The Performance of Characteristic-Sorted Portfolios: Evaluating the Past and Predicting the Future

AYDOĞAN ALTI, TRAVIS L. JOHNSON, and SHERIDAN TITMAN*

February 2023

Abstract

We study persistent fluctuations in characteristic-sorted portfolio returns through the lens of a statistical model. The model provides a simple formula for adjusting the standard errors of expected return estimates. With plausible parameter values, adjusted standard errors double, casting doubt on the interpretation that the historical performance of characteristic-sorted portfolios represents unconditional return premia. Similarly, maximum likelihood estimates indicate that the historical data are consistent with a wide range of return processes. Finally, our Bayesian analysis shows that investor posteriors about expected returns are highly dependent on their priors about persistence, even after observing close to 60 years of data.

*All three authors are with the Department of Finance, the University of Texas at Austin. Contact information: aydogan.alti@mccombs.utexas.edu, travis.johnson@mccombs.utexas.edu, and sheridan.titman@mccombs.utexas.edu. We thank Svetlana Bryzgalova (discussant), Scott Cederburg, Eugene Fama, Kenneth French, Amit Goyal (discussant), Narasimhan Jegadeesh, Lubos Pastor, Allan Timmermann, and seminar participants at the Asian Bureau of Finance and Economic Research (ABFER) Conference, Financial Intermediation Research Society (FIRS) Conference, Peking University, and the University of Texas at Austin for helpful comments, and the Q Group for awarding us the 2022 Jack Treynor prize.

A large and growing literature links firm characteristics, such as valuation ratios, to expected stock returns. While the evidence documented in this literature convincingly rejects the CAPM, going beyond this rejection and interpreting alternatives has proven to be challenging. In particular, it is an open question whether the historical links between characteristics and returns represent permanent economic forces that will continue to shape returns in the future, or transitory forces that will gradually dissipate.

A case in point is the value premium. The historical tendency of value stocks to outperform growth stocks, shown by Fama and French (1992) and others, has weakened in the most recent period. Yet as Fama and French (2021) point out in a recent study, we cannot confidently conclude that the value premium post-1991 is different than the value premium pre-1991. Although the value premium is quite high in the early period, and is not statistically significant in the latter period, the returns are sufficiently noisy that we cannot reject the hypothesis that they are drawn from identical distributions.[1]

This paper examines the returns of characteristic-sorted portfolios through the lens of a statistical model that allows expected returns to vary over time. The central feature of our model, persistent fluctuations in expected returns, are likely to arise because the underlying forces that link returns to characteristics change over time.[2] Our objective is to use this model to analyze how these fluctuations influence our inferences about characteristic-sorted portfolio returns. As we show, accounting for these fluctuations have important implications for how we evaluate the historical performance of characteristic-sorted portfolios, as well as how information from historical returns are incorporated into portfolio strategies.

In our model, small but persistent shocks accumulate over time and generate conditional expected returns that deviate substantially from unconditional expected returns. These deviations lead to positive serial correlation in returns, implying that there are effectively fewer independent observations relative to the i.i.d. case. As a result, inferences about unconditional expected returns become less precise. Put differently, persistent time variations make it difficult to distinguish between a high unconditional expected return and the realizations of conditional expected returns that happened to be relatively high in the sample period.

[1]Another example of long-term fluctuations in characteristic-sorted portfolio returns is the strengthening profitability effect, documented by Novy-Marx (2013).

[2]We provide further motivation for the theoretical underpinnings of our statistical model in Section 1.1.

Our analysis formalizes this intuition and provides three main contributions. First, we use our model to derive a simple formula that adjusts the standard errors of unconditional expected return estimates to account for persistent variation in returns. We show that, given plausible persistence parameters, this adjustment can double the standard errors of unconditional expected return estimates. Second, we estimate the full model, including the persistence parameters, using maximum likelihood. We find that the persistence parameters are estimated with very little precision. In particular, the historical data are consistent with very different return processes, ranging from those with conditional expected returns exhibiting persistent fluctuations around zero unconditional means to those with i.i.d. returns and large unconditional means. Given these diverse possibilities, it is natural to ask how Bayesian investors with different prior beliefs interpret the historical time series and forecast future returns. This leads to our third main finding, which is that the posterior beliefs and portfolio choices of Bayesian investors are highly sensitive to their priors about the degree of persistence, even after observing close to 60 years of data.

The main focus of our empirical analysis is on the returns of the value, investment, profitability, and size portfolios studied in Fama and French (2015).[3] As a first step we report return autocorrelation estimates for these portfolios. We find that all four exhibit positive autocorrelation over yearly horizons, and while only the size portfolio's autocorrelation estimate is significant on its own, a joint test of the four portfolios strongly rejects the zero-autocorrelation null hypothesis. These autocorrelation estimates, however, are sufficiently imprecise that they are consistent with a wide variety of return processes, with persistence lasting from months to decades.

We then use our model to assess how return autocorrelation patterns affect inferences about expected returns. Specifically, we derive a closed-form formula for adjusting the standard errors of unconditional expected return estimates as a function of the model-implied first-order return autocorrelation and its decay rate over time. This adjustment results in higher standard errors when returns exhibit more persistent variation – that is, when the first-order autocorrelation is more positive and decays more slowly.

[3]Throughout the paper, we analyze returns of market-neutral portfolios that are constructed using value-weighted quintiles of the individual characteristics.

As our estimates illustrate, different assumptions about the magnitude of persistent variation in characteristic-sorted portfolio returns, all consistent with the historical data, generate substantially different inferences about unconditional expected returns. If returns are assumed to be i.i.d., meaning they have no persistence, standard errors are low and the hypothesis of zero unconditional expected returns is strongly rejected. However, if returns are assumed to exhibit plausible degrees of persistence, the model-adjusted standard errors are substantially higher and the hypothesis is often not rejected. For example, standard errors nearly double, relative to the case where returns are i.i.d., when return autocorrelation is 5% at one-quarter lag and decays with a half-life of five years. In contrast, we find that the commonly-used Newey and West (1987) procedure generates negligible standard error adjustments when applied to persistent return time series.

In addition to estimating standard errors using assumed parameters, we use maximum likelihood to estimate the model's parameters, including those that determine the magnitude of persistent variation in returns. Consistent with the highly noisy autocorrelation estimates in our reduced-form regressions, we find that the persistence parameters are imprecisely estimated. Given these imprecise estimates, we cannot rule out a wide variety of a plausible alternative explanations for the historical performance of the characteristic-sorted portfolios. Indeed, likelihood ratio tests fail to strongly reject the hypothesis that unconditional expected returns are zero, but conditional expected returns are both variable and highly persistent.[4]

While the results discussed so far are expressed from a frequentist statistical perspective, our focus on how assumptions about time-variation affect inferences about unconditional expected returns has a natural Bayesian interpretation. For example, our OLS estimates are similar to a Bayesian analysis with dogmatic priors about the degree of persistent variation in expected returns, while the maximum likelihood estimates are similar to a Bayesian analysis with agnostic priors. This analogy is limited, however, to priors about persistence, as the frequentist analysis assumes fully agnostic priors about unconditional expected returns.

To assess how priors about unconditional expected returns and return persistence interact, we embed our statistical model into a Bayesian framework. Specifically, we consider an

[4]With time-variation, p-values for the null of zero unconditional means are 15.7%, 3.4%, 4.4%, and 36.7%, for value, investment, profitability, and size portfolios, respectively. Without time-variation, p-values are 5.5%, 0.1%, 0.1%, and 93.5%.

investor with prior beliefs about the model parameters that he updates based on the observed historical return series. We analyze a number of economically plausible prior beliefs. For example, an investor guided by predictions of the CAPM may have the prior belief that the expected returns of characteristic-sorted portfolios cannot deviate substantially from zero. In contrast, an investor with less confidence in the CAPM may allow for larger and more persistent fluctuations in expected returns.

We find that prior beliefs about persistence substantially affect how investors update their beliefs about unconditional expected returns after observing the return history. If investors have strong priors that expected returns fluctuate very little, then their posterior beliefs about unconditional expected returns tend to be relatively tight and not highly sensitive to priors about unconditional expected returns. However, if investors' priors put more weight on the possibility of persistent fluctuations, then their posterior beliefs about unconditional expected returns become more diffuse and more sensitive to their priors. Thus, investors learn less from data and rely more on their priors when returns exhibit more persistence.

We also use our Bayesian approach to generate estimates of conditional expected returns and Sharpe ratios at each point in time in our sample. We find that Bayesian investors who allow for the possibility of time-varying returns have very different views of conditional versus unconditional expected returns. The reason is that while data from the early and later parts of the sample are equally important for estimating unconditional expected returns, conditional expected return estimates at any point in time put more weight on the more recent observations when returns are persistent. For example, because the value portfolio performed particularly poorly towards the end of the sample, the conditional value premium in 2022, measured as the mean of the posterior, is generally around a quarter of the unconditional value premium, and is close to zero in many specifications. The profitability portfolio exhibits the opposite pattern: because the returns were stronger in recent decades, the posteriors for the conditional expected return in 2022 are higher than the posteriors for the unconditional expected return.

We repeat the Bayesian estimation procedure at the end of each calendar year using only prior observations to form posterior beliefs and compute optimal portfolio weights for an investor. This out-of-sample exercise relates to the performance of factor-timing strategies discussed in the recent literature.[5] We find that investors with different priors about unconditional expected returns choose substantially different portfolios early in the sample but quickly converge to similar choices. However, investors with different priors about persistence have ongoing differences in portfolio choices driven by the extent of market timing they pursue. This timing behaviour is particularly strong for the size portfolio, which despite having near-zero average returns in the full sample has large variations in conditional expected returns.[6]

Our final analysis contributes to the ongoing debate about what Cochrane (2011) termed the 'factor zoo' by applying our approach to a broad set of 174 characteristic-sorted portfolios described in Chen and Zimmermann (2021). We find that admitting the possibility of time-variation in conditional expected returns significantly reduces the number of these portfolios that exhibit strong evidence against zero unconditional expected returns. Specifically, only four (42) of the 174 portfolio have p-values below 0.1% (1%) using our maximum likelihood estimator, compared to 58 (81) using the standard OLS approach. Failure to meet these lower p-value thresholds is particularly relevant since p-values between 1% and 5% can easily arise because of data mining (Harvey, Liu, and Zhu, 2016).

While we are among the first to analyze time variation in the expected returns of characteristic-sorted portfolios, there is a well-established literature that explores a number of related issues within the context of the aggregate market portfolio. For example, Ferson, Sarkissian, and Simin (2003) consider the predictability of aggregate market returns using predictor variables such as price/dividend ratios. They present simulations that show that OLS regressions can overstate the significance of such relationships in finite samples when expected returns are persistent, even when the Newey and West (1987) standard errors adjustment is used. Although our application is different, our framework is similar and we also show that the Newey and West (1987) correction is not effective in dealing with the problem. In addition, we explore frequentist remedies, such as OLS standard error corrections when the level of persistence in expected returns is known, and maximum likelihood when the level of persistence is estimated.

[5] See Gupta and Kelly (2019) and Ehsani and Linnainmaa (2022).

[6] Brown, Kleidon, and Marsh (1983), Conrad and Kaul (1988), and Guidolin and Timmermann (2008) also find evidence of time-variation in conditional size portfolio returns.

Pástor and Stambaugh (2009, 2012) and Avramov, Cederburg, and Lučivjanská (2018) also conduct Bayesian analyses of time-varying expected returns and, like us, find that the priors about the return generating process substantially affect the posteriors about expected returns. Our study differs from these papers in both application and focus. Specifically, our analysis concerns characteristic-sorted portfolios rather than the aggregate market portfolio, and we put greater emphasis on prior beliefs about persistence.[7]

We are aware of only two studies that use Bayesian methods to study time variation in characteristic-sorted portfolio returns: Pástor (2000) and Smith and Timmermann (2022). In contrast to our analysis, persistence plays no role in Pástor (2000) as expected returns are assumed to be constant. Smith and Timmermann (2022) assume that there are occasional structural breaks where return premia of all characteristic-sorted portfolios, as well as the market factor, change at the same time and then remain constant until the next structural break occurs. Our approach complements Smith and Timmermann (2022) by examining continuous variations in return premia that are specific to an individual characteristic-sorted portfolio. Continuous variations permit out-of-sample forecasting – structural breaks can only be identified in hindsight – as well as standard error corrections for in-sample inference.

Finally, our analysis is related to a number of recent papers that combine evidence from several characteristic-sorted portfolios to show that there is persistent time-variation in conditional expected returns, e.g., Lewellen (2002), McLean and Pontiff (2016), Avramov et al. (2017), Gupta and Kelly (2019), Arnott et al. (2021a), and Ehsani and Linnainmaa (2022). We contribute to this literature by providing model-based estimates of the magnitude of persistent variation in returns, and by studying the implications of persistence for inferences on expected returns.

The remainder of the paper is organized as follows. Section 1 describes our statistical model of the return-generating process. Section 2 documents the historical performance and return autocorrelations of the four characteristic-sorted portfolios we study. Section 3 presents the frequentist estimations of the model. Section 4 presents the Bayesian analysis. Section 5 summarizes results for 174 characteristic-sorted portfolios. Section 6 concludes.

[7]Other papers in the literature that employ Bayesian methods to study aggregate market returns include Kandel and Stambaugh (1996), Barberis (2000), Wachter and Warusawitharana (2009), and Johannes, Korteweg, and Polson (2014).

1. Statistical Model

Our empirical analyses of characteristic-sorted portfolios apply a statistical model in which conditional expected returns exhibit persistent fluctuations. In this section, we first provide a brief discussion of the theoretical motivation for our model. We then present the model specification and describe the return patterns that the model generates.

1.1. Model Motivation

In traditional asset pricing tests, such as tests of the CAPM, the null hypothesis is that expected excess returns are *always* zero – that is, at each point in time – not just on average. Thus, under the null, returns are serially uncorrelated. As we discuss in the Introduction, these tests, which appropriately assume independent residuals, convincingly reject the CAPM.

The traditional tests provide less guidance, however, if we want to distinguish between various alternatives to the CAPM null. For instance, leading behavioral explanations of the value premium are based on the idea that investor overconfidence (Daniel, Hirsleifer, and Subrahmanyam, 1998) or optimism in evaluating new technologies (Shiller, 2000) cause growth stocks to be overpriced. However, as modeled in Altı and Titman (2019), behavioral biases can also generate persistent cycles whereby the value effect is positive in some episodes and negative in others. More generally, characteristic-sorted portfolio returns may exhibit both unconditional and conditional deviations from the CAPM benchmark.[8]

The statistical model described below allows for not just deviations in unconditional expected returns from the CAPM benchmark, but also allows for time-variation in expected returns. Estimates of the model thus capture a broader set of alternatives to the CAPM.

[8] Rational theories of characteristic-based return predictability can also generate persistent fluctuations in expected returns. For instance, Gârleanu, Kogan, and Panageas (2012) and Kogan, Papanikolaou, and Stoffman (2020) provide theories where rational investors hold lower-returning growth stocks to hedge technology shocks. One might expect the economic fundamentals that drive these rational explanations to fluctuate over time, causing time-variation in the value premium.

1.2. Model Specification

We assume that the time-series of the zero-cost portfolio returns r_t satisfies:

$$r_{t+1} = \mu_t + \epsilon_{t+1}, \tag{1}$$

$$\mu_{t+1} = \mu + \lambda(\mu_t - \mu + \delta_{t+1}), \tag{2}$$

where μ_t and μ are the conditional and unconditional expected returns, respectively. The shocks to unexpected returns ϵ_t and the shocks to expected returns δ_t are i.i.d. and follow a joint normal distribution with variances σ_ϵ and σ_δ, respectively, and correlation $\rho \in (-1, 1)$.[9] We expect ρ to be negative, since positive shocks to expected returns, ceteris paribus, reduce an investment's value.[10]

The parameter $\lambda \geq 0$ in Equation (2) determines the persistence of shocks to μ_t. To facilitate interpretation of economic magnitudes, we express λ in our empirical analyses in terms of the annualized half-life of shocks to expected returns, H:

$$H = \frac{\log(0.5)}{\log(\lambda)} \frac{1}{N}, \tag{3}$$

where N is the number of periods per year (e.g. four for quarterly data).

The econometrician does not observe μ_t, but can estimate it – along with other model parameters – from the observed return realizations $\mathbf{R} = [r_1, r_2, \ldots, r_T]'$. Conditional on parameters $\Omega = [\mu, \lambda, \sigma_\epsilon, \sigma_\delta, \rho,]$, $\mathbf{R}$ has the following mean and covariance matrix:

$$\mathbb{E}(\mathbf{R}|\Omega) = \mu, \tag{4}$$

$$\mathrm{Cov}\,(\mathbf{R}|\Omega) = \mathbf{\Sigma}(\Omega), \qquad \mathbf{\Sigma}(\Omega)_{i,j} = \begin{cases} \frac{\lambda^2\sigma_\delta^2}{1-\lambda^2} + \sigma_\epsilon^2 & \text{if } i = j \\ \lambda^{|i-j|}\left(\frac{\lambda^2\sigma_\delta^2}{1-\lambda^2} + \rho\sigma_\delta\sigma_\epsilon\right) & \text{if } i \neq j \end{cases}. \tag{5}$$

Equation (5) shows that shocks to both the expected and unexpected returns contribute to

[9] Conrad and Kaul (1988) uses the same specification but assume $\rho = 0$.

[10] In Equation (2) we multiply the shock δ_{t+1} by λ so that returns are i.i.d. when $\lambda = 0$. With δ_{t+1} outside the parenthesis and $\lambda = 0$, $cov(r_{i,t}, r_{i,t-1}) = \rho\sigma_\delta^2$, meaning that one would also need to assume $\rho = 0$ for returns to be independent across time.

the volatility of returns (the terms σ_δ^2 and σ_ϵ^2, respectively). Note also that the covariance between r_i and r_j decays at a constant rate λ as $|i-j|$ grows.

1.3. *Identification*

The model is over-parameterized in the sense that multiple values of Ω lead to the same predicted moments $\mathbb{E}(\mathbf{R}|\Omega) = \mu$ and $\mathrm{Cov}(\mathbf{R}|\Omega) = \mathbf{\Sigma}(\Omega)$. To see this, consider the variance and one-lag autocorrelation of returns:

$$\sigma_r^2(\Omega) = \mathrm{Var}(r_t) = \frac{\lambda^2 \sigma_\delta^2}{1-\lambda^2} + \sigma_\epsilon^2, \tag{6}$$

$$\gamma(\Omega) = \mathrm{Corr}(r_{t+1}, r_t) = \lambda \frac{\lambda^2\sigma_\delta^2 + (1-\lambda^2)\rho\sigma_\delta\sigma_\epsilon}{\lambda^2\sigma_\delta^2 + (1-\lambda^2)\sigma_\epsilon^2}. \tag{7}$$

Using this alternative notation, the covariance matrix becomes:

$$\mathbf{\Sigma}(\Omega)_{i,j} = \begin{cases} \sigma_r^2 & \text{if } i = j, \\ \lambda^{|i-j|-1}\gamma\sigma_r^2 & \text{if } i \neq j. \end{cases} \tag{8}$$

Inspecting Equation (8), we see that any two parameterizations Ω and $\tilde{\Omega}$ which yield the same λ, σ_r, and γ will result in the same covariance matrix $\mathbf{\Sigma}$ for returns.

To better understand identification in our model, note that we use the sample mean of returns to estimate the unconditional expected return μ (Equation (4)), and the average rate at which the covariance between r_i and r_j decays as $|i-j|$ grows to estimate the persistence parameter λ (Equation (5)). The identification problem arises because we have three other model parameters to be identified (σ_ϵ, σ_δ, and ρ), but only two other moments that can be estimated: the variance of returns σ_r^2 in Equation (6) and the one-lag autocorrelation γ in Equation (7).[11] Intuitively, the identification problem arises because one cannot distinguish between different channels that generate return variance and autocorrelation. An increase in the volatility of expected return shocks σ_δ increases both return variance and autocorrelation,

[11]Note from Equation (8) that return covariances at longer lags do not provide any additional information about the model parameters, because all these covariance terms are scaled by $\gamma\sigma_r^2$.

but the same increases can also be generated from increases in the volatility of unexpected return shocks σ_ϵ and the correlation parameter ρ.

We address this identification problem in our frequentist analysis by estimating the four moments $\theta = [\mu, \lambda, \sigma_r, \gamma]$, which we can identify, rather than the full set of underlying parameters Ω. In doing so, we apply the constraint that there must be a parameterization Ω which is consistent with θ and satisfies $\sigma_\epsilon > 0$, $\sigma_\delta > 0$, $\lambda \geq 0$, and $\rho \in (-1, 1)$. The identification problem does not arise in our Bayesian analysis because we compute a posterior distribution for parameter values, which is unique given a set of priors and observed data, rather than a single point estimate, which is not unique.

1.4. The Sign of Return Autocorrelations

Although our model can generate both positive and negative return autocorrelations, negative autocorrelations of material magnitude tend to occur only when expected returns exhibit very little persistence. Time variation in expected returns is a source of positive return autocorrelation because current and recent past returns have similar expected returns. A negative return autocorrelation requires a negative correlation between realized returns and *changes* in expected returns. However, unless shocks to expected returns are quickly mean-reverting, the information in past returns about a single period's change in expected return is less relevant than the information about the ongoing level of expected returns.

We can see this formally using Equation (7), which specifies the first-order return autocorrelation γ as a function of λ, ρ, σ_ϵ, and σ_δ. The first term in the numerator, $\lambda^2\sigma_\delta^2$, represents the effect of persistent expected return variations and is always positive. The second term in the numerator, $(1-\lambda^2)\rho\sigma_\delta\sigma_\epsilon$, represents the effect of the correlation between shocks to expected and unexpected returns and is negative when $\rho < 0$. Comparing these two terms, we see that the sign of γ is determined by the relative magnitudes of λ and $|\rho\sigma_\epsilon/\sigma_\delta|$. In particular, when λ is close to one, as is the case for the highly-persistent variations that motivate our analysis, $\gamma < 0$ only if $\sigma_\epsilon/\sigma_\delta$ is sufficiently large – that is, if shocks to expected returns exhibit very little variation relative to shocks to unexpected returns.

In Appendix Table 1, we quantify how small λ (or, equivalently, H) needs to be for γ to be negative. We assume $\rho = -1$ to provide an upper bound on the prevalence of negative autocorrelations, and consider various combinations of H and $\sigma_\epsilon/\sigma_\delta$. We find that materially negative γ values obtain only when H is half a year or less.

2. Characteristic-Sorted Portfolios

We apply our statistical model to study the portfolios that are formed by sorting stocks based on value, investment, profitability, and size. We focus on these four characteristic-sorted portfolios because they are the basis of the Fama and French (2015) five-factor model and have been extensively studied elsewhere. In Section 5 we extend our analysis to a broader set of 174 portfolios that have also been analyzed in previous literature.

2.1. Data and Characteristic Definitions

We use data on the historical returns of characteristic-sorted portfolios from Ken French's website.[12] Each portfolio combines a long position in a value-weighted portfolio of firms in one extreme quintile of the characteristic with a short position in the other extreme.

The characteristics are defined following Fama and French (2015). *Value* is the ratio of the book value of equity ($B_{i,y}$) to the market value of equity ($M_{i,y}$) as of the end of the prior fiscal year y. *Investment* is the growth rate in the book value of assets ($\text{Assets}_{i,y}/\text{Assets}_{i,y-1}$). *Profitability* is revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses in year y divided by book equity in year $y-1$. *Size* is $M_{i,y}$.

In contrast to most of the literature, which examines the monthly returns of characteristic sorted portfolios, we analyze quarterly returns. As we show in Appendix A, the monthly returns of three of the four portfolios we study exhibit strong positive first-order autocorrelations.[13] While these short-term autocorrelations are consistent with time-varying expected returns, they could also be driven by lead-lag effects and other short-term microstructure effects. To focus our analysis on longer-term autocorrelations driven by persistent variations in expected returns, we estimate our model using quarterly returns.

[12] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

[13] This is consistent with the evidence in Gupta and Kelly (2019) that 47 of 65 characteristic-based portfolios have significantly positive first-order autocorrelation.

Since the market risk premium is not our focus, we use market-neutral versions of each portfolio's returns throughout, calculated as:

$$r_{i,t}^{\beta=0} = r_{i,t} - \hat{\beta}_i(r_{m,t} - r_{f,t}), \tag{9}$$

where $r_{i,t}$ is the quarterly return of the long-short portfolio i, $r_{m,t} - r_{f,t}$ is the quarterly excess market return, and $\hat{\beta}_i$ is the full-sample market beta. The average return and Sharpe ratio of this market-neutral portfolio are equivalent to the alpha and information ratio, respectively, of the underlying long-short portfolio.

2.2. Historical Performance of Characteristic-Sorted Portfolios

Table 1 summarizes the historical performance of the characteristic-sorted portfolios. The value, investment, and profitability portfolios have annualized mean returns above 4% and Sharpe ratios between 0.3 and 0.5, while the size portfolio's mean return is close to zero. Based on standard errors calculated under the assumption of i.i.d. returns, we strongly reject the null hypothesis of zero expected returns for the first three portfolios.[14]

We also examine the historical performance of the portfolios in the first and second halves of our sample period, 1963–1992 and 1993–2021. As discussed in McLean and Pontiff (2016), Linnainmaa and Roberts (2018), and Fama and French (2021), the value strategy's average returns are smaller in the second half of the sample and are not statistically different from zero. However, as emphasized by Fama and French (2021), the difference between the two halves of the sample is not statistically significant.[15]

The investment, profitability, and size portfolios each show different return patterns across subsamples. The investment portfolio's returns are largely consistent over time and statistically significant in both halves of the sample. The profitability portfolio follows the

[14] We compute the i.i.d. standard errors by taking the standard deviation across simulated samples formed by re-sampling historical data with replacement.

[15] Recent studies provide two potential explanations for the decline of the value premium: book value may have worsened as a proxy for the value of assets in place (Choi, So, and Wang, 2021; Eisfeldt, Kim, and Papanikolaou, 2022; Goncalves and Leonard, 2023), or a series of shocks may have widened the difference in multiples between growth and value stocks (Israel, Laursen, and Richardson, 2020; Arnott et al., 2021b). Both imply time variation in expected returns and thus are consistent with our model.

opposite pattern as the value portfolio, performing better in the second half of the sample than the first, though again the difference is statistically insignificant. The size portfolio has small and statistically insignificant returns in both halves of the sample.[16]

2.3. Autocorrelation Estimates in Model Simulations and Historical Data

As discussed in the Introduction, a number of recent studies document short-term autocorrelation in characteristic-sorted portfolio returns, or more generally analyze portfolio timing strategies that are premised upon time-variation in expected returns. Before we estimate our model, we present similar reduced-form analyses using both model-simulated return data and the historical returns of the four characteristic-sorted portfolios that we analyze.

Estimates of the autocorrelation structure for individual portfolios are inherently imprecise because realized returns are quite volatile relative to the plausible variations in expected returns. Furthermore, autocorrelation estimates have a well-document downward bias in small samples (Kendall, 1954; Marriott and Pope, 1954). To illustrate these issues, we first present autocorrelation estimates that we generate with model-simulated data samples under a variety of assumptions about H and γ. In these simulations, we fix the model parameters and generate 50,000 samples with 234 observations, which is the number of quarters in our empirical sample.[17] For each simulated return series, we estimate return autocorrelations using regressions of quarterly returns on averages of returns over the previous L quarters:

$$r_t = a + b_L \left(\frac{1}{L} \sum_{l=1}^{L} r_{t-l} \right) + \epsilon_t. \tag{10}$$

Panel A of Table 2 presents average values, as well as 95% confidence intervals, for the autocorrelation coefficient $\hat{b}_L$ across samples that are simulated under a variety of parametric

[16]The absence of a significant size effect in the early part of the sample is in part due to our use of value weights in portfolio construction.

[17]The remaining model parameters we use for the simulations are $\mu = 0$ and $\sigma_r = 7.57\%$, which is the full-sample return standard deviation for the value portfolio. These choices have no effect on the results we report in this section as autocorrelation estimates from regressions with intercepts are invariant to linear transformations of r_t.

assumptions. Specifically, we consider (H, γ) combinations where the half-life of shocks H equals 2.5, five, and ten years, and the first-order return autocorrelation γ equals 2.5%, 5%, and 10%. For each parameterization we also compute $\hat{\sigma}_{sr}$, which denotes the volatility of the annualized Sharpe ratio conditional on past returns. More specifically, $\hat{\sigma}_{sr}$ is the volatility of the conditional Sharpe ratio computed by an investor who knows the model parameters and observes an infinite history of past return realizations, but not the realizations of the conditional expected returns μ_t. As Table 2 shows, the volatilities of conditional Sharpe ratios generated by these parameterizations are roughly comparable to the historical Sharpe ratios of characteristic-sorted portfolios reported in Table 1.

The first row of Panel A in Table 2 reports the autocorrelation estimates when $H = 0$ and thus returns are distributed i.i.d. As expected, the average estimated autocorrelation coefficients are negative due to the aforementioned downward bias: because the in-sample mean is used to calculate the autocorrelation coefficient, the data appears to be mean-reverting even when there is no true mean reversion. The bias is stronger the longer the past return window because there is a smaller effective sample size. To take an extreme example, if one were to divide the sample into two observations, one half would appear above-average and the other below-average, resulting in apparent mean reversion in every random sample.

The other rows in Panel A of Table 2 show autocorrelation estimates when expected returns exhibit persistent variation, i.e., $H > 0$ and $\gamma > 0$. Two observations emerge from the reported estimates. First, the downward bias continues to affect autocorrelation estimates even when expected returns are persistent, especially with longer past return windows. Second, and more importantly, the confidence intervals for the autocorrelation estimates are quite wide and include negative values in every parameterization – even those with large γ.

Panel B of Table 2 presents estimates of Equation (10) for the historical samples of the quarterly returns of the value, investment, profitability, and size portfolios. The first column shows that all four portfolios have positive b_4, indicating that past-year returns positively predict next-quarter returns. The second and third columns show that longer past-return windows have point estimates with varying signs.

We do not report standard errors or bias corrections in Panel B of Table 2 because Panel A shows that both depend heavily on the magnitude and persistence of the variations in expected returns. In most cases, the confidence intervals for different parameterizations in Panel A include all of the point estimates in Panel B, which means that one cannot reject any of the posited autocorrelation structures. Even economically large regression coefficients, such as $\hat{b}_{20} = -0.30$ for the investment portfolio, lie in the 95% confidence interval for every parameterization from i.i.d. to $H = 10$ and $\gamma = 5\%$.

For three out of the four characteristic-sorted portfolios, the autocorrelation estimates do not reject any reasonable parameterization of our model. The size portfolio is the exception; we convincingly reject the i.i.d. null in the regression with both the one-year and five-year lags. In addition, in a pooled regression that combines the returns of the four portfolios, the i.i.d. null hypothesis is rejected for one-year and five-year returns with p-values of 0.0% and 2.2%, respectively.[18] In a pooled regression with only value, investment, and profitability, we reject the i.i.d null with the one year lag with a p-value of 2.8%. The reason we can jointly but not individually reject the null is that the coefficients for the value, investment, and profitability portfolios are each above the i.i.d. benchmark, but not by enough to reject on an individual basis.

3. OLS and Maximum Likelihood Estimations

As the previous section shows, the autocorrelation patterns observed in the historical returns of characteristic-sorted portfolios are consistent with a range of assumptions about the magnitude of persistent variation in conditional expected returns. In this section, we formally analyze the impact of such variation on the estimates of unconditional expected returns using OLS regressions and maximum likelihood estimations.

3.1. OLS Estimation With Model-Corrected Standard Errors

We start by estimating the unconditional expected return μ and its standard error using OLS regressions of observed returns r_t on a constant. The OLS estimate $\hat{\mu}^{OLS}$ is a consistent

[18] We conduct this test comparing the sum of the individual estimates from observed data to the distribution of this sum in samples simulated under the i.i.d. assumption.

estimator of μ, even when conditional expected returns vary, as in our model. The correct standard errors for $\hat{\mu}^{OLS}$ depend on the covariance matrix of the residuals $\psi_t = r_t - \mu$.[19]

The typical approaches used to adjust standard errors are the White (1980) correction for potential heteroskedasticity and Newey and West (1987), which corrects for both heteroskedasticity and autocorrelations up to a small number of lags. When expected returns are time-varying and persistent, both of the standard approaches produce understated standard errors. The reason is that persistent variations in expected returns generate small but long-lasting correlations in ψ_t that extend beyond the windows considered by Newey and West (1987). Formally, our model generates the residuals

$$\psi_t = \mu_{t-1} - \mu + \epsilon_t, \tag{11}$$

which implies that ψ_t has long-lasting autocorrelation due to persistent variations in μ_t. As we show below, even if we extend the number of lags in Newey and West (1987) to match or exceed the half-lives of shocks to expected returns, standard errors remain under-estimated because the lags are too large a fraction of the observed time series for the asymptotic results in Newey and West (1987) to hold.

If the first-order return autocorrelation γ and the half-life of shocks H are known, we can correct the OLS standard errors for the resulting autocorrelation in the residuals ψ_t using the structure of our model.[20] Specifically, the asymptotic standard error of $\hat{\mu}^{OLS}$ satisfies

$$\begin{aligned} \text{SE}(\hat{\mu}^{OLS}) &= \sqrt{\frac{\mathbf{1}'\boldsymbol{\Sigma}\mathbf{1}}{T^2}} \\ &= \left(\frac{\sigma_r}{\sqrt{T}}\right)\sqrt{1 + 2\gamma\left[\frac{\lambda^T + T(1-\lambda) - 1}{T(1-\lambda)^2}\right]}. \end{aligned} \tag{12}$$

In Equation (12), T is the number of observations, $\mathbf{1}$ is a $T \times 1$ vector of ones, and $\boldsymbol{\Sigma}$ is the

[19] We use 'correct standard errors' as an informal shorthand for the correct specification of the asymptotic distribution of $\hat{\mu}$.

[20] Note that the model structure can also be used to obtain Generalized Least Squares (GLS) estimates. As detailed in Appendix B, we find that the GLS estimates differ only slightly from the OLS estimates for the four characteristic-sorted portfolios we study.

covariance matrix of returns. The first line is the general formula for computing standard errors with a known covariance matrix (see Section 4.5 of Cameron and Trivedi (2005)). In our model, $\mathbf{\Sigma}$ is fully specified by γ and λ, as shown in Equation (8). Substituting from Equation (8) results in the formula in the second line of Equation (12) after some algebraic manipulation. Note that the first term in this formula, $\sigma_r/\sqrt{T}$, is the unadjusted OLS standard error. Thus, the second term in square roots is the adjustment factor, which is a function of T, γ, and λ. For large T, Equation (12) approximates to:

$$\mathrm{SE}(\hat{\mu}^{OLS}) \approx \left(\frac{\sigma_r}{\sqrt{T}}\right)\sqrt{1+\frac{2\gamma}{1-\lambda}}. \tag{13}$$

The standard error adjustments in Equations (12) and (13) can also be stated in terms of the annualized half-life H instead of λ by substituting $\lambda = 0.5^{\frac{1}{HN}}$, which follows from Equation (3), where N is the number of return observation periods per year.

Table 3 shows how different assumptions about γ and H affect the t-statistics for unconditional expected return estimates that obtain with model-adjusted standard errors given by Equation (12). For comparison, we also report t-statistics corrected using Newey and West (1987) with the number of lags matching the half-life of shocks H. Across all four portfolios, we find that Newey-West t-statistics differ very little from the unadjusted t-statistics. Model-implied t-statistics, by contrast, are substantially lower. For instance, the model adjustment cuts t-statistics approximately by half when $H = 5$ years and $\gamma = 5\%$. Overall, the results in Table 3 indicate that accounting for plausible magnitudes of persistent variation in expected returns can result in inferences about unconditional expected returns that differ materially from the inferences that obtain with the assumption of i.i.d. returns.

3.2. Maximum Likelihood Tests

This subsection describes maximum likelihood estimates of our statistical model. In contrast to the ordinary least squares regressions described in the previous subsection, maximum likelihood estimation requires distributional assumptions, but allows us to estimate all of the model parameters. In particular, we estimate, rather than assume, the model parameters that determine the magnitude of persistent variations in expected returns.

3.2.1. Tests for $\mu = 0$

We test the null hypothesis that the unconditional expected return μ equals zero under a variety of assumptions about the structure of time-variation in conditional expected returns. For each assumption, we estimate the model using maximum likelihood twice, first with no restrictions on μ and then under the restriction of $\mu = 0$. In both cases we restrict H to be less than or equal to 20 years because μ is not identified when H approaches infinity. Using these estimates, we compute the p-value for the $\mu = 0$ null using a likelihood ratio test.

The first three columns in Panel A of Table 4 test whether $\mu = 0$ assuming no time variation in expected returns (i.e., $H = 0$ and thus returns are i.i.d.). Testing $\mu = 0$ under this assumption is analogous to using OLS with no standard error correction. Not surprisingly, the likelihood-ratio p-values strongly reject the null for the investment and profitability portfolios, and reject with a p-value of 5.5% for the value portfolio.

The next five columns in Panel A of Table 4 relax the i.i.d. assumption and estimate the values of H and γ that maximize the likelihood of observing the historical data.[21] We find that when we allow for the possibility of time-varying means, the p-value for the hypothesis that $\mu = 0$ increases for the three previously-significant portfolios. As the panel shows, when μ is restricted to be zero, the model fits the data by using large values of H combined with positive values of γ. This combination explains the historical returns as arising from positive realizations of *persistent* expected returns that eventually dissipate. Because such persistent variation is more-difficult to reject empirically, it offers a plausible alternative to large μ, increasing the p-value for rejecting the $\mu = 0$ hypothesis.

3.2.2. Restrictions on H and γ

Next, we use maximum likelihood estimates to assess the plausibility of the various assumptions about H and γ that we made in our OLS analyses in Section 3. For each assumption, we re-estimate the model by restricting H and γ to their assumed values, and

[21] As discussed in Section 1, we estimate the parameters summarizing the covariance matrix of returns $\theta = [\mu, H, \sigma_r, \gamma]$ directly rather than the underlying parameters $\Omega = [\mu, \lambda, \sigma_\epsilon, \sigma_\delta, \rho,]$ because the later are not fully identified. We restrict our estimates of θ to the range for which there exists at least one possible Ω yielding the same covariance matrix of returns.

calculate likelihood ratios relative to the unrestricted model to test whether we can reject the restriction. We also use likelihood ratios to test whether we can reject the hypothesis that $\mu = 0$ given the restrictions on H and γ.

Panel B of Table 4 presents the results. For value, investment, and profitability portfolios, we cannot reject *any* of the restrictions on the structure of time-variation in expected returns, including the i.i.d. hypothesis and the possibility of extremely-persistent and economically large shocks ($H = 10$ years, $\gamma = 5\%$). Similar to the results reported in Table 3, Panel B of Table 4 also shows that these alternative assumptions have material effects on our inferences about $\mu = 0$. For instance, p-values for $\mu = 0$ increase from 0.1% in the i.i.d. case to above 10% in some specifications for both investment and profitability portfolios.

While the results for value, investment, and profitability portfolios suggest that the data offer very little guidance about the magnitude of persistent variations in expected returns, the results for the size portfolio show that this is not always the case. Consistent with the reduced-form evidence in Table 2, our estimates for the size portfolio in Table 4 suggest strong positive autocorrelation in returns with a half-life between one and two years. As a result, the maximum likelihood estimations strongly reject both the i.i.d. restriction and the restriction that $H = 10$. The contrast between the results for the size portfolio and the other characteristic-sorted portfolios shows that the model tests can have power, but the return patterns for value, investment, and profitability portfolios are particularly inconclusive.

4. Bayesian Analysis

The analysis in the previous section shows that the historical data are consistent with a variety of substantially different return generating processes. Given this, one might expect that investors learn slowly about the parameters governing the return generating process and put relatively more weight on their priors. A natural next step, therefore, is to ask how investors with different prior beliefs interpret the historical data, and how this in turn influences their investment decisions. We examine these normative questions in this section. Specifically, we pursue a Bayesian analysis that specifies prior likelihoods of different model parameterizations and uses the observed data to calculate posterior likelihoods. We also compute posteriors for moments that affect investors' portfolio timing decisions, such as the conditional Sharpe ratios at each point in time.

4.1. Prior Beliefs About Model Parameters

To facilitate economic intuition, our Bayesian analysis specifies prior beliefs on a transformation of the model parameters expressed in terms of Sharpe ratios rather than expected returns.[22] Specifically, we specify priors over μ_{sr}, the unconditional Sharpe ratio of portfolio returns; σ_r, the volatility of portfolio returns; H, the half-life of shocks to expected returns; σ_{sr}, the volatility of conditional Sharpe ratios; and ρ, the correlation between unexpected and expected return shocks. The mapping between these annualized parameters in annualized terms and the underlying model parameters $\Omega = [\mu, \lambda, \sigma_\epsilon, \sigma_\delta, \rho,]$ is given by the following equations, where $N = 4$ is the number of periods per year:

$$\mu_{sr} = \frac{\mu}{\sigma_\epsilon}\sqrt{N}, \qquad \sigma_r = \sqrt{\left(\frac{\lambda^2\sigma_\delta^2}{1-\lambda^2} + \sigma_\epsilon^2\right) N}, \tag{14}$$

$$H = \frac{\log(0.5)}{\log(\lambda)}\frac{1}{N}, \qquad \sigma_{sr} = \sqrt{\left(\frac{\lambda^2\sigma_\delta^2/(1-\lambda^2)}{\sigma_\epsilon^2}\right) N}, \qquad \rho = \rho.$$

We consider a variety of priors on μ_{sr} and H, which are summarized in Panel A of Table 5. For μ_{sr}, we first consider normal prior distributions centered at -0.4, 0, and 0.4, all with a standard deviation of 0.4.[23] For the value portfolio, these three investors can be viewed as having growth, neutral, or value inclinations. We also examine an uninformative prior where μ_{sr} is distributed uniformly between -2 and 2.[24]

[22] Another advantage of this transformation is that the same Sharpe ratio priors can be applied across all characteristic-sorted portfolios regardless of their volatility levels.

[23] We use ± 0.4 as the center for the unconditional Sharpe ratio distributions to roughly match the US equity market's estimated Sharpe ratio.

[24] Uniform distributions over wider supports give nearly-identical results because the data strongly reject unconditional Sharpe ratios above 2 or below -2 for for the portfolios we study when H is less than 20 years.

To illustrate how prior beliefs about H influence Bayesian inferences about expected returns, we consider three dogmatic priors and one agnostic prior. The dogmatic priors assert that $H = 0$ (making returns i.i.d.), $H = 2.5$ years, or $H = 5$ years with certainty. The agnostic prior, by contrast, views H as unknown and uniformly distributed between 0 and 10 years.

We consider uniform priors over wide ranges for the remaining parameters. The prior for the volatility of annual returns, σ_r, is uniformly distributed between 10% and 20%. The prior for the volatility of conditional Sharpe ratios, σ_{sr}, is uniformly distributed between 0 and 1. The correlation between unexpected and expected return shocks, ρ, is likely to be negative given the inverse relation between prices and expected returns. In contrast to aggregate market returns, however, characteristic-sorted portfolio returns should be driven primarily by cash-flow news rather than discount rate news. Based on these observations, we specify the prior on ρ to be uniformly distributed in the interval $[-0.5, 0]$.

To provide intuition for economic magnitudes, Panel B of Table 5 reports summary statistics for a number of moments that are implied by the priors specified in Panel A. We calculate these moments by simulating 50,000 draws from each prior and calculating the value of each moment implied by each parameter draw. The first set of columns shows that μ, the unconditional expected return, has a prior mean that equals -5.3%, zero, or 5.3%, depending on the prior specification. The middle set of columns show that the prior mean values of one-lag return autocorrelation, γ, are positive in all cases, indicating that the positive effect due to persistence of expected returns typically outweighs the negative effect due to $\rho < 0$, although the 95% confidence intervals do include negative values.

The parameter σ_{sr} governs the volatility of the true conditional Sharpe ratio, defined as the conditional expected return μ_t in Equation (2) divided by the volatility of unexpected return shocks σ_ϵ. This Sharpe ratio is unlikely to be perfectly observable to investors in practice. If an investor forecasts expected returns based on past return realizations, combined with his beliefs about the model parameters, then the relevant measure of the time variation in conditional Sharpe ratios is $\hat{\sigma}_{sr}$, defined earlier in Section 2.3. The last set of columns in Panel B in Table 5 report summary statistics for $\hat{\sigma}_{sr}$ for each prior specification we consider. Note that, while the Sharpe ratio conditional on past returns can be quite volatile, it is not as volatile as the unobserved conditional Sharpe ratio. In particular, both the means and the 95% confidence intervals for $\hat{\sigma}_{sr}$ are about half as large as those for σ_{sr}.

4.2. Posteriors for Unconditional Expected Returns and Sharpe Ratios

We compute posterior distributions given each (H, μ_{sr}) prior specification and each of the four characteristic-based portfolios, resulting in 64 prior-data pairs. For each pair, we characterize the posterior distributions by generating 50,000 posterior draws using the M.C.M.C. procedure detailed in Appendix C. We compute the posterior distributions for the full parameter vector Ω, but for brevity we report the results only for a subset of parameters and moments that are of economic interest.

The results are summarized in Table 6, which reports summary statistics for the posterior distributions of a number of model parameters and moments, and Figure 1, which plots summary statistics for posterior distributions of μ_{sr} in the black lines on the left. As the table and the figure show, different priors about H result in substantially different posterior beliefs about unconditional expected returns μ and Sharpe ratios μ_{sr}. For instance, the 95% confidence intervals for μ and μ_{sr} become wider as priors for H increase, making the possibility of zero or negative unconditional expected returns much more likely. The intuition for this result is the same as in the frequentist analysis above: the data do not strongly reject the possibility that the historical return performance of characteristic-sorted portfolios is explained by persistent but dissipating positive shocks to conditional expected returns.

The Bayesian analysis also produces an insight that is distinct from the frequentist analysis: the extent to which priors about μ_{sr} affect posteriors about μ and μ_{sr} depends on the prior about H. Bayesian investors who believe $H = 0$ have similar posteriors about μ and μ_{sr} despite large differences in priors. On the other hand, Bayesian investors whose priors are that H is, or might be, large have substantial differences in their posterior beliefs about μ and μ_{sr} despite observing 58 years of data. For example, means of posteriors about the value portfolio's μ_{sr} are clustered between 0.19 and 0.27 for the $H = 0$ prior, but vary from 0.11 to 0.27 for the $H \sim U(0, 10)$ prior.

The reason the sensitivity of posteriors to priors depends on H is that Bayesian investors 'shrink' observed in-sample average returns towards the mean of their priors, and the extent of this adjustment depends on H. If an investor believes H equals zero and thus returns are i.i.d., the data are more informative about unconditional expected returns and thus the posterior hews closer to the in-sample average. If the investor believes H is or may be larger than zero, the return data are less informative about unconditional expected returns and so the posterior depends more on the prior.

Table 6 also shows that the posterior distributions for H, γ, and $\hat{\sigma}_{sr}$ for the value, investment, and profitability portfolios differ very little from the corresponding prior distributions reported in Table 5. This finding is consistent with the evidence in Tables 2 and 4 that the data offer little guidance on the autocorrelation patterns of these portfolios. The size portfolio, by contrast, has posterior distributions for H with means well below the prior mean of five years, indicating that the data push Bayesian investors towards lower H. Furthermore, unlike the other three portfolios, the posteriors for size portfolio's γ and $\hat{\sigma}_{sr}$ have much higher means compared to priors and confidence intervals that exclude zero, suggesting that the evidence tilts in favor of positive autocorrelation. Still, posteriors for the size portfolio's H, γ, and $\hat{\sigma}_{sr}$ are quite wide, leaving room for many potential interpretations of the data.

Just as priors about persistence affect posteriors about unconditional expected returns, priors about unconditional expected returns can also affect posteriors about persistence. For example, investors with negative priors on μ_{sr} are more amenable to interpreting positive historical returns as arising from large and persistent variations in conditional expected returns – i.e., higher posteriors for H, γ, and $\hat{\sigma}_{sr}$ – relative to investors with positive priors on μ_{sr}. Table 6 shows this is indeed the case for the value, investment, and profitability portfolios, which exhibited substantial average return performance during our sample period.

4.3. Posteriors for Conditional Sharpe Ratios

In addition to unconditional expected returns and Sharpe ratios, the Bayesian analysis allows us to compute posterior distributions of conditional Sharpe ratios at each point in time during our sample period.[25] Figure 2 plots the time series of posterior means of these condi-

[25] The conditional Sharpe ratio distributions reported in this subsection are the posterior beliefs that obtain after observing the full historical sample of return data. Thus, the magnitudes should be interpreted as measuring in-sample economic significance rather than informing real-time portfolio choices. We consider out-of-sample portfolio choices in the next subsection.

tional Sharpe ratios that obtain with the 'agnostic' prior specification with $\mu_{sr} \sim U(-2, 2)$ and $H \sim U(0, 10)$ for the four characteristic-sorted portfolios we study. As the figure shows, the fluctuations in conditional Sharpe ratios are large, generally varying between zero and 0.8 on an annualized basis. Despite having unconditional Sharpe ratio estimates near zero, the conditional Sharpe ratios of the size portfolio are especially volatile, varying from -0.75 to above one. These relatively larger fluctuations are a reflection of the more positive return autocorrelations that the size portfolio exhibits during our sample period.

We also compute posterior distributions of forward-looking conditional Sharpe ratios for the quarter following the end of our sample period (Q1 of 2022). These posteriors differ from the unconditional posteriors because they rely on recent trends to extrapolate future performance. When $\gamma > 0$, the extrapolation is positive, meaning that conditional expected returns are higher (lower) than unconditional expected returns when recent returns are higher (lower) than the full-sample average. When $\gamma < 0$, the extrapolation is negative, meaning that conditional expectations are inversely related to recent trends.

The grey lines on the right in Figure 1 present the means and 95% confidence intervals for posterior beliefs about the 2022 conditional Sharpe ratio of each portfolio. As the first rows show, conditional and unconditional Sharpe ratios are by definition the same when $H = 0$. When $H > 0$, however, both the value and investment portfolios have smaller conditional Sharpe ratios compared to their unconditional Sharpe ratios. For the value portfolio, which had particularly poor recent performance, the pessimistic or neutral Bayesian investors believe that the conditional Sharpe ratios in 2022 are centered near zero and could even be quite negative. Because the profitability portfolio performed better in recent years than earlier in the sample, we find the opposite effect in Panel C of Figure 1: posteriors about 2022 Sharpe ratios are *higher* than posteriors about the unconditional Sharpe ratio. As with the bearish view of value and investment, the bullish view for profitability is stronger for larger values of H than smaller ones.

4.4. Out-of-Sample Forecasts

In the final part of our Bayesian analysis, we consider the portfolio choice problem of a risk-averse investor whose beliefs about the distribution of future returns are influenced by his prior beliefs as well as past return data. To facilitate this analysis, we repeat the Bayesian estimation procedure at the end of each calendar year in our sample, using only past data available up to that point in time. This expanding-window out-of-sample approach allows us to quantify how different prior beliefs about persistence and unconditional expected returns affect Bayesian investors' portfolio choices through the sample period.

For each calendar year y in our sample, 1963–2021, we calculate posterior belief distributions about model parameters based on quarterly data available through y and given each of the prior beliefs described in Section 4.1.[26] While most out-of-sample forecasting exercises wait for a 'training' period to avoid over-fitting extremely short samples, we start in 1963 after observing only two quarterly observations because Bayesian investors rely on their prior distributions – and thus do not 'over-fit' – even with very few data observations.

Next, we generate 50,000 random draws from the posterior parameter distribution and for each parameter draw, generate 10 random observations from the implied return distribution for the next-year. Combined, we get 500,000 draws from the posterior distribution of next-year returns $\tilde{r}_{y+1}$. Given these draws, we compute out-of-sample portfolio weights w_y^* allocated to the (zero-cost) characteristic-sorted portfolio for an investor with CRRA utility:

$$w_y^* = \arg\max_w \mathbb{E}_y\left(\frac{(1 + r_{f,y} + w \cdot \tilde{r}_{y+1})^{1-\gamma}}{1-\gamma}\right), \tag{15}$$

where $r_{f,y}$ is the risk-free rate as of year y. To avoid adding covariance parameters to our Bayesian estimation, we assume that the investor optimizes only over the amount allocated to a single characteristic-sorted portfolio and not all four portfolios simultaneously. Our portfolio weights should therefore be interpreted as economically meaningful summaries of the posterior return distribution for each characteristic-sorted portfolio rather than weights for an optimal multi-asset portfolio.

Figure 3 illustrates the main results of the out-of-sample analysis by plotting differences in portfolio weights for investors with different prior beliefs. The solid lines in the figure show

[26]We repeat the exercise once per year instead of each quarter to save computational time. Given the slow-moving changes in expected returns we are interested in, we would not expect large quarter-to-quarter variations in conditional distributions of either model parameters or returns.

how learning about the unconditional expected return μ affects portfolio choices. Specifically, the solid lines plot the difference in weights chosen by an investor with a bullish prior ($\mu_{sr} \sim N(0.4, 0.4)$) and an investor with a neutral prior ($\mu_{sr} \sim N(0.0, 0.4)$), where both investors believe $H = 0$. While the bullish investor initially puts much more weight in each characteristic-sorted portfolio than the neutral investor, this difference gradually dissipates over time as more data becomes available and the investors place less weight on their priors.

The dotted lines in Figure 3 show how differences in H priors lead to differences in the aggressiveness of market timing. Specifically, the dotted lines plot the difference in weights chosen by an investor who accounts for the possibility of persistent variations in returns ($H \sim U(0, 10)$) and an investor who believes returns are i.i.d. ($H = 0$), where both investors have neutral priors about μ. Because the investor with $H \sim U(0, 10)$ priors uses recent past returns to 'time' the characteristic-sorted portfolio, while the $H = 0$ investor does not, there are large variations in the differences between these two investors' portfolio weights. These differences do not dissipate over time because, as discussed above, the data offer very little guidance on the true value of H.

5. Broader Analysis and Implications

Our analysis up to this point focuses on the four prominent examples of characteristic-sorted portfolios. As we show, estimates of the unconditional expected returns of these portfolios become less precise once we account for the possibility of persistent variations in conditional expected returns. We also find that there is relatively low power to estimate the magnitude of persistent variations for the value, investment and profitability portfolios, while there is stronger evidence of persistent variation for the size portfolio.

In this section we consider a broader set of portfolios in order to assess the prevalence of the findings discussed above. Specifically, we apply our model to a set of 174 long-short equity portfolios studied in the prior literature. In addition to the four portfolios we have considered so far, these portfolios include all strategies described in the March 2022 update of the "Open Source Cross-Sectional Asset Pricing" dataset (Chen and Zimmermann, 2021) with at least 120 months of historical data. Our analysis continues to focus on quarterly market-neutral returns using value-weighted quintiles.

5.1. *Frequentist Analysis of the "Factor Zoo"*

The apparent ease with which research uncovers significant relationships between firm characteristics and average stock returns is inconsistent with traditional asset pricing theories (Cochrane, 2011). As Harvey, Liu, and Zhu (2016) and others discuss, this evidence may be partly due to p-hacking; i.e., researchers testing many different characteristics and reporting only those that turn out to have a statistically-significant relation to future returns. Harvey, Liu, and Zhu (2016) suggests addressing p-hacking by requiring higher t-statistic (or lower p-value) thresholds for rejecting the no-predictability null hypothesis. As Chen and Zimmermann (2021) show, however, higher significance thresholds do not necessarily eliminate the anomaly. For instance, 58 of the 174 portfolios in our sample have p-values below 0.1%, which is extremely unlikely under the null even with p-hacking.

We offer an alternative interpretation of the factor zoo: by ignoring the possibility of persistent variations in expected returns, the published analyses may overstate the statistical significance of estimates of unconditional expected returns. Figure 4 summarizes our main findings in this regard.[27] As the figure shows, relative to OLS estimates that assume i.i.d. returns, rejections of the $\mu = 0$ null hypothesis are much weaker in our maximum likelihood estimates that allow for time-variation in expected returns. The gap between the MLE and OLS tests are particularly large for p-values of 0.1% and 1%, which MLE achieve for only 4 and 42 portfolios, respectively, compared to 58 and 81 using OLS.

5.2. *Bayesian Analysis of Time-Variation in Conditional Sharpe Ratios*

Our next set of results focus on the magnitude of persistent return variation for the broader set of 174 portfolios. To facilitate this analysis, we employ the full-sample Bayesian approach described in Section 4. We report results that obtain with the neutral unconditional

[27] The complete results for our frequentist and Bayesian analyses of the 174 portfolios are reported in Appendix Table 3.

Sharpe ratio prior $\mu_{sr} \sim N(0, 0.4)$ and the agnostic half-life prior $H \sim U(0, 10)$.[28]

Panel A of Figure 5 presents the frequency distribution of the posterior mean for $\hat{\sigma}_{sr}$, the volatility of the Sharpe ratio conditional on past returns. As the panel shows, there are substantial differences across portfolios in terms of time-variation in returns. While the posterior means for $\hat{\sigma}_{sr}$ for many of the portfolios are similar to value, investment, and profitability and cluster near or below the center of the prior distribution (0.23, reported in Table 5), a substantial minority are similar to size and have posterior means that are much larger than the prior mean. In unreported analysis, we also find that portfolios that exhibit higher (absolute) return correlation with the size portfolio tend to have higher posterior means for $\hat{\sigma}_{sr}$. Examples include portfolios sorted on illiquidity, share price, and idiosyncratic volatility. This result indicates that understanding what drives variations in conditional size premia may also help explain time-variations in a variety of other portfolios.

Panel B of Figure 5 presents the frequency distribution of the difference between the posterior means of 2022 conditional Sharpe ratio and the unconditional Sharpe ratio. The main result from this panel is that there does not appear to be a broad negative time trend in portfolio returns: if anything, the difference between 2022 Sharpe ratios and unconditional Sharpe ratios is slightly positive on average. This result may seem surprising given the evidence in McLean and Pontiff (2016) and Smith and Timmermann (2022) that return anomalies have weakened over time. Two differences between our analysis and prior work explain the discrepancy. First, the posterior means of unconditional Sharpe ratios tend to be below in-sample Sharpe ratios, as these Bayesian posteriors are influenced by prior beliefs that are centered around zero.[29] Second, the average portfolio in our sample performed well toward the end of the sample period, which receives the most weight in Bayesian forecasts of 2022 returns.[30]

[28]Results using other priors are available upon request.

[29]The across-portfolio average of posterior means for unconditional Sharpe ratios is 0.27, while the average in-sample Sharpe ratio is 0.33.

[30]For instance, realized returns in 2021 averaged 7.54% compared to the full-sample average of 4.14%.

6. Conclusion

Traditional tests of the CAPM assume independently distributed residuals. As we show in this paper, these tests overstate the statistical significance of estimates of unconditional expected returns when conditional expected returns vary over time. Our analysis, however, should not be viewed as a critique of traditional tests of the CAPM. Under the CAPM null, expected excess returns are always zero, implying that the residuals are independently distributed. Thus, ignoring benchmark problems (i.e., Roll (1977)), traditional tests appropriately reject the CAPM null. Having established this rejection, however, it is natural to allow for the possibility of time variation in expected returns. We propose a statistical model that allows for this possibility and conduct both frequentist and Bayesian tests based on the model.

Our results have implications for practitioners as well as academics who are interested in characteristic-sorted portfolios. Financial institutions now offer a multitude of relatively passive investment products, such as ETFs and mutual funds, that aim to exploit the long-term links between returns and characteristics that have been identified in academic research. At the same time, there exist active hedge funds that attempt to time the variations in characteristic return premia that are described in the more recent literature. Our analysis offers guidance for both groups as to how much they should use past performance while forming their near-term performance outlooks for these portfolios.

References

Altı, Aydoğan and Sheridan Titman, 2019, A dynamic model of characteristic-based return predictability, *Journal of Finance* 74, 3187–3216.

Arnott, Robert D, Mark Clements, Vitali Kalesnik, and Juhani T Linnainmaa, 2021a, Factor momentum, Available at SSRN 3116974 .

Arnott, Robert D, Campbell R Harvey, Vitali Kalesnik, and Juhani T Linnainmaa, 2021b, Reports of values death may be greatly exaggerated, *Financial Analysts Journal* 77, 44–67.

Avramov, Doron, Scott Cederburg, and Katarína Lučivjanská, 2018, Are stocks riskier over the long run? taking cues from economic theory, *Review of Financial Studies* 31, 556–594.

Avramov, Doron, Si Cheng, Amnon Schreiber, and Koby Shemer, 2017, Scaling up market anomalies, *Journal of Investing* 26, 89–105.

Barberis, Nicholas, 2000, Investing for the long run when returns are predictable, *Journal of Finance* 55, 225–264.

Brown, Philip, Allan W Kleidon, and Terry A Marsh, 1983, New evidence on the nature of size-related anomalies in stock prices, *Journal of Financial Economics* 12, 33–56.

Cameron, A Colin and Pravin K Trivedi, *Microeconometrics: methods and applications* (Cambridge University Press 2005).

Chen, Andrew Y and Tom Zimmermann, 2021, Open source cross-sectional asset pricing, *Critical Finance Review, Forthcoming* .

Choi, Ki-Soon, Eric C So, and Charles CY Wang, 2021, Going by the book: Valuation ratios and stock returns, SSRN working paper 3854022 .

Cochrane, John H, 2011, Presidential address: Discount rates, *Journal of Finance* 66, 1047–1108.

Conrad, Jennifer and Gautam Kaul, 1988, Time-variation in expected returns, *Journal of Business* 409–425.

Daniel, Kent, David Hirsleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* 53, 1839–1885.

Ehsani, Sina and Juhani T Linnainmaa, 2022, Factor momentum and the momentum factor, *Journal of Finance* 77, 1877–1919.

Eisfeldt, Andrea L, Edward Kim, and Dimitris Papanikolaou, 2022, Intangible value, *Critical Finance Review* 11, 299–332.

Fama, Eugene F and Kenneth R French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, Eugene F and Kenneth R French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F and Kenneth R French, 2021, The value premium, *Review of Asset Pricing Studies* 11, 105–121.

Ferson, Wayne E, Sergei Sarkissian, and Timothy T Simin, 2003, Spurious regressions in financial economics?, *Journal of Finance* 58, 1393–1413.

Gârleanu, Nicolae, Leonid Kogan, and Stavros Panageas, 2012, Displacement risk and asset returns, *Journal of Financial Economics* 105, 491–510.

Goncalves, Andrei and Gregory Leonard, 2023, The fundamental-to-market ratio and the value premium decline, *Journal of Financial Economics* 147, 382–405.

Guidolin, Massimo and Allan Timmermann, 2008, Size and value anomalies under regime shifts, *Journal of Financial Econometrics* 6, 1–48.

Gupta, Tarun and Bryan Kelly, 2019, Factor momentum everywhere, *Journal of Portfolio Management* 45, 13–36.

Harvey, Campbell R, Yan Liu, and Heqing Zhu, 2016, and the cross-section of expected returns, *Review of Financial Studies* 29, 5–68.

Israel, Ronen, Kristoffer Laursen, and Scott Richardson, 2020, Is (systematic) value investing dead?, *Journal of Portfolio Management* 47, 38–62.

Johannes, Michael, Arthur Korteweg, and Nicholas Polson, 2014, Sequential learning, predictability, and optimal portfolio returns, *Journal of Finance* 69, 611–644.

Kandel, Shmuel and Robert F Stambaugh, 1996, On the predictability of stock returns: an asset-allocation perspective, *Journal of Finance* 51, 385–424.

Kendall, Maurice G, 1954, Note on bias in the estimation of autocorrelation, *Biometrika* 41, 403–404.

Kogan, Leonid, Dimitris Papanikolaou, and Noah Stoffman, 2020, Left behind: Creative destruction, inequality, and the stock market, *Journal of Political Economy* 128, 855–906.

Lewellen, Jonathan, 2002, Momentum and autocorrelation in stock returns, *Review of Financial Studies* 15, 533–564.

Linnainmaa, Juhani T and Michael R Roberts, 2018, The history of the cross-section of stock returns, *Review of Financial Studies* 31, 2606–2649.

Marriott, FHC and JA Pope, 1954, Bias in the estimation of autocorrelations, *Biometrika* 41, 390–402.

McLean, R David and Jeffrey Pontiff, 2016, Does academic research destroy stock return predictability?, *Journal of Finance* 71, 5–32.

Newey, Whitney K and Kenneth D West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1–28.

Pástor, L'uboš, 2000, Portfolio selection and asset pricing models, *Journal of Finance* 55, 179–223.

Pástor, L'uboš and Robert F Stambaugh, 2009, Predictive systems: Living with imperfect predictors, *Journal of Finance* 64, 1583–1628.

Pástor, L'uboš and Robert F Stambaugh, 2012, Are stocks really less volatile in the long run?, *Journal of Finance* 67, 431–478.

Roll, Richard, 1977, A critique of the asset pricing theory's tests part i: On past and potential testability of the theory, *Journal of Financial Economics* 4, 129–176.

Shiller, Robert J, *Irrational exuberance* (Princeton university press 2000).

Smith, Simon and Allan Timmermann, 2022, Have risk premia vanished?, *Journal of Financial Economics* 145, 553–576.

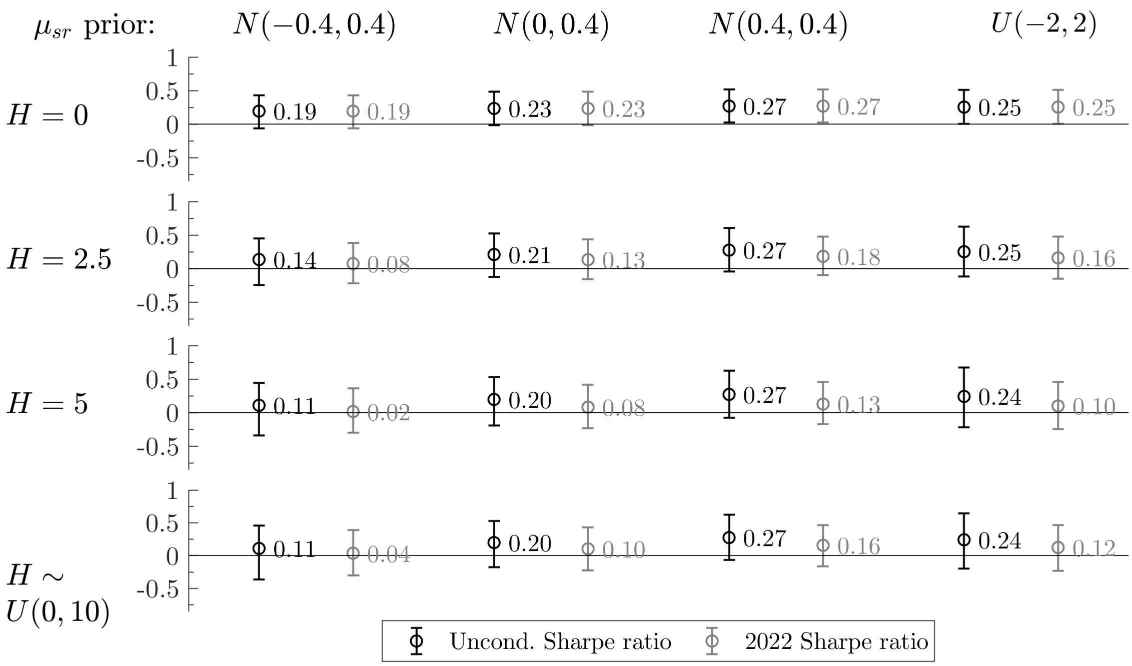
Wachter, Jessica A and Missaka Warusawitharana, 2009, Predictable returns and asset allocation: Should a skeptical investor time the market?, *Journal of Econometrics* 148, 162–178.

White, Halbert, 1980, A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica* 817–838.

Figure 1: Posterior Sharpe Ratios

This figure reports the means and 95% confidence intervals for posterior distributions of Sharpe ratios for value, investment, profitability, and size portfolios, as defined in the header of Table 1. Posteriors for the unconditional Sharpe ratio are in black on the left, and the conditional Sharpe ratio as of Q1 of 2022 are in grey on the right. Each panel shows the posteriors for 16 different (H, μ_{sr}) combinations of prior beliefs, where H is the half-life of shocks to expected returns in years, and μ_{sr} is the unconditional Sharpe ratio. All Sharpe ratios are annualized. Our sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

Panel A: Value



Panel B: Investment

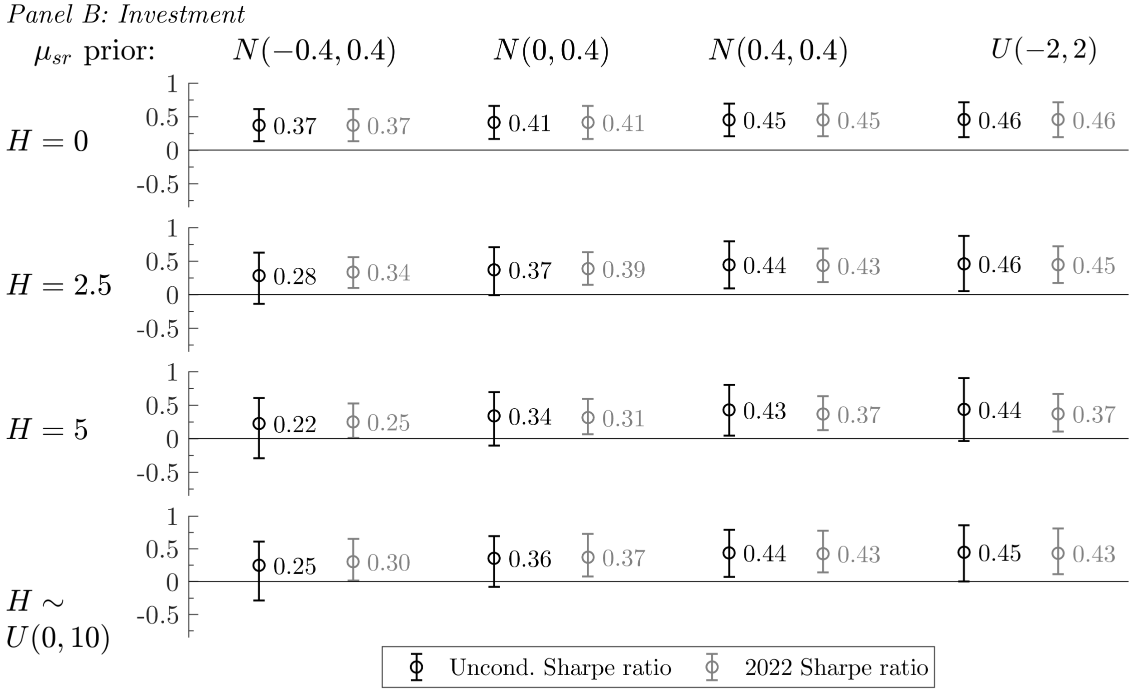
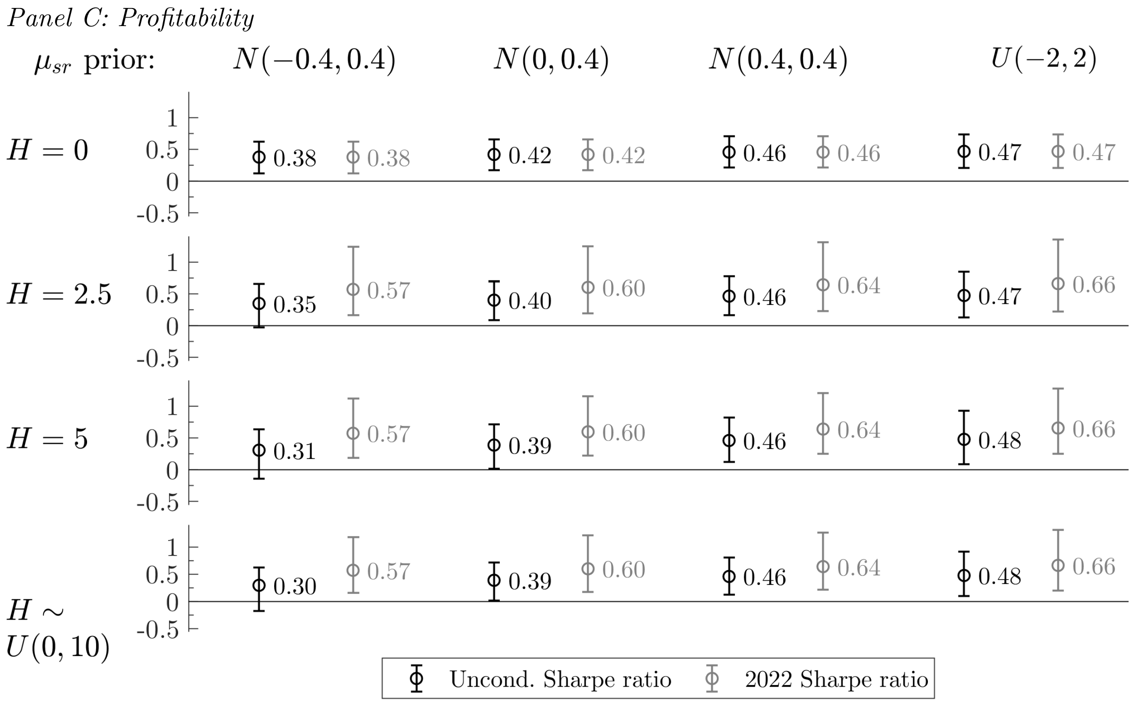


Figure 1: Posterior Sharpe Ratios (continued)



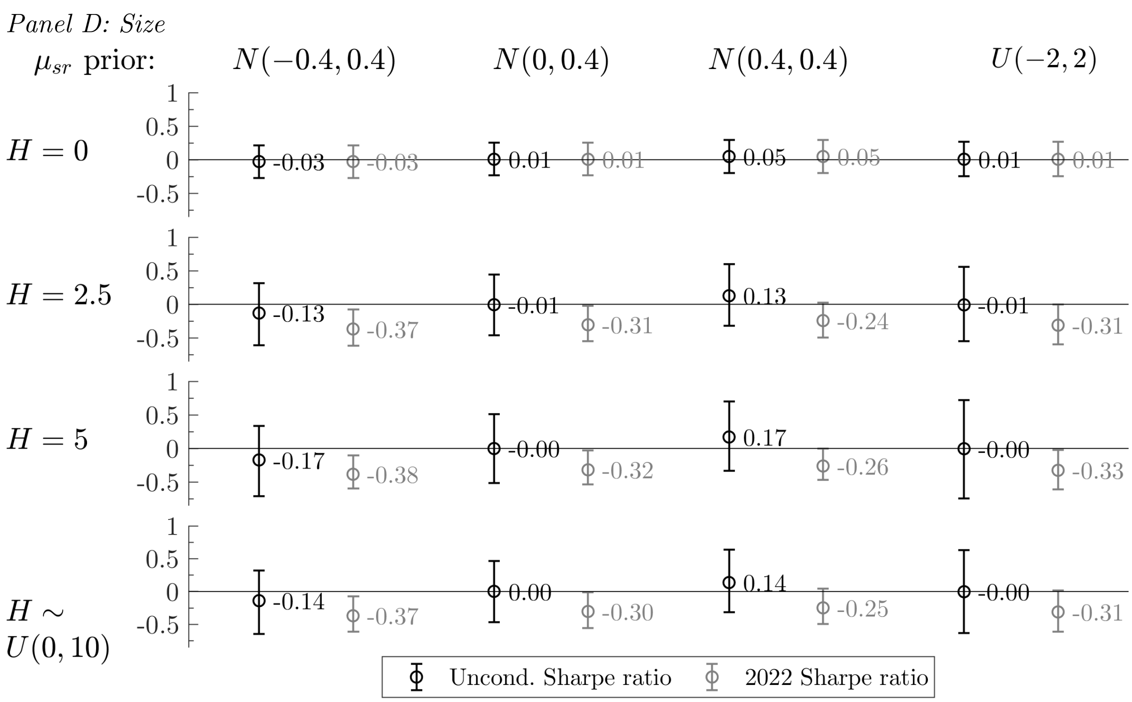
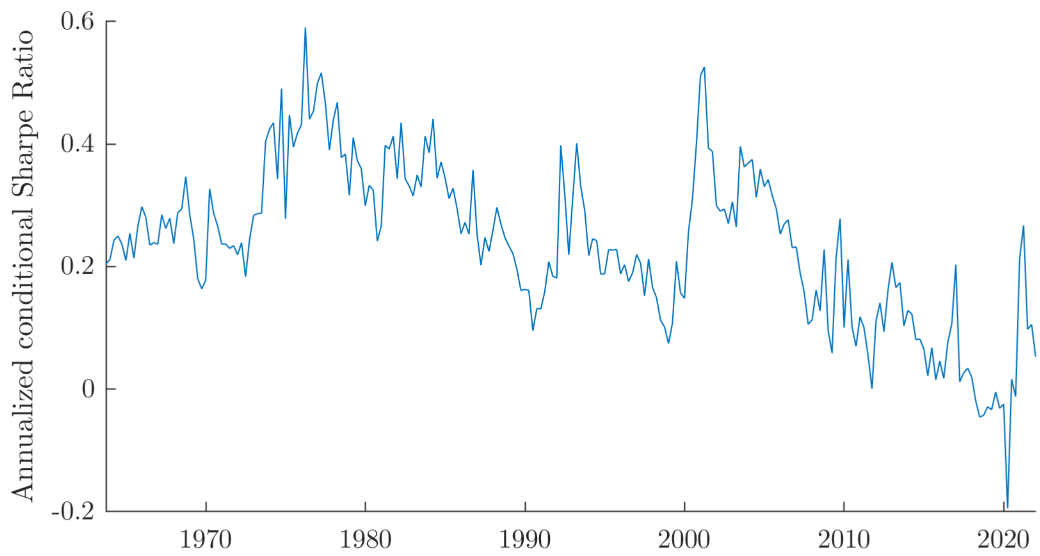


Figure 2: Posteriors on Conditional Sharpe Ratios Across Time

This figure shows means of posterior belief distributions for conditional Sharpe ratios of value, investment, profitability, and size portfolios, as defined in the header of Table 1. Posterior distributions are computed using the full sample of historical data and the prior beliefs that the unconditional Sharpe ratio μ_{sr} is distributed $N(0, 0.4)$ and the half-life of shocks to expected returns H is distributed $U(0, 10)$. Our sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

Panel A: Value



Panel B: Investment

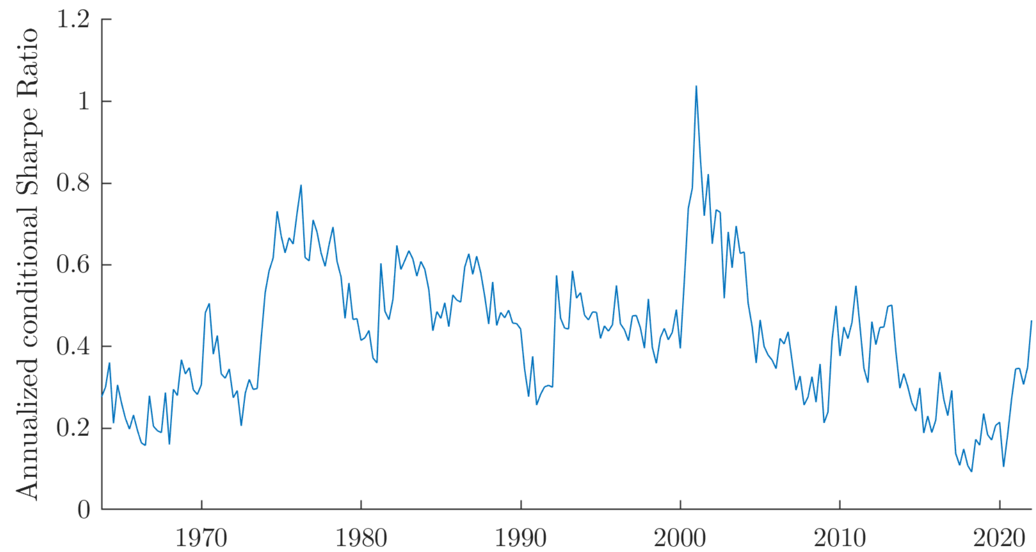
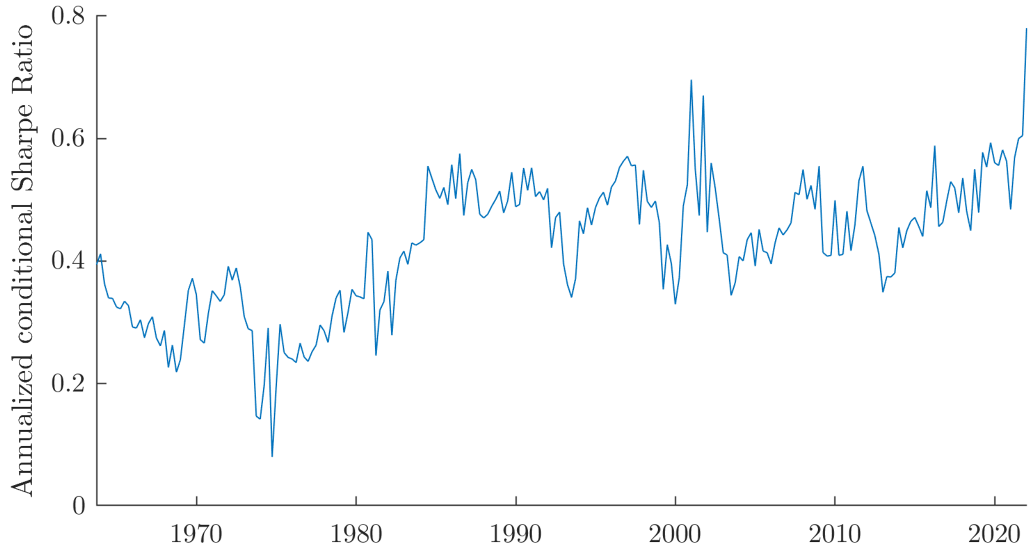


Figure 2: Posteriors on Conditional Sharpe Ratios Across Time (continued)

Panel C: Profitability



Panel D: Size

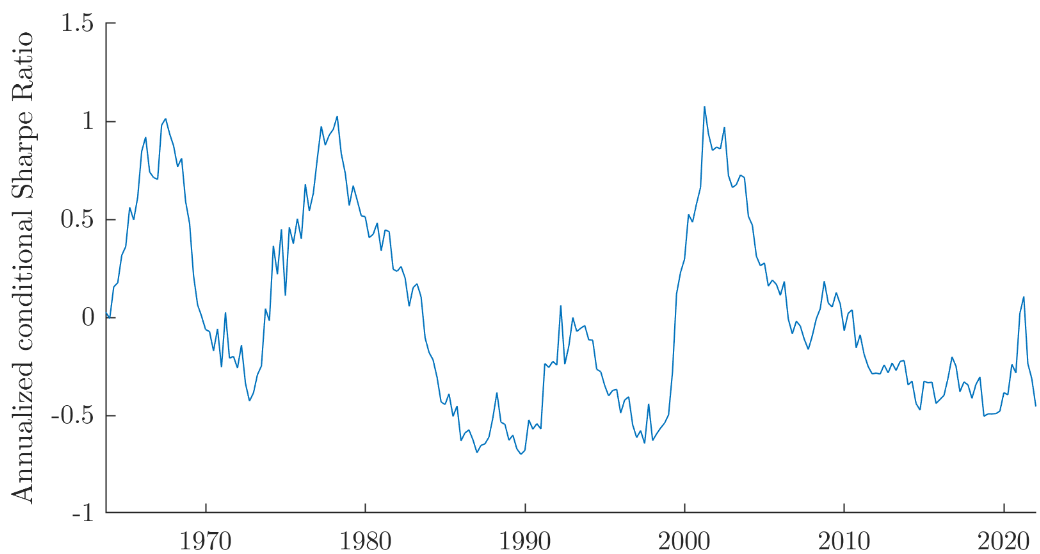
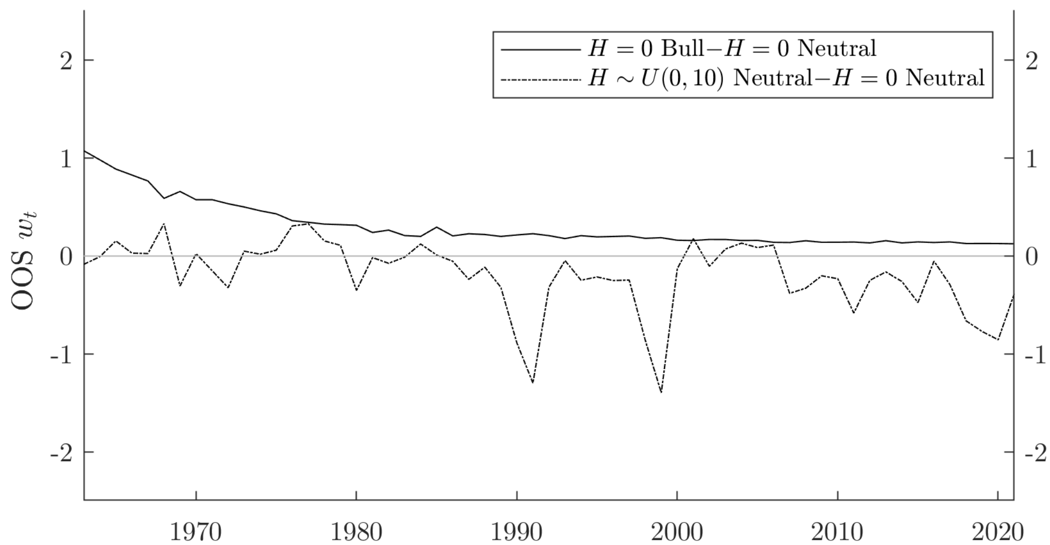


Figure 3: Out-of-Sample Portfolio Choices

This figure presents differences in portfolio weights w_t chosen by Bayesian investors with different priors who calculate posteriors using only past data available at each point in time. The solid line shows the difference in w_t chosen by an investor with priors that the half-life of shocks to expected returns $H = 0$ and that the unconditional Sharpe ratio μ_{sr} is distributed $N(0.4, 0.4)$ ('Bull'), and an investor with priors that $H = 0$ and μ_{sr} is distributed $N(0, 0.4)$ ('Neutral'). The dotted line shows the difference in w_t chosen by an investor with priors that H is distributed $U(0, 10)$ and that μ_{sr} is distributed $N(0, 0.4)$, and an investor with priors that $H = 0$ and that μ_{sr} is distributed $N(0, 0.4)$. For each prior and characteristic-sorted portfolio, we compute the posterior distribution of next-year returns and the corresponding optimal portfolio weights in the risk-free asset and the characteristic-sorted portfolio for an investor with CRRA utility and relative risk aversion of 2.

Panel A: Value



Panel B: Investment

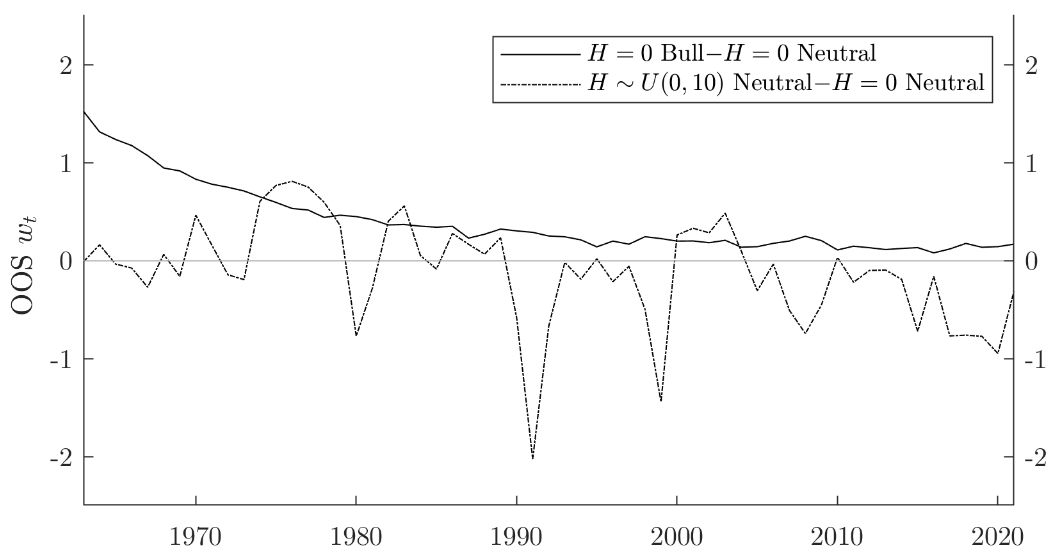
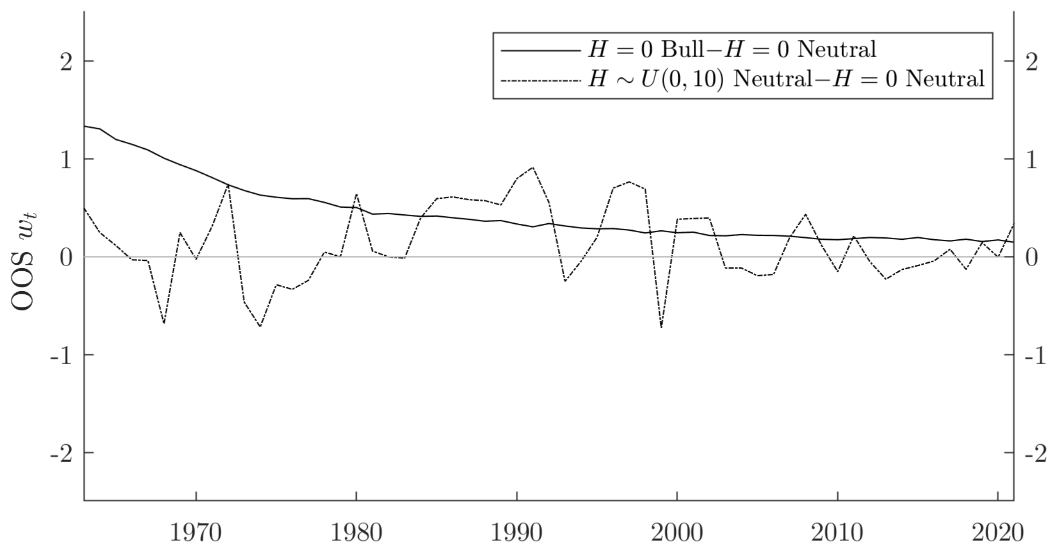


Figure 3: Out-of-Sample Portfolio Choices (continued)

Panel C: Profitability



Panel D: Size

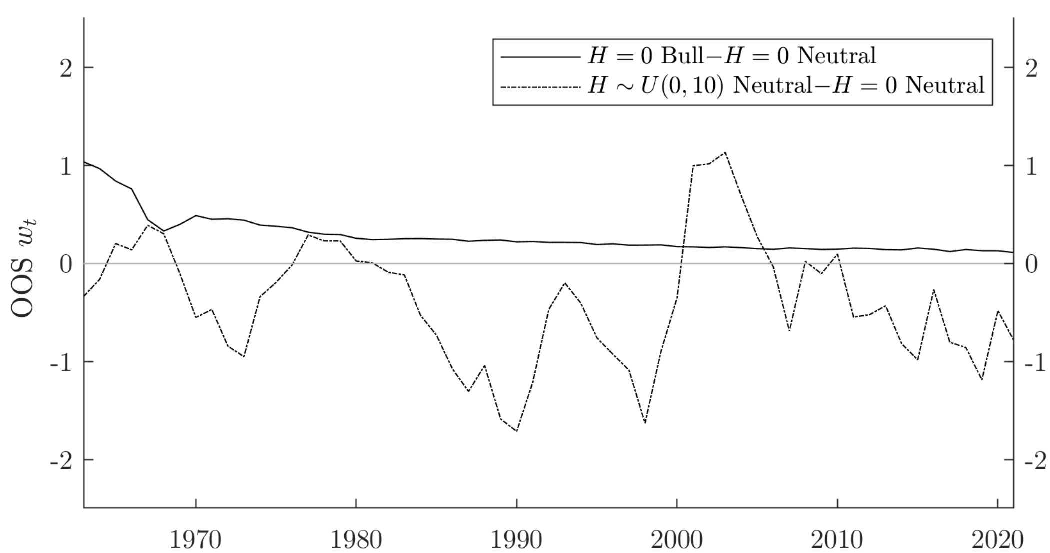


Figure 4: Unconditional Expected Return Inferences for 174 Portfolios

This figure reports the number of characteristic-sorted portfolios for which the null hypothesis that the unconditional expected return μ is zero is rejected at different p-value thresholds. The blue bars use OLS with White (1980) standard errors. The red bars use likelihood ratio tests based on the maximum likelihood estimates of our statistical model, as described in Section 3.2.1. We conduct these tests for quarterly returns of 174 long-short equity portfolios, which include the four portfolios that we focus on in our main analysis and 170 portfolios analyzed by Chen and Zimmermann (2021). The sample period varies across portfolios and consists of at most 234 quarterly observations from Q3 1963 through Q4 2021.

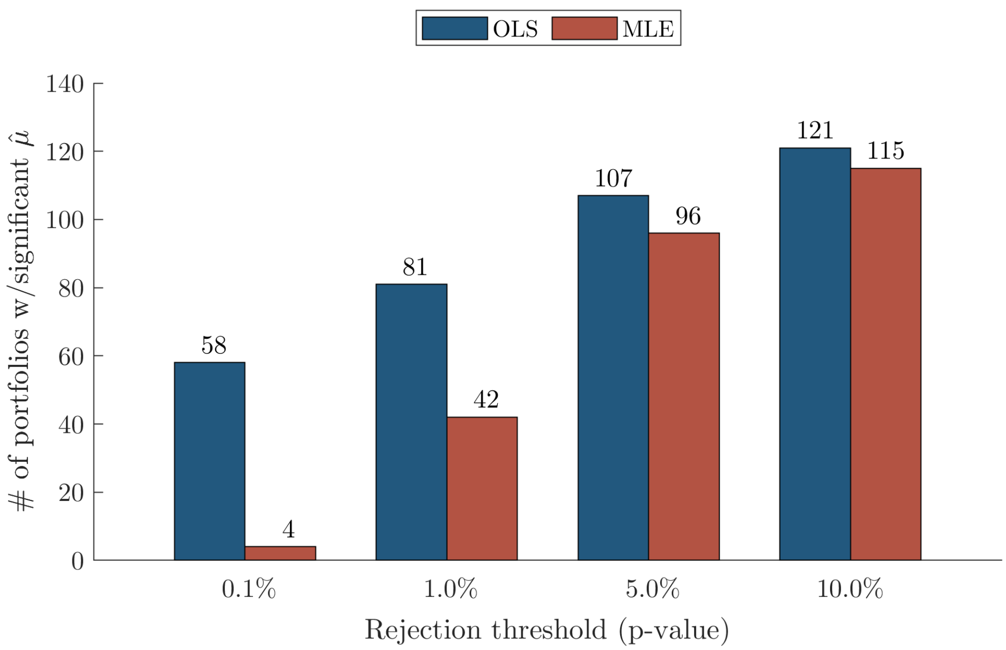
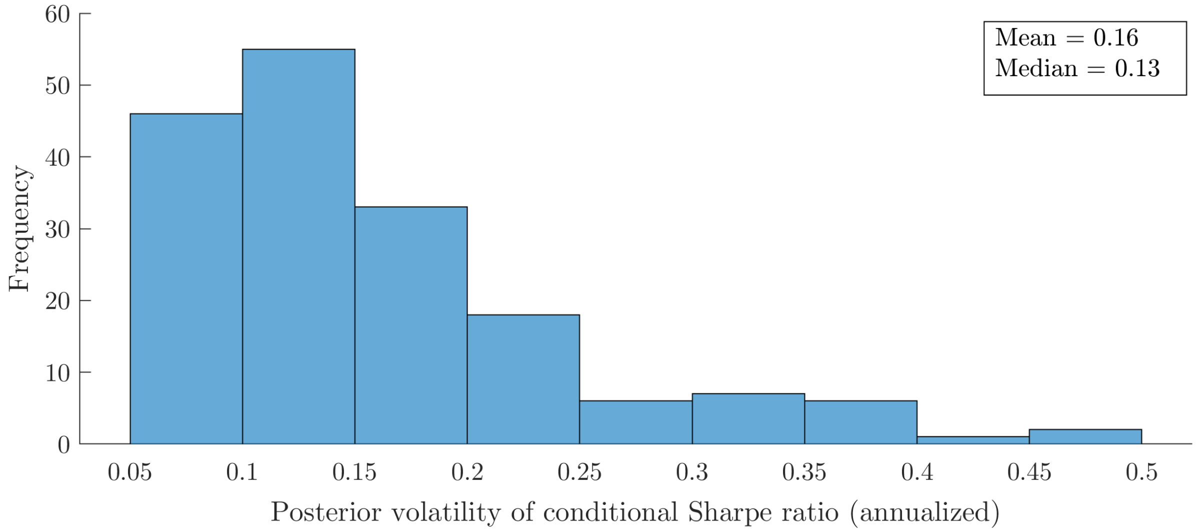


Figure 5: Bayesian Posteriors for 174 Portfolios

This figure presents frequency distributions that summarize the Bayesian inferences about the conditional Sharpe ratios of 174 long-short equity portfolios, which include the four portfolios that we focus on in our main analysis and 170 portfolios analyzed by Chen and Zimmermann (2021). Panel A plots the frequency distribution of the posterior mean for $\hat{\sigma}_{sr}$, the volatility of the Sharpe ratio conditional on past returns. Panel B plots the frequency distribution of the difference between the posterior means for the 2022 conditional Sharpe ratio and the unconditional Sharpe ratio. All Sharpe ratios are annualized. The sample period varies across portfolios and consists of at most 234 quarterly observations from Q3 1963 through Q4 2021.

Panel A: Volatility of conditional Sharpe ratios



Panel B: Conditional Sharpe ratios in 2022 relative to unconditional Sharpe ratios

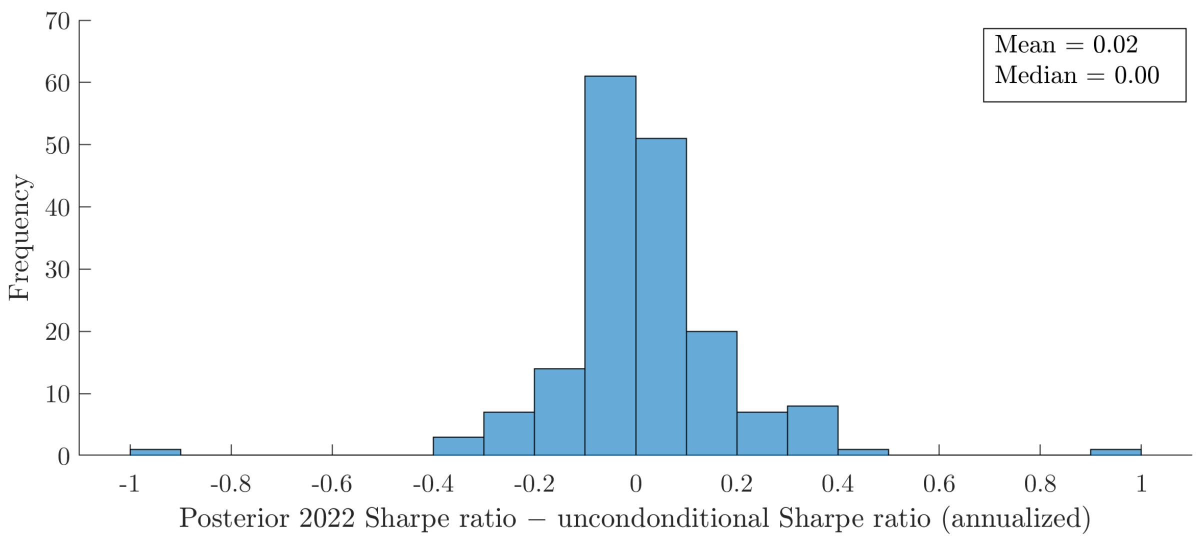


Table 1: Historical Performance of Characteristic-Sorted Portfolios

This table presents statistics summarizing the historical performance of value-weighted quintile portfolios formed on value, investment, profitability, and size characteristics. The value portfolio is based on sorting firms by their book-to-market ratios, the investment portfolio on sorting by the annual growth rate of total assets, the profitability portfolio on sorting by operating profits divided by book equity, and the size portfolio on sorting by market capitalization, as in Fama and French (2015). The investment portfolio is long firms in the lowest quintile and short firms in the highest quintile, while the other three portfolios are long the highest quintile and short the lowest. For each portfolio, we compute market-neutral returns by hedging out market risk using the full-sample market β. We report the mean annualized quarterly return in percentage terms and the annualized Sharpe ratio for the full sample, two subsamples, and the difference between the subsamples. t-statistics based on iid re-sampling of the calendar quarters in each sample are in parenthesis. Our sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

	Mean (annualized %)				Sharpe ratio (annualized)			
	All	1963–1992	1993–2021	Diff	All	1963–1992	1993–2021	Diff
Value	3.80	6.09	1.47	-4.62	0.25	0.45	0.09	-0.36
	(1.92)	(2.19)	(0.52)	-(1.17)	(1.91)	(2.40)	(0.47)	-(1.35)
Investment	4.75	4.81	4.68	-0.12	0.45	0.48	0.41	-0.07
	(3.39)	(2.46)	(2.35)	-(0.04)	(3.36)	(2.59)	(2.18)	-(0.26)
Profitability	4.91	3.40	6.45	3.05	0.45	0.36	0.54	0.18
	(3.47)	(1.71)	(3.20)	(1.08)	(3.45)	(1.92)	(2.86)	(0.69)
Size	0.16	1.97	-1.68	-3.65	0.01	0.12	-0.12	-0.24
	(0.08)	(0.69)	-(0.58)	-(0.90)	(0.08)	(0.64)	-(0.62)	-(0.89)

Table 2: Autocorrelation Estimates with Simulated and Historical Data

This table presents statistics summarizing autocorrelation estimates with model-simulated and historical portfolio returns. For each portfolio, we run overlapping time-series regressions of quarterly returns r_t on a constant and rolling averages of past returns from quarters $t-L$ through $t-1$:

$$r_t = a + b_L \left(\frac{1}{L} \sum_{l=1}^{L} r_{t-l} \right) + \epsilon_t. \tag{16}$$

Panel A presents average coefficients b_L and 95% confidence intervals from 50,000 simulated 234-quarter samples under various parameterizations of the model. The parameter H is the half-life of shocks to expected returns denoted in years, γ is the first-order autocorrelation of quarterly returns, and $\hat{\sigma}_{sr}$ is the volatility of the annualized Sharpe ratio conditional on past realized returns. Panel B show estimates for the value, investment, profitability, and size portfolios, as defined in the header of Table 1. Standard errors based on iid re-sampling of the calendar months in our sample are in parenthesis. The joint significance row presents the fraction of i.i.d. simulations for which the sum of the $\hat{b}$ across the four portfolios exceeds the sum in observed data. The historical sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

Panel A: Model Simulations

Parameterization			Average coefficient [95% confidence interval]		
H	γ	$\hat{\sigma}_{sr}$	b_4	b_{10}	b_{20}
IID		0.00	-0.03	-0.09	-0.20
			[-0.31,0.22]	[-0.59,0.30]	[-1.04,0.38]
2.5	2.5%	0.12	0.04	0.06	0.01
			[-0.24,0.29]	[-0.44,0.43]	[-0.76,0.52]
2.5	5%	0.22	0.11	0.16	0.14
			[-0.18,0.36]	[-0.32,0.52]	[-0.59,0.61]
2.5	10%	0.38	0.22	0.32	0.31
			[-0.07,0.47]	[-0.13,0.63]	[-0.32,0.71]
5	2.5%	0.15	0.04	0.06	0.03
			[-0.25,0.29]	[-0.45,0.44]	[-0.76,0.55]
5	5%	0.26	0.10	0.16	0.18
			[-0.19,0.36]	[-0.33,0.54]	[-0.57,0.66]
10	2.5%	0.18	0.03	0.03	0.01
			[-0.26,0.28]	[-0.48,0.43]	[-0.81,0.56]
10	5%	0.30	0.08	0.13	0.15
			[-0.22,0.35]	[-0.38,0.53]	[-0.64,0.66]

Panel B: Estimates for historical data

	b_4	b_{10}	b_{20}
Value	0.12	-0.07	0.11
Investment	0.21	0.07	-0.24
Profitability	0.15	-0.18	-0.30
Size	0.42	0.48	0.18
Pooled	0.25	0.20	0.07
iid p-value	0.0%	2.2%	16.7%
Pooled (without size)	0.15	-0.05	-0.07
iid p-value	2.8%	48.9%	40.4%

Table 3: OLS with Time-Varying Expected Returns

This table presents estimates of unconditional expected returns of value, investment, profitability, and size portfolios, as defined in the header of Table 1, along with t-statistics calculated using a variety of approaches. The first row reports the point estimate of the unconditional expected return, $\hat{\mu}$, in annualized percentage terms. The 'unadjusted t-statistic' row reports the typical OLS standard error, which is calculated under the assumption of independently distributed returns. The next three rows report standard errors calculated using the Newey and West (1987) adjustment with 10, 20, or 40 quarterly lags. The remaining rows report OLS standard errors corrected for autocorrelation using Equation (12) with different values of H, the half-life of shocks to expected returns in years, and γ, the first-order autocorrelation of quarterly returns. Our sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

	Value	Investment	Profitability	Size
$\hat{\mu}$ (annualized %)	3.80	4.75	4.91	0.16
Unadjusted t-statistic	(1.93)	(3.42)	(3.49)	(0.08)
Newey-West t-statistic (10 lags)	(1.86)	(3.01)	(3.45)	(0.06)
Newey-West t-statistic (20 lags)	(1.87)	(3.20)	(3.81)	(0.05)
Newey-West t-statistic (40 lags)	(1.89)	(3.48)	(3.60)	(0.06)
Model t-statistic ($H = 2.5$, $\gamma = 2.5\%$)	(1.47)	(2.62)	(2.67)	(0.06)
Model t-statistic ($H = 2.5$, $\gamma = 5\%$)	(1.24)	(2.21)	(2.25)	(0.05)
Model t-statistic ($H = 2.5$, $\gamma = 10\%$)	(0.99)	(1.75)	(1.79)	(0.04)
Model t-statistic ($H = 5$, $\gamma = 2.5\%$)	(1.27)	(2.26)	(2.30)	(0.05)
Model t-statistic ($H = 5$, $\gamma = 5\%$)	(1.02)	(1.81)	(1.84)	(0.04)
Model t-statistic ($H = 10$, $\gamma = 2.5\%$)	(1.07)	(1.91)	(1.94)	(0.05)
Model t-statistic ($H = 10$, $\gamma = 5\%$)	(0.83)	(1.47)	(1.50)	(0.03)

Table 4: Maximum Likelihood Hypothesis Tests

This table presents parameter estimates and hypothesis tests based on maximum-likelihood estimates of our model for value, investment, profitability, and size portfolios, as defined in the header of Table 1. Panel A reports estimates of μ, the unconditional expected return in annualized percentage terms; σ_r, the volatility of returns in annualized percentage terms; H, the half-life of shocks to expected returns in years; and γ, the first-order autocorrelation of quarterly returns. The rows labelled $\mu = 0$ re-estimate the model with μ restricted to be zero, and report the p-value for this restriction based on a likelihood ratio test. Panel B presents a variety of hypothesis tests for different restrictions on H and γ. In each case, we report the likelihood ratio p-value of the (H, γ) restriction (Rest. p-value), and the likelihood ratio p-value of the $\mu = 0$ restriction given the (H, γ) restriction. Our sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

Panel A: Tests for $\mu = 0$

	IID			Time-varying means				
	μ (%)	σ_r (%)	p-value (%)	μ (%)	σ_r (%)	H (years)	γ (%)	p-value (%)
Value	3.80	15.10		3.96	15.10	20.00	-0.43	
$\mu = 0$	0.00	15.22	5.5%	0.00	15.21	20.00	2.03	15.7%
Investment	4.75	10.60		5.15	10.60	8.95	-0.96	
$\mu = 0$	0.00	10.87	0.1%	0.00	10.80	20.00	4.27	3.4%
Profitability	4.91	10.77		4.92	10.77	19.57	-0.44	
$\mu = 0$	0.00	11.05	0.1%	0.00	10.97	20.00	4.18	4.4%
Size	0.16	15.43		0.15	15.43	1.23	14.01	
$\mu = 0$	0.00	15.43	93.5%	0.00	15.43	2.07	13.01	36.7%

Panel B: Restrictions on H and γ

Restrictions	H (years):		2.5			5			10		
	γ (%):	IID	-	2.5	5	-	2.5	5	-	2.5	5
Value	Rest. p-value (%)	50.8	52.1	67.9	44.3	51.0	57.9	34.1	55.0	52.5	31.5
	$\mu = 0$ p-value (%)	5.5	12.4	15.4	23.9	15.4	24.1	36.8	18.2	35.1	51.2
Investment	Rest. p-value (%)	27.1	30.1	56.9	47.1	36.2	39.2	26.7	90.7	30.0	19.3
	$\mu = 0$ p-value (%)	0.1	3.6	1.1	3.3	3.4	3.4	9.8	2.8	8.8	21.6
Profitability	Rest. p-value (%)	51.4	52.3	55.2	32.3	51.8	50.5	27.4	58.3	49.1	28.0
	$\mu = 0$ p-value (%)	0.1	0.7	0.7	2.0	1.4	1.8	5.1	2.9	4.5	11.0
Size	Rest. p-value (%)	0.1	25.4	4.2	13.2	4.6	2.1	4.4	1.0	1.0	1.5
	$\mu = 0$ p-value (%)	93.5	100.0	96.1	98.9	100.0	95.8	97.6	100.0	95.0	96.6

Table 5: Prior Beliefs about Model Parameters

This table presents summary statistics for the prior distributions that we use for our Bayesian analyses. Panel A lists the possible priors we consider for μ_{sr}, the annualized unconditional Sharpe ratio; H, the half-life of shocks to expected returns in years; σ_r, the volatility of returns in annualized percentage terms; σ_{sr}, the volatility of the annualized conditional Sharpe ratio; and ρ, the correlation between shocks to unexpected and expected returns. The term $N(\textit{mean}, \textit{standard deviation})$ indicates a normal distribution, $U(\textit{lower bound}, \textit{upper bound})$ indicates a uniform distribution, and a number stated without a distribution indicates a dogmatic prior that the parameter value equals that number. Panel B presents the means and 95% confidence intervals implied by each (H, μ_{sr}) prior parameterization for prior distributions of μ, the unconditional expected return in annualized percentage terms; γ, the first-order autocorrelation of quarterly returns; and $\hat{\sigma}_{sr}$, the volatility of the annualized Sharpe ratio conditional on past realized returns.

Panel A: Priors on Transformed Parameters

μ_{sr}	H (years)	σ_r (%)	σ_{sr}	ρ
$N(-0.4, 0.4)$	0	$U(10, 20)$	$U(0, 1)$	$U(-0.5, 0)$
$N(0, 0.4)$	2.5			
$N(0.4, 0.4)$	5			
$U(-2, 2)$	$U(0, 10)$			

Panel B: Moments of Priors

		μ (%)		γ (%)		$\hat{\sigma}_{sr}$	
H	μ_{sr}	mean	95% CI	mean	95% CI	mean	95% CI
0	$N(-0.4, 0.4)$	-5.30	[-15.90,5.16]	-	-	-	-
0	$N(0, 0.4)$	-0.01	[-10.47,10.42]	-	-	-	-
0	$N(0.4, 0.4)$	5.32	[-5.10,16.05]	-	-	-	-
0	$U(-2, 2)$	-0.02	[-25.84,25.70]	-	-	-	-
2.5	$N(-0.4, 0.4)$	-5.34	[-16.07,5.07]	4.81	[-0.58,15.44]	0.19	[0.00,0.52]
2.5	$N(0, 0.4)$	-0.04	[-10.42,10.37]	4.76	[-0.59,15.41]	0.19	[0.00,0.52]
2.5	$N(0.4, 0.4)$	5.29	[-5.10,15.98]	4.78	[-0.58,15.43]	0.19	[0.00,0.52]
2.5	$U(-2, 2)$	-0.05	[-25.88,25.72]	4.76	[-0.58,15.45]	0.19	[0.00,0.52]
5	$N(-0.4, 0.4)$	-5.30	[-15.94,5.11]	5.58	[-0.25,16.49]	0.25	[0.00,0.61]
5	$N(0, 0.4)$	0.01	[-10.47,10.47]	5.60	[-0.26,16.47]	0.25	[0.00,0.61]
5	$N(0.4, 0.4)$	5.23	[-5.18,15.94]	5.61	[-0.26,16.54]	0.25	[0.00,0.61]
5	$U(-2, 2)$	0.00	[-25.75,25.93]	5.56	[-0.25,16.41]	0.25	[0.00,0.61]
$U(0, 10)$	$N(-0.4, 0.4)$	-5.32	[-16.03,4.99]	4.91	[-1.91,16.37]	0.23	[0.00,0.63]
$U(0, 10)$	$N(0, 0.4)$	-0.02	[-10.48,10.38]	4.90	[-1.79,16.37]	0.23	[0.00,0.63]
$U(0, 10)$	$N(0.4, 0.4)$	5.29	[-5.14,15.98]	4.92	[-1.76,16.40]	0.23	[0.00,0.63]
$U(0, 10)$	$U(-2, 2)$	-0.01	[-25.81,25.82]	4.90	[-1.83,16.38]	0.23	[0.00,0.63]

Table 6: Posterior Beliefs

This table presents summary statistics for posterior distributions that obtain given the historical return data for the value, investment, profitability, and size portfolios, as described in the header of Table 1. The first two columns describe the prior belief specification (H, μ_{sr}), where H is the half-life of shocks to expected returns and μ_{sr} is the unconditional Sharpe ratio. The remaining columns report the means and 95% confidence intervals for the posterior distributions of μ, the unconditional expected return; H; γ, the first-order autocorrelation of quarterly returns; and $\hat{\sigma}_{sr}$, the volatility of the Sharpe ratio conditional on past realized returns. The variables μ, H, and $\hat{\sigma}_{sr}$ are annualized. We compute the posterior beliefs using an M.C.M.C. procedure described in Appendix C. Our sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

Panel A: Value

Prior		μ (%)		H (years)		γ (%)		$\hat{\sigma}_{sr}$	
H	μ_{sr}	mean	95% CI	mean	95% CI	mean	95% CI	mean	95% CI
0	$N(-0.4, 0.4)$	2.86	[-0.94,6.40]	-	-	-	-	-	-
0	$N(0, 0.4)$	3.46	[-0.23,7.13]	-	-	-	-	-	-
0	$N(0.4, 0.4)$	3.99	[0.41,7.64]	-	-	-	-	-	-
0	$U(-2, 2)$	3.76	[0.05,7.53]	-	-	-	-	-	-
2.5	$N(-0.4, 0.4)$	2.02	[-3.53,6.67]	-	-	3.04	[-0.60,12.07]	0.13	[0.00,0.44]
2.5	$N(0, 0.4)$	3.11	[-1.85,7.65]	-	-	2.77	[-0.59,11.73]	0.12	[0.00,0.43]
2.5	$N(0.4, 0.4)$	4.05	[-0.59,8.74]	-	-	2.67	[-0.60,11.51]	0.12	[0.00,0.42]
2.5	$U(-2, 2)$	3.72	[-1.72,9.10]	-	-	2.99	[-0.61,12.26]	0.13	[0.00,0.44]
5	$N(-0.4, 0.4)$	1.62	[-4.94,6.56]	-	-	3.12	[-0.28,12.62]	0.16	[0.00,0.51]
5	$N(0, 0.4)$	2.91	[-2.82,7.85]	-	-	2.70	[-0.29,11.85]	0.14	[0.00,0.49]
5	$N(0.4, 0.4)$	4.02	[-1.17,9.15]	-	-	2.70	[-0.28,11.78]	0.14	[0.00,0.49]
5	$U(-2, 2)$	3.57	[-3.21,9.78]	-	-	3.00	[-0.29,12.52]	0.15	[0.00,0.51]
$U(0, 10)$	$N(-0.4, 0.4)$	1.63	[-5.25,6.73]	4.83	[0.33,9.73]	3.29	[-0.68,12.68]	0.16	[0.00,0.51]
$U(0, 10)$	$N(0, 0.4)$	2.93	[-2.59,7.80]	4.65	[0.30,9.73]	2.81	[-0.87,11.70]	0.14	[0.00,0.47]
$U(0, 10)$	$N(0.4, 0.4)$	4.05	[-0.93,9.11]	4.64	[0.28,9.72]	2.72	[-0.87,11.71]	0.13	[0.00,0.47]
$U(0, 10)$	$U(-2, 2)$	3.56	[-2.84,9.35]	4.79	[0.34,9.76]	3.00	[-0.63,12.24]	0.15	[0.00,0.49]

Table 6: Posterior Beliefs (continued)

Panel B: Investment

Prior		μ (%)		H (years)		γ (%)		$\hat{\sigma}_{sr}$	
H	μ_{sr}	mean	95% CI	mean	95% CI	mean	95% CI	mean	95% CI
0	$N(-0.4, 0.4)$	3.86	[1.42,6.29]	-	-	-	-	-	-
0	$N(0, 0.4)$	4.31	[1.74,6.84]	-	-	-	-	-	-
0	$N(0.4, 0.4)$	4.69	[2.12,7.22]	-	-	-	-	-	-
0	$U(-2, 2)$	4.75	[2.06,7.38]	-	-	-	-	-	-
2.5	$N(-0.4, 0.4)$	2.91	[-1.38,6.42]	-	-	4.65	[-0.52,13.96]	0.19	[0.00,0.49]
2.5	$N(0, 0.4)$	3.79	[-0.13,7.19]	-	-	4.08	[-0.52,13.47]	0.17	[0.00,0.47]
2.5	$N(0.4, 0.4)$	4.56	[0.94,8.03]	-	-	3.98	[-0.53,13.50]	0.17	[0.00,0.47]
2.5	$U(-2, 2)$	4.70	[0.53,8.79]	-	-	4.09	[-0.54,13.51]	0.17	[0.00,0.47]
5	$N(-0.4, 0.4)$	2.34	[-2.91,6.31]	-	-	4.50	[-0.22,14.64]	0.21	[0.00,0.56]
5	$N(0, 0.4)$	3.52	[-1.06,7.19]	-	-	3.62	[-0.27,13.79]	0.18	[0.00,0.54]
5	$N(0.4, 0.4)$	4.44	[0.48,8.16]	-	-	3.29	[-0.30,13.41]	0.16	[0.00,0.53]
5	$U(-2, 2)$	4.51	[-0.38,9.24]	-	-	3.61	[-0.26,13.79]	0.18	[0.00,0.54]
$U(0, 10)$	$N(-0.4, 0.4)$	2.57	[-2.95,6.25]	4.75	[0.38,9.72]	4.46	[-0.44,14.14]	0.20	[0.00,0.55]
$U(0, 10)$	$N(0, 0.4)$	3.65	[-0.83,7.11]	4.33	[0.36,9.71]	3.86	[-0.62,13.41]	0.17	[0.00,0.52]
$U(0, 10)$	$N(0.4, 0.4)$	4.51	[0.71,8.00]	4.20	[0.33,9.64]	3.60	[-0.65,12.81]	0.16	[0.00,0.50]
$U(0, 10)$	$U(-2, 2)$	4.58	[0.02,8.65]	4.19	[0.30,9.65]	3.77	[-0.66,13.63]	0.17	[0.00,0.52]

Panel C: Profitability

Prior		μ (%)		H (years)		γ (%)		$\hat{\sigma}_{sr}$	
H	μ_{sr}	mean	95% CI	mean	95% CI	mean	95% CI	mean	95% CI
0	$N(-0.4, 0.4)$	4.01	[1.31,6.49]	-	-	-	-	-	-
0	$N(0, 0.4)$	4.44	[1.82,6.97]	-	-	-	-	-	-
0	$N(0.4, 0.4)$	4.83	[2.25,7.45]	-	-	-	-	-	-
0	$U(-2, 2)$	4.93	[2.21,7.71]	-	-	-	-	-	-
2.5	$N(-0.4, 0.4)$	3.70	[-0.24,6.90]	-	-	2.38	[-0.62,11.69]	0.11	[0.00,0.43]
2.5	$N(0, 0.4)$	4.25	[0.92,7.35]	-	-	2.28	[-0.62,11.02]	0.10	[0.00,0.41]
2.5	$N(0.4, 0.4)$	4.90	[1.72,8.12]	-	-	2.19	[-0.64,11.00]	0.10	[0.00,0.41]
2.5	$U(-2, 2)$	5.01	[1.31,8.78]	-	-	2.35	[-0.63,11.46]	0.11	[0.00,0.42]
5	$N(-0.4, 0.4)$	3.26	[-1.42,6.80]	-	-	2.73	[-0.31,11.67]	0.14	[0.00,0.48]
5	$N(0, 0.4)$	4.11	[0.19,7.49]	-	-	2.37	[-0.30,11.11]	0.13	[0.00,0.47]
5	$N(0.4, 0.4)$	4.88	[1.30,8.61]	-	-	2.34	[-0.30,10.99]	0.13	[0.00,0.46]
5	$U(-2, 2)$	5.05	[0.91,9.58]	-	-	2.45	[-0.30,11.45]	0.13	[0.00,0.48]
$U(0, 10)$	$N(-0.4, 0.4)$	3.15	[-1.73,6.58]	5.01	[0.29,9.78]	2.81	[-0.79,12.25]	0.14	[0.00,0.51]
$U(0, 10)$	$N(0, 0.4)$	4.12	[0.19,7.52]	4.74	[0.27,9.77]	2.41	[-1.12,11.45]	0.12	[0.00,0.47]
$U(0, 10)$	$N(0.4, 0.4)$	4.85	[1.34,8.44]	4.74	[0.27,9.72]	2.32	[-1.20,11.01]	0.12	[0.00,0.46]
$U(0, 10)$	$U(-2, 2)$	5.03	[1.06,9.45]	4.71	[0.29,9.72]	2.50	[-0.99,11.48]	0.13	[0.00,0.48]

Table 6: Posterior Beliefs (continued)

Panel D: Size

Prior		μ (%)		H (years)		γ (%)		$\hat{\sigma}_{sr}$	
H	μ_{sr}	mean	95% CI	mean	95% CI	mean	95% CI	mean	95% CI
0	$N(-0.4, 0.4)$	-0.42	[-4.16,3.28]	-	-	-	-	-	-
0	$N(0, 0.4)$	0.10	[-3.55,3.84]	-	-	-	-	-	-
0	$N(0.4, 0.4)$	0.73	[-3.04,4.50]	-	-	-	-	-	-
0	$U(-2, 2)$	0.11	[-3.70,4.02]	-	-	-	-	-	-
2.5	$N(-0.4, 0.4)$	-1.86	[-8.62,4.57]	-	-	10.85	[3.20,17.08]	0.40	[0.15,0.56]
2.5	$N(0, 0.4)$	-0.09	[-6.56,6.25]	-	-	10.66	[3.04,16.96]	0.39	[0.14,0.56]
2.5	$N(0.4, 0.4)$	1.81	[-4.57,8.45]	-	-	10.70	[3.03,16.85]	0.39	[0.14,0.56]
2.5	$U(-2, 2)$	-0.12	[-7.83,7.88]	-	-	10.93	[3.23,16.95]	0.40	[0.15,0.56]
5	$N(-0.4, 0.4)$	-2.46	[-10.12,4.84]	-	-	10.90	[1.88,17.67]	0.45	[0.12,0.64]
5	$N(0, 0.4)$	-0.03	[-7.41,7.28]	-	-	10.63	[1.40,17.59]	0.44	[0.09,0.64]
5	$N(0.4, 0.4)$	2.44	[-4.87,9.92]	-	-	10.86	[1.82,17.59]	0.45	[0.12,0.64]
5	$U(-2, 2)$	-0.07	[-10.66,10.28]	-	-	11.00	[1.67,17.70]	0.45	[0.11,0.64]
$U(0, 10)$	$N(-0.4, 0.4)$	-2.00	[-9.14,4.55]	3.39	[0.78,8.98]	10.49	[2.27,17.05]	0.39	[0.11,0.62]
$U(0, 10)$	$N(0, 0.4)$	0.04	[-6.71,6.65]	3.32	[0.74,8.99]	10.31	[2.00,17.08]	0.38	[0.10,0.62]
$U(0, 10)$	$N(0.4, 0.4)$	1.96	[-4.53,9.16]	3.47	[0.83,9.13]	10.47	[2.08,17.05]	0.39	[0.10,0.63]
$U(0, 10)$	$U(-2, 2)$	-0.04	[-9.04,9.00]	3.51	[0.78,9.12]	10.64	[2.35,17.30]	0.40	[0.11,0.63]

Appendix A. Autocorrelations in Monthly Returns

As described in Section 2.1, we analyze quarterly returns of characteristic-sorted portfolios rather than the monthly returns that are typically studied in the literature. We do so because monthly returns exhibit strong first-order autocorrelations, which may be caused by lead-lag effects, under-reaction, or some other transitory source of persistence. While interesting on its own, this form of autocorrelation differs from the main focus of this paper, namely, slow-moving but persistent variations in expected returns. To avoid biasing our estimates towards large but quickly-reverting variations, we use quarterly rather than monthly data.

Appendix Figure 1 illustrates the magnitude of the autocorrelations at monthly lags $l = 1$ through $l = 60$ for the four portfolios we study, as well as the autocorrelations estimated in a pooled regression including all four portfolios. The first-order autocorrelation is the single largest coefficient for any of the 60 months for value, investment, and profitability, as well as in pooled regressions. Quarterly data do not exhibit a strong first-order autocorrelation (see Table 2) because the two- and three-month autocorrelations in Appendix Figure 1 are much smaller and statistically insignificant.

Appendix B. Generalized Least Squares (GLS) Estimations

In addition to the OLS standard error correction described in Section 3.1, we estimate unconditional expected returns μ in GLS regressions that use the covariance matrix Σ implied by γ and H. These GLS estimations adjust both the point estimates $\hat{\mu}$ and the standard errors. While the OLS estimates are based on an equally-weighted average of the returns in the sample, the GLS estimates utilize an average weighted by the amount of orthogonal information each observation contains about the unconditional expected return. When conditional expected returns exhibit persistent variations, the observations in the middle of the sample are relatively more redundant because they 'over-sample' the same epoch of conditional expected returns. As illustrated by Appendix Figure 2, GLS therefore over-weights observations at the beginning and end of the sample. This effect is larger for larger values of H, and reverses when γ is negative.

Appendix Table 2 shows the GLS point estimates and t-statistics, along with the corresponding OLS statistics, for a variety of assumptions about γ and H. We find that the GLS corrections to point estimates are generally small, but somewhat more negative in some specifications for the value and the investment portfolios. The reason for this negative effect is that these portfolios had unusually low returns at the beginning and/or the end of the sample, which GLS infers as providing more independent information relative to the observations in the middle. Overall, the main conclusion from the OLS analysis, that t-statistics can be half as large for reasonable values of H and γ relative to uncorrected t-statistics, remains unchanged when using GLS.

Appendix C. Sampling Bayesian Posteriors

We draw samples of $N = 50,000$ observations from the posterior distribution of model parameters Ω^{post} using the following procedure:

1. Draw N observations Ω_i^{prior}, $i \in [1, N]$ from the prior distribution.
2. Accept Ω_1^{prior} as the first observation of the posterior distribution $\Omega_1^{\text{posterior}}$.
3. For observations $i = 2 \ldots N$:
   (a) Evaluate the conditional likelihood of the data D given the ith draw from the prior parameters as well as the $i-1$st draw from the posterior parameters:

$$\mathcal{L}^{\text{propose}} = \mathcal{L}(D|\Omega = \Omega_i^{\text{prior}}), \tag{17}$$

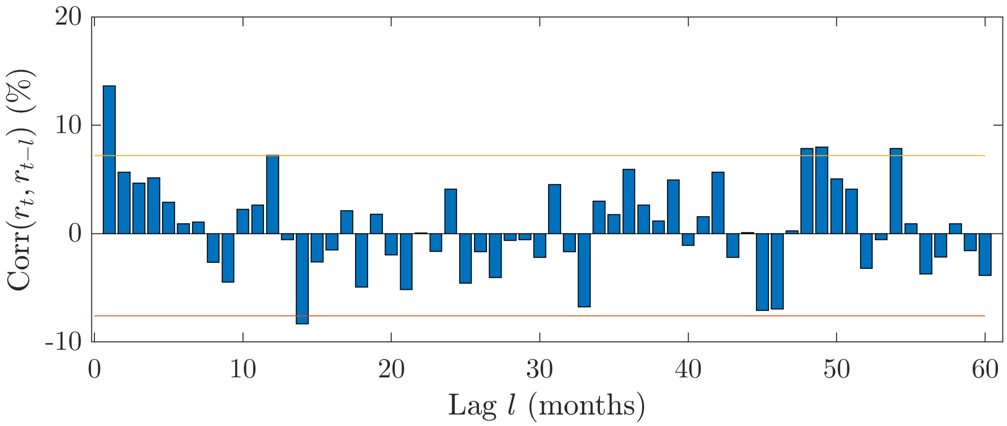
$$\mathcal{L}^{\text{previous}} = \mathcal{L}(D|\Omega = \Omega_{i-1}^{\text{posterior}}). \tag{18}$$

   (b) If $\mathcal{L}^{\text{propose}} \geq \mathcal{L}^{\text{previous}}$, accept $\Omega_i^{\text{posterior}} = \Omega_i^{\text{prior}}$.
   (c) If $\mathcal{L}^{\text{propose}} \leq \mathcal{L}^{\text{previous}}$, accept $\Omega_i^{\text{posterior}} = \Omega_i^{\text{prior}}$ with probability $\frac{\mathcal{L}^{\text{propose}}}{\mathcal{L}^{\text{previous}}}$, and otherwise retain $\Omega_i^{\text{posterior}} = \Omega_{i-1}^{\text{prior}}$.

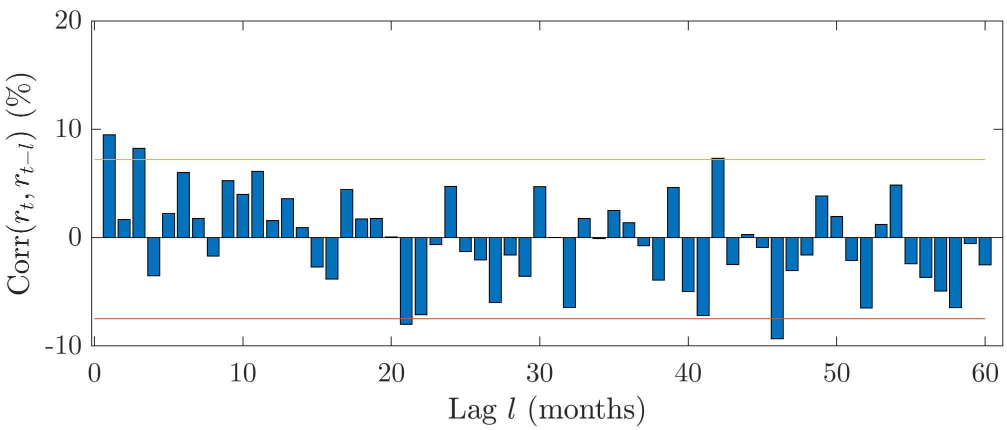
Appendix Figure 1: Monthly Autocorrelograms

This figure presents autocorrelations of monthly returns for value, investment, profitability, and size portfolios, as defined in the header of Table 1, as well the autocorrelation estimated in a pooled regression containing all four portfolios. We estimate the autocorrelation for each lag l independently. The horizontal lines represent the 95% confidence interval for autocorrelation coefficients under the zero-autocorrelation null hypothesis. Our sample consists of 678 monthly observations from Q3 1963 through Q4 2021.

Panel A: Value

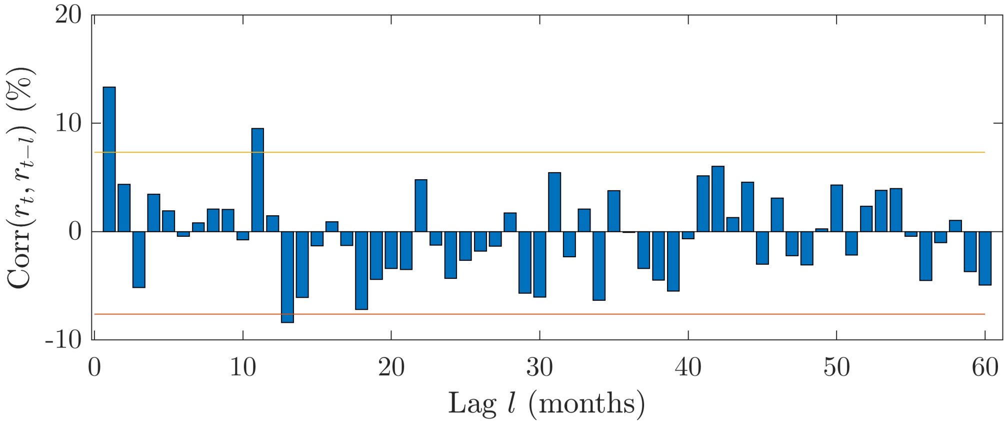


Panel B: Investment

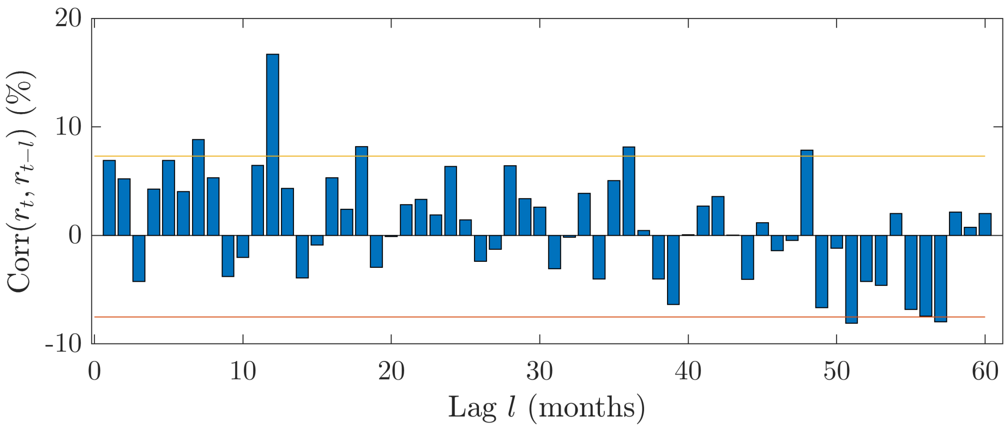


Appendix Figure 1: Monthly Autocorrelograms (continued)

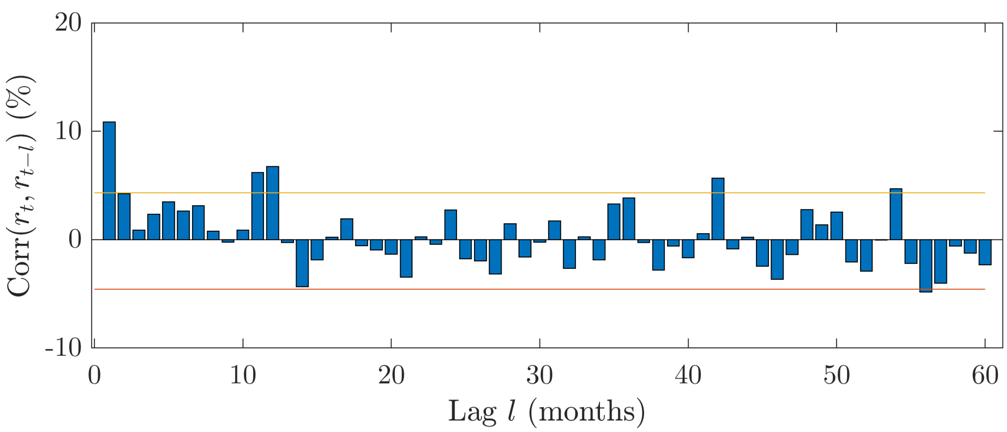
Panel C: Profitability



Panel D: Size

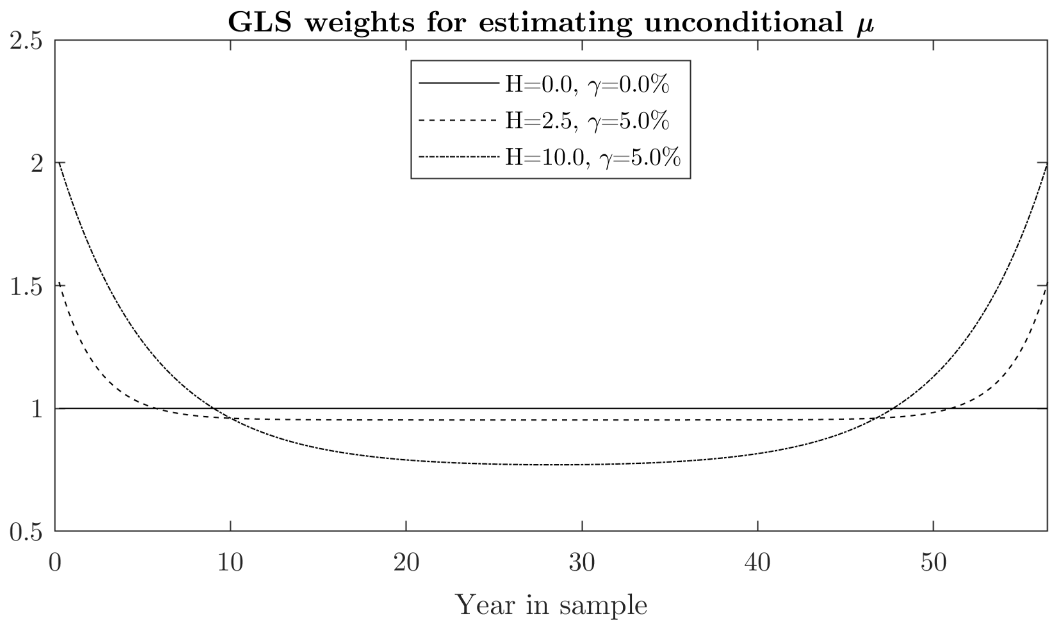


Panel E: Pooled



Appendix Figure 2: GLS Influence Functions and Return Autocorrelations

This figure presents the GLS weights that apply in the estimation of the unconditional expected return μ under different assumptions about the autocorrelation structure of returns. The parameter H is the half-life of shocks to expected returns in years, and γ is the first-order autocorrelation of quarterly returns.



Appendix Table 1: Return Autocorrelations for Different Parameterizations

This table shows the first-order autocorrelation of quarterly returns in percentage terms, $\gamma = corr(r_t, r_{t-1})$, implied by various combinations of H, the half-life of shocks to expected returns, and $\sigma_\delta / \sigma_\epsilon$, the ratio of the volatilities of expected and unexpected return shocks. We assume $\rho = -1$ throughout to provide an upper bound on the magnitude negative autocorrelations.

H	$\frac{\sigma_\delta}{\sigma_\epsilon}$						
	0.001	0.01	0.1	0.25	0.5	0.75	1
0.1	-0.0	-0.2	-1.8	-4.4	-8.6	-12.7	-16.6
0.25	-0.0	-0.5	-4.8	-11.2	-19.2	-23.7	-25.0
0.5	-0.1	-0.7	-6.3	-12.5	-14.1	-8.5	0.0
1	-0.1	-0.8	-6.2	-7.2	5.4	21.7	34.8
2.5	-0.1	-0.9	-2.9	11.2	41.1	59.2	69.1
5	-0.1	-0.8	3.3	32.1	64.3	77.5	83.7
10	-0.1	-0.7	14.1	54.0	80.1	88.1	91.6
25	-0.1	-0.3	35.7	76.6	91.4	95.1	96.6

Appendix Table 2: GLS with Time-Varying Expected Returns

This table presents estimates of unconditional expected returns of value, investment, profitability, and size portfolios, as defined in the header of Table 1, under a variety of assumptions about the magnitude and persistence of variations in conditional expected returns. The model-implied autocorrelation structure of returns are summarized by H, the half-life of shocks to expected returns in years, and γ, the first-order autocorrelation of quarterly returns. We estimate unconditional expected returns μ using both OLS and GLS and calculate t-statistics using the model-implied correlation matrix of the regression error terms. Our sample consists of 234 quarterly observations from Q3 1963 through Q4 2021.

Panel A: Value

	H (years)	2.5			5		10	
	γ (%)	2.5	5	10	2.5	5	2.5	5
OLS	$\hat{\mu}$ (%)	3.80	3.80	3.80	3.80	3.80	3.80	3.80
	Model t-stat	(1.47)	(1.24)	(0.99)	(1.27)	(1.02)	(1.07)	(0.83)
GLS	$\hat{\mu}$ (%)	3.68	3.62	3.57	3.50	3.36	3.28	3.03
	Model t-stat	(1.43)	(1.19)	(0.93)	(1.18)	(0.91)	(0.94)	(0.67)

Panel B: Investment

	H (years)	2.5			5		10	
	γ (%)	2.5	5	10	2.5	5	2.5	5
OLS	$\hat{\mu}$ (%)	4.75	4.75	4.75	4.75	4.75	4.75	4.75
	Model t-stat	(2.62)	(2.21)	(1.75)	(2.26)	(1.81)	(1.91)	(1.47)
GLS	$\hat{\mu}$ (%)	4.65	4.62	4.63	4.46	4.35	4.23	4.00
	Model t-stat	(2.57)	(2.15)	(1.72)	(2.14)	(1.68)	(1.72)	(1.26)

Panel C: Profitability

	H (years)	2.5			5		10	
	γ (%)	2.5	5	10	2.5	5	2.5	5
OLS	$\hat{\mu}$ (%)	4.91	4.91	4.91	4.91	4.91	4.91	4.91
	Model t-stat	(2.67)	(2.25)	(1.79)	(2.30)	(1.84)	(1.94)	(1.50)
GLS	$\hat{\mu}$ (%)	5.04	5.16	5.36	5.07	5.23	5.04	5.23
	Model t-stat	(2.74)	(2.37)	(1.96)	(2.39)	(1.98)	(2.02)	(1.62)

Panel D: Size

	H (years)	2.5			5		10	
	γ (%)	2.5	5	10	2.5	5	2.5	5
OLS	$\hat{\mu}$ (%)	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	Model t-stat	(0.06)	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)	(0.03)
GLS	$\hat{\mu}$ (%)	0.16	0.13	0.08	-0.04	0.16	0.12	0.23
	Model t-stat	(0.05)	(0.03)	-(0.01)	(0.05)	(0.03)	(0.06)	(0.04)

Appendix Table 3: Inference and Posteriors for 174 Portfolios

This table presents summary statistics and estimation results for the four portfolios that we focus on in our main analysis (the first four rows) and 170 long-short equity portfolios analyzed by Chen and Zimmermann (2021) (the remaining rows). The first two columns report $E(r_t)$, the in-sample average of annualized quarterly returns, and SR, the in-sample annualized Sharpe ratio. The next two columns report the p-values for the hypothesis that the unconditional expected return μ equals zero using OLS regressions that assume i.i.d. returns and maximum likelihood estimations (MLE) that allow for time-varying expected returns. The next four columns report the means of Bayesian posterior distributions for $P(\mu > 0)$, the probability that $\mu > 0$; μ_{sr}, the unconditional Sharpe ratio; $\mu_{sr,2022}$, the conditional Sharpe ratio for Q1 of 2022; and $\hat{\sigma}_{sr}$, the volatility of the Sharpe ratio conditional on past returns. The last column reports ρ(size), which denotes the correlation of each portfolio's returns with the size portfolio's return.

			$\mu = 0$ p-values		Bayesian Posterior Means				
Portfolio	$E(r_t)$	SR	OLS	MLE	$P(\mu > 0)$	μ_{sr}	$\mu_{sr,2022}$	$\hat{\sigma}_{sr}$	ρ(size)
Value (FF)	3.80	0.25	5.4%	10.4%	87.8%	0.20	0.10	0.14	0.40
Investment (FF)	4.75	0.45	0.1%	1.4%	95.6%	0.36	0.37	0.17	0.04
Profitability (FF)	4.91	0.45	0.0%	2.4%	98.0%	0.39	0.60	0.12	-0.52
Size (FF)	0.16	0.01	93.5%	97.5%	50.9%	0.00	-0.30	0.38	1.00
AM	1.34	0.08	54.6%	61.4%	65.6%	0.06	-0.01	0.12	0.18
AOP	3.88	0.34	2.0%	3.7%	93.3%	0.28	0.39	0.12	-0.26
AbnormalAccruals	4.92	0.53	0.0%	0.1%	99.0%	0.45	0.53	0.08	0.04
Accruals	4.59	0.47	0.0%	0.0%	99.5%	0.42	0.43	0.05	-0.19
Activism1	-0.59	-0.10	68.7%	66.9%	38.3%	-0.07	-0.13	0.19	0.20
AdExp	5.79	0.38	0.7%	4.4%	89.0%	0.27	0.13	0.22	0.11
AgeIPO	13.77	0.59	0.0%	0.5%	97.5%	0.46	0.50	0.12	-0.30
AnalystRevision	5.40	0.57	0.0%	1.9%	97.9%	0.47	0.57	0.13	-0.14
AnalystValue	5.39	0.32	3.1%	9.1%	90.9%	0.25	0.28	0.15	-0.11
AnnouncementReturn	8.37	1.05	0.0%	0.4%	99.3%	0.82	0.99	0.18	-0.07
AssetGrowth	4.38	0.40	0.2%	5.5%	91.5%	0.29	0.24	0.21	0.17
BM	2.30	0.11	41.7%	48.6%	71.1%	0.08	0.00	0.12	0.36
BMdec	3.78	0.27	3.9%	10.9%	82.3%	0.18	-0.07	0.25	0.33
BPEBM	1.75	0.25	5.5%	4.0%	92.9%	0.23	0.32	0.10	0.22
Beta	-7.13	-0.36	0.6%	3.7%	6.5%	-0.29	-0.49	0.19	0.60
BetaFP	-7.70	-0.53	0.0%	1.3%	3.5%	-0.41	-0.41	0.16	0.31
BetaLiquidityPS	1.56	0.16	24.9%	18.8%	74.5%	0.11	-0.03	0.15	0.02
BetaTailRisk	-0.14	-0.01	93.3%	93.0%	47.7%	-0.01	-0.22	0.24	0.42
BetaVIX	8.38	0.72	0.0%	2.3%	91.3%	0.41	0.53	0.34	-0.29
BidAskSpread	-10.93	-0.46	0.0%	7.9%	4.9%	-0.36	-0.51	0.19	0.68
BookLeverage	-0.48	-0.03	80.3%	82.6%	42.7%	-0.02	-0.03	0.08	0.11
BrandInvest	-0.86	-0.06	70.2%	13.0%	38.9%	-0.04	-0.07	0.09	0.41
CBOperProf	6.87	0.74	0.0%	0.8%	99.4%	0.62	0.93	0.15	-0.52
CF	6.35	0.44	0.1%	0.7%	97.6%	0.37	0.38	0.10	0.09
Cash	2.84	0.20	16.2%	4.9%	86.3%	0.17	0.15	0.08	0.21
CashProd	1.56	0.12	34.4%	43.4%	75.6%	0.10	0.04	0.10	0.08
Cfp	6.85	0.50	0.0%	0.4%	98.7%	0.42	0.45	0.07	-0.53
ChAssetTurnover	2.06	0.26	4.6%	4.4%	91.4%	0.22	0.16	0.10	0.03
ChEQ	5.32	0.50	0.0%	0.2%	99.0%	0.43	0.46	0.08	0.06
ChInv	5.78	0.70	0.0%	0.3%	98.7%	0.56	0.59	0.17	-0.09
ChInvIA	3.30	0.41	0.2%	0.1%	99.2%	0.38	0.58	0.13	-0.09
ChNNCOA	3.01	0.42	0.1%	0.6%	98.0%	0.36	0.34	0.08	-0.02
ChNWC	2.89	0.39	0.3%	2.3%	95.5%	0.32	0.29	0.14	-0.06
ChTax	3.68	0.38	0.4%	14.2%	95.6%	0.32	0.24	0.14	-0.02
ChangeInRecommendation	2.15	0.28	13.8%	24.9%	80.2%	0.20	0.39	0.22	-0.06
CompEquIss	3.85	0.30	2.3%	0.5%	95.8%	0.26	0.30	0.07	-0.09
CompositeDebtIssuance	1.86	0.25	5.9%	10.4%	84.4%	0.18	0.03	0.18	0.10
CoskewACX	0.67	0.06	65.5%	34.4%	65.7%	0.05	0.04	0.09	0.20
Coskewness	1.71	0.18	18.0%	8.0%	82.8%	0.15	0.13	0.11	-0.02
CustomerMomentum	9.93	0.39	1.3%	6.7%	94.0%	0.31	0.31	0.12	0.09
DNoa	6.26	0.72	0.0%	0.2%	96.6%	0.51	0.45	0.28	-0.01
DelBreadth	1.90	0.19	21.6%	3.7%	83.4%	0.16	0.20	0.08	0.06
DelCOA	2.70	0.27	4.0%	8.5%	91.4%	0.22	0.20	0.11	-0.13
DelCOL	-0.91	-0.08	53.2%	23.1%	29.2%	-0.08	-0.10	0.06	-0.02
DelDRC	4.51	0.40	7.5%	5.2%	84.6%	0.25	0.30	0.16	-0.15

Appendix Table 3: Inference and Posteriors for 174 Portfolios (continued)

Portfolio	$E(r_t)$	SR	$\mu = 0$ p-values OLS	MLE	Bayesian Posterior Means $P(\mu > 0)$	μ_{sr}	$\mu_{sr,2022}$	$\hat{\sigma}_{sr}$	ρ(size)
DelEqu	3.47	0.30	2.3%	0.8%	94.8%	0.26	0.29	0.09	0.15
DelFINL	3.91	0.63	0.0%	2.2%	97.8%	0.50	0.55	0.18	0.08
DelLTI	1.75	0.33	1.2%	2.8%	95.2%	0.28	0.44	0.12	0.04
DelNetFin	1.56	0.22	9.3%	13.8%	86.7%	0.18	0.06	0.12	-0.08
DolVol	1.25	0.09	47.9%	49.8%	66.0%	0.06	-0.10	0.18	0.56
EBM	2.61	0.22	8.7%	13.5%	87.2%	0.18	0.15	0.13	0.24
EP	3.47	0.25	5.3%	1.9%	92.0%	0.22	0.25	0.09	0.22
EarnSupBig	3.43	0.25	5.5%	2.2%	94.5%	0.22	0.22	0.06	-0.14
EarningsConsistency	3.42	0.27	4.0%	4.4%	94.5%	0.24	0.21	0.10	0.10
EarningsForecastDisparity	3.68	0.25	12.0%	2.0%	89.7%	0.20	0.20	0.08	-0.03
EarningsStreak	8.37	0.93	0.0%	0.1%	98.9%	0.71	0.82	0.14	-0.17
EarningsSurprise	2.43	0.33	1.3%	3.2%	96.5%	0.29	0.29	0.09	-0.05
EntMult	5.81	0.39	0.3%	1.5%	95.3%	0.32	0.26	0.13	0.08
EquityDuration	7.81	0.48	0.0%	1.5%	95.7%	0.37	0.36	0.18	0.01
ExclExp	2.68	0.54	0.1%	0.6%	97.2%	0.42	0.46	0.10	-0.33
FEPS	11.88	0.77	0.0%	0.1%	98.8%	0.61	0.87	0.15	-0.63
FR	1.19	0.15	35.3%	50.1%	71.2%	0.14	0.54	0.37	-0.17
Fgr5yrLag	7.67	0.43	0.7%	2.6%	96.1%	0.35	0.42	0.12	-0.31
FirmAge	-1.19	-0.14	28.4%	31.3%	22.4%	-0.12	-0.30	0.15	0.50
FirmAgeMom	18.00	0.78	0.0%	1.0%	98.6%	0.60	0.95	0.24	0.00
ForecastDispersion	6.11	0.38	1.0%	0.9%	95.5%	0.32	0.42	0.10	-0.56
Frontier	1.80	0.09	51.2%	14.0%	67.4%	0.07	-0.04	0.12	0.52
GP	4.47	0.39	0.3%	1.6%	96.1%	0.34	0.58	0.15	-0.16
GrAdExp	3.47	0.29	4.6%	3.2%	92.0%	0.24	0.28	0.11	-0.08
GrLTNOA	1.71	0.21	10.6%	0.7%	90.0%	0.18	0.19	0.07	-0.16
GrSaleToGrInv	3.02	0.37	0.5%	12.2%	89.3%	0.28	0.29	0.29	0.07
GrSaleToGrOverhead	0.63	0.07	61.8%	51.3%	66.1%	0.06	0.04	0.06	0.07
Grcapx	4.08	0.41	0.2%	1.7%	95.2%	0.33	0.28	0.16	0.16
Grcapx3y	2.93	0.29	2.8%	6.4%	91.7%	0.24	0.14	0.13	0.18
Herf	0.77	0.07	59.2%	67.5%	65.3%	0.06	0.03	0.10	0.25
HerfAsset	0.85	0.08	54.1%	64.4%	68.3%	0.07	0.07	0.11	0.31
HerfBE	0.79	0.08	56.4%	68.1%	67.7%	0.07	0.04	0.13	0.35
High52	15.21	0.76	0.0%	0.6%	99.5%	0.64	0.70	0.12	-0.44
Hire	2.22	0.22	10.0%	17.3%	84.8%	0.17	0.10	0.13	-0.06
IdioRisk	15.92	0.78	0.0%	1.6%	95.8%	0.51	0.91	0.39	-0.65
IdioVol3F	15.71	0.74	0.0%	0.6%	96.8%	0.52	0.86	0.34	-0.68
IdioVolAHT	13.94	0.56	0.0%	1.6%	95.4%	0.41	0.60	0.25	-0.71
Illiquidity	3.01	0.25	5.1%	46.9%	69.8%	0.12	-0.85	0.45	0.77
IndMom	2.88	0.19	13.8%	18.1%	82.9%	0.16	0.19	0.18	0.16
IndRetBig	20.46	1.26	0.0%	6.3%	98.0%	0.66	0.48	0.46	0.06
IntMom	11.92	0.62	0.0%	4.7%	95.5%	0.44	0.16	0.24	-0.09
IntanBM	0.31	0.02	89.5%	89.3%	52.7%	0.01	-0.17	0.15	0.41
IntanCFP	2.43	0.14	28.7%	10.4%	79.0%	0.11	0.05	0.08	0.22
IntanEP	1.92	0.12	36.2%	10.3%	77.0%	0.10	0.07	0.07	0.31
IntanSP	-1.14	-0.06	66.8%	71.0%	37.8%	-0.05	-0.13	0.15	0.56
InvGrowth	5.14	0.49	0.0%	6.1%	96.1%	0.39	0.49	0.19	-0.03
InvestPPEInv	4.91	0.49	0.0%	0.7%	94.4%	0.37	0.21	0.22	0.10
Investment	1.16	0.12	34.0%	40.8%	79.0%	0.11	0.08	0.07	0.37
LRreversal	1.68	0.09	51.3%	61.2%	67.1%	0.07	0.04	0.15	0.48
Leverage	1.83	0.10	43.5%	51.2%	72.2%	0.08	0.05	0.11	0.10
MRreversal	3.59	0.24	6.5%	18.4%	85.2%	0.19	0.10	0.20	0.20
MaxRet	11.38	0.65	0.0%	0.3%	97.3%	0.49	0.75	0.30	-0.59
MeanRankRevGrowth	3.62	0.35	0.7%	2.7%	95.2%	0.29	0.25	0.10	0.05
Mom12m	15.26	0.68	0.0%	1.0%	99.4%	0.58	0.56	0.09	-0.15
Mom12mOffSeason	8.86	0.44	0.1%	0.2%	99.3%	0.39	0.40	0.05	-0.06
Mom6m	10.45	0.55	0.0%	0.7%	98.8%	0.47	0.58	0.12	-0.08
Mom6mJunk	5.33	0.30	6.5%	7.4%	86.4%	0.22	0.23	0.15	0.14
MomOffSeason	5.28	0.30	2.0%	13.5%	91.8%	0.24	0.17	0.13	0.20
MomOffSeason06YrPlus	8.79	0.73	0.0%	1.8%	98.5%	0.58	0.69	0.17	0.07
MomOffSeason11YrPlus	3.70	0.35	0.7%	1.1%	96.0%	0.30	0.27	0.09	0.04
MomOffSeason16YrPlus	2.81	0.31	1.9%	0.6%	96.5%	0.27	0.27	0.07	-0.01
MomSeason	4.91	0.42	0.1%	16.4%	83.9%	0.26	0.32	0.48	-0.06
MomSeason06YrPlus	6.15	0.58	0.0%	3.5%	94.7%	0.41	0.38	0.33	0.00

Appendix Table 3: Inference and Posteriors for 174 Portfolios (continued)

			$\mu = 0$ p-values		Bayesian Posterior Means				
Portfolio	$E(r_t)$	SR	OLS	MLE	$P(\mu > 0)$	μ_{sr}	$\mu_{sr,2022}$	$\hat{\sigma}_{sr}$	ρ(size)
MomSeason11YrPlus	5.76	0.61	0.0%	0.3%	99.5%	0.53	0.52	0.08	-0.04
MomSeason16YrPlus	3.09	0.32	1.5%	0.4%	98.0%	0.29	0.27	0.09	-0.02
MomSeasonShort	5.76	0.46	0.0%	17.2%	91.2%	0.32	0.01	0.26	-0.06
NOA	4.23	0.41	0.2%	5.4%	93.1%	0.32	0.34	0.24	-0.06
NetDebtFinance	3.55	0.54	0.0%	1.1%	97.2%	0.43	0.50	0.14	0.00
NetDebtPrice	2.41	0.15	25.3%	2.9%	82.0%	0.13	0.11	0.07	0.17
NetEquityFinance	5.35	0.55	0.0%	0.3%	98.6%	0.46	0.52	0.10	-0.34
NetPayoutYield	7.01	0.53	0.0%	0.4%	97.4%	0.43	0.74	0.17	-0.35
NumEarnIncrease	2.68	0.47	0.0%	0.6%	98.4%	0.40	0.49	0.10	-0.06
OPLeverage	3.41	0.28	2.9%	4.4%	95.4%	0.25	0.27	0.07	0.20
OperProf	6.19	0.46	0.0%	0.9%	97.8%	0.40	0.58	0.15	-0.47
OperProfRD	7.04	0.58	0.0%	1.3%	94.4%	0.42	1.34	0.37	-0.56
OptionVolume1	3.35	0.28	15.7%	32.5%	68.7%	0.12	-0.09	0.31	0.12
OptionVolume2	2.95	0.38	5.6%	8.5%	84.7%	0.24	0.19	0.18	0.01
OrderBacklog	-1.08	-0.07	59.6%	65.3%	36.6%	-0.05	0.06	0.12	-0.09
OrderBacklogChg	-0.52	-0.04	77.8%	33.7%	42.7%	-0.03	-0.06	0.09	-0.05
OrgCap	5.12	0.55	0.0%	0.1%	99.3%	0.48	0.48	0.08	-0.02
PS	8.39	0.42	0.3%	0.6%	96.3%	0.34	0.29	0.09	-0.20
PayoutYield	4.31	0.33	1.1%	2.7%	94.5%	0.28	0.44	0.13	-0.09
PctAcc	2.34	0.26	5.5%	2.9%	94.0%	0.23	0.22	0.08	-0.25
PctTotAcc	0.83	0.10	56.0%	27.8%	67.7%	0.09	0.07	0.10	-0.14
PredictedFE	2.88	0.19	24.9%	33.1%	82.2%	0.16	0.20	0.10	-0.13
Price	-5.90	-0.21	10.7%	31.7%	24.2%	-0.15	-0.35	0.33	0.72
PriceDelayRsq	-1.76	-0.16	22.5%	4.1%	17.8%	-0.14	-0.20	0.11	0.23
PriceDelaySlope	-0.90	-0.09	49.6%	42.7%	31.7%	-0.07	-0.08	0.11	0.32
PriceDelayTstat	0.23	0.03	79.2%	27.6%	58.1%	0.03	0.02	0.09	-0.32
ProbInformedTrading	15.75	1.21	0.0%	1.4%	95.2%	0.62	0.81	0.29	-0.26
RD	1.50	0.09	53.8%	61.0%	64.9%	0.07	0.08	0.20	0.39
RDAbility	5.35	0.26	9.3%	4.8%	87.5%	0.21	0.28	0.13	-0.02
RDS	2.01	0.35	1.9%	5.5%	90.7%	0.26	0.21	0.15	0.13
RDcap	1.63	0.07	64.6%	81.6%	57.6%	0.03	-0.36	0.23	0.47
REV6	4.34	0.20	18.4%	25.2%	75.8%	0.13	-0.06	0.19	-0.19
Realestate	3.22	0.31	2.6%	4.5%	90.1%	0.24	0.19	0.15	-0.19
ResidualMomentum	5.96	0.57	0.0%	1.8%	98.9%	0.48	0.43	0.10	-0.04
RetConglomerate	6.44	0.46	0.3%	3.8%	96.1%	0.37	0.35	0.12	-0.04
ReturnSkew	-0.09	-0.01	91.2%	95.3%	50.3%	0.01	0.32	0.25	-0.20
ReturnSkew3F	-1.77	-0.33	1.1%	18.8%	9.1%	-0.25	-0.19	0.16	-0.23
RevenueSurprise	1.51	0.20	12.4%	7.7%	88.6%	0.18	0.19	0.09	-0.04
RoE	2.88	0.26	4.3%	6.4%	91.1%	0.23	0.34	0.13	-0.53
Roaq	6.65	0.40	0.5%	2.4%	93.2%	0.31	0.51	0.16	-0.50
SP	4.76	0.27	3.8%	8.9%	90.0%	0.22	0.17	0.14	0.39
STreversal	-0.18	-0.01	92.8%	100.0%	46.4%	-0.01	-0.15	0.14	0.16
Sfe	10.93	0.56	0.0%	0.2%	98.4%	0.47	0.49	0.10	-0.30
ShareIss1Y	6.51	0.83	0.0%	0.2%	99.1%	0.65	0.70	0.20	-0.19
ShareIss5Y	4.95	0.59	0.0%	0.1%	98.3%	0.48	0.47	0.17	-0.02
ShortInterest	4.40	0.41	0.4%	0.7%	97.5%	0.35	0.40	0.09	-0.26
Size	-0.05	0.00	98.6%	97.6%	50.5%	0.01	-0.15	0.31	0.82
Skew1	5.89	0.47	1.8%	3.9%	91.4%	0.33	0.41	0.17	-0.10
SmileSlope	13.06	1.34	0.0%	3.2%	95.9%	0.64	0.88	0.40	-0.12
Std$_turn$	11.35	0.69	0.0%	4.1%	94.3%	0.44	0.49	0.36	-0.50
Tang	0.60	0.05	71.5%	38.8%	62.4%	0.04	0.07	0.07	0.08
Tax	4.34	0.53	0.0%	0.3%	96.7%	0.43	0.78	0.22	-0.27
TotalAccruals	2.48	0.29	2.8%	6.3%	92.8%	0.25	0.34	0.13	0.10
TrendFactor	11.88	0.74	0.0%	11.9%	96.4%	0.50	0.21	0.24	0.15
VarCF	1.48	0.10	44.9%	57.0%	67.9%	0.09	0.30	0.23	-0.71
VolMkt	4.78	0.33	1.1%	1.1%	94.9%	0.28	0.34	0.12	-0.42
VolSD	4.09	0.52	0.0%	0.5%	98.6%	0.44	0.46	0.10	0.17
VolumeTrend	4.68	0.42	0.1%	4.8%	92.4%	0.32	0.49	0.26	0.21
XFIN	9.91	0.70	0.0%	1.6%	97.5%	0.53	0.91	0.23	-0.50
Zerotrade	5.04	0.51	0.0%	0.8%	96.7%	0.40	0.49	0.17	-0.26
ZerotradeAlt1	4.29	0.38	0.3%	6.8%	89.4%	0.28	0.56	0.30	-0.38
ZerotradeAlt12	5.17	0.52	0.0%	1.5%	96.3%	0.41	0.46	0.18	-0.21