Moral Hazard and the Corporate Information Environment*

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Abstract

Managerial incentives are substantially related to a firm's market value, so how information is revealed to the market affects managerial behavior. We analyze a model in which the manager needs to exert costly effort to implement a risky, long-term project and the project may generate verifiable information revealing its value. The optimal disclosure rule to motivate managerial effort is the manager's strategic disclosure because it protects the manager from the downside of the project and induces the rational market to punish nondisclosure. A more transparent information regime is not always preferred because it may reduce the manager's discretion over disclosure. We also derive the optimal disclosure when both effort stimulation and project selection are considered.

Keywords: moral hazard, information design, strategic disclosure, corporate governance

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1 Introduction

In practice, managerial incentives are substantially related to a firm's market value. Stock and options account for over 50% of CEO compensations (Frydman and Jenter, 2010). The market value also affects managers' non-pecuniary benefits such as reputation and influence. The rationale behind such prevalence is that the information contained in the market value is useful for assessing managers' performance. The stock market is forward-looking and can price firms very efficiently based on all available information. In the literature, many papers treat the information structure underlying the market value as exogenous and study how to design managerial compensation to deal with managerial moral hazard problems.

However, the information structure underlying the market value is not exogenous or fixed. It depends on the corporate information environment, which intuitively consists of two elements. The first element, the information regime, determines how much information is revealed exogenously and how much is disclosed voluntarily by firms. The information regime is affected by various information mechanism like regulatory requirement, accounting standard, analyst coverage, and so on. The second element, firms' voluntary disclosure, determines how firms disclose information beyond regulatory requirement. In practice, voluntary disclosure is largely affected by corporate governance.

In this paper, we consider the classical corporate governance problem of inducing managerial effort from a different angle: taking contracts as given and studying the design of the corporate information environment. In the model, the firm intends to motivate the manager to undertake a risky, long-term project. However, to increase the probability of having the project, the manager has to exert more effort. A key assumption is that if the project is implemented, information about its value may arrive and can be revealed through exogenous revelation or the firm's voluntary disclosure. The rational market updates the market value based on the revealed information. Since managerial compensation depends on the market value, information revelation affects managerial effort through the market. In this setup, I derive the optimal rule for the firm's voluntary disclosure to motivate managerial effort and evaluate information regimes given that the optimal rule is implemented.

Why should risky, long-term projects be considered especially? First, they are an important source of economic growth. A concern for managerial short-termism in public companies has been prevailing for many years. In 2018 Wall Street Journal, Jamie Dimon and Warren Buffet shouted out, "short-termism is harming the economy."[1] The main consequence of short-termism is the underinvestment of long-term projects, including innovation activities. Second, risky, long-term projects are accompanied by severe information imperfection in the relatively short term. On the one hand, the firm knows more about the project than the market, so the market relies heavily on the firm's disclosure to obtain information. On the other hand, the firm often lacks disclosure credibility and needs to resort to external information mechanisms (such as patenting and third-party testing) to gain more. These informational features of risky, long-term projects imply a potentially important role of the corporate information environment.

We first derive the optimal disclosure rule to motivate managerial effort. One might first think that the best way to motivate managerial effort is to require that all information be disclosed, as this would make the market value most informative about this effort. However, this is not the case. In fact, as long as the managerial compensation is strictly increasing in the market value, the unique optimal disclosure rule is to give the manager full discretion to disclose strategically. Note that what motivates the manager to exert effort is the difference between the expected of having the project and not. Since disclosure is only attached to the project, the goal is essentially maximizing the difference between disclosure and nondisclosure. With complete discretion, the manager would reveal good news and withhold bad news to maximize his utility. Such revealing and withholding strategy naturally reallocates the downside risk of the project from disclosure to nondisclosure and thus maximizes their difference. On the one hand, strategic disclosure protects the manager from the downside of the project. This is the commonly-known *Protection* effect of strategic disclosure, which makes having the project more attractive. On the other hand, the downside withheld by the manager mixes into the nondisclosure cases and decreases the nondisclosure market value (Acharya et al., 2011). This is the *Punishment* effect of strategic disclosure, which lowers the manager's utility upon nondisclosure. Because disclosure is attached to the project, the *Punishment* effect affects the case of no project more, so it also increases the relative attractiveness of having the project. Interestingly, although strategic disclosure conveys only partial information to the market, it provides the most incentive for the manager to exert effort.

Next, we evaluate information regimes, assuming that the firm's voluntary disclosure follows the manager's strategic disclosure. We are especially concerned about the transparency of information regimes, which refers to the amount of information revealed to the market in equilibrium. The optimality of strategic disclosure implies a non-monotone relationship between transparency

[1] https://www.wsj.com/articles/short-termism-is-harming-the-economy-1528336801

and managerial effort. As a result, the impact of a more transparent information regime is ambiguous. More transparency has positive effects if it results from more information being verifiable. It has negative effects if it results from firms having less discretion over disclosure. When choosing between information regimes, the firm or the social planner may face a tradeoff between the two effects. The paper provides criteria to compare information regimes.

Having established the baseline model, we consider a natural extension incorporating project selection. As pointed out by the recent literature (Ben-Porath et al., 2018; DeMarzo et al., 2019), under strategic disclosure, the manager may inefficiently choose the project with a lower expectation but higher dispersion. Therefore, in terms of disclosure, there is tension between project selection and effort stimulation. We derive the optimal disclosure rule that maximizes managerial effort while ensuring efficient project selection. Unlike strategic disclosure, the optimal disclosure rule discloses the bad news that weakens the attractiveness of the inefficient project relative to the efficient one most. Under some common distributional assumptions, this rule can be implemented by requiring the manager to disclose the information when it implies potentially large loss.

Finally, we provide some discussion about the model. First, we discuss the role of disclosure in a joint design with contracting. A common intuition is that due to the vast flexibility of contracts, contract design may largely play down or even eliminate the role of information design in the sense that more information is always desirable (Holmstrom, 1979). However, in realistic corporate settings, managers usually multitask, so the managerial compensation based on the firm's market value has to deal with different agency problems at the same time. Such incompleteness of contract design leaves room for information design to improve outcomes. Therefore, the paper's results are meaningful, even if contract design is allowed. Second, we modify the baseline setup to accommodate the classical corporate short-termism problem (Stein, 1989). We find that strategic disclosure could be effective in mitigating short-termism as well. Third, we discuss related empirical studies on how the corporate information environment affects innovations and R&D activities, which are the best embodiment of risky, long-term projects in reality. Empirical findings support the role of strategic disclosure in motivating innovations and R&D activities and the ambiguous impacts of transparency implied by the model.

The paper is organized as follows. The remainder of this section reviews the related literature. Section 2 describes the baseline setup. Section 3 characterizes the equilibrium of the game and derives the optimal disclosure rule. Section 4 evaluates information regimes, assuming that the optimal disclosure rule is implemented. Section 5 extends the baseline setup to incorporate project selection. Section 6 discusses the role of disclosure in a joint design, short-termism, and related empirical studies. Section 7 concludes the paper. All proofs are given in the Appendix.

Related Literature

Our paper is built upon the large literature on strategic disclosure. Dye (1985) and Jung and Kwon (1988) invented the approach of uncertain information endowment to prevent unraveling under strategic disclosure. Many papers follow this approach and study the consequences of strategic disclosure. The strand of literature that studies the real effects of strategic disclosure on corporate decisions is particularly related to ours. Fishman and Hagerty (1990) consider a "persuasion game" in which a party with private information can verifiably disclose some of his information and study the optimal amount of discretion granted to the informed party. Shavell (1994) analyzes incentives to acquire valuation information before sales transactions when the seller's disclosure is strategic or mandatory. He finds that strategic disclosure results in socially excessive incentives to acquire information and mandatory disclosure is socially desirable for sellers. Pae (1999), Pae (2002) and Guttman and Meng (2021) have similar findings in different settings. Beyer and Guttman (2012) consider a model in which managers' disclosure and investment decisions are both endogenous and focus on the signaling effects stemming from private information about the exogenous quality of investment opportunities. Wen (2013) considers a model in which a firm can only disclose if it invests, so it may undertake unprofitable investments in order to have the opportunity to disclose. Our paper also assumes that disclosure opportunities are attached to the project. Ben-Porath et al. (2018) finds that under strategic disclosure, the manager may inefficiently select the project with a lower expectation but higher dispersion. They characterize the agent's worst-case equilibrium payoff in the presence of the inefficient project selection. DeMarzo et al. (2019) analyze test design and certification standards when an uninformed seller has the option to generate and disclose information. They demonstrate a similar intuition. Our paper departs from this literature by considering the optimal disclosure rather than taking certain disclosure as given. In this sense, Fishman and Hagerty (1990) is closest to ours since they also consider a design problem. Moreover, the literature mainly documents the inefficiencies brought about by strategic disclosure. Our paper is the first to point out that strategic disclosure can be efficient in motivating managerial effort.

Our paper is also related to the literature on the corporate information environment. Gigler and Hemmer (2004), in an agency model that allows for contract renegotiation, study when it is advantageous to improve corporate transparency by requiring disclosure instead of allowing the manager's strategic disclosure. Gigler et al. (2014) develop a cost-benefit tradeoff to shed light on the frequency with which firms should be required to report the results of their operations to the capital market. Friedman et al. (2018) study the optimal regulation of mandatory disclosure to provide the most information to investors. They consider the interactions between mandatory reporting and voluntary disclosure and establish a rationale for imperfect reporting standards. Versano (2020) analyzes the role of disclosure enforcement mechanisms in directing the disclosure practices of managers when shareholders use the information to monitor the manager. Frenkel et al. (2020) analyzes how a firm's voluntary disclosure is affected by third-party disclosure. Our paper studies how the corporate information environment affects managerial effort to initiate risky, long-term projects. Based on the optimality of strategic disclosure, we uncover a tradeoff between more credible disclosure and less disclosure discretion and highlight the ambiguous impact of transparency. Our theoretical findings are consistent with the recent empirical literature on innovations and R&D activities discussed in Section 6.3.

Finally, our paper echoes the finance theory literature on innovations. Manso (2011) derives the innovation-motivating incentive scheme, which suggests that to effectively motivate innovation, managers must be protected from short-term failures. In our paper, the intuition behind the optimality of strategic disclosure is also providing tolerance for adverse short-term outcomes. Since the managerial compensation needs to deal with various agency problems, it usually poses some downside risk to the manager. Therefore, strategic disclosure can provide further protection and thus motivate innovations.

2 The Baseline Setup

The model has three players: the market, the firm's manager, and the firm's board[2]. They are all risk neutral. The firm has a potential new project, F. The project has stochastic net present value x. x has a expectation of μ and follows a continuous distribution $f(x)$ with full support over $[\underline{x}, \bar{x}]$. $\underline{x} < 0 < \mu < \bar{x}$, which means it is efficient to implement Project F ex ante but the project is risky and may incur a loss to the firm ex post. The firm value without Project F is normalized to $V = 0$, so $V = x$ with the project.

The model has two dates, 0, 1. At date 0, the board first specifies the firm's disclosure rule.

[2]We regard the board as a symbol or a representative of the firm's shareholders.

The manager then exerts effort to implement Project F. The effort is represented by e, which incurs a private cost $C(e)$ to the manager. e is only observable to the manager. With probability e, the manager implement Project F; otherwise, he does not implement Project F. At date 1, some information may be revealed, and the market updates the firm's market value based on it.

2.1 Information at date 1

At date 1, if Project F is implemented, information revealing x arrives.[3] We regard Project F as a symbol of projects and activities that affect the firm value in the long run. Hence, in the relatively short run, which is date 1, Project F does not generate any concrete financial information such as revenue. Instead, the information here is new data or new testing results about new products or technology developed by Project F.

Two features about the information are important. The first is its verifiability. Only verifiable information can be credibly revealed to the market and thus affect it. The second is the firm's discretion over the disclosure of the information. Some verifiable information is required to be disclosed or may be revealed by third parties, while the other can be disclosed voluntarily by the firm. Therefore, if Project F is implemented, there are three possible scenarios:

1. *Exogenous revelation*: with probability p, the information is verifiable and exogenously revealed.
2. *Voluntary disclosure*: with probability q, the information is verifiable, and the firm decides its disclosure.
3. *Nonverifiable information*: with probability $1-p-q$, the information is nonverifiable.

Intuitively, p represents the amount of verifiable information revealed exogenously. Because of the exogeneity, the revelation is independent of the information content. q represents the amount of verifiable information that the firm privately observes and can disclose at its discretion. Depending on the firm's disclosure rule, the disclosure of this information potentially depends on its content. Throughout the paper, (p,q) is referred to as an information regime. For the time being, we treat (p,q) as exogenous. In Section 4, we compare information regimes.

If Project F is not implemented, no information arrives. We assume that when no verifiable information is disclosed, the market cannot distinguish whether Project F is implemented or whether the firm withholds verifiable information. This assumption is motivated by the fact that in large

[3] Whether the information reveals x or it is just a noisy signal of x does not matter.

firms with many projects, the manager can easily implement a fictional project that looks like Project F but does not significantly change the firm value. As made clear later, the board would like to motivate the manager to exert effort, and punishing him for not implementing any project is a straightforward strategy. A fictional project serves as a camouflage to exempt the manager from the punishment. This assumption implies that besides affecting the firm value, Project F also endows the firm with the possibility to distinguish its state from nondisclosure states.

The market is rational and competitive, so the firm's market value is always equal to the expected value of the firm conditional on the information revealed to the market. Let P_1 be the market value at date 1. If verifiable information is revealed to the market, P_1 equals x. Otherwise, since the market cannot distinguish the states of the firm, P_1 equals a constant nondisclosure market value $\hat{x}$.

2.2 The manager

The manager's utility is his compensation minus the private cost of effort. Since the board relies on the market to assess the manager's performance, the managerial compensation consists of a base salary, stock, and options. The manager's utility is

$$\alpha(P_1) - C(e),$$

where $\alpha(\cdot)$ represents the manager's compensation. Unless otherwise specified, $\alpha(\cdot)$ is given exogenously. $\alpha(\cdot)$ is continuously differentiable, and its first-order derivative is always positive and smaller than 1. This assumption is common in the literature of optimal contracting. $\alpha'(\cdot) > 0$ prevents the manager from diverting the firm's assets,[4] while $\alpha'(\cdot) < 1$ prevents the manager from injecting his money into the firm and inflating the firm value. We don't assume any particular form of $\alpha(\cdot)$, so it can potentially include other incentive of the manager that is related to the market value. For example, the possibility of being fired and career concern.

An implicit assumption about the manager's utility is that since Project F is a risky, long-term project, its value may not be fully revealed when the manager gets compensated. Therefore, the manager's compensation is affected by the information available to the market in the relatively short term. As long as the manager's compensation has this feature, the paper's main results hold even if part of his compensation depends on the actual firm value revealed in the long term. We omit the manager's long-term compensation in our analysis for simplicity.

[4] Please see Section 6.1 for more discussion.

The effort e is bounded between $\underline{e}$ and $\overline{e}$, where $0 < \underline{e} < \overline{e} < 1$. We assume that the cost of effort $C(e)$ satisfies:

1. $C(\underline{e}) = 0$, $\lim_{e \to \overline{e}} C(e) = +\infty$;
2. $C'(\underline{e}) = 0$, $\lim_{e \to \overline{e}} C'(e) = +\infty$;
3. $C''(e)$ has a positive lower bound $\frac{\overline{\alpha}|\underline{x}|}{\min\{\underline{e}(1-\underline{e}),\overline{e}(1-\overline{e})\}} + \underline{C}$, where $\underline{C}$ is positive and $\overline{\alpha} \equiv \sup\{\alpha'(\cdot)\}$ is the upper bound of the derivative of the short-term compensation.

The first two assumptions are commonly used to guarantee interior solutions. The third assumption requires the cost function to be sufficiently convex. It is used to guarantee the uniqueness of the equilibrium.

2.3 The board

The board intends to maximize the firm's expected value, which is proportional to managerial effort. To maximize managerial effort, the board specifies the firm's disclosure rule. It is easy to see that the disclosure rule matters only in the scenario of voluntary disclosure. In this case, the only relevant state variable for disclosure is x. Therefore, any disclosure rule can be represented by a real-valued function, $D : [\underline{x}, \overline{x}] \mapsto [0,1]$, which means x is disclosed with probability $D(x)$. Denote the set of all disclosure rules like this as $\mathscr{D}$.

A natural question here is how the disclosure rule specified by the board is enforced. The answer to this question also determines what disclosure rule can be effectively enforced. As suggested by the literature (Marinovic and Varas, 2016), public investors' ex post litigation is an important and effective mechanism to discipline firms' disclosure of relevant information. When public investors think the stock trades at misleading prices since the manager withholds relevant information, they can request the board or the regulator to investigate. The manager will be punished if he is found to withhold information in violation of the disclosure rule. Therefore, due to public investors' ex post litigation, the board can enforce the manager to disclose information but cannot enforce him to withhold. This enforcement constraint is called "partial enforcement" in the paper.

In the rest of the paper, we will derive the firm's optimal disclosure rule to motivate managerial effort in different situations. When deriving the optimal disclosure rule, we first assume that any disclosure rule in $\mathscr{D}$ can be enforced. Although this assumption is not realistic, it allows us to better illustrate the economic intuition behind disclosure without being affected by enforcement issues. Then we derive the optimal disclosure rule subject to partial enforcement and discuss its implementation in practice. Note that the manager's strategic disclosure is always enforceable because it naturally arises in equilibrium if the board lets the manager disclose at his discretion.

2.4 The equilibrium concept

The equilibrium concept we use here is Perfect Bayesian equilibrium. The equilibrium is represented by $(D(\cdot), e^*, \hat{x})$ and satisfies that

- the market, observing $D(\cdot)$ and anticipating the manager's response, sets P_1 based on available information;
- the manager, observing $D(\cdot)$ and anticipating the market's price setting behavior, chooses effort to maximize his expected utility

$$e^* \in \arg\max E[\alpha(P_1)] - C(e);$$

- the board, anticipating the market's price-setting behavior and the manager's response, chooses $D(\cdot)$ to maximize e^*.

To avoid triviality, we assume under strategic disclosure, the equilibrium effort is strictly greater than $\underline{e}$. Since strategic disclosure will prove to be most effective in motivating effort, if this assumption does not hold, the equilibrium effort is $\underline{e}$ under any disclosure rule.

3 The Optimal Disclosure Rule

In this section, we consider the disclosure rule that maximizes managerial effort. First, we characterize the equilibrium under any disclosure rule $D(\cdot)$. Second, we compare the two most common disclosure rules: strategic disclosure and full disclosure. Through this comparison, we illustrate the rationale behind the superiority of strategic disclosure. Third, we formally prove that strategic disclosure is uniquely optimal among all disclosure rules. Finally, we discuss the assumptions that leads to the optimality of strategic disclosure.

3.1 The equilibrium under any disclosure

We derive the equilibrium under any disclosure $D(\cdot)$. First, we consider the manager's decision problem, taking the nondisclosure market value $\hat{x}$ as given. With probability $ep + eqD(x)$, Project F is implemented, and its value is revealed to the market through either exogenous revelation or voluntary disclosure, so the market value is x. Otherwise, no verifiable information is revealed, so the market value is $\hat{x}$. Taken together, the manager's expected utility by choosing the effort e is

$$epE[\alpha(x)] + eqE[\alpha(x)D(x)] + (1 - ep - eqE[D(x)])\alpha(\hat{x}) - C(e).$$

For the manager, the marginal benefit of exerting more effort is the difference between having Project F and not. When the project value is revealed, the manager receives $\alpha(x)$ instead of $\alpha(\hat{x})$, so

$$MB = pE[\alpha(x) - \alpha(\hat{x})] + qE[(\alpha(x) - \alpha(\hat{x}))D(x)].$$

For a given $\hat{x}$, the first-order condition for managerial effort e is

$$MB \begin{cases} = C'(e), & e > \underline{e} \\ \leq 0, & e = \underline{e} \end{cases}. \tag{MFC}$$

On the other hand, the market rationally sets the nondisclosure market value to be the expected value of the firm conditional on nondisclosure. Given the disclosure $D(\cdot)$ and the effort e, nondisclosure implies three possibilities:

1. With probability $1 - e$, Project F is not implemented, so the expected value of the firm is 0.
2. With probability $e(1 - p - q)$, Project F is implemented, but no verifiable information arrives, so the expected value of the firm is μ.
3. With the probability $eq \cdot E[1 - D(x)]$, Project F is implemented, and the firm does not disclose verifiable information, so the expected value of the firm is $E[x(1 - D(x))]/E[1 - D(x)]$.

Then the nondisclosure market value is their weighted average, i.e.,

$$\begin{aligned} \hat{x} &= \frac{(1-e)\cdot 0 + e(1-p-q)\mu + eqE[x(1-D(x))]}{(1-e) + e(1-p-q) + eqE[1-D(x)]} \\ &= \frac{e(1-p)\mu - eqE[xD(x)]}{1 - ep - eqE[D(x)]}. \end{aligned} \tag{NDP}$$

The equilibrium is pinned down by the manager's first-order condition (MFC) and the equation for the nondisclosure market value (NDP). Due to the assumption that the second derivative of the cost of effort is sufficiently large, the equilibrium exists and is unique under any disclosure $D(\cdot)$.

Lemma 1. *Under any disclosure $D(\cdot)$, there exists a unique equilibrium.*

3.2 Strategic disclosure and full disclosure

To obtain the intuition of the optimal disclosure rule, we first compare full disclosure with the manager's strategic disclosure. Under full disclosure, the manager discloses verifiable information whenever it arrives, so full disclosure is represented by $D(x) = 1$. Under strategic disclosure, the manager discloses to maximizes P_1. Given the nondisclosure market value $\hat{x}$, the manager discloses x only when $x \geq \hat{x}$. Therefore, strategic disclosure is represented by $D(x) = 1\{x \geq \hat{x}\}$. The following lemma implies that under strategic disclosure, the equilibrium exists and is unique. Here, the uniqueness cannot be implied by Lemma 1 because strategic disclosure itself is endogenous. In addition, the nondisclosure market value under strategic disclosure is strictly between $\underline{x}$ and μ, which implies that strategic disclosure always differ from full disclosure.

Lemma 2. *Under strategic disclosure, there exists a unique equilibrium $(e^{str}, \hat{x}^{str})$. And $\underline{x} < \hat{x}^{str} < \mu$.*

Proposition 1. *Strategic disclosure dominates full disclosure in motivating effort, i.e., $e^{str} > e^{full}$.*

Figure 1 illustrates the intuition of the superiority of strategic disclosure over full disclosure. The simple key idea is that what motivates the manager to exert effort is the difference between the expected payoff of having Project F and not. The upper half demonstrates what happens under full disclosure. As long as verifiable information arrives, the project value is revealed. The solid rectangles represent that the market can see all realizations of x and the manager receives $\alpha(x)$. When no verifiable information arrives or Project F is not implemented, the market cannot distinguish the state, so the nondisclosure market value is the expected value in the two cases, which is represented by the red line.

The lower half demonstrates strategic disclosure. In the scenario of voluntary disclosure, the manager can receive the upside of the project by revealing good news and avoid its downside by withholding bad news. Instead of the downside, he receives the nondisclosure market value, which is represented by the long red line. This is the first effect of strategic disclosure, the *Protection* effect. The *Protection* effect reflects the option value contained in strategic disclosure and raises

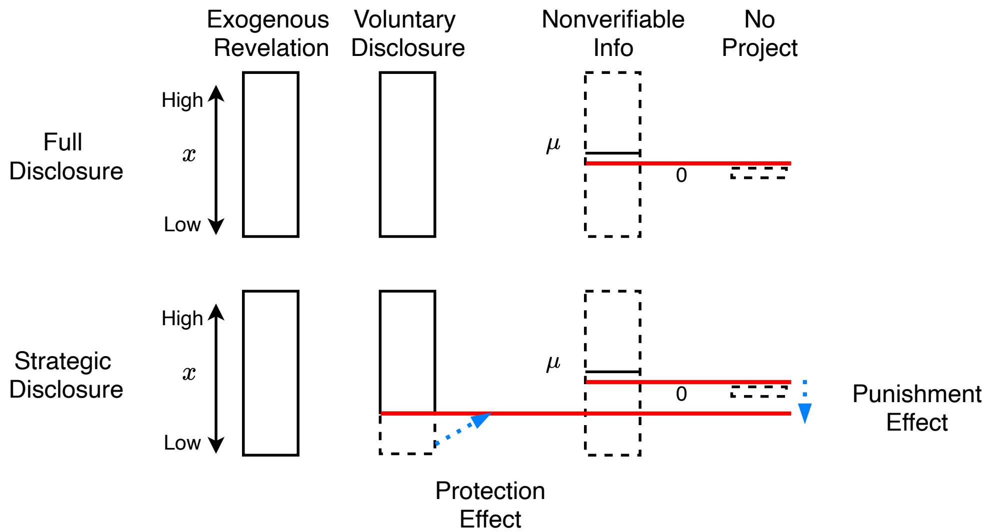

Figure 1: Full disclosure vs. strategic disclosure

the manager's expected utility when he has Project F. On the other hand, the invisible downside withheld by the manager does not disappear. It mixes into nondisclosure cases and decreases the nondisclosure market value from the short red line to the long red line. This is the second effect of strategic disclosure, the *Punishment* effect, which lowers the manager's utility upon nondisclosure. Since Project F allows the possibility for information revelation, the *Punishment* effect hurts the manager more when he does not have Project F, so it increases the relative attractiveness of having Project F.

Under full disclosure, the worst thing is implementing a bad project, which dampens the manager's incentive to exert effort. Under strategic disclosure, the worst thing becomes not being able to disclose, and the manager has to exert more effort to get rid of it. Essentially, by mixing the downside of Project F into nondisclosure cases, strategic disclosure converts fear of a bad project to fear of nondisclosure.

3.3 The optimality of strategic disclosure

Now, we look at the two effects for general cases and prove the optimality of strategic disclosure. Since the cost function of effort is exogenous, the optimal disclosure rule maximizes

$$MB = pE[\alpha(x) - \alpha(\hat{x})] + qE[(\alpha(x) - \alpha(\hat{x}))D(x)].$$

MB is affected by disclosure rules through three channels.

The first is the direct effect on MB, given $\hat{x}$. This effect pushes towards maximizing the second term in MB,

$$E[(\alpha(x)-\alpha(\hat{x}))D(x)],$$

and implies strategic disclosure $D(x)=1\{x\geq\hat{x}\}$. It corresponds to the *Protection* effect.

The second is the indirect effect on MB through $\hat{x}$, given the effort e. It corresponds to the *Punishment* effect: the lower $\hat{x}$ is, the higher MB is. If we rewrite the equation for $\hat{x}$ in the following way,

$$\hat{x}=\frac{e(1-p-q)\mu+eqE[x(1-D(x))]}{1-ep-eq+eqE[1-D(x)]}\Rightarrow(1-ep)\hat{x}=e(1-p)\mu-eqE[(x-\hat{x})D(x)],$$

we can see that this effect essentially pushes towards maximizing

$$E[(x-\hat{x})D(x)].$$

By the minimum principle (Acharya et al., 2011), this effect also implies strategic disclosure.[5]

The third is the equilibrium effect of e on $\hat{x}$. If this effect is too strong, it may lead to multiple equilibria. We assume a sufficiently large second derivative of the cost of effort to rule out multiple equilibria. This assumption turns out to be enough to prevent this effect from overturning the first two effects. Therefore, the *Protection* effect and the *Punishment* effect lead to the optimality of strategic disclosure.

Proposition 2. *The optimal disclosure rule is uniquely the manager's strategic disclosure,* $D(x)=1\{x\geq\hat{x}^{str}\}$. *The equilibrium under strategic disclosure,* $(e^{str},\hat{x}^{str})$*, is pinned down by*

$$\begin{cases} C'(e^{str})=pE[\alpha(x)-\alpha(\hat{x}^{str})]+qE[(\alpha(x)-\alpha(\hat{x}^{str}))\cdot 1\{x\geq\hat{x}^{str}\}], \\ \hat{x}^{str}=\frac{e(1-p)\mu-eqE[x\cdot 1\{x\geq\hat{x}^{str}\}]}{1-ep-eqE[1\{x\geq\hat{x}^{str}\}]}. \end{cases}$$

It takes three steps to derive this result. The first is to capture the two effects. Consider two disclosures and the equilibria under them, $(D,e,\hat{x})$ and $(D',e',\hat{x}')$. Suppose $e>\underline{e}$. Let $\delta(\cdot)\equiv D'(\cdot)-D(\cdot)$ be the difference of the two disclosures. In the following Lemma 3, the first (second)

[5] Given the effort e, the expected value of the firm is fixed. The minimum principle claims that strategic disclosure minimizes the nondisclosure market value in this case.

inequality means that $D'(\cdot)$ has a stronger *Protection* (*Punishment*) effect than $D(\cdot)$. So, if the two inequalities hold, $D'(\cdot)$ strictly dominates $D(\cdot)$.

Lemma 3. *If* $E[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)]>0$ *and* $E[(x-\hat{x})\cdot\delta(x)]>0$*, then* $e'-e>0$*.*

If we let D' be such a cutoff disclosure, $D'(x)\equiv 1\{x\geq\hat{x}\}$, whose cutoff is the nondisclosure market value $\hat{x}$ under D, then

$$\delta(x)\equiv(1-D(x))\cdot 1\{x\geq\hat{x}\}-D(x)\cdot 1\{x<\hat{x}\}.$$

So, $\delta(x)$ is positive for x greater than $\hat{x}$ and negative for x smaller than $\hat{x}$. By Lemma 3, we know that $D'(\cdot)$ strictly dominates $D(\cdot)$ unless $D(\cdot)$ is strategic disclosure. This second step is summarized in Lemma 4. Lemma 4 have two implications. First, the optimal disclosure must be a cutoff one: $D(x)=1\{x\geq k\}$ where $k\in[\underline{x},\overline{x}]$. Second, any cutoff disclosure substantially different from strategic disclosure is strictly dominated.

Lemma 4. *For any* $(D,e,\hat{x})$*, define* $D'(x)\equiv 1\{x\geq\hat{x}\}$*. If* $D\neq 1\{x\geq\hat{x}^{str}\}$*, then* $D'(\cdot)$ *strictly dominates* $D(\cdot)$*.*

The final step is to confirm the existence of the optimal cutoff disclosure. Lemma 5 establishes a continuity of the effort in disclosures. In addition, the set of cutoff disclosures is compact because it is essentially a closed, bounded set in $\mathbb{R}^1$. Then we obtain the existence of the optimal disclosure and finish the proof.

Lemma 5. *There exists* $K>0$ *such that* $|e'-e|\leq K\cdot E[|\delta(x)|]$ *always holds.*

3.4 Discussion

Finally in this section, we discuss the assumptions that lead to the optimality of strategic disclosure. We assume that the compensation α is strictly increasing. If α is just weakly increasing, strategic disclosure is still optimal but not necessarily unique, so strategic disclosure may not strictly dominate full disclosure. Section 6.1 argues that strictly increasing compensation should be expected in a realistic corporate setting.

Another important assumption of the model is that unless verifiable information is revealed, the market cannot discern whether Project F is implemented. It effectively ensures that the no-project case has full exposure to nondisclosure. So, the *Punishment* effect, which punishes nondisclosure, essentially makes the no-project case relatively less attractive. To see the importance of this assumption, suppose that the market can discern no project and the set the market value to be 0 in that case. Then under a disclosure rule $D(\cdot)$, the manager's marginal benefit of effort is

$$pE[\alpha(x)] + qE[\alpha(x)D(x)] + (1 - p - qE[D(x)])\alpha(\hat{x}) - \alpha(0).$$

Here, the nondisclosure market value $\hat{x}$ does not imply any possibility of no project and satisfies

$$(1 - p - qE[D(x)])\hat{x} + pE[x] + qE[xD(x)] = E[x].$$

In this case, the optimal disclosure rule depends on the compensation α. For example, for convex compensation, full disclosure is optimal; for linear compensation, disclosure rules do not affect managerial effort.

On the other hand, for the *Punishment* effect to make the no-project case relatively less attractive, we only need that the manager's payoff has more exposure to nondisclosure when he does not have Project F than when he does. Following this logic, Proposition 3 indicates that weaker versions of the assumption can also guarantee the optimality of strategic disclosure in different situations.

Proposition 3. *Suppose that the market can discern no project with probability p_{no} and the equilibrium is unique under any disclosure. For general compensation, strategic disclosure is uniquely optimal if $p_{no} \leq p$. For linear compensation, strategic disclosure is uniquely optimal if $p_{no} < 1$.*

4 Information Regimes

Besides the firm's disclosure rule for the voluntary disclosure, the other element of corporate information environment is the information regime. In the model, the information regime is characterized by (p, q). Recall that p is the probability that the information is verifiable and revealed exogenously; q is probability that the information is verifiable and can revealed through firms' voluntary disclosure. Therefore, we can interpret $p + q$ as information verifiability and q as firms' disclosure discretion. In reality, the information regime can be affected by various information mechanisms. For example, disclosure requirement, accounting standard, and analyst coverage can increase information verifiability but may also reduce firms' disclosure discretion ($\Delta p + \Delta q > 0$, $\Delta q < 0$), while patenting and optional testing can increase both ($\Delta p = 0$, $\Delta q > 0$). Examining the information regime can shed light on the impact of these information mechanisms.

As discussed in Section 6.3, many papers has studied transparency of corporate information environment and its impact on firms' investment and innovation. To relate to this topic, we establish the concept of transparency in the model's framework. An information regime is said to be more transparent if more verifiable information is revealed in equilibrium through exogenous revelation or voluntary disclosure.

To examine the impact of the information regime, this section compares different information regimes in motivating effort. Suppose the current regime is (p_0, q_0) and its equilibrium is $(e_0, \hat{x}_0)$. There is an alternative regime (p_1, q_1). We focus on the following question: if the goal is to maximize managerial effort, should the alternative regime be adopted? For this exercise, we assume that the firm's voluntary disclosure follows the manager's strategic disclosure.

4.1 General Characterization

To avoid triviality, we assume that under strategic disclosure, the equilibrium efforts are both strictly greater than $\underline{e}$ in the two information regimes. The two equilibria in the two information regimes can be characterized by the following equation system indexed by η.

$$\begin{aligned} C'(e) = & \{p_0 E[\alpha(x) - \alpha(\hat{x}_0)] + q_0 E[\max\{\alpha(x) - \alpha(\hat{x}_0), 0\}]\}(1 - \eta) \\ & + \{p_1 E[\alpha(x) - \alpha(\hat{x}_1)] + q_1 E[\max\{\alpha(x) - \alpha(\hat{x}_1), 0\}]\}\eta, \end{aligned}$$

where

$$\hat{x}_0 = \frac{e(1 - p_0 - q_0) \cdot \mu + eq_0 \cdot E[x \cdot 1\{x < \hat{x}_0\}]}{1 - ep_0 - eq_0 + eq_0 \cdot E[1\{x < \hat{x}_0\}]}$$

and

$$\hat{x}_1 = \frac{e(1 - p_1 - q_1) \cdot \mu + eq_1 \cdot E[x \cdot 1\{x < \hat{x}_1\}]}{1 - ep_1 - eq_1 + eq_1 \cdot E[1\{x < \hat{x}_1\}]}.$$

When $\eta = 0$, $(e, \hat{x}_0)$ represents the equilibrium in the current regime (p_0, q_0); when $\eta = 1$, $(e, \hat{x}_1)$ represents the equilibrium in the alternative regime (p_1, q_1).

For the equilibrium indexed by η, we define

$$\begin{aligned}\Sigma_\eta =& p_1 E[\alpha(x)-\alpha(\hat{x}_1)] + q_1 E[\max\{\alpha(x)-\alpha(\hat{x}_1),0\}] \\ & - p_0 E[\alpha(x)-\alpha(\hat{x}_0)] - q_0 E[\max\{\alpha(x)-\alpha(\hat{x}_0),0\}].\end{aligned}$$

Lemma 6 implies that Σ_η determines how the equilibrium effort e varies with the index η.

Lemma 6. *$sign(de/d\eta) = sign(\Sigma_\eta)$.*

The intuition of Lemma 6 is straightforward. We can imagine η as the probability that the firm stays in the alternative information regime. Note that what motivates the manager to exert effort is the difference between the expected payoff of having Project F and that of not having. In the alternative regime, the difference is

$$p_1 E[\alpha(x)-\alpha(\hat{x}_1)] + q_1 E[\max\{\alpha(x)-\alpha(\hat{x}_1),0\}],$$

while in the current one, the difference is

$$p_0 E[\alpha(x)-\alpha(\hat{x}_0)] + q_0 E[\max\{\alpha(x)-\alpha(\hat{x}_0),0\}].$$

Therefore, Σ_η captures the net effect of an increase in η.

The next step is to determine the sign of Σ_η. The following Lemma 7 shows that the sign of Σ_η is determined exogenously by the model primitives, independent of η.

Lemma 7. *$sign(\Sigma_\eta)$ is independent of η.*

Lemma 7 is based on a simple observation that if $\Sigma_\eta = 0$ holds for any η, then $(e, \hat{x}_0, \hat{x}_1)$ are constant for all η, so $\Sigma_\eta = 0$ always holds. Moreover, since Σ_η is continuous in η, Σ_η does not cross 0 if it is nonzero. Therefore, the sign of Σ_η stays unchanged as η varies. The independence implies that knowing the sign of Σ_0, we can readily determine which information regime dominates.

Proposition 4. *If $\Sigma_0 < 0$, the current regime strictly dominates. If $\Sigma_0 > 0$, the alternative regime strictly dominates. If $\Sigma_0 = 0$, the two regimes are equivalent.*

In the rest of the section, we use Proposition 4 as the starting point to derive implications about comparison between information regimes.

4.2 Implications for convex compensation

Proposition 5. *Suppose that $\alpha(\cdot)$ is convex.*

- *If $p_1 + q_1 > p_0 + q_0$ and $q_1 \geq q_0$, the alternative regime strictly dominates.*
- *If $p_1 + q_1 \leq p_0 + q_0$ and $q_1 < q_0$, the current regime strictly dominates.*

To see the intuition of Proposition 5, we can view an information regime as a bundle of disclosures. An information regime (p, q) essentially contains p of full disclosure, q of strategic disclosure, and $(1 - p - q)$ of nondisclosure. The comparison between information regimes depends on the comparison between the three disclosures. Note that for convex compensation, strategic disclosure dominates full disclosure in motivating effort and full disclosure dominates nondisclosure. Such non-monotone relationship between information revelation and effort implies that more transparency is not always preferred. More transparency can have positive effects if it results from higher information verifiability. That is because when increasing information verifiability, we are essentially replacing nondisclosure with full disclosure or strategic disclosure. More transparency can also have negative effects if it results from more information being exogenously revealed and the firm having less disclosure discretion, which is equivalent to replacing strategic disclosure with full disclosure. To sum up, the optimality of strategic disclosure implies that the impact of more transparency is ambiguous in general and depends on what induces the additional transparency.

4.3 Implications for linear compensation

Proposition 5 does not address the case where information verifiability and firms' disclosure discretion change in opposite directions. To shed more light on this ambiguous case, we further focus on linear compensation, which is commonly assumed in the literature.

Define ρ as the ratio of the increase in transparency to the increase in the overall information, i.e.,

$$\rho \equiv \frac{q_0 - q_1}{(p_1 + q_1) - (p_0 + q_0)},$$

and define σ_p as

$$\sigma_p \equiv \frac{\mu - \hat{x}_0}{E[\max\{\hat{x}_0 - x, 0\}]}.$$

Proposition 6 implies that the relationship between the two objects determines which information regime dominates.

Proposition 6. *Suppose that the compensation is linear and consider the case with* $p_1 + q_1 > p_0 + q_0$ *and* $q_1 < q_0$.

- *When* $\rho = \sigma_p$*, the two regimes are equivalent.*
- *When* $\rho < \sigma_p$*, the alternative regime strictly dominates.*
- *When* $\rho > \sigma_p$*, the current regime strictly dominates.*

Intuitively, a smaller ρ means that the decrease in disclosure discretion is smaller, so the alternative regime is more favorable. Proposition 6 claims that σ_p is the simple threshold for ρ. Under linear compensation, the fluctuation of the project value translates proportionally into that of the compensation. The manager's expected payoff is proportional to $\hat{x}_0$ under nondisclosure, μ under full disclosure, and $E[\max\{x, \hat{x}_0\}]$ under strategic disclosure. Therefore, $\mu - \hat{x}_0$ is proportional to the difference between full disclosure and nondisclosure and thus captures the impact of higher information verifiability; $E[\max\{\hat{x}_0 - x, 0\}]$ is proportional to the difference between strategic disclosure and full disclosure and thus captures the impact of higher disclosure discretion.

Notice that $E[\max\{\hat{x}_0 - x, 0\}]$ represents the downside risk of implementing Project F for the manager in the current regime. Proposition 6 implies that for projects with higher downside risk, disclosure discretion is more effective in motivating managerial effort and thus more valuable.

5 Effort Stimulation and Project Selection

The previous analysis demonstrates the optimality of strategic disclosure in motivating the manager to exert effort. However, as pointed out by Ben-Porath et al. (2018) and DeMarzo et al. (2019), under strategic disclosure, the manager may inefficiently choose the project with a lower expectation but higher dispersion. The simple intuition behind this finding is that strategic disclosure gives the manager an option value and the option value increases with the dispersion of the project. Therefore, in terms of information disclosure, there is tension between effort stimulation and project selection.

In this section, we consider an extension of the baseline setup in which the manager can choose one project to implement from two potential ones. Taking the manager's compensation as given, we derive the optimal disclosure rule that maximizes managerial effort while ensuring efficient project selection. Here, project efficiency is measured by its expected value. Suppose that in addition to Project F, the manager can choose to implement Project G. The value of Project G

follows a continuous distribution $g(\cdot)$ with $E_G[x] = \nu$. Compared to $F(\cdot)$, $G(\cdot)$ is weakly more dispersed in the sense that $\tilde{G}(x) \equiv G(x+\nu-\mu)$ is a mean-preserving spread of $F(x)$, i.e., for any y,

$$\int_{-\infty}^{y} \tilde{G}(x)dx \geq \int_{-\infty}^{y} F(x)dx.$$

To simplify the derivation of the optimal disclosure, we make two technical assumptions about Project G:

1. $g(\cdot)$ has full support over $[\underline{x}, \overline{x}]$, the same as $f(\cdot)$.
2. For any z, $\{x \in [\underline{x}, \overline{x}] | g(x)/f(x) = z\}$ has zero measure.

Following Ben-Porath et al. (2018), we assume linear compensation, so α represents its slope.

5.1 $\nu \geq \mu$

When $\nu \geq \mu$, Project G is the efficient one. Our goal is to derive the disclosure that maximizes the effort while ensuring that the manager chooses G. Define

$$MB(F, D, \hat{x}) \equiv \alpha p(E_F[x] - \hat{x}) + \alpha q E_F[(x - \hat{x}) \cdot D(x)].$$

Given the disclosure $D(x)$ and the nondisclosure market value $\hat{x}$, for any effort e, if the manager chooses Project F, his utility is

$$e \cdot MB(F, D, \hat{x}) + \alpha \hat{x} - C(e).$$

Similarly, if he chooses Project G, his utility is

$$e \cdot MB(G, D, \hat{x}) + \alpha \hat{x} - C(e).$$

Therefore, the condition for the manager to choose G is

$$MB(F, D, \hat{x}) \leq MB(G, D, \hat{x}). \tag{EPG}$$

Proposition 7. *If $\nu \geq \mu$, strategic disclosure results in efficient project selection and maximizes the equilibrium effort.*

When Project G is the only one that the manager can implement, by Proposition 2, we know strategic disclosure maximizes the equilibrium effort. Therefore, as long as the manager chooses Project G under strategic disclosure, strategic disclosure is optimal. By proving that (EPG) holds under strategic disclosure for any nondisclosure market value $\hat{x}$, Proposition 7 confirms the optimality of strategic disclosure.

5.2 $\nu < \mu$

When $\nu < \mu$, Project F is the efficient one. If $D(\cdot)$ results in Project F being chosen, the equilibrium is characterized as follows.

$$\begin{aligned} C'(e) &= \alpha p(\mu - \hat{x}) + \alpha q E_F[(x-\hat{x}) \cdot D(x)], \\ \hat{x} &= \frac{e(1-p)\mu - eqE_F[xD(x)]}{1-ep-eqE_F[D(x)]}, \\ MB(F,D,\hat{x}) &\geq MB(G,D,\hat{x}). \end{aligned} \tag{EPF}$$

The first two equations determine the equilibrium effort and the nondisclosure market value, given that F is chosen. The third constraint is the condition for F to be chosen. We call any disclosure rule satisfying these conditions *eligible*. Further, a disclosure rule is called just (strictly) *eligible* if it is *eligible* and the (EPF) constraint holds with equality (inequality). Hence, the optimal disclosure rule is the eligible one that maximizes the equilibrium effort. It is easy to see that strategic disclosure is optimal if it is eligible.

Next, we derive the optimal disclosure rule, assuming that strategic disclosure is not eligible. First, we consider the scenario of strong enforcement.

Proposition 8. *In the scenario of strong enforcement, the optimal disclosure rule is almost surely*

$$D(x) = 1\{x|x < \hat{x}, g(x)/f(x) > k\} + 1\{x|x > \hat{x}, g(x)/f(x) < k\},$$

where $\hat{x}$ is its corresponding nondisclosure market value and k makes $D(\cdot)$ just eligible.

To see the intuition of this result, we can use strategic disclosure as the starting point to consider the optimal disclosure rule. Strategic disclosure can maximize managerial effort, but it results in inefficient project selection. A natural idea is to modify strategic disclosure in some ways such that efficient project selection can be sustained. That means, unlike strategic disclosure, eligible disclosures may disclose some bad news of the project and withhold some good news. However, such deviation from strategic disclosure renders the two projects less attractive to the manager and thus disincentivizes effort. Intuitively, the optimal disclosure rule should reduce the attractiveness of Project G sufficiently while preserving that of Project F as much as possible. Therefore, the optimal disclosure rule must have two features. First, it is just eligible. Otherwise, we can make the disclosure a bit more like strategic disclosure and increase the effort without changing the choice of projects. Second, it discloses the bad news that weakens the attractiveness of Project G relative to Project F most and withholds the good news that strengthens it most. What defines bad news and good news? In equilibrium, it depends on whether the project value is lower or higher than the nondisclosure market value. What determines the magnitude of the relative attractiveness of Project G to Project F? It is the ratio of their probability density functions, $g(x)/f(x)$. Then we naturally obtain that the project value should be disclosed only when it falls in the two areas characterized by Proposition 8.

The disclosure rule characterized by Proposition 8 requires the manager to disclose some bad news and withhold some good news, which violates partial enforcement. Partial enforcement implies that only the eligible disclosure $D(x)$ that satisfies $\forall x > \hat{x}, D(x) = 1$ is feasible. Proposition 9 characterizes the optimal disclosure rule subject to partial enforcement. It largely follows the idea of Proposition 8.

Proposition 9. *Suppose the firm can only force the manager to disclose information. The optimal disclosure rule exists. If $D(\cdot)$ is optimal and $\hat{x}$ is its corresponding nondisclosure market value, then $D(x) = 1\{x|x < \hat{x}, g(x)/f(x) > k\} + 1\{x|x > \hat{x}\}$ and $D(\cdot)$ is just eligible.*

5.3 Numerical examples and implementation

To provide more concrete implications, we use a numerical example to illustrate the optimal disclosure rule subject to partial enforcement and discuss its implementation. Regarding the project values, we assume $\underline{x} = -0.8$, $\bar{x} = 1.2$, $f(\cdot)$ and $g(\cdot)$ are respectively the normal distributions $N(0.2, 0.15)$ and $N(0.17, 0.3)$ truncated over $[-0.8, 1.2]$. Regarding the information regime, we assume $p = 0$ and $q = 0.9693$. Then according to Proposition 9, the optimal disclosure $D(\cdot)$ is characterized by $\hat{x} = 0$ and $k = 2.1452$.[6]

[6] We deliberately pick $p = 0$ and $q = 0.9693$ such that $\hat{x} = 0$. The advantage of $\hat{x} = 0$ is to exempt us from specifying $C(e)$ and solving the highly nonlinear equation system.

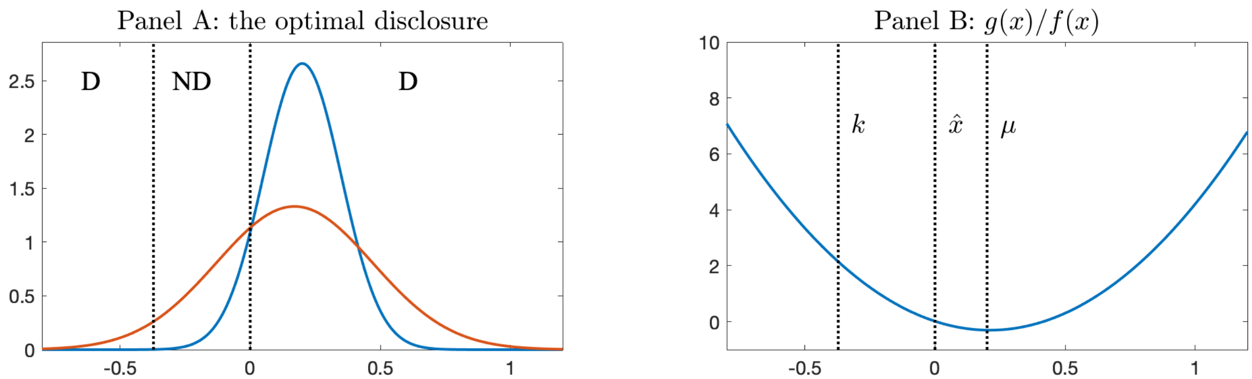

Figure 2: Partial Enforcement

Truncated normal distributions have an important feature. That is, $g(x)/f(x)$ first strictly decreases, attains its minimum at a point greater than μ, and then strictly increases. This pattern is presented in Panel B of Figure 2. Since $\hat{x} < \mu$ under the optimal disclosure,[7] $g(x)/f(x)$ is strictly decreasing over $[\underline{x}, \hat{x}]$. This implies that when the project value is low, it falls in the area $\{x|x < \hat{x}, g(x)/f(x) > k\}$ and thus is disclosed; when the project value is medium but below the nondisclosure market value, it falls in the area $\{x|x < \hat{x}, g(x)/f(x) \leq k\}$ and thus is not disclosed. When the project value is above the nondisclosure market value, it is disclosed by the manager. Panel A of Figure 2 illustrates such a three-area optimal disclosure rule.

The implementation of the optimal disclosure rule here is straightforward. The board only needs to require the manager to disclose when the potential loss is substantially large.

6 Discussion

6.1 Disclosure in a joint design

Throughout the paper, we assume that compensation is strictly increasing. When the compensation is strictly increasing, full disclosure poses downside risk to the manager, so strategic disclosure is strictly better. A natural concern is that if we allow some kinds of contract design, strictly

[7] It follows

$$\hat{x} = \frac{(1-e)\cdot 0 + e(1-p-q)\mu + eqE[x\cdot 1\{x|x<\hat{x}, g(x)/f(x)\leq k\}]}{(1-e)+e(1-p-q)+eqE[1\{x|x<\hat{x}, g(x)/f(x)\leq k\}]} \leq \frac{(1-e)\cdot 0 + e(1-p-q)\mu}{(1-e)+e(1-p-q)} < \mu.$$

increasing compensation may not be the optimal, and optimal compensation may work well with full disclosure, which renders the discussion so far meaningless. The validity of this concern depends on the scope of the problem we are considering. If we focus on the simple baseline setup, it is valid. For example, one optimal joint design is full disclosure plus a compensation with full tolerance for loss.

However, if we consider a more complicated and realistic setting, such a not-strictly-increasing compensation may not be feasible or desirable for several reasons. First, the board may not know the state of the world and the project exactly. What exacerbates the situation is that the board usually needs to determine the compensation several years before they are aware of the project. Second, the board cannot fully determine the manager's utility. For example, the board cannot commit to not firing the manager if the outcome is bad; the board cannot control career or reputation concern either. The third reason, also probably the most important one, is that top executives usually multitask, so the compensation is designed to deal with different agency problems simultaneously. Depending on the nature and the importance of other agency problems, optimal compensation for all agency problems may deviate a lot from that for purely effort stimulation we are considering. Next, we illustrate this point using a simple example.

Suppose that the firm has an ongoing business besides the potential new project in the baseline setup. The ongoing business generates exogenous stochastic cash flow $y \in (-\infty, +\infty)$ at date 1. The manager can choose to divert $y - z$, which is part of y, out of the firm. Due to certain cost associated with diversion, the manager can earn $\kappa(y - z)$ $(\kappa < 1)$. Another interpretation of κ is the effort that the manager needs to exert to maintain the operation of the ongoing business. The cash flow after diversion, z, is disclosed to the public through financing reporting. The market sets the firm's market value to be the expected value of the new project plus z. The board can specify a compensation plan that is based on the firm's market value at date 1 and the disclosure rule for the new project, $D(\cdot)$.

In this setup, two different agency problems are present. Regarding the new project, the manager needs to exert costly effort; regarding the ongoing business, the manager can divert cash flow. To simplify the illustration, we assume that the board's goal is to maximize managerial effort while preventing any diversion.

Proposition 10. *The optimal compensation must be strictly increasing in the market value, and the optimal disclosure rule is uniquely strategic disclosure.*

The basic intuition here is that pure contract design is "incomplete" in many cases, as shown

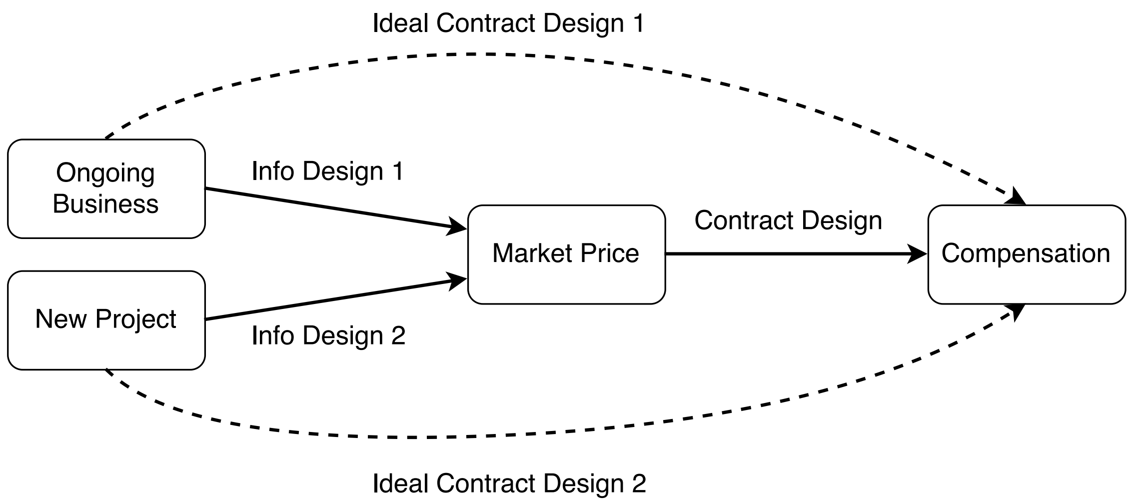

Figure 3: Information design and contract design

in Figure 3. Since projects may have different major agency problems, the board would like the compensation to be affected by project values differently. For mature business, the board wants pay-for-performance; for innovation activities, the board wants tolerance for failure. Ideally, the board would like to write different contracts on their values separately and implement full disclosure. However, in many cases, only the market value of the whole firm is available for contracting. As a result, the board has to use a unidimensional contract to deal with different problems. One way to attenuate this incompleteness is to impose different disclosure rules on different types of information. Therefore, even if managerial compensation can be jointly designed, a careful design of information disclosure still has its own value for corporate governance.

6.2 Short-termism

Short-termism is widely discussed among both practitioners and researchers. As first shown by Stein (1989), the cost of long-term projects is rapidly reflected in the short term while their benefit is not, so a manager who cares about the short-term firm value underinvest in long-term projects. Short-termism is blamed by many people for public firms' insufficient innovations nowadays. This subsection shows that our main intuition about the optimal disclosure rule carries over to the classical short-termism problem. We modify the baseline setup to capture short-termism and show that strategic disclosure can effectively mitigate its impact.

To implement the new project, the manager does not need to exert any personal effort but the firm needs to pay a R&D expense. At date 0, the manager chooses a R&D expense $C_{RD}(e)$. Then with probability e, the manager implements the project; otherwise, he fails. Apart from the R&D expense, the firm also pays a stochastic operating expense y. So, the firm value equals $V-(C_{RD}(e)+y)$. As in the baseline setup, the market may receive information about x exogenously or by the manager's disclosure. The market also observes the total expense $TE=C_{RD}(e)+y$ but not each of them. This assumption captures the idea that the market cannot observe R&D activities precisely and thus does not fully recognize their value. To be consistent with the literature on short-termism, we assume that the manager's compensation depends linearly on the firm's short-term market value and actual value as follows,

$$\alpha \cdot P_1 + \beta \cdot V.$$

We consider pure-strategy perfect Bayesian equilibria of the game. Now, consider any disclosure $D(x)$, and suppose the market expects the R&D expense to be $C_{RD}(e^{RD})$[8], so the nondisclosure market value is

$$\hat{x} = \frac{e^{RD}(1-p)\mu - e^{RD}qE[xD(x)]}{1-e^{RD}p-e^{RD}qE[D(x)]}.$$

Then the manager's expected utility by choosing $C_{RD}(e)$ is

$$ep\alpha\mu + eq\alpha E[xD(x)] + (1-ep-eqE[D(x)])\alpha\hat{x} + e\beta\mu + (1-e)\beta\cdot 0 - (\alpha+\beta)\left(C_{RD}(e)+E[y]\right).$$

His marginal benefit of increasing e is

$$\alpha\left\{p(\mu-\hat{x}) + qE[(x-\hat{x})D(x)] - C'_{RD}(e)\right\} + \beta\left\{\mu - C'_{RD}(e)\right\}.$$

Note that $\mu - C'_{RD}(e)$ is also the firm's marginal benefit, so the long-term compensation provides the manager with the proper incentive. The manager bears no private cost, so if information is complete and the compensation is linear, he is fully aligned with the firm. The key to the equilibrium R&D expense is the information environment in the short term. First, we confirm the existence of short-termism in this setting.

[8] The market thinks that the manager always chooses the same $C_{RD}(e)$ and thus attributes any change in the observable TE to y.

Lemma 8. *Under full disclosure, the R&D expense is too low.*

Because of information incompleteness in the short term, even full disclosure does not ensure the value of R&D to be fully incorporated in the market value. To see this, suppose $1-p-q>0$, which represents the extent of information incompleteness. Under full disclosure $D(x)=1$,

$$\hat{x}=\frac{e^{RD}(1-p-q)\mu}{1-e^{RD}p-e^{RD}q}>0.$$

So,

$$p(\mu-\hat{x})+qE[(x-\hat{x})]=(p+q)(\mu-\hat{x})<\mu.$$

Next, we derive the optimal disclosure rule, which intends to give the manager more incentive to increase the R&D expense than full disclosure. According to Proposition 2, we know that strategic disclosure maximizes $p(\mu-\hat{x})+qE[(x-\hat{x})D(x)]$ and thus maximizes the R&D expense among all possible disclosure rules.

Proposition 11. *If* $(1-p)/q \geq E[\max\{x,0\}]/\mu$*, strategic disclosure is uniquely optimal. If* $(1-p)/q<E[\max\{x,0\}]/\mu$*, any disclosure that satisfies* $E[xD(x)]=\mu(1-p)/q$ *is optimal.*

Unlike effort in the baseline setup, the R&D expense is not the manager's private cost and born by the firm, so there exists an optimal level of the R&D expense. If short-termism due to information incompleteness is sufficiently severe, strategic disclosure is optimal. Otherwise, it leads to excessive R&D expense. To constrain the manager's excessive incentive under strategic disclosure, the firm can require the manager to disclose when potential loss is large as discussed in Section 5.3.

6.3 Related empirical studies

The main focus of the theory is to design the corporate information environment to motivate risky, long-term projects, which correspond to innovations and R&D activities in reality. Here, we discuss related empirical studies.

The mechanisms of strategic disclosure The key mechanism that makes strategic disclosure superior to all others is that it protects the manager from downside risk while punishing him for nondisclosure. The protection effect is widely recognized. Though not directly testing the protection effect of strategic disclosure, Chen et al. (2015) find that firms with CEO contractual protection are less likely to cut R&D expenditures to avoid earnings decreases. The punishment effect relies on rational expectation of the market. Glaeser et al. (2020) provides the first evidence of it. They examine the relation between managerial horizon and strategic disclosure, using patenting as a measure of disclosure. They find that managers with short horizon are thought to be more eager to use strategic disclosure to boost stock prices; correspondingly, investors discount the value of their firms more upon nondisclosure. They also find that these managers have a higher patent value per dollar of R&D spending, which may imply higher efficiency in R&D activities.

Transparency A more transparent corporate information environment may result in more information being disclosed via mandatory disclosure instead of strategic disclosure. As a result, improving transparency may render managers less eager to initiate innovative projects. This possibility is supported by several studies in the U.S. setting. He and Tian (2013) find that firms covered by a larger number of analysts generate fewer patents and patents with lower impact. Wies and Moorman (2015) find that compared to their private counterparts, public firms innovate at higher levels but these innovations are less risky, characterized by fewer breakthrough innovations and fewer innovations in new-to-the-firm categories. Kraft et al. (2018) find that increased reporting frequency is associated with an economically large decline in investments. Fu et al. (2019) find that higher reporting frequency significantly reduces treatment firms' innovation output. On the other hand, a more transparent corporate information environment may also increase the credibility of corporate disclosure and the overall information available to the market. Studying the firms in 29 countries, Zhong (2018) finds that transparency boosts innovative effort and increases innovative efficiency. The difference between the findings in the U.S. and across countries could be ascribed to the initial quality of the corporate information environment. The U.S. already has comprehensive investor protection, efficient accounting systems, and high-quality patenting systems. Transparency largely means forcing the firm to disclose, which has negative effects. But in other countries, transparency means higher information verifiability, which has positive effects.

7 Concluding Remark

Information design has not yet received much attention in corporate governance. An intuition that prevails for long is that more information or more disclosure is always better because it provides more informative signals for contract design (Hölmstrom, 1979). If we consider a joint design of contracts and information, the flexibility of contract design renders information design trivial in many cases. However, the intuition ignores that in a realistic and complicated corporate setting, the contract based on the firm's market value needs to deal with various agency problems. Such incompleteness of contract design leaves room for information design to improve outcomes.

This paper is built upon the incompleteness of contract design and explores the role information design in corporate governance. Fortunately, the optimality of strategic disclosure in motivating effort is robust to any strictly increasing compensation. This allows us to abstract away from the exact nature of the complicated corporate setting and exempts us from handling the joint design explicitly. This optimality result makes two issues trickier than they were considered before. First, a more transparent information regime is not obviously desirable. Although transparency may increase the amount of credible information available to the market, it may also decrease the firm's disclosure discretion. Therefore, when choosing between different information regimes, the decision maker needs to examine their exact nature besides transparency. Second, recent papers have pointed out inefficient project selection under strategic disclosure, and the implied optimal disclosure is full disclosure. However, this paper implies that regarding information disclosure, there is tension between project selection and effort stimulation. To strike a balance, the optimal disclosure rule should still give the manager some discretion over disclosure.

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Appendix

Proof of Lemma 1

Let
$$x(e) \equiv \frac{e(1-p)\mu - eqE[xD(x)]}{1-ep-eqE[D(x)]},$$
which equals the nondisclosure market value under $D(\cdot)$. Since $x(e)$ is unique given e, it suffices to show e is unique.

Let
$$a(e) = C'(e) - pE[\alpha(x) - \alpha(x(e))] - qE[(\alpha(x) - \alpha(x(e)))D(x)].$$
First, we show that $a(e)$ is strictly increasing. By
$$\frac{dx(e)}{de} = \frac{(1-p)\mu - qE[xD(x)]}{\{1-ep-eqE[D(x)]\}^2} = \frac{x(e)}{e\{1-ep-eqE[D(x)]\}},$$

$$\begin{aligned}
a'(e) &= C''(e) - \frac{d\{pE[\alpha(x)-\alpha(x(e))] + qE[(\alpha(x)-\alpha(x(e)))D(x)]\}}{dx(e)} \cdot \frac{dx(e)}{de} \\
&= C''(e) + \{p + qE[D(x)]\}\,\alpha'(x(e)) \cdot \frac{x(e)}{e\{1-ep-eqE[D(x)]\}} \\
&\geq C''(e) - \frac{\alpha'(x(e))(p+q)|\underline{x}|}{e(1-ep-eq)} \\
&\geq C''(e) - \frac{\overline{\alpha}|\underline{x}|}{\min\{\underline{e}(1-\underline{e}), \overline{e}(1-\overline{e})\}}.
\end{aligned}$$

The last line follows that $e(1-e)$ is concave so its minimum is attained at either the maximum of e or the minimum. $C''(e) > \frac{\overline{\alpha}|\underline{x}|}{\min\{\underline{e}(1-\underline{e}),\overline{e}(1-\overline{e})\}}$ can guarantee that $a(e)$ is strictly increasing for any $(\alpha(\cdot), p, q)$.

If $a(\underline{e}) \geq 0$, then $a(e) > 0$ for any $e \in (\underline{e}, \overline{e})$, which implies that $e > \underline{e}$ cannot be an equilibrium. So, the unique equilibrium is $e = \underline{e}$. If $a(\underline{e}) < 0$, then $e = \underline{e}$ cannot be an equilibrium. Since $a(\overline{e}) > 0$, there exists a unique $e \in (\underline{e}, \overline{e})$ such that $a(e) = 0$. Then it is the unique equilibrium.

Proof of Lemma 2

First, we show that for any given $e \in [\underline{e}, \overline{e}]$, there exists a unique $x(e) \in (\underline{x}, \overline{x})$ such that

$$x(e) = \frac{e(1-p)\mu - eqE[x \cdot 1\{x \geq x(e)\}]}{1 - ep - eqE[1\{x \geq x(e)\}]}.$$

Let

$$b(y) = y - \frac{e(1-p)\mu - eqE[x \cdot 1\{x \geq y\}]}{1 - ep - eqE[1\{x \geq y\}]}$$

Since

$$b(\overline{x}) = \overline{x} - \frac{e(1-p)\mu - eqE[x \cdot 1\{x \geq \overline{x}\}]}{1 - ep - eqE[1\{x \geq \overline{x}\}]} \geq \overline{x} - \mu > 0$$

and

$$b(\underline{x}) = \underline{x} - \frac{e(1-p)\mu - eqE[x \cdot 1\{x \geq \underline{x}\}]}{1 - ep - eqE[1\{x \geq \underline{x}\}]} = \underline{x} - e\mu\frac{1-p-q}{1-eq-eq} \leq \underline{x} < 0,$$

by the continuity of $b(y)$, $b(y)$ has a root in $(\underline{x}, \overline{x})$. Suppose x_1 and x_2 both satisfy the condition, and $x_1 < x_2$. Then

$$\begin{aligned} x_2 &= \frac{e(1-p-q)\mu + eqE[x \cdot 1\{x < x_2\}]}{1 - ep - eq + eqE[1\{x < x_2\}]} \\ &= \frac{e(1-p-q)\mu + eqE[x \cdot 1\{x < x_1\}] + eqE[x \cdot 1\{x_1 \leq x < x_2\}]}{1 - ep - eq + eqE[1\{x < x_1\}] + eqE[1\{x_1 \leq x < x_2\}]} \end{aligned}$$

Since

$$\frac{e(1-p-q)\mu + eqE[x \cdot 1\{x < x_1\}]}{1 - ep - eq + eqE[1\{x < x_1\}]} = x_1 < x_2$$

and

$$\frac{eq \cdot E[x \cdot 1\{x_1 \leq x < x_2\}]}{eq \cdot E[1\{x_1 \leq x < x_2\}]} \leq x_2,$$

we obtain $x_2 < x_2$. Contradiction! $x(e)$ is unique.

To prove the uniqueness of the equilibrium, it suffices to show that e is unique. Let

$$a(e) = C'(e) - pE[\alpha(x) - \alpha(x(e))] - qE[\max\{\alpha(x) - \alpha(x(e)), 0\}],$$

where

$$x(e) = \frac{e(1-p)\mu - eqE[x\cdot 1\{x \geq x(e)\}]}{1-ep-eqE[1\{x \geq x(e)\}]} \Leftrightarrow e = \frac{x(e)}{\mu - p(\mu - x(e)) - qE[(x-x(e))\cdot 1\{x \geq x(e)\}]}.$$

First, we show that $a(e)$ is strictly increasing.

$$\begin{aligned} a'(e) &= C''(e) - \frac{d\{pE[\alpha(x)-\alpha(x(e))] + qE[\max\{\alpha(x)-\alpha(x(e)),0\}]\}}{dx(e)} \cdot \frac{dx(e)}{de} \\ &= C''(e) + \{p + q[1-F(x(e))]\}\,\alpha'(x(e)) \cdot \frac{dx(e)}{de}. \end{aligned}$$

Note

$$\begin{aligned} \frac{de}{dx(e)} &= \frac{(1-p)\mu - qE[x\cdot 1\{x \geq x(e)\}]}{\{\mu - p(\mu - x(e)) - qE[(x-x(e))\cdot 1\{x \geq x(e)\}]\}^2} \\ &= \{(1-p)\mu - qE[x\cdot 1\{x \geq x(e)\}]\}\,\frac{e^2}{x(e)^2} \end{aligned}$$

By

$$\begin{aligned} \frac{x(e)}{(1-p)\mu - qE[x\cdot 1\{x \geq x(e)\}]} &= \frac{e}{1-ep-eqE[1\{x \geq x(e)\}]}, \\ \frac{de}{dx(e)} &= \{1-ep-eqE[1\{x \geq x(e)\}]\}\,\frac{e}{x(e)}. \end{aligned}$$

We obtain

$$\begin{aligned} a'(e) &= C''(e) + \{p + q[1-F(x(e))]\}\,\alpha'(x(e)) \cdot \frac{x(e)}{e\{1-ep-eqE[1\{x \geq x(e)\}]\}} \\ &\geq C''(e) - \frac{\alpha'(x(e))(p+q)|\underline{x}|}{e(1-ep-eq)} \\ &\geq C''(e) - \frac{\overline{\alpha}|\underline{x}|}{\min\{\underline{e}(1-\underline{e}), \overline{e}(1-\overline{e})\}} \end{aligned}$$

The rest of the proof follows the proof of Lemma 1.

Finally, we prove that $\underline{x} < \hat{x}^{str} < \mu$.

$$\begin{aligned}\hat{x}^{str} &= \frac{(1-e^{str})\cdot 0 + e^{str}(1-p-q)\cdot\mu + e^{str}q\cdot E[x\cdot 1\{x<\hat{x}^{str}\}]}{(1-e^{str}) + e^{str}(1-p-q) + e^{str}q\cdot E[1\{x<\hat{x}^{str}\}]} \\ &\geq \frac{(1-e^{str})\cdot 0 + e^{str}(1-p-q)\cdot\underline{x} + e^{str}q\cdot E[1\{x<\hat{x}^{str}\}]\cdot\underline{x}}{(1-e^{str}) + e^{str}(1-p-q) + e^{str}q\cdot E[1\{x<\hat{x}^{str}\}]}.\end{aligned}$$

Since $\underline{x} < 0$ and $e^{str} < 1$, $\hat{x}^{str} > \underline{x}$. By

$$\frac{e^{str}q\cdot E[x\cdot 1\{x<\hat{x}^{str}\}]}{e^{str}q\cdot E[1\{x<\hat{x}^{str}\}]} \leq \hat{x}^{str},$$

$$\hat{x}^{str} \leq \frac{(1-e^{str})\cdot 0 + e^{str}(1-p-q)\cdot\mu}{(1-e^{str}) + e^{str}(1-p-q)} < \mu.$$

Proof of Proposition 1

If $\hat{x}^{str} \leq \hat{x}^{full}$,

$$\begin{aligned}E[\alpha(x)-\alpha(\hat{x}^{str})] &\geq E[\alpha(x)-\alpha(\hat{x}^{full})] \\ E[\max\{\alpha(x),\alpha(\hat{x}^{str})\}] - \alpha(\hat{x}^{str}) > E[\alpha(x)] - \alpha(\hat{x}^{str}) &\geq E[\alpha(x)] - \alpha(\hat{x}^{full}).\end{aligned}$$

Then

$$\begin{aligned}&(p+q)E[\alpha(x)-\alpha(\hat{x}^{full})] \\ <&pE[\alpha(x)-\alpha(\hat{x}^{str})] + qE[\max\{\alpha(x)-\alpha(\hat{x}^{str}),0\}].\end{aligned}$$

According to (MFC), this implies that $e^{str} > e^{full}$.

Consider $\hat{x}^{str} > \hat{x}^{full}$. Notice

$$\hat{x}^{str} = \frac{(1-e^{str})\cdot 0 + e^{str}(1-p-q)\cdot\mu + e^{str}q\cdot E[x\cdot 1\{x<\hat{x}^{str}\}]}{(1-e^{str}) + e^{str}(1-p-q) + e^{str}q\cdot E[1\{x<\hat{x}^{str}\}]}$$

and

$$\frac{e^{str}qE[x\cdot 1\{x<\hat{x}^{str}\}]}{e^{str}qE[1\{x<\hat{x}^{str}\}]} < \hat{x}^{str},$$

so

$$\frac{(1-e^{str})\cdot 0+e^{str}(1-p-q)\cdot\mu}{(1-e^{str})+e^{str}(1-p-q)} \geq \hat{x}^{str} > \hat{x}^{full} = \frac{(1-e^{full})\cdot 0+e^{full}(1-p-q)\cdot\mu}{(1-e^{full})+e^{full}(1-p-q)},$$

which implies $e^{str} > e^{full}$.

Proof of Lemma 3

The difference between the two nondisclosure market value is

$$\begin{aligned}
\hat{x}' - \hat{x} &= \frac{e'(1-p)\mu - e'qE[x\cdot D(x)+x\cdot\delta(x)]}{1-e'p-e'qE[D(x)+\delta(x)]} - \frac{e(1-p)\mu - eqE[x\cdot D(x)]}{1-ep-eqE[D(x)]} \\
&= \frac{(1-p)\mu - qE[x\cdot D(x)]}{1/e'-p-qE[D(x)+\delta(x)]} - \frac{(1-p)\mu - qE[x\cdot D(x)]}{1/e-p-qE[D(x)]} - \frac{e'qE[x\cdot\delta(x)]}{1-e'p-e'qE[D'(x)]} \\
&= \frac{\{(1-p)\mu - qE[x\cdot D(x)]\}(1/e-1/e'+qE[\delta(x)])}{\{1/e'-p-qE[D'(x)]\}\{1/e-p-qE[D(x)]\}} - \frac{e'qE[x\cdot\delta(x)]}{1-e'p-e'qE[D'(x)]} \\
&= \frac{\hat{x}(1/e-1/e'+qE[\delta(x)])}{1/e'-p-qE[D'(x)]} - \frac{e'qE[x\cdot\delta(x)]}{1-e'p-e'qE[D'(x)]} \\
&= \frac{\hat{x}}{e\{1-e'p-e'qE[D'(x)]\}}(e'-e) - \frac{e'qE[(x-\hat{x})\cdot\delta(x)]}{1-e'p-e'qE[D(x)]}.
\end{aligned}$$

So, the difference between the marginal costs of effort under the two disclosures is

$$\begin{aligned}
C'(e') - C'(e) &= -p(\alpha(\hat{x}') - \alpha(\hat{x})) + qE[(\alpha(x)-\alpha(\hat{x}'))\cdot D'(x)] - qE[(\alpha(x)-\alpha(\hat{x}))\cdot D(x)] \\
&= -\left\{p+qE[D'(x)]\right\}(\alpha(\hat{x}')-\alpha(\hat{x})) + qE[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)].
\end{aligned}$$

By the Mean Value Theorem, there exists $\tilde{x}$ between $\hat{x}$ and $\hat{x}'$ such that $\alpha'(\tilde{x})(\hat{x}'-\hat{x}) = \alpha(\hat{x}')-\alpha(\hat{x})$, and there exists $\tilde{e}$ between e' and e such that $C'(e')-C'(e) = C''(\tilde{e})(e'-e)$. Therefore,

$$\begin{aligned}
& C''(\tilde{e})(e'-e) \\
=& -\left\{p+qE[D'(x)]\right\}\alpha'(\tilde{x})(\hat{x}'-\hat{x}) + qE[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)] \\
=& -\frac{\alpha'(\tilde{x})\{p+qE[D'(x)]\}\hat{x}}{e\{1-e'p-e'qE[D'(x)]\}}(e'-e) + \frac{\alpha'(\tilde{x})\{p+qE[D'(x)]\}e'qE[(x-\hat{x})\cdot\delta(x)]}{1-e'p-e'qE[D'(x)]} \\
& + qE[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)].
\end{aligned}$$

Combining similar terms,

$$\begin{aligned}
& \left\{C''(\tilde{e}) + \frac{\alpha'(\tilde{x})\{p+qE[D'(x)]\}\hat{x}}{e\{1-e'p-e'qE[D'(x)]\}}\right\}(e'-e) \\
=& \frac{\alpha'(\tilde{x})\{p+qE[D'(x)]\}e'q}{1-e'p-e'qE[D'(x)]}E[(x-\hat{x})\cdot\delta(x)] + qE[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)].
\end{aligned}$$

Since

$$C''(\tilde{e}) + \frac{\alpha'(\tilde{x})\{p+qE[D'(x)]\}\hat{x}}{e\{1-e'p-e'qE[D'(x)]\}} \geq C''(\tilde{e}) - \frac{\overline{\alpha}\,|\underline{x}|}{\min\{\underline{e}(1-\underline{e}),\overline{e}(1-\overline{e})\}} \geq \underline{C} > 0,$$

if $E[(x-\hat{x})\cdot\delta(x)] > 0$ and $E[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)] > 0$, $e'-e > 0$.

Proof of Lemma 4

It is easy to see that

$$\int_{\hat{x}}^{+\infty}(1-D(x))f(x)dx + \int_{-\infty}^{\hat{x}}D(x)f(x)dx \geq 0.$$

The equality holds if and only if $D(x) = 1\{x \geq \hat{x}^{str}\}$ almost surely.

Since

$$\delta(x) \equiv (1-D(x))\cdot 1\{x \geq \hat{x}\} - D(x)\cdot 1\{x < \hat{x}\},$$

$$\begin{aligned}
E[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)] &= E[(\alpha(x)-\alpha(\hat{x}))\cdot(1-D(x))\cdot 1\{x > \hat{x}\} - (\alpha(x)-\alpha(\hat{x}))\cdot D(x)\cdot 1\{x < \hat{x}\}] \\
&= \int_{\hat{x}}^{+\infty}(\alpha(x)-\alpha(\hat{x}))(1-D(x))f(x)dx + \int_{-\infty}^{\hat{x}}(\alpha(\hat{x})-\alpha(x))D(x)f(x)dx.
\end{aligned}$$

So, if $D \neq 1\{x \geq \hat{x}^{str}\}$, $E[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)] > 0$ and $E[(x-\hat{x})\cdot\delta(x)] > 0$.

Proof of Lemma 5

$$\begin{aligned}
\underline{C}\left|e'-e\right| &\leq \left\{C''(\tilde{e}) + \frac{\alpha'(\tilde{x})\{p+qE[D'(x)]\}\hat{x}}{e\{1-e'p-e'qE[D'(x)]\}}\right\}\left|e'-e\right| \\
&\leq \frac{\alpha'(\tilde{x})\{p+qE[D'(x)]\}e'q}{1-e'p-e'qE[D'(x)]}E[|x-\hat{x}|\cdot|\delta(x)|] + qE[|\alpha(x)-\alpha(\hat{x})|\cdot|\delta(x)|] \\
&\leq \frac{\overline{\alpha}\overline{e}}{(1-\overline{e})}\cdot E[|x-\hat{x}|\cdot|\delta(x)|] + E[|\alpha(x)-\alpha(\hat{x})|\cdot|\delta(x)|].
\end{aligned}$$

So,

$$\begin{aligned}
\left|e'-e\right| &\leq \frac{\overline{\alpha}\overline{e}}{\underline{C}(1-\overline{e})}\cdot E[|x-\hat{x}|\cdot|\delta(x)|] + \frac{1}{\underline{C}}E[|\alpha(x)-\alpha(\hat{x})|\cdot|\delta(x)|] \\
&\leq \left[\frac{\overline{\alpha}\overline{e}}{\underline{C}(1-\overline{e})}(\overline{x}-\underline{x}) + \frac{1}{\underline{C}}(\alpha(\overline{x})-\alpha(\underline{x}))\right]E[|\delta(x)|] \\
&\leq K\cdot E[|\delta(x)|],
\end{aligned}$$

which holds for $K \geq \left[\frac{\overline{\alpha}\overline{e}}{\underline{C}(1-\overline{e})}(\overline{x}-\underline{x}) + \frac{1}{\underline{C}}(\alpha(\overline{x})-\alpha(\underline{x}))\right]$.

Proof of Proposition 2

By Lemma 4 we know that if the optimal disclosure exists, it must be a cutoff one: $D(x) = 1\{x \geq k\}$, $k \in [\underline{x}, \overline{x}]$. Second, any nonstrategic cutoff disclosure is strictly dominated. We only need to prove the existence of the optimal cutoff disclosure.

Consider two cutoff disclosures, $D(x) = 1\{x \geq k\}$ and $D'(x) = 1\{x \geq k+\Delta\}$. By Lemma 5,

$$\left|e'-e\right| \leq K\cdot E[|\delta(x)|] = K\cdot E[|1\{x \geq k\} - 1\{x \geq k+\Delta\}|] = K[F(k+\Delta)-F(k)].$$

Since F is atomless, as $\Delta \to 0$, $F(k+\Delta)-F(k) \to 0$, so $e'-e \to 0$. This implies the equilibrium effort is continuous in the cutoff. On the other hand, the set of cutoffs, $[\underline{x}, \overline{x}]$, is compact. So, the optimal cutoff disclosure exists.

Proof of Proposition 3

The first-order condition for managerial effort is

$$C'(e) = pE\left[\alpha(x) - \alpha(\hat{x})\right] + qE\left[\left(\alpha(x) - \alpha(\hat{x})\right)D(x)\right] - p_{no}.\left(\alpha(0) - \alpha(\hat{x})\right).$$

The nondisclosure market value satisfies

$$\begin{aligned}\hat{x} &= \frac{e(1-p)\mu - eqE[xD(x)]}{1-(1-e)p_{no} - ep - eqE[D(x)]} \\ &= \frac{e(1-p)\mu - eqE[xD(x)]}{1-p_{no} - e(p-p_{no}) - eqE[D(x)]}\end{aligned}$$

Following the proof of Lemma 3, we consider two disclosure rules $D(\cdot)$ and $D'(\cdot)$. Then The difference between the two nondisclosure market value is

$$\hat{x}' - \hat{x} = \frac{\hat{x}}{e\{1-p_{no} - e'(p-p_{no}) - e'qE[D'(x)]\}}(e'-e) - \frac{e'qE[(x-\hat{x})\cdot\delta(x)]}{1-p_{no} - e'(p-p_{no}) - e'qE[D(x)]}.$$

The difference between the marginal costs of effort under the two disclosure rules is

$$C'(e') - C'(e) = -\left\{p - p_{no} + qE[D'(x)]\right\}\left(\alpha(\hat{x}') - \alpha(\hat{x})\right) + qE[(\alpha(x) - \alpha(\hat{x}))\cdot\delta(x)].$$

Combining them, we obtain

$$e' - e \propto \frac{e'p - e'p_{no} + e'qE[D'(x)]}{1-p_{no} - e'(p-p_{no}) - e'qE[D'(x)]}\alpha'(\tilde{x})qE[(x-\hat{x})\cdot\delta(x)] + qE[(\alpha(x) - \alpha(\hat{x}))\cdot\delta(x)].$$

For general compensation and $p_{no} \leq p$, if $E[(x-\hat{x})\cdot\delta(x)] > 0$ and $E[(\alpha(x)-\alpha(\hat{x}))\cdot\delta(x)] > 0$, $e'-e > 0$. For linear compensation and $p_{no} < 1$

$$e' - e \propto \frac{1-p_{no}}{1-p_{no} - e'(p-p_{no}) - e'qE[D'(x)]}qE[(x-\hat{x})\cdot\delta(x)],$$

so if $E[(x-\hat{x})\cdot\delta(x)] > 0$, $e'-e > 0$.

The rest of the proof follows the proof of Lemma 4, Lemma 5, and Proposition 2.

Proof of Lemma 6

Differentiating the equation system with respect to e, $\hat{x}_1$, $\hat{x}_2$, and η, we obtain

$$C''(e)de = \Sigma_\eta d\eta - (1-\eta)\{p_0 + q_0[1-F(\hat{x}_0)]\}\alpha'(\hat{x}_0)d\hat{x}_0 - \eta\{p_1 + q_1[1-F(\hat{x}_1)]\}\alpha'(\hat{x}_1)d\hat{x}_1,$$

and

$$d\hat{x}_0 = \frac{1}{1-ep_0-eq_0[1-F(\hat{x}_0)]}\frac{\hat{x}_0}{e}de,$$

and

$$d\hat{x}_1 = \frac{1}{1-ep_1-eq_1[1-F(\hat{x}_1)]}\frac{\hat{x}_1}{e}de.$$

Combining them,

$$\left\{C''(e) + \frac{(1-\eta)\alpha'(\hat{x}_0)\{p_0+q_0[1-F(\hat{x}_0)]\}}{1-ep_0-eq_0[1-F(\hat{x}_0)]}\frac{\hat{x}_0}{e} + \frac{\eta\alpha'(\hat{x}_1)\{p_1+q_1[1-F(\hat{x}_1)]\}}{1-ep_1-eq_1[1-F(\hat{x}_1)]}\frac{\hat{x}_1}{e}\right\}de = \Sigma_\eta d\eta$$

Since

$$C''(e) > \frac{\overline{\alpha}\,|\underline{x}|}{\min\{\underline{e}(1-\underline{e}),\overline{e}(1-\overline{e})\}},$$

$$C''(e) + \frac{(1-\eta)\alpha'(\hat{x}_0)\{p_0+q_0[1-F(\hat{x}_0)]\}}{1-ep_0-eq_0[1-F(\hat{x}_0)]}\frac{\hat{x}_0}{e} + \frac{\eta\alpha'(\hat{x}_1)\{p_1+q_1[1-F(\hat{x}_1)]\}}{1-ep_1-eq_1[1-F(\hat{x}_1)]}\frac{\hat{x}_1}{e} > 0.$$

We obtain

$$\text{sign}(de/d\eta) = \text{sign}(\Sigma_\eta).$$

Proof of Lemma 7

Suppose there exists an η such that the corresponding equilibrium $(e,\hat{x}_0,\hat{x}_1)$ has $\Sigma_\eta = 0$. That means $(e,\hat{x}_0,\hat{x}_1)$ satisfies

$$C'(e) = \{p_0 E[\alpha(x) - \alpha(\hat{x}_0)] + q_0 E[\max\{\alpha(x)-\alpha(\hat{x}_0),0\}]\},$$

where

$$\hat{x}_i = \frac{e(1-p_i-q_i)\cdot\mu + eq_i\cdot E[x\cdot 1\{x<\hat{x}_i\}]}{1-ep_i-eq_i+eq_i\cdot E[1\{x<\hat{x}_i\}]}, i=0,1.$$

It is easy to see that this equation system is independent of η. Then for any η, $(e, \hat{x}_0, \hat{x}_1)$ solves the equation system. Since the equilibrium is always unique, $(e, \hat{x}_0, \hat{x}_1)$ is the equilibrium, which implies $\Sigma_\eta = 0$ always holds.

Suppose there exists an η such that its corresponding equilibrium has $\Sigma_\eta > (<)0$. Notice that Σ_η is continuous in $(\hat{x}_0, \hat{x}_1)$ and $(\hat{x}_0, \hat{x}_1)$ is continuous in η. If there exists another η' such that its corresponding equilibrium has $\Sigma_{\eta'} \leq (\geq)0$, then there must also exist an η'' such that its corresponding equilibrium has $\Sigma_{\eta''} = 0$. By the above analysis, this implies $\Sigma_\eta = 0$. Contradiction! So, such η' does not exist.

To sum up, $\text{sign}(\Sigma_\eta)$ is independent of η.

Proof of Proposition 5

Note that when $(p_1, q_1) = (p_0, q_0)$, $\Sigma_0 = 0$.

Consider the effect of an increase in p_1.

$$\frac{\partial \Sigma_0}{\partial p_1} = E[\alpha(x) - \alpha(\hat{x}_1)] - \{p_1 + q_1 [1 - F(\hat{x}_1)]\} \alpha'(\hat{x}_1) \frac{\partial \hat{x}_1}{\partial p_1}$$

By

$$\frac{\partial \hat{x}_1}{\partial p_1} = -\frac{e(\mu - \hat{x}_1)}{1 - ep_1 - eq_1[1 - F(\hat{x}_1)]},$$

$$\frac{\partial \Sigma_0}{\partial p_1} = E[\alpha(x) - \alpha(\hat{x}_1)] + \frac{\alpha'(\hat{x}_1) \{p_1 + q_1 [1 - F(\hat{x}_1)]\}}{1 - ep_1 - eq_1[1 - F(\hat{x}_1)]} e(\mu - \hat{x}_1).$$

Since $\alpha(\cdot)$ is convex, $E[\alpha(x)] \geq \alpha(\mu)$, so

$$\begin{aligned} \frac{\partial \Sigma_0}{\partial p_1} &\geq \alpha(\mu) - \alpha(\hat{x}_1) + \frac{\alpha'(\hat{x}_1) \{p_1 + q_1 [1 - F(\hat{x}_1)]\}}{1 - ep_1 - eq_1[1 - F(\hat{x}_1)]} e(\mu - \hat{x}_1) \\ &> 0. \end{aligned}$$

The last inequality follows that $\hat{x}_1 < \mu$ under strategic disclosure.

Consider the effect of an increase in q_1.

$$\frac{\partial \Sigma_0}{\partial q_1} = E[\max\{\alpha(x) - \alpha(\hat{x}_1), 0\}] - \{p_1 + q_1 [1 - F(\hat{x}_1)]\} \alpha'(\hat{x}_1) \frac{\partial \hat{x}_1}{\partial q_1}$$

By

$$\frac{\partial \hat{x}_1}{\partial q_1} = -\frac{eE[\max\{x-\hat{x}_1,0\}]}{1-ep_1-eq_1[1-F(\hat{x}_1)]},$$

$$\begin{aligned}\frac{\partial \Sigma_0}{\partial q_1} &= E[\max\{\alpha(x)-\alpha(\hat{x}_1),0\}] + \frac{\alpha'(\hat{x}_1)\{p_1+q_1[1-F(\hat{x}_1)]\}}{1-ep_1-eq_1[1-F(\hat{x}_1)]} eE[\max\{x-\hat{x}_1,0\}] \\ &> 0.\end{aligned}$$

Consider that p_1 increases and p_1+q_1 remains the same,

$$\begin{aligned}\frac{d\Sigma_0}{dp_1} &= \frac{\partial \Sigma_0}{\partial p_1} - \frac{\partial \Sigma_0}{\partial q_1} \\ &= E[\min\{\alpha(x)-\alpha(\hat{x}_1),0\}] + \frac{\alpha'(\hat{x}_1)\{p_1+q_1[1-F(\hat{x}_1)]\}}{1-ep_1-eq_1[1-F(\hat{x}_1)]} eE[\min\{x-\hat{x}_1,0\}] \\ &< 0.\end{aligned}$$

Proof of Proposition 6

Assume $\alpha(x) = ax$,

$$\Sigma_0 = ap_1(\mu-\hat{x}_1) + aq_1E[\max\{x-\hat{x}_1,0\}] - ap_0(\mu-\hat{x}_0) - aq_0E[\max\{x-\hat{x}_0,0\}].$$

Consider the $\eta-$family of (p_1,q_1) that satisfies $p_1 = p_0+t$ and $q_1 = q_0-\eta t$ for $t>0$.

Suppose $\hat{x}_1(t)$ satisfies

$$\begin{aligned}\hat{x}_1(t) &= \frac{e(1-p_1-q_1)\cdot\mu + eq_1\cdot E[x\cdot 1\{x<\hat{x}_1(t)\}]}{1-ep_1-eq_1+eq_1\cdot E[1\{x<\hat{x}_1(t)\}]} \\ &\Leftrightarrow e\mu - \hat{x}_1(t) - ep_1(\mu-\hat{x}_1(t)) - eq_1E[\max\{x-\hat{x}_1(t),0\}] = 0.\end{aligned}$$

From this, we readily obtain two things for any t. First,

$$\Sigma_0 = -a\frac{\hat{x}_1(t)}{e} + a\frac{\hat{x}_0}{e}.$$

Second,

$$\frac{d\hat{x}_1(t)}{dt} = -\frac{e(\mu - \hat{x}_1(t) - \eta E[\max\{x - \hat{x}_1(t), 0\}])}{1 - ep_1 - eq_1[1 - F(\hat{x}_1(t))]}.$$

If $\mu - \hat{x}_0 - \eta E[\max\{x - \hat{x}_0, 0\}] = 0$, then

$$\begin{aligned} & e\mu - \hat{x}_0 - ep_1(\mu - \hat{x}_0) - eq_1 E[\max\{x - \hat{x}_0, 0\}] \\ =& e\mu - \hat{x}_0 - ep_0(\mu - \hat{x}_0) - eq_0 E[\max\{x - \hat{x}_0, 0\}] \\ =& 0. \end{aligned}$$

By the uniqueness of $\hat{x}_1(t)$, $\hat{x}_1(t) = \hat{x}_0$. So,

$$\begin{aligned} \Sigma_0 &= ap_1(\mu - \hat{x}_0) + aq_1 E[\max\{x - \hat{x}_0, 0\}] - ap_0(\mu - \hat{x}_0) - aq_0 E[\max\{x - \hat{x}_0, 0\}] \\ &= 0. \end{aligned}$$

Then any (p_1, q_1) in η-family is equivalent to (p_0, q_0).

Suppose $\mu - \hat{x}_0 - \eta E[\max\{x - \hat{x}_0, 0\}] > 0$. If there exists a $\tilde{t}$ such that $\mu - \hat{x}_1(\tilde{t}) - \eta E[\max\{x - \hat{x}_1(\tilde{t}), 0\}] = 0$, then

$$\begin{aligned} & e\mu - \hat{x}_1(\tilde{t}) - e(p_0 + \tilde{t})(\mu - \hat{x}_1(\tilde{t})) - e(q_0 - \eta\tilde{t}) E[\max\{x - \hat{x}_1(\tilde{t}), 0\}] = 0 \\ & \Rightarrow e\mu - \hat{x}_1(\tilde{t}) - ep_0(\mu - \hat{x}_1(\tilde{t})) - eq_0 E[\max\{x - \hat{x}_1(\tilde{t}), 0\}] = 0. \end{aligned}$$

By the uniqueness of $\hat{x}_0$, this implies $\hat{x}_1(\tilde{t}) = \hat{x}_0$ and further $\mu - \hat{x}_0 - \eta E[\max\{x - \hat{x}_0, 0\}] = 0$. Contradiction! So, such $\tilde{t}$ does not exist. Note that $\hat{x}_1(t)$ is continuous in t, so is $\mu - \hat{x}_1(t) - \eta E[\max\{x - \hat{x}_1(t), 0\}]$. So,

$$\begin{aligned} & \operatorname{sign}(\mu - \hat{x}_1(t) - \eta E[\max\{x - \hat{x}_1(t), 0\}]) \\ =& \operatorname{sign}(\mu - \hat{x}_0 - \eta E[\max\{x - \hat{x}_0, 0\}]) \\ =& 1. \end{aligned}$$

This implies $d\hat{x}_1(t)/dt < 0$. Hence, $\hat{x}_1(t) < \hat{x}_1(0) = \hat{x}_0 \Rightarrow \Sigma_0 > 0$. That means, any (p_1, q_1) in η-family strictly dominates (p_0, q_0).

For $\mu - \hat{x}_0 - \eta E[\max\{x - \hat{x}_0, 0\}] < 0$, following a similar step, we obtain that (p_0, q_0) strictly dominates any (p_1, q_1) in η-family.

Finally, since

$$
\begin{aligned}
&\operatorname{sign}(\mu-\hat{x}_0-\eta E[\max\{x-\hat{x}_0,0\}]) \\
=&\operatorname{sign}\left(\frac{\mu-\hat{x}_0}{E[\max\{x-\hat{x}_0,0\}]}-\eta\right)=\operatorname{sign}\left(\frac{1}{1+\frac{E[\max\{\hat{x}_0-x,0\}]}{\mu-\hat{x}_0}}-\eta\right) \\
=&\operatorname{sign}\left(\frac{1}{1+\frac{1}{\sigma_p}}-\frac{1}{1+\frac{1-\eta}{\eta}}\right)=\operatorname{sign}\left(\sigma_p-\frac{\eta}{1-\eta}\right) \\
=&\operatorname{sign}(\sigma_p-\rho)
\end{aligned}
$$

we obtain Proposition 6.

Proof of Proposition 7

F has the same expectation as $\tilde{G}$ and dominates it in the sense of second–order stochastic dominance. Following Theorem 1 in Ben-Porath et al. (2018), we can readily obtain

$$MB(\tilde{G},1\{x>\hat{x}\},\hat{x})\geq MB(F,1\{x>\hat{x}\},\hat{x}).$$

Therefore, it suffices to prove

$$MB(G,1\{x>\hat{x}\},\hat{x})\geq MB(\tilde{G},1\{x>\hat{x}\},\hat{x}).$$

Since $\nu \geq \mu$, for any $\hat{x}$,

$$
\begin{aligned}
MB(G, 1\{x>\hat{x}\}, \hat{x}) &= \alpha p(E_G[x]-\hat{x}) + \alpha q E_G[\max\{x-\hat{x}, 0\}] \\
&= \alpha p(\nu - \hat{x}) + \alpha q \int_{\hat{x}}^{\overline{x}} (x-\hat{x}) dG(x) \\
&\geq \alpha p(\nu - \hat{x}) + \alpha q \int_{\hat{x}+\nu-\mu}^{\overline{x}} (x-\nu+\mu-\hat{x}) dG(x) \\
&\geq \alpha p(\mu - \hat{x}) + \alpha q \int_{\hat{x}}^{\overline{x}} (x-\hat{x}) d\tilde{G}(x) \\
&= MB(\tilde{G}, 1\{x>\hat{x}\}, \hat{x}).
\end{aligned}
$$

Proof of Proposition 8

Let $D(x;k,k) \equiv 1\{x|x<\hat{x}, g(x)/f(x)>k\} \cup \{x|x>\hat{x}, g(x)/f(x)<k\}$ and $(e(k), \hat{x}(k))$ be the equilibrium under $D(x;k,k)$.

Since $f(x)$ and $g(x)$ are continuous, $g(x)/f(x)$ is continuous. Over $[\underline{x}, \overline{x}]$, $g(x)/f(x)$ has the maximum and the minimum. Let $\overline{k} \equiv \max_{x\in[\underline{x},\overline{x}]} \{g(x)/f(x)\}$ and

$$
\mathscr{K} \equiv \left[0, \max_{x\in[\underline{x},\overline{x}]} \{g(x)/f(x)\}\right] \cap \{k|D(x;k,k) \text{ is eligible}\}.
$$

When considering $D(x;k,k)$, we can focus on $k \in \mathscr{K}$.

The proof have three main steps. The first step is to show that there exists k^* that maximizes $e(k)$ among all $k \in \mathscr{K}$. The second step is to show that $D(x;k^*,k^*)$ must be just eligible. The third step is to show that if a disclosure is different from any $D(x;k,k)$ for a positive measure, there must exist k such that $D(x;k,k)$ results in the same equilibrium effort and is strictly eligible. These three steps imply that the optimal disclosures must be $D(x;k^*,k^*)$ almost surely.

Step 1

Consider $D(x;k,k)$ and $D(x;k+dk,k+dk)$, where dk is a small positive number. Denote $(e(k), \hat{x}(k))$ as $(e, \hat{x})$ and $(e(k+dk), \hat{x}(k+dk))$ as $(e', \hat{x}')$. We conjecture that $e'-e$ and $\hat{x}'-\hat{x}$ are at most infinitesimals of the same order as dk and verify it later.

Consider an instrumental disclosure

$$D'(x) = 1\{x|x < \hat{x}, g(x)/f(x) > k+dk\} \cup \{x|x > \hat{x}, g(x)/f(x) < k+dk\}.$$

The difference between $D(x;k+dk,k+dk)$ and $D'(x)$ rests on $\hat{x}' - \hat{x}$. In the interval $(\min\{\hat{x}',\hat{x}\},\max\{\hat{x}',\hat{x}\})$, they have complementary probability of disclosure, so

$$D(x;k+dk,k+dk) - D'(x) = [2D(x;k+dk,k+dk)-1] \cdot 1\{\min\{\hat{x}',\hat{x}\} \leq x \leq \max\{\hat{x}',\hat{x}\}\}$$

The difference between $D'(x)$ and $D(x;k,k)$ rests on dk, so

$$D'(x) - D(x;k,k) = -1\{x|x < \hat{x}, k < g(x)/f(x) \leq k+dk\} + 1\{x|x > \hat{x}, k < g(x)/f(x) \leq k+dk\}.$$

Then the difference between between $D(x;k+dk,k+dk)$ and $D(x;k,k)$ is

$$\begin{aligned}\delta(x) =& -1\{x|x < \hat{x}, k < g(x)/f(x) \leq k+dk\} + 1\{x|x > \hat{x}, k < g(x)/f(x) \leq k+dk\} \\ &+ [2D(x;k+dk,k+dk)-1] \cdot 1\{\min\{\hat{x}',\hat{x}\} \leq x \leq \max\{\hat{x}',\hat{x}\}\}.\end{aligned}$$

Further,

$$E_F[(x-\hat{x})\delta(x)] = \int_{-\infty}^{\infty} |x-\hat{x}| \cdot 1\{k < g(x)/f(x) \leq k+dk\} f(x)dx + o(dk)$$

and

$$\begin{aligned}E_G[(x-\hat{x})\delta(x)] &= \int_{-\infty}^{\infty} |x-\hat{x}| \cdot 1\{k < g(x)/f(x) \leq k+dk\} g(x)dx + o(dk) \\ &\leq \bar{k} E_F[(x-\hat{x})d(x)] + o(dk)\end{aligned}$$

The second term of $d(x)$ can be ignored in this expectation because in the interval $(\min\{\hat{x}',\hat{x}\},\max\{\hat{x}',\hat{x}\})$, $x-\hat{x}$ and the measure of the interval are both at most infinitesimals of the same order as dk. Since $\{x \in [\underline{x},\overline{x}] | g(x)/f(x) = z\}$ has measure zero for any z, as $dk \to 0$, $E_F[(x-\hat{x})d(x)]$ and $E_G[(x-\hat{x})d(x)]$ converge to 0.

According to the proof of Lemma 3,

$$\left[C''(\tilde{e})+\frac{\alpha\{p+qE_F[D(x;k+dk,k+dk)]\}\hat{x}}{e\{1-e'p-e'qE_F[D(x;k+dk,k+dk)]\}}\right](e'-e)$$
$$=\frac{\alpha q}{\{1-e'p-e'qE_F[D(x;k+dk,k+dk)]\}}E_F[(x-\hat{x})\cdot\delta(x)],$$
$$\hat{x}'-\hat{x}=\frac{\hat{x}}{e\{1-e'p-e'qE_F[D(x;k+dk,k+dk)]\}}(e'-e)-\frac{e'qE_F[(x-\hat{x})\cdot\delta(x)]}{\{1-e'p-e'qE_F[D(x;k+dk,k+dk)]\}}.$$

Therefore, we have verified that $e'-e$ and $\hat{x}'-\hat{x}$ are at most infinitesimals of the same order as dk.

Since

$$MB(F,D(x;k+dk,k+dk),\hat{x}')-MB(F,D(x;k,k),\hat{x})$$
$$=\alpha qE_F[(x-\hat{x})\delta(x)]+\alpha q(\hat{x}-\hat{x}')E_F[D(x;k+dk,k+dk)],$$

$MB(F,D(x;k,k),\hat{x}(k))$ is continuous in k. So is $MB(F,D(x;k,k),\hat{x}(k))-MB(G,D(x;k,k),\hat{x}(k))$. This implies that the set, $\{k|D(x;k,k)\text{ is eligible}\}$, is closed, so $\mathscr{K}$ is compact. Since $e(k)$ is continuous in k, there exists k^* that maximizes $e(k)$ among all $k\in\mathscr{K}$.

Step 2

An important observation is

$$\max_{x\in[\underline{x},\overline{x}]}\left\{\frac{g(x)}{f(x)}\right\}\cdot\lim\inf_{dk\to 0}\frac{E_F[(x-\hat{x})\delta(x)]}{dk}\geq\lim\inf_{dk\to 0}\frac{E_G[(x-\hat{x})\delta(x)]}{dk}\geq 0$$

This implies that as k increases, there are two possibilities.

1. $\liminf_{dk\to 0}E_F[(x-\hat{x})d(x)]/dk=0$. So, $e(k)$, $\hat{x}$, $MB(F,D(x;k,k),\hat{x}(k))-MB(G,D(x;k,k),\hat{x}(k))$ stay unchanged.
2. $\liminf_{dk\to 0}E_F[(x-\hat{x})d(x)]/dk>0$. So, $e(k)$ increases.

Suppose that $D(x;k^*,k^*)$ is strictly eligible. Note that $D(x;\overline{k},\overline{k})$ is strategic disclosure and not eligible. Since $MB(F,D(x;k,k),\hat{x}(k))-MB(G,D(x;k,k),\hat{x}(k))$ is continuous in k, there must exists $\tilde{k}\in(k^*,\overline{k})$ such that $D(x;\tilde{k},\tilde{k})$ is just eligible.

Notice that as k increases from k^* to $\tilde{k}$, $MB(F,D(x;k,k),\hat{x}(k))-MB(G,D(x;k,k),\hat{x}(k))$ decreases to 0. This implies that $\liminf_{dk\to 0}E_F[(x-\hat{x})\delta(x)]/dk>0$ for a positive measure of k, so e

increases. However, $e(k)$ is maximized at k^*. Contradiction! $D(x;k^*,k^*)$ must be just eligible.

Step 3

Consider $D(x)$ that is eligible and different from any $D(x;k,k)$ for a positive measure. Suppose its equilibrium is $(e,\hat{x})$. Then for any k,

$$\int_{-\infty}^{\hat{x}}(1-D(x))\cdot 1\{g(x)/f(x)>k\}f(x)dx+\int_{-\infty}^{\hat{x}}D(x)\cdot 1\{g(x)/f(x)<k\}f(x)dx$$
$$+\int_{\hat{x}}^{\infty}D(x)\cdot 1\{g(x)/f(x)>k\}f(x)dx+\int_{\hat{x}}^{\infty}(1-D(x))\cdot 1\{g(x)/f(x)<k\}f(x)dx>0.$$

The difference between between $D(x;k,k)$ and $D(x)$ is

$$\begin{aligned}\delta(x)\equiv&(1-D(x))\cdot 1\{x<\hat{x},g(x)/f(x)>k\}-D(x)\cdot 1\{x<\hat{x},g(x)/f(x)\leq k\}\\&-D(x)\cdot 1\{x>\hat{x},g(x)/f(x)>k\}+(1-D(x))\cdot 1\{x>\hat{x},g(x)/f(x)\leq k\}.\end{aligned}$$

Then

$$\begin{aligned}&E_F[(x-\hat{x})\cdot\delta(x)]\\=&\int_{-\infty}^{\hat{x}}(x-\hat{x})(1-D(x))\cdot 1\{g(x)/f(x)>k\}f(x)dx-\int_{-\infty}^{\hat{x}}(x-\hat{x})D(x)\cdot 1\{g(x)/f(x)\leq k\}f(x)dx\\&-\int_{\hat{x}}^{\infty}(x-\hat{x})D(x)\cdot 1\{g(x)/f(x)>k\}f(x)dx+\int_{\hat{x}}^{\infty}(x-\hat{x})(1-D(x))\cdot 1\{g(x)/f(x)\leq k\}f(x)dx.\end{aligned}$$

Since $\{x\in[\underline{x},\overline{x}]|g(x)/f(x)=z\}$ has measure zero for any z, $E_F[(x-\hat{x})\cdot\delta(x)]$ is continuous in k. Note that if $k=0$,

$$\begin{aligned}&E_F[(x-\hat{x})\cdot\delta(x)]\\=&\int_{-\infty}^{\hat{x}}(x-\hat{x})(1-D(x))\cdot 1\{g(x)/f(x)>k\}f(x)dx-\int_{\hat{x}}^{\infty}(x-\hat{x})D(x)\cdot 1\{g(x)/f(x)>k\}f(x)dx\\<&0;\end{aligned}$$

if $k = \overline{k}$,

$$
\begin{aligned}
&E_F[(x-\hat{x}) \cdot d(x)] \\
=&-\int_{-\infty}^{\hat{x}} (x-\hat{x})D(x) \cdot 1\{g(x)/f(x) \leq k\} f(x)dx + \int_{\hat{x}}^{\infty} (x-\hat{x})(1-D(x)) \cdot 1\{g(x)/f(x) \leq k\} f(x)dx \\
>&0.
\end{aligned}
$$

Therefore, there exists a $\tilde{k} \in (0, \overline{k})$ such that

$$
E_F[(x-\hat{x}) \cdot \delta(x)] = 0.
$$

This also implies $D(x;\tilde{k},\tilde{k})$ and $D(x)$ have the same effort and nondisclosure market value in equilibrium.

On the other hand, for $\tilde{k}$,

$$
\begin{aligned}
&E_G[(x-\hat{x}) \cdot \delta(x)] \\
=&\int_{-\infty}^{\hat{x}} (x-\hat{x})(1-D(x)) \cdot 1\{g(x)/f(x) > k\} g(x)dx - \int_{-\infty}^{\hat{x}} (x-\hat{x})D(x) \cdot 1\{g(x)/f(x) \leq k\} g(x)dx \\
&-\int_{\hat{x}}^{\infty} (x-\hat{x})D(x) \cdot 1\{g(x)/f(x) > k\} g(x)dx + \int_{\hat{x}}^{\infty} (x-\hat{x})(1-D(x)) \cdot 1\{g(x)/f(x) \leq k\} g(x)dx \\
=&\int_{-\infty}^{\hat{x}} (x-\hat{x})(1-D(x)) \cdot 1\{g(x)/f(x) > k\} \frac{g(x)}{f(x)} f(x)dx - \int_{-\infty}^{\hat{x}} (x-\hat{x})D(x) \cdot 1\{g(x)/f(x) \leq k\} \frac{g(x)}{f(x)} f(x)dx \\
&-\int_{\hat{x}}^{\infty} (x-\hat{x})D(x) \cdot 1\{g(x)/f(x) > k\} \frac{g(x)}{f(x)} f(x)dx + \int_{\hat{x}}^{\infty} (x-\hat{x})(1-D(x)) \cdot 1\{g(x)/f(x) \leq k\} \frac{g(x)}{f(x)} f(x)dx \\
<&k\int_{-\infty}^{\hat{x}} (x-\hat{x})(1-D(x)) \cdot 1\{g(x)/f(x) > k\} f(x)dx - k\int_{-\infty}^{\hat{x}} (x-\hat{x})D(x) \cdot 1\{g(x)/f(x) \leq k\} f(x)dx \\
&-k\int_{\hat{x}}^{\infty} (x-\hat{x})D(x) \cdot 1\{g(x)/f(x) > k\} f(x)dx + k\int_{\hat{x}}^{\infty} (x-\hat{x})(1-D(x)) \cdot 1\{g(x)/f(x) \leq k\} f(x)dx \\
=&kE_F[(x-\hat{x}) \cdot \delta(x)] \\
=&E_F[(x-\hat{x}) \cdot \delta(x)].
\end{aligned}
$$

Let's check the eligibility of $D(x;\tilde{k},\tilde{k})$.

$$\begin{aligned} MB(F,D(x;\tilde{k},\tilde{k}),\hat{x}) &= \alpha p(E_F[x]-\hat{x})+\alpha qE_F[(x-\hat{x})\cdot D(x;\tilde{k},\tilde{k})] \\ &= \alpha p(E_F[x]-\hat{x})+\alpha qE_F[(x-\hat{x})\cdot D(x)]+\alpha qE_F[(x-\hat{x})\cdot \delta(x)] \\ &> MB(F,D(x),\hat{x})+\alpha qE_G[(x-\hat{x})\cdot \delta(x)]. \end{aligned}$$

Since $D(x)$ is eligible,

$$MB(F,D(x),\hat{x}) \geq MB(G,D(x),\hat{x}),$$

so

$$MB(F,D(x;\tilde{k},\tilde{k}),\hat{x}) > MB(G,D(x),\hat{x})+\alpha qE_G[(x-\hat{x})\cdot d(x)] = MB(G,D(x;\tilde{k},\tilde{k}),\hat{x}).$$

This implies that $D(x;\tilde{k},\tilde{k})$ is strictly eligible. Since $D(x;\tilde{k},\tilde{k})$ is strictly dominated by $D(x;k^*,k^*)$, so is $D(x)$.

Proof of Proposition 9

Let $D(x;k) \equiv 1\{x|x<\hat{x}, g(x)/f(x)>k\} \cup \{x|x>\hat{x}\}$. The proof basically follows that of Proposition 8, replacing everything with the counterpart of $D(x;k)$.

Proof of Proposition 10

We only need to show that the sufficient and necessary condition for preventing any diversion is $\forall u>v$, $\alpha(u)-\alpha(v) \geq \kappa(u-v)$.

Consider the case that no project is implemented. The manager's payoff at date 1 is $\alpha(z)+\kappa(y-z)$. No cash flow diverted requires that $\forall y>z$,

$$\alpha(z)+\kappa(y-z) \leq \alpha(y).$$

Therefore, the condition is necessary. Its sufficiency in this case is obvious.

Consider the case that the project is implemented and its value is x. If the manager diverts $y-z$, his expected payoff at date 1 is

$$[p+qD(x)]\,\alpha(x+z)+[1-p-qD(x)]\,\alpha(\hat{x}+z)+\kappa(y-z),$$

which is weakly smaller than

$$[p+qD(x)]\,\alpha(x+y)+[1-p-qD(x)]\,\alpha(\hat{x}+y)$$

if the condition holds. Therefore, the condition is sufficient to prevent any diversion in this case.

Proof of Proposition 11

If $\frac{1-p}{q} \geq \frac{E[\max\{x,0\}]}{\mu}$,

$$(1-p)\mu - qE[x\cdot 1\{x\geq \hat{x}\}] \geq (1-p)\mu - qE[x\cdot 1\{x\geq 0\}] \geq 0,$$

so $\hat{x}\geq 0$. Then

$$p\,(\mu-\hat{x})+qE[\max\{x-\hat{x},0\}] \leq p\mu + qE[\max\{x,0\}] \leq \mu.$$

This implies that strategic disclosure does not result in too high R&D expense.

If $\frac{1-p}{q} < \frac{E[\max\{x,0\}]}{\mu}$, since

$$E[x\cdot 1\{x\geq 0\}] = E[\max\{x,0\}] > \frac{(1-p)\mu}{q} > E[x\cdot 1\{x\leq 0\}],$$

there exists $D(\cdot)$ such that $E[xD(x)] = \frac{(1-p)\mu}{q}$. Under such $D(\cdot)$, $\hat{x}=0$ and

$$p(\mu-\hat{x})+qE[(x-\hat{x})D(x)] = p\mu + qE[xD(x)] = \mu.$$

Therefore, $D(\cdot)$ results in the efficient R&D expense.