Debt Maturity and Commitment

Abstract

If firms can issue debt only at discrete dates, as opposed to on a continuous-time basis, debt maturity is an effective device against the commitment problem on future debt issuances that leads to leverage ratcheting in equilibrium. With a shorter debt maturity, leverage becomes less persistent because its upward adjustments are faster and its long-run level lower, which increases the value of future debt dynamics. A shorter maturity also alleviates the underinvestment problem by increasing the ex ante value of installed capital. In a decomposition of the credit spread consistent with equilibrium, we show that the component due to the commitment problem on future debt issuances is sizeable when leverage and default risk are low, and it is reduced by a shorter maturity.

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Keywords: debt-equity agency conflicts, leverage ratchet effect, financial contracting, debt maturity.

1 Introduction

Maturity, as shown by an extensive theoretical and empirical literature, is a key aspect of a debt contract.[1] Debt maturity affects the interest tax shield, credit risk, future refinancing costs, and agency conflicts on future investments. DeMarzo and He (2021) set a higher benchmark for the value contribution of debt policy by showing that a static tradeoff may not hold in a dynamic setting: Even when debt would be advantageous, the shareholders' continuous attempt to gain from trading it undermines their credibility on restraining future debt issuances, making debt financing irrelevant for firm value. As a consequence, debt maturity also is ex ante irrelevant.

In a dynamic setting without commitment, analyzing whether debt maturity is relevant to financial contracting requires that some frictions be either added to (or removed from) the model considered by DeMarzo and He (2021). There are now several analyses of the role of maturity under no commitment: Dangl and Zechner (2021), reinstate shareholders' commitment to future debt policies by adding a debt covenant and find that debt maturity is relevant because a shorter maturity reduces credit risk and increases the benefit of debt financing. Benzoni et al. (2022) add debt issuance costs to DeMarzo and He (2021), and find that in equilibrium the choice of debt maturity trades off tax benefits against issuance costs. Finally, Malenko and Tsoy (2021) consider a time-invariant equilibrium concept based on a trigger strategy, which is different from the one in DeMarzo and He (2021), and show that a short maturity creates value by committing the shareholders to repay the debt when the firm, after a negative shock, is in financial distress and there is no other credible actions the shareholders can take.

To date, there has been no research that shows whether and how the choice of debt maturity can alleviate *per se* the shareholders' commitment problem to future debt policies. For this purpose, we develop a parsimonious dynamic discrete-time model of a firm, exposed to i.i.d productivity shocks (in an extended version of the model there is also a persistent productivity shock), which over time invests in depreciating capital stock, subject to adjustment costs, and issues debt and equity in an efficient and frictionless financial market. Debt financing is favored by the tax shield on interest payments. The debt is an unprotected long-term contract with given (average) maturity gauged by the contractual amortization parameter, ξ.

[1] A detailed literature review is offered at the end of the Introduction.

We assume two commitment problems: shareholders cannot commit to *repay the debt* and cannot commit to *future debt and investment policies.* As per the first, if shareholders default on debt, there is no recovery by the debt holders (in an extended version we analyze also the case with non-zero recovery and pari passu debt). As per the second commitment problem, debt issuances and repurchases are at the current debt price. Debt trades and investment or liquidation of assets can occur only at discrete dates and only once per period and aim at maximizing the current equity value. The price of debt immediately reflects the current action and the expectation of future actions by the shareholders. In line with Aguiar et al. (2019) and DeMarzo and He (2021), the solution of the game between the shareholders and the debt holders is based on the Markov Perfect Equilibrium (MPE), in which debt price depends only on fundamentals.

The model allows to make several novel contributions. First, debt maturity is *per se* able to limit the shareholders' incentives to implement time-inconsistent debt issuances. Even if upward ratcheting incentives are in principle strong, as in DeMarzo and He (2021), their effects are mitigated when debt maturity is short, restoring a dynamic trade-off and a target leverage for the firm. This is because, absent other frictions that may favor a longer debt maturity, such as as debt issuance costs and refinancing costs, the shorter the maturity the lower the long-run leverage of the firm, the faster the upward leverage adjustments following a positive profitability shock, and the less persistent the leverage dynamics. Therefore the shorter the maturity the higher the ex ante value of dynamic capital structure for shareholders. These predictions are in line with the empirical literature documenting an ex ante positive value of debt financing (e.g., Korteweg (2010) and van Binsbergen et al. (2010)) and the existence of a target leverage from which firms deviate in response to investment shocks and to which firms slowly mean-revert (e.g., DeAngelo et al. (2011)).

Second, the MPE in our model is such that, absent offsetting forces, for any debt maturity longer or equal to one year, the shareholders of an unlevered firm are always better off issuing debt as long as the tax benefits are higher than the expected bankruptcy costs. This is different from DeMarzo and He (2021), who find two MPE's: the permanently unlevered equilibrium for a currently unlevered firm, and the leverage ratcheting equilibrium for a firm that has already debt in place.

Third, we show that, for a highly levered firm, the leverage ratchet effect induces underinvestment not only through the well known debt overhang channel of Myers (1977) and Hennessy (2004), but also by reducing the marginal q owing to a lower value of debt financing. A shorter debt maturity alleviates the problem by increasing the marginal q (and Tobin's Q as well) and lowering the correlation of investment with negative productivity shocks. Indeed, we show that if debt maturity is sufficiently short, shareholders decide to finance positive NPV projects for the firm using equity, despite debt overhang.

Fourth, we show how leverage dynamics are affected by the relative maturities of debt and asset, gauged respectively by the debt amortization rate and the asset depreciation rate. In particular, if debt maturity is higher than asset maturity, the limited commitment equilibrium displays ever increasing leverage dynamics: underinvestment becomes increasingly more severe (and is accompanied by cashing out) and the rapid decline of capital stock accelerates the increases in leverage. Overall, our analysis shows that models with constant investment policy underestimate the full extent of the leverage ratchet effect, and therefore the contribution of debt maturity to firm value.

Our analysis shows that debt maturity plays a role in alleviating the commitment problem on debt repayments and the commitment problem on future debt issuances. Given this dual role, we separate the component of credit spread that is solely due to shareholders' lack of commitment to debt repayment, from the component that is solely driven by time-inconsistent debt issuances. This allows us to determine under what circumstances one component is bigger than the other, and how and if debt maturity is a more or less effective device against either commitment problems. We show that, everything else equal, the agency component of credit spread is higher when the leverage is low, and that a shorter debt maturity can significantly reduce it both in absolute and relative terms.

The paper has the following structure: In Section 2 we position our contribution in the debt maturity literature and the recent literature on commitment in leverage decisions. In Section 3 the model is introduced. In Section 4 we analyze the MPE. In Section 5 we derive the main predictions of the model, focussing on leverage policy, investment, and firm dynamics. In Section 6 we show how the cost of debt can be decomposed into a part that reflects the commitment problem on debt repayment and a part that depends on the commitment problem on future debt and investment policies. Section 7 presents the concluding remarks. All derivations and proofs are in Appendix, where we offer also additional benchmark models, additional discussions and robustness checks.

2 Literature review

There is an extensive theoretical and empirical literature on corporate debt maturity. As for the theoretical contributions, in Leland (1994) and Leland and Toft (1996) a longer debt maturity is optimal, as it allows the shareholders to capture a higher interest tax shield. Myers (1977) predicts that debt-equity agency conflicts make shorter maturity preferable for firms with valuable growth options. However, Diamond and He (2014) show that in some conditions a shorter maturity can lead to more severe debt overhang. Diamond (1991, 1993) argues that a shorter maturity allows firms with valuable future projects to benefit from the expected improvement of their credit standing, but exposes them to higher refinancing risk and sensitivity of financing costs to new information. Refinancing risk is the focus of the models by He and Xiong (2012) and He and Milbradt (2014). Recently, Hu et al. (2022) show how long-term debt creates a natural tool to share risk between shareholders and long-term debt holders, in a downturn, if the firm is far from default, whereas short-term debt mitigates the leverage ratchet effect because it is constantly repriced. Long-term debt has therefore a risk-management role, as it reduces default risk, which is valuable ex ante in the presence of bankruptcy costs. As for the empirical literature on corporate debt maturity, Guedes and Opler (1996), Barclay and Smith (1995), Stohs and Mauer (1996) study the determinants of debt maturity choices, and find that larger firms with more growth opportunities tend to have shorter debt maturity to reduce Myers (1977) the debt overhang issues, also firms with higher credit quality borrow more at the short end of the maturity spectrum. Differently with this literature, our analysis focusses on the effect that debt maturity has on the commitment to future debt and investment decisions.

Relative to the recent theoretical literature on leverage policies without commitment, we depart from the setting in DeMarzo and He (2021) by assuming that debt trades occur only in predetermined discrete dates, with only one trade of arbitrary size occurring in each of these dates. Hence, our setting is in discrete time, similarly to Aguiar et al. (2019) for sovereign debt, although differently from them we do not allow the firm to trade in one-year and long term debt simultaneously, but restrict all the debt to have the same maturity.

Different from Benzoni et al. (2022) and Malenko and Tsoy (2021), our solution concept is MPE, which is the equilibrium giving the lowest possible equity value. This choice suites us, because if we show that debt maturity matters in our setting, it will do so also under equilibrium concepts in which the ex ante value of equity is higher. Our choice of the equilibrium concept has also an empirical motivation: while a time-consistent equilibrium which satisfies a credibility constraint, like the one in Benzoni et al. (2022) and Malenko and Tsoy (2021), is an important benchmark for the value that a dynamic capital structure can create, the relevance of a MPE is confirmed by the fact that in practice several types of commitment devices, like covenants, collateral, and credit risk insurance, are used. That means that, absent such devices, the commitment problem cannot be easily address solely based on reputation concerns arising from a repeated interaction of the firm with the capital markets.

We analyze debt maturity as a device against commitment problems on future debt issuances. Such a role in our model is in addition to the role, already analyzed by Dangl and Zechner (2021), played against shareholders' lack of commitment to debt repayment, whereby a shorter debt maturity increases the likelihood of debt repayments.[2] Because Dangl and Zechner (2021) remove the shareholders' commitment problem to debt policies by introducing a covenant that forces shareholders to repay all outstanding debt when they want to increase the debt, the question of whether the debt maturity can control the commitment problem to the debt policy cannot be addresses in their model.

In Dangl and Zechner (2021) and Benzoni et al. (2022), an interior solution to the optimal debt maturity problem emerges because they assume an offsetting force, debt issuance costs, which makes short debt maturity costly. Because we are not interested in finding an interior optimal debt maturity, we do not make the same assumption of costly debt issuance, although we would get an interior solution if we also added such a cost.

Benzoni et al. (2022) study the role played by debt issuance costs as a commitment device in the setting by DeMarzo and He (2021). As we show that the choice of debt maturity in such a setting is per se an important device against shareholders' lack of commitment to future policies, our analysis is in a way complementary to the one by Benzoni et al. (2022). In Malenko and Tsoy (2021), the role of debt maturity is to discipline debt reductions when the firm is in financial distress. In our model the mitigating

[2]In a static setting, the value implication of debt maturity is solely due to the effect of lack of shareholders' commitment on debt repayment. To analyze the commitment problem on future debt issuances require a dynamic setting.

effect of debt maturity is always present, also when there is an upturn and shareholders issue new debt.

3 The model

The model of the firm is in discrete time, with dates $t = 0, 1, \ldots$, and we assume that the investors (shareholders and debt holders) are risk neutral. The firm has capital stock k, with law of motion $k' = I + (1 - \delta)k$, where $\delta > 0$ the depreciation rate and I is the amount invested, which has unrestricted sign. When the firm invests, the adjustment cost is $\Psi(k, k') = \varphi k(I/k)^2/2$, with $\varphi > 0$. The after corporate tax cash flow function is $(1 - \tau)(y + x)k$, where x is an idiosyncratic i.i.d. shock with density $\phi(x)$, support $[\underline{x}, \overline{x}]$ with $\underline{x} < 0 < \overline{x}$ such that $\mu_x = \int_{\underline{x}}^{\overline{x}} x\phi(x)dx = 0$, and $y > 0$ is a persistent shock, which follows a Markov chain on a compact support.

We assume the firm can issue long-term debt with face value b and coupon rate $r = 1/\beta - 1$, where β is the risk-free discount factor.[3] Debt financing is incentivized by tax-deduction of coupon payments. Each period, the firm commits to repay a fraction ξ of the debt at face value. The parameter ξ can therefore be interpreted as the inverse of the average debt maturity (if the debt remains unchanged and there is no default).

The equity holders optimally decide when to default on the debt payments by maximizing the equity value. At default, the absolute priority rule applies and equity holders get nothing. As done in similar contributions, to focus on the dilutive effect of debt issuance via the default probability, rather than via reduction of the recovery at default, in the baseline model we assume that nothing is recovered at default.[4] At all future periods and *only once per period*,[5] given the current state (b, k), equity holders maximize their value by deciding to change the debt level to b' (i.e., issue new debt or repurchase outstanding debt) at the resulting market price of debt, p. All debt issuances

[3]This ensures that the debt trades at the face value when it is risk free.

[4]We will also illustrate the case with non-zero recovery at default, under the assumption of *pari passu debt* issuance.

[5]Given our discrete-time setting, this is a key assumption. The alternative scenario would be that multiple debt trades are possible in each period, which would create the issues described in Bizer and DeMarzo (1992), ultimately making debt amortization, ξ, ineffective as a commitment device.

in the model use the same debt contract.[6] We assume there are no frictions on debt and equity trades to raise (disburse) capital from (to) the financial market. Therefore, like other contributions in this literature (e.g., Aguiar et al. (2019) for sovereign debt and DeMarzo and He (2021) for corporate debt) our model has two main frictions: lack of commitment to repay the debt and lack of commitment to future debt issuances.

Conditional on the firm being solvent at (b, k, y), the levered equity value is[7]

$$V(b,k,y) = \max_{(b',k')} -[k' - (1-\delta)k] + \tau\delta k - \Psi(k,k') + [b' - (1-\xi)b]\, p(b',k',y) + \beta\mathbb{E}_y\left[\int_{x_d(b',k',y')}^{\overline{x}} \{(1-\tau)(y'+x')k' - [(1-\tau)r + \xi]\, b' + V(b',k',y')\}\, \phi(x')dx'\right], \quad (1)$$

with default threshold

$$x_d(b',k',y') = r\frac{b'}{k'} + \frac{\xi b' - V(b',k',y')}{(1-\tau)k'} - y'. \quad (2)$$

The program in (1) reflects the fact that the equity holders cannot credibly commit to future decisions on debt and capital stock, rather at each date these are decided to maximize the current equity value.

The price of debt is

$$p(b,k,y) = \beta\mathbb{E}_y\left[\{r + \xi + (1-\xi)p(b',k',y')\} \int_{x_d(b,k,y')}^{\overline{x}} \phi(x')dx'\right], \quad (3)$$

where $(b', k') = G(b, k, y')$ is the optimal policy of the equity program in (1).

3.1 A stationary program

The model in (1)-(3), which is based on state variables (b, k), is non-stationary. However, given the linearity of the production function and the assumption made on capital

[6] As pointed out by Benzoni et al. (2022), shareholders may not be able to commit to the maturity of debt contract issued in the future. Because we want to analyze the effect of debt maturity choice on the shareholders' lack of commitment to the leverage policy, for tractability, like many other contributions in the literature, we assume that ξ is set at $t = 0$ and never changed thereafter.

[7] Here and thereafter, the lower limit of integration is actually $x_d(b', k', y') \vee \underline{x}$, which we shorten for the convenience of exposition.

adjustment cost, V is homogeneous of degree one and p of degree zero in (b, k). Because we must calculate the Markov perfect equilibrium numerically, we need a *stationary* program that can be solved numerically based on value iteration techniques. Stationarity is achieved by using book leverage, $\ell = b/k$, as state variable.

Therefore, instead of the equity program in (1), for the optimal k' described later on we solve the auxiliary *leverage program*,

$$v(\tilde{\ell}, y) = \max_{\ell'} \left[\ell' - (1-\xi)\tilde{\ell}\right] p(\ell', y) + \beta \mathcal{V}(\ell', y), \tag{4}$$

where $\tilde{\ell} = b/k'$, $\ell' = b'/k'$, and with

$$\mathcal{V}(\ell', y) = \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \{(1-\tau)(y' + x' - r\ell') - \xi\ell' + V(\ell', y')\} \phi(x') dx' \right] \tag{5}$$

denoting the continuation value of equity. In this expression, $x_d(\ell, y) = r\ell + (\xi\ell - V(\ell, y))/(1-\tau) - y$ is the default threshold expressed as a function of current leverage, ℓ, and $V(\ell, y) = V(b, k, y)/k$ with a small abuse of notation. From (4), we find the optimal leverage policy, $\ell' = g(\tilde{\ell}, y)$, and the equity value per unit of capital stock, $v(\tilde{\ell}, y)$.

The debt price for given current leverage, ℓ, can be written as $p(b, k, y) = p(b/k, 1, y) = p(\ell, y)$, where

$$p(\ell, y) = \beta \mathbb{E}_y \left[\{r + \xi + (1-\xi)p(\ell', y')\} \int_{x_d(\ell, y')}^{\overline{x}} \phi(x') dx' \right]. \tag{6}$$

This price is based on the end-of-period leverage decision $\ell' = g(\ell/\kappa, y') = g(b/k', y')$, where the investment decision $k'/k = \kappa = h(\ell, y')$ is described below. Given $V(\cdot, \cdot)$ and $\kappa = h(\cdot, \cdot)$, we solve (4) and (6) simultaneously to determine the debt price, $p(\ell, y)$, the optimal leverage policy function, $\ell' = g(\tilde{\ell}, y)$, and $v(\tilde{\ell}, y)$.

To determine the optimal investment policy, $\kappa = h(\ell, y)$, we solve the program

$$V(\ell, y) = \max_{\kappa} \; -[\kappa - 1 + (1-\tau)\delta] - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 + \kappa \, v(\ell/\kappa, y). \tag{7}$$

The details of the derivation of the recursive system (4)-(7) are in Appendix A. From the equilibrium solution of (4)-(7) we then recover the equilibrium for (1)-(3).

4 Markov perfect equilibrium

Like Aguiar et al. (2019) and DeMarzo and He (2021), among others, we focus on an equilibrium concept, Markov perfect equilibrium (MPE), whereby the firm policy depends only on the payoff-relevant state variables, and the policy is time-consistent, in that it is a fixed point in which future policies are the same as this period's policy, and depends only on the state.

Specifically, the MPE of the firm model is defined by security prices, $V(b,k,y) = kV(b/k,y)$ and $p(b,k,y) = p(b/k,y)$, and optimal policy $(b',k') = G(b,k,y')$, such that $k' = kh(b/k,y)$ and $b' = k'g(b/k',y)$, where the security prices reflect the expectations regarding the optimal policy, and given the security prices the equity holders will not deviate from said policy.

The MPE is calculated by solving equations (4) and (6) simultaneously, using a value function iteration approach. At a given step of the iterative procedure, based on $V(\ell,y)$ and κ from the previous step, we find $v(\ell/\kappa,y)$ and $\ell' = g(\ell/\kappa,y)$ from (4), and we calculate $p(\ell,y)$ in (6). Then, in (7) we solve the first-order condition for optimal investment, which requires a numerical approach, as κ is defined implicitly, and the functions in the equation are defined numerically. This is done by using a spline interpolation of $v(\ell/\kappa,y)$, which allows the calculation of its first derivative. This step finds $\kappa = h(\ell,y)$ and $V(\ell,y)$, from which a new iteration is started. The iterative procedure is halted when the maximum between the improvement of value function and the improvement of debt price is lower than a given tolerance. More details of the numerical algorithm are in Appendix F.

We state the following proposition. The proof is in Appendix E.

Proposition 1. *1. For a given y, ℓ, and κ:*[8]

(a) $v(\ell/\kappa,y)$ is convex in ℓ/κ and $V(\ell,y)$ is convex in ℓ;

(b) assuming $v(\cdot,y)$ is differentiable, $p(\ell,y)$ is a decreasing function of ℓ, that is $\partial_\ell p < 0$;

[8] The notation $\partial_x f(\hat{x},y)$ for the partial derivative is to indicate the variable, x, with respect to which the derivative of the function, f, is calculated and the point, $(\hat{x},y)$, in which it is evaluated.

(c) if the objective function in (4) *is concave with respect to* ℓ',[9] *there is a unique equilibrium policy function,* $\ell' = g(\ell/\kappa, y)$ *that satisfies condition*

$$\left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] \partial_\ell p(\ell', y) + \beta\tau r \; \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \phi(x')dx'\right] = 0. \tag{8}$$

The function $g(\ell/\kappa, y)$ *is strictly increasing in* ℓ/κ*, that is* $\partial_\ell g > 0$.

(d) The equilibrium investment policy, $\kappa = h(\ell, y)$*, satisfies condition*

$$v(\ell/\kappa, y) + \frac{\ell}{\kappa} p(\ell', y) = 1 + \varphi\left[\kappa - (1-\delta)\right], \tag{9}$$

where the left-hand side of (9) *is marginal* q*. The function* $h(\ell, y)$ *is decreasing in* ℓ*, that is* $\partial_\ell h < 0$.

2. *When the debt is risk free (i.e.,* $x_d < \underline{x}$*), the equilibrium investment and leverage policy in the MPE maximizes the value of the firm (equity plus debt). Hence, there is no agency conflict.*

3. *For one-year debt,* $\xi = 1$*, the equilibrium investment and leverage policy in the MPE maximizes the value of the firm (equity plus debt). Hence, there is no agency conflict.*

To comment, Point 1.c describes a characteristic of the equilibrium leverage policy with limited commitment defined by Admati et al. (2018) as *leverage ratchet effect* (LRE). The first-order condition (8) is indeed the same as the one by DeMarzo and He (2021) in a continuous-time setting, with the generalization here to endogenous investment, although our setting is in discrete time, similarly to Aguiar et al. (2019) model for sovereign debt.[10] Point 1.c shows that our MPE features the same behavior.

The first order condition for investment, in Point 1.d, has the usual neoclassical structure, with a positive marginal q on the left-hand side. On the one hand, the debt policy has a positive first order effect on investment because part of the marginal value derives from the tax shields allowed by the positive cash flows generated by physical capital. On the other hand, condition $\partial_\ell h < 0$ shows that investment is negatively

[9]In the baseline case of zero recovery at default, this condition is satisfied if $p(\ell, y)$ is a concave function of ℓ.

[10]In Appendix E, we generalize it to the case of non-zero recovery at default.

Table 1: Base case parameters

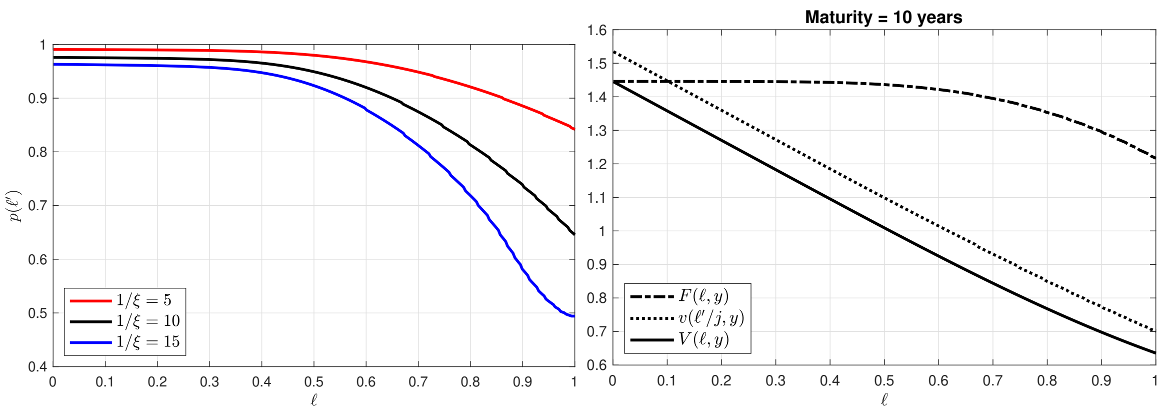
symbol	description	value
β	discount factor	0.95
X	iid profitability parameter	1.6
τ	corporate tax rate	0.20
φ	capital adjustment cost	6
δ	depreciation rate	0.085
μ_y	average persistent profitability	0.21
ν	autocorrelation persistent profitability	0.9
σ	volatility persistent profitability	0.025

affected by current leverage, as expected. However, the underinvestment mechanism in our model is different from debt overhang, in the sense of Myers (1977) and Hennessy (2004), which is owing to truncation of equity horizon due to default. In our model, the higher the leverage the lower the value of the interest tax shield, as reflected by a lower debt price. Therefore the lower investment is due to lower marginal q. A second effect of leverage on investment is via debt maturity: as we will see later, for a given ℓ, investment is higher the higher ξ (i.e., the shorter the maturity), because for higher ξ, the price of debt is higher, everything else equal. Overall, underinvestment in our model is driven by the leverage ratchet effect.

Point 2 confirms, in our setting, the known fact that debt-equity agency conflicts are present only when debt is risky. Hence, the risk of default is necessary for the equity holders to have an incentive to adopt a self-serving leverage policy, which then triggers underinvestment. Finally, the intuition of Point 3 is immediate: because with single-period debt the price of new debt always reflects the possible end-of-period conflicts regarding investment and financing, the cost of such distortions is fully paid by shareholders, who therefore refrain from deviating from a value maximizing policy. Hence, firm value maximization coincides with equity value maximization when the debt is single-period, $1/\xi = 1$.

Figure 1: MPE - value functions

The figure plots against leverage, $\ell = b/k$, (left panel) the debt price, $p(\ell', y)$, for maturities $1/\xi = 5, 10, 15$ years; (right panel) the value of equity, $V(\ell, y)$, the debt price, $p(\ell', y)$, and the corresponding total firm value, $F(\ell, y) = V(\ell, y) + (1-\xi)\ell p(\ell', y)$, for maturity $1/\xi = 10$ years. The debt price is at the optimal leverage policy, $\ell' = g(\ell/\kappa, y)$, and $\kappa = h(\ell, y)$ is optimal investment. The MPE is calculated assuming that y is non-stochastic. The calculation is based on $k = 1$, and the parameters in Table 1.



5 Results

In this section, we illustrate various aspects of the equilibrium quantities and policies. We set the model on an annual basis and choose a baseline calibration as in Table 1. The discount factor is in line with similar discrete-time settings (e.g., Cooley and Quadrini (2001)). The depreciation rate is 0.85, at the middle point between the estimation by Hennessy and Whited (2005) and the value chosen by Cooley and Quadrini (2001). The corporate tax rate, τ, is set at 20%, as in Benzoni et al. (2022). As for the investment side of the model, given δ, τ, and $r = 1/\beta - 1$, the capital adjustment cost parameter, φ, and the average of the persistent component of profitability, μ_y are jointly chosen to satisfy condition (22), which ensures identity of the analytic solution and numerical solution for the unlevered version of the model, as explained in Appendix B. The parameters ν and σ of the persistent component of profitability, which will be relevant only when simulating economies, are chosen in line with Gomes and Schmid (2021). Finally, X is chosen given the other parameters to have a reasonable one-year default probability of 0.7% for the debt with 10 year maturity.

In this part, to isolate the main features of the equilibrium, we assume initially that y is non-stochastic, and relax this assumption later on. Figure 1 shows the equilibrium values of corporate securities against current leverage, $\ell = b/k$. The left panel shows that the debt price is decreasing and concave with respect to current leverage, and for a given leverage it is decreasing in the maturity, $1/\xi$. On the right panel, for a maturity $1/\xi = 10$,[11] equity and total firm value are strictly decreasing in ℓ similarly to DeMarzo and He (2021), which suggests the firm would be better off with less debt, contrary to the prediction of traditional models based on commitment to the debt policy (e.g., Goldstein et al. (2001)). As stated in Proposition 1, equity price is a convex function of ℓ.

Figure 2 shows the equilibrium policies against ℓ, for three different maturities, $1/\xi = 5, 10, 15$ years. Investment is positive, $\kappa > 1 - \delta$, and as anticipated in Proposition 1, it declines for higher leverage and longer debt maturities, in response to a lower debt price. Although investment and leverage policies are decided simultaneously and linked to one the other via the price of debt, to simplify the exposition we first focus on the leverage policy, assuming investment as given, and next we will show the investment policy.

5.1 Leverage policy

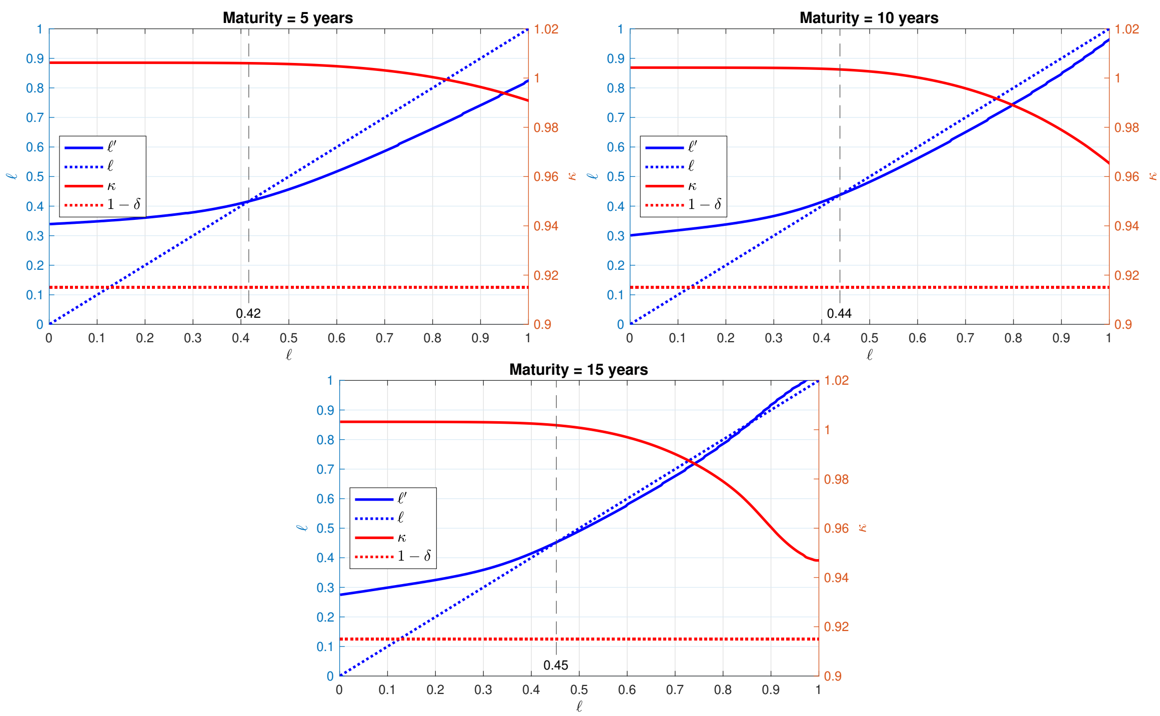
Given the investment decision made at ℓ, denoted by $\kappa = k'/k$, in equilibrium there is always an incentive to increase the leverage. That is ℓ' is increasing in $\ell/\kappa = b/k'$, as noted in Proposition 1. In Figure 2, by plotting where g crosses the 45 degree line, we single out the equilibrium long-run leverage of the firm, denoted by $\hat{\ell}$, as the fixed point of g, that is the solution of equation $\ell = g(\ell/\kappa, y)$. Depending on $1/\xi$, there can be more than one solution: for instance, for $1/\xi = 15$, in the interval $[0, 1]$, there are two fixed points: one at .45 and another at .85.[12] The two fixed points are quite different because at the lower one the function $g(\ell/\kappa, y)$ is convex in ℓ/κ, whereas at the higher one g is concave, which has implications for the leverage dynamics.

[11]This plot is qualitatively the same for different maturities. Hence, we show only $1/\xi = 10$ for brevity.

[12]For high $1/\xi$, there can be more fixed points when extending the interval of ℓ above 1. We do not consider those cases, to limit our analysis to plausible levels of leverage.

Figure 2: MPE - policy functions

The figure plots against $\ell = b/k$, the optimal leverage policy, $\ell' = g(\ell/\kappa, y)$, on the left axis and the equilibrium investment policy, $\kappa = h(\ell, y)$, on the right axis. The equilibrium is calculated assuming that y is non-stochastic. We plot also the 45 degree line, to determine the long-run leverage, $\hat{\ell}$, as the fixed point of g, that is the solution of $\ell = g(\ell/\kappa, y)$. The calculation is based on $k = 1$, the parameters in Table 1, and maturities $1/\xi = 5, 10, 15$ years.



Indeed, depending on whether there is one or more such fixed points (or none), and depending on current leverage, the leverage dynamics of the firm can be of three types: (1) for low current leverage (lower than the lowest $\hat{\ell}$), the leverage adjusts towards the long-run average; (2) for high leverage and low maturity, so there is only one $\hat{\ell}$ and the current leverage is higher than $\hat{\ell}$, the leverage adjusts downward towards $\hat{\ell}$; (3) for high leverage and high maturity, there are two fixed points, say $\hat{\ell}_1 < \hat{\ell}_2$. If the current leverage is lower than $\hat{\ell}_2$, the leverage adjusts downward towards $\hat{\ell}_1$, whereas if the ℓ is higher than $\hat{\ell}_2$, the leverage diverges to 100%.[13]

We compare the leverage policy for different debt maturities in Figure 2. When maturity is low, the speed of mean reversion towards $\hat{\ell}$ is higher (i.e. the slope of $g(\ell/\kappa)$ is lower). To appreciate the result, some comments are in order. It is well understood that the incentives that give rise to leverage ratcheting are *asymmetric*: shareholders actively increase leverage when it is lower than $\hat{\ell}$, but debt reductions towards $\hat{\ell}$ occur only via debt amortization, which is mechanically faster for shorter debt maturities. The figure shows that, for short maturity, $g(\ell/\kappa, y)$ is still an increasing function of ℓ/κ for $\ell > \hat{\ell}$, but ℓ' is lower than what needed to offset debt amortization, ξ, over the year. Surprisingly, Figure 2 shows that also the speed of *upward* adjustment is increasing in ξ. This is a consequence of the higher marginal value of new debt issuance when commitment increases for higher ξ. While also Dangl and Zechner (2021) highlight the effect of maturity on downward debt adjustments, the positive effect of shorter maturity on upward adjustments, which is a consequence of the maturity's role as a commitment device against time-inconsistent debt policies, is unique to our setting.[14]

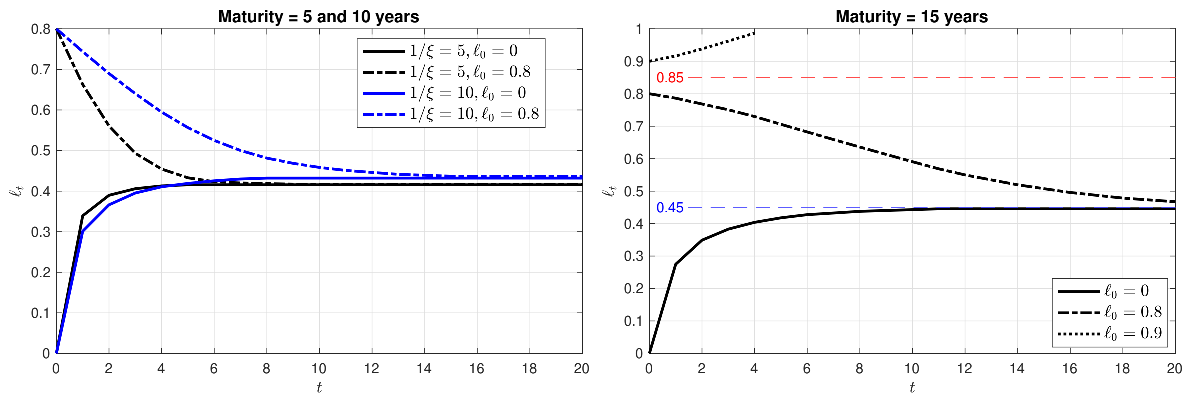
Figure 2 shows also that $\hat{\ell}$ (when there is only one fixed point) is increasing in $1/\xi$. That is, the long-run leverage is higher the higher the debt maturity. Indeed, in the equilibria exemplified in Figure 2, for $1/\xi = 5$, $\hat{\ell} = .42$ and for $1/\xi = 10$, $\hat{\ell} = .44$. This can be explained by the incentives on the leverage policy described before. As noted above, for higher $1/\xi$, the slope of ℓ' is higher, which makes downward mean reversion

[13] In unreported results, for very high debt maturity (in the current calibration, $1/\xi > 40$) the equilibrium leverage policy is such that $g(\ell/\kappa, y) > \ell$ for all $\ell \in [0, 1]$, and there is no fixed point. Also in this case, the leverage diverges to 100%.

[14] Dangl and Zechner (2021) exclude time-inconsistent debt policies from their setting by forcing the firm to repurchase all existing debt before any leverage increase.

Figure 3: MPE - leverage dynamics

The figure plots, in the left panel, for the cases $1/\xi = 5, 10$, the dynamics of leverage for a firm with initial leverage equal to zero (solid lines), or 0.8 (dashed-dotted); in the right panel, for the case $1/\xi = 15$, the dynamics of leverage for a firm with initial leverage equal to zero (solid lines), or 0.8 (dashed-dotted), or 0.9 (dotted). These paths are conditional on the firm being solvent at the beginning of each period. The simulations are based on $k_0 = 1$, and the parameters in Table 1, assuming that y is non-stochastic and constant.



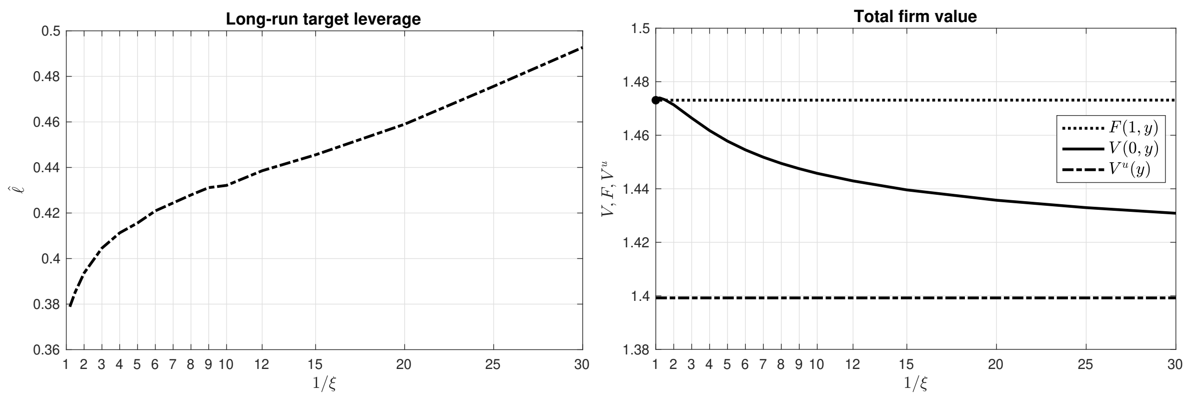
slower and only partially offsetting the leverage increases. The result is a higher long-run leverage, $\hat{\ell}$.[15]

To further illustrate these points, Figure 3 shows the dynamics of leverage, starting from different leverage, for the cases $1/\xi = 5, 10$ (left panel), in which there is only one fixed point of $\hat{\ell} = g(\hat{\ell}/\kappa, y)$, and for the case $1/\xi = 15$ (left panel), in which there are two fixed points. The left panel shows that for lower $1/\xi$, in which there is only one fixed point, the speed of mean-reversion is lower as a shorter maturity is equivalent to a faster debt amortization. The figure confirms also that for lower maturity, the upward speed of adjustment is higher. The resulting long-run leverage is higher the higher $1/\xi$. The right panel confirms that the leverage converges to $\hat{\ell}$ if $\ell_0 < .85$, the highest solution, but diverges if $\ell_0 > .85$. The left panel in Figure 4 confirms, for a large set of debt maturities, that the long-run leverage for a firm starting at zero-leverage, is monotonically increasing in $1/\xi$.

[15] In unreported numerical results we show that for very high debt maturity, including $\xi = 0$, corresponding to infinite maturity debt, we find $\hat{\ell} = 1$ under the same assumptions as in Figure 2, which is explained by the fact that for very low $\xi = 0$ downward mean reversion of leverage becomes very weak or absent.

Figure 4: MPE - effect of debt maturity

The left panel shows the long-run target leverage for a firm starting at zero leverage, $\hat{\ell}$ against average debt maturity $1/\xi$. The right panel shows the value of unlevered equity, $V(0, y)$, which coincides with total firm value, under no commitment (solid), commitment to value maximizing leverage policy (dotted), and in the case with permanently no debt, $V^u(y)$, (dashed) against debt maturity $1/\xi$. We assume y is non-stochastic and constant, $k_0 = 1$, and the parameters in Table 1.



The continuous-time equilibrium found by DeMarzo and He (2021) displays a non-monotonic relation between debt maturity and target leverage. If we were to show the relation between end-of-period target leverage (that is $(1-\xi)\hat{\ell}$) versus $1/\xi$, we would also find a U-shaped curve. The non-monotonic relation is due in our case to the dual role of the debt maturity parameter, ξ, which sets both the ex ante incentives and the speed of debt amortization: For high ξ the commitment to debt repayment is high, which increases debt capacity. For $\xi < 1$ such a commitment is reduced and the speed of downward adjustment noted above is quite strong, which results in a relatively lower target leverage. For even lower ξ, that is higher maturity, the debt capacity is lower, the dominant effect is the reduced downward adjustment via amortization, which produces a higher target leverage.

DeMarzo and He (2021) prove that in continuous time, where it is only optimal to adopt a smooth leverage policy, the MPE value of equity is independent of debt and equal to the value of a firm permanently excluded from the debt market. Because of the irrelevance of debt for the ex ante equity value, then also the debt maturity is irrelevant ex ante. As DeMarzo and He (2021) explain,[16] the debt irrelevance result is an artifact of the continuous-time setting, which cannot be a realistic benchmark for how real-life firms operate. In our discrete-time setting, we can assess the role that maturity plays on the ex ante value of the firm, before any debt is issued. In the right panel of Figure 4, we show that, in a world in which y is non-stochastic and constant, with no commitment to the leverage policy, the value of equity (and of the firm) currently at zero leverage, $V(0, y)$, is decreasing with debt maturity. The highest value is found for $1/\xi = 1$ (denoted by *), the limiting case in which full commitment is restored, as stated in Proposition 1. This shows that, in a model with lack of commitment to the leverage policy, the maximum ex ante value is achieved when the debt amortization parameter, the commitment device in our model, is at the maximum level.[17] A corner solution with respect to debt maturity emerges because in our model no other frictions, such as refinancing costs, offset the positive effect of shorter debt maturity. This result is consistent with the findings in Benzoni et al. (2022) in continuous-time, who show that, in the case with vanishing fixed debt issuance cost, the optimal maturity is zero.

Figure 1 showed that if debt is already in place, equity and firm value are decreasing in leverage. Can we then conclude that the shareholders of a currently unlevered firm are better off not issuing any debt? In the right panel of Figure 4 we report the value of the firm that remains *permanently* unlevered, $V^u(y)$.[18] The figure shows that the opposite conclusion is true, as the shareholders of an unlevered firm would benefit from issuing debt. Indeed, the benefit is maximum if the (lack of) commitment problem to the debt policy is absent, which happens if $1/\xi = 1$, but it remains positive for the range of debt maturities that we consider in the figure, for which only partial commitment is possible, although diminished for higher $1/\xi$.[19]

[16]This is confirmed also by DeMarzo (2019), who shows that the ex ante gain from trade in discrete time is completely dissipated in the continuous time limit.

[17]The case $\xi = 1$ has full commitment because we assume that only one debt issuance can take place in each period. The solution for the limiting case $\xi = 1$ is based on a modified algorithm based on the version of the model presented in Appendix E in the proof of Proposition 1. The case $\xi = 1$ in our setting is equivalent to the case of the debt amortization parameter going to infinity (i.e., zero maturity) in continuous time, as they both correspond to complete repayment of the outstanding debt.

[18]This value is derived analytically in Appendix B, and is different from $V(0, y)$, which is the value of a firm that is at the moment unlevered, but given it has access to the debt market, it will issue debt right after.

[19]Only for infinite-maturity debt, corresponding to $\xi = 0$ (we do not report it in the figure for scale reasons), in which the commitment problem is maximum, the value is $V(0, y) = 1.408$, which shows that shareholders's benefit of issuing any debt remains positive but negligible. Although $V(0, y) - V^u \approx 0$ when $\xi = 0$, we cannot rule out that the positive difference is only due to numerical reasons.

The fact that, even if the commitment problem is severe, shareholders of an unlevered firm are better off start issuing debt is in contrast with DeMarzo and He (2021), who find two MPEs: the permanently unlevered equilibrium for a currently unlevered firm; the leverage ratchet effect equilibrium for a firm that has already debt in place.[20] To understand this result, recall Figure 3 which shows that an unlevered firm will not jump to the long-run leverage at once, but will gradually build up debt, the upward adjustment being slower the higher $1/\xi$. Along this path, while increasing the leverage net of the committed debt repayment, the shareholders will capture the incremental tax shield, but will not endogenize the increased bankruptcy costs for legacy debt. The present value of the benefits captured by the shareholders is the difference between $V(0, y)$ and $V^u(y)$. Figure 4 shows that such benefits are lower the longer the maturity and the bigger the commitment problem, because for higher $1/\xi$ the accrual of tax shield via new debt issuances is slower. Yet, these benefits are strictly positive.[21]

In the right panel of Figure 4, we plot also the total ex ante firm value in the case with full commitment to the firm value maximizing debt policy, denoted by $F(k, y)$, which equals $V(0, y)$ for $1/\xi = 1$.[22] While the value of the firm is much less sensitive to debt maturity with commitment than without commitment, the right panel of Figure 4 shows it is slightly increasing with respect to maturity. This happens in our model because with commitment a higher $1/\xi$ reduces the default threshold, which has a positive effect on firm value.[23]

5.2 Investment policy

Figure 5 plots the investment policy, the leverage policy, and the value of corporate securities against current leverage, for different debt maturities, $1/\xi$, for the baseline case with no commitment, as already reported in Figure 1, and for three different benchmark models. In the first, the firm is excluded from the debt market and remains permanently

[20]In DeMarzo and He (2021), the switch between the first and the second equilibrium occurs only via a suboptimal debt issuance decision.

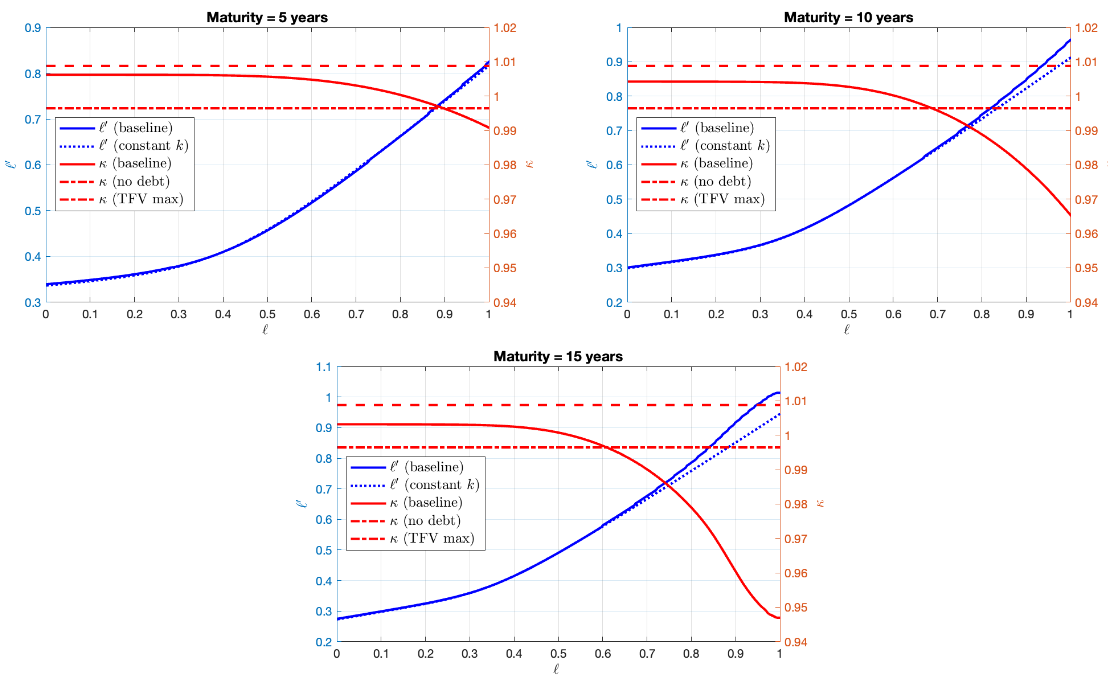
[21]When these benefits become small, as it happens with very long debt maturity, they can be easily offset by a small one-time initial debt issuance cost, which we do not model. This prediction is different from the one in Benzoni et al. (2022), where transaction costs are incurred at any time the firm refinances, as their model shows that a firm optimally remains unlevered for combinations of high issuance costs and relatively short debt maturity.

[22]The model for this case is presented in Appendix D.

[23]However, we cannot rule out that this effect is only due to numerical reasons.

Figure 5: MPE - Investment policy

The figure plots, against $\ell = b/k$, investment (red lines) and leverage (blue lines) policies. Four cases are presented in this figure: the baseline model with no commitment (solid lines), the case with constant investment ($k' = k = 1$) (dotted); the case with permanently no debt (dash-dotted), and the total firm value (TFV) maximizing case (dashed). The calculations are based on the assumption that y is non-stochastic and constant, $k = 1$, using the parameters in Table 1, and maturities $1/\xi = 5, 10, 15$ years.

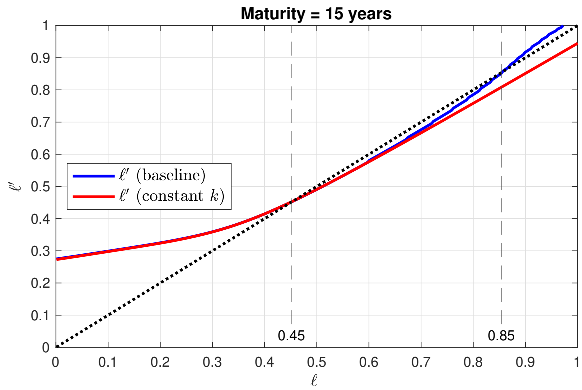


unlevered, in the second the firm has a fixed investment policy so that $k' = k = 1$ at all dates, and in the third the case with full commitment illustrated above, with firm value maximizing policy. The equilibria for these three benchmark cases are described in Appendix B, C, and D, respectively. The results are presented for three debt maturities: $1/\xi = 5, 10, 15$ years.

Relative to the permanently unlevered firm equilibrium (which has constant $\kappa = .9965$), Figure 5 (red lines) shows that access to debt with limited commitment allows for higher investment when current leverage is low, but leads to underinvestment when current leverage is high. This is because a higher leverage makes both $v(\tilde{\ell})$ and $p(\ell)$ small under limited commitment, hence reducing the marginal q in (9). There is a second,

Figure 6: MPE - Underinvestment and leverage policy

The figure plots, against $\ell = b/k$, the leverage policy for the baseline model (blue line) and the model with constant investment. The calculations are based on the assumption that y is non-stochastic and constant, $k = 1$, using the parameters in Table 1, and debt maturity $1/\xi = 15$ years.



distinct channel to underinvestment with limited commitment, gauged by $1/\xi$, due to the leverage ratchet effect, which reduces the tax benefits of debt. Given ℓ, this effect is transmitted to investment via a lower $p(\ell)$ when $1/\xi$ is high, as seen in the left panel of Figure 1, and therefore has a further negative effect on marginal q, everything else equal. Finally, for the same current leverage, investment with limited commitment is never as high as in the firm value maximizing equilibrium, because in that case the commitment problem is absent and the value of the tax shield can be fully achieved at all ℓ.

The commitment problem on leverage policy is more severe when coupled to underinvestment, as seen by comparing the baseline equilibrium to the one with fixed investment in Figure 5 (blue lines). Indeed, if ℓ is high, $\kappa = h(\ell)$ is low due to underinvestment, as shown above, and therefore ℓ/κ is higher than in the case with fixed investment. In Proposition 1 we showed that, because of the leverage ratchet effect, g is increasing, which makes $\ell' = g(\ell/\kappa)$ higher in the case with endogenous investment than in the case with fixed investment. Figure 5 shows that this interaction of underinvestment with the commitment problem on the leverage policy is stronger for higher $1/\xi$, when the commitment problem is more severe.

To better illustrate the effect of underinvestment on the leverage policy, in Figure 6 we show, for $1/\xi = 15$ years, that while for the baseline leverage policy there are two solutions to equation $\ell = g(\ell, y)$, as shown before, for the case with fixed investment there is only one. The implication is clear: in the baseline model, if current leverage is high (that is, higher than $\hat{\ell}_2$), the equilibrium dynamics will display a leverage diverging to 100% over time, whereas in the model with fixed investment the equilibrium dynamics will always be mean reverting to $\hat{\ell}_1$. In other words, models with constant investment policy *underestimate* the leverage ratchet effect, because the full extent of the leverage ratchet effect on firms dynamics and security pricing can be appreciated only in a model with endogenous investment.

Despite the fact that the shareholders' choice of investment can take both signs, as they can freely increase or decrease the firm's asset in each period, the overall effect of the commitment problem on investment is *asymmetric*. Indeed, the speed of *upward* adjustment towards long-run leverage is unchanged by the presence of endogenous investment. However, for high leverage, underinvestment accelerates the debt issuance and reduces the speed of *downward* adjustment. As a result, the firm will have a higher long-run leverage.[24]

The joint effect of underinvestment and leverage ratcheting is to reduce the price of debt and increase the equity value in the baseline model, relative to the case with fixed investment, and this effect is mostly visible for high current leverage, as showed in Figure 7.[25] Overall, the value of the firm is lower for the combined effects of increased bankruptcy costs and underinvestment as a consequence of these agency costs.

5.3 Firm dynamics

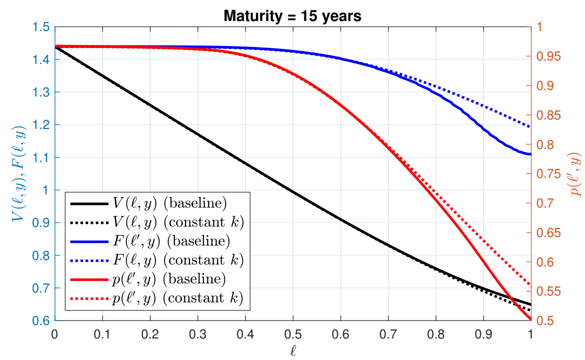
We use the model to derive implications for the dynamics of the firm policies and security values. In particular, we will focus on the dynamics of the investment and debt policy, and the related dynamics of cash flow and, through the flow of funds equation, of the

[24] These results are not comparable to the ones in DeMarzo and He (2021), because in their case the firm is endowed with an option to reduce the scale of investment, which is more valuable in downturns, reducing default risk. This increases the price of debt and the speed of debt issuance in downturns. However, also in their case, debt issuance is slower with fixed investment for high current leverage.

[25] The effect is present but less visible also for $1/\xi = 5, 10$ years. For this reason we present only the case $1/\xi = 15$.

Figure 7: MPE - Underinvestment and value functions

The figure plots, against $\ell = b/k$, the equilibrium value of equity, debt price, total firm value. Two cases are presented in this figure: the baseline case with no commitment (solid lines), and the case with constant investment ($k' = k = 1$) (dotted lines). The calculations are based on the assumption that y is non-stochastic and constant, $k = 1$, using the parameters in Table 1, and debt maturity $1/\xi = 15$ years.



payout policy. To isolate the effects of limited commitment on the debt policy, we first consider the case with non-stochastic y.

Given the current state $(b_t, k_t, y_t) = (\ell, 1, y)$, and the related optimal policy (κ, ℓ'), where $\kappa = h(\ell, y)$ and $\ell' = g(\ell/\kappa, y)$, the cash flow from investment policy is

$$cf^k(\ell, y) = -(\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi(\kappa - 1 + \delta)^2$$

and the cash flow from the debt policy is

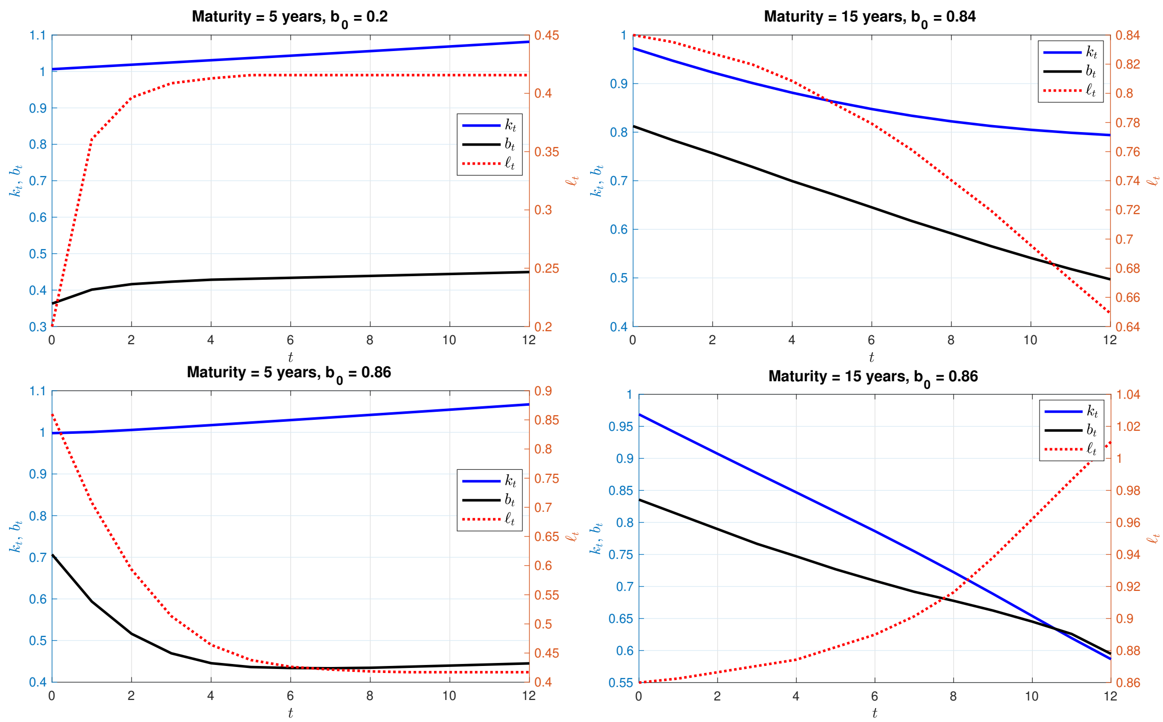
$$cf^d(\ell, y) = \kappa \left[\ell' - (1 - \xi)\frac{\ell}{\kappa}\right] p(\ell', y).$$

Given k_t, we derive the respective dollar values of the two cash flows as $CF_t^k = k_t\, cf^k(\ell, y)$ and $CF_t^d = k_t\; cf^d(\ell, y)$. The (expected) dividend for $k = 1$ over the period $[t, t + 1]$ is

$$\begin{aligned} d(\ell, y) = &-(\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi(\kappa - 1 + \delta)^2 + \kappa \left[\ell' - (1 - \xi)\frac{\ell}{\kappa}\right] p(\ell', y) \\ &+ \kappa\beta\mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \{(1 - \tau)(y' + x') - ((1 - \tau)r + \xi)\, \ell'\}\, \phi(x')dx'\right], \end{aligned}$$

Figure 8: MPE - Asset and debt dynamics

The figure plots, for the cases with maturity $1/\xi = 5$ (left column) and $1/\xi = 15$ (right column) years, the paths of k_t and b_t (and of $\ell_t = b_t/k_t$) for a firm with $k_0 = 1$ and $b_0 = 0.2$ (top left), $b_0 = 0.84$ (top right), and $b_0 = 0.86$ (bottom row). These paths are conditional on the firm being solvent at the beginning of each period. The simulations are based on the parameters in Table 1, assuming that y is non-stochastic and constant.



from which we calculate the dollar dividend $D_t = k_t \ d(\ell, y)$. Finally, the continuation value of the firm at t for $k = 1$ is

$$cv(\ell, y) = \kappa\beta\mathbb{E}_y \left[\int_{x_d(\ell',y')}^{\overline{x}} V(\ell', y')\phi(x')dx' \right],$$

from which we determine the dollar continuation value $CV_t = k_t \ cv(\ell, y)$.

The dynamics of the firm is determined by the initial state (b_0, k_0), and the relation between ξ and δ. In Figures 8 and 9, for a given k_0, based on calibration in Table 1 with δ kept constant at 8.5%, we compare the case $\xi > \delta$ (maturity 5 years) to the case $\xi < \delta$ (maturity 15 years), at high or low b_0. Starting from low b_0, in the top left panel of

Figure 9: MPE - Cash flow and payout dynamics

The figure plots, for the cases with maturity $1/\xi = 5$ (left column) and $1/\xi = 15$ (right column) years, the paths of CF_t^k, CF_t^d, D_t, and CV_t for a firm with $k_0 = 1$ and $b_0 = 0.2$ (top left), $b_0 = 0.84$ (top right), and $b_0 = 0.86$ (bottom row). These paths are conditional on the firm being solvent at the beginning of each period. The simulations are based on the parameters in Table 1, assuming that y is non-stochastic and constant.

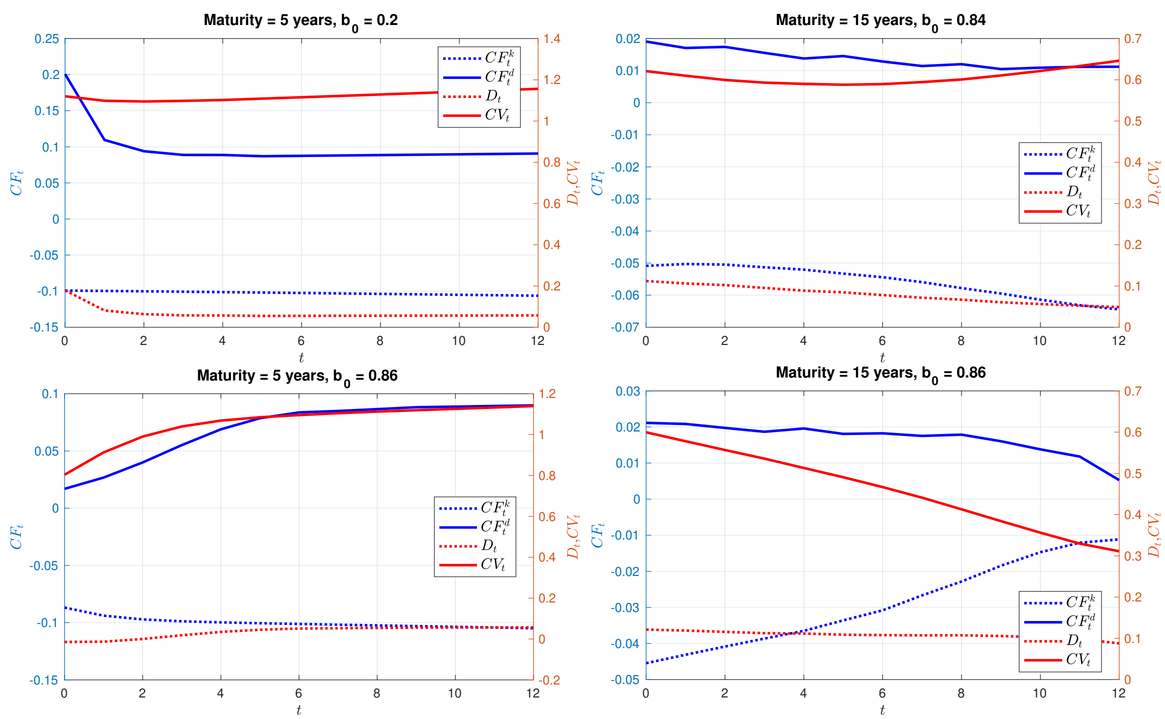


Figure 8,[26] the firm's capital stock will grow together with the debt, eventually reaching the long term leverage $\hat{\ell}$ (or $\hat{\ell}_1$ if there are multiple solutions). The corresponding cash flow and payout dynamics are reported in Figure 9 (top left panel): as the commitment problem on debt policy is kept at bay by a relatively low leverage, the price of debt remains relatively high, making the cash flow from debt issuance large enough to finance investment and allow a positive, albeit declining, (dollar) dividend. Overall, the (dollar) value of the firm, measured by the continuation value, is growing over time.

If b_0 is high, the firm's history is quite different depending on whether ξ is higher or lower than δ. If $\xi > \delta$, the bottom left panel in Figure 8 shows that an initial high debt is progressively reduced via amortization, which is significant. In the corresponding panel of Figure 9, although investment is initially hindered by high leverage, it later becomes healthier as the leverage reverts downwards. Quite remarkably, in the initial phase, the proceeds from debt issuance are relatively low, to the point that investment must be financed mainly by equity injection. This shows that, if the commitment device is effective (that is ξ is high), debt overhang, whereby shareholders refrain from funding positive NPV projects, can be overcome. Hence, if commitment is restored via low debt maturity, shareholders finance investment by injecting equity.

If $\xi < \delta$,[27] as in the right column of Figure 8, the firm's history depends on whether b_0 is such that ℓ_0 is higher or lower than $\hat{\ell}_2$. If $\ell_0 < \hat{\ell}_2$, we know from Figure 2 that the leverage will revert to $\hat{\ell}_1$. In stark contrast to the case with high ξ considered above, this is done via a reduction of both k_t and b_t over time by a reduced investment and debt issuance, which remain positive but insufficient to offset capital depreciation and debt amortization, respectively. As a manifestation of the agency issues created by limited commitment on the debt policy, the shareholders pay themselves a positive (although declining) dividend throughout. Along this path, the (dollar) value of the firm is first declining, while the debt is being reduced, and then grows again when the leverage is approaching the long-run target.

Finally, if $\xi < \delta$ and $\ell_0 < \hat{\ell}_2$ the leverage will diverge to 100% as a consequence of a rapid decline of capital stock, as underinvestment becomes increasingly severe and capital

[26]Here and in Figure 9, we only report the case $1/\xi = 5$ because the plot for $1/\xi = 10$ is very similar.

[27]For illustration, we choose ξ such that there are multiple roots to equation $\ell = g(\ell, y)$ for $\ell \in [0, 1]$. In the case there is a single root, the results are qualitatively the same as in the case $\xi > \delta$.

depreciation is faster than debt amortization. In this decline, shareholders manage to pay themselves a positive dividend by issuing debt.

Next, we analyze the leverage and credit risk dynamics in response to the evolution of the persistent component of productivity, y_t. We first analyze one path of y_t, starting from $y_0 = \overline{y}$. In Figure 10, we illustrate the evolution of capital stock, debt, leverage, and credit spread

$$cs(b', k', y) = cs(\ell', y) = (r + \xi)\left(\frac{1}{p(\ell', y)} - 1\right), \tag{10}$$

for a firm starting with unit capital, $k_0 = 1$,[28] and unlevered ($b_0 = 0$), and then implementing the optimal policy from the MPE. We show the evolution for two possible debt maturities: $1/\xi = 5$ and $1/\xi = 15$. Clearly, although y_t follows a stationary process, (k_t, b_t) do not, due to capital accumulation. However, stationarity is recovered when considering ℓ_t as state variable.

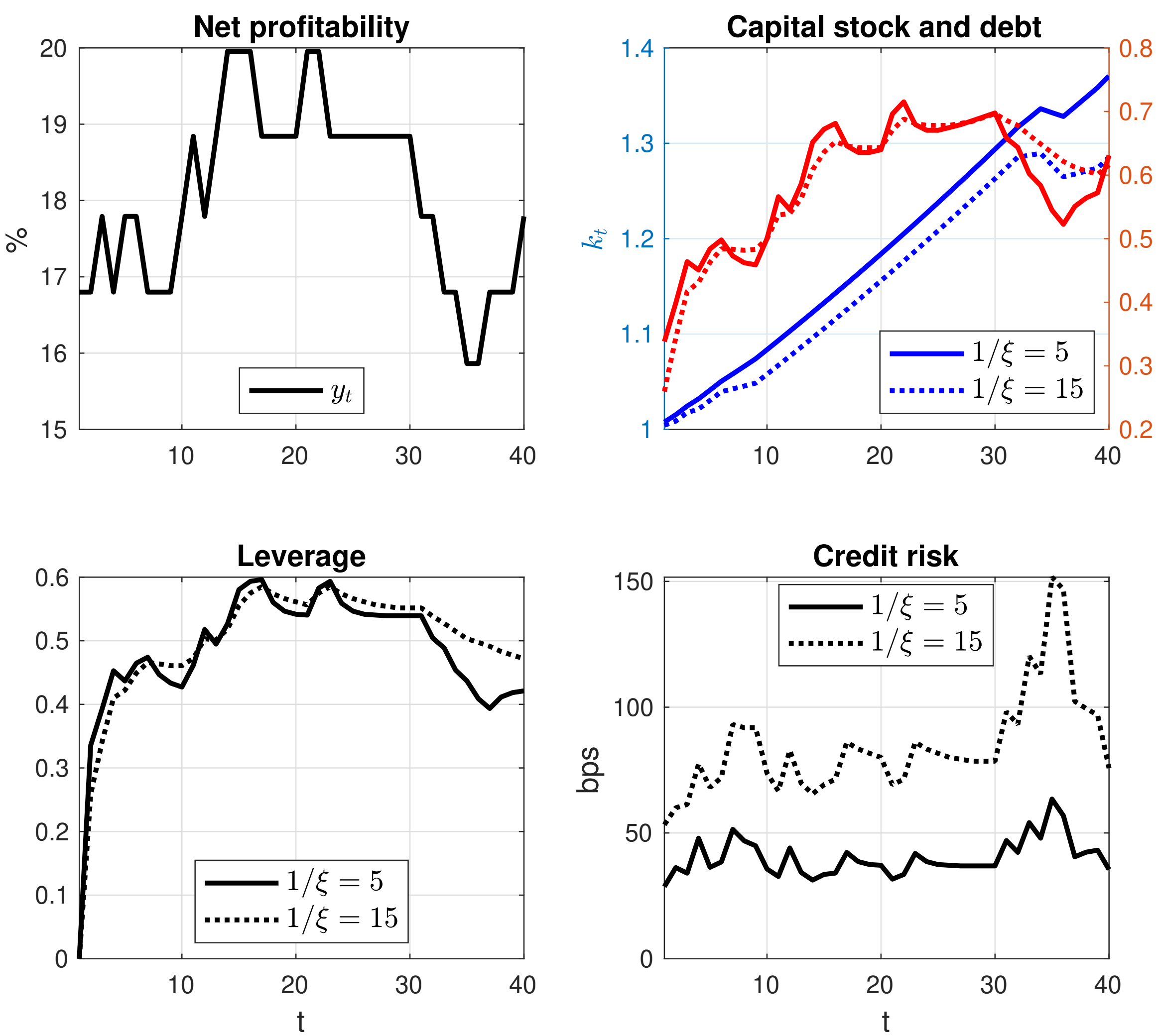
By construction, given our assumption of zero debt adjustment costs and positive adjustment cost for capital stock, in Figure 10 the debt policy is more sensitive than investment to changes in profitability. In fact, it is only when there is a significant downturn in y_t, as it happens towards the end of the sample period, that the firm's asset deviates from the growth path and is downsized, whereas the reaction of the debt policy to productivity shocks is immediate. Given the LRE described in Proposition 1, the debt reduction following a negative profitability shock occurs only via debt amortization, ξ. In other words, the response of the debt policy to a change in y_t depends on debt maturity, with a shorter maturity debt case showing a higher speed of downward adjustment, as already noted in Section 5.1.

A few new facts emerge from Figure 10. First, the response of investment to a negative shock is faster if the debt maturity is longer, as underinvestment is more severe in this case. The resulting leverage dynamics shows that with shorter debt maturity the correlation of new leverage and profitability is higher, whereas for longer maturity leverage is more persistent.

[28]Because of the homogeneity property of the model, the dynamics would be the same if we chose a different k_0.

Figure 10: Firm policies and credit risk dynamics with time-varying profitability

The figure shows one path of $(1-\tau)y_t$ and the corresponding evolution of k_t and b_t, starting from $y_0 = \overline{y}$, $b_0 = 0$ and $k_0 = 1$ and using the optimal policy in the MPE. This produces new leverage, $\ell_t = b_t/k_t$ and credit spread, cs_t. These paths are conditional on the firm being solvent at the beginning of each period. We present with solid lines the case with maturity $1/\xi = 5$, and with dotted lines the case with maturity $1/\xi = 15$. The equilibrium is based on the parameters in Table 1.



Secondly, Figure 10 shows that the credit spread is affected by the leverage dynamics, and it is lower when the debt has shorter maturity. This is the result of two effects. First, a lower $1/\xi$ reduces the shareholders' commitment problem to debt repayment, as noted also by Dangl and Zechner (2021), and therefore the credit spread for short maturity debt is lower and the spike of the credit spread during the period of economic decline of the firm is contained.[29] The second effect, unique to our setting, is that a lower $1/\xi$ increases the shareholders' commitment to the leverage policy, and therefore it will reduce the incentive to increase the leverage in those future states in which the firm is solvent. This has a positive ex ante effect on the debt price which reduces the credit spread even further.[30]

Thirdly, capital accumulation in Figure 10 is slower for longer debt maturity, a consequence of the fact that for higher $1/\xi$ the commitment problem to future debt issuances is increased. Indeed, as clarified in Section 5.2, the more severe such a commitment problem, the lower the ex ante value of the tax shield, and therefore of the marginal q. This has the effect of reducing investment in each period, and ultimately of reducing the real size of the firm in the long run.

A more detailed picture of the dynamics of the firm's policies and security prices can be seen in Table 2, in which we report several summary statistics of interesting quantities from a simulated economy under different scenarios for the debt maturity, $1/\xi$. Within each maturity scenario, given the MPE based on the parameters in Table 1, we simulate an economy comprising 1000 firms, each starting unlevered from an initial random state (k_0, y_0), and implement the equilibrium investment and leverage policy for 200 years.[31] While a firm may default over each year, we record the relevant beginning-of-year quantities conditional on the firm being solvent.

In particular, for each (b, k, y) visited with positive probability, given the equilibrium policy (b', k'), where $k' = kh(b/k, y)$ and $b' = k'g(b/k', y)$, we calculate new book leverage

[29] In unreported results, 1-year default probability follows a similar path to the credit spread.

[30] In Section 6 we separate the effect of ξ on the credit spread due to the first commitment problem, as analyzed already by Dangl and Zechner (2021), from the one due to the second commitment problem, which is unique to this setting.

[31] We simulate 250 years, and then drop the first 50 years to eliminate the dependence from the initial state.

Table 2: Debt maturity and firm dynamics

Summary statistics based on a simulation of 1000 firms (starting from random initial states), for 200 years. The paths for y and the initial states are the same for all ξ. $\mu(\cdot)$ indicates average, $ac(\cdot)$ autocorrelation, $\rho(\cdot,\cdot)$ correlation. bl is new book leverage, ml is new market leverage, Q is Tobin's q, i investment, cs credit spread (in bps), and dp default probability (in %). The parameters are from Table 1.

$1/\xi$	1	5	10	15
$\mu(bl)$	0.34	0.43	0.45	0.48
$ac(bl)$	0.88	0.94	0.96	0.96
$\rho(bl, y)$	1.00	0.98	0.92	0.74
$\mu(ml)$	0.23	0.29	0.30	0.31
$\mu(Q)$	1.47	1.46	1.45	1.44
$\mu(i)$	0.10	0.09	0.09	0.09
$\rho(i, y)$	1.00	0.85	0.90	0.92
$\mu(cs)$	1.85	46.24	76.42	104.09
$\mu(dp)$	0.02	0.43	0.71	1.00

as b'/k', new market leverage as $b'p(b',k',y)/F(b,k,y)$, investment as $i = k'/k - (1-\delta)$, Tobin's Q as $F(b,k,y)/k'$, where

$$F(b,k,y) = V(b,k,y) + (1-\xi)bp(b',k',y)$$

is total firm value and $V(b,k,y) = kV(\ell,y)$ with $V(\ell,y)$ is from (7), and $p(b',k',y) = p(\ell',y)$ from (6). We calculate also the 1-year default probability

$$dp(b',k',y) = \mathbb{E}_y\left[\int_{\underline{x}}^{x_d(b',k',y')} \phi(x')dx'\right].$$

The statistics presented in Table 2 are calculated as time-series average of cross-sectional averages of the specific statistics. To ease comparison, in the first column we present also the case with $\xi = 1$, for which the commitment problem to the leverage policy is absent. We can make several important remarks from Table 2. The first is that, for the cases with limited commitment, $1/\xi > 1$, optimal market leverage, as well as optimal book leverage, is increasing in the debt maturity, as shown in Section 5.1.

The second main remark is that, for higher maturity, book and market leverage become more persistent, which is in line with the intuition from Figure 10 that shorter maturity increases the speed of adjustment of the leverage towards the long-run level in response to profitability shocks. This is confirmed by the observation that the correlation of contemporaneous changes in leverage and profitability shocks is almost perfect for shorter maturity, but it declines significantly for longer maturities.[32]

The third point is that the different leverage policies for different debt maturities have real effects, as Tobin's Q is higher (and more persistent, and less sensitive to shocks) when the commitment problem is less severe for shorter debt maturities. This is reflected also by investment, which is instead driven by marginal q, although the effect is less visible on average. There is an interesting U-shaped pattern of the correlation of investment with y. This can be easily explained by considering that, for $\xi = 1$ and full commitment on the debt policy, investment is first best and fully reacts to shocks on y, but with high $1/\xi$, the correlation of investment with negative productivity shocks increase due to more severe underinvestment.[33]

The final point is that the debt maturity has an important effect on credit risk and the cost of debt of the firm, with default probability and and credit spread growing two-fold when maturity is 15 vis-à-vis 5 years.

6 Implications for the cost of debt

From the analysis of the MPE in Section 5, it is clear that the two commitment problems (on debt repayments and on future debt issuances) have a major impact on the leverage policy, the investment policy, and ultimately the cost of debt. Both commitment problems are controlled by the debt maturity, $1/\xi$. On the one hand, a bigger ξ forces the equity holders to repay at face value a larger fraction of the outstanding debt. Because this directly determines the leverage threshold by increasing default risk,

[32] The model cannot replicate the empirical stylized fact that profitability and leverage are negatively correlated, because we assumed no debt issuance costs, as in DeMarzo and He (2021). To generate negative correlation via hysteresis we would need to add debt issuance costs to our model, which is left for future research.

[33] These results are qualitatively confirmed for other set of parameters gauging the credit risk of the firm, as shown in the appendix, Table 3.

a higher ξ leads to lower leverage and therefore lower credit spread. On the other hand, a lower ξ, that is a longer debt maturity, hinders the incentive to reduce the debt in future economic downturns, which creates an agency conflict between debt and equity holders in the form of underinvestment and debt dilution. As shown in Section 5, this reduces the price of debt, with a positive effect on the credit spread.

To analyze under what circumstances one commitment problem is more severe than the other, or equivalently what effect of debt maturity on credit spread is stronger, we decompose the cost of debt financing into a *default component*, motivated by the commitment problem on debt repayments, and an *agency component*, motivated by the commitment problem on debt issuances.

Given the MPE, the credit spread in (10) can be seen as the difference between the yield of the firm's debt and the yield of a debt contract not affected by default, but that is otherwise identical (same maturity, same face value). Indeed, denoting by $p^d(b', k', y)$ the price of the same contract without default, given the equilibrium policy (b', k') in (b, k, y), and noting that the coupon rate is $r = 1/\beta - 1$, equation (10) is equivalent to

$$(r + \xi) \left(\frac{1}{p(b', k', y)} - \frac{1}{p^d(b', k', y)} \right).$$

If we want to separate the agency component from the default component of such a spread, we would need to single out the price reduction due to agency conflicts in future states in which the firm is solvent. Denoting by $p^a(b', k', y)$ the price of the debt contract that is neither affected by default nor by agency conflicts, but is otherwise identical to the current debt contract, the agency spread would be $(r+\xi)/p^d(b', k', y) - (r+\xi)/p^a(b', k', y)$. However, Proposition 1, Point 2 shows that in the MPE the exclusion of default implies also that the agency cost vanishes, and $p^d \equiv p^a \equiv 1$. Thus, in the credit spread defined in (10), the default and the agency component cannot be separated in the sense that the credit spread comprises only the default component, because over an infinite horizon the ultimate effect of agency conflicts is to cause the firm's default.

However, based on the MPE, over any finite horizon H the separation of the credit spread in a default and an agency component is possible. The first is due to the cost of default (according to the equilibrium policy) up to H. The second derives from the agency conflicts in *non-default states*, at all dates before H. Because the equilibrium policy is expressed in a dynamic programming setting as one-period decisions, and prices are for infinite-lived securities (although with finite average maturity $1/\xi$), we can only define the decomposition in a recursive fashion.[34]

We start from the one-year horizon case. We first define the price at t of the same debt contract as in the baseline case equation (3), but *without default* at $(t+1)$, as

$$p_1^d(b,k,y) = \beta \mathbb{E}_y \left[r + \xi + (1-\xi) p(b',k',y') \right]. \tag{11}$$

Notably, in (11) we exclude the default event at $t+1$, but the firm's policy at that date is the equilibrium one, (b',k'). We define the one-year *default spread*, which is the increment to the cost of debt for default occurring at $(t+1)$ as

$$ds_1(b',k',y) = (r+\xi) \left(\frac{1}{p(b',k',y)} - \frac{1}{p_1^d(b',k',y)} \right).$$

Intuitively, the one-year default spread is the difference in yield of the original debt and the yield of the same debt without the effect of default in one year. Although it is derived from infinite-lived debt (with average maturity $1/\xi$), because the two debt contracts with price p and p_1^d are identical after $(t+1)$, the above spread captures only the cost of default occurring at $(t+1)$.

Next, we define the price at t of the same debt contract but without the effects of default and of *agency conflicts* on the continuation value at $(t+1)$ as

$$p_1^a(b,k,y) = \beta \mathbb{E}_y \left[r + \xi + (1-\xi) \max\{ p(b',k',y'), p(b,k,y') \} \right]. \tag{12}$$

The modified continuation value in (12), which excludes the effects of agency conflicts at $(t+1)$, is key to our analysis and is based on the assumption that from $(t+2)$ onward, the debt holders will be fully exposed to the effects of the commitment problem (default and agency conflicts) deriving from the equilibrium policy, as reflected by the dependence on the equilibrium debt price $p(b',k',y')$. However, the *negative effects* of changes from (b,k) to (b',k') at $t+1$ on the debt price, $p(b',k',y') < p(b,k,y')$, are excluded, while the debt can benefit from the same positive effects of those changes as the original debt.

[34] In Appendix H, we discuss our decomposition of the credit spread vis-à-vis alternative approaches to decompose the credit spread into a pure default component vs an agency component. Our approach has the key property of being consistent with the MPE, whereas other approaches result in an off-equilibrium decomposition.

Similarly to what done for the default spread, the one-year *agency spread* is the difference between the yield of the debt in (11) and the yield of the debt in (12),[35]

$$as_1(b', k', y) = (r + \xi) \left(\frac{1}{p_1^d(b', k', y)} - \frac{1}{p_1^a(b', k', y)} \right). \tag{13}$$

Importantly, this definition is based on the assumption that the equilibrium policies *at all states* are exactly the ones defined by the MPE, and only their negative effect on the continuation value of the debt at $(t+1)$ is eliminated. As before, although the spread is derived from the price of infinite-lived debt contract, because the two contracts are identical after $(t+1)$, the spread reflects only the agency cost at that date.[36]

Finally, once the default and the agency components have been defined, we calculate the *credit spread* for one-year exposure to the commitment problem as

$$cs_1(b', k', y) = ds_1(b', k', y) + as_1(b', k', y).$$

We recursively derive the same decomposition of the credit spread for an arbitrary H-year horizon, $H > 1$ and generate the term-structure of the credit spreads and of their constituents. Given p_{n-1}^d, we define the debt price without the effects of default for n years as

$$p_n^d(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1 - \xi) p_{n-1}^d(b', k', y') \right]. \tag{14}$$

This recursive pricing equation is updated for $n = 1, \ldots, H$, starting from $p_0^d = p$. Hence, the spread for a H-year exposure to default is

$$ds_H(b', k', y) = (r + \xi) \left(\frac{1}{p(b', k', y)} - \frac{1}{p_H^d(b', k', y)} \right), \tag{15}$$

[35] From t to $t+1$, there can be a reduction of the price of debt, $p(b', k', y') < p(b, k, y')$, if an unfavorable (non-default) state y' occurs. However, this reduction is per se neither the effect of default nor of agency conflicts at $(t+1)$, and therefore it is not included in as_1. The agency spread comprises the effect of the transition from y to y' only if it changes the policy (b', k') adversely for the debt holders.

[36] Also Jungherr and Schott (2021) quantify the cost of corporate debt due to lack of commitment. Differently from our decomposition of the equilibrium credit spread, they calculate this cost in a counterfactual experiment, comparing the credit spread in an economy without commitment to the credit spread in an economy with commitment. While this may give an idea of the increased cost of capital, in equilibrium the average leverage is lower under commitment, which in part drives the difference of credit spread. Our approach, by using the MPE policies and prices, overcomes this issue.

which is the cost of debt due to the chance that the firm defaults in any of the following H years. In a similar fashion, given p^a_{n-1}, the price of debt without the effect of default and of agency issues for n years is

$$p^a_n(b,k,y) = \beta \mathbb{E}_y \left[r + \xi + (1-\xi) \max\{p^a_{n-1}(b',k',y'), p^a_{n-1}(b,k,y')\} \right]. \tag{16}$$

where the recursive pricing equation is updated for $n = 1, \ldots, H$, starting from $p^a_0 = p$. Then, the spread for a H-year exposure to agency conflicts is

$$as_H(b',k',y) = (r+\xi) \left(\frac{1}{p^d_H(b',k',y)} - \frac{1}{p^a_H(b',k',y)} \right), \tag{17}$$

which is the cost of debt solely due to agency conflicts in non-default states in any year from 1 to H. Finally, the credit spread due to exposure to default and agency conflicts for H years is $cs_H(b',k',y) = ds_H(b',k',y) + as_H(b',k',y)$.

Proposition 2. *Given the MPE, we have*

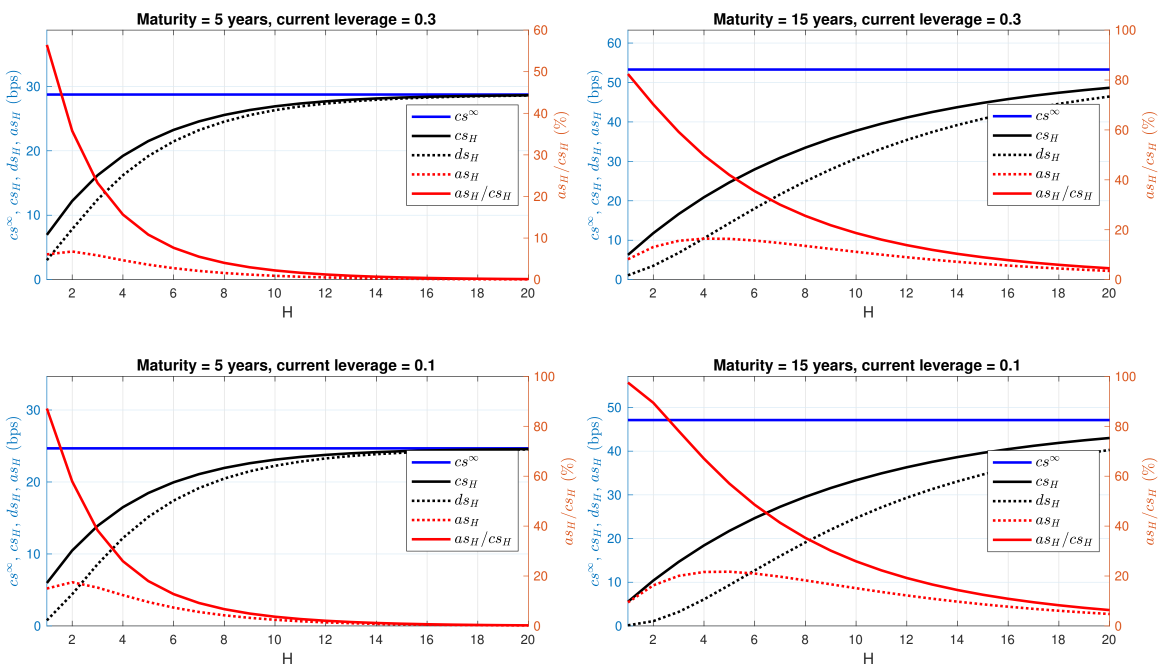
1. *for all* $H \geq 1$, $ds_H \geq 0$, $as_H \geq 0$, *and* $cs_H \geq 0$*;*
2. *for all* $H \geq 1$, $cs_{H-1} \leq cs_H \leq cs$, *and for* $H \to \infty$, $cs_H \to cs$*;*
3. *for* $H \to \infty$, $ds_H \to cs$ *and* $as_H \to 0$.

The proof is in Appendix G. The intuition is that the longer the horizon, H, over which the creditors are exposed to the two commitment problems, the closer cs_H is to the credit spread in (10), and asymptotically the two are the same. Also, at longer horizons the agency spread becomes negligible and the default spread dominant.

Figure 11 shows how the credit spread at a given horizon H is decomposed into the above defined default and agency components, for debt maturities, $1/\xi = 5, 15$ years, and current leverage ℓ. As a benchmark, we report also the credit spread defined in (10), denoted c^∞. A few remarks are in order. First, c^∞ is higher for higher $1/\xi$. This is intuitive, being ξ a parameter gauging the commitment of the equity holders to debt repayment at par. Hence, the smaller it is (i.e., the longer the maturity and the smaller the mean reversion of the leverage) the higher the leverage in future economic downturns, which increases the credit spread.

Figure 11: Decomposition of credit spread

The figure plots the decomposition of the credit spread into a default spread and agency spread, as defined respectively in equations (15) and (17), for different horizons H, which represent the number of periods for which debt holders are exposed to the negative effects of equity holders' lack of commitment. Panels on the left have maturity $1/\xi = 5$ years and panel on the right $1/\xi = 15$ years. The calculation is based on $k = 1$, and $\ell = 0.3$ in the top panels, and $\ell = 0.1$ in the bottom panels. The MPE is based on parameters in Table 1 assuming that y is non-stochastic and constant.



Second the anticipation by debt holders to be exposed to equity-maximizing leverage decisions leads to a sizable agency spread of about 8-10bps at least for horizons of up 8 years, when debt maturity is long. This is because the agency spread reflects the expectation of future transfers of value from debt to equity due to the LRE. The fraction of credit spread that can be attributed solely to agency conflicts related to the leverage policy (right y-axis) is substantial at short horizons. As transfers that are far in the future weigh increasingly less, the agency spread vanishes when longer horizons are considered, and only the default spread remains. In the limit for very long horizons, as seen in Proposition 2, the credit spread only comprises the default component.

Third, the absolute and relative value of the agency spread is strongly influenced by the parameter ξ, being higher at all horizons if debt maturity is longer. As noted by DeMarzo and He (2021), a model based on lack of commitment to future debt issuances, which has negative effect on the investment policy, can generate positive yield spreads for firms even in the presence of small default risk. In the figure, for the case $\ell = 0.3$, the proportion of credit spread attributable to agency conflicts for $H = 1$ grows from about 50% of the credit spread for $1/\xi = 5$ to about 80% for $1/\xi = 15$. The agency proportion is even larger when credit risk is lower, as shown by the case $\ell = 0.1$. These results confirm the intuition by Myers (1977) that a longer debt maturity exacerbates (debt dilution and underinvestment) agency conflicts.

Overall, a credit risk model based on lack of commitment on future debt policies can succeed where traditional structural models have been unsuccessful (see Huang and Huang (2012)), because it allows for an additional channel of credit risk, the agency channel, besides the pure default one, which is sizeable in particular with low leverage firms.

7 Conclusion

Debt maturity has a known role in mitigating default risk, by committing the firm to repay a given amount of debt at a predetermined date. We show that debt maturity plays an important role in alleviating the negative effects of shareholders' lack of credibility in regard to future debt issuances and investment decisions.

Our analysis is based on the model by DeMarzo and He (2021), to which we add the friction that the firm can trade their own debt at discrete dates. To this framework, we also add dynamic investment. We show that debt maturity *per se* can restore a dynamic trade-off theory, so that a leverage ratchet effect model can have realistic leverage dynamics, investments, and a sizeable tax shield. Debt maturity mitigates also underinvestment via a channel, different from debt overhang, whereby a lower maturity increases the marginal q of capital stock. Because debt maturity addresses both the commitment problem to debt repayment and the commitment problem to future policies, we decompose the cost of debt into an agency component and a default component. We show that the agency component is more important when the leverage is low and default far, and a lower debt maturity directly affects this part.

Our paper is focussed on the positive contribution of debt maturity in alleviating the commitment problem to future debt and investment policies. Admittedly, we have excluded any negative effects of a shorter maturity, like debt issuance costs and refinancing risk, which have been analyzed among others by Benzoni et al. (2022), Dangl and Zechner (2021), He and Xiong (2012), and He and Milbradt (2014). Hence, our results cannot be interpreted in a quantitative sense, because no force in the model is offsetting the positive effect of shorter maturity. We leave such a quantitative analysis of the maturity as a commitment device against the leverage ratchet effect for future work.

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Online Appendix

A Derivation of equations (4)-(7)

Proof: Given the linearity of the production function and the assumed capital adjustment cost, it can be proved that $V(b,k,y) = kV(b/k,1,y)$ (homogeneity of degree one) and $p(b,k,y) = p(b/k,1,y)$ and $x_d(b',k',y') = x_d(b'/k',1,y')$ (homogeneity of degree zero). Therefore, we can write the equity program in (1) as

$$\begin{aligned} kV(b/k,1,y) = k \max_{(b',k')} \; & -\left[\frac{k'}{k} - (1-\delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\frac{k'}{k} - (1-\delta)\right]^2 \\ & + \left[\frac{b'}{k} - (1-\xi)\frac{b}{k}\right] p(b',k',y) + \beta\mathbb{E}_y\left[\int_{x_d(b'/k',y')}^{\overline{x}} \left\{(1-\tau)(y'+x')\frac{k'}{k}\right.\right. \\ & \left.\left. - \left[(1-\tau)r + \xi\right]\frac{b'}{k} + \frac{k'}{k}V(b'/k',1,y')\right\}\phi(x')dx'\right]. \end{aligned}$$

Defining $\kappa = k'/k$, $\ell = b/k$, $\ell' = b'/k'$, and $\eta = b'/k$, simplifying k from both sides, and dropping the capital stock dependence (with a small abuse of notation), $V(\cdot,y) = V(\cdot,1,y)$ and $p(\cdot,1,y) = p(\cdot,y)$, from the expressions above we have

$$\begin{aligned} V(\ell,y) = \max_{(\eta,\kappa)} \; & -\left[\kappa - (1-\delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 \\ & + \left[\eta - (1-\xi)\ell\right] p(\ell',y) + \beta\mathbb{E}_y\left[\int_{x_d(\ell',y')}^{\overline{x}} \left\{(1-\tau)(y'+x')\kappa\right.\right. \\ & \left.\left. - \left[(1-\tau)r + \xi\right]\eta + \kappa V(\ell',y')\right\}\phi(x')dx'\right], \end{aligned}$$

where

$$x_d(\ell,y) = r\ell + \frac{\xi\ell - V(\ell,y)}{1-\tau} - y.$$

Focussing on the last two lines of the equity value equation, we have

$$\begin{aligned} V(\ell,y) = \max_{(\ell',\kappa)} \; & -\left[\kappa - (1-\delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 \\ & + \kappa\left[\frac{\eta}{\kappa} - (1-\xi)\frac{\ell}{\kappa}\right] p(\ell',y) + \kappa\beta\mathbb{E}_y\left[\int_{x_d(\ell',y')}^{\overline{x}} \left\{(1-\tau)(y'+x')\right.\right. \\ & \left.\left. - \left[(1-\tau)r + \xi\right]\frac{\eta}{\kappa} + V(\ell',y')\right\}\phi(x')dx'\right]. \end{aligned}$$

We note that $\eta/\kappa = \ell'$. The program above can be decomposed into two parts: the first, for given ℓ and κ (and therefore, the state variable for this program is ℓ/κ) optimizes the last two lines with respect to ℓ'; the second part optimizes with respect to κ. More explicitly, we can write the program above as

$$
\begin{aligned}
V(\ell, y) = \max_{\kappa} \; & -[\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 \\
& + \kappa \max_{\ell'} \left\{ \left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] p(\ell', y) + \beta \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \{(1-\tau)(y' + x') \right.\right. \\
& \left.\left. - \left[(1-\tau)r + \xi\right]\ell' + V(\ell', y')\} \phi(x')dx' \right] \right\}.
\end{aligned}
$$

From this we have

$$
\begin{aligned}
V(\ell, y) = \max_{\kappa} \; & -[\kappa - 1 + \delta - \tau\delta] - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 \\
& + \kappa \max_{\ell'} \left\{ \left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] p(\ell', y) + \beta(1-\tau)\mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} x'\phi(x')dx' \right] \right. \\
& \left. + \beta \mathbb{E}_y \left[\left[(1-\tau)(y' - r\ell') - \xi\ell' + V(\ell', y')\right] \int_{x_d(\ell', y')}^{\overline{x}} \phi(x')dx' \right] \right\},
\end{aligned}
$$

where it is easy to recognize this gives (7) for the outer maximization with respect to κ, and (4) (together with (5)) for the inner maximization with respect to ℓ'.

The price of debt is (for increased generality, we derive here the price for the general case of non-zero recovery at default)

$$
\begin{aligned}
p(b, k, y) = \beta \mathbb{E}_y & \left[\int_{x_d(b,k,y')}^{\overline{x}} \{r + \xi + (1-\xi)p(b', k', y')\} \phi(x')dx' \right. \\
& \left. + \int_{\underline{x}}^{x_d(b,k,y')} \frac{(1-\tau)(y' + x')k + R(b, k, y')}{b} \phi(x')dx' \right]. \qquad (18)
\end{aligned}
$$

For given (b, k), the debt holders' recovery at default is $(1-\tau)(y+x)k + R(b, k, y)$, where $R(b, k, y) = \min\{(1 - \delta - \alpha)k, \tau rb + V(b, k, y)\}$ and α is a bankruptcy cost parameter. With this specification, the recovery is never higher than the asset value at default,

$(1-\tau)(y+x)k+\tau rb+V(b,k,y)$, and therefore it is never higher than the debt obligation, $(r+\xi)b$. Hence, given $p(b,k,y)=p(\ell,y)$, we have

$$p(\ell,y)=\beta\mathbb{E}_y\left[\{r+\xi+(1-\xi)p(\ell',y')\}\int_{x_d(\ell,y')}^{\overline{x}}\phi(x')dx' \right.$$
$$\left. +\int_{\underline{x}}^{x_d(\ell,y')}\frac{(1-\tau)(y'+x')+R(\ell,y')}{\ell}\phi(x')dx'\right],$$

where $R(\ell,y')=\min\{1-\delta-\alpha,\tau r\ell+V(\ell,y')\}$. By excluding the recovery at default part in the second line, (6) obtains. □

B Model of unlevered firm

This is a first benchmark model of a firm with no debt. The value of equity (and of the firm) is defined by the recursive program

$$V^u(k,y)=\max_{k'}-[k'-(1-\delta)k]+\tau\delta k-\Psi(k,k')$$
$$+\beta\mathbb{E}_y\left[\int_{\underline{x}}^{\overline{x}}\{(1-\tau)(y'+x')k'+V^u(k',y')\}\phi(x')dx'\right].$$

The linearity of the production function and the assumed capital adjustment cost ensure that $V^u(k,y)=kV^u(1,y)$ (homogeneity of degree one). We denote by $\kappa=k'/k$, and (with a small abuse of notation) $V^u(y)=V^u(1,y)$. We have

$$V^u(y)=\max_{\kappa}-[\kappa-(1-\delta)]+\tau\delta-\frac{1}{2}\varphi[\kappa-(1-\delta)]^2+\kappa\beta\{(1-\tau)\mu_y+\mathbb{E}_y[V^u(y')]\},\tag{19}$$

where $\mu_y=\mathbb{E}_y[y']$. In the general case, $V^u(\cdot)$ is found using the value iteration algorithm together with the first-order condition $1+\varphi[\kappa-(1-\delta)]=\beta\{(1-\tau)\mu_y+\mathbb{E}_y[V^u(y')]\}$.

Under the assumption that y is non-stochastic and constant through time, and denoting the value function by $V^u=V^u(y)$, we have

$$V^u=\max_{\kappa}-[\kappa-(1-\delta)]+\tau\delta-\frac{1}{2}\varphi[\kappa-(1-\delta)]^2+\kappa\beta[(1-\tau)y+V^u].\tag{20}$$

The first-order condition is

$$1+\varphi[\kappa-(1-\delta)]=\beta[(1-\tau)y+V^u].$$

By using this equation to eliminate V^u in (20), and after a few manipulations we find the optimal solution of the investment problem:[37]

$$\kappa^* = 1 + r - \sqrt{(r+\delta)^2 + \frac{2}{\varphi}\left[r - (1-\tau)(y-\delta)\right]}, \tag{21}$$

where $r = 1/\beta - 1$, as long as

$$\delta + \frac{r}{1-\tau} < y < \delta + \frac{r}{1-\tau} + \frac{\varphi}{2}\frac{(r+\delta)^2}{1-\tau}. \tag{22}$$

By using the optimal investment solution in the first-order condition above, we find the value function:

$$V^u = (1+r)\left[1 + \varphi(r+\delta)\right] - (1-\tau)y - \varphi(1+r)\sqrt{(r+\delta)^2 + \frac{2}{\varphi}\left[r - (1-\tau)(y-\delta)\right]}. \tag{23}$$

If the condition in (22) holds true, the solution in (23) is equal to the solution of (20) when using the value iteration algorithm.

In the case y is a Markov process, if (22) is true for all possible values in the ergodic set of y, the fixed point problem in (19) has solution (i.e., the value iteration algorithm converges). This sets a restriction on the parameters of the y process.

C Model of levered firm with constant capital stock

This is a second benchmark model of a firm that can manage its leverage while the capital stock remains constant, $k' = k = 1$, and therefore investment equals depreciation. This model holds for $\xi < 1$, that is debt maturity strictly higher then one year.[38] The value of equity is

$$\begin{aligned} V(b,y) = &-(1-\tau)\delta - \frac{1}{2}\varphi\delta^2 + \max_{b'} \ \left[b' - (1-\xi)b\right] p(b') \\ &+ \beta\mathbb{E}_y\left[\int_{x_d(b',y')}^{\overline{x}} \left\{(1-\tau)(y' + x' - rb') - \xi b' + V(b',y)\right\}\phi(x')dx'\right], \end{aligned} \tag{24}$$

[37] Of the two roots of the quadratic equation, the solution is the one in the text because of the condition $\kappa < 1 + r$.

[38] We will show later on a general model for $\xi = 1$, of which the model with no investment is a subcase.

where $x_d(b', y') = rb' + [\xi b' - V(b')]/(1-\tau) - y'$. The debt price in the general case with non-zero recovery at default is

$$p(b, y) = \beta \mathbb{E}_y \left[\{r + \xi + (1-\xi)p(b', y')\} \int_{x_d(b,y')}^{\overline{x}} \phi(x')dx' \right.$$
$$\left. + \int_{\overline{x}}^{x_d(b,y')} \frac{(1-\tau)(y'+x') + R(b', y')}{b} \phi(x')dx' \right], \quad (25)$$

where $R(b, y) = \min\{1 - \delta - \alpha, \tau rb + V(b, y)\}$ and $b' = g(b, y')$ is the optimal policy function from (24). The solution of the model is found using value function iteration based simultaneously on equations (24) and (25).

D Total firm value maximization

In this section, we derive a third benchmark model, in which investment and debt decision are maximizing the value of the firm and therefore avoids all agency issues. For increased generality, we consider the case of non-zero recovery at default.

Lemma 3. *The default/investment/financing MPE in the case equity holders commit to maximize total firm value is found by solving the dynamic program*

$$F(k, y) = \max_{(b',k')} \; -[k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + \beta(1-\tau)\mu_y k'$$
$$+ \beta \mathbb{E}_y \left[\int_{\underline{x}}^{x_d(b',k',y')} R(b', k', y')\phi(x')dx' + \int_{x_d(b',k',y')}^{\overline{x}} \{\tau rb' + F(k', y')\} \phi(x')dx' \right], \quad (26)$$

where the default threshold, $x_d(b', k', y')$*, is defined in* (2)*, with* $V(b', k', y') = F(k', y') - (1-\xi)b'p(b'', k'', y')$*, and* $p(b, k, y)$ *is defined in* (18)*, based on the optimal policy resulting from* (26)*.*

Proof. At state (b, k, y) in t, total firm value, F, for an arbitrary (b', k') is

$$V(b, k, y) + (1-\xi)b\, p(b', k', y) = -[k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + b'p(b', k', y)$$
$$+ \beta \mathbb{E}_y \left[\int_{x_d(b',k',y')}^{\overline{x}} \{(1-\tau)(y'+x')k' - [(1-\tau)r + \xi]\, b' + V(b', k', y')\} \phi(x')dx' \right],$$

which, by replacing $p(b', k', y)$ from (3) and after a few simplifications becomes

$$
\begin{aligned}
-[k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + \beta\mathbb{E}_y \left[\int_{\underline{x}}^{\overline{x}} (1-\tau)(y'+x')k' + \phi(x')dx'\right] \\
+ \beta\mathbb{E}_y \left[\int_{\underline{x}}^{x_d(b',k',y')} R(b', k', y')\phi(x')dx' \right. \\
\left. + \int_{x_d(b',k',y')}^{\overline{x}} \{\tau r b' + V(b', k', y') + (1-\xi)b'p(b'', k'', y')\}\,\phi(x')dx'\right],
\end{aligned}
$$

where (b'', k'') denotes the choice at $t+1$. By replacing $V(b', k', y') + (1-\xi)b'p(b'', k'', y')$, with the total firm value at (b', k', y') and choosing (b', k') that maximizes the expression above we define the recursive program in (26), from which we determine F and the related optimal policy.

Notably, the program

$$
\begin{aligned}
\max_{(b',k')} \; -[k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + \beta\mathbb{E}_y \left[\int_{\underline{x}}^{\overline{x}} (1-\tau)(y'+x')k' + \phi(x')dx'\right] \\
+ \beta\mathbb{E}_y \left[\int_{\underline{x}}^{x_d(b',k',y')} R(b', k', y')\phi(x')dx' + \int_{x_d(b',k',y')}^{\overline{x}} \{\tau r b' + F(b', k', y')\}\,\phi(x')dx'\right]
\end{aligned}
$$

does not depend on b, and therefore the optimal choice, $(b', k') = G(k, y)$, and the value function, $F(k, y)$, are independent of b, which by a recursive application gives (26). The debt price is from equation (3) using the optimal policy in (26), and the value of equity required to pin down the recovery value, R, and the default threshold, x_d, is $V(b', k', y') = F(k', y') - (1-\xi)b'p(b'', k'', y')$. □

Lemma 4. *The MPE for the case equity holders maximize total firm value is found by solving simultaneously the outer investment program*

$$
F(y) = \max_{\kappa} - [\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 + \beta\kappa\left[(1-\tau)\mu_y + f(y)\right], \tag{27}
$$

and the inner leverage program

$$
f(y) = \max_{\ell'} \; \mathbb{E}_y\left[R(\ell', y')\int_{\underline{x}}^{x_d(\ell',y)} \phi(x')dx' + \{\tau r\ell' + F(y')\}\int_{x_d(\ell',y)}^{\overline{x}} \phi(x')dx'\right], \tag{28}
$$

where the debt price is

$$p(y) = \beta \mathbb{E}_y \left[\{r + \xi + (1-\xi)p(y')\} \int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx' + \int_{\underline{x}}^{x_d(\ell',y')} \frac{(1-\tau)(y'+x') + R(\ell',y')}{\ell'} \phi(x')dx' \right], \quad (29)$$

with default threshold

$$x_d(\ell', y') = r\ell' + \frac{\xi\ell'(1-p(y')) + \ell' p(y') - F(y')}{1-\tau} - y'$$

and recovery at default

$$R(\ell', y') = \min\{1 - \delta - \alpha, \tau r \ell' + F(y') - (1-\xi)\ell' p(y')\}.$$

The optimal solution of the outer investment program is

$$\kappa^* = \frac{\beta[(1-\tau)\mu_y + f(y)] - 1}{\varphi} + (1-\delta).$$

Proof. The auxiliary program to solve (26), following the approach in Appendix A, is

$$F(y) = \max_{\kappa} -[\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi[\kappa - (1-\delta)]^2 + \beta\kappa \max_{\ell'} \mathbb{E}_y \left[R(\ell', y') \int_{\underline{x}}^{x_d(\ell',y')} \phi(x')dx' + \{\tau r \ell' + F(y')\} \int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx' \right]$$

where we used $F(k, y) = kF(1, y)$ and dropped the first argument from F. Also, $R(\ell', y') = \min\{1 - \delta - \alpha, \tau r\ell' + V(\ell', y')\}$ and $x_d(\ell', y') = r\ell' + (\xi\ell' - V(\ell', y'))/(1-\tau) - y'$, where $V(\ell', y')$ is the related equity value. From this, we can define the outer investment program in (27), and the inner leverage program in (28). The optimal policy in the leverage program depends only on the current y, that is $\ell' = g(y)$. Therefore, the relevant price of debt at the current date is $p(y) = p(g(y), y)$, which only depends on y and not on current leverage, as shown in (29). Hence, the equity value required to determine the default threshold and the recovery at default is $V(\ell', y') = F(y') - (1-\xi)\ell' \, p(y')$. Once $f(y)$ and the leverage policy $\ell' = g(y)$ are determined, the outer investment program has analytic solution κ^*. □

Numerically, $f(y)$ (and therefore investment) and $\ell' = g(y)$ are (slightly) decreasing in ξ (increasing in debt maturity). This means that with full commitment to the value maximizing leverage policy, the longer the maturity the higher the value of the firm, investment, and the leverage.[39] A priori, it is difficult to determine the sign of such a dependence because in the objective function of the leverage program both R and x_d are increasing in ξ, and therefore, for a given ℓ', on the one hand the recovery at default is lower for a longer maturity, but on the other default becomes less likely for for a longer maturity. Which of the two effects is dominant must be determined numerically.

E Proof of Proposition 1

1. (a) Convexity of $v(\cdot, y)$ is a consequence of its optimality, as per equation (4). Indeed, for any $\ell' \neq \ell$, shareholders can adjust the leverage to ℓ with proceeds $(\ell - \ell')p$ from the issuance.[40] Given the optimality of v, the shareholders cannot get a higher value than $v(\ell'/\kappa, y)$, that is $v(\ell'/\kappa, y) \geq v(\ell/\kappa, y) - (\ell' - \ell)p/\kappa$, which means v is (weakly) convex. Exactly the same argument can be used to show the convexity of $V(\cdot, y)$, given $\kappa = \kappa^*$, the optimal investment policy.

   (b) To show that $p(\cdot, y)$ is decreasing,

$$\partial_\ell p(\ell, y) = \beta \mathbb{E}_y \left[(1-\xi)\partial_\ell p(\ell, y')\partial_\ell g(\ell, y') \int_{x_d(\ell,y')}^{\overline{x}} \phi(x')dx' \right.$$
$$\left. - \left[r + \xi + (1-\xi)p(\ell', y')\right] \phi(x_d(\ell, y'))\partial_\ell x_d(\ell, y') \right]. \quad (30)$$

   Because

$$\partial_\ell V(\ell, y) = \kappa^* \frac{1}{\kappa^*} \partial_\ell v(\ell/\kappa^*, y) = \partial_\ell v(\tilde{\ell}, y) = -p(\ell', y), \quad (31)$$

   where κ^* is the optimal solution of the program in (7), then $\partial_\ell^2 v(\tilde{\ell}, y) = -\partial_\ell p(\ell', y)\partial_\ell g(\tilde{\ell}, y)$, is positive, given $v(\cdot, y)$ is convex. Hence, $\partial_\ell p \partial_\ell g < 0$ in (30). Also, using (31),

$$\partial_\ell x_d(\ell, y) = r + \frac{\xi - \partial_\ell V(\ell, y)}{1-\tau} = r + \frac{\xi + p(\ell, y)}{1-\tau},$$

   which is clearly positive. Hence, the right-hand side of (30) is negative.

[39] This is in contrast with Proposition B.3 in Xiang (2019) stating that, under commitment, the value of the firm for *any* debt maturity is the same as in the case of single-period debt.

[40] This is equivalent to adjusting the debt from b' to b, with proceeds $(b - b')p/k$.

(c) The first-order condition of (4) is

$$p(\ell', y) + \left[\ell' - (1-\xi)\tilde{\ell}\right] \partial_\ell p(\ell', y)$$
$$+ \beta \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \left\{ -(1-\tau) r + \xi + (1-\xi) \partial_\ell V(\ell', y') \right\} \phi(x') dx' \right] = 0.$$

Using (31), the first-order condition becomes (we consider here the case with non-zero recovery at default)

$$\left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] \partial_\ell p(\ell', y)$$
$$+\beta \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \tau r \phi(x') dx' + \int_{\underline{x}}^{x_d(\ell', y')} \frac{(1-\tau)(y+x') + R(\ell', y')}{\ell'} \phi(x') dx' \right] = 0.$$

and $\ell' = g(\tilde{\ell}, y)$ is its unique solution, given the concavity of the objective function. Denoting by $\Gamma(\ell', \tilde{\ell})$ the left-hand side of the equation above, and using the implicit function theorem, the derivative of $g(\tilde{\ell}, y)$ is $\partial_{\tilde{\ell}} g = -\partial_{\tilde{\ell}} \Gamma / \partial_{\ell'} \Gamma$. Because $\partial_{\tilde{\ell}} \Gamma = -(1-\xi)\partial_\ell p(\ell', y) > 0$, then the sign of $\partial_{\tilde{\ell}} g$ depends on the sign of $\partial_{\ell'} \Gamma$, which is negative if the objective function in (4) is assumed concave.[41]

(d) The derivation of (9) is straightforward. To determine the sign of $\partial_\ell h$, we use the implicit function theorem again, although based on the first-order condition for optimal investment in (9), which gives

$$\partial_\ell h = -\frac{-\partial^2_{\tilde{\ell}} v\,(\ell/\kappa)\,\ell/\kappa^2}{\partial^2_{\tilde{\ell}} v\,(\ell/\kappa)\,\ell^2/\kappa^3 - \varphi}.$$

The denominator of the ratio is negative under the assumption that the objective function in (7) is concave. Hence, the sign of $\partial_\ell h$ is the opposite of the sign of $\partial^2_{\tilde{\ell}} v$, which is positive, given v is convex. Hence, investment is decreasing in leverage.

2. If $x_d < \underline{x}$ for all (b, k, y'), then from (3) we have

$$p(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1-\xi) p(b', k', y') \right]. \tag{32}$$

[41]If there is zero recovery at default, the partial derivative of Γ with respect to ℓ' becomes

$$\partial_{\ell'} \Gamma = \partial_\ell p(\ell', y) + \left[\ell' - (1-\xi)\tilde{\ell}\right] \partial^2_\ell p(\ell', y) - \beta \tau r \mathbb{E}_y \left[\partial_{\ell'} x_d \phi(x_d)\right],$$

which is negative if $\partial^2_\ell p$, the second partial derivative of p with respect to ℓ, is negative.

Because $r = 1/\beta - 1$, by conjecturing the solution $p(b', k', y') = 1$ on the right-hand side, then the debt price is $p(b, k, y) = 1$, which confirms this is a solution. It is easy to show this is the only solution of (32). Under the no-default assumption, the right-hand side of (1) becomes

$$\max_{(b',k')} - [k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') - \tilde{b}$$
$$+ \beta \mathbb{E}_y \left[(1-\tau)y'k' + \tau r b' + (1-\xi)b' + V(\tilde{b}', k', y') \right] =$$
$$-\tilde{b} + \max_{(b',k')} - [k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + \beta \mathbb{E}_y \left[(1-\tau)y'k' + \tau r b' + F(\tilde{b}', k', y') \right],$$

where we took out of the objective function the constant term, $-\tilde{b}$, which does not affect the optimal policy. Hence, the optimal policy, (b', k'), maximizes the value of the firm (for the no-default case). The equity value resulting from this program, $V(\tilde{b}, k, y)$, plus $\tilde{b}$, gives $F(\tilde{b}, k, y)$. Hence, solving the fixed point of the above program gives the optimal firm value and the firm value maximizing investment and debt policies, as shown in the previous lemma. □

3. For $\xi = 1$, the right-hand side of (1) becomes

$$\max_{(b',k')} - [k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + b'p(b', k', y)$$
$$+ \beta \mathbb{E}_y \left[\int_{x_d(b',k',y')}^{\overline{x}} \left\{ (1-\tau)(y' + x')k' - [1 + (1-\tau)r]\, b' + V(\tilde{b}', k', y') \right\} \phi(x') dx' \right]. \tag{33}$$

By replacing (for more generality, we provide the proof for the case with non-zero recovery at default)

$$b'p(b', k', y) = \beta \mathbb{E}_y \left[\int_{x_d(b',k',y')}^{\overline{x}} (1+r) b' \phi(x') dx' + \int_{\underline{x}}^{x_d(b',k',y)} \{ (1-\tau)(y' + x')k' + R(b', k', y') \} \phi(x') dx' \right]$$

in (33), and simplifying, we find

$$\max_{(b',k')} - [k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + \beta(1-\tau)\mu_y k'$$
$$+ \beta \mathbb{E}_y \left[R(b', k', y') \int_{\underline{x}}^{x_d(b',k',y)} \phi(x') dx' + \left\{ \tau r b' + V(\tilde{b}', k', y') \right\} \int_{x_d(b',k',y')}^{\overline{x}} \phi(x') dx' \right].$$

This program coincides with the one in the right-hand side of (26), except for the value function. However, because current leverage is identically zero for $\xi = 1$, equity and firm value coincide and the programs yield recursively the same optimal value and policy. Hence, the fixed point from this program is the same as the one for (26).

To show that also investment is firm value maximizing, from (26) and following the same steps as in Lemma 4, with $V(y)$ in place of $F(y)$, we have an outer investment program

$$V(y) = \max_{\kappa} - [\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi [\kappa - (1-\delta)]^2 + \kappa\beta [(1-\tau)\mu_y + v(y)] \quad (34)$$

and the inner leverage program

$$v(y) = \max_{\ell'} \mathbb{E}_y \left[R(\ell', y') \int_{\underline{x}}^{x_d(\ell', y')} \phi(x')dx' + \{\tau r \ell' + V(y')\} \int_{x_d(\ell', y')}^{\underline{x}} \phi(x')dx' \right].$$

and the corresponding debt price is

$$p(y) = \beta \mathbb{E}_y \left[(1+r) \int_{x_d(\ell', y')}^{\overline{x}} \phi(x')dx' + \int_{\underline{x}}^{x_d(\ell', y')} \frac{(1-\tau)(y'+x') + R(\ell', y')}{\ell'} \phi(x')dx' \right],$$

where $\ell' = g(y)$ is the optimal leverage from the leverage program. □

F Numerical approach

The algorithm to solve the general model is based on a discretized version of the leverage interval, $[0, \overline{\ell}]$. Although the numerical results are presented for $\ell \in [0, 1]$, we solve the model for $\overline{\ell} > 1$ to avoid the effect of the upper bound on the numerical solution. As for the distribution of x, we set $[\underline{x}, \overline{x}] = [-X, X]$, with $X > 0$, and use the specification

$$\phi(x) = \frac{3}{4X} \left[1 - \left(\frac{x}{X} \right)^2 \right].$$

With this specification of ϕ, the integrals involved in the calculations of the iterative algorithm have analytic expressions:

$$\int_{x_d}^{\overline{x}} \phi(x)dx = \frac{1}{2} - \frac{3x_d}{4X} + \frac{x_d^3}{4X^3}, \qquad \int_{x_d}^{\overline{x}} x\phi(x)dx = \frac{3X}{16} - \frac{3x_d^2}{8X} + \frac{3x_d^4}{16X^3},$$

$$\int_{\underline{x}}^{x_d} \phi(x)dx = \frac{1}{2} + \frac{3x_d}{4X} - \frac{x_d^3}{4X^3}, \qquad \int_{\underline{x}}^{x_d} x\phi(x)dx = -\frac{3X}{16} + \frac{3x_d^2}{8X} - \frac{3x_d^4}{16X^3}.$$

The process for y is

$$\log y' = (1-\nu)\log \overline{y} + \nu \log y + \sigma\varepsilon,$$

with $\varepsilon \sim \mathcal{N}(0,1)$, that we discretize with seven points the compact interval

$$\left[-3\sigma/\sqrt{1-\nu^2}, +3\sigma/\sqrt{1-\nu^2}\right]$$

using Gauss-Hermite quadrature.

The optimization in (4) with respect to ℓ' is done with an exhaustive search on a grid of 7500 points in $[0, \overline{b}]$. The cubic spline interpolation of $v(\ell/\kappa, y)$ is done with 100 points. The tolerance set for maximum of the improvement of the value function and of the debt price is 10^{-6}.

G Proof of Proposition 2

For increased generality, we provide the proof under the assumption of non-zero recovery at default.

Lemma. For all $n \geq 0$, $p^j_{n+1} \geq p^j_n$, where $p^j_0 = p$, for $j = d, a$.

Proof. The lemma can be proved by induction. We first consider the case $j = d$. For $n = 0$, the statement is $p^d_1(b,k,y) \geq p(b,k,y)$, which is true because in equation (3) in default $(x' < x_d(b,k,y'))$

$$(1-\tau)(y'+x')k + R(b,k,y') \leq (r+\xi)b.$$

Next, assuming the inequality is true for a given n, that is $p^d_n(b',k',y') \geq p^d_{n-1}(b',k',y')$, because the firm's policy is invariant with respect to n and using equation (14), from this inequality we have $p^d_{n+1}(b,k,y) \geq p^d_n(b,k,y)$, which proves the lemma.

We then consider the case $j = a$. For $n = 0$ the statement is $p^a_1(b,k,y) \geq p(b,k,y)$. We first notice that $p^a_1(b,k,y) \geq p^d_1(b,k,y)$ because using (11) and (16) this is equivalent to $\max\{p(b',k',y'), p(b,k,y')\} \geq p(b',k',y')$, which is obviously true. We have proved above that $p^d_1(b,k,y) \geq p(b,k,y)$, from which we conclude $p^a_1(b,k,y) \geq p(b,k,y)$. Next, assuming the statement is true for given n, that is $p^a_n \geq p^a_{n-1}$, this implies

$$\max\{p^a_n(b',k',y'), p^a_n(b,k,y')\} \geq \max\{p^a_{n-1}(b',k',y'), p^a_{n-1}(b,k,y')\}.$$

Because the firm's policy is invariant with respect to n and using (16), from the above inequality we have $p^a_{n+1}(b,k,y) \geq p^a_n(b,k,y)$, which proves the lemma. □

1. The proof is by induction. As for the non-negativity of the agency spread, for the case $H = 1$ the statement $as_1(b',k',y) \geq 0$ is equivalent to $p^a_1(b',k',y) \geq p^d_1(b',k',y)$. Given the specification of the price of the debt without default in (11) and without default and agency conflicts in (12), $p^a_1(b,k,y) \geq p^d_1(b,k,y)$ is equivalent to $\max\{p(b',k',y'), p(b,k,y)\} \geq p(b',k',y')$, which is obviously true. As for the induction step, assuming the statement $p^a_{H-1}(b',k',y') \geq p^d_{H-1}(b',k',y')$ is true for $H-1$, for the case $H > 1$, $as_H(b',k',y) \geq 0$ is equivalent to $p^a_H(b',k',y') \geq p^d_H(b',k',y')$. Using the same argument as above, comparing (16) to (14) we have

   $$\max\{p^a_{H-1}(b',k',y'), p^a_{H-1}(b,k,y')\} \geq p^a_{H-1}(b',k',y') \geq p^d_{H-1}(b',k',y'),$$

   which is assumed true. Hence the agency spread is always non-negative.

   To prove that $ds_H \geq 0$, which is equivalent to $p^d_H \geq p$, for all $H \geq 1$ we use the lemma above, and note that $p^d_H \geq p^d_{H-1} \geq \ldots \geq p$.

2. The statement $cs_{H-1} \leq cs_H$ is equivalent to $p^a_H \geq p^a_{H-1}$, which has been proved in the lemma above. The statement $cs \geq cs_H$ is equivalent to $p^a_H \leq 1$, which is true for all $H \geq 1$ as p^a_H is bounded by the risk-free price of \$1. Also, $cs_H \to cs$ for $H \to \infty$, because the sequence $\{p^a_H\}$ is increasing and bounded above, hence it has limit and $p^a_H \to 1$, which concludes the proof.

3. To prove that $ds_H \to cs$ for $H \to \infty$, we observe that $ds_H \leq cs_H \leq cs$ for all H. Also, $ds_H \leq cs$ is equivalent to $p^d_H \leq 1$ for all H. As before, the sequence $\{p^d_H\}$ is increasing and bounded above by 1, hence $p^d_H \to 1$.

   Point (1) above states that $as_H \geq 0$, which is equivalent to $p^a_H \geq p^d_H$, for all H. Because $p^d_H \to 1$ and $p^a_H \to 1$, this proves that $as_H \to 0$. □

H Discussion on the proposed approach to decompose the credit spread in the agency vs the default component

The proposed decomposition of the credit spread into a default component and an agency component is motivated by the purpose of separating the affect of the two commitment problems, to repay the debt and to future debt policies, which are both controlled by the same maturity parameter, ξ. Also, the decomposition is for a *given* investment and financing policy in the MPE. We argue that any alternative decomposition approach to the one introduced above would entail an interaction between the policy effects and the price effects of the agency issues, which would cause a deviation from the equilibrium policy. Hence, the resulting separation in two components would depend on the extent of such an interaction and would be arbitrary.

This can be better understood by thinking of two (seemingly plausible) decomposition approaches. In the first approach, the effect of agency would be neutralized because the value losses from dilution are 'rebated' to legacy bond holders by shareholders. Alternatively, the 'rebate' may be paid by an external party, who would need to be financed by the shareholders anyway. Because of this provision, the shareholders' policies would change relative to the equilibrium ones. A second approach is to assume the shareholders are subject to a 'blanket' protective covenant that precludes them from taking any actions that lowers bond values at the respective horizons. While such a covenant would eliminate the effects of agency on the continuation value on the debt, it would also constrain the shareholders' future decisions, affecting their policy. For both approaches, we would then define the agency spread as the difference between the credit spread obtained in the benchmark model and the one resulting from the approach. Either way, the resulting decomposition into an agency and a default spread would not separate the pure default component from the agency component under the MPE, exactly because this definition entails a deviation from that equilibrium.

Our decomposition of the credit spread naturally follows by applying the same approach used to define the default spread to the case of a debt price reduction due to agency conflicts. Given the equilibrium policy, there is no other way to define the credit spread but $(r+\xi)/p-(r+\xi)/p^d$ (or $(r+\xi)/p-(r+\xi)/p^d_H$ for finite horizons), because the yield $(r+\xi)/p^d$ is for a debt which is identical to the one in the baseline model, except for default. Notably, to define the latter debt we do not need to assume that the debt holders get a 'rebate' or there is a protective covenant (or collateral) that eliminates default risk. In fact, if the debt contract had this provision, we would be calculating the credit spread for a policy different from the one in the MPE.

Finally, the definition in (17) provides a *on-the-equilibrium-path* decomposition of the credit spread into a default component and an agency component exactly because we do not allow the equilibrium policy to be affected. An alternative definition, which entails a deviation from the equilibrium policy, would be off the equilibrium path at any state of the baseline model.

I Sensitivity analysis

In this appendix we check the robustness of the numerical results presented in the main body of the paper by presenting the results of two alternative specifications of the model. The fundamental difference with the specification in the main text is that

Table 3: Debt maturity and firm dynamics - Sensitivity analysis

These moments are based on a simulation of 1000 firms (starting from random initial states), for 250 years, from which we dropped the first 50 years. The paths for y and the initial states are the same for all ξ. $\mu(bl)$ and $ac(bl)$ are the average and autocorrelation of b/k, $\rho(y, bl)$ is the correlation of the of y with b/k, $\mu(ml)$ is the average of market leverage, $\mu(q)$ and $ac(q)$ are the average and autocorrelation of Tobin's Q, $m(cs)$ is the median credit spread, and $\mu(dp)$ is the average 1-y default probability. The left panel is for the same credit risk as in the base case, gauged by $X = 1.6$, and positive recovery at default with bankruptcy cost $\alpha = 0.3$. The right panel is for lower credit risk than in the base case, gauged by $X = 1.4$, and lower bankruptcy cost, $\alpha = 0.15$. All the other parameters are from Table 1.

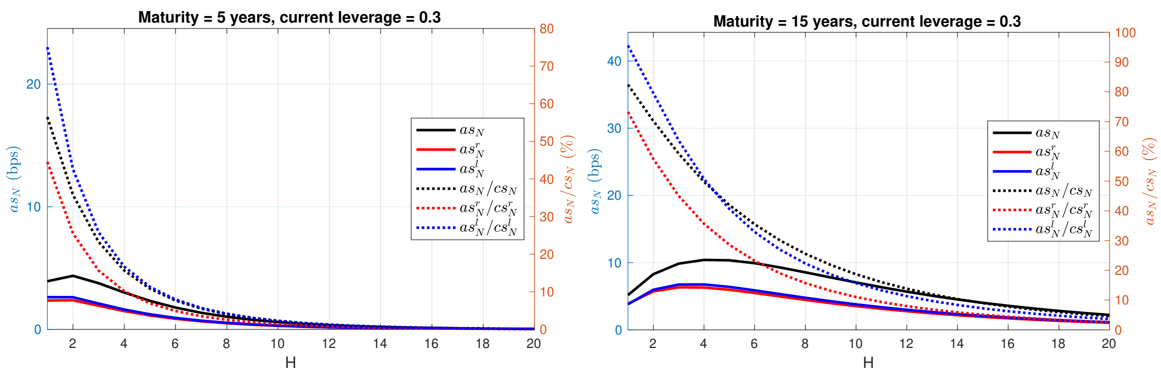
	Recovery at default				Lower credit risk			
$1/\xi$	1	5	10	15	1	5	10	15
$\mu(bl)$	0.37	0.39	0.40	0.41	0.57	0.58	0.59	0.60
$ac(bl)$	0.88	0.93	0.95	0.96	0.88	0.92	0.95	0.96
$\rho(bl, y)$	1.00	0.99	0.96	0.94	1.00	0.99	0.97	0.93
$\mu(ml)$	0.25	0.26	0.26	0.27	0.38	0.38	0.38	0.39
$\mu(Q)$	1.47	1.46	1.45	1.44	1.52	1.50	1.48	1.47
$\mu(i)$	0.10	0.09	0.09	0.09	0.11	0.09	0.09	0.09
$\rho(i, y)$	1.00	0.85	0.90	0.91	1.00	0.77	0.80	0.83
$\mu(cs)$	7.71	35.70	58.00	74.38	5.14	26.77	47.35	63.62
$\mu(dp)$	0.03	0.15	0.25	0.33	0.04	0.20	0.35	0.48

here we consider non-zero recovery at default, under the assumption of pari passu debt issuance. In particular, at default the debt holders receive $(1-\tau)(y+x)k + R(b, k, y)$, where $R(b, k, y) = \min\{(1-\delta-\alpha)k, \tau rb + V(b, k, y)\}$, and α is a proportional bankruptcy cost.

Given the parameters in Table 1, we analyze two cases: one with the same credit risk as in the base case, but with recovery at default, with $\alpha = 0.3$; the other with low credit risk, gauged by $X = 1.4$, and $\alpha = 0.15$. The first case is to show that the consideration of a non-zero debt recovery at default does not qualitatively change our conclusions. The second is to offer a comparison to a case with lower credit risk. Table 3 shows moments from the two simulated economies. While the same patterns seen in Table 2 are also found here (optimal leverage and leverage persistence are increasing with debt maturity, Tobin's Q and investment decreasing with debt maturity, higher correlation of investment with negative productivity shocks for longer debt maturity), Table 3 shows that, compared to the model with zero debt recovery at default, book and market leverage and credit risk are lower, and the correlation of book leverage with productivity shocks is higher. This means that a positive recovery at default makes the

Figure 12: Agency spread - Sensitivity analysis

The figure plots the agency spread in absolute (left y-axis, solid lines) and relative (right y-axis, dotted lines) terms, for three cases: the base case with zero recovery at default (black lines), the case with positive recovery at default and $\alpha = 0.3$ (red lines), and the case with recovery at default with $\alpha = 0.15$ and lower credit risk ($X = 1.4$). These rates are presented for $k = 1$, $\ell = 0.3$, and different horizons H, which represent the number of periods for which debt holders are exposed to the negative effects of equity holders' lack of commitment. Panels on the left have maturity $1/\xi = 5$ years and panel on the right $1/\xi = 15$ years. The MPE is based on parameters in Table 1 assuming that y is non-stochastic and constant.



debt policy less stiff and reduces the severity of the leverage ratchet effect, and hence the contribution of debt maturity as a commitment device is less visible. The reduced severity can be seen also in Figure 12, where the agency component of the credit spread, vis-à-vis the base case considered in the paper, is lower both in absolute and relative terms.

Compared to the case with $X = 1.6$ and $\alpha = 0.3$, Table 3 shows that with lower credit risk, book and market leverage and Tobin's Q are obviously higher, and the correlation of investment with negative productivity shocks is lower, but also the effect of debt maturity as a commitment device is smaller, which indicates that the severity of the leverage ratchet effect increases with credit risk. This intuition is confirmed in Figure 12, which shows that although the agency component of credit spread in absolute terms is lower in this case, in relative terms it constitutes a larger part of the credit spread.