The Consequences of Index Investing on Managerial Incentives

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Abstract

I develop a model that analyzes how index investing impacts optimal managerial contracts. Index investors are constrained to purchase all risky assets in a fixed proportion, so information that affects their demand for the index gets reflected in all the constituent stock prices. Suppose a manager takes a value-enhancing action. Index investors now expect higher index payoffs and buy more of the index, which boosts the stock prices of all index firms. Thus, the prices of other index firms are positively related to and contain unique information about the manager's effort. The optimal contract puts a positive weight on the index to increase the effort sensitivity of the manager's pay, not to reduce his risk. Relative performance evaluation is, therefore, not optimal in this setting.

1 Introduction

Recent estimates suggest that the percentage of US equities owned by index funds or index ETFs has doubled over the past decade from 8% to 16%.[1] Such a shift in the composition of investors is bound to affect the informational content of stock prices because active and index investors gather and trade upon information differently. Active investors are concerned with both the aggregate and relative performance of stocks while index investors only care about the former. Changes in the informational content of stock prices are important to examine because stock prices affect the decisions made by firms. For example, managers often have part of their compensation tied to their firm's stock price to incentivize them to act in the best interest of shareholders. Therefore, if the growth of index investing changes how managerial actions are reflected in financial markets, the manager's optimal contract will also change.

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[1]See 2022 Investment Company Factbook page 30. However, a recent academic study (Chinco and Sammon (2022)) suggests that this number is an understatement and that the true percentage was around 38% in 2020.

I develop a joint model of contracting with moral hazard and financial markets with index investors to examine how optimal managerial compensation changes as the fraction of index investors increases. To my knowledge, this is the first paper to study the impact of index investing on managerial compensation. Suppose there are two firms and index investors exist. The optimal contract for manager 1 always puts a positive weight on firm 2's stock price and this weight is increasing in the fraction of index investors. Manager 1's effort increases expectations about the payoffs of firm 1 and the index (because firm 1 is part of the index). Index investors buy more of the index after receiving this good news, which pushes up both stock prices. As a result, *both* stock prices reflect the index investors' information about manager 1's effort. The positive relationship between manager 1's effort and firm 2's stock price suggests that it should get a positive weight in manager 1's contract to increase the effort sensitivity of his pay.

The optimal contract includes the other firm's stock price to increase effort sensitivity of the manager's pay. This motivation differs from the relative performance evaluation (RPE) literature which argues that the purpose of including the performance of other firms in the manager's contract is to hedge common risks. To see the difference between the effort sensitivity and hedging motives, suppose the stock prices of the two firms are positively correlated and the manager is risk-averse. The classic RPE result (e.g., Holmstrom (1982)) is to put a negative weight on the other firm's stock price to reduce the volatility of the manager's pay. This outcome allows the principal to introduce stronger incentives without violating the manager's participation constraint. In contrast, my paper puts a positive weight on the other firm's stock price because it is increasing in the manager's effort. In other words, I find that the benefits of increasing the effort sensitivity of the manager's pay outweigh the costs of increased pay volatility. I should note that these two motivations are not always at odds with one another, but when they are, effort sensitivity dominates. This result is robust to an arbitrary number of firms.

We must understand how stock prices respond to a change in the fraction of index investors to see the intuition behind the above result. Over reasonable parameters, an increase in the fraction of index investors makes a stock less sensitive to its own cash flow, more sensitive to the other firm's cash flow, have a greater covariance with the other firm's stock price, and have a greater variance. The first three effects come directly from the synchronized asset demands of indexers. Suppose one firm has a high cash flow realization. Indexers buy less of the firm that has the high realization and more of the firm that does not than active traders do because they are constrained to buy all stocks in the same proportion. This reasoning leads to the additional result that firms place less weight on their stock price in the optimal contract as the fraction of index investors increases. The higher volatility comes from noise traders playing a greater role in determining relative prices when there are fewer active traders.

The two price sensitivity effects imply placing more weight on the other firm's stock price in the optimal contract because it is relatively more informative about managerial effort. The effects of the higher covariance and variance are conflicting. Contracts can more effectively hedge common risk with a greater covariance, but any given hedge becomes riskier with greater variance. Thus, the greater variance caused by index investors dilutes the hedging benefits of a higher covariance. I show that the dilution is severe enough to guarantee that the effort sensitivity effect dominates.

My model assumes that the goal of each firm's principal is to maximize the expected dividends of its firm, but the common ownership literature (e.g., Azar, Schmalz, and Tecu (2018)) suggests that the growth of indexing may change that objective. Index investors typically invest through an index fund managed by a large asset manager (e.g., Vanguard, BlackRock, or Fidelity), so ownership tends to become more concentrated as indexing increases. These diversified asset managers may then govern their firms to maximize portfolio profits, which is not necessarily equivalent to maximizing the profits of each individual firm in their portfolio. I include an extension where the principal sets each manager's contract to maximize expected aggregate dividends and I find that the optimal contract parameters have the same sign as the single owner case; all the common owner does is alter the magnitudes.

My paper contributes to the literature on how index investing impacts equilibrium asset prices. Most theoretical papers in this literature rely on noisy rational expectations equilibrium (REE) models of financial markets as developed by Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985). I use the Admati (1985) framework (a multi-asset version of Hellwig (1980)) for my financial market and add index investors and endogenous cash flows driven by managerial effort.

The three index investing papers most similar to mine are Bond and Garcia (2022), Buss and Sundaresan (2020), and Liu and Wang (2018). Bond and Garcia (2022) use a Diamond and Verrecchia (1981) style financial market to analyze the welfare consequences of a decline in the cost of index investing. They find that as the fraction of index investors increases, the price efficiency of the index declines and the price efficiency of individual stocks increases. The latter result is because the marginal trader who switches from active to index investing following a decline in the cost of index investing is less informed than the remaining active traders. Thus, the average information quality for individual stocks increases when they leave. Buss and Sundaresan (2020) develop a model of feedback between price informativeness and corporate investment. Essentially, an increase in the fraction of passive investors lowers the firm's cost of capital because active investors hold fewer shares in equilibrium – and thus less risk. A lower cost of capital encourages investment, which makes the firm's cash flows more volatile and therefore incentivizes active investors to gather more information. Liu and Wang (2018) develop a model of index investors with endogenous information acquisition and find that the effect of index investors on price efficiency depends on how gathering information on one stock affects the costs of gathering information on additional stocks.

As the above papers suggest, there is no theoretical consensus on how an increase in indexing impacts price efficiency. Other papers on the subject include Lee (2020) which finds a positive relationship between indexing and price efficiency, Baruch and Zhang (2021) which finds a negative relationship, and Coles, Heath, and Ringgenberg (2022) which finds no relationship. Davies (2021) and Sammon (2022) also contribute to this literature. The latter uses theoretically motivated measures of price informativeness to find that price efficiency has declined over the last 30 years. This result suggests a negative relationship between index investing and price efficiency. My simple model of index investors also implies a negative relationship between index investing and price efficiency, but that is not the focus of my paper. One of my main contributions to this literature is to endogenize stock payoffs with a managerial moral hazard problem.

My paper also contributes to the literature on the real effects of financial markets; specifically, on how financial market developments impact managerial incentives. See Bond, Goldstein, and Edmans (2012) for a survey. Two classic papers on how stock prices can be used to learn about managerial effort are Holmstrom and Tirole (1993) and Diamond and Verrecchia (1982). Holmstrom and Tirole (1993) uses a Kyle (1985) style financial market to examine whether a firm's stock price is a useful tool for monitoring managers. Unlike my model, the firm will optimally contract on both the final payoffs and the stock price because they contain different, but correlated, pieces of information. My model simplifies the analysis by making output a sufficient statistic for managerial effort and then assumes the principal can only contract on prices. I make this choice because I am primarily interested in how stock market developments impact incentives, so allowing contracts to also depend on output would unnecessarily complicate the analysis. Diamond and Verrecchia (1981) examines a moral hazard model where the stock price contains some unique information for the principal above and beyond output. My main contribution to this literature is to examine how well markets can monitor managerial effort when index traders impact the informativeness of prices.

My paper makes extensive use of two fundamental results in contract theory: the informativeness principle of Holmstrom (1979) and the hedging benefits of relative performance evaluation as seen in Holmstrom (1982). The informativeness principle states that a performance measure should be included in a contract if and only if it provides additional information beyond the pre-existing contracting variables. I find that firm 1's stock price is not a sufficient statistic for manager 1's effort whenever there is a positive fraction of index investors; firm 2's stock price also contains valuable information. Index investors are constrained to purchase equal amounts of both stocks, so any information that they have about one manager's effort gets divided between the two stocks. Therefore, the other firm's stock price should be included in the optimal contract. I also find that my results go against the basic intuition of relative performance evaluation models. As discussed earlier, the benefits of improving the pay sensitivity of effort always outweigh the costs of increased pay variance. This result could perhaps explain the so-called "RPE puzzle" as defined in Murphy (1999) and further studied in DeAngelis and Grinstein (2016) and Gong, Li, and Shin (2011). The puzzle is the lack of evidence for firms using RPE despite the well-known theoretical benefits. My model suggests that "negative RPE" is optimal in a world with index investors because other prices are informative of effort.

The above result concerning the optimally of "negative RPE" is also found in the common ownership literature. The idea is that the owner of two firms may want to place a positive weight on the other firm's stock price in each manager's compensation to reduce competition between the two firms and increase joint profits. Gordon (1990) and Aggarwal and Samwick (1999) are two well-known papers that demonstrate this result. The relationship between my paper and this literature will be discussed more fully in Section 7.

The rest of the paper is as follows: in Section 2 I present a toy model to motivate my analysis, in Section 3 I present my full model, in Section 4 I solve my model, in Section 5 I extend my analysis to $N \geq 2$ firms, in Section 6 I discuss alternative ways to model index investors, in Section 7 I examine how common ownership impacts my results, in Section 8 I discuss the empirical implications of my results, and I conclude in Section 9.

2 Toy Model

To fully see the difference between my contracting motivation and that of the RPE literature, consider the following contracting problem. There are two firms ($k \in \{1, 2\}$) whose outputs (x_k) depend on their manager's effort (e_k) and noise (η_k). Formally, $x_k = e_k + \eta_k$ with $\eta_1, \eta_2 \sim N(0, 1)$ and $corr(\eta_1, \eta_2) = 0$.

Firms cannot contract directly on output but must rely instead on a performance measure $P_k = e_k + \alpha e_j + \eta_k + \phi_k$ with $\alpha \in [0, 1]$, $\phi_k \sim N(0, 1)$, $corr(\phi_1, \phi_2) = \rho$. Assume ϕ_k is independent of η_1 and η_2. The performance measure for firm 1 equals its output plus a term that depends on the effort of manager 2 (αe_j) and an additional noise term that is correlated across performance measures (ϕ). So even though fundamentals are completely independent, the performance measures of the two firms are linked by effort (through α) and a correlated noise term (through ρ).

The principal is risk-neutral, the manager is risk-averse (specifically, CARA utility[2]), and contracts are linear in the two performance measures. For firm 1, the contract is $w_1 = l_1 + m_1 P_1 + n_1 P_2$. The formal contracting problem for firm 1 is then:

$$\max_{l_1, m_1, n_1} E(x_1 - w_1)$$

$$\text{s.t. } E(w_1) - \frac{1}{2}e_1^2 - \frac{1}{2}Var(w_1) \geq 0 \quad \text{(IR)}$$

$$e_1 \in \underset{e_1}{\text{argmax}}\, E(w_1) - \frac{1}{2}e_1^2 - \frac{1}{2}Var(w_1) \quad \text{(IC)}$$

The principal maximizes expected output minus the manager's wage. The manager requires his expected utility to be non-negative and chooses effort to maximize his expected utility given the contract. The manager's expected utility equals the expected wage minus a quadratic cost of effort term and a risk-aversion term. I will drop the firm subscripts in the following discussion because the solutions are identical across firms.

Lemma 1 *When $\alpha = 0$, the optimal contract sets $m^* > 0$ and has n^* have the opposite sign of $\rho = corr(\eta_1, \eta_2)$.*

We can interpret Lemma 1 as the classic RPE result. The positive sign on m^* – the contract weight on the firm's own stock price – is straightforward, but the sign of n^* is the main RPE result; we set the sign of n^* to be opposite that of ρ to hedge out noise in the performance measure. By doing so, we reduce the volatility of the risk-averse manager's pay which allows the principal to put stronger incentives in place.

[2] I set the manager's coefficient of absolute risk aversion to 1 for simplicity.

Lemma 2 *When $\alpha > \frac{\rho}{2}$, the optimal contracts sets $m^* > 0$ and $n^* > 0$.*

We still get $m^* > 0$ when $\alpha > 0$. However, we are now guaranteed to get $n^* > 0$ when $\alpha > \frac{\rho}{2}$. Note that α can be interpreted as the sensitivity of firm 1's stock price to manager 2's effort divided by the sensitivity of firm 1's stock price to manager 1's effort. We can also interpret $\frac{\rho}{2}$ as the covariance of the two performance measures divided by their common variance $Var(P) \equiv Var(P_1) = Var(P_2) = 2$.

$$\alpha = \left(\frac{dP_k}{de_j}\right) / \left(\frac{dP_k}{de_k}\right) \text{ and } \frac{\rho}{2} = \frac{cov(P_1, P_2)}{Var(P)}$$

In the RPE model, we always set $n^* < 0$ when $\rho > 0$, but now we set $n^* > 0$ whenever $\alpha > \frac{\rho}{2} > 0$ even though doing so increases the manager's risk. The benefits of increasing the effort sensitivity of the manager's pay outweigh the costs of increased pay volatility.

In my full model, I endogenize the parameters α and ρ through a model of financial markets with index investors. Specifically, the synchronized demands of index investors create both the correlation between the two performance measures (ρ) and the presence of the other manager's effort (α) in each firm's performance measure. I then demonstrate that the magnitudes of these parameters are such that $\alpha > \frac{\rho}{2}$, so $n^* > 0$.

3 Full Model

3.1 Firms

There are two firms: 1 and 2.[3] Each firm is controlled by a risk-neutral principal and managed by a risk-averse manager. The manager privately exerts costly effort to increase the mean cash flow of his firm. Denote firm $k's$ cash flow as x_k. The manager of firm k is paid a wage w_k as compensation for his effort. Each firm pays out the entirety of its cash flow net of managerial compensation (i.e., $x_k - w_k$) to its shareholders as a dividend. Financial market participants trade shares of each firm at the equilibrium price P_k. Assume each firm has one share outstanding. The details of the contracting problem and financial market will be explained in the following subsections.

3.2 Contracting Problem

Each firm's principal faces a moral hazard problem. The risk-neutral principal wants to maximize the expected dividends of the firm: $E(x_k - w_k)$. Each risk-averse manager chooses his firm's mean cash flow e_k at a

[3] All results generalize to an arbitrary number N firms. See Section 5 for details.

personal cost of $\frac{1}{2}e_k^2$. The manager's choice of e_k is his private information. I assume the cash flows of the two firms are normally distributed, independent, and have identical variances. Specifically, let $x_k \sim N(e_k, 1)$.[4]

The principal induces effort with a contract. I focus on linear incentive schemes of the following form:

$$w_k = l_k + m_k P_k + n_k P_j$$

That is, the contract has a fixed component l_k, a component based upon the firm's own stock price m_k, and a component based upon the other firm's stock price n_k. I assume that the principal cannot contract on the actual cash flow x_k. This assumption can be rationalized by supposing that the payoffs are for a long-term project and the manager must be paid immediately for current consumption.

The manager has CARA utility over $w_k - \frac{1}{2}e_k^2$ with risk aversion coefficient 1. He also requires an expected utility of at least 0 to participate. Therefore, the principal of firm k chooses (l_k, m_k, n_k) to maximize expected dividends subject to the manager participating and choosing the effort level that maximizes his expected utility given his contract.

3.3 Financial Market

The financial market consists of three assets: a risk-free asset, the stock of firm 1, and the stock of firm 2. I assume the risk-free asset has a guaranteed price and payoff of $1. There is an infinite supply of the risk-free asset and 1 share of each stock outstanding.

There exist a unit mass of agents $i \in [0, 1]$ with CARA utility over terminal wealth and a common coefficient of absolute risk aversion 1. All agents have the same starting wealth W_0. Agents choose portfolios to maximize their expected utility. I also assume there exist noise traders with random demand for each asset, z_1, z_2, which are i.i.d random variables with distribution $N(0, \tau_z^{-1})$. These random variables are independent of every other random variable in the model.

However, for mathematical convenience, I follow Bond and Garcia (2022) and analyze the financial market

[4] When the cash flow's variance gets too small, market clearing restricts the principal to contracts that make the stock price resemble a risk-free asset. This restriction generates all sorts of bizarre results such as putting a negative contract weight on the firm's own stock price. To avoid these unrealistic complications, I set the variance to 1 in my base model.

in terms of two synthetic assets rather than the original stock prices. Define the following synthetic asset prices:

$$\begin{bmatrix} P_{IN} \\ P_{LS} \end{bmatrix} = T \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \text{with } T = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{1}$$

I divide by $\sqrt{2}$ to keep the variances of the synthetic cash flows and noise demands equal to the originals. P_{IN} is the price of an index asset (proportional to $P_1 + P_2$) and P_{LS} is the price of a long-short asset (proportional to $P_1 - P_2$). We can similarly transform all other variables in the model: variables with subscript IN are for the index asset and variables with subscript LS are for the long-short asset. Also, observe that $Cov(x_{IN}, x_{LS}) = 0, Cov(z_{IN}, z_{LS}) = 0$, and $Cov(x_{IN}, z_{LS}) = Cov(x_{LS}, z_{IN}) = 0$. As a result, the two synthetic asset prices will also be uncorrelated.

The purpose of this transformation in both Bond and Garcia (2022) and here is to simplify the traders' asset demands. Index investors generate multiple sources of correlation in the original stock prices which makes each trader's information set quite large. However, because the two synthetic assets have zero correlation, focusing on them simplifies the calculations considerably.

I assume an exogenous fraction of traders λ are active and the remaining $1 - \lambda$ are indexed. I endogenize this fraction in Appendix 2, but it does not impact my results at all. Active traders can purchase as much of the index and long-short assets as they want while index investors are constrained to only trade the index asset. This is equivalent to saying active traders can freely trade stocks 1 and 2 while index investors can only purchase them in the same proportion.

Active traders receive an independent noisy signal about both the index and long short asset. Let $s_{i1} = x_1 + \epsilon_{i1}$ be a signal for the stock 1 and $s_{i2} = x_2 + \epsilon_{i2}$ be a signal for stock 2 and $s_i = (s_{i1}, s_{i2})'$. Active investors see the two synthetic signals ($s_{i,IN}$ and $s_{i,LS}$) generated by Ts_i. Observe that these two synthetic signals are uncorrelated. Passive investors only receive a signal about the index asset; that is, passive trader i only observes the signal $s_{i,IN} = \frac{s_{i1}+s_{i2}}{\sqrt{2}}$. This is equivalent to saying that active traders see signals about both of the original assets while index traders only see a signal about aggregate cash flows. Let $\epsilon_{ik} \sim N(0, \tau_\epsilon^{-1})$ be the distribution of noise and assume that noise realizations are independent across assets k and agents i. [5] Further assume signal noise is independent of all other random variables in the model.

[5] Note that the composite signals have the same variance as the signals for the two base assets

Equilibrium prices are arrived at through market clearing. Once the synthetic asset prices are discovered, it is straightforward to invert equation 1 to find the original stock prices .

3.4 Timing

1. The principal and manager agree on a contract and publicly announce it. The manager exerts effort.

2. Investors make a rational inference about e_k given the contract. Denote this belief e_k^*.

3. Investors trade based upon their information sets, the managers' contracts, and their type (active or indexed). Equilibrium prices are generated.

4. The manager of each firm k receives wage w_k.

5. Gross payoffs x_k are realized.

Note that this timeline has the manager being paid before payoffs are realized. Thus, the manager's contract cannot depend on the actual realization of payoffs. Rather, it must depend on stock prices, a noisy signal for payoffs. One can interpret the manager being paid before x_k is realized as the firm borrowing amount w_k at the risk-free rate once equilibrium prices are realized.

3.5 Definition of Equilibrium

An equilibrium in this model is a variation on the standard noisy rational expectations equilibrium.

1. I conjecture an equilibrium price function for each risky asset.

2. Given the conjectured prices, the principal and agent find the optimal moral hazard contract.

3. Given the conjectured prices, managerial contracts, their information, and their type, each trader selects their optimal portfolio.

4. Given asset demands, the financial market clears and the resulting price matches the form of the initial conjecture.

4 Model Solutions

4.1 Financial Market Equilibrium

I make the following conjectures for the prices:

$$P_1 = A_1 + Bx_1 + Cx_2 + Dz_1 + Ez_2 \tag{2}$$
$$P_2 = A_2 + Bx_2 + Cx_1 + Dz_2 + Ez_1$$

Traders also form correct in equilibrium beliefs about the effort chosen by the managers given contracts w_1 and w_2. Denote these beliefs as e_1^* and e_2^*. These conjectures imply that all traders have common prior beliefs about the payoffs of the assets. I allow the constant terms to be different because they depend on the equilibrium effort conjectures.

We now must confirm that equilibrium prices will indeed be of the conjectured form. As discussed in Section 3, it will be convenient to change basis and find the prices of two synthetic assets: an index and a long-short asset. I will then use the law of one price to find the prices of the original assets. As a reminder, I define the synthetic prices as

$$\begin{bmatrix} P_{IN} \\ P_{LS} \end{bmatrix} = T \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \text{with } T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{3}$$

All other transformed variables are defined similarly: variables with subscript IN are for the index asset and variables with subscript LS are for the long-short asset. I conjecture that the prices of the synthetic assets will be linear in their own payoffs and noise demands.

$$P_{IN} = A_{IN} + B_{IN}x_{IN} + C_{IN}z_{IN} \tag{4}$$
$$P_{LS} = A_{LS} + B_{LS}x_{LS} + C_{LS}z_{LS}$$

This conjecture implies that the original prices are of the form specified in equation 2.

Because the two synthetic assets have zero covariance, traders do not need to include information about the long-short asset in their demand for the index asset and vice-versa. Trader i's demand for the index asset will then be:

$$D(i,IN) = \frac{E(x_{IN}|s_{i,IN},P_{IN}) - w_{IN} - P_{IN}}{Var(x_{IN}|s_{i,IN},P_{IN})} \tag{5}$$

$$= e^*_{IN} + s_{i,IN}\tau_\epsilon + \rho^2_{IN}\tau_z(x_{IN} + \rho^{-1}_{IN} z_{IN}) - (1 + \tau_\epsilon + \rho^2_{IN}\tau_z)(w_{IN} + P_{IN})$$

where $\rho_{IN} = \frac{B_{IN}}{C_{IN}}$ and e^*_{IN} is the traders' correct in equilibrium belief about the mean payoff of the index. Notice that the wage is a constant to traders because they are conditioning on the price and know all the contract parameters. Demand for the long-short asset is defined similarly. Market clearing for each asset then requires:

$$\int_0^1 D(i,IN)di + z_{IN} = \frac{2}{\sqrt{2}} \tag{6}$$

$$\int_0^\lambda D(i,LS)di + z_{LS} = 0 \tag{7}$$

Index asset demand from the rational traders integrates from zero to one because both active and index investors trade it. Long-short asset demand integrates to λ because only active traders trade it. The supply of each synthetic asset comes from:

$$T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{2}} \\ 0 \end{bmatrix} \tag{8}$$

I can then verify that synthetic prices are of the form conjectured in equation 4. I then pre-multiply the synthetic price vector by T^{-1} to confirm my conjecture from equation 2. Importantly, the price coefficients are functions of the contract parameters $(l_1, l_2, m_1, m_2, n_1, n_2)$, so I will denote the coefficients as, for example, $B(l,m,n)$ to emphasize this fact.

4.2 Normalization of Prices

Contracts impact prices in two ways: directly through effort's effect on the mean payoff and indirectly through the price coefficients. It will be convenient to follow Holmstrom and Tirole (1993) and normalize the prices found in the previous section so that they only depend on the direct effect of contracting. Focusing on the direct effect highlights the key economic forces in my model and allows for sharper conclusions. I will discuss results for the original prices in Section 4.4, but as a preview, the signs on the optimal contract parameters are unchanged by this normalization.

From the prices (P_1, P_2) and contracting parameters $(l_1, l_2, m_1, m_2, n_1, n_2)$, we can create the following normalized prices:

$$\tilde{P_1} = (1 + m_1)P_1 + n_1 P_2 + l_1$$
$$\tilde{P_2} = (1 + m_2)P_2 + n_2 P_2 + l_2 \tag{9}$$

Observe that the normalized prices are constructed from public information and equal the asset prices that would exist in an economy without the contracting problem (taking the mean payoff e_k^* as given). As a result, this normalization allows us to separate the financial market and contracting problem, so the principal can take prices as given when choosing a contract. The details about these normalized prices are given below.

$$\tilde{P_1} = \tilde{A}_1 + \tilde{B}x_1 + \tilde{C}x_2 + \tilde{D}z_1 + \tilde{E}z_2$$
$$\tilde{P_2} = \tilde{A}_2 + \tilde{B}x_2 + \tilde{C}x_1 + \tilde{D}z_2 + \tilde{E}z_1 \text{ with}$$
$$\tilde{A}_1 = \frac{e_1^* - 2 + e_2^*}{(\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2} + \frac{e_1^* - e_2^*}{(\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2}$$
$$\tilde{A}_2 = \frac{e_1^* - 2 + e_2^*}{(\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2} - \frac{e_1^* - e_2^*}{(\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2}$$
$$\tilde{B} = \frac{\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon}{(\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2} + \frac{\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon}{(\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2}$$
$$\tilde{C} = \frac{\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon}{(\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2} - \frac{\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon}{(\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2}$$
$$\tilde{D} = \frac{\tau_\epsilon\, \tau_z + 1}{(\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2} + \frac{\tau_\epsilon\, \tau_z\, \lambda^2 + 1}{\lambda\, (\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2}$$
$$\tilde{E} = \frac{\tau_\epsilon\, \tau_z + 1}{(\tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2} - \frac{\tau_\epsilon\, \tau_z\, \lambda^2 + 1}{\lambda\, (\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)\, 2}$$

It can be shown that $\tilde{B}, \tilde{C}$, and $\tilde{D} \geq 0$ and $\tilde{E} \leq 0$. The signs of $\tilde{B}$ and $\tilde{D}$ are straightforward. When there is positive cash flow news or positive noise demand for firm 1, its stock price increases. The signs of $\tilde{C}$ and $\tilde{E}$ are consequences of indexing. Suppose firm 1 has a large realization of x_1, so index investors receive a positive signal about the index asset. They suspect that $x_{IN} = \frac{x_1 + x_2}{\sqrt{2}}$ is large, but don't know which firm is the driving force. As a result, they buy more of both firms through the index. Therefore, a large realization of x_1 leads to greater index demand, which increases firm 2's stock price. Similarly, suppose there is a lot of noise demand for stock 1. Index investors see the price of the index go up without receiving any good news about the index's cash flows. Thus, they buy less of both stocks in the index, which drives down firm 2's stock price.

It can be shown that contracting on performance measures $\tilde{P_1}, \tilde{P_2}$ is equivalent to contracting on P_1, P_2 with the correct adjustments. Define the normalized contract for firm 1 as $w_1 = \tilde{l_1} + \tilde{m_1}\tilde{P_1} + \tilde{n_1}\tilde{P_2}$ with

$$\begin{aligned}\tilde{l_1} &= \frac{l_1 + l_1\, m_2 - l_2\, n_1}{m_1 + m_2 + m_1\, m_2 - n_1\, n_2 + 1}\\ \tilde{m_1} &= \frac{m_1 + m_1\, m_2 - n_1\, n_2}{m_1 + m_2 + m_1\, m_2 - n_1\, n_2 + 1}\\ \tilde{n_1} &= \frac{n_1}{m_1 + m_2 + m_1\, m_2 - n_1\, n_2 + 1}\end{aligned} \tag{10}$$

The compensation for manager 2 is defined similarly (just swap all the 1's and 2's). Notice that there are no strategic interactions between the two principals because the normalized market prices are independent of both contracts; all the interactions seen in the original contracts come from undoing the normalization. Both principals take the performance measures $\tilde{P_1}$ and $\tilde{P_2}$ as given when deciding on the contracting weights (except through the total amount of effort incentivized).

4.3 Contracts

4.3.1 Normalized Performance Measures

The principal's problem with the normalized performance measures is:

$$\max_{\tilde{l_k}, \tilde{m_k}, \tilde{n_k}} E(x_k - w_k) \tag{11}$$

$$\text{s.t. } E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \geq 0 \tag{IR}$$

$$e_k \in \underset{e_k}{\operatorname{argmax}}\, E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \tag{IC}$$

Taking the manager's first-order condition in constraint IC and solving gives us the following optimal effort:

$$e_k^* = \tilde{m_k}\tilde{B} + \tilde{n_k}\tilde{C} \tag{12}$$

Optimal effort equals the weight the contract puts on the manager's stock price times the sensitivity of his stock price to his effort plus the weight the contract puts on the other firm's stock price times the sensitivity of the other stock price to his effort. Note the positive incentive effect of putting a positive weight on the other firm's stock price. We know that $\tilde{C} > 0$, so greater effort by manager 1 boosts the stock price of firm 2 through $\tilde{C}$. Thus, setting $\tilde{n_k} > 0$ generates more effort, all else equal. With this choice of effort, we can plug back into the optimization problem. Specifically, we can take e_k^* and insert it into the objective function and IR constraint.

We then bind the IR and substitute into the objective function. The principal's new problem is

$$\max_{\tilde{m_k},\tilde{n_k}} e_k^* - \frac{1}{2}e_k^{*2} - \frac{1}{2}\left((\tilde{m_k}^2 + \tilde{n_k}^2)Var(\tilde{P}) + 2\tilde{m_k}\tilde{n_k}Cov(\tilde{P_1},\tilde{P_2})\right) \tag{13}$$

where $Var(\tilde{P}) = Var(\tilde{P_1}) = Var(\tilde{P_2})$ and does not depend on $\tilde{l_k}, \tilde{m_k}$, or $\tilde{n_k}$. Solving this maximization problem then gives us:

$$\tilde{m_k}^* = \Gamma * \left(\tilde{B}Var(\tilde{P}) - \tilde{C}Cov(\tilde{P_1},\tilde{P_2})\right) \geq 0 \tag{14}$$

$$\tilde{n_k}^* = \Gamma * \left(\tilde{C}Var(\tilde{P}) - \tilde{B}Cov(\tilde{P_1},\tilde{P_2})\right) \tag{15}$$

$$\text{with } \Gamma \equiv \frac{1}{Var(\tilde{B}\tilde{P_1} - \tilde{C}\tilde{P_2}) + Var(\tilde{P})^2 - Cov(\tilde{P_1},\tilde{P_2})^2} > 0$$

Note that these contract parameters will be the same for both firms, which implies that $e_1^* = e_2^*$. Thus, the equilibrium asset prices will have identical constant terms (i.e., $\tilde{A_1} = \tilde{A_2} \equiv \tilde{A}$). Also, $\tilde{l_k}^*$ will be the value that binds the agent's participation constraint given $\tilde{m_k}^*, \tilde{n_k}^*$.

Even before plugging in, we can tell that $\tilde{m_k}^*$ will be non-negative because $\tilde{B} \geq \tilde{C}$ and $Var(\tilde{P}) \geq Cov(\tilde{P_1},\tilde{P_2})$. This result is intuitive: the manager's own stock price is increasing in his effort, so putting a positive weight on $\tilde{m}$ is good for incentives.

The sign of $\tilde{n}$ is unclear at this stage, so we will need to plug in the price coefficients. However, we can say that $\tilde{n}$ will be positive if and only if the following condition is met:

Lemma 3 *The optimal choice of $\tilde{n}$ will be greater than or equal to zero if and only if* $\frac{\tilde{C}}{\tilde{B}} \geq \frac{Cov(\tilde{P_1},\tilde{P_2})}{Var(\tilde{P})}$

Lemma 3 is a direct application of the condition $\alpha > \frac{\rho}{2}$ (i.e., Lemma 2) from the toy model in Section 2 with $\frac{\tilde{C}}{\tilde{B}}$ representing α and $\frac{Cov(\tilde{P_1},\tilde{P_2})}{Var(\tilde{P})}$ representing $\frac{\rho}{2}$. There are two reasons to tie the manager's wage to the other stock price: it provides additional evidence of the manager's effort (effort sensitivity effect) and it can be used to eliminate noise from the manager's wage (hedging effect). In my model, the effort sensitivity effect is captured by $\frac{\tilde{C}}{\tilde{B}}$ and the hedging effect by $\frac{Cov(\tilde{P_1},\tilde{P_2})}{Var(\tilde{P})}$. The effort sensitivity effect measures how efficient (in terms of price responsiveness) it is to incentivize effort via the other firm's stock price relative to the manager's own stock price. The hedging effect measures how providing these incentives affects the manager's risk. If $\tilde{C}$ is large relative to $\tilde{B}$, it will be efficient to incentivize effort by putting more weight on the other firm's stock price rather than just on the manager's own stock price because it provides additional evidence of managerial effort. Additionally, suppose $Cov(\tilde{P}_1, \tilde{P}_2) > 0$. Then a larger covariance induces the principal to lower $\tilde{n}$ (and potentially make it negative) in order to hedge out common sources of noise in the manager's pay. The reverse will be true if the covariance is negative. If $\text{Var}(\tilde{P})$ is large, then the principal will rely less on the other stock for any level of covariance because tying the manager's compensation to another volatile stock will increase his risk. After plugging in for all the relevant expressions, I get the following result.

Proposition 1 *When there are index investors (i.e., $\lambda \neq 1$), $\tilde{n}^* > 0$. When there are only active investors, $\tilde{n}^* = 0$.*

4.3.2 Intuition for Optimal Contract

Proposition 1 claims that the effort sensitivity effect always dominates the hedging effect. The two effects are working in the same direction when $Cov(\tilde{P}_1, \tilde{P}_2) < 0$, so the claim is only in doubt when the covariance is positive. [6] The reason why the effort sensitivity effect dominates when the covariance is positive is because noise traders dilute the benefits of hedging.

To see this result, note that the hedging effect depends on both the covariance and variance of the normalized prices. I will address each of those terms in turn. The covariance can be expressed as:

$$Cov(\tilde{P}_1, \tilde{P}_2) = 2 * \left(\tilde{B}\tilde{C} + \frac{\tilde{D}\tilde{E}}{\tau_z}\right) \tag{16}$$

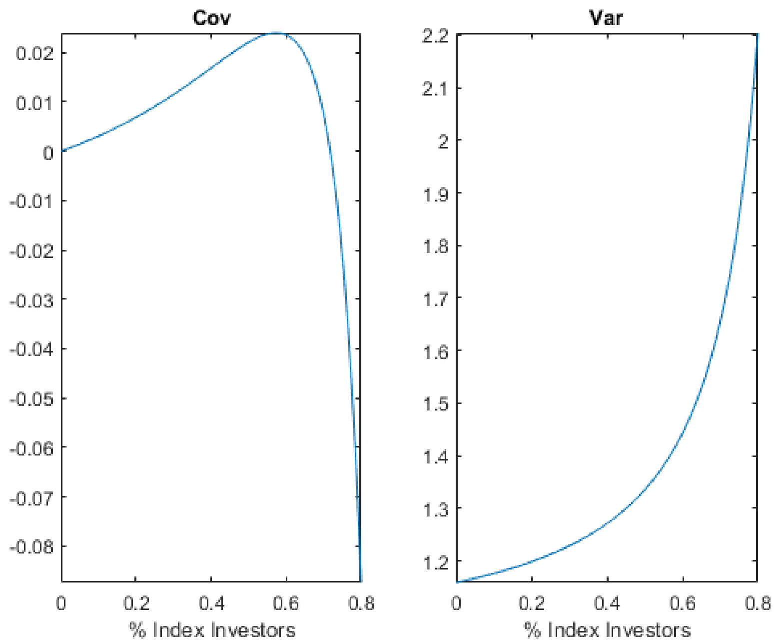
Recall that $\tilde{B}, \tilde{C}, \tilde{D} \geq 0$ and $\tilde{E} \leq 0$. These signs us that the first term in the covariance is positive and the second term is negative. The first term is the cash flow component of the covariance and it is increasing in the fraction of index investors. The second term is the noise trader component of the covariance and it is decreasing in the fraction of index investors. Thus, the two terms somewhat offset each other which prevents the covariance from ever getting too positive. As a result, the benefits of hedging are capped at a relatively low value. On the other hand, the variance is strictly increasing in the fraction of index investors. This result is due to noise traders playing a greater role in determining relative prices when there are fewer active traders.

This result can be seen in Figure 1 [7]. The covariance equals 0 when there are only active investors, and then starts to slowly increase. However, the noise trader component ($\frac{\tilde{D}\tilde{E}}{\tau_z}$) keeps it from ever getting too big.

[6] Whether the covariance is positive or negative depends on the parameters in my model - both can be achieved

[7] This example sets $\tau_\epsilon = 2$, and $\tau_z = 6$

Figure 1: Normalized Price Covariance and Variance

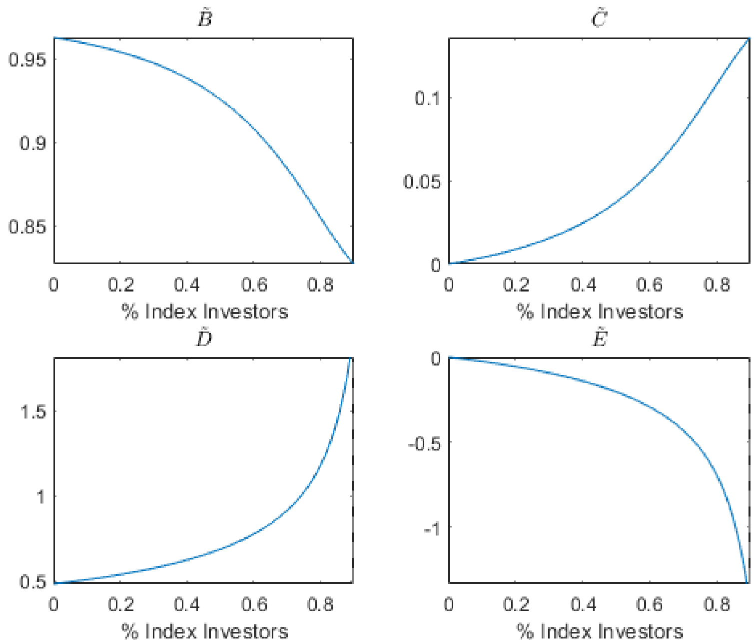


We can also see the variance increasing with the fraction of index investors. Putting these effects together, we see that noise traders prevent the benefits of hedging from ever getting too large.

Thus, the increased prominence of noise traders dilutes the benefits of hedging which allows the effort sensitivity effect, which is strictly increasing in the fraction of index investors, to dominate.

We can see the effort sensitivity effect increasing with the fraction of index investors by looking at a graph of the price coefficients in Figure 2. We see that as the fraction of index investors increases, $\tilde{B}$ decreases while $\tilde{C}$ increases. As a result, the ratio $\frac{\tilde{C}}{\tilde{B}}$ is increasing in the fraction of index investors. Intuitively, with more indexers, prices should reflect the firm's own cash flow less and the cash flow of the other firm more. We can also see that $\tilde{D}$ heads to infinity as and $\tilde{E}$ heads to negative infinity as the fraction of indexers increases. This result follows from the fact that when the fraction of active traders approaches zero, there is no one to correct the "mispricing" generated by the noise traders. Thus, to make the market clear, equilibrium prices become extremely sensitive to noise demand in order to induce the remaining active traders to take on large opposite positions. It is these effects that cause the covariance and variance to head to negative and positive infinity respectively as the fraction of index investors approaches one in Figure 1.

Figure 2: Normalized Price Coefficients



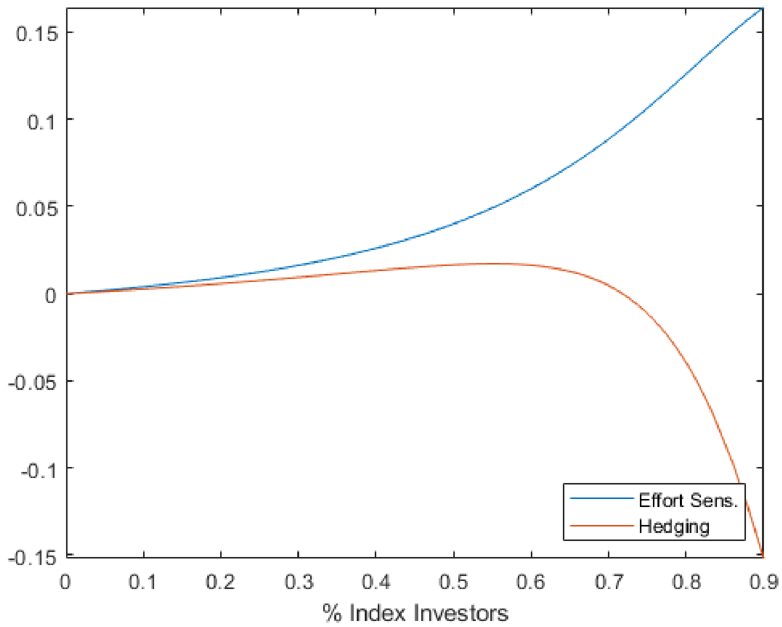
Putting all this together, we can see the effort sensitivity effect always being above the hedging effect in Figure 3. Lastly, we can see the optimal normalized contracting coefficients in Figure 4. When there are only index investors, we put equal weight on both the firm's own price and the other firm's price. This is because when there are only index investors, the market is unable to distinguish the two firms in any way. Then as more active traders appear, contract start putting a larger weight on the firm's own price and a lower weight on the other firm's price because of the changes in how the manager's effort is reflected in financial markets.

I can also derive the following analytical results whose empirical implications will be discussed in Section 8:

Proposition 2 *Additional analytical results:*

- *Coefficients* $\tilde{B}, \tilde{C}, \tilde{D} \geq 0$ *and* $\tilde{E} \leq 0$
- $\tilde{n}^* > 0$ *and is increasing in the fraction of index investors.*
- $\tilde{m}^* > 0$. *Can be increasing or decreasing in the fraction of index investors.*
- *Wage covariance is positive and increasing in the fraction of index investors.*
- *Effort is positive and is decreasing in the fraction of index investors.*
- *Principal's profits are positive and decreasing in the fraction of index investors.*

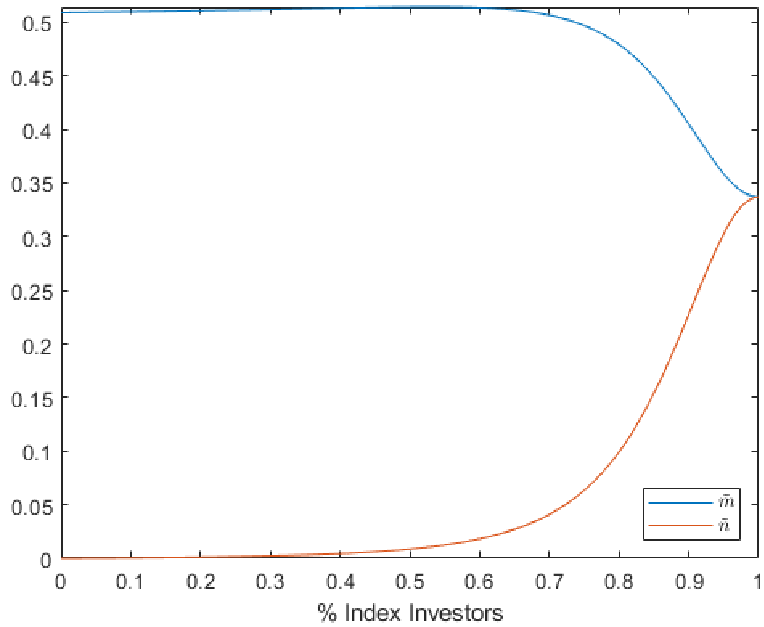
Figure 3: Effort Sensitivity and Hedging Effects



The first two points have been discussed at length. The third point regarding the comparative statics of $\tilde{m}$ with respect to λ merits more discussion. Most of the time, $\tilde{m}$ is increasing in the fraction of active investors because with more active traders, the firm's own stock price better reflects managerial effort. However, price variance can also increase with the amount of active traders under certain parameters due to the magnitude of $\tilde{B}$; notice how much larger the magnitude of $\tilde{B}$ is compared to the other price coefficients when the fraction of index investors is low in Figure 2. Thus, a slightly lower level of $\tilde{m}$ can be optimal for high levels of active investors because stocks are riskier. Nevertheless, I wish to emphasize that under most parameters, $\tilde{m}$ is decreasing in the fraction of index investors.

Increased wage covariance is a direct consequence of firms more closely tying their manager's wage to the stock performance of the other firms. The final two points are a result of price efficiency decreasing with the amount of index investors in my model. With more index investors, the manager's effort is reflected less in his own price, more in the other firm's price, and less overall. Thus, some information is "lost" with more index investors, which hurts incentives and profits.

Figure 4: Normalized Contracting Parameters



4.4 Original Performance Measures

We now want to map $\tilde{m}, \tilde{n}$ back into the original wage. Inverting the mappings given by equation 10 gives us the following characterization of m, n in terms of $\tilde{m}, \tilde{n}$.

$$m_1^* = m_2^* \equiv m^* = \frac{\tilde{m}^2 - \tilde{m} - \tilde{n}^2}{2\tilde{m} - \tilde{m}^2 + \tilde{n}^2 - 1} \tag{17}$$

$$n_1^* = n_2^* \equiv n^* = \frac{-\tilde{n}}{2\tilde{m} - \tilde{m}^2 + \tilde{n}^2 - 1} \tag{18}$$

By undoing the normalization, we get the following proposition.

Proposition 3 *Optimal contract parameters based on the original prices have the same sign as those based on the normalized prices.*

The intuition for the original contract is largely the same as for the normalized contract.

We can also rewrite the price coefficients as functions of the normalized price coefficients and the normalized contract parameters.

$$
\begin{aligned}
B &= \tilde{B} - \tilde{m}\tilde{B} - \tilde{n}\tilde{C} \\
C &= \tilde{C} - \tilde{m}\tilde{C} - \tilde{n}\tilde{B} \\
D &= \tilde{D} - \tilde{m}\tilde{D} - \tilde{n}\tilde{E} \\
E &= \tilde{E} - \tilde{m}\tilde{E} - \tilde{n}\tilde{D} \qquad (19)
\end{aligned}
$$

The price coefficients can be interpreted as the sum of the following three components. I'll use B as an example, but the rest of the coefficients follow the same pattern.

1. The normalized price coefficient ($\tilde{B}$)
2. The reduction in dividends caused by placing incentives on the firm's own stock price ($-\tilde{m}\tilde{B}$)
3. The reduction in dividends caused by placing incentives on the other firm's stock price ($-\tilde{n}\tilde{C}$)

How does a marginal increase in firm 1's cash flow affect P_1? First, there is the basic effect of higher cash flows leading to higher dividends. This positive effect is captured by $\tilde{B}$ and measures how sensitive the price is to dividends in a world without the contracting problem. Second, because higher cash flows means higher prices, dividends are reduced because the manager is paid more. The manager's compensation is related to his firm's cash flow through $\tilde{B}\tilde{m} + \tilde{C}\tilde{n}$. This term negatively affects dividends. Therefore, higher cash flows have both positive and negative effects on the stock price, but in the end $B > 0$ so the positives outweigh the negatives. This result does not necessarily follow for C, however, because the positive cash flow effect of $\tilde{C}$ is relatively small and the negative cash flow effect of $\tilde{m}\tilde{C} + \tilde{n}\tilde{B}$ is large. Thus, C can be positive or negative depending on the parameters. Nevertheless, this outcome does not overturn my result about the effort sensitivity effect dominating the hedging effect in the most empirically relevant cases because of the links between C and $Cov(P_1, P_2)$. The covariance between the two original prices is

$$
Cov(P_1, P_2) = 2 * \left(BC + \frac{DE}{\tau_z} \right) \qquad (20)
$$

We also still have $D > 0$ and $E < 0$, so a necessary condition for a positive covariance given the other parameters is $C > 0$. Thus, if we focus on parameters that generate a positive covariance, the effort sensitivity still dominates the hedging effect because n^* is guaranteed to be positive by Proposition 3. This is admittedly a less universal result than with the normalized contracts, but the parameters that generate a positive covariance are the most relevant ones because there is widespread consensus in the empirical literature that indexing increases the correlation between stocks in the same index (e.g., Barberis et. al (2005) for S&P 500 index funds and Da and Shive (2017) for ETFs).

5 N Firms

It is tempting to think that the preeminence of the effort sensitivity effect is a consequence of there only being two firms. With two firms, good aggregate cash flow signals can only come from two places, so indexed investors buy relatively large amounts of both firms. As a result, firm $1's$ stock price will be fairly sensitive to firm $2's$ cash flows. It stands to reason that with more firms, indexed demand will be spread out over more assets. However, the effort sensitivity effect always dominates, even with an arbitrarily large number of firms.

Proposition 4 *Both $\tilde{m^*}, \tilde{n^*} \geq 0$ for all number of firms $N \geq 1$.*

The condition outlined in Lemma 3 is still decisive even with N firms. Although the sensitivity of firm $1's$ stock price with respect to x_2 declines as the number of firms increases, so does the covariance-variance ratio and the sensitivity of firm $1's$ stock price to its own cash flow. In the end, the effort sensitivity effect still dominates. However, even though $\tilde{n}^*$ is always positive, its magnitude is decreasing in N; the optimal contract puts less weight on each individual firm in the index, but that weight is always positive.

Despite the decrease in magnitude for each individual firm, it turns out that the combined contract weight that is put on all other firms is increasing in N. Since all firms are ex-ante identical, the principal will put the same weight $\tilde{n}^*$ on all other index firms in his contract. Therefore, the weight that is put on all other firms is $(N-1)\tilde{n^*}$. My claim is then formally expressed in the following proposition:

Proposition 5 *The combined weight that the optimal contract puts on all other index firms, $(N-1)\tilde{n^*}$, is increasing in N.*

The intuition for Proposition 5 comes from how the demands of the index investors are split between index firms. Suppose the manager of firm 1 takes good action and increases his firm's payoff. Index investors expect higher index payoffs and buy more of all index firms. When there are N=2 firms in the index, this demand is split between the two firms, so index investors essentially assign 50% of "the credit" for this signal to stock 1 and 50% to stock 2. When there are N=100 firms in the index, the demand is split among one hundred firms, so index investors assign 1% of "the credit" to stock 1 and 99% to the remaining stocks. As a result, relatively more of "the credit" is assigned to other firms in the index as N increases.

We can also see this result by looking at the stock prices when there are N firms.

$$\tilde{P_k} = \tilde{A} + \tilde{B}x_k + \tilde{C}(x_1 + ...x_{k-1} + x_{k+1} + ...x_N) + \tilde{D}z_k + \tilde{E}(z_1 + ...z_{k-1} + z_{k+1} + ...z_N) \tag{21}$$

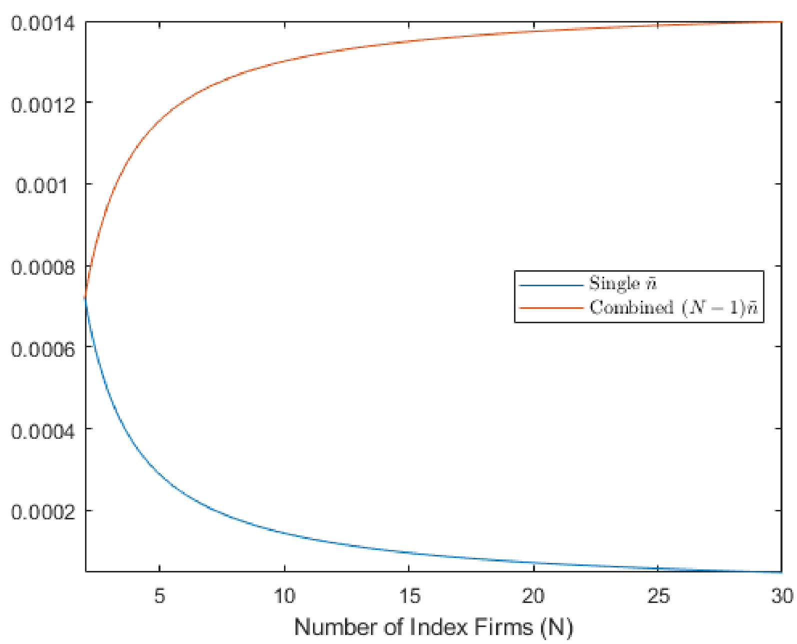
Therefore, the sensitivity of a firm $k's$ stock price to its cash flow is $\tilde{B}$, while the combined sensitivity of all other firms' stock prices to firm k's cash flow is $(N-1)\tilde{C}$.

These terms will end up being:

$$\begin{aligned} \tilde{B} &= \frac{\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1} + \frac{1}{N}\left(\frac{\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1} - \frac{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1}\right) \\ (N-1)\tilde{C} &= \frac{N-1}{N}\left(\frac{\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1} - \frac{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1}\right) \end{aligned} \tag{22}$$

Since $\frac{\tau_z\tau_\epsilon^2+\tau_\epsilon}{\tau_z\tau_\epsilon^2+\tau_\epsilon+\tau_x} \geq \frac{\lambda^2\tau_z\tau_\epsilon^2+\tau_\epsilon}{\lambda^2\tau_z\tau_\epsilon^2+\tau_\epsilon+\tau_x}$, it is easy to see that $\tilde{B}$ is decreasing in N while $(N-1)\tilde{C}$ is increasing in N. That is, firm k's stock price becomes less sensitive to its cash flow and combined sensitivities of all other firms' stock prices become more sensitive to firm k's cash flow as N increases. These results can be seen graphically in Figure 5 below where the fraction of index investors is set at 16% to match the Investment Company Institute's 2022 Factbook.

Figure 5: Single $\tilde{n}$ and Combined $(N-1)\tilde{n}$



6 Modeling Index Investors

The theoretical literature has not yet reached a consensus about the best way to model the information sets of index investors. Some papers (e.g. Bond and Garcia (2022)) let index investors condition their index demand on both private information and equilibrium prices, while on the other extreme, Coles et. al (2022) models index investors as only using prior information. The essential question for this line of research therefore is: how passive are index investors? In my view, the minimum level of passivity that must be required for an investor to be an index investor is that they must only trade stocks in some fixed proportion. That is, whatever their information set, index investors only use their information to decide how to divide their portfolio between a fixed index of risky assets and a risk-free asset. This condition appears consistent with virtually every paper in the literature. The different modeling assumptions in this literature are also very dependent on which classic model of financial markets they are extending. Indexing papers based on Grossman and Stiglitz (1980) tend to assign index investors the role of the "uninformed" investors in that paper with the additional restriction that they must only trade the index. On the other hand, models based on Grossman (1976), Hellwig (1980), or Diamond and Verrecchia (1981) allow index investors to receive private signals. Thus, in a sense, the debate about what is the best way to model index investors is similar to the older debate about which classic model of financial markets is most useful. As all the above models provide useful insights into the price formation process, it is a worthwhile endeavor to examine index investing under each of them.

Nevertheless, I wish to justify why I chose to model index investors as having private information both intuitively and empirically. First, index investors in my model only receive private information about the cash flows of the index while active investors receive private information about both index cash flows and relative cash flows. Gathering private information on index cash flows can be thought of as focusing attention only on broad economic developments rather than on individual firms. One can therefore view an index investor in my model as someone who after deciding to become an indexer, does some research on the state of the overall economy and arrives at some private conclusion. For example, a private view on the economy could be shaped by an individual's news choice, the performance of their employer, opinions of friends and relatives, etc. This assumption seems like a plausible description of how index investors divide their wealth between an index and a risk-free asset. The alternative of having indexers condition their demands only on public information would lead to them all having identical demand for the index, which ignores the fact that investors tend to have heterogenous views about where the market is going. For example, Giglio et. al (2021) analyze a survey administered to Vanguard retail traders (an asset manager whose clients are known to favor passive funds) on their expectations for various macroeconomic and financial indicators. The authors demonstrate that there exists substantial cross-sectional dispersion in investor beliefs about where the market or economy is heading and that these beliefs get reflected in the investors' portfolios. These results suggest that it is reasonable to model indexers as having private information that gets reflected in their portfolio choice.

However, despite my preference for modeling index investors as having private information, I will now examine what happens to my results if they do not. Suppose we model passive investors as having no private information, but rationally incorporating the public information in the price. Their demand would then be:

$$D(i,1) = \frac{E(x_1|P) - P_1}{Var(x_1|P)}$$

Shocks to firm 1 should still impact the price of firm 2 through the demands of index investors. Suppose firm 1 had a good realization of x_1. Index investors will see a higher index price as a result, but not know whether it is due to increased cash flows or increased noise demand. One can show that the net result will be reduced demand for the index, which will lower the stock price of firm 2. Therefore, the stock price of firm 2 will still reflect the effort of firm 1's manager (albeit in the opposite direction), so the optimal contract will condition on it. The magnitude and sign of n will almost certainly be different in this case, but it should still be non-zero; the informativeness principle still applies. However, the optimal contract will have a different sign than before. Now the effort sensitivity effect suggests placing a negative weight on the other firm's stock price while the hedging effect always implies a positive weight due to the covariance now being guaranteed to be negative. It turns out that the hedging effect will dominate in this case because both terms of the covariance are now negative, which makes the covariance more extreme and thus increases the benefits of hedging.

The above line of reasoning can be taken a step further. The minimum condition required for each firm's stock to reflect the fundamentals [8] of both firms is passive investors not having perfectly inelastic demand for the index asset. As long as their demand is price sensitive, each firm's stock price will somewhat reflect the fundamentals of the other firm through the changing demands of the index traders. Private information, optimally incorporating all public information, or even downward sloping demand is not necessary, just that index demand is price sensitive.

The "no private information" conclusions also lead to some results that go against the empirical evidence.

[8] By fundamentals I mean x and z.

As I just mentioned, a model where index investors have no private information leads to a negative correlation between the two stock prices, whereas nearly all empirical work on index investing finds that indexing leads to a positive correlation (e.g., Barberis et. al (2005) for S&P 500 index funds and Da and Shive (2017) for ETFs). A necessary condition for a positive correlation in my model is that index investors buy more of the index following an increase in the index's cash flow. Again, this condition fails in the model with no private information because investors cannot tell index price increases caused by cash flows versus noise demand apart. Providing index investors with a private signal about index cash flows can be enough to increase their demand for the index following an increase in one firm's cash flows. My results also do not depend on index investors having the same aggregate signal quality as active investors. I only require that the indexers' signals are not *too* much worse.

7 Relationship to the Common Ownership Literature

A topic ignored in this paper is how the growth of index investing may change the principal's objective function. Index investors typically purchase an index through an index fund managed by a large asset manager such as Vanguard or BlackRock. The common ownership literature examines how corporate governance changes as these asset managers build increasingly large stakes in many firms due to increased index fund flows. One common view is that these asset managers try to maximize the value of their entire portfolio, which may be inconsistent with maximizing the value of each constituent firm. For example, individual firm value maximization may lead to inefficiently aggressive competition between firms from the asset manager's perspective. One hypothesized way to prevent such things from happening is to rely less on relative performance evaluation for managerial incentives or even to put a positive weight on a competing firm's stock price (Gordon (1990) and Aggarwal and Samwick (1999)).

My paper, therefore, offers a separate rationale for placing a positive weight on competitors' stock prices: it helps incentivize managerial effort. In my model, introducing a common principal who aims to maximize

$$E(x_1 + x_2 - w_1 - w_2)$$

would not alter the optimal contracts because the $x's$ only depend on their own manager's effort. I will now provide a short extension where each manager's effort has a direct impact on the other firm's payoff. Define firm k's new mean payoff as

$$\bar{e_k} = e_k + \beta e_j, \text{ for } k, j \in \{1, 2\}, k \neq j \tag{23}$$

So now the mean of x_k is $\bar{e_k}$ which is impacted by both manager k's effort (through e_k) and manager j's effort (through e_j). The parameter β signifies the degree to which each manager's effort affects the other firm's payoff. My base model is therefore a specialized case with $\beta = 0$. A positive β describes a situation where the two managers' efforts complement each other. Perhaps this describes a case with a manufacturer and a retailer. If the manufacturer makes higher quality products and the retailer becomes more effective at sales, both firms benefit. A negative β can be interpreted as a situation where the two firms are rivals competing for the same market share.

This modeling change only affects the equilibrium asset prices $(\tilde{P_1}, \tilde{P_2})$ through changed mean payoffs. Importantly, price coefficients $\tilde{B}, \tilde{C}, \tilde{D}$, and $\tilde{E}$ are unchanged because they only depend on exogenous parameters (τ_ϵ, τ_z, and λ). Consequently, the variances and covariances of the prices are also unchanged.

Given the unchanged asset prices, we can jump directly to the contracting problem.

7.1 Single Principal

A single principal maximizing his own firm's expected dividends solves:

$$\max_{\tilde{l_k}, \tilde{m_k}, \tilde{n_k}} E(x_k - w_k) \tag{24}$$

$$\text{s.t. } E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \geq 0 \tag{IR}$$

$$e_k \in \underset{e_k}{\operatorname{argmax}}\, E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \tag{IC}$$

where now:

$$E(x_k) = e_k + \beta e_j$$

$$E(w_k) = E(\tilde{l_k} + \tilde{m_k}\tilde{P_k} + \tilde{n_k}\tilde{P_j})$$

$$= \tilde{l_k} + \tilde{m_k}(\tilde{A} + \tilde{B}(e_k + \beta e_j) + \tilde{C}(e_j + \beta e_k)) + \tilde{n_k}(b_{0j} + \tilde{B}(e_j + \beta e_k) + \tilde{C}(e_k + \beta e_j)) \tag{25}$$

Manager k's optimal choice of effort is then

$$e_k^* = \tilde{m_k}(\tilde{B} + \beta\tilde{C}) + \tilde{n_k}(\tilde{C} + \beta\tilde{B}) \tag{26}$$

The manager's effort is now reflected in two additional places relative to the base model: through the x_j terms in both his own and the other firm's stock prices. Holding the contract fixed with $\tilde{m}, \tilde{n} > 0$, a positive β induces more effort from the manager because his effort is more strongly reflected in the stock market. Conversely, a negative β generates less effort because effort lowers the payoff and expected stock price of the other firm. The signs of the contracting parameters are described by a condition analogous to Lemma 3:

Proposition 6 *The optimal choice of $\tilde{m}$ will be greater than or equal to zero if and only if $\frac{\tilde{B}+\beta\tilde{C}}{\tilde{C}+\beta\tilde{B}} \geq \frac{Cov(\tilde{P}_1,\tilde{P}_2)}{Var(\tilde{P})}$. The optimal choice of $\tilde{n}$ will be greater than or equal to zero if and only if $\frac{\tilde{C}+\beta\tilde{B}}{\tilde{B}+\beta\tilde{C}} \geq \frac{Cov(\tilde{P}_1,\tilde{P}_2)}{Var(\tilde{P})}$.*

Proof: *Solution method is identical to Lemma 3. The only change is that the sensitivity of firm 2's stock price to manager 1's effort is now represented by $\tilde{C} + \beta\tilde{B}$ rather than $\tilde{C}$ and similarly for the sensitivity of firm 1's stock price to manager 1's effort (i.e., $\tilde{B} + \beta\tilde{C}$ rather than $\tilde{B}$).*

The hedging effect is the same as before, but now the effort sensitivity effect takes into account how effort affects the other firm's cash flows. I assume throughout that the exogenous parameters are such that $Cov(\tilde{P}_1, \tilde{P}_2) > 0$ because that is the most empirically relevant case.

Suppose $\beta > 0$; then both $\tilde{m}$ and $\tilde{n}$ remain positive. In fact, the effort sensitivity effect is even stronger than in the base case with $\beta = 0$. This result follows because a positive β increases the effort sensitivity of the other firm's stock price more than the firm's own stock price; mathematically, $\frac{\tilde{C}+\beta\tilde{B}}{\tilde{B}+\beta\tilde{C}} > \frac{\tilde{C}}{\tilde{B}}$ because $\beta > 0$ and $\tilde{B} > \tilde{C} > 0$. Intuitively, because stock prices are more sensitive to their own cash flow than the other firm's cash flow (i.e., $\tilde{B} \geq \tilde{C}$), the impact that effort has on the other firm's cash flow through β is more salient in the other firm's stock price than the firm's own.

The results are less clear cut when $\beta < 0$. I assume that $\beta \in (-\frac{\tilde{B}}{\tilde{C}}, 0)$ because if it were more negative, the manager's effort would be so destructive for the other firm that an increase would actually reduce *his own* expected stock price, which seems unrealistic. If $\beta \in (-\frac{\tilde{B}}{\tilde{C}}, -\frac{\tilde{C}}{\tilde{B}})$, $\tilde{n}$ will trivially be negative because both the effort sensitivity and hedging effects call for a negative sign. The effort sensitivity effect wants a negative sign in this region because effort reduces the expected stock price of the other firm and the hedging effect also wants a negative sign because the correlation between the two stock prices is positive. One can then show that if $\beta \in \left(-\frac{\tilde{C}}{\tilde{B}}, \frac{\tilde{C}var(\tilde{P})-\tilde{B}Cov(\tilde{P}_1,\tilde{P}_2)}{\tilde{C}Cov(\tilde{P}_1,\tilde{P}_2)-\tilde{B}var(\tilde{P})}\right)$, $\tilde{n}$ will also be negative and if $\beta \in \left(\frac{\tilde{C}var(\tilde{P})-\tilde{B}Cov(\tilde{P}_1,\tilde{P}_2)}{\tilde{C}Cov(\tilde{P}_1,\tilde{P}_2)-\tilde{B}var(\tilde{P})}, 0\right)$, $\tilde{n}$ will be positive. Basically, for a fixed hedging effect, a lower β makes a negative $\tilde{n}$ increasingly optimal as the manager's effort is reflected less positively in the other firm's stock price.

7.2 Common Owner

A common owner who maximizes joint dividends solves:

$$\max_{\hat{l},\tilde{m},\tilde{n}} E(x_1 + x_2 - w_1 - w_2) \tag{27}$$

$$\text{s.t. } E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \geq 0 \text{ for } k \in \{1,2\} \tag{IR}$$

$$e_k \in \underset{e_k}{\operatorname{argmax}} \, E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \text{ for } k \in \{1,2\} \tag{IC}$$

Each manager's optimal effort choice is still described by equation 26. The only substantive difference between the common owner and single principal problems is that the common owner takes into account how each manager's effort impacts the other firm's payoff in the $E(x_1+x_2)$ portion of his objective function. Thus, when deciding on manager 1's contract, the common owner considers $(1+\beta)e_1^*$ as opposed to just e_1^* in his objective function. Let $\tilde{m}_s$ and $\tilde{n}_s$ be the optimal contract for the single principal (as solved for in Section 7.1) and $\tilde{m}_c$ and $\tilde{n}_c$ be the optimal common owner contract. It turns out that there is a simple relation between the two:

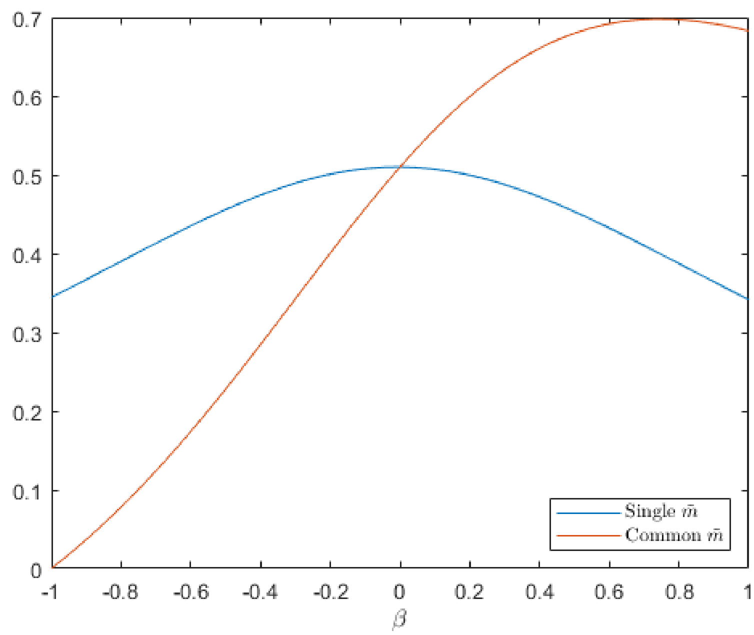
Proposition 7 *The optimal common owner contract parameters are the optimal single owner contract parameters multiplied by* $1+\beta$*. That is,* $\tilde{m}_c = (1+\beta)\tilde{m}_s$ *and* $\tilde{n}_c = (1+\beta)\tilde{m}_c$*.*

Thus, for $\beta > -1$, the common owner simply scales the single owner contract up or down without changing the sign. A positive β means that each manager's effort generates a positive externality for the other firm that the single owner does not care about. The common owner wants to take advantage of this positive externality and thus makes each contract more performance sensitive in the positive direction. The opposite happens when $\beta \in (-1,0)$. Each manager's effort generates a negative externality for the other firm so the common owner makes the optimal contract less performance sensitive, but still with the same sign. In the extreme case where $\beta < -1$, the negative externality is so severe that it outweighs the positive effect that the manager's effort has on his own firm's performance and the common owner flips the signs on the optimal contract parameters.

In sum, all common ownership does in the realistic case where $\beta \in (-1,1)$ is change the magnitude of the single owner contract without altering the sign. Common ownership is therefore not the driving force behind a positive sign on the other firm's stock price in the optimal contract; rather, the key is still the relative sizes of the effort sensitivity and the hedging effects. Thus, the results of my paper caution against interpreting the presence of a positive association between managerial compensation and the stock performance of peers as evidence of anti-competitive activities. Such a result can also be generated by a model in which each firm is a strict profit maximizer that wants to incentivize more managerial effort.

Plots of the optimal $\tilde{m}$ and $\tilde{n}$ for the single and common owner as a function of β can be seen below. In these plots, the fraction of index investors is set at 16% to match the Investment Company Institute's 2022 Factbook. In Figure 6, we see that the optimal $\tilde{m}$ is positive everywhere, but the common owner sets it higher when $\beta > 0$ and lower when $\beta < 0$. The optimal $\tilde{n}$ can be positive or negative depending on β, but the sign on the common owner's contract is always the same as the single owner's.

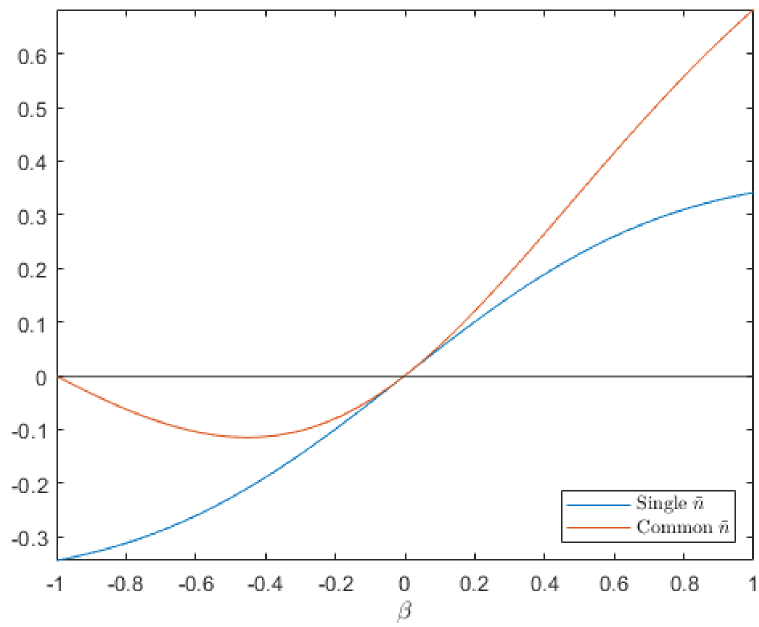
Figure 6: Single and Common Owner $\tilde{m}$



8 Empirical Implications

A few recent empirical papers (Jochem et al. (2021) and Cabezon (2021)) note that CEO pay has become more uniform over time. In particular, Jochem et al. (2021) claims that increased reciprocal benchmarking is the primary cause of this trend and that reciprocal benchmarking is in turn driven by the growth of passive investing. This result is consistent with my model. However, their rationale for why passive investing increases benchmarking is that passive investors are more likely to rely on proxy advisors — who carefully analyze pay relative to peers — for voting recommendations. My paper suggests an additional channel for this phenomenon: index investing causes stocks in the same index to reflect the effort of all managers in the index, so linking a

Figure 7: Single and Common Owner $\tilde{n}$



manager's compensation to the performance of other stocks in the same index is good for incentives. In other words, there are two ways through which index investing can make contracts more similar: (1) the governance channel identified by Jochem et al. (2021) and (2) the asset pricing channel implied by my model. Which channel is more important is an interesting empirical question that has not yet been addressed.

A second empirical implication concerns my main result that a firm should place a positive weight on the stock performance of other firms in the same index. There is some evidence for this result, but the proposed mechanism is common ownership rather than changes in equilibrium asset prices. Anton et al. (2016) and Liang (2016) find that higher levels of common ownership are associated with greater sensitivity of managerial pay to the performance of industry peers. However, the growth of common ownership is in large part driven by the increased popularity of index investing because most index investors invest through a handful of asset managers. Additionally, Anton et al. (2022) finds that increases in common ownership are associated with a decline in the performance sensitivity of managerial pay, which is also consistent with my model. In other words, the decline in pay sensitivity to the firm's performance and the increase in pay sensitivity to peer performance is consistent with both frameworks. Future papers can examine the relative importance of each view for the observed results by integrating a more realistic financial market in models of common ownership. I should also mention that Kwon (2016) finds the opposite result: increases in common ownership are associated with more relative performance pay, so the empirical evidence is not unanimous.

9 Conclusion

I develop a joint model of financial markets with index investors and contracting with moral hazard to examine how changes in the prevalence of indexing impact optimal managerial contracts. Index investors are constrained to purchase all risky assets in the same proportion, so information that affects their demand for the index is transmitted to all stocks in the index. For example, suppose firm 1 is in a stock index and its manager takes a value-enhancing action. Index investors expect higher index payoffs and purchase more of the index. This trade distributes the index investors' information about the manager's effort to all stocks in the index. Thus, the prices of other index firms are positively related to and contain unique information about the manager of firm 1's effort. The synchronized demand of index investors is what makes it optimal for the principal of each firm to tie their manager's compensation to the performance of other index firms.

Under some mild assumptions, the principal will always put a positive contract weight on other index firms' stocks. There are two (possibly conflicting) reasons to condition the manager's compensation on the other firms' stocks: (1) to increase the effort sensitivity of the manager's pay and (2) to hedge out the manager's risks. The former reason is a consequence of the manager's effort being reflected in other firms' stocks. The latter reason comes from the relative performance evaluation literature: the principal should make use of the non-zero correlation between the stocks to reduce the manager's exposure to common risks. I find that the effort sensitivity channel always dominates the hedging channel because the increased volatility brought about by indexers reduces the benefits of hedging.

I believe my results are most empirically relevant for smaller indexes with dominant companies. One good example is an industry index. The CEO of an industry leader should have their compensation positively linked to the performance of their industry's stock index because their actions are likely to be strongly reflected in that index's performance. It seems unlikely that an S&P CEO would have a comparable impact on S&P 500 performance due to that index's size. Section 5 shows that my results hold with an arbitrary number of firms, but the efficiency losses from not following the optimal contract are quantitatively small for companies that are a tiny percentage of their index's total market capitalization. Additionally, individuals who trade industry index funds or ETFs seem to be more likely to have good private information about the state of an industry than the typical S&P 500 investor does about the state of the aggregate market.

Stock prices are a useful tool for managerial compensation because they are an easy-to-access and high-frequency performance measure. Thus, assuming prices are relatively efficient, they can be used to learn about actions privately taken by a manager. However, financial markets are always evolving, so firms need to keep track of how these changes impact the incentive schemes that they design for their manager.

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11 Appendix

Proof of Lemmas 1 and 2 Setting the first order condition for IC equal to 0 and solving for e_1 yields

$$e_1^* = m_1 + n_1\alpha \tag{28}$$

We can then bind the IR condition and plug it and e_1^* into the objective function to get (after some simplification):

$$\max_{m_1,n_1} = e_1^* - \frac{1}{2}e_1^{*2} - \frac{1}{2}Var(w_1) \tag{29}$$

where $Var(w_1) = m_1^2 Var(P_1) + n_1^2 Var(P_2) + 2Cov(P_1, P_2) = 2m_1^2 + 2n_1^2 + 2m_1 n_1 \rho$.
We can then set the two first-order conditions with respect to m_1 and n_1 equal to zero to get a system of two equations for m_1 and n_1. Solving gives us:

$$m_1^* = \frac{2 - \alpha\rho}{\Gamma} > 0 \tag{30}$$

$$n_1^* = \frac{2\alpha - \rho}{\Gamma} \tag{31}$$

where $\Gamma \equiv 2\alpha^2 - 2\alpha\rho - \rho^2 + 6 > 0$. Clearly, m_1^* is always positive given the restrictions on α and ρ. When $\alpha = 0$, we see that n_1^* will have the opposite sign of ρ which proves Lemma 1. Additionally, whenever $\alpha > \frac{\rho}{2}$ we are guaranteed to get $n_1^* > 0$ which proves Lemma 2.

Proof of Lemma 3 Follows immediately from equation (15).

Proof of Proposition 1 Plug the relevant price coefficients into equation (15) and simplify. This gives us:

$$\tilde{n^*} = \frac{\tau_\epsilon\, \tau_z \left(1 - \lambda^2\right) \left(\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1\right)}{\left(\tau_\epsilon\, \tau_z\, \lambda^2 + 1\right) \left(\tau_\epsilon\, \tau_z + 1\right) \left(4\, \lambda^2\, {\tau_\epsilon}^4\, {\tau_z}^2 + 3\, \lambda^2\, {\tau_\epsilon}^2\, \tau_z + 3\, {\tau_\epsilon}^2\, \tau_z + 2\right)} \tag{32}$$

Since $\lambda \in [0, 1]$, it must be the case that $\tilde{n^*} > 0$ when $\lambda < 1$ and $\tilde{n^*} = 0$ when $\lambda = 1$.

Proof of Proposition 3 Apply the mapping characterized by equations (17) and (18) and simplify the resulting expressions. The expressions are too long to fit on this page, but m^* is a ratio of two entirely positive terms and n^* is a ratio of two entirely positive terms multiplied by $(1-\lambda^2)$. Thus, they are both positive.

Proof of Proposition 4 Following the same procedure as with $N = 2$ firms, we get the following price conjecture and resulting equilibrium coefficients:

$$\tilde{P}_k = b_0 + \tilde{B}x_k + \tilde{C}(x_1 + ...x_{k-1} + x_{k+1} + ...x_N) + \tilde{D}z_k + \tilde{E}(z_1 + ...z_{k-1} + z_{k+1} + ...z_N) \tag{33}$$
$$\tilde{B} = \frac{\tau_\epsilon\,(\tau_\epsilon\,\tau_z + 1)}{N\,(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)} + \frac{\tau_\epsilon\left(\tau_\epsilon\,\tau_z\,\lambda^2 + 1\right)(N-1)}{N\,(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)}$$
$$\tilde{C} = \frac{\tau_\epsilon\,(\tau_\epsilon\,\tau_z + 1)}{N\,(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)} - \frac{\tau_\epsilon\left(\tau_\epsilon\,\tau_z\,\lambda^2 + 1\right)}{N\,(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)}$$
$$\tilde{D} = \frac{\tau_\epsilon\,\tau_z + 1}{N\,(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)} + \frac{\left(\tau_\epsilon\,\tau_z\,\lambda^2 + 1\right)(N-1)}{N\,\lambda\,(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)}$$
$$\tilde{E} = \frac{\tau_\epsilon\,\tau_z + 1}{N\,(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)} - \frac{\tau_\epsilon\,\tau_z\,\lambda^2 + 1}{N\,\lambda\,(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + \tau_x)}$$

The manager's choice of effort will then be

$$e_k^* = \tilde{m}\tilde{B} + (N-1)\tilde{n}\tilde{C} \tag{34}$$

The above equation has the principal put the same weight on each of the other firms in the contract. Due to the fact that each firm is ex-ante identical and independent, it is straightforward to prove that this result is optimal.

Then by binding the participation constraint and plugging into the objective, the principal's problem becomes

$$\max_{\tilde{m},\tilde{n}} e_k^* - \frac{1}{2}e_k^{*2} - \frac{1}{2}\left((\tilde{m}^2 + (N-1)\tilde{n}^2)var + 2*((N-1)\tilde{m}\tilde{n} + \binom{N-1}{2}\tilde{n}^2)cov\right) \tag{35}$$

where

$$var = \frac{\tilde{B}^2 + (N-1)\tilde{C}^2}{\tau_x} + \frac{\tilde{D}^2 + (N-1)\tilde{E}^2}{\tau_z} \tag{36}$$
$$cov = \frac{(N-2)\tilde{C}^2 + 2\tilde{B}\tilde{C}}{\tau_x} + \frac{(N-2)\tilde{E}^2 + 2\tilde{D}\tilde{E}}{\tau_z}$$

We can then take the first order conditions and solve the resulting system of equations for $\tilde{n}$. For and N, we can see that the optimal contract weight will be

$$\tilde{n} = \frac{\tau_\epsilon \, \tau_x{}^2 \, \tau_z \left(1 - \lambda^2\right) \left(\tau_z \, \lambda^2 \, \tau_\epsilon{}^2 + \tau_z \, \tau_\epsilon{}^2 + \tau_\epsilon + \tau_x\right)}{\left(\tau_\epsilon \, \tau_z \, \lambda^2 + 1\right) \left(\tau_\epsilon \, \tau_z + 1\right) \Gamma} \tag{37}$$

with $\Gamma = \left(N \, \lambda^2 \, \tau_\epsilon{}^4 \, \tau_x \, \tau_z{}^2 + N \, \lambda^2 \, \tau_\epsilon{}^4 \, \tau_z{}^2 + (N-1) \, \lambda^2 \, \tau_\epsilon{}^2 \, \tau_x{}^2 \, \tau_z + N \, \lambda^2 \, \tau_\epsilon{}^2 \, \tau_x \, \tau_z + \tau_\epsilon{}^2 \, \tau_x{}^2 \, \tau_z + N \, \tau_\epsilon{}^2 \, \tau_x \, \tau_z + N \, \tau_x{}^2\right)$

Proof of Proposition 5 Multiply equation (37) by $N - 1$ and take the partial derivitive of the resulting expression with respect to N. It is straightforward to verify that every term in this partial derivative is positive.

Proof of Proposition 7 The manager's optimal effort is the same as equation (26). Binding the IR constraint and plugging into the common owner's objective function gives us:

$$\max_{\tilde{l}, \tilde{m}, \tilde{n}} (1+\beta)e_1^* + (1+\beta)e_2^* - \frac{1}{2}{e_1^*}^2 - \frac{1}{2}{e_2^*}^2 - \frac{1}{2}Var(w_1) - \frac{1}{2}Var(w_2) \tag{38}$$

with $e_k^* = \tilde{m_k}(\tilde{B} + \beta\tilde{C}) + \tilde{n_k}(\tilde{C} + \beta\tilde{B})$. Take the first-order conditions with respect to $\tilde{m_1}, \tilde{m_2}, \tilde{n_1}$, and $\tilde{n_2}$ to get a pair of symmetric systems of equations: one for $\tilde{m_1}$ and $\tilde{n_1}$, and another for $\tilde{m_2}$ and $\tilde{n_2}$.

$$\begin{aligned} \tilde{m_k} &= \frac{B_1 + B_1 \, \beta - \text{cov} \, \tilde{n_1} - B_1 \, B_2 \, \tilde{n_1}}{{B_1}^2 + \text{var}} \\ \tilde{n_k} &= \frac{B_2 + B_2 \, \beta - \text{cov} \, \tilde{m_1} - B_1 \, B_2 \, \tilde{m_1}}{{B_2}^2 + \text{var}} \\ &\text{with } B_1 = \tilde{B} + \beta\tilde{C} \text{ and } B_2 = \tilde{C} + \beta\tilde{B} \end{aligned} \tag{39}$$

Solving then gives us answers analogous to (14) and (15) with B_1 in place of $\tilde{B}$ and B_2 in place of $\tilde{C}$ and the entire thing multiplied by $(1 + \beta)$. This is equivalently, the single owner solution multiplied by $(1 + \beta)$.

12 Appendix 2: Endogenous Fraction of Index Investors

To endogenize the fraction of index investors, I will include a cost of being active $c > 0$ to my model. This cost can be thought of as the extra trading, information gathering, or information processing costs of dealing with multiple assets rather than just the index. Each investor must be indifferent between being an indexer and active ex-ante in order to have a positive fraction of both index and active traders ($\lambda \in (0, 1)$). Define the ex-ante (T=2 in the timeline) expected utilities of being an active and passive investor as $E(U_a)$ and $E(U_p)$ respectively. To avoid complications with correlated prices, I will stick to using the synthetic assets in this analysis. My proof is similar to Theorem 3 in Grossman and Stiglitz (1980) and can be viewed as a special case of Proposition 3.1 in Admati and Pfleiderer (1987).

Proposition 8 *The expected utilities of active and passive traders can be expressed as:*

$$E(U_a) = -\sqrt{\frac{Var(\hat{x_1} - \hat{P_1}|\hat{s},\hat{P})}{Var(\hat{x_1} - \hat{P_1})}}\sqrt{\frac{Var(\hat{x_2} - \hat{P_2}|\hat{s},\hat{P})}{Var(\hat{x_2} - \hat{P_2})}}\exp\left(-(W_0 - c) - \frac{E(\hat{x_1} - \hat{P_1})^2}{2Var(\hat{x_1} - \hat{P_1})} - \frac{E(\hat{x_2} - \hat{P_2})^2}{2Var(\hat{x_2} - \hat{P_2})}\right) \tag{40}$$

$$E(U_p) = -\sqrt{\frac{Var(\hat{x_1} - \hat{P_1}|\hat{s},\hat{P})}{Var(\hat{x_1} - \hat{P_1})}}\exp\left(-W_0 - \frac{E(\hat{x_1} - \hat{P_1})^2}{2Var(\hat{x_1} - \hat{P_1})}\right) \tag{41}$$

Proof of Proposition 8 First, let's write down the expected utility of an active agent at the trading stage.

$$E(U_a|\hat{s},\hat{P}) = -\exp\left(-\alpha(w_0 - c) - \frac{(E(\hat{x_1}|\hat{s},\hat{P}) - \hat{P_1})^2}{2Var(\hat{x_1}|\hat{s},\hat{P})} - \frac{(E(\hat{x_2}|\hat{s},\hat{P}) - \hat{P_2})^2}{2Var(\hat{x_2}|\hat{s},\hat{P})}\right) \tag{42}$$

We will now integrate over $\hat{s}$ and use the fact that $(\hat{x},\hat{\epsilon},\hat{z})$ are all independent to get:

$$E(U_a|\hat{P}) = -\exp(\alpha(w_0 - c))E\left(\exp\left(\frac{-(E(\hat{x_1}|\hat{s},\hat{P}) - \hat{P_1})^2}{2Var(\hat{x_1}|\hat{s},\hat{P})}\right)|\hat{P}\right)E\left(\exp\left(\frac{-(E(\hat{x_2}|\hat{s},\hat{P}) - \hat{P_2})^2}{2Var(\hat{x_2}|\hat{s},\hat{P})}\right)|\hat{P}\right) \tag{43}$$

Now define

$$Y_k \equiv \frac{E(\hat{x_k} - \hat{P_k}|\hat{s},\hat{P})}{\sqrt{Var(E(\hat{x_k} - \hat{P_k}|\hat{s},\hat{P}))}} \tag{44}$$

Note that Y_k is a normal random variable with variance 1. Also define:

$$t \equiv \frac{Var(E(\hat{x_k} - \hat{P_k}|\hat{s},\hat{P}))}{2E(Var(\hat{x_k} - \hat{P_k}|\hat{s},\hat{P}))} \tag{45}$$

Then observe that

$$E\left(\exp\left(\frac{-(E(\hat{x_k}|\hat{s},\hat{P}) - \hat{P_k})^2}{2Var(\hat{x_k}|\hat{s},\hat{P})}\right)|\hat{P}\right) = E(\exp(-tY_k^2)|\hat{P}) \tag{46}$$

Because Y_k is normal with variance 1, Y_k^2 is a non-central chi square random variable. The above equation then defines the moment generating function, which any statistics text can tell you equals:

$$E(\exp(-tY_k^2)|\hat{P}) = \frac{1}{\sqrt{1+2t}}\exp\left(\frac{-E(Y_k|\hat{P})^2 t}{1+2t}\right) \tag{47}$$

Following this process for both $k = 1, 2$ and plugging into equation 48 gives us:

$$E(U_a|\hat{P}) = -\sqrt{\frac{Var(\hat{x_1} - \hat{P}_1|\hat{s}, \hat{P})}{Var(\hat{x_1} - \hat{P}_1|\hat{P})}}\sqrt{\frac{Var(\hat{x_2} - \hat{P}_2|\hat{s}, \hat{P})}{Var(\hat{x_2} - \hat{P}_2|\hat{P})}}\exp\left(-\alpha(W_0 - c) - \frac{E(\hat{x_1} - \hat{P}_1|\hat{P})^2}{2Var(\hat{x_1} - \hat{P}_1|\hat{P})} - \frac{E(\hat{x_2} - \hat{P}_2|\hat{P})^2}{2Var(\hat{x_2} - \hat{P}_2|\hat{P})}\right) \tag{48}$$

Now we want to integrate out prices, so follow basically the same process as before and the result is equation (37) for the ex-ante expected utility of the active agent. The proof is identical for the passive agent except they do not trade synthetic asset 2. These results are of the form given in Admati and Pfleiderer (1987).

Corollary 1 *The ratio of expected utilities is*

$$\Lambda(\lambda) \equiv \frac{E(U_a)}{E(U_p)} = \sqrt{\frac{Var(\hat{x_2} - \hat{P}_2|\hat{s}, \hat{P})}{Var(\hat{x_2} - \hat{P}_2)}}\exp\left(c - \frac{E(\hat{x_2} - \hat{P}_2)^2}{2Var(\hat{x_2} - \hat{P}_2)}\right) \tag{49}$$

Proof of Corollary 1 Follows immediately from dividing equation (37) by equation (38).

It will be helpful to note the following lemma.

Lemma 4 $\Lambda(\lambda)$ *is an increasing function of* λ

Proof of Lemma 4 Note that $E(\hat{x_2} - \hat{P}_2) = 0$ because synthetic asset 2 is defined as the long short asset. Thus, its expected payoff and supply are both 0, making its expected return 0. So the only term in $\Lambda(\lambda)$ involving λ is

$$\sqrt{\frac{Var(\hat{x_2} - \hat{P}_2|\hat{s}, \hat{P})}{Var(\hat{x_2} - \hat{P}_2)}} = \frac{\lambda^2\,\tau_z\left(\alpha^2\,\tau_\epsilon + \tau_x\,\alpha^2 + \tau_z\,\lambda^2\,{\tau_\epsilon}^2\right)}{\alpha^4 + 2\,\alpha^2\,\lambda^2\,\tau_\epsilon\,\tau_z + \tau_x\,\alpha^2\,\lambda^2\,\tau_z + \lambda^4\,{\tau_\epsilon}^2\,{\tau_z}^2} \tag{50}$$

It is straightforward to confirm that the RHS of equation 54 is increasing in λ by taking the partial derivative with respect to λ.

That is, active traders become relatively worse off as the fraction of active traders increases. This result should be intuitive: relative prices become more efficient as the number of active traders increases, so there are less profits to be made from trading synthetic asset 2. Also, recall that utilities are negative in this model, so $\Lambda(\lambda) < 1 (> 1)$ means active traders are better (worse) off. Then, similar to Grossman and Stiglitz (1980), I can define different types of financial market equilibrium:

Theorem 1 *If* $\lambda \in (0, 1)$ *and* $\Lambda(\lambda) = 1$*, then we have the interior equilibrium described in Section 6. If* $\lambda = 1$ *and* $\Lambda(1) \leq 1$*, then we have the no-indexing equilibrium described in Section 5. There is no equilibrium where everyone is an index investor. All equilibrium are unique.*

The proof follows from Lemma 1 and the fact that higher levels of $\Lambda()$ indicate that active traders are worse off. In general, the final sentence of the previous theorem would instead be: if $\lambda = 0$ and $\Lambda(1) > 1$, then we have an equilibrium where everyone is an index investor. However, when $\lambda = 0$, it can be shown that $Var(\hat{x_2} - \hat{P}_2)^{-1} = 0$, so $\Lambda(0) = 0$ always. Correcting relative prices will become increasingly profitable as λ approaches zero, making it optimal for someone to always be active.

Thus, the type of equilibrium is completely determined by $\Lambda(1)$. If $\Lambda(1) \leq 1$, we have an equilibrium with only active traders. If $\Lambda(1) > 1$, then we have an interior equilibrium with a positive fraction of both active and passive investors. The equilibrium percentage of passive investors can then be manipulated by changing the cost of active investing c. Therefore, we can essentially talk about λ as if it is exogenous because changing the exogenous c amounts to changing the equilibrium λ that results.