Tail risk and asset prices in the short-term*

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Abstract

We combine high-frequency stock returns with risk-neutralization to extract the daily common component of tail risks perceived by investors in the cross-section of firms. Our tail risk measure significantly predicts the equity premium and variance risk premium at short-horizons. Furthermore, a long-short portfolio built by sorting stocks on their recent exposure to tail risk generates abnormal returns with respect to standard factor models and helps explain the momentum anomaly. Incorporating investors' preferences via risk-neutralization is fundamental to our findings.

Keywords: Left tail risk, return predictability, factor models, risk-neutralization, high-frequency data.

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1 Introduction

Left tail risk is a pervasive feature of financial markets. As such, a large body of work has investigated its role in determining asset prices. Taken together, the empirical evidence indicates that compensation required by investors for bearing tail risk is fundamental to explain aggregate market risk premia and the cross-section of stock returns at relatively low frequencies (monthly or longer). This evidence is based on a number of different tail measures. In particular, information can be extracted either from stock prices (e.g., Bali et al., 2009; Kelly and Jiang, 2014), reflecting risk under the physical or statistical measure under which prices are observed, or from option prices (e.g., Andersen et al., 2015; Bollerslev et al., 2015), capturing tail risk under the risk-neutral measure incorporating investors' preferences.

In this paper, we propose a new tail measure available at a daily frequency, which allows us to investigate the short-term effects of tail risk on asset prices. We first estimate the common tail risk component of a cross-section of intra-day stock returns on day t, $\lambda_t^{\mathbb{P}}$, using the Hill (1975) power law estimator. This essentially adapts the tail index by Kelly and Jiang (2014) to a high-frequency environment. Then, we introduce a novel version of the Hill estimator, $\lambda_t^{\mathbb{Q}}$, that relies on risk-neutralized returns. More specifically, we apply a nonparametric adjustment to the pooled cross-section of stock returns on day t where "bad" states of nature, represented by states of high marginal utility, are overweighted to reflect investors' compensation for risk. The dynamics of the physical and risk-neutral Hill estimators differ substantially as compensation for risk varies over time.

Our approach overcomes two main challenges. First, extreme events are infrequently observed by definition. This limits the information available from the time series of a single asset such as the market index. Second, option maturities are relatively long compared to daily events, which makes it difficult to measure the tail risk specific to day t using option prices. By using high-frequency data on a large cross-section of stock returns, we are able to extract information about the level of tail risk at day t from the individual extreme events experienced by different stocks. Furthermore, our risk-neutralization allows to obtain a tail measure incorporating the economic valuation of tail risks by investors, which otherwise would only be possible using option prices (see, e.g., Aït-Sahalia and Lo, 2000).

Our empirical analysis is conducted considering each of our tail measures ($\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$) in order to assess the information content of investors' economic valuation of tail risk. In particular, we compute the difference between the two estimators at a given point in time to capture the additional thickness of the left tail coming from the compensation demanded by investors for bearing tail risk. We call this difference the tail risk premium (TRP) and also investigate its implications for asset prices.

A distinctive feature of the tail measures we estimate is that they tend to decrease in periods of market distress. This in contrast with the usual increase that volatility-based risk measures exhibit during crises. In fact, the Hill estimator captures the thickness of the left tail after taking into account the effect of volatility, such that tail risk and volatility can move in different directions. This suggests that crisis periods are more often associated with bursts in volatility rather than more activity in the left tail.[1] Even so, we find that the tail risk premium increases during crisis periods. This indicates that, even though financial crises are not associated with higher tail risk, they are associated with a higher compensation demanded to bear tail risk.

We start by examining the short-term relation between the tail measures and the equity premium with one-day, one-week and one-month ahead daily predictive regressions. We find that the risk-neutral tail risk positively predicts excess market returns. This is consistent with the idea that investors are averse to tail risk, such that they require higher returns to hold the market when tail risk increases. Such positive relation is statistically significant and holds at the one-month horizon, while there is a negative predictive relation at the one-day horizon. That is, at first an increase in tail risk signals a moment of distress followed by an immediate negative realization of the market, which reverses to a positive relation one month later due to compensation for bearing tail risk. Patterns are similar for the tail risk premium, whereas $\lambda_t^{\mathbb{P}}$ has no predictive power for excess market returns regardless of the horizon. In other words, only the economic perception of tail risk carries

[1]This is consistent with the evidence by Christensen et al. (2014), Kelly and Jiang (2014) and Chapman et al. (2018), and the fact that realized kurtosis also decreases during financial crises.

a premium in the short-term. This is robust to controlling for many alternative predictors.

We also analyze the predictive power of the tail measures for the market variance risk premium (Bekaert and Hoerova, 2014; Bollerslev et al., 2009). Our baseline specification includes as control the left tail volatility of Bollerslev et al. (2015), which they show captures an important component of this premium related to compensation for downside risk. Our risk-neutral tail measures strongly predict one-week and one-month ahead variance risk premium, where investors demand a higher compensation for bearing market variance risk when tail risk is higher. This indicates that the economic perception of tail risk in the cross-section of firms offers additional information beyond that contained in the option-implied left tail volatility in the short-term. In contrast, the physical tail risk is statistically insignificant in the predictive regressions. These results are again robust to accounting for several relevant controls, including the lagged variance risk premium.

We then investigate whether short-term tail risk is priced in the cross-section of stocks. To do so, for each of our tail measures, we build a long-short portfolio by sorting stocks each month on their recent exposure to the measure, based on contemporaneous daily regressions. The tail risk factors constructed from $\lambda_t^{\mathbb{Q}}$ and TRP generate statistically significant average returns that cannot be explained by standard factor models, where stocks with high exposure to tail risk (or tail risk premium) have high hedging capacity and are thus highly priced, yielding subsequent low returns. In contrast, the tail factor associated with $\lambda_t^{\mathbb{P}}$ leads to insignificant spreads in returns. In other words, only the short-term exposure to tail risk as perceived by investors (that is, its economic valuation) explains differences in expected returns across stocks.

The findings discussed above suggest that our tail risk factor may potentially be useful in explaining anomalies that standard factor models fail to explain. Perhaps the most prominent example of such an anomaly is momentum (Fama and French, 2016). Motivated by the evidence from Daniel and Moskowitz (2016) that momentum strategies are exposed to crash risk, we test if including our tail risk factor improves the explanatory power for this anomaly. We find that the average return of the momentum strategy can be explained by its statistically significant loading on the risk-neutral tail risk factor. The same is not true considering the physical tail risk factor. This suggests that momentum is priced because it captures short-term exposure to tail risk as perceived by investors.

The remainder of the paper is organized as follows. After a brief discussion of the related literature, Section 2 describes the methodology to construct our tail measures. Section 3 presents the data and the estimated tail measures, while Section 4 contains our empirical analysis. Section 5 concludes the paper. Lastly, Appendices A, B and C contain the main figures and tables, robustness results and variables definitions, respectively.

1.1 Related literature

Our paper is mainly related to an evolving literature investigating the effects of left tail risk and investors' compensation for such risk on financial markets. Bollerslev and Todorov (2011), Bollerslev et al. (2015) and Andersen et al. (2015, 2017) provide evidence that tail risk is an important determinant of the equity and variance risk premia using option-implied tail measures. Extracting information from observed stock prices, Bali et al. (2009), Kelly and Jiang (2014), Almeida et al. (2017), Weller (2019) and Almeida et al. (2022) show that tail risk strongly predicts future market returns and macroeconomic activity. Computing tail risk at the firm-level, Bali et al. (2014), Chabi-Yo et al. (2018) and Atilgan et al. (2020) document significant cross-sectional relations between tail risk and future stock returns. International evidence on the effects of tail risk beyond the U.S. market is provided by Andersen et al. (2020), Andersen et al. (2021) and Freire (2021). We contribute to this literature by proposing a novel method to estimate the tail risk specific to each day t. We document new short-term return predictability for the aggregate market and the cross-section of stocks, with particular focus on the role of incorporating investors' preferences towards tail risk.[2]

The closest work to ours is by Kelly and Jiang (2014), who propose the Hill estimator to estimate the common tail risk component of a cross-section of stocks at a monthly frequency. We adapt their estimator to a daily frequency using intra-day stock returns and put forward a new version of the Hill estimator based on risk-neutralized returns.

[2]For an early contribution on the role of taking economic valuation into account for computing risk measures, see Aït-Sahalia and Lo (2000).

We use both physical and risk-neutral estimators and their difference to study the relation between tail risk (and its economic valuation) and risk premia at horizons up to a month. In this context, we find that the economic perception of tail risk, as opposed to the physical tail risk, is an important determinant of the equity premium, variance risk premium and the cross-section of returns.

Also on a closely related work, Almeida et al. (2022) introduce a daily tail measure based on the expected shortfall of risk-neutralized intra-day market returns.[3] While we also use high-frequency data and risk-neutralization to estimate tail risk at a daily frequency, there are important differences between our approach and theirs. First, we extract information about the tail from extreme events of a cross-section of stocks, while they only consider the market index. Second, the tail measures in the two papers are inherently different, as their measure is closely related to volatility whereas ours is to higher moments such as kurtosis. Third, while Almeida et al. (2022) focus on predicting market risk premia, we also investigate how short-term exposure to tail risk is priced in the cross-section of stocks.

Our paper is also related to the extensive literature identifying factors that are relevant to explain differences in the cross-section of stock returns, including Carhart (1997), Pástor and Stambaugh (2003), Fama and French (2015, 2016), among many others. Using our risk-neutral tail measure, we construct a tradable tail risk factor by sorting stocks based on their recent exposure to tail risk. This factor produces significant spreads in stock returns that cannot be explained by exposures to standard factors. We also show that our tail risk factor significantly explains the average returns of the momentum anomaly (Jegadeesh and Titman, 1993), offering a risk based explanation for momentum that is in line with previous evidence by Daniel and Moskowitz (2016).[4] The risk-neutralization and daily frequency of our tail measure are fundamental to this finding.

[3] Almeida et al. (2022) build on the method by Almeida et al. (2017), who were the first to incorporate risk-neutralization in the estimation of tail risk, albeit at a lower monthly frequency.

[4] Kelly et al. (2021) find that a sizable fraction of momentum can be explained by conditional exposure to priced latent factors. Alternatively, we show that controlling for the static exposure of momentum to our tail risk factor accounts for the momentum premium.

2 Methodology

In this section, we describe the approach we take to estimate left tail risk at a daily frequency. Using a cross-section of intra-day stock returns, we first extract information about the common component of the tail risks of individual firms using the Hill estimator. Then, we introduce a new version of the Hill estimator that relies on risk-neutralized stock returns, thus incorporating the investors' perception of risk in the estimation of extreme event risk. The difference between the two estimators at a given point in time captures the additional thickness of the left tail distribution that comes from the compensation demanded by investors for bearing tail risk. We call this difference the tail risk premium.

2.1 Hill estimator

Extreme events in financial markets are rare by definition. This makes it challenging to construct an aggregate measure of tail risk relying on a single asset such as the market index, since informative observations for the tail are infrequent. To overcome this issue, we follow Kelly and Jiang (2014) by adopting a panel estimation approach capturing common tail behavior in the cross-section of individual stock returns. The identifying assumption is that the dynamics of the tail distributions of the firms are similar, so that extreme events in the cross-section allow us to extract the common component of their tail risk at each point in time.

More specifically, we assume that the left tail of the return distribution of asset i follows a power law structure.[5] That is, its day t conditional left tail distribution, defined as the set of extreme returns below some negative threshold u_t, obeys the following:

$$P(R^i_{t+1} < r | R^i_{t+1} < u_t \text{ and } \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_i/\lambda_t}, \tag{1}$$

where $r < u_t < 0$ and $\mathcal{F}_t$ is the conditioning information set.[6] The parameter a_i/λ_t

[5] See Kelly and Jiang (2014) for a detailed motivation of the use of a power law structure to model the left tail distribution of returns. In sum, for a large class of heavy-tailed distributions, the left tail converges to a generalized power law distribution.

[6] $r < u_t < 0$ and $a_i/\lambda_t > 0$ guarantee that the probability $(r/u_t)^{-a_i/\lambda_t}$ is always between zero and one.

is the tail exponent which determines the shape of the tail distribution of asset i. The constant a_i may be different across assets in the cross-section, implying that they can have different levels of tail risk. However, their dynamics are driven by a common time-varying component, λ_t. The higher the λ_t, the thicker the stock returns' left tails and the higher the probabilities of extreme negative returns in the cross-section. Therefore, we refer to λ_t as our measure of aggregate tail risk.[7]

For each day t in our sample, we estimate the common tail risk component λ_t by applying the standard Hill (1975) power law estimator to the pooled cross-section of intra-day returns:[8]

$$\lambda_t^{\mathbb{P}} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t}, \tag{2}$$

where $R_{k,t}$ is the kth high-frequency return that is below the threshold u_t on day t, K_t is the total number of returns that fall below this threshold within day t and the superscript $\mathbb{P}$ denotes that returns are observed under the physical probability measure.[9] The threshold u_t represents an extreme quantile determining that the observed returns below u_t belong to the left tail and follow the power law structure. We follow Kelly and Jiang (2014) by defining u_t to be the fifth percentile of the return cross-section for each time period, which makes the threshold time-varying as the pooled intra-day return distribution changes from day to day.[10]

2.2 Risk-neutral Hill estimator

The Hill estimator extracts the common tail risk component from the pooled cross-section of returns observed under the physical probability measure, where all observations are deemed equally likely to happen. In that sense, $\lambda_t^{\mathbb{P}}$ does not incorporate the true risks that are perceived by investors in financial markets. In particular, if investors are risk averse, then "bad" states of the world where marginal utility is high should be

[7] In extreme value theory, the parameter λ_t is also often called the shape parameter, and its inverse $1/\lambda_t$ the tail index (see, e.g., Danielsson, 2011).

[8] While Kelly and Jiang (2014) use daily returns to estimate tail risk at a monthly frequency, we rely on intra-day returns to obtain the tail risk specific to day t.

[9] In the Hill formula, returns that fall below threshold u_t are treated as the first K_t entries of R_t. This is without loss of generality since in the pooled cross-section the elements of R_t are exchangeable.

[10] Our empirical results are qualitatively similar if we define u_t to be the first or tenth percentile.

overweighted to reflect compensation for risk. Since such states are precisely the ones that matter for the estimation of tail risk, the economic perception of the left tail of returns may be underestimated by the physical Hill estimator. Moreover, its dynamics can also differ from that captured by $\lambda_t^{\mathbb{P}}$, as compensation for risk demanded by investors may vary over time depending on business conditions.

In order to incorporate investors' compensation for risk in the estimation of left tail risk, we propose a new version of the Hill estimator coupled with risk-neutralization. The idea is to tilt the physical measure such that systematic risk in the cross-section of stock returns is corrected for. This is possible by weighting observations with a pricing kernel, or stochastic discount factor (SDF), that correctly prices systematic factors of high-frequency stock returns. Motivated by Kozak et al. (2020), we rely on principal component analysis (PCA) to identify the systematic factors. They show that the absence of near-arbitrage opportunities implies that the SDF can be represented as a function of a few dominant principal components (PCs) of returns.

More specifically, we consider as the factors the top-five PCs driving most of the intra-day return variation on day t. This is consistent with Pelger (2020), who uses PCA to document the presence of five systematic factors explaining the intra-day returns of individual stocks. In following Ait-Sahalia and Xiu (2017) and Pelger (2020), the factor loadings Λ_t are obtained as the eigenvectors associated with the 5 largest eigenvalues of the realized covariance $R_t^{\intercal} R_t$, where R_t denotes the panel matrix of the high-frequency log-returns of the stocks. The matrix of intra-day factor returns is then given by $F_t = R_t \Lambda_t$.

Let the vector $F_{n,t}$ denote the return over the n-th intra-daily time interval on day t of the top-five PCs of stock returns on day t. We work with an SDF that satisfies the Euler equations for the systematic factors:

$$\frac{1}{N} \sum_{n=1}^{N} m_{n,t} F_{n,t} = 0_5, \tag{3}$$

where 0_5 is a conformable vector of zeros and N denotes the total number of intra-daily observations. We normalize the mean of the SDF to be one ($\frac{1}{N} \sum_{n=1}^{N} m_{n,t} = 1$).[11] The

[11] This implies an implicit gross risk-free rate of one such that we can treat the net stock returns in

pricing kernel tilts the physical measure $1/N$ to produce risk-neutral probabilities $m_{n,t}/N$ that overweight states with high marginal utility to reflect higher compensation for risk demanded by risk averse investors. That is, the SDF corrects for risk by risk-neutralizing assets returns with $\tilde{R}^i_{n,t} = m_{n,t} R^i_{n,t}$. We discuss how to identify the pricing kernel in the next subsection.

To derive the risk-neutral Hill estimator $\lambda_t^{\mathbb{Q}}$, we posit that the left tail of the risk-neutral return distribution of each asset i in the cross-section also follows a power law structure. The estimator is then obtained by using the pooled cross-section of risk-neutralized returns in equation (2):

$$\lambda_t^{\mathbb{Q}} = \frac{1}{\tilde{K}_t} \sum_{k=1}^{\tilde{K}_t} \ln \frac{\tilde{R}_{k,t}}{u_t}. \tag{4}$$

Due to the risk-neutralization, negative stock returns observed during states of high (low) marginal utility get properly overweighted (downweighted) by values of the pricing kernel above (below) its mean one, reflecting compensation for risk. The difference between $\lambda_t^{\mathbb{Q}}$ and $\lambda_t^{\mathbb{P}}$ captures the additional tail thickness coming from investors' risk preferences towards extreme negative events. Throughout the paper, we call this difference the tail risk premium ($TRP_t = \lambda_t^{\mathbb{P}} - \lambda_t^{\mathbb{Q}}$).[12]

2.3 Risk-neutralization

The exact distortion of the physical measure, or correction for systematic risk in the cross-section of stocks, depends on the pricing kernel considered.[13] Besides correctly pricing the factor returns, there are two important properties that the SDF must satisfy. First, it must be nonnegative in order to be consistent with no-arbitrage. This guarantees that the tilted risk-neutral probabilities $m_{n,t}/N$ constitute a proper probability measure. Second, it should incorporate information about higher moments of the return distribu-

the cross-section as excess returns.

[12]The wording "premium" here comes with a slight abuse of notation as it does not refer to the usual difference between a physical and risk-neutral expectation, but rather to the difference between an estimator obtained under the physical measure ($\lambda_t^{\mathbb{P}}$) and another under the risk-neutral measure ($\lambda_t^{\mathbb{Q}}$).

[13]We consider the realistic case of an incomplete market, where there exists an infinity of pricing kernels that correctly price the systematic factors under no-arbitrage.

tion. This is important for modeling tail risk, since investors' aversion to downside risk is related to negative skewness aversion (see, e.g., Schneider and Trojani, 2015).

We follow the nonparametric approach developed by Almeida and Garcia (2017) to obtain a nonlinear pricing kernel satisfying the properties above. Their method consists in estimating SDFs minimizing a family of discrepancy loss functions (Cressie and Read, 1984) subject to correctly pricing a set of returns. This approach is a generalization of Hansen and Jagannathan (1991), who show how to obtain a minimum variance SDF from data on asset returns. Almeida and Garcia (2017) consider more general loss functions that take into account higher moments and imply nonnegative SDFs. Adapted to our context, the minimum discrepancy problem is given by:

$$\begin{gathered}\min_{\{m_{1,t},\ldots,m_{N,t}\}} \tfrac{1}{N}\sum_{n=1}^{N} \tfrac{m_{n,t}^{\gamma+1}-1}{\gamma(\gamma+1)}, \\ \text{s.t. } \tfrac{1}{N}\sum_{n=1}^{N} m_{n,t}F_{n,t} = 0_5,\ \tfrac{1}{N}\sum_{n=1}^{N} m_{n,t} = 1,\ m_{n,t} \geq 0\ \ \forall n,\end{gathered} \tag{5}$$

where the parameter $\gamma \in \mathbb{R}$ indexes the convex loss function in the Cressie and Read (1984) discrepancy family. This family captures as particular cases several loss functions in the literature, such as the Hansen and Jagannathan (1991) quadratic loss function when $\gamma = 1$ and the Kullback Leibler Information Criterion adopted by Stutzer (1995) when $\gamma \to 0$.

Under the assumption of no-arbitrage in the observed sample, Almeida and Garcia (2017) show that solving (5) is equivalent to solving the simpler dual problem below, for $\gamma < 0$:[14]

$$\lambda_\gamma^* = \arg\max_{\lambda \in \Lambda_\gamma} \frac{1}{N}\sum_{n=1}^{N} -\frac{1}{\gamma+1}\left(1-\gamma\lambda F_{n,t}\right)^{\left(\frac{\gamma+1}{\gamma}\right)}, \tag{6}$$

where $\Lambda_\gamma = \{\lambda \in \mathbb{R}^5 : \text{for } n = 1, \ldots, N,\ (1-\gamma\lambda F_{n,t}) > 0\}$. The minimum discrepancy SDF can then be recovered from the first-order condition of (6) with respect to the row-

[14] For $\gamma > 0$, the problem is unconstrained with an indicator function in the objective function: $\frac{1}{N}\sum_{n=1}^{N} -\frac{1}{\gamma+1}\left(1-\gamma\lambda F_{n,t}\right)^{\left(\frac{\gamma+1}{\gamma}\right)} I_{\Lambda_\gamma(F_{n,t})}(\lambda)$, where $\Lambda_\gamma(F_{n,t}) = \{\lambda \in \mathbb{R}^5 : (1-\gamma\lambda F_{n,t}) > 0\}$ and $I_A(x) = 1$ if $x \in A$, and 0 otherwise. For $\gamma \to 0$, the problem is unconstrained and the objective function is exponential: $\frac{1}{N}\sum_{n=1}^{N} -e^{-\lambda F_{n,t}}$.

vector λ, evaluated at λ_γ^*:

$$m_{\gamma,n,t}^* = \left(1 - \gamma\lambda_\gamma^* F_{n,t}\right)^{\frac{1}{\gamma}}, \quad n = 1, ..., N. \tag{7}$$

The dual problem can be economically interpreted as an optimal portfolio problem for an investor maximizing hyperbolic absolute risk aversion (HARA) utility, where $\lambda_\gamma^* F_{n,t}$ is the endogenous optimal portfolio of the systematic factors. The SDF $m_{\gamma,n,t}^*$ is the marginal utility of the investor and will be higher for "bad" states of nature represented by negative realizations of the optimal portfolio of the factors.

For each γ, the solution λ_γ^* of the dual problem (6) leads to a different minimum discrepancy SDF. While by construction they all correctly price the systematic factor returns, they do so by representing distinct risk preferences. In particular, Almeida and Freire (2022) show that positive absolute prudence (Kimball, 1990), which is related to aversion to downside risk and a convex marginal utility, is captured by $\gamma < 1$. Moreover, the smaller the γ, the more aversion to downside risk is embedded in the SDF, where the pricing kernel gets more convex, putting more weight on extreme negative observations of the optimal portfolio returns.[15] They also show that, for extreme negative γs (usually below -5), the constrained maximization in the dual problem (6) may not have a solution. In order to successfully identify a pricing kernel capturing aversion to downside risk, we choose the one associated with $\gamma = -3$ to calculate the risk-neutral Hill estimator.[16]

3 Data description and implementation details

3.1 Data

Our sample consists of 5-minute returns for a panel of 100 stocks that were in the S&P 500 for the entire period between 2000 and 2020. This implies that the total number of intra-daily observations for each stock on a given day is $N = 78$. The data is obtained

[15] Since the mean of the pricing kernel continues to be the same, this means that less weight is given to intermediary return observations.

[16] Considering pricing kernels minimizing loss functions indexed by alternative γs associated with aversion to downside risk (such as -2 or -1) leads to similar conclusions.

from TickData Inc. Although our approach does not require a balanced panel, it does require liquid assets as otherwise the estimation may be impacted by the presence of zero returns (see, e.g., Bandi et al., 2020, 2017) and liquidity-related microstructure noise (see, e.g., Aït-Sahalia and Yu, 2009; Hansen and Lunde, 2006).[17] Therefore, we consider 100 highly liquid assets that were traded continuously over the sample period and work with a balanced panel for transparency and ease of exposition.

Throughout the paper, we use data on market returns, risk factors, and uncertainty measures. The popular five factors of Fama and French (2015), the momentum factor and the risk-free rate are obtained from Kenneth French's website. The liquidity factor of Pástor and Stambaugh (2003) is available from Lubos Pastor's website. The VIX index and the left tail variation (LTV) proposed by Bollerslev et al. (2015) are respectively obtained from the Chicago Board Options Exchange (CBOE) and from the tailindex website, which is made available by Torben Andersen and Viktor Todorov. The LTV captures the option-implied risk-neutral expectation of return volatility stemming from large negative price jumps.

We also construct a number of variables that are used as controls in our analysis. Using high-frequency market returns sampled every 5 minutes obtained from TickData Inc, we compute measures of the realized variance (RV), realized skewness (RSK), realized kurtosis (RK) and jump variation (JV) of the S&P 500 index (Amaya et al., 2015; Andersen et al., 2001, 2003; Barndorff-Nielsen and Shephard, 2004). Using daily market returns, we further calculate the reversal (REV) of Jegadeesh (1990) and Lehmann (1990), momentum (MoM) of Jegadeesh and Titman (1993) and maximum (Max) and minimum (Min) daily return (Bali et al., 2011) for the market.

Finally, we compute the variance risk premium (VRP) as the difference between the risk-neutral and physical expectations of the market return variance (e.g. Bekaert and Hoerova, 2014; Bollerslev et al., 2009). We define the VRP on day t as the squared VIX index (scaled to the daily level) minus the realized variance of day t. Appendix C

[17] To formally exclude the impact of microstructure noise, we have performed the Hausman tests for microstructure noise and first-order serial correlation of Aït-Sahalia and Xiu (2019), for each stock and each day. The tests reject any significant presence of microstructure noise and first-order serial autocorrelation in the returns.

contains the detailed definitions of the variables we use.

3.2 Principal components and risk-neutral estimates

As described in Section 2, for a given day t in our sample, we first extract the top-five PCs explaining most of the variation in the high-frequency panel of stock returns. The PCs are themselves returns of portfolios of the original stocks. Figure 1 plots the average over each day of our sample of the percentage of variance explained by each of the PCs. The first PC (PC1) explains nearly 30% of the variation in the stock returns. In our data, PC1 is always a level factor with long positions of similar magnitude across stocks. In other words, as it is usually the case, PC1 can be interpreted as a market factor. The remaining PCs are long-short portfolios of the original stocks, which together add 30% to the overall explained variation. That is, the top-five PCs explain around 60% of the return variation across the 100 stocks.

Then, we estimate the SDF for each day t using the 78 intra-day returns of the top-five PCs. Given that PC1 can be seen as the market factor, we impose the economic restriction of a 5% lower bound on the annualized equity premium, following Almeida and Freire (2022).[18] While results are similar compared to those where this restriction is not imposed, we keep it because it is economically sound to consider a lower bound on the equity premium (Campbell and Thompson, 2008; Martin, 2017; Pettenuzzo et al., 2014). This restriction is only imposed for the estimation of the SDF. For the remaining PCs, we do not impose restrictions as they do not have straightforward interpretations.

To illustrate how the SDF distorts the physical measure, Figure 2 plots the estimated risk-neutral probabilities ($m_{\gamma,n,t}/N$) for various values of γ and the physical probabilities ($1/N$) for a random day in our sample. The observed patterns are representative of other dates. As can be seen, the risk-neutral measures give more probability weight to negative returns of the optimal portfolio of PCs and less weight to positive returns compared to the physical measure. This reflects agents' risk aversion: investors require more compensation

[18]More specifically, for each day t, we impose that the average return of PC1 is at least 5% above the risk-free rate, in annualized terms. That is, we shift the mean of PC1 to the lower bound when the bound is binding.

(i.e., the SDF is higher) for "bad" states of the world. In the estimation of tail risk, this is such that negative stock returns, observed during intra-daily intervals for which the optimal portfolio of systematic factors experiences negative (positive) returns, get overweighted (downweighted) to reflect more (less) compensation for risk. The relative compensation for risk in the left tail of the optimal portfolio returns depends, in turn, on the aversion to downside risk. The smaller the γ, the more averse to downside risk (or, equivalently, the more prudent) is the investor and the greater are the weights to negative returns under the risk-neutral measure. As previously mentioned, we use the SDF associated with $\gamma = -3$ for the estimation of the risk-neutral Hill estimator.

3.3 Tail risk estimates

We estimate the tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ as detailed in Section 2 using the set of intra-day return observations for all stocks for each day t. The upper panels of Figure 3 plot their one-month moving averages, for ease of exposition. The measures share some similarities, with a correlation of 45.8%. In particular, both measures tend to decrease in periods of market distress. This is in contrast to the usual increase that standard risk measures based on volatility exhibit during crises. To understand this pattern, the left lower panel of Figure 3 reports the time-varying threshold u_t (in absolute value), that determines where the left tail begins in the Hill estimator. As can be seen, u_t resembles a volatility measure, peaking during financial crises. The tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ can thus be thought of as capturing the thickness of the left tail after taking into account the effect of volatility. In fact, as Kelly and Jiang (2014) note, a fixed percentile is used to define u_t exactly for this reason: if volatility increases but the shape of the return left tail is unchanged, an increase of the threshold (in absolute value) absorbs the effect of volatility changes and leaves estimates of the tail exponent unaffected.[19] Therefore, Figure 3 effectively shows that financial crises are more often associated with bursts in

[19]In unreported tests, we calculate $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ with a constant threshold $u_t = u$ and find that both measures behave like volatility-type measures. This indicates that defining u_t as a fixed percentile of the return cross-section is instrumental to isolate the effects of volatility from the shape of the left tail.

volatility rather than more activity in the left tail.[20]

Even though the tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ display similarities, they are still fundamentally different. The right lower panel of Figure 3 plots the absolute value of the tail risk premium, $|TRP_t|$. As would be expected, $\lambda_t^{\mathbb{Q}}$ is always above $\lambda_t^{\mathbb{P}}$, indicating that the left tail of the pooled stock return distribution is thicker under the risk-neutral measure incorporating investors' preferences. However, the additional thickness of the tail coming from the risk compensation required by investors varies substantially over time. In particular, $|TRP_t|$ tends to peak during crisis periods. This suggests that, even though financial crises are not associated with higher tail risk, they are associated with a higher compensation demanded to bear tail risk.

3.4 Comparison with other risk measures

Table 1 reports the correlation between $\lambda_t^{\mathbb{P}}$, $\lambda_t^{\mathbb{Q}}$, u_t, TRP_t and several risk measures. Both tail risk measures are negatively correlated with volatility measures (RV_t and VIX_t^2), whereas (the absolute value of) u_t has a strong positive correlation with these measures. This is consistent with the fact that the time-varying threshold u_t controls for the effect of volatility in the calculation of $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$. In contrast, both tail risk measures are positively related to realized higher-order moments such as skewness and kurtosis. As for the TRP_t, it is strongly negatively related to $\lambda_t^{\mathbb{Q}}$, indicating that the additional thickness of the tail coming from investors' risk compensation increases (i.e., TRP_t gets more negative) when risk-neutral tail risk increases.

Figure 4 provides further details on the relation between $\lambda_t^{\mathbb{Q}}$ and risk variables (the plots are similar for $\lambda_t^{\mathbb{P}}$). The upper panels make clear that tail risk, as measured as the shape parameter of the left tail of stock returns, is lower during periods of high volatility. On the other hand, the lower panels show that realized skewness and kurtosis co-move considerably with $\lambda_t^{\mathbb{Q}}$. This is especially true for kurtosis, which is often regarded as a measure of tail thickness. As can be seen, like $\lambda_t^{\mathbb{Q}}$, realized kurtosis tends to be lower during periods of market distress. This suggests that measures of tail thickness, such as

[20] This is in line with previous findings in the literature (see, e.g., Chapman et al., 2018; Christensen et al., 2014; Kelly and Jiang, 2014).

the ones we propose here, are more closely related to higher-order return moments like kurtosis than to second moments such as volatility.

4 Empirical results

This section provides empirical evidence of the information content of our tail risk measures for asset prices in the short-term. We document the predictive power of the tail measures in forecasting the equity premium and the variance risk premium. In particular, we consider one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) ahead daily predictive regressions. In addition, we investigate how tail risk is priced in the cross-section of stocks. To do so, we construct monthly long-short portfolios by sorting stocks on their recent exposure to the tail risk measures.

4.1 Predicting the equity premium

There is extensive empirical evidence that, at relatively low frequencies (monthly or longer), measures of tail risk strongly predict future excess market returns, with a positive relation (see, e.g., Almeida et al., 2017; Bollerslev et al., 2015; Kelly and Jiang, 2014). This is consistent with the idea that investors are averse to tail risk, such that they require a higher return to hold the market when tail risk increases. To shed light on this relation in the short-term, we investigate whether and how our tail risk measures predict the equity premium at short horizons with daily regressions.

Table 2 contains our main results for predicting excess market returns, i.e., the equity premium. The reported coefficients of the predictive regressions are scaled to be interpreted as the effect of a one standard deviation increase in the regressor on future excess market returns. Focusing first on Panel A, we find a negative but insignificant relation between the physical tail risk measure $\lambda_t^{\mathbb{P}}$ and the equity premium, irrespective of the forecasting horizon. In contrast, the risk-neutral tail measure $\lambda_t^{\mathbb{Q}}$ switches from negatively predicting one-day ahead market returns to positively predicting one-month ahead returns, with statistical significance at the 5% and 10% level, respectively. This can be interpreted as follows. At first, an increase in risk-neutral tail risk signals a moment of distress followed by a negative realization of the market. However, since investors are averse to aggregate tail risk, they require a higher return to hold the market after a tail risk shock, where this compensation appears over the horizon of one month. Importantly, the different patterns observed for $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ suggest that only the economic perception of tail risk carries a premium in the short-term.

To further assess the role of risk-neutralization in predicting the equity premium, we consider bivariate regressions based on $\lambda_t^{\mathbb{P}}$ and the tail risk premium (TRP). These results corroborate those for $\lambda_t^{\mathbb{Q}}$. Over the one-day horizon, an increase in the wedge between the risk-neutral and physical tail risk (i.e., a decrease of TRP_t) is associated with a negative market return. In contrast, over the one-month horizon, investors require a higher return to hold the market when the tail risk premium is higher (i.e., TRP_t is more negative). This relation is statistically significant at the 5% level. The incremental information content afforded by risk-neutralization is also clear from the R^2 of the regressions. For instance, for one-month ahead predictions, the R^2 associated with $\lambda_t^{\mathbb{Q}}$ and the bivariate regression is 0.07% and 0.11%, respectively, while for $\lambda_t^{\mathbb{P}}$ it is almost negligible (0.004%).

To isolate the effect of the negative relation between tail risk and one-day ahead market returns, Panel B of Table 2 reports results for one-week and one-month ahead regressions where the excess market return is accumulated from $t+2$ to $t+h$. When doing so, the switch to a positive sign of the coefficient of $\lambda_t^{\mathbb{Q}}$ already appears at the one-week horizon, albeit it is still insignificant. More importantly, the effect of an increase in risk-neutral tail risk one month later is much larger, as observed by a positive coefficient that is significant at the 5% level and an R^2 twice as large as in Panel A. The results for TRP are even stronger, while $\lambda_t^{\mathbb{P}}$ continues to be insignificant. This reinforces the idea that the immediate negative effect of risk-neutral tail risk on excess market returns reverses to a significant positive relation reflecting compensation for tail risk in the short-term.

In Appendix B, we show that our predictability results for the equity premium are robust to controlling for several alternative predictors. In fact, the risk-neutral tail risk and the tail risk premium are the strongest predictors of one-month ahead market returns among the controls. The only variable with similar predictive power is the VRP. This indicates that the variance risk premium helps explain variation in future excess market returns not only at lower frequencies (Bollerslev et al., 2009) but also in the short-term.

In sum, our results indicate that the economic perception of tail risk is an important determinant of the equity premium in the short-term. Investors require a significantly higher market return after one month following an increase in tail risk. The effect is even larger if we ignore the immediate negative relation between tail risk and market returns. In particular, accounting for investors' aversion to downside risk in computing tail risk provides fundamental information about the equity premium that is not contained in the physical tail risk measure.

4.2 Predicting the variance risk premium

The variation of volatility is often associated with time-varying economic uncertainty. In particular, the variance risk premium (VRP) captures investors' compensation for variance risk and is usually regarded as a proxy for aggregate risk aversion (see, e.g., Bekaert et al., 2013; Campbell and Cochrane, 1999). Bollerslev et al. (2015) show that a large fraction of the variance risk premium comes from compensation demanded by investors for bearing left tail risk. Motivated by that, we examine the predictive relation between our tail risk measures and the VRP at short horizons, with particular focus on the role of incorporating investors' aversion to downside risk with risk-neutralization.

Table 3 reports the main predictability results for the variance risk premium. Our baseline specification includes the LTV of Bollerslev et al. (2015) as a control, given that it captures an important component of the VRP associated with tail risk. Coefficients are scaled to be interpreted as the effect of a one standard deviation increase in the regressor on future VRP. At the one-day horizon, only LTV is significant, where a higher expected volatility stemming for negative price jumps leads to a higher VRP on the next day. For one-week and one-month horizons, the positive relation between LTV and future VRP is even stronger. However, now the risk-neutral tail risk and the TRP are also statistically significant. A higher perception of tail risk in the cross-section of firms leads to a higher variance risk premium. This shows that our risk-neutral tail measures contain complementary information to LTV about the VRP. In contrast, after controlling for LTV, the physical tail risk has no predictive power for the VRP.

In Appendix B, we show that the results above are robust to controlling for a number of alternative predictors beyond the LTV. In particular, in multivariate regressions including all controls, the only significant predictors of the variance risk premium at the one-week horizon are the $\lambda_t^{\mathbb{Q}}$ (or TRP), the lagged VRP and JV, and at the one-month horizon the $\lambda_t^{\mathbb{Q}}$ (or TRP), the lagged VRP and VIX^2 (or RV). In these regressions, we do not include the LTV as it is not available for the whole sample.

In sum, we document that our risk-neutral tail measures possess strong predictive power for the variance risk premium in the short-term. Investors require a higher compensation to bear variance risk when their perception of tail risk increases. These effects are robust to several measures of volatility, jump risk and the LTV of Bollerslev et al. (2015), indicating that the thickness of the left tail of the pooled cross-section of returns under the risk-neutral distribution provides complementary information about the VRP. In particular, physical tail risk is not related to the VRP once we consider those controls.

4.3 Predicting the cross-section of stock returns

So far, we have shown that the economic perception of tail risk by investors is an important determinant of aggregate market risk premia at short-horizons. This section investigates whether recent exposure to tail risk is priced in the cross-section of stock returns through portfolio sorts. To do so, at the end of each month in our sample, we measure the insurance value of our 100 individual stocks with daily regressions over the previous 7 months, i.e., we estimate contemporaneous betas with respect to our tail measures: $R_{i,t} = \mu_i + \beta_i TR_{i,t}$, where $TR_{i,t} \in \{\lambda_t^{\mathbb{P}}, \lambda_t^{\mathbb{Q}}, TRP_t\}$.[21] Then, we form equally-weighted portfolios over the next month by sorting the 100 stocks into portfolios using quintile breakpoints calculated based on the given sorting variable.

The first three panels of Table 4 report the results for $\lambda_t^{\mathbb{P}}$, $\lambda_t^{\mathbb{Q}}$ and TRP_t, respectively.

[21] Our results are robust to different estimation windows for the betas, such as 3, 5, 9, 12 and 24 months. These results are available upon request.

In addition to the average returns of the quintile portfolios, we also report the portfolios alphas (i.e., intercepts) from regressions of portfolio excess returns on the Fama-French three and five factors as well as extended models controlling for momentum (Carhart, 1997) and liquidity (Pástor and Stambaugh, 2003) factors. The last two columns report the average returns and alphas of the high minus low zero net investment portfolio and associated t-statistics, which are estimated using Newey-West robust standard errors. Panel D presents the p-values from various tests of the monotonicity (Patton and Timmermann, 2010) of average returns across the five quintile portfolios reported in Panels A–C. All tests have a null hypothesis of a flat pattern (no relation). While the MR Up and MR Down tests have alternative hypotheses of an increasing and decreasing pattern, the MR test alternative hypothesis is unrestricted. The tests are estimated using 10,000 bootstrap replications and a block length equal to 10 months.

Several conclusions can be drawn from these results. First, stocks that are more positively related to tail risk in the short-term earn lower returns. This is economically sound, as stocks with high β_i provide hedging opportunities against tail risk and are thus highly priced, yielding subsequent low returns. This relation is monotonic across quintile portfolios for $\lambda_t^{\mathbb{Q}}$, which is formally confirmed by the rejection of the flat pattern using the MR and MR Down tests. In contrast, a flat pattern cannot be rejected for $\lambda_t^{\mathbb{P}}$. For the TRP_t, there is a monotonic increasing pattern across quintile portfolios, which is due to the negative sign of tail risk premium as $\lambda_t^{\mathbb{P}}$ is always smaller than $\lambda_t^{\mathbb{Q}}$. Stocks with more negative β_i with respect to TRP_t pay well when compensation for bearing tail risk is high, such that they are highly priced and yield subsequent low returns. This relation is confirmed by the rejection of the MR and MR Up tests.

Second, exposure to physical tail risk generates insignificant average returns for the high minus low portfolio. On the other hand, the return spreads associated with exposures to $\lambda_t^{\mathbb{Q}}$ and TRP_t are both statistically and economically significant, where the corresponding high minus low strategies earn an average monthly return of -0.76% and 0.80%, respectively. This shows that recent exposure to tail risk as perceived by investors is strongly priced in the cross-section. To further illustrate, Figure 5 plots the cumulative returns of the quintile portfolios based on TRP_t. As can be seen, while the high portfolio performs reasonably well on its own, the robust profitability of the high minus low strategy is mainly driven by selling the stocks that pay well when compensation for bearing tail risk is high.

Third, the average high minus low returns in Table 4 are generally larger (in absolute value) after controlling for standard factor models in the literature. For instance, controlling for the Fama-French 3 factors results in an average monthly excess return of -1.25% (t-statistic -3.55) and 1.27% (t-statistic 3.47) for $\lambda_t^{\mathbb{Q}}$ and TRP_t, respectively. The reason is that the tail factors are negatively correlated with the market, size and value factors. This can be seen from Table 5, which provides further details on the regressions of our high minus low portfolio returns on factors models. Again, only the tail measures incorporating investors' preferences are able to generate statistically significant alphas. In particular, the large alphas of the high minus low $\lambda_t^{\mathbb{Q}}$ and TRP_t portfolios hold with significant factor exposure and high adjusted R^2. This suggests that our risk-neutral tail factors capture risk premium that is not reflected in firms' exposures to the market, size, value, profitability and momentum factors. By contrast, the tail factor based on $\lambda_t^{\mathbb{P}}$ holds no relation with standard factors, as it is only significantly exposed to the market factor and the adjusted R^2 of the regressions are low.

The results above unambiguously show that only the tail measures incorporating investors' preferences drive risk premium in the cross-section of stocks. To illustrate the differences between the physical and risk-neutral tail measures, Figure 6 plots, for each measure, the time series of the average β_i within each quintile portfolio and its difference between the high and low portfolios. During financial crises (e.g., the dot-com bubble, the global financial crisis and the Covid-19 pandemic), stocks' exposures to risk-neutral tail risk and tail risk premium generally increase, as would be expected.[22] In contrast, the β_is with respect to physical tail risk either decrease or fail to increase by the same magnitude. This suggests that the additional information content of the economic valuation of tail risk for the cross-section of returns is especially relevant during periods of market distress.

[22]Note that the TRP_t is negative by construction and therefore a more negative β_i implies a higher sensitivity to tail risk premium.

In sum, we find that the investors' perception of tail risk and the compensation for such risk in the short-term is strongly priced in the cross-section of stocks. High minus low portfolios based on the recent exposure to tail risk generate statistically and economically significant average returns, which are even larger after controlling for standard factor models. The information content of risk-neutralization beyond that contained in physical tail risk is especially relevant during financial crises.

4.4 Explaining the momentum anomaly

Since Jegadeesh and Titman (1993), momentum has been one of the most widely studied anomalies in the cross-section of returns. Even so, there is still no consensus on how to explain it. As documented by Fama and French (2016), momentum remains one of the few anomalies for which predominant factor models such as Fama and French (2015) hold no explanatory power. More recently, Kelly et al. (2021) show that a sizable fraction of momentum can be explained by conditional risk exposure, as stocks' past performance can be seen as a noisy proxy for their time-varying loadings to priced factors. In this section, we alternatively investigate whether the momentum strategy remains profitable after controlling for its static exposure to our tail risk factors. Our motivation comes from the fact that there is a crash risk component in momentum strategies as they experience large negative returns during financial crises (Daniel and Moskowitz, 2016), such that the compensation for such risk can potentially be captured by our factors.

Table 6 conveys the regression results of the momentum high minus low returns on the Fama-French five factor model plus the liquidity factor of Pástor and Stambaugh (2003), as well as extended models including the tail risk factors. The first column shows that, in our sample, momentum generates a positive but insignificant alpha over the Fama-French and liquidity factors.[23] Further controlling for the physical tail risk factor does not help in explaining momentum, as noted by a larger alpha, an insignificant loading on the tail factor and the decrease in the adjusted R-squared. In contrast, after adding the tail risk factor based on $\lambda_t^{\mathbb{Q}}$ or TRP_t, the alpha of the momentum strategy becomes

[23] In our sample, the average return of the momentum strategy is positive but statistically insignificant.

highly negative, and the adjusted R-squared increases substantially. In other words, the significant exposure of the momentum strategy to risk-neutral tail risk helps explain the spreads in returns it generates, which is aligned with our initial motivation.

In sum, we find that the risk premium associated with the momentum anomaly is in large part coming from the significant exposure of this strategy to risk captured by our tail factors. That is, short-term tail risk helps explain momentum. Importantly, this holds true only when investors' preferences are incorporated in the tail measures.

5 Conclusion

In this paper, we introduce a new tail risk measure at a daily frequency by combining high-frequency returns of a cross-section of stocks with a risk-neutralization algorithm. We use our measure to shed light on the effects of tail risk on asset prices at short-horizons and investigate to what extent these effects depend on information coming from the physical measure, under which asset prices are observed, and the risk-neutral measure, which incorporates investors' preferences.

We find that the compensation required by investors for bearing tail risk is an important determinant of the equity premium and the variance risk premium at horizons up to a month. In addition, tail risk is priced in the cross-section of stocks. A tradable tail factor built by sorting stocks on their recent exposure to tail risk produces significant spreads in stock returns that cannot be explained by standard factor models. Using our tail factor, we show that exposure of momentum strategies to tail risk helps explain the momentum anomaly. Incorporating investors' preferences in the estimation of tail risk is fundamental to our findings.

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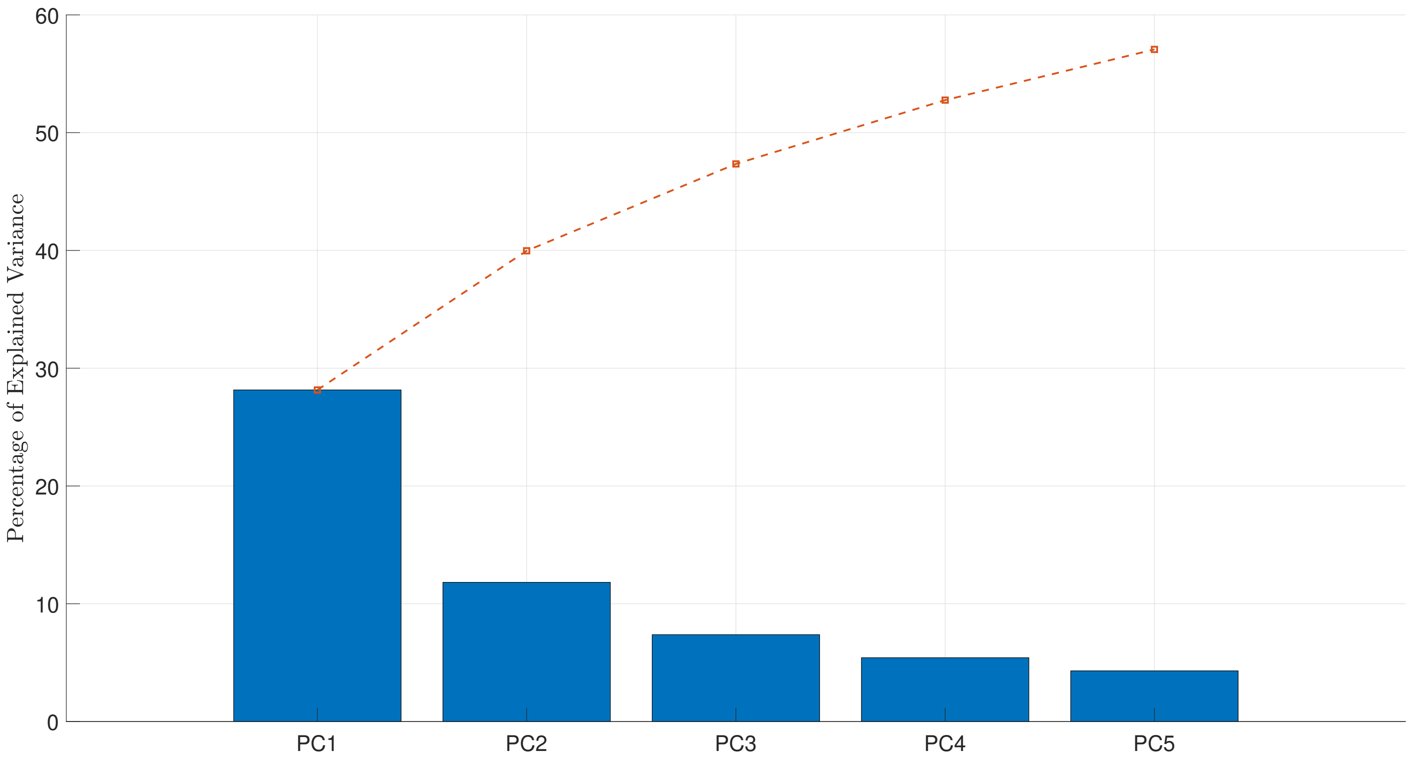
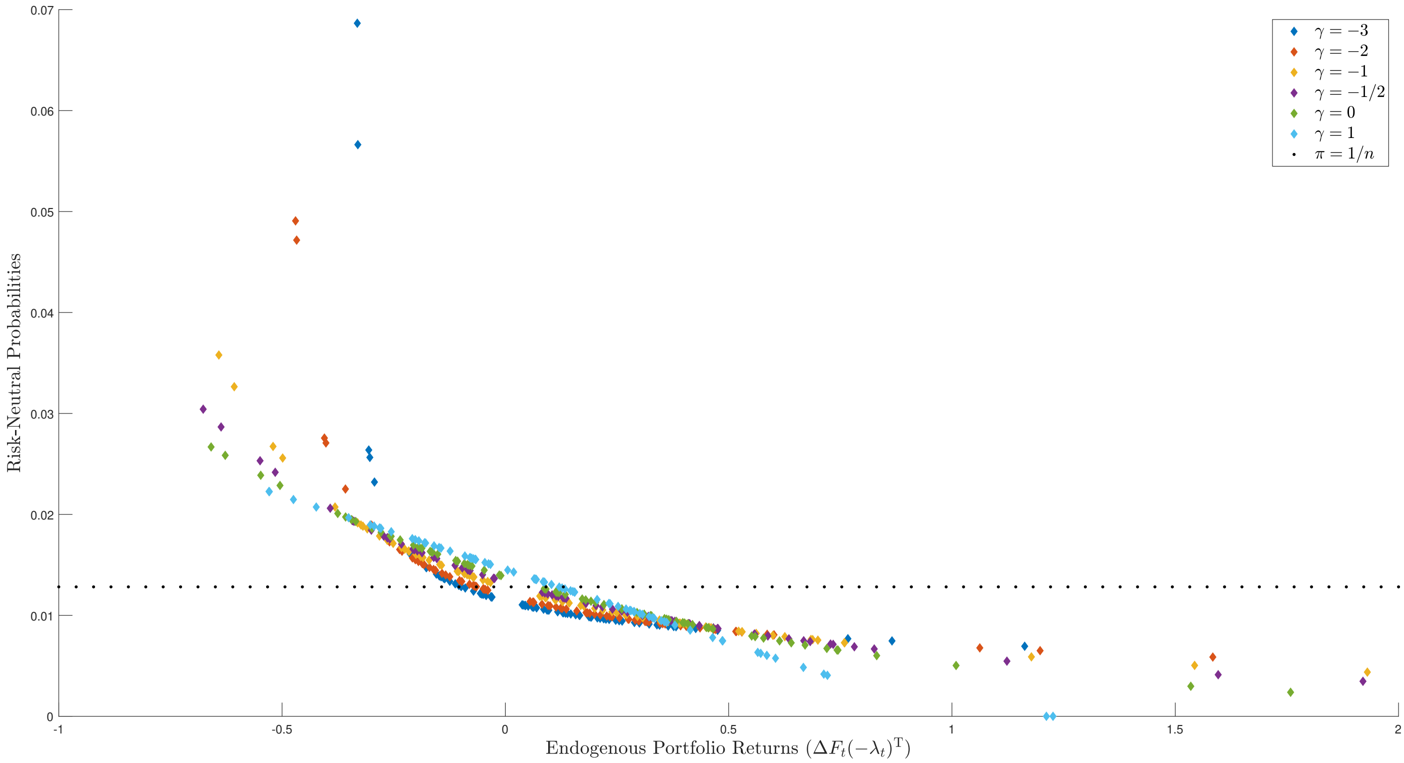
A Figures and tables

Figure 1: Explained variation of Principal Components



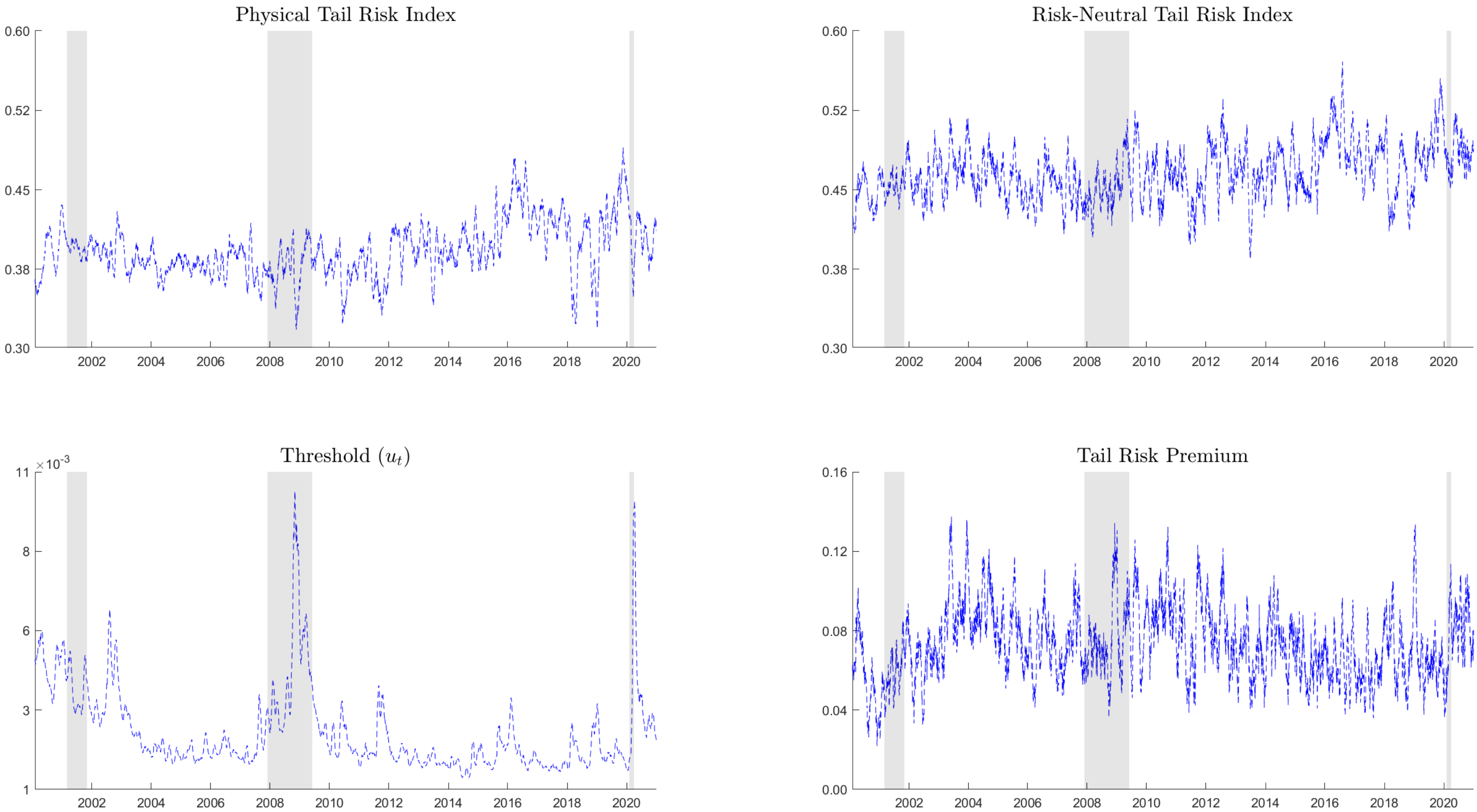
Note: The figure depicts the average over each day of our sample of the percentage of explained variance in the panel of intra-day stock returns by the top-five PCs (in blue bars) and the accumulated percentage of explained variance (in red). The sample ranges from January, 2000 to December, 2020.

Figure 2: Minimum dispersion risk-neutral probabilities



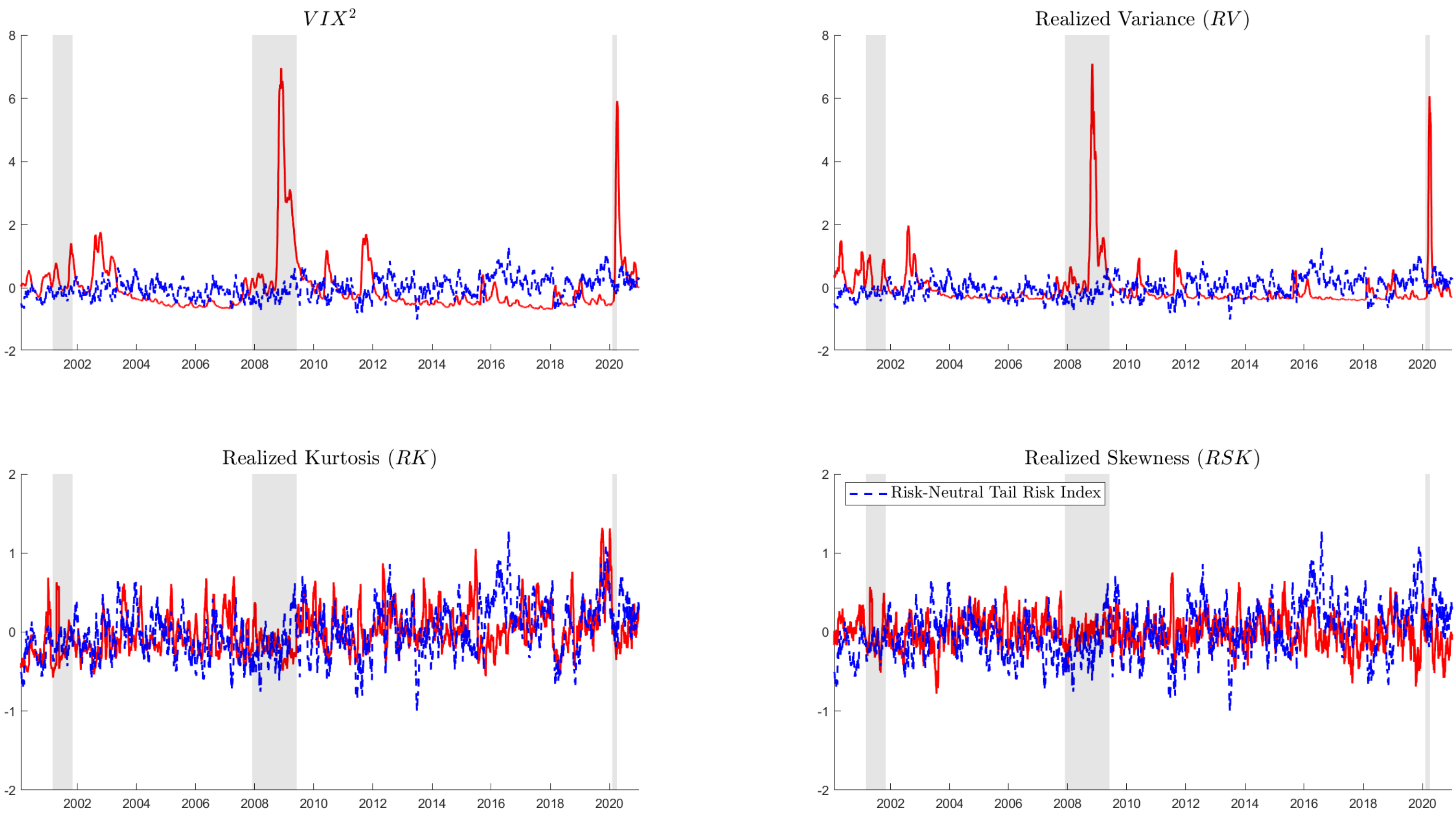
Note: The figure depicts the minimum dispersion risk-neutral probabilities for various values of γ and the physical measure ($\pi = 1/N$) for the 78 intra-daily endogenous portfolio returns ($\lambda_\gamma^* F_{n,t}$) for a random day in our sample.

Figure 3: Tail risk index measures



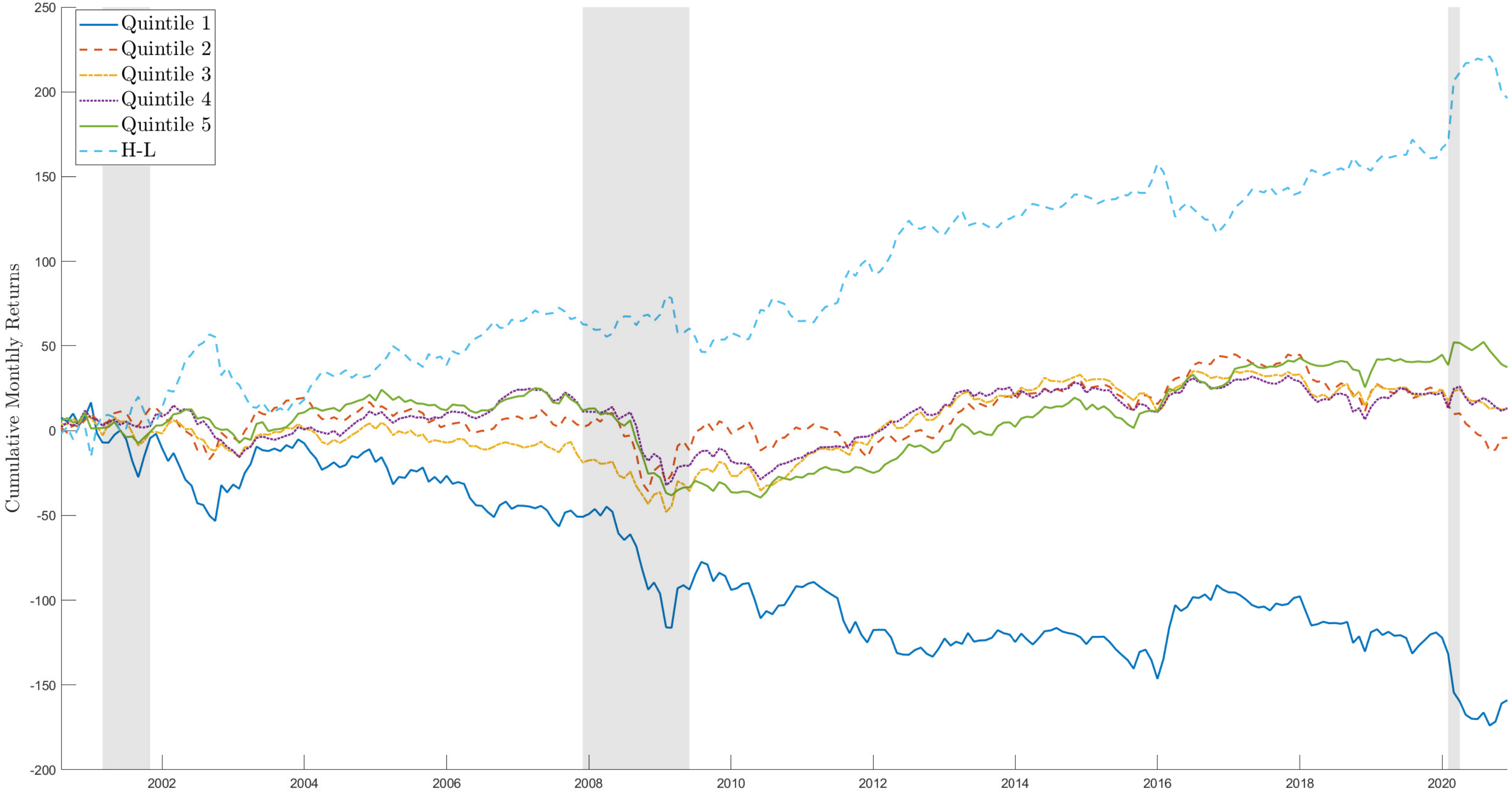
Note: The figure plots, in the upper panels, the 1-month moving average of the physical and risk-neutral tail risk indices. In the bottom panels, the corresponding moving averages for the threshold and the tail risk premium are depicted. For illustration purposes, the bottom panels plot the absolute value of both the threshold and the tail risk premium. Shaded areas depict NBER recession dates. The sample ranges from January, 2000 to December, 2020.

Figure 4: Tail risk index and risk measures



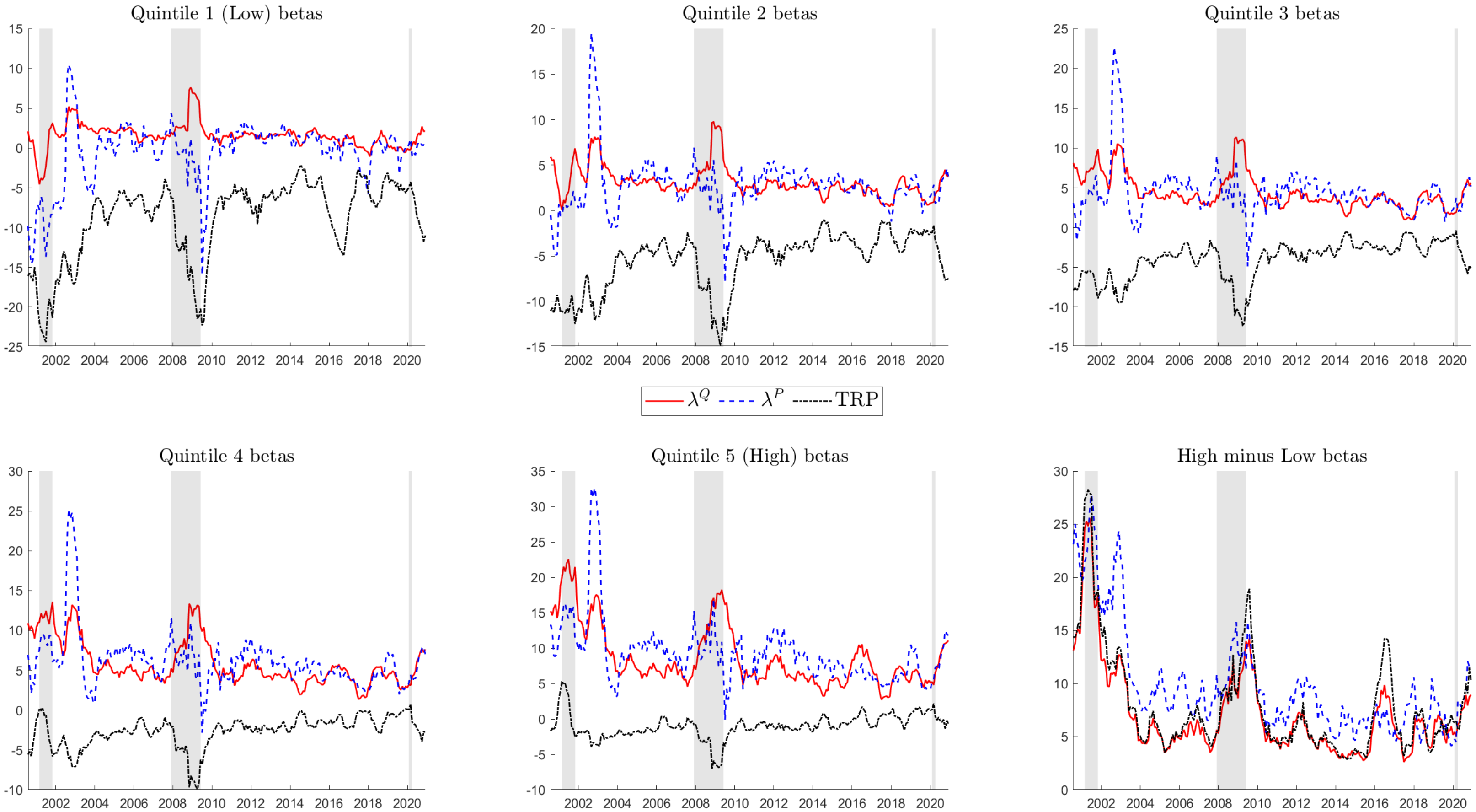
Note: The figure plots, in the upper panels, the 1-month standardized moving average of the squared VIX and realized variance of the S&P 500 index. Similarly, the bottom panels depict the corresponding stardandized moving averages for the realized kurtosis and realized skewness. For comparison, we also plot the 1-month standardized moving average of the risk-neutral tail risk index (blue dotted line). Shaded areas depict NBER recession dates. The sample ranges from January, 2000 to December, 2020.

Figure 5: Cumulative monthly quintile portfolio returns formed by sorting on TRP



Note: The figure depicts the cumulative monthly returns for each quintile portfolio and the high minus low zero net investment portfolio formed by sorting on the tail risk premium (TRP). Shaded areas depict NBER recession dates. The sample ranges from January, 2000 to December, 2020.

Figure 6: Portfolio quintile βs



Note: The figure depicts the time series average sensitivity to tail risk for all stocks within each quintile portfolio and the high minus low zero net investment portfolio. Shaded areas depict NBER recession dates. The sample ranges from January, 2000 to December, 2020.

Table 1: Correlation and AR(1) coefficients

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	$\lvert u_t \rvert$	RV_t	VIX_t^2	VRP_t	RSK_t	RK_t
$\lambda_t^{\mathbb{P}}$	0.393	0.458	0.116	−0.196	−0.148	−0.198	0.029	0.020	0.206
$\lambda_t^{\mathbb{Q}}$		0.183	−0.830	−0.145	−0.070	−0.078	0.030	0.285	0.169
TRP_t			0.112	0.039	−0.014	−0.037	−0.015	−0.243	−0.059
$\lvert u_t \rvert$				0.914	0.802	0.831	−0.399	−0.071	−0.100
RV_t					0.682	0.761	−0.768	0.051	0.051
VIX_t^2						0.964	−0.169	−0.008	−0.085
VRP_t							0.325	−0.085	−0.162
RSK_t								−0.041	0.131
RK_t									0.059

Note: The table reports in the off-diagonal the correlation of each pair of variables and in the main diagonal the AR(1) coefficient. The sample ranges from January, 2000 to December, 2020.

Table 2: Predicting excess market returns

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
Panel A: $t + 1 : t + h$									
$\lambda_t^{\mathbb{P}}$	−0.021		−0.023	−0.021		−0.024	−0.026		−0.011
t-tstat	(−1.554)		(−1.748)	(−0.539)		(−0.636)	(−0.233)		(−0.095)
$\lambda_t^{\mathbb{Q}}$		−0.030			−0.038			0.107	
t-tstat		(−2.352)			(−1.303)			(1.685)	
TRP_t			0.023			0.032			−0.135
t-tstat			(1.811)			(1.167)			(−2.075)
R^2	0.05	0.106	0.113	0.011	0.037	0.037	0.004	0.071	0.115
Panel B: $t + 2 : t + h$									
$\lambda_t^{\mathbb{P}}$				−0.009		−0.007	−0.001		0.020
t-tstat				(−0.227)		(−0.179)	(−0.011)		(0.179)
$\lambda_t^{\mathbb{Q}}$					0.011			0.162	
t-tstat					(0.358)			(2.232)	
TRP_t						−0.017			−0.184
t-tstat						(−0.590)			(−2.810)
R^2				0.002	0.003	0.009	0.000	0.162	0.206

Note: The table reports in two panels the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess market returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, Panel A considers the excess market returns from $t + 1$ to $t + h$, while Panel B considers the excess market returns from $t + 2$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The sample ranges from January, 2000 to December, 2020.

Table 3: Predicting variance risk premium (VRP)

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	−0.010		−0.009	−0.022		−0.014	−0.036		−0.031
t-tstat	(−0.422)		(−0.377)	(−0.926)		(−0.597)	(−1.530)		(−1.333)
$\lambda_t^{\mathbb{Q}}$		0.003			0.053			0.027	
t-tstat		(0.128)			(2.651)			(2.145)	
TRP_t			−0.009			−0.071			−0.049
t-tstat			(−0.368)			(−3.559)			(−3.724)
LTV	0.396	0.398	0.396	0.582	0.590	0.581	0.556	0.564	0.555
t-stat	(4.432)	(4.473)	(4.430)	(5.418)	(5.491)	(5.420)	(6.582)	(6.661)	(6.587)
R^2	7.047	7.043	7.051	20.441	20.581	20.736	38.783	38.821	39.076

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$). For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The LTV is the left tail variation of Bollerslev et al. (2015). The sample ranges from January, 2000 to December, 2019.

Table 4: Monthly sorted portfolios

	Low	2	3	4	High	High−Low	t-stat
Panel A: $\lambda_t^{\mathbb{P}}$							
Average Return	−0.089	0.061	−0.105	0.118	−0.392	−0.304	−1.002
CAPM alpha	−0.400	−0.200	−0.414	−0.220	−0.846	−0.446	−1.361
FF3 alpha	−0.429	−0.201	−0.411	−0.228	−0.874	−0.445	−1.492
FF5 alpha	−0.498	−0.369	−0.465	−0.284	−0.846	−0.348	−1.237
FF5 + Mom alpha	−0.494	−0.369	−0.461	−0.282	−0.842	−0.349	−1.239
FF5 + Mom + Liq alpha	−0.472	−0.340	−0.434	−0.264	−0.839	−0.367	−1.336
Panel B: $\lambda_t^{\mathbb{Q}}$							
Average Return	0.184	0.058	0.043	−0.114	−0.579	−0.763	−1.957
CAPM alpha	0.020	−0.191	−0.247	−0.520	−1.141	−1.162	−3.070
FF3 alpha	0.042	−0.181	−0.254	−0.541	−1.208	−1.250	−3.557
FF5 alpha	−0.152	−0.392	−0.406	−0.510	−1.003	−0.851	−2.734
FF5 + Mom alpha	−0.153	−0.394	−0.403	−0.505	−0.994	−0.841	−2.737
FF5 + Mom + Liq alpha	−0.121	−0.366	−0.378	−0.500	−0.985	−0.864	−2.861
Panel C: TRP_t							
Average Return	−0.649	−0.017	0.052	0.054	0.152	0.801	2.101
CAPM alpha	−1.211	−0.407	−0.245	-0.182	-0.034	1.177	3.036
FF3 alpha	−1.283	−0.430	−0.249	-0.173	-0.007	1.276	3.475
FF5 alpha	−1.029	−0.471	−0.360	-0.375	-0.228	0.801	2.414
FF5 + Mom alpha	−1.020	−0.465	−0.357	-0.375	-0.231	0.790	2.443
FF5 + Mom + Liq alpha	−1.013	−0.458	−0.332	-0.344	-0.203	0.809	2.493

Panel D: Monotonocity Test	MR	MR Up	MR Down
Avg. return $\lambda_t^{\mathbb{P}}$	0.779	0.226	0.054
Avg. return $\lambda_t^{\mathbb{Q}}$	**0.025**	0.962	**0.035**
Avg. return TRP	**0.019**	**0.013**	0.947

Note: The table reports the results of univariate portfolio analyses of the relation between the tail risk measures and the cross-section of returns. Monthly portfolios are formed by sorting the 100 stocks into portfolios using quintile breakpoints calculated based on the given sort variable using the 100 stocks. The table also reports portfolios alphas from regressions of portfolio excess returns using the Fama-French three and five factors as well as extended models controlling for momentum (Carhart, 1997) and liquidity (Pástor and Stambaugh, 2003) factors. Returns and alphas are in percentage terms. The last two columns report the high minus low zero net investment portfolio and associated t-statistics, which are estimated using Newey-West robust standard errors with a lag length equal to 5. Panel D presents the p-values from various tests of the monotonicity (Patton and Timmermann, 2010) of average returns across the 5 quintile portfolios reported in Panels A–C. All tests have a null hypothesis of a flat pattern (no relation). While the MR Up and MR Down tests have alternative hypotheses of an increasing and decreasing pattern, the MR test is unrestricted. Bold p-values indicate significance at the 5% or better. The tests are estimated using 10,000 bootstrap replications and a block length equal to 10 months. The sample ranges from August, 2000 to December, 2020.

Table 5: High minus low tail risk factor regressions

	$\lambda^{\mathbb{P}}$			$\lambda^{\mathbb{Q}}$			TRP_t		
α	−0.445	−0.348	−0.367	−1.250	−0.851	−0.864	1.276	0.801	0.809
t-stat	−1.492	−1.237	−1.336	−3.557⋆	−2.734⋆	−2.861⋆	3.475⋆	2.414⋆	2.493⋆
MKT	0.245	0.207	0.192	0.568	0.415	0.295	−0.515	−0.334	−0.207
t-stat	2.950	2.412	2.077	8.030	4.934	3.522	−5.880	−4.377	−2.684
SMB	−0.042	−0.066	−0.111	0.534	0.443	0.392	−0.607	−0.492	−0.450
t-stat	−0.423	−0.629	−1.068	4.074	3.476	3.422	−5.183	−4.265	−4.410
HML	0.234	0.307	0.321	0.205	0.537	0.412	−0.168	−0.539	−0.397
t-stat	1.656	1.799	1.735	1.212	2.486	1.952	−0.816	−2.249	−1.662
RMW		−0.122	−0.154		−0.454	−0.318		0.577	0.418
t-stat		−0.695	−0.826		−2.129	−1.720		2.616	2.534
CMA		−0.109	−0.097		−0.552	−0.426		0.570	0.434
t-stat		−0.423	−0.355		−1.655	−1.485		1.539	1.383
Mom			0.014			−0.300			0.334
t-stat			0.153			−3.638			4.490
Liq			10.207			12.741			−10.950
t-stat			1.213			1.310			−1.217
Adj R^2	8.592	8.329	8.306	36.081	40.220	45.389	35.267	41.330	47.893

Note: The table reports the regression results of the high minus low tail factor on the Fama-French three and five factor models, as well as extended models controlling for momentum (Carhart, 1997) and liquidity (Pástor and Stambaugh, 2003) factors. The t-statistics are estimated using Newey-West robust standard errors with a lag length equal to 5, and a star (⋆) besides the α's t-statistic denotes statistical significance at the 5% or better. The sample ranges from August, 2000 to December, 2020.

Table 6: Momentum anomaly and the tail risk factor

	FF5 + Liq	FF5 + Liq+$\lambda^{\mathbb{P}}$	FF5 + Liq+$\lambda^{\mathbb{Q}}$	FF5 + Liq+TRP
α	0.022	0.028	−0.233	−0.259
t-stat	0.078	0.099	−0.719	−0.814
MKT	−0.328	−0.331	−0.213	−0.219
t-stat	−3.533⋆	−3.394⋆	−2.309⋆	−2.490⋆
SMB	−0.012	−0.010	0.104	0.144
t-stat	−0.081	−0.069	0.802	1.106
HML	−0.443	−0.449	−0.284	−0.256
t-stat	−3.088⋆	−2.901⋆	−2.056⋆	−1.958
RMW	0.540	0.543	0.400	0.334
t-stat	2.441	2.476	1.997	1.678
CMA	0.365	0.367	0.209	0.174
t-stat	1.124	1.120	0.806	0.720
Liq	6.530	6.356	9.686	9.548
t-stat	0.685	0.689	0.999	1.034
Tail Factor		0.017	−0.293	0.344
t-stat		0.154	−2.623⋆	3.054⋆
R^2_{Adj}	25.339	25.042	31.602	33.644

Note: The table reports the regression results of the momentum factor on the Fama-French five factor models plus the liquidity factor (Pástor and Stambaugh, 2003), as well as extended models controlling for the tail risk factor. A star (⋆) denotes statistical significance at the 5% or better. The t-statistics are estimated using Newey-West robust standard errors with a lag length equal to 5. The sample ranges from August, 2000 to December, 2020.

B Robustness results

Table 7: One-day ahead predictive excess market return regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	Rev	RK	JV	MoM	Max	Min	RV	RSK	VRP	R^2
I.I	−0.021												0.050
	(−1.554)												
I.II	−0.013			−0.031									0.155
	(−0.985)			(−1.767)									
I.III	−0.025				0.019								0.093
	(−1.837)				(1.563)								
I.IV	−0.020					0.002							0.050
	(−1.549)					(0.090)							
I.V	−0.020						0.015						0.078
	(−1.485)						(1.008)						
I.VI	−0.022							−0.024					0.117
	(−1.670)							(−1.340)					
I.VII	−0.020								−0.001				0.050
	(−1.469)								(−0.048)				
I.VIII	−0.022									−0.009			0.061
	(−1.650)									(−0.459)			
I.IX	−0.020										−0.010		0.063
	(−1.538)										(−0.871)		
I.X	−0.020											0.015	0.076
	(−1.484)											(0.710)	
I.XI	−0.016			−0.049	0.022	0.011	0.021	−0.003	0.029	0.010	−0.004	0.044	0.382
	(−1.125)			(−1.480)	(1.554)	(0.408)	(1.196)	(−0.072)	(0.790)	(0.262)	(−0.365)	(1.623)	
II.I		−0.030											0.106
		(−2.352)											
II.II		−0.023		−0.028									0.195
		(−1.774)		(−1.626)									
II.III		−0.033			0.020								0.152
		(−2.588)			(1.615)								
II.IV		−0.030				0.002							0.107
		(−2.351)				(0.110)							
II.V		−0.029					0.015						0.134
		(−2.303)					(1.008)						
II.VI		−0.029						−0.021					0.157
		(−2.255)						(−1.169)					
II.VII		−0.030							−0.024				0.175
		(−2.388)							(−1.367)				
II.VIII		−0.030								−0.008			0.114
		(−2.400)								(−0.412)			
II.IX		−0.029									−0.004		0.108
		(−2.185)									(−0.329)		
II.X		−0.030										0.016	0.135
		(−2.345)										(0.762)	
II.XI		−0.025		−0.046	0.023	0.010	0.022	−0.003	0.027	0.012	0.001	0.045	0.420
		(−1.810)		(−1.392)	(1.601)	(0.378)	(1.275)	(−0.079)	(0.734)	(0.301)	(0.047)	(1.677)	
III.I	−0.023		0.023										0.113
	(−1.748)		(1.811)										
III.II	−0.016		0.019	−0.028									0.196
	(−1.188)		(1.766)	(−1.578)									
III.III	−0.028		0.025		0.022								0.166
	(−2.074)		(1.944)		(1.740)								
III.IV	−0.023		0.023			0.002							0.113
	(−1.743)		(1.812)			(0.098)							
III.V	−0.022		0.023				0.015						0.140
	(−1.677)		(1.797)				(0.986)						
III.VI	−0.024		0.021					−0.022					0.168
	(−1.836)		(1.693)					(−1.219)					
III.VII	−0.023		0.023						0.000				0.113
	(−1.666)		(1.811)						(−0.012)				
III.VIII	−0.025		0.023							−0.010			0.124
	(−1.843)		(1.814)							(−0.463)			
III.IX	−0.023		0.022								−0.005		0.116
	(−1.725)		(1.688)								(−0.395)		
III.X	−0.022		0.024									0.015	0.140
	(−1.678)		(1.843)									(0.740)	
III.XI	−0.019		0.020	−0.046	0.024	0.010	0.021	−0.003	0.028	0.011	0.000	0.044	0.424
	(−1.319)		(1.761)	(−1.381)	(1.654)	(0.370)	(1.207)	(−0.081)	(0.754)	(0.287)	(−0.008)	(1.643)	

Note: The table reports in three panels the one-day ahead daily predictive regression coefficients and robust t-statistics (in parentheses) for the excess market returns. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant equity return predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 8: One-week ahead predictive excess market return regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	Rev	RK	JV	MoM	Max	Min	RV	RSK	VRP	R^2
I.I	−0.021												0.011
	(−0.539)												
I.II	0.001			−0.093									0.222
	(0.030)			(−1.519)									
I.III	−0.020				−0.003								0.011
	(−0.516)				(−0.127)								
I.IV	−0.022					−0.046							0.067
	(−0.578)					(−1.433)							
I.V	−0.017						0.072						0.146
	(−0.429)						(1.144)						
I.VI	−0.026							−0.086					0.204
	(−0.696)							(−1.634)					
I.VII	−0.031								0.041				0.051
	(−0.829)								(0.540)				
I.VIII	−0.032									−0.074			0.152
	(−0.854)									(−1.254)			
I.IX	−0.020										−0.038		0.049
	(−0.520)										(−1.633)		
I.X	−0.017											0.060	0.105
	(−0.450)											(1.222)	
I.XI	0.000			−0.254	0.011	0.020	0.077	0.086	0.202	−0.008	−0.008	0.128	0.937
	(0.001)			(−2.600)	(0.347)	(0.374)	(1.151)	(0.929)	(1.679)	(−0.070)	(−0.301)	(1.392)	
II.I		−0.038											0.037
		(−1.303)											
II.II		−0.016		−0.089									0.229
		(−0.582)		(−1.448)									
II.III		−0.037			−0.001								0.037
		(−1.295)			(−0.048)								
II.IV		−0.038				−0.046							0.092
		(−1.316)				(−1.416)							
II.V		−0.035					0.072						0.170
		(−1.206)					(1.139)						
II.VI		−0.033						−0.083					0.214
		(−1.134)						(−1.557)					
II.VII		−0.039							−0.079				0.201
		(−1.358)							(−1.202)				
II.VIII		−0.043								−0.073			0.173
		(−1.503)								(−1.222)			
II.IX		−0.030									−0.032		0.062
		(−1.009)									(−1.292)		
II.X		−0.037										0.061	0.133
		(−1.292)										(1.234)	
II.XI		−0.009		−0.252	0.012	0.019	0.076	0.086	0.202	−0.008	−0.006	0.128	0.939
		(−0.313)		(−2.580)	(0.386)	(0.359)	(1.146)	(0.926)	(1.682)	(−0.070)	(−0.226)	(1.391)	
III.I	−0.024		0.032										0.037
	(−0.636)		(1.167)										
III.II	−0.002		0.019	−0.090									0.231
	(−0.043)		(0.682)	(−1.461)									
III.III	−0.024		0.032		−0.001								0.037
	(−0.627)		(1.162)		(−0.027)								
III.IV	−0.026		0.032			−0.046							0.092
	(−0.674)		(1.155)			(−1.426)							
III.V	−0.020		0.031				0.072						0.170
	(−0.522)		(1.135)				(1.136)						
III.VI	−0.029		0.024					−0.084					0.218
	(−0.763)		(0.867)					(−1.584)					
III.VII	−0.035		0.033						0.042				0.079
	(−0.937)		(1.207)						(0.551)				
III.VIII	−0.035		0.032							−0.074			0.178
	(−0.955)		(1.176)							(−1.255)			
III.IX	−0.023		0.024								−0.033		0.063
	(−0.595)		(0.829)								(−1.316)		
III.X	−0.021		0.033									0.061	0.133
	(−0.550)		(1.219)									(1.237)	
III.XI	−0.001		0.010	−0.252	0.012	0.020	0.077	0.086	0.201	−0.008	−0.005	0.129	0.940
	(−0.037)		(0.356)	(−2.579)	(0.368)	(0.363)	(1.153)	(0.926)	(1.673)	(−0.066)	(−0.203)	(1.395)	

Note: The table reports in three panels the one-week ahead daily predictive regression coefficients and robust t-statistics (in parentheses) for the excess market returns. The market return is the cumulative return from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant equity return predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 9: One-month ahead predictive excess market return regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	Rev	RK	JV	MoM	Max	Min	RV	RSK	VRP	R^2
I.I	−0.026												0.004
	(−0.233)												
I.II	−0.042			0.068									0.031
	(−0.354)			(0.493)									
I.III	−0.038				0.056								0.023
	(−0.329)				(0.820)								
I.IV	−0.033					−0.229							0.328
	(−0.295)					(−1.707)							
I.V	−0.009						0.302						0.565
	(−0.078)						(1.131)						
I.VI	−0.038							−0.182					0.207
	(−0.340)							(−0.972)					
I.VII	−0.106								0.301				0.523
	(−0.810)								(1.207)				
I.VIII	−0.072									−0.308			0.579
	(−0.624)									(−1.656)			
I.IX	−0.024										−0.118		0.090
	(−0.213)										(−2.309)		
I.X	−0.009											0.294	0.537
	(−0.082)											(1.858)	
I.XI	−0.033			−0.245	0.126	−0.051	0.387	0.033	0.557	0.337	−0.071	0.619	2.209
	(−0.254)			(−1.185)	(1.606)	(−0.467)	(1.349)	(0.141)	(1.752)	(1.114)	(−1.391)	(2.216)	
II.I		0.107											0.071
		(1.685)											
II.II		0.099		0.034									0.077
		(1.393)		(0.258)									
II.III		0.102			0.031								0.077
		(1.400)			(0.479)								
II.IV		0.105				−0.227							0.390
		(1.736)				(−1.692)							
II.V		0.118					0.307						0.650
		(1.582)					(1.154)						
II.VI		0.118						−0.186					0.283
		(1.573)						(−1.001)					
II.VII		0.069							0.263				0.487
		(0.921)							(1.111)				
II.VIII		0.087								−0.292			0.595
		(1.196)								(−1.594)			
II.IX		0.142									−0.151		0.203
		(1.810)									(−2.679)		
II.X		0.109										0.295	0.609
		(1.484)										(1.866)	
II.XI		0.138		−0.278	0.099	−0.039	0.403	0.037	0.552	0.338	−0.096	0.628	2.307
		(1.802)		(−1.337)	(1.330)	(−0.347)	(1.419)	(0.157)	(1.765)	(1.123)	(−1.694)	(2.246)	
III.I	−0.011		−0.135										0.115
	(−0.095)		(−2.075)										
III.II	−0.023		−0.128	0.048									0.128
	(−0.192)		(−2.114)	(0.350)									
III.III	−0.020		−0.131		0.045								0.127
	(−0.178)		(−2.042)		(0.664)								
III.IV	−0.017		−0.137			−0.230							0.441
	(−0.155)		(−2.099)			(−1.716)							
III.V	0.007		−0.139				0.303						0.681
	(0.065)		(−2.105)				(1.140)						
III.VI	−0.021		−0.154					−0.196					0.350
	(−0.192)		(−2.276)					(−1.048)					
III.VII	−0.090		−0.127						0.297				0.621
	(−0.697)		(−2.009)						(1.196)				
III.VIII	−0.056		−0.134							−0.308			0.689
	(−0.493)		(−2.087)							(−1.662)			
III.IX	−0.003		−0.175								−0.161		0.265
	(−0.027)		(−2.524)								(−2.893)		
III.X	0.006		−0.128									0.291	0.636
	(0.050)		(−1.990)									(1.849)	
III.XI	−0.008		−0.165	−0.272	0.114	−0.043	0.386	0.036	0.567	0.328	−0.108	0.614	2.361
	(−0.064)		(−2.496)	(−1.313)	(1.465)	(−0.385)	(1.346)	(0.153)	(1.786)	(1.087)	(−1.944)	(2.206)	

Note: The table reports in three panels the one-month ahead daily predictive regression coefficients and robust t-statistics (in parentheses) for the excess market returns. The market return is the cumulative return from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant equity return predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 10: One-day ahead predictive variance risk premium regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	VRP	RV	VIX^2	JV	Max	Min	R^2
I.I	−0.053									0.112
	(−1.885)									
I.II	−0.020			0.568						12.868
	(−0.790)			(5.145)						
I.III	−0.051				0.013					0.118
	(−1.984)				(0.145)					
I.IV	0.033					0.435				7.347
	(1.329)					(3.970)				
I.V	−0.049						0.132			0.808
	(−1.744)						(2.279)			
I.VI	0.009							0.471		8.789
	(0.360)							(6.701)		
I.VII	0.034								−0.433	7.261
	(1.386)								(−5.806)	
I.VIII	0.023			0.558	−0.080		0.295	0.212	−0.092	19.576
	(0.975)			(4.797)	(−0.459)		(3.249)	(2.109)	(−1.515)	
I.IX	0.023			0.612		−0.076	0.295	0.212	−0.092	19.576
	(0.975)			(6.785)		(−0.459)	(3.249)	(2.109)	(−1.515)	
II.I		−0.020								0.016
		(−0.797)								
II.II		−0.017		0.569						12.864
		(−0.707)		(5.156)						
II.III		−0.019			0.019					0.031
		(−0.748)			(0.213)					
II.IV		0.013				0.430				7.312
		(0.539)				(3.972)				
II.V		−0.019					0.134			0.727
		(−0.750)					(2.290)			
II.VI		−0.012						0.470		8.792
		(−0.480)						(6.676)		
II.VII		0.010							−0.427	7.221
		(0.404)							(−5.750)	
II.VIII		−0.011		0.554	−0.087		0.297	0.216	−0.088	19.561
		(−0.437)		(4.771)	(−0.496)		(3.270)	(2.150)	(−1.439)	
II.IX		−0.011		0.612		−0.081	0.297	0.216	−0.088	19.561
		(−0.437)		(6.778)		(−0.496)	(3.270)	(2.150)	(−1.439)	
III.I	−0.053		−0.004							0.113
	(−1.870)		(−0.173)							
III.II	−0.021		0.009	0.568						12.872
	(−0.827)		(0.370)	(5.145)						
III.III	−0.051		−0.005		0.013					0.119
	(−1.959)		(−0.174)		(0.145)					
III.IV	0.033		0.002			0.435				7.347
	(1.309)		(0.077)			(3.971)				
III.V	−0.049		−0.004				0.132			0.809
	(−1.727)		(−0.139)				(2.280)			
III.VI	0.007		0.018					0.472		8.802
	(0.278)		(0.745)					(6.714)		
III.VII	0.033		0.006						−0.433	7.262
	(1.345)		(0.247)						(−5.809)	
III.VIII	0.020		0.024	0.557	−0.082		0.296	0.214	−0.092	19.597
	(0.841)		(0.953)	(4.789)	(−0.470)		(3.253)	(2.127)	(−1.516)	
III.IX	0.020		0.024	0.612		−0.077	0.296	0.214	−0.092	19.597
	(0.841)		(0.953)	(6.782)		(−0.470)	(3.253)	(2.127)	(−1.516)	

Note: The table reports in three panels the one-day ahead daily predictive regression coefficients and robust t-statistics (in parentheses) for the market variance risk premium. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 11: One-week ahead predictive variance risk premium regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	VRP	RV	VIX^2	JV	Max	Min	R^2
I.I	−0.092									0.441
	(−2.703)									
I.II	−0.053			0.674						23.993
	(−1.937)			(5.933)						
I.III	−0.070				0.148					1.555
	(−2.250)				(1.057)					
I.IV	0.039					0.660				22.222
	(1.323)					(4.416)				
I.V	−0.088						0.145			1.530
	(−2.621)						(2.285)			
I.VI	−0.008							0.640		21.399
	(−0.300)							(7.040)		
I.VII	0.029								−0.601	18.463
	(1.033)								(−5.506)	
I.VIII	0.029			0.791	0.225		0.238	0.200	−0.042	39.377
	(1.139)			(6.746)	(1.255)		(2.543)	(1.740)	(−0.454)	
I.IX	0.029			0.639		0.212	0.238	0.200	−0.042	39.377
	(1.139)			(6.417)		(1.255)	(2.543)	(1.740)	(−0.454)	
II.I		0.026								0.035
		(1.106)								
II.II		0.030		0.677						23.892
		(1.438)		(5.946)						
II.III		0.037			0.161					1.374
		(1.593)			(1.141)					
II.IV		0.077				0.658				22.455
		(3.573)				(4.484)				
II.V		0.027					0.147			1.166
		(1.161)					(2.283)			
II.VI		0.037						0.642		21.468
		(1.790)						(7.054)		
II.VII		0.068							−0.600	18.664
		(3.168)							(−5.577)	
II.VIII		0.054		0.792	0.228		0.238	0.196	−0.041	39.487
		(2.430)		(6.785)	(1.276)		(2.556)	(1.704)	(−0.451)	
II.IX		0.054		0.638		0.215	0.238	0.196	−0.041	39.487
		(2.430)		(6.429)		(1.276)	(2.556)	(1.704)	(−0.451)	
III.I	−0.083		−0.077							0.746
	(−2.497)		(−3.148)							
III.II	−0.046		−0.061	0.672						24.183
	(−1.724)		(−2.743)	(5.934)						
III.III	−0.062		−0.078		0.148					1.862
	(−1.997)		(−3.222)		(1.061)					
III.IV	0.046		−0.068			0.659				22.455
	(1.583)		(−3.389)			(4.415)				
III.V	−0.079		−0.076				0.144			1.827
	(−2.410)		(−3.146)				(2.279)			
III.VI	−0.003		−0.047					0.638		21.509
	(−0.107)		(−2.301)					(7.023)		
III.VII	0.036		−0.063						−0.599	18.664
	(1.284)		(−3.030)						(−5.504)	
III.VIII	0.034		−0.047	0.792	0.229		0.237	0.195	−0.042	39.488
	(1.333)		(−2.283)	(6.767)	(1.270)		(2.544)	(1.696)	(−0.453)	
III.IX	0.034		−0.047	0.638		0.215	0.237	0.195	−0.042	39.488
	(1.333)		(−2.283)	(6.425)		(1.270)	(2.544)	(1.696)	(−0.453)	

Note: The table reports in three panels the one-week ahead predictive regression coefficients and robust t-statistics (in parentheses) for the market variance risk premium. The variance risk premium is the cumulative VRP from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 12: One-month ahead predictive variance risk premium regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	VRP	RV	VIX^2	JV	Max	Min	R^2
I.I	−0.119									1.544
	(−2.870)									
I.II	−0.093			0.448						23.357
	(−2.648)			(3.605)						
I.III	−0.068				0.344					14.149
	(−2.038)				(3.389)					
I.IV	0.021					0.704				53.516
	(0.883)					(7.779)				
I.V	−0.115						0.153			4.090
	(−2.830)						(2.535)			
I.VI	−0.041							0.595		39.497
	(−1.731)							(7.995)		
I.VII	0.000								−0.595	38.523
	(0.015)								(−7.477)	
I.VIII	0.018			0.712	0.590		0.066	0.055	0.003	62.294
	(0.872)			(7.999)	(7.122)		(1.595)	(0.792)	(0.035)	
I.IX	0.018			0.315		0.555	0.066	0.055	0.003	62.294
	(0.872)			(5.979)		(7.122)	(1.595)	(0.792)	(0.035)	
II.I		0.001								0.000
		(0.038)								
II.II		0.003		0.454						22.417
		(0.236)		(3.627)						
II.III		0.025			0.356					13.721
		(1.656)			(3.457)					
II.IV		0.056				0.705				53.806
		(5.025)				(7.958)				
II.V		0.002					0.156			2.662
		(0.113)					(2.505)			
II.VI		0.011						0.601		39.334
		(0.908)						(8.048)		
II.VII		0.043							−0.598	38.723
		(3.162)							(−7.659)	
II.VIII		0.047		0.714	0.594		0.066	0.051	0.002	62.501
		(4.134)		(8.052)	(7.166)		(1.587)	(0.734)	(0.029)	
II.IX		0.047		0.314		0.559	0.066	0.051	0.002	62.501
		(4.134)		(5.996)		(7.166)	(1.587)	(0.734)	(0.029)	
III.I	−0.112		−0.063							1.965
	(−2.791)		(−3.562)							
III.II	−0.087		−0.052	0.447						23.646
	(−2.575)		(−3.321)	(3.598)						
III.III	−0.061		−0.063		0.344					14.580
	(−1.876)		(−3.991)		(3.407)					
III.IV	0.026		−0.052			0.704				53.810
	(1.172)		(−4.264)			(7.782)				
III.V	−0.108		−0.062				0.152			4.497
	(−2.747)		(−3.558)				(2.529)			
III.VI	−0.037		−0.034					0.594		39.621
	(−1.614)		(−2.565)					(7.984)		
III.VII	0.006		−0.048						−0.594	38.773
	(0.205)		(−3.593)						(−7.498)	
III.VIII	0.023		−0.044	0.714	0.594		0.066	0.051	0.003	62.502
	(1.149)		(−3.675)	(8.019)	(7.111)		(1.592)	(0.732)	(0.036)	
III.IX	0.023		−0.044	0.314		0.559	0.066	0.051	0.003	62.502
	(1.149)		(−3.675)	(5.995)		(7.111)	(1.592)	(0.732)	(0.036)	

Note: The table reports in three panels the one-month ahead daily predictive regression coefficients and robust t-statistics (in parentheses) for the market variance risk premium. The variance risk premium is the cumulative VRP from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 13: One-week ahead predictive excess market return regressions from $t+2$ to $t+h$

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	Rev	RK	JV	MoM	Max	Min	RV	RSK	VRP	R^2
I.I	−0.009												0.002
	(−0.227)												
I.II	0.007			−0.067									0.113
	(0.173)			(−1.132)									
I.III	−0.005				−0.019								0.011
	(−0.127)				(−0.717)								
I.IV	−0.011					−0.058							0.090
	(−0.273)					(−1.219)							
I.V	−0.005						0.072						0.135
	(−0.122)						(1.139)						
I.VI	−0.015							−0.094					0.230
	(−0.389)							(−1.728)					
I.VII	−0.025								0.062				0.094
	(−0.643)								(0.835)				
I.VIII	0.005									−0.087			0.197
	(0.157)									(−1.462)			
I.IX	−0.008										−0.061		0.100
	(−0.198)										(−2.584)		
I.X	−0.005											0.070	0.128
	(−0.125)											(1.303)	
I.XI	0.006			−0.203	−0.003	0.020	0.075	0.051	0.191	0.007	−0.032	0.136	0.886
	(0.157)			(−2.136)	(−0.096)	(0.386)	(1.131)	(0.553)	(1.578)	(0.062)	(−1.272)	(1.374)	
II.I		0.011											0.003
		(0.358)											
II.II		0.028		−0.072									0.131
		(0.954)		(−1.214)									
II.III		0.014			−0.022								0.015
		(0.483)			(−0.849)								
II.IV		0.010				−0.058							0.090
		(0.344)				(−1.207)							
II.V		0.013					0.072						0.139
		(0.445)					(1.154)						
II.VI		0.016						−0.094					0.231
		(0.529)						(−1.715)					
II.VII		0.003							0.055				0.079
		(0.094)							(0.754)				
II.VIII		0.005								−0.087			0.197
		(0.157)								(−1.462)			
II.IX		0.026									−0.068		0.115
		(0.845)									(−2.715)		
II.X		0.011										0.070	0.130
		(0.371)										(1.309)	
II.XI		0.045		−0.211	−0.008	0.023	0.077	0.052	0.192	0.006	−0.041	0.137	0.931
		(1.495)		(−2.224)	(−0.275)	(0.443)	(1.163)	(0.563)	(1.591)	(0.054)	(−1.566)	(1.384)	
III.I	−0.007		−0.017										0.009
	(−0.179)		(−0.590)										
III.II	0.011		−0.027	−0.071									0.131
	(0.278)		(−0.972)	(−1.193)									
III.III	−0.003		−0.018		−0.020								0.019
	(−0.066)		(−0.648)		(−0.776)								
III.IV	−0.009		−0.017			−0.058							0.098
	(−0.223)		(−0.604)			(−1.222)							
III.V	−0.003		−0.018				0.072						0.143
	(−0.071)		(−0.619)				(1.143)						
III.VI	−0.012		−0.026					−0.096					0.248
	(−0.317)		(−0.916)					(−1.766)					
III.VII	−0.023		−0.015						0.061				0.100
	(−0.600)		(−0.536)						(0.830)				
III.VIII	−0.020		−0.016							−0.090			0.216
	(−0.536)		(−0.583)							(−1.527)			
III.IX	−0.004		−0.034								−0.070		0.128
	(−0.095)		(−1.155)								(−2.809)		
III.X	−0.003		−0.015									0.069	0.133
	(−0.082)		(−0.529)									(1.297)	
III.XI	0.013		−0.046	−0.210	−0.006	0.023	0.074	0.052	0.194	0.005	−0.043	0.134	0.936
	(0.333)		(−1.671)	(−2.213)	(−0.206)	(0.432)	(1.124)	(0.562)	(1.602)	(0.042)	(−1.616)	(1.358)	

Note: The table reports in three panels the one=week ahead daily predictive regression coefficients and robust t-statistics (in parentheses) for the excess market returns. The market return is the cumulative return from $t+2$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant equity return predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 14: One-month ahead predictive excess market return regressions from $t+2$ to $t+h$

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	TRP_t	Rev	Size	Illiq	MoM	Max	Min	RV	RSK	VRP	R^2
I.I	−0.001												0.000
	(−0.011)												
I.II	−0.029			0.118									0.081
	(−0.246)			(0.875)									
I.III	−0.011				0.047								0.013
	(−0.095)				(0.672)								
I.IV	−0.008					−0.235							0.341
	(−0.073)					(−1.666)							
I.V	0.016						0.299						0.549
	(0.142)						(1.118)						
I.VI	−0.011							−0.146					0.132
	(−0.097)							(−0.782)					
I.VII	−0.085								0.315				0.568
	(−0.652)								(1.287)				
I.VIII	0.140									−0.310			0.753
	(1.971)									(−1.674)			
I.IX	0.001										−0.103		0.065
	(0.006)										(−1.965)		
I.X	0.017											0.318	0.622
	(0.157)											(1.921)	
I.XI	−0.013			−0.216	0.115	−0.043	0.392	0.070	0.554	0.306	−0.061	0.621	2.251
	(−0.103)			(−1.046)	(1.405)	(−0.373)	(1.362)	(0.298)	(1.799)	(1.016)	(−1.183)	(2.180)	
II.I		0.162											0.162
		(2.232)											
II.II		0.144		0.077									0.196
		(2.089)		(0.592)									
II.III		0.159			0.018								0.163
		(2.217)			(0.272)								
II.IV		0.160				−0.234							0.499
		(2.215)				(−1.657)							
II.V		0.173					0.304						0.731
		(2.349)					(1.143)						
II.VI		0.171						−0.155					0.310
		(2.312)						(−0.836)					
II.VII		0.122							0.274				0.616
		(1.680)							(1.187)				
II.VIII		0.140								−0.310			0.753
		(1.971)								(−1.674)			
II.IX		0.196									−0.147		0.288
		(2.548)									(−2.593)		
II.X		0.164										0.318	0.785
		(2.271)										(1.926)	
II.XI		0.186		−0.256	0.085	−0.027	0.408	0.074	0.552	0.306	−0.096	0.629	2.438
		(2.519)		(−1.230)	(1.094)	(−0.238)	(1.431)	(0.318)	(1.819)	(1.017)	(−1.699)	(2.206)	
III.I	0.020		−0.184										0.206
	(0.179)		(−2.810)										
III.II	−0.003		−0.171	0.092									0.254
	(−0.028)		(−2.822)	(0.684)									
III.III	0.013		−0.182		0.032								0.212
	(0.115)		(−2.806)		(0.456)								
III.IV	0.013		−0.186			−0.236							0.551
	(0.118)		(−2.832)			(−1.680)							
III.V	0.038		−0.188				0.301						0.763
	(0.336)		(−2.841)				(1.130)						
III.VI	0.011		−0.200					−0.166					0.373
	(0.097)		(−2.916)					(−0.883)					
III.VII	−0.063		−0.176						0.310				0.756
	(−0.492)		(−2.774)						(1.272)				
III.VIII	−0.028		−0.183							−0.327			0.852
	(−0.250)		(−2.836)							(−1.741)			
III.IX	0.027		−0.224								−0.158		0.350
	(0.246)		(−3.208)								(−2.796)		
III.X	0.037		−0.177									0.314	0.812
	(0.341)		(−2.735)									(1.909)	
III.XI	0.018		−0.209	−0.250	0.100	−0.031	0.390	0.073	0.567	0.296	−0.108	0.615	2.493
	(0.141)		(−3.170)	(−1.206)	(1.232)	(−0.273)	(1.358)	(0.314)	(1.840)	(0.982)	(−1.946)	(2.168)	

Note: The table reports in three panels the one-month ahead daily predictive regression coefficients and robust t-statistics (in parentheses) for the excess market returns. The market return is the cumulative return from $t+2$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The first regression, in the first and second panel, presents the univariate results for the tail risk measures. In the last panel, the first regression presents the bivariate regression ($\lambda_t^{\mathbb{P}} + TRP_t$). The subsequent regressions control for relevant equity return predictors, which are defined in Appendix C. The Sample ranges from January, 2000 to December, 2020.

Table 15: Predicting excess market returns with tail risk measures and LTV

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
Panel A: $t+1 : t+h$									
$\lambda_t^{\mathbb{P}}$	−0.025		−0.028	−0.056		−0.058	−0.121		−0.107
t-tstat	(−1.908)		(−2.064)	(−1.491)		(−1.543)	(−1.055)		(−0.937)
$\lambda_t^{\mathbb{Q}}$		−0.029			−0.041			0.060	
t-tstat		(−2.287)			(−1.403)			(0.798)	
TRP_t			0.020			0.018			−0.129
t-tstat			(1.562)			(0.635)			(−1.937)
LTV	−0.015	−0.013	−0.015	−0.060	−0.053	−0.060	−0.246	−0.221	−0.248
t-stat	(−0.878)	(−0.741)	(−0.864)	(−0.926)	(−0.818)	(−0.922)	(−1.359)	(−1.230)	(−1.374)
R^2	0.090	0.117	0.138	0.149	0.112	0.149	0.428	0.360	0.536
Panel B: $t+2 : t+h$									
$\lambda_t^{\mathbb{P}}$				−0.043		−0.040	−0.098		−0.079
t-tstat				(−1.105)		(−1.029)	(−0.853)		(−0.692)
$\lambda_t^{\mathbb{Q}}$					0.006			0.109	
t-tstat					(0.197)			(1.476)	
TRP_t						−0.028			−0.173
t-tstat						(−0.980)			(−2.551)
LTV				−0.068	−0.060	−0.069	−0.245	−0.220	−0.247
t-stat				(−0.990)	(−0.880)	(−0.996)	(−1.387)	(−1.260)	(−1.408)
R^2				0.147	0.101	0.168	0.404	0.421	0.597

Note: The table reports in two panels the regression coefficients and robust t-statistics (in parentheses) for daily predictive regressions of excess market returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$). For forecasting horizons larger than 1 day, Panel A considers the excess market returns from $t+1$ to $t+h$, while Panel B considers the excess market returns from $t+2$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared. The LTV is the left tail variation of Bollerslev et al. (2015). The sample ranges from January, 2000 to December, 2019.

C Variables Definitions

- Reversal (REV): following Jegadeesh (1990) and Lehmann (1990), the short-term reversal variable is defined as the weekly market return over the previous week from Tuesday to Monday.

- Momentum (MoM): following Jegadeesh and Titman (1993), the momentum variable at the end of day t is defined as the compound gross market return from day $t - 252$ through day $t - 21$, skipping the short-term reversal month.

- Variance Risk Premium (VRP): we compute the variance risk premium as a short position in a variance swap, namely, as the difference between risk-neutral and physical expectations of the variance of market returns (e.g. Bekaert and Hoerova, 2014; Bollerslev et al., 2009):

$$VRP_t = \left(\frac{VIX_t}{\sqrt{365}} \right)^2 - RV_t,$$

  where VIX_t is the CBOE volatility index, and RV_t is the realized variance estimated using 5-minute market returns.

- Maximum daily return (Max): the Max variable is defined as the largest total daily market return observed over the previous week (see Bali et al., 2011).

- Minimum daily return (Min): the Min variable is defined as the smallest total daily market return observed over the previous week (see Bali et al., 2011).

- Realized Variance (RV): the realized variance is defined as the sum of the intraday squared returns (e.g., Andersen et al., 2001, 2003):

$$RV_t = \sum_{n=1}^{N} R_{n,t}^2,$$

  where $R_{n,t}$ denotes the log-return on the S&P 500 index over the n-th intra-daily time interval on day t.

- Realized Skewness (RSK): the RSK is the ex-post daily realized skewness based on intra-day market returns standardized by the realized variance (e.g., Amaya et al., 2015):

$$RSK_t = \frac{\sqrt{N} \sum_{n=1}^{N} R_{n,t}^3}{RV_t^{3/2}}.$$

- Realized Kurtosis (RK): the RK is the ex-post daily realized kurtosis based on intra-day market returns standardized by the variance (e.g., Amaya et al., 2015):

$$RSK_t = \frac{N \sum_{n=1}^{N} R_{n,t}^4}{RV_t^2}.$$

- Jump Variation (JV): the jump variation is defined as the difference between the RV and a consistent measure of the integrated variance, such as the bipower variation (BV) of Barndorff-Nielsen and Shephard (2004):

$$JV_t = \max\left(RV_t - BV_t, 0\right),$$

where $BV_t = \pi/2(N/(N-1)) \sum_{n=2}^{N} |R_{n,t}||R_{n-1,t}|$.

- Volatility Index (VIX): the VIX is the CBOE volatility index expressed in variance form (VIX^2) and scaled to the daily level.

- Left Tail Variation (LTV): the left tail variation proposed by Bollerslev et al. (2015) is an option implied measure of short-horizon downside tail risk obtained from short-dated out-of-the-money put options. The measure is obtained from www.tailindex.com.