Central Bank's Balance Sheet and Treasury Markets Disruptions

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Abstract

This paper presents a dynamic model of the Treasury market, accounting for recent disruptions. We investigate the impact of various shocks on repo rates and Treasury yields and examine policy implications. Our findings highlight the crucial role of the reserves-to-outstanding Treasury securities ratio as a predictor of market disruptions and emphasize the importance of central bank balance sheet policies in maintaining stability. The model offers valuable insights for policymakers and serves as a foundation for future research on regulation and policy interventions in government securities markets.

Keywords: Repo Markets, Liquidity Risk, Cash-future Basis, Shadow Banks, Balance Sheet Cost, Intraday Liquidity Requirements

JEL Classifications: E43, E44, E52, G12

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1 Introduction

Recent years have witnessed a series of disruptions in government securities markets across various jurisdictions, raising concerns about the stability of these markets and their implications for the broader financial system. Notable episodes include the September 2019 overnight repo rate volatility, the March 2020 Covid-19 crisis-induced Treasury market stress, and the September 2022 turmoil in the UK sovereign bond market. These events, characterized by unusual surges in yields, have significant implications for governments' funding capacity and financial stability.

Although a growing body of literature has examined these specific market dislocation events,[1] a unified framework encompassing these episodes and presenting a coherent picture of modern Treasury market fragility is still lacking. Such a framework is necessary for understanding the fundamental mechanisms behind these market disruptions and study the associated policy-relevant questions.

In this paper, we propose a dynamic and balance-sheet-consistent model of the Treasury market that accounts for the various disruptions observed in recent years, emphasizing the role of central banks' balance sheets and equilibrium portfolio allocations. Our model features three agents participating in repo markets (households, banks, and shadow banks), a treasury, a central bank, and two regulatory constraints imposed on banks: capital regulation, which increases the cost of their balance sheets, and an intraday reserves requirement, which necessitates the maintenance of positive reserve balances at all times during the day. Households provide repo through a dealer intermediary to leveraged non-bank investors, such as hedge funds, insurance companies, or pension funds. Due to the costly nature of banks' balance sheets, shadow banks optimally hold Treasuries financed in repo in equilibrium, which creates liquidity risk for those institutions as a consequence of a combination of transaction costs and the existence of repo supply shocks. We incorporate several empirically-relevant shocks to the Treasury and repo markets and analyze the response of repo rates and sovereign yields depending on agents' balance sheet positions at the time of the shock. Our dynamic setting ensures that portfolio allocations are consistent with shock and policy intervention anticipations.

The first contribution of this paper is to emphasize the combined impact of both the asset and liability sides of the central bank's balance sheet on the propensity for Treasury market distortions. On the asset side, a large portfolio of Treasury securities held by the

[1] See, for example, He, Nagel, and Song (2022), Avalos, Ehlers, and Eren (2019), Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin (2020), Correa, Du, and Liao (2020), and Copeland, Duffie, and Yang (2022) for studies on the repo market turmoil in September 2019 and March 2020, and Bank of England (2022) for the September 2022 turmoil in the UK sovereign bond market.

central bank diminishes the demand for repo financing from shadow banks, subsequently reducing the likelihood of a rate spike. Furthermore, on the liability side, a large central bank balance sheet with abundant reserves enables banks to utilize those reserves for repo lending when necessary, further decreasing the probability of a spike. Consequently, when the central bank undertakes a "Quantitative Tightening" (QT) operation to reduce the overall size of its balance sheet, it exerts simultaneous pressure on both demand and supply of repo financing. In brief, we identify the ratio of reserves to outstanding Treasury securities as the primary determinant of Treasury yields and repo rates, as well as a robust predictor of Treasury market disruptions.

Our second contribution involves examining how equilibrium Treasury holdings are influenced by the interplay of regulatory frictions and expectations of repo supply shocks. Specifically, we demonstrate that as repo supply shocks become less probable, shadow banks allocate a larger portion of their portfolio to Treasuries, thereby exploiting their lower balance sheet cost.[2] Notably, we also establish that this mechanism further implies that a decrease in shock probability leads to an increasing of the shock intensity as a result of agents' anticipations. This outcome is analogous to the "paradox of prudence" described in Brunnermeier and Sannikov (2014), where agents react to an exogenous decrease in risk probability by modifying their portfolios to return to the initial risk level. In this context, there exists a trade-off between the extensive and intensive margins of risk.

The third contribution of this paper is to characterize how the impact of a repo supply shock is distributed between its effects on repo rates and Treasury yields. In our model, since shadow banks are the marginal holders of Treasury securities, which they finance in repo markets, a shock to the cost of repo financing also influences Treasury yields. This mechanism is present in He, Nagel, and Song (2022) but does not account for why Treasury yields appear more reactive to shocks in some episodes such as March 2020, while the repo market absorbs the majority of the shock in other episodes like September 2019. Our model demonstrates that the existence of fixed trading costs implies that the market most affected depends on agents' expectations regarding the shock's duration. In particular, we find that short-lived shocks will have a greater impact on the repo market than on the Treasury market, as shadow banks are willing to pay high interest rates for a brief period to avoid incurring round-trip transaction costs on their Treasury holdings. In contrast, when shadow banks anticipate a long-lasting shock, it becomes less costly to

[2]This mechanism aligns with the empirical observations from and Avalos et al. (2019) finding an increased participation of relative-value hedge funds in Treasury markets, engaging in cash-future basis trades, or essentially providing warehousing for Treasuries while financing themselves overnight in the bilateral repo market.

pay a fixed transaction cost rather than a high repo rate over an extended period.[3]

The fourth contribution of this paper is to provide a unified framework that concurrently explains several prominent features of Treasury and repo markets, enabling a more precise comprehension of the mechanisms at play. Specifically, the model is capable of capturing the unique characteristics of intermediation shocks reflecting quarter-end events in repo markets, such as increased repo intermediation spreads, elevated reverse repo facility volumes, banks augmenting their net lending, and a decrease in bank reserves. The model also accounts for tax deadline shocks representing money market fund outflows around tax deadlines, which, in contrast, lead to stable intermediation spreads, no increase in reverse repo facility volumes, a rise in the TGA account, and an increase in net bank lending in repo markets accompanied by a decrease in bank reserves. In both scenarios, the model generates the empirical observation from Pozsar (2019) and Correa, Du, and Liao (2020) that banks act as a "lender-of-next-to-last-resort" by covering up for the gap in repo supply by draining their reserves down until reaching their intraday reserves constraint.

In addition to the aforementioned contributions, we explore the mechanisms behind these disruptions, allowing for the precise identification of the frictions necessary to explain the disruptions. For instance, we find that repo rates and Treasury yields can only increase beyond the interest on reserves following a repo supply shock with the strict combination of three frictions: banks' balance sheet costs, an intraday reserves requirement, and an active reverse repo facility. To put it differently, eliminating any of these three frictions would prevent repo rates from spiking upward. Another crucial insight derived from the model is that the drawdown of reserves following various types of shocks can be both helpful and an hindrance. Until reaching the intraday constraint, these reductions in reserves are beneficial, as they free up space on the balance sheet of banks, allowing them to lend in repo the exact amount needed for markets to clear without a surge in rates. However, the same drawdown becomes problematic once the intraday constraint is binding, as beyond that point, banks cannot lend more in repo. This duality highlights the complex interplay of factors driving market dynamics and emphasizes the need for a nuanced understanding of the mechanisms at work.

Lastly, the granularity of the framework also allows us to study how, depending on its design, a repo facility may help alleviate some of these shocks but not necessarily all of them. Our model highlights that the effectiveness of central bank operations depends on the interplay between the type of repo supply shock and the counterpart involved in the

[3]This insight supports the observation that repo markets were primarily affected in September 2019 around the tax-deadline shock, whereas Treasury market yields were more impacted in March 2020 following the Covid shock.

operation. Specifically, we find that a repo facility available only to banks is not effective in addressing repo intermediation shocks, which were common during quarter-ends in the latter half of the 2010s when foreign dealer banks reduced their balance sheets for window-dressing purposes. This outcome occurs because the increase in repo rates in this scenario stems from a growth in dealers' balance sheets and marginal intermediation costs, rather than a decrease in net repo supply. As a result, a repo facility proves helpful only if directly accessible to shadow banks, allowing the central bank to effectively act as an intermediary by simultaneously borrowing repo from households at the reverse repo facility and lending to shadow banks at the repo facility. Conversely, we find that repo rate spikes caused by a sharp decline in repo supply due to large corporations paying taxes, such as the event in mid-September 2019, can be effectively mitigated by a repo facility open solely to banks. In this case, banks can utilize their balance sheets to intermediate repos from the central bank to shadow banks, bridging the gap in supply.

Related Literature Our paper complements the literature highlighting the costly nature of intermediation and the frictions it introduces in Treasury markets. Munyan (2015) and Du, Tepper, and Verdelhan (2018) demonstrate that these frictions manifest as quarter-end effects in repo and FX swap rates, as foreign dealers reduce balance sheet size to comply with leverage ratio regulations. Andersen, Duffie, and Song (2019) reveal the implications of such regulations on funding value adjustments (FVAs) for major dealers, identifying debt overhang costs for shareholders. Correa, Du, and Liao (2020) illustrate how banks engage in "reserves-draining" intermediation to lend in money markets following quarter-end shocks to bypass these constraints. Klingler and Syrstad (2021) provide a comprehensive empirical inquiry across the many factors influencing repo rates. Relevant to our object of study, these constraints have also been found to impact the pricing of Treasury-based arbitrage trades, as seen in cash-future basis (Barth and Kahn, 2021), swap spreads Jermann (2020), and CIP violations Du, Tepper, and Verdelhan (2018). Boyarchenko, Giannone, and Santangelo (2018) further show how dealer balance sheet costs affect repo pricing and arbitrage funding for non-banks. He, Nagel, and Song (2022) connect these findings to the extraordinary increase in Treasury yields observed in March 2020 through a preferred habitat model whereby dealers incur an increased cost of holding Treasuries when absorbing fire sales from other sectors.

In addition, another strand of the literature has pointed to intraday liquidity stress tests and their effect on banks' ability to serve as a stabilizing force in repo markets or "lender-of-next-to-last-resort" and increased effectively increased the financial system's reliance on reserves. Pozsar (2019) identifies potential liquidity concerns related to Treasury settlements and excess balance sheet normalization. Gagnon and Sack (2019) discuss policy options to address these issues, such as a standing repo facility, higher reserve levels, and explicit directives to control the repo rate. In particular, the repo turmoil of September 2019 has been partially attributed to hedge funds' use of repo to finance Treasury holdings by Avalos, Ehlers, and Eren (2019). Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin (2020) provide a detailed account of the event, highlighting the role of reserves and interbank market frictions, while Anbil, Anderson, and Senyuz (2021) emphasize the role of trading relationships. d'Avernas and Vandeweyer (2022) and Yang (2022) model the impact of intraday liquidity constraints on money market dislocations, finding that non-linearities can generate significant spikes in repo rates. Copeland, Duffie, and Yang (2022) emphasize the role of reserves in alleviating intraday repo payment timing stresses. Acharya and Rajan (2022) and Acharya, Chauhan, Rajan, and Steffen (2023) identify a ratchet effect on banks' liquidity, implying that removing reserves during Quantitative Tightening exposes banks to increased liquidity risk.

Our study distinguishes itself from the cited literature by presenting a comprehensive framework that encompasses both capital and liquidity regulation in a dynamic context, which includes a complete role for the central bank balance sheet. This framework enables us to understand the mechanisms connecting frictions to policy and offers a unique set of implications absent in existing work. In particular, our framework highlights the important role of both sides of the central bank balance as a stabilizing force in Treasury markets, as well as how anticipation of shocks and policy interventions may affect steady-state portfolio choices and, ultimately, the likelihood and magnitude of these shocks as detailed previously.

2 Model

This section presents our model of repo and Treasury bond markets. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions and assume that all stochastic processes are adapted. The economy evolves in continuous time with $t \in [0, \infty)$ and is populated by a continuum of traditional banks (with a dealer subsidiary), shadow banks, and households, as well as the Treasury and a central bank. Figure 1 depicts the balance sheets of the different sectors in the economy. The Treasury issues Treasury bonds against future tax liabilities and maintains a balance in the Treasury General Account (TGA); the central bank holds outstanding Treasury bonds and issues reserves to the banking sector; and households invest their wealth and future tax in repo and deposits. Traditional banks hold securities, reserves, and some Treasury bonds leveraged with deposits. Traditional banks can also either lend repo or lever with repo. Shadow banks hold Treasury bonds financed with repo.

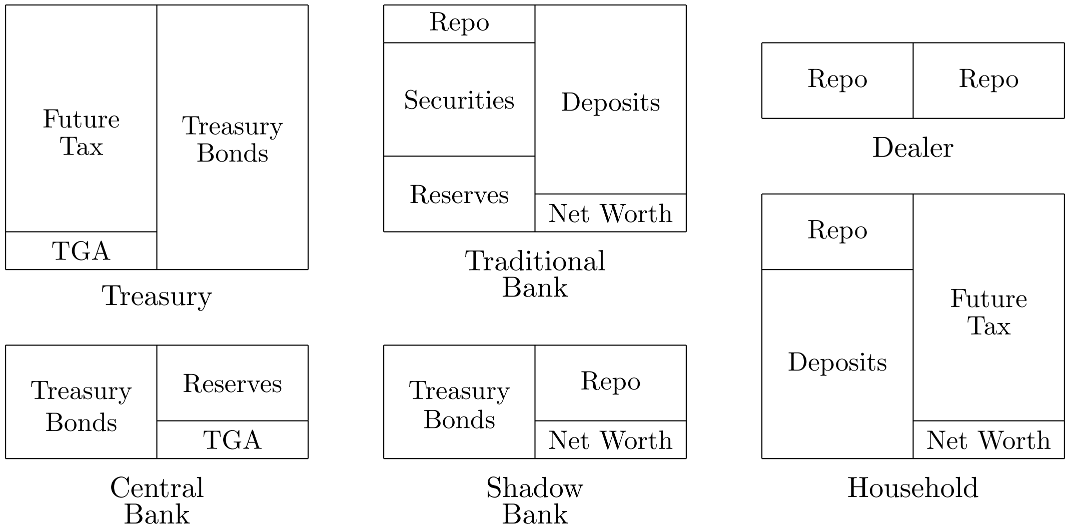

Figure 1: Chart of Sectors' Balance Sheets

Main Frictions Four economic forces and frictions play an important role in our framework. First, households, which includes firms investing in money market funds, have preferences regarding the composition of their portfolio of liquid assets, such as repos and traditional bank deposits. Second, traditional banks and their dealer subsidiaries are subject to a balance sheet cost, which could arise as debt-overhang costs when issuing additional equity in a setting where bank debt carries some credit risk (Andersen, Duffie, and Song, 2019). Thirdly, Liquidity Stress Test (LST) regulations mandate that traditional banks maintain a buffer of reserves at all times during the day, which limits their ability to lend in repo markets to shadow banks.[4] Fourth, buying or selling Treasuries incurs a transaction cost. Our analysis aims at understanding how these forces interact together to explain recent events in Treasury and repo markets.

2.1 Environment

Preferences Bankers have logarithmic preferences over their consumption rate c_t of their net worth n_t with a time preference ρ:

$$\mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} \log(c_u n_u) du \right]. \tag{1}$$

[4]See d'Avernas and Vandeweyer (2022).

Households further value liquidity services provided by holding repo and deposits,

$$\mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} \left(\log(c_u n_u) + \beta \log(h(w_u^p, w_u^d, \alpha_u) n_u) \right) du \right], \tag{2}$$

where h is a Cobb-Douglas aggregator of deposits and repo portfolio weights w_t^d and w_t^h,

$$h(w_t^p, w_t^d, \alpha_t) = (w_t^d)^{\alpha_t} (w_t^p)^{1-\alpha_t}. \tag{3}$$

The parameter α_t corresponds to the preference of households for holding repo relative to deposits and follows

$$d\alpha_t = \begin{cases} (\alpha^s - \alpha_t) dN_t & \text{if} \quad \alpha_t \neq \alpha_s, \\ (\alpha' - \alpha_t) dN_t & \text{if} \quad \alpha_t = \alpha_s, \end{cases} \tag{4}$$

where dN_t is a Poisson process with intensity $\lambda_t > 0$ and α' is a random variable independent and identically distributed according to a continuous uniform distribution between $\underline{\alpha}$ and 1. Thus, the economy features a steady state preference parameter α^s. Upon the arrival of a Poisson shock in state $\alpha_t = \alpha^s$, the household preference parameter α_t takes on a new random value $\alpha' \in [0, 1]$. Upon the arrival of a Poisson shock in state $\alpha_t \neq \alpha^s$, the α_t reverts to α^s. The Poisson intensity λ_t is equal to λ if $\alpha_t = \alpha^s$ and equal to λ' otherwise. Hence, λ represents the likelihood of a repo supply shock while λ' determines the expected duration of the shock. All agents have perfect information on the Poisson process.

Technology There is a unit of risk-free capital producing a flow of real output, y, with constant productivity. We assume that capital can only be held by traditional banks to instead focus our analysis on the transactions of Treasuries, repos, and reserves.

Overlapping Generations In order to abstract from sectorial wealth dynamics, we assume that agents are short-lived in an overlapping generations framework such that the wealth shares of the traditional bank sector, shadow bank sector, and household sector—respectively given by $n_t/N_t, \overline{n}_t/N_t$, and n_t^h/N_t, where $N_t = n_t + \overline{n}_t + n_t^h$—are constant over time.[5]

Treasury The Treasury issues bonds against the future tax liabilities of households and is responsible for administrating redistributive lump-sum tax policies. The net present

[5]Thus, ρ is the effective time discount rate net of the death rate.

value of future tax liabilities must equal the outstanding amount of Treasuries: $\tau_t^h n_t^h + a_t = b_t$, where b_t is the quantity of bonds issued, a_t is the size of the TGA account, and τ_t^h is the future tax liability of households per unit of wealth.

Central Bank The central bank holds Treasury bonds, $\underline{b}_t$, financed by reserves held by banks m_t and in the Treasury General Account (TGA) a_t. The underline notation differentiates the central bank's holdings of Treasury bonds $\underline{b}_t$ from the bonds issued by the Treasury b_t. The central bank can also lend repo at the repo facility, rp_t, and borrow repo at the reverse repo facility, rrp_t. Thus, the balance sheet constraint for the central bank is given by

$$\underline{b}_t + rp_t = m_t + a_t + rrp_t. \tag{5}$$

The central bank determines the interest rates at which the central bank lends at the repo facility and at which it borrows at the reverse repo facility. These rates are denoted by $r_t^{rp} > r_t^m$ and $r_t^{rrp} < r_t^m$, respectively. Thus, in the presence of facilities, it is the net quantities lent $rp_t - rrp_t$ that determine the quantity of reserves in the banking sector m_t as a residual. In addition, for simplicity, we assume that the central bank always operates with zero net worth and instantaneously transfers all seigniorage revenues to the Treasury.

Dealers We make the assumption that the repo market is fully intermediated, meaning that households exclusively invest in repos through bank dealer subsidiaries rather than directly with traditional or shadow banks.[6] In order to account for fluctuations in intermediation capacities, our model includes a foreign dealer possessing a balance sheet size denoted by f_t. This foreign dealer operates alongside the traditional bank dealer subsidiary to intermediate repos between households and banks. We interpret variations in the foreign dealer's balance sheet size as a representation of phenomena such as the window-dressing practices observed at quarter ends among foreign dealers, as described by (Munyan, 2015). In this way, our model seeks to provide a clear and comprehensive understanding of the underlying mechanisms in the repo market.

[6] This assumption aligns with the actual institutional framework in the US, where the vast majority of repo transactions are effectively intermediated by securities dealers. For further institutional details on repo markets, refer to the work of Copeland, Martin, and Walker (2014).

2.2 Agent Problems

Traditional Banks Traditional banks face a Merton's (1969) portfolio choice problem augmented with transaction costs and a balance sheet cost. Traditional banks maximize their lifetime expected logarithmic utility:

$$\max_{\{c_u \geq 0, w_u^k \geq 0, w_u^b \geq 0, w_u^m \geq 0, w_u^p, w_u^x \geq 0, w_u^d \geq 0\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} \log(c_u n_u) du \right], \tag{6}$$

subject to the law of motion of wealth:

$$\begin{aligned} dn_t = & \big(w_t^k r_t^k + w_t^b r_t^b + w_t^m r_t^m + w_t^p r_t^p + w_t^x (r_t^p - r_t^{pt}) - w_t^d r_t^d - c_t \big) n_t dt \\ & - \frac{\chi}{2} \ell_t^2 n_t dt + (e^{-\nu(|dw_t^b|)} - 1) n_t, \end{aligned} \tag{7}$$

the balance sheet constraint:

$$w_t^k + w_t^b + w_t^m + w_t^p = 1 + w_t^d, \tag{8}$$

and the LST constraint:

$$w_t^p \leq \kappa w_t^m. \tag{9}$$

Traditional bankers choose their consumption rate c_t, their portfolio weights for capital w_t^k, Treasury bonds w_t^b, reserves w_t^m, and deposits w_t^d given their respective interest rates r_t^k, r_t^b, r_t^m, and r_t^d. Traditional banks choose the portfolio weight for repo w_t^p given the interest rates in the bilateral repo market r_t^p. Traditional banks select the size of their dealer balance sheet w_t^x to profit from the spread between the bilateral and triparty repo rates, but incur a balance sheet cost. The balance sheet cost is quadratic in bank leverage ℓ_t with a cost parameter χ where

$$\ell_t \equiv w_t^d - \min\{0, w_t^p\} + w_t^x. \tag{10}$$

Banks are subject to transaction costs when trading Treasury bonds. The wealth after a transaction n_t is equal to

$$n_t = n_{t\text{-}} e^{-\nu |dw_t^b|}, \tag{11}$$

where ν is the transaction cost and $t-$ is the time prior to the transaction.[7][8]

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[7]Thus, $dw_t^b = w_t^b - w_{t\text{-}}^b$.

[8]We abstract from potential transaction costs on capital as capital can only be held by traditional

Finally, traditional banks are subject to the LST constraint, which limits repo lending to a fraction κ of reserve holdings.

Shadow Banks Shadow banks face the same problem as traditional banks, but without balance sheet cost. Shadow banks maximize their lifetime expected logarithmic utility:

$$\max_{\{\overline{c}_u \geq 0, \overline{w}_u^b \geq 0, \overline{w}_u^p\}_{u=t}^{\infty}} \left[\int_t^{\infty} e^{-\rho(u-t)} \log(\overline{c}_u \overline{n}_u) du \right], \tag{12}$$

subject to the law of motion of wealth:

$$d\overline{n}_t = \left(\overline{w}_t^b r_t^b - \overline{w}_t^p r_t^p - \overline{c}_t \right) \overline{n}_t dt + (e^{-\nu |d\overline{w}_t^b|} - 1)\overline{n}_t, \tag{13}$$

and the balance sheet constraint:

$$\overline{w}_t^b = 1 + \overline{w}_t^p. \tag{14}$$

Shadow banks choose holdings of Treasury bonds $\overline{w}_t^b$ and repo financing $\overline{w}_t^p$ given the respective interest rates r_t^b and r_t^p. As traditional banks, shadow banks also incur a similar transaction cost when purchasing or selling Treasury bonds.

Households Households maximize lifetime utility of consumption and liquidity benefits:

$$\max_{\{c_u^h \geq 0, w_u^{h,d} \geq 0, w_u^{h,p} \geq 0\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} \left(\log(c_u^h n_u^h) + \beta \log(h(w_u^{h,p}, w_u^{h,d}, \alpha_u)) \right) du \right], \tag{15}$$

subject to the law of motion of wealth:

$$dn_t^h = \left(w_t^{h,p} r_t^{p,t} + w_t^{h,d} r_t^d - c_t^h - r_t^\tau \right) n_t^h dt, \tag{16}$$

and the balance sheet constraint:

$$w_t^{h,p} + w_t^{h,d} = 1 + \tau_t^h. \tag{17}$$

Households choose consumption c_t^h and their portfolio holdings of repo $w_t^{h,p}$ and $w_t^{h,d}$ given their liquidity preference α_t and pay lump-sum taxes $r_t^\tau n_t^h$.

banks and focus our analysis on the transaction of Treasury bonds. Allowing only banks to hold capital is without loss of generality as long as the transaction cost on capital is larger than the transaction cost on Treasury bonds.

Treasury Budget Constraint The budget constraint for the Treasury is given by

$$r_t^b b_t = r_t^\tau n_t^h + r_t^b \underline{b}_t + r_t^{rp} rp_t - r_t^m m_t - r_t^{rrp} rrp_t. \tag{18}$$

To pay interest on Treasury bonds, the Treasury collects taxes from households and seigniorage revenues from the central bank.

2.3 Solving

We provide a definition for a Markov equilibrium, make further assumptions to restrict the set of equilibria we are interested in, and derive first order conditions. We assume that central bank policies $\{\bar{b}, r^{rp}, r^{rrp}\}$, Treasury policies $\{b, a\}$, and foreign dealer balance sheet size f are constant over time. The state space of the economy is given by household liquidity preference α_t and, because there is a transaction cost to sell or purchase Treasury bonds, the portfolio weights on Treasuries $\{w_{t\text{-}}^b, \overline{w}_{t\text{-}}^b\}$. Thus, we define the state space vector as $\mathbf{x}_t \equiv \{\alpha_t, w_{t\text{-}}^b, \overline{w}_{t\text{-}}^b\}$. In Section 4, we solve for the static equilibrium for different monetary policy decisions assuming that $\mathbf{x}_t$ is constant. In Section 3 and 5, we solve for the dynamic equilibrium given the law of motion for the household liquidity preference parameter α_t.

Definition 1. *Given central bank policies $\{\bar{b}, r^{rp}, r^{rrp}\}$, Treasury policies $\{b, a\}$, and foreign dealer balance sheet size f, a **Markov equilibrium** $\mathcal{M}$ in x_t is a set of functions $g_t = g(x_t)$ for (i) interest rates $\{r_t^k, r_t^b, r_t^m, r_t^p, r_t^{tp}, r_t^d\}$; (ii) individual controls for traditional banks $\{w_t^k, w_t^b, w_t^m, w_t^p, w_t^x, w_t^d, c_t\}$, shadow banks $\{\overline{w}_t^b, \overline{w}_t^p, \bar{c}_t\}$, and households $\{w_t^{h,p}, w_t^{h,d}, c_t^h\}$ such that:*

1. *Agents' optimal controls (ii) solve their respective problems given prices (i).*
2. *The balance sheet constraint of the central bank is satisfied.*
3. *The balance sheet constraint and budget constraint of the Treasury are satisfied.*

4. Markets clear:

(a) output:	$c_t n_t + \overline{c}_t \overline{n}_t + c_t^h n_t^h = y,$
(b) capital:	$w_t^k n_t = 1,$
(c) Treasury bonds:	$\overline{w}_t^b \overline{n}_t + w_t^b n_t = b - \underline{b},$
(d) reserves:	$w_t^m n_t = m_t,$
(e) triparty repo:	$w_t^{h,p} n_t^h = w_t^x n_t + rrp_t + f,$
(f) bilateral repo:	$(w_t^x + w_t^p) n_t + rp_t + f = \overline{w}_t^p \overline{n}_t,$
(g) deposits:	$w_t^d n_t = w_t^{h,d} n_t^h.$

5. The law of motion for x_t is consistent with agents' perceptions.

Due to logarithmic preferences, all agents of the same type choose the same set of control variables when stated as a proportion of their net worth, irrespective of the anticipated future realizations of the state variables. Hence, we only have to track the distribution of wealth between agent types and not within types at a given point in time.

Equilibrium Restrictions We restrict our analysis to central bank and Treasury policies such that, in equilibrium, traditional and shadow banks are leveraged: $\ell_t > 0$ and $\overline{w}_t^p > 0$; the traditional bank dealer subsidiary has a positive balance sheet size: $w_t^x > 0$; and reserves are in strict positive supply: $m > 0$.

First-order Conditions Applying the maximum principle, we derive the first-order conditions for all agents. With logarithmic preferences, every agent always consumes a fixed proportion ρ: $c_t = \overline{c}_t = c_t^h = \rho$.

The first-order conditions for reserves, bilateral repo, and triparty repo of traditional banks are given by

$$r_t^k - r_t^d = \chi \ell_t, \tag{19}$$

$$r_t^k - r_t^m = \kappa \vartheta^m, \tag{20}$$

$$r_t^k - r_t^p \begin{cases} = -\vartheta^m & \text{if } w_t^p > 0, \\ \in [-\vartheta^m, \chi \ell_t] & \text{if } w_t^p = 0, \\ = \chi \ell_t & \text{if } w_t^p < 0, \end{cases} \tag{21}$$

$$r_t^p - r_t^{pt} = \chi \ell_t, \tag{22}$$

where ϑ^m is the shadow price of the LST constraint. In Equation (19), traditional banks equalize the marginal benefits of issuing deposits (return on capital) to its marginal cost (the marginal increase in the balance sheet cost). As shown in Equation (20), the marginal cost is the forgone interest of holding a unit of reserves, which must equate to the marginal benefit of loosening the LST constraint. Similarly, in Equations (21), if a traditional bank invest in repo, the bilateral repo rate needs to compensate for the tightening of the LST constraint. If a traditional bank funds itself in repo, the repo rate must be sufficiently low to compensate for the increase in the balance sheet cost, as for deposits. In Equation (22), traditional banks require a spread between bilateral and triparty repo to further compensate for the balance sheet cost incurred by intermediating repo from households to shadow banks at the dealer subsidiary.

Next, the households' first-order condition for their relative holdings of triparty repo and deposit is given by

$$r_t^{pt} - r_t^d = \rho \frac{\beta}{1+\beta} \left(\frac{\alpha_t}{w_t^{h,d}} - \frac{1-\alpha_t}{w_t^{h,p}} \right). \tag{23}$$

Finally, traditional and shadow banks trade off the marginal benefit of holding Treasury bonds funded with deposits or repos, given that they must pay a cost on these transactions. This decision ultimately depends on current and future rates and the stochastic process for the state variables. We postpone these considerations and provide a full characterization of the dynamic problem in Section 5.

State Space Partitioning We define four disjoint sets of equilibria corresponding to changes in the pricing of the bilateral repo.

Definition 2. *Let $\mathcal{A}$ be the set of* **arbitraged** *repo market equilibria, defined as* $\{\mathcal{M}(x) \in \mathcal{A} \mid w_t^p < 0\}$.

Definition 3. *Let $\mathcal{S}$ be the set of* **segmented** *repo market equilibria, defined as* $\{\mathcal{M}(x) \in \mathcal{S} \mid w_t^p = 0 \text{ and } r_t^p < r_t^m\}$.

Definition 4. *Let $\mathcal{U}$ be the set of* **unconstrained** *repo market equilibria, defined as* $\{\mathcal{M}(x) \in \mathcal{U} \mid r_t^p = r_t^m\}$.

Definition 5. *Let $\mathcal{C}$ be the set of* **constrained** *repo market equilibria, defined as* $\{\mathcal{M}(x) \in \mathcal{C} \mid r_t^p > r_t^m\}$.

We define as *arbitraged* the set of equilibria in which traditional banks are borrowing in bilateral repos; as *segmented* the ones in which traditional banks are not marginal

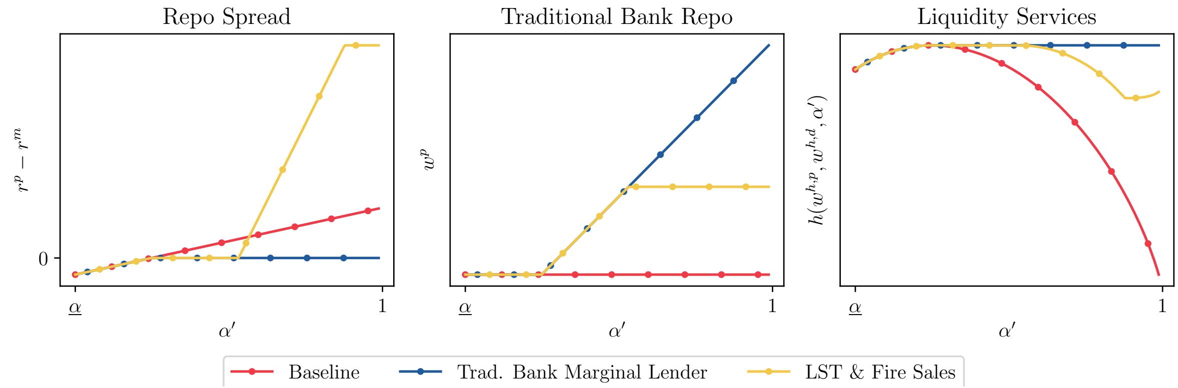

Figure 2: Simple Dynamic Model: Repo Supply Shocks. The Figure provides a graphical representation of repo spreads, traditional banks repo lending, and households liquidity services as a function of the shock parameter α' for a numerical example of a given equilibrium under three different restrictions: (1) in red is a baseline case with no feasible repo lending or Treasury sale, (2) in blue is a model allowing banks to lend in repos without a LST constraint (3) in yellow introduces the LST constraint and a finite transaction cost on Treasuries.

in bilateral repos due to the balance sheet cost; as *unconstrained* the ones in which traditional banks are net lenders of repos and the LST constraint is not binding; and as *constrained* the ones in which traditional banks are constrained by the LST regulation. All proofs of lemmas and propositions are relegated to Appendix A.

3 Dynamics in a Simple Setting

To better understand the model's implications for Treasury markets, we first present a set of results within a simplified framework, where we replace the balance sheet cost assumption with the stricter constraint that banks cannot borrow in repo. Consequently, the size of the dealers' balance sheet is irrelevant and we do not need to distinguish between the Triparty and bilateral repo markets. In Section 5, we gradually relax these assumptions and demonstrate that the key findings persist in more a realistic setting. In the full model, the balance sheet cost plays a similar role to the strict no-repo-borrowing assumption assumed here as it renders borrowing in repo for financing Treasury inventories increasingly expensive for banks.

Assumption 1. *Traditional banks are not subject to a balance sheet cost and are unable to borrow in repo:* $\chi = 0$ *and* $w^p_t \geq 0$.

Given the monetary policy $\underline{b}$ (without repo or reverse repo facilities) and fiscal policies b, a, we analyze the dynamic equilibrium of the economy in response to shocks to the liquidity preference parameter α of households. When α increases, households prefer to hold less repo, leading to either traditional banks covering the resulting funding gap of shadow banks by providing more repo, or an increase in the repo-deposit spreads $r^p - r^d$ to accommodate households retaining its original portfolio despite the change in preferences.

Banks as Lender-of-Next-to-Last-Resort Figure section 3 illustrates the adjustments in repo spread $r^p - r^m$, the quantity of repo lending by traditional banks w^p, and the value of liquidity services h as a function of the shocked state α'. The red lines represent a benchmark case where traditional banks are unable to adjust their quantity of repo, necessitating rate adjustments to compensate households for maintaining the fixed composition of repo and deposits. The blue line scenario allows banks to become marginal lenders in the repo market without being subject to the LST constraint. When traditional bank repo lending is not constrained by liquidity stress testing, the economy responds to the negative repo preference shock by having banks borrow more in deposits from households and intermediate those funds to shadow banks through repo, thus optimizing the composition of households' portfolios. However, once the LST constraint becomes binding, banks can no longer lend in repos, and the equilibrium requires households to absorb a suboptimal mix of repos and deposits, leading to increased repo spreads. For a finite transaction cost, repo spreads can increase up to a point where it becomes more profitable for shadow banks to pay the transaction cost and sell Treasuries to traditional banks to reduce their balance sheet size, rather than paying the prohibitive repo rate. This Treasury fire sales provides an outside option for shadow banks and caps the level at which repo rates can move. We discuss below the dynamics of this fire-sale decision, which on agents' portfolio decisions in anticipation of equilibrium shock frequency, intensity, and duration.

Paradox of Prudence in Treasury Markets In our model, the intensity of repo and Treasury yield movements is endogenous and depends on ex-ante portfolio allocations. We find this feedback loop connects the frequency of the shock to its intensity. Proposition proposition 1 formalizes this insight.

Proposition 1. *Lower shock frequency λ results in a higher probability of a Treasury yield spike and larger expected Treasury yield spikes, conditional on the arrival of a repo supply shock:*

$$\frac{\partial}{\partial \lambda}\mathcal{P}(\alpha' \in \mathcal{F}) < 0, \tag{24}$$

$$\frac{\partial}{\partial \lambda}\left(\mathbb{E}[r^b(\alpha')] - r^b(\alpha^s)\right) < 0. \tag{25}$$

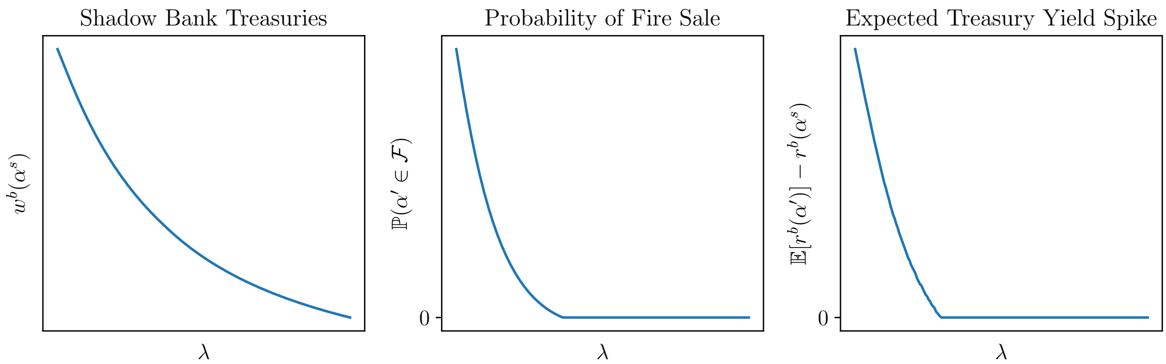

Figure 3: Simple Dynamic Model: Repo Supply Shock Frequency. Repo supply shock frequency is denoted by λ, the intensity of the Poisson process of shocks from the normal state. As supply shock frequency decreases, the risk for rising repo rates due to lower repo supply decreases, to which shadow banks respond by increasing Treasury holdings in the normal state. These increased shadow bank Treasury bond holdings lead to higher probabilities for fire sales and high expected Treasury yield spikes.

Furthermore, these increases are facilitated by larger Treasury bond holdings by shadow banks in the normal state:

$$\frac{\partial \overline{w}^b(\alpha^s)}{\partial \lambda} < 0. \tag{26}$$

Proposition 1 demonstrates that when the frequency of shocks decreases, shadow banks optimally react by increasing their leveraged Treasury holdings in the steady state, which eventually results in a larger expected Treasury yield surge once the Poisson shock hits. This result is akin to the "Paradox of Prudence" in Brunnermeier and Sannikov (2014) with the additional feature that agents are trading off risk frequency for risk intensity. These findings are illustrated in Figure 3.

Shock Duration: Repo Rates vs Treasury Yields We next consider how expectation about the duration of the shock affects equilibrium prices upon entering in the shock state. In particular, Proposition 2 shows that short-lived shocks result in high repo rate spikes and low Treasury yield surges in expectation, while long-lived shocks result in high Treasury yield spikes and low repo spikes. When a repo supply shock is expected to be short-lived, shadow banks are willing to pay a high repo rate for a short period of time to avoid paying costly round-trip transaction fees, which reduces the likelihood of a fire sale. In contrast, if a shock is expected to last for a long period of time, shadow banks will prefer to sell Treasuries rather than aving to pay high repo rates for a potentially long period of time. Shadow banks reducing Treasury bond holdings reduces repo demand, allowing repo rates to decline in the shocked states. This asymmetry might help elucidate why repo rates experienced a dramatic spike in September 2019 while Treasury yields remained relatively stable, as opposed to the events of 2020 when Treasury yields rose sharply but repo rates did not surge significantly. The September 2019 spike was likely highly transitory and attributable to the tax deadline, while the March 2020 shock occurred amid the COVID-19 pandemic, characterized by considerably greater uncertainty regarding its duration and long-term impact.

Proposition 2. *The expected duration of the repo supply shock affects equilibrium. In particular, shorter shock duration leads to a reduced probability of fire sales, conditional on Poisson supply shock arrival:*

$$\frac{\partial}{\partial \lambda'} \mathcal{P}(\alpha' \in \mathcal{F}) < 0. \tag{27}$$

Furthermore, shorter shock duration leads to lower expected Treasury yield spikes and higher expected repo spikes:

$$\frac{\partial}{\partial \lambda'} \mathbb{E}[r^b(\alpha')] - r^b(\alpha^s) < 0, \tag{28}$$

$$\frac{\partial}{\partial \lambda'} \mathbb{E}[r^p(\alpha')] - r^p(\alpha^s) > 0. \tag{29}$$

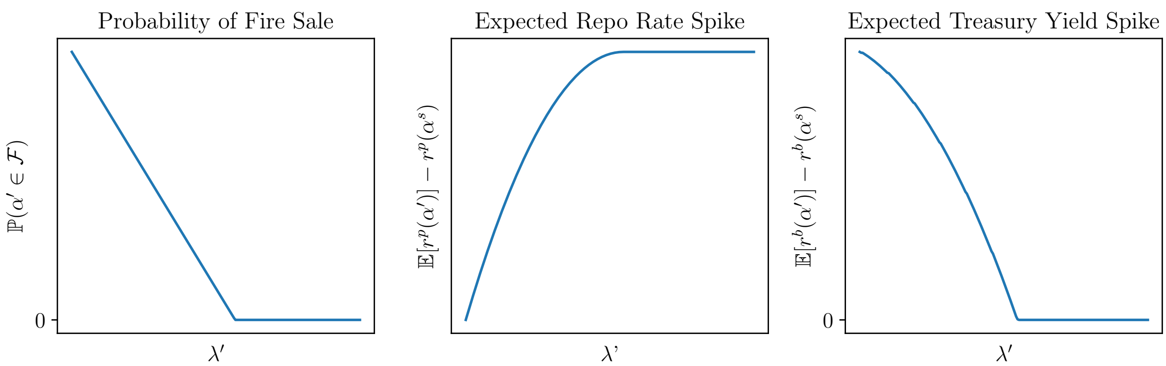

Figure 4: Simple Dynamic Model: Repo Supply Shock Duration. Shock duration is inverse to λ', the intensity of the Poisson process determining the return from a shock to the normal state. In particular, the expected duration of a shock is equal to $1/\lambda'$. Hence shock duration increases from the right to the left of the subplots. When shocks are short-lived, shadow banks are willing to take short-lived negative profits to avoid paying costly round-trip transaction costs. However, if shocks are expected to be very long lived, then a transaction becomes optimal to avoid negative spreads for long periods of time. Fire sales decrease repo demand within shocked states and hence the expected repo rate spike.

4 Comparative Statics Analysis

In this section, we present key insights from the model as a comparative statics exercise that abstracts from dynamics: state variables are constant over time ($\mathbf{x}_t = \mathbf{x} \ \ \forall t$).[9] Our primary focus is to understand how equilibrium prices and holdings evolve as a function of variables typically implicated in causing Treasury market disruptions, such as the size of foreign repo intermediation, the size of the TGA account, the size of the central bank balance sheet, and Treasury bond issuance. This approach enables an initial exploration of the effects of shocks, such as quarter ends, tax deadlines, quantitative tightening or easing, and fiscal expansion. In Section 5, we incorporate dynamic shocks back into the model and examine the impact of the expected severity and duration of these shocks on repo and Treasury markets in a fully dynamic setting. We begin by introducing a baseline without repo or reverse repo facilities, and subsequently demonstrate how the introduction of these facilities alters the baseline equilibria.

4.1 Demand and Supply in Repo Markets

In this section, we provide an initial examination of the influence of repo demand and supply imbalances on determining equilibrium interest rates across various markets, both with and without central bank facilities. These findings will be used below when investigating the impact of specific shocks on equilibrium prices and allocations.

Without Central Bank Facilities We first characterize equilibrium interest rates in the absence of central bank facilities. That is, the central bank set interest rates for its facilities that are never binding $r^{rp} = \infty$ and $r^{rrp} = -\infty$.

Lemma 1. *In an economy without facilities, liquidity services are maximized if and only if traditional banks are net lenders of repo and the LST constraint is not binding; that is,* $\ell(x^\star) > \ell(x) \ \forall x, x^\star$ *if and only if* $\mathcal{M}(x^\star) \in \mathcal{U}$ *and* $\mathcal{M}(x) \notin \mathcal{U}$*.*

Proposition 1 shows that whenever traditional banks are net borrowers of repos or the LST constraint is binding, market forces cannot adjust in order to supply the first-best allocation of liquid assets to households. In that case, repo rates deviate from the interest on reserves in order to compensate households for having to provide more or less repo funding than their optimal portfolio composition. Lemma 2 demonstrates that high quantities of Treasury bonds outstanding $b_t - \underline{b}_t$, low future tax liabilities $\tau^h n^h$, low

[9]Implicitly, we assume that α_t is constant over time and the transaction cost κ is sufficiently high to keep the allocation of Treasury bonds constant over time ($w^b_t = w^b_{t\text{-}} = w^b$ and $\overline{w}^b_t = \overline{w}^b_{t\text{-}} = \overline{w}^b$).

supply of reserves m, or high household preference for deposits α can lead to an upward deviation of repo rates from the interest on reserves, and vice versa.

Lemma 2. *In the absence of repo and reverse repo facilities:*

(i) $r^p > r^m$ *if and only if*

$$\underbrace{b - \underline{b} - w^b n - \overline{n}}_{\textit{s-bank repo demand}} > \underbrace{(1-\alpha)(1+\tau^h)n^h + \kappa m}_{\textit{highest repo supply at optimum}}; \tag{30}$$

(ii) $r^p < r^m$ *if and only if*

$$\underbrace{b - \underline{b} - w^b n - \overline{n}}_{\textit{s-bank repo demand}} < \underbrace{(1-\alpha)(1+\tau^h)n^h}_{\textit{lowest repo supply at optimum}}. \tag{31}$$

Lemma 2 tells us that when the demand for repo from shadow banks, $b - \underline{b} - w^b n - \overline{n}$, is high, traditional can provide the marginal funds, up to the LST constraint (κm). When the LST constraint is binding, repo rates must increase to incentivize households to provide more repo than their optimal portfolio allocation, which is given by $(1 - \alpha)(1 + \tau^h)n^h$. Conversely, when the supply of repo is too high, repo rates must drop to either incentivize households to hold less repo or compensate the balance sheet cost for traditional banks to fund themselves with repo.

With Central Bank Facilities We now characterize equilibria in settings with central bank repo and reverse repo facilities. For parsimony, we rule out the two extreme cases in which households would find it profitable to hold all their assets entirely at the reverse repo facility and in which traditional banks would be funded entirely at the repo facility. To provide policy-relevant insights on the design of facilities, we consider two types of standing repo facilities: one open to all agents (including shadow banks) and one open to banks only.

Lemma 3. *A reverse repo facility with rate* r^{rrp} *acts as a floor in the triparty repo market:* $r^{pt} \geq r^{rrp}$*. A repo facility with rate* r^{rp} *that is open only to traditional banks acts as a ceiling in the triparty repo market:* $r^{pt} \leq r^{rp}$*. A broad-access repo facility open to both traditional and shadow banks acts as a ceiling in the bilateral repo market:* $r^p \leq r^{rp}$*.*

Lemma 3 outlines specific outcomes for different central bank facility designs. First, a reverse repo facility offers a fixed rate of return option for households when investing in repos that is independent of the rate provided by traditional banks at the dealer subsidiary. Since the dealer's funding rate (triparty repo rate r^{pt}) is consistently lower than the dealer's lending rate (bilateral rate r^p) to compensate for balance sheet costs, r^{rrp} functions as a floor for the triparty repo market rate. Likewise, a repo facility accessible only to traditional banks serves as a ceiling on the dealer's funding rate (triparty rate), but not on the shadow banks' funding rate (bilateral rate). A bilateral rate higher than the repo facility rate would not incentivize traditional banks to borrow at the facility unless the triparty rate is also higher than the repo facility rate. However, if the repo facility is open to both traditional and shadow banks, shadow banks choose to borrow at the facility instead of traditional banks, as shadow banks are not subject to costly balance sheet constraints. Consequently, a broad access repo facility acts as a rate ceiling on the bilateral repo rate rather than the triparty repo rate.

4.2 Comparative Statics

In this section, we present our c omparative findings. We examine the behavior of repo rates and Treasury yields for various institutional settings under the following shocks: intermediation, tax deadline, fiscal expansion, and quantitative tightening. Our analysis allows a precise decomposition of the mechanisms leading a specific shock to a specific outcome and demonstrates that the effectiveness of facilities depends on the type of shock being considered. As previously mentioned, we assume that Treasury holdings remain fixed while varying other model parameters one by one. For the sake of simplicity in presenting this section's results, we set $w^b = 0$ without loss of generality.[10]

Foreign Intermediation Shock We begin by analyzing comparative static changes in the foreign dealer intermediation volumes f. This experiment is designed to capture the withdrawal from foreign dealer intermediaries at quarter-ends for window dressing their balance sheets, as studied by (Munyan, 2015) but could also represent any outright increase in repo intermediation cost, as could arise from changes in regulation for example or a negative shock to bank equity. Figure 5 presents the comparative statics outcomes for different levels of foreign dealer intermediation under various institutional frameworks. In red, we illustrate our baseline scenario without the RRP facility or LST constraint. As the foreign dealer sector contracts (i.e., moving leftward on the graphs), the triparty repo spread to IOR, $r^{pt} - r^m$, declines, allowing for an increase in intermediation spread $r^p - r^{pt}$ as compensation for a larger marginal balance sheet cost of banks $\chi\ell$ while maintaining the bilateral repo rate equal to IOR, $r^p = r^m$. This no-arbitrage condition is driven by

[10]We show in the online appendix that the results in this section are unaffected by setting w^b to another fixed value for comparative statics. As discussed in Section 3, with dynamic shocks, Treasury bond holdings w^b and $\overline{w}^b$ are determined endogenously as functions of the shock probability and the anticipated shock duration.

Equation (22) in the absence of a binding LST constraint. Remarkably, in this baseline case, all other variables remain constant, including households' liquidity benefits, because the shock does not prompt any portfolio rebalancing from any agents.

The blue lines illustrate the situation when the Fed has a reverse repo facility in place, as has been the case since 2014 in the US. Following Lemma 3, the RRP facility establishes a lower bound on the triparty repo spread to IOR, $r^{pt} - r^m$. Upon reaching this limit, households exercise their option to lend repo directly to the Fed at the RRP rate, r^{rrp}, as shown in the Panel RRP Quantity. As seen in the third graph of Figure 5 , when the triparty repo rate reaches its RRP floor, banks begin lending in repo to shadow banks, thereby preventing bilateral repo rates from rising above IOR. This adjustment is made possible by the shift of households into RRP with the Fed, which reduces the reserve quantity on bank balance sheets and creates room for banks to lend. As noted by Diamond, Jiang, and Ma (2022), by occupying space on banks' balance sheets, reserves crowd out potential lending opportunities, so a decrease in reserves can benefit repo markets. In other words, our model clarifies that the "reserves-draining" repo lending from banks observed at quarter-end by Correa, Du, and Liao (2020) is a side-product of the concurrent surge in reverse repo facility volumes also documented in the same article. Examining the first diagram, which displays the spread between the bilateral repo rate and IOR, $r^p - r^m$, and traditional banks' repo lending positions, we observe that banks only carry out these operations until reaching the point f^{LST}. This point corresponds to the moment when the LST constraint becomes binding. Beyond that point, the reserve quantity on banks' balance sheets limits banks' ability to lend in repo, causing the bilateral repo rate to rise above IOR. This yield surge is the consequence of households having to adjust their portfolios to hold more repo, resulting in reduced liquidity benefits from their liquidity assets.

Following this reasoning, Proposition 3 proves that a repo spike requires the combination of three previously mentioned frictions: a binding LST, a positive balance sheet cost, and an RRP facility. In other words, relaxing any of those assumptions would make a repo spike impossible following a reduction in foreign intermediation.

Proposition 3. *Given*

$$b - m - a - w^b n - \overline{n} \leq (1-\alpha)(1+\tau^h)n^h + \kappa m, \tag{32}$$

the bilateral repo rate is above the interest on reserves, $r^p > r^m$, if and only if (i) LST is binding $\vartheta^m > 0$, (ii) balance sheet cost is positive $\chi > 0$, and (iii) RRP facility is binding $rrp > 0$.

Condition (32) is akin to Condition (30), but accounting for the presence of facilities, and guarantees that the baseline demand for repo is not already above the capacity of the system unconstrained by LST. The role of the RRP facility is particularly noteworthy; in its absence, the intermediation shock is absorbed by a continuous decrease in triparty rates, and the portfolio allocation of households remains close to the optimum. By establishing a lower bound on the triparty rates, the central bank introduces a market distortion by subsidizing triparty repo markets through direct provision of repo assets. This subsidy is a necessary condition for causing excessive household portfolio allocation to repo when the LST constraint becomes binding, as seen in Panel Liquidity Services. It is important to note that this misallocation does not necessarily result in a welfare loss, as the RRP facility also economizes on the balance sheet cost, which represents a deadweight loss in this economy.

We proceed to explore the implications of introducing a repo facility. Our analysis reveals that the facility's design is key to its efficacy. Notably, a repo facility that remains inaccessible to shadow banks, as by the current repo facility design at the Fed, is not effective in this scenario since a bank-intermediated repo facility fails to alleviate a spread increase caused by a congested dealer balance sheet (see Lemma 3). In contrast, a broad-access repo facility, when combined with a reverse repo facility, allows the central bank to effectively act as an intermediary in the repo markets, thereby preventing the repo rate from rising beyond the repo facility rate. This result echos the arguments from Duffie, Geithner, Parkinson, and Stein (2022) arguing in favor of broadening the access of the Fed's standing repo facility.

This impact of a repo facility accessible to banks can be observed with the yellow lines in Figure 5, where we verify that the bilateral repo rate does not surge above the RP facility rate. This cap is made feasible in this situation because the Fed effectively serves as an intermediary in the repo market by concurrently borrowing from households in triparty repo markets and lending to shadow banks in the bilateral repo, thereby economizing on dealers' balance sheet utilization.

Tax Deadline Shock We further investigate a scenario involving a repo supply shock, such as during tax deadlines when corporations utilize their cash balances in money market funds to meet their tax obligations. These tax payments are deposited into the TGA, as shown in Figure 6.

In contrast to the repo intermediation shock, a tax deadline shock does not lead to an increase in the repo intermediation spread (i.e., the difference between bilateral and triparty repo rates $r^p - r^{pt}$) as dealers' balance sheets do not expand. Instead, the diminished repo supply exerts simultaneous upward pressure on both bilateral and triparty repo rates, causing them to move in tandem. Additionally, the inflow of reserves into the TGA reduces the supply of reserves accessible to banks, leading to tighter intraday regulatory restrictions, limiting traditional banks' capacity to lend in repo, and exacerbating the repo supply shock. This mechanism corresponds with the events of September 2019 and aligns with the findings of Correa, Du, and Liao (2020), which establish a connection between the TGA and repo rates. In this scenario, due to the net reduction in repo supply from households and the subsequent increase in triparty repo rates, the reverse repo facility does not come into play.

We also examine the introduction of a standing repo facility under this scenario with various access designs. Contrasting with the intermediation shock, we find that a repo facility, even if accessible only to banks, is sufficient to prevent repo rates from exceeding the facility rate (blue lines of Figure 6). Under this setting, traditional banks borrow repos from the central bank and lend them to shadow banks via dealer subsidiaries without an increase in the size of their balance sheet. The essential difference between the intermediation shock and the tax deadline shock lies in the fact that, for the latter, when banks borrow from the central bank to intermediate repos to shadow banks, they merely compensate for the diminished repo funding from households, so their balance sheets do not need to expand beyond the baseline case. With a broad access repo facility, shadow banks directly borrow from the central bank, thereby further economizing the traditional banks' balance sheets, making it a more efficient tool in our model.

Central Bank Balance Sheet Shock We examine the impact of central bank balance sheet shocks on repo markets. Figure 7 illustrates that a reduction in the central bank balance sheet (moving leftward on the x-axes) simultaneously influences repo demand from shadow banks through Treasury supply and potential repo supply from traditional banks through reserves supply.[11]

Initially, as the central bank sells Treasuries, shadow banks increase their holdings of Treasuries and their demand for repo. As long as traditional banks hold enough reserves to satisfy their LST constraint, this increase in shadow banks' portfolios does not affect repo spreads ($r^p - r^m$), because the concurrent rundown of reserves frees up space on banks' balance sheets, enabling them to provide the necessary repo to shadow banks. Similar to previous shocks, issues arise once banks reach their LST constraint at point b^{LST} and cannot further lend in repo to shadow banks. Beyond this point, any additional

[11]Although we interpret comparative statics as a reduction of the central bank balance sheet (QT), we stress that all insights have an inverse interpretation when the central bank increases its balance sheet size (QE), and acting as buyer-of-last-resort.

reduction in the central bank's balance sheet results in an increase of both repo spreads $r^p - r^m$ and $r^{pt} - r^m$ to compensate households for shifting their portfolio away from the optimal portfolio composition and toward more repo lending (see Panel T-Banks Repo). In line with the tax deadline shock, we find that a reverse repo facility does not come into play as a result of a smaller central bank balance sheet size (blue lines of Figure 7). Instead, the RRP facility plays a significant role when the central bank balance sheet is large, transforming excess reserves into scarce repos that are in high demand from households.[12] Lastly, as with the tax deadline shock, a standing repo facility accessible only to banks is sufficient to prevent bilateral repo rates from spiking above the repo facility rate, but at a higher balance sheet cost compared to a broad-access facility (yellow lines of Figure 7).

Fiscal Shock In Figure 8, we investigate the influence of fiscal expansion shocks on repo and Treasuries markets. When not accompanied by additional purchases from the central bank, an increase in outstanding Treasuries shifts the balance towards higher repo demand, as Treasury holdings are primarily held by shadow banks and financed with repo. Consequently, this shock puts further upward pressure on repo rates and Treasury yields in a manner similar to the quantitative tightening shock discussed previously. Moreover, following the sale of issued Treasury securities, the TGA balance expands, displacing reserves and consequently tightening the LST constraint. As with the prior discussion, the reverse repo facility will come into play as a consequence of a reduction of Treasury supply rather than an increase.[13] Moreover, a repo facility available exclusively to traditional banks can help prevent repo spikes. However, similar to the central bank shock case, the most efficient approach involves opening the facility directly to shadow banks. This strategy economizes on bank balance sheet space and prevents repo rates from exceeding the repo facility rate.

5 Dynamics Analysis

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[12]This mechanism is investigated by d'Avernas and Vandeweyer (2023).

[13]Note that our focus is on longer-term Treasury securities that are financed through repo, as opposed to T-bills, which are directly held by money funds and serve as direct substitutes for repo. For an analysis of how the supply of T-bills impacts triparty repo rates, refer to the study conducted by d'Avernas and Vandeweyer (2023).

6 Conclusion

This article proposes a dynamic model of the Treasury market that captures the various disruptions observed in recent years. We emphasize the central bank's balance sheet, portfolio allocations, and regulatory frictions in shaping market stability. Our framework identifies the necessary frictions to explain disruptions, highlights the dual nature of reserve drawdowns, and investigates the effectiveness of repo facilities. To allow for a tractable exposition, our framework is nonetheless leaving out some elements that are likely to interact with the aforementioned results, including the absence of interest rate risk on long-term Treasuries, more realistic wealth dynamics, and additional regulatory pieces such as the liquidity coverage ratio. Exploring those interactions is left for future research. Overall, this study contributes to the policy debate and provides a foundation for future research on the sources of government securities market instability and the impact of regulation on government funding costs.

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Appendices

A Proofs

Given our assumption on the law of motion of α_t and the functional form of the transaction cost, equilibrium prices are only a function of α_t. The portfolio allocation $w^b_{t\text{-}}$ and $\overline{w}_{t\text{-}}$ are only relevant for the quantity of transaction costs paid by traditional and shadow banks that occur whenever $w^b_{t\text{-}} \neq w^b_t$. Thus, we abuse notation and define the state space as α_t instead of $\{\alpha, w^b_{t\text{-}}, \overline{w}_{t\text{-}}\}$. In the following proofs, we rewrite agents' problems in recursive form and drop the time subscript for ease of notation.

First, we guess and verify that the value functions have the following form:

$$V(n, w^b; \alpha) = \xi(\alpha) + \frac{\log(n)}{\rho} + \frac{\theta(\alpha) w^b}{\rho}, \tag{33}$$

$$\overline{V}(\overline{n}, \overline{w}^b; \alpha) = \overline{\xi}(\alpha) + \frac{\log(\overline{n})}{\rho} + \frac{\overline{\theta}(\alpha)\overline{w}^b}{\rho}, \tag{34}$$

$$V^h(n^h; \alpha) = \xi^h(\alpha) + (1+\beta)\frac{\log(n^h)}{\rho}. \tag{35}$$

Shadow Banks We can write the HJB for shadow banks as

$$\begin{aligned}\overline{V}(\overline{n}_-, \overline{w}^b_-, \alpha) = \max_{\overline{c}, \overline{w}^b, \overline{w}^p} \Big\{ & \log(\overline{c}\overline{n})dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_t\big[\overline{V}(\overline{n}+d\overline{n}, \overline{w}^b, \alpha + d\alpha)|dN = 0\big] \\ & + (1-\rho dt)\lambda dt \mathbb{E}\big[\overline{V}(\overline{n}+d\overline{n}, \overline{w}^b, \alpha + d\alpha)|dN = 1\big]\Big\}\end{aligned} \tag{36}$$

such that $\overline{w}^b = 1 + \overline{w}^p$ and

$$d\overline{n} = \left(\overline{w}^b r^b - \overline{w}^p r^p - \overline{c}\right)\overline{n}dt + (e^{-\nu|d\overline{w}^b|} - 1)\overline{n}. \tag{37}$$

Using Ito's lemma, the law of motion for α, and the law of motion for $\overline{n}$, we can rewrite the HJB in equation (38) as

$$\begin{aligned}&(\rho + \lambda(\alpha))\overline{V}(\overline{n}, \overline{w}^b(\alpha), \alpha) \\ &\quad = \max_{\overline{c}} \Big\{ \log(\overline{c}(\alpha)\overline{n}) + \left(\overline{w}^b(\alpha) r^b(\alpha) + (1-\overline{w}^b(\alpha))r^p(\alpha) - \overline{c}(\alpha)\right)\overline{n} \\ &\qquad + \lambda(\alpha)\int \overline{V}(\overline{n}e^{-\nu|\overline{w}^b(\alpha') - \overline{w}^b(\alpha)|}, \overline{w}^b(\alpha'), \alpha')d\alpha' \Big\}.\end{aligned} \tag{38}$$

Substitute with the guess for $\overline{V}$ obtains

$$
\begin{aligned}
&(\rho+\lambda(\alpha))\overline{V}(\overline{n},\overline{w}^b(\alpha),\alpha)\\
&\qquad = \max_{\overline{c}}\Bigg\{\log(\overline{c}(\alpha)\overline{n}) + \frac{\overline{w}^b(\alpha)r^b(\alpha)+(1-\overline{w}^b(\alpha))r^p(\alpha)-\overline{c}(\alpha)}{\rho}\\
&\qquad + \lambda(\alpha)\int\left(\overline{\xi}(\alpha') + \frac{\log(\overline{n}e^{-\nu|\overline{w}^b(\alpha')-\overline{w}^b(\alpha)|})}{\rho} + \frac{\overline{\theta}(\alpha')\overline{w}^b(\alpha')}{\rho}\right)da'\Bigg\}. \qquad (39)
\end{aligned}
$$

As bankers can adjust their holdings of treasuries instantaneously by paying the transaction cost, the value function given $\overline{w}^b_-$ must be equal to the value that would obtain by changing the debt level to the optimum, that we denote $\overline{w}^{b\star}(\overline{w}^b_-,\alpha)$:

$$
\begin{aligned}
\overline{V}(\overline{n}_-,\overline{w}^b_-,\alpha) &= \max_{\overline{w}^b}\left\{\overline{V}(\overline{n}_-\overline{\iota}(\overline{w}^b_-,\overline{w}^b),\overline{w}^b,\alpha)\right\} \qquad (40)\\
&= \overline{V}(\overline{n}_-\overline{\iota}(\overline{w}^b_-,\overline{w}^{b\star}(\overline{w}^b_-,\alpha)),\overline{w}^{b\star}(\overline{w}^b_-,\alpha),\alpha). \qquad (41)
\end{aligned}
$$

For ease of notation, we now use the short notation $\overline{w}^{b\star} \equiv \overline{w}^{b\star}(\overline{w}^b_-,\alpha)$. Thus, $\overline{w}^{b\star}$ is determined by

$$
\overline{V}_n(\overline{n}_-\overline{\iota}(\overline{w}^b_-,\overline{w}^{b\star}),\overline{w}^{b\star},\alpha)\overline{\iota}_w(\overline{w}^b_-,\overline{w}^{b\star})\overline{n}_- + \overline{V}_w(\overline{n}_-\overline{\iota}(\overline{w}^b_-,\overline{w}^{b\star}),\overline{w}^{b\star},\alpha) = 0, \qquad (42)
$$

where we use the notation $f_x = \partial f/\partial x$ for partial derivatives. Substituting for the guess for $\overline{V}$, we get

$$
-\nu\,\mathrm{sign}(\overline{w}^{b\star}-\overline{w}^b_-)+\overline{\theta}=0. \qquad (43)
$$

Then we can write the optimal weight on treasuries as follows:

$$
\overline{w}^{b\star}(\overline{w}^b,\alpha) = \begin{cases} 1 & \text{if } \;\overline{\theta} < -\nu \\ [1,\overline{w}^b] & \text{if } \;\overline{\theta} = -\nu, \\ \overline{w}^b & \text{if } \;-\nu < \overline{\theta} < \nu, \\ [\overline{w}^b,\infty] & \text{if } \;\overline{\theta} = \nu. \end{cases} \qquad (44)
$$

We omit the case for $\overline{\theta} > \nu$ as this would lead to an infinite holding of treasuries which is infeasible in equilibrium.

The first order-condition yields $\overline{c} = \rho$. If $|\overline{w}^b(\alpha') - \overline{w}^b(\alpha)| > 0$, using the envelope condition with respect to w^b, we get

$$
(\rho+\lambda(\alpha))\overline{\theta}(\alpha) = r^b(\alpha) - r^p(\alpha) + \lambda(\alpha)\frac{\partial}{\partial\overline{w}^b(\alpha)}\int \log(\overline{n}e^{-\nu|\overline{w}^b(\alpha')-\overline{w}^b(\alpha)|})\,da' \qquad (45)
$$

For $\alpha = \alpha^s$, this simplifies to

$$
(\rho+\lambda)\overline{\theta}(\alpha^s) = r^b(\alpha^s) - r^p(\alpha^s) + \lambda\nu\int \mathrm{sign}(\overline{w}^b(\alpha') - \overline{w}^b(\alpha^s))\,da'. \qquad (46)
$$

For $\alpha \neq \alpha^s$, since after a Poisson shock α is guaranteed to be α^s, we get

$$(\rho + \lambda')\overline{\theta}(\alpha) = r^b(\alpha) - r^p(\alpha) + \lambda' \nu \,\mathrm{sign}(\overline{w}^b(\alpha^s) - \overline{w}^b(\alpha)). \tag{47}$$

However, if $|\overline{w}^b(\alpha') - \overline{w}^b(\alpha)| = 0$, using the envelope condition with respect to w^b, we get

$$(\rho + \lambda(\alpha))\overline{\theta}(\alpha) = r^b(\alpha) - r^p(\alpha) + \lambda(\alpha) \int \overline{\theta}(\alpha')da'. \tag{48}$$

For $\alpha = \alpha^s$, this simplifies to

$$(\rho + \lambda)\overline{\theta}(\alpha^s) = r^b(\alpha^s) - r^p(\alpha^s) + \lambda \int \overline{\theta}(\alpha')da'. \tag{49}$$

For $\alpha \neq \alpha^s$, since after a Poisson shock α is guaranteed to be α^s, we get

$$(\rho + \lambda')\overline{\theta}(\alpha) = r^b(\alpha) - r^p(\alpha) + \lambda'\overline{\theta}(\alpha^s). \tag{50}$$

Traditional Banks Similarly, we can write the HJB for traditional banks as

$$\begin{aligned} V(n_-,&w^b_-(\alpha), \alpha) \\ &= \max_{c,w^s,w^b,w^m,w^d,w^p} \Big\{ \log(c(\alpha)n)dt \\ &\quad + (1-\rho dt)(1-\lambda(\alpha)dt)\mathbb{E}_t\big[V(n+dn, w^b(\alpha), \alpha + d\alpha)|dN = 0\big] \\ &\quad + (1-\rho dt)\lambda(\alpha)dt\mathbb{E}\big[V(n+dn, w^b(\alpha), \alpha + d\alpha)|dN = 1\big]\Big\} \end{aligned} \tag{51}$$

such that $w^k + w^b + w^m + w^p = 1 + w^d$,

$$\begin{aligned} dn = \big(&w^k(\alpha)r^k(\alpha) + w^b(\alpha)r^b(\alpha) + w^m(\alpha)r^m(\alpha) + w^p(\alpha)r^p(\alpha) \\ &- w^d(\alpha)r^d(\alpha) - c(\alpha)\big)\, ndt + (e^{-\nu|dw^b|} - 1)n, \end{aligned} \tag{52}$$

and

$$w^p \leq \kappa w^m. \tag{53}$$

As before, the value function given w^b_- must be equal to the value that would obtain by changing the Treasury bond holdings to the optimum, that we denote $w^{b\star}(w^b_-, \alpha)$. For ease of notation, we use the short notation $w^{b\star} \equiv w^{b\star}(w^b_-, \alpha)$, and $\iota^\star \equiv \iota(w^b_-, w^{b\star}(w^b_-, \alpha))$. That is,

$$V(n_-, w^b_-, \alpha) = \max_{w^b \geq 0} \big\{\; V(n_- \iota(w^b), w^b, \alpha)\big\} \tag{54}$$

$$= V(n_-\iota^\star, w^{b\star}, \alpha). \tag{55}$$

Thus, $w^{b\star}$ is determined by

$$V_n(n_-\iota^\star, w^{b\star}, \alpha)\iota^\star_{w^b} n_- + V_{w^b}(n_-\iota^\star, w^{b\star}, \alpha) = 0. \tag{56}$$

Substituting for the guess for V, we get

$$-\nu \,\mathrm{sign}(w^{b\star} - w^b_-) + \theta = 0 \tag{57}$$

Thus,

$$w^{b\star}(w^b, \alpha) = \begin{cases} 0 & \text{if} \quad \theta < -\nu \\ [0, w^b] & \text{if} \quad \theta = -\nu, \\ w^b & \text{if} \quad -\nu < \theta < \nu, \\ [w^b, \infty] & \text{if} \quad \theta = \nu. \end{cases} \tag{58}$$

Using the same steps as for the shadow banks and substitute with the guess for V obtains

$$\begin{aligned}(\rho + \lambda(\alpha))V(n, w^b(\alpha), \alpha) = \max_{c, w^k, w^m, w^d} \Bigg\{ & \log(c(\alpha)n) + \frac{\mu^n(\alpha)}{\rho} \\ & + \lambda(\alpha) \int \left(\xi(\alpha') + \frac{\log(ne^{-\nu|w^b(\alpha')-w^b(\alpha)|})}{\rho} + \frac{\theta(\alpha')w^b(\alpha')}{\rho} \right) da' \\ & + \vartheta^m(\kappa w^m(\alpha) - w^p(\alpha)) + \vartheta^p w^p(\alpha) \Bigg\}, \end{aligned} \tag{59}$$

where

$$\mu^n(\alpha) \equiv w^k(\alpha)\mu^k(\alpha) + w^b(\alpha)r^b(\alpha) + w^m(\alpha)r^m(\alpha) + w^p(\alpha)r^p(\alpha) - w^d(\alpha)r^d(\alpha) - c(\alpha) \tag{60}$$

and where $\vartheta^m(\alpha)$ is the Lagrange multiplier on the constraint $\kappa w^m(\alpha) \geq w^p(\alpha)$ and $\vartheta^p(\alpha)$ is the Lagrange multiplier on the non-negativity constraint $w^p(\alpha) \geq 0$. Thus, given that in equilibrium $w^d > 0$, the first order condition for c, w^k, w^m and w^p are given by

$$c = \rho, \tag{61}$$

$$r^k(\alpha) - r^d(\alpha) = 0, \tag{62}$$

$$r^m(\alpha) - r^d(\alpha) = -\kappa\vartheta^m(\alpha), \tag{63}$$

$$r^p(\alpha) - r^d(\alpha) = \vartheta^m(\alpha) - \vartheta^p(\alpha). \tag{64}$$

Thus,

$$w^p(\alpha) \begin{cases} 0 & \text{if } r^p(\alpha) - r^d(\alpha) < 0, \\ \in [0, \kappa w^m(\alpha)] & \text{if } r^p(\alpha) - r^d(\alpha) = 0, \\ = \kappa w^m(\alpha) & \text{if } r^p(\alpha) - r^d(\alpha) > 0. \end{cases} \tag{65}$$

As before, if $|\overline{w}^b(\alpha') - \overline{w}^b(\alpha)| > 0$, the envelope condition yields

$$(\rho + \lambda)\theta(\alpha) = r^b(\alpha) - r^d(\alpha) + \lambda \frac{\partial}{\partial w^b(\alpha)} \int \log(n e^{-\nu|w^b(\alpha') - w^b(\alpha)|}) da'. \tag{66}$$

For $\alpha = \alpha^s$, this simplifies to

$$(\rho + \lambda)\theta(\alpha^s) = r^b(\alpha^s) - r^d(\alpha^s) + \lambda\nu \int \text{sign}(w^b(\alpha') - w^b(\alpha^s)) da'. \tag{67}$$

For $\alpha \neq \alpha^s$, since after a Poisson shock α is guaranteed to be α^s, we get

$$(\rho + \lambda')\theta(\alpha) = r^b(\alpha) - r^d(\alpha) + \lambda'\nu \, \text{sign}(w^b(\alpha^s) - w^b(\alpha)). \tag{68}$$

However, if $|\overline{w}^b(\alpha') - \overline{w}^b(\alpha)| = 0$, the envelope condition yields

$$(\rho + \lambda)\theta(\alpha) = r^b(\alpha) - r^d(\alpha) + \lambda \int \theta(\alpha') da'. \tag{69}$$

For $\alpha = \alpha^s$, this simplifies to

$$(\rho + \lambda)\theta(\alpha^s) = r^b(\alpha^s) - r^d(\alpha^s) + \lambda \int \theta(\alpha') da'. \tag{70}$$

For $\alpha \neq \alpha^s$, since after a Poisson shock α is guaranteed to be α^s, we get

$$(\rho + \lambda')\theta(\alpha) = r^b(\alpha) - r^d(\alpha) + \lambda'\theta(\alpha^s). \tag{71}$$

Households Similarly, we can write the HJB for households as

$$\begin{aligned} V^h(n^h, \alpha) = \max_{c^h, w^{h,i}, w^{h,d}, w^{h,p}} \Big\{ & \log(c^h n^h) dt + \beta \log(h(w^{h,p}, w^{h,d}, \alpha) n^h) dt \\ & + (1 - \rho dt)(1 - \lambda dt)\mathbb{E}_t\big[V^h(n^h + dn^h, \alpha + d\alpha) | dN = 0\big] \\ & + (1 - \rho dt)\lambda dt \mathbb{E}\big[V^h(n^h + dn^h, \alpha + d\alpha) | dN = 1\big] \Big\} \end{aligned} \tag{72}$$

where

$$h(w^{h,p}, w^{h,d}, \alpha) = (w^{h,d})^\alpha (w^{h,p})^{1-\alpha} \tag{73}$$

and such that $w^{h,p} + w^{h,d} = 1 + \tau^h$ and

$$dn^h = (w^{h,d}(\alpha) r^d(\alpha) + w^{h,p} r^p(\alpha) - c(\alpha)) n^h dt. \tag{74}$$

We can rewrite the HJB as follows:

$$(\rho + \lambda(\alpha))V^h(n^h, \alpha) = \max_{c^h, w^{h,p}, w^{h,d}} \left\{ \log(c^h n^h) + \beta \log(h(w_u^{h,p}, w_u^{h,d}, \alpha)n^h) + (1+\beta)\frac{\mu^{h,n}(\alpha)}{\rho} + \lambda(\alpha) \int \left(\xi^h(\alpha') + \frac{\log(n^h)}{\rho} \right) da' \right\}, \tag{75}$$

where

$$\mu^{h,n}(\alpha) = w^{h,d}(\alpha)r^d(\alpha) + w^{h,p}r^p(\alpha) - c(\alpha). \tag{76}$$

The first-order conditions for households are given by

$$c^h = \rho, \tag{77}$$

$$r^p - r^d = \rho \frac{\beta}{1+\beta} \left(\frac{\alpha}{w^{h,d}} - \frac{1-\alpha}{w^{h,p}} \right). \tag{78}$$

Solving

Assumption 2.

$$\overline{n} < b - \underline{b}$$

Assumption 2 ensures that shadow banks must be leveraged in order to hold all Treasury bonds in the economy.

Definition 6. *Let $\mathcal{F}$ be the set of* **firesale** *equilibria, a subset of constrained repo market equilibria $\mathcal{C}$, defined as $\{\mathcal{M}(\alpha) \in \mathcal{F} \mid \overline{w}^{b\star}(\alpha) < \overline{w}^{b\star}(\alpha^s)\}$.*

We define as *firesale* the set of equilibria in which shadow banks fire-sell Treasury bonds to traditional banks.

Assumption 3.

$$\mathbb{P}(\alpha \in \mathcal{F}) > 0.$$

Assumption 3 ensures that some liquidity preference shocks trigger a firesale by shadow banks.

Furthermore, we restrict our analysis to preference shocks such that traditional banks never choose to fire-sell Treasury bonds to shadow banks and none of the constraints are binding in the normal state.

Assumption 4. *The lower bound of the liquidity preference parameter $\underline{\alpha}$ is such that $\overline{w}^b(\underline{\alpha}) \geq \overline{w}^b(\alpha^s)$.*

Assumption 5. *The normal state of the liquidity preference parameter α^s is such that $\vartheta^m(\alpha^s) = 0$ and $\overline{w}^b(\alpha^s) > 1$.*

Lemma A1. *If there exists α' such that $0 < w^b(\alpha^s) < w^b(\alpha')$, then $\theta(\alpha^s) \leq -\overline{\theta}(\alpha^s) = -\nu$, $\theta(\alpha') = -\overline{\theta}(\alpha') = \nu$, $\overline{w}^b(\alpha^s) > \overline{w}^b(\alpha')$, and there does not exist $w^b(\alpha') < w^b(\alpha^s)$ or $\overline{w}^b(\alpha^s) < \overline{w}^b(\alpha')$.*

Proof. From the envelope condition for w^b in equation (69), we have that for the traditional bank to be incentivized to increase its holding of treasuries and pay the adjustment cost when moving from α^s to α' and vice versa, then it must be that $\theta(\alpha') = \nu$ and $\theta(\alpha^s) \leq -\nu$. Thus, there cannot exist another state such that $w^b(\alpha') < w^b(\alpha^s)$.

The market clearing condition for the treasury market is given by

$$w^b(\alpha)n + \overline{w}^b(\alpha)\overline{n} + \underline{b} = b \quad \forall\alpha. \tag{79}$$

Thus, the reverse must be true for $\overline{w}^b(\alpha)$ and $\overline{\theta}(\alpha)$. □

Lemma A2. *It must be that* $w^b(\alpha^s) \leq w^b(\alpha')$ *for all* $\alpha' \in [0, 1]$.

Proof. Assume the contrary: there exists α' such that $w^b(\alpha') < w^b(\alpha^s)$. For $\alpha = \alpha^s$, Equation (69) yields

$$(\rho + \lambda)\nu = r^b(\alpha^s) - r^d(\alpha^s) + \lambda\nu \int \text{sign}(w^b(\alpha') - w^b(\alpha^s))da'. \tag{80}$$

From the first-order condition for w^p and $\vartheta^m(\alpha^s) = 0$, it must be that $r^p(\alpha^s) \leq r^d(\alpha^s)$. Note that $\int \text{sign}(w^b(\alpha') - w^b(\alpha^s))da' \leq 1$. Thus, we get

$$r^b(\alpha^s) \geq \rho\nu + r^d(\alpha^s) \geq \rho\nu + r^p(\alpha^s) > r^p(\alpha^s). \tag{81}$$

From the market clearing condition for the treasury market, we get $\overline{w}^b(\alpha') > \overline{w}^b(\alpha^s)$. Thus, equation (45), together with Assumption 5, yields

$$-(\rho + \lambda)\nu = r^b(\alpha^s) - r^p(\alpha^s) + \lambda\nu \int \text{sign}(\overline{w}^b(\alpha') - \overline{w}^b(\alpha^s))da'. \tag{82}$$

Since $\int \text{sign}(\overline{w}^b(\alpha') - \overline{w}^b(\alpha^s))da' \geq -1$, we get

$$r^b(\alpha^s) \leq -\rho\nu + r^p(\alpha^s) < r^p(\alpha^s), \tag{83}$$

which is a contradiction with condition (81). □

Normal State: $\alpha = \alpha^s$. Assume $w^b(\alpha^s) > 0$. Given Lemma A1, the envelope conditions for traditional and shadow banks lead to the following expressions for rates relative to $r^d(\alpha^s)$:

$$r^b(\alpha^s) - r^d(\alpha^s) = -(\rho + \lambda)\nu - \lambda\nu\mathbb{P}(\alpha \in \mathcal{F}), \tag{84}$$
$$r^b(\alpha^s) - r^p(\alpha^s) = (\rho + \lambda)\nu + \lambda\nu\mathbb{P}(\alpha \in \mathcal{F}). \tag{85}$$

Thus,

$$r^p(\alpha^s) - r^d(\alpha^s) = -2\nu(\rho + \lambda) - 2\lambda\nu\mathbb{P}(\alpha \in \mathcal{F}). \tag{86}$$

For ease of notation, define

$$\Theta^s \equiv -2\nu(\rho+\lambda) - 2\lambda\nu\mathbb{P}(\alpha \in \mathcal{F}). \tag{87}$$

Given that $\vartheta^m(\alpha^s) = 0$, $\Theta^s < 0$.

Then, $\vartheta^p(\alpha^s) > 0$ and from the traditional banks first-order conditions, we get $w^p(\alpha^s) = 0$. Furthermore, from the repo market clearing condition, we obtain $w^{h,p}(\alpha^s)n^h = \overline{w}^p(\alpha^s)\overline{n}$ and $w^{h,p}$ is determined by $r^p(\alpha^s) - r^d(\alpha^s) = \Theta^s$. For ease of notation, we define $\mathcal{H}$ as

$$\mathcal{H}(s,\alpha) \equiv \frac{s(1+\beta)(1+\tau^h) - \rho\beta + \mathcal{G}(s,\alpha)}{2s(1+\beta)} \tag{88}$$

where

$$\mathcal{G}(s,\alpha) \equiv \sqrt{(\rho\beta)^2 + s^2(1+\beta)^2(1+\tau^h)^2 + 2\rho\beta s(1+\beta)(1+\tau^h)(1-2\alpha)}. \tag{89}$$

Then $\mathcal{H}(r^p - r^d, \alpha^s)$ is the solution[14] of Equation 78 for $w^{h,p}$ in terms of a spread $r^p - r^d$. Then, we have

$$w^{h,p}(\alpha^s) = \mathcal{H}(\Theta^s, \alpha^s) \tag{90}$$

which, combined with the repo market clearing condition, yields

$$\overline{w}^p(\alpha^s)\overline{n} = \mathcal{H}(\Theta^s, \alpha^s)n^h. \tag{91}$$

Then, combined with the the shadow bank balance sheet constraint and Treasury bond market clearing condition, we get

$$\overline{w}^b(\alpha^s)\overline{n} = \overline{n} + \mathcal{H}(\Theta^s, \alpha^s)n^h \tag{92}$$

and

$$w^b(\alpha^s)n = b - \underline{b} - \overline{n} - \mathcal{H}(\Theta^s, \alpha^s)n^h. \tag{93}$$

States with the LST Constraint Not Binding: $\alpha'|\vartheta^m(\alpha') = 0$**.** Consider some shock $\alpha' \in (\underline{\alpha}, 1)$ such that the LST constraint does not bind. First, let us assume that there are no fire sale: $w^b(\alpha') = w^b(\alpha^s)$ and $\overline{w}^b(\alpha') = \overline{w}^b(\alpha^s)$. From the envelope conditions of traditional and shadow banks, we get

$$(\rho+\lambda')\theta(\alpha') = r^b(\alpha') - r^d(\alpha') - \lambda'\nu, \tag{94}$$

$$(\rho+\lambda')\overline{\theta}(\alpha') = r^b(\alpha') - r^p(\alpha') + \lambda'\nu, \tag{95}$$

and

$$(\rho+\lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) = r^p(\alpha') - r^d(\alpha') - 2\lambda'\nu. \tag{96}$$

[14]We need only care about the upper root of the quadratic as the lower root always gives $w^{h,p} < 0$, which is infeasible since households cannot fund themselves with repo.

Assume that $(\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu > 0$ such that $r^p(\alpha') > r^d(\alpha')$. Then, from the traditional bank first-order conditions, we get $\vartheta^m(\alpha') > 0$. Thus, it must be that $(\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu \leq 0$.

Define $\Theta' \equiv (\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu$. Since $\overline{w}^b(\alpha') = \overline{w}^b(\alpha^s)$, we get

$$\overline{w}^p(\alpha')\overline{n} = \mathcal{H}(\Theta^s, \alpha^s)n^h, \tag{97}$$

$$w^{h,p}(\alpha') = \mathcal{H}(\Theta', \alpha'). \tag{98}$$

The repo market clearing condition then yields

$$w^p(\alpha')n = (\mathcal{H}(\Theta^s, \alpha^s) - \mathcal{H}(\Theta', \alpha'))\, n^h. \tag{99}$$

We have two cases.

- Case $\Theta' < 0$. Then $r^p(\alpha') < r^d(\alpha')$ and $w^p(\alpha') = 0$ by traditional bank first-order condition. Then $\mathcal{H}(\Theta^s, \alpha^s) = \mathcal{H}(\Theta', \alpha')$, which pins down the value of Θ'. Thus, $\mathcal{M}(\alpha') \in \mathcal{S}$.
- Case $\Theta' = 0$. Thus, $r^p(\alpha') - r^d(\alpha') = 0$ and $w^p(\alpha') \in [0, \kappa w^m(\alpha')]$. Thus, $\mathcal{M}(\alpha') \in \mathcal{U}$.

In the first case, the spread between repo and deposit rates change from Θ^s to Θ' to compensate households for holding the same level of repo as in the normal state despite the preference shock. In the second case we have traditional banks acting as the marginal lenders of repo and household reach their optimal portfolio allocation.

States with the LST Constraint Binding: $\alpha''|\vartheta^m(\alpha'') > 0$**.** Consider some shock $\alpha'' \in (\underline{\alpha}, 1)$ such that the LST constraint binds. Then we have $w^p(\alpha'') = \kappa w^m(\alpha'')$. Therefore, from the first-order conditions of the traditional banks, it must be that $r^p(\alpha'') > r^d(\alpha'')$. Assuming that no transaction occurs, the envelope conditions yield

$$r^b(\alpha'') - r^d(\alpha'') = (\rho + \lambda')\theta(\alpha'') + \lambda'\nu, \tag{100}$$

$$r^b(\alpha'') - r^p(\alpha'') = (\rho + \lambda')\overline{\theta}(\alpha'') - \lambda'\nu. \tag{101}$$

Thus,

$$r^p(\alpha'') - r^d(\alpha'') = (\rho + \lambda')(\theta(\alpha'') - \overline{\theta}(\alpha'')) + 2\lambda'\nu. \tag{102}$$

For ease of notation, define

$$\Theta'' \equiv (\rho + \lambda')(\theta(\alpha'') - \overline{\theta}(\alpha'')) + 2\lambda'\nu. \tag{103}$$

Given that $\vartheta^m(\alpha'') > 0$ and $\vartheta^p(\alpha'') = 0$, $r^p(\alpha'') - r^d(\alpha'') > 0$ and $\Theta'' > 0$. Since $\overline{w}^b(\alpha'') = \overline{w}^b(\alpha^s)$, then

$$\overline{w}^b(\alpha'')\overline{n} = \overline{n} + \mathcal{H}(\Theta^s, \alpha^s)n^h, \tag{104}$$

$$\overline{w}^p(\alpha'')\overline{n} = \mathcal{H}(\Theta^s, \alpha^s)n^h. \tag{105}$$

From the household first-order conditions, we get

$$w^{h,p}(\alpha'') = \mathcal{H}(\Theta'', \alpha''), \tag{106}$$

which, combined with the repo market clearing condition, yields

$$w^p(\alpha'')n = (\mathcal{H}(\Theta^s, \alpha^s) - \mathcal{H}(\Theta'', \alpha''))\, n^h. \tag{107}$$

Since $w^p(\alpha'') = \kappa m/n$, we can solve for Θ''.

If the solution gives $(\Theta'' - 2\lambda'\nu)/(\rho + \lambda') = \theta^b(\alpha'') - \overline{\theta}^b(\alpha'') > 2\nu$, then a fire sale occurs. Guess that $\overline{w}^b(\alpha'') > 1$. In this case, we get

$$r^p(\alpha'') - r^d(\alpha'') = 2\rho\nu + 4\lambda'\nu. \tag{108}$$

From the household first-order conditions, we get

$$w^{h,p}(\alpha'') = \mathcal{H}(2\rho\nu + 4\lambda'\nu, \alpha''), \tag{109}$$

Since the LST constraint is binding, we get

$$w^p(\alpha'') = \kappa m/n, \tag{110}$$

which, combined with the repo market clearing condition, yields

$$\overline{w}^p(\alpha'')\overline{n} = \mathcal{H}(2\rho\nu + 4\lambda'\nu, \alpha'')n^h + \kappa m \tag{111}$$

and

$$\overline{w}^b(\alpha'')\overline{n} = \overline{n} + \mathcal{H}(2\rho\nu + 4\lambda'\nu, \alpha'')n^h + \kappa m. \tag{112}$$

Thus, $\overline{w}^b(\alpha'') > 1$. Then by the Treasury bond market clearing condition we have

$$w^b(\alpha'')n = b - \underline{b} - \overline{n} - \mathcal{H}(2\rho\nu + 4\lambda'\nu, \alpha'')n^h - \kappa m. \tag{113}$$

Let us solve for the threshold: $(\Theta'' - 2\lambda'\nu)/(\rho + \lambda') = 2\nu$. Thus, $\Theta'' = 2\nu(\rho + \lambda') + 2\lambda'\nu$ and there is no discontinuity.

In Section 4, we solve for equilibria assuming that the state variables are constant over time. Implicitly, we assume that α_t is constant over time and the transaction cost κ is sufficiently high to keep the allocation of Treasury bonds constant over time ($w^b_t = w^b_{t\text{-}} = w^b$ and $\overline{w}^b_t = \overline{w}^b_{t\text{-}} = \overline{w}^b$). Thus, for ease of notation, we drop the time subscript.

A.1 Proof of Lemma 1

Liquidity services $h(w^{h,p}, w^{h,d}, \alpha)$, given the budget constraint (17), are maximized when $w^{h,p} = (1-\alpha)(1+\tau^h)$ and $w^{h,d} = \alpha(1+\tau^h)$. Given households' first-order condition for triparty repo in Equation 23, this occurs if and only if $r^{pt} = r^d$. Combining Equations 19 and 22 obtains $r^p - r^k = r^{pt} - r^d$. Thus, $r^{pt} = r^d$ if and only if $r^p = r^k$.

Assume that $r^p = r^k$. If $\vartheta^m > 0$, then $w^p = \kappa w^m > 0$ since $m > 0$ and $r^p > r^k$ given the first-order condition for bilateral repo of traditional banks in Equation 21. Thus, if $r^p = r^k$, then $\vartheta^m = 0$ and $r^k = r^m$ and $r^p = r^m$. Thus, $\mathcal{M}(x) \in \mathcal{U}$.

Assume that $\mathcal{M}(x) \in \mathcal{U}$. Then $\vartheta^m = 0$. Assume the contrary: $\vartheta^m > 0$. Then $w^p = \kappa w^m > 0$ since $m > 0$ and $r^k = r^m + \kappa\vartheta^m = r^p - \vartheta^m$. Thus, $r^m < r^p$, a contradiction. Thus, $r^k = r^m = r^p$.

Therefore, liquidity services are at the optimum if and only if $\mathcal{M}(x) \in \mathcal{U}$.

Lemma A3. *Combining all the market clearing conditions, we get*

$$w^{h,p}n^h = b - \underline{b} - w^b n - \overline{n} - w^p n.$$

Proof. From the Treasury bond market clearing condition, we have $\overline{w}^b\overline{n} = b - \underline{b} - w^b n$. Then, substituting $\overline{w}^p$ by the shadow bank balance sheet constraint, we get $\overline{w}^p\overline{n} = b - \underline{b} - w^b n - \overline{n}$. From the bilateral repo market clearing condition, we get $w^x n + w^p n + f = b - \underline{b} - w^b n - \overline{n}$. Combined with the triparty repo market clearing condition, we get $w^{h,p}n^h + w^p n = b - \underline{b} - w^b n - \overline{n}$. □

Lemma A4. *In the absence of repo and reverse repo facilities, the LST constraint binds ($\vartheta^m > 0$) if and only if $b - \underline{b} - w^b n > \overline{n} + (1-\alpha)(1+\tau^h)n^h + w^p n$.*

Proof. Assume that $b - \underline{b} - w^b n > \overline{n} + (1-\alpha)(1+\tau^h)n^h + \kappa m$. Using Lemma A3, we get $w^{h,p}n^h > (1-\alpha)(1+\tau^h)n^h + \kappa m - w^p n \geq (1-\alpha)(1+\tau^h)n^h$. Then, by Lemma 1, $r^{pt} > r^d$ and by Equations 19, 20, 21, and 22, $\vartheta^m > 0$.

Assume that $\vartheta^m > 0$. Then, $w^p = \kappa w^m > 0$ and by Equations 19, 21, and 22, $r^{pt} > r^d$. Thus, by Lemma 1, $w^{h,p}n^h > (1-\alpha)(1+\tau^h)n^h$. Using Lemma A3, we get $b - \underline{b} - w^b n - \overline{n} - w^p n > (1-\alpha)(1+\tau^h)n^h$. Thus, $b - \underline{b} - w^b n > \overline{n} + (1-\alpha)(1+\tau^h)n^h + w^p n$. □

Lemma A5. *In the absence of repo and reverse repo facilities, $r^{pt} < r^d$ if and only if $b - \underline{b} - w^b n < \overline{n} + (1-\alpha)(1+\tau^h)n^h$.*

Proof. Assume that $r^{pt} < r^d$. Thus, by Lemma 1, $w^{h,p}n^h < (1-\alpha)(1+\tau^h)n^h$. Using Lemma A3, we get $b - \underline{b} - w^b n - \overline{n} < (1-\alpha)(1+\tau^h)n^h + w^p n$. Furthermore, if $r^{pt} < r^d$, then $w^p \leq 0$ by Equations 19, 21, and 22. Thus, $b - \underline{b} - w^b n < \overline{n} + (1-\alpha)(1+\tau^h)n^h$.

Assume that $b - \underline{b} - w^b n < \overline{n} + (1-\alpha)(1+\tau^h)n^h$. Using Lemma A3, we get $w^{h,p}n^h + w^p n < (1-\alpha)(1+\tau^h)n^h$. We have two cases.

- Case $w^p \geq 0$. Then, $w^{h,p}n^h < (1-\alpha)(1+\tau^h)n^h$. Thus, by Equation 23, $r^{pt} < r^d$.
- Case $w^p < 0$. Then, $\vartheta^m = 0$ and $r^k = r^m > r^p$ and $r^{pt} < r^d$.

□

A.2 Proof of Lemma 2

Proof. If $\vartheta^m > 0$, then $w^p = \kappa w^m > 0$. Thus, by Equation 20 and 21, $r^p > r^m$. If $r^p > r^m$, by Equation 20 and 21, $\vartheta^m > 0$.

Thus, given Lemma A 4, $r^p > r^m$ if and only if $b - \underline{b} - w^b n > \overline{n} + (1-\alpha)(1+\tau^h)n^h + \kappa m$.

Lemma A5 provides the proof of the second inequality. □

A.3 Proof of Lemma 3

Assume by way of contradiction that $r^{rrp} > r^{pt}$. Then households have a better investment opportunity than the market rate for triparty repo.

Assume by way of contradiction that $r^{rp} < r^{pt}$ with a repo facility open only to traditional banks. Then traditional bank dealers have a cheaper funding option available than the triparty repo rate.

Assume by way of contradiction that $r^{rp} < r^{p}$ with a broad access repo facility. Then shadow banks have a better funding rate available than the market rate of bilateral repo.

Assume $rp = rrp = 0$. Then,

$$\begin{aligned} w^h n^h &= b - \underline{b} - w^b n - \overline{n} - w^p n + rrp - rp && (114)\\ &= b - \underline{b} - \kappa(\underline{b} - a) - \overline{n} - w^p n + rrp - rp && (115)\\ &= b - (1+\kappa)\underline{b} + \kappa a - \overline{n} - w^p n && (116)\\ & && (117)\end{aligned}$$

A.4 Proof of Proposition 3

From the traditional bank first-order conditions, we have that

$$r^p = \begin{cases} r^m + \kappa\vartheta^m - \chi\ell & \text{if } w_t^p < 0,\\ \in [r^m + \kappa\vartheta^m - \chi\ell, r^m + (1+\kappa)\vartheta^m] & \text{if } w_t^p = 0,\\ r^m + (1+\kappa)\vartheta^m & \text{if } w_t^p > 0. \end{cases} \quad (118)$$

It is direct to see that if $\vartheta^m = 0$, then $r^p \le r^m$. Thus, we only consider cases with $\vartheta^m > 0$. Then $w^p n = \kappa m > 0$ and $r^p = \min\{r^{rp}, r^m + (1+\kappa)\vartheta^m\}$.

Assume $\chi = 0$. Then $r^d = r^k = r^m + \kappa\vartheta^m$ and $r^p = r^{pt}$. Thus, $r^{pt} = \min\{ r^r p, r^d + \vartheta^m\} > r^m > r^{rrp}$. Thus, $rrp = 0$. We have two cases.

- Case $r^d + \vartheta^m \le r^{rp}$. Then $r^{pt} > r^d$. For markets to clear, we need $\mathcal{H}(r^{pt} - r^d, \alpha) = b - \underline{b} - \kappa(\underline{b} - a) - \overline{n} - rp$. Since $rp \ge 0$, $\mathcal{H}(r^{pt} - r^d, \alpha) \le b - \underline{b} - \kappa(\underline{b} - a) - \overline{n}$. Given that $\mathcal{H}(r^{pt} - r^d, \alpha) > \mathcal{H}(0, \alpha) = (1-\alpha)(1+\tau^h)n^h$, we have a contradiction.
- Case $r^d + \vartheta^m > r^{rp} > r^d$. Then $r^{pt} > r^d$ and we can apply the same argument than the previous case.
- Case $r^d + \vartheta^m > r^{rp} = r^d$. Then $r^{pt} = r^d$ and we can apply the same argument than the previous case.

For markets to clear, we need $\mathcal{H}(r^{pt} - r^d, \alpha) = b - m - a - w^b n - \overline{n} - \kappa m$. Since $\mathcal{H}(r^{pt} - r^d, \alpha) > \mathcal{H}(0, \alpha) = (1-\alpha)(1+\tau^h)n^h$, we have a contradiction.

Assume $rrp = 0$. Then, $m + a \ge \underline{b}$. Furthermore, $r^d = r^k - \chi\ell = r^p - \vartheta^m - \chi\ell = r^{pt} - \vartheta^m$ and $r^{pt} > r^d$. For markets to clear, we need $\mathcal{H}(r^{pt} - r^d, \alpha) = b - m - a - w^b n - \overline{n} - \kappa m$. Since $\mathcal{H}(r^{pt} - r^d, \alpha) > \mathcal{H}(0, \alpha) = (1-\alpha)(1+\tau^h)n^h$, we have a contradiction.

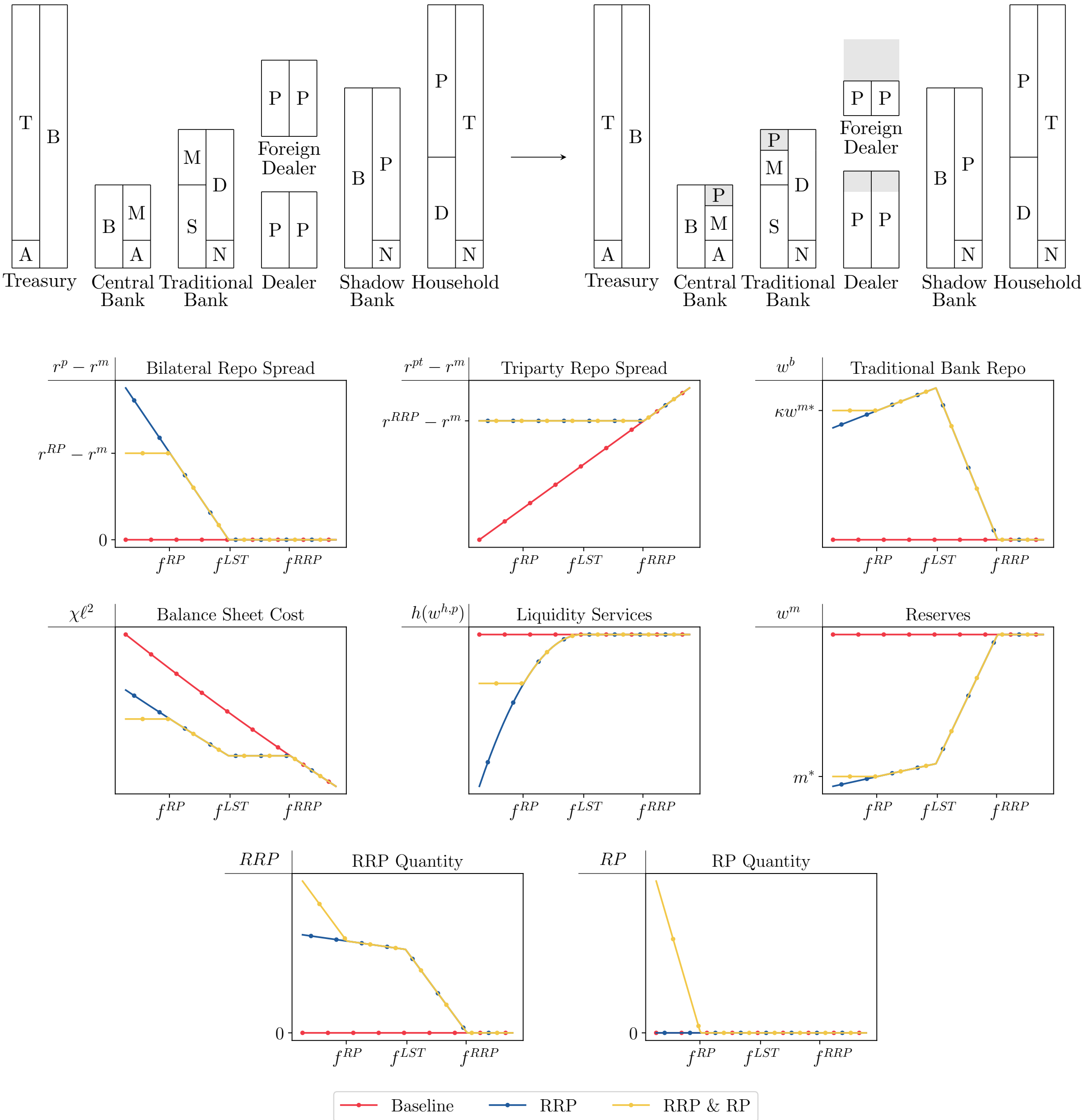

Figure 5: Intermediation Shock. The top panels shows the impact of an intermediation shock, such as foreign dealer window-dressing on quarter-ends, on the balance sheets of the economy. T denotes the present value of future tax, B denotes Treasury bonds, N denotes net worth, M denotes reserves, K denotes capital, P denotes repo, and D denotes deposits. Comparative statics are displayed below the balance sheet diagrams. A negative shock to intermediation would be a move from the right side of a chart (high foreign dealer intermediation) to the left (low foreign dealer intermediation). f^{RRP} refers to the quantity of foreign repo intermediation at which $r^{pt} = r^{RRP}$ with no facilities, f^{LST} refers to the quantity of foreign repo intermediation at which the left and right hand sides of Equation 30 are equalized with a central bank reverse repo facility in place, and f^{RP} refers to the quantity of foreign repo intermediation at which $r^p = r^{RP}$ with a reverse repo facility in place.

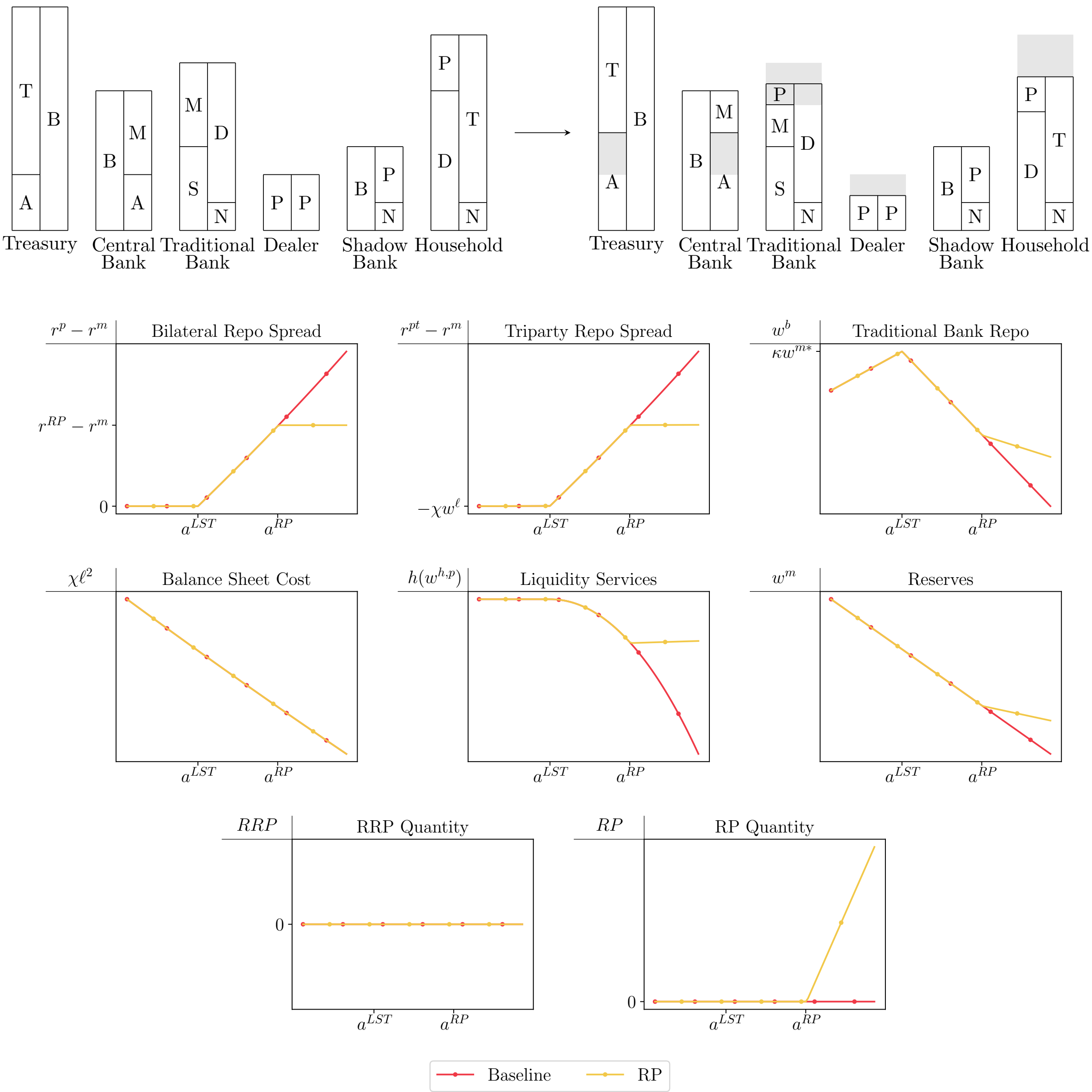

Figure 6: Tax Deadline Shock. Balance sheets prior to and after a tax deadline shock are shown on the top left and top right, respectively. T denotes the present value of future tax, B denotes Treasury bonds, N denotes net worth, M denotes reserves, K denotes capital, P denotes repo, and D denotes deposits. Comparative statics are displayed below the balance sheet diagrams. A positive shock to the Treasury General Account (TGA) would be a move from the left side of a chart to the right. a^{LST} refers to the size of the TGA at which the left and right hand sides of Equation 30 are equalized with a central bank reverse repo facility in place.

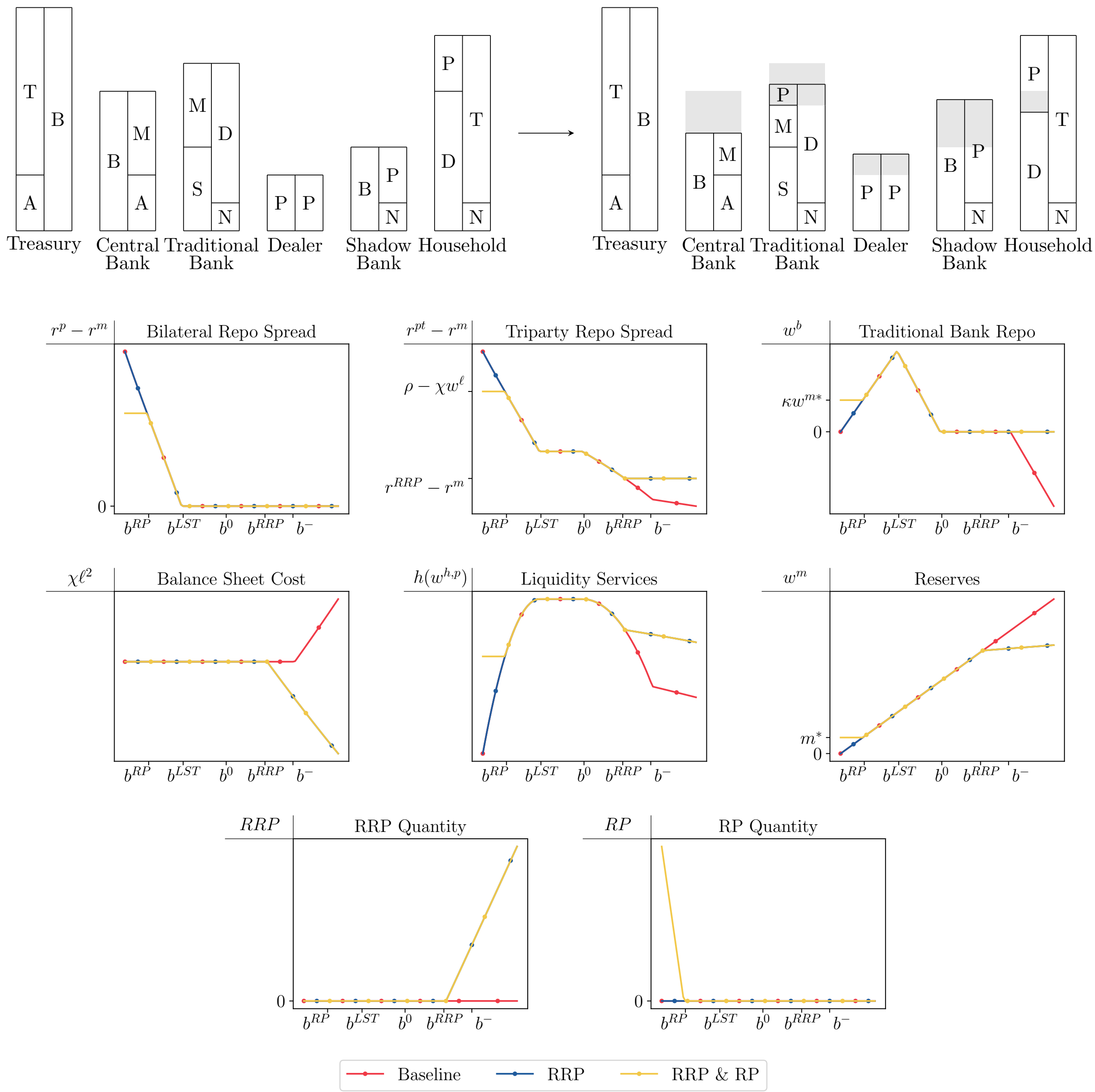

Figure 7: Central Bank Balance Sheet Shock. Balance sheets prior to and after a central bank balance sheet reduction are shown on the top left and top right, respectively. T denotes the present value of future tax, B denotes Treasury bonds, N denotes net worth, M denotes reserves, K denotes capital, P denotes repo, and D denotes deposits. Comparative statics are displayed below the balance sheet diagrams. A decrease in the central bank balance sheet would be a move from the right side of a chart to the left. b^{RRP} refers to the quantity of Treasury bonds held by the central bank at which $r^{p,t} = r^{RRP}$ with no facilities, b^{LST} refers to the quantity of Treasury bonds held by the central bank at which the left and right hand sides of Equation 30 are equalized with a central bank reverse repo facility in place, and b^{RP} refers to the quantity of Treasury bonds held by the central bank at which $r^p = r^{RP}$ with a reverse repo facility in place.

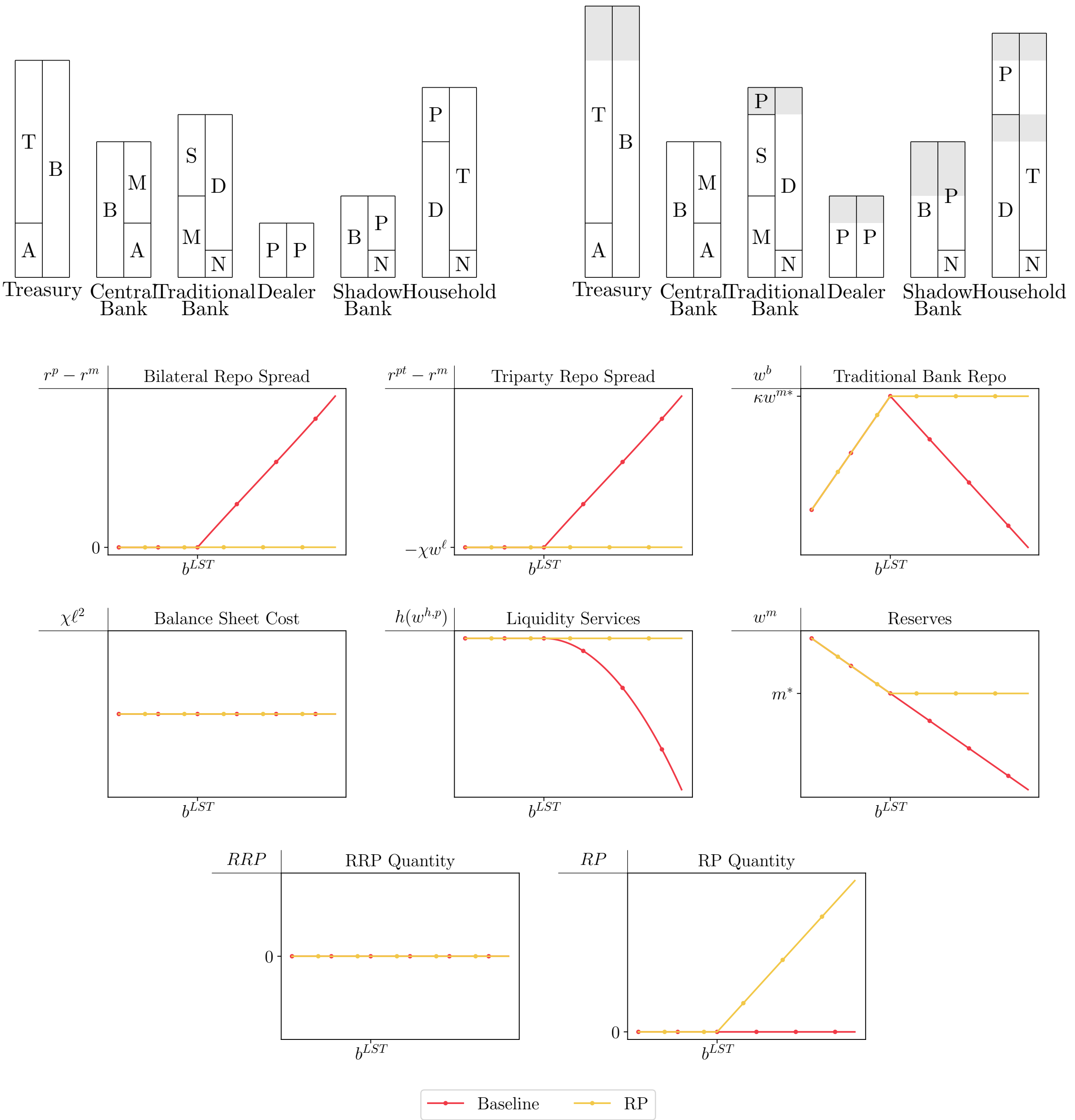

Figure 8: Fiscal Shock. Balance sheets prior to and after a fiscal shock, such as increased issuance of Treasuries, are shown on the top left and top right, respectively. T denotes the present value of future tax, B denotes Treasury bonds, N denotes net worth, M denotes reserves, K denotes capital, P denotes repo, and D denotes deposits. Comparative statics are displayed below the balance sheet diagrams. An increase in Treasuries issued would be a move from the left side of a chart to the right. b^* refers to the quantity of Treasury bonds issued by the Treasury at which the left and right hand sides of Equation 30 are equalized with a central bank reverse repo facility in place.