.

Proposition 6 (**Effect of Market Power on Bond Risk Premia**): *Suppose $\theta_0(\tau) > 0$ and $\bar{r} > 0$. There exists $\bar{\alpha} > 0$ such that, for every demand slope $\alpha(\tau)$ satisfying $0 \le \alpha(\tau) < \bar{\alpha}$ for all $\tau \in [0, T]$, the marginal effect of market power on bond risk premia, $\partial(\mu_t^{(\tau)} - \bar{r})/\partial\mathscr{M}$, is positive and increasing in τ.*

The proof is in Appendix A.5. Moreover, market power affects the response of bond risk premia to interest rate shocks. A large empirical literature on bond pricing, starting from Fama and Bliss (1987), documents that the slope of the yield curve is a strong predictor of bond risk premia. Fama and Bliss (1987) run the following predictive regression:

$$\frac{1}{\Delta\tau}\log\left(\frac{P_{t+\Delta\tau}^{(\tau-\Delta\tau)}}{P_t^{(\tau)}}\right) - y_t^{(\Delta\tau)} = a_{\text{FB}} + b_{\text{FB}}\left(f_t^{(\tau-\Delta\tau,\tau)} - y_t^{(\Delta\tau)}\right) + e_{t+\Delta\tau} \tag{45}$$

One prominent explanation for this predictability is the preferred-habitat model of the term structure with demand slope of preferred-habitat investors $\alpha(\tau) > 0$, as in Vayanos and Vila (2021). A negative shock on the short rate steepens the yield curve and raises long-term bond prices, which induces preferred-habitat investors to demand fewer long-maturity bonds. Dealers must absorb this additional supply, increasing their exposure to interest rate risk and therefore requiring higher risk premia. This bond-holding adjustment channel generates the well-known positive relationship between the yield curve slope and bond excess returns documented in Fama and Bliss (1987). Dealers' market power amplifies the relation between bond risk premia and the slope of the yield curve through the volatility channel. Since market power leads to more volatile bond returns, the same position adjustment by dealers in response to a short-rate shock commands a larger effect on risk premia. Therefore, the

Fama-Bliss coefficient b_{FB} in equation (45) is increasing in dealers' market power $\mathscr{M}$. The following proposition formalizes this intuition.

Proposition 7 (**Slope of the Yield Curve and Bond Risk Premia**): *For $\Delta\tau \to 0$ and for all τ, the coefficient b_{FB} in FB regression in equation (45) is given by*

$$b_{\text{FB}} = 1 - \frac{\kappa_r}{\kappa_r^*} \tag{46}$$

where κ_r^* is implicitly defined as a function of $\mathscr{M}$ in equation (41). When $\gamma > 0$ and $\alpha(\tau) > 0$, b_{FB} is positive and strictly increasing in dealers' market power $\mathscr{M}$, i.e., $b_{\text{FB}} > 0$ and $\frac{\partial b_{\text{FB}}}{\partial \mathscr{M}} > 0$.

The proof of Proposition 7 is provided in Appendix A.6. To see how volatility channel works through in equation (46), substituting equation (39) into equation (41) gives

$$\kappa_r^* = \kappa_r + \gamma \int_0^T \alpha(\tau) A_r(\tau)^2 \sigma_r^2 \, d\tau. \tag{47}$$

Market power $\mathscr{M}$, through rent extraction from dealers, increases the sensitivity of bond prices to short-rate shocks $A_r(\tau)$, which raises the volatility of bond returns $A_r(\tau)^2 \sigma_r^2$ and therefore κ_r^*. Hence $b_{\text{FB}} = 1 - \frac{\kappa_r}{\kappa_r^*}$ rises with it.

The predictability of bond returns by the yield curve slope is one of the most important empirical facts in fixed income markets, and dealer market power determines its strength. The model implies a conditional forecasting pattern: bond excess returns load more heavily on the yield curve slope when dealer market power is elevated, producing a larger Fama-Bliss coefficient, and the loading weakens as competitive pressure erodes market power. I examine this prediction quantitatively in Section V.C.

This bond risk premium result is, at its core, a statement about the term premium's response to short-rate shocks: when short rates fall, the dealers must absorb more long-duration bonds from preferred-habitat investors, which increases their exposure to interest rate risk and the term premium rises in response. The volatility channel amplifies this term premium response to dealers' position adjustments. This amplified term premium response to position adjustment is a key channel through which dealer market power shapes monetary policy transmission, examined next in Section III.A.4.

A.4. Monetary Policy Transmission

This section examines how dealer market power shapes the transmission of three canonical monetary policy actions to the yield curve. A short-rate shock is the surprise component of a single policy decision, the gap between the short rate the central bank sets at a given FOMC meeting and what markets expected going in, and is naturally a short-horizon object, since on its own it carries no signal about any lasting change in policy. Forward guidance and quantitative easing, by contrast, are longer-horizon interventions closer to a regime announcement than to a single meeting's surprise: forward guidance is an unanticipated and persistent shift in the short rate's long-run target itself, and quantitative easing is an unanticipated and persistent shift in the level of habitat demand dealers must absorb. I analyze all three in turn, starting with the short-rate shock.

Impact of a short-rate shock. The Federal Reserve implements its monetary policy by adjusting the short rate r_t that shapes the repo borrowing rate for dealers. Dealers borrow short-term funds through repos and invest in longer-term bonds. Therefore, dealers transmit short-rate shocks to bond yields through their carry trades. When there is a shock to the short rate r_t, the impact on the long bond yield can be decomposed into two channels. The first is the effect through the expected future short rate, as described by the expectations hypothesis of the term structure. The second is the effect through the term premium, which depends on how dealers adjust their bond holdings in response to short-rate shocks.

Following Vayanos and Vila (2021), I measure the impact of short-rate shocks on long end of the yield curve by analyzing the reaction of forward rates. The advantage of using forward rates is that it is easy to decompose the effect of short-rate shocks into the expectations and term-premium channels. I evaluate how a time-t shock to the short rate r_t affects the expected τ-period-ahead short rate $E_t(r_{t+\tau})$ at time $t+1$ and the instantaneous forward rate for maturity τ, $f_t^{(\tau)}$. The latter rate is defined as

$$f_t^{(\tau)} \equiv \lim_{\Delta\tau \to 0} f_t^{(\tau-\Delta\tau,\tau)} = -\frac{\partial \log P_t^{(\tau)}}{\partial \tau} = A_r'(\tau) r_t + C'(\tau). \tag{48}$$

where the second equality follows from taking the limit of $\Delta\tau \to 0$ of forward rate definition in equation (9) and the last equality follows from the bond price conjecture in equation (17). It follows that the sensitivity of the instantaneous forward rate and expected future short rate to a unit shock to the

short rate r_t are given by

$$\frac{\partial f_t^{(\tau)}}{\partial r_t} = A_r'(\tau) = (1+\mathscr{M})e^{-\kappa_r^*\tau}, \quad \frac{\partial E_t(r_{t+\tau})}{\partial r_t} = e^{-\kappa_r\tau} \tag{49}$$

When the expectations hypothesis of the term structure holds, forward rates move one-to-one with expected future short rates:

$$\frac{\partial f_t^{(\tau)}}{\partial r_t} = \frac{\partial E_t(r_{t+\tau})}{\partial r_t} = e^{-\kappa_r\tau}. \tag{50}$$

This is indeed the case in the benchmark with no market power in the repo market, $\mathscr{M} = 0$, and no risk aversion, $\gamma = 0$, so that $\kappa_r^* = \kappa_r$. Introducing risk aversion, $\gamma > 0$, while keeping the market power channel muted, $\mathscr{M} = 0$, raises κ_r^* above κ_r, and the sensitivity of the forward rate to the short rate becomes

$$\frac{\partial f_t^{(\tau)}}{\partial r_t} = e^{-\kappa_r^*\tau} < e^{-\kappa_r\tau}. \tag{51}$$

In this case, I replicate the result in Vayanos and Vila (2021) that the forward rate underreacts to short-rate shocks because of a negative term premium response. With demand slope from preferred-habitat investors $\alpha(\tau) > 0$, the decline in bond prices following a short-rate hike induces preferred-habitat investors to buy long-term bonds from dealers, and the term premium falls as dealers take on less exposure to interest rate risk.

Introducing the market-power channel, $\mathscr{M} > 0$, changes this picture: forward rate sensitivity is now affected by dealers' market power and can be increasing in $\mathscr{M}$, particularly at modest maturities. Recall from equation (49) that forward rate sensitivity to short-rate shocks is $\partial f_t^{(\tau)}/\partial r_t = (1+\mathscr{M})e^{-\kappa_r^*\tau}$: market power operates through both the direct multiplier $(1+\mathscr{M})$ and the risk-neutral mean reversion speed κ_r^*, pulling forward-rate sensitivity in two opposite directions.

The direct channel operates through the multiplier $(1+\mathscr{M})$ via rent extraction: a marginal rise in the short rate raises dealers' marginal shadow funding cost by $(1+\mathscr{M})$, so forward rates react more when dealers have more market power. This positive effect declines with maturity because the shock is transitory, as captured by the term $e^{-\kappa_r^*\tau}$, which itself declines with τ.

The indirect channel operates through κ_r^* in the exponential term $e^{-\kappa_r^*\tau}$ and is the same underreac-

tion channel as in Vayanos and Vila (2021): since $\alpha(\tau) > 0$, a rise in r_t lowers bond prices and induces preferred-habitat investors to absorb more bonds from dealers, so dealers require a smaller price of interest rate risk as they take on less interest rate risk exposure, dampening the forward rate's response to the short-rate shock. As explained in Section III.A.3, market power amplifies this dampening effect through the volatility channel: higher market power raises the volatility of bond returns, reflected in a higher κ_r^*, which strengthens the negative term premium response captured by the exponential term $e^{-\kappa_r^* \tau}$. This change in κ_r^* has a larger effect on forward rate sensitivity at longer maturities, because the long term bonds have larger exposure to interest rate risk and hence larger term premium effect. Specifically, the volatility channel operates through the response of the price of interest rate risk $|\Delta\lambda_{r,t}|$ to dealers' position adjustments, and the magnitude of term premium effect at maturity τ is $A_r(\tau) \times |\Delta\lambda_{r,t}|$, where the exposure to interest rate risk $A_r(\tau)$ is increasing in τ. The negative term premium response to a short-rate shock is therefore larger at longer maturities.

Putting the two channels together: the direct channel's contribution is positive and shrinks toward zero with maturity, while the indirect channel's contribution starts at zero and grows in magnitude in the opposite direction. Market power therefore amplifies forward rate sensitivity at short maturities, where the direct channel dominates, and diminishes it at long maturities, where the indirect channel dominates.

Proposition 8 (**Market Power and Over/Underreaction Relative to the Expectations Hypothesis**): *Suppose $\mathcal{M} > 0$. There exists a unique threshold maturity*

$$\tau^{**} \equiv \frac{\ln(1+\mathcal{M})}{\kappa_r^* - \kappa_r} \tag{52}$$

such that the forward rate overreacts to a short-rate shock relative to the expectations hypothesis, for every $\tau < \tau^{**}$, and underreacts, for every $\tau > \tau^{**}$.

The proof is in Appendix A.7. Moreover, the turning point is not the end of the story: the short-rate shock itself is transitory, decaying at rate κ_r, so the entire forward-rate response, $(1+\mathcal{M})e^{-\kappa_r^* \tau}$, must vanish as $\tau \to \infty$, pulling the net effect of market power back toward zero at the very long end. Market power's effect on forward rate sensitivity is thus hump-shaped in sign: positive at short maturities, turning negative once the indirect channel dominates, and closing back toward zero at the very long end. The following proposition formalizes this intuition and characterizes the effect of market power

on forward rate sensitivity to short-rate shocks.

Proposition 9 (**Market Power and Reaction of Forward Rates**): *Let* $\mathscr{S}_t^{(\tau)} \equiv \frac{\partial}{\partial \mathscr{M}}\left(\frac{\partial f_t^{(\tau)}}{\partial r_t}\right)$ *denote the impact of dealers' market power* $\mathscr{M}$ *on the forward rate sensitivity to short-rate shocks. It satisfies the following properties:*

1. *There exists a threshold level of maturity* $\hat{\tau} \in (0,\infty)$ *such that* $\mathscr{S}_t^{(\tau)} > 0$ *if and only if* $\tau \in (0,\hat{\tau})$.
2. *There exists a threshold level of maturity* $\bar{\tau} \in (0,\infty)$ *such that* $\frac{\partial}{\partial \tau}\mathscr{S}_t^{(\tau)} < 0$ *if and only if* $\tau < \bar{\tau}$. *The effect is therefore U-shaped.*
3. *The above two properties jointly imply there exists a threshold level of maturity* $\tau^* = \min\{\hat{\tau},\bar{\tau}\}$ *such that* $\mathscr{S}_t^{(\tau)} > 0$ *and* $\frac{\partial}{\partial \tau}\mathscr{S}_t^{(\tau)} < 0$ *for all* $\tau \in (0,\tau^*)$. *Combining both properties: for maturities up to* τ^*, *market power unambiguously raises forward-rate sensitivity, by an amount that shrinks with maturity.*

The proof of Proposition 9 is provided in Appendix A.8.

Impact of forward guidance. Forward guidance is a persistent shift $\Delta\bar{r}$ in the short rate's long-run mean $\bar{r}$, unanticipated at time zero. The resulting yield-curve sensitivity is

$$\mathrm{FG}(\tau;\mathscr{M}) \equiv \frac{\partial y_t^{(\tau)}}{\partial \bar{r}}\Big|_{r_t=\bar{r}} = \frac{A_r(\tau)}{\tau} + \frac{\kappa_r}{1+K}\cdot\frac{1}{\tau}\int_0^\tau A_r(u)\,du, \qquad K \equiv \gamma\sigma_r^2\int_0^T \alpha(\tau)A_r(\tau)\left[\int_0^\tau A_r(u)\,du\right]d\tau, \quad (53)$$

with $K \geq 0$ is a scalar and $K = 0$ if and only if $\alpha(\tau) \equiv 0$. Forward guidance reaches yields through the same two channels as a short-rate shock, a direct effect and a term-premium effect, but its persistence changes both channels' behavior at long maturities. First, because forward guidance shifts the short rate's long-run mean, not its current level, the direct effect does not decay with τ the way it does for a transitory shock; it remains large even at long maturities, so it is never overtaken by the term-premium effect the way it is in the transitory case. Market power's effect on forward guidance's pass-through is therefore always positive, unlike the transitory shock, where the indirect channel eventually dominates and flips the sign negative. Second, and for the same reason, the impact of market power no longer has to vanish as $\tau \to \infty$: recall that in the transitory case, the hump shape arose precisely because the response had to decay back to zero at the very long end, forcing the effect to turn from negative back toward zero. With forward guidance's persistent effect on the long end, that pull-back-to-zero force is absent, so the effect declines monotonically instead of reversing course. Market power's effect on forward guidance's pass-through is thus always positive and largest at the short end of the curve, where the term-premium effect is weakest. The following proposition formalizes this intuition.

Proposition 10 (**Market Power's Effect on Forward Guidance**): *Let* $FG_{\mathscr{M}}(\tau) \equiv \partial FG(\tau;\mathscr{M})/\partial\mathscr{M}$. $FG_{\mathscr{M}}(\tau)$ *is strictly positive and strictly decreasing in* τ. *Market power therefore amplifies forward guidance's pass-through to yields at every maturity, an amplification that is largest at the short end of the curve.*

The proof is in Appendix A.11.

Impact of quantitative easing. Quantitative easing is an unanticipated and persistent shift ε in the level of habitat demand, applied uniformly across every maturity: $\theta_0(\tau) \to \theta_0(\tau) + \varepsilon$ for every τ. The resulting yield-curve sensitivity is

$$\mathrm{QE}(\tau;\mathscr{M}) \equiv \frac{\partial y_t^{(\tau)}}{\partial \varepsilon}\Big|_{r_t=\bar{r}} = \frac{(1+\mathscr{M})\Psi_r^*}{1+K} \cdot \frac{1}{\tau}\int_0^\tau A_r(u)\,du, \qquad \Psi_r^* \equiv \gamma\sigma_r^2 \int_0^T \frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\,d\tau > 0, \tag{54}$$

where Ψ_r^* reduces to Ψ_r of equation (44) when $\kappa_r^* = \kappa_r$. Quantitative easing's entire pass-through runs through the term-premium channel, and with the opposite sign from forward guidance's: a large habitat supply shock leaves dealers holding more of the bond, so they demand more compensation for bearing it, raising yields. Market power strengthens this response through the same volatility channel as above, and since the exposure to interest rate risk, $A_r(\tau)$, increases with maturity, so does the term-premium response. Market power's impact on quantitative easing's pass-through therefore grows with maturity.

Proposition 11 (**Market Power's Effect on QE**): *Let* $QE_{\mathscr{M}}(\tau) \equiv \partial QE(\tau;\mathscr{M})/\partial\mathscr{M}$. $QE_{\mathscr{M}}(\tau)$ *is strictly increasing in* τ, *with* $\lim_{\tau\to 0^+} QE_{\mathscr{M}}(\tau) = 0$ *and* $\lim_{\tau\to\infty} QE_{\mathscr{M}}(\tau) > 0$. *Market power's effect on quantitative easing's pass-through to yields is therefore positive and largest at the long end of the curve.*

The proof is in Appendix A.11.

Market power affects both policies through the volatility channel: it raises the change in price of interest rate risk dealers demand for a given change in their holdings, strengthening the magnitude of the term-premium response to any policy shock. Forward guidance and quantitative easing differ only in the sign of that response. Forward guidance's term-premium response is negative, offsetting a positive direct effect that market power generates equally at every maturity; quantitative easing's is positive and accounts for all of the policy's pass-through. As the term-premium effect always grows with maturity, market power's effect on forward guidance is largest at the short end of the curve, while its effect on quantitative easing is largest at the long end. Section V.C verifies both patterns

numerically at the paper's full calibration.

B. Demand Risk

This section specializes the general model of Section II to the case with the demand factor active ($\sigma_\beta > 0$). The central question is how dealer market power $\mathscr{M}$ shapes the pass-through of the quantity shock β_t to the yield curve. I first characterize the joint equilibrium loadings and then isolate the feedback through which an initial increase in short-rate exposure is compounded into a larger price response to quantity shocks.

B.1. Equilibrium

Proposition 3 above already gives the ODE system needed here: with $\sigma_\beta > 0$ kept general, equations (35) and (36) are a coupled, linear, constant-coefficient system in (A_r, A_β), with $A_r(0) = A_\beta(0) = 0$, and $C(\tau)$ follows by direct quadrature from equation (37).

Solution. Writing $A(\tau) \equiv (A_r(\tau), A_\beta(\tau))^\top$, $\mathscr{E} \equiv (1,0)^\top$, and

$$\Xi \equiv \begin{pmatrix} \kappa_r + \Omega_{rr} & \Omega_{\beta r} \\ -\Omega_{r\beta} & \kappa_\beta - \Omega_{\beta\beta} \end{pmatrix}, \tag{55}$$

equations (35)–(36) are $A'(\tau) + \Xi A(\tau) = (1+\mathscr{M})\mathscr{E}$, with solution given in the following proposition.

Proposition 12 (**Equilibrium Yield Curve With Demand-Shock Risk**): *Suppose* Ξ *in equation* (55) *is invertible and its eigenvalues have strictly positive real parts. Then*

$$A(\tau) = (1+\mathscr{M})\,\Xi^{-1}\left(I - e^{-\Xi\tau}\right)\mathscr{E}, \tag{56}$$

where I *is the* 2×2 *identity matrix and* $e^{-\Xi\tau}$ *is the matrix exponential.*

The proof is in Appendix A.9. Ξ itself depends on (A_r, A_β, C) through the Ω integrals, so equation (56) is a fixed point. I solve it numerically with the dedicated closed-form-affine solver described in Section V.B.

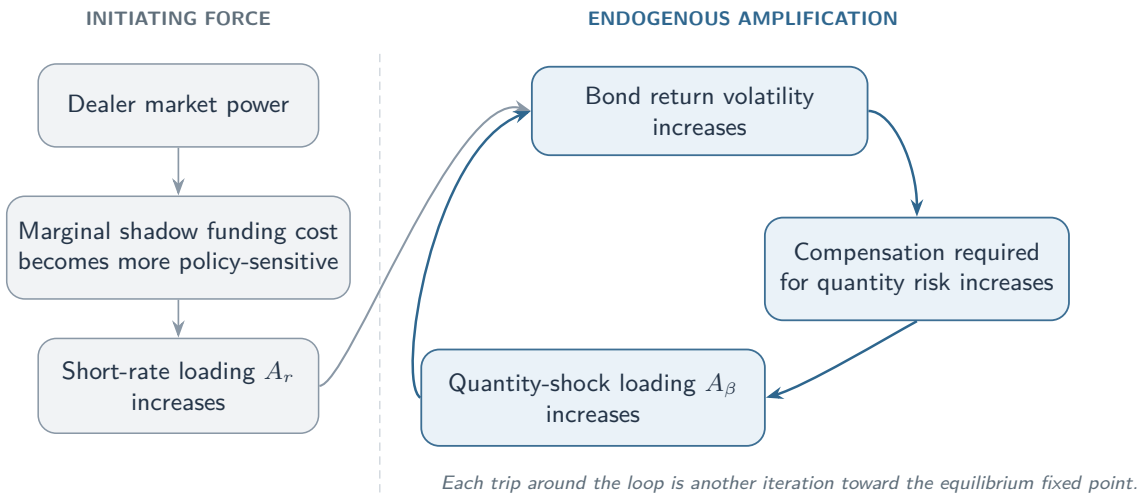


Figure 1. The Volatility Spiral

Notes: Dealer market power is the initiating force, not part of the feedback loop itself. It raises the sensitivity of the marginal shadow funding cost to the policy rate and thereby increases the bond-price loading on short-rate risk, $A_r(\tau)$. The endogenous loop runs from return volatility to the compensation required for quantity risk, to the quantity-shock loading $A_\beta(\tau)$, and back to return volatility. Each trip around the loop corresponds to another iteration of the equilibrium fixed-point map.

B.2. Volatility Spiral and Quantity-Shock Pass-Through

The key result of this section is that dealer market power amplifies quantity shocks even though the market-power rent is tied directly to the funding rate, not to the quantity shock. The reason is an equilibrium feedback. Market power first raises bonds' exposure to short-rate risk, $A_r(\tau)$, making their returns more volatile. Higher volatility increases the compensation dealers require to absorb a given quantity shock, so the shock produces a larger price response, $A_\beta(\tau)$. The larger response itself adds to return volatility, which further increases the required compensation for quantity risk and magnifies $A_\beta(\tau)$ again. The process continues until the equilibrium fixed point is reached. I refer to this self-reinforcing but convergent feedback as the *volatility spiral*. Figure 1 illustrates the mechanism. A friction originating in the funding market therefore magnifies shocks originating entirely in the bond market.

The pass-through of the quantity shock to yields at maturity τ is

$$\frac{\partial y_t^{(\tau)}}{\partial \beta_t} = \frac{A_\beta(\tau)}{\tau}. \tag{57}$$

I now show the two stages of amplification formally. Consider the special case $\psi = 0$ and $\alpha(\tau) \equiv 0$, so

dealers' holdings are fixed. Under $\alpha(\tau) \equiv 0$, $\Omega_{rr} = \Omega_{\beta r} = 0$ by equation (31), so $\kappa_r^* = \kappa_r$. With $\psi = 0$, $d_r = 1$ and $d_\beta = d_0 = 0$ by Proposition 2. In the equilibrium ODE system of Proposition 3, the market power term $\mathscr{M}$ enters only through the forcing term of the A_r equation (28):

$$A_r'(\tau) + \kappa_r A_r(\tau) = 1 + \mathscr{M}, \tag{58}$$

solved by

$$A_r(\tau) = \frac{(1+\mathscr{M})(1-e^{-\kappa_r \tau})}{\kappa_r} \tag{59}$$

as in Proposition 4. Market power's initial effect is therefore to raise $A_r(\tau)$ by the multiplier $(1+\mathscr{M})$, which raises the short-rate component of return volatility by the same factor. This initial amplification then enters the price impact of quantity shocks through the $A_r(\tau)\Omega_{r\beta}$ term in equation (29) for $A_\beta(\tau)$. Here $\Omega_{r\beta}$ is given by

$$\Omega_{r\beta} \equiv \gamma\sigma_r^2 \int_0^T A_r(u)\big(\theta(u) - \alpha(u)A_\beta(u)\big)du = \gamma\sigma_r^2 \int_0^T \theta(u)A_r(u)\,du = (1+\mathscr{M})\Theta_r, \tag{60}$$

where the second equality comes from $\alpha(\tau) = 0$ and

$$\Theta_r \equiv \gamma\sigma_r^2 \int_0^T \theta(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r}\,d\tau. \tag{61}$$

Substituting $\Omega_{r\beta}$ from equation (60) and $d_\beta = 0$ into equation (29) gives a single linear ODE with constant coefficients governing $A_\beta(\tau)$:

$$A_\beta'(\tau) + \kappa_\beta^* A_\beta(\tau) = A_r(\tau)\Omega_{r\beta} = (1+\mathscr{M})^2\,\Theta_r\,\frac{1-e^{-\kappa_r\tau}}{\kappa_r}, \qquad A_\beta(0) = 0, \tag{62}$$

where

$$\kappa_\beta^* \equiv \kappa_\beta - \Omega_{\beta\beta} = \kappa_\beta - \gamma\sigma_\beta^2 \int_0^T A_\beta(\tau)\theta(\tau)d\tau. \tag{63}$$

Equation (62) displays the first compounding directly. One factor $(1+\mathscr{M})$ arises because market power raises the short-rate loading $A_r(\tau)$; the second arises because the same amplified loading enters $\Omega_{r\beta}$. Hence the forcing term $A_r(\tau)\Omega_{r\beta}$ scales with $(1+\mathscr{M})^2$, not $(1+\mathscr{M})$. This square captures the first cross-

market round of amplification. The subsequent rounds enter through κ_β^*: because A_β appears on the right-hand side of equation (63), a larger quantity-shock loading changes the equilibrium coefficient that determines A_β itself. The following lemma gives a closed-form solution for $A_\beta(\tau)$ and establishes existence and uniqueness of the fixed point κ_β^*.

Lemma 3 (**Market Power Amplification of Quantity Shocks**): *Suppose* $\alpha(\tau) \equiv 0$, $\theta(\tau) \geq 0$ *not identically zero, and* $\sigma_\beta > 0$. *If*

$$\gamma \sigma_\beta^2 (1+\mathscr{M})^2 \Theta_r \cdot \frac{1}{2} \int_0^T \theta(u) \frac{1-e^{-\kappa_r u}}{\kappa_r} u^2 \, du < 1, \tag{64}$$

then there exists a unique κ_β^* *solving equation* (63), *and, with* Θ_r *as in equation* (61),

$$A_\beta(\tau) = (1+\mathscr{M})^2 \frac{\Theta_r}{\kappa_r} \left(\frac{1-e^{-\kappa_\beta^* \tau}}{\kappa_\beta^*} - \frac{e^{-\kappa_r \tau} - e^{-\kappa_\beta^* \tau}}{\kappa_\beta^* - \kappa_r} \right). \tag{65}$$

Moreover, the quantity-shock pass-through $A_\beta(\tau)$ *is positive and increasing in market power* $\mathscr{M}$ *for every* $\tau > 0$.

The proof is in Appendix A.10. Condition (64) is a Banach contraction-mapping bound on the fixed-point map defining κ_β^* in equation (63). Each iteration of this map corresponds to another round of the volatility spiral: a larger A_β raises return variance, which changes the compensation for quantity risk and maps into a new A_β. This interpretation parallels the feedback-loop representations in Brunnermeier and Pedersen (2009) and Brunnermeier and Sannikov (2014). The same inequality also provides the regularity needed to establish the sign of κ_β^*'s response to $\mathscr{M}$. At the calibrated values of κ_r, κ_β, and σ_β, I numerically verify the condition across a range of magnitudes and shapes for $\theta(\tau)$. It holds comfortably at the calibrated value $\mathscr{M} = 1.0$, remaining slack up to roughly $\mathscr{M} \approx 2.4$ before the bound binds.

Numerical solution of the full model also confirms that positivity and monotonicity of $A_\beta(\tau)$ in $\mathscr{M}$ survive at the paper's calibrated $\alpha > 0$, well outside the $\alpha \equiv 0$ case Lemma 3 covers analytically, and considerably beyond the calibration in either direction. Thus, the special case separates the two layers of the mechanism cleanly: the product $A_r \Omega_{r\beta}$ generates the $(1+\mathscr{M})^2$ first-round amplification, and the fixed point in κ_β^* captures the subsequent rounds of the volatility spiral.

IV. Empirical Patterns

This section presents suggestive aggregate evidence consistent with the model's predictions. The model's quantity-risk channel is a market-wide equilibrium effect: dealer market power changes bond-return volatility and, consequently, the compensation required to absorb aggregate quantity risk. Because this feedback operates through prices common to the entire Treasury market, it does not generate clean cross-sectional treatment variation across individual dealers. Nor is repo-market concentration exogenous. Regulatory and institutional changes capable of shifting market-wide concentration typically affect Treasury markets through other channels as well, precluding clean causal identification. I therefore ask a narrower question: whether periods of greater repo concentration exhibit the joint patterns in repo intermediation, Treasury risk premia, and monetary-policy sensitivity predicted by the model.

A. Data and Measurement

The sample runs from January 2011 through September 2025, with its start determined by the availability of money market funds' repo holdings. Since October 2010, money market funds have reported their portfolio holdings to the SEC through Form N-MFP. I use the Office of Financial Research's Money Market Monitor, which aggregates these filings to the dealer-counterparty level, and construct the Herfindahl–Hirschman Index of dealers' repo borrowing from money market funds,

$$HHI_t = \sum_{i=1}^{N_t} \left(\frac{\text{RepoBorrowing}_{i,t}}{\sum_{j=1}^{N_t} \text{RepoBorrowing}_{j,t}} \right)^2 \times 100, \tag{66}$$

where N_t is the number of dealer counterparties reporting repo borrowing from money market funds in month t. Following Huber (2023), I allow three months after mandatory reporting began before starting the sample. Higher HHI_t means that repo borrowing is more concentrated among a small number of dealers. I use it as an empirical proxy for time variation in market-wide dealer market power. Treasury zero-coupon yields, forward rates, and bond returns come from Gürkaynak, Sack, and Wright (2007). I also use the ex-ante term-premium estimates of Adrian et al. (2013) and Kim and Wright (2011).

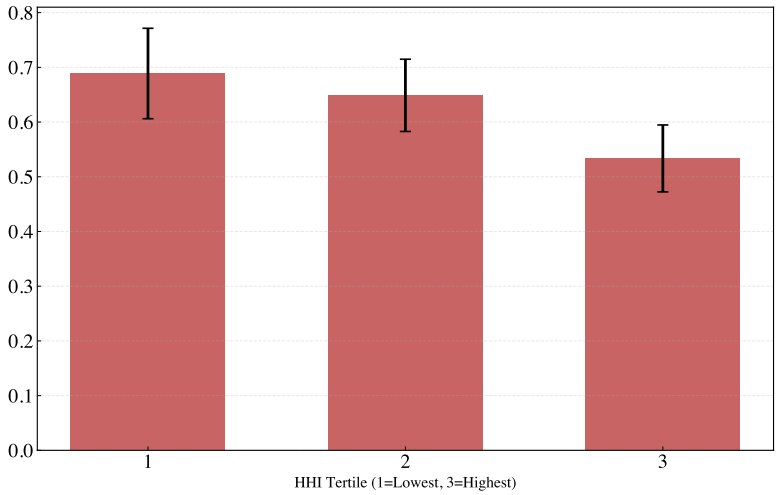


Figure 2. Repo Concentration and Private Repo Intermediation

Notes: The figure plots the mean private repo ratio by tercile of the Herfindahl–Hirschman Index of dealers' repo borrowing from money market funds, with 90% confidence intervals. The private repo ratio is the share of money market funds' total repo lending allocated to private dealers instead of the Federal Reserve's ON-RRP facility. Sample: September 2013 to July 2025.

B. Repo Concentration and Intermediation

The most direct observable implication of the initiating friction concerns the level of repo intermediation. Market power lowers dealers' carry positions in levels even though, as Section III.A shows, it makes those positions more sensitive to movements in the policy rate. I measure private repo intermediation by the share of money market funds' total repo lending allocated to private dealers instead of the Federal Reserve's overnight reverse-repurchase facility. This private repo ratio is an aggregate counterpart to the quantity of repo-financed carry positions in the model. I restrict this measure to the period after the ON-RRP facility began operating in September 2013: before then, money market funds had no alternative to private dealers, so the ratio is mechanically at its ceiling of one for reasons of monetary-policy implementation rather than dealer market power.

Figure 2 plots the mean private repo ratio by tercile of HHI_t. The ratio declines monotonically with concentration. In the underlying monthly data, its correlation with HHI is -0.29 ($t = -3.56$). It matches the model's prediction that the level of repo-financed intermediation is lower when dealers possess greater market power.

Figure 3 plots the mean tri-party repo spread by HHI tercile. I construct the tri-party repo spread as the difference between dealers' tri-party borrowing rate and the ON-RRP rate, the money funds' outside option. From August 2014 onward, the tri-party rate is the Federal Reserve Bank of New

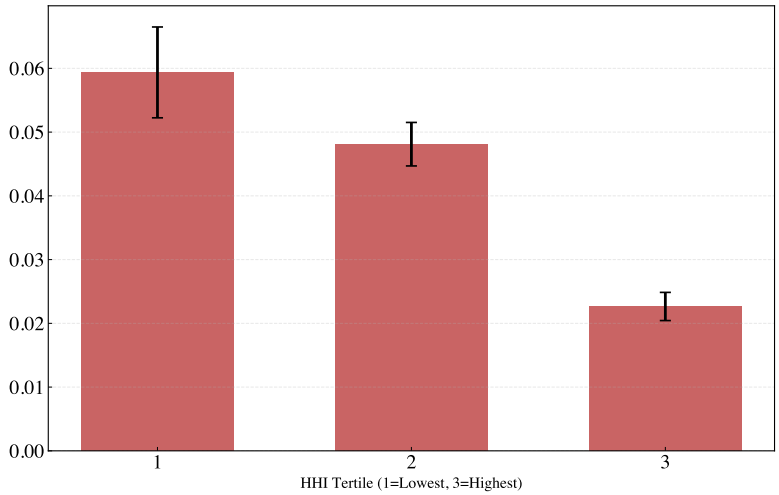


Figure 3. Repo Concentration and the Tri-Party Repo Spread

Notes: The figure plots the mean spread between the tri-party Treasury repo rate and the Federal Reserve's ON-RRP rate by HHI tercile, with 90% confidence intervals. Sample: September 2013 to July 2025.

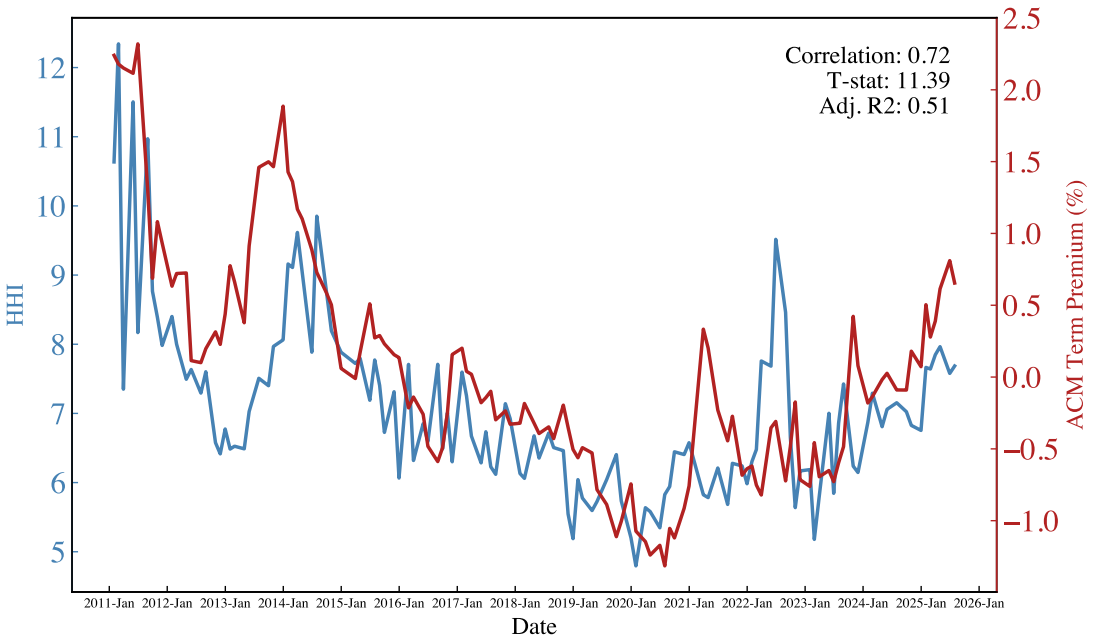
York's Tri-Party General Collateral Rate; from August 2012 to August 2014, it is the Bank of New York Mellon's volume-weighted median rate on overnight Treasury repo, following He et al. (2022). In the model, the spread satisfies $r_{d,t} - r_t = \psi D_t/Q$, so lower aggregate dealer borrowing is associated with a narrower spread. The spread in Figure 3 falls with concentration; its correlation with HHI in the underlying monthly data is -0.28 ($t = -3.54$).

C. Repo Concentration and Treasury Risk Premia

I next examine two related pricing implications: greater repo concentration should be associated with higher compensation for bearing Treasury risk and with a stronger mapping from the yield-curve slope into future excess returns. The first pattern appears in commonly used ex-ante term-premium estimates. The correlation between HHI_t and the Adrian et al. (2013) 10-year term premium is 0.72. A monthly regression of the term premium on HHI, with Newey–West standard errors using four lags, gives a slope of 0.45 ($t = 9.51$) for the Adrian et al. (2013) measure and 0.13 ($t = 4.72$) for the Kim and Wright (2011) measure. The sample standard deviation of HHI is 1.25. A one-standard-deviation increase in HHI_t is therefore associated with a 56 basis-point higher Adrian et al. (2013) term premium and a 16 basis-point higher Kim and Wright (2011) term premium.

Moreover, I examine the relation between repo concentration and realized bond excess returns. For

Panel A: Adrian et al. (2013) 10-Year Term Premium



Panel B: Kim and Wright (2011) 10-Year Term Premium

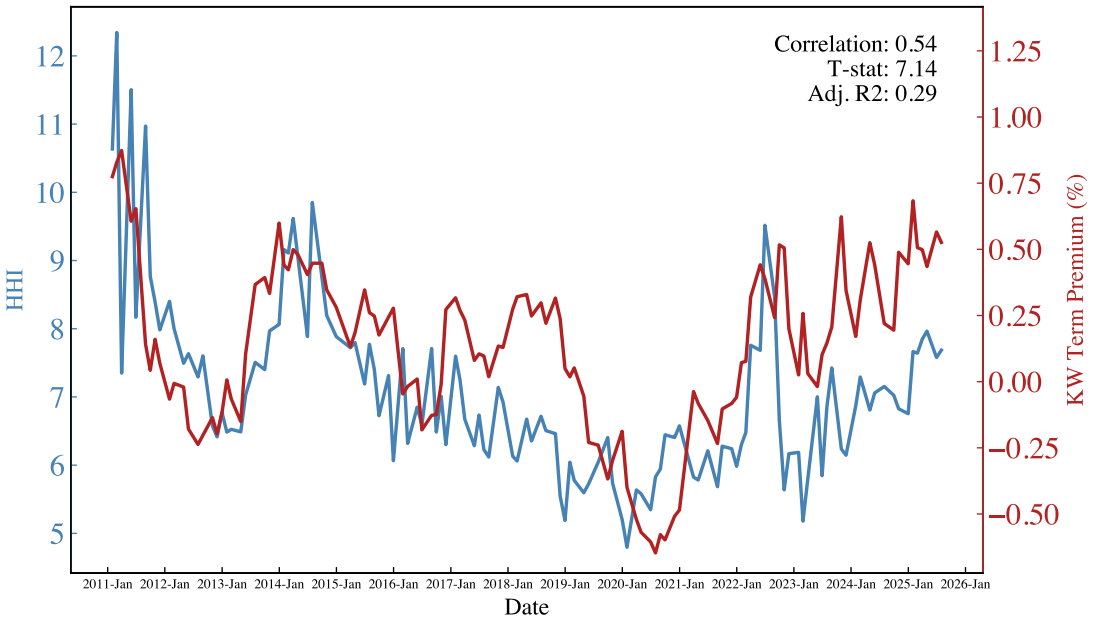


Figure 4. Repo Concentration and Ex-Ante Term Premium

Notes: The blue line plots HHI_t (left axis); the red line plots the ex-ante 10-year term premium (right axis). Panel A uses Adrian et al. (2013); Panel B uses Kim and Wright (2011). Sample: January 2011 to September 2025, monthly, aligned to the repo data's reporting dates.

each maturity $\tau = 2, \ldots, 10$, I estimate

$$rx_{t+1}^{(\tau)} = \alpha^{(\tau)} + \beta^{(\tau)} HHI_t + \varepsilon_{t+1}^{(\tau)}, \tag{67}$$

where $rx_{t+1}^{(\tau)}$ is the one-year log excess return on a τ-year bond. Panel A of Figure 5 plots $\hat{\beta}^{(\tau)}$. The coefficients are positive and generally increase with maturity, reaching 2.56 at ten years ($t = 2.33$). Thus, a one-standard-deviation increase in concentration is associated with a 3.2 percentage point higher predicted one-year excess return on a 10-year bond.

The model also predicts that concentration strengthens Fama–Bliss return predictability. I estimate

$$\begin{aligned} rx_{t+1}^{(\tau)} = a^{(\tau)}\left(f_t^{(\tau-1,\tau)} - y_t^{(1)}\right) + b^{(\tau)} HHI_t \\ + c^{(\tau)} HHI_t \left(f_t^{(\tau-1,\tau)} - y_t^{(1)}\right) + d^{(\tau)} + \varepsilon_{t+1}^{(\tau)}, \end{aligned} \tag{68}$$

transforming HHI into decile ranks for the interaction's interpretation. Panels B and C show that the HHI level coefficient and its interaction with the forward-rate spread are positive across maturities. Moving up one HHI decile is associated with a 0.3 to 1 increase in the Fama–Bliss coefficient, depending on maturity. Panel D shows that the market-structure terms also increase the regression's adjusted R^2. Standard errors in these annual-return regressions are Newey–West adjusted with 36 lags, following Greenwood and Vayanos (2014b).

D. Repo Concentration and Forward-Rate Sensitivity

The short-rate channel yields a third aggregate implication: forward rates should co-move more strongly with the short end of the curve when repo concentration is high, particularly at short and intermediate maturities. I first estimate the sensitivity of daily changes in each τ-year forward rate to daily changes in the one-year forward rate separately above and below the median HHI. I then estimate the interaction specification

$$\Delta f_t^{(\tau)} = \alpha^{(\tau)} + \beta^{(\tau)} \Delta f_t^{(1)} + \gamma^{(\tau)} D_t + \varphi^{(\tau)} D_t \, \Delta f_t^{(1)} + \varepsilon_t^{(\tau)}, \tag{69}$$

where $D_t \in \{-\frac{1}{2}, \frac{1}{2}\}$ is a centered dummy equal to $\frac{1}{2}$ when HHI_t is above its sample median and $-\frac{1}{2}$ otherwise. I interpret $\varphi^{(\tau)}$ as the difference in forward-rate sensitivity between high- and low-HHI periods. Panel A of Figure 6 shows that forward-rate sensitivity is higher in the high-HHI subsample at short maturities and that the gap narrows along the curve. Panel B reports $\hat{\varphi}^{(\tau)}$ from equation (69). The estimates rise from 0.11 at two years to a hump peaking at 0.17 at four years, before declining and turning slightly negative by ten years (-0.02), matching the shape predicted by Proposition 9.

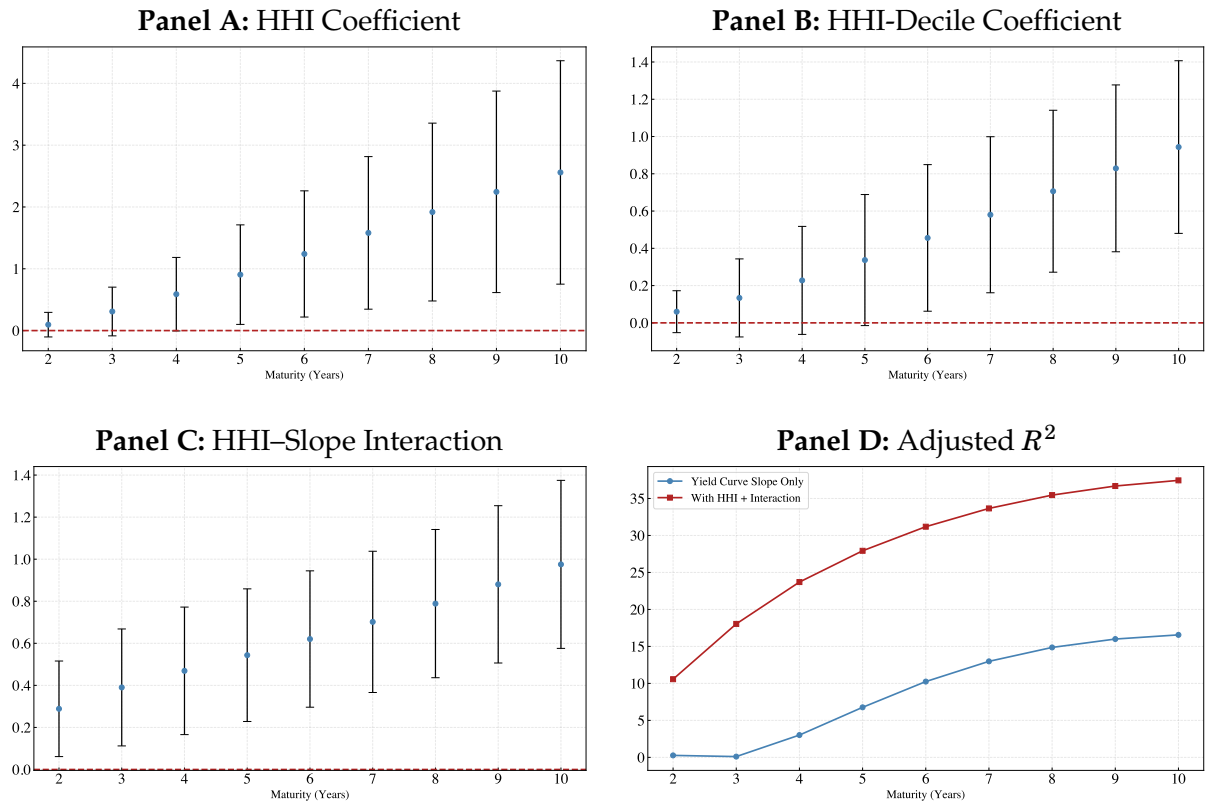


Figure 5. Repo Concentration and Bond Excess Returns

Notes: Panel A plots $\hat{\beta}^{(\tau)}$ from equation (67). Panels B and C plot $\hat{b}^{(\tau)}$ and $\hat{c}^{(\tau)}$ from equation (68). Panel D compares the adjusted R^2 of the standard Fama–Bliss regression against the specification augmented with HHI and its interaction with the forward-rate spread. Bars are 90% confidence intervals. Sample: January 2011 to September 2025.

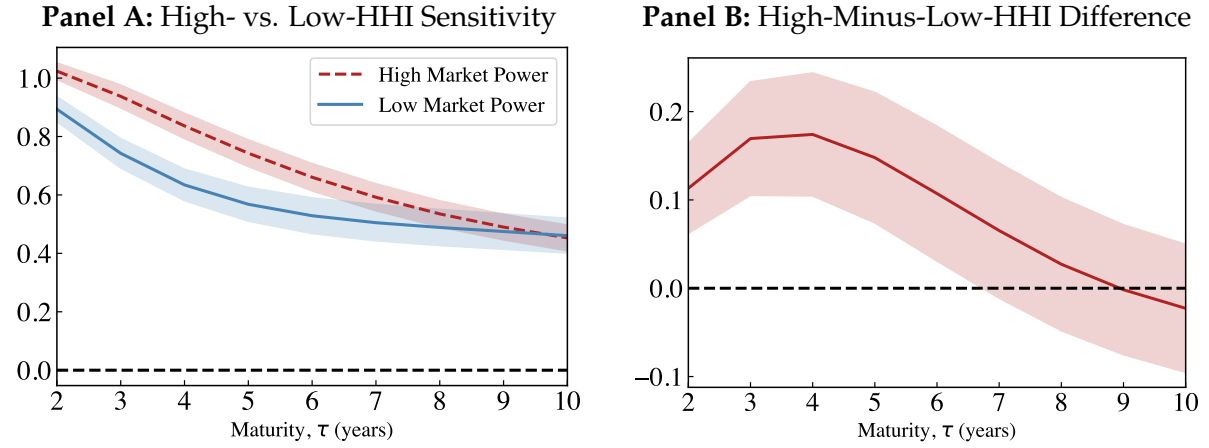


Figure 6. Repo Concentration and Forward-Rate Sensitivity

Notes: Panel A estimates the sensitivity of daily changes in each forward rate to changes in the one-year forward rate separately above and below the sample median of HHI. Panel B plots $\hat{\varphi}^{(\tau)}$ from equation (69), the high-minus-low-HHI difference in forward-rate sensitivity. Shaded regions are 90% confidence intervals. Sample: January 2011 to September 2025.

Taken together, these results document that private repo intermediation, Treasury risk premia, return predictability, and forward-rate sensitivity move jointly with concentration in the directions implied by the model. The quantitative analysis next asks whether a calibrated version of the mechanism can reproduce the shape and magnitude of these untargeted patterns and evaluates its implications for quantity shocks and monetary-policy transmission.

V. Quantitative Analysis

This section calibrates the model to U.S. Treasury and repo market data and assesses the quantitative importance of dealer market power. I first ask whether the full calibrated model preserves the analytical results derived under the special cases of Section III. I then compare its untargeted implications with the aggregate patterns documented in Section IV and use the model to run counterfactual policy experiments.

A. Setup and Equilibrium

The setup is exactly as in Section II. I now restore the regulation-cost parameter to $\psi \neq 0$, so the funding rate dealers face is once again $r_{d,t} = r_t + \psi D_t/Q$. The equilibrium is then exactly Proposition 2 of Section II: a fixed point in the nine scalars $\{\Omega_{rr}, \Omega_{\beta r}, \Omega_{r\beta}, \Omega_{\beta\beta}, \Omega_r, \Omega_\beta, d_r, d_\beta, d_0\}$. I solve it numerically and the details of the numerical solution are in Appendix A.12.

B. Calibration

I calibrate the model to monthly U.S. nominal Treasury zero-coupon yields over January 2011 – September 2025, the same sample period used throughout the paper; the sample start is limited by the availability of the repo market data underlying my measure of dealer market power as money market funds' repo lending records begin in late 2010. The model has eleven parameters, which I discipline in two groups: dealer market power $\mathscr{M}$, set externally from Huber (2023); and ten common parameters, calibrated jointly by matching the model's stationary moments to their *unconditional* data counterparts.

I fix $\mathscr{M} = 1.0$ throughout, taken from Huber (2023), who models money market funds' concentration aversion when allocating cash across dealers and estimates the elasticity $\nu_i \equiv \partial \ln D_{i,t}/\partial \ln r_{i,t}$, exactly the object equation (4) defines as $1/\mathscr{M}$. Two independent estimates in the paper both point to $\mathscr{M} \approx 1.0$.

The first uses a structural markdown estimate. In this paper's model, the rent a dealer extracts from her market power, equation (22)'s $\mathscr{M} r_{i,t}$ term, is exactly what Huber (2023) calls the markdown: the gap between a dealer's actual funding rate and the marginal value of the funds to her. Huber (2023) estimates a markdown of 26 basis points; the average tri-party repo rate over their estimation sample is $\bar{r} \approx 27$ basis points in my own repo-rate series, giving

$$\mathscr{M} = \frac{26}{27} \approx 0.96.$$

The second uses Huber (2023)'s Treasury-auction instrumental-variable estimate, which identifies a semi-elasticity, not ν_i itself: a one-basis-point rate increase raises a dealer's funding by about 3.6%, i.e., $\partial \ln D / \partial r \approx 360$ with the rate in decimal terms. Converting this semi-elasticity into ν_i requires multiplying by the rate level, since $\partial \ln r = \partial r / r$,

$$\nu_i \equiv \frac{\partial \ln D}{\partial \ln r} = \bar{r} \times \frac{\partial \ln D}{\partial r} \approx 0.0027 \times 360 \approx 0.98$$

at the same $\bar{r}$. This implies

$$\mathscr{M} = \frac{1}{\nu_i} \approx \frac{1}{0.98} \approx 1.02.$$

Therefore, I take $\mathscr{M} = 1.0$ for the rest of the analysis.

For the quantitative implementation I specialize the general demand schedule of equation (25) to an exponential-decay functional form, following Vayanos and Vila (2021) and Kekre, Lenel, and Mainardi (2026), who use the same specification for the maturity profile of habitat demand:

$$\alpha(\tau) = \alpha\, e^{-\delta_\alpha \tau}, \qquad \theta_0(\tau) = \theta_0\, \alpha(\tau), \qquad \theta(\tau) = \theta_0(\tau), \tag{70}$$

so a single decay rate δ_α governs how both the price elasticity $\alpha(\tau)$ and the demand intercept $\theta_0(\tau)$ taper with maturity, and the quantity-shock loading $\theta(\tau)$ shares $\theta_0(\tau)$'s functional form exactly, with no separate free parameter of its own. Table 1 reports the calibrated parameters and the target moments.

The long-run mean $\bar{r}$ and mean-reversion speed κ_r of the short rate have their largest effect on the one-year yield: $\bar{r}$ sets its unconditional level, while κ_r governs how persistent it is. The one-year yield's unconditional level and its own monthly autocorrelation therefore identify $\bar{r}$ and κ_r, respectively, fol-

lowing Vayanos and Vila (2021). The short-rate diffusion σ_r governs yield volatility, and I identify it off the one-year yield's own monthly volatility. Risk aversion γ and the habitat elasticity scale α both shape the unconditional slope of the yield curve, and I identify them off two different segments of that slope, the twenty-year-minus-one-year and thirty-year-minus-one-year spreads respectively, following Kekre et al. (2026)'s use of an unconditional slope moment to identify the analogous risk-aversion parameter in a closely related segmented-markets term structure model. The demand intercept θ_0 sets the unconditional level of the thirty-year yield, and the habitat elasticity's decay rate δ_α, governing how quickly the demand-driven markup fades with maturity, is identified off the ten-year-minus-one-year slope. The demand-shock parameters play an analogous role for the yield curve's second and third moments: the demand-shock magnitude σ_β governs the correlation between the ten- and one-year yields, and the demand factor's mean-reversion speed κ_β governs the twenty-year yield's own monthly volatility. Finally, I match the funding wedge ψ_Q directly to the average spread between dealers' repo rates and the ONRRP rate offered by the Federal Reserve. Table 1 reports the calibrated parameters and the corresponding target moments and Figure 7 presents the model's fit to the data across the maturity curve.

Table 1 Calibration of the Quantitative Model

Parameter	Target moment	Value	Model	Data
$\mathcal{M}$ (market power)	Huber (2023), external	1.0	–	–
$\bar{r}$ (long-run short rate)	$\mathbb{E}[y^{(1)}]$	0.730%	1.78%	1.58%
κ_r (short-rate mean rev.)	AutoCorr$[y^{(1)}]$ (monthly)	0.227	0.981	0.994
σ_r (short-rate vol.)	Std$[y^{(1)}]$ (monthly)	0.0081	2.15%	1.74%
γ (risk aversion)	$\mathbb{E}[y^{(20)} - y^{(1)}]$	1.008	1.53%	1.54%
α (demand scale)	$\mathbb{E}[y^{(30)} - y^{(1)}]$	1.555	1.72%	1.74%
θ_0 (demand intercept)	$\mathbb{E}[y^{(30)}]$	1.631	3.50%	3.32%
δ_α (habitat decay)	$\mathbb{E}[y^{(10)} - y^{(1)}]$	0.250	1.09%	1.02%
σ_β (demand vol.)	Corr$[y^{(10)}, y^{(1)}]$	0.322	0.801	0.798
κ_β (demand mean rev.)	Std$[y^{(20)}]$ (monthly)	0.097	0.99%	0.97%
ψ_Q (funding wedge)	$\mathbb{E}[r_d - r_t]$	4.8e-5	4.4bp	4.4bp

Notes: The table reports the calibrated parameters, their target moments, and the model's fit to those moments, against monthly U.S. nominal Treasury zero-coupon yields, January 2011 – September 2025.

C. Quantitative Results

This section quantifies the calibrated model's implications for the bond price's sensitivity to short-rate and quantity shocks, the resulting bond risk premia, the yield curve, and monetary policy trans-

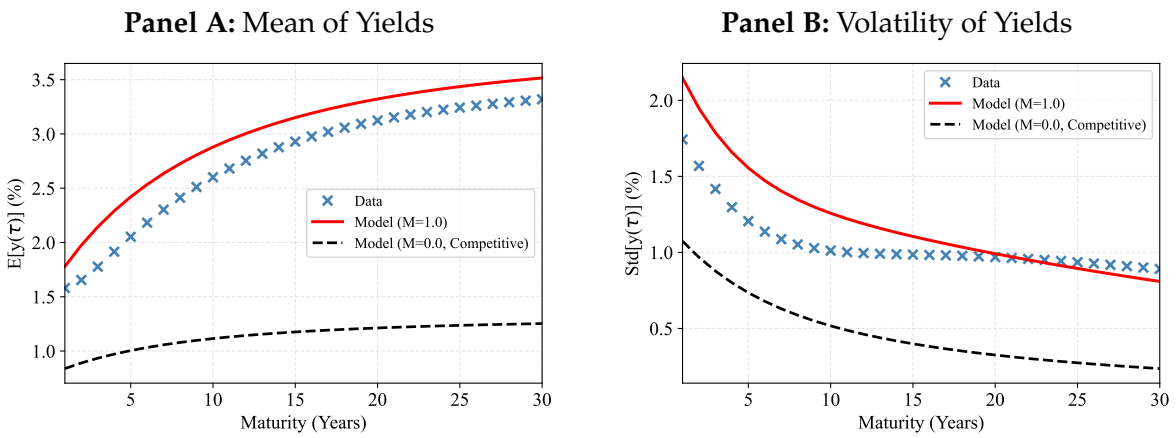


Figure 7. Calibration Fit Across the Maturity Curve

Notes: This figure compares the model's fit to the data across the maturity curve. Crosses are data moments from monthly U.S. nominal Treasury zero-coupon yields, January 2011 through September 2025. Red solid lines are the corresponding yield curve moments from the calibrated baseline model, $\mathscr{M} = 1$, at the calibration of Table 1. Black dashed lines are the same moments from the competitive counterfactual, $\mathscr{M} = 0$, at the same common parameters.

mission. Throughout, "Baseline" denotes the calibrated model at $\mathscr{M} = 1.0$ and "Competitive" the same common parameters solved at $\mathscr{M} = 0$, a counterfactual with no dealer market power. For the illustrative model–data comparison, I split the empirical sample into high- and low-concentration regimes using the HHI measure defined in equation (66). I compare the high-minus-low-HHI difference against the model's own market-power gap, solving the equilibrium at $\mathscr{M} = 1.5$ and $\mathscr{M} = 0.5$ and differencing the two. $\mathscr{M} = 1.5$ and $\mathscr{M} = 0.5$ are hypothetical high- and low-market-power scenarios, respectively, chosen to bracket the calibrated $\mathscr{M} = 1.0$, not to map HHI mechanically into the structural parameter $\mathscr{M}$. The comparison is therefore illustrative, asking whether the market-power channel can plausibly account for the magnitude of the regime difference observed empirically.

Bond price sensitivity. Proposition 5 and Lemma 3 show analytically, in each channel's tractable special case, that market power raises the bond price's sensitivity to the short rate and to the quantity shock, respectively. Figure 8 confirms both results survive at the full quantitative calibration, with $\sigma_\beta > 0$ and $\psi_Q > 0$ restored. Panel A plots the raw short-rate loading $A_r(\tau)$: the baseline sits above the competitive counterfactual at every maturity, and the gap widens with τ, since longer-maturity bonds carry proportionally more of the amplified interest rate risk. Panel B plots the raw quantity-shock loading $A_\beta(\tau)$: the baseline again sits above the competitive counterfactual throughout, with a gap strictly increasing in τ. Taken together, the panels verify the two observable legs of the mechanism in Figure 1: market power first raises $A_r(\tau)$, and the resulting increase in return volatility is accompanied

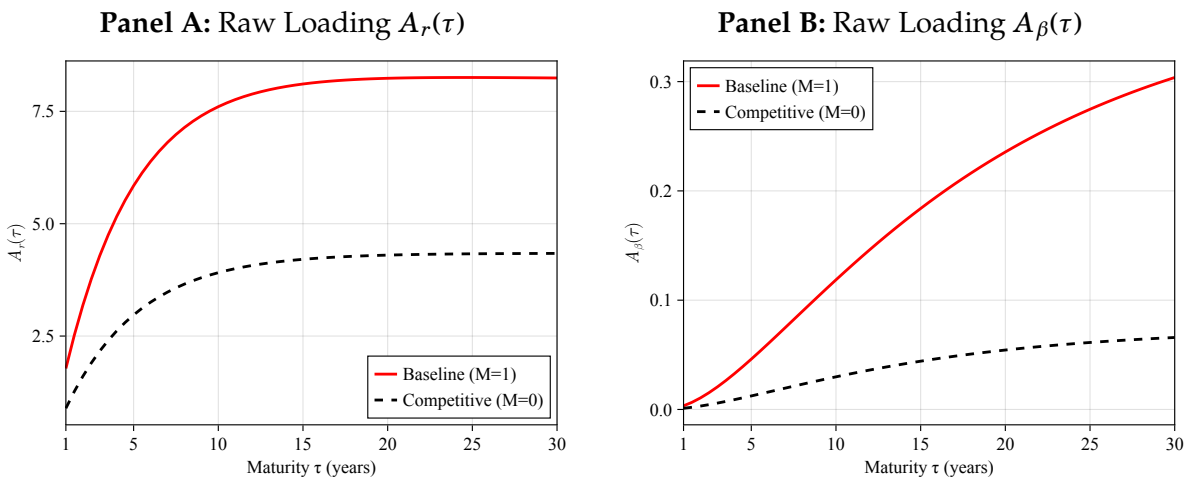


Figure 8. Bond Price Sensitivity to Short-Rate and Quantity Shocks

Notes: Panel A plots the bond-price loading on the short rate, $-\partial \log P(\tau)/\partial r_t = A_r(\tau)$, the object Proposition 5 characterizes. Panel B plots the bond-price loading on the demand shock, $-\partial \log P(\tau)/\partial \beta_t = A_\beta(\tau)$, the object Lemma 3 characterizes. Both panels compare the baseline calibration ($\mathscr{M} = 1$) against the competitive benchmark ($\mathscr{M} = 0$), with all other common parameters held at the full calibration of Table 1.

by a larger $A_\beta(\tau)$. I turn next to the corresponding bond risk premia.

Impulse responses to a monetary shock. Figure 9 shows the impulse responses to a monetary shock. Panel A shows that, through the rent-extraction channel, dealers' carry positions are more sensitive to the monetary shock under market power, even though market power reduces their positions in levels. This larger response makes bond prices move more for the same shock, which is exactly the amplified price sensitivity of Proposition 5, and also increases the volatility of bond returns. With both the size of the position adjustment and the quantity of risk each unit of bond carries now larger, the resulting change in the price of risk is correspondingly larger, for both interest-rate and quantity-shock risk, $\lambda_{r,t}$ and $\lambda_{\beta,t}$, as Panels C and D show. Market power's impact on these prices of risk is substantial: the market-power gap in $\lambda_{r,t}$ and $\lambda_{\beta,t}$ dwarfs, in relative terms, the corresponding gap in the yield curve slope itself, shown in Panel B, which also widens under market power but far less sharply. This is, at its core, why market power raises the Fama-Bliss coefficient, formalized in Proposition 7: the same change in slope is now associated with a substantially larger change in the compensation dealers require, so a given slope movement predicts a larger excess return.

The yield curve. Figure 7's black dashed line plots the competitive ($\mathscr{M} = 0$) counterfactual yield curve alongside the baseline, in Panel A, both evaluated at the unconditional steady state of the short

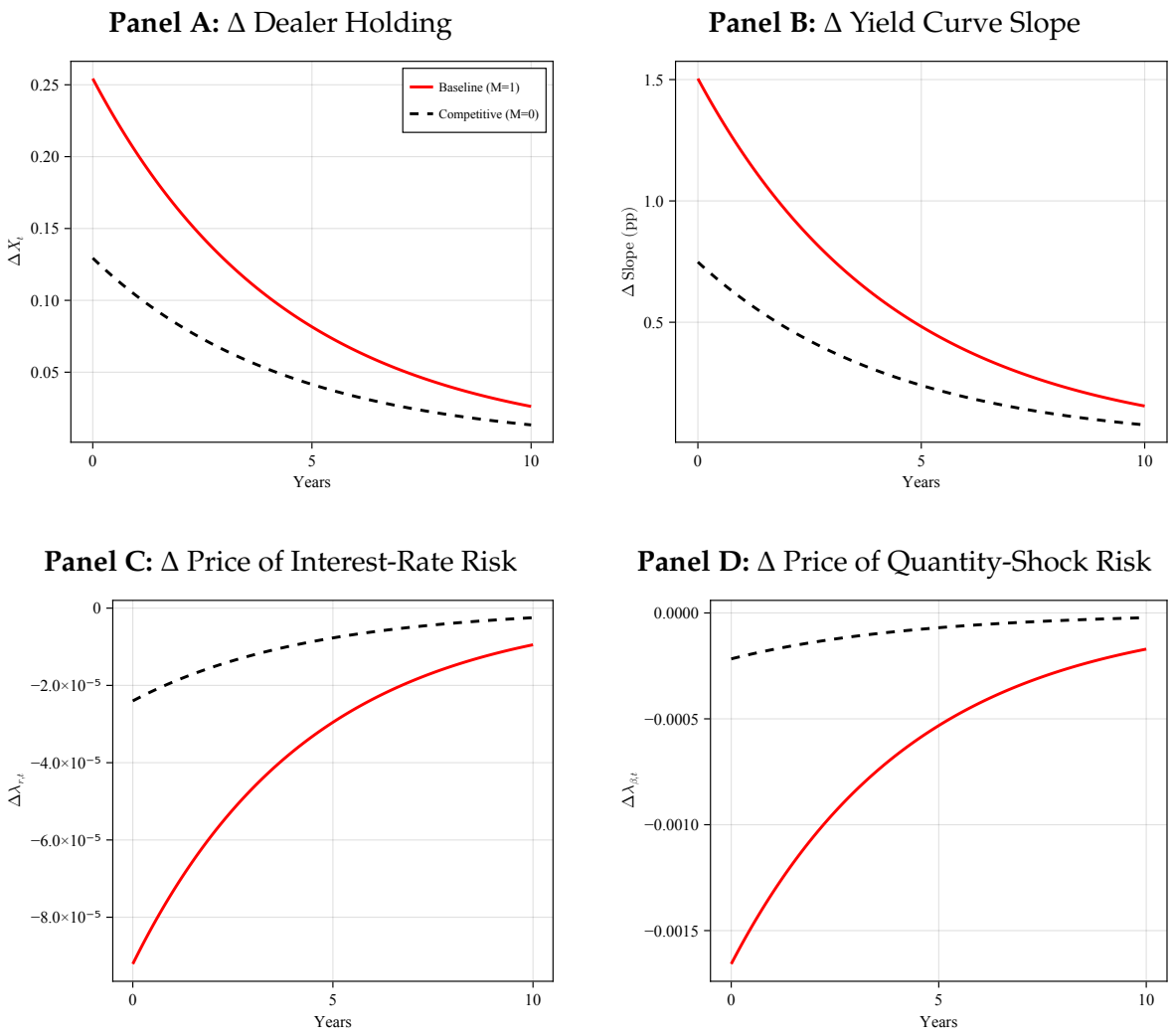


Figure 9. Impulse Response to an Interest-Rate Shock, Baseline vs. Competitive Counterfactual

Notes: Response over calendar time to a −100bp shock to r_t, decaying at rate κ_r, relative to the unconditional (no-shock) steady state. Panel A: aggregate dealer holding $X_t = \int_0^T X_t^{(\tau)} d\tau$. Panel B: yield curve slope $y_t^{(30)} - y_t^{(1)}$. Panels C–D: the prices of interest-rate and quantity-shock risk, $\lambda_{r,t}$ and $\lambda_{\beta,t}$. Parameters as in Table 1.

rate and demand factor ($r_t = \bar{r}$, $\beta_t = 0$). The baseline curve sits above the competitive curve at every maturity, and the gap widens with maturity, from 94 basis points at one year to 176 basis points at ten years and 226 basis points at thirty years, consistent with market power raising the risk premium that dealers require for absorbing duration risk at every point on the curve. Higher yields mechanically imply lower bond prices, so this gap translates directly into a dollar valuation of dealer market power's effect on the Treasury market. I discount the maturity profile of outstanding U.S. marketable Treasury debt ($30 trillion as of September 2025, with 33% maturing within one year, 35% between one and five years, and the remaining 32% at longer maturities) against each curve's zero-coupon-equivalent dis-

count curve.[2] Under the competitive counterfactual, this debt is worth approximately \$27.9 trillion; under the baseline, approximately \$25.3 trillion. Dealer market power therefore accounts for a gap of \$2.6 trillion, roughly 9%, in the market value of outstanding U.S. Treasury debt, holding its face value and maturity structure fixed.

Sensitivity to short-rate shocks. Proposition 9 shows analytically that market power amplifies the pass-through of short-rate shocks to yields. Figure 10 compares the calibrated model with the forward-rate-sensitivity patterns documented in Section IV.D. Panel A compares the pooled sensitivity with the baseline and competitive model economies. Panel B compares the model's own $\mathscr{M} = 1.5$-minus-$\mathscr{M} = 0.5$ difference against $\hat{\varphi}^{(\tau)}$ from equation (69), the same high-minus-low-HHI difference plotted in Figure 6's Panel B.

Panel A compares levels: the data and the calibrated baseline match well, both decaying with maturity, with the model starting above the data at two years, nearly coinciding by four years, and falling below the data by ten years. This close fit is notable given that the model is not calibrated to match the level of forward-rate sensitivity at all. The competitive counterfactual sits uniformly below the baseline, confirming that market power amplifies the level of pass-through throughout. Panel B compares the market-power gap: the data's High-minus-Low-HHI difference rises from 0.11 at two years to a hump peaking at 0.17 at four years, before declining and turning slightly negative by ten years (-0.02), while the model's difference decays monotonically, from its largest value of 0.60 at two years to a small positive value of 0.05 by ten years, matching the data's overall declining pattern at short and intermediate maturities.

Monetary policy transmission. I now compare the baseline and competitive counterfactual's responses to two canonical monetary policy experiments, forward guidance and QE, to quantify the effect of market power on policy transmission. The first policy I analyze is forward guidance: a persistent shock $\Delta\bar{r} = -100$ basis points to the long-run mean of the short rate, unanticipated at time zero. The second is QE: a persistent reduction in habitat demand, $\Delta\int\theta_0(\tau)d\tau = -0.12$, also unanticipated at time zero. I compute both experiments by fully re-solving the nine-scalar equilibrium fixed point

[2]Marketable debt outstanding: NY Fed Liberty Street Economics, "How Has Treasury Market Liquidity Fared in 2025?", November 2025, https://libertystreeteconomics.newyorkfed.org/2025/11/how-has-treasury-market-liquidity-fared-in-2025/. Maturity distribution: U.S. Government Accountability Office, *Federal Debt Management: Treasury Is Meeting Borrowing Needs but the Deteriorating Fiscal Outlook Poses Risks*, GAO-26-107529, March 2026, https://files.gao.gov/reports/GAO-26-107529/index.html.

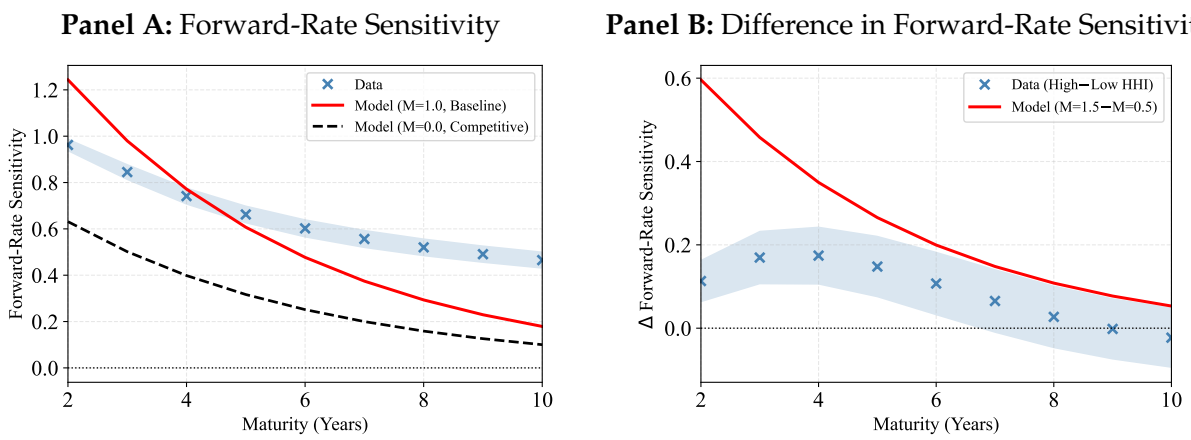


Figure 10. Forward Rate Sensitivity

Notes: Panel A plots the pooled, unconditional (no HHI split) forward-rate-sensitivity coefficient $\hat{\beta}^{(\tau)}_{\text{pool}}$, shown as blue markers with a shaded 90% confidence band, against the model-implied $\partial f^{(\tau)}/\partial r$ at the baseline calibration, $\mathscr{M} = 1.0$, shown as a red solid line, and the competitive counterfactual, $\mathscr{M} = 0$, shown as a black dashed line. Panel B plots $\hat{\varphi}^{(\tau)}$ from equation (69), the High-minus-Low-HHI difference in forward-rate sensitivity, shown as blue markers with a shaded 90% confidence band, against the model's $\mathscr{M} = 1.5$-minus-$\mathscr{M} = 0.5$ difference, shown as a red solid line. Parameters as in Table 1.

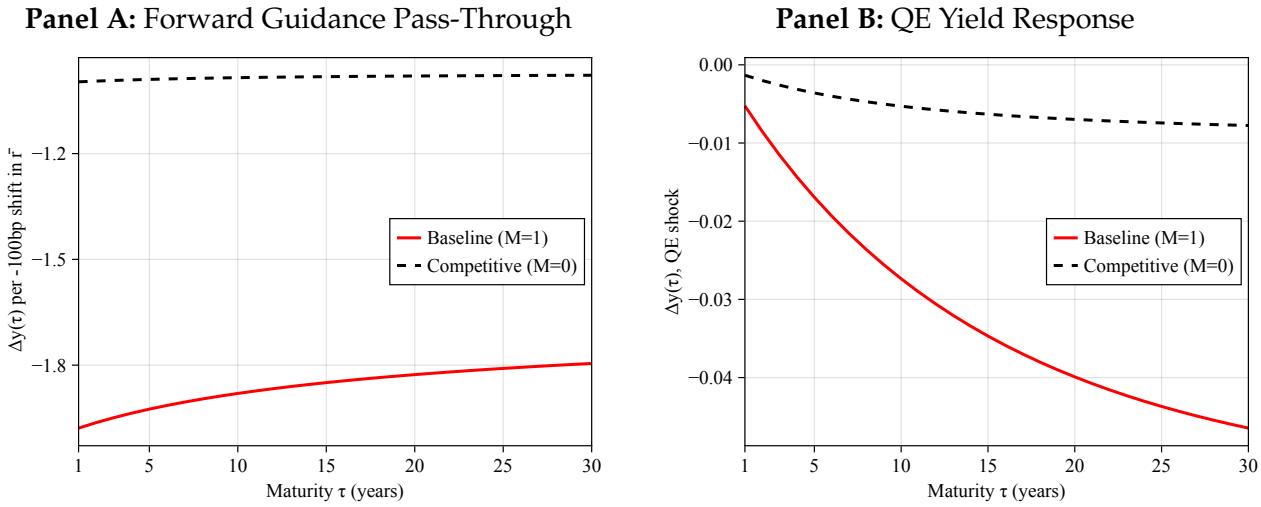


Figure 11. Policy Transmission, Baseline vs. Competitive Counterfactual

Notes: Panel A plots the yield response per unit of a persistent −100 basis point shift in the long-run short rate $\bar{r}$. Panel B plots the yield response to a reduction in habitat demand $\Delta\int\theta_0(\tau)d\tau = -0.12$. Both curves share the common structural parameters of Table 1, calibrated independently of these two experiments.

of Appendix A.12 at the shocked parameter value, $\bar{r}$ for forward guidance and θ_0 for QE, and differencing the resulting yield curve against the unshocked baseline, holding fixed the common structural parameters of Table 1 throughout. Appendix A.13 gives the full procedure for each experiment.

Figure 11 plots the results. Panel A reports the forward-guidance experiment. At every maturity, the baseline's pass-through exceeds the competitive counterfactual's by a ratio of roughly 1.8–2.0×,

falling gently from 1.99× at one year to 1.84× at thirty years. Panel B reports the QE experiment: at every maturity, the baseline's yield decline exceeds the competitive counterfactual's, and the ratio rises monotonically with maturity, from 3.9× at one year to 6.0× at thirty years, the amplification largest at the long end. Both experiments show dealer market power strengthening the transmission of monetary policy to the term structure, consistent with the amplification results derived analytically for short-rate shocks in Section III.A and for quantity shocks in Section III.B.

VI. Conclusion

This paper shows that the market structure of repo financing matters for the term structure of interest rates. I incorporate dealer market power in repo markets into a preferred-habitat model in which dealers finance Treasury carry trades with short-term borrowing from money market funds. With market power, an individual dealer faces an upward-sloping supply of repo funding and strategically restricts carry trades to preserve a funding rent. This rent enters the dealer's first-order condition as a marginal shadow funding cost that moves with the policy rate. Market power therefore reduces carry positions in levels but makes them more sensitive to monetary shocks, increasing Treasury price sensitivity to short-rate risk. The resulting rise in bond volatility raises the compensation dealers require for absorbing quantity shocks; the larger price impact adds to volatility again through the equilibrium feedback I call the volatility spiral.

This mechanism links repo-market structure to several central features of Treasury pricing. Dealer market power raises bond risk premia and strengthens the predictive relation between the yield curve's slope and future excess returns. It also amplifies the transmission of both interest-rate and quantity shocks, with distinct signatures across maturity. For forward guidance, the amplification is strongest at short maturities and declines along the curve as the term-premium channel increasingly offsets the direct funding-rent channel. For quantitative easing, the same term-premium channel instead reinforces the effect of reduced bond supply, so the amplification grows with maturity instead of fading. The joint patterns in the data point in the directions implied by the model: periods of greater repo concentration exhibit less private repo intermediation, higher term premia, stronger return predictability, and greater forward-rate sensitivity at short and intermediate maturities.

The quantitative effects are substantial. At a level of dealer market power disciplined independently by estimates from the repo market, the model implies that dealer market power raises Trea-

sury yields throughout the maturity structure. Holding the face value and maturity distribution of outstanding Treasury debt fixed, the resulting valuation difference relative to a competitive repo market is approximately $2.6 trillion, or 9% of outstanding Treasury market value. Market power also materially changes monetary-policy multipliers: the same forward-guidance shock produces a yield response about 1.8–2.0 times as large as under competition, while the response to quantitative easing is 3.9–6.0 times as large, with the gap widening toward the long end of the curve.

These results speak to ongoing reforms of Treasury funding markets. In December 2023 the Securities and Exchange Commission adopted rules requiring central clearing for most U.S. Treasury repo transactions, a reform SEC Chair Gary Gensler framed as making the market "more efficient, competitive, and resilient." Through the lens of the model, such entry-promoting reform pushes the repo market toward the competitive counterfactual of my calibration, and the gap between that counterfactual and the baseline quantifies what it could achieve: a lower cost of capital for the Treasury, but weaker monetary policy transmission. Market-structure reform is hence a trade-off, and policymakers evaluating it should weigh its effect on the entire yield curve, not only on the repo market itself.

REFERENCES

Adrian, Tobias, Richard K Crump, and Emanuel Moench, 2013, Pricing the term structure with linear regressions, *Journal of Financial Economics* 110, 110–138.

Brunnermeier, Markus K, and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *The review of financial studies* 22, 2201–2238.

Brunnermeier, Markus K, and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Carboni, Giacomo, and Martin Ellison, 2022, Preferred habitat and monetary policy through the looking-glass, Technical Report 2697, European Central Bank Working Paper.

Cavaleri, Filippo, 2023, A preferred-habitat model with a corporate sector, *Available at SSRN 4603103* .

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2017, The deposits channel of monetary policy, *The Quarterly Journal of Economics* 132, 1819–1876.

Fama, Eugene F, and Robert R Bliss, 1987, The information in long-maturity forward rates, *The American Economic Review* 680–692.

Fleming, Michael J, Giang Nguyen, and Joshua V Rosenberg, 2008, How do treasury dealers manage their positions?, *FRB of New York Staff Report* .

Gourinchas, Pierre-Olivier, Walker Ray, and Dimitri Vayanos, 2025, A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers, *American Economic Review* 115, 3788–3824.

Greenwood, Robin, Samuel Hanson, Jeremy C Stein, and Adi Sunderam, 2023a, A Quantity-Driven Theory of Term Premia and Exchange Rates, *The Quarterly Journal of Economics* 138, 2327–2389.

Greenwood, Robin, Samuel Hanson, and Dimitri Vayanos, 2023b, Supply and demand and the term structure of interest rates, *Annual Review of Financial Economics* 16.

Greenwood, Robin, and Dimitri Vayanos, 2014a, Bond Supply and Excess Bond Returns, *Review of Financial Studies* 27, 663–713.

Greenwood, Robin, and Dimitri Vayanos, 2014b, Bond supply and excess bond returns, *The Review of Financial Studies* 27, 663–713.

Gürkaynak, Refet S, Brian Sack, and Jonathan H Wright, 2007, The us treasury yield curve: 1961 to the present, *Journal of monetary Economics* 54, 2291–2304.

Hanson, Samuel G., 2014, Mortgage convexity, *Journal of Financial Economics* 113, 270–299.

Hanson, Samuel G, David O Lucca, and Jonathan H Wright, 2021, Rate-amplifying demand and the excess sensitivity of long-term rates, *The Quarterly Journal of Economics* 136, 1719–1781.

Hanson, Samuel G, and Jeremy C Stein, 2015, Monetary policy and long-term real rates, *Journal of Financial Economics* 115, 429–448.

He, Zhiguo, Stefan Nagel, and Zhaogang Song, 2022, Treasury inconvenience yields during the COVID-19 crisis, *Journal of Financial Economics* 143, 57–79.

Huber, Amy Wang, 2023, Market power in wholesale funding: A structural perspective from the tri-party repo market, *Journal of Financial Economics* .

Kekre, Rohan, Moritz Lenel, and Federico Mainardi, 2026, Monetary Policy, Segmentation, and the Term Structure, Technical report, Working Paper.

Khetan, Umang, Jian Li, Ioana Neamțu, and Ishita Sen, 2023, The market for sharing interest rate risk: Quantities and asset prices, *Available at SSRN 4517795* .

Kim, Don H, and Jonathan H Wright, 2011, An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates, Technical report, Federal Reserve Board Finance and Economics Discussion Series.

Malkhozov, Aytek, Philippe Mueller, Andrea Vedolin, and Gyuri Venter, 2016, Mortgage risk and the yield curve, *The Review of Financial Studies* 29, 1220–1253.

Vayanos, Dimitri, and Jean-Luc Vila, 2021, A Preferred-Habitat Model of the Term Structure of Interest Rates, *Econometrica* 89, 77–112.

Wang, Yifei, Toni M Whited, Yufeng Wu, and Kairong Xiao, 2022, Bank market power and monetary policy transmission: Evidence from a structural estimation, *The Journal of Finance* 77, 2093–2141.

Xiao, Kairong, 2020, Monetary transmission through shadow banks, *The Review of Financial Studies* 33, 2379–2420.

A Proofs of Propositions

1. Proof of Proposition 1

Proof. The lender solves the following optimization problem

$$\max_{\{D_{i,t}\}} U_D = \left(\int_0^1 (D_{i,t} r_{i,t})^{1-\frac{1}{\eta}} di\right)^{\frac{1}{1-\frac{1}{\eta}}}, \quad \text{s.t.} \int_0^1 D_{i,t} di = D_t. \tag{A1}$$

The Lagrangean of the above problem is given by

$$\mathscr{L} = \left(\int_0^1 (D_{i,t} r_{i,t})^{1-\frac{1}{\eta}} di\right)^{\frac{1}{1-\frac{1}{\eta}}} + \lambda \left(D_t - \int_0^1 D_{i,t} di\right), \tag{A2}$$

where λ denotes the Lagrange multiplier associated with the constraint. The first-order conditions with respect to $D_{i,t}$ yield

$$\frac{\partial \mathscr{L}}{\partial D_{i,t}} = 0 \Longrightarrow U_D^{\frac{1}{\eta}} D_{i,t}^{-\frac{1}{\eta}} r_{i,t}^{1-\frac{1}{\eta}} = \lambda \tag{A3}$$

Multiplying both sides of equation (A3) by $D_{i,t}$ and integrating over i from 0 to 1, I obtain

$$U_D^{\frac{1}{\eta}} \underbrace{\int_0^1 D_{i,t}^{1-\frac{1}{\eta}} r_{i,t}^{1-\frac{1}{\eta}} di}_{=U_D^{1-\frac{1}{\eta}}} = \lambda \underbrace{\int_0^1 D_{i,t} di}_{D_t} \Longrightarrow U_D = \lambda D_t \tag{A4}$$

Hence, λ is the aggregate return from investing on private dealer sectors. I let $r_d \equiv \lambda$ denote the return of the dealer sector. Replacing U_D with λD_t in equation (A3), we obtain the optimal allocation

$$D_{i,t} = \left(\frac{r_{i,t}}{r_d}\right)^{\eta-1} D_t. \tag{A5}$$

Finally, replacing the above expression into the budget constraint, I obtain the expression of r_d

$$r_d = \left(\int_0^1 r_{i,t}^{\eta-1} di\right)^{\frac{1}{\eta-1}}. \tag{A6}$$

□

2. Proof of Proposition 4

Proof. First, I define

$$\kappa_r^* \equiv \kappa_r + \Omega_{rr} = \kappa_r + \gamma\sigma_r^2 \int_0^T \alpha(\tau) A_r(\tau)^2 d\tau. \tag{A7}$$

and rewrite the ODE for $A_r(\tau)$ in equation (35) as

$$A_r(\tau)\kappa_r^* + A_r'(\tau) - (1 + \mathscr{M}) = 0. \tag{A8}$$

With the boundary condition $A_r(0) = 0$, this ODE has the solution in equation (39). Substituting this solution back into the definition of κ_r^* in equation (A7), I obtain the fixed point equation for κ_r^* in equation (41). Next, taking the integral from 0 to τ on both sides of the ODE for $C(\tau)$ in equation (37), using the boundary condition $C(0) = 0$, and defining $\bar{r}^*$ to satisfy

$$\kappa_r^* \bar{r}^* = \kappa_r \bar{r} + \Omega_r = \kappa_r \bar{r} + \gamma\sigma_r^2 \int_0^T (\theta_0(\tau) - \alpha(\tau) C(\tau)) A_r(\tau) d\tau, \tag{A9}$$

this ODE has the solution in equation (40). Now, substituting the solution for $C(\tau)$ back into the definition of $\bar{r}^*$ in equation (A9), I obtain

$$\begin{aligned} &\kappa_r^* \bar{r}^* \left[1 + \gamma\sigma_r^2 \int_0^\infty \alpha(\tau) \left[\int_0^\tau A_r(u) du \right] A_r(\tau) d\tau \right] \\ &= \kappa_r \bar{r} + \gamma\sigma_r^2 \int_0^\infty \theta_0(\tau) A_r(\tau) d\tau + \frac{\gamma\sigma_r^4}{2} \int_0^\infty \alpha(\tau) \left[\int_0^\tau A_r(u)^2 du \right] A_r(\tau) d\tau \end{aligned} \tag{A10}$$

Notice that

$$\kappa_r \bar{r} = \kappa_r^* \bar{r} + \left(\kappa_r - \kappa_r^*\right) \bar{r} + \kappa_r^* \bar{r} \gamma\sigma_r^2 \int_0^\infty \alpha(\tau) \left[\int_0^\tau A_r(u) du \right] A_r(\tau) d\tau - \kappa_r^* \bar{r} \gamma\sigma_r^2 \int_0^\infty \alpha(\tau) \left[\int_0^\tau A_r(u) du \right] A_r(\tau) d\tau \tag{A11}$$

where the equation holds because first two terms on the right-hand side equal to the left-hand side and the last two terms cancel out. Merging the first and third terms on the right-hand side, I have

$$\begin{aligned} \kappa_r \bar{r} =& \kappa_r^* \bar{r} \left[1 + \gamma\sigma_r^2 \int_0^\infty \alpha(\tau) \left[\int_0^\tau A_r(u) du \right] A_r(\tau) d\tau \right] \\ &+ \left(\kappa_r - \kappa_r^*\right) \bar{r} - \kappa_r^* \bar{r} \gamma\sigma_r^2 \int_0^\infty \alpha(\tau) \left[\int_0^\tau A_r(u) du \right] A_r(\tau) d\tau. \end{aligned} \tag{A12}$$

Next, I simplify the last two terms on the right-hand side above:

$$\begin{aligned}
&\left(\kappa_r - \kappa_r^*\right)\bar{r} - \kappa_r^*\bar{r}\gamma\sigma_r^2\int_0^\infty \alpha(\tau)\left[\int_0^\tau A_r(u)du\right]A_r(\tau)d\tau \\
&= -\bar{r}\gamma\sigma_r^2\int_0^\infty \alpha(\tau)A_r(\tau)^2 d\tau - \kappa_r^*\bar{r}\gamma\sigma_r^2\int_0^\infty \alpha(\tau)\left[\int_0^\tau A_r(u)du\right]A_r(\tau)d\tau \\
&= -\bar{r}\gamma\sigma_r^2\int_0^\infty \alpha(\tau)\left[A_r(\tau) + \kappa_r^*\int_0^\tau A_r(u)du\right]A_r(\tau)d\tau
\end{aligned} \tag{A13}$$

where the first step follows from substituting $\kappa_r - \kappa_r^*$ in equation (A7) and the second step is rearranging the terms. Moreover, notice that

$$A_r(\tau) + \kappa_r^*\int_0^\tau A_r(u)du = \int_0^\tau \left[A_r'(u) + \kappa_r^* A_r(u)\right]du = \int_0^\tau (1+\mathscr{M})du = (1+\mathscr{M})\tau \tag{A14}$$

where the second last step follows from the ODE for $A_r(\tau)$ in equation (A8). Therefore, substituting this into equation (A13), I have

$$\left(\kappa_r - \kappa_r^*\right)\bar{r} - \kappa_r^*\bar{r}\gamma\sigma_r^2\int_0^\infty \alpha(\tau)\left[\int_0^\tau A_r(u)du\right]A_r(\tau)d\tau = -\bar{r}\gamma\sigma_r^2(1+\mathscr{M})\int_0^\infty \alpha(\tau)\tau A_r(\tau)d\tau \tag{A15}$$

Substituting this back into equation (A12), I find

$$\kappa_r\bar{r} = \kappa_r^*\bar{r}\left[1 + \gamma\sigma_r^2\int_0^\infty \alpha(\tau)\left[\int_0^\tau A_r(u)du\right]A_r(\tau)d\tau\right] - \bar{r}\gamma\sigma_r^2(1+\mathscr{M})\int_0^\infty \alpha(\tau)\tau A_r(\tau)d\tau \tag{A16}$$

Substituting this back into equation (A10), I obtain

$$\begin{aligned}
&\kappa_r^*\bar{r}^*\left[1 + \gamma\sigma_r^2\int_0^\infty \alpha(\tau)\left[\int_0^\tau A_r(u)du\right]A_r(\tau)d\tau\right] \\
&= \kappa_r^*\bar{r}\left[1 + \gamma\sigma_r^2\int_0^\infty \alpha(\tau)\left[\int_0^\tau A_r(u)du\right]A_r(\tau)d\tau\right] - \bar{r}\gamma\sigma_r^2(1+\mathscr{M})\int_0^\infty \alpha(\tau)\tau A_r(\tau)d\tau \\
&\quad + \gamma\sigma_r^2\int_0^\infty \theta_0(\tau)A_r(\tau)d\tau + \frac{\gamma\sigma_r^4}{2}\int_0^\infty \alpha(\tau)\left[\int_0^\tau A_r(u)^2 du\right]A_r(\tau)d\tau
\end{aligned} \tag{A17}$$

Finally, substituting the solution for $A_r(\tau)$ in equation (39) into the above equation and rearranging the terms yield the fixed point equation for $\bar{r}^*$ in equation (42). □

3. Proof of Proposition 5

Proof. Differentiating the closed form for $A_r(\tau;\mathscr{M})$ in equation (39) with respect to $\mathscr{M}$, holding τ fixed and treating $\kappa_r^* = \kappa_r^*(\mathscr{M})$,

$$\frac{\partial A_r(\tau;\mathscr{M})}{\partial \mathscr{M}} = a(\tau) + (1+\mathscr{M})\left[\frac{\tau e^{-\kappa_r^*\tau}}{\kappa_r^*} - \frac{a(\tau)}{\kappa_r^*}\right]\frac{d\kappa_r^*}{d\mathscr{M}} = \left[1 - \frac{(1+\mathscr{M})}{\kappa_r^*}\frac{d\kappa_r^*}{d\mathscr{M}}\right]a(\tau) + \frac{(1+\mathscr{M})}{\kappa_r^*}\frac{d\kappa_r^*}{d\mathscr{M}}\tau e^{-\kappa_r^*\tau}, \quad \text{(A18)}$$

where $a(\tau) \equiv (1-e^{-\kappa_r^*\tau})/\kappa_r^* > 0$ for every $\tau > 0$. As shown in the proof of Propositions 10 and 11 (Appendix A.11), the second and fourth of the four conditions defining $\bar{\mathscr{M}}$ there give $(1+\mathscr{M})\,d\kappa_r^*/d\mathscr{M} \leq \kappa_r^*$ for every $\mathscr{M} \in [0,\bar{\mathscr{M}}]$. Since $d\kappa_r^*/d\mathscr{M} > 0$ always, established in the proof of Proposition 7, the second term above is strictly positive for every $\tau > 0$, and the first term is non-negative for every $\mathscr{M} \in [0,\bar{\mathscr{M}}]$. The sum of a non-negative term and a strictly positive term is strictly positive, so $\partial A_r(\tau;\mathscr{M})/\partial\mathscr{M} > 0$ for every $\tau > 0$ and every $\mathscr{M} \in [0,\bar{\mathscr{M}}]$. □

4. Proof of Lemma 2

Proof. Substituting bond market clearing, equation (25) with $\sigma_\beta = 0$, into the definition of $\lambda_{r,t}$ gives

$$\lambda_{r,t} = -\gamma\sigma_r^2 \int_0^T \big(\theta_0(u) - \alpha(u)C(u)\big)A_r(u)\,du + \gamma\sigma_r^2 r_t \int_0^T \alpha(u)A_r(u)^2\,du = -\Omega_r + r_t\Omega_{rr}. \tag{A19}$$

Evaluating at $r_t = \bar{r}$ and using $\Omega_{rr} = \kappa_r^* - \kappa_r$ (equation (A7)) and $\Omega_r = \kappa_r^*\bar{r}^* - \kappa_r\bar{r}$ (equation (A9)) gives

$$\lambda_{r,t}\big|_{r_t=\bar{r}} = \bar{r}(\kappa_r^* - \kappa_r) - (\kappa_r^*\bar{r}^* - \kappa_r\bar{r}) = \kappa_r^*(\bar{r} - \bar{r}^*). \tag{A20}$$

Separately, multiplying equation (39) by κ_r^* gives $A_r(\tau)\kappa_r^* = (1+\mathscr{M})(1-e^{-\kappa_r^*\tau})$. Substituting both into the first-order condition, equation (34) with $A_\beta \equiv 0$, evaluated at $r_t = \bar{r}$,

$$\mu_t^{(\tau)} - \bar{r} = -A_r(\tau)\lambda_{r,t}\big|_{r_t=\bar{r}} + \mathscr{M}\bar{r} = A_r(\tau)\kappa_r^*(\bar{r}^* - \bar{r}) + \mathscr{M}\bar{r}, \tag{A21}$$

and substituting $A_r(\tau)\kappa_r^* = (1+\mathscr{M})(1-e^{-\kappa_r^*\tau})$ yields equation (43). □

5. Proof of Proposition 6

Proof. When $\alpha(\tau) \equiv 0$, $\Omega_{rr} = \gamma\sigma_r^2\int_0^T \alpha(\tau)A_r(\tau)^2\,d\tau = 0$, so $\kappa_r^* = \kappa_r$ by equation (A7), independent of $\mathscr{M}$. Bond market clearing, equation (25) with $\sigma_\beta = 0$ and $\alpha \equiv 0$, reduces to $X_t^{(\tau)} = \theta_0(\tau)$, so

$$\Omega_r = \gamma\sigma_r^2\int_0^T \theta_0(\tau)A_r(\tau)\,d\tau = \gamma\sigma_r^2\int_0^T \theta_0(\tau)(1+\mathscr{M})\frac{1-e^{-\kappa_r\tau}}{\kappa_r}\,d\tau = (1+\mathscr{M})\Omega_r^0, \tag{A22}$$

using $A_r(\tau) = (1+\mathscr{M})(1-e^{-\kappa_r\tau})/\kappa_r$ (equation (39) with $\kappa_r^* = \kappa_r$) and the definition of Ω_r^0. Substituting into $\kappa_r^*\bar{r}^* = \kappa_r\bar{r} + \Omega_r$ (equation (A9)) and dividing by $\kappa_r^* = \kappa_r$ gives

$$\bar{r}^*(\mathscr{M}) = \bar{r} + (1+\mathscr{M})\frac{\Omega_r^0}{\kappa_r}. \tag{A23}$$

Substituting $\kappa_r^* = \kappa_r$ and this expression for $\bar{r}^*(\mathscr{M})$ into Lemma 2's equation (43) gives

$$\mu_t^{(\tau)} - \bar{r} = (1+\mathscr{M})^2\left(\frac{1-e^{-\kappa_r\tau}}{\kappa_r}\right)\Omega_r^0 + \mathscr{M}\bar{r}. \tag{A24}$$

Differentiating with respect to $\mathscr{M}$ yields equation (44). Since $\theta_0(\tau) > 0$ for all τ, $\Omega_r^0 > 0$; since $\bar{r} > 0$ too, both terms on the right-hand side of equation (44) are strictly positive, proving positivity at $\alpha \equiv 0$. Differentiating equation (44) with respect to τ gives $2(1+\mathscr{M})\Omega_r^0 e^{-\kappa_r\tau} > 0$, proving that the marginal effect is strictly increasing in τ at $\alpha \equiv 0$.

I now extend both properties to a neighborhood $[0,\bar{\alpha})$ of $\alpha \equiv 0$. Write $D(\tau,\mathscr{M};\alpha) \equiv \partial(\mu_t^{(\tau)} - \bar{r})/\partial\mathscr{M}$, so $D(\tau,\mathscr{M};0)$ is exactly the right-hand side of equation (44), shown above to be strictly positive with strictly positive τ-derivative for every $\tau \in [0,T]$. Since $[0,T]$ is compact and both $D(\cdot,\mathscr{M};0)$ and $\partial D(\cdot,\mathscr{M};0)/\partial\tau$

are continuous in τ, the extreme value theorem gives strictly positive margins

$$\delta \equiv \min_{\tau\in[0,T]} D(\tau,\mathscr{M};0) > 0, \qquad \delta' \equiv \min_{\tau\in[0,T]} \frac{\partial D(\tau,\mathscr{M};0)}{\partial\tau} > 0. \tag{A25}$$

For general $\alpha(\cdot)$, differentiating the Lemma 2 identity with respect to $\mathscr{M}$ gives

$$D(\tau,\mathscr{M};\alpha) = (1-e^{-\kappa_r^*\tau})(\bar{r}^*-\bar{r}) + (1+\mathscr{M})\left[\tau e^{-\kappa_r^*\tau}\frac{d\kappa_r^*}{d\mathscr{M}}(\bar{r}^*-\bar{r}) + (1-e^{-\kappa_r^*\tau})\frac{d\bar{r}^*}{d\mathscr{M}}\right] + \bar{r}, \tag{A26}$$

a continuous function of $(\kappa_r^*, \bar{r}^*, d\kappa_r^*/d\mathscr{M}, d\bar{r}^*/d\mathscr{M})$, uniformly in $\tau \in [0,T]$. These four equilibrium objects are themselves continuous in $\alpha(\cdot)$ near $\alpha \equiv 0$. Both κ_r^* and $\bar{r}^*$ solve the scalar fixed points of equations (41) and (42), in which $\alpha(\cdot)$ enters only through integrals against continuous functions of τ. The non-degeneracy condition verified in the proof of Proposition 7 ($\partial\Phi/\partial\kappa_r^* > 0$ there, evaluated at $\alpha \equiv 0$) puts the implicit function theorem's hypotheses in force at $\alpha \equiv 0$, so $d\kappa_r^*/d\mathscr{M}$ and $d\bar{r}^*/d\mathscr{M}$, as derivatives of continuously-parameterized fixed points, inherit this continuity. Consequently $D(\tau,\mathscr{M};\alpha)$ and $\partial D(\tau,\mathscr{M};\alpha)/\partial\tau$ converge to $D(\tau,\mathscr{M};0)$ and $\partial D(\tau,\mathscr{M};0)/\partial\tau$ uniformly over $\tau \in [0,T]$ as $\sup_\tau \alpha(\tau) \to 0$. Choose $\bar{\alpha} > 0$ small enough that this convergence keeps $D(\tau,\mathscr{M};\alpha)$ within δ of $D(\tau,\mathscr{M};0)$ and $\partial D(\tau,\mathscr{M};\alpha)/\partial\tau$ within δ' of $\partial D(\tau,\mathscr{M};0)/\partial\tau$ for every $\tau \in [0,T]$, whenever $0 \le \alpha(\tau) \le \bar{\alpha}$ for all τ. Then $D(\tau,\mathscr{M};\alpha) > 0$ and $\partial D(\tau,\mathscr{M};\alpha)/\partial\tau > 0$ for every such $\alpha(\cdot)$ and every $\tau \in [0,T]$, establishing strict positivity and strict τ-monotonicity of the marginal effect on this neighborhood. □

6. Proof of Proposition 7

Proof. For convenience, I denote the dependent variable in the FB regression in equation (45) by Y_t^τ and the independent variable by X_t. Using equations (8), (9), and (17), I can rewrite the dependent and independent variables in the FB regression in equation (45) as follows:

$$Y_t^\tau = \frac{1}{\Delta\tau}\Big(A_r(\tau)r_t + C(\tau) - \big[A_r(\tau-\Delta\tau)r_{t+\Delta\tau} + C(\tau-\Delta\tau)\big] - \big[A_r(\Delta\tau)r_t + C(\Delta\tau)\big]\Big) \tag{A27}$$

$$X_t = \frac{1}{\Delta\tau}\Big(A_r(\tau)r_t + C(\tau) - \big[A_r(\tau-\Delta\tau)r_t + C(\tau-\Delta\tau)\big] - \big[A_r(\Delta\tau)r_t + C(\Delta\tau)\big]\Big) \tag{A28}$$

Substituting these expressions into $b_{\text{FB}} \equiv \lim_{\Delta\tau\to 0}\frac{\text{Cov}(Y_t^\tau, X_t)}{\text{Var}(X_t)}$, I obtain

$$b_{\text{FB}} = \lim_{\Delta\tau\to 0}\frac{[A_r(\tau)-A_r(\Delta\tau)]\text{Var}(r_t) - A_r(\tau-\Delta\tau)\text{Cov}(r_{t+\Delta\tau}, r_t)}{[A_r(\tau)-A_r(\tau-\Delta\tau)-A_r(\Delta\tau)]\text{Var}(r_t)} \tag{A29}$$

Next, I calculate $\text{Cov}(r_{t+\Delta\tau}, r_t)$. I first calculate $r_{t+\Delta\tau}$. I define $X_s := r_s - \bar{r}$ and it satisfies

$$dX_s = -\kappa_r X_s\, ds + \sigma_r\, dB_{r,s}. \tag{A30}$$

Multiply both sides by the integrating factor $e^{\kappa_r s}$ and apply Itô's lemma to $e^{\kappa_r s}X_s$:

$$d\big(e^{\kappa_r s}X_s\big) = e^{\kappa_r s}dX_s + \kappa_r e^{\kappa_r s}X_s\, ds = \sigma_r e^{\kappa_r s}dB_{r,s}. \tag{A31}$$

Integrating from $s = t$ to $s = t+\Delta\tau$ yields

$$e^{\kappa_r(t+\Delta\tau)}X_{t+\Delta\tau} - e^{\kappa_r t}X_t = \sigma_r\int_t^{t+\Delta\tau} e^{\kappa_r s}\, dB_{r,s}. \tag{A32}$$

Rearranging gives

$$X_{t+\Delta\tau} = e^{-\kappa_r\Delta\tau}X_t + \sigma_r\int_t^{t+\Delta\tau} e^{-\kappa_r(t+\Delta\tau-s)}dB_{r,s}. \tag{A33}$$

Substituting back $X_s = r_s - \bar{r}$, I obtain

$$r_{t+\Delta\tau} = \bar{r}(1-e^{-\kappa_r\Delta\tau}) + r_t e^{-\kappa_r\Delta\tau} + \sigma_r\int_t^{t+\Delta\tau} e^{-\kappa_r(t+\Delta\tau-s)}dB_{r,s}. \tag{A34}$$

As the first term and the last term are independent of r_t, I have

$$\text{Cov}(r_{t+\Delta\tau}, r_t) = \text{Var}(r_t)e^{-\kappa_r\Delta\tau}, \tag{A35}$$

Substituting this expression back into the expression for b_{FB}, I obtain

$$b_{\text{FB}} = \lim_{\Delta\tau\to 0}\frac{A_r(\tau) - A_r(\tau-\Delta\tau)e^{-\kappa_r\Delta\tau} - A_r(\Delta\tau)}{A_r(\tau)-A_r(\tau-\Delta\tau)-A_r(\Delta\tau)}. \tag{A36}$$

Dividing the numerator and denominator of b_{FB} by $\Delta\tau$ and rearranging terms, I obtain

$$b_{\text{FB}} = \lim_{\Delta\tau\to 0} \frac{\frac{A_r(\tau)-A_r(\tau-\Delta\tau)}{\Delta\tau} - \frac{A_r(\Delta\tau)}{\Delta\tau} + \frac{A_r(\tau-\Delta\tau)(1-e^{-\kappa_r\Delta\tau})}{\Delta\tau}}{\frac{A_r(\tau)-A_r(\tau-\Delta\tau)}{\Delta\tau} - \frac{A_r(\Delta\tau)}{\Delta\tau}}. \tag{A37}$$

Applying L'Hôpital's rule, I have

$$\lim_{\Delta\tau\to 0} \frac{A_r(\tau)-A_r(\tau-\Delta\tau)}{\Delta\tau} = A_r'(\tau) \tag{A38}$$

$$\lim_{\Delta\tau\to 0} \frac{A_r(\tau-\Delta\tau)(1-e^{-\kappa_r\Delta\tau})}{\Delta\tau} = \kappa_r A_r(\tau) \tag{A39}$$

$$\lim_{\Delta\tau\to 0} \frac{A_r(\Delta\tau)}{\Delta\tau} = \lim_{\Delta\tau\to 0} \frac{(1+\mathscr{M})(1-e^{-\kappa_r^*\Delta\tau})}{\kappa_r^*\Delta\tau} = 1+\mathscr{M}, \tag{A40}$$

Substituting these limits back into the expression for b_{FB}, I obtain

$$b_{\text{FB}} = \frac{A_r'(\tau) - (1+\mathscr{M}) + \kappa_r A_r(\tau)}{A_r'(\tau) - (1+\mathscr{M})}. \tag{A41}$$

Finally, substituting $A_r(\tau) = \frac{(1+\mathscr{M})(1-e^{-\kappa_r^*\tau})}{\kappa_r^*}$ in equation (39), and its derivative $A_r'(\tau) = (1+\mathscr{M})e^{-\kappa_r^*\tau}$ into the above expression yields

$$b_{\text{FB}} = \frac{(1+\mathscr{M})(e^{-\kappa_r^*\tau}-1)\left(\frac{\kappa_r^*-\kappa_r}{\kappa_r^*}\right)}{(1+\mathscr{M})(e^{-\kappa_r^*\tau}-1)} = \frac{\kappa_r^*-\kappa_r}{\kappa_r^*}. \tag{A42}$$

I first show that $b_{\text{FB}} > 0$. From equation (41), I have

$$\kappa_r^* = \kappa_r + \gamma\sigma_r^2(1+\mathscr{M})^2 \int_0^T \alpha(\tau)\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right)^2 d\tau. \tag{A43}$$

Since $\gamma > 0$, $\sigma_r^2 > 0$, $\alpha(\tau) > 0$, and $(1+\mathscr{M}) > 1$, the integral term is strictly positive. Therefore, $\kappa_r^* > \kappa_r$, which implies that $b_{\text{FB}} > 0$. I then show that b_{FB} is strictly increasing in $\mathscr{M}$. As $\frac{\partial b_{\text{FB}}}{\partial \mathscr{M}} = \frac{\kappa_r}{\kappa_r^{*2}}\frac{d\kappa_r^*}{d\mathscr{M}}$, it suffices to show that κ_r^* is strictly increasing in $\mathscr{M}$. Define the implicit function $\Phi : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ by:

$$\Phi(\kappa_r^*, \mathscr{M}) = \kappa_r^* - \kappa_r - \gamma\sigma_r^2(1+\mathscr{M})^2 \int_0^T \alpha(\tau)\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right)^2 d\tau = 0. \tag{A44}$$

By the implicit function theorem:

$$\frac{d\kappa_r^*}{d\mathscr{M}} = -\frac{\frac{\partial\Phi}{\partial\mathscr{M}}}{\frac{\partial\Phi}{\partial\kappa_r^*}}. \tag{A45}$$

I now compute $\frac{\partial \Phi}{\partial \mathscr{M}}$:

$$\frac{\partial \Phi}{\partial \mathscr{M}} = -2\gamma\sigma_r^2(1+\mathscr{M})\int_0^T \alpha(\tau)\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right)^2 d\tau < 0, \tag{A46}$$

where the inequality holds since all parameters are positive. Next, I compute $\frac{\partial \Phi}{\partial \kappa_r^*}$:

$$\frac{\partial \Phi}{\partial \kappa_r^*} = 1 - 2\gamma\sigma_r^2(1+\mathscr{M})^2\int_0^T \alpha(\tau)\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\frac{\partial}{\partial \kappa_r^*}\left[\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right] d\tau. \tag{A47}$$

I have:

$$\frac{\partial}{\partial \kappa_r^*}\left[\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right] = \frac{\tau\kappa_r^* e^{-\kappa_r^*\tau} - (1-e^{-\kappa_r^*\tau})}{\kappa_r^{*2}} = \frac{\psi(\kappa_r^*\tau)}{\kappa_r^{*2}}, \tag{A48}$$

where $\psi(x) = xe^{-x} - (1-e^{-x})$. For $x > 0$, I have $\psi'(x) = -xe^{-x} < 0$ and $\psi(0) = 0$, so $\psi(x) < 0$ for all $x > 0$. Therefore:

$$\frac{\partial \Phi}{\partial \kappa_r^*} = 1 - \frac{2\gamma\sigma_r^2(1+\mathscr{M})^2}{\kappa_r^{*3}}\int_0^T \alpha(\tau)(1-e^{-\kappa_r^*\tau})\psi(\kappa_r^*\tau)d\tau. \tag{A49}$$

Since $(1-e^{-\kappa_r^*\tau}) > 0$ and $\psi(\kappa_r^*\tau) < 0$, the integral is negative. Hence:

$$\frac{\partial \Phi}{\partial \kappa_r^*} > 1 > 0. \tag{A50}$$

Combining the results above, I have $\frac{\partial \Phi}{\partial \mathscr{M}} < 0$ and $\frac{\partial \Phi}{\partial \kappa_r^*} > 0$, which implies:

$$\frac{d\kappa_r^*}{d\mathscr{M}} = -\frac{\frac{\partial \Phi}{\partial \mathscr{M}}}{\frac{\partial \Phi}{\partial \kappa_r^*}} > 0. \tag{A51}$$

Therefore, κ_r^* is strictly increasing in $\mathscr{M}$ and hence b_{FB} is strictly increasing in $\mathscr{M}$:

$$\frac{\partial b_{\text{FB}}}{\partial \mathscr{M}} = \frac{\kappa_r}{\kappa_r^{*2}}\frac{d\kappa_r^*}{d\mathscr{M}} > 0. \tag{A52}$$

□

7. Proof of Proposition 8

Proof. The ratio of the model's forward-rate sensitivity to the expectations-hypothesis benchmark is, from equation (49),

$$R(\tau) = \frac{\partial f_t^{(\tau)}/\partial r_t}{\partial E_t(r_{t+\tau})/\partial r_t} = \frac{(1+\mathscr{M})e^{-\kappa_r^* \tau}}{e^{-\kappa_r \tau}} = (1+\mathscr{M})e^{-(\kappa_r^* - \kappa_r)\tau}. \tag{A53}$$

Since $\kappa_r^* = \kappa_r + \gamma\sigma_r^2(1+\mathscr{M})^2 \int_0^T \alpha(\tau)A_r(\tau)^2\, d\tau$, and $\gamma > 0$ and $\sigma_r^2 > 0$ by assumption, the integral is strictly positive whenever $\alpha(\tau)$ is not identically zero and $A_r(\tau)$ is not identically zero, the latter holding whenever $\sigma_r > 0$. Hence $\kappa_r^* - \kappa_r > 0$ strictly, for every $\mathscr{M} \geq 0$.

$R(\tau)$ is therefore a continuous, strictly decreasing function of τ on $[0,\infty)$, since its derivative $-(\kappa_r^* - \kappa_r)R(\tau)$ is strictly negative everywhere, with $R(0) = 1+\mathscr{M}$ and $\lim_{\tau\to\infty} R(\tau) = 0$. Since $\mathscr{M} > 0$ by assumption, $R(0) = 1+\mathscr{M} > 1$. By the intermediate value theorem, since R is continuous and strictly decreasing from a value above 1 to 0, there exists a unique $\tau^{**} \in (0,\infty)$ such that $R(\tau^{**}) = 1$. Solving $(1+\mathscr{M})e^{-(\kappa_r^* - \kappa_r)\tau^{**}} = 1$ for τ^{**} gives

$$\tau^{**} = \frac{\ln(1+\mathscr{M})}{\kappa_r^* - \kappa_r}, \tag{A54}$$

establishing equation (52). Since R is strictly decreasing, $R(\tau) > R(\tau^{**}) = 1$ for every $\tau < \tau^{**}$, and $R(\tau) < R(\tau^{**}) = 1$ for every $\tau > \tau^{**}$, establishing the proposition. □

8. Proof of Proposition 9

Proof. I establish the properties of $\mathscr{S}_t^{(\tau)} \equiv \frac{\partial}{\partial \mathscr{M}}\left(\frac{\partial f_t^{(\tau)}}{\partial r_t}\right)$ through a series of steps. I first verify equation (**??**). From the bond pricing formula $P_t^{(\tau)} = e^{-A_r(\tau)r_t - C(\tau)}$, the forward rate is given by:

$$f_t^{(\tau)} = -\frac{\partial \log P_t^{(\tau)}}{\partial \tau} = A_r'(\tau) r_t + C'(\tau) \tag{A55}$$

Taking the derivative with respect to r_t:

$$\frac{\partial f_t^{(\tau)}}{\partial r_t} = A_r'(\tau) = (1+\mathscr{M}) e^{-\kappa_r^* \tau} \tag{A56}$$

This establishes equation (**??**).

I now derive the expression for $\mathscr{S}_t^{(\tau)}$. From equation (**??**):

$$\mathscr{S}_t^{(\tau)} = \frac{\partial}{\partial \mathscr{M}}\left[(1+\mathscr{M}) e^{-\kappa_r^* \tau}\right] = e^{-\kappa_r^* \tau} + (1+\mathscr{M}) e^{-\kappa_r^* \tau}\left(-\tau \frac{d\kappa_r^*}{d\mathscr{M}}\right) \tag{A57}$$

Factoring out $e^{-\kappa_r^* \tau} > 0$:

$$\mathscr{S}_t^{(\tau)} = e^{-\kappa_r^* \tau}\left[1 - (1+\mathscr{M})\tau \frac{d\kappa_r^*}{d\mathscr{M}}\right] \tag{A58}$$

To establish Property 1, I define:

$$\phi(\tau) \equiv 1 - (1+\mathscr{M})\tau \frac{d\kappa_r^*}{d\mathscr{M}} \tag{A59}$$

κ_r^* depends on the entire function $A_r(\cdot)$ over $[0,T]$ but not on the specific point τ, so it is independent of τ, and I have:

$$\frac{d\phi}{d\tau} = -(1+\mathscr{M})\frac{d\kappa_r^*}{d\mathscr{M}} < 0 \tag{A60}$$

Therefore, $\phi(\tau)$ is a strictly decreasing linear function of τ with $\phi(0) = 1 > 0$ and $\frac{d\phi}{d\tau} < 0$ for all $\tau \geq 0$. This implies there exists a unique $\hat{\tau} > 0$ such that $\phi(\hat{\tau}) = 0$:

$$\hat{\tau} = \frac{1}{(1+\mathscr{M})\frac{d\kappa_r^*}{d\mathscr{M}}} \tag{A61}$$

Since $\phi(\tau) > 0$ for $\tau < \hat{\tau}$ and $\phi(\tau) < 0$ for $\tau > \hat{\tau}$, and since $e^{-\kappa_r^* \tau} > 0$ for all τ, I have from equation (A58):

$$\mathscr{S}_t^{(\tau)} > 0 \text{ for all } \tau \in (0, \hat{\tau}) \quad \text{and} \quad \mathscr{S}_t^{(\tau)} < 0 \text{ for all } \tau \in (\hat{\tau}, \infty)$$

This establishes Property 1. In particular, if $T \leq \hat{\tau}$, then $\mathscr{S}_t^{(\tau)} > 0$ for all $\tau \in (0, T]$, implying that market power increases the forward rate sensitivity for all maturities. I next establish Property 2 by analyzing the monotonicity of $\mathscr{S}_t^{(\tau)}$ with respect to τ. I compute the derivative:

$$\frac{\partial \mathscr{S}_t^{(\tau)}}{\partial \tau} = \frac{\partial}{\partial \tau}\left[e^{-\kappa_r^* \tau}\phi(\tau)\right] = e^{-\kappa_r^* \tau}\left[-\kappa_r^* \phi(\tau) - (1+\mathscr{M})\frac{d\kappa_r^*}{d\mathscr{M}}\right] \tag{A62}$$

Since $e^{-\kappa_r^*\tau} > 0$, the sign of $\frac{\partial \mathscr{S}_t^{(\tau)}}{\partial \tau}$ is determined by:

$$-\kappa_r^* \phi(\tau) - (1+\mathscr{M})\frac{d\kappa_r^*}{d\mathscr{M}} = -\kappa_r^* + (1+\mathscr{M})\frac{d\kappa_r^*}{d\mathscr{M}}(\kappa_r^*\tau - 1) \tag{A63}$$

Define:

$$h(\tau) \equiv -\kappa_r^* + (1+\mathscr{M})\frac{d\kappa_r^*}{d\mathscr{M}}(\kappa_r^*\tau - 1) \tag{A64}$$

This is a linear function of τ with:

$$h(0) = -\kappa_r^* - (1+\mathscr{M})\frac{d\kappa_r^*}{d\mathscr{M}} < 0 \tag{A65}$$

$$\frac{dh}{d\tau} = (1+\mathscr{M})\kappa_r^*\frac{d\kappa_r^*}{d\mathscr{M}} > 0 \tag{A66}$$

Therefore, there exists a unique threshold $\bar{\tau} > 0$ such that $h(\bar{\tau}) = 0$:

$$\bar{\tau} = \frac{1}{\kappa_r^*} + \frac{1}{(1+\mathscr{M})\kappa_r^*\frac{d\kappa_r^*}{d\mathscr{M}}} \tag{A67}$$

For $\tau < \bar{\tau}$, I have $h(\tau) < 0$, hence $\frac{\partial \mathscr{S}_t^{(\tau)}}{\partial \tau} < 0$. For $\tau > \bar{\tau}$, I have $h(\tau) > 0$, hence $\frac{\partial \mathscr{S}_t^{(\tau)}}{\partial \tau} > 0$. This establishes that $\mathscr{S}_t^{(\tau)}$ is strictly decreasing on $[0, \bar{\tau})$ and strictly increasing on $(\bar{\tau}, \infty)$, reaching its minimum at $\tau = \bar{\tau}$. Therefore, if $T \leq \bar{\tau}$, then $\mathscr{S}_t^{(\tau)}$ is monotonically decreasing on $[0, T]$. If $T > \bar{\tau}$, then $\mathscr{S}_t^{(\tau)}$ exhibits a U-shape on $[0, T]$. This establishes Property 2.

Finally, I establish Property 3. From Properties 1 and 2, I know $\mathscr{S}_t^{(\tau)} > 0$ for all $\tau \in (0, \hat{\tau})$ and $\frac{\partial \mathscr{S}_t^{(\tau)}}{\partial \tau} < 0$ for all $\tau \in (0, \bar{\tau})$. Define $\tau^* \equiv \min\{\hat{\tau}, \bar{\tau}\}$. For any $\tau \in (0, \tau^*)$, I have $\tau < \hat{\tau}$ and $\tau < \bar{\tau}$, which implies both $\mathscr{S}_t^{(\tau)} > 0$ and $\frac{\partial \mathscr{S}_t^{(\tau)}}{\partial \tau} < 0$. This establishes Property 3, showing that for short maturities (up to τ^*), market power always increases the forward rate sensitivity, and this positive impact is decreasing in maturity. □

9. Proof of Proposition 12

Proof. Equations (35)–(36) of Section III.B can be written in vector form as the linear, constant-coefficient system

$$A'(\tau) + \Xi A(\tau) = (1+\mathscr{M})\mathscr{E}, \qquad A(0) = \mathbf{0}, \tag{A68}$$

where Ξ is given in equation (55). Premultiplying by the integrating factor $e^{\Xi\tau}$ gives

$$\frac{d}{d\tau}\left[e^{\Xi\tau}A(\tau)\right] = e^{\Xi\tau}A'(\tau) + \Xi e^{\Xi\tau}A(\tau) = e^{\Xi\tau}(1+\mathscr{M})\mathscr{E}, \tag{A69}$$

using $e^{\Xi\tau}\Xi = \Xi e^{\Xi\tau}$, since any matrix commutes with its own exponential. Integrating both sides from 0 to τ and using $A(0) = \mathbf{0}$,

$$e^{\Xi\tau}A(\tau) = (1+\mathscr{M})\int_0^\tau e^{\Xi s}\,ds\,\mathscr{E}. \tag{A70}$$

Since Ξ is invertible by assumption, $\int_0^\tau e^{\Xi s}\,ds = \Xi^{-1}\left(e^{\Xi\tau} - I\right)$ (differentiate the right-hand side in τ to verify, using again that Ξ and $e^{\Xi\tau}$ commute). Hence

$$e^{\Xi\tau}A(\tau) = (1+\mathscr{M})\Xi^{-1}\left(e^{\Xi\tau} - I\right)\mathscr{E}. \tag{A71}$$

Left-multiplying both sides by $e^{-\Xi\tau}$, and using that $e^{-\Xi\tau}$ commutes with Ξ^{-1} and with $e^{\Xi\tau}$, since both are power series in Ξ, yields

$$A(\tau) = (1+\mathscr{M})\Xi^{-1}\left(I - e^{-\Xi\tau}\right)\mathscr{E}, \tag{A72}$$

which is equation (56). The hypothesis that Ξ's eigenvalues have strictly positive real parts ensures $e^{-\Xi\tau} \to 0$ as $\tau \to \infty$, so $A(\tau)$ is bounded on $[0,\infty)$ and converges to the finite limit $(1+\mathscr{M})\Xi^{-1}\mathscr{E}$, as required for the loadings of a well-defined infinite-maturity bond price. □

10. Proof of Lemma 3

Proof. At $\alpha(\tau) \equiv 0$, $\Omega_{rr} = \gamma\sigma_r^2\int_0^T \alpha(\tau)A_r(\tau)^2 d\tau = 0$ and $\Omega_{\beta r} = \gamma\sigma_\beta^2\int_0^T \alpha(\tau)A_\beta(\tau)A_r(\tau)d\tau = 0$ exactly, for every σ_β, since both carry an explicit $\alpha(\tau)$ factor. Hence $\kappa_r^* = \kappa_r$ exactly and $A_r(\tau) = (1+\mathscr{M})(1-e^{-\kappa_r\tau})/\kappa_r$ by Proposition 4. Since $\Omega_{r\beta}$ carries no $\alpha(\tau)$ factor,

$$\Omega_{r\beta} = \gamma\sigma_r^2\int_0^T A_r(u)\big(\theta(u) - \alpha(u)A_\beta(u)\big)du = \gamma\sigma_r^2\int_0^T \theta(u)A_r(u)\,du = (1+\mathscr{M})\Theta_r, \tag{A73}$$

exactly, using $\alpha \equiv 0$ and $A_r(u) = (1+\mathscr{M})(1-e^{-\kappa_r u})/\kappa_r$. Only $\Omega_{\beta\beta} = \gamma\sigma_\beta^2\int_0^T A_\beta(\tau)\theta(\tau)d\tau$ retains a nontrivial dependence on A_β, giving $\kappa_\beta^* \equiv \kappa_\beta - \Omega_{\beta\beta}$ as in equation (63) and reducing equation (36) to

$$A_\beta'(\tau) + \kappa_\beta^* A_\beta(\tau) = (1+\mathscr{M})\Theta_r A_r(\tau), \qquad A_\beta(0) = 0. \tag{A74}$$

To establish existence and uniqueness of κ_β^*, I fix a trial value $\kappa_\beta > 0$ and let $A_\beta(\tau;\kappa_\beta) \equiv (1+\mathscr{M})^2\frac{\Theta_r}{\kappa_r}\varphi(\tau,\kappa_\beta)$,

where

$$\varphi(\tau,\kappa_\beta) \equiv \frac{1-e^{-\kappa_\beta \tau}}{\kappa_\beta} - \frac{e^{-\kappa_r \tau} - e^{-\kappa_\beta \tau}}{\kappa_\beta - \kappa_r}, \tag{A75}$$

denote the solution of equation (A74) obtained by integrating factor $e^{\kappa_\beta \tau}$ (multiplying by $e^{\kappa_\beta \tau}$, integrating from 0 to τ, and evaluating the resulting integral yields exactly $\varphi(\tau,\kappa_\beta)$ above). Define $\Omega_{\beta\beta}(\kappa_\beta) \equiv \gamma\sigma_\beta^2 \int_0^T A_\beta(\tau;\kappa_\beta)\theta(\tau)d\tau$ and $G(\kappa_\beta) \equiv \kappa_\beta - \Omega_{\beta\beta}(\kappa_\beta)$, so κ_β^* is a fixed point $\kappa_\beta^* = G(\kappa_\beta^*)$ using the exogenous κ_β as the initial value. Writing $\varphi(\tau,\kappa_\beta) = \kappa_r \cdot \hat{\varphi}(\tau,\kappa_\beta)$ where $\hat{\varphi}(\tau,\kappa_\beta) \equiv \int_0^\tau e^{-\kappa_\beta(\tau-s)} \frac{1-e^{-\kappa_r s}}{\kappa_r} ds$ is the convolution form of the same function (differentiating the closed form in κ_β and matching against the integral form verifies the two coincide),

$$\frac{\partial \hat{\varphi}}{\partial \kappa_\beta}(\tau,\kappa_\beta) = -\int_0^\tau (\tau - s) e^{-\kappa_\beta(\tau-s)} \frac{1-e^{-\kappa_r s}}{\kappa_r} ds < 0 \tag{A76}$$

strictly, for every $\tau > 0$, $\kappa_\beta \geq 0$, since $(\tau - s) \geq 0$ and $(1-e^{-\kappa_r s})/\kappa_r > 0$ on $(0,\tau)$. Bounding $e^{-\kappa_\beta(\tau-s)} \leq 1$ and $(1-e^{-\kappa_r s})/\kappa_r \leq (1-e^{-\kappa_r \tau})/\kappa_r$ for $s \leq \tau$, since the map $u \mapsto (1-e^{-\kappa_r u})/\kappa_r$ is increasing,

$$\left| \frac{\partial \hat{\varphi}}{\partial \kappa_\beta}(\tau,\kappa_\beta) \right| \leq \frac{1-e^{-\kappa_r \tau}}{\kappa_r} \int_0^\tau (\tau - s) ds = \frac{1-e^{-\kappa_r \tau}}{\kappa_r} \cdot \frac{\tau^2}{2}. \tag{A77}$$

Since $\Omega_{\beta\beta}(\kappa_\beta) = \gamma\sigma_\beta^2(1+\mathscr{M})^2\Theta_r \int_0^T \theta(\tau)\hat{\varphi}(\tau,\kappa_\beta)d\tau$ (using $\varphi = \kappa_r\hat{\varphi}$ and the κ_r in the denominator of Θ_r/κ_r canceling against it),

$$|G'(\kappa_\beta)| = \left|\Omega'_{\beta\beta}(\kappa_\beta)\right| \leq \gamma\sigma_\beta^2(1+\mathscr{M})^2\Theta_r \cdot \frac{1}{2}\int_0^T \theta(u) \frac{1-e^{-\kappa_r u}}{\kappa_r} u^2 du, \tag{A78}$$

which is strictly less than 1 by condition (64). Hence G is a contraction on $\mathbb{R}_+$, and the Banach fixed-point theorem gives existence and uniqueness of $\kappa_\beta^* = G(\kappa_\beta^*)$. Substituting $\kappa_\beta = \kappa_\beta^*$ into $A_\beta(\tau;\kappa_\beta)$ above yields equation (65).

It remains to show $A_\beta(\tau)$ is positive and increasing in $\mathscr{M}$. Since $\varphi(\tau,\kappa_\beta) = \kappa_r\hat{\varphi}(\tau,\kappa_\beta)$ and $\hat{\varphi}(\tau,\kappa_\beta) = \int_0^\tau e^{-\kappa_\beta(\tau-s)}\frac{1-e^{-\kappa_r s}}{\kappa_r}ds > 0$ for $\tau > 0$, a positive kernel integrated against a strictly positive function on $(0,\tau)$, and $\Theta_r > 0$ given $\theta(\tau) \geq 0$ not identically zero, $A_\beta(\tau;\mathscr{M}) = (1+\mathscr{M})^2\frac{\Theta_r}{\kappa_r}\varphi(\tau,\kappa_\beta^*) > 0$ for every $\tau > 0$.

For the sign of $d\kappa_\beta^*/d\mathscr{M}$, let $F(\kappa_\beta,\mathscr{M}) \equiv \kappa_\beta - \kappa_\beta + \Omega_{\beta\beta}(\kappa_\beta,\mathscr{M})$ (using κ_β for both the fixed exogenous parameter and the trial variable, as above) so κ_β^* solves $F(\kappa_\beta^*,\mathscr{M}) = 0$. Then

$$\frac{\partial F}{\partial \mathscr{M}} = \frac{2}{1+\mathscr{M}}\Omega_{\beta\beta}(\kappa_\beta^*,\mathscr{M}) > 0, \qquad \frac{\partial F}{\partial \kappa_\beta^*} = 1 - G'(\kappa_\beta^*) > 0 \tag{A79}$$

by condition (64) and the contraction bound $|G'(\kappa_\beta^*)| < 1$ established above (noting $G'(\kappa_\beta) = -\Omega'_{\beta\beta}(\kappa_\beta) > 0$, since $\hat{\varphi}_{\kappa_\beta} < 0$, so $G'(\kappa_\beta^*) = |G'(\kappa_\beta^*)| < 1$, giving $1 - G'(\kappa_\beta^*) > 0$). By the implicit function theorem,

$$\frac{d\kappa_\beta^*}{d\mathscr{M}} = -\frac{\partial F/\partial \mathscr{M}}{\partial F/\partial \kappa_\beta^*} < 0. \tag{A80}$$

Write $\zeta \equiv \Theta_r/\kappa_r$, a constant independent of $\mathscr{M}$ since κ_r does not depend on $\mathscr{M}$ at $\alpha \equiv 0$ (unlike the short-rate case, where the analogous constant is evaluated at the $\mathscr{M}$-dependent κ_r^*). Then $A_\beta(\tau;\mathscr{M}) = (1+\mathscr{M})^2\zeta\,\varphi(\tau,\kappa_\beta^*(\mathscr{M}))$, and differentiating with respect to $\mathscr{M}$,

$$\frac{\partial A_\beta(\tau;\mathscr{M})}{\partial \mathscr{M}} = (1+\mathscr{M})\zeta\left[2\varphi(\tau,\kappa_\beta^*) + (1+\mathscr{M})\,\varphi_{\kappa_\beta}(\tau,\kappa_\beta^*)\frac{d\kappa_\beta^*}{d\mathscr{M}}\right], \tag{A81}$$

where $\varphi_{\kappa_\beta} \equiv \partial\varphi/\partial\kappa_\beta = \kappa_r\,\hat{\varphi}_{\kappa_\beta} < 0$. Both terms in the bracket are strictly positive: the first because $\varphi(\tau,\kappa_\beta^*) > 0$ (shown above), the second because it is the product of $(1+\mathscr{M}) > 0$, $\varphi_{\kappa_\beta}(\tau,\kappa_\beta^*) < 0$, and $d\kappa_\beta^*/d\mathscr{M} < 0$, two negative factors and one positive. Hence $\partial A_\beta(\tau;\mathscr{M})/\partial\mathscr{M} > 0$ for every $\tau > 0$, with both channels pushing in the same direction, unlike Proposition 6's short-rate analogue, where the corresponding two channels can have opposite signs. □

11. Proofs of Propositions 10 and 11

Proof. Write $a \equiv 1+\mathscr{M}$, $\kappa \equiv \kappa_r^*$, $\rho_r \equiv \kappa_r/(\kappa(1+K))$, $\rho_\theta \equiv a\Psi_r^*/(\kappa(1+K))$, $\eta \equiv a\kappa\rho_\theta = a^2\Psi_r^*/(1+K)$, and $I(\tau) \equiv \int_0^\tau A_r(u)\,du$, so that $\mathrm{FG}(\tau;\mathscr{M}) = F_r(\tau)/\tau$ with $F_r(\tau) \equiv A_r(\tau) + \kappa\rho_r I(\tau)$, and $\mathrm{QE}(\tau;\mathscr{M}) = F_\theta(\tau)/\tau$ with $F_\theta(\tau) \equiv \kappa\rho_\theta I(\tau)$. Since $\kappa_r^* \geq \kappa_r$ always, $\rho_r \leq \kappa_r/\kappa_r^* \leq 1$, with $\rho_r = 1$ if and only if $K = 0$ if and only if $\alpha(\tau) \equiv 0$. As shown in the proof of Proposition 7, $\kappa'(\mathscr{M}) \equiv d\kappa_r^*/d\mathscr{M} > 0$ for any $\alpha(\tau) \geq 0$. Define $\eta(\mathscr{M}) \equiv a\kappa\rho_\theta = a^2\Psi_r^*/(1+K)$, $D(\mathscr{M}) \equiv \kappa(\mathscr{M})(1+K(\mathscr{M}))$, so that $\rho_r = \kappa_r/D$, and let $\bar{\mathscr{M}}$ be the supremum of values below which

$$\rho_r(\mathscr{M}) \geq \tfrac{1}{2}, \qquad K'(\mathscr{M}) \geq 0, \qquad \eta'(\mathscr{M})\kappa(\mathscr{M}) \geq 2\eta(\mathscr{M})\kappa'(\mathscr{M}), \qquad (1+\mathscr{M})D'(\mathscr{M}) \leq D(\mathscr{M}) \tag{A82}$$

all hold. At $\mathscr{M} = 0$, K is generally strictly positive, so $\rho_r(0) < 1$; as $\mathscr{M}$ grows, ρ_r falls and K rises, so the first condition in equation (A82) tightens only gradually, while the other three hold with a wide margin throughout, since η, driven by the multiplier a^2, grows far faster in $\mathscr{M}$ than κ does, and D itself grows far more slowly in $\mathscr{M}$ than the multiplier a does. Table 2 in Appendix A.11.1 confirms $\bar{\mathscr{M}}$ exceeds 3, above every value of $\mathscr{M}$ this paper's calibration considers (Section V.C); $\rho_r \geq \frac{1}{2}$ is the binding constraint throughout.

The second and fourth conditions in equation (A82) also justify Proposition 5's hypothesis: since $D = \kappa(1+K)$, $D'/D = \kappa'/\kappa + K'/(1+K)$, so $K' \geq 0$ gives $\kappa'/\kappa \leq D'/D$, and $(1+\mathscr{M})D' \leq D$ gives $D'/D \leq 1/(1+\mathscr{M})$; together these give $(1+\mathscr{M})\kappa' \leq \kappa$ for every $\mathscr{M} \leq \bar{\mathscr{M}}$, the same threshold used throughout this section.

Forward guidance. Substituting $A_r(\tau) = a(1-e^{-\kappa\tau})/\kappa$ and $I(\tau) = (a/\kappa)[\tau - (1-e^{-\kappa\tau})/\kappa]$ and simplifying,

$$\mathrm{FG}'(\tau) \equiv \frac{\partial \mathrm{FG}(\tau;\mathscr{M})}{\partial\tau} = \frac{F_r'(\tau)\tau - F_r(\tau)}{\tau^2} = \frac{\varphi\,\psi(\tau)}{\tau^2}, \qquad \varphi \equiv a(1-\rho_r), \qquad \psi(\tau) \equiv \tau e^{-\kappa\tau} - \frac{1-e^{-\kappa\tau}}{\kappa}. \tag{A83}$$

$\psi(0) = 0$ and $\psi'(\tau) = -\kappa\tau e^{-\kappa\tau} < 0$ for $\tau > 0$, so $\psi(\tau) < 0$ for every $\tau > 0$; whenever $\alpha(\tau) > 0$ on a positive-measure subset of $(0,T]$, $K > 0$ and $\rho_r < 1$ strictly, so $\varphi > 0$ and $\mathrm{FG}'(\tau) < 0$, recovering that $\mathrm{FG}(\tau;\mathscr{M})$ itself

is strictly decreasing in τ. To sign $\mathrm{FG}_{\mathscr{M}}(\tau) \equiv \partial \mathrm{FG}(\tau;\mathscr{M})/\partial\mathscr{M}$, differentiate $\mathrm{FG}'(\tau) = \varphi\,\psi(\tau)/\tau^2$ in $\mathscr{M}$. Since

$$\frac{\partial\psi(\tau)}{\partial\kappa} = -\tau^2 e^{-\kappa\tau} - \frac{\psi(\tau)}{\kappa}, \tag{A84}$$

$$\tau^2\,\frac{\partial \mathrm{FG}'(\tau)}{\partial\mathscr{M}} = \varphi'\psi(\tau) + \varphi\kappa'\frac{\partial\psi(\tau)}{\partial\kappa} = \psi(\tau)\left[\varphi' - \frac{\varphi\kappa'}{\kappa}\right] - \varphi\kappa'\tau^2 e^{-\kappa\tau}. \tag{A85}$$

The second term is strictly negative for $\tau > 0$, and since $\psi(\tau) < 0$, the first term is non-positive whenever $\varphi'\kappa \geq \varphi\kappa'$. Differentiating $\varphi = \alpha(1-\rho_r)$ and using $\rho_r' = -\rho_r\left[\kappa'/\kappa + K'/(1+K)\right]$,

$$\varphi'\kappa - \varphi\kappa' = \kappa(1-\rho_r) + \alpha\kappa'(2\rho_r - 1) + \frac{\alpha\rho_r\kappa K'}{1+K}. \tag{A86}$$

The first term is non-negative since $\rho_r \leq 1$; the second is non-negative whenever $\rho_r \geq 1/2$, since $\kappa' > 0$; the third is non-negative whenever $K' \geq 0$. So, for every $\mathscr{M} \leq \bar{\mathscr{M}}$, $\varphi'\kappa \geq \varphi\kappa'$, and hence $\partial \mathrm{FG}'(\tau)/\partial\mathscr{M} < 0$ for every $\tau > 0$, proving $\mathrm{FG}_{\mathscr{M}}(\tau)$ is strictly decreasing in τ. As $\tau \to 0^+$, $\mathrm{FG}(\tau;\mathscr{M}) \to \alpha$ for every $\mathscr{M}$ (shown above), so $\mathrm{FG}_{\mathscr{M}}(0^+) = 1$; as $\tau \to \infty$, $\mathrm{FG}(\tau;\mathscr{M}) \to \alpha\rho_r$, so $\mathrm{FG}_{\mathscr{M}}(\infty) = \rho_r + \alpha\rho_r' = \rho_r\left[1 - \alpha\left(\kappa'/\kappa + K'/(1+K)\right)\right] = \rho_r\left(1 - \frac{\alpha D'}{D}\right)$, using $\kappa'/\kappa + K'/(1+K) = D'/D$; this is strictly less than 1 by the strict monotonicity just established. Since $\rho_r > 0$ always, $\mathrm{FG}_{\mathscr{M}}(\infty) \geq 0$ whenever $\alpha D' \leq D$, the fourth condition in equation (A82). For every $\mathscr{M} \leq \bar{\mathscr{M}}$, then, $\mathrm{FG}_{\mathscr{M}}(\tau)$ strictly decreases from $\mathrm{FG}_{\mathscr{M}}(0^+) = 1$ to a non-negative floor $\mathrm{FG}_{\mathscr{M}}(\infty) \geq 0$, so $\mathrm{FG}_{\mathscr{M}}(\tau) > 0$ for every finite $\tau > 0$: market power strictly amplifies forward guidance's pass-through at every maturity.

Quantitative easing. Write $b(\tau) \equiv (1 - e^{-\kappa\tau})/\kappa$, $B(\tau) \equiv \int_0^\tau b(u)\,du$, so $A_r(\tau) = \alpha b(\tau)$ and $I(\tau) = \alpha B(\tau)$. Since $F_\theta'(\tau)\tau - F_\theta(\tau) = \kappa\rho_\theta\left[A_r(\tau)\tau - I(\tau)\right] = \eta H(\tau)$ with $H(\tau) \equiv \tau b(\tau) - B(\tau)$, and $b(\cdot)$ is strictly increasing, $H(\tau) > 0$ for every $\tau > 0$, so

$$\mathrm{QE}'(\tau) = \frac{\eta H(\tau)}{\tau^2}, \tag{A87}$$

recovering that $\mathrm{QE}(\tau;\mathscr{M})$ itself is strictly increasing in τ. To sign $\mathrm{QE}_{\mathscr{M}}(\tau) \equiv \partial \mathrm{QE}(\tau;\mathscr{M})/\partial\mathscr{M}$, differentiate $H(\tau)$ in κ. Using $\partial b(\tau)/\partial\kappa = \psi(\tau)/\kappa$,

$$\frac{\partial H(\tau)}{\partial\kappa} = \frac{\tau\psi(\tau) - \Psi(\tau)}{\kappa} \equiv \frac{\widehat{B}(\tau)}{\kappa}, \qquad \Psi(\tau) \equiv \int_0^\tau \psi(u)\,du. \tag{A88}$$

$\widehat{B}(0) = 0$ and $\widehat{B}'(\tau) = \tau\psi'(\tau) = -\kappa\tau^2 e^{-\kappa\tau} < 0$ for $\tau > 0$, so $\widehat{B}(\tau) < 0$ for every $\tau > 0$. Define $\widetilde{H}(\tau) \equiv \tau^2 e^{-\kappa\tau} - \int_0^\tau u e^{-\kappa u}\,du$; since $\psi(\tau) = \tau e^{-\kappa\tau} - b(\tau)$, direct substitution gives $\widehat{B}(\tau) = \widetilde{H}(\tau) - H(\tau)$. Moreover, $(\widetilde{H} + H)(0) = 0$, $(\widetilde{H} + H)'(\tau) = \tau e^{-\kappa\tau}(2 - \kappa\tau)$ is positive for $\tau < 2/\kappa$ and negative for $\tau > 2/\kappa$, and $(\widetilde{H} + H)(\tau) \to 0$ as $\tau \to \infty$: a function that rises from 0, peaks once, and returns to 0 without crossing back through it is non-negative throughout, so $\widetilde{H}(\tau) + H(\tau) \geq 0$ for every $\tau \geq 0$, with equality only at $\tau = 0$ and in the limit $\tau \to \infty$. Combined with $\widehat{B} = \widetilde{H} - H$, this gives

$$|\widehat{B}(\tau)| = -\widehat{B}(\tau) \leq 2H(\tau) \qquad \text{for every } \tau \geq 0, \tag{A89}$$

strictly for every finite $\tau > 0$. Now,

$$\tau^2 \frac{\partial \mathrm{QE}'(\tau)}{\partial \mathscr{M}} = \eta' H(\tau) + \eta\kappa' \frac{\widehat{B}(\tau)}{\kappa} = \eta' H(\tau) - \frac{\eta\kappa'}{\kappa} \left|\widehat{B}(\tau)\right| \geq H(\tau) \left[\eta' - \frac{2\eta\kappa'}{\kappa}\right], \tag{A90}$$

which is strictly positive for every $\tau > 0$ and every $\mathscr{M} \leq \bar{\mathscr{M}}$, since $H(\tau) > 0$ and the bound on $|\widehat{B}(\tau)|$ is strict for finite τ. This proves $\mathrm{QE}_{\mathscr{M}}(\tau)$ is strictly increasing in τ. As $\tau \to 0^+$, $\mathrm{QE}(\tau;\mathscr{M}) \to 0$ for every $\mathscr{M}$, so $\mathrm{QE}_{\mathscr{M}}(0^+) = 0$; as $\tau \to \infty$, $\mathrm{QE}(\tau;\mathscr{M}) \to \eta/\kappa$, so $\mathrm{QE}_{\mathscr{M}}(\infty) = (\eta'\kappa - \eta\kappa')/\kappa^2$, which is strictly positive for every $\mathscr{M} \leq \bar{\mathscr{M}}$, since $\eta'\kappa \geq 2\eta\kappa' > \eta\kappa'$ there. □

Internet Appendix 11.1. Numerical Verification of the Threshold $\bar{\mathscr{M}}$

Table 2 evaluates the four conditions in equation (A82) at this paper's calibration, for a grid of $\mathscr{M}$ values spanning several multiples of the calibrated $\mathscr{M} = 1.0$ used throughout Section V.C. $K(\mathscr{M})$ is strictly increasing, $\eta'(\mathscr{M})\kappa(\mathscr{M})$ exceeds $2\eta(\mathscr{M})\kappa'(\mathscr{M})$, and $(1+\mathscr{M})D'(\mathscr{M})$ stays below $D(\mathscr{M})$ throughout the range shown; the binding constraint is $\rho_r(\mathscr{M}) \geq \frac{1}{2}$, which fails only once $\mathscr{M}$ climbs past $\bar{\mathscr{M}} \approx 6.4$, more than six times the calibrated value. Since $K'(\mathscr{M}) \geq 0$ and $(1+\mathscr{M})D'(\mathscr{M}) \leq D(\mathscr{M})$ hold throughout this same range, the table also verifies Proposition 5's hypothesis up to $\bar{\mathscr{M}}$.

Table 2 Threshold conditions for Propositions 5, 10, and 11

$\mathscr{M}$	κ_r^*	K	ρ_r	η	$\eta'\kappa / 2\eta\kappa'$	$(1+\mathscr{M})D'/D$
0.25	0.2311	0.0187	0.966	0.0112	–	–
0.50	0.2327	0.0267	0.952	0.0160	25.2	0.08
0.75	0.2346	0.0359	0.936	0.0214	18.3	0.11
1.00	0.2367	0.0464	0.918	0.0274	13.9	0.14
1.25	0.2390	0.0580	0.899	0.0341	11.0	0.18
1.50	0.2416	0.0707	0.879	0.0412	8.9	0.21
2.00	0.2473	0.0989	0.837	0.0567	6.8	0.27
3.00	0.2607	0.1644	0.749	0.0909	4.4	0.39
4.00	0.2759	0.2387	0.665	0.1272	2.9	0.53
5.00	0.2925	0.3182	0.590	0.1636	2.2	0.66
6.00	0.3098	0.4007	0.524	0.1993	1.7	0.77
6.40	0.3169	0.4342	0.500	0.2132	1.5	0.83

Notes: The table evaluates, at each value of $\mathscr{M}$ shown, the four conditions of equation (A82) that jointly define the threshold $\bar{\mathscr{M}}$, at this paper's calibration (Table 1) with $\mathscr{M}$ itself left free. κ_r^* is the risk-neutral mean-reversion fixed point of equation (41); K and η are the auxiliary objects defined alongside equation (A82); $\rho_r \equiv \kappa_r/(\kappa_r^*(1+K))$ is the binding constraint, required to stay at or above $\frac{1}{2}$; the last two columns report the two remaining conditions as ratios, each required to stay above (respectively below) one. A dash indicates the ratio is not evaluated at $\mathscr{M} = 0.25$, where the denominator is not yet numerically distinguishable from zero. All four conditions hold throughout the range shown; $\rho_r \geq \frac{1}{2}$ is the first to fail, at $\bar{\mathscr{M}} \approx 6.4$, reported in the table's final row.

Internet Appendix 11.2. Robustness of the Threshold $\bar{\mathscr{M}}$ to the Calibration

Since $\kappa_r^*(\mathscr{M})$'s fixed point, and hence all four conditions defining $\bar{\mathscr{M}}$ in equation (A82), depend only on five primitives, γ, σ_r, κ_r, α, and δ_α, and not at all on $\bar{r}$, θ_0, σ_β, κ_β, or ψ_Q, I check how sensitive

$\bar{\mathscr{M}}$ is to each of these five primitives individually.

I perturb each of the five primitives by ±10%, ±20%, ±30%, and ±40% from its calibrated value, holding the other four fixed, and recompute $\kappa_r^*(\mathscr{M})$, $K(\mathscr{M})$, $D(\mathscr{M})$, $\rho_r(\mathscr{M})$, and $\eta(\mathscr{M})$ for each perturbed parameter vector, using the same tractable-case objects as Table 2. For each perturbed vector, I find the smallest $\mathscr{M}$ at which any of the four conditions in equation (A82) first fails, a ceteris-paribus, one-at-a-time sensitivity check around the existing point estimate.

Table 3 reports the results. Across all forty perturbations, $\rho_r \geq \frac{1}{2}$ remains the binding constraint throughout: none of the other three conditions ever binds first, confirming the same constraint identified in Table 2. $\bar{\mathscr{M}}$ moves substantially with the calibration: it falls as low as 3.25 when δ_α is 40% below its calibrated value, the slowest-decaying demand slope tested, and rises as high as 11.5 when σ_r is 40% below its calibrated value, but never drops below 3.25, more than three times the calibrated $\mathscr{M} = 1.0$ used throughout the paper's quantitative results. The threshold tightens when γ, σ_r, or α rise, or when δ_α falls, a more persistent demand slope, and relaxes under the opposite moves; κ_r works in the other direction, with $\bar{\mathscr{M}}$ rising as κ_r rises. This pattern is intuitive: each of γ, σ_r, α scales up how strongly κ_r^* responds to market power, while a higher physical mean-reversion κ_r provides a larger base against which that same absolute response is a smaller relative move.

Table 3 Robustness of $\bar{\mathscr{M}}$ to One-at-a-Time Perturbations of the Calibration

Parameter	−40%	−30%	−20%	−10%	+10%	+20%	+30%	+40%
γ	8.75	7.75	7.25	6.75	6.25	5.75	5.25	5.25
σ_r	11.50	9.75	8.25	7.25	5.75	5.25	4.75	4.25
κ_r	3.75	4.75	5.25	5.75	7.25	7.75	8.25	8.75
α	8.75	7.75	7.25	6.75	6.25	5.75	5.25	5.25
δ_α	**3.25**	3.75	4.75	5.75	7.25	8.25	9.25	10.50

Notes: Each row perturbs one primitive by the column's percentage from its calibrated value (Table 1), holding the other four of $\{\gamma, \sigma_r, \kappa_r, \alpha, \delta_\alpha\}$ fixed, and reports the resulting $\bar{\mathscr{M}}$; the unperturbed calibration gives $\bar{\mathscr{M}} = 6.25$ on this grid, consistent with Table 2's finer bisection search. $\rho_r \geq \frac{1}{2}$ is the binding constraint throughout; boldface marks the single lowest value reached across all forty perturbations. Grid resolution is 0.5 near the baseline, coarsening at higher $\mathscr{M}$, so reported values are the midpoint between the last passing and first failing grid point.

12. Numerical Solution of the Quantitative Model (Section V)

To obtain a numerical solution of the quantitative model, I solve for the nine scalars that satisfy Proposition 2's fixed point, $\{\Omega_{rr}, \Omega_{\beta r}, \Omega_{r\beta}, \Omega_{\beta\beta}, \Omega_r, \Omega_\beta, d_0, d_r, d_\beta\}$, for a given set of structural parameters and market power $\mathscr{M}$. I now describe the steps used to produce the results in Section V.

Step 1. I first solve a simplified one-factor sub-problem to obtain a starting guess. Setting $\Omega_{\beta r} = \Omega_{\beta\beta} = \Omega_\beta = d_\beta = 0$ and $d_r = 1$, a close approximation since ψ_Q is small throughout this paper's calibration, the nine-scalar system collapses to the single scalar fixed point

$$\kappa_r^* = \kappa_r + \gamma\sigma_r^2(1+\mathscr{M})^2 \int_0^T \alpha(\tau) A_r(\tau;\kappa_r^*)^2 \, d\tau, \tag{A91}$$

which I solve by bisection. Bisection is unconditionally robust here because the right-hand side is monotonic in κ_r^* with a unique root, unlike direct forward iteration on the same map, which is unstable at the large values of $\mathscr{M}$ this calibration requires.

Step 2. Given a guess for the nine scalars, I solve the linear, constant-coefficient two-dimensional ODE system

$$A_r(\tau)(\kappa_r + \Omega_{rr}) + A_r'(\tau) + A_\beta(\tau)\Omega_{\beta r} = (1+\mathscr{M})d_r, \tag{A92}$$

$$A_\beta(\tau)(\kappa_\beta - \Omega_{\beta\beta}) + A_\beta'(\tau) - A_r(\tau)\Omega_{r\beta} = (1+\mathscr{M})d_\beta, \tag{A93}$$

equations (28)–(29), for $A_r(\tau), A_\beta(\tau)$. Writing the system as $x'(\tau) = \mathrm{Mmat}\, x(\tau) + b$ with $x = (A_r, A_\beta)$, the exact solution is $x(\tau) = \mathrm{Mmat}^{-1}\big(e^{\mathrm{Mmat}\,\tau} - I\big)b$, which I evaluate directly via the matrix exponential instead of numerically stepping the ODE: explicit Runge-Kutta integration is unconditionally unstable at this calibration's large values of $\mathscr{M}$, since κ_r^* can reach 15–20 and Runge-Kutta's finite stability region is violated at any practical grid step. The matrix exponential is exact and unconditionally stable at any τ, provided Mmat itself is stable, and I fall back to a fine-substep Runge-Kutta scheme only in the rare case where Mmat is near-singular. I obtain $C(\tau)$ by direct quadrature of equation (30).

Step 3. Using $A_r(\tau), A_\beta(\tau), C(\tau)$ from Step 2, I recompute the nine scalars from their definitions: $\Omega_{rr}, \Omega_{\beta r}, \Omega_{r\beta}, \Omega_{\beta\beta}, \Omega_r, \Omega_\beta$ from equation (33), and d_0, d_r, d_β by aggregating market clearing into the repo-spread equation (Section II). Writing $x \equiv (\Omega_{rr}, \Omega_{\beta r}, \Omega_{r\beta}, \Omega_{\beta\beta}, \Omega_r, \Omega_\beta, d_0, d_r, d_\beta)$ for the guess and $\hat{x}(x)$ for the value each of the nine scalars recomputes to, I then check whether

$$\hat{x}(x) = x. \tag{A94}$$

Step 4. If condition (A94) is not satisfied, I update the guess x and repeat Steps 2–3 until convergence. I do not update by direct (damped) iteration on $\hat{x}(x)$, since this is unstable at the large values of $\mathscr{M}$ this calibration requires, for the same reason plain iteration fails in Step 1's one-factor case: the $(1+\mathscr{M})^2$ scaling in Ω_{rr} makes naive iteration diverge well before it can converge. Instead, I update x using L-BFGS to minimize

$$\|\hat{x}(x) - x\|^2, \tag{A95}$$

the sum of squared residuals of the nine-dimensional map, since each evaluation of $\hat{x}(x)$ is cheap, one closed-form matrix-exponential evaluation, and the map is smooth.

Step 5. Because the fixed point's location is essentially resolution-independent, a converged answer at a coarse maturity grid differs from a fine-grid answer only by quadrature error, not by where the fixed point sits, while the L-BFGS search cost in Step 4 scales with grid resolution, I first solve Steps 1–4 at a coarse grid, then refine once at the finer grid used for the reported results.

Step 6. I use multiple L-BFGS restarts from independent perturbations of the Step 1 warm start, since a single run can stall on flat or stiff directions in the nine-dimensional residual surface. I judge convergence by the residual norm falling below a strict numerical tolerance; a relative-change stopping rule can trigger prematurely while the residual is still economically large.

13. Computing the Forward-Guidance and Quantitative-Easing Experiments (Section V.C)

Section V.C's Figure 11 compares the baseline and competitive counterfactual's yield response to forward guidance and quantitative easing. This appendix documents how those two experiments are computed: both by fully re-solving the nine-scalar fixed point of Appendix A.12 at the shocked parameter value and differencing the resulting yield curve against the unshocked baseline, holding the ten common structural parameters of Table 1 fixed throughout.

Step 1. I first solve the unshocked model twice, at the baseline market power $\mathscr{M} = 1.0$ and at the competitive counterfactual $\mathscr{M} = 0$, using the procedure of Appendix A.12. This gives the two unconditional-mean yield curves $y_{\text{base}}(\tau)$ and $y_{\text{cf}}(\tau)$, each evaluated at $r_t = \bar{r}$, that also appear in Figure 8.

Step 2 (forward guidance). I shift the short rate's long-run mean by $\Delta\bar{r} = -100$ basis points, holding every other structural parameter fixed, and re-solve the fixed point at $\bar{r} + \Delta\bar{r}$ for both $\mathscr{M} = 1.0$ and $\mathscr{M} = 0$. Forward guidance's reported response is the resulting yield curve's change relative to the unshocked baseline of Step 1,

$$\Delta y_B(\tau) = y_{\text{base}}(\tau; \bar{r} + \Delta\bar{r}) - y_{\text{base}}(\tau; \bar{r}), \qquad \Delta y_C(\tau) = y_{\text{cf}}(\tau; \bar{r} + \Delta\bar{r}) - y_{\text{cf}}(\tau; \bar{r}), \tag{A96}$$

plotted as Panel A of Figure 11. A full re-solve is necessary because $\bar{r}$ enters the equilibrium in two places: directly, through $C(\tau)$'s own ODE (equation (37)), and indirectly, through the risk prices λ_r, λ_β, since preferred-habitat investors' net demand $\theta_0(\tau) - \alpha(\tau)[A_r(\tau)\bar{r} + C(\tau)]$ depends on $\bar{r}$. I verified that this recovers equation (53)'s closed-form comparative static exactly: $\Delta y_B(\tau)/\Delta\bar{r}$ from the full re-solve matches $\text{FG}(\tau; \mathscr{M})$ evaluated directly from the formula to three decimal places, at every maturity and at both values of $\mathscr{M}$.

Step 3 (quantitative easing). I model a persistent reduction in habitat demand as a shift in the scalar level θ_0, holding the demand-slope decay $\alpha(\tau)$ fixed, so that $\theta_0(\tau) = (\theta_0 + \Delta\theta_0)\alpha(\tau)$ retains its exponential shape at every maturity. I choose $\Delta\theta_0 = -0.12\,\delta_\alpha/\alpha$ so that the resulting change in aggregate habitat demand, $\int_0^\infty \Delta\theta_0(\tau)\,d\tau = \Delta\theta_0 \int_0^\infty \alpha(\tau)\,d\tau = \Delta\theta_0\,\alpha/\delta_\alpha$, equals -0.12, matching the shock size stated

in Section V.C. I re-solve the fixed point at $\theta_0 + \Delta\theta_0$ for both $\mathscr{M} = 1.0$ and $\mathscr{M} = 0$, and construct

$$\Delta y_B(\tau) = y_{\text{base}}(\tau; \theta_0 + \Delta\theta_0) - y_{\text{base}}(\tau; \theta_0), \qquad \Delta y_C(\tau) = y_{\text{cf}}(\tau; \theta_0 + \Delta\theta_0) - y_{\text{cf}}(\tau; \theta_0), \tag{A97}$$

plotted as Panel B of Figure 11. Unlike forward guidance, quantitative easing has no shortcut to rule out: a shift in θ_0 moves every one of the nine fixed-point scalars, including $A_r(\tau)$ and $A_\beta(\tau)$ themselves, so a full re-solve is the only way to compute it.

Step 4. Both experiments vary only $\mathscr{M}$, baseline versus competitive, and the shocked parameter itself, $\bar{r}$ for forward guidance and θ_0 for quantitative easing, holding every other structural parameter at its calibrated value from Table 1. The ratio $\Delta y_B(\tau)/\Delta y_C(\tau)$ reported in Section V.C is therefore attributable entirely to dealer market power, not to any difference in the underlying calibration.