

Anomaly Predictability with the Mean-Variance Portfolio^{*}

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Abstract

Conditional mean-variance efficient (CMVE) portfolios should feature unpredictable pricing errors. This paper shows that a wide range of heuristic CMVE portfolios lead to a novel set of asset-specific predictors, which are endogenous to the model and identify potential conditional misspecification. Using several prominent factor models and a large cross-section of test assets, we document that past pricing errors predict future risk-adjusted anomaly returns and that a zero-cost strategy exploiting this pattern generates positive alphas. These findings provide direct evidence against the notion of no-arbitrage, posing a challenge for asset pricing models.

Keywords: Factor Models, Return Predictability, Conditional Misspecification, Mean-Variance Portfolio, SDF

JEL codes: C38, G12, G17.

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1 Introduction

In this paper, we provide novel evidence of anomaly return predictability that is inconsistent with the standard no-arbitrage condition and, thus challenges prominent linear factor models. Our predictability is endogenous to any chosen model, i.e. we construct a predictor for asset *returns* based on the common trend in the *price* dynamics of the test asset and of the risk factors at hand, without relying on information outside the model (like sentiment, accounting signals, or other macro-based state variables). We then show how the presence of such endogenous predictability, or lack thereof, can be linked to the conditional misspecification of the mean-variance efficient portfolio constructed using different factor models. Whereas a large and important literature tries to understand whether factor models are misspecified, i.e. if a set of candidate factors span the stochastic discount factor, our unique contribution is to show how a conditional model misspecification translates in anomaly return predictability and to quantify the economic value for an investor that trades based on such predictability.

Our analysis starts from the expected return-beta representation which posits a linear relationship between expected returns of any asset and the expected return on the conditional mean-variance efficient portfolio. Such representation implies that no variable should forecast future risk-adjusted returns when the model is correctly specified. Of course, the choice of the predictor is critical, and subject to data snooping (Lo and MacKinlay, 1990). Our proposed predictor builds on the intuition that if there is persistent mispricing, this will show up in the price level (Shiller, 1981). In a similar spirit, Chernov et al. (2021) argue that long-term returns, which we will use to construct price levels, convey information about possible model misspecification beyond one-period returns. The novelty in our approach is that we use standard factor models to determine the fundamental value to which the price level of

a test asset should be compared to. Thus, our predictor only leverages the conditioning information used in the construction of the model’s factors, i.e. it is endogenous to the model at hand (Chernov et al., 2021).

Our test is simple. The loading on the deviations of a given test asset price from the target price implied by the mean-variance portfolio should be zero when the factor model is well specified, i.e. if factors describe asset return dynamics accurately. On the other hand, the loading should be negative if factors do not span the conditional mean-variance efficient portfolio. The intuition for the negative sign is simple: When an asset’s price is above its fundamental price level (measured by the long-run, cumulative factor return), future asset returns are low, and vice versa.

Our test is formulated in terms of implications for the conditional mean-variance efficient (CMVE) portfolio. To construct the CMVE we use several versions of the Fama-French (2015, FF5) model. As a benchmark case, we employ a standard combination of the FF5 factors based on their unconditional first and second moments. Then, to account for conditioning information about the factors’ mean and volatility in the construction of the mean-variance efficient portfolio, we implement a version with factor timing (Haddad et al., 2020), and a version with volatility timing (Moreira and Muir, 2017). We also use the characteristic-efficient factors of Daniel, Mota, Rottke and Santos (2020) since Kozak and Nagel (2022) show that hedging the unpriced components of heuristic factor returns makes them more likely to span the stochastic discount factor. Finally, we perform several robustness tests by replacing the FF5 factors with the Hou et al. (2015) q -factors in the construction of the mean-variance portfolio.

We refer to the difference between cumulative (log) asset returns and the cumulative

returns on the mean-variance efficient portfolio (built from one of the factor models described above) as to price deviations. As test assets, we use 90 portfolios from the long and short sides of 45 well-known and widely used characteristic-based strategies (Haddad et al., 2020; Kelly et al., 2020). Independently from how we construct the mean-variance portfolio and from the test asset considered, we show that these deviations forecast future anomaly returns with a negative sign, thus rejecting the restriction from the beta-representation. The negative loading of future portfolio returns on the current price deviation implies that when asset prices are higher (lower) than the long-run price level implied by the factor model, we expect lower (higher) returns in the next period so that the deviations are corrected. Thus, it is natural to interpret the price deviations as the level of under- or over-pricing of a given asset relative to the price implied by the mean-variance portfolio.

The evidence in favor of anomalies' predictability is obtained by taking an out-of-sample perspective, i.e. by constructing price deviations in real time. Also, importantly, our documented predictability already accounts for the possibility that the exposure of a given test asset to the mean-variance portfolio is time-varying. We do so in two ways: by using a classic fixed-length rolling window approach not subject to overconditioning bias (e.g., Fama and French, 1997; Boguth et al., 2011) and by using the non-parametric method proposed by Ang and Kristensen (2012). The latter allows for tighter windows when there is more portfolio variation that can be picked up with greater precision. Despite these attempts, we continue to find evidence of sizable asset return predictability implied by the price deviations.

Averaging across different factor models, we find that a value of the test asset above the target value implied by the mean-portfolio signals future negative returns over the next two to three years, at which point the price deviation is washed away. Interestingly, the long spell of time it takes for returns to revert toward their target value is in line with the evidence in

[Daniel et al. \(2022\)](#) who show that the beliefs of optimistic agents (who overreact to positive information) decay towards rational beliefs over a roughly 5-year period.

Importantly, the predictive content of our price deviations survives after controlling for the test asset’s book-to-market ratio, for the asset momentum or reversal effects captured by the 1- and 5-year past returns, respectively, and for measures of aggregate sentiment ([Baker and Wurgler, 2006](#); [Huang et al., 2014](#)). The result that our price deviations predict returns negatively and survive after controlling for the portfolio reversal based on long-term (5-years) past returns is interesting. After all, our price deviations are obtained from the cumulative past returns *relative* to the cumulative mean-variance efficient ones. Thus, the fact that the price deviations series remains statistically significant after controlling for the (absolute) 5-years past returns, suggests that there is more information content in relative (to a given factor model) mispricing than in absolute mispricing as captured by the stand-alone past return series.

Although the out-of-sample R^2 from a forecasting model is a commonly used metric in the return predictability literature (see, e.g., [Rapach and Zhou, 2022](#)), [Kelly et al. \(2022\)](#) pointed out that it is an incomplete measure of the model economic value. Thus, we also implement a portfolio exercise to quantify the economic magnitude of the documented no-arbitrage rejections. Specifically, we form a long-short portfolio that buys anomalies with the highest one-year-ahead expected returns (decile 10) and sells anomalies with the lowest one-year-ahead expected returns (decile 1) based on the signal provided by the asset price deviations. Such a long-short investment strategy generates an out-of-sample annualized Sharpe ratio of 0.53 and 0.55 when the deviations are relative to the Fama-French five-factor model or to the [Daniel et al. \(2020\)](#) hedged factors, respectively. Thus, the misspecification of the return dynamics in state-of-the-art models of the stochastic discount factors are quantitatively large.

We also verify that the performance of our mispriced portfolio cannot be explained by other factor models, in particular those behavioral models that have been proposed to capture temporary, long- and short-horizon deviations of prices from fundamental values (Daniel et al., 2020), as well as models where factors are constructed to capture aggregate mispricing (e.g., Stambaugh and Yuan, 2016; Bartram and Grinblatt, 2018). We find that our price deviations convey different information from that captured by the Daniel et al. (2020) behavioral factors and from the mispricing factor of Bartram and Grinblatt (2018), as testified by a large and statistically significant alpha induced by our strategy relative to these models. Interestingly, independently of the candidate mean-variance portfolio, the lowest alpha and largest time-series R^2 obtain in correspondence of the mispricing factor model of Stambaugh and Yuan (2016). This suggests that indeed our strategy captures under/over reaction of asset price levels and, to capture such price dynamics, one needs additional mispricing factors outside those included in the candidate SDF model (which we use to infer the target price level).

Price divergences could be related to frictions that prevent rational traders from eliminating such deviations or to irrational behaviour of agents, or both. On the one hand, idiosyncratic risk can be interpreted as an holding cost that makes temporary deviations of market prices from equilibrium prices possible (e.g., Pontiff, 1996, 2006). Transitory price dislocations can also be caused by the limited, and therefore slow-moving, capital of the currently available investors (e.g., Duffie, 2010). On the other hand, temporary price discrepancies can be the consequence of over-reaction to news caused by an exaggeration of probability of states that are objectively more likely (e.g., Bordalo et al., 2019). Thus, inspired by the literature on “diagnostic expectations” (e.g., Bordalo et al., 2019; Gennaioli and Shleifer, 2018), we conclude by linking our predictive framework to a model where the price

deviation captures agents' over-reaction to news in prices that are subsequently corrected in return dynamics.¹

Related Literature. Our analysis builds upon, and relates to, the large empirical literature that studies temporary deviations of asset values from fundamentals. In an early contribution, [Poterba and Summers \(1988\)](#) find positive autocorrelation in returns over short horizons and negative autocorrelation over longer horizons which can be explained by persistent, but transitory, divergences between prices and fundamental values. Concurrently, [Fama and French \(1988\)](#) argue that the observed U-shaped pattern of the regression slope from forward h -period industry returns $r_{t,t+h}$ on past returns $r_{t-h,t}$ is consistent with the view that prices have a slowly decaying stationary component. Our finding that the deviations of a portfolio price from a given factor model forecast the portfolio returns is consistent with the permanent-transitory decomposition of prices proposed by [Fama and French \(1988\)](#).

Importantly, our evidence is that price deviations forecast reversal, not continuation, of returns. Thus our evidence complements that in [Ehsani and Linnainmaa \(2021\)](#) about positive auto-correlations in anomalies.

Recently, [Dong et al. \(2021\)](#) show that returns of the short- and (to a lesser extent) the long-leg of anomaly portfolios are positively related to the next period's market return. To explain this finding, the authors also exploit the permanent-transitory decomposition for the prices of an anomaly portfolio. However, their approach does not require direct computation of the transitory component of prices. Differently, we provide a framework to compute the

¹Behavioral models in which investors overreact to, e.g., news about firms' prospects have a long tradition; see, e.g., [Barberis et al. \(1998\)](#), [Daniel et al. \(1998, 2001\)](#) and [Gervais and Odean \(2001\)](#). We choose diagnostic expectation as a modeling framework since it yields overreaction to not just private but also public information, unlike model of investor overconfidence ([Daniel et al., 1998](#)) where decision makers exaggerate the precision of private information.

transitory price component, and use it to time characteristics-sorted portfolio returns.

Our paper is related to a recent and rapidly growing literature that aims at explaining multi-period (cumulative) portfolio returns and portfolio price level.² The paper closest to ours is [Chernov et al. \(2021\)](#). These authors propose to use multi-horizon returns to test over-identifying restrictions of a given factor model. Using their novel test, [Chernov et al. \(2021\)](#) find that popular factor pricing models are unable to price their own factors at multiple return horizons even when one allows for state-of-the-art SDF sensitivities. We share a similar interest in (misspecification of) conditional dynamics. The conditional model misspecification documented in our paper is complementary to that analyzed by [Chernov et al. \(2021\)](#). Whereas [Chernov et al. \(2021\)](#) focus on the pricing of factors at multiple horizons, we instead test for misspecification in the risk-adjusted *short-run* dynamics of a test asset by exploiting information in *long-run* (cumulative) asset and factor returns.

Our paper contributes to the debate on factor timing ([Cohen et al., 2003](#); [Haddad et al., 2020](#); [Baba-Yara et al., 2020](#)). In particular, we provide evidence that price deviations predict a vast array of portfolio returns. Also, we show that such predictability derives naturally from portfolio prices being anchored to factor prices.

Our paper is also related to the literature that aims at linking the time-series and cross-sectional predictability. For example, [Maio and Santa-Clara \(2012\)](#) and [Boons \(2016\)](#) employ the I-CAPM to study the consistency between time-series and cross-sectional behavior of state variables and factors. [Lettau and Pelger \(2020\)](#) discuss the tension between the time-series and cross-sectional objectives when designing a factor model. We contribute to this important discussion by showing that cross-sectional models should incorporate the in-

²See, e.g., [Cohen et al. \(2009\)](#), [Brennan and Wang \(2010\)](#), [Keloharju et al. \(2019\)](#), [Baba-Yara et al. \(2020\)](#), [Hendershott et al. \(2020\)](#), [Van Binsbergen and Opp \(2019\)](#), [Cho and Polk \(2020\)](#), and [Boons et al. \(2021\)](#).

formation in the limit multi-period returns (i.e. prices) in order to capture portfolio returns time-series dynamics.

[Kozak and Nagel \(2022\)](#) study under which conditions the factors constructed with heuristic methods (in particular, OLS factors and dimension-reduction methods) span the conditional mean-variance frontier.

Recently, [Lopez-Lira and Roussanov \(2022\)](#) show how to construct a portfolio that exploits individual stock return predictability while hedging all undiversifiable risk; they document that such portfolio delivers a Sharpe ratio above one. Similar to them, our paper challenges the notion of a trade-off between systematic risk and expected returns. Whereas [Lopez-Lira and Roussanov \(2022\)](#) exploit a wide range of characteristics to forecast stock returns, we instead show how to construct a predictor that is endogenous to the factor model under scrutiny, and link this predictability to the conditional pricing ability of the model.

Finally, despite the popularity of factor models in asset pricing (e.g., [Ang, 2014](#)), the literature on the relationship between the choice of factors and the investment horizon is less developed. Specifically, the factor-based approach to portfolio allocation and risk management has concentrated almost exclusively on modeling one-period returns, devoting less attention to the long-run relation between the performance of assets and factors.³ In this paper, we propose a methodology that exploits long-horizon returns to test the short-run dynamic properties of asset pricing models.

³[Hansen and Scheinkman \(2009\)](#) and [Backus et al. \(2014\)](#) have developed tools allowing researchers to characterize properties of equilibrium models at different horizons.

2 Mean-variance returns, prices and predictability

Let R_{t+1}^e be the vector collecting the return on asset i in excess of the risk-free rate, R_{t+1}^{ei} .

The conditional mean-variance efficient (CMVE) portfolio is given by⁴

$$R_{t+1}^{mv} = \left(k_t^{-1} V_t (R_{t+1}^e)^{-1} E_t [R_{t+1}^e] \right)^\top R_{t+1}^e, \quad (1)$$

where $V_t (R_{t+1}^e)$ and $E_t [R_{t+1}^e]$ are the conditional first and second moments of excess returns, and k_t is a time-varying scalar, known at time t , governing the leverage of the portfolio.

The no-arbitrage condition

$$E_t [R_{t+1}^e] = - \frac{Cov_t (M_{t+1}, R_{t+1}^e)}{E_t [M_{t+1}]}, \quad (2)$$

implies the conditional beta-pricing representation:⁵

$$E_t [R_{t+1}^e] = \beta_{i,t} E_t [R_{t+1}^{mv}] \quad (3)$$

For any return i included in the portfolio, the validity of Equation (3) requires that in a time series regression of the form:

$$R_{t+1}^{ei} = \beta_{i,t} R_{t+1}^{mv} + \varepsilon_{i,t+1} \quad (4)$$

the error terms should be unpredictable, i.e. $E_t [\varepsilon_{i,t+1}] = 0$ (see, e.g., [Ferson and Harvey, 1991, 1999](#); [Ferson and Korajczyk, 1995](#)). Otherwise, one would buy (sell) the hedged portfolio

⁴See [Hansen and Richard \(1987\)](#); [Ferson and Siegel \(2001\)](#); [Cochrane \(2005\)](#); [Chernov et al. \(2021\)](#). For completeness, we show the derivations of the CMVE portfolio in Appendix A.

⁵Consider the linear SDF $M_t = 1 - b_t (R_{t+1}^{mv} - E_t [R_{t+1}^{mv}])$. Applying the no-arbitrage condition to the minimum variance efficient portfolio we have: $E_t [M_{t+1}] = b_t \frac{Var_t [R_{t+1}^{mv}]}{E_t [R_{t+1}^{mv}]}$. By substituting this expression and $Cov_t (M_{t+1}, R_{t+1}^e) = -b_t Cov_t (R_{t+1}^{mv}, R_{t+1}^e)$ into (2), one obtains the desired expression.

lio $R_{t+1}^{ei} - \beta_{i,t} R_{t+1}^{mv}$ when the error is expected to be positive (negative), making a risk-adjusted profit and violating the fact that the SDF prices conditionally the given asset.

We propose to test for conditional misspecification of the SDF implied by (1) by generating a return predictor that is endogenous to the model (i.e. it depends solely on the candidate CMVE). We start by log-linearizing the Euler condition (2):⁶

$$E_t r_{i,t+1}^e + \frac{1}{2} \text{Var}_t r_{i,t+1}^e = \beta_{i,t} E_t r_{t+1}^{mv} \quad (5)$$

where $r_{i,t+1}^e = r_{i,t+1} - r_{f,t+1}$, and the conditional variance of the risky asset return on the left hand side of (5) is a Jensen's inequality correction that appears because we are working with logs.

Our test for conditional mispecification involves the coefficient δ_i in the following model-implied regression specification

$$r_{i,t+1}^e = c_{i,t} + \beta_{i,t} r_{t+1}^{mv} + \delta_i u_{i,t} + \varepsilon_{i,t+1} . \quad (6)$$

where $c_{i,t}$ is a (possibly time-varying) constant that captures the Jensen's effect. If the portfolio is CMVE and, hence, the associated SDF is correctly specified, one should have $\delta_i = 0$. One has, of course, many choices for $u_{i,t}$. We construct a predictor that is endogenous to the model as follows:

$$u_{i,t} = u_{i,t-1} + \underbrace{\left(r_{i,t}^e - c_{i,t-1} - \beta_{i,t-1} r_t^{mv} \right)}_{\tilde{\varepsilon}_{i,t}} \quad (H_0)$$

i.e. our predictor is the cumulative sum of risk-adjusted returns. To interpret $u_{i,t}$, it is conve-

⁶This expression holds exactly if the SDF and the asset i returns have a joint conditional lognormal distribution.

nient to define the log price of asset i as the cumulative log return: $\ln P_{i,t+1} = \ln P_{i,t} + r_{i,t+1}$. Similarly, we have $\ln P_{mv,t+1} = \ln P_{mv,t} + r_{mv,t+1}$ for the CMVE portfolio, and $\ln P_{rf,t+1} = \ln P_{rf,t} + r_{rf,t+1}$ for the risk-free asset. Note now that if $\beta_{i,t} \simeq \beta_{i,t-1}$, then

$$u_{i,t} = \ln P_{i,t} - \ln P_{rf,t} - \sum c_{i,t} - \beta_{i,t} \ln P_{mv,t} , \quad (7)$$

i.e. $u_{i,t}$ captures deviations of test asset prices from the price warranted by the CMVE portfolio (adjusted for a possible time-varying trend captured by $\sum c_{i,t}$).⁷ The intuition behind the proposed predictor is that if there is persistent mispricing, it will show up in the price level (Shiller, 1981). Equation (7) suggests to compute the mispricing by comparing the portfolio price level to the value implied by the mean-variance portfolio ($\beta_{i,t} \ln P_{mv,t}$).

The interpretation of $u_{i,t}$ as price deviations rest on the assumption $\beta_{i,t} \simeq \beta_{i,t-1}$, i.e. the betas for our portfolios vary slowly and smoothly over time. This assumption is consistent with several economic models. E.g., Gomes et al. (2003) suggest that betas are a function of productivity shocks, which are often calibrated with an autocorrelation of 0.95 at the quarterly horizon. This translates into a monthly autocorrelation of conditional betas above 0.98. Similarly, in Santos and Veronesi (2006), stock betas change as the ratio of labor income to total consumption changes, which is also a highly persistent variable. Also, many

⁷Practically, by cumulating log excess returns on asset i we abstract from any source of long-run nominal comovement between the asset prices and the mean-variance efficient prices. To see this, consider for ease of exposition the CAPM model (the market is always included in the factor models studied in this paper) and constant betas. Under the null, we have: $(r_{i,t+1} - r_{rf,t+1}) = \beta_i (r_{m,t+1} - r_{rf,t+1}) + \varepsilon_{i,t+1}$. Compounding the left- and right-hand side yields:

$$\begin{aligned} \ln P_{i,t+1} - \ln P_{rf,t+1} &= \beta_i (\ln P_{m,t+1} - \ln P_{rf,t+1}) + u_{i,t+1}, \\ \text{or, equivalently,} \\ \ln P_{i,t+1} &= \beta_i \ln P_{m,t+1} + (1 - \beta_i) \ln P_{rf,t+1} + u_{i,t+1}. \end{aligned}$$

The term $(1 - \beta_i) \ln P_{rf,t+1}$ effectively removes inflation-related trends that are common to the market factor and the asset prices.

previous empirical studies (see, e.g., [Jagannathan and Wang, 1996](#); [Lettau and Ludvigson, 2001](#); [Petkova and Zhang, 2005](#); [Lewellen and Nagel, 2006](#); [Ang and Chen, 2007](#); [Pelger, 2020](#); [Lopez-Lira and Roussanov, 2022](#)) find that conditional betas are stable within short time window.

Our test $\delta_i = 0$ is intuitive. In fact, in a correctly specified factor model there is an unpredictable error term $\tilde{\varepsilon}_{i,t}$. This error term leads to a random walk component in test asset prices, when prices are defined as cumulative returns. Hence, under the null of the model, $u_{i,t}$ in (H_0) is a martingale and the deviation of an asset price $\ln P_{i,t}$ from the price implied by the mean-variance portfolio are permanent. This implies that price deviations should not forecast risk-adjusted excess returns ($E_t[u_{i,t+1} - u_{i,t}] = E_t[\tilde{\varepsilon}_{i,t+1}] = 0$).

The alternative hypothesis, which we entertain in this paper, is that these price deviations are persistent instead. To be specific, we assume that the price deviations are mean reverting:

$$u_{i,t} = \rho_i u_{i,t-1} + \tilde{\varepsilon}_{i,t} \tag{H_1}$$

which implies $\delta_i = \rho_i - 1 < 0$ in Eq. (6).⁸ In words, if asset prices are above the target value implied by the mean-variance portfolio price, and if these price deviations are persistent but mean-reverting (i.e. $\rho_i < 1$), then future expected returns are lower (higher) on a risk-adjusted basis (i.e. after controlling for $\beta_{i,t} r_{t+1}^{mv}$).

Finally, note that under the null (H_0) , $u_{i,t}$ is a martingale, i.e. $\rho_i = 1$ and $\delta_i = 0$ in Eq. (6). Thus one can view (H_1) as the unrestricted model, and (H_0) as the restricted model.

⁸The result obtains by first differencing $u_{i,t} = (\ln P_{i,t} - \ln P_{rf,t}) - \sum c_{i,t} - \beta_{i,t} \ln P_{mv,t}$ (see Eq. (7)), using the autoregressive dynamics for $u_{i,t}$ under (H_1) and our definition of log prices as cumulative log returns, and using the assumption that betas are changing slowly over time, i.e. $\beta_{i,t+1} \simeq \beta_{i,t}$ which implies $\beta_{i,t+1} \ln P_{mv,t+1} - \beta_{i,t} \ln P_{mv,t} \simeq \beta_{i,t} r_{t+1}^{mv}$.

2.1 Construction of Price Deviations

A large literature (e.g., [Goyal and Welch, 2007](#), [Rapach and Zhou, 2013](#), [Martin and Nagel, 2020](#), [Boudoukh et al., 2021](#)) documents that out-of-sample tests provide the most rigorous and relevant evidence on stock return predictability. Therefore, to construct our predictor $u_{i,t}$ and to test the null $\delta_i = 0$ in (6), we take an out-of-sample perspective. In particular, we show that the price deviations captured by $u_{i,t}$ can be exploited in real time to predict asset returns.

First, following e.g., [Fama and French \(1997\)](#) and [Ferson and Harvey \(1999\)](#), we estimate the conditional betas using a regression over a 60-month rolling window:⁹

$$r_{\tau+1}^e = c_{i,t} + \beta_{i,t} r_{\tau+1}^{mv} + \varepsilon_{\tau+1}, \quad \tau = t - 60 : t - 1. \quad (8)$$

We then construct the risk-adjusted return at time $t + 1$ as:

$$\widehat{\varepsilon}_{i,t+1} = r_{i,t+1}^e - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r_{t+1}^{mv}$$

where the beta and the constant are obtained from the rolling window regression (using information up to time t only, as denoted by the subscript). We repeat this same steps at time $t + 2$ and construct $\widehat{\varepsilon}_{i,t+2}$ based on betas (and constant) from a rolling regressions over

⁹Using rolling windows to estimate conditional loadings gets around the problem of instrumenting time-varying factor loadings with the “right” state variables (e.g., [Shanken, 1990](#); [Jagannathan and Wang, 1996](#); [Lettau and Ludvigson, 2001](#). See also discussion in [Lewellen and Nagel, 2006](#)). As we use a rolling window of 60-months, our conditional betas estimates are not subject to overconditioning bias ([Boguth et al., 2011](#)). While the choice of the rolling window length is arbitrary, we document that our results hold when using the optimal nonparametric technique developed in [Ang and Kristensen \(2012\)](#) to estimate time-varying betas.

the period $t - 60 + 1$ to t . Our predictor is given by:

$$\widehat{u}_{i,t} = \sum_{\tau=0}^t \widehat{\varepsilon}_{i,\tau} \tag{9}$$

and, importantly, it is obtainable in real time. We then run the predictive regression

$$r_{i,t+1} - r_{f,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r_{t+1}^{mv} = \delta_i \widehat{u}_{i,t} + \epsilon_{i,t+1} . \tag{10}$$

and test the null hypothesis $\delta_i = 0$ in Section 3.2. A rejection of the null, and in particular a negative δ_i , suggests that underpricing ($u_{i,t} < 0$) is followed by positive returns. We exploit this insight, and the fact that $u_{i,t}$ can be obtained in real time, to develop a trading strategy based on mispricing in Section 3.3. As we will see, the price deviations take time to be reabsorbed, which implies that our trading strategy does not require high-frequency rebalancing, reducing possible concerns about trading costs.

Note that our empirical analysis is using conditional betas since, as shown by Hansen and Richard (1987), assuming constant betas is not innocuous. For example, with constant betas, price deviations could simply be a byproduct of time-varying loadings. Although our benchmark approach employs betas estimated over a rolling window, Appendix E.2 repeats our analysis when we estimate time-varying betas using the kernel method proposed by Ang and Kristensen (2012). The advantage of this method is that it allows the bandwidth of the kernel to vary across portfolios, i.e. to use tighter windows when there is more variation to be picked up with greater precision. Importantly, we will see that our findings continue to hold when we use this alternative approach.

To summarize, our assumptions lead to the following approach to test for conditional

misspecification and engage in anomaly timing:

1. Start from a factor model.
2. Construct the CMVE portfolio R_{t+1}^{mv} given in equation (1).
3. Estimate using a rolling window equation (8) to construct risk-adjusted returns $\widehat{\varepsilon}_{t+1}$ and price deviations \widehat{u}_t .
4. Run the predictive regression (10). An estimate of $\delta \neq 0$ leads to a rejection of the null (H_0), thus implying that the model is misspecified.

3 Empirical Results

3.1 Data

Our analysis focuses on characteristics-based factors. Specifically, we implement the mean-variance efficient portfolio using the following factor model representation:

$$R_{t+1}^{mv} = b_t^\top C_t R_{t+1}^e = b_t^\top \mathbf{f}_{t+1} , \quad (11)$$

where C_t is a $K \times N_t$ matrix of stock-level characteristics which define a set of K factors, $\mathbf{f}_{t+1} = C_t R_{t+1}^e$; and b_t is a $K \times 1$ timing vector that optimally combines these factors over time to get to the minimum variance portfolio (see, e.g., [Haddad et al., 2020](#); [Moreira and Muir, 2017](#)). Theoretically, the variation in the minimum variance portfolio weights must be driven by factor and volatility timing: $b_t \propto V_t(\mathbf{f}_{t+1})^{-1} E_t[\mathbf{f}_{t+1}]$. We use the [Fama and French \(2015, FF5, henceforth\)](#) as factors, i.e. $\mathbf{f}_t' = [MKT_t \text{ SIZE}_t \text{ HML}_t \text{ RMW}_t \text{ CMW}_t]$

in equation (11), and entertain a version of FF5 with either factor return (factor-timing, henceforth) or volatility timing (vol-timing, henceforth). Mindful that standard factors are contaminated with unpriced components,¹⁰ we also employ the hedging approach of Daniel, Mota, Rottke and Santos (2020, DMRS) that aims at removing unpriced risks from the original factors. We call the residualized (with respect to the hedge portfolio returns) FF5 factors, FF5-DMRS.¹¹ This gives a total of four candidate SDFs. In addition, in Appendix D we repeat our main analysis when we use the Hou et al. (2015) q -factors to construct the mean-variance efficient portfolio.

Given our factors (e.g., the FF5), we estimate b such that the single-horizon monthly returns to the factors themselves are priced without error. For the volatility timing version, we follow Moreira and Muir (2017) and use $b_{i,t} = b_i V_t^{-1} (f_{i,t+1})$ which is computed using squared realized daily factor returns. For the factor timing version, we follow Haddad et al. (2020) and use $b_{i,t} = b_i E_t (f_{i,t+1})$ where the out-of-sample expectations for the factors are constructed using each factor's book-to-market ratio. In all cases, we estimate the constants of proportionality b_i for each factor i by matching the in-sample average returns to the timed factors in the model at hand, analogous to how we estimated the vector b in the baseline FF5 models.

We focus on U.S. data—NYSE, AMEX, and Nasdaq stocks from the Center for Research in Security Prices (CRSP) and Compustat data required for sorting – for the sample 1967–2019. Throughout we use monthly observations but we focus on 1-year holding-period excess return. Indeed, the holding period for asset returns must be sufficiently long to allow for a reaction of returns at time $t+1$ to the asset price deviations from the mean-variance portfolio

¹⁰For example, Gerakos and Linnainmaa (2017) find that the HML value factor is contaminated with unpriced components.

¹¹We are grateful to Simon Rottke for sharing the up-to-date hedged FF5 factors.

prices at time t . Without further qualification, r_{t+1} will always denote the one-year-ahead log excess returns.¹² However, we repeat the relevant tests also with monthly returns.

To investigate the validity of a given SDF, we consider as test assets a large cross-section of anomaly portfolios based on single-sorts of 45 different characteristics. These test assets, or a subset of it, have been used by [Kozak et al. \(2020\)](#), [Kelly et al. \(2020\)](#), [Haddad et al. \(2020\)](#), and [Lettau and Pelger \(2020\)](#), among others.¹³

3.2 Conditional Mispecification and Price Deviations

To test for the conditional validity of a given SDF, we run the following predictive regression:

$$\tilde{r}_{i,t+1} = a + \delta \hat{u}_{i,t} + \varepsilon_{i,t+1} \quad (12)$$

where $\tilde{r}_{i,t+1} = r_{i,t+1} - r_{f,t+1} - \hat{c}_{i,t} - \hat{\beta}_{i,t} r_{mv,t+1}^{mv}$ is the log excess return of test asset i at time $t+1$ net of the exposure to the log return on the mean-variance efficient portfolio $r_{mv,t+1}$; and $\hat{u}_{i,t}$ measures the deviations of asset i prices from the mean-variance portfolio ones. The null is $H_0 : \delta = 0$ against $H_1 : \delta < 0$.

We start by describing the properties of the price deviations $u_{i,t}$. In particular, [Table 1](#) shows the half-life ([Panel A](#)) and the magnitude of $u_{i,t}$ ([Panel B](#)) across our test assets

¹²In our sample (see discussion of [Figure 2](#)) there is statistical evidence in favor of return predictability for one- up to twenty month ahead, i.e. $\hat{u}_{i,t}$ manifests forecasting ability for $r_{t+h/12}$ with $h = 1, \dots, 20$. We leave open the question of the economic determinants of the timing of return reaction to price deviations, and decide to focus on 1-year holding-period returns in line with recent empirical studies on time-variation in anomaly returns (e.g., [Lochstoer and Tetlock, 2020](#)) and on the dynamics of equity portfolios (e.g., [Kelly, Kozak and Giglio, 2020](#)).

¹³We thank Serhiy Kozak for making his data available at <https://sites.google.com/site/serhiykozak/data?authuser=0>. [Appendix Table C.1](#) lists the categories and the portfolios included in each category.

$i = 1, \dots, 90$. The price deviations are persistent but mean-reverting with an average half-life of about 2.5 years for all the models considered. Comparing the FF5 model to its timed or hedged versions, we observe very similar half-life distributions. In particular, for all models prices *temporarily* drift away from their mean-variance target level. Panel B, investigates the size of the price deviations which we proxy with the volatility of $u_{i,t}$. All models display price-deviations that are economically sizable, with a volatility of about 20% on average. Finally, in Panel C, we observe that price deviations from the FF5 SDF have a high correlation of 0.8 or more with those obtained from its factor-timed or hedged versions.

Table 2, column (1), shows the results from the pooled regression. Each panel refers to a different candidate SDFs, namely the FF5 model, its factor timed and volatility managed versions, and the FF5 residualized with respect to the DMRS hedge portfolio. Independently from the candidate SDF, we find a negative and statistically significant coefficient on the price deviations. The coefficient is economically large: for example, for the FF5-DMRS we find that a 1% positive deviation of (log) portfolio prices from the model-implied SDF value, lead to an expected return that is lower by 27 bps over the next year (on average, across portfolios). Also, note that the R^2 associated with the predictability induced by the price-deviations are about 10%, or larger, and thus comparable to the R^2 found in the aggregate market return predictability literature (e.g., Cochrane, 2008, 2011).

Figure 1 reports the asset specific $\hat{\delta}_i$, along with its standard error, obtained from estimating equation (12) for each top decile portfolio. Figure E.1 displays the analogous analysis for each bottom decile portfolio. The figure shows that $\hat{\delta}_i$ is negative and significantly different from zero for all the portfolios and all the SDFs considered. Hence, the evidence points to an ubiquitous rejection of the null in favor of price deviations that are persistent but mean reverting ($0 < \rho_i < 1$). Appendix Figure E.2 shows similar conclusions when we use

nonparametric conditional beta as in [Ang and Kristensen \(2012\)](#).

Next, we discuss the predictive ability of price deviations over alternative horizons. Recall that so far we have used annual returns in equation (12). Figure 2 shows the estimates of δ from a pooled regression when we forecast h -period ahead monthly returns, with $h = 1, \dots, 60$ (i.e. returns are not compounded). For ease of exposition, we multiply the estimated coefficients by twelve so to make their magnitude comparable to the coefficient reported in Table 2 (which is based on annual returns). Across all models, there is statistical evidence in favor of return predictability for each of the future twenty months. Moreover, the magnitude of the coefficient is negative, similar across models, and decaying to zero only slowly as we increase the forecasting horizon. Comparing different SDFs, we observe that the price deviations from the FF5 and FF5-DMRS predict returns quite persistently, for up to forty months. On the other hand, the adoption of factor timing and, to a lesser extent, volatility management make the price deviations more transient as confirmed by a faster decay pattern in the coefficients, which become insignificant between 1.5 and 3 years. In sum, a value of the test asset above the target value implied by the mean-portfolio signals future negative returns over the next two to three years (in line with the average half-life reported in Table 1), at which point the price deviation is washed away. Interestingly, the long spell of time it takes for returns to revert toward their target value is in line with the evidence in [Daniel et al. \(2022\)](#) who show that the beliefs of optimistic agents (who overreact to positive information) decay towards rational beliefs over a roughly 5-year period.

Recall that our price deviations signal is obtained in real time, using only information up-to-time t . Thus, we now evaluate its predictive ability for each portfolio using the out-of-sample (OOS) R^2 metric proposed by [Campbell and Thompson \(2008\)](#). Table 3 shows the results for each test asset. Each panel refers again to a given SDF model. On average

(across models), we document positive OOS R^2 for more than 70% of anomaly portfolios. Most importantly, we find significant out-of-sample R^2 for relevant characteristics-sorted portfolios such as value, duration, and investment. In Appendix Table D.2, we confirm these results for alternative factor models. Appendix Table E.1 shows similar, and sometimes stronger, results when we estimate time-varying betas with the non-parametric [Ang and Kristensen \(2012\)](#) approach. Although the out-of-sample R^2 from a forecasting model is a commonly used metric in the return predictability literature (see, e.g., [Rapach and Zhou, 2022](#)), [Kelly et al. \(2022\)](#) pointed out that it is an incomplete measure of the model economic value. Thus, in Section 3.3 we implement a portfolio exercise to quantify the economic profits of a market timer that exploits the price deviations implied by a given SDF model. Before doing so, in the next subsection we make sure that the predictive power of the price deviations is not subsumed by well known predictors.

3.2.1 The information content of price deviations

Our predictor $u_{i,t}$ is endogenous to the model: it accounts for the conditioning information (characteristics and possible timing variables) used in the construction of the SDF, and it allows to test conditional aspects of the model, namely the dynamics of future returns. However, one may wonder how it relates to other portfolio return predictors.

To address this question, we run the following pooled regression:

$$\tilde{r}_{i,t+1} = a + \delta \hat{u}_{i,t} + \gamma X_{i,t} + \epsilon_{i,t} \quad (13)$$

where $X_{i,t}$ is an alternative candidate predictor for the anomaly portfolio i . Columns (2) to (5) of Table 2 show the estimates when we control for (the portfolio) long-term reversal, past

one-year returns, the book-to-market ratios, and aggregate sentiment as measured by the [Baker and Wurgler \(2006\)](#) investor sentiment index. Each panel refers to a specific SDF. We focus on results from the pooled regression only for ease of exposition, but all our conclusions hold when we run asset-specific individual regressions.

[Insert Table 2 about here]

In column (2) we consider the reversal signal based on past 5-year returns (skipping the most recent year) as an additional anomaly portfolio predictor. After all, our price deviations are obtained from the cumulative past returns relative to the cumulative mean-variance efficient ones. We see that the series of past returns *relative to* the mean-variance portfolio remains statistically significant after controlling for the (absolute) 5-years past return series. Moreover, the loading $\hat{\delta}$ is always negative and of similar magnitude to the value reported in column(1). This result suggests that there is more information content in relative (to a given factor model) mispricing than in absolute mispricing as captured by the stand-alone past return series.

In column (3), we report results for regression (13) when we include the portfolio’s performance over the prior year from month $t - 12$ to $t - 1$ along with the price deviations. In a recent paper, [Ehsani and Linnainmaa \(2021\)](#) document that most factors are positively autocorrelated, and propose a factor that bets on the continuation in factor returns. Contrary to their work, our framework focuses on price deviations that forecast reversal, not continuation, of returns. It is then not surprising to see that our price deviations (capturing reversal) continue to be statistically significant after controlling for the portfolio momentum (capturing continuation).

Column (4) reports results for regression (13) when the control variable $X_{i,t}$ is the port-

folio’ book-to-market ratio. Indeed, valuation ratios are often used in return forecasting regressions (e.g., [Cochrane, 2005](#); [Campbell, 2017](#)) as they represent a natural predictor according to the Campbell-Shiller ([1988](#)) log-linear present value model. Even after controlling for the book-to-market ratio, the coefficient on the price deviations is statistically significant, and negative: 1% positive price deviations for the test portfolio in this period imply an expected return that is, over the next year, lower by about 22 bps.

Finally, [Shen et al. \(2017\)](#) document a negative predictive relation between the returns to portfolios sorted on macro-related risk factors and investor sentiment proxied by [Baker and Wurgler \(2006\)](#) index. Related, [Avramov et al. \(2019\)](#) show that mispricing occurs across financial distressed firms during periods of high market sentiment because in these times both retail and institutional investors are overly optimistic about the likelihood and consequences of financial distress. The sluggish investors’ response to correct overpricing leads to a wide range of anomalies in the cross-section of stocks and bonds. Column (4) of [Table 2](#) displays the results from a predictive regression that controls for sentiment. Once again, we find that the predictive content of the price deviations is not driven away by aggregate sentiment. This result continue to hold true when we use the improved aggregate sentiment of [Huang et al. \(2014\)](#).

Overall, our evidence suggests that price deviations convey information about the time-series dynamics of risk-adjusted returns $r_{i,t+1} - \beta_i r_{mv,t+1}$ for a wide range of portfolios. The predictive informative content of these price deviations is not subsumed by valuation ratios, momentum or reversal in individual factors, or aggregate sentiment.¹⁴

¹⁴Appendix [Table D.1](#) confirms these conclusions hold when we employ deviations of asset prices from the mean-variance portfolios implied by the HXZ factors or by their volatility managed version. In particular, for all these models, we find a negative and significant loading controlling for well-known predictors.

3.3 Return Dynamics, Mispricing, and Trading Strategy

In this section, we study the performance of a strategy that goes long portfolios with prices below the model-implied target ($u_{i,t} < 0$), and goes short those with prices higher than what the mean-variance portfolio would suggest ($u_{i,t} > 0$). We proceed as follows. First, recall that the term $u_{i,t}$ measures the deviation of the portfolio price i from the mean-variance target price, and it is obtained in real-time using only information up to time t . We focus on a large cross-section of anomaly portfolios based on single-sorts of 45 different characteristics (see [Kozak et al., 2020](#)), for a total of 90 portfolios. We sort these 90 anomaly portfolios in deciles once per year (in December) according to the portfolio-specific price deviation, $u_{i,t}$. We then hold the position in the top and bottom deciles for one year, at which point we repeat the sorting procedure.

Figure 3 shows the performance of the top and bottom deciles sorted on the price deviations, along with the aggregate market returns. The top left panel refer to the results obtained when we compute price deviations relative to the FF5 mean-variance portfolio.¹⁵ The next two panels refer to the results for the factor and volatility timed version of the FF5. The bottom right panel refers to DMRS hedged version of FF5. As expected, the long leg which contains underpriced test assets outperforms the market, whereas the short leg with overpriced portfolios underperforms. A strategy that goes long underpriced test assets and short overpriced assets generates an annualized average excess return of 3.2% and 3.4% for the FF5 model and its version that hedges unpriced risks. The associated annualized Sharpe Ratios are 0.53 and 0.55, respectively. A version of the mean-variance portfolio that times factors' returns obtains even stronger performance with annualized average excess return of

¹⁵Appendix Figure E.3 shows the performance of the strategy based on the FF5 mean-variance portfolio and nonparametric conditional betas ([Ang and Kristensen, 2012](#)).

4.3% and a Sharpe ratio of 0.66. The performance of a strategy based on deviations from a volatility-timed mean-variance portfolio attains an average return of 3.6% and a Sharpe ratio of 0.61.

It is important to understand whether our mispriced portfolio display alphas relative to factor models, in particular those behavioral models aiming at capturing temporary, long- and short-horizon deviations of prices from fundamental values (Daniel et al., 2020), as well as models where factors are constructed to capture aggregate mispricing (e.g., Stambaugh and Yuan, 2016; Bartram and Grinblatt, 2018). Table 4 shows exposures to standard factors of our long-short strategy based on model-implied price deviations, along with its alpha. Several observations stand out. First, the average return of such strategy is not captured by standard characteristics-based or behavioral factors. In particular, our price deviations convey different information from that captured by the Daniel et al. (2020) behavioral factors and from the mispricing factor of Bartram and Grinblatt (2018): In correspondence of these two models (see rightmost two columns of Table 4), we observe that the constant remains large and statistically, for every panel (i.e. independently of the factors used to construct the mean-variance portfolio). Second, we observe that, independently of the candidate SDF, the lowest alpha and largest R^2 obtain in correspondence of the mispricing factor model of Stambaugh and Yuan (2016). This suggests that indeed our strategy captures under/over reaction of asset price levels and, to capture such price dynamics, one needs additional mispricing factors outside those included in the candidate SDF model (which dictates the target price level). For example, when we use the DMRS FF5 factors to obtain the mean-variance portfolio (see Panel (d)), we observe a sizable R^2 of about 30% in correspondence of the Stambaugh and Yuan (2016) model, which is larger than that obtained after adjusting for “rational” model like the q -factor one. The alpha remains nevertheless significant at 1.8%

per year despite a positive and statistically significant loading on the MNGT and PERF factors of [Stambaugh and Yuan \(2016\)](#).

We provide a battery of robustness tests. Using our cross-section of 45 characteristics, and the underlying 90 portfolios from the long and short sides of these strategies, we show in [Table D.3](#) the performance of our strategy when the price deviations are computed relative to either an SDF that employs the q -factors of [Hou et al. \(2015\)](#) (Panel (a)) or a volatility-timed version of the same q -model (Panel (b)). The annualized return and Sharpe ratio of the strategy which uses deviations of prices from the q -factors are 3.7% and 0.63. Importantly, the alpha remains significant after controlling for behavioral as well as other prominent factors proposed in the literature. [Table E.2](#) shows, for the case of the FF5 mean-variance portfolio, the performance of our strategy when the time-varying exposures are computed with the non-parametric approach of [Ang and Kristensen \(2012\)](#). Note that this approach adjusts the length of the window (over which to compute betas) based on how much variation there is in portfolio betas. For example, the growth portfolio does not exhibit much variation in beta so the window estimation procedure picks a long bandwidth, corresponding to (a windows of about) 60 months. In contrast, we find significant time variation in beta for the value portfolio and the procedure picks a relatively tighter windows that allow this variation to be picked up with greater precision. Despite this more challenging set-up, we confirm the presence of statistically significant alphas for our strategy that longs portfolio with negative price deviations, and shorts portfolio with positive price deviations. [Table E.3](#) shows the performance of a rank-based strategy that invest in all portfolios rather than just the top and bottom quintiles.¹⁶ Despite the fact that now we take less extreme positions, the alphas

¹⁶Simply using ranks of the signals as portfolio weights helps mitigate the influence of outliers. Specifically, the weight on portfolio i at time t is: $w_{i,t} \propto \text{rank}(u_{i,t}) - \sum_i \text{rank}(u_{i,t})/N$.

remain economically large and statistically significant. For example, when we use the FF5 factors to compute the target price level, rank-weighted strategy has a Sharpe ratio of 0.65 and an annualized alpha of 2.4% (relative to the [Stambaugh and Yuan \(2016\)](#) model). Finally, Appendix Table [E.4](#) repeats our analysis when we rebalance our portfolio monthly. In this case, the strategy that uses asset value deviations from the FF5 mean-variance portfolio has a Sharpe ratio of 0.57 and an annualized alpha of 3% relative to the [Stambaugh and Yuan \(2016\)](#) model.

Overall, our analysis suggests that the deviations of a portfolio price from its long-term level implied by the mean-variance contain timely information to predict anomaly returns out-of-sample. Our conclusions is robust to alternative factors used to construct the SDF (HXZ or FF5-factor models, and their timed version), to the use of non-parametric procedure for the computation of the time-varying exposures, to the universe of test assets used, and to alternative ways to construct the strategy.

[Insert Figure [3](#) and Table [4](#) about here]

4 Discussion

4.1 A Statistical Interpretation

In their seminal contribution, [Fama and French \(1988\)](#) argue that the (log of) stock price, $\ln P_{i,t}$, is composed of two parts: a permanent component $q_{i,t}$, modeled as a random walk with drift, and a temporary component $u_{i,t}$, modeled as a stationary AR(1) process,

$$\ln P_{i,t} = q_{i,t} + u_{i,t} \tag{14}$$

$$q_{i,t} = q_{i,t-1} + \alpha_i + \eta_{i,t}$$

$$u_{i,t} = \rho_i u_{i,t-1} + v_{i,t}$$

where $\eta_{i,t}$ and $v_{i,t}$ are independent processes with zero mean and constant variance and $|\rho_i| < 1$. [Fama and French \(1988\)](#) argue that the slowly mean reverting temporary component induces predictability in returns.

It is easy to map our alternative hypothesis (H_1) in the [Fama and French \(1988\)](#) framework: just assume that the permanent component for the (log of) stock price is $q_{i,t} = \beta_i \ln P_{mv,t}$; i.e. the permanent component is common across assets. Thus, our analysis uncovers the return predictability induced by the deviation of asset prices from their (common) permanent trend captured by the mean-variance portfolio.

4.2 An Economic Interpretation

In the introductory session of our paper we have stated the economic mechanism behind the return predictability from price deviations could be generated by either slow adjustment of prices to new information or by the presence of Diagnostic Expectations. In this section we develop these two alternative interpretations in the light of our empirical results.

Consider first the case of slow adjustment of prices to new information as considered in the model proposed by [Amihud and Mendelson \(1987\)](#). Let P_t be the observed log asset price, and V_{t+1} its intrinsic value. Prices adjust slowly towards their intrinsic value; specifically, P_t

evolves according to the following dynamics:

$$P_{t+1} = P_t + k(V_{t+1} - P_t) \quad (15)$$

where k is a parameter controlling the adjustment of prices towards the asset intrinsic value. If the adjustment parameter satisfies $0 < k < 1$, then the observed asset price P_t adjusts slowly to the fundamental price V_t :

$$P_{t+1} = kV_{t+1} + (1 - k)P_t \quad (16)$$

In our language, V_t is the price of the CMVE portfolio and the difference $V_{t+1} - V_t = R_{t+1}^V$ is the CMVE portfolio return. For $0 < k < 1$, Eq. (16) describes the dynamics of a security that manifests temporary deviations from its intrinsic value.

We calibrate R_t^V to the CMVE portfolio return constructed using the [Fama and French \(2015\)](#) five-factor model over the period 1967–2019. Specifically, R_t^V is normally distributed with an annualized mean of 1.23% and an annualized volatility of 1.12%. The price vector is constructed as $V_{t+1} = V_t + R_{t+1}^V$. We then simulate a sample of 636 observations of P_{t+1} using equation (16). Using simulated prices, we construct returns. Then, we run regressions (12) and store the estimated δ . We repeat the simulation 10'000 times.

Figure [F.1](#) reports the distribution of δ for three different calibrations of the adjustment parameter. For $k = 0.5$ (top panel), the simple partial-adjustment model features a significant and negative coefficient on price deviations. The average δ is about -0.25 , which is comparable to the mean value across the 90 anomaly portfolios reported in [Table 2 Panel A](#). As the adjustment parameter gets closer to one (i.e. full price adjustment to information),

price deviation loadings get closer to zero (c.f., bottom panel with $k = .95$). Indeed, the extreme case of an economy without slow adjustments (i.e. $k = 1$), features a δ centered exactly at zero.

Our empirical evidence could also be interpreted with an expectation formation mechanism close to a model of belief formation based on representativeness heuristic where temporary discrepancies are a consequence of over-reaction to news (e.g., [Bordalo et al., 2019](#)). Diagnostic Expectations have been proposed to model transitory deviations from Rational Expectations for stationary univariate processes. Agents with Diagnostic Expectations extrapolate into the future current news about the generic univariate process of interest, x_t :

$$E_t^\theta [x_{t+1}] = E_t [x_{t+1}] + \theta [E_t [x_{t+1}] - E_{t-1} [x_{t+1}]], \quad (17)$$

where the parameter θ controls the size of the deviations from rational expectations. Expectations are diagnostic when $\theta > 0$, and are rational when $\theta = 0$. Diagnostic Expectations converge to Rational Expectations in the long-run but in the short-run current news in x_t (e.g., excess returns) are extrapolated into the future. Our framework in [Section 2](#) is related albeit with important differences. First, our framework is bi-variate, i.e. it requires a description of a given test asset and of the mean-variance portfolio. Second, our framework requires non-stationary processes for the prices (of the individual asset and of the mean-variance portfolio). In particular, expectations at time t for asset returns at time $t+1$ depend on their expectations conditional upon the returns on the CMVE portfolio and on the deviations of prices from their expectations conditional upon the price of the CMVE portfolio at time t :

$$E_t [r_{i,t+1}] = E_t [r_{i,t+1}|r_{mv,t+1}] + \delta_i (\ln P_{i,t} - E [\ln P_{i,t} | \ln P_{mv,t}]), \quad (18)$$

where negative values for δ_i imply that returns increase (decrease) when prices are below (above) their conditional expectations at time t . In other words, while prices are non stationary, the deviations of prices $\ln P_{i,t}$ from their projection on the minimum-variance portfolio prices ($E[\ln P_{i,t} | \ln P_{mv,t}]$) are temporary (i.e. stationary); in this sense, the price deviations play a similar role as the news term in the DE framework (17).

5 Conclusion

Standard asset pricing theory establishes that risk-adjusted returns should be unpredictable. Instead, this paper documents that deviations of portfolio prices from the value dictated by leading factor models predict future risk-adjusted returns. This predictability is endogenous to the model, i.e. it does not need any conditioning variables beyond those used in the construction of the conditional mean-variance efficient portfolio. We also show how such a predictability can be used to test the conditional validity of any given SDF. A real-time strategy that exploits mean-reverting price deviations generates Sharpe ratios of about 0.6. Finally, we show that both an economy featuring slow price adjustment or models of belief formation based on representativeness heuristic can be used to rationalize our empirical evidence. Our results have relevant implications for the practical implementation of asset allocation, risk measurement, and risk management based on the parsimonious factor representation of large cross-sections of assets.

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Tables and Figures

Table 1: Price Deviations: Summary Statistics

This table reports descriptive statistics for price deviations computed using different CMVE portfolios. Price deviations \hat{u} are computed as in equation (9). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak et al. (2020); see Appendix Table C.1 for a description of the anomalies. As factor models to compute the mean-variance efficient portfolios, we employ Fama and French (2015, FF5), its factor return and volatility timed versions, and its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Panel A reports the half-life (in months) distribution of price deviations for each factor model. Panel B reports the standard deviation (in percentage) distribution of price deviations for each factor model. Panel C reports the average correlation across price deviations for each factor model. The half-life is calculated as $\log(0.5)/\log(|\rho|)$, where ρ is the estimated first-order autoregressive parameter for price deviations. Monthly observations. The sample period is 1967 to 2019.

Panel A: Half-Life

Model	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
FF5	30.1	14.4	23.6	27.4	36.1	62.7
factor-timing	34.3	17.0	24.4	32.7	40.4	76.7
vol-timing	30.4	16.6	23.6	28.5	34.3	86.3
FF5-DMRS	30.3	14.1	25.3	28.5	34.3	53.0

Panel B: Standard Deviation

Model	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
FF5	21.0	13.2	17.8	20.5	23.6	33.2
factor-timing	26.3	18.6	23.3	25.9	28.6	34.6
vol-timing	23.6	10.7	20.0	22.9	27.1	39.7
FF5-DMRS	25.2	15.1	21.6	25.2	27.8	40.4

Panel C: Correlation Matrix

	FF5	factor-timing	vol-timing	FF5-DMRS
FF5	1	0.797	0.593	0.841
factor-timing		1	0.272	0.851
vol-timing			1	0.553
FF5-DMRS				1

Table 2: Predicting Anomaly Returns with Price Deviations

This table reports pooled estimates for δ_i from predictive regression (13). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak et al. (2020). Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using different mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Our panel features: $n = 565$, $T = 90$, $N = 50850$. Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: FF5

	(1)	(2)	(3)	(4)	(5)
δ	-0.222*** (0.029)	-0.183*** (0.028)	-0.229*** (0.033)	-0.217*** (0.028)	-0.223*** (0.028)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R ²	0.097	0.131	0.098	0.102	0.128

Panel B: FF5 with Factor Timing

	(1)	(2)	(3)	(4)	(5)
δ	-0.218*** (0.031)	-0.173*** (0.033)	-0.202*** (0.029)	-0.202*** (0.027)	-0.197*** (0.028)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R ²	0.112	0.143	0.116	0.119	0.131

Panel C: Volatility-managed FF5

	(1)	(2)	(3)	(4)	(5)
δ	-0.273*** (0.059)	-0.258*** (0.053)	-0.279*** (0.056)	-0.285*** (0.058)	-0.307*** (0.052)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R ²	0.125	0.219	0.125	0.151	0.203

Panel D: FF5-DMRS

	(1)	(2)	(3)	(4)	(5)
δ	-0.265*** (0.040)	-0.188*** (0.042)	-0.267*** (0.041)	-0.252*** (0.039)	-0.245*** (0.037)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R ²	0.118	0.173	0.118	0.125	0.169

Table 3: Out-of-Sample Predictability

This table reports the out-of-sample R^2 (R_{OOS}^2) for the predictive regression $\tilde{r}_{i,t+1} = a_i + b_i \hat{u}_{i,t} + \epsilon_{i,t}$, where $\tilde{r}_{i,t+1}$ is the test asset i log risk-adjusted return at time $t + 1$ and price deviations \hat{u} are computed as in equation (9). Test assets are the long legs for the 45 anomalies constructed in Kozak et al. (2020). We report results for price deviations computed using different CMVE portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. The R_{OOS}^2 is computed as in Campbell and Thompson (2008); p -values for R_{OOS}^2 are computed as in Clark and West (2007). The burn-in sample starts in Jan 1967 and ends in Dec 1987, we then use an expanding window for estimating the predictive regressions. Monthly observations of annual returns.

Panel A: FF5

Anomaly	R_{OOS}^2	Anomaly	R_{OOS}^2	Anomaly	R_{OOS}^2
accruals	17.02***	indmom	2.87***	price	-19.42
age	5.21***	indmomrev	6.55***	prof	-17.56
aturnover	-7.48	indrrev	12***	roaa	-12
betaarb	-11.68	indrrevlv	-0.56	roea	-11.01
cfp	0.49***	inv	16.1***	season	8.94***
ciss	-1.32	invcap	10.65***	sgrowth	18.28***
divg	13.49***	ivol	-14.82	shvol	-10.73
divp	1.38***	lev	-6.82	size	7.48***
dur	7.35***	lrrev	12.02***	sp	18.31***
ep	9.61***	mom	5.88***	strev	9.75***
exchsw	-3.44	mom12	7.16***	valmom	11.3***
fscore	2.03***	momrev	12.6***	valmomprof	7.82***
gmargins	-14.57	nissa	-5.53	valprof	20.24***
growth	11.78***	nissm	1.69***	value	0.97***
igrowth	8.7***	noa	-18.42	valuem	-1.6

Panel B: FF5 with factor-timing

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	5.93***	indmom	-9.04	price	-15.97
age	10.68***	indmomrev	-10.2	prof	-13.76
aturnover	-6.23	indrrev	4.17***	roaa	-21.12
betaarb	-10.55	indrrevlv	-15.69	roea	-18.27
cfp	15.24***	inv	3.14***	season	-10.99
ciss	3.75***	invcap	3.46***	sgrowth	7.85***
divg	-8.55	ivol	-5.43	shvol	-0.53
divp	-1.52	lev	-2.02	size	8.17***
dur	11.2***	lrrev	-3.49	sp	12.91***
ep	-5.49	mom	1.31***	strev	3.94***
exchsw	-9.81	mom12	-5.81	valmom	-1.35
fscore	-5.61	momrev	-3.79	valmomprof	-8.12
gargins	-22.75	nissa	-16.46	valprof	14.95***
growth	9.1***	nissm	-4.38	value	16.51***
igrowth	-2.34	noa	-17.56	valuem	-3.04

Panel C: Volatility-Managed FF5

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	4.49***	indmom	12.74***	price	9.1***
age	9.28***	indmomrev	3.37***	prof	16.11***
aturnover	-5.07	indrrev	2.89***	roaa	12.5***
betaarb	12.36***	indrrevlv	-5.41	roea	12.19***
cfp	-0.24	inv	8.79***	season	7.74***
ciss	15.23***	invcap	16.45***	sgrowth	14.11***
divg	15.27***	ivol	10.58***	shvol	6.58***
divp	-0.01	lev	-4.73	size	9.28***
dur	-10.07	lrrev	7.85***	sp	2.31***
ep	2.3***	mom	18.06***	strev	5.25***
exchsw	15.9***	mom12	15.05***	valmom	1.76***
fscore	18.9***	momrev	15.39***	valmomprof	10.15***
gargins	8.65***	nissa	6.19***	valprof	-2.7
growth	11.62***	nissm	8.27***	value	-1.43
igrowth	13.84***	noa	6.37***	valuem	5.78***

Panel D: FF5-DMRS

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	11.55***	indmom	1.22***	price	-13.99
age	6.76***	indmomrev	-1.87	prof	-17.42
aturnover	-5.7	indrrev	18.94***	roaa	-19.36
betaarb	4.69***	indrrevlv	-0.56	roea	-16.73
cfp	17.1***	inv	15.95***	season	1.2***
ciss	0.9***	invcap	14.27***	sgrowth	12.52***
divg	8.49***	ivol	-12.15	shvol	-5.88
divp	5.21***	lev	7.29***	size	9.28***
dur	12.1***	lrrev	6.71***	sp	17.18***
ep	13.08***	mom	8.61***	strev	18.87***
exchsw	-2.54	mom12	10.07***	valmom	10.29***
fscore	0.84***	momrev	6.59***	valmomprof	-2.63
gmargins	-16.49	nissa	-10.15	valprof	8.23***
growth	14.67***	nissm	-0.43	value	21.74***
igrowth	13.17***	noa	-19.09	valuem	13.4***

Table 4: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) in quintiles according to their price deviation level. The zero-cost strategy goes long on the quintile associated with the lowest levels of price deviations and short on the bottom quintile associated with the highest levels. Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using different CMVE portfolios. Panel A reports results for the [Fama and French \(2015, FF5\)](#) factor model, Panels B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in [Daniel et al. \(2020\)](#), dubbed FF5-DMRS. We control for the following factor models: [Carhart \(1997\)](#) (C4), [Fama and French \(2018\)](#) (FF6), [Hou et al. \(2015\)](#) (q), [Stambaugh and Yuan \(2016\)](#) (SY4), [Daniel et al. \(2020\)](#) (DHS3), [Bartram and Grinblatt \(2018\)](#) (BG3). Values in parenthesis are [Newey and West \(1987\)](#) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

Panel A: FF5

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.26*** (0.09)	0.25** (0.10)	0.23** (0.11)	0.20** (0.08)	0.34*** (0.11)	0.27*** (0.10)
MKT	-0.04 (0.03)	-0.04 (0.03)	-0.04 (0.04)	-0.01 (0.03)	-0.03 (0.04)	-0.06* (0.03)
SMB	0.09 (0.09)	0.10 (0.07)		0.13 (0.09)		0.06 (0.09)
HML	0.17** (0.07)	0.16** (0.06)				
Mom	0.03 (0.04)	0.03 (0.04)				
RMW		0.02 (0.11)				
CMA		0.01 (0.08)				
ME			0.11 (0.07)			
IA			0.16* (0.10)			
ROE			0.03 (0.06)			
Mgmt				0.20*** (0.07)		
Perf				-0.01 (0.04)		
PEAD					-0.15* (0.08)	
FIN					0.06 (0.09)	
BG						0.10** (0.05)
Observations	384	384	384	384	384	384
Adjusted R ²	0.09	0.09	0.06	0.11	0.06	0.07

Panel B: FF5 with factor-timing

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.39*** (0.09)	0.32*** (0.09)	0.32*** (0.10)	0.27*** (0.08)	0.39*** (0.11)	0.39*** (0.10)
MKT	-0.11*** (0.03)	-0.07** (0.03)	-0.08* (0.04)	-0.05* (0.03)	-0.06 (0.05)	-0.13*** (0.04)
SMB	-0.05 (0.07)	-0.03 (0.07)		0.01 (0.07)		-0.10* (0.06)
HML	0.27*** (0.07)	0.17*** (0.06)				
Mom	0.01 (0.04)	-0.00 (0.03)				
RMW		0.07 (0.08)				
CMA		0.18** (0.08)				
ME			-0.03 (0.06)			
IA			0.38*** (0.10)			
ROE			0.01 (0.05)			
Mgmt				0.34*** (0.07)		
Perf				-0.02 (0.05)		
PEAD					-0.16** (0.06)	
FIN					0.19*** (0.07)	
BG						0.14*** (0.04)
Observations	384	384	384	384	384	384
Adjusted R ²	0.28	0.30	0.24	0.34	0.29	0.18

Panel C: Volatility-managed FF5

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.25** (0.11)	0.27** (0.11)	0.28** (0.13)	0.17 (0.10)	0.36*** (0.12)	0.32*** (0.12)
MKT	0.01 (0.04)	0.00 (0.04)	-0.02 (0.04)	0.04 (0.04)	-0.04 (0.04)	-0.03 (0.04)
SMB	0.16** (0.06)	0.12* (0.06)		0.20*** (0.08)		0.16** (0.07)
HML	0.05 (0.05)	0.03 (0.05)				
Mom	0.09*** (0.03)	0.09*** (0.02)				
RMW		-0.11 (0.07)				
CMA		0.10 (0.07)				
ME			0.16** (0.07)			
IA			0.05 (0.10)			
ROE			0.00 (0.06)			
Mgmt				0.14** (0.06)		
Perf				0.09* (0.05)		
PEAD					-0.01 (0.06)	
FIN					-0.07 (0.07)	
BG						-0.02 (0.05)
Observations	384	384	384	384	384	384
Adjusted R ²	0.11	0.14	0.07	0.11	0.01	0.06

Panel D: FF5-DMRS

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.31*** (0.09)	0.21** (0.08)	0.22** (0.09)	0.15* (0.08)	0.33*** (0.10)	0.38*** (0.10)
MKT	-0.12*** (0.04)	-0.06* (0.03)	-0.09** (0.04)	-0.04 (0.03)	-0.10** (0.04)	-0.15*** (0.05)
SMB	-0.01 (0.06)	0.01 (0.06)		0.07 (0.06)		-0.02 (0.05)
HML	0.17** (0.07)	0.02 (0.06)				
Mom	0.07*** (0.03)	0.06*** (0.02)				
RMW		0.11* (0.06)				
CMA		0.29*** (0.06)				
ME			0.04 (0.06)			
IA			0.27*** (0.08)			
ROE			0.15*** (0.05)			
Mgmt				0.30*** (0.06)		
Perf				0.11*** (0.04)		
PEAD					-0.06 (0.06)	
FIN					0.13** (0.06)	
BG						0.03 (0.04)
Observations	384	384	384	384	384	384
Adjusted R ²	0.21	0.26	0.24	0.29	0.20	0.13

Panel A: FF5

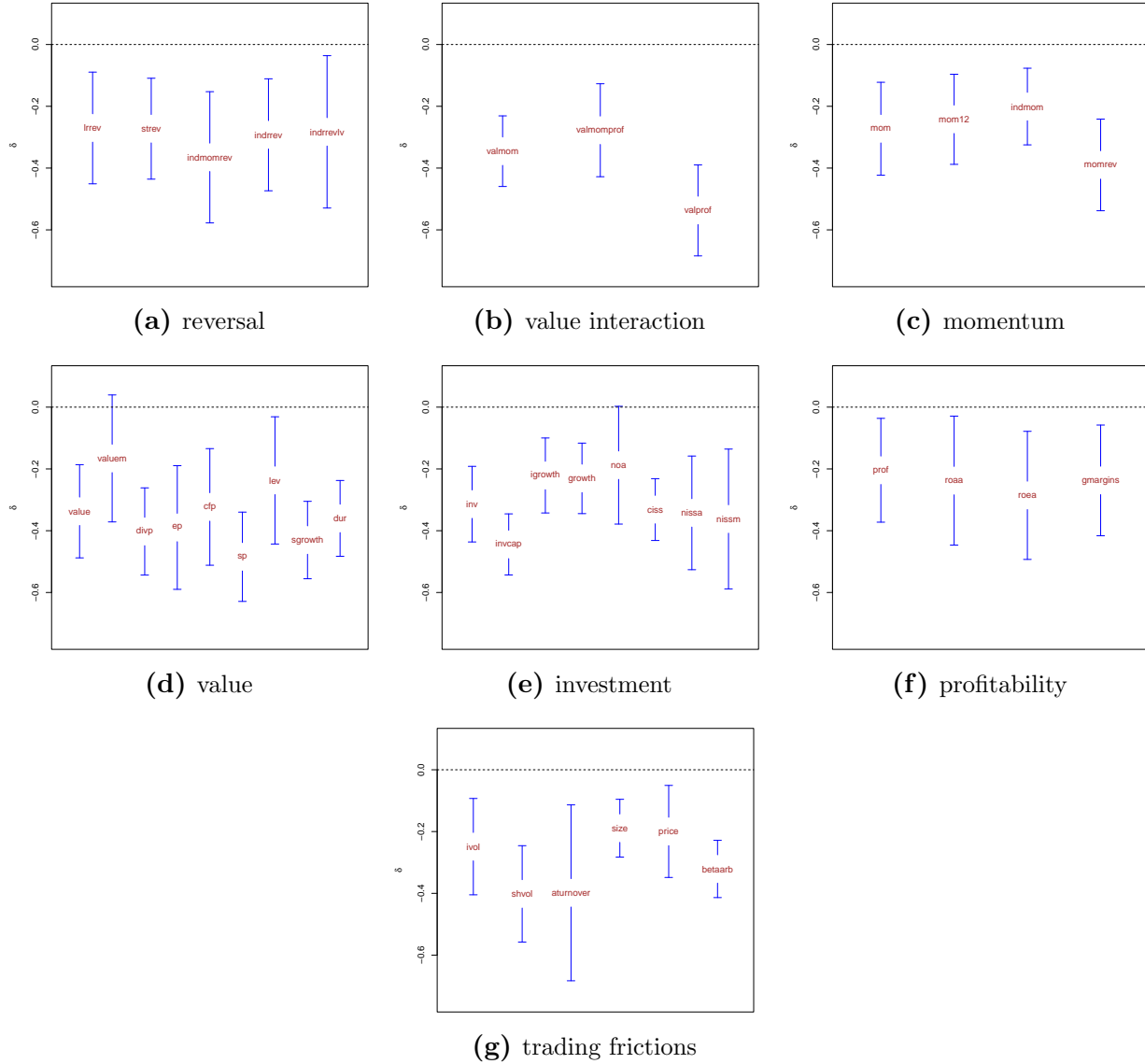
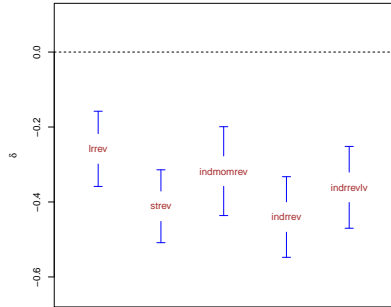
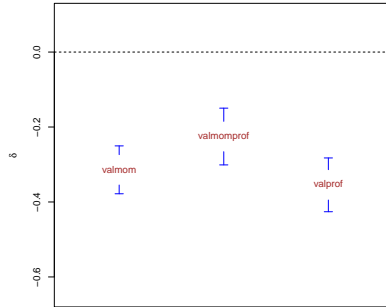


Figure 1: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (12) with respective confidence intervals at 5% level of significance. Test assets are the 45 top anomaly portfolios constructed in Kozak et al. (2020). Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using different CMVE portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

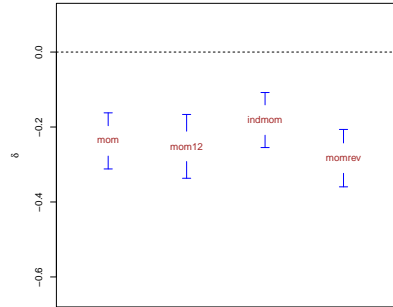
Panel B: FF5 with factor-timing



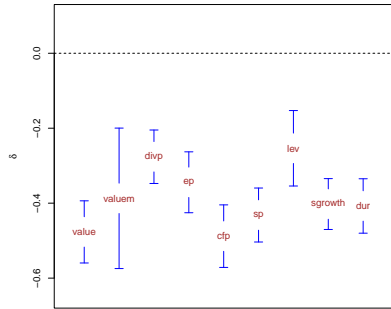
(a) reversal



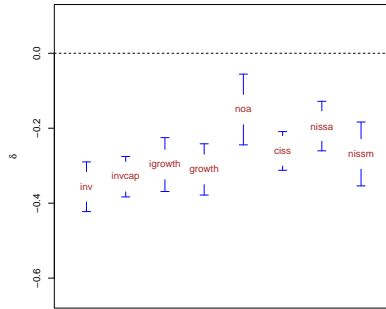
(b) value interaction



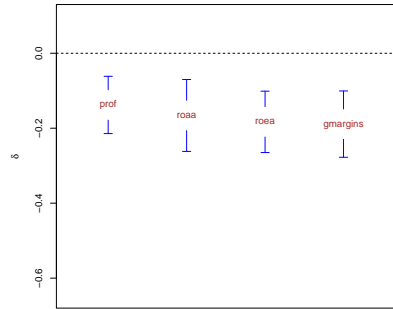
(c) momentum



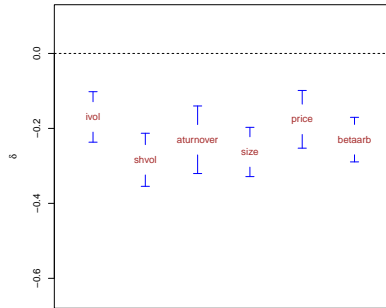
(d) value



(e) investment

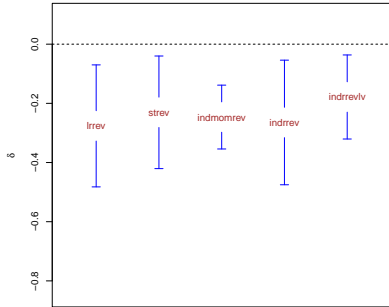


(f) profitability

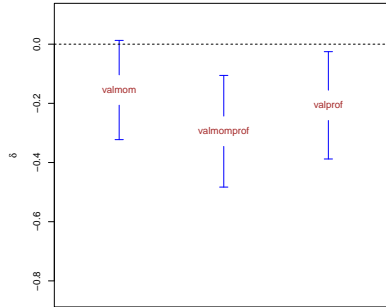


(g) trading frictions

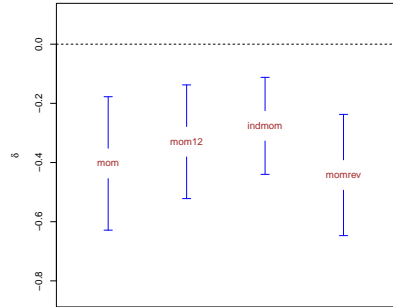
Panel C: Volatility-Managed FF5



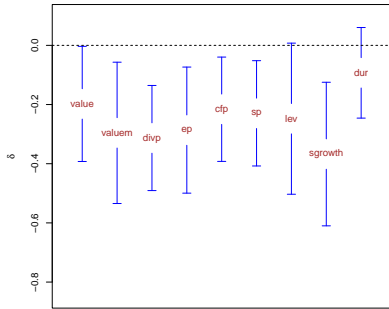
(a) reversal



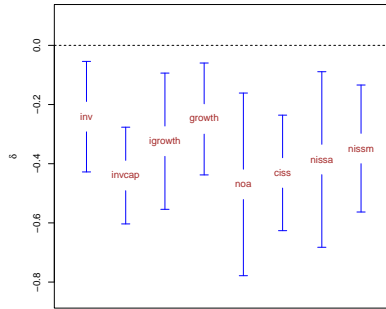
(b) value interaction



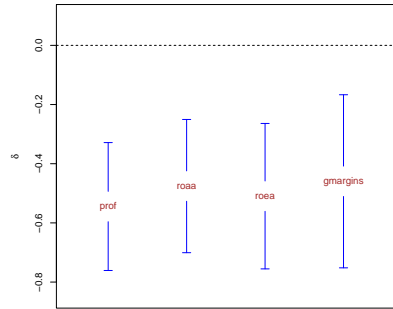
(c) momentum



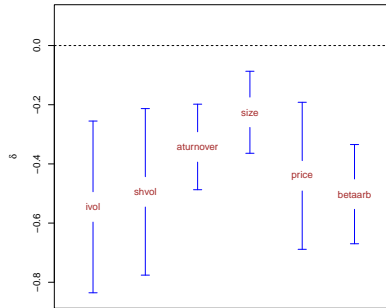
(d) value



(e) investment

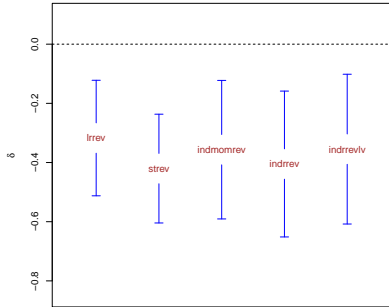


(f) profitability

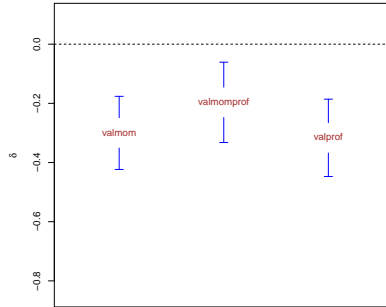


(g) trading frictions

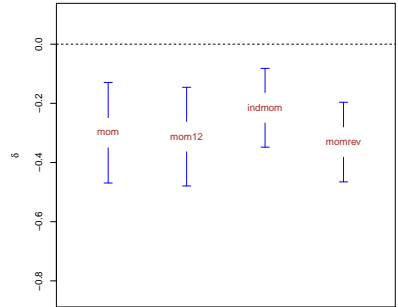
Panel D: FF5-DMRS



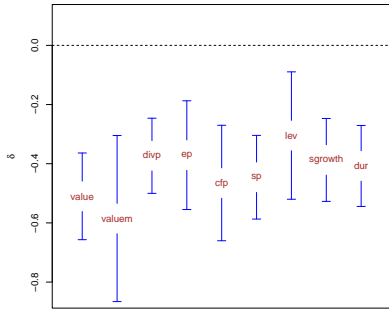
(a) reversal



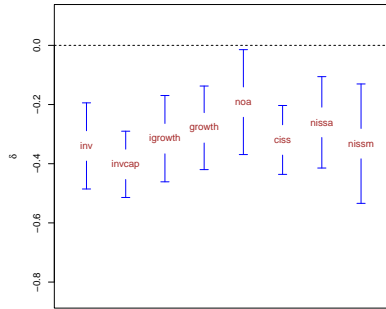
(b) value interaction



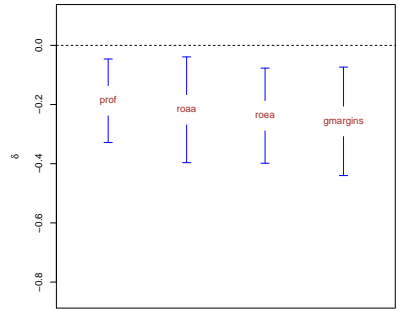
(c) momentum



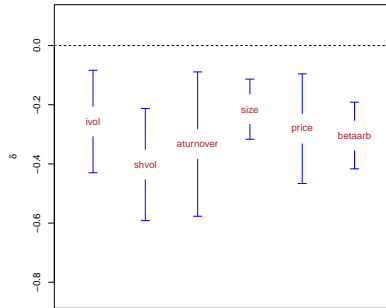
(d) value



(e) investment



(f) profitability



(g) trading frictions

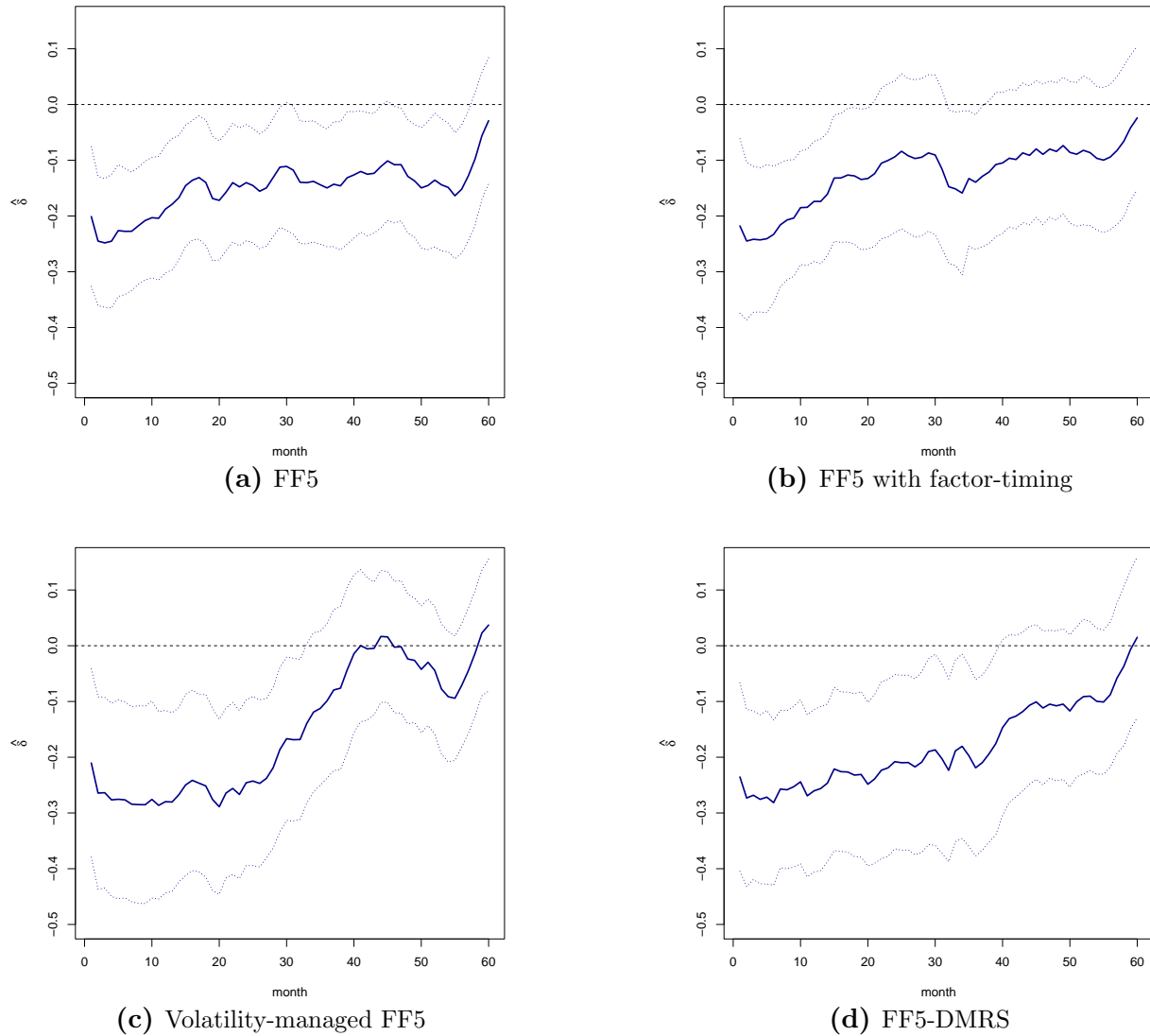
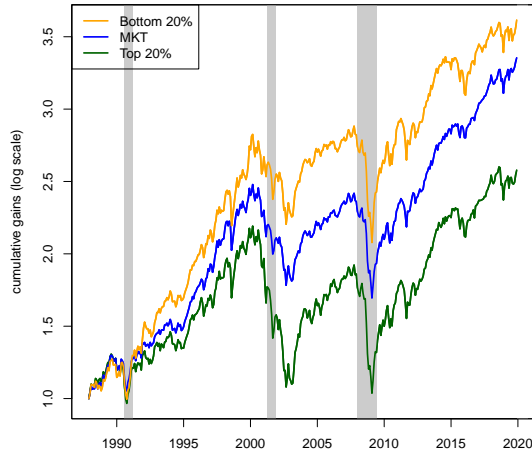
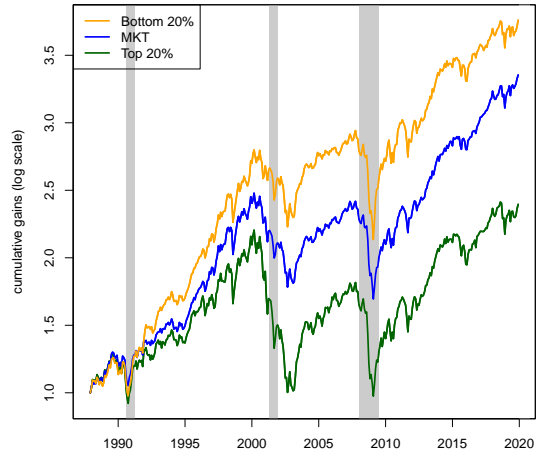


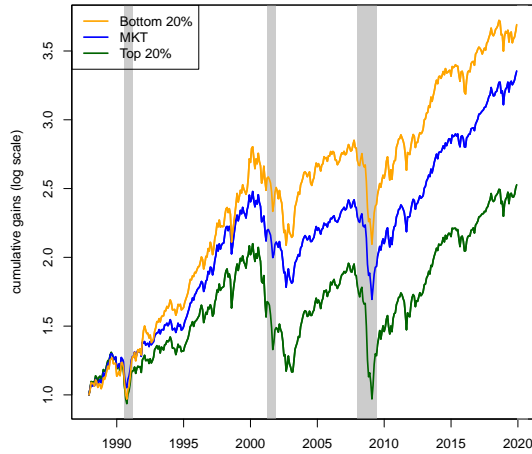
Figure 2: Price Deviations and Horizon of Predictability. This figure shows pooled regression estimates of δ_i for equation (12) for h -period ahead monthly returns ($h = 1, \dots, 60$). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak et al. (2020). Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using different CMVE portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Standard errors are computed as in Newey and West (1987) with automatic bandwidth selection procedure as described in Newey and West (1994). Non-overlapping monthly observations. The sample period is 1967 to 2019.



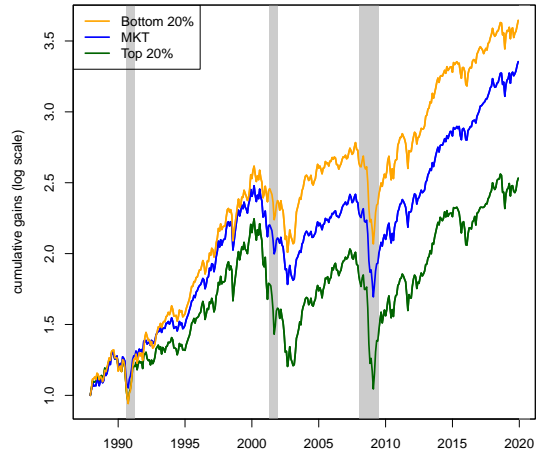
(a) FF5



(b) FF5 with factor-timing



(c) Volatility-Managed FF5



(d) FF5-DMRS

Figure 3: Anomaly Portfolios Rotation using Real-Time Price Deviations. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) according their price deviation level. Bottom (top) 20% corresponds to the cumulative gains of a dynamic strategy that goes long on the 18 portfolios associated with the lowest (highest) level of price deviation. MKT is the performance of a static buy-and-hold strategy on the market portfolio in excess of the risk-free rate. Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using different CMVE portfolios. Panel A reports results for the [Fama and French \(2015, FF5\)](#) factor model, Panel B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in [Daniel et al. \(2020\)](#), dubbed FF5-DMRS. Shaded areas are NBER recessions. Monthly observations.

Appendix

A Mean-variance efficient portfolio and the SDF

Define $\Sigma_t = V_t [R_{t+1}^e]$ and $\mu_t = E_t (R_{t+1}^e)$ and consider the following portfolio:

$$R_{t+1}^C = w_t^\top R_{t+1}^e \tag{A.1}$$

$$w_t = k_t^{-1} \Sigma_t^{-1} \mu_t \tag{A.2}$$

Next, we show that the SDF

$$\boxed{M_{t+1}^C = 1 - k_t (R_{t+1}^C - E_t [R_{t+1}^C])}$$

prices all assets conditionally:

$$\begin{aligned} E_t [M_{t+1}^C R_{t+1}^e] &= E_t [1 - k_t (R_{t+1}^C - E_t [R_{t+1}^C]) R_{t+1}^e] \\ &= E_t [R_{t+1}^e] - k_t E_t [(w_t^\top R_{t+1}^e - w_t^\top E_t [R_{t+1}^e]) R_{t+1}^e] \\ &= E_t [R_{t+1}^e] - k_t w_t^\top E_t [(R_{t+1}^e - E_t [R_{t+1}^e]) R_{t+1}^e] \\ &= E_t [R_{t+1}^e] - k_t k_t^{-1} \mu_t^\top \Sigma_t^{-1} \Sigma_t \\ &= 0 \end{aligned} \tag{A.3}$$

The parameter k_t is found by pricing the portfolio R_{t+1}^C itself:

$$\begin{aligned} E_t [M_{t+1}^C R_{t+1}^C] &= E_t [1 - k_t (R_{t+1}^C - E_t [R_{t+1}^C]) R_{t+1}^C] \\ &= E_t [R_{t+1}^C] - k_t E_t [(R_{t+1}^C - E_t [R_{t+1}^C]) R_{t+1}^C] \\ &= E_t [R_{t+1}^C] - V_t [R_{t+1}^C] k_t \\ &= 0 \Leftrightarrow k_t = (V_t [R_{t+1}^C])^{-1} E_t [R_{t+1}^C] \end{aligned}$$

B Price deviations when factor returns are i.i.d.

This example is inspired by Section 2.4 in [Chernov et al. \(2021\)](#).

Suppose that the true model is given by:

$$M_{t+1} = 1 - \mathbf{b}^\top (\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]) \quad , \quad \mathbf{b} = V(\mathbf{f}_{t+1})^{-1} E[\mathbf{f}_{t+1}]$$

where the factors \mathbf{f}_{t+1} are excess returns to traded portfolios.

Suppose also that the factor returns are i.i.d. Thus, the model prices the factors both conditionally and unconditionally.

Despite the factors being i.i.d., our predictive model (12) implies that test assets' returns are not, since their dynamics feature the (persistent) $u_{i,t}$ term:

$$\begin{aligned} r_{i,t+1} &= \beta'_i \mathbf{f}_{t+1} + \underbrace{\Delta u_{i,t+1}}_{\delta_i u_{i,t} + v_{i,t+1}} \quad . \\ u_{i,t} &= (\ln P_{i,t} - \ln P_{f,t}) - \beta_i \ln P_{f,t}, \\ u_{i,t} &= \rho_i u_{i,t-1} + \varepsilon_{i,t} \end{aligned}$$

where $\delta_i = 1 - \rho_i$ and for simplicity we have omitted the α_i .

Note that the SDF prices r_{t+1}^i *unconditionally*:

$$\begin{aligned} E[M_{t+1} r_{t+1}^i] &= E[(1 - \mathbf{b}^\top (\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}])) (\beta'_i \mathbf{f}_{t+1} + \Delta u_{i,t+1})] \\ &= (\beta'_i \underbrace{E[(1 - \mathbf{b}^\top (\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}])) \mathbf{f}_{t+1}]}_{=0 \text{ using the definition of } \mathbf{b}}) + E[(1 - \mathbf{b}^\top (\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}])) \Delta u_{i,t+1}] \end{aligned}$$

where the last term is zero given our assumption of factors being independent over time and the price deviations $u_{i,t}$ being zero mean. However, the SDF does not price r_{t+1}^i

conditionally:

$$\begin{aligned}
E_t [M_{t+1} r_{t+1}^i] &= E_t [(1 - \mathbf{b}^\top (\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}])) (\beta_i' \mathbf{f}_{t+1} + \Delta u_{i,t+1})] \\
&= \beta_i' \underbrace{E_t [(1 - \mathbf{b}^\top (\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}])) \mathbf{f}_{t+1}]}_{=0 \text{ using the definition of } b \text{ and factors being iid}} \\
&\quad + \underbrace{E_t [(\mathbf{b}^\top (\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}])) \Delta u_{i,t+1}]}_{=0 \text{ since factors are i.i.d and property of } u_{i,t}} + E_t [\Delta u_{i,t+1}] \\
&= \delta_i u_{i,t}
\end{aligned}$$

where in the last step we exploit the AR(1) dynamics for $u_{i,t}$.

Furthermore, we have that

$$\text{Cov}(u_{i,t-1}, u_{i,t}) \neq 0$$

In words, through our predictive system we document that test assets feature persistent pricing errors.¹⁷

¹⁷In the notation of Chernov et al. (2021), $\delta_i u_{i,t}$ is the conditional pricing error at horizon $h = 1$.

C Test Assets

Table C.1: Categories

We group anomaly portfolios constructed in [Kozak et al. \(2020\)](#) following [Lettau and Pelger \(2020\)](#). This table lists the categories and the portfolios that we include in each category. Differently from [Lettau and Pelger \(2020\)](#), we allocate some of the portfolios in the category “others” across the other categories. In total, we consider 8 categories and 45 anomaly portfolios. Anomalies are defined in [Kelly et al. \(2020\)](#), [Haddad et al. \(2020\)](#), and [Kozak et al. \(2020\)](#).

Category	Anomaly Portfolios
reversal	indmomrev, indrrev, indrrevlv, lrrev, strev
value interaction	valmom, valmomprof, valprof
momentum	indmom, mom, mom12, momrev
value	cfp, divp, dur, ep, lev, sgrowth, sp, value, valuem
investment	ciss, inv, invcap, igrowth, growth, nissa, nissm, noa
profitability	gmargins, prof, roaa, roea
trading frictions	aturnover, betaarb, ivol, price, shvol, size
others	accruals, age, divg, exchsw, fscore, season

Notes: lrrev is long-term reversal calculated as in [De Bondt and Thaler \(1985\)](#). strev is short-term reversal calculated as in [Jegadeesh \(1990\)](#). indmomrev is industry momentum-reversal reversal calculated as in [Moskowitz and Grinblatt \(1999\)](#). indrrev is industry relative reversal calculated as in [Da et al. \(2014\)](#). indrrevlv is industry relative reversal low volatility calculated as in [Da et al. \(2014\)](#). valmom is value-momentum calculated as in [Novy-Marx \(2013\)](#). valmomprof is value-momentum-profitability calculated as in [Novy-Marx \(2013\)](#). valprof is value-profitability calculated as in [Novy-Marx \(2013\)](#). mom is 6-months momentum calculated as in [Jegadeesh and Titman \(1993\)](#). mom12 is 12-months momentum calculated as in [Jegadeesh and Titman \(1993\)](#). indmom is long-term reversal calculated as in [Moskowitz and Grinblatt \(1999\)](#). momrev is momentum-reversal calculated as in [Jegadeesh and Titman \(1993\)](#). value is annual value calculated as in [Fama and French \(1993\)](#). valuem is monthly value calculated as in [Asness and Frazzini \(2013\)](#). divp is dividend yield calculated as in [Naranjo et al. \(1998\)](#). ep is earnings/price calculated as in [Basu \(1977\)](#). cfp is cash-flow/market value of equity calculated as in [Lakonishok et al. \(1994\)](#). sp is sales-to-price calculated as in [Barbee Jr et al. \(1996\)](#). lev is leverage calculated as in [Bhandari \(1988\)](#). sgrowth is sales growth calculated as in [Lakonishok et al. \(1994\)](#). inv is investment calculated as in [Chen et al. \(2011\)](#). invcap is investment-to-capital calculated as in [Xing \(2008\)](#). igrowth is investment growth calculated as in [Xing \(2008\)](#). growth is asset growth calculated as in [Cooper et al. \(2008\)](#). noa is net operating asset calculated as in [Hirshleifer et al. \(2004\)](#). ciss is composite issuance calculated as in [Daniel and Titman \(2006\)](#). prof is ross profitability calculated as in [Novy-Marx \(2013\)](#). roaa is annual return on assets calculated as in [Chen et al. \(2011\)](#). roea is annual return on equity calculated as in [Haugen et al. \(1996\)](#). gmargins is gross margins calculated as in [Novy-Marx \(2013\)](#). ivol is idiosyncratic volatility calculated as in [Ang et al. \(2006\)](#). shvol is share volume calculated as in [Datar et al. \(1998\)](#). aturnover is asset turnover calculated as in [Soliman \(2008\)](#). size is size calculated as in [Fama and French \(1993\)](#).

D Alternative Factor Models: Robustness

D.1 Predictive regressions

Table D.1: Pooled Regressions for Alternative Factor Models

This table reports pooled estimates for δ_i from predictive regression (13). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak et al. (2020). Price deviations \hat{u} are computed as in equation (9). We report results for different CMVE portfolios. Panel A reports results for the Hou et al. (2015, HXZ) factor model, Panel B reports results for its volatility timed version, and Panel C reports results for the principal component model employed in Kelly et al. (2020). Our panel features: $n = 565$, $T = 90$, $N = 50850$. Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: HXZ

	(1)	(2)	(3)	(4)	(5)
δ	-0.347*** (0.049)	-0.300*** (0.052)	-0.352*** (0.046)	-0.332*** (0.046)	-0.340*** (0.046)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R ²	0.159	0.185	0.159	0.167	0.183

Panel B: Volatility-managed HXZ

	(1)	(2)	(3)	(4)	(5)
δ	-0.377*** (0.065)	-0.316*** (0.066)	-0.405*** (0.063)	-0.371*** (0.063)	-0.365*** (0.059)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R ²	0.176	0.226	0.181	0.198	0.227

D.2 Out-Of-Sample R^2

Table D.2: Out-of-Sample Predictability for Alternative Factor Models

This table reports the out-of-sample R^2 (R_{OOS}^2) for the predictive regression $\tilde{r}_{i,t+1} = a_i + b_i \hat{u}_{i,t} + \epsilon_{i,t}$, where $\tilde{r}_{i,t+1}$ is the test asset i log risk-adjusted return and price deviations \hat{u} are computed as in equation (9). Test assets are the long legs for the 45 anomalies constructed in Kozak et al. (2020). See Appendix Table C.1 for a description of the anomalies. We report results for different mean-variance efficient portfolios. Panel A reports results for the Hou et al. (2015, HXZ) factor model, Panel B reports results for its volatility timed version, and Panel C reports results for the principal component model employed in Kelly et al. (2020). The R_{OOS}^2 is computed as in Campbell and Thompson (2008); p -values for R_{OOS}^2 are computed as in Clark and West (2007). The burn-in sample starts in Jan 1967 and ends in Dec 1987, we then use an expanding window for estimating the predictive regressions. Monthly observations of annual returns.

Panel A: HXZ

Anomaly	R_{OOS}^2	Anomaly	R_{OOS}^2	Anomaly	R_{OOS}^2
accruals	14.71***	indmom	11.64***	price	-2.61
age	10.88***	indmomrev	1.96***	prof	3.17***
aturnover	10.12***	indrrev	18.07***	roaa	-3.22
betaarb	12.37***	indrrevlv	-7.58	roea	-3.4
cfp	16.86***	inv	27.41***	season	5.49***
ciss	18.04***	invcap	15.59***	sgrowth	29.52***
divg	18.59***	ivol	2.59***	shvol	15.42***
divp	21.39***	lev	4.26***	size	12.32***
dur	3.01***	lrrev	20.48***	sp	32.9***
ep	21.87***	mom	11.89***	strev	17.54***
exchsw	9.17***	mom12	17.12***	valmom	11.31***
fscore	9.67***	momrev	19.18***	valmomprof	10.83***
gmargins	-3.67	nissa	6.63***	valprof	32.43***
growth	24.42***	nissm	16.87***	value	24.63***
igrowth	20.3***	noa	-1.29	valuem	6.74***

Panel B: Vol-managed HXZ

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	-1.78	indmom	18.46***	price	0.75***
age	8.06***	indmomrev	-0.6	prof	15.01***
aturnover	17.93***	indrrev	6.16***	roaa	4.33***
betaarb	3.15***	indrrevlv	-19.48	roea	6.09***
cfp	23.58***	inv	19.8***	season	3.03***
ciss	18.64***	invcap	3.96***	sgrowth	25.54***
divg	19.11***	ivol	-2.24	shvol	-0.75
divp	19.83***	lev	11.16***	size	19.7***
dur	6.17***	lrrev	23.44***	sp	29.72***
ep	15.09***	mom	22.99***	strev	15.72***
exchsw	12.82***	mom12	21.83***	valmom	5.54***
fscore	14.66***	momrev	30.72***	valmomprof	17***
gmargins	-1.85	nissa	9.77***	valprof	23.06***
growth	22.26***	nissm	14.98***	value	21.95***
igrowth	25.88***	noa	3.58***	valuem	25.64***

D.3 Anomaly Rotation using Price Deviations

Table D.3: Long-Short Anomaly Portfolio Alphas for Alternative Factor Models

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) in quintiles according their price deviation level. The zero-cost strategy goes long on the quintile associated with the lowest levels of price deviations and short on the bottom quintile associated with the highest levels. Price deviations \hat{u} are computed as in equation (9). We report results for different CMVE portfolios. Panel A reports results for the [Hou et al. \(2015, HXZ\)](#) factor model, Panel B reports results for its volatility timed version, and Panel C reports results for the principal component model employed in [Kelly et al. \(2020\)](#). We control for the following factor models: [Carhart \(1997\)](#) (C4), [Fama and French \(2018\)](#) (FF6), [Hou et al. \(2015\)](#) (q), [Stambaugh and Yuan \(2016\)](#) (SY4), [Daniel et al. \(2020\)](#) (DHS3), [Bartram and Grinblatt \(2018\)](#) (BG3). Values in parenthesis are [Newey and West \(1987\)](#) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

Panel A: HXZ

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.29*** (0.10)	0.26** (0.11)	0.27** (0.12)	0.19** (0.09)	0.38*** (0.11)	0.28*** (0.11)
MKT	-0.01 (0.03)	0.01 (0.03)	-0.00 (0.04)	0.04 (0.03)	-0.00 (0.04)	-0.02 (0.04)
SMB	0.08 (0.07)	0.07 (0.06)		0.13* (0.07)		0.05 (0.06)
HML	0.17*** (0.07)	0.11** (0.05)				
Mom	-0.00 (0.03)	-0.01 (0.03)				
RMW		-0.02 (0.07)				
CMA		0.16*** (0.06)				
ME			0.06 (0.05)			
IA			0.24*** (0.08)			
ROE			-0.06 (0.04)			
Mgmt				0.25*** (0.06)		
Perf				-0.01 (0.05)		
PEAD					-0.16*** (0.06)	
FIN					0.05 (0.06)	
BG						0.10** (0.04)
Observations	384	384	384	384	384	384
Adjusted R ²	0.09	0.11	0.08	0.14	0.05	0.06

Panel B: Volatility managed HXZ

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.25* (0.13)	0.20* (0.11)	0.23* (0.13)	0.10 (0.11)	0.30** (0.12)	0.31** (0.13)
MKT	0.02 (0.05)	0.04 (0.04)	0.01 (0.05)	0.09** (0.04)	-0.01 (0.05)	-0.02 (0.06)
SMB	0.05 (0.06)	0.05 (0.07)		0.12* (0.06)		0.06 (0.05)
HML	0.06 (0.08)	-0.02 (0.08)				
Mom	0.08*** (0.03)	0.08** (0.03)				
RMW		0.02 (0.09)				
CMA		0.17* (0.10)				
ME			0.07 (0.06)			
IA			0.09 (0.11)			
ROE			0.07 (0.07)			
Mgmt				0.20*** (0.08)		
Perf				0.15*** (0.05)		
PEAD					0.01 (0.06)	
FIN					-0.00 (0.06)	
BG						-0.02 (0.06)
Observations	384	384	384	384	384	384
Adjusted R ²	0.04	0.06	0.01	0.11	-0.01	0.00

E Fama and French (2015): Further Results

E.1 Bottom Deciles

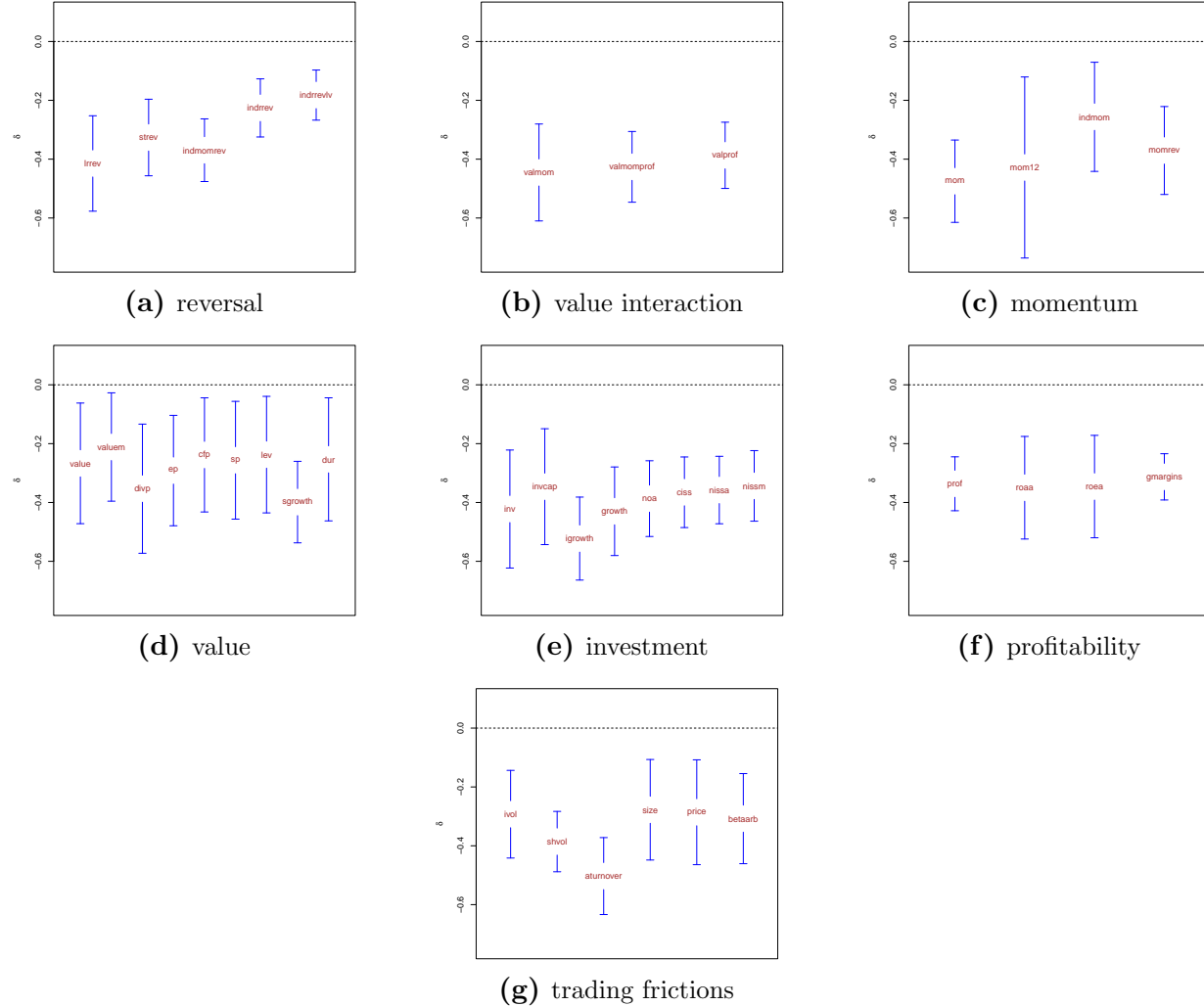


Figure E.1: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (12) with respective confidence intervals at 5% level of significance. Test assets are the 45 bottom anomaly portfolios constructed in Kozak et al. (2020). Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using the Fama and French (2015) factor model to calculate the CMVE portfolios. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

E.2 Nonparametric Conditional Betas (Ang and Kristensen, 2012)

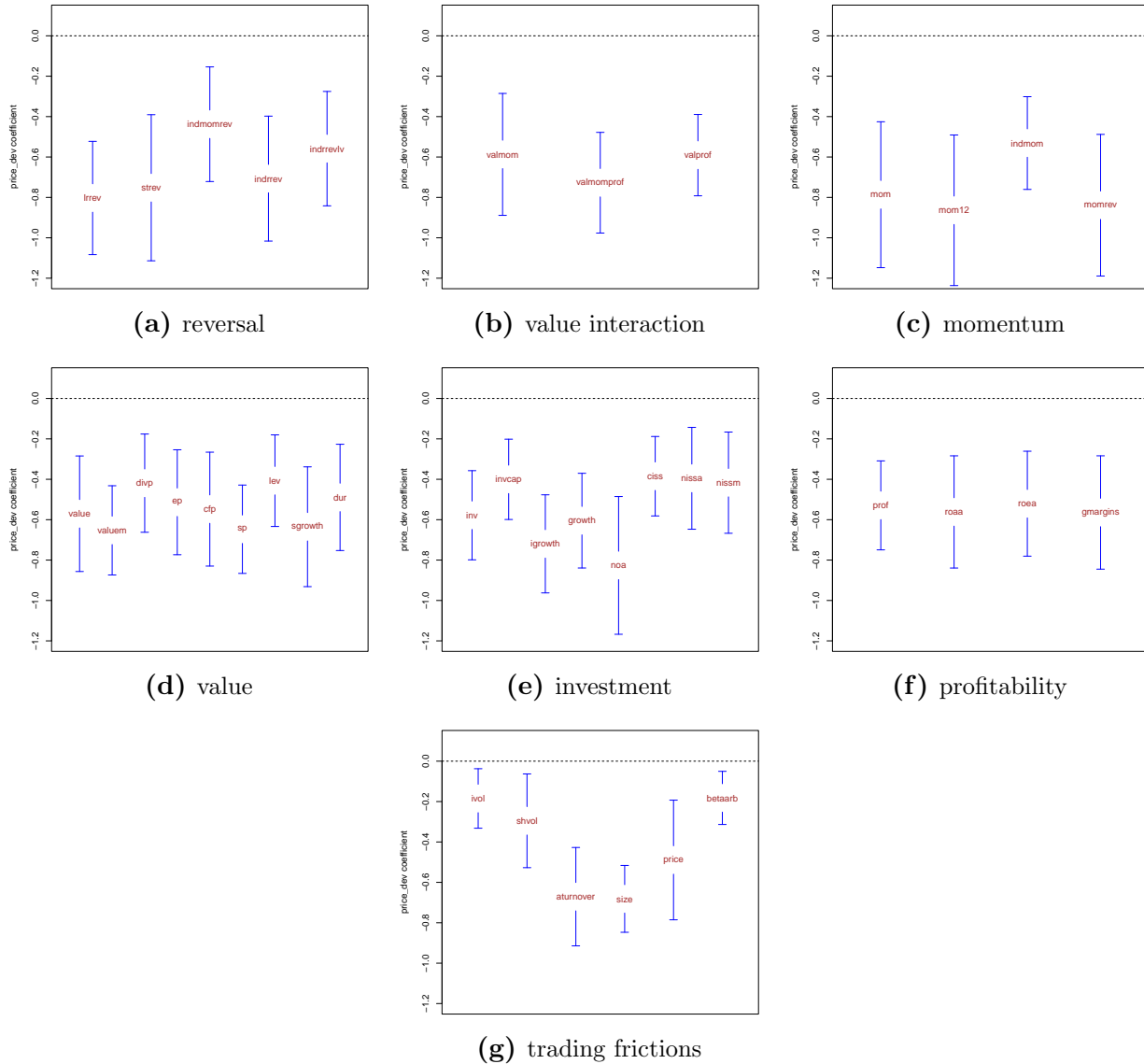


Figure E.2: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (12) with respective confidence intervals at 5% level of significance. Test assets are the 45 top anomaly portfolios constructed in Kozak et al. (2020). Price deviations \hat{u} are computed as in equation (9) using the methodology proposed by Ang and Kristensen (2012) to calculate time-varying parameters. We report results for price deviations computed using the Fama and French (2015) factor model to calculate the CMVE portfolio. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

Table E.1: Out-of-Sample Predictability

This table reports the out-of-sample R^2 (R_{OOS}^2) for the predictive regression $\tilde{r}_{i,t+1} = a_i + b_i \hat{u}_{i,t} + \epsilon_{i,t}$, where $\tilde{r}_{i,t+1}$ is the test asset i log risk-adjusted return at time $t + 1$ and price deviations \hat{u} are computed as in equation (9) using the methodology proposed by [Ang and Kristensen \(2012\)](#) to calculate time-varying parameters. Test assets are the long legs for the 45 anomalies constructed in [Kozak et al. \(2020\)](#). We report results for price deviations computed using the [Fama and French \(2015\)](#) factor model to calculate the CMVE portfolio. The R_{OOS}^2 is computed as in [Campbell and Thompson \(2008\)](#); p -values for R_{OOS}^2 are computed as in [Clark and West \(2007\)](#). The burn-in sample starts in Jan 1967 and ends in Dec 1987, we then use an expanding window for estimating the predictive regressions. Monthly observations of annual returns.

Anomaly	R_{OOS}^2	Anomaly	R_{OOS}^2	Anomaly	R_{OOS}^2
accruals	32.2***	indmom	23.25***	price	7.86***
age	28.24***	indmomrev	2.23***	prof	17.21***
aturnover	14.53***	indrrev	17.59***	roaa	19.12***
betaarb	2.4***	indrrevlv	7.57***	roea	9.78***
cfp	20.2***	inv	16.52***	season	29.97***
ciss	7.18***	invcap	13.54***	sgrowth	27.44***
divg	20.18***	ivol	-7.62	shvol	-8.44
divp	7.63***	lev	3.84***	size	26.29***
dur	10.36***	lrrev	32.56***	sp	23.75***
ep	19.28***	mom	36.67***	strev	20.75***
exchsw	9.8***	mom12	41.28***	valmom	19.04***
fscore	11.09***	momrev	34.48***	valmomprof	27.78***
gmargins	12.82***	nissa	1.79***	valprof	16.46***
growth	24.91***	nissm	5.1***	value	17.26***
igrowth	21.67***	noa	32.41***	valuem	19.79***

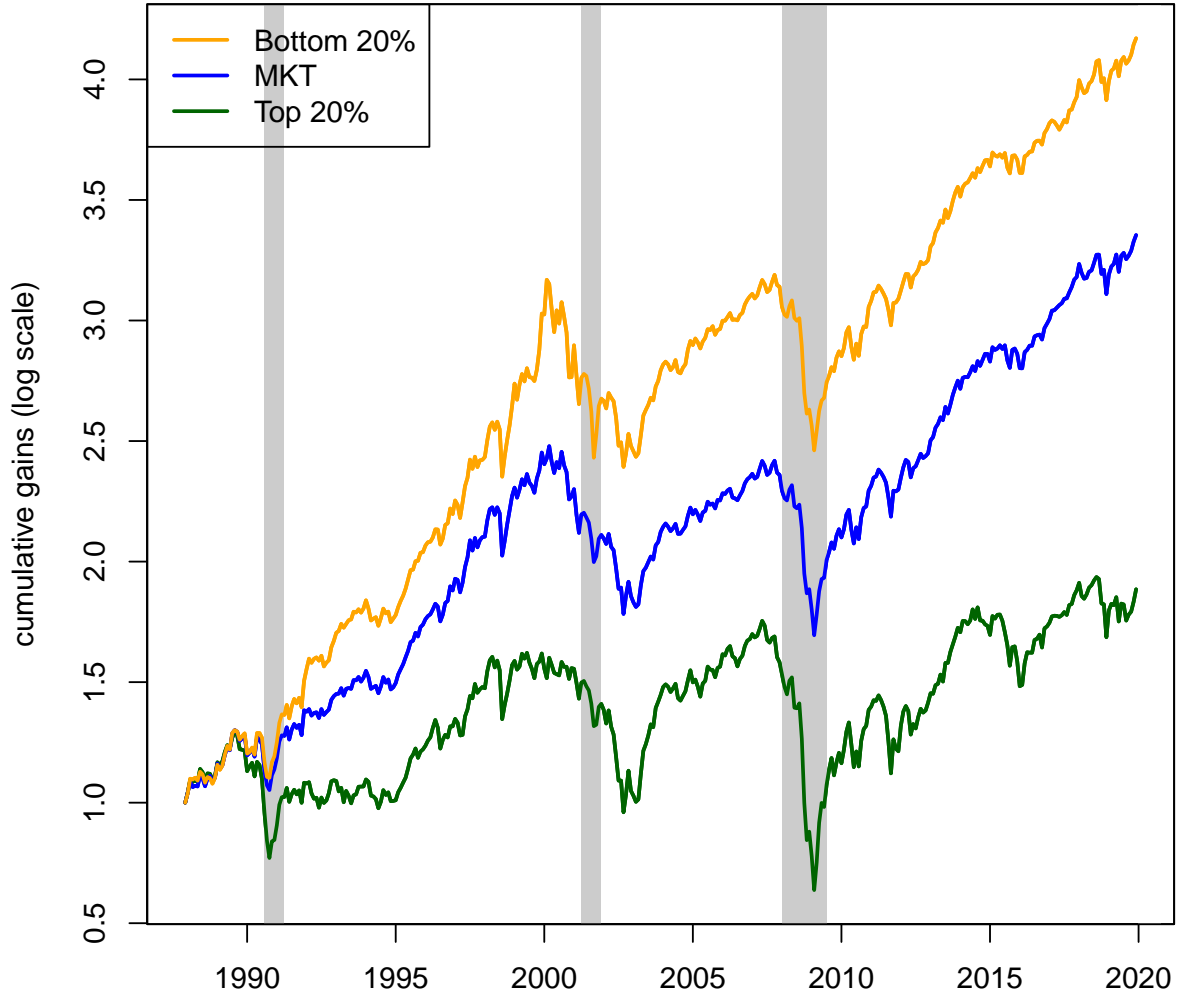


Figure E.3: Anomaly Portfolios Rotation using Real-Time Price Deviations. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) according their price deviation level. Bottom (top) 20% corresponds to the cumulative gains of a dynamic strategy that goes long on the 18 portfolios associated with the lowest (highest) level of price deviation. MKT is the performance of a static buy-and-hold strategy on the market portfolio in excess of the risk-free rate. Price deviations \hat{u} are computed as in equation (9) using the methodology proposed by [Ang and Kristensen \(2012\)](#) to calculate time-varying parameters. We report results for price deviations computed using the [Fama and French \(2015\)](#) factor model to calculate the CMVE portfolio. Shaded areas are NBER recessions. Monthly observations.

Table E.2: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) in quintiles according to their price deviation level. The zero-cost strategy goes long on the quintile associated with the lowest levels of price deviations and short on the bottom quintile associated with the highest levels. Price deviations \hat{u} are computed as in equation (9) using the methodology proposed by [Ang and Kristensen \(2012\)](#) to calculate time-varying parameters. We report results for price deviations computed using the [Fama and French \(2015\)](#) factor model to calculate the CMVE portfolio. Values in parenthesis are [Newey and West \(1987\)](#) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	FF3	C4	FF5	FF6	q	SY4	DHS3
Constant	0.73*** (0.15)	0.64*** (0.15)	0.94*** (0.15)	0.86*** (0.16)	0.95*** (0.18)	0.81*** (0.17)	0.60*** (0.13)
MKT	-0.13** (0.05)	-0.08* (0.04)	-0.22*** (0.05)	-0.18*** (0.05)	-0.21*** (0.07)	-0.16*** (0.05)	-0.18*** (0.06)
SMB	0.00 (0.15)	0.00 (0.13)	-0.14 (0.10)	-0.14* (0.08)		-0.05 (0.18)	
HML	-0.50*** (0.14)	-0.44*** (0.15)	-0.33*** (0.11)	-0.23*** (0.09)			
Mom		0.14 (0.09)		0.16** (0.07)			
RMW			-0.45*** (0.16)	-0.48*** (0.13)			
CMA			-0.15 (0.17)	-0.22 (0.15)			
ME					-0.06 (0.17)		
IA					-0.62*** (0.19)		
ROE					-0.21 (0.16)		
Mgmt						-0.41** (0.20)	
Perf						0.10 (0.11)	
PEAD							0.39*** (0.11)
FIN							-0.26* (0.14)
Observations	384	384	384	384	384	384	384
Adjusted R ²	0.23	0.27	0.32	0.38	0.17	0.15	0.19

E.3 Rank-Weighted Strategy

Table E.3: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) in quintiles according their price deviation level. The zero-cost strategy goes long on the quintile associated with the lowest levels of price deviations and short on the bottom quintile associated with the highest levels. Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using the [Fama and French \(2015\)](#) factor model to calculate the CMVE portfolio. Values in parenthesis are [Newey and West \(1987\)](#) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	FF3	C4	FF5	FF6	q	SY4	DHS3
Constant	0.26*** (0.08)	0.25*** (0.07)	0.25*** (0.08)	0.24*** (0.08)	0.23*** (0.08)	0.20*** (0.06)	0.30*** (0.09)
MKT	-0.04* (0.02)	-0.04* (0.02)	-0.04 (0.03)	-0.03 (0.03)	-0.03 (0.03)	-0.01 (0.02)	-0.03 (0.03)
SMB	0.05 (0.07)	0.05 (0.07)	0.06 (0.06)	0.06 (0.06)		0.07 (0.07)	
HML	0.09* (0.05)	0.10** (0.05)	0.09 (0.05)	0.10** (0.05)			
Mom		0.02 (0.03)		0.02 (0.03)			
RMW			0.03 (0.08)	0.03 (0.08)			
CMA			-0.01 (0.06)	-0.02 (0.06)			
ME					0.06 (0.06)		
IA					0.08 (0.07)		
ROE					0.04 (0.05)		
Mgmt						0.14*** (0.05)	
Perf						-0.00 (0.03)	
PEAD							-0.10* (0.06)
FIN							0.05 (0.07)
Observations	384	384	384	384	384	384	384
Adjusted R ²	0.06	0.06	0.06	0.06	0.04	0.09	0.06

E.4 Monthly (Non-Overlapping) Observations

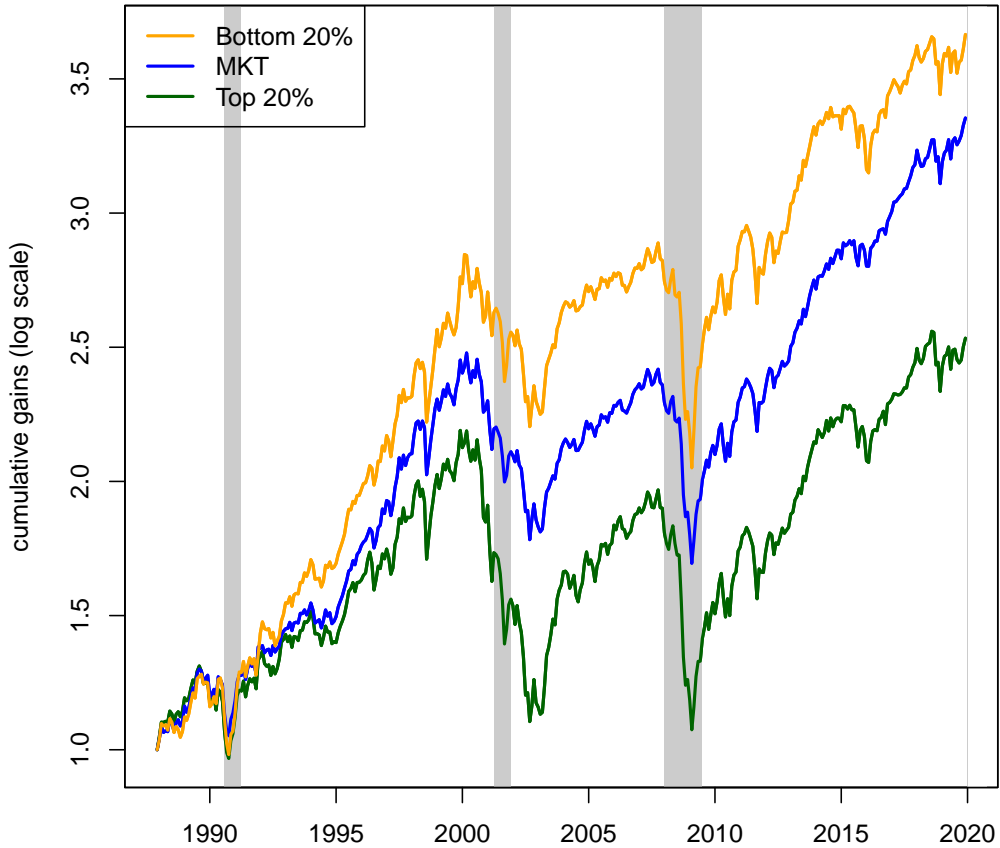


Figure E.4: FF5

Figure E.5: Anomaly Portfolios Rotation using Real-Time Price Deviations. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) according to their price deviation level. Bottom (top) 20% corresponds to the cumulative gains of a dynamic strategy that goes long on the 18 portfolios associated with the lowest (highest) level of price deviation. MKT is the performance of a static buy-and-hold strategy on the market portfolio in excess of the risk-free rate. Price deviations \hat{u} are computed as in equation (9). We report results for price deviations computed using the [Fama and French \(2015\)](#) factor model to calculate the CMVE portfolio. Shaded areas are NBER recessions. Monthly observations.

Table E.4: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in [Kozak et al. \(2020\)](#) in deciles according to their price deviation level computed in real-time using an expanding window from 1987 to 2018. The zero-cost strategy goes long on the top decile and short on the bottom decile. Values in parenthesis are [Newey and West \(1987\)](#) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	C4	FF6	q	SY4	DHS3
Constant	0.30*** (0.09)	0.31*** (0.10)	0.28*** (0.10)	0.25*** (0.09)	0.35*** (0.11)
MKT	-0.03 (0.03)	-0.04 (0.03)	-0.03 (0.04)	-0.01 (0.03)	-0.02 (0.05)
SMB	0.09 (0.12)	0.08 (0.09)		0.11 (0.11)	
HML	0.16** (0.08)	0.18** (0.07)			
Mom	-0.01 (0.04)	-0.01 (0.04)			
RMW		-0.02 (0.13)			
CMA		-0.03 (0.10)			
ME			0.10 (0.09)		
IA			0.14 (0.10)		
ROE			-0.01 (0.07)		
Mgmt				0.18** (0.07)	
Perf				-0.06 (0.05)	
PEAD					-0.13 (0.09)
FIN					0.05 (0.10)
Observations	384	384	384	384	384
Adjusted R ²	0.08	0.08	0.04	0.11	0.04

F A Model of Slow Adjustment to Information

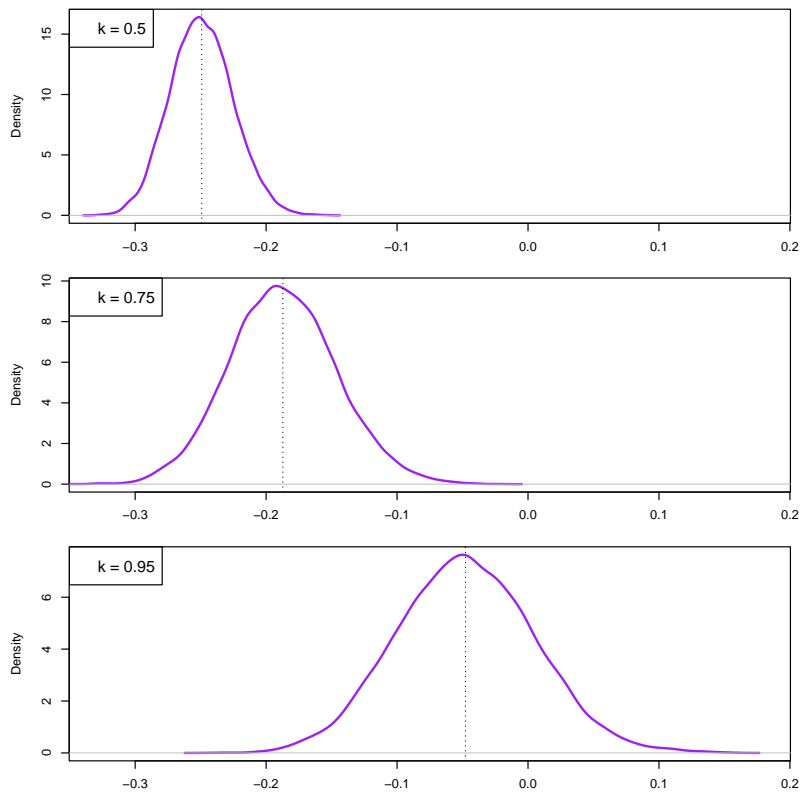


Figure F.1: Predictability Using Price Deviations and Slow Adjustment to Information. This figure shows ex-post densities for δ coefficients in specification (12) for different calibrations of the adjustment parameter k in equation (16). We calibrate R_t^V to the CMVE portfolio return over the period 1967–2019, with an annualized (percentage) mean of 1.23% and an annualized volatility of 1.12%. Prices are constructed as $V_{t+1} = V_t + R_{t+1}^V$. We then simulate 10000 times a sample of 636 observations of P_{t+1} using equation (16). The case $k = 1$ is full price adjustment to information.