A Necessary Lie: Conflict of Interest in Investor-Paid Ratings

Filipp Prokopev*

July 31, 2026

Abstract

Investor payment is widely thought to eliminate conflicts of interest in ratings, or information sale more broadly. I show that the conflicts persist when investors have market power and are themselves delegated asset managers with agency frictions. In contrast to dispersed small investors, a large fund with a legacy position can purchase a favorable report that inflates its market value. The provider prefers to sell distorted ratings to the fund instead of dispersed investors in illiquid markets where information leakage destroys rents from broad sales. Although the bias lowers information *precision*, the fund demanding it can expand the *access* to informed trading, so tougher regulation of providers can *reduce* price informativeness.

JEL: D82, G14, G23, G24
Keywords: delegated asset management, monopolistic information sales, investor-paid ratings, conflicts of interest, information rationing, price informativeness

*Stephen M. Ross School of Business, University of Michigan, e-mail: prokopev@umich.edu

1 Introduction

Financial markets rely on specialized information providers – credit rating agencies, sustainability data vendors – whose recommendations affect capital allocation. A central concern about the sale of information is a potential conflict of interest: when issuers pay, providers inflate ratings to win business.[1] Conventional wisdom suggests that if investors who value accuracy compensate the rating agency directly, the conflict should disappear – all the more so if the ratings are private, observed only by the subscribers. This reasoning features prominently in regulatory debates and has helped drive the recent growth of investor-paid credit ratings and ESG scores.[2] However, a growing body of evidence suggests that these ratings are *not free* from bias.[3] In this paper, I demonstrate how the conflict of interest emerges in investor-paid ratings under *market concentration* and *agency frictions* that characterize much of modern asset management. Paradoxically, financial markets' efficiency can *improve* even as the information itself becomes less accurate.

I consider a market for a financial asset in which a monopolistic information provider sells information (rating) about the fundamental value of the asset. The information provider can either sell to atomistic investors with asset endowments or to a large fund with a legacy position. Importantly, under each model of sale, the provider can distort the rating at a cost. When it sells to individual investors, there is no bias in equilibrium. However, when it sells to a large fund, the rater distorts the rating to allow the fund to circumvent trading constraints.[4]

The model delivers three novel results about delegated asset managers. First, portfolio constraints intended to protect investors can backfire. A rule written on a third-party assessment stops the manager from deviating in trading, but not from paying to distort the assessment. In other words, conflict moves from trading to information acquisition. Second, equilibrium prices can be *more informative* under rating distortion that favors the fund (catering) than under direct sale of unbiased ratings to individual investors. The distortion rent grows with the capital under management, so the fund *extends* informed trading

[1] This pattern has been extensively documented since 2008. Empirical evidence for corporate bonds includes Becker and Milbourn (2011), Cornaggia and Cornaggia (2013), and Xia (2014). For theoretical surveys, see Jeon and Lovo (2013) and Sangiorgi and Spatt (2017a).

[2] Most ESG data providers have adopted an investor-paid model of sales and their revenues are estimated at approximately US$2 billion in 2024 (see "The Market for ESG Data in 2024." Opimas, March 11.). In credit markets, Egan-Jones, an investor-paid rating agency, has become a prominent provider of credit and private debt ratings (see SEC raises questions over rating agency behind private credit boom).

[3] Agrawal et al. (2025), Berg et al. (2022) for ESG scores, and Bonsall et al. (2024) for credit ratings.

[4] U.S. equity ownership has become increasingly intermediated. Large institutional investors' share of the market rose from 27 percent in 1980 to 52 percent in 1996. See Gompers and Metrick (2001). The same pattern is even more pronounced in credit markets, where three-quarters of U.S. corporate bonds are held by institutional investors (Koijen and Yogo (2023)).

to investors who the rater would have rationed out. Third, the model endogenizes the organization of the information market: the rater chooses whom to sell to. Catering to the fund wins when markets are illiquid and information leaks into prices quickly under the direct sale. In this case, the information rent gets smaller while the *distortion* rent does not. Catering is also preferred when the fund weights its legacy position more heavily, when it is more opaque, and when distortion is cheaper; under these conditions, the distortion rent is significant.

Model Overview and Main Results. A monopolistic information provider (rating agency, a.k.a rater) observes a noisy signal about the payoff of a risky asset and chooses how to sell it. A continuum of individual investors with a prior asset endowment trade in the market and face heterogeneous transaction costs, leading some to delegate to the fund. The fund can trade on investors' behalf for a fee, but it has legacy asset holdings, introducing a conflict with new delegating investors. The rater can sell its signal either to dispersed investors or to the fund. In either case, the rater can distort the reported signal at a cost to win business.

The buyer's market power is essential to the mechanism. Empirically, institutional orders move prices (Koijen and Yogo, 2019), and each of the "Big Three" holds more than five percent of the shares of most public companies (Bebchuk and Hirst, 2019). However, the market power alone does not explain why *private* ratings are distorted, since the payer and the user appear to coincide in this case. Therefore, the second ingredient is that these buyers are delegated managers, whose mandates are frequently written on assessments they purchase – investment-grade-only bond mandates, rating-based capital requirements, ESG screens. Unable to deviate from the constraints, funds instead pay to distort their input.

Under direct sale to dispersed investors, the rater's optimal bias is zero in equilibrium. Intuitively, each individual investor's actions have a measure-zero impact on the aggregate signal and the asset price. Even though we allow the rater to price-discriminate based on endowment, the aggregate bias is not a function of any individual investor's decision, so the rater cannot extract the wealth gain from distorted signals. In other words, when clients are atomistic, they cannot affect other agent's beliefs and only use the information in the signal. Similarly, a large fund with price impact in the asset market purchases an unbiased signal when its trading is unconstrained, and so the fund is the ultimate user of the acquired information. In the asset market, it still trades in the direction of its legacy holding, injecting non-fundamental noise into the price, i.e., the distortion happens at the trading stage, not at the signal acquisition stage.

When selling to the fund *with a portfolio constraint*, there is a bias in the signal in

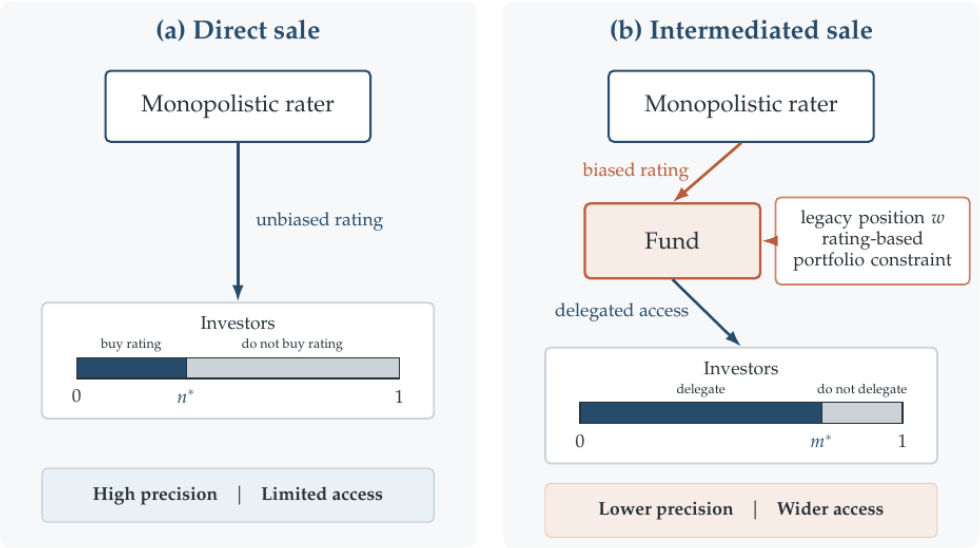


Figure 1: Organization of information distribution.

equilibrium. In the model, individual investors demand return, but the fund manager also cares about the market value of its existing position. The fund cannot deviate from a committed trading rule in order to pump the value of its position, but it can bias the signal underlying the rule to implement the favorable position. Two features sustain the bias in equilibrium: (i) a *private* relational contract lets the rater and fund split the gains from distortion; (ii) investors cannot unwind the bias because they are uncertain about exact incentives to inflate, i.e., they do not observe the legacy position.[5] Hence, when the buyer is a delegated intermediary with hidden exposures and market power, the same distortions that plague issuer-paid ratings reappear.

An important distinction of my framework from the issuer-paid model is that the intermediary who pays for the signal controls *the distribution of information*. The monopolistic provider optimally rations information in direct sales to prevent a rapid leak into prices and preserve the value of information (Figure 1). Delegation changes this trade-off. Because the fund's gain from distortion increases with the capital it manages, it has an incentive to attract a larger investor base and extend informed trading to individual investors excluded under direct sale. This expansion in access can more than offset the loss in signal accuracy, raising price informativeness – and, under some conditions, investor welfare.

Thus, the model endogenizes the organization of information markets. Specifically, we let the rater choose the optimal way of selling signals and *characterize when* it prefers to cater

[5]In the market for ESG ratings, opaque payments and tailored pricing are ubiquitous. For example, the major provider Morgan Stanley Capital International (MSCI), tailors its pricing strategies to the specific characteristics of its clients, based on size and usage. Moreover, these payments are unobserved by other market participants.

to the large fund by distorting signals: (i) when markets are illiquid, information leakage through the asset price makes catering to the large fund more attractive; (ii) when the fund is less myopic or more transparent to market participants, fund catering pays less. This characterization generates clear empirical predictions about when we should expect conflict of interest to re-appear in information sales to investors.

Empirical Predictions and Policy Implications. The model rationalizes the finding that subscriber-paid agencies inflate ratings for firms held by their institutional clients (Bonsall et al., 2024), and sharpens it in three directions. First, what triggers the bias is the constraint the client trades under, not its size: an equally large and equally opaque manager free to trade as it likes buys an unbiased rating and pumps the price instead. The two distortions are therefore substitutes, and tougher regulation of raters displaces the conflict toward the trading margin rather than removing it. Second, the model predicts that because the distortion rent requires uncertainty about the fund's exposure, inflation should attenuate when holdings are disclosed alongside ratings. It should not vanish, however, since the incentive to inflate also depends on the manager's compensation and on positions held elsewhere in the fund family: bias should persist for funds with opaque compensation or family affiliation. Third, raising the cost of distortion improves accuracy while contracting informed access, so it can *lower* price informativeness.

These results shift the regulatory question from *who pays* for information to *who acts on it, and through what contract it is sold.* Since demand for distortion originates with portfolio constraints, supervision should extend beyond raters to the funds that write third-party assessments into their mandates. Aligning holdings disclosure with rating releases attacks the rent at its source, and is less blunt than penalizing raters, which withdraws informed capital as a side effect.

Related Literature. The paper relates most directly to the information sales literature (Admati and Pfleiderer, 1986, 1990; Cespa, 2008; Garcia and Sangiorgi, 2011; Chen and Sun, 2024; Lovo and Olivier, 2026). Most papers in this literature assume there is no reliability problem. In contrast, my paper focuses entirely on the seller's incentives to distort the signal in some direction ex post, as opposed to introducing a non-directional noise ex ante as in Admati and Pfleiderer (1986). Relatedly, Admati and Pfleiderer (1990) consider the intermediated sale of information where the seller sets up a fund to control the information use through trading. Here, the intermediary is *uninformed* and has *agency frictions*. Lovo and Olivier (2026) show that a monopolistic provider sells the signal to the issuer when it improves the firm valuation, i.e., when the firm has a high score and when investors price

it in.[6] Relatedly, in this work, the seller earns more when it inflates the client's legacy position, but this is achieved through *signal distortion*. Importantly, I can focus on how the intermediary controls the distribution of information (Figure 1) while the issuer-paid signal becomes available to everyone. Finally, Chen and Sun (2024) consider information distortion by *a firm manager* and an offsetting action by the information provider. In my work, the provider might distort the signal herself when it is profitable for catering to the fund, which seems to be supported by empirical evidence.

There is a growing empirical evidence of distortions in subscriber-paid ratings (Agrawal et al. (2025) and Berg et al. (2021) for ESG scores, Bonsall et al. (2024) for credit ratings), which this paper helps rationalize. In particular, Bonsall et al. (2024) provide evidence that a major subscriber-paid credit rating agency, Egan Jones, inflates its ratings for firms held by its institutional client relative to Moody's ratings. We analyze the conditions under which the bias in investor-paid sales can emerge: the buyers must be large delegated asset managers, and markets must be relatively less liquid (like corporate bond markets). However, as we show, this distortion does not have to harm market efficiency.

Several papers at the intersection of contract theory and trading are devoted to the sale of information by agents with conflicts of interest (Biais and Germain, 2002; Dybvig et al., 2010; Kyle et al., 2011; He and Xiong, 2013). Biais and Germain (2002) studies optimal contracts for an information seller who can also trade on their own account. Kyle et al. (2011) focus on contracts motivating a delegated asset manager to exert more effort in producing signals. In both, return-sharing rule aligns the manager's incentives. In my work, however, the delegated asset manager acquires information from a third party, and the return-sharing with investors is an imperfect instrument to solve the managerial myopia.

This study contributes to the literature exploring the implications of increased *delegated portfolio management* for the quality of asset prices. Bhattacharyya and Nanda (2013) develop a model in which an informed fund manager inflates net asset value (NAV) by purchasing securities already held in the portfolio, thereby exerting upward price pressure and temporarily increasing the portfolio's marked value. Wang (2024) provides empirical evidence of portfolio pumping within mutual fund families. The mechanism distorts prices through non-informational trading. My paper nests that mechanism. Section 3.2 shows that a large fund with full trading *discretion* pumps the price directly and purchases an unbiased signal: pumping is what obtains when no portfolio requirement binds. Such requirements are themselves the subject of work on the contracting value of ratings (Donaldson and Piacentino, 2018; Parlour and Rajan, 2020), which emphasizes how rating-contingent restrictions support contracting between investors and managers. Here the binding requirement migrates the

[6] Kashyap and Kovrijnykh (2016) and Bongaerts (2014) also compare issuer-paid and investor-paid ratings.

distortion from the portfolio to the information on which the portfolio is based: the same contractibility that makes a rating useful makes it worth corrupting.

Some studies analyze how delegated fund management affects price informativeness. Gârleanu and Pedersen (2018) model a market for *informed* asset managers, whose incentives are aligned with investors, with search cost and show that the market becomes more efficient when this cost decreases. This paper is focused on agency frictions between the asset manager and her investors and shows that surprisingly they can improve market efficiency as well. Buss and Sundaresan (2023) show how an increase in *passive* ownership can lead to higher incentives to acquire information and, thus, to more informative prices. However, Bond and Garcia (2022) deliver the opposite predictions in a noisy rational expectation equilibrium framework without endogenous information acquisition or investment. While these papers assume that delegated asset manager is an informed party (which corresponds to in-house analysis), my work describes the information acquisition by funds.

This paper relates to the literature on conflicts of interest in credit ratings. The vast majority of work has been focused on *issuer-paid* ratings. Research documents its vulnerabilities to rating shopping (Becker and Milbourn (2011); Bolton et al. (2012)), regulatory distortions (Opp et al. (2013)), and lack of transparency (Sangiorgi and Spatt (2017b)). The contribution of this paper is to show that selling to investors as opposed to firms does not resolve the conflicts of interest.

A final strand shows that biased information provision can dominate unbiased provision. Dewatripont and Tirole (1999) argue that enfranchising partisan advocates elicits more information than a neutral investigator, and Che and Kartik (2009) show that a decision maker may prefer a biased adviser precisely because the bias sharpens the incentive to acquire and reveal information. This paper shows that a similar mechanism can work in asset markets: when large intermediaries demand distorted signals, they may end up alleviating information rationing rather than undermining the information market.

2 Model

Assets. There are two assets in the economy. One is risk-free, with a normalized return of zero. The other is a risky asset whose terminal payoff at $t = 1$ is $v \sim N(0, \sigma_v^2)$. The risky asset is in stochastic supply $z \sim N(0, \sigma_z^2)$, where fluctuations around the mean represent noise trading. In principle we can think of v as including non-pecuniary component to think about ESG investing.

Information. A monopolistic information provider–hereafter the rating agency –privately observes a noisy signal about the fundamental value $s = v + \varepsilon$, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, independent of v and z. We denote the aggregate signal volatility as $\sigma_s^2 \equiv \sigma_v^2 + \sigma_\varepsilon^2$, and the signal-to-noise ratio as $\tau \equiv \frac{\sigma_v^2}{\sigma_s^2}$.

The rater can introduce a bias $b \in \mathbb{R}$ into the reported signal $\hat{s} = s + b$ ex post. A positive (negative) b causes the signal to overstate (understate) the asset's expected payoff. Bias is costly, reflecting reputational damage or regulatory penalties (Lacker and Weinberg, 1989), with quadratic cost: $C(b) = \frac{c_b}{2} b^2$.

Individual Investors. A unit mass of individual investors are endowed with W_0 in cash, and $e_i = \epsilon_i$ shares of the risky asset, where $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ is idiosyncratic. By the law of large numbers, the aggregate investor endowment is $\int e_j \, dj = 0$, a.s.[7]

Each individual investor is a price-taker.Trading independently entails a fixed transaction cost c, drawn independently from $U[0, \bar{c}]$. Alternatively, investors may delegate to a fund at a management fee f, thereby avoiding this cost.

Preferences over the risky position x, before transaction costs, take the form:

$$\mathbb{E}[U_i(W_1) \mid \mathcal{I}_i] = \mathbb{E}\left[-\exp(-\gamma W_1) \mid \mathcal{I}_i\right] \tag{1}$$

where terminal wealth is $W_1 = W_0 + Pe_i + (v - P)x - c_i$, P is the asset price, $\gamma > 0$ captures risk aversion, $\mathcal{I}_i$ is investor i's information set, and c is transaction cost for *active investors* and f is the management fee for *fund investors*.

Fund. A single *uninformed* fund may trade on behalf of individual investors.

At the beginning of the game, the fund announces a *management fee* f, proportional to the assets under management at $t = 1$, and commitment to a trading rule (specified later). An endogenous fraction m of investors delegates.

The fund holds a pre-existing position w, unobserved by individual investors (formed between reporting periods [8]). However, investors know its distribution: $w \sim N(0, \sigma_w^2)$. The fund manager's total payoff is:

$$\pi_F = \underbrace{fm \, \{W_0 + (v - P)x_F\}}_{\text{management fees}} + \underbrace{\alpha \, P \, w}_{\text{legacy position}}, \tag{2}$$

[7] Alternatively, we could have specified a common *random* endowment shock, i.e. $e_i = w + \epsilon_i$ with $w \sim N(0, \sigma_w^2)$. While this specification does not change results on conflicts of interest (see Appendix B.1), it makes the asset market equilibrium more complicated since e_i becomes correlated with the aggregate supply $z + w$ and hence the equilibrium risky asset price.

[8] Alternatively, we can think of it as asset exposure at the fund-family level (Wang, 2024)

where the bracketed sum shows the assets under management at $t = 1$, while x_F is the share position per fund investor and $\alpha \geq 0$ is the weight placed on the interim market value of the prior position, which can depend on managerial compensation or myopia, or the mass of earlier cohort of fund investors. The term $\alpha\, P\, w$ generates the fund's incentive to seek a favorable signal distortion.

When individual investors acquire the signal, the fund is uninformed and acts as a trade facilitator: each delegated dollar is invested in the risky asset, and investors choose their own allocation between the risky and riskless assets. A natural example of such an arrangement is an investor who purchases research, chooses an allocation to the risky portfolio, and implements it through a passive index fund, like the Vanguard 500 Index Fund.

When the fund acquires the signal, it can commit to a rule-based position that maximizes its investors' expected utility,

$$x_F(\hat{s}, P) \;=\; \arg\max_{x}\; \mathbb{E}\big[-\exp\big(-\gamma\,(W_0 + (v-P)x)\big)\,\big|\,\mathcal{I}_i\big] \;=\; \frac{\mathbb{E}[v \mid \hat{s}] - P}{\gamma\,\mathbb{V}(v \mid \hat{s})}. \tag{3}$$

This trading rule is simply an optimal trading quantity from the perspective of a price-taking investor i with information set $\mathcal{I}_i$. The fund cannot deviate from this rule, but it can distort the input $\hat{s}$ itself, or rather have the rating agency distort it. A natural example is an ESG fund that purchases sustainability ratings and allocates its portfolio according to pre-specified rating thresholds or weights.

Rating Agency. The rater operates can either sell the signal directly to individual investors, or through the fund.

(i) Direct sale. The rater sells the signal directly to individual investors via a private relational contract discriminating based on the size of self-reported endowment:

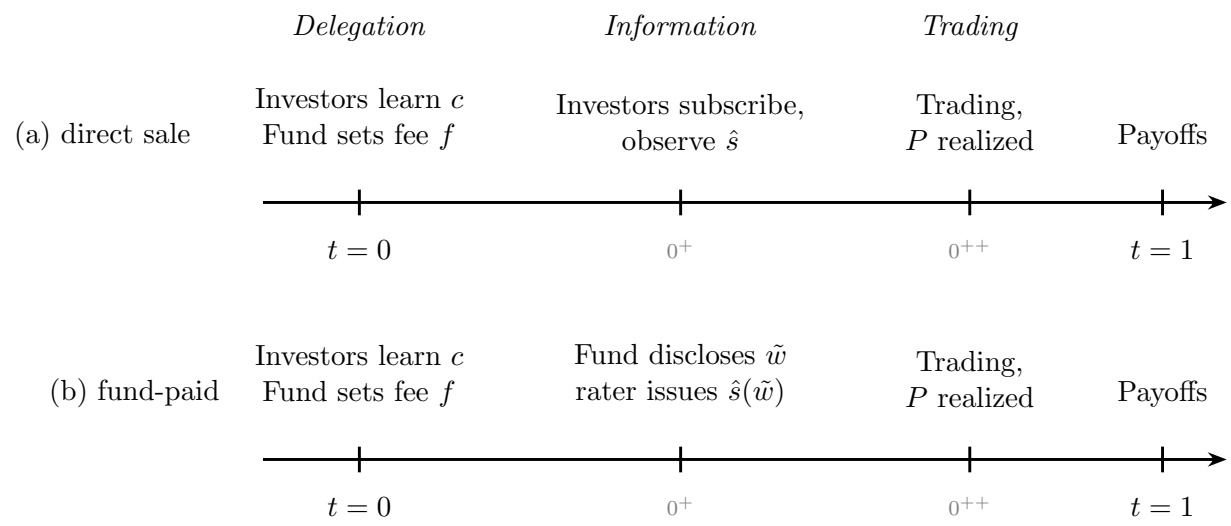
$$\phi(\hat{s},\, \tilde{e}) = F + \tilde{e}\, P(\hat{s}), \tag{4}$$

where F is a fixed component, and $\tilde{e}\, P(\hat{s})$ is a premium that funds are willing to pay based on their self-reported position $\tilde{e}$. Subscribers then observe the signal $\hat{s}$ while non-subscribers learn from the risky asset price P.

(ii) Intermediated sale. The rater sells the signal to the fund, who then trades on individual investors' behalf. Non-fund investors learn only through prices under their conjecture about the distribution of $b \sim N(\bar{b}, \bar{\sigma}_b^2)$.

Under the fund-paid model, the rater's compensation is tied to the interim market value of the fund's reported prior position. Specifically, if the fund privately discloses a position

Figure 2: Timelines under the two models of sale



Panel (a): individual-investor-paid model. Panel (b): fund-paid model .

$\tilde{w}$, the rater receives:

$$\phi(\hat{s},\, \tilde{w}) = F + \tilde{w}\, P(\hat{s}), \tag{5}$$

where F is a fixed component determined as an outcome of Nash bargaining over the fund's expected surplus from contracting, $\mathcal{V} \equiv \mathbb{E}[\pi_F] - \mathbb{E}\left[\pi_F^{uninformed}\right]$.

Equilibrium. We focus on perfect Bayesian Nash Equilibria that consists of: (i) subscription decisions and portfolio choices by individual investors that maximize expected utility based on their information sets; (ii) a bias b and a sale-model choice by the rater that maximize its expected payoff; and (iii) a fee f and signal-acquisition decision by the fund manager; (iv) market clearing; (v) investors' beliefs $(\bar{b}, \bar{\sigma}_b^2)$ that coincide with the equilibrium moments.

Figure 2 summarizes the timing of events under each benchmark.

2.1 Discussion of Assumptions

Trading and Information Intermediation. The fund performs two conceptually distinct functions. In both regimes, it intermediates trading: it implements trades on their behalf more efficiently than they could trade individually. This role is independent of who acquires the signal and creates the baseline demand for delegation. Under the fund-paid regime, the fund additionally intermediates information. It purchases the third-party signal,

incorporates it into a common portfolio rule, and passes its informational value to its clients through the resulting portfolio. Investors therefore obtain both execution and signal access through delegation rather than purchasing the information separately.

This arrangement is common when delegated portfolios rely on vendor-supplied inputs. ESG and index funds implement portfolios constructed using proprietary index and rating data, while bond funds and insurance portfolios use external credit ratings to satisfy risk-management requirements. For example, the iShares ESG Aware MSCI USA ETF tracks an MSCI index whose portfolio weights depend on MSCI's issuer-level ESG assessments, so shareholders obtain exposure to this information through the fund rather than purchasing MSCI data themselves.[9]

Legacy Position and Myopia. The legacy position w captures an exposure whose interim value enters the manager's objective but is not internalized by the investors whose capital it currently allocates. It may represent holdings accumulated between disclosure dates, positions of an earlier investor cohort, or overlapping holdings of other funds within the same family. The term αPw is a reduced-form representation of the manager's compensation or fund flows being linked to the marked-to-market performance of this exposure.

This creates an intertemporal or cross-fund agency conflict. A favorable report raises the value of the legacy position, benefiting the manager or related investors, while the current fund investors bear the consequences of a portfolio chosen using distorted information. Because the relevant exposure is not observed when investors contract with the fund, they cannot condition the mandate or fee on the manager's incentive to inflate its value. Such incentives are consistent with evidence that fund families trade to support the performance of affiliated portfolios (Ma et al., 2019; Wang, 2024).

Alternative conflicts of interest There are alternative channels that could generate biased ratings. First, large investors might demand distorted *public ratings* to induce smaller investors to trade favorably, i.e., buying assets the large investor already holds. This is less plausible in our setting, since the relevant ratings are sold predominantly *privately* (most ESG scores and some credit ratings like Egan Jones). Second, inflated ratings can facilitate "reaching for yield" in both public and private markets: although private-credit instruments often lack observable prices, their ratings determine regulatory treatment and eligibility for rating-sensitive institutions, so favorable ratings let investors, especially insurers, obtain higher-yielding exposures while economizing on capital.[10] Section 6.2 discusses both.

[9] See the fund's prospectus. In credit markets, Egan-Jones distributes its ratings primarily through subscriptions purchased by institutional investors (Bonsall et al., 2024).

[10] "SEC raises questions over rating agency behind private credit boom," Financial Times.

Information Structure. There is a sole specialized owner of information which motivated by two considerations. First, informed *insiders* may be unable to communicate credibly, making a third party the sole source of *verifiable* information. Second, rating agencies aggregate data across many firms and can produce cross-sectional rankings that no individual firm can generate. Both mechanisms are consistent with evidence that asset prices react to rating announcements (Jorion et al., 2005; Bonsall et al., 2024), although this sensitivity has diminished with greater market transparency (Badoer and Demiroglu, 2019).

Since the risky asset is in stochastic supply, prices do not fully reveal the signal, and the sole channel for information spillover among non-subscribers is the equilibrium price. Investors neither communicate directly nor resell acquired information. Rational inference from prices is a standard benchmark and is consistent with empirical evidence of price-based learning (Banerjee, 2011; Bai et al., 2016).

Tailored Pricing. The tailored pricing (5) reflects the customized, non-public pricing terms observed in financial data markets. Major providers such as MSCI charge non-transparent licensing fees, increasing in client assets under management, making the rater's effective revenue an increasing function of the fund's assets. Accordingly, $\tilde{w}\,P(\hat{s})$ captures the portion of client surplus the rater extracts, where a favorable distortion raises the value of the fund's reported legacy position. Consistent with this mechanism, Bonsall et al. (2024) document that subscriber-paid agencies systematically inflate ratings for firms held by a larger fraction of their clients who pay higher fees.

The current regulation of ESG rating providers does not prevent funds from privately disclosing portfolio exposures to ESG data providers. Most of these providers operate outside Nationally Recognized Statistical Rating Organizations (NRSROs) regulatory framework which bars ratings agencies from using material non-public information like client holdings in their assessments.

3 Benchmarks: When No One Demands Distortions

The rater has no independent taste for distortion. Given a conjectured linear price, its problem is linear-quadratic in b, so in every regime below it solves the same first-order condition:

$$b^* \;=\; \frac{\beta_1}{c_b} \;\times\; \big(\text{aggregate exposure reported by paying clients}\big). \tag{6}$$

The rater supplies exactly as much bias as it is demanded and paid for.

This section characterizes two environments in which buyers do not demand bias, so (6) returns $b^* = 0$. In the first, dispersed investors are too small for any individual report to move the aggregate bias or the price. In the second, the buyer is a large fund with market power and a hidden legacy position, but *with unconstrained trading*. Hence, the fund can pump the price by trading directly and will demand a precise signal.

3.1 Dispersed Investors

In this section, the rater sells the signal *directly* to individual investors. This benchmark is close to Admati and Pfleiderer (1986) and we keep it brief. Interestingly, we show that the bias is zero in equilibrium, i.e., $b^* = 0$, despite small investors' asset endowment.

Asset market equilibrium. There is a unique asset market equilibrium that is linear $P = \beta_1 \hat{s} + \beta_2 z$ (Grossman and Stiglitz (1980)) and yields a price informativeness of $\bar{\tau}_P = \frac{\sigma_v^2}{\sigma_{\hat{s}}^2 + \bar{\sigma}_b^2 + \rho^2 \sigma_z^2}$ where $\rho \equiv \frac{\beta_2}{\beta_1} = -\frac{\gamma \sigma_v^2 (1-\bar{\tau})}{n \frac{1-f(1-m)}{1-f} \bar{\tau}}$ is the noise-to-signal ratio, $\bar{\tau} = \frac{\sigma_v^2}{\sigma_{\hat{s}}^2 + \bar{\sigma}_{\hat{v}}^2}$ is the perceived precision of $\hat{s}$, n is the mass of informed investors and m is the mass of delegating investors at the management fee f.

The optimal trade depends on both the conditional mean and variance of v, so in principle, the rater's change in signal precision σ_ε^2 would have a non-trivial optimum effect on trading aggressiveness and price informativeness as in Admati and Pfleiderer (1986), but it is held explicitly fixed in this model. The bias in this paper and the noise σ_ε^2 operate as different instruments: noise is a symmetric ex-ante distortion of the second moment, while bias is a deterministic shift of the realized signal. Under any preference, they move different objects.

Information acquisition. We can express the value of information in dollar certainty equivalents. An individual investor with information set $\mathcal{I}$ values it ex ante as $\mathrm{CE}(\mathcal{I}) = \frac{1}{2\gamma} \log \frac{\mathbb{V}(v-P)}{\mathbb{V}(v|\mathcal{I})}$, which corresponds to the Shannon's mutual information, i.e. entropy reduction (Shannon (1948)): $\mathrm{CE}(\mathcal{I}) = \mathbb{I}(v - P, \mathcal{I})$.

Under delegation, realized returns are attenuated by management fees. However, investors can scale their investment to mitigate the effect of fees. In equilibrium, when mass n investors acquire information, the value of information for an individual investor corresponds to a reduction in entropy, independent of the management fee:

$$\phi(n) = \Delta_S \equiv \mathrm{CE}(\hat{s}, P) - \mathrm{CE}(P) = \frac{1}{2\gamma} \log \frac{1 - \bar{\tau}_P(n)}{1 - \bar{\tau}} = \mathbb{I}(v - P, \{\hat{s}, P\}) - \mathbb{I}(v - P, P), \quad (7)$$

which is a standard object in the literature (Admati and Pfleiderer (1986)).

Recall that investor i can privately report position $\tilde{e}_i$, while his true position is e_i. After a report-contingent payment to the rater, his payoff shifts by $(e_i - \tilde{e}_i)\mathbb{E}[P \mid \tilde{e}_i] = (e_i - \tilde{e}_i)\mathbb{E}[P]$. Since $\mathbb{E}[P] = 0$, the optimal reported endowment is indeterminate, but robust selection (to the rater's deviations $b - \bar{b} > 0$) gives $\tilde{e}_i^* = 0$ and the aggregate bias is $b^* = \frac{1}{c_b}\beta_1 \int \tilde{e}_j \, dj = 0$.

To clarify, this result does not rely on the distributional assumption about endowment e_i, namely its aggregation into zero $\int e_i di = 0$. Instead, the reason why individual investors are not willing to pay a premium for distortion is because they are too small to impact the bias b^* and the price P. Indeed, in the Appendix B.1, we consider a distribution of endowment with a common shock: $e_i = w + \epsilon_i$, where $w \sim N(0, \sigma_w^2)$ and $\epsilon_i \sim N(0, \sigma_\epsilon^2)$, and show that the price is still insensitive to an individual investor's actions: $(e_i - \tilde{e}_i)\mathbb{E}[P \mid \tilde{e}_i] = (e_i - \tilde{e}_i)\mathbb{E}[P]$.[11]

Given demand (7), the rater maximizes profit over the mass of buyers n:

$$\pi_{RA}(n) \;=\; n \cdot \phi(n) = n \cdot \frac{1}{2\gamma} \log\left(\frac{1 + \tilde{\rho}(n)}{1 + (1-\tau)\,\tilde{\rho}(n)} \right), \qquad \tilde{\rho}(n) \equiv \rho^2(n) \tfrac{\sigma_z^2}{\sigma_\varepsilon^2}, \tag{8}$$

where $\tilde{\rho}$ is a scaled noise-to-signal ratio. The following Proposition states the solution:

Proposition 1 *In the individual-investor-paid model, the rater sells unbiased scores (i.e., $b^* = 0$). The optimal client mass is $n^* = \min\left\{1, \; \frac{\gamma\, \sigma_\varepsilon\, \sigma_z}{\frac{1-f(1-m)}{1-f}\sqrt{\tilde{\rho}^*}}\right\}$, where the noise-to-signal ratio $\tilde{\rho}^*$ is the unique positive solution to:*

$$\log\left(\frac{1 + \tilde{\rho}^*}{1 + (1-\tau)\tilde{\rho}^*} \right) \;=\; \frac{2\tau\, \tilde{\rho}^*}{(1 + \tilde{\rho}^*)\,(1 + (1-\tau)\tilde{\rho}^*)}. \tag{9}$$

The Proposition establishes one of the main results of the paper. When investors are atomistic, the rater has no incentive to distort the signal in order to cater to its clients. Thus, the conflict of interest generating strategic signal distortions does not arise simply because investors have endowment; it requires that information buyers have *market power*. This condition is increasingly relevant in modern financial markets but is absent from much of the theoretical literature on information sales (Admati and Pfleiderer, 1986, 1990; Bongaerts, 2014; Lovo and Olivier, 2026). Section 4 relaxes the atomistic-investor assumption and shows how conflicts of interest re-emerge.

Note that the equilibrium signal-to-noise ratio $\tilde{\rho}^*$, given by Equation (9), depends exclusively on the precision of the rater's signal τ. Then, the price informativeness also depends on τ only, and is invariant to market liquidity σ_z^2 or management fees f. Intuitively, the information provider adjusts the fee ϕ to control how much information leaks into prices

[11] In contrast, in the fund-paid case in Section 4, as we will see later, both the price impact and distortion b are internalized by the manager, and as a result, it prefers a non-trivial report $\tilde{w}$.

(Grossman and Stiglitz (1980), Cespa (2008)). The provider thus rations information to extract a larger surplus. As we show in Section 5.2, the intermediated sale of information alleviates this rationing.

3.2 Large Fund without a Portfolio Requirement

Now, consider a large fund acquiring the information. Similar to individual investors, the fund has an unobserved legacy position w, which could be inflated by the rater to subsequently share the surplus, but *no portfolio constraints*. We show that in this case, the fund *does not* demand distorted signal.

Specifically, we let the fund choose the number of shares x_F strategically to maximize its total profits (2), which include both the management fees and the market value of the legacy position (the source of conflict). In the absence of portfolio constraints, the fund's trading depends only on the available *information*, which is fully captured by s. Hence, without the portfolio constraint (3), there is no reason to demand distorted information.

The fund is the only strategic trader who internalizes its price impact, while active individual investors are uninformed price-takers. The following Lemma characterizes the asset-market equilibrium.

In the appendix, we show that the equilibrium is unique and linear: $P = \frac{\tau}{2}s + \frac{\alpha\lambda}{2fm}w - \frac{\lambda}{2m}z$, and the fund's trading is given by

$$x_F^N(s, w, P) = \frac{\tau}{2\lambda}s + \frac{\alpha}{2fm}w + \frac{1}{2m}z. \tag{10}$$

where λ is the price impact, and the price informativeness is $\tau_P^N < \tau/2$.

The three components of x_F^N are informed trading, portfolio pumping, and liquidity provision. The fund trades in the direction of its legacy position. The pumping is driven by the manager's myopia α and is tempered by management fees she collects from new investors. The resulting trade injects w-noise into the price and reduces its informativeness.

Before paying the fixed information fee, the fund's expected payoff is

$$\Pi_N = \mathbb{E}[\pi_F] = fmW_0 + \underbrace{\frac{fm\,\tau\sigma_v^2}{4\lambda}}_{\text{informed trading}} + \underbrace{\frac{f\lambda\sigma_z^2}{4m}}_{\text{liquidity provision}} + \underbrace{\frac{\alpha^2\lambda\sigma_w^2}{4fm}}_{\text{pumping}}. \tag{11}$$

Delegation under discretion. Portfolio pumping benefits the manager at the expense of its new investors. Recall that Δ_S^N is the certainty-equivalent gain from delegating to the unconstrained fund that acquires information. The marginal delegating investor's indifference

condition equilibrates the extra management fees and expected gains from informed trade

$$fW_0 - \bar{c}(1-m) = \Delta_S^N(m, f). \tag{12}$$

A marginal increase in pumping, $p \equiv \left(\frac{\alpha}{2fm}\right)^2 \sigma_w^2$, introduces noise to the fund's informed trade, and thus strictly lowers investors' willingness to pay for delegation: $\partial \Delta_S^N / \partial p < 0$. Indeed, a marginal increase in pumping both lowers expected return on their position and adds non-fundamental risk that delegating investors have to bear. Therefore, the fund must charge a lower fee to implement a given delegated mass.

Commitment raises the fee investors are willing to pay, but forces the manager to give up its private pumping rent. To compare the benefits and the cost of commitment, we next characterize the fund's profits under perfect commitment.

Perfect commitment. Suppose the fund adopts the portfolio requirement (3), and the signal distortion is prohibitively costly, $c_b \to \infty$, i.e., the commitment is perfect. We further show that, starting from a sufficiently high distortion cost, the manager's expected profit increases as c_b declines. Thus, our analysis here will extends to an economically relevant range of finite, but high, distortion costs.

Let $\Delta_S^P(m, f)$ denote the information gain from delegating under perfect commitment. The fee $f^P(m)$ that implements a delegating mass m satisfies the following indifference condition for the marginal delegating investor:

$$f^P(m)W_0 - \bar{c}(1-m) = \Delta_S^P\big(m, f^P(m)\big). \tag{13}$$

Then, an increase in the fund manager's profit per delegating investor evaluated at the same delegating mass m is

$$\frac{\Pi_P - \Pi_N}{m} = \left(f^P - f^N\right) W_0 + \left(f^P \Gamma^P - f^N \Gamma^N\right) - f^N \lambda p, \tag{14}$$

where Γ^i denotes an expected trading gain in equilibrium i:

$$\Gamma^N \equiv \frac{\mathbb{V}(\mathbb{E}[v \mid s] - P^N|_{p=0})}{\lambda} \quad \text{and} \quad \Gamma^P \equiv \frac{\mathbb{V}(\mathbb{E}[v \mid s] - P^P)}{\gamma \mathbb{V}(v \mid s)}.$$

Let m_N^* denote the optimal delegated mass under *discretion*. Define the increase in delegation surplus from perfect commitment, evaluated at the *discretionary fee* f^N, as $G_N \equiv \Delta_S^P(m_N^*, f^N) - \Delta_S^N(m_N^*, f^N)$. If $G_N > 0$, commitment supports a higher fee. Let $\rho_f \in (0,1)$ denote the fraction of this surplus gain captured through the fee. Appendix A.1.1 shows

that $(f^P - f^N)W_0 = \rho_f G_N$.

Proposition 2 *Suppose the discretionary price is sufficiently more informative than the mandated one to cover the fee wedge,*

$$\left(1 - \tau_P^P\right)\left(1 - f^2\mathcal{R}_P^2\right) \;\geq\; 1 - \tau_P^N(0), \tag{15}$$

which holds whenever price impact is low,

$$\lambda < \gamma\sigma_v^2(1 - \tau), \tag{16}$$

And suppose the fee satisfies

$$\frac{f^N}{1 - f^N} \;<\; \rho_f\left(1 - \mathcal{R}_N^2\right), \tag{17}$$

where $\mathcal{R}_i^2$ is the predictable share of return variance, i.e., $\mathcal{R}_i^2 \equiv \mathbb{V}(\mathbb{E}[v \mid s] - P_i)/\mathbb{V}(v - P_i)$ for $i \in \{N, P\}$. Then, perfect commitment is strictly preferred by the fund manager: $\Pi_P > \Pi_N$.

Commitment exchanges a private rent for a higher fee, and the exchange is favorable under two conditions: (i) when pumping destroys the information value, i.e. leaks too much information into the equilibrium price, which happens due to the low price impact and aggressive trading (Condition 16); (ii) when the equilibrium fee the fund can charge without commitment is small, which renders pumping less constrained and limits the fee component of the fund profit.

The price-impact condition holds when the fund is small relative to the active sector and the price is noisy: with $\beta_s = \tau/2$, the condition $\lambda \leq \lambda^{PC}$ reduces to

$$2m\left(1 - \tau_P^N\right) \;\leq\; (1 - m)(1 - \tau)\left(1 - \frac{2\tau_P^N}{\tau}\right),$$

which is guaranteed by $m \leq (1 - \tau)/(3 - \tau)$ when the price carries little information, and is relaxed by liquidity trading and by the very pumping that a small fee invites, so that conditions (i) and (ii) reinforce one another.

This cross-sectional prediction lines up with where portfolio pumping is actually observed. Quarter- and year-end NAV inflation is concentrated in small-cap funds (Carhart et al., 2002), whose holdings are illiquid and whose trades are large relative to the float — in the model's notation, precisely the high-λ, high-m case in which the pumping rent $f\lambda p$ is largest and condition (i) fails, so that discretion dominates and pumping survives in equilibrium.

4 Delegation under a Rating-Contingent Constraint

Now we characterize the equilibrium when the signal is purchased *with a rating-contingent portfolio constraint*. Like atomistic investors, the fund has a legacy position (denoted as w), and its payment to the rating agency is exposure-contingent. The key distinction from the individual-investor-paid benchmark is that the fund is large and can move both prices and signal distortion. This creates the capacity to benefit from a distorted signal when the distortion raises the value of the fund's legacy position.

The distortion in the fund-paid model arises because the fund is constrained in the use of proceeds from investors. The fund is subject to a binding portfolio requirement (3). After the trade, investors observe both the fund's position x_F and the signal $\hat{s}$ that justifies this position. Hence, any deviation from the required trading rule is observable ex post.[12]

This disclosure requirement prevents the fund from directly distorting its portfolio choice. However, it does not prevent the fund from distorting the signal on which the required trade is based. Investors observe the reported signal $\hat{s}$, but they do not observe the raw signal s. A biased report $\hat{s} = s + b$, therefore, allows the fund to implement a distorted position while remaining formally compliant with the portfolio constraint. The fund's position is optimal given $\hat{s}$, even though it is not the position investors would have chosen had they observed the unbiased signal s.

Figure 2, Panel (b) shows the timing of the events. First, investors see the contract offered by the fund $(f, x_F(\hat{s}))$ and decide whether to delegate their trading to the fund. Second, the fund sends a message $\tilde{w}$ to the rater about its position. Third, the rater observes the private signal s and discloses a modified signal $\hat{s}$. Solving the model backwards, we start from the trading equilibrium.

Asset market equilibrium. In the fund-paid model, delegation and signal access are bundled: the informed group is exactly the delegating investors.

The following Lemma describes the asset market equilibrium:

Lemma 1 *There is a unique linear equilibrium $P = \beta_0 + \beta_1 \hat{s} + \beta_2 z$. The perceived precisions is $\bar{\tau} = \frac{\sigma_v^2}{\sigma_s^2 + \bar{\sigma}_b^2}$, while the price informativeness is given by*

$$\bar{\tau}_P = \frac{\sigma_v^2}{\sigma_s^2 + \bar{\sigma}_b^2 + \bar{\rho}^2 \sigma_z^2}, \quad \textit{where} \quad \bar{\rho} \equiv \frac{\beta_2}{\beta_1} = -\frac{\gamma \sigma_v^2 (1 - \bar{\tau})}{m\, \bar{\tau}} \quad \textit{is noise-to-signal ratio} \tag{18}$$

[12]For example, ESG funds need to demonstrate that their positions comply with the green mandate at disclosure dates, while on other dates their trading might deviate (Parise and Rubin (2025))

Moreover, the ratio $\frac{1-\bar{\tau}_P}{1-\bar{\tau}}$ *is increasing in* $\bar{\tau}$ *for* $\bar{\tau} \in [0, \tau^*)$ *and decreasing for* $\bar{\tau} \in [\tau^*, 1]$, $\tau^* \equiv \frac{\sqrt{\gamma^2\sigma_v^2\sigma_z^2}}{m+\sqrt{\gamma^2\sigma_v^2\sigma_z^2}}$.

See the closed-form expressions for the coefficients in the appendix. The Lemma states that the signal noise has a non-monotone effect on the delegation rent. Starting from a sufficiently precise fund signal, i.e., $\bar{\tau} \in [\tau^*, 1]$, additional noise can increase the relative value of delegation because noise limits the trading intensity of informed agents and prices become less informative faster than the direct signal, so the value of the signal $\frac{1-\bar{\tau}_P}{1-\bar{\tau}}$ increases. But once the fund signal is sufficiently noisy, further noise reduces the value of delegation. In other words, when the signal is very precise, informed investors trade much more aggressively and impound more information into the asset price.

Delegation. When the fund provides information to its investors, delegation and information acquisition are bundled. The mass of fund investors is pinned down by the certainty-equivalent indifference between delegating (informed) and trading actively (uninformed). For the indifferent investor with the cost c^*, the incremental cost in management fees should be offset by the value of information:

$$fW_0 = \bar{c}(1-m) + \underbrace{\Delta_S + \frac{1}{2\gamma}\log\left(1 - f^2\hat{\mathcal{R}}^2\right)}_{\equiv \Delta_D(m,f)}, \tag{19}$$

where $\hat{\mathcal{R}}^2 \equiv \mathbb{V}\left(\mathbb{E}\left[v \mid \hat{s}\right] - P\right)/\mathbb{V}\left(v - P\right)$ is the predictable share of excess return variance based on the biased signal $\hat{s}$. Since the fee is paid as a share of the realized return, f lowers the information rent retained by investors. Investors with $c_i \geq c^*$ delegate–their saved transaction cost plus the information rent dominates the fee. The term Δ_D captures the information-rent bonus from bundling signal access into delegation. [13] with a management fee discount.

Δ_D depends on m and f, so (19) defines $f(m)$ implicitly. The dependence runs through the information rent ($\partial\Delta_D/\partial f < 0$), which compresses the informed investor's advantage. As a result, there is a unique fee $f(m)$ which solves (19). Moreover, it decreases with m, i.e. $df/dm < 0$. In other words, to attract a higher mass of fund investors, the fund manager must lower the fee.

[13] ESG funds charge meaningfully higher fees than passive equity benchmarks of comparable risk profile, with much of the spread typically attributed to higher operating costs. See, e.g., Morningstar's annual *U.S. Sustainable Funds Landscape* reports. As this result suggests, the fee differential reflects not only cost differences, but also the rents that delegated intermediaries can extract when they provide some additional proprietary information from rating agencies.

Notice that the value of delegating(19) depends on the investors' beliefs about the rater's bias $\bar{\sigma}_b^2$. This belief will be consistent with the rater's strategy in equilibrium, but is taken as given by the fund and the rater.

Information acquisition. Next, the fund and the rater arrange the sale of information. The payment 5 is contingent on the reported legacy position $\tilde{w}$ and realized $\hat{s}$.

The fund's expected payoff with endowment is w and the message $\tilde{w}$ is:

$$\mathbb{E}[\pi_F \mid w, \tilde{w}] = \underbrace{fm\left\{W_0 + \mathbb{E}[(v-P)x_F \mid w, \tilde{w}]\right\}}_{\text{management fees}} + \underbrace{\alpha w\, \mathbb{E}[P \mid w, \tilde{w}]}_{\text{legacy position}} - \underbrace{\tilde{w}\, \mathbb{E}[P \mid w, \tilde{w}] - F}_{\text{payment to the rater}}, \quad (20)$$

The first term is the management fee from new investors, and the second term is the market value of the fund's position in the risky asset, w. The last term reflects the payment to the rating agency for the signal.

We will solve this optimization problem of the fund and the ratings agency backwards. First, conditional on s and $\tilde{w}$, the rater maximizes its payment given by:

$$\max_b \; F + \tilde{w}\, \mathbb{E}[P(\hat{s}) \mid s, b] - \frac{c_b}{2}\, b^2, \quad (21)$$

which, given the conjectured linear price, admits the closed-form solution: $b^*(\tilde{w}) = \frac{\beta_1}{c_b}\, \tilde{w}$. The bias is independent of the actual signal s because P is linear in $\hat{s}$, and therefore s could be separated from b, and the cost of bias is quadratic.

Second,the fund chooses $\tilde{w}$, knowing the rater's response, and before observing s. The following Lemma describes the optimal report $\tilde{w}$ of the manager.

Lemma 2 (Optimal report) *In equilibrium, the fund's optimal disclosure to the rater is*

$$\tilde{w}^*(w) \;=\; \frac{\alpha}{2(1+\mu)}\, w, \quad (22)$$

where $\mu \equiv \frac{fm\beta_1(\bar{\tau}-\beta_1)}{\gamma(1-f)\sigma_v^2(1-\bar{\tau})c_b}$ *captures the impact of bias on management fees.*

The optimal report is linear in w and independent of s. Linearity stems from the quadratic structure of $\mathbb{E}[\pi_F \mid w, \tilde{w}]$ in $\tilde{w}$. The slope $\alpha/(2(1+\mu))$ measures how aggressively the fund translates legacy exposure into requested distortion: α scales the marginal benefit of inflating the legacy position, while $1+\mu$ scales the marginal cost, combining the rater's cost and the lost management fees from new investors due to price distortion μ. As a result, the equilibrium bias is given by:

Proposition 3 (Bias) *In equilibrium, the rater chooses the following bias:*

$$b^*(w) \;=\; \frac{\alpha\,\beta_1}{2c_b\,(1+\mu)}\,w \tag{23}$$

The equilibrium moments of b^* are pinned down by

$$\mathbb{E}[b^*] = 0, \qquad \sigma_b^2 \;=\; \frac{\alpha^2\beta_1(\sigma_b^2)^2\sigma_w^2}{4\left[c_b + f(\sigma_b^2)m\dfrac{\beta_1(\sigma_b^2)(\bar{\tau}-\beta_1(\sigma_b^2))}{\gamma\sigma_v^2(1-\bar{\tau})}\right]^2}, \tag{24}$$

Denote the function on the RHS of Equation 24 showing the sensitivity of the bias to the legacy position as $\mathcal{B}(\sigma_b^2, m)$. It reflects the underlying tradeoffs of the fund and the rater. The numerator reflects the benefits from inflating the market value of the legacy position, which plays a role as long as the fund manager values it, i.e. $\alpha > 0$. The more sensitive the price to the signal is, i.e. the higher β_1, the more effective the signal distortion becomes. However, distortion itself reduces price sensitivity: $\partial\beta_1/\partial\sigma_b^2 = (\partial\beta_1/\partial\bar{\tau}) \cdot (\partial\bar{\tau}/\partial\sigma_b^2) < 0$. Thus, the marginal benefits of distortion are decreasing.

The denominator shows the marginal cost of distortion: (i) c_b is the explicit cost; (ii) $\bar{\mu} = \frac{fm\beta_1(\bar{\tau}-\beta_1)}{\gamma(1-f)\mathbb{V}(v|\hat{s})}$ is the implicit cost, i.e., the management fees lost due to signal distortion. Note that the implicit cost also depends on the price sensitivity to the signal, which becomes smaller as σ_b^2 increases. Hence, higher bias attenuates price sensitivity and erodes the marginal benefits of distortion, but at the same time, it lowers the implicit cost of distortion μ. As a result, there could be multiple equilibria, characterized by a fixed point (Equation 24), with high or low bias.

Under reasonable and empirically plausible conditions, we can get a unique perfect Bayesian Nash Equilibrium (PBNE). Specifically, one sufficient condition requires the explicit cost of bias (reputation or litigation risks) c_b to be sufficiently high as the following result establishes. Intuitively, the multiplicity arises due to self-fulfilling beliefs, and the higher explicit cost c_b discipline the feedback and guaranties uniqueness:

Lemma 3 (Unique PBNE) *Define the elasticities with respect to the bias,*

$$\varepsilon_{\beta_1}(\sigma_b^2) \equiv \frac{\partial \log \beta_1}{\partial \log \sigma_b^2}, \qquad \varepsilon_{\bar{\mu}}(\sigma_b^2) \equiv \frac{\partial \log \bar{\mu}}{\partial \log \sigma_b^2}, \tag{25}$$

Suppose the benefit-side elasticity is dominated by the cost-side response on the domain:

$$\varepsilon_{\beta_1}(\sigma_b^2) - \tfrac{\bar{\mu}}{c_b+\bar{\mu}}\,\varepsilon_{\bar{\mu}}(\sigma_b^2) \;<\; \frac{\sigma_b^2}{2\,\mathcal{B}(\sigma_b^2, m)} \qquad \textit{for all } \sigma_b^2 \in (0,\infty). \tag{26}$$

Then the equilibrium bias dispersion $\sigma_b^{2*}(m)$ that solves (24) exists and is unique.

The following parameter constraint given uniqueness:

$$c_b \;>\; \sqrt[3]{\frac{\alpha^2\sigma_w^2\,\sigma_v^6}{8\,\gamma\,\sigma_\varepsilon^4\,\sigma_s^6}\left(1+\frac{\tau}{2\gamma W_0(1-\bar{f}^2)}\right)}, \tag{27}$$

Intuitively, (26) is a stability requirement on investors' beliefs about distortion. This elasticity decomposition identifies the forces that make the beliefs self-correcting. A fixed point is a belief that reproduces itself: a bias slightly above (below) the belief $\bar{\sigma}_b^2$ pulls this belief back. The first term $\varepsilon_{\beta_1}(\sigma_b^2)$ represents the *benefit channel*: a higher anticipated bias lowers the price sensitivity to the signal β_1 ($\varepsilon_{\beta_1} < 0$) and the rater is encouraged to distort less. The *cost channel* works through the management fees: when anticipated bias lowers the equilibrium fee ($\varepsilon_{\bar{\mu}} < 0$), the implicit cost falls and the rater is encouraged to distort more. This destabilizing channel works only to the extent that the implicit fee $\bar{\mu}$ rather than the explicit cost c_b disciplines the rater. Thus, when $\frac{\bar{\mu}}{c_b+\bar{\mu}}$ is low, i.e., the rater exposed to meaningful regulatory penalties (Equation 27), the equilibrium bias is uniquely determined.

A large explicit cost of bias (c_b) is plausible in regulated rating markets. Since Dodd–Frank, NRSROs have faced internal-control requirements and conflict-of-interest rules. Enforcement actions further show that the costs of misconduct can be substantial, including legal settlements. [14] As a result, in rating markets, the magnitude of distortions is not large. Existing evidence points to economically meaningful but incremental bias: rating agencies issue more favorable ratings and delay downgrades when exposed to stronger client-side incentives, rather than abandoning rating discipline altogether (e.g., Bonsall et al., 2024; Cornaggia and Cornaggia, 2013; Xia, 2014). This pattern fits the small-bias equilibrium. The rater may shade information toward valuable clients, but the magnitude of distortion is constrained by regulatory scrutiny and reputation.

Next, recall that the rater solves its optimization problem when the management fees are already set. As a result, Proposition (3) specifies the level of bias in equilibrium for a given mass of fund investors m. The following Lemma identifies when $\sigma_b^{2*}(m)$ is increasing.

Lemma 4 *At a locally stable equilibrium $\sigma_b^{2*}(m)$, suppose the price loading dominates the implicit cost response to varying mass of delegating investors:*

$$\frac{\partial \log \beta_1}{\partial m} \;>\; \frac{\partial \log \bar{\mu}}{\partial m} \tag{28}$$

[14] See, e.g., Financial Times, SEC raises questions over rating agency behind private credit boom"; SEC, SEC Charges Egan-Jones Ratings Co. and CEO with Conflict of Interest Violations"; and Wall Street Journal, "Credit-Rating Providers Settle With SEC Over Off-Channel Communication Violations."

Then equilibrium bias dispersion rises with the delegated mass, $d\sigma_b^{2*}/dm > 0$.

The endogenous condition (28) *can be guaranteed by a simple primitive bound:*

$$c_b > \frac{\tau^2}{4\gamma\sigma_v^2(1-\tau)}, \tag{29}$$

Intuitively, the larger mass of investors makes the fund bigger. A bigger fund then can move market prices in the favorable direction. This increases the benefits from manipulation by the price more sensitive to signal changes (β_1 increases). This is illustrated with the increasing solid blue line in Figure 3a.

Management fees. The fund's equilibrium payoff before the fixed payment F to the rating agency is

$$\Pi \equiv \mathbb{E}[\pi_F] = fm\left\{W_0 + \Gamma^F\right\} + \frac{\alpha^2\beta_1^2}{4c_b(1+\mu)}\,\sigma_w^2 \tag{30}$$

where Γ^F is the mean-variance trading rent of a fund investor: $\Gamma^F = \frac{(\bar{\tau}-\beta_1)[\sigma_v^2-\beta_1\sigma_s^2]+\beta_2^2\sigma_z^2}{\gamma\sigma_v^2(1-\bar{\tau})}$.

Two economic channels generate the fund's surplus from information acquisition. First, the fund can charge higher *intermediation rents*: information makes delegation more attractive, raising both the willingness to pay of fund investors relative to the no-information benchmark. Second, the fund can earn additional *signal distortion rent* from inflating its legacy position.

When $\sigma_w \to 0$ legacy positions are transparent, and rational individual investors can unwind the bias. As a result, the distortion rent collapses. When $\alpha \to 0$, the fund places no weight on legacy-position appreciation, and the bias mechanism is shut down: $b^* \to 0$, $\bar{\sigma}_b^2 \to 0$. In both cases, the fund's willingness to pay depends solely on the gain from information, rather than manipulation.

Define:

$$\mathcal{M}(\sigma_b^2) \;\equiv\; \arg\max_m \Pi(m,\, \sigma_b^2) \tag{31}$$

The following Lemma states the sufficient conditions for $\mathcal{M}$ to increase in σ_b^2.

Lemma 5 (Delegation contraction due to bias) *Suppose that the fund's optimum is interior and satisfies $\Pi_{mm} < 0$. If*

$$(W_0 + \Gamma^F)(mf)_{m\,\bar{\tau}} \;>\; \left|(mf)_m\Gamma^F_{\bar{\tau}} + (mf)_{\bar{\tau}}\Gamma^F_m + (mf)\Gamma^F_{m\bar{\tau}} + \left(\tfrac{\alpha^2\beta_1^2}{4c_b(1+\mu)}\,\sigma_w^2\right)_{m\bar{\tau}}\right|, \tag{32}$$

then $\frac{d\mathcal{M}}{d\sigma_b^2} < 0$. *Moreover, Condition* (32) *is guaranteed for sufficiently large* W_0 *if*

$$\gamma^2 \sigma_v^2 \sigma_z^2 (1-\tau)^2 \geq 10\tau, \tag{33}$$

$$c_b \geq \frac{126\gamma\alpha^2\sigma_w^2\tau}{(1-\tau)^4}. \tag{34}$$

In particular, there exists a finite $\bar{W}_0$*, such that* $W_0 \geq \bar{W}_0$ *implies* $d\mathcal{M}/d\sigma_b^2 < 0$.

The conditions isolate three forces. First, sufficiently strong noise trading prevents prices from immediately revealing the fund's signal. Precision then raises the marginal value of delegated access,

$$\partial_{m\bar{\tau}} \left[m \log \left(\frac{1-\bar{\tau}_P}{1-\bar{\tau}} \right) \right] > 0.$$

Because anticipated bias lowers precision, it reduces the fund's incentive to expand access.

Second, precision changes the marginal legacy value of a larger fund through $\partial_{m\bar{\tau}}\beta_1^2$, whose sign is ambiguous. A sufficiently high c_b ensures that any adverse legacy effect cannot dominate the access-value effect. Because $\bar{\tau}$ is held fixed in the best response $\mathcal{M}(x)$, the equilibrium response of $\bar{\tau}$ to c_b cannot relax this restriction.

Finally, a sufficiently large W_0 makes the remaining fee-dependent forces small. Since

$$fW_0 \leq \bar{c}_\tau, \qquad f \leq \frac{\bar{c}_\tau}{W_0},$$

the fee-retention correction is $O(f^2)$, while the gross-trading-gain and implicit-cost corrections are $O(f)$. Thus, $W_0 \geq \bar{W}_0$ places the equilibrium in a small-fee region where these ambiguous forces cannot overturn the leading effect.

This region is empirically plausible. Asset-weighted expense ratios in 2025 were approximately 0.40% for equity mutual funds, 0.36% for bond mutual funds, and 0.09–0.14% for index ETFs, while U.S.-registered investment companies managed approximately $45.1 trillion.[15] However, the wealth condition should be read as requiring a large asset base relative to transaction costs and information rents, not a particular absolute dollar amount.

Full equilibrium. The equilibrium bias is jointly determined with the size of the delegated sector. To illustrate this fixed-point logic, Figure 3 plots two graphs in the (σ_b^2, m) space. The rater-side optimum for a given mass m is obtained from the consistency condition for

[15] Investment Company Institute, "Trends in the Expenses and Fees of Funds, 2025" and *2026 Investment Company Fact Book: Quick Facts Guide*.

bias dispersion,

$$\sigma_b^2(m) \;=\; \mathcal{B}(\sigma_b^2,\, m), \tag{35}$$

A larger mass of fund investors m affects this optimum through two opposing forces: it raises the price impact of the report through β_1, but it also changes the equilibrium fee and the effective cost of inducing bias. Under the conditions of Lemma 4, the positive effect dominates and $\mathcal{B}$ is increasing in m.

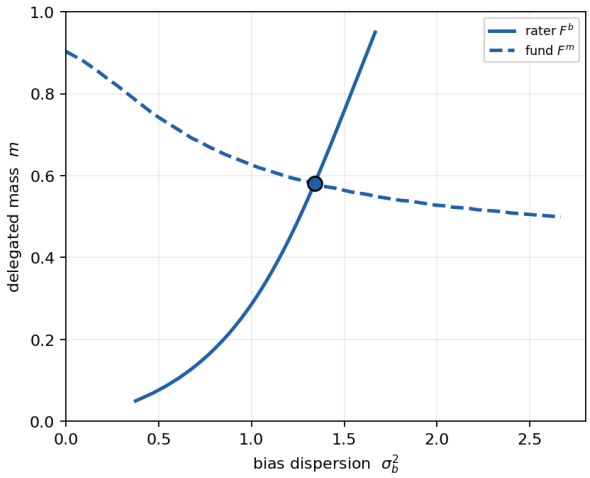
The other part of the equilibrium is the fund side optimum: for each candidate level of anticipated bias dispersion $\bar{\sigma}_b^2$, the fund chooses the delegated mass m that maximizes the surplus from operating the fund-paid information regime. In equilibrium, this best response is given by

$$m(\sigma_b^2) \;=\; \mathcal{M}(\sigma_b^2).$$

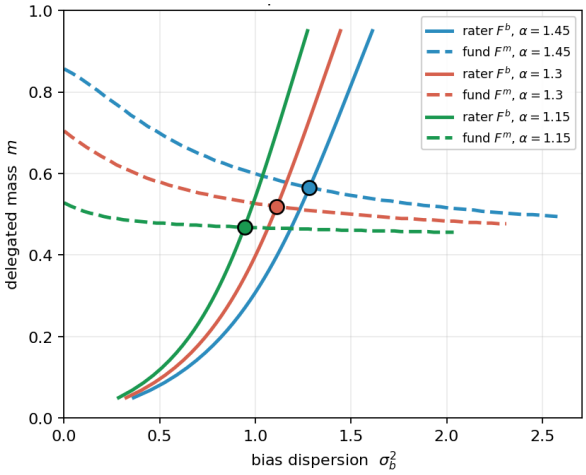
The intersection of the two optimums therefore pins down the fixed point (σ_b^{2*}, m^*): the amount of distortion the rater is willing to supply is exactly the amount consistent with the delegated sector that the fund can sustain.

The two mappings have different economic interpretations. The rater-side optimum can be interpreted as a *distortion supply*, although it is a consistency condition rather than a conventional supply curve. By contrast, $\mathcal{M}(\sigma_b^2, \alpha)$ is the fund's desired delegated mass when anticipated bias is taken as given by individual investors. It can be interpreted as a *demand* for informed trading. Lemma 5 shows that this optimum is decreasing for empirically relevant parameters (small bias and small management fees f^*): greater anticipated bias reduces the informational value of delegation and therefore the mass the fund wishes to serve. The full equilibrium is the intersection of an upward-sloping distortion supply and a downward-sloping information demand.

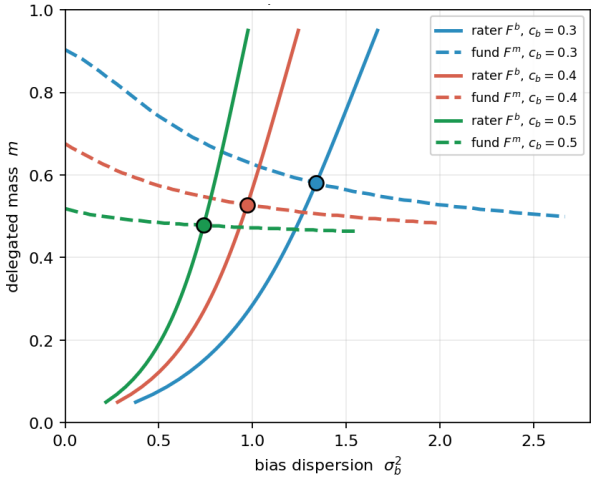
An increase in the conflict between the fund and its new investors α shifts both curves upward. At a fixed delegated mass, it raises the value of distorting the report and hence the bias supplied by the rater, i.e. $\partial\mathcal{B}/\partial\alpha > 0$. At a fixed level of anticipated bias, it raises the legacy-position return from operating a *larger* fund and hence the fund's desired scale, i.e. $\partial\mathcal{M}/\partial\alpha > 0$. Distortion therefore rises unambiguously, but the effect on delegated access is not immediate: the direct expansion in desired scale is partly offset by the contraction in access caused by the resulting increase in bias. By similar logic, the explicit cost of distortion c_b moves both curves downward: $\partial\mathcal{B}/\partial c_b < 0$ and $\partial\mathcal{M}/\partial c_b < 0$, which lowers the equilibrium bias, but has an ex-ante ambiguous effect on the equilibrium mass m^*. The following Proposition characterizes which force dominates.



(a) Baseline



(b) Comparative statics in α



(c) Comparative statics in c_b

Figure 3: Joint determination of bias and delegated capital.

The solid lines are the rater-side bias optimum, which imposes consistency of the equilibrium bias variance σ_b^2 given the delegated mass m. The dashed lines are the fund-side delegation optimum, which gives the delegated mass that maximizes the fund's participation surplus for each candidate level of bias dispersion. Panel (a) shows the baseline fixed point. Panel (b) varies the payoff sensitivity to the fund's prior position, α. Panel (c) varies the marginal cost of bias, c_b.

Proposition 4 (Access–distortion co-movement) *Suppose the conditions of Lemmas 4 and 5 hold. Then there exists a unique interior solution to the system*

$$\begin{aligned} \sigma_b^2(m) &= \mathcal{B}(\sigma_b^2, m), \\ m(\sigma_b^2) &= \mathcal{M}(\sigma_b^2) \end{aligned}$$

Moreover,

$$\frac{d\sigma_b^{2*}}{d\alpha} > 0, \qquad \frac{d\sigma_b^{2*}}{dc_b} < 0,$$

while greater sensitivity to the legacy position ***raises*** *equilibrium delegation,* $\frac{dm^*}{d\alpha} < 0$*, iff*

$$\frac{\partial \mathcal{M}}{\partial \alpha}\left(1 - \frac{\partial \mathcal{B}}{\partial \sigma_b^2}\right) > -\frac{\partial \mathcal{M}}{\partial \sigma_b^2}\frac{\partial \mathcal{B}}{\partial \alpha}. \tag{36}$$

and a lower cost of distortion ***raises*** *delegated access,* $\frac{dm^*}{dc_b} > 0$*, iff*

$$-\frac{\partial \mathcal{M}}{\partial c_b}\left(1 - \frac{\partial \mathcal{B}}{\partial \sigma_b^2}\right) > \frac{\partial \mathcal{M}}{\partial \sigma_b^2}\frac{\partial \mathcal{B}}{\partial c_b}. \tag{37}$$

Intuitively, a stronger legacy motive or a lower distortion cost necessarily raise equilibrium bias. They also expand delegation precisely when its direct positive effect on the fund's desired scale dominates the contraction in access caused by the increase in anticipated bias.

The conditions in Proposition 4 compare these two forces directly. Rewriting (36),

$$\underbrace{\frac{\partial \mathcal{M}}{\partial \alpha}}_{\text{direct expansion in access}} > \underbrace{-\frac{\partial \mathcal{M}}{\partial \sigma_b^2}\frac{\partial \mathcal{B}/\partial \alpha}{1 - \partial \mathcal{B}/\partial \sigma_b^2}}_{\text{contraction caused by additional bias}}.$$

The fraction on the right is the increase in equilibrium bias generated by α when m is held fixed. Thus, access and distortion rise together when the direct scale benefit of a stronger legacy motive exceeds the loss of access generated by the additional distortion.

The same comparison can be expressed in elasticity terms. Evaluated at the equilibrium, (36) is equivalent to

$$\frac{\frac{\alpha}{m^*}\frac{\partial \mathcal{M}}{\partial \alpha}}{-\frac{\sigma_b^{2*}}{m^*}\frac{\partial \mathcal{M}}{\partial \sigma_b^2}} > \frac{\frac{\alpha}{\sigma_b^{2*}}\frac{\partial \mathcal{B}}{\partial \alpha}}{1 - \frac{\partial \mathcal{B}}{\partial \sigma_b^2}}$$

The left-hand side measures the elasticity of desired access to the legacy motive relative to its sensitivity to anticipated bias. The right-hand side measures the elasticity of distortion to the legacy motive, adjusted for the feedback from anticipated to realized bias. Co-movement is therefore more likely when the legacy motive has a strong direct effect on desired fund scale, when access is relatively insensitive to bias, or when the bias fixed point is strongly self-correcting.

Rater's profits. The rater's payoff at the realized state (w, s, z) is

$$\pi_{RA}(w, s, z) \;=\; F + \tilde{w}^*(w)\, P(s, b^*(w), z) \;-\; \frac{c_b}{2}\, b^*(w)^2. \tag{38}$$

Substituting the equilibrium choices $\tilde{w}^*(w) = \alpha w/(2c_b(1+\mu))$ and $b^*(w) = \alpha\beta_1 w/[2c_b(1+\mu)]$ into (38) delivers the following decomposition.

Let $\mathcal{V} \equiv \mathbb{E}[\pi_F] - \mathbb{E}[\pi_F^U]$ denote the incremental surplus generated by providing the fund with information, excluding the gains and costs associated with signal distortion. Nash bargaining assigns the rater a share η of this informational surplus, i.e., $F = \eta\,\mathcal{V} = \eta$. In addition, the rater earns the distortion rent generated by the state-contingent component of the contract.

Proposition 5 (Rater's profit) *The rater's ex-post profit decomposes as*

$$\pi_{RA}(w, s, z) \;=\; \eta\mathcal{V} + \frac{\alpha\beta_1\, w}{2(1+\mu)} \left(s + b^* + \frac{\beta_2}{\beta_1}\, z - \frac{b^*}{2} \right). \tag{39}$$

Taking unconditional expectations,

$$\mathbb{E}[\pi_{RA}] \;=\; \eta\mathcal{V} + \frac{\alpha^2\beta_1^2\,\sigma_w^2}{8c_b(1+\mu)^2} \;>\; 0, \tag{40}$$

The rater's ex-post profit is negative whenever the realized price-signal component $s_P \equiv \frac{P_1}{\beta_1} = s + b^* + (\beta_2/\beta_1)z$ falls below $b^*(w)/2 - 2(1+\mu)\eta\mathcal{V}/\alpha\beta_1 w$ when $w > 0$ or exceeds $b^*(w)/2 - 2(1+\mu)\eta\mathcal{V}/\alpha\beta_1 w$ when $w < 0$.

We can make several observations about the rater profit. First, the contract can produce losses with some probability ex-post. The loss condition $s_P < b^*(w)/2 - 2(1+\mu)\eta\mathcal{V}/\alpha\beta_1 w$ ($s_P > b^*(w)/2 - 2(1+\mu)\eta\mathcal{V}/\alpha\beta_1 w$) when $w > 0$ ($w < 0$) has a clean intuition: the rater loses money precisely when the realized price signal is too small to "validate" the bias the rater introduced.

Second, cross-sectional dispersion σ_w^2 is an important source of rents to both contracting parties. If $\sigma_w \to 0$, the fund's distortion-rent term vanishes. The fund-paid model relies fundamentally on uncertainty in fund-side legacy exposures to generate the surplus. Otherwise, investors can perfectly unwind the bias.

5 Comparison between the Models of Sales

5.1 The Rating Agency's Choice

The preceding sections treated the identity of the information buyer as given. We now endogenize this choice. In other words, we consider the optimal choice of who the rating agency should sell the information.

Under individual-investor-paid sale, the provider earns an *information rent*. It sells signal precision to dispersed investors, but wider access causes information to leak into the price and reduces its value. Under fund-paid sale, the provider can additionally earn a *distortion rent*. By tailoring the released signal to the fund's hidden legacy exposure, it helps the fund raise the marked-to-market value of that exposure and splits the resulting surplus.

The regime choice therefore determines whether the information sale is organized around broad access to an unbiased signal or around a bilateral relationship with a strategic institutional client. The first arrangement resembles standardized investment research sold on common terms to a broad subscriber base: retail equity research or widely distributed financial-data products (such as Morningstar subscriptions).

The second arrangement resembles investor-paid credit ratings, private-credit assessments, and customized ESG index licenses sold to a few large asset managers. In these markets, contract terms are often bilateral and non-public, and an institutional client's willingness to pay may depend on how the provider's signal affects the value of assets already in its portfolio (Bonsall et al. (2024)).

Let

$$\Pi_R^* \equiv n^* \phi^* = \frac{n^*}{2\gamma} \log\left(\frac{1+\tilde{\rho}^*}{1+(1-\tau)\tilde{\rho}^*} \right) \tag{41}$$

denote the provider's equilibrium profit under retail sale, and let

$$\Pi_F^* \equiv \eta \mathcal{V} + \frac{\alpha^2 \beta_1^2 \sigma_w^2}{8 c_b (1+\mu)^2} \quad \text{where} \quad \mathcal{V} \equiv \mathbb{E}[\pi_F] - \mathbb{E}[\pi_F^U] \quad \text{is the fund's added value} \tag{42}$$

denote its equilibrium expected profit under fund-paid sale. All objects in (42) are evaluated at the fund-paid equilibrium.

Proposition 6 (Information-market organization) *The rating agency chooses the fund-paid regime if and only if*

$$\eta \mathcal{V} + \frac{\alpha^2 \beta_1^2 \sigma_w^2}{8 c_b (1+\mu)^2} \geq \frac{n^*}{2\gamma} \log\left(\frac{1+\tilde{\rho}^*}{1+(1-\tau)\tilde{\rho}^*} \right). \tag{43}$$

If $\alpha\sigma_w > 0$, the expected distortion rent converges to

$$\underline{\Pi}_D \equiv \frac{\alpha^2\sigma_w^2}{8c_b}\bar{\tau}_0^2 > 0, \tag{44}$$

whereas equilibrium profit under individual-investor-paid sale converges to zero. Consequently, there exists $\bar{\sigma}_z > 0$ such that the fund-paid relationship generates strictly more surplus than retail sale whenever $\sigma_z < \bar{\sigma}_z$.

Intuitively, the individual-investor-paid result reflects the standard Grossman-Stiglitz logic that information is more valuable in more liquid markets. In this case, the rater monetizes the precision τ and tries to minimize the information leakage through limiting access to the signal: $n^* \to 0$. Under the fund-paid model, the rater monetizes its ability to inflate prices on behalf of conflicted intermediaries: profits scale in α^2 (the fund's stake in price), σ_w^2 (fund exposure opacity), and $1/c_b$ (the inverse of bias cost).

Based on this Proposition, we can characterize when the fund-paid sale is more lucrative for the information provider. In particular, higher α (fund's price-level stake) and σ_w^2 (cross-sectional dispersion of fund positions), i.e., higher agency frictions, favor fund-paid sale. Second, lower c_b, i.e., weaker reputational or regulatory penalties on distortion, favors fund-paid sale. [16] Third, it can explain why illiquid markets with little noise trading, such as corporate bond markets, tilt toward a concentrated access to the signal through intermediaries. Indeed, roughly three-quarters of U.S. corporate bonds are held by institutional investors (Koijen and Yogo (2023)).

5.2 Price Informativeness

This section compares the equilibrium informativeness of asset prices under the individual-investor-paid and fund-paid models. Price informativeness is a natural welfare statistic for any agent whose decisions depend on prices: regulators monitoring market quality, non-participating investors learning from prices, and firms making real investment decisions based on observed valuations. The standard measure is $\mathcal{PI} \equiv -\mathbb{V}(v \mid P)$, the negative of the conditional variance of the asset's terminal payoff given the equilibrium price. Higher $\mathcal{PI}$ corresponds to more informative prices.

[16] The well-documented shift of NRSROs toward issuer-paid models in the early 2000s, followed by post-Dodd-Frank tightening of disclosure rules, can be read as movements along this dimension: deregulation reduced c_b; subsequent re-regulation raised c_b. Moreover, recent surge in the private credit area and widespread concerns about the reliability of private ratings could be viewed through the lens of insufficient regulation, reputational concerns as well as agency frictions. See The new crop of rating agencies behind the private credit boom

Both regimes deliver price informativeness through the common form, $\bar{\tau}_P = \sigma_v^2/(\sigma_s^2 + \bar{\sigma}_b^2 + \bar{\rho}^2\sigma_z^2)$. The comparison is cleanest for the reciprocal $1/\tau_P$, the price noise, which decomposes into two additive pieces: a term in the effective signal precision and a term in the noise that leaks through the price. Using $\rho = -\gamma\sigma_v^2(1-\bar{\tau})/(m\bar{\tau})$ under the fund-paid model (Lemma 1) and $\rho = -\gamma\sigma_v^2(1-\tau)/(n^*M\tau)$ under the retail model (Lemma **??**, with $b^* = 0$ so $\hat{\sigma}_b^2 = 0$), the two price-noise expressions are

$$\frac{1}{\tau_P^F} = \frac{1}{\bar{\tau}} + \gamma^2\sigma_v^2\sigma_z^2 \left(\frac{1-\bar{\tau}}{\bar{\tau}}\right)^2 \cdot \frac{1}{m^2}, \tag{45}$$

$$\frac{1}{\tau_P^R} = \frac{1}{\tau} + \gamma^2\sigma_v^2\sigma_z^2 \left(\frac{1-\tau}{\tau}\right)^2 \frac{1}{(n^*M)^2}. \tag{46}$$

In each expression the first term is the noise that would remain if the price impounded all informed trading; bias enters here raising it from $1/\tau$ to $1/\bar{\tau} \geq 1/\tau$. The second term is the residual noise-trading dispersion that survives because the informed sector–mass m (fund) or q^*M (retail)–cannot fully reveal its information.

A useful simplification on the retail side follows from Proposition 1: at the rater's optimum the retail price noise is a function of τ alone, $\frac{1}{\tau_P^R} = \frac{1+\tilde{\rho}^*(\tau)\,(1-\tau)}{\tau}$, independent of noise σ_z^2, risk-aversion γ, and all other parameters, except for the signal precision τ. Intuitively, the rater scales q^* so that any liquidity change that would result in higher information leakage is exactly offset. In Figure 4, the price informativeness τ_P^R is a horizontal line.

The price informativeness decomposition for the fund-paid case is also useful for emphasizing the trade-off between signal distortion and access to informed trading. Under the conditions of Proposition 4, the equilibrium mass of fund investors m^* is positively correlated with the signal distortion distortion $(\sigma_b^2)^*$. Hence, when the equilibrium bias increases due to higher importance of legacy position α, its opacity σ_w^2 or lower cost of distortion c_b, this is also followed by a decrease in management fees f resulting in a higher fund base m^*. Specifically, the fund-paid expression separates the effects of access and distortion through its total differential:

$$d\tau_P^F = (\tau_P^F)^2 \Bigg[\underbrace{\frac{2\gamma^2\sigma_v^2\sigma_z^2}{m^3}\left(\frac{1-\bar{\tau}}{\bar{\tau}}\right)^2 dm}_{\text{access effect}} + \underbrace{\left(\frac{1}{\bar{\tau}^2} + \frac{2\gamma^2\sigma_v^2\sigma_z^2(1-\bar{\tau})}{m^2\bar{\tau}^3}\right) d\bar{\tau}}_{\text{distortion effect}} \Bigg]. \tag{47}$$

Greater access raises price informativeness through $dm > 0$. Greater distortion reduces perceived signal precision, so $d\bar{\tau} < 0$, and lowers price informativeness both directly and by weakening the fund's informed trading. Thus, when access and distortion increase together, prices become more informative if and only if the access effect in (47) dominates the distortion

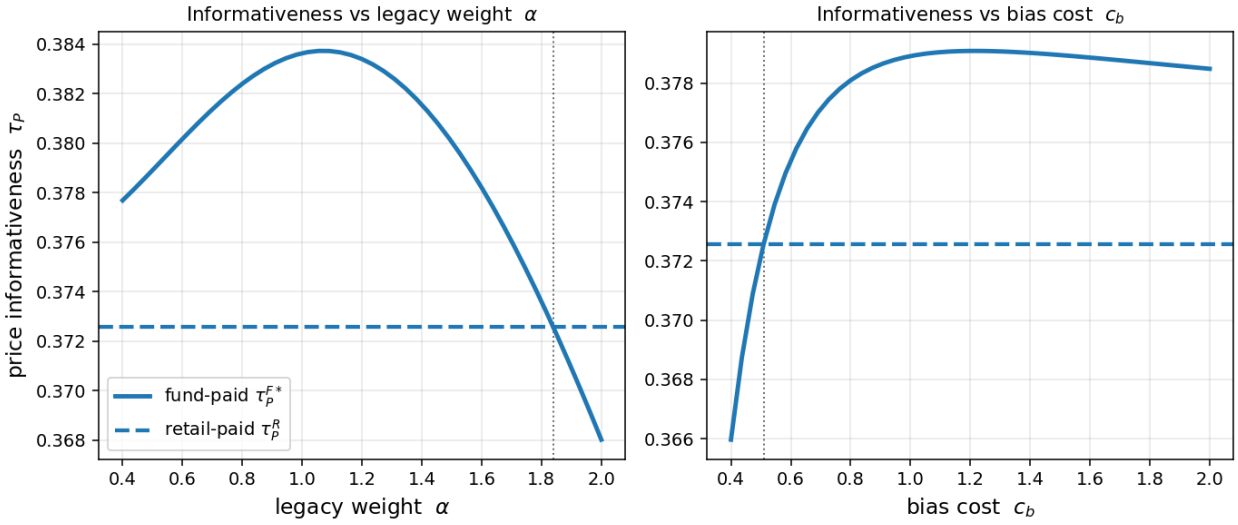


Figure 4: Price informativeness

effect.

$$\frac{dm^*}{-d\bar{\tau}} > \frac{m^*}{\bar{\tau}(1-\bar{\tau})} + \frac{(m^*)^3}{2\gamma^2\sigma_v^2\sigma_z^2(1-\bar{\tau})^2}.$$

This condition explains the hump shape in Figure 4. The marginal contribution of access declines as the fund expands, whereas the marginal loss from a further reduction in signal precision becomes larger. Thus, an increase in α initially raises informativeness because the expansion in access dominates, but eventually lowers it as access saturates and distortion accumulates. The c_b comparative static reflects the same forces in reverse: increasing c_b initially produces a large precision gain by reducing bias, but once bias is small, further increases mainly contract delegated access.

The following Proposition characterizes when information distribution through the delegated asset manager leads to more informative prices:

Proposition 7 (Fund vs. retail informativeness) *The information sale through the intermediary leads to more informative prices than sale to individual investors, $\tau_P^F > \tau_P^R$, iff*

$$\gamma^2\sigma_v^2\sigma_z^2\left[\frac{(1-\tau)^2}{(n^*M)^2\tau^2} - \frac{(1-\bar{\tau})^2}{m^2\bar{\tau}^2}\right] > \frac{1}{\bar{\tau}} - \frac{1}{\tau} \geq 0. \tag{48}$$

Holding the fund equilibrium $(\bar{\tau}, m)$ fixed, fund-paid prices are more informative precisely in illiquid markets, $\sigma_z^2 < \sigma_z^{2*}$, where σ_z^{2*} solves (48)

Inequality (48) states the trade-off in primitives. The right-hand side is the *precision the*

fund loses when manipulating, $1/\bar{\tau} - 1/\tau$. The left-hand side is the *leakage it saves* by trading through a sector of mass m rather than the investors subscriber base q^*M. Fund-paid prices are more informative exactly when the second effect dominates. Three comparative statics follow.

The contrast with the canonical information-sales literature is instructive. To defend its rents against the leakage of information into prices, the monopolist of Admati and Pfleiderer (1986) and Admati and Pfleiderer (1990) must *ration* quality—selling noisier signals as markets thin—so that in the Grossman and Stiglitz (1980) limit the information market unravels. Here a second, leakage-proof rent operates on the interim price *level* demanded by a conflicted intermediary, and survives exactly where the precision rent vanishes.

6 Concluding Remarks

6.1 Empirical Predictions and Policy Implications.

The model rationalizes the central finding of Bonsall et al. (2024), that a subscriber-paid agency issues more favorable ratings for firms held by its institutional clients, and it explains why the documented magnitudes are economically meaningful but incremental: the small-bias equilibrium of Lemma 3 obtains precisely when the explicit cost of distortion c_b is large, as it is under post-Dodd–Frank supervision. The model also delivers predictions on when this catering is likely to persist in equilibrium.

(i) Portfolio constraints can generate bias. Section 3.2 shows that a large, opaque, conflicted fund with trading discretion purchases an *unbiased* rating. Conditional on client size and holdings, therefore, inflation should concentrate among clients whose portfolios are governed by rating-contingent rules—investment-grade-only bond mandates, rating-based capital requirements, and ESG funds with screen-based strategies—and should be absent for unconstrained clients of the same provider. This is a within-size test: it does not predict that bigger clients get better ratings, but that *constrained* clients do.

Additionally, this implies that studies examining the ESG funds on firms' ESG performance and use ESG scores as outcome variables (e.g., (Chen et al., 2020)) may conflate real improvements in firm behavior with endogenous changes in measurement.

(ii) The two distortions are substitutes. The conflict between the fund and its clients is the same in both regimes; only its instrument differs. An unconstrained manager expresses its legacy motive through the portfolio, pumping the price of names it already holds (Ben-David et al., 2013; Wang, 2024); a constrained manager cannot, and expresses it through the rating instead. Across clients, quarter-end pumping and rating tilt in the same assets should

therefore be negatively related. The prediction also has a time-series form: an increase in c_b, i.e., higher enforcement penalties, lowers the equilibrium bias and can push managers toward the trading distortion. Regulating the rater displaces the conflict rather than removing it.

(iii) Tougher regulation of raters need not improve price informativeness. Proposition 4 and equation (47) imply that raising c_b improves signal precision while contracting delegated access. Because informativeness is hump-shaped in the equilibrium bias (Figure 4), the two effects trade off differently depending on where a market starts. In asset classes with substantial pre-existing distortion, a tightening should *raise* price informativeness, since the precision gain dominates; in asset classes where bias is already small, the same tightening should *lower* it, since it mainly withdraws informed capital.

(v) Bias requires uncertainty about the client's exposure. Proposition 5 shows that the distortion rent vanishes as $\sigma_w \to 0$: with transparent legacy positions, investors unwind the bias. Inflation should therefore be concentrated in ratings released when client holdings are stale, i.e., late in the disclosure cycle. Similarly, the rent requires bilateral contracting: distortion should be more pronounced where provider pricing is non-transparent and the provider–client relationship is long-lived.

(vi) Market structure, not only bias, varies with liquidity. Proposition 6 predicts how the information market is *organized*, and this can be tested without measuring bias at all. Providers serving illiquid asset classes should derive revenue from a concentrated set of institutional subscribers on bilateral, non-public terms; providers serving liquid asset classes should sell broadly on transparent and unified terms.

(vii) Disagreement across providers is predictable from client relationships. The oligopoly extension in Appendix **??** implies that competition among raters expands access but does not discipline catering, since the distortion rent is bilateral and not contestable. Cross-provider dispersion in ESG scores, widely documented but largely treated as noise (Berg et al., 2021), should therefore carry a directional component: for a given name, the optimistic provider should be the one whose institutional client holds it, and disagreement should widen with the concentration of institutional ownership.

The model also implies that managers with a stronger legacy motive charge lower fees and operate larger funds, monetizing through the distortion rent rather than through the fee. This prediction depends on the manager retaining pricing power over f, and is therefore less robust than the preceding ones.

Together these results shift the regulatory question from *who pays* to *who buys, under what mandate, and through what contract.* Three implications follow. First, supervision

should extend beyond NRSROs to the funds that write third-party assessments into their portfolio rules: the demand for distortion originates with the mandate. Second, aligning the timing of holding disclosure with rating releases attacks the rent at its source, since the contract is worthless when σ_w is small; this is a less blunt instrument than raising c_b, which contracts informed access as a side effect. Third, transparency of provider–client payment terms is valuable precisely because the rent is relational: it survives competition among providers, so entry is not a substitute for contract transparency.

6.2 Alternative Conflicts of Interest

The paper focuses on circumventing portfolio constraints as a source of bias, but investor-paid ratings can become distorted through other channels. More generally, any large buyer with a *price-level exposure* may pay for a favorable *public* signal. An institutional investor with a large legacy position benefits when a positive rating induces others to buy the asset. Because the investor's own message may lack credibility, certification by an ostensibly independent provider can make this strategy effective (Benabou and Laroque, 1992). Similarly, a benchmark-tracking fund benefits when an index provider's signal raises the value of index constituents.[17] These channels differ from the baseline constraint mechanism, but share its central feature: the buyer values the signal's price effect as well as its accuracy.

The underlying friction is managerial myopia induced by the standard assets-under-management fee structure: investors care about returns over the evaluation period, whereas managers' fee revenue depends on interim portfolio values.[18] Figure 5 illustrates this wedge. The two portfolio paths have identical initial and terminal values, and hence identical investor returns, but generate different fee revenue. The path with higher interim valuations supports a larger average fee base. Managers therefore benefit from signals that raise the interim value of existing positions, even without improving investors' terminal wealth.

A conceptually distinct channel is risk shifting, or "reaching for yield". This channel requires neither legacy holdings nor AUM-based managerial myopia. A manager with limited downside may prefer more risk than capital providers do. This concern is salient among insurers, which reach for yield within regulatory rating categories (Becker and Ivashina, 2015), corporate-bond mutual funds, which tilt toward higher-yielding bonds to improve performance and attract flows (Choi and Kronlund, 2018), and private-credit funds in which investors absorb portfolio losses. [19] When investment eligibility or regulatory capital treat-

[17] Agrawal et al. (2025) provide evidence of such index-alignment distortion in MSCI ESG ratings.

[18] Section 205(a)(1) of the Investment Advisers Act of 1940 generally prohibits registered investment advisers from receiving a share of capital gains or appreciation. Mutual fund compensation is therefore typically based on assets under management rather than a one-sided performance fee.

[19] See, e.g., Financial Times, SEC raises questions over rating agency behind private credit boom"

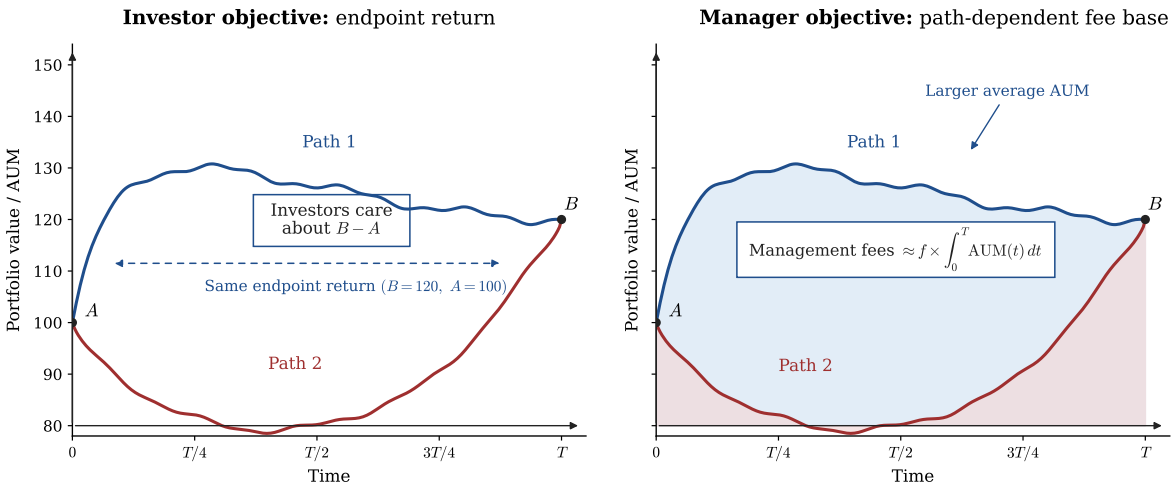


Figure 5: Investor and manager objectives.

ment depends on a third-party rating, asset managers may pay to have credit quality overstated, making a high-yield asset appear compliant or less capital intensive. The distortion creates value even if it has no effect on the asset's price: it relaxes the constraint on risk taking and shifts downside risk to investors.

6.3 Conclusion

This paper shows that investor payment does not by itself eliminate conflicts of interest in information provision. When a rating agency sells information to dispersed, price-taking investors, it supplies an unbiased signal. Market power alone is not sufficient to overturn this result: an unconstrained fund also purchases an unbiased signal and pursues its legacy-position motive directly through portfolio pumping. Bias emerges when a large delegated manager is subject to a signal-contingent portfolio requirement. Because the fund cannot deviate observably from the prescribed trading rule, it pays the information provider to distort the input on which that rule is based. The mandate therefore relocates the conflict from portfolio choice to information production, even though it can also serve as a commitment device that limits direct pumping and supports a larger investor base.

The model also endogenizes how the information market is organized. Under individual-investor-paid sale, the provider earns a precision rent that is eroded as informed trading reveals the signal through prices. Under fund-paid sale, it additionally earns a distortion rent by tailoring the report to the fund's hidden legacy exposure. This rent is larger when the manager places more weight on interim portfolio values, its exposures are more opaque, and distortion is less costly. It also survives in illiquid markets in which information leakage causes broad retail sales to unravel. Consequently, sufficiently illiquid markets are organized

around a bilateral relationship with a strategic institutional client rather than broad access to an unbiased signal.

Finally, biased information need not produce less informative prices. Catering reduces signal precision, but selling through a fund can expand the mass of investors whose trades incorporate the signal. When this access effect dominates the loss of precision, fund-paid prices are more informative than individual-paid prices, particularly in illiquid markets. Regulation that raises the cost of distortion can therefore improve signal quality while contracting informed access and, in some cases, reducing price informativeness.

These results shift the regulatory question from simply *who pays* for information to *who buys it, under what mandate, and through what contract.* Relevant safeguards include greater transparency of provider–client payments, timely disclosure of institutional exposures, and scrutiny of portfolio rules that rely on information controlled by a fund's own vendor. Such rules may discipline direct trading distortions while simultaneously creating demand for distorted ratings. The central policy trade-off is therefore not merely bias versus accuracy, but signal precision versus access to informed trading.

References

Anat R Admati and Paul Pfleiderer. A monopolistic market for information. *Journal of Economic Theory*, 39(2):400–438, 1986.

Anat R Admati and Paul Pfleiderer. Direct and indirect sale of information. *Econometrica: Journal of the Econometric Society*, pages 901–928, 1990.

Sonakshi Agrawal, Lisa Yao Liu, Shivaram Rajgopal, Suhas A Sridharan, Yifan Yan, and Teri Lombardi Yohn. Esg ratings of esg index providers. Technical report, New Working Paper Series, 2025.

Dominique C Badoer and Cem Demiroglu. The relevance of credit ratings in transparent bond markets. *The Review of Financial Studies*, 32(1):42–74, 2019.

Jennie Bai, Thomas Philippon, and Alexi Savov. Have financial markets become more informative? *Journal of Financial Economics*, 122(3):625 – 654, 2016. ISSN 0304-405X.

Snehal Banerjee. Learning from prices and the dispersion in beliefs. *The Review of Financial Studies*, 24(9):3025–3068, 2011.

Lucian A Bebchuk and Scott Hirst. The specter of the giant three. Technical report, National Bureau of Economic Research, 2019.

Bo Becker and Victoria Ivashina. Reaching for yield in the bond market. *The Journal of Finance*, 70(5):1863–1902, 2015.

Bo Becker and Todd Milbourn. How did increased competition affect credit ratings? *Journal of financial economics*, 101(3):493–514, 2011.

ITZHAK Ben-David, Francesco Franzoni, Augustin Landier, and Rabih Moussawi. Do hedge funds manipulate stock prices? *The Journal of Finance*, 68(6):2383–2434, 2013.

Roland Benabou and Guy Laroque. Using privileged information to manipulate markets: Insiders, gurus, and credibility. *The Quarterly Journal of Economics*, 107(3):921–958, 1992.

Florian Berg, Kornelia Fabisik, and Zacharias Sautner. Is history repeating itself? the (un) predictable past of esg ratings. *SSRN Electronic Journal*, 3722087, 2021.

Florian Berg, Julian F Kölbel, and Roberto Rigobon. Aggregate confusion: The divergence of esg ratings. *Review of Finance*, 26(6):1315–1344, 2022.

Sugato Bhattacharyya and Vikram Nanda. Portfolio pumping, trading activity and fund performance. *Review of Finance*, 17(3):885–919, 2013.

Bruno Biais and Laurent Germain. Incentive-compatible contracts for the sale of information. *The Review of Financial Studies*, 15(4):987–1003, 2002.

Patrick Bolton, Xavier Freixas, and Joel Shapiro. The credit ratings game. *The Journal of Finance*, 67(1):85–111, 2012.

Philip Bond and Diego Garcia. The equilibrium consequences of indexing. *The Review of Financial Studies*, 35(7):3175–3230, 2022.

Dion Bongaerts. *Alternatives for issuer-paid credit rating agencies*. Number 1703. ECB Working Paper, 2014.

Samuel B Bonsall, Jacquelyn R Gillette, Gabriel Pundrich, and Eric So. Conflicts of interest in subscriber-paid credit ratings. *Journal of Accounting and Economics*, 77(1):101614, 2024.

Adrian Buss and Savitar Sundaresan. More risk, more information: How passive ownership can improve informational efficiency. *The Review of Financial Studies*, 36(12):4713–4758, 2023.

Mark M Carhart, Ron Kaniel, David K Musto, and Adam V Reed. Leaning for the tape: Evidence of gaming behavior in equity mutual funds. *The Journal of Finance*, 57(2): 661–693, 2002.

Giovanni Cespa. Information sales and insider trading with long-lived information. *The Journal of Finance*, 63(2):639–672, 2008.

Yeon-Koo Che and Navin Kartik. Opinions as incentives. *Journal of Political Economy*, 117 (5):815–860, 2009.

Hui Chen and Jian Sun. Market for manipulable information. 2024.

Tao Chen, Hui Dong, and Chen Lin. Institutional shareholders and corporate social responsibility. *Journal of Financial Economics*, 135(2):483–504, 2020.

Jaewon Choi and Mathias Kronlund. Reaching for yield in corporate bond mutual funds. *The Review of Financial Studies*, 31(5):1930–1965, 2018.

Jess Cornaggia and Kimberly J Cornaggia. Estimating the costs of issuer-paid credit ratings. *The Review of Financial Studies*, 26(9):2229–2269, 2013.

Mathias Dewatripont and Jean Tirole. Advocates. *Journal of Political Economy*, 107(1): 1–39, 1999.

Jason Roderick Donaldson and Giorgia Piacentino. Contracting to compete for flows. *Journal of Economic Theory*, 173:289–319, 2018.

Philip H Dybvig, Heber K Farnsworth, and Jennifer N Carpenter. Portfolio performance and agency. *The Review of Financial Studies*, 23(1):1–23, 2010.

Diego Garcia and Francesco Sangiorgi. Information sales and strategic trading. *The Review of Financial Studies*, 24(9):3069–3104, 2011.

Nicolae Gârleanu and Lasse Heje Pedersen. Efficiently inefficient markets for assets and asset management. *The Journal of Finance*, 73(4):1663–1712, 2018.

Paul A Gompers and Andrew Metrick. Institutional investors and equity prices. *The quarterly journal of Economics*, 116(1):229–259, 2001.

Sanford J Grossman and Joseph E Stiglitz. On the impossibility of informationally efficient markets. *The American economic review*, 70(3):393–408, 1980.

Zhiguo He and Wei Xiong. Delegated asset management, investment mandates, and capital immobility. *Journal of Financial Economics*, 107(2):239–258, 2013.

Doh-Shin Jeon and Stefano Lovo. Credit rating industry: A helicopter tour of stylized facts and recent theories. *International Journal of Industrial Organization*, 31(5):643–651, 2013.

Philippe Jorion, Zhu Liu, and Charles Shi. Informational effects of regulation fd: evidence from rating agencies. *Journal of financial economics*, 76(2):309–330, 2005.

Anil K Kashyap and Natalia Kovrijnykh. Who should pay for credit ratings and how? *The Review of Financial Studies*, 29(2):420–456, 2016.

Ralph SJ Koijen and Motohiro Yogo. A demand system approach to asset pricing. *Journal of Political Economy*, 127(4):1475–1515, 2019.

Ralph SJ Koijen and Motohiro Yogo. Understanding the ownership structure of corporate bonds. *American Economic Review: Insights*, 5(1):73–91, 2023.

Albert S Kyle, Hui Ou-Yang, and Bin Wei. A model of portfolio delegation and strategic trading. *The Review of Financial Studies*, 24(11):3778–3812, 2011.

Jeffrey M Lacker and John A Weinberg. Optimal contracts under costly state falsification. *Journal of Political Economy*, 97(6):1345–1363, 1989.

Stefano Lovo and Jacques Olivier. Who should pay for esg ratings? *Review of Finance*, 30 (2):597–637, 2026.

Linlin Ma, Yuehua Tang, and Juan-Pedro Gomez. Portfolio manager compensation in the us mutual fund industry. *The Journal of Finance*, 74(2):587–638, 2019.

Christian C Opp, Marcus M Opp, and Milton Harris. Rating agencies in the face of regulation. *Journal of financial Economics*, 108(1):46–61, 2013.

Gianpaolo Parise and Mirco Rubin. Green window dressing. *The Journal of Finance*, 80(6): 3555–3588, 2025.

Christine A Parlour and Uday Rajan. Contracting on credit ratings: Adding value to public information. *The Review of Financial Studies*, 33(4):1412–1444, 2020.

Francesco Sangiorgi and Chester Spatt. The economics of credit rating agencies. *Foundations and Trends® in Finance*, 12(1):1–116, 2017a.

Francesco Sangiorgi and Chester Spatt. Opacity, credit rating shopping, and bias. *Management Science*, 63(12):4016–4036, 2017b.

Claude E. Shannon. A mathematical theory of communication. *The Bell System Technical Journal*, 27:379–423, 1948.

Pingle Wang. Portfolio pumping in mutual fund families. *Journal of Financial Economics*, 156:103839, 2024.

Han Xia. Can investor-paid credit rating agencies improve the information quality of issuer-paid rating agencies? *Journal of Financial Economics*, 111(2):450–468, 2014.

A Proofs

A.1 Proofs for Section 3.1

Trading equilibrium. Conjecture a linear price of the form $P = \beta_0 + \beta_1 \hat{s} + \beta_2 z$. and denote $\bar{P} \equiv \mathbb{E}[P] = \beta_0 + \beta_1 \bar{b}$.

Subscribers observe $\hat{s}$ and update their beliefs: $\mathbb{E}[v \mid \hat{s}] = \bar{\tau}(\hat{s} - \bar{b})$, $\mathbb{V}(v \mid \hat{s}) = \sigma_v^2(1 - \bar{\tau})$, where $\bar{\tau} = \frac{\sigma_v^2}{\sigma_s^2 + \bar{\sigma}_b^2}$. Non-subscribers observe P, or equivalently $s_P \equiv \frac{P - \bar{P}}{\beta_1} = \hat{s} - \bar{b} + \frac{\beta_2}{\beta_1} z$, and update: $\mathbb{E}[v \mid P] = \bar{\tau}_P s_P$, $\mathbb{V}(v \mid P) = \sigma_v^2(1 - \bar{\tau}_P)$, where the price informativeness $\bar{\tau}_P = \frac{\sigma_v^2}{\sigma_s^2 + \bar{\sigma}_b^2 + \rho^2 \sigma_z^2}$, and the noise-to-signal ratio is $\rho \equiv \frac{\beta_2}{\beta_1}$. Define the *delegation-scaling factor*

$$M \;\equiv\; (1 - m) + \frac{m}{1 - f} \;=\; \frac{1 - f(1 - m)}{1 - f} \;\geq\; 1.$$

M measures the aggregate trading intensity relative to a market with no delegation; $M = 1$ when $m = 0$ or $f = 0$.

Markets clear at prices that equilibrate aggregate supply and demand: $\int x_j \, dj = z$:

$$\frac{M}{\gamma \, \sigma_v^2} \Big[\frac{n}{1 - \bar{\tau}} \big(\bar{\tau}(\hat{s} - \bar{b}) - P \big) + \frac{1 - n}{1 - \bar{\tau}_P} \big(\bar{\tau}_P s_P - P \big) \Big] = z$$

Now match the coefficients. The constant term gives $\bar{P} = 0$, so $\beta_0 = -\beta_1 \bar{b}$

Matching the coefficient on $\hat{s}$ gives

$$\beta_1 = \frac{n \, \bar{\tau}/(1 - \bar{\tau}) + (1 - n) \, \bar{\tau}_P/(1 - \bar{\tau}_P)}{n/(1 - \bar{\tau}) + (1 - n)/(1 - \bar{\tau}_P)}, \tag{49}$$

Finally, matching the coefficient on z gives:

$$\rho A_S \bar{\tau} = -\frac{\gamma}{M} \quad \Rightarrow \quad \rho = \frac{\beta_2}{\beta_1} = -\frac{\gamma \sigma_v^2 (1 - \bar{\tau})}{n M \bar{\tau}}.$$

Proof of Proposition 1. *(i)* $b^* = 0$. Under the contract (4), investor i's expected payoff under report $\tilde{e}_i$ is reduced by $(e_i - \tilde{e}_i)\mathbb{E}[P(\hat{s})] - F$. It holds that $\mathbb{E}[P(\hat{s})] = 0$ on the equilibrium path, so investor i is indifferent between $\tilde{e}_i$. Robustness to off-equilibrium path deviations of the rater toward $b > \bar{b}$, which would yield $\mathbb{E}[P \mid b] = \beta_1(b - \bar{b}) > 0$ pins down $\tilde{e}_i^* = 0$. Aggregating, $C \equiv \int \tilde{e}_j \, dj = 0$ a.s. The rater's bias FOC is $\beta_1 C = c_b b$, giving $b^* = 0$. Beliefs $(\bar{b}, \bar{\sigma}_b^2) = (0, 0)$ are consistent ex post.

(ii) WTP under CARA. The certainty equivalent of trading optimally on info set $\mathcal{I}$ at

price P, evaluated ex ante, is the standard result (Admati and Pfleiderer, 1986):

$$\mathrm{CE}(\mathcal{I}) = \frac{1}{2\gamma} \log \frac{\mathbb{V}(v - P)}{\mathbb{V}(v \mid \mathcal{I})},$$

R is identical across active and Delegating investors. Willingness to pay for subscribing:

$$\Delta_S(n) \equiv \mathrm{CE}(\hat{s}, P) - \mathrm{CE}(P) = \frac{1}{2\gamma} \log \frac{\mathbb{V}(v \mid P)}{\mathbb{V}(v \mid \hat{s}, P)} = \frac{1}{2\gamma} \log \frac{1 - \bar{\tau}_P(n)}{1 - \bar{\tau}},$$

the same for both investor.

(iii) Rater's optimum. The rater sets the uniform fee $\phi = \Delta_S(n)$ and chooses n to maximize $\pi_{RA}(n) = n\,\Delta_S(n)$. With $b^* = 0$, $\bar{\sigma}_b^2 = 0$, and $\sigma_v^2(1 - \bar{\tau})/\bar{\tau} = \sigma_\varepsilon^2$ we simplify:

$$\rho^2 \sigma_z^2 = \sigma_v^2 \cdot \tilde{\rho} \cdot \frac{1 - \bar{\tau}}{\bar{\tau}}, \quad \tilde{\rho} = \frac{\gamma^2 \sigma_\varepsilon^2 \sigma_z^2}{(nM)^2}.$$

Hence, $\bar{\tau}_P = \frac{\bar{\tau}}{1+\tilde{\rho}(1-\bar{\tau})}$, $\frac{1-\bar{\tau}_P}{1-\bar{\tau}} = \frac{1+\tilde{\rho}}{1+(1-\bar{\tau})\tilde{\rho}}$, so $\Delta_S(n) = \frac{1}{2\gamma} \log \frac{1+\tilde{\rho}}{1+(1-\bar{\tau})\tilde{\rho}}$ and $\pi_{RA}(n) = n\,\Delta_S(n)$ (8).

Using, $\frac{d}{d\tilde{\rho}} \log \frac{1+\tilde{\rho}}{1+(1-\bar{\tau})\tilde{\rho}} = \frac{1}{1+\tilde{\rho}} - \frac{1-\bar{\tau}}{1+(1-\bar{\tau})\tilde{\rho}} = \frac{\bar{\tau}}{(1+\tilde{\rho})(1+(1-\bar{\tau})\tilde{\rho})}$, the interior FOC:

$$\log \frac{1 + \tilde{\rho}^*}{1 + (1 - \bar{\tau})\tilde{\rho}^*} = \frac{2\bar{\tau}\,\tilde{\rho}^*}{(1 + \tilde{\rho}^*)(1 + (1 - \bar{\tau})\tilde{\rho}^*)},$$

Then, the second derivative:

$$\frac{\partial^2 \pi_{RA}}{\partial n\, \partial \tilde{\rho}} = \frac{\bar{\tau}[3(1 - \bar{\tau})\tilde{\rho}^2 + (2 - \bar{\tau})\tilde{\rho} - 1]}{[(1 + \tilde{\rho})(1 + (1 - \bar{\tau})\tilde{\rho})]^2},$$

the numerator is strictly increasing in $\tilde{\rho}$ for $\tilde{\rho} > 0$ and $\tau \in [0, 1]$. Hence, $\frac{\partial \pi_{RA}}{\partial n}$ is decreasing in $\tilde{\rho}$ at first and then increasing. Since $\frac{\partial \pi_{RA}}{\partial n}(0) = 0$, there is a unique crossing at some $\tilde{\rho}^* > 0$.

Inverting $\tilde{\rho}^* = \frac{\gamma^2 \sigma_\varepsilon^2 \sigma_z^2}{(n^* M)^2}$ gives $n^* = \frac{\gamma \sigma_\varepsilon\, \sigma_z}{M\sqrt{\tilde{\rho}^*}}$ or $n^* = 1$: whatever is smaller.

A.1.1 Discretionary equilibrium and the delegation surplus

Fix $m = m_N^*$ throughout. Let τ_P^N denote the price informativeness under discretion, and τ_P^P – under commitment

Market Clearing. Conjecture $P = \beta_s^N s + \beta_w^N w + \beta_z^N z$ and set $s_P = P/\beta_s^N$. Gaussian projection gives $\mathbb{E}[v \mid P] = \tau_P^N s_P$ with $\tau_P^N = \sigma_v^2/[\sigma_s^2 + (\beta_w^N/\beta_s^N)^2\sigma_w^2 + (\beta_z^N/\beta_s^N)^2\sigma_z^2]$, so an uninformed active investor demands $x_A = (\tau_P^N/\beta_s^N - 1)P/[\gamma\sigma_v^2(1 - \tau_P^N)]$. Market clearing $mx_F + (1 - m)x_A = z$ gives the residual supply faced by the fund, $P = \lambda x_F - \frac{\lambda}{m} z$, with

per-investor price impact

$$\lambda \equiv m\Lambda = \frac{\partial P}{\partial x_F} = \frac{m\gamma\sigma_v^2(1-\tau_P^N)}{(1-m)(1-\tau_P^N/\beta_s^N)}. \tag{50}$$

The fund submits a demand schedule and therefore optimizes against realized residual supply: differentiating $fm(\tau s - P)x_F + \alpha w P$ in x_F gives $fm[(\tau s - P) - \lambda x_F] + \alpha w \lambda = 0$. Combining with the uninformed schedule, which by (50) is $x_A^N = -\frac{1}{\lambda}\frac{m}{1-m}P$,

$$x_F^N = \frac{\tau s - P}{\lambda} + \frac{\alpha}{fm} w \implies P = \frac{\tau}{2}s + \frac{\alpha\lambda}{2fm}w - \frac{\lambda}{2m}z, \quad x_F^N = \frac{\tau s}{2\lambda} + \frac{z}{2m} + \frac{\alpha w}{2fm}. \tag{51}$$

To argue *uniqueness*, note that the price informativeness:

$$\tau_P^N(\lambda) = \frac{\sigma_v^2}{\sigma_s^2 + \frac{\alpha^2\sigma_w^2/f^2 + \sigma_z^2}{m^2\tau^2}\lambda^2},$$

falls strictly from τ to 0. Substituting $\beta_s^N = \tau/2$ into (50), $\lambda\frac{1-2\tau_P^N(\lambda)/\tau}{1-\tau_P^N(\lambda)} = \frac{m\gamma\sigma_v^2}{1-m}$; the left-hand side is strictly increasing in λ, so the root is unique, and $\lambda > 0$ implies $\tau_P^N < \tau/2$. Substituting (51) into the objective and using $s \perp w \perp z$,

$$\Pi_N = \mathbb{E}[\pi_F] = fmW_0 + \underbrace{\frac{fm\,\tau\sigma_v^2}{4\lambda}}_{\text{informed trading}} + \underbrace{\frac{f\lambda\sigma_z^2}{4m}}_{\text{liquidity provision}} + \underbrace{\frac{\alpha^2\lambda\sigma_w^2}{4fm}}_{\text{pumping}}. \tag{52}$$

The delegated ex-ante utility. Fix an information set $\mathcal{I}$ and write the returns:

$$r \equiv v - P, \qquad \hat{r} \equiv \mathbb{E}[v \mid \mathcal{I}] - P, \tag{53}$$

Terminal wealth of *delegating investors* is $W_1 = (1-f)(W_0 + rx_F)$, so $\mathrm{CE} = (1-f)W_0 - \gamma^{-1}\log\mathbb{E}[e^{-\gamma(1-f)\,rx_F}]$. Writing $rx_F = \frac{1}{2}\zeta' A\zeta$ with $\zeta = (r, x_F)'$, $A = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$ and covariance Σ, the Gaussian quadratic-form formula $\mathbb{E}[e^{\frac{t}{2}\zeta' A\zeta}] = \det(I - t\Sigma A)^{-1/2}$ at $t = -\gamma(1-f)$ gives

$$\mathbb{E}\big[e^{-\gamma(1-f)\,rx_F}\big] = \Big[\big(1 + \gamma(1-f)\,\mathbb{C}(r, x_F)\big)^2 - (\gamma(1-f))^2\mathbb{V}(r)\mathbb{V}(x_F)\Big]^{-1/2}, \tag{54}$$

Hence, $\mathrm{CE} = (1-f)W_0 + \frac{1}{2\gamma}\log\mathcal{K}$ with

$$\mathcal{K} \;=\; 1 + 2\gamma(1-f)\,\mathbb{C}(r, x_F) - (\gamma(1-f))^2\big[\mathbb{V}(r)\mathbb{V}(x_F) - \mathbb{C}(r, x_F)^2\big]. \tag{55}$$

Since $v - \mathbb{E}[v \mid \mathcal{I}] \perp \mathcal{I}$ while $\hat{r}$ and x_F are $\mathcal{I}$-measurable,

$$\mathbb{V}(r) = \mathbb{V}(\hat{r}) + \mathbb{V}(v \mid \mathcal{I}), \qquad \mathbb{C}(r, x_F) = \mathbb{C}(\hat{r}, x_F), \tag{56}$$

so with $\mathcal{R}^2 \equiv \mathbb{V}(\hat{r})/\mathbb{V}(r)$ the predictable share of excess-return variance, $\mathbb{V}(\hat{r}) = \mathcal{R}^2\mathbb{V}(r)$ and $\mathbb{V}(v \mid \mathcal{I}) = (1 - \mathcal{R}^2)\mathbb{V}(r)$. Projecting the delegated position on the wedge,

$$x_F = a\,\hat{r} + u, \qquad u \perp \hat{r}, \quad \mathbb{E}[u] = 0, \qquad a = \frac{\mathbb{C}(\hat{r}, x_F)}{\mathbb{V}(\hat{r})}. \tag{57}$$

Then $\mathbb{C}(r, x_F) = a\mathbb{V}(\hat{r})$, $\mathbb{V}(x_F) = a^2\mathbb{V}(\hat{r}) + \mathbb{V}(u)$, and the bracket in (55) reduces, the $a^2\mathbb{V}(\hat{r})^2$ terms cancelling, to

$$\mathbb{V}(r)\mathbb{V}(x_F) - \mathbb{C}(r, x_F)^2 = a^2\,\mathbb{V}(v \mid \mathcal{I})\,\mathbb{V}(\hat{r}) + \mathbb{V}(r)\,\mathbb{V}(u). \tag{58}$$

The resulting expression is concave quadratic in a with vertex $a^* = 1/[\gamma(1-f)\,\mathbb{V}(v \mid \mathcal{I})]$, at which its value is $1 + \mathbb{V}(\hat{r})/\mathbb{V}(v \mid \mathcal{I}) = (1 - \mathcal{R}^2)^{-1}$. Hence

$$\mathcal{K} = \underbrace{\frac{1}{1 - \mathcal{R}^2}}_{\text{price-taking}} - \underbrace{(\gamma(1-f))^2\,\mathbb{V}(v \mid \mathcal{I})\,\mathbb{V}(\hat{r})\,(a - a^*)^2}_{\text{mis-scaling}} - \underbrace{(\gamma(1-f))^2\,\mathbb{V}(r)\,\mathbb{V}(u)}_{\text{contamination}} \tag{59}$$

The ceiling is obtained by an investor choosing her own position: a^* is the fee-adjusted price-taking rule, so delegation can only subtract from it. It does so in two ways: (i) the manager loads the correct signal by the wrong amount, i.e. $a \neq a^*$, (ii) the delegated position contains noise, i.e. $\mathbb{V}(u) > 0$. By (56) and Cauchy–Schwarz, $\mathrm{corr}(r, x_F) \leq \mathcal{R}$ with equality iff $\mathbb{V}(u) = 0$. All objects in (59) are evaluated at the prevailing equilibrium price; the decomposition therefore isolates portfolio quality, and the price channel is treated separately below.

Let $p \equiv \left(\frac{\alpha}{2fm}\right)^2 \sigma_w^2$, and define the pumping-off wedge and position

$$\hat{y} \equiv \tau s - P^0 = \frac{\tau}{2}s + \frac{\lambda}{2m}z, \qquad x_0 \equiv \frac{\hat{y}}{\lambda}, \qquad r_0 \equiv v - P^0, \qquad P^0 \equiv P^N\big|_{p=0},$$

so $x_F^N = x_0 + \frac{\alpha}{2fm}\,w$ and $r_N = r_0 - \frac{\alpha\lambda}{2fm}\,w$ with $w \perp (r_0, x_0)$. The pumping-off moments are:

$$\mathbb{V}(x_0) = \frac{\mathbb{V}(\hat{y})}{\lambda^2}, \qquad \mathbb{C}(r_0, x_0) = \frac{\mathbb{V}(\hat{y})}{\lambda}, \qquad \mathbb{V}(r_0) = \mathbb{V}(\hat{y}) + \mathbb{V}(v \mid s).$$

Because the fund submits a schedule, x_0 is proportional to $\hat{y}$. In the notation of (59) the pumping-off position has $u = 0$ and $a = 1/\lambda$, so the only deviation at $p = 0$ is mis-

scaling. Adding the legacy tilt, $\mathbb{C}(r_N, x_F^N) = \mathbb{V}(\hat{y})/\lambda - \lambda p$, $\mathbb{V}(x_F^N) = \mathbb{V}(\hat{y})/\lambda^2 + p$ and $\mathbb{V}(r_N) = \mathbb{V}(\hat{y}) + \mathbb{V}(v \mid s) + \lambda^2 p$, so (58) equals $\mathbb{V}(v \mid s)\mathbb{V}(\hat{y})/\lambda^2 + p(\mathbb{V}(v \mid s) + 4\mathbb{V}(\hat{y}))$: the p^2 terms cancel and $\mathcal{K}_N$ is affine in p,

$$\mathcal{K}_N(p) = \mathcal{K}_N^0 - \mathcal{A}_N\, p, \qquad \mathcal{K}_N^0 = 1 + \frac{2\gamma(1-f)\mathbb{V}(\hat{y})}{\lambda} - \frac{(\gamma(1-f))^2\,\mathbb{V}(v \mid s)\mathbb{V}(\hat{y})}{\lambda^2}, \tag{60}$$

$$\mathcal{A}_N \equiv 2\gamma(1-f)\lambda + (\gamma(1-f))^2\,\mathbb{V}\Big(v + \tfrac{\lambda}{m} z\Big) > 0, \tag{61}$$

using $\mathbb{V}(v \mid s) + 4\mathbb{V}(\hat{y}) = \mathbb{V}(r_0 + \lambda x_0)$. Pumping moves all three terms of (59): it raises the ceiling, since distortion enlarges the wedge $\tau s - P$; but the legacy tilt is orthogonal to that wedge and loads it with the wrong sign, so the additional opportunity is forfeited. The ceiling gain cancels exactly against the mis-scaling loss, leaving $\mathcal{A}_N > 0$.

Delegation surplus. The active investor trades optimally on the price, and her ex ante trading value is the mutual information between the excess return and the price, $\mathrm{CE}^A = \gamma^{-1}\mathbb{I}(v - P; P)$ with $\mathbb{I}(X;Y) \equiv \frac{1}{2}\log[\mathbb{V}(X)/\mathbb{V}(X \mid Y)]$. Hence the delegation surplus, net of fee and saved transaction cost, is

$$\Delta_S^N(m, f) = \frac{1}{2\gamma}\Big[\log \mathcal{K}_N - 2\,\mathbb{I}\big(v - P^N; P^N\big)\Big]. \tag{62}$$

Since P^N loads $\tau/2$ on the price signal while the posterior loads τ_P^N,

$$\mathbb{I}(v - P^N; P^N) = \log\left(1 + \frac{\big(\tau/2 - \tau_P^N\big)^2}{\tau_P^N\big(1 - \tau_P^N\big)}\right), \tag{63}$$

which is strictly decreasing in τ_P^N on $\tau_P^N < \tau/2$: the residual uncertainty about the return decreases as prices become more informative. Since $\frac{1}{\tau_P^N(p)} = \frac{1}{\tau} + \frac{\lambda^2\left(\sigma_z^2/m^2 + 4p\right)}{\tau^2\sigma_v^2}$ is increasing in p, pumping lowers τ_P^N and therefore raises the outside option. Thus, $\partial\Delta_S^N/\partial p < 0$: *pumping lowers the delegation value.* Combining with (60), pumping lowers the surplus by

$$\Delta_S^N\big|_{p=0} - \Delta_S^N(p) = \frac{1}{2\gamma}\log\frac{\mathcal{K}_N^0}{\mathcal{K}_N^0 - \mathcal{A}_N p} + \frac{1}{\gamma}\Big[\mathbb{I}(p) - \mathbb{I}(0)\Big] \;>\; 0, \tag{64}$$

Signing the commitment surplus. Both regimes are instances of (59): (i) the mandate sets $(a, u) = (a^P, 0)$ with $a^P = 1/(\gamma\mathbb{V}(v \mid s))$ at price P^P, (ii) discretion sets $(a, u) = (1/\lambda, u_p)$ at $P^N(p)$. Two evaluations are immediate. At the investor's own optimum the penalties

vanish and $\mathcal{K}/e^{2\mathbb{I}} = \mathbb{V}(v \mid P)/\mathbb{V}(v \mid s)$, so

$$\Delta_S(P; a^*, 0) = \frac{1}{2\gamma} \log \frac{1-\tau_P}{1-\tau} = \frac{1}{\gamma}\, \mathbb{I}(v; s \mid P), \tag{65}$$

the signal premium at that price. And since $a^P - a^* = -f/[(1-f)\gamma\mathbb{V}(v \mid s)]$, the mandate incurs mis-scaling $f^2\mathcal{R}^2/(1-\mathcal{R}^2)$, giving $\mathcal{K}^P = (1 - f^2\mathcal{R}_P^2)/(1-\mathcal{R}_P^2)$.

Changing one argument at a time along

$$(P_p^N; \tfrac{1}{\lambda}, u_p) \to (P_0^N; \tfrac{1}{\lambda}, 0) \to (P_0^N; a^*, 0) \to (P^P; a^*, 0) \to (P^P; a^P, 0)$$

decomposes the commitment surplus as $G_N = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4$,

$$\Delta_1 = \Delta_S^N\big|_{p=0} - \Delta_S^N(p) \;>\; 0 \qquad \text{(pumping)}, \tag{66}$$

$$\Delta_2 = \frac{1}{2\gamma} \log \frac{\mathcal{K}_N^0 + (\gamma(1-f))^2 \mathbb{V}(v|s)\mathbb{V}(\hat{y}) \left(\frac{1}{\lambda} - a^*\right)^2}{\mathcal{K}_N^0} \;\geq\; 0, \qquad \text{(mis-scaling)}, \tag{67}$$

$$\Delta_3 = \frac{1}{2\gamma} \log \frac{1-\tau_P^P}{1-\tau_P^N(0)} \qquad \text{(price informativeness)}, \tag{68}$$

$$\Delta_4 = \frac{1}{2\gamma} \log\big(1 - f^2\mathcal{R}_P^2\big) \;\leq\; 0 \qquad \text{(fee wedge)}. \tag{69}$$

Δ_2 is nonnegative and reflects a welfare improvement through a better position alignment: $a = \frac{1}{\lambda} \to a^*$.

Δ_3 compares prices at a common position, $\Delta_3 > 0$ iff

$$\tau_P^P < \tau_P^N(0) \qquad \Longleftrightarrow \qquad \lambda < \lambda^\ddagger \equiv \gamma\sigma_v^2(1-\tau). \tag{70}$$

Δ_4 is the only negative contribution and satisfies $|\Delta_4| \leq f^2\mathcal{R}_P^2/[2\gamma(1-f^2\mathcal{R}_P^2)] = O(f^2)$. We obtain $G_N > 0$ throughout the small-fee region of Lemma 5. Commitment is valuable when price impact is low: shading then leaves the price no more informative than the mandate would, so the manager's strategic restraint buys the investor nothing and costs her a mis-scaled position.

Fees and profits. The two participation conditions $f^P W_0 = \bar{c}(1-m) + \Delta_S^P(f^P)$ and $f^N W_0 = \bar{c}(1-m) + \Delta_S^N(f^N)$ share the same right-hand constant. Subtracting, adding and subtracting $\Delta_S^P(f^N)$, and applying the mean value theorem gives, for some $\tilde{f} \in (f^N, f^P)$,

$$(f^P - f^N)W_0 = \rho_f\, G_N, \qquad \rho_f \equiv \frac{W_0}{W_0 - \Delta_S^{P,f}(\tilde{f})} \in (0,1), \tag{71}$$

with $\rho_f < 1$ because $\Delta_S^{P,f} < 0$: a higher fee compresses the information rent. A positive commitment surplus therefore passes through to a strictly higher fee. Combining with (14), in which $\Gamma^N = \mathbb{V}(\hat{y})/\lambda = \frac{\tau\sigma_v^2}{4\lambda} + \frac{\lambda\sigma_z^2}{4m^2}$ is the discretionary trading gain gross of the pumping leg and $\Gamma^P = \mathbb{V}(\hat{r}^P)/(\gamma\mathbb{V}(v \mid s))$ its committed counterpart,

$$\frac{\Pi_P - \Pi_N}{m} = \rho_f G_N + f^N\big(\Gamma^P - \Gamma^N\big) + \underbrace{(f^P - f^N)\,\Gamma^P}_{\geq 0} - f^N\lambda p. \tag{72}$$

With $G_N > 0$ the marked term is nonnegative, so the comparison reduces to the fundamental-rent gap $\Gamma^P - \Gamma^N$ against the forgone pumping rent $f^N\lambda p$. Substituting G_N into (72) yields the sufficient condition

$$\rho_f(\Delta_2 + \Delta_3 + \Delta_4) + f^N\big(\Gamma^P - \Gamma^N\big) + p\left(\frac{\rho_f\,\mathcal{A}_N}{2\gamma\,\mathcal{K}_N^0} - f^N\lambda\right) > 0,$$

whose last term is nonnegative once the fee revenue recovered by removing a unit of pumping exceeds the manager's private return on it.

Both terms in the bracket are linear in p, so the sign does not depend on the intensity of pumping. Using $\mathcal{A}_N > 2\gamma(1-f)\lambda$ and discarding the positive variance component, the price impact cancels and the bracket is nonnegative whenever $\rho_f(1-f^N) > f^N\mathcal{K}_N^0$. Since $\mathcal{K}_N^0 = 1 + S_N(1-\delta_N^2) \leq (1-\mathcal{R}_N^2)^{-1}$, a sufficient condition is

$$\frac{f^N}{1-f^N} \;<\; \rho_f\big(1-\mathcal{R}_N^2\big). \tag{73}$$

Per unit of p the manager nets $f^N\lambda$ while the investor loses at least $(1-f^N)\lambda/\mathcal{K}_N^0$: the price markup is a transfer the manager collects on its fee share alone, whereas the investor bears it on the whole portfolio and additionally bears the unrewarded risk in the second component of $\mathcal{A}_N$. Pumping is therefore a highly inefficient transfer, and commitment pays as soon as a nontrivial fraction ρ_f of the investor's gain is recaptured through fees.

A.2 Proofs for Section 4

Proof of Lemma 1. Conjecture $P = \beta_0 + \beta_1\hat{s} + \beta_2 z$ and write it in centered form $P = \bar{P} + \beta_1(\hat{s} - \bar{b}) + \beta_2 z$ with $\bar{P} \equiv \mathbb{E}[P]$. Let $(\hat{s} - \bar{b}) \equiv \hat{s} - \bar{b}$ denote the centered reported signal and $\rho \equiv \beta_2/\beta_1$.

Informed (delegated) investors observe $\hat{s}$ and form

$$\mathbb{E}[v \mid \hat{s}] = \bar{\tau}\,(\hat{s} - \bar{b}), \qquad \mathbb{V}(v \mid \hat{s}) = \sigma_v^2(1-\bar{\tau}), \qquad \bar{\tau} = \frac{\sigma_v^2}{\sigma_s^2 + \bar{\sigma}_b^2}.$$

Uninformed (active) investors observe P, or equivalently $s_P \equiv P/\beta_1 = (\hat{s}-\bar{b})+\rho z$, and form

$$\mathbb{E}[v \mid P] = \bar{\tau}_P\, s_P, \qquad \mathbb{V}(v \mid P) = \sigma_v^2(1-\bar{\tau}_P), \qquad \bar{\tau}_P = \frac{\sigma_v^2}{\sigma_s^2+\bar{\sigma}_b^2+\rho^2\sigma_z^2}.$$

Writing A_F and A_A as $A_F \equiv \frac{m}{\sigma_v^2(1-\bar{\tau})}$, $A_A \equiv \frac{1-m}{\sigma_v^2(1-\bar{\tau}_P)}$, aggregate informed and uninformed demands are

$$m\tfrac{\bar{\tau}(\hat{s}-\bar{b})-P}{\gamma\sigma_v^2(1-\bar{\tau})} = \gamma^{-1}A_F[\bar{\tau}(\hat{s}-\bar{b})-P] \qquad \text{and} \qquad \gamma^{-1}A_A[\bar{\tau}_P s_P - P]$$

respectively.

With $\bar{e}=0$, aggregate demand equals the noise supply, $\int x_j\, dj = z$:

$$\gamma^{-1}\Big[A_F\big(\bar{\tau}(\hat{s}-\bar{b})-P\big)+A_A\big(\bar{\tau}_P s_P - P\big)\Big] = z.$$

Substituting $s_P = (\hat{s}-\bar{b})+\rho z$ and $P-\bar{P} = \beta_1(\hat{s}-\bar{b})+\beta_2 z$ and matching coefficients:

Constant. $(A_F+A_A)\bar{P}=0$, so $\bar{P}=0$ and $\beta_0 = -\beta_1\bar{b}$.

Coefficient on $(\hat{s}-\bar{b})$*:* $(A_F+A_A)\beta_1 = A_F\bar{\tau}+A_A\bar{\tau}_P \implies \beta_1 = \frac{A_F\bar{\tau}+A_A\bar{\tau}_P}{A_F+A_A}$, which is (**??**).

Coefficient on z*:* $\rho\, A_F\bar{\tau} = -\gamma \implies \rho = \frac{\beta_2}{\beta_1} = -\frac{\gamma\sigma_v^2(1-\bar{\tau})}{m\,\bar{\tau}}$, matching (**??**).

Define $K \equiv \frac{\gamma^2\sigma_v^2\sigma_z^2}{m^2} > 0$. Since $\sigma_s^2+\bar{\sigma}_b^2 = \frac{\sigma_v^2}{\bar{\tau}}$, we can rewrite:

$$\bar{\tau}_P = \frac{\sigma_v^2}{\sigma_v^2/\tau + \left(\dfrac{\gamma\sigma_v^2(1-\bar{\tau})}{m\tau}\right)^2\sigma_z^2} = \frac{1}{1/\bar{\tau}+K(1-\tau)^2/\bar{\tau}^2} = \frac{\bar{\tau}^2}{\bar{\tau}+K(1-\bar{\tau})^2}. \tag{74}$$

Therefore,

$$R(\tau) \equiv \frac{1-\bar{\tau}_P}{1-\bar{\tau}} = \frac{1-\dfrac{\bar{\tau}^2}{\bar{\tau}+K(1-\bar{\tau})^2}}{1-\bar{\tau}} = \frac{\bar{\tau}+K(1-\bar{\tau})^2-\bar{\tau}^2}{(1-\bar{\tau})\left[\bar{\tau}+K(1-\bar{\tau})^2\right]} = \frac{\bar{\tau}+K(1-\bar{\tau})}{\bar{\tau}+K(1-\bar{\tau})^2}. \tag{75}$$

Differentiating (75) gives

$$R'(\bar{\tau}) = \frac{K\left[K(1-\bar{\tau})^2-\bar{\tau}^2\right]}{\left[\bar{\tau}+K(1-\bar{\tau})^2\right]^2}. \tag{76}$$

Thus $R'(\bar{\tau})>0$ if and only if $\bar{\tau} < \frac{\sqrt{K}}{1+\sqrt{K}}$.

Conditional certainty equivalents. The Delegating investor's terminal wealth is scaled by the fee, $W_1^D = (1-f)\big(W_0+(v-P)x_F\big)$, so her effective risk aversion is $\gamma(1-f)$.

Conditional on $(\hat{s}, P)$, the trading gain is Gaussian and the certainty equivalent is

$$\mathcal{CE}^D = (1-f)W_0 + (1-f)\left(\mathbb{E}[v \mid \hat{s}] - P\right) x_F - \tfrac{\gamma}{2}(1-f)^2 \mathbb{V}(v \mid \hat{s})\, x_F^2 \tag{77}$$

Substituting $x_F = \left(\mathbb{E}[v \mid \hat{s}] - P\right)/\gamma \mathbb{V}(v \mid \hat{s})$ and using $(1-f) - \frac{1}{2}(1-f)^2 = \frac{1}{2}(1-f^2)$,

$$\mathcal{CE}^D = (1-f)W_0 + \frac{(1-f^2)}{2\gamma \mathbb{V}(v \mid \hat{s})}\left(\mathbb{E}[v \mid \hat{s}] - P\right)^2. \tag{78}$$

The active uninformed investor trades $x_A = (\mathbb{E}[v \mid P] - P)/(\gamma \mathbb{V}(v \mid P))$, giving

$$\mathcal{CE}^A = W_0 - c + \frac{1}{2\gamma \mathbb{V}(v \mid P)}(\mathbb{E}[v \mid P] - P)^2$$

.

Ex-ante certainty equivalents. The conditional CEs are quadratic in the mean-zero Gaussian forecast errors $\mathbb{E}[v \mid \hat{s}] - P$ and $\mathbb{E}[v \mid P] - P$. Using $\mathbb{E}[\exp(-aX^2)] = (1+2aV)^{-1/2}$ for $X \sim N(0, V)$ and $\mathrm{CE} = -\gamma^{-1} \log \mathbb{E}[\exp(-\gamma\, \mathcal{CE})]$, the ex-ante certainty equivalents are

$$\mathrm{CE}^D = (1-f)W_0 + \frac{1}{2\gamma}\log\big(1 + (1-f^2)V_D\big), \qquad V_D \equiv \frac{\mathbb{V}\big(\mathbb{E}[v \mid \hat{s}] - P\big)}{\mathbb{V}(v \mid \hat{s})}, \tag{79}$$

$$\mathrm{CE}^A = W_0 - c + \frac{1}{2\gamma}\log\big(1 + V_A\big), \qquad V_A \equiv \frac{\mathbb{V}\big(\mathbb{E}[v \mid P] - P\big)}{\mathbb{V}(v \mid P)}. \tag{80}$$

The marginal delegator has cost c^* at which $\mathrm{CE}^D = \mathrm{CE}^A$:

$$fW_0 - c^* = \frac{1}{2\gamma}\log\left[\frac{1 + (1-f^2)V_D}{1 + V_A}\right]. \tag{81}$$

Then,

$$\frac{1+V_D}{1+V_A} = \frac{\mathbb{V}(v-P)/\mathbb{V}(v \mid \hat{s})}{\mathbb{V}(v-P)/\mathbb{V}(v \mid P)} = \frac{\mathbb{V}(v \mid P)}{\mathbb{V}(v \mid \hat{s})} = \frac{1-\bar{\tau}_P}{1-\bar{\tau}}. \tag{82}$$

We can rewrite(81) as

$$\frac{1 + (1-f^2)V_D}{1+V_A} = \frac{(1+V_D) - f^2 V_D}{1+V_A} = \frac{1+V_D}{1+V_A}\left(1 - f^2 \frac{V_D}{1+V_D}\right). \tag{83}$$

Substituting (82) yields:

$$fW_0 - c^* = \frac{1}{2\gamma}\log\left[\frac{1-\bar{\tau}_P}{1-\bar{\tau}}\left(1 - f^2\,\frac{\mathbb{V}\left(\mathbb{E}[v \mid \hat{s}] - P\right)}{\mathbb{V}(v-P)}\right)\right] \;\equiv\; \Delta R(m,f), \tag{84}$$

Note that $\frac{\mathbb{V}(\mathbb{E}[v|\hat{s}]-P)}{\mathbb{V}(v-P)} = \frac{\mathbb{V}(\mathbb{E}[v|\hat{s}]-P)}{\mathbb{V}(\mathbb{E}[v|\hat{s}]-P)+\sigma_v^2(1-\bar{\tau})} = 1 - \frac{\sigma_v^2(1-\bar{\tau})}{\mathbb{V}(\mathbb{E}[v|\hat{s}]-P)+\sigma_v^2(1-\bar{\tau})}$, and $\mathbb{V}\left(\mathbb{E}[v \mid \hat{s}] - P\right) = (\bar{\tau}-\beta_1)^2\left(\sigma_s^2 + \bar{\sigma}_b^2\right) + \beta_2^2\,\sigma_z^2$ decreases with m, since the price reflects more information. Then, $\frac{\mathbb{V}(\mathbb{E}[v|\hat{s}]-P)}{\mathbb{V}(v-P)}$ decreases with m as well.

Proof of Lemma 2 Conditional on s and $\tilde{w}$, the rater's optimization (21) has linear-quadratic structure with $\mathbb{E}[P(\hat{s}) \mid s, b] = \beta_1(s + b - \bar{b})$, so the first-order condition yields $b^*(\tilde{w}) = \beta_1\tilde{w}/c_b \equiv \frac{\beta_1}{c_b}\tilde{w}$. The bias is independent of s because P is linear in $\hat{s}$.

Given the conjectured price, the constraint (3) delivers

$$\gamma\sigma_v^2(1-\bar{\tau})x_F \;=\; (\bar{\tau}-\beta_1)\left(s + \frac{\beta_1}{c_b}\tilde{w}\right) - \beta_2 z.$$

Using $\mathbb{E}[s] = \mathbb{E}[z] = 0$, $\mathbb{V}(s) = \sigma_s^2$, $\mathbb{E}[vs] = \sigma_v^2$, the expected management fees are:

$$\begin{aligned}
&\gamma\sigma_v^2(1-\bar{\tau})\,\mathbb{E}[(v-P)x_F \mid w, \tilde{w}] \\
&= \mathbb{E}\left[\left(v - \beta_1 s - \beta_1\frac{\beta_1}{c_b}\tilde{w} - \beta_2 z\right)\left((\bar{\tau}-\beta_1)\left(s + \frac{\beta_1}{c_b}\tilde{w}\right) - \beta_2 z\right) \mid w, \tilde{w}\right] \\
&= (\bar{\tau}-\beta_1)\left[\sigma_v^2 - \beta_1\sigma_s^2 - \beta_1\left(\frac{\beta_1}{c_b}\tilde{w}\right)^2\right] + \beta_2^2\sigma_z^2.
\end{aligned}$$

Then, the payoff of the fund manager is:

$$\mathbb{E}[\pi_F \mid \tilde{w}, w] = fm\left\{W_0 + \left[\gamma\sigma_v^2(1-\bar{\tau})\right]^{-1}\left((\bar{\tau}-\beta_1)\left[\sigma_v^2 - \beta_1\sigma_s^2\right] + \beta_2^2\sigma_z^2\right)\right\} \tag{85}$$

$$- \left[\frac{fm}{\gamma\sigma_v^2(1-\bar{\tau})}\beta_1\left(\bar{\tau}-\beta_1\right)\right]\left(\frac{\beta_1}{c_b}\tilde{w}\right)^2 + (\alpha w - \tilde{w})\,\beta_1\,\frac{\beta_1}{c_b}\,\tilde{w}. \tag{86}$$

First-order condition. Denote $V \equiv \mathbb{E}[\pi_F \mid w, \tilde{w}]$. Differentiating V in $\tilde{w}$:

$$\frac{\partial V}{\partial \tilde{w}} \;=\; -\frac{2fm\,\beta_1(\bar{\tau}-\beta_1)\frac{\beta_1}{c_b}}{\gamma\sigma_v^2(1-\bar{\tau})}\frac{\beta_1}{c_b}\tilde{w} - 2\,\beta_1\frac{\beta_1}{c_b}\tilde{w} + \alpha w\beta_1\frac{\beta_1}{c_b} = 0 \implies \tilde{w}^*(w) \;=\; \frac{\alpha}{2(1+\mu)}w$$

where μ denotes the marginal effect on the management fees:

$$\mu \equiv \frac{fm\,\beta_1\left(\bar{\tau}-\beta_1\right)}{\gamma\sigma_v^2(1-\bar{\tau})c_b} = \frac{f(1-m)}{\gamma\sigma_v^2}\frac{\beta_1}{c_b}\frac{\beta_1-\bar{\tau}_P}{1-\bar{\tau}_P}$$

.

Then, the expected payoff of the manager is

$$\max \mathbb{E}[\pi_F \mid \tilde{w}, w] = fm \left\{ W_0 + \frac{(\bar{\tau} - \beta_1)[\sigma_v^2 - \beta_1 \sigma_s^2] + \beta_2^2 \sigma_z^2}{\gamma \sigma_v^2 (1 - \bar{\tau})} \right\} + \frac{\alpha^2 \beta_1^2}{4 c_b (1 + \mu)} w^2 \tag{87}$$

The bias has the mean of $\mathbb{E}[\tilde{w}^*] = \mathbb{E}[w] = 0$. and the variance of $\mathbb{V}(b) = \alpha^2 \beta_1^2 \sigma_w^2 / (2(1 + \mu) c_b)^2$. Then, the belief consistency requires that $\bar{b} = 0$, $\bar{\sigma}_b^2 = \sigma_b^2 = \alpha^2 \beta_1^2 \sigma_w^2 / (2(1 + \mu) c_b)^2$.

Proof of Proposition 3 Write $x \equiv \sigma_b^2$ and treat m as fixed, so β_1, f, μ, η_b, and $\mathcal{B}$ are functions of x alone. Every object enters through $\bar{\tau} = \sigma_v^2 / (\sigma_s^2 + x)$, strictly decreasing in x. The belief-consistency requirement is the scalar fixed-point equation $x = \mathcal{B}(x) = \sigma_w^2 \, \eta_b(x)^2$.

From (**??**), $\log \eta_b = \log \beta_1 - \log(c_b + \bar{\mu})$, so

$$\frac{d \log \eta_b}{d \log x} = \varepsilon_{\beta_1} - \frac{\bar{\mu}}{c_b + \bar{\mu}} \, \varepsilon_{\bar{\mu}}, \tag{88}$$

Converting to a level derivative gives

$$\frac{d\mathcal{B}}{dx} = \frac{2\mathcal{B}(x)}{x} \Big[\varepsilon_{\beta_1}(x) - s_T(x) \varepsilon_{\mu}(x) \Big]. \tag{89}$$

By hypothesis (26), the right-hand side of (89) is strictly below one for all $x > 0$. Hence $\Phi'(x) = (\mathcal{B})'(x) - 1 < 0$ on the entire domain, so Φ is strictly decreasing.

The maps $\beta_1, \bar{\tau}, \bar{\tau}_P, \bar{\mu}$ are continuous in x and the denominator of η_b exceeds $c_b > 0$, so Φ is continuous. At $x = 0$, beliefs are degenerate and the rater's optimal response to any conjectured positive legacy dispersion yields a strictly positive bias variance whenever $\alpha \sigma_w > 0$, so $\Phi(0) = \mathcal{B}(0) > 0$. Because $\beta_1 \leq \bar{\tau} \leq 1$ and $c_b + \bar{\mu} \geq c_b$,

$$\mathcal{B}(x) \leq \frac{\alpha^2 \sigma_w^2 \sigma_v^4}{4 c_b^2 (\sigma_s^2 + x)^2} \xrightarrow[x \to \infty]{} 0, \tag{90}$$

so $\Phi(x) < 0$ for x large. A continuous, strictly decreasing function that is positive at 0 and eventually negative has exactly one zero. That zero is the unique PBNE bias dispersion $\sigma_b^{2*}(m)$.

Write $t \equiv \bar{\tau}(x)$. We first record the primitive fee-response bound used below. From the delegation indifference condition,

$$f W_0 = \bar{c}(1 - m) + \Delta R(t, f),$$

and since

$$\Delta R(t,f) \le \frac{1}{2\gamma}\log\frac{1-\bar{\tau}_P}{1-t} \le \frac{1}{2\gamma}\log\frac{1}{1-\tau} = \frac{1}{2\gamma}\log\left(1+\frac{\tau}{1+\tau}\right) < \frac{1}{2\gamma}\frac{\tau}{1+\tau},$$

we have $f \le \bar{f}$.

From 75 and 76, we obtain:

$$(1-t)\left(\frac{R'}{R}\right)^{+} = \frac{K(1-t)\left[K(1-t)^2 - t^2\right]^{+}}{\left[t+K(1-t)^2\right]\left[t+K(1-t)\right]} \le 1. \tag{91}$$

Now write

$$\Theta \equiv \frac{\mathbb{V}(\mathbb{E}[v \mid \hat{s}] - P)}{\mathbb{V}(v-P)}, \qquad g \equiv 1-\Theta = \frac{\mathbb{V}(v \mid \hat{s})}{\mathbb{V}(v-P)}.$$

Taking the partial derivative in t holding f fixed gives

$$(1-t)\partial_t \log(1-f^2\Theta) = \frac{f^2(1-t)g'}{(1-f^2)+f^2 g} \le \frac{f^2}{1-f^2}(1-t)g'. \tag{92}$$

The price system implies the bound $(1-t)(g')^{+} \le \frac{1}{4}$.

Indeed, with $a \equiv 1-t$ and

$$g(t) = \frac{1}{1+Q(a)}, \qquad Q(a) \equiv \frac{\mathbb{V}(\mathbb{E}[v \mid \hat{s}] - P)}{\mathbb{V}(v \mid \hat{s})} \ge 0,$$

we have

$$g'(t) = \frac{Q_a(a)}{(1+Q(a))^2}.$$

Using the closed-form price loading

$$\beta_1(t) = tu(t), \qquad u(t) = \frac{1}{1+h(t)}, \qquad h(t) = \frac{(1-m)K(1-t)^2}{t+mK(1-t)},$$

one obtains $aQ_a \le Q$ (since $d\,(n(a)/a)\,/d\,a$). Therefore,

$$(1-t)g' \le \frac{Q}{(1+Q)^2} \le \frac{1}{4}.$$

Combining this with (92) and $f \le \bar{f}$ yields

$$(1-t)\Big(\partial_t \log(1-f^2\Theta)\Big) < \frac{\bar{f}^2}{1-\bar{f}^2}.$$

Therefore,

$$2\gamma(1-t)(\partial_t \Delta R) \leq 1 + \frac{\bar{f}^2}{1-\bar{f}^2} = \frac{1}{1-\bar{f}^2}.$$

Implicitly differentiating the delegation condition gives

$$\frac{df}{dt} = \frac{\partial_t \Delta R}{W_0 - \partial_f \Delta R} \leq \frac{1}{2\gamma W_0 (1-t)(1-\bar{f}^2)} \tag{93}$$

We now bound the slope of the bias-consistency map at a candidate fixed point. Let $\bar{x} = \mathcal{B}(\bar{x})$. Throughout this part, elasticities are with respect to t: $\varepsilon_y \equiv \frac{\partial \log y}{\partial \log t}$.

Since $t = \sigma_v^2/(\sigma_s^2 + x)$,

$$\frac{\partial \log t}{\partial x} = -\frac{1}{\sigma_s^2 + x}.$$

Therefore, at any fixed point,

$$F^{b\prime}(\bar{x}) = \frac{2\bar{x}}{\sigma_s^2 + \bar{x}} \left(s_T \varepsilon_{\bar{\mu}} - \varepsilon_{\beta_1} \right), \qquad s_T \equiv \frac{\bar{\mu}}{c_b + \bar{\mu}}. \tag{94}$$

The implicit marginal cost is

$$\bar{\mu} = \frac{f\, m\, \beta_1 (t - \beta_1)}{\gamma \sigma_v^2 (1-t)} \implies \log \bar{\mu} = \log f + \log \beta_1 + \log(t - \beta_1) - \log(1-t)$$

Hence,

$$\varepsilon_{\bar{\mu}} = \varepsilon_f + \varepsilon_{\beta_1} + \varepsilon_{t-\beta_1} + \frac{t}{1-t}. \tag{95}$$

Substituting (95) into (94),

$$s_T \varepsilon_{\bar{\mu}} - \varepsilon_{\beta_1} = -(1 - s_T)\varepsilon_{\beta_1} + s_T \left(\varepsilon_f + \varepsilon_{t-\beta_1} + \frac{t}{1-t} \right). \tag{96}$$

Now $\varepsilon_{\beta_1} > 0$. Moreover, writing $\beta_1 = t/(1+h)$ with

$$h(t) = \frac{(1-m)K(1-t)^2}{t + mK(1-t)},$$

we have $h'(t) < 0$, so

$$t - \beta_1 = \frac{th}{1+h} \quad \Rightarrow \quad \varepsilon_{t-\beta_1} = 1 + \frac{th'}{h(1+h)} \leq 1.$$

Therefore,

$$s_T\varepsilon_{\bar{\mu}} - \varepsilon_{\beta_1} \le s_T\varepsilon_f + s_T\left(1 + \frac{t}{1-t}\right) = s_T\frac{f'}{f}t + \frac{s_T}{1-t} \tag{97}$$

Next, $s_T = \frac{\bar{\mu}}{c_b+\bar{\mu}} \le \frac{\bar{\mu}}{c_b}$. Using $f \le 1$, $m \le 1$, $\beta_1(t-\beta_1) \le t^2/4$, and $t \le \tau$,

$$s_T \le \frac{\tau^2}{4\gamma\sigma_v^2(1-\tau)c_b} = \frac{\sigma_v^2}{4\gamma\sigma_\varepsilon^2\sigma_s^2 c_b}.$$

At any fixed point $\bar{x} = \mathcal{B}(\bar{x})$,

$$\bar{x} = \mathcal{B}(\bar{x}) \le \frac{\alpha^2\sigma_w^2\tau^2}{4c_b^2} = \frac{\alpha^2\sigma_w^2\sigma_v^4}{4c_b^2\sigma_s^4}.$$

Therefore the non-fee term is bounded by

$$F^{b\prime}(\bar{x}) = \frac{2\bar{x}}{\sigma_s^2+\bar{x}}\frac{s_T}{1-t} \le \frac{2\bar{x}s_T}{\sigma_\varepsilon^2} \le \frac{\alpha^2\sigma_w^2\sigma_v^6}{8\gamma c_b^3\sigma_\varepsilon^4\sigma_s^6}.$$

For the fee term, the factor f cancels:

$$s_T\frac{f_t^+}{f}t \;\le\; \frac{\bar{\mu}}{c_b}\frac{f_t^+}{f}t = \frac{m\,t\,\beta_1(t-\beta_1)}{\gamma\sigma_v^2(1-t)c_b}f_t^+ \;\le\; \frac{m\,t\,\beta_1(t-\beta_1)}{2\gamma^2\sigma_v^2W_0c_b(1-t)^2(1-\bar{f}^2)}$$

Hence

$$\frac{2\bar{x}}{\sigma_s^2+\bar{x}}s_T\frac{f_t^+}{f}t \le \frac{\alpha^2\sigma_w^2\sigma_v^6}{8\gamma c_b^3\sigma_\varepsilon^4\sigma_s^6}\cdot\frac{\tau}{2\gamma W_0(1-\bar{f}^2)}.$$

Combining the two terms,

$$dF^b(\bar{x})/dx \le \frac{\alpha^2\sigma_w^2\sigma_v^6}{8\gamma c_b^3\sigma_\varepsilon^4\sigma_s^6}\left(1 + \frac{\tau}{2\gamma W_0(1-\bar{f}^2)}\right).$$

Thus it is sufficient that

$$c_b^3 > \frac{\alpha^2\sigma_w^2\sigma_v^6}{8\gamma\sigma_\varepsilon^4\sigma_s^6}\left(1 + \frac{\tau}{2\gamma W_0(1-\bar{f}^2)}\right).$$

Proof of Lemma 4. Let $\Phi(m,\sigma_b^2) \equiv \mathcal{B}(m,\sigma_b^2) - \sigma_b^2$, so the branch solves $\Phi = 0$. Local stability is $\partial\mathcal{B}/\partial\sigma_b^2 < 1$ (Prop. 3), i.e. $\partial\Phi/\partial\sigma_b^2 < 0$. The implicit function theorem gives

$$\frac{d\sigma_b^{2*}}{dm} = \frac{\partial\mathcal{B}/\partial m}{1-\partial\mathcal{B}/\partial\sigma_b^2}, \tag{98}$$

whose denominator is positive; hence $\text{sign}(d\sigma_b^{2*}/dm) = \text{sign}(\partial_m \mathcal{B})$.

Since $\mathcal{B} = \frac{\alpha^2\sigma_w^2}{4}[\beta_1/(c_b+\bar{\mu})]^2$ with $\beta_1, (c_b+\bar{\mu}) > 0$, we have $\text{sign}(\partial_m \mathcal{B}) = \text{sign}(\partial_m \log \beta_1 - \partial_m \log(c_b+\bar{\mu}))$. Writing $s_T \equiv \bar{\mu}/(c_b+\bar{\mu}) \in [0,1)$ for the fee-erosion share of the marginal cost of bias (as in Prop. 3) and using $\partial_m \log \bar{\mu} = \partial_m \log f + \partial_m \log g$,

$$\partial_m \log\left[\frac{\beta_1}{c_b+\bar{\mu}}\right] = \partial_m \log \beta_1 - s_T(\partial_m \log f + \partial_m \log g)\,. \tag{99}$$

A primitive sufficient condition. The endogenous condition (28) can be guaranteed by a simple primitive bound:

$$c_b > \frac{\tau^2}{4\gamma\sigma_v^2(1-\tau)}, \tag{100}$$

To see this, note

$$\text{sign}(\partial_m \mathcal{B}) = \text{sign}\left\{\partial_m \log \beta_1 - s_T\, \partial_m \log \bar{\mu}\right\}.$$

Now define $G(m) \equiv \frac{m\beta_1(\bar{\tau}-\beta_1)}{\gamma\sigma_v^2(1-\bar{\tau})}$. Since $(f_m \le 0)$, we have

$$\partial_m \log \bar{\mu} = \partial_m \log f + \partial_m \log G \le \partial_m \log G.$$

At fixed beliefs, the fund-paid price loading can be written as

$$\beta_1(m) = \bar{\tau}\frac{m\,(T+(1-\bar{\tau})m)}{T(1-\bar{\tau}+\bar{\tau}m)+(1-\bar{\tau})m^2}, \qquad T \equiv \frac{\gamma^2\sigma_v^2\sigma_z^2(1-\bar{\tau})^2}{\bar{\tau}} > 0.$$

We can show that

$$m\bar{\tau}\beta_1' - \beta_1(\bar{\tau}-\beta_1) = \frac{Tm^2\bar{\tau}^2(1-\bar{\tau})(T+1-\bar{\tau})}{\left[T(1-\bar{\tau}+\bar{\tau}m)+(1-\bar{\tau})m^2\right]^2} > 0 \iff \frac{1}{m} - \frac{\beta_1'}{\bar{\tau}-\beta_1} < \frac{\beta_1'}{\beta_1}$$

Therefore,

$$\partial_m \log G = \frac{1}{m} + \frac{\beta_1'}{\beta_1} - \frac{\beta_1'}{\bar{\tau}-\beta_1} \le 2\frac{\beta_1'}{\beta_1} \implies \partial_m \log\left(\frac{\beta_1}{c_b+\bar{\mu}}\right) \ge (1-2s_T)\,\partial_m \log \beta_1.$$

Hence it is enough to guarantee $s_T < 1/2$, or equivalently $\bar{\mu} < c_b$:

$$\bar{\mu} \;=\; f(m)\frac{m\beta_1(\bar{\tau}-\beta_1)}{\gamma\sigma_v^2(1-\bar{\tau})} \le \frac{\bar{\tau}^2}{4\gamma\sigma_v^2(1-\bar{\tau})} \le \frac{\tau^2}{4\gamma\sigma_v^2(1-\tau)} < c_b,$$

where the first inequality uses $(f \le 1)$, $(m \le 1)$, and $(\beta_1(\bar{\tau}-\beta_1) \le \bar{\tau}^2/4)$, while the second

uses ($\bar{\tau} \leq \tau$). Therefore ($\partial_m \mathcal{B} > 0$), and local stability then implies

$$\frac{d\sigma_b^{2*}}{dm} = \frac{\partial_m \mathcal{B}}{1 - \partial_{\sigma_b^2} \mathcal{B}} > 0.$$

Proof of Lemma 5 Because

$$t_x = -\frac{t^2}{\sigma_v^2} < 0,$$

it is enough to show that $\Pi_{mt} > 0$. Differentiating (**??**) gives

$$\Pi_{mt} = \underbrace{(W_0 + \Gamma^F) A_{mt}}_{\text{marginal fee-revenue effect}} + \underbrace{A_m R^F_{I,t} + A_t R^F_{I,m} + A R^F_{I,mt}}_{\text{gross-trading-gain effect}} + \underbrace{L_{mt}}_{\text{legacy-distortion effect}}. \tag{101}$$

The first term measures whether precision raises the marginal fee revenue generated by expanding delegated access. The second block captures all interactions between access, precision, and the fund's gross trading gain. Unlike the first term, this block does not have a uniform sign: precision can improve the fund's trading opportunity, but it can also cause more information to be incorporated into the price. The last term captures how precision changes the marginal legacy-position value of expanding the fund; its sign is also generally ambiguous.

Condition (32) requires the positive marginal fee-revenue effect to dominate the largest adverse contribution of the other two blocks. It therefore implies $\Pi_{mt} > 0$. Since $\Pi_{mm} < 0$,

$$\frac{d\mathcal{M}}{dx} = -\frac{\Pi_{mx}}{\Pi_{mm}} = -\frac{\Pi_{mt} t_x}{\Pi_{mm}} < 0.$$

We next establish the primitive guarantee. The delegation condition is

$$f W_0 = \bar{c}(1 - m) + \Delta R(m, f, t).$$

Multiplying by m and substituting into the objective gives

$$\Pi(m, t) = m \left[\bar{c}(1 - m) + \Delta R(m, f, t)\right] + A(m, t)\Gamma^F(m, t) + L(m, t). \tag{102}$$

The information rent can be decomposed as

$$\Delta R = \frac{1}{2\gamma} \log \mathcal{R}(m, t) + \frac{1}{2\gamma} \log\left(1 - f^2 \Theta(m, t)\right),$$

where

$$\mathcal{R}(m,t) = \frac{t + K(m)(1-t)}{t + K(m)(1-t)^2}, \qquad K(m) = \frac{\gamma^2 \sigma_v^2 \sigma_z^2}{m^2}.$$

Define

$$\bar{c}_\tau \equiv \bar{c} + \frac{1}{2\gamma} \log \frac{1}{1-\tau}, \qquad W_{\min} \equiv \max\left\{1,\, 2\bar{c}_\tau\right\}. \tag{103}$$

Because

$$\log(1 - f^2\Theta) \leq 0 \qquad \text{and} \qquad \mathcal{R}(m,t) \leq \frac{1}{1-t} \leq \frac{1}{1-\tau},$$

the delegation condition implies

$$fW_0 = \bar{c}(1-m) + \Delta R(m,f,t) \leq \bar{c}_\tau.$$

Hence, for $W_0 \geq W_{\min}$,

$$0 \leq f \leq \frac{\bar{c}_\tau}{W_0} \leq \frac{1}{2}, \qquad 1 - f^2\Theta \geq \frac{3}{4}.$$

Implicit differentiation of the delegation condition gives

$$f_m = \frac{-\bar{c} + \Delta_S^m}{W_0 - \Delta_S^f}, \qquad f_t = \frac{\Delta_S^t}{W_0 - \Delta_S^f}.$$

Since $\Delta_S^f \leq 0$, the denominator is at least W_0. All partial derivatives of ΔR are bounded on the compact domain when $f \leq 1/2$. A further differentiation therefore yields, uniformly on $\mathcal{D}$,

$$f,\ |f_m|,\ |f_t|,\ |f_{mt}| \ = \ O(W_0^{-1})$$

It follows that

$$\partial_{mt}\left[\frac{m}{2\gamma}\log(1 - f^2\Theta)\right] \longrightarrow 0, \tag{104}$$

$$\partial_{mt}\left[mf\Gamma^F\right] \longrightarrow 0, \tag{105}$$

$$\frac{\alpha^2\sigma_w^2}{4c_b}\partial_{mt}\left[\frac{\beta_1^2}{1+\mu} - \beta_1^2\right] \longrightarrow 0, \tag{106}$$

uniformly on $\mathcal{D}$.

Define the combined remainder

$$\varepsilon(W_0) \equiv \sup_{(m,t)\in\mathcal{D}} \left| \partial_{mt}\left[\frac{m}{2\gamma}\log(1-f^2\Theta)\right] + \partial_{mt}\left[mf\Gamma^F\right] + \frac{\alpha^2\sigma_w^2}{4c_b}\partial_{mt}\left[\frac{\beta_1^2}{1+\mu} - \beta_1^2\right] \right|. \tag{107}$$

The preceding uniform convergence implies

$$\varepsilon(W_0) \longrightarrow 0 \qquad \text{as } W_0 \longrightarrow \infty.$$

The fund's payoff therefore satisfies

$$\Pi_{mt} \geq \frac{1}{2\gamma}\partial_{mt}\left[m\log\mathcal{R}(m,t)\right] - \frac{\alpha^2\sigma_w^2}{4c_b}\left|\partial_{mt}\beta_1^2\right| - \varepsilon(W_0). \tag{108}$$

Condition (33) gives

$$\partial_{mt}\left[m\log\mathcal{R}(m,t)\right] \geq \frac{1}{3},$$

while condition (34) gives

$$\frac{\alpha^2\sigma_w^2}{4c_b}\left|\partial_{mt}\beta_1^2\right| \leq \frac{1}{12\gamma}.$$

Consequently,

$$\Pi_{mt} \geq \frac{1}{12\gamma} - \varepsilon(W_0).$$

Because $\varepsilon(W_0) \to 0$, there exists a finite $\bar{W}_0 \geq W_{\min}$ such that

$$W_0 \geq \bar{W}_0 \quad \Longrightarrow \quad \varepsilon(W_0) \leq \frac{1}{24\gamma}.$$

Therefore,

$$\Pi_{mt} \geq \frac{1}{24\gamma} > 0.$$

Since $t_x < 0$ and $\Pi_{mm} < 0$,

$$\frac{d\mathcal{M}}{d\sigma_b^2} = -\frac{\Pi_{mt}t_x}{\Pi_{mm}} < 0.$$

Proof of Proposition 4. All derivatives are evaluated at the equilibrium. The preceding lemmas give

$$1 - \frac{\partial\mathcal{B}}{\partial\sigma_b^2} > 0, \qquad \frac{\partial\mathcal{B}}{\partial m} > 0, \qquad \frac{\partial\mathcal{M}}{\partial\sigma_b^2} < 0.$$

The rater-side locus is therefore increasing, whereas the fund-side locus is decreasing, so an interior intersection is unique.

Holding (m, σ_b^2) fixed,

$$\frac{\partial \mathcal{B}}{\partial \alpha} = \frac{2\mathcal{B}}{\alpha} > 0, \qquad \frac{\partial \mathcal{B}}{\partial c_b} = -\frac{2\mathcal{B}}{c_b + \bar{\mu}} < 0.$$

The part of the fund's payoff that depends directly on (α, c_b) is

$$\frac{\alpha^2 \sigma_w^2}{4} \frac{\beta_1^2}{c_b + \bar{\mu}}.$$

Because $\partial \mathcal{B}/\partial m > 0$ and $\partial \beta_1/\partial m > 0$, it follows that

$$\Pi_{m\alpha} > 0, \qquad \Pi_{mc_b} < 0.$$

Since $\Pi_{mm} < 0$, the fund's first-order condition implies

$$\frac{\partial \mathcal{M}}{\partial \alpha} > 0, \qquad \frac{\partial \mathcal{M}}{\partial c_b} < 0.$$

For either parameter $p \in \{\alpha, c_b\}$, total differentiation of the two equilibrium conditions gives

$$\frac{d\sigma_b^{2*}}{dp} = \frac{\dfrac{\partial \mathcal{B}}{\partial p} + \dfrac{\partial \mathcal{B}}{\partial m}\dfrac{\partial \mathcal{M}}{\partial p}}{1 - \dfrac{\partial \mathcal{B}}{\partial \sigma_b^2} - \dfrac{\partial \mathcal{B}}{\partial m}\dfrac{\partial \mathcal{M}}{\partial \sigma_b^2}}, \tag{109}$$

$$\frac{dm^*}{dp} = \frac{\dfrac{\partial \mathcal{M}}{\partial \sigma_b^2}\dfrac{\partial \mathcal{B}}{\partial p} + \left(1 - \dfrac{\partial \mathcal{B}}{\partial \sigma_b^2}\right)\dfrac{\partial \mathcal{M}}{\partial p}}{1 - \dfrac{\partial \mathcal{B}}{\partial \sigma_b^2} - \dfrac{\partial \mathcal{B}}{\partial m}\dfrac{\partial \mathcal{M}}{\partial \sigma_b^2}}. \tag{110}$$

The common denominator is positive. The signs established above imply $d\sigma_b^{2*}/d\alpha > 0$ and $d\sigma_b^{2*}/dc_b < 0$. The numerator of the second expression is positive for $p = \alpha$ exactly under (36), and negative for $p = c_b$ exactly under (37).

A.3 Proofs of Section 5

Proof of Proposition 6. The first statement follows directly from comparing the rater's equilibrium profits under the two regimes. Under individual-investor-paid sale, Proposition 1

gives

$$\Pi_R^* = \frac{n^*}{2\gamma} \log\left(\frac{1+\tilde{\rho}^*}{1+(1-\tau)\tilde{\rho}^*} \right). \tag{111}$$

Under fund-paid sale, Proposition 5 gives

$$\Pi_F^* = \eta \mathcal{V} + \frac{\alpha^2 \beta_1^2 \sigma_w^2}{8 c_b (1+\mu)^2}. \tag{112}$$

Hence, the rater chooses fund-paid sale if and only if $\Pi_F^* \geq \Pi_R^*$, which is condition (43).

We next study the limit as noise trading disappears. For sufficiently small σ_z, the direct sale solution is interior, so $n^* = \frac{\gamma \sigma_\varepsilon \sigma_z}{M\sqrt{\tilde{\rho}^*}}$. Because $\tilde{\rho}^*$ depends only on the primitive signal precision τ, substitution into (111) gives

$$\Pi_R^* = \frac{\sigma_\varepsilon \sigma_z}{2M\sqrt{\tilde{\rho}^*}} \log\left(\frac{1+\tilde{\rho}^*}{1+(1-\tau)\tilde{\rho}^*} \right) \longrightarrow 0 \qquad \text{as} \quad \sigma_z \to 0 \tag{113}$$

Now consider the fund-paid equilibrium. By Lemma 1,

$$\bar{\tau}_P = \frac{\sigma_v^2}{\sigma_s^2 + \sigma_b^2 + \rho^2 \sigma_z^2}, \qquad\qquad \rho = -\frac{\gamma \sigma_v^2 (1-\bar{\tau})}{m \bar{\tau}}. \tag{114}$$

Under the maintained nondegeneracy condition $\sigma_z / m^* \to 0$, we have $\rho^2 \sigma_z^2 \to 0$ and therefore $\bar{\tau}_P \to \bar{\tau}$. Because the equilibrium price loading β_1 is a weighted average of $\bar{\tau}$ and $\bar{\tau}_P$, it follows that $\beta_1 \to \bar{\tau}_0$, where $\bar{\tau}_0$ denotes the limiting effective precision. Moreover, since the price reveals the reported signal, distortion no longer reduces the fund's informational trading rent, so its implicit management-fee cost disappears $\mu = \frac{f m \beta_1 (\bar{\tau} - \beta_1)}{\gamma \sigma_v^2 (1-\bar{\tau}) c_b} \to 0$.

The equilibrium bias-consistency condition consequently becomes

$$\sigma_{b,0}^2 = \frac{\alpha^2 \bar{\tau}_0^2 \sigma_w^2}{4 c_b^2} \quad \Longrightarrow \quad \bar{\tau}_0 = \frac{\sigma_v^2}{\sigma_s^2 + \sigma_{b,0}^2}. \tag{115}$$

Substituting (115) yields

$$\frac{\alpha^2 \sigma_w^2}{4 c_b^2} \bar{\tau}_0^3 + \sigma_s^2 \bar{\tau}_0 - \sigma_v^2 = 0. \tag{116}$$

The left-hand side is strictly increasing in $\bar{\tau}_0$, is negative at zero, and is positive at $\tau = \sigma_v^2 / \sigma_s^2$ whenever $\alpha \sigma_w > 0$. Therefore, (116) has a unique solution $\bar{\tau}_0 \in (0, \tau)$.

The rater's expected distortion rent thus converges to

$$\underline{\Pi}_D \equiv \lim_{\sigma_z \to 0} \frac{\alpha^2 \beta_1^2 \sigma_w^2}{8c_b(1+\mu)^2} = \frac{\alpha^2 \sigma_w^2}{8c_b} \bar{\tau}_0^2 = \frac{c_b}{2} \sigma_{b,0}^2 > 0. \tag{117}$$

Continuity then implies that there exists $\bar{\sigma}_z > 0$ such that $\Pi_F^* > \Pi_R^*$ whenever $\sigma_z < \bar{\sigma}_z$.

B Extensions

B.1 Individual-Paid Model with a Common Endowment Shock

The individual benchmark of Section 3.1 assumes purely idiosyncratic endowments, $e_i = \epsilon_i$ with $\int e_j \, dj = 0$ a.s. Because the assumption pins down the aggregate endowment mechanically, it is worth isolating *which* conclusion it carries. This subsection replaces it with a common component,

$$e_i \;=\; w + \epsilon_i, \qquad w \sim N(0, \sigma_w^2), \quad \epsilon_i \sim N(0, \sigma_\epsilon^2), \tag{118}$$

The aggregate endowment is now $\int e_j \, dj = w \neq 0$ a.s. Two objects change relative to the baseline. First, once investors liquidate, effective supply becomes $z + w$. Second, each own endowment e_i is a private signal about w, hence about the supply component of the equilibrium price, so e_i now enters non-subscribers' inference. We show that the zero-bias result of Proposition 1 is unaffected.

The bias result survives via atomicity. In the baseline the aggregate reported endowment $C \equiv \int \tilde{e}_j \, dj$ is zero *two ways at once*: truthful reporting already gives $\int \epsilon_j \, dj = 0$ by the law of large numbers, and the atomistic investor has no incentive to report otherwise. Under (118) the first channel disappears—truthful reporting would give $C = n\,w \neq 0$ and, through the rater's first-order condition $b^* = \beta_1 C / c_b$, a state-contingent bias $b^* = \beta_1 q w / c_b$, positive precisely when the commonly held exposure w is positive. What still delivers $b^* = 0$ is the atomicity argument alone.

Proposition 8 (Zero bias with a common endowment shock) *Under the individual-investor-paid contract* (4) *and the common-shock endowment* (118)*, the rater sells an unbiased signal, $b^* = 0$, and the beliefs $(\bar{b}, \bar{\sigma}_b^2) = (0, 0)$ are consistent on the equilibrium path.*

Proof. Under (4) investor i's terminal wealth is $W_1 = W_0 + P e_i + (v - P) x_i - c_i - F - \tilde{e}_i P$, so the report $\tilde{e}_i$ enters only through $-\tilde{e}_i P$; the CARA trade x_i is independent of e_i and $\tilde{e}_i$. The report therefore shifts i's subscription value by $(e_i - \tilde{e}_i)\,\mathbb{E}[P \mid \tilde{e}_i] = (e_i - \tilde{e}_i)\,\mathbb{E}[P]$,

where the last equality also uses the atomicity of investors: a single report $\tilde{e}_i$ does not move the aggregate bias b^*. On the conjectured path $\bar{b} = 0$, so $\beta_0 = -\beta_1 \bar{b} = 0$ and $\mathbb{E}[\hat{s}] = \mathbb{E}[z] = \mathbb{E}[w] = 0$, thus $\mathbb{E}[P] = 0$ and i is indifferent over $\tilde{e}_i$. Robustness to a rater deviation $b > \bar{b}$, which would raise $\mathbb{E}[P \mid b] = \beta_1(b - \bar{b}) > 0$ and expose any $\tilde{e}_i \neq 0$ to an extra payment $\tilde{e}_i \beta_1 (b - \bar{b})$, selects $\tilde{e}_i^* = 0$. Aggregating, $C = \int \tilde{e}_j \, dj = 0$, and the rater's bias first-order condition $\beta_1 C = c_b b$ gives $b^* = 0$. The realized distribution of b is degenerate at zero, so $(\bar{b}, \bar{\sigma}_b^2) = (0, 0)$ is consistent. ■

The economic content is that the zero-bias benchmark holds even when the aggregate endowment is a non-degenerate random variable. The mechanical, law-of-large-numbers route is gone; the strategic route–price-taking plus the private-contract robustness refinement–remains.