Why is Asset Demand Inelastic?*

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Abstract

In many frictionless asset-pricing models, investor demand curves are virtually flat. Koijen and Yogo (2019), in contrast, estimate surprisingly inelastic demand. In this paper, we identify the source of this discrepancy and show that low demand elasticity estimates for individual stocks are not puzzling if price movements are *not* entirely associated with short-term discount rate changes. In a standard portfolio choice framework, we show that the demand elasticity is primarily determined by how expected returns impact investor portfolio weights in response to price movements. If prices only drop due to next-period discount rates (as most theoretical models assume), we show that demand elasticity will be high. But, if price movements are not entirely driven by next-period expected returns (as empirical estimates of elasticity measure), demand will be inelastic. Consistent with inelastic demand estimates, we find evidence of weak reversals or momentum at different horizons.

Keywords: Price elasticity, demand system asset pricing, price reversals.

JEL Classification: G11, G12, G14.

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1 Introduction

A central question in the demand-based asset pricing literature (e.g., Koijen and Yogo, 2019; Koijen, Koulischer, Nguyen, and Yogo, 2021) is how investors' portfolio responds to changes in prices and asset fundamentals. As emphasized in Brunnermeier, Farhi, Koijen, Krishnamurthy, Ludvigson, Lustig, Nagel, and Piazzesi (2021), "[i]dentifying demand elasticities is a central goal in this literature." In many frictionless asset pricing models, investor demand curves are virtually flat, implying high demand elasticities. A key new fact in asset pricing is that demand curves (at the individual stock and aggregate levels) are surprisingly inelastic compared to theories (Koijen and Yogo, 2019; Gabaix and Koijen, 2021). Models featuring elastic demand fail to generate many salient asset pricing facts such as price impact of fund flows and retail trades, excess volatility puzzle, behavioral biases, etc.

In this paper, we show that inelastic demand estimates for individual stocks are not puzzling if price movements are *not* entirely associated with short-term discount rate changes. We first define the demand elasticity of an investor as the percent change in the number of shares they hold when prices move by 1%. Since the number of shares held is the product of investor wealth and portfolio weight divided by the asset price, if we assume the portfolio weight is a function of expected returns and other residual variables (e.g., asset volatility), we can decompose demand elasticity into three components: the mean (or expected return) component, the residual component, and the wealth effects. The mean component is the impact of changes in the expected return on portfolio weights as prices change, the residual component is the effect of the residual determinants of portfolio weights (e.g., volatility) aside from the expected returns, and the wealth effect is the change in investor wealth due to price changes.

We first show that in a standard portfolio choice framework, the expected return component is the primary determinant of demand elasticity, and its contribution is much more significant than those from the residual component and wealth effects. We then further decompose the mean component of elasticity into the product of two parts: first, the change in the (log) number of shares held, S, in response to the changes in the discount rate, μ, and second the change in expected returns in response to movements in the (log) price, P:

$$\eta = -\frac{\partial \log S}{\partial \log P} = \frac{\partial \log S}{\partial \tilde{\mu}} \times \left(-\frac{\partial \tilde{\mu}}{\partial \log P}\right) \tag{1}$$

We show demand curves are inelastic primarily due to slow reversals from the second component in Equation (1).

In computing demand elasticity, it is crucial to consider the sources of price movement. From the Campbell and Shiller decomposition, price changes are associated with changes in future dividends, changes in future discount rates, or a combination of both. Moreover, price movements due to changes in discount rates can be related to short-term or long-term expected returns or a combination of the two. Given these facts, we then consider two definitions of demand elasticity associated with different sources of price movements. In the first definition of elasticity, which we label D1, we consider price changes only due to the next-period discount rate movements, holding everything else constant. A price drop due only to the next-period discount rates implies one-for-one reversals. Therefore, under D1, we essentially assume the second component in the definition of elasticity in Equation (1) is equal to one.

Estimating D1 elasticity in the stock market is extremely challenging. This is because it requires an instrument that identifies prices movements associated with one-for-one price reversals within a single period (e.g., monthly, quarterly, etc.). Investors aware of this instrument could rationally trade aggressively on the predictions this instrument delivers, weakening the predictive power of the instrument itself. If the instrument predicts one-for-one price movements but investors do not have access to it in real time, then it is no surprise that investors do not aggressively trade against these one-for-one price movements given that it is outside of their information set. Thus to estimate D1 elasticity in the stock market requires an instrument that predicts one-for-one single-period reversals, it is in the investor's information set, but investors have not traded aggressively on this instrument enough to eliminate its predictive power. This is a high bar indeed.

Moreover, as mentioned above, the assumption of full reversal next period, creates near arbitrage opportunities. Therefore, we arrive at our second definition of elasticity, D2, which considers responses to price movements that are *not* entirely driven by the next-period discount rates. Depending on the sign and magnitude of the second term in Equation (1), definition D2 covers a broad range of elasticity estimates: upward-sloping demand if it is negative, perfectly inelastic if it is zero, and D1 if it is equal to one.

Our main contribution is to identify weak reversals as the source of the inelastic demand puzzle in the stock market. As we show later, almost all empirical estimates measure D2 elasticity while most theoretical models only study the response of portfolio weights to changes in the discount rates, i.e., D1.[1] We empirically estimate D1 in the bond market where it is arguably more plausible to do so, and, consistent with theory, we find high demand elasticities.

Given weak reversals, it is important to understand whether reversals are weak enough to deliver low estimated elasticity values comparable to estimates from the literature and whether the residual component of demand is small. To answer both of these questions, we estimate the components of a classic portfolio choice problem with Epstein-Zin preferences from Campbell, Chan, and Viceira (2003). We decompose the residual components of elasticity into a covariance component, a variance component, and a consumption-to-wealth ratio hedging component. We find that these

[1]A notable exception is the model in Gabaix and Koijen (2021) where it considers price movements that do not revert back next period. Consistent with these definitions, Petajisto (2009) measures very high D1 demand elasticity of more than 6,000 in his calibration. This number is almost three orders of magnitude larger than the empirical estimates that measure D2.

components are relatively small, meaning that the entire residual component is relatively small. We also find that weak reversals are enough to deliver relatively inelastic demand, comparable to literature estimates, through the expected return component of elasticity.

While weak reversals are enough to deliver inelastic demand and large price impacts from flow, a compelling theoretical reason for weak price reversals is still needed. Why are there weak price reversals? We provide two different models that deliver weak price reversals: first, a model of arbitrage coordination with "intuitive expectations" in the style of Fuster, Laibson, and Mendel (2010), and an asymmetric information model.

The coordination model relies on the fundamental insight that trade against noise-induced price movements is only profitable if other investors also trade against these price changes. Prices are then corrected, earning trading profits for arbs. With rational expectations, arbitrageurs correctly perceive that other investors will trade based on mispricing, and thus there is no coordination, and demand is relatively elastic. However, when expectations of future willingness to trade against mispricing are formed by simply taking the average of historical willingness to trade against mispricing, i.e., intuitive expectations are used instead of rational expectations in the spirit of Fuster et al. (2010), if demand is initially inelastic, then it continues to be so throughout time. This model has the interesting property that the initial expectations of future elasticity values largely determine the elasticity. Only mass coordination of traders is required to switch from one equilibrium to another.

The asymmetric information model is reminiscent of the no-trade theorem of Milgrom and Stokey (1982), where price movements are only driven by private information. This implies that uninformed investors are rationally unwilling to trade when prices change because it means they are trading against better-informed investors. In our model, price movements are partly driven by noise trades. Thus uninformed investors are willing to trade when price changes are induced by noise but not against private information-induced price movements. Given the difficulty of disentangling the sources of price movements, uninformed investors' demand elasticity is determined by the mix of noise and private-information price movements in equilibrium. Thus this model can deliver extremely elastic or perfectly inelastic demand depending on the degree of private information and noise.

Whether these two or other frameworks explain the primary source of weak reversals is unclear. However, weak reversals appear to be a strong theoretical and empirical source of inelastic demand.

Related literature

Our paper is about stock market micro elasticity which examines the change in the relative price of two stocks if one buys \$1 of one and sells \$1 of the other (e.g., Shleifer, 1986; Harris and Gurel, 1986).[2] There is a range methodologies to estimate demand elasticities at the individual stock level: index exclusion (Chang, Hong, and Liskovich, 2015; Pavlova and Sikorskaya, 2020), dividend payments (Schmickler, 2020), mutual fund flows (Lou, 2012), and trade-level price impacts (Frazzini, Israel, and Moskowitz, 2018; Bouchaud, Bonart, Donier, and Gould, 2018). There are also structural approaches using asset demand systems (Koijen and Yogo, 2019; Haddad, Huebner, and Loualiche, 2021). The estimates of micro price multipliers (inverse of micro elasticities) range from 0.3 to 15, much higher than what existing models predict.[3] So, demand curves are much more inelastic compared to existing theories. In this paper, we provide a microfoundation for inelastic

[2]This is in contrast with the literature on macro elasticity that studies how the aggregate stock market's value changes if one buys \$1 worth of stocks by selling \$1 worth of bonds (e.g., Johnson, 2006; Deuskar and Johnson, 2011; Da, Larrain, Sialm, and Tessada, 2018; Gabaix and Koijen, 2021; Li, Pearson, and Zhang, 2020; Hartzmark and Solomon, 2022). A more recent literature studies factor-level multipliers which is the price impact if an investor buys a fraction of the outstanding shares of a cross-sectional pricing factor such as size or value (e.g., Peng and Wang, 2021; Li, 2021). The evidence in the literature suggests that the micro elasticity is much larger than the aggregate elasticity given that different stocks are closer substitutes than the stock and bond market indices (Gabaix and Koijen, 2021).

[3]See Table 1 and Figure 2 of Gabaix and Koijen (2021) for more details.

demand based on investor beliefs about discount rates and cash flows.

Our finding that most stock prices movements exhibit weak, if any, reversals is consistent with extant findings in the cross-section of stock returns. Stock returns typically exhibit reversals within a month (Jegadeesh, 1990), momentum over quarterly to annual frequency (Jegadeesh and Titman, 1993), and reversals over multiple years (De Bondt and Thaler, 1985). In our framework, weak price reversals leads to low demand elasticity.

Finally, we relate to the large literature on the role of information in financial markets (e.g., Grossman and Stiglitz, 1980; Hellwig, 1980, Kyle, 1989; Van Nieuwerburgh and Veldkamp, 2010, and many others). In a noisy rational expectation equilibrium model with private and public information, we show that uninformed traders are price inelastic, and more precise public information leads to more elastic demand curves. Our model on coordination among arbitrageurs is related to earlier work on risks associated with arbitrageur chains (Dow and Gorton, 1994), arbitrage herding (Froot, Scharfstein, and Stein, 1992), and coordination risk in short-selling (Abreu and Brunnermeier, 2002, 2003).

2 Calibration of Demand Elasticity

Petajisto (2009) considers a simple calibration with CARA utility where supply decreases 10% for an asset, and the price increases only 16 basis points. This implies an elasticity of about 6,000 ($\approx 10/0.0016$).[4]

We consider two definitions of demand elasticity and corresponding calibrations. Suppose there are N assets indexed by i, each with excess returns $r_{i,t}$ at time t. Consider an investor whose demand in terms of shares of asset i is written as $S_{i,t}$. We can write $S_{i,t} = A_t w_{i,t}/P_{i,t}$, where A_t is

[4]See Appendix A for details of the model in Petajisto (2009) and its calibration.

the wealth or value of all assets of the investor, $w_{i,t}$ are the portfolio weights, and $P_{i,t}$ is the share price of the asset. Let $D_{i,t}$ be the dividend per share of the asset, and $R_{f,t}$ be the gross risk-free rate. Let $\tilde{\mathbb{E}}_t(\cdot)$ denote the subjective conditional expectation of the investor and let $\tilde{\mu}_{i,t} \equiv \tilde{\mathbb{E}}_t[r_{i,t+1}]$ denote the conditional expected excess return. We can express the portfolio weight in asset i as a function of expected return of the asset and other components:

$$w_{i,t} = g_{i,t}(\tilde{\mu}_{i,t}, \tilde{v}_{i,t}) \tag{2}$$

where $g_{i,t}$ is a function and $v_{i,t}$ consists of the residual determinants of portfolio weights aside from subjective expected returns that are a potentially function of prices, namely risk.

Assuming differentiability and positive demand, we can write the demand elasticity as:

$$\eta_{i,t} \equiv -\frac{\partial \log(S_{i,t})}{\partial \log(P_{i,t})} = \underbrace{1 - \frac{\partial \log(w_{i,t})}{\partial \tilde{\mu}_{i,t}} \frac{\partial \tilde{\mu}_{i,t}}{\partial \log(P_{i,t})}}_{\text{mean component}} \underbrace{- \frac{\partial \log(w_{i,t})}{\partial \tilde{v}_{i,t}} \frac{\partial \tilde{v}_{i,t}}{\partial \log(P_{i,t})}}_{\text{residual component}} \underbrace{- \frac{\partial \log(A_t)}{\partial \log(P_{i,t})}}_{\text{wealth effect}}. \tag{3}$$

Thus an elasticity is reduction in demand in percent terms when prices in crease by 1%.

Note that Koijen and Yogo (2019) assume investor wealth A_t is exogenous or not a function of prices. In a macroeconomic setting, this is an unrealistic assumption. However, with a large N (many assets in a diversified portfolio with relatively small weights), a 1% price movement changes the value of the portfolio much less than 1%. Thus quantitatively, these effects should be small in a microeconomic setting. We likewise consider exogenous wealth, thus effectively setting these wealth effects to zero when considering elasticity values. In reality, $\partial A_t / \partial \log(P_{i,t})$ is typically tiny, positive, and much less than 1. This makes the demand even more inelastic, and as we show, we can explain empirically estimated inelastic demand without relying on these wealth effects.

In computing demand elasticity, it is critical to note that we cannot simply operate off Equation (3), as there are many *types* of price changes. From Campbell and Shiller (1988), a price change is either associated with future dividends, future discount rates, or some combination of both. Furthermore, price movements due to changes in discount rates can be related to short-term or long-term expected returns or a combination of the two. In other words, price changes associated with discount rate movements may affect various parts of the term structure of equities (van Binsbergen and Koijen, 2017). Given these facts, we consider different definitions of the price elasticity of demand, associated with different types of price movements.

The first definition of the elasticity is from movements in demand associated *only* with short term discount rates changes:

Definition 1 (D1). *This is the elasticity $\eta_{i,t}$ in Equation (3) ceteris paribus. In particular, we consider a price movement such that*

$$\frac{\partial \tilde{\mathbb{E}}_t[P_{i,t+1}]}{\partial \log(P_{i,t})} = 0, \quad \frac{\partial \tilde{\mathbb{E}}_t[D_{i,t+1}]}{\partial \log(P_{i,t})} = 0, \quad \textit{and} \quad \frac{\partial \tilde{v}_{i,t}}{\partial \log(P_{i,t})} = 0.$$

In this definition of elasticity, future expectations of payouts are fixed, and the risk is assumed to be constant as well. Definition 1 corresponds to the calibration in Petajisto (2009) discussed above, where future payments and the variance-covariance structures are assumed to be exogenous. Fundamentally, in the definition of demand elasticity in D1, we consider a price change associated with only a short-term (i.e., the next period) discount rate change.

We can write the investor's subjective expected return as:

$$\begin{aligned} \tilde{\mu}_{i,t} &= \frac{\tilde{\mathbb{E}}_t[P_{i,t+1}] + \tilde{\mathbb{E}}_t[D_{i,t+1}]}{P_{i,t}} - R_{f,t} \\ &= \left(\tilde{\mathbb{E}}_t[P_{i,t+1}] + \tilde{\mathbb{E}}_t[D_{i,t+1}]\right) \times \exp\left(-\log(P_{i,t})\right) - R_{f,t} \end{aligned} \tag{4}$$

Under Definition 1, we have:

$$\begin{aligned}\frac{\partial \tilde{\mu}_{i,t}}{\partial \log(P_{i,t})} &= -\left(\tilde{\mathbb{E}}_t[P_{i,t+1}] + \tilde{\mathbb{E}}_t[D_{i,t+1}]\right) \times \exp\left(-\log(P_{i,t})\right) \\ &= -\frac{\tilde{\mathbb{E}}_t[P_{i,t+1}] + \tilde{\mathbb{E}}_t[D_{i,t+1}]}{P_{i,t}} \\ &= -(\tilde{\mu}_{i,t} + R_{f,t}). \end{aligned} \tag{5}$$

This means that the demand elasticity is:

$$\eta_{i,t} = 1 + \left(\tilde{\mu}_{i,t} + R_{f,t}\right) \frac{\partial \log(w_{i,t})}{\partial \tilde{\mu}_{i,t}} = 1 + \left(\tilde{\mu}_{i,t} + R_{f,t}\right) \frac{1}{w_{i,t}} \frac{\partial w_{i,t}}{\partial \tilde{\mu}_{i,t}} \tag{6}$$

From Equation 6, a calibration of the elasticity under D1 requires a value for $\partial w_{i,t} / \partial \tilde{\mu}_{i,t}$. To do this, we consider a standard CARA utility model in which the investor maximizes:

$$\tilde{\mathbb{E}}_t\left[-\exp\{-\gamma A_t\left(w_t' r_{t+1} + R_{f,t}(1 - \iota' w_t)\right)\}\right], \tag{7}$$

where γ is the absolute risk aversion parameter, w_t is an N dimensional vector of portfolio weights, r_t is an N dimensional vector of excess returns, and ι is an N dimensional vector of ones. The FOC is:

$$w_t = \frac{1}{\gamma A_t} \tilde{\Sigma}_t^{-1} \tilde{\mu}_t, \tag{8}$$

where $\tilde{\Sigma}_t$ is the subjective beliefs about the covariance matrix. Thus we can write:

$$\frac{\partial w_i}{\partial \tilde{\mu}_i} = \frac{\tilde{\tau}_{i,t}}{\gamma A_t}, \tag{9}$$

where $\tilde{\tau}_{i,t}$ is the i^{th} term along the diagonal of the precision matrix $\tilde{\Sigma}_t^{-1}$.

To keep the calibration simple, we consider identical expected returns, standard deviations, and correlations for all N assets. We set this subjected expected excess return to 0.06, the average subjective correlation to 0.3, and the subjective volatility to 0.3.[5] We consider $N = 2{,}000$ assets. We set the CARA risk aversion coefficient times wealth, γA_t, to be 2.2, simply because this allows portfolio weights to sum to one implying a zero-net supply (demand) of the risk-free asset. We set the risk-free rate to zero, which is conservative because a larger risk-free rate yields a higher elasticity. This yields $\tilde{\tau}_i \approx 15.9$. We plug in the average portfolio weight of $1/N$ to compute the demand elasticity.[6] Plugging these values into Equation (6) yields an elasticity of about 15,000.[7] Of course this elasticity varies as the parameter values change and we consider assets with higher and lower weights. Trying a range of parameter values seems to yield an average elasticity across assets that is at least three orders of magnitude above unity. Notice that with more assets, the average elasticity tends to be higher. This is because more assets create more substitutability, thus naturally generating higher elasticity values.

In reality, a price drop associated with only next-period discount rates is a relatively high-return low-risk proposition. In other words, this is a price movement with essentially one-for-one reversals in expectation, which generates a compelling investment opportunity. In our next definition of price elasticity, we consider a price elasticity for an average price movement.

D1 is difficult in the stock market to estimate, because it is difficult to obtain an instrument that predicts price movements with one-for-one reversals that is in a standard investor's information set. We would expect investors to use such instruments to aggressively trade against such price movements, potentially eroding the ability of the instrument to generate one-for-one price reversals

[5] Pollet and Wilson (2010) report average daily correlations to be 0.237, and longer-horizon correlations are higher due to autocorrelations across days. Thus, we use 0.3 as a reasonable parameter value.

[6] This does not mean we consider only an equal-weighted portfolio. We consider the elasticity for an asset with average weights.

[7] From Equations (6) and (9), the elasticity is $1 + 1.06 \times 2000 \times (15.9/2.2) \approx 15000$.

in the first place. Thus there are good economic reason to indicate that D1 may be difficult, if not impossible, to estimate.

Definition 2 (D2). *This is the elasticity $\eta_{i,t}$ in Equation (3), where the variation in $\log(P_{i,t})$ corresponds to an average price movement, and not just a price movement associated with only the next period discount rates.*

Calibrating demand elasticity according to Definition 2 is more complicated and requires a calibration of the subjective expectation of a price reversal, i.e., the size of $\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t})$. It similarly requires knowing how subjective expectations of the residual component of demand in Equation (2), $\tilde{v}_{i,t}$, vary with prices and how important this is for demand.

In what follows, we first show that in a standard portfolio choice framework, the mean component in Equation (3) dominates the residual and wealth effect components of elasticity. We then measure the size of price reversals in the stock market and show they weak reversals is the source of inelastic demand.

3 Data

We use the standard CRSP-Compustat merged dataset for returns and asset characteristics. We follow Koijen and Yogo (2019) in calculating profitability, book-to-market ratios, investment, dividend-to-book ratios, and beta. We download monthly and daily frequency data. To form returns for the quarterly and annual frequencies, we cumulate returns from the monthly stock data. To form weekly frequency stock data, we cumulate returns from the daily frequency.

We use the Treasury bill rates for the respective frequency from Ken French's website as the risk-free rate for both the monthly, weekly, and daily frequency. For the quarterly and annual frequencies, we use the 3-month and 1-year treasury bill rates from the Federal Reserve Economic Data (FRED). They have codes TB3MS and GS1, respectively.

We download daily, weekly, and monthly Fama and French (2015) 5 factor and momentum returns from Ken French's website. We cumulate these to obtain quarterly and annual factor returns.

We download the log consumption to wealth deviations as in Lettau and Ludvigson (2001) from Martin Lettau's website. These data are quarterly. We also download quarterly 13F institutional holdings data from Thomson Reuters when replicating the price instrument in Koijen and Yogo (2019).

4 Elasticity Decomposition

In this section, we address two important questions. First, do weak reversals translate into inelastic demand in a standard portfolio choice framework? Second, how important is the residual component of demand?

First, we consider the case of Epstein-Zin multivariate demand for N available assets from Campbell et al. (2003). They show, after loglineariztion, portfolio weights are given by:

$$w_t = \frac{1}{\gamma}\Sigma_t^{-1}\left[\mathbb{E}_t[y_{t+1}] + \frac{1}{2}\sigma_t^2 - \frac{\theta}{\psi}\sigma_{c-w,t}\right], \tag{10}$$

where y_t is an N dimensional vector of log returns minus the log risk-free rate, Σ_t is the $N \times N$ conditional covariance matrix of y_{t+1}, σ_t^2 is the N dimensional vector containing the diagonal elements of Σ_t, $\sigma_{c-w,t}$ is the N dimensional vector of conditional covariance of the log consumption to wealth ratio and y_{t+1}, $\gamma > 0$ is the relative risk aversion coefficient, $\psi > 0$ is the elasticity of intertemporal substitution, and $\theta \equiv (1-\gamma)/(1-\psi^{-1})$.[8]

We consider only quarterly data in this section, since cay is available at the quarterly level and not any higher frequencies. Thus all the results in this section are quarterly.

We need to model both the 1) conditional expectation of y_t as a function of prices as well and then 2) the conditional covariance of y_t itself and the log-consumption to wealth ratio as a function of prices as well. We do this by simply running predictive regressions for the relevant terms, as we describe below.

4.1 Conditional expectation model

The first regression is simply log returns on lagged log price changes. Let $\Delta p_{i,t} \equiv \log(P_{i,t}) - \log(P_{i,t-1})$. This is a simple panel regression:

$$y_{t+1} = \beta_0 + \beta_1 \Delta p_{i,t} + \epsilon_{i,t+1}. \tag{11}$$

Thus with this information set, we have a simple partial effect:

$$\frac{\partial \tilde{E}_t[y_{i,t+1}]}{\partial p_{i,t}} = \beta_1. \tag{12}$$

This regression corresponds to an information set consisting of only previous period price movements. We refer to this as the simple model.

We consider an expanded information set, where we include the log book to price ratio, $b_{i,t} - p_{i,t}$. We also consider the log of market capitalization, cross-sectionally normalized, $(p_{i,t} - \bar{\mu}_t)/\bar{\sigma}_t$. We include other regressors that are not functions of prices, including profitability, investment, dividend

[8] See equation (20) of Campbell et al. (2003). Note that in their equation, there are some additional terms because they also consider y_t to be log return of the asset minus a benchmark with potential covariance terms. We consider just the risk-free rate, which eliminates some of these extra terms.

to book ratio, and market beta, stacked into a column vector of controls X_t^c. The regression is written as:

$$\log(R_{i,t+1}) = \beta_0 + \beta_1 \Delta p_{i,t} + \beta_2 (b_{i,t} - p_{i,t}) + \beta_3 \frac{p_{i,t} - \bar{\mu}_t}{\bar{\sigma}_t} + (X_t^c)' \beta + \epsilon_{i,t+1}. \tag{13}$$

where β is a vector of regression parameters. Thus the average estimated price derivative is simply:

$$\frac{\partial \tilde{E}_t[y_{t+1}]}{\partial p_t} \equiv \text{Mean}\left(\frac{\partial \tilde{E}_t[y_{i,t+1}]}{\partial p_{i,t}}\right) = \beta_1 - \beta_2 + \beta_3 \times \text{Mean}\left(1/\bar{\sigma}_t\right) \tag{14}$$

We refer to this as the cross sectional model, because it uses a standard set of cross sectional variables to predict variation in expected returns.

We consider a greatly expanded information set, where stock fixed effects are added to the regression in (13). We refer to this as the fixed effects model. The regression equation is the same, except β_0 is now changed to $\beta_{i,0}$—a stock fixed effect. This specification corresponds to a greatly expanded information set, where an investor has a stock-specific estimate of the valuation and return of each asset instead of relying on cross-sectional relationships. In other words, the regression in Equation (13) corresponds to an investor who forecasts with rational expectations using all returns projected onto the space of asset characteristics. This means a high price predicts a low return when other assets with similar characteristics have a similarly high price/low return relationship. If one does not find this relationship in the cross-section, this supposed investor does not believe there is a strong price/return relationship.

In a regression with stock fixed effects, the corresponding investor knows if the price of any asset is high or low. The investor does not rely on the cross-section and has an estimate of whether the price is high or low in the time series of the asset's own returns. This corresponds to a relatively expansive information set.

The marginal price effect is calculate for each stock in each period, and the average effect across stocks and periods is shown in the second column of Table 1. Note that with the simple model, the marginal effect is identical across stocks and time. Standard errors are shown below the estimates, which are double clustered by quarter and stock. Note that both the simple and cross sectional models have *positive* average marginal effects, indicating potentially rational upward sloping demand if the elasticity is determined solely by the mean effects. The fixed effects average marginal effect is negative, but only -0.035, which means a 1% price increase decreases expected returns by only 3.5 basis points (bps). This is far from a one-for-one reversal.

4.2 Conditional covariance terms model

We consider a standard factor structure for the covariance matrix:

$$\Sigma_t = \beta_t \Omega \beta_t' + \zeta I, \tag{15}$$

where Ω is the $F \times F$ matrix of factor returns, β_t is a $N \times F$ vector corresponding to F factors, and $\zeta > 0$ is a scalar that dictates the size of the idiosyncratic variance. For the F factors, we use the Fama-French 5 factors along with momentum, i.e., $F = 6$. We estimate Ω as the estimated covariance matrix. If we estimate β_t with rolling regressions of historical returns, then the covariance matrix would, mechanically, not be a function of prices. If the $\sigma_{c-w,t}$ component is estimated similarly, then the residual component of demand would mechanically be zero. We consider an alternative specification that allows the residual component of demand elasticity to be non-zero. This involves simply running predictive regressions of the "realized" covariance terms on (log) prices and other variables in order to determine how well price fluctuations predict these covariance terms.

We parameterize β_t as

$$\beta_t = \begin{bmatrix} \sigma_{1,t} & \sigma_{2,t} & \dots & \sigma_{F,t} \end{bmatrix} \Omega^{-1}, \tag{16}$$

where $\sigma_{j,t}$ is the N dimensional column vector of conditional covariance terms of y_{t+1} and factor return $f_{j,t}$. We parameterize $\sigma_{j,t}$ as linear in characteristics X_t, i.e., $\sigma_{j,t} = X_t \beta_j^f$, where β_j^f is a vector of regression parameters. We fit these coefficients by running the following regression:

$$(f_{j,t+1} - \mu_j^f)\epsilon_{t+1} = X_t \beta_j^f + \nu_{j,t+1}, \tag{17}$$

where μ_j^f is the average return for the j^{th} factor. Note that the left-hand side variable is essentially the "realized covariance," meaning that the conditional expectation of this variable is the conditional covariance as long as the model for the conditional mean is correct. Since we have three different models of the mean above, we plug in three different regression residuals ϵ_{t+1} into the regression above, and we obtain similar results in terms of the size of the residual elasticity term. We run this regression separately for each factor $f = 1, \dots, F$. If we define $\Gamma = \left(\beta_1^f, \beta_2^f, ..., \beta_F^f\right)$, which is a $K \times F$ matrix, then we can write:

$$\beta_t = X_t \Gamma \Omega^{-1}. \tag{18}$$

Thus conditional betas are functions of characteristics, some of which include prices. This characterization allows the covariance matrix Σ_t to be a function of characteristics X_t, potentially allowing price to affect demand through channels outside just the expected return channel—i.e., the residual elasticity channel in Equation (3) discussed above. Notice that this characterization follows both Pástor and Stambaugh (2003) and Kelly, Pruitt, and Su (2019) by settings the beta of the assets equal to a simple linear function of asset characteristics. This specification implies that the covariance matrix of individual asset log excess returns is:

$$\Sigma_t = \beta_t \Omega \beta_t' + \zeta I = X_t \Gamma \Omega^{-1} \Gamma' X_t' + \zeta I. \tag{19}$$

We estimate ζ as the variance (across assets and time) of $y_{t+1} - \beta_t f_{t+1}$.

We follow a similar approach to obtain an estimate of $\sigma_{c-w,t}$. We fit the following regression:

$$\text{cay}_{t+1} \cdot \epsilon_{t+1} = X_t \beta_{c-w} + \nu_{t+1}, \tag{20}$$

where cay_{t+1} is the deviation of the log consumption to wealth ratio from the average from Lettau and Ludvigson (2001), downloaded from Martin Lettau's website. Note that the cay data is quarterly; thus, we consider elasticity only at the quarterly horizon in this section.

4.3 Results

With these parameterizations, we can rewrite Equation (10) as:

$$\begin{aligned} w_t &= \frac{1}{\gamma}(X_t \Gamma \Omega^{-1} \Gamma' X_t' + \zeta I)^{-1} \left[X_t \beta + \frac{1}{2} \text{Diag}\left(X_t \Gamma \Omega^{-1} \Gamma' X_t' + \zeta I \right) - \frac{\theta}{\psi} X_t \beta_{c-w} \right] \\ &= \frac{1}{\gamma \zeta} \left(I - \underbrace{X_t \Gamma (\zeta \Omega + \Gamma' X_t' X_t \Gamma)^{-1} \Gamma' X_t'}_{\text{covariance}} \right) \Bigg[\underbrace{X_t \beta}_{\text{mean}} + \underbrace{\frac{1}{2} \text{Diag}(X_t \Gamma \Omega^{-1} \Gamma' X_t' + \zeta I)}_{\text{variance}} - \underbrace{\frac{\theta}{\psi} X_t \beta_{c-w}}_{\text{cay}} \Bigg], \end{aligned} \tag{21}$$

where $\text{Diag}(\cdot)$ is the function that has a square matrix as an argument and outputs a column vector containing the diagonal of the matrix.

In Equation (21), we highlight four different components or channels through which price changes can affect demand: (1) the mean component, (2) the covariance component, (3) the variance component, and (4) the consumption to wealth component labeled cay. Note that with this demand specification, components (2), (3), and (4) together account for the residual component discussed above in Equation (3).

The purpose of the exercise in this section is to determine if the overall optimal Epstein-Zin demand yields inelastic demand and how important the four components of the residual elasticity are for the overall demand elasticity. In order to do this, we show the decomposition of demand elasticity into these five components. Let A_t be an $N \times J$ matrix with elements $Y_{i,j,t}$ and let p_t be the $N \times 1$ vector of log prices. Assume that each element of $A_{i,j,t}$ is a differentiable function of $p_{i,t}$. Define the following:

$$\nabla_{p_t} A_t = \begin{bmatrix} \frac{\partial A_{1,1,t}}{\partial p_{1,t}} & \frac{\partial A_{1,2,t}}{\partial p_{1,t}} & \cdots & \frac{\partial A_{1,J,t}}{\partial p_{1,t}} \\ \frac{\partial A_{2,1,t}}{\partial p_{2,t}} & \frac{\partial A_{2,2,t}}{\partial p_{2,t}} & \cdots & \frac{\partial A_{2,J,t}}{\partial p_{2,t}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial A_{N,1,t}}{\partial p_{N,t}} & \frac{\partial A_{N,2,t}}{\partial p_{N,t}} & \cdots & \frac{\partial A_{N,J,t}}{\partial p_{N,t}} \end{bmatrix}. \tag{22}$$

Then as shown above, we can write:

$$\nabla_{p_t} X_t = \begin{bmatrix} 0 & 1 & -1 & 1/\bar{\sigma}_t & 0 & \dots & 0 \\ 0 & 1 & -1 & 1/\bar{\sigma}_t & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & -1 & 1/\bar{\sigma}_t & 0 & \dots & 0 \end{bmatrix}, \tag{23}$$

where elements in the first column are zero because of the intercept, elements in the second column are one because this is the reversals column, elements in the third column are -1 because this corresponds to the log book-to-market ratio, elements in the fourth column are $1/\bar{\sigma}_t$ because this corresponds to the size column. The rest of columns are zero because their corresponding characteristics are not functions of price (investment, profitability, dividend to book, and market beta). Using this notation, we can define the elasticity, following Koijen and Yogo (2019) for only assets with positive weights, as:

$$\eta_t = \eta_t^m + \eta_t^c + \eta_t^v + \eta_t^{\text{cay}}, \tag{24}$$

where η_t^m, η_t^c, η_t^v, and η_t^{cay} correspond to the elasticity components from mean, covariance, variance, and cay in Equation (21), respectively. These components are defined as:

$$\eta_t^m = \vec{1} - \frac{1}{\gamma\zeta}\text{diag}(w_t)^{-1}((\vec{1} - \text{Diag}(X_t\Gamma\Lambda_t\Gamma' X_t')) \circ ((\nabla_{p_t} X_t)\beta)), \tag{25}$$

$$\begin{aligned}\eta_t^c = &\frac{1}{\gamma\zeta}\text{diag}(w_t)^{-1}((\nabla_{p_t} X_t)\Gamma\Lambda_t\Gamma' X_t'\mu_t \\ &- \frac{1}{\gamma\zeta}\text{diag}(w_t)^{-1}(\text{Diag}(X_t\Gamma\Lambda_t((\nabla_{p_t} X_t)\Gamma)') \circ (X_t\Gamma\Lambda_t\Gamma' X_t'\mu_t)) \\ &- \frac{1}{\gamma\zeta}\text{diag}(w_t)^{-1}(\text{Diag}(X_t\Gamma\Lambda_t\Gamma' X_t') \circ (((\nabla_{p_t} X_t)\Gamma)\Lambda_t\Gamma' X_t'\mu_t)) \\ &+ \frac{1}{\gamma\zeta}\text{diag}(w_t)^{-1}(\text{Diag}(X_t\Gamma\Lambda_t((\nabla_{p_t} X_t)\Gamma)') \circ \mu_t),\end{aligned} \tag{26}$$

$$\eta_t^v = -\frac{1}{\gamma\zeta}\text{diag}(w_t)^{-1}((\vec{1} - \text{Diag}(X_t\Gamma\Lambda_t\Gamma' X_t')) \circ (((X_t\Gamma\Omega^{-1}\Gamma') \circ (\nabla_{p_t} X_t))\vec{1})), \tag{27}$$

$$\eta_t^{\text{cay}} = \frac{\theta}{\psi}\frac{1}{\gamma\zeta}\text{diag}(w_t)^{-1}((\vec{1} - \text{Diag}(X_t\Gamma\Lambda_t\Gamma' X_t')) \circ ((\nabla_{p_t} X_t)\beta_{c-w})), \tag{28}$$

where $\Lambda_t = (\zeta\Omega + \Gamma' X_t' X_t\Gamma)^{-1}$, $\mu_t = X_t\beta + \frac{1}{2}\text{Diag}(X_t\Gamma\Omega^{-1}\Gamma' X_t' + \zeta I) - \frac{\theta}{\psi}X_t\beta_{c-w}$, $\circ$ is the Hadamard product (element-wise), and $\vec{1}$ is a vector of ones.

In Table 1, we present the mean elasticity component, η^m, in column (1) for the three models for the conditional expectation discussed above. We then decompose η^m into two parts: first, the change in expected returns in response to (log) price movements in column (2), and second, the change in the (log) portfolio weight in response to the changes in the discount rate in column (3). We find that for all three cases, the reversal component in column (2) is much smaller than the impact of discount rates on the portfolio weight, leading to low demand elasticities. For the fixed effects model in the third row, the reversal component in negative, i.e., there is momentum at the

	(1) η^m	(2) $\frac{\partial \mathbb{E}_t[r_{t+1}]}{\partial p_t}$	(3) $\frac{\partial \log(w_t)}{\partial \mathbb{E}_t[r_{t+1}]}$
Simple Reversal Model	0.982***	0.046***	0.382***
	(0.001)	(0.000)	(0.011)
Cross Sectional Model	0.095***	0.031***	29.230***
	(0.012)	(0.000)	(0.381)
Fixed Effects Model	2.581***	-0.035***	45.878***
	(0.021)	(0.000)	(0.627)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 1. Mean Elasticity Decomposition. In this table we decompose the mean component of demand elasticity, η_m, in column (1) into the reversal component in column (2) and the response of portfolio weights to discount rate changes in column (3). As shown above, $\eta_m = 1 - \frac{\partial \mathbb{E}_t[r_{t+1}]}{\partial p_t} \times \frac{\partial \log(w_t)}{\partial \mathbb{E}_t[r_{t+1}]}$.

quarterly horizon.

Note that because the weights have a $1/\gamma$ term, then $\text{diag}(w_t)^{-1}/gamma$ is only a function of gamma due to the cay component. In other words, for the elasticity, γ only matters for the component θ/ψ term that multiplies the cay component, and does not enter the elasticity through any other way. We need to pick reasonable Epstein-Zin values of γ and ψ in order to estimate the size of this cay component. To do this, we pick the largest possible value for $|\theta/\psi|$ with a reasonable range for γ and ψ. We consider $\gamma \leq 10$ and $\psi \in [1.5, 2]$. These values of γ and ψ imply preference for the early resolution of uncertainty and have been used extensively in the asset pricing literature to address a number of asset pricing puzzles (e.g., Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008). An EIS value greater than one implies a decline in asset prices when the effective risk aversion in the economy increases. Given this range, the largest possible $|\theta/\psi|$ is 18, with $\gamma = 10$ and $\psi = 1.5$. Using a smaller value of $|\theta/\psi|$ of course simply shrinks the already small cay elasticity term to zero even further.

In Table 2, we decompose the demand elasticity in column (1) into mean and residual components in columns (2) and (3). From Equation (24), the residual component in column (3) is the

	(1) Elasticity	(2) Mean	(3) Residual	(4) Covariance	(5) Variance	(6) cay
Simple Reversal Model	1.304***	0.982***	0.322***	0.456***	-0.132***	-0.002***
	(0.023)	(0.001)	(0.023)	(0.042)	(0.021)	(0.000)
Cross Sectional Model	-0.058***	0.095***	-0.153***	$< 10^{-5}$	$< 10^{-5}$	-0.153***
	(0.014)	(0.012)	(0.002)	(0.000)	(0.000)	(0.002)
Fixed Effects Model	2.434***	2.581***	-0.147***	$< 10^{-5}$	$< 10^{-5}$	-0.147***
	(0.019)	(0.021)	(0.002)	(0.000)	(0.000)	(0.002)
Note:						*p<0.1; **p<0.05; ***p<0.01

Table 2. Elasticity Decomposition. In this table, we decompose demand elasticity into mean and residual components. The elasticity in column (1) is the sum of the mean and residual components in columns (2) from Table 1 and (3). From Equation (24), the residual component in column (3) is the sum of covariance, variance, and cay components in columns (4), (5), and (6).

sum of covariance, variance, and cay components in columns (4), (5), and (6). We find that the elasticity is primarily determined by the mean component.

These results justify using Equation (1) as an approximation for the demand elasticity in Equation (3), where the elasticity is primarily set by the expected return due to a price change. From hereon, we focus on this mean effect. Note that classic portfolio choice models take the covariance structure as exogenous to prices; thus, we find this a reasonable approximation. Indeed, after accounting for the mean effects, price changes predict the covariance terms only weakly, consistent with the elasticity of demand being primarily determined by the mean component. If the mean effects are relatively weak, then demand is inelastic.

While the model above is just one exercise in computing demand elasticity, one may wonder whether other models estimated with reasonable reversals deliver much larger elasticity values. Davis (2023) estimates the demand elasticity of twelve quantitative portfolio choice models and shows that demand is either inelastic or even upward-sloping (corresponding to momentum trading). Thus across a wide variety of models, weak reversals imply inelastic demand.

5 Price Reversals

If elasticity is mostly determined by the mean effects, i.e. $\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t})$, then it is important to understand how large these mean effects are. In the previous section we showed that the average marginal effect for prices on expected returns was much smaller than one-for-one and even positive across some models. Under Definition 1, holding expectations of future payouts constant, we had $-\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t}) = \tilde{\mu}_{i,t} + R_{f,t}$, which indicates a strong reversal by assumption. Fundamentally, for Definition 2, to estimate the size of $-\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t})$, we need to make assumptions about both how expectations are formed and the information set.

In this section, we estimate reversals, $-\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t})$, using a variety of methods. Section 5.1 estimates price reversals associated with average price movements. To speak to the demand elasticities estimated in Koijen and Yogo (2019), Section 5.2 estimates $-\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t})$ when specializing to the price movements induced by their instrument. Citing and interpreting several existing studies, Section 5.3 argues that when examining price movements that will surely revert in the near term, results do indicate very high demand elasticities (D1).

5.1 Price reversals associated with average price movements

How should investors respond to an *average* price movement when they are not given any additional information? To answer this, we estimate the simple reversal regression in Equation (11). Table 3 shows the estimated slope coefficient β_1, which corresponds to an estimate of $\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t})$ under rational expectations with this limited information set. Standard errors are double clustered at the asset and time period levels. Strikingly, this is *positive* at the annual, quarterly, and monthly horizons. Thus instead of finding reversals or no effects, we find a momentum effect. Therefore, when not using information beyond recent prices, this can generate

	Dependent variable: r_{t+1}				
	(1) Annual	(2) Quarterly	(3) Monthly	(4) Weekly	(5) Daily
Δp_t	0.103**	0.046**	0.011	-0.049***	-0.136***
	(0.041)	(0.019)	(0.011)	(0.005)	(0.003)
Observations	186986	740624	2217863	9633930	46520654
R^2	0.007	0.001	0.000	0.002	0.014
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 3. Simple Reversal Regressions. This table presents estimated coefficients from the regression in Equation (11). This regression corresponds to an information set consisting of only previous period price movements. The slope coefficient corresponds to an estimate of $\partial\tilde{\mu}_{i,t}/\partial\log(P_{i,t})$ under rational expectations with this limited information set. Standard errors are double clustered at the asset and time period levels.

upward sloping demand curves at these horizons (see, e.g., Stein, 2009). The annual and quarterly coefficients are statistically significant. The weekly and daily estimates correspond to reversals of 4.9 and 13.6 basis points (bps) for a 1% change in prices.

We show these regression results corresponding to the cross sectional model in Table 4. As we see from the estimates of $\frac{\partial\tilde{\mu}_{i,t}}{\partial\log(P_{i,t})}$, with this expanded information set, price variations do not predict future returns in a statistically significant way at the annual, quarterly, and monthly horizons. At the weekly and daily horizons, a 1% price increase predicts a 5 and 13.6 bps drop in returns over the next period, respectively. This drop is due almost solely to the reversals component (Δp_t), and not to the book-to-market component ($b_t - p_t$) nor the size component $(p_{i,t} - \bar{\mu}_t)/\bar{\sigma}_t$.

The regression results for the fixed effects model are shown in Table 5. In terms of point estimates, $\frac{\partial\tilde{\mu}_{i,t}}{\partial\log(P_{i,t})}$ is negative across all investment horizons. The annual, weekly, and daily horizons have statistically significant estimates. A 1% price increase corresponds to anywhere between a 2 to 14 bps decrease in the next period expected returns, depending on the horizon.[9]

Thus across annual, quarterly, monthly, weekly, and daily investment horizons, price predictabil-

[9]This range is a bit broader when confidence intervals are considered.

	Dependent variable: r_{t+1}				
	(1) Annual	(2) Quarterly	(3) Monthly	(4) Weekly	(5) Daily
Δp_t	0.097**	0.038**	0.007	-0.050***	-0.136***
	(0.040)	(0.019)	(0.011)	(0.005)	(0.003)
$b_t - p_t$	0.065***	0.013***	0.003***	0.001**	0.000***
	(0.011)	(0.004)	(0.001)	(0.000)	(0.000)
$\frac{p_t - \mu_t}{\sigma_t}$	0.043***	0.012***	0.004***	0.001***	0.000***
	(0.010)	(0.003)	(0.001)	(0.000)	(0.000)
profitability	0.219***	0.080***	0.030***	0.008***	0.002***
	(0.029)	(0.009)	(0.003)	(0.001)	(0.000)
investment	-0.226***	-0.063***	-0.022***	-0.005***	-0.001***
	(0.036)	(0.012)	(0.004)	(0.001)	(0.000)
dividend to book	0.539	0.072	0.018	0.003	0.001
	(0.399)	(0.104)	(0.029)	(0.006)	(0.001)
beta	-4.288**	-0.590	-0.152	-0.034	-0.007
	(2.109)	(0.557)	(0.166)	(0.033)	(0.006)
$\partial \mathbb{E}_t[r_{t+1}]/\partial p_t$	0.053	0.031	0.006	-0.05	-0.136
χ^2 Test Statistic	2.06	2.806	0.255	93.695	2030.941
Joint Test p-value	0.151	0.094	0.614	0	0
Observations	186986	740624	2217863	9633930	46520654
R^2	0.044	0.014	0.005	0.003	0.014
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 4. Main Reversal Regressions. This table presents estimated coefficients from the regression in Equation (13). Relative to Equation (11), this specification corresponds to an expanded information set which includes the log book to price ratio, $b_{i,t} - p_{i,t}$, as well as the log of market capitalization, cross-sectionally normalized, $(p_{i,t} - \bar{\mu}_t)/\bar{\sigma}_t$. We also include other regressors that are not functions of prices: profitability, investment, dividend to book ratio, and market beta, stacked into a column vector X_t. Standard errors are double clustered at the asset and time period levels.

ity are much weaker than one-for-one reversals.

5.2 Price reversals associated with the Koijen and Yogo (2019) instrument

Given the most frequently discussed demand elasticity estimate comes from Koijen and Yogo (2019), we examine the price reversals associated with their instrument. We estimate the following

	Dependent variable: r_{t+1}				
	(1) Annual	(2) Quarterly	(3) Monthly	(4) Weekly	(5) Daily
Δp_t	0.042	0.007	-0.006	-0.053***	-0.136***
	(0.037)	(0.019)	(0.011)	(0.005)	(0.003)
$b_t - p_t$	0.100***	0.025***	0.007***	0.001**	0.000***
	(0.020)	(0.007)	(0.002)	(0.001)	(0.000)
$\frac{p_t - \mu_t}{\sigma_t}$	-0.170***	-0.034***	-0.013***	-0.004***	-0.001***
	(0.042)	(0.010)	(0.003)	(0.001)	(0.000)
profitability	0.140***	0.060***	0.023***	0.006***	0.001***
	(0.026)	(0.008)	(0.002)	(0.000)	(0.000)
investment	-0.144***	-0.049***	-0.016***	-0.004***	-0.001***
	(0.034)	(0.012)	(0.004)	(0.001)	(0.000)
dividend to book	-0.114	-0.056	-0.031	-0.010	-0.001
	(0.309)	(0.091)	(0.028)	(0.006)	(0.001)
beta	-4.427	-0.288	-0.022	-0.005	-0.002
	(3.019)	(0.703)	(0.221)	(0.045)	(0.008)
$\partial \mathbb{E}_t[r_{t+1}]/\partial p_t$	-0.141	-0.034	-0.02	-0.056	-0.137
χ^2 Test Statistic	15.397	3.46	3.147	118.936	2070.72
Joint Test p-value	0	0.063	0.076	0	0
Observations	186986	740624	2217863	9633930	46520654
R^2	0.037	0.008	0.003	0.003	0.014
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 5. Reversal Regressions with Fixed Effects. This table presents estimated coefficients a greatly expanded information set, where stock fixed effects are added to the regression in (13). The regression equation is the same, except β_0 is now changed to $\beta_{i,0}$—a stock fixed effect. This specification corresponds to a greatly expanded information set, where an investor has a stock-specific estimate of the valuation and return of each asset instead of relying on cross-sectional relationships. Standard errors are double clustered at the asset and time period levels.

panel regression for multiple horizons h:

$$r_{i,t+1\to t+h} = \beta_0 + \beta_1 \log\left(\frac{M_t}{B_t}\right) + (X_t^c)'\beta + \epsilon_{i,t+1}, \tag{29}$$

where the main independent variable $\log\left(\frac{M_t}{B_t}\right)$, log market-to-book ratio, is instrumented using $\log\left(\frac{\widehat{M}_t}{B_t}\right)$, where $\widehat{M}_t$ is the holdings-based instrument in Koijen and Yogo (2019). The coefficient estimate of $-\beta_1$ can be directly interpreted as the key theoretical quantity $-\frac{\partial \tilde{\mu}}{\partial \log(P)}$. We also follow Koijen and Yogo to control for the same vector of stock characteristics X_t^c. To focus on cross-sectional results, we add time-fixed effects. We calculate Driscoll-Kraay standard errors (Driscoll and Kraay, 1998) with 8 lags which control for both time-series and cross-sectional correlations.

The results are reported in Table 6. The first four columns report equal-weighted results for forecasting returns for the subsequent one, two, four, and eight quarters, and we see reversal coefficients of 0.131 and 0.352, respectively, at the quarterly and annual frequencies, which is statistically significant but also much lower than the $-\frac{\partial \tilde{\mu}}{\partial \log(P)} = 1$ benchmark in frictionless models. More importantly, this predictability is significantly weaker in larger stocks. To obtain more economically relevant results from the perspective of large institutions, columns (5) to (8) estimate value-weighted regressions. In order to account for the fact that market size has grown dramatically over time, we normalize the sum of weights in each period to one. The results are now only statistically significant at the 5% level but not the 1% level, and the degrees of reversions are much smaller, with $-\frac{\partial \tilde{\mu}}{\log(P)}$ only 0.018 and 0.062 at the quarterly and annual frequencies.

In conclusion, while there is evidence of price reversals associated with the Koijen and Yogo (2019) instrument, the reversals are quite weak, especially when considering large-cap stocks that institutions more heavily hold. This finding is consistent with the low institutional demand elasticities estimated in Koijen and Yogo (2019).

5.3 High estimates of D1 in bond markets

As discussed earlier, it is very difficult to estimate D1 in the stock market, but there are existing estimates that we would argue are close to estimating D1 in the bond market. Recall that estimating D1 requires price shocks that are sure to revert in the short term. This is only satisfied by bonds with high credit quality and near-term maturity. For instance, consider U.S. Treasury bills that will

	Dependent variable: $r_{t+1\to t+h}$							
	Equal-weighted				Value-weighted			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$h = 1$	2	4	8	$h = 1$	2	4	8
$\log(M_t/B_t)$	-0.131^{***}	-0.224^{***}	-0.352^{***}	-0.537^{***}	-0.018^{**}	-0.033^{**}	-0.062^{**}	-0.114^{**}
	(0.010)	(0.015)	(0.027)	(0.061)	(0.008)	(0.014)	(0.025)	(0.049)
beta	0.035^{***}	0.059^{***}	0.088^{***}	0.127^{***}	0.000	−0.001	−0.004	−0.013
	(0.008)	(0.015)	(0.027)	(0.047)	(0.004)	(0.007)	(0.012)	(0.021)
investment	0.000	0.001	0.004	0.017	−0.002	−0.003	−0.005	0.001
	(0.002)	(0.004)	(0.007)	(0.010)	(0.003)	(0.005)	(0.007)	(0.012)
profitability	0.077^{***}	0.133^{***}	0.218^{***}	0.346^{***}	0.024^{**}	0.043^{***}	0.076^{**}	0.132^{**}
	(0.007)	(0.013)	(0.024)	(0.052)	(0.009)	(0.016)	(0.031)	(0.063)
div/book	0.007	0.009	0.003	−0.026	-0.013^{**}	-0.023^{**}	-0.032^{**}	-0.053^{*}
	(0.005)	(0.008)	(0.014)	(0.023)	(0.005)	(0.009)	(0.016)	(0.032)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	602,377	590,955	562,106	506,480	602,377	590,955	562,106	506,480
R^2	0.011	0.021	0.031	0.041	0.001	0.011	0.011	0.021
Note:								*p<0.1; **p<0.05; ***p<0.01

Table 6. Predicting Reversals with the Koijen and Yogo (2019) instrument. This table presents estimated coefficients from the regression in Equation (29). The main independent variable $\log(M_t/B_t)$, is instrumented using $\log(\widehat{M}_t/B_t)$ where $\widehat{M}_t$ is computed using the Koijen and Yogo (2019) instrument. We follow Koijen and Yogo (2019) to control for a number of other characteristics that are cross-sectionally transformed to be uniformly distributed between 0 and 1. All regressions control for time fixed effects, and we compute Driscoll-Kraay standard errors with 8 lags, which controls for both time-series and cross-sectional correlations. Columns (1) to (4) estimate equal-weighted forecasting regressions. Columns (5) to (8) estimate value-weighted regressions where, to account for the fact that total market size went up over time, we standardized the sum of weights in each period to unity.

mature within a year. Because U.S. Treasuries are considered risk-free, any price dislocations must revert by the time of maturity.

Similar logic can also apply, to a large extent, to very highly rated corporate bonds with short maturity. Sufficiently highly rated bonds have a close-to-zero chance of defaulting within a few months. For instance, S&P's 2021 Annual Global Corporate Default And Rating Transition Study shows that, while bonds rated at A- may default in the long run, the probability that they default within a year is only 0.07%. Bonds rated higher have even lower default probabilities.[10] Since 1980, no corporate bond rated AA+ or AAA has ever defaulted within a year. Therefore, it is reasonable to

[10] See Table 9 in 2021 Annual Global Corporate Default And Rating Transition Study, available here.

think that investors would largely recognize price movements in these bonds as reflecting short-term discount rate changes that are sure to revert quickly. The same, of course, is not true for lower-rated bonds for which price movements may be associated with higher default probabilities.

The theory in Section 6 indicates that D1 elasticity is large. Taking the reciprocal of that, price multipliers in these securities should be very small. Extant estimates are consistent with this and we summarize them briefly below.

1. **Short-term U.S. Treasury bonds.** Lou, Yan, and Zhang (2013) document that yields of two-, five-, and ten-year U.S. Treasury bonds rise temporarily around U.S. Treasury bond auctions. The price impacts in their study imply demand elasticities in the range of 10 to 30. However, they do not find measurable price impacts for shorter-term Treasuries, indicating that demand elasticities in those securities are much higher than 30.

2. **Short-term developed country government bonds.** Koijen and Yogo (2020) examine cross-country demand for stocks and government bonds using a structural framework. For short-term government bond debt, they estimate demand elasticities of 42.

3. **Highly rated short-term corporate bonds.** Using mutual fund flow-induced trading as instruments, Li, Fu, and Chaudhary (2022) find close to zero price impact of trading in short-term investment grade bonds, which is consistent with a very high D1.

Overall, existing results indicate that in settings closer to truly estimating D1, the results all indicate significantly larger demand elasticities than that estimated in Koijen and Yogo (2019). Of course, this comparison needs to be heavily caveated as these estimates are not in the stock market.

6 Theoretical Framework

While weak reversals imply rationally inelastic demand and large price impacts from flows, the question remains: why are price reversals weak? We present two different theoretical frameworks that both produce weak predictive power of prices on returns and inelastic demand. We acknowledge that there are likely other explanations beyond these two.

We first present a model of arbitrage coordination, where it is only profitable for an arbitrageur to trade on price fluctuations if others trade on price fluctuations. Otherwise, prices never correct, and trading on these variations is not profitable. In this model, we consider expectations about future arbitrage activity formed based on past arbitrage activity. Thus there is a continuum of equilibria, and initially, inelastic demand results in demand that is inelastic from then onward.

We then present a noisy rational expectation equilibrium model in which informed agents have heterogeneous signal qualities. In this simple framework, demand becomes more inelastic as the signals become less precise. When there is no noise, uninformed traders are infinitely reluctant to trade against price movements and have perfectly inelastic demand.

6.1 A model of arbitrage coordination

Asset. There is a single risky asset with zero net supply being traded at times $t = \cdots, -1, 0, 1, \cdots$. The asset's fundamental value is normalized to zero. The risk-free rate is also normalized to zero. Thus, any price deviation from zero constitute a mispricing.

Trading. There are noise traders who submit market orders and have desired positions Z_t that is perfectly observable when trading at time t. We assume Z_t is follows an AR(1) process with persistence parameter $\rho < 1$:

$$Z_t = \rho Z_{t-1} + u_t,$$

where $u_t \sim \mathcal{N}(0, \sigma_u^2)$ is i.i.d. over time.

There is an overlapping generation (OLG) of arbitrageurs (arbs) born in each period t, and establish position x_t at time t and liquidate it at time $t+1$ at price P_{t+1}.[11] We assume there is a mass n of arbs who have mean-variance preferences with unit risk aversion. The OLG structure is important, because we want generation t's behavior to depend on their anticipation of generation $t+1$.

Equilibrium. We restrict attention to "linear equilibria" in which each generation of arbs submits demand curves of the form $D_t(P_t) = -\eta_t \cdot P_t$. Note that each generation can have a different η_t. From market-clearing, the equilibrium price at time t is

$$P_t = \frac{Z_t}{n\eta_t}.$$

Note that when n is large and the arbitrage capacity is high, the mispricing disappears.

Portfolio optimization. Consider generation t of arbs. They take η_{t+1} as given but do not observe noise trades Z_t. The return (price change) from time t to $t+1$ is

$$\begin{aligned} R_{t+1} &= P_{t+1} - P_t \\ &= \frac{Z_{t+1}}{n\eta_{t+1}} - P_t. \end{aligned}$$

[11] The total market order submitted is exactly Z_t. Specifically, the liquidating arbs of generation $t-1$ submit an order of size $-x_{t-1} = Z_{t-1}$, while noise traders submit an order of $Z_t - Z_{t-1}$, and these two sum to Z_t at time t.

So, the expected return and volatility of the asset at time t is

$$
\begin{aligned}
\mathbb{E}_t(R_{t+1}) &= \frac{\rho Z_t}{n\eta_{t+1}} - P_t \\
&= -\left(1 - \frac{\rho\eta_t}{\eta_{t+1}}\right) \cdot P_t, \\
\mathbb{V}\mathrm{ar}_t(R_{t+1}) &= \frac{\sigma_u^2}{n^2\eta_{t+1}^2}.
\end{aligned}
$$

Thus, we verify the linear demand assumption:

$$
\begin{aligned}
D_t(P_t) &= \frac{\mathbb{E}_t(R_{t+1})}{\mathbb{V}\mathrm{ar}_t(R_{t+1})} \\
&= -\underbrace{\left[\frac{n^2\eta_{t+1}^2}{\sigma_u^2} \cdot \left(1 - \frac{\rho\eta_t}{\eta_{t+1}}\right)\right]}_{\equiv\tilde{\eta}_t} \cdot P_t,
\end{aligned}
\tag{30}
$$

where we used $\tilde{\eta}_t$ to denote the choice of a specific arbitrageur who takes the behavior of other arbs. i.e., (η_t, η_{t+1}), as given.

If we consider a symmetric equilibrium, where $\tilde{\eta}_t = \eta_t$, then we have:

$$
D_t(P_t) = -\left(\frac{n^2\eta_{t+1}^2}{\sigma_u^2 + \rho n^2\eta_{t+1}}\right) \cdot P_t. \tag{31}
$$

Coordination with rational expectations. We see from Equation (30) that $\tilde{\eta}_t$ is decreasing in η_t. This is intuitive: the more other arbs trade on mispricing at time t, the lower the remaining profits, and thus the less an arbitrageur should trade on the mispricing.

A more interesting coordination relationship exists. $\tilde{\eta}_t$ is increasing in η_{t+1}. This means the more the *next* generation of arbs trades on mispricing, the more the current generation will profit, so the more they should trade on it.

If we focus on a symmetric stationary equilibrium, so that $\eta = \eta_t = \tilde{\eta}_t = \eta_{t+1}$, we have:

$$\eta = \frac{\sigma_u^2}{(1-\rho)\cdot n^2}$$

If we examine the demand curve of *all* arbs which determines price multipliers, then we have:

$$n\eta = \frac{\sigma_u^2}{(1-\rho)n} = \frac{\sigma_Z^2(1+\rho)}{n}, \tag{32}$$

where $\sigma_Z^2 = \sigma_u^2/(1-\rho^2)$ is the unconditional variance of noise trader's demand.

We are interested in what makes $n\eta$ small and price multipliers large (or demand elasticities low). In the symmetric stationary equilibrium where all arbs coordinate, more extreme demand from noise trades (higher σ_Z and ρ) leads to more arbitrage opportunities for arbs leading to a higher price impact. Why is this? Higher σ_Z means large orders from noise traders and, therefore, more mispricing to trade against. In the extreme case where $\sigma_Z = 0$, there is no noise driving prices, and thus no reason to trade against prices. In other words, demand is perfectly inelastic in this case. We find something similar in the model below: eliminating noise creates perfectly inelastic demand.

Also, demand elasticity is increasing in the persistence of demand from noise traders, i.e., when ρ is high. A higher ρ creates more predictable and potentially profitable trading opportunities when arbs rely on the other arbs to trade against price movements. In the case below, where an arbitrageur cannot necessarily rely on the arbitrage activity of other traders, demand elasticity is *decreasing* in ρ. Thus the demand response to price changes increases with higher ρ precisely when arbs rely on each other to trade against future mispricing. Conditional on this coordinated response, demand is more elastic with more persistent noise.

Coordination with intuitive expectations. Note that in Equations (30) and (31), η_t is a function of η_{t+1}. We assumed above that at time t arbs knew the future η_{t+1}. Consider the case in which instead of η_{t+1}, its subjective expectation, $\tilde{\mathbb{E}}_t[\eta_{t+1}]$, is plugged into Equations (30) and (31). In particular, we assume that arbs form *intuitive expectations* as in Fuster et al. (2010). In Fuster et al. (2010), intuitive expectations correspond to setting expectations with simple historical linear regressions. Our model is even simpler: expectations are set using the historical average of η_t. Given that prices are observed, equilibrium still means that η_t is perfectly known.

Plugging in intuitive expectations into Equation (31), we have:

$$D_t(P_t) = -\left[\frac{n^2\tilde{\mathbb{E}}_t[\eta_{t+1}]^2}{\rho n^2\tilde{\mathbb{E}}_t[\eta_{t+1}] + \sigma_u^2}\right] \cdot P_t. \tag{33}$$

Consider the extreme case where $\tilde{\mathbb{E}}_0[\eta_1] = 0$, i.e., arbs expect perfectly inelastic demand initially. In this case, next period demand is perfectly inelastic, and this continues for eternity. Thus there is a continuum of equilibria, determined by the initial expectations of elasticity values.

This is an oddly circular model. If demand is inelastic initially, then demand continues to be inelastic indefinitely. If demand is initially elastic, then demand continues to be elastic forever. Thus this explanation still does not explain why it was inelastic initially. This model has another odd property: coordination en mass can change the equilibrium. If investors coordinate to change their expectations, then expectations shift the elasticity of demand.

6.2 A noisy rational expectation model

The model in this section is quite different from the one in Section 6.1. This model is fully rational, and does not depend on initially inelastic demand to deliver inelastic demand. The model below delivers inelastic demand with private information. This model is based on Hellwig

(1980). This setting is slightly different from Grossman and Stiglitz (1980) in that each agent observes a different signal and tries to back out the signals of other agents from the price. Let $d \sim \mathcal{N}(\mu, v_d)$ denote public information that all investors observe. Suppose an asset has a payoff δ that, conditional on the public information, is normally distributed with mean d and variance v_δ: $\delta \mid d \sim \mathcal{N}(d, v_\delta)$. Thus, it must be the case that the unconditional distribution of δ can be written as $\delta \sim \mathcal{N}(\mu, v_d + v_\delta)$.

Assume each agent i observes $\delta + \epsilon_i$, where ϵ's are iid normal with mean zero and variance v_ϵ, and $\epsilon_i \mid d \sim \mathcal{N}(0, v_\epsilon)$. There are N informed agents each having CARA utility with risk aversion parameter γ. The noisy supply is denoted by Z, with normally-distributed per capita supply $z \equiv Z/N$: $z \mid d \sim \mathcal{N}(\mu_z, v_z)$.

Each agent's demand is:

$$X_i = \frac{\mathbb{E}(\delta \mid \delta + \epsilon_i, P, d) - P}{\gamma \mathbb{V}\text{ar}(\delta \mid \delta + \epsilon_i, P, d)}.$$

The conditional expectation in the numerator can be written as:

$$\mathbb{E}(\delta \mid \delta + \epsilon_i, P, d) = a_0 + a_\delta (\delta + \epsilon_i) + a_P P.$$

Denote the conditional variance in the denominator, $\mathbb{V}\text{ar}(\delta \mid \delta + \epsilon_i, P, d)$, as v. Conjecture that the price P can be written as:

$$P = k_0 + k_\delta \frac{\sum_i (\delta + \epsilon_i)}{N} + k_z \left(\frac{Z}{N}\right).$$

We can write the market clearing condition as:

$$\sum_i \frac{a_0 + a_\delta (\delta + \epsilon_i) + a_P P - P}{\gamma v} = Z.$$

Solving for price we get:

$$P = \frac{a_0}{1-a_P} + \frac{a_\delta}{1-a_P}\left(\delta + \frac{1}{N}\sum_i \epsilon_i\right) - \frac{\gamma v}{1-a_P}\frac{Z}{N},$$

which implies

$$k_0 = \frac{a_0}{1-a_P}, \quad k_\delta = \frac{a_\delta}{1-a_P}, \quad \text{and} \quad k_z = -\frac{\gamma v}{1-a_P}. \tag{34}$$

Since ϵ_i's are iid and have zero mean, by the law of large numbers in a large market (as $N \to \infty$), we have:[12]

$$\frac{1}{N}\sum_i \epsilon_i = 0. \tag{35}$$

Explicitly calculating the conditional expectation to get a_0, a_δ, and a_P and substituting in (34), we have

$$\begin{aligned} k_0 &= \frac{\gamma v_\epsilon \left(d\gamma v_z v_\epsilon + v_\delta \mu_z\right)}{\gamma^2 v_z v_\epsilon^2 + v_\delta\left(1+\gamma^2 v_z v_\epsilon\right)}, \\ k_\delta &= \frac{v_\delta\left(1+\gamma^2 v_z v_\epsilon\right)}{\gamma^2 v_z v_\epsilon^2 + v_\delta\left(1+\gamma^2 v_z v_\epsilon\right)}, \\ k_z &= -\frac{\gamma v_\delta v_\epsilon\left(1+\gamma^2 v_z v_\epsilon\right)}{\gamma^2 v_z v_\epsilon^2 + v_\delta\left(1+\gamma^2 v_z v_\epsilon\right)}. \end{aligned} \tag{36}$$

where we can then calculate a_0, a_δ, and a_P by substituting in Equation (34).

The price is $P = k_0 + k_\delta\delta + k_z z$ which completes the solutions of the equilibrium. The X_i's can be backed out given a_0, a_δ, and a_P, v, and P.

[12] Alternatively, one can assume instead that there is a unit mass of investors and replace $\frac{1}{N}\sum_i \epsilon_i \approx 0$ with $\int_i \epsilon_i = 0$ (e.g., Van Nieuwerburgh and Veldkamp, 2010).

Demand elasticity

To derive the demand elasticity, we take the derivative of the demand with respect to the equilibrium price:

$$\frac{\partial X_i}{\partial P} = -\frac{1 - a_P}{\gamma v} = \frac{1}{k_z}.$$

From Equation (36),

$$\frac{\partial X_i}{\partial P} = -\frac{1}{\gamma v_\epsilon} - \frac{\gamma v_z v_\epsilon}{v_\delta + \gamma^2 v_z v_\delta v_\epsilon}.$$

If we assume the variance of per capita noisy supply, v_z, is small, we get

$$\frac{\partial X_i}{\partial P} \approx -\frac{1}{\gamma v_\epsilon}. \tag{37}$$

Thus the demand in this case is more inelastic when the variance of signals is high.

How would this case compare to a model without private or public information? In that case, agent i demand would be:

$$X_i = \frac{\mathbb{E}[\delta] - P}{\gamma \mathbb{V}\text{ar}(\delta)} = \frac{\mu - P}{\gamma v_\delta}.$$

So the elasticity can be written as:[13]

$$\frac{\partial X_i}{\partial P} = -\frac{1}{\gamma v_\delta}.$$

Therefore, aggregate demand is more inelastic (approximately) if $v_\epsilon > v_\delta$.

The partial derivative in the elasticity term, $\partial X_i / \partial P$, captures the change in demand *holding all other terms fixed*, in particular, public information. However, empirically, we might be interested in how a change in prices predicts changes in demand *unconditionally*. In other words, we might

[13]The elasticity is defined as: $-\frac{\partial \log(X_i)}{\partial \log(P)} = -\frac{P}{X_i}\frac{\partial X_i}{\partial P}$, but we ignore the P/X_i term to simplify the exposition in this section. Clearly, if $\frac{\partial X_i}{\partial P} \approx 0$, then $\frac{\partial \log(X_i)}{\partial \log(P)} \approx 0$.

consider:

$$\eta \equiv -\frac{\mathrm{Cov}(X_i, P)}{\mathbb{V}\mathrm{ar}(P)}, \tag{38}$$

which has the interpretation of a regression slope coefficient. The difference between $-\partial X_i/\partial P$ and η is that the former describes how much a unit change in price affects demand *holding all other terms fixed*, while η describes how much a unit change in price affects demand *without holding other terms fixed*.

To calculate η, note that after plugging in the equilibrium price, demand is:

$$X_i = \frac{\epsilon_i}{\gamma v_\epsilon} + \frac{Z}{N}. \tag{39}$$

The variance of price in the denominator of (38) is:

$$\mathrm{Var}(P) = v_d + \underbrace{\frac{v_\delta^2 \left(1 + \gamma^2 v_z v_\epsilon\right)^2 \left(v_\delta + \gamma^2 v_z v_\epsilon^2\right)}{\left(\gamma^2 v_z v_\epsilon^2 + v_\delta \left(1 + \gamma^2 v_z v_\epsilon\right)\right)^2}}_{\equiv V} = v_d + V.$$

Thus from (38), we have:[14]

$$\eta = -\frac{k_z v_z}{v_d + V}. \tag{40}$$

Importantly, if the variance of public information is large (i.e., as $v_d \to \infty$), we can show that η approaches zero, indicating inelastic demand. This means that economically, if public information is very volatile but not controlled for appropriately in an elasticity regression estimation, demand will appear quite inelastic.

We emphasize that although prices change with public information d, from Equation (39), investors do *not* adjust their demand as public information arrives. Thus when prices move in

[14] In Appendix B, we write out the general expression for η and calculate the two limits discussed below.

response to public information, demand will appear very inelastic. As shown in Figure 1, consistent with the model, demand becomes more elastic as the public information becomes more precise, moving from the solid to the dashed line.

Again consider the case where v_z is small. In this case, we can show that

$$\lim_{v_z \to 0} \eta = 0, \tag{41}$$

that is, if per capita noisy supply term is small, then total variation in prices does not predict changes in demand at all. In other words, this naive approach to elasticity estimation would uncover a perfectly inelastic demand.

This section, with only one type of informed agent, nicely illustrates how private information can generate inelastic demand *in aggregate*. However, the heterogeneity in demand elasticities is partly driven by varying degrees of asymmetric information among agents. In the next section, we show that a model of heterogeneous signal quality can generate a dispersion of elasticity terms across investors.

In Figure 1, we show the impact of private and public information on demand elasticity. As mentioned above, demand becomes more elastic as the public signal becomes more precise, moving from the solid to the dashed line. Moreover, given the precision of the public signal (on the solid or dashed lines) demand becomes less elastic as investors become less privately informed, which we show in the next section.

7 Conclusion

In this paper, we identify weak reversals as the source of the inelastic demand puzzle in the stock market. We first decompose demand elasticity into three parts: the mean, residual, and

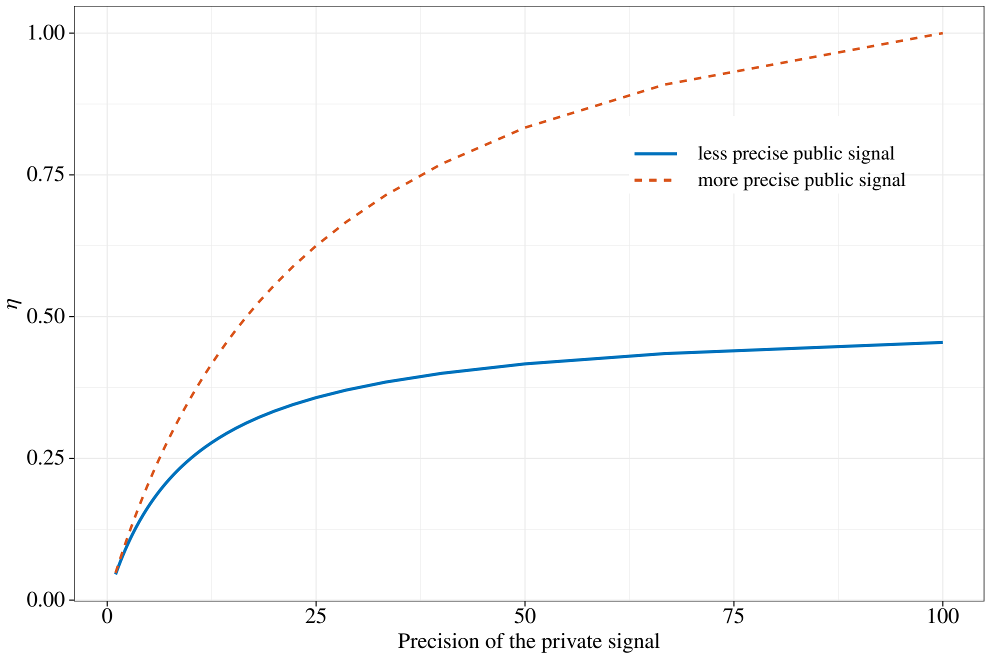

Figure 1. Information and demand elasticity. This figure qualitatively shows that demand becomes more elastic as the public signal becomes more precise. Moreover, given the precision of the public signal, demand becomes less elastic as investors become less informed.

wealth effect components. In a standard portfolio choice model, we then show the mean component primarily determines demand elasticity.

We further decompose the mean component of elasticity into the product of two parts: the change in portfolio weights in response to the change in expected returns and the change in expected returns due to price movements, i.e., reversals. Given the Campbell and Shiller decomposition, we then consider two definitions of demand elasticity associated with different sources of price movements. In the first definition of elasticity, D1, we consider price changes only due to the next-period discount rate movements, holding everything else constant, i.e., when we assume one-for-one reversals. Given that estimating D1 in the stock market is highly challenging and creates near arbitrage opportunities, we introduce the second definition, D2, which considers price movements not entirely driven by next-period discount rates.

We highlight that almost all empirical estimates measure D2 elasticity while most theoretical models only study D1. We empirically estimate D1 for investment-grade bonds where it is arguably more plausible to do so, and, consistent with theory, we find high demand elasticities. As discussed above and consistent with prior literature, we find low D2 elasticity at different horizons driven by weak reversals.

Finally, we present two theoretical frameworks that are consistent with inelastic demand due to weak reversals, one based on coordinated arbitrage and the other based on asymmetric information. Although other models can potentially deliver our empirical findings, we show that weak reversals are a strong source of inelastic demand, both empirically and theoretically.

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Appendix

A Demand Elasticity in a Standard Asset Pricing Model

To fix ideas, consider a simple static partial equilibrium model.[1] Suppose there are N assets, indexed by n, each with supply u_n. Assume the risk-free rate is constant, normalized to 0. Dividends for stock n is assumed to have the following form:

$$D_n = a_n + b_n F + e_n,$$

where $F \sim \mathcal{N}\left(0, \sigma_m^2\right)$ is the common factor and $e_n \sim \mathcal{N}\left(0, \sigma_e^2\right)$ represents the idiosyncratic risk.

There is a representative investor with constant absolute risk aversion (CARA) preferences, with wealth W and risk aversion γ who chooses portfolio weights for stocks $n = 1, \ldots, N$ to maximize her utility subject to the budget constraint:

$$\max_{\alpha_1,\ldots,\alpha_N} \mathbb{E}\left[-\exp(-\gamma W)\right],$$

$$\text{subject to} \quad W = W_0 + \sum_{n=1}^{N} \alpha_n \left(D_n - P_n\right),$$

where P_n is the price of stock n. From the first-order condition for stock n and market-clearing, we have:

$$P_n = a_n - \gamma \left[\sigma_m^2 \left(\sum_{m \neq n} u_m b_m\right) b_n + \left(\sigma_m^2 b_n^2 + \sigma_e^2\right) u_n\right]$$

Consider the following calibration. Suppose there are $N = 1000$ stocks, each with unit supply $u_n = 1$. Also let $a_n = 105$, $b_n = 100$, $\sigma_m^2 = 0.04$, $\sigma_e^2 = 900$, and $\gamma = 1.25 \times 10^{-5}$. These parameters imply a market risk premium of 5%, all stocks having a price of 100, a market beta of 1, and a standard deviation of idiosyncratic return of 30%.

Consider a supply shock of -10% ($u_n = 0.9$) for one stock. This leads to a price increase of only 0.1621 bps. Part of this increase is due to the reduction in the aggregate market risk premium (there is less aggregate risk and all stocks increase by 0.05 bps.) So the differential impact is only

[1] This simple model and its standard calibration is from Section II.A. of Petajisto (2009). It was also discussed during the Workshop on Demand System Asset Pricing organized by Ralph Koijen and Motohiro Yogo in May, 2022.

0.11 bps, meaning the demand curve is virtually flat. In this setting, the micro price elasticity of demand is very large:[2]

$$-\frac{\Delta Q/Q}{\Delta P/P} = \frac{0.10}{1.621e-5} \approx 6168,$$

implying a negligible *micro* multiplier (the inverse of micro demand elasticity). Thus, in standard asset pricing models demand curves are virtually flat.

Macro multipliers implied from frictionless asset pricing models are usually quite small as well.[3] As discussed in Gabaix and Koijen (2021), "in traditional, elastic asset pricing models the macro elasticity is around 10 to 20."[4]

Empirically, demand curves are surprisingly inelastic compared to standard models both at the micro (Koijen and Yogo, 2019) and macro (Gabaix and Koijen, 2021) levels.[5] As mentioned above, estimates of micro and macro demand elasticities, around 1 and 0.2 respectively, are much lower than what standard frictionless theories suggest.

B Homogeneous Signal Quality Model with Public Information

To reiterate the main text, there is a signal $s_i = \delta + \epsilon_i$. The agent knows his signal quality λ_i, but $\epsilon_i \sim \mathcal{N}(0, v_\epsilon)$ are iid across investors.

We conjecture that price is linear in fundamental and per-capita noisy supply:

$$P = k_0 + k_\delta \delta + k_z \frac{Z}{N}.$$

Define

$$\begin{bmatrix} \delta \\ s_i \\ P \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} d \\ d \\ k_0 + k_\delta d + k_z \mu_z \end{bmatrix}, \begin{bmatrix} v_\delta & v_\delta & k_\delta v_\delta \\ v_\delta & v_\delta + v_\epsilon & k_\delta v_\delta \\ k_\delta v_\delta & k_\delta v_\delta & k_\delta^2 v_\delta + k_z^2 v_z \end{bmatrix} \right)$$

[2]In a general equilibrium setting, Johnson (2006) perturbs the risky asset supply and finds finite *macro* elasticity even in the frictionless Lucas economy.

[3]One notable exception is the general equilibrium model in Johnson (2006). He studies the equilibrium price change in response to a perturbation in the risky asset supply, allowing for the interest rate to vary when stock prices change.

[4]Appendix F.4 in Gabaix and Koijen (2021) provides a detailed discussion.

[5]Li and Lin (2022) show that prices are more inelastic when demand is less diversifiable.

Thus

$$
\begin{aligned}
\mathbb{E}[\delta \mid s_i, P] &= d + \begin{bmatrix} v_\delta & k_\delta v_\delta \end{bmatrix} \begin{bmatrix} v_\delta + v_\epsilon & k_\delta v_\delta \\ k_\delta v_\delta & k_\delta^2 v_\delta + k_z^2 v_z \end{bmatrix}^{-1} \begin{bmatrix} \delta + \epsilon_i - d \\ P - k_0 - k_\delta d - k_z \mu_z \end{bmatrix} \\
&= \frac{(p - k_0)\, k_\delta v_\delta v_\epsilon + k_z^2 v_z \left((\delta + \epsilon_i) v_\delta + d v_\epsilon\right) - k_z k_\delta v_\delta v_\epsilon \mu_z}{k_\delta^2 v_\delta v_\epsilon + k_z^2 v_z \left(v_\delta + v_\epsilon\right)} \\
\mathbb{V}\mathrm{ar}(\delta \mid s_i, P) &= v_\delta - \begin{bmatrix} v_\delta & k_\delta v_\delta \end{bmatrix} \begin{bmatrix} v_\delta + v_\epsilon & k_\delta v_\delta \\ k_\delta v_\delta & k_\delta^2 v_\delta + k_z^2 v_z \end{bmatrix}^{-1} \begin{bmatrix} v_\delta \\ k_\delta v_\delta \end{bmatrix} \\
&= \frac{k_z^2 v_z v_\delta v_\epsilon}{k_\delta^2 v_\delta v_\epsilon + k_z^2 v_z \left(v_\delta + v_\epsilon\right)}
\end{aligned}
$$

So

$$
\delta \mid s_i, P \sim \mathcal{N}\left(\mathbb{E}\left[\delta \mid s_i, P\right], \mathbb{V}\mathrm{ar}(\delta \mid s_i, P)\right)
$$

The CARA demand is:

$$
\begin{aligned}
X_i &= \frac{\mathbb{E}[\delta \mid s_i, P] - P}{\gamma \mathbb{V}\mathrm{ar}(\delta \mid s_i, P)} \\
&= \frac{P\left(-\frac{(-1+k_\delta)k_\delta}{k_z^2 v_z} - \frac{1}{v_\delta} - \frac{1}{v_\epsilon}\right)}{\gamma} + \frac{\delta}{\gamma v_\epsilon} + \frac{\epsilon_i}{\gamma v_\epsilon} + \frac{-\frac{k_0 k_\delta}{k_z^2 v_z} + \frac{d}{v_\delta} - \frac{k_\delta \mu_z}{k_z v_z}}{\gamma}.
\end{aligned}
$$

We can write average demand as

$$
\frac{1}{N} \sum_i X_i = b_0 + b_p p + b_\delta \delta
$$

where

$$
\begin{aligned}
b_0 &= \frac{-\frac{k_0 k_\delta}{k_z^2 v_z} + \frac{d}{v_\delta} - \frac{k_\delta \mu_z}{k_z v_z}}{\gamma} \\
b_p &= \frac{\left(-\frac{(-1+k_\delta)k_\delta}{k_z^2 v_z} - \frac{1}{v_\delta} - \frac{1}{v_\epsilon}\right)}{\gamma} \\
b_\delta &= \frac{1}{\gamma v_\epsilon}
\end{aligned}
$$

To solve the model completely, we must solve the following equations:

$$
\begin{aligned}
k_0 &= -\frac{b_0}{b_p} \\
k_\delta &= -\frac{b_\delta}{b_p} \\
k_z &= \frac{1}{b_p}
\end{aligned}
$$

Solving this system of equations yields

$$
\begin{aligned}
k_0 &= \frac{\gamma v_\epsilon \left(d\gamma v_z v_\epsilon + v_\delta \mu_z\right)}{\gamma^2 v_z v_\epsilon^2 + v_\delta \left(1 + \gamma^2 v_z v_\epsilon\right)}, \\
k_\delta &= \frac{v_\delta \left(1 + \gamma^2 v_z v_\epsilon\right)}{\gamma^2 v_z v_\epsilon^2 + v_\delta \left(1 + \gamma^2 v_z v_\epsilon\right)}, \\
k_z &= -\frac{\gamma v_\delta v_\epsilon \left(1 + \gamma^2 v_z v_\epsilon\right)}{\gamma^2 v_z v_\epsilon^2 + v_\delta \left(1 + \gamma^2 v_z v_\epsilon\right)}.
\end{aligned} \tag{B.1}
$$

We can calculate η as shown in the text. If we calculate it out fully, we have, once we plug in all the constants:

$$
\eta = \frac{\gamma v_z v_\delta v_\epsilon \left(1 + \gamma^2 v_z v_\epsilon\right)\left(\gamma^2 v_z v_\epsilon^2 + v_\delta \left(1 + \gamma^2 v_z v_\epsilon\right)\right)}{v_\delta^2 \left(1 + \gamma^2 v_z v_\epsilon\right)^2 \left(v_\delta + \gamma^2 v_z v_\epsilon^2\right) + v_d \left(\gamma^2 v_z v_\epsilon^2 + v_\delta \left(1 + \gamma^2 v_z v_\epsilon\right)\right)^2} \tag{B.2}
$$

thus it's clear that

$$\lim_{v_d \to \infty} \eta = 0 \quad \text{and} \quad \lim_{v_z \to 0} \eta = 0. \tag{B.3}$$