Regulatory Model Secrecy and Bank Reporting Discretion

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Abstract

This paper studies how banking regulators should disclose the models they use to assess banks that have reporting discretion. In my setting, assessments depend on both economic conditions and the fundamental of banks' asset. The regulatory models provide signals about economic conditions, while banks report information about their asset fundamentals. On the one hand, disclosing the models helps banks understand how their assets perform in different economic environments. On the other hand, it induces banks with socially undesirable assets to manipulate reports and obtain favorable assessments. While the regulator can partially deter manipulation by designing the assessment rule optimally, the disclosure of regulatory models remains necessary. The optimal disclosure policy is to disclose the regulatory models when the assessment rule is more likely to induce manipulation and keep them secret otherwise. In this way, disclosure complements the assessment rule by reducing manipulation in cases that harm the regulator more. The analyses speak directly to the supervisory stress test and climate risk stress test.

Keywords: Stress test, Regulatory secrecy, Disclosure, Accounting discretion

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1 Introduction

Regulators assess banks on a regular basis to ensure the stability and sustainability of banking industry. In order to do this, regulators rely on models which capture various features of the economy and banks. These models are not always disclosed to banks, making the process of regulatory assessment opaque and its implications unclear. One important reason for not disclosing the models is to prevent banks from gaming the regulatory assessment (Flannery 2019; Clark and Li 2022). A common way for banks to game is to provide uninformative reports that do not represent the underlying risks (Huizinga and Laeven 2012; Bushman and C. D. Williams 2012; Bushman 2016). However, regulatory models contain valuable information which can help banks to understand how their assets perform under different economic conditions. By disclosing the models, regulators enable banks to make more informed decisions. In this paper, I study how regulators should disclose the models they use to assess banks, when banks have reporting discretion.

This study is especially relevant for supervisory stress test and climate risk stress test. The supervisory stress test employs a batch of regulatory models to evaluate the resilience of large banks to adverse macroeconomic shocks. The regulatory models translate the macroeconomic shocks into the risk parameters at banks' level and assign losses to particular positions. While some countries disclose the models transparently, some other countries choose not to disclose them. For example, in the past, the Federal Reserve only provided the broad framework and methodology used in the supervisory stress test. In recent years, it has moved towards more disclosure about the models, including key variables and certain equations. Whereas in Europe, comprehensive disclosure about the stress test methodologies becomes common practice.[1] While disclosing the models helps banks understand the impact of macroeconomic shocks on their business activities, it also facilitates banks to manipulate information, which then benefits banks at the expense of compromising the reliability of the stress test results. The severity of such manipulations, which is governed by banks' reporting discretion, crucially affects the tradeoff of disclosing the stress test models.

In the case of climate risk stress tests, the tradeoff of disclosing the regulatory models is even more pertinent. Several countries have conducted climate risk stress tests in recent years.[2]

[1] For example, the European Banking Authority (EBA) discloses the details of models used in stress test (see https://www.eba.europa.eu/eba-launches-2023-eu-wide-stress-test-0), the Federal Reserve instead discloses high-level information about the models underlying Dodd-Frank Act Stress Test (DFAST) and demonstrates how these models work on hypothetical loan portfolios (See https://www.federalreserve.gov/publications/files/2022-march-supervisory-stress-test-methodology.pdf).

[2] For example, the Bank of England describes the climate risk stress test scenarios in June 2021. See https://www.bankofengland.co.uk/climate-change. The ECB has conducted the climate stress test

To fully capture the climate risk factors, the time horizons for climate risk stress tests usually range between 30 and 50 years. Such long time horizons considerably increase the uncertainty about the implication of any business activities.[3] Nevertheless, the regulatory models used in the test remain confidential. Concurrently, there has been a growing focus on banks' reporting of climate related issues. Governments and market watchdogs have proposed several reporting rules for banks to follow.[4] Despite these developments, a standardized reporting framework has yet to be established, resulting in substantial reporting discretion for banks.

I develop a tractable model to study the optimal disclosure policy about the regulatory models. The model features one bank and one regulator. The bank has an existing asset whose payoff depends on the economic conditions and the asset's fundamental which can be either high or low. When the economic conditions improve and/or the fundamental value is high, the asset produces higher payoff. To evaluate the asset, the regulator conducts stress test which proceeds as follows. The regulator uses the regulatory models to obtain a signal about the economic conditions. The regulator then discloses the signal to the bank according to the disclosure choice (discussed later). In this paper, disclosing the regulatory models is equivalent to disclosing the signal generated by the models. Subsequently, the bank, which has reporting discretion, reports the fundamental of the asset. Based on the signal and the bank's report, the regulator makes a pass/fail decision. Passing the test allows the bank to retain the asset, while failing requires the bank to liquidate the asset.[5]

The bank has large private benefit when retaining the asset such that it prefers to hold the asset regardless of its payoff. But the regulator prefers to keep the asset only when the fundamental is high. This conflict of interests motivates the bank to engage in report manipulation. By manipulating the report, the bank aims to make the low fundamental asset and the high fundamental asset appear more similar, reducing the informativeness of the report. Manipulation is costly to the bank, and the cost is determined by the amount of reporting discretion that the bank has.

I first show that the regulator's pass/fail decision is characterized by a cutoff rule based on

since 2021 and published the results. See results for 2022 https://www.bankingsupervision.europa.eu/ecb/pub/pdf/ssm.climate_stress_test_report.20220708~2e3cc0999f.en.pdf.

[3]See https://www.bis.org/fsi/publ/insights34.htm

[4]For example, in Europe, the Corporate Sustainability Reporting Directive (CSRD) entered into force on 5 January 2023, which requires large companies and listed SMEs to disclose social and environmental related information. See https://finance.ec.europa.eu/capital-markets-union-and-financial-markets/company-reporting-and-auditing/company-reporting/corporate-sustainability-reporting_en. The U.S. Securities and Exchange Commission (SEC) also proposed rule changes which require registrants to include certain climate-related disclosures. See https://www.sec.gov/news/press-release/2022-46.

[5]I assume that liquidation is the only possible remedial action after failing the stress test. Further discussions on this point are in Section 2.

the bank's report. Specifically, a high report is indicative of a high fundamental asset, leading the regulator to pass the bank if the report exceeds a threshold, and to fail otherwise. While the bank's report is informative about the asset's fundamental, it may contain manipulation. Hence, when choosing the passing threshold, the regulator trades off the cost of passing a low fundamental asset (i.e., inefficient continuation) against the cost of failing a high fundamental asset (i.e., inefficient liquidation), determined by the payoffs of the low and high fundamental asset. As manipulation increases, it becomes more difficult for the regulator to distinguish between low and high fundamental asset. Consequently, the regulator is more likely to face both inefficient continuation and inefficient liquidation. Depending on the relative cost of the two inefficiencies, the regulator adjusts the passing threshold in order to address the one that is more costly.

However, even when the passing threshold is chosen optimally, I find that the regulator passes the low fundamental asset (inefficient continuation) too often. This occurs because the optimal passing threshold is an ex post response to the bank's manipulation choice. That is, when choosing the passing threshold, the regulator takes the bank's manipulation choice as given. As a result, from the ex ante perspective, the optimal passing threshold is too lenient, leading to excessive inefficient continuation. Therefore, the regulator requires additional tools to tackle this issue.

The disclosure of regulatory models complements the pass/fail decision as an ex ante approach to mitigate the issue of excessive inefficient continuation caused by the bank's manipulation. The bank's manipulation incentive is driven by the desire to have low fundamental assets pass the test. The level of manipulation is then determined by two factors: the extent to which manipulation increases the probability of passing the test for the low fundamental asset and the gain from passing the test with the low fundamental asset. The former is determined by the passing threshold, while the latter depends on the payoff of the low fundamental asset.

Disclosing the regulator's signal enables the bank to observe the economic conditions which determines both the passing threshold and the payoff of the low fundamental asset. The effect of this disclosure on the bank's manipulation is two-fold. First, it induces the bank's manipulation to be contingent on the economic conditions. I show that when the cost of inefficient continuation and that of inefficient liquidation are comparable, the bank's manipulation choice varies with the payoff of the low fundamental asset. In this case, the bank manipulates less when payoff is low. However, when the cost of one inefficiency outweighs the other, the bank's manipulation choice changes based on the extent to which manipulation can increase the probability of passing the test for the low fundamental asset. Specifically, the bank manipulates more when manipulation increases the passing probability by larger magnitude. The intuition of the bank's manipulation choice is as follows. When the costs of the two inefficiencies are comparable, the regulator's choice of the passing threshold is less responsive to the bank's manipulation. Because any adjustment in the threshold would affect both inefficient liquidation and inefficient continuation which are equally costly. As a result, the bank can manipulate to increase the probability of passing the test for the low fundamental asset without triggering the regulator to adjust the passing threshold. Given that manipulation is effective in increasing the passing probability for the low fundamental asset, the bank's manipulation choice is then driven by the gain after passing the test with the low fundamental asset, which is determined by the payoff of the low fundamental asset. However, when the cost of one inefficiency dominates, the regulator's choice of the passing threshold becomes sensitive to the bank's manipulation, restricting the extent to which manipulation can increase the passing probability for the low fundamental asset. In this case, the bank manipulates more when manipulation increases the passing probability by larger magnitude. Hence, the disclosure of the regulator's signal introduces variability into the bank's manipulation, which fluctuates in response to the economic conditions. Without the disclosure of the regulator's signal, the bank's manipulation remains constant.

The second effect of disclosing the regulator's signal on the bank's manipulation is that it affects the bank's expected amount of manipulation. When the bank's manipulation incentive is driven by the gain from passing the test with the low fundamental asset, i.e., the payoff of the low fundamental asset, disclosing the regulator's signal reduces the expected amount of manipulation. Conversely, if the bank manipulates to increase the probability of passing the test for the low fundamental asset, disclosing the regulator's signal increases the expected amount of manipulation. This additional effect arises from the interaction between the bank's manipulation and the regulator's choice of the passing threshold, amplifying the first effect of the disclosure of regulator's signal.

The optimal disclosure policy is to disclose the regulator's signal when the cost of inefficient continuation and that of inefficient liquidation are comparable, and keep the signal secret otherwise. When the costs of the two inefficiencies are comparable and the regulator's signal is disclosed, the bank manipulates less when the payoff of the low fundamental asset is low. This manipulation choice benefits the regulator as it reduces the bank's manipulation when passing the low fundamental asset results in larger losses. Hence, disclosing of the signal divers the bank's manipulation away from cases where the regulator suffers more losses from manipulation. Moreover, disclosing the signal in this case reduces the expected amount of manipulation. In contrast, when the cost of one inefficiency dominates and the regulator still discloses the signal, the bank manipulates more when manipulation results in larger increase in the passing probability for the low fundamental asset, exacerbating the regulator's expected losses from passing the low fundamental asset. In addition, disclosing the signal further increases the expected amount of manipulation. Hence, the regulator should not disclose the signal in this case.

The optimal disclosure policy crucially depends on the bank's private benefit when passing the test and its reporting discretion. If the regulator could benefit from increased disclosure but is concerned about potential manipulation, lower reporting discretion and lower private benefit for the bank lead to more disclosure from the regulator. However, if the regulator could benefit from decreased disclosure, lower reporting discretion and lower private benefit for the bank reduce disclosure from the regulator.

The remainder of the paper is organized as follows. The rest of the introduction discusses the relevant literature. Section 2 presents the model. Section 3 studies the optimal pass/fail decision and the bank's manipulation response. Section 4 analyzes the optimal disclosure policy about the regulatory models. Section 5 conducts comparative statics and demonstrates how the bank's reporting discretion affects the optimal disclosure policy. Section 6 discusses the model assumptions. Section 7 concludes. All proofs are included in Appendix A.

1.1 Related literature

The growing literature on stress test design has focused on disclosure about the results (Goldstein and Sapra 2013; Goldstein and Leitner 2018; Corona, Nan, and Zhang 2019; Quigley and Walther 2020) and scenario design (Parlatore and Philippon 2022). Instead, I focus on, before conducting the stress test, whether the regulator should communicate with the bank about the stress test models. Similar to my paper, Leitner and B. Williams (2023) also study the disclosure policy about the regulatory models. In their paper, revealing the regulatory models induces the bank to always invest in risky asset even when the value is low, but not revealing may lead to underinvestment. While their focus is on the riskiness of bank's investment, I examine the role of bank's information input in the stress test and study how reporting discretion affects the disclosure policy about regulatory models.

Several papers study the impact of stress test assessment on policy design (Agarwal and Goel 2020) and on the bank's opaqueness (Petrella and Resti 2013). In this paper, I show that the disclosure policy about the regulatory models affect the bank's reporting incentive which further influence the accuracy and reliability of stress test results.

The disclosure literature (e.g., Verrecchia (1983) and Dye (1985)) focuses on the disclosure of firms' (in my case, the bank's) information and its impact on the market's expectation. Instead, I focus on the disclosure of the regulator's private information, and I study how it affects the interactions between the regulator and the bank.

Regarding the bank, I study its reporting incentive when reporting discretion exists. The bank's reporting discretion determines how much information the regulator can communicate with the bank. The role of reporting discretion is also analyzed in Gao and Jiang (2018) in the context of bank run. In their paper, the reporting discretion reduces the panic-based runs, but it may also reduce the fundamental-based run. In general, this paper contributes to the literature on the determinants of reporting quality (e.g., Leuz, Nanda, and Wysocki 2003; Barth, Landsman, and Lang 2008; Holthausen 2009; Leuz and Wysocki 2016). The consensus is that the reporting quality depends on various factors. Among others, the regulatory environment and the development of capital market are crucial. This paper shows that the stress test design can affect the reporting quality of banks.

More broadly, this paper contributes to the discussion about the interplay between prudential and accounting regulation. Bertomeu, Mahieux, and Sapra (2020) show that accounting measurement complements capital requirements to affect the level and efficiency of banks' credit decisions. Corona, Nan, and Zhang (2015) examine the impact of accounting information quality on banks' risk-taking incentives, taking into account the interbank competition. This paper shows that the stress test design should be coherent with the prevailing accounting regulation to achieve informative assessment.

2 The model

Consider a risk-neutral economy with no discounting. There is one regulator and one bank. The regulator conducts stress test on the bank. I model the stress test as a four-period game.

At $t = 1$, the stressed scenarios are exogenously given and observed by everyone. The regulator uses regulatory models to predict the impact of the macroeconomic variables included in these stressed scenarios on the banking industry. The output of the regulatory models is summarized in a signal $s \in S = [\underline{s}, \bar{s}]$ with a cumulative distribution function F and density f. The density f has full support. The regulator privately observes s. Throughout the paper, I refer to the signal s as the economic condition. The signal s could represent the probability of a liquidity shock in the interbank market during a given macroeconomic stress, or the aggregate amount of deposit withdrawals from a specific industry due to supply chain disruptions.

The focus of this paper is to study the optimal disclosure policy about the signal s. At $t = 0$, the regulator commits to a disclosure policy before conducting the stress test. The disclosure policy is defined by the disclosure set $D \subseteq S$ and the no-disclosure sets $N_n \subseteq S$, where $n \in [1, +\infty)$ denotes the number of no-disclosure sets, and, for simplicity, the first no-disclosure set is denoted by $N \equiv N_1$. For any signal $s \in D$, the regulator communicates it truthfully to the bank. For any signal $s \in N_n$, the regulator informs the bank that the signal falls within N_n. I restrict the analyses of no-disclosure sets to monotone disclosure rule in which N_n pools signals from connected intervals.

The bank's asset has continuation value $X(s, \omega)$ and liquidation value $L(s, \omega)$, which depend on a state variable ω and the economic condition s. The variable ω represents the fundamental of the asset. It is either ω_h with probability q_h or ω_l with probability $q_l \equiv 1 - q_h$ and $\omega_h > \omega_l$. Let $x(s, \omega)$ denote the relative gains from continuing the asset. That is,

$$x(s, \omega) = X(s, \omega) - L(s, \omega).$$

For the remainder of the paper, the solutions are derived in terms of $x(s, \omega)$.

To define efficient liquidation and efficient continuation, I assume that $x(s, \omega_l) \leq 0 \leq x(s, \omega_h)$. This implies that the asset should only continue if its fundamental value is high. I assume that $x(s, \omega)$ is increasing and weakly concave in s. Moreover, for high fundamental asset, the relative gain from continuation increases weakly faster and is less concave in s. Formally, it is assumed that $x''(s, \omega_h) \geq x''(s, \omega_l)$ and $x'(s, \omega_h) \geq x'(s, \omega_l)$. I also assume that $x(s, \omega_l)$ is weakly log-concave in s to ensure that $x(s, \omega_l)$ is not too concave in s.[6] I make the following further assumption about the bank's asset.

Assumption 1. *The bank's asset is ex ante worth continuing:* $\mathbb{E}_\omega \left[x(s, \omega)\right] \in [0, q_h x(\bar{s}, \omega_h)]$ *for* $s \in [\underline{s}, \bar{s}]$.

This assumption also suggests that the expected continuation value of the bank's asset exceeds its liquidation value for any signal s.[7] Consequently, failing high fundamental asset (inefficient liquidation) is more costly than passing low fundamental asset (inefficient continuation). This assumption helps to characterize the optimal disclosure policy, but my main results can be extended to cases where this condition is violated. I discuss this assumption in Section 6.1.

The fundamental of the bank's asset ω is not observable to anyone but it is measured by the report. More specifically, the fundamental of the bank's asset determines the report distribution. If the fundamental is ω_i, then the report t follows a distribution with density $g^i(t)$ over $t \in [\underline{t}, \bar{t}]$,

[6]Notice that $x(s, \omega_h)$ is assumed to be non-negative and concave in s, which imply that $x(s, \omega_h)$ is also weakly log-concave in s. However, given that $x(s, \omega_l)$ is assumed to be non-positive, such implication does not hold for $x(s, \omega_l)$. Hence, I impose the weakly log-concavity assumption on $x(s, \omega_l)$ only.

[7]Notice that $x(s, \omega_l)$ is assumed to be non-positive, hence, the maximum value of $\mathbb{E}_\omega \left[x(s, \omega)\right]$ is $q_h x(\bar{s}, \omega_h)$.

where $i = \{h, l\}$. The density functions have full support and satisfy the monotone likelihood ratio property (MLRP), i.e., $\frac{g^l(t)}{g^h(t)}$ is decreasing in t. This assumption implies that the report t is informative about the asset fundamental. Moreover, I assume that the ratio $\frac{g^l(t)}{g^h(t)}$ is concave in t. I impose the regularity condition that the hazard rate $\frac{g^h(t)}{1-G^h(t)}$ and $\frac{g^l(t)}{1-G^l(t)}$ are decreasing on the support of t. This assumption means that the probability that the reported value will be below t conditional on the reported value is already t is decreasing in t. In other words, when the bank receives a high-value report, it becomes more probable to receive a higher report.

At $t = 2$, the bank may engage in costly manipulation to affect the report generating process. I follow Gao and Jiang (2020) to model bank's manipulation as ex ante manipulation. That is, the bank chooses the manipulation level before observing the fundamental of the asset. Specifically, the bank chooses manipulation $m \in [0, 1]$ to change the report distribution from $g^i(t)$ to

$$g^i_m(t) = g^i(t) + m\big(g^h(t) - g^i(t)\big). \tag{1}$$

If $m = 0$, the report generating process is not affected by manipulation. If $m = 1$, then the report is always generated from the distribution of high fundamental asset $g^h(t)$. If $m \in (0, 1)$, then manipulation improves the distribution in the sense of first-order stochastic dominance. The cost of manipulation m is $kc(m)$ for the bank, where $k \in (0, +\infty)$ measures the degree of reporting discretion determined by rules and regulations and the cost function $c(m)$ is increasing and convex with $c(0) = c'(0) = 0$. I also assume that $\frac{c'(m)}{c''(m)}$ is weakly increasing in m, or, equivalently, that $c'(m)$ is weakly log-concave. The conditions are often used in the literature (see Gao and Jiang (2020)) and are satisfied for common convex functions, e.g. $c(m) = m^q$ for $q \geq 2$.

After observing the economic condition s and receiving the report t from the bank, the regulator makes a pass/fail decision a at $t = 3$. In particular, the regulator passes ($a = 1$) or fails ($a = 0$) the bank to maximize the payoff u given by

$$u \equiv ax(s, \omega). \tag{2}$$

The bank's payoff v is

$$v \equiv a\big(x(s, \omega) + B\big) - kc(m). \tag{3}$$

Where B is the bank's private benefit from continuing the asset. I assume $x(s, \omega) + B > 0$ for all s and ω, meaning that the private benefit B is sufficiently large for the bank to prefer continuation regardless of the value of x. The private benefit introduces a conflict of interest between the regulator and the bank.

The timeline of the model is as follows,

At $t = 0$, the regulator commits to a disclosure policy about the signal s.

At $t = 1$, the regulatory models generate a signal s. The regulator privately observes s and discloses it to the bank according to the disclosure policy.

At $t = 2$, the bank chooses the level of manipulation m to affect the report generating process.

At $t = 3$, the state ω is realized, and the bank's report t is generated. Based on the signal s and report t, the regulator passes or fails the bank. And payoffs are realized.

The equilibrium is characterized by the regulator's disclosure policy about s, the pass/fail decision a, and the bank's manipulation m. I solve the model by backward induction. I first solve for the regulator's pass/fail decision a for given manipulation level m and disclosure policy about s. Anticipating the pass/fail decision rule, the bank then chooses the manipulation m for given disclosure policy about s. Lastly, the regulator chooses the disclosure policy about s, taking into account its impact on the bank's manipulation choice and, consequently, the pass/fail decision.

3 Manipulation and pass/fail decision

In this section, I discuss the bank's manipulation choice and the regulator's pass/fail decision, taking the disclosure policy as given.

At $t = 3$, the regulator forms expectation of the relative continuation value x based on the signal s and the bank's report t. The regulator passes $(a = 1)$ the bank if and only if

$$\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}] \geq 0. \tag{4}$$

Where $\hat{m}$ is the regulator's conjecture about the bank's manipulation.[8] Since $g^h(t)$ is a monotone likelihood ratio improvement of $g^l(t)$, the expected relative continuation value $\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}]$ is increasing in the report t. As a result, the pass/fail decision follows a cutoff rule

Lemma 1. *For a given signal s, a bank's report t, and a conjecture about the bank's manipulation $\hat{m}$, the regulator passes the bank if and only if $t \geq t_p(s,\hat{m})$, where the passing threshold $t_p(s,\hat{m})$*

[8]The conditional expectation is

$$\begin{aligned}\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}] &= x(s,\omega_h)\Pr(\omega=\omega_h|t,\hat{m}) + x(s,\omega_l)\Pr(\omega=\omega_l|t,\hat{m})\\ &= x(s,\omega_h)\frac{q_h g^h(t)}{q_h g^h(t)+q_l g^l_{\hat{m}}(t)} + x(s,\omega_l)\frac{q_l g^l_{\hat{m}}(t)}{q_h g^h(t)+q_l g^l_{\hat{m}}(t)}.\end{aligned}$$

solves

$$\mathbb{E}_\omega[x(s,\omega)|t_p,\hat{m}] = 0.$$

All proofs are included in Appendix A. The passing threshold $t_p(s,\hat{m})$ is defined by the regulator's indifferent condition. That is, the regulator is indifferent between passing and failing the bank when the report is $t_p(s,\hat{m})$. This passing threshold is chosen to equalize the expected cost of failing high fundamental asset (inefficient liquidation) and the expected cost of passing low fundamental asset (inefficient continuation) *for a given signal s and a given conjecture about manipulation* $\hat{m}$. The following lemma characterizes the passing threshold $t_p(s,\hat{m})$.

Lemma 2. *For a given level of manipulation m, the passing threshold $t_p(s,m)$ is decreasing in s. For a given signal s, the passing threshold $t_p(s,m)$ is decreasing in m.*

The intuition of this lemma follows from how the cost of failing high fundamental asset and that of passing low fundamental asset change with the signal s and the manipulation m. For a given manipulation level m, the relative gain from continuing the asset $x(s,\omega)$ is increasing in s, implying that failing high fundamental asset becomes more costly relative to passing low fundamental asset. In response to the rising cost of inefficient liquidation, the regulator is willing to lower the passing threshold and pass the bank more often. The second result captures how manipulation affects the relative cost of failing high fundamental asset and passing low fundamental asset. Assumption 1 assumes that in absence of the report, the regulator's expectation of the relative gain from continuing the asset is non-negative. This implies that inefficient liquidation (failing high fundamental asset) is more costly than inefficient continuation (passing low fundamental asset) in expectation for all signal s. Manipulation makes the report distribution of low and high fundamental asset more similar, making it more difficult for the regulator to differentiate between the two types of assets. In order to preserve the high fundamental asset, the regulator needs to decrease the passing threshold.

At $t = 2$, the bank anticipates the passing threshold $t_p(s,\hat{m})$ and chooses the manipulation m to maximize the expected payoff. The bank's expected payoff depends on the disclosure of s. If the bank does not observe the regulator's signal s, the expected payoff is

$$V(\hat{m},m) = \mathbb{E}_s\left[q_h\big(x(s,\omega_h)+B\big)\int_{t\geq t_p(s,\hat{m})} g^h(t)dt + q_l\big(x(s,\omega_l)+B\big)\int_{t\geq t_p(s,\hat{m})} g^l_m(t)dt \,\middle|\, s\in N_n\right]$$
$$- kc(m).$$

Where N_n is the no disclosure set containing signals s that are not disclosed to the bank. For ease of exposition, I introduce the following definition.

$$\Delta\big(t_p(s,\hat{m})\big) \equiv \int_{t\geq t_p(s,\hat{m})} \big(g^h(t) - g^l(t)\big)dt. \tag{5}$$

This term is the difference in passing probability between high and low fundamental asset. It also measures the increases in passing probability for low fundamental asset if the bank manipulates the report distribution. Taking derivative of $V(\hat{m},m)$ with respect to m, I obtain the following first-order condition of the bank's manipulation m,

$$\mathbb{E}_s\left[q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,\hat{m})\big)\,\big|\, s\in N_n\right] - kc'(m) = 0.$$

In equilibrium, the regulator's conjecture about the manipulation $\hat{m}$ is consistent with the bank's choice. The equilibrium manipulation m_{N_n} solves

$$\mathbb{E}_s\left[q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m_{N_n})\big)\,\big|\, s\in N_n\right] - kc'(m_{N_n}) = 0. \tag{6}$$

This condition suggests that no disclosure of s forces the bank's manipulation m_{N_n} to be constant over the regulator's signal s.

If the bank observes the regulator's signal s, the expected payoff is

$$V(s,\hat{m},m) = q_h\big(x(s,\omega_h)+B\big)\int_{t\geq t_p(s,\hat{m})} g^h(t)dt + q_l\big(x(s,\omega_l)+B\big)\int_{t\geq t_p(s,\hat{m})} g^l_m(t)dt - kc(m).$$

The first-order condition of the bank's manipulation response m is as follows,

$$q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,\hat{m})\big) - kc'(m) = 0.$$

Similar to no disclosure case, the regulator's conjecture about the manipulation is consistent with the bank's choice in equilibrium. The equilibrium manipulation $m_D(s)$ is determined by

$$q_l\big(x(s,\omega_l)+B\big)\Delta\Big(t_p\big(s,m_D(s)\big)\Big) - kc'\big(m_D(s)\big) = 0. \tag{7}$$

I make the following notation for ease of exposition

$$MB_b\big(s,t_p(s,m)\big) \equiv q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big). \tag{8}$$

Where "MB" stands for "marginal benefit" and "b" represents "bank". $MB_b\big(s,t_p(s,m)\big)$ is the bank's marginal benefit of manipulation for given regulator's signal s and manipulation level

m. It consists of two components. The first component is the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. Given that the relative gain from continuing the asset $x(s,\omega_l)$ is increasing in the signal s, the expected gain after passing the test with manipulation is increasing in s. All else equal, the bank manipulates more when the signal s is high.

The second component $\Delta\big(t_p(s,m)\big)$ represents the increases in the passing probability if the bank changes the report distribution from $g^l(t)$ to $g^h(t)$. This term crucially depends on the passing threshold $t_p(s,m)$. Lemma 2 shows that the passing threshold $t_p(s,m)$ is decreasing in s, since the relative cost of failing the high fundamental asset is rising. As the passing threshold decreases, the test becomes more lenient in the sense that low fundamental asset is more likely to pass the test without manipulation. In other words, the difference in the passing probability between $g^h(t)$ and $g^l(t)$ shrinks. The following lemma summarizes the impact of the signal s on the difference in passing probability $\Delta\big(t_p(s,m)\big)$.

Lemma 3. *For given manipulation level m, $\Delta\big(t_p(s,m)\big)$ is decreasing in s.*

This lemma implies that manipulation is less effective in increasing the passing probability as s increases. Therefore, the bank manipulates less when the signal s is high. When evaluating the bank's manipulation incentive MB_b, the differences in passing probability $\Delta\big(t_p(s,m)\big)$ acts as a counterforce to the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. The magnitude of the two forces then determines how the manipulation $m_D(s)$ responds to the signal s.

Proposition 1. *When s is disclosed, the level of manipulation $m_D(s)$ is unique and it is increasing in s for $s < s_D$ and it is decreasing in s for $s > s_D$, where $s_D \in (\underline{s}, \bar{s})$ is the unique solution for $\frac{\partial MB_b\left(s, t_p(s, m_D)\right)}{\partial s} = 0$.*

This result identifies the forces that determines the bank's manipulation $m_D(s)$ when s is disclosed, and it highlights the effect of passing threshold $t_p(s,m)$ on the bank's manipulation $m_D(s)$. When the signal is relatively low, i.e., $s < s_D$, the cost of inefficient liquidation compared to that of inefficient continuation is moderate. Hence, the passing threshold is set at a medium level to prevent both types of error, and it is insensitive to the bank's manipulation. This choice of passing threshold leads to a substantial difference in the passing probability between high and low fundamental asset, and it also implies that manipulation is very effective in increasing the likelihood of passing the test. Given the high effectiveness of manipulation in increasing the likelihood of passing the test, the bank focuses on the expected gain after manipulation. As a result, the bank's manipulation $m_D(s)$ follows the changes in the expected gain after passing the test with manipulation, and it is increasing in s. When $s > s_D$, the inefficient liquidation becomes more costly than inefficient continuation. Hence, the passing threshold is set relatively low to prevent inefficient liquidation. Consequently, the low fundamental asset is more likely to pass the test even without manipulation. In this case, the bank manipulates more when manipulation still has incremental effect on increasing the passing probability. Hence, the bank's manipulation $m_D(s)$ changes with the difference in passing probability between high and low fundamental asset, and it is increasing in s.

Disclosure of s affects how manipulation changes with s. When s is not disclosed, the bank's manipulation m_{N_n} is constant over the signal s. When s is disclosed, the bank's manipulation incentive changes with both the expected gain after passing the test with manipulation $q_l(x(s,\omega_l)+B)$ and the increases in passing probability of low fundamental asset after manipulation $\Delta\Big(t_p\big(s,m_D(s)\big)\Big)$. Consequently, the manipulation $m_D(s)$ varies with s and such variation further affects the expected level of manipulation. The following proposition compares the expected level of manipulation when s is disclosed with the one when s is not disclosed.

Proposition 2. *$\mathbb{E}_s\big[m_D(s)|s\in N\big]\leq m_N$ if $N\subseteq[\underline{s},s_D]$ and $\mathbb{E}_s\big[m_D(s)|s\in N\big]\geq m_N$ if $N\subseteq[s_D,\bar{s}]$.*

This result shows the additional effect of disclosing s. When $s\leq s_D$, the bank's manipulation $m_D(s)$ is driven by the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$ and it is increasing in the signal s. In response, the regulator decreases the passing threshold $t_p\big(s,m_D(s)\big)$, which makes the bank more likely to pass the test regardless of the fundamental value. Such endogenous response of the regulator's pass/fail decision then decreases the magnitude of passing probability that can be increased by manipulation, leaving manipulation less useful and decreases the bank's manipulation incentive. Such endogenous response is absent if s is not disclosed. Hence, the expected level of manipulation is less if s is disclosed. However, when $s>s_D$, the bank manipulates to increase the passing probability and the manipulation level $m_D(s)$ is decreasing in s. In response, the regulator increases the passing threshold $t_p\big(s,m_D(s)\big)$ to make the test more difficult. Such endogenous response of the regulator's pass/fail decision then widens the difference of passing probability between low and high fundamental asset. More importantly, such response makes the manipulation useful in increasing the passing probability for low fundamental asset, amplifying the bank's manipulation incentive. Hence, the expected level of manipulation when s is disclosed is larger compared to the case when s is not disclosed.

4 Disclosure

In this section, I discuss the optimal disclosure policy about the regulator's signal s, taking into account the bank's manipulation response and its impact on the regulator's pass/fail decision. I show that disclosure and passing threshold are complementary tools for the regulator to minimize the adverse consequence of the bank's manipulation.

For given signal s, the regulator's expected payoff at $t = 1$ is obtained by integrating all reports value that are higher than the passing threshold $t_p(s, m^*)$,

$$\begin{aligned} u(s, m^*) &= \int_{t \geq t_p(s,m^*)} \mathbb{E}_\omega[x(s,\omega)|t, m^*] g_{m^*}(t) dt \\ &= \int_{t \geq t_p(s,m^*)} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m^*}(t)\big) dt. \end{aligned} \tag{9}$$

Where $m^* = \{m_D(s), m_{N_n}\}$ is the equilibrium manipulation choice of the bank and $g_{m^*}(t)$ is the unconditional distribution of report t when the manipulation is m^*. That is,

$$g_{m^*}(t) = q_h g^h(t) + q_l g^l_{m^*}(t).$$

At $t = 0$, the regulator chooses disclosure policy D and N_n to maximize the ex ante payoff

$$\begin{aligned} U =& \int_{s \in D} u\big(s, m_D(s)\big) dF(s) + \sum_n \left(\int_{s \in N_n} u(s, m_{N_n}) dF(s) \right) \\ =& \int_{s \in D} \left(\int_{t \geq t_p\left(s, m_D(s)\right)} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m_D(s)}(t)\big) dt \right) dF(s) \\ &+ \sum_n \left(\int_{s \in N_n} \left(\int_{t \geq t_p(s, m_{N_n})} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m_{N_n}}(t)\big) dt \right) dF(s) \right). \end{aligned} \tag{10}$$

As briefly discussed in Lemma 2, manipulation increases the similarity between the report of low fundamental asset and that of the high fundamental asset, making it more likely that the regulator fails the high fundamental asset (inefficient liquidation) and passes the low fundamental asset (inefficient continuation). The regulator is able to use the pass/fail decision to control this adverse consequence of manipulation, but only partially. Because when choosing the passing threshold $t_p(s, m)$, the regulator trades off the cost of inefficient liquidation against the cost of inefficient continuation *for given level of manipulation* m. However, since the bank prefers to continue the asset regardless of its payoff, the bank's manipulation increases the likelihood of inefficient continuation ex ante. Such inefficient continuation cannot be prevented by using the optimal pass/fail rule. I first define the additional losses caused by manipulation for given signal

s. Taking derivative of $u(s, m)$ in (9) with respect to m, I obtain[9]

$$ML_r\big(s, t_p(s, m)\big) \equiv q_l x(s, \omega_l)\Delta\big(t_p(s, m)\big). \tag{11}$$

Where "ML" stands for "marginal loss" and "r" represents "regulator". Given that the asset should be liquidated when fundamental is low, i.e., $x(s, \omega_l) \leq 0$ for all s, this term is non-positive. It captures the regulator's marginal losses from continuing the low fundamental asset due to manipulation.

The additional losses caused by manipulation $ML_r\big(s, t_p(s, m)\big)$ consists of two components. The first component is the expected loss of passing the low fundamental asset $q_l x(s, \omega_l)$. The second component $\Delta\big(t_p(s, m)\big)$ is the increases in passing probability after the bank changes the report distribution from $g^l(t)$ from $g^h(t)$. This component captures the regulator's inability to distinguish the low fundamental asset and the high fundamental asset due to the bank's manipulation.

Lemma 4. *For any disclosure set D or no-disclosure set N_n, $ML_r\big(s, t_p(s, m^*)\big)$ is increasing in s for $m^* = \{m_D(s), m_{N_n}\}$.*

This lemma suggests that, regardless of the disclosure of s, the regulator bears less additional losses from manipulation as the signal s increases. The intuition is as follows. Since the relative gain from continuing the asset $x(s, \omega_l)$ is increasing in s, the regulator's loss from passing the low fundamental asset is ameliorated. In addition, the passing threshold $t_p(s, m^*)$ is decreasing in the signal s, shrinking the difference in passing probability between the low fundamental asset and high fundamental $\Delta\big(t_p(s, m^*)\big)$. This means that as the signal s increases, the increase in passing probability decreases even if the bank shifts the report distribution from $g^l(t)$ to $g^h(t)$. Consequently, the regulator is less likely to pass low fundamental asset, reducing the additional losses caused by manipulation.[10]

The regulator needs additional tool to control $ML\big(s, t_p(s, m)\big)$. Lemma 4 shows that the regulator's additional loss caused by manipulation $ML_r\big(s, t_p(s, m)\big)$ is increasing in the signal s. To minimize the regulator's expected loss from manipulation, the regulator should distribute

[9]Notice that the passing threshold $t_p(s, m)$ is chosen optimally for given signal s and manipulation m, hence, the derivative with respect to $t_p(s, m)$ is zero and does not appear in ML_r, i.e., $\frac{\partial u(s,m)}{\partial t_p(s,m)} \frac{\partial t_p(s,m)}{\partial m} = 0$.

[10]Notice that when the passing threshold is very low, the regulator is more likely to pass the low fundamental asset. However, this continuation is already considered when the regulator chooses the optimal passing threshold $t_p(s, m^*)$, which balances the tradeoff between inefficient liquidation and inefficient continuation. The term $ML_r\big(s, t_p(s, m^*)\big)$ does not capture such continuation and it only reflects the *additional* inefficient continuation caused by the bank's manipulation. And such additional inefficient continuation is reduced as the signal s increases.

more manipulation to cases where the marginal loss $ML_r\big(s, t_p(s,m)\big)$ is small and reduce the overall level of manipulation. Recall that Proposition 1 and Proposition 2 state that disclosure not only affect how manipulation distributes across the signal s but also affect the expected amount of manipulation across all signal s. Hence, the regulator can leverage the disclosure of the regulatory signal s to minimize the expected loss from manipulation.

To pin down the optimal disclosure policy about the regulatory signal s, I first discuss the cost and benefit of disclosure for the regulator. For given expected amount of manipulation, the disclosure of the regulatory signal s affects how manipulation distributes across the regulator's marginal loss ML_r. Disclosing s reveals $\Delta\big(t_p(s,m)\big)$ which is the increases in the passing probability after changing the report distribution from $g^l(t)$ to $g^h(t)$. As captured by $MB_b(s, t_p(s,m))$, all else equal, the bank's gain from manipulation is higher when $\Delta\big(t_p(s,m)\big)$ is large. A large $\Delta\big(t_p(s,m)\big)$ also means that the regulator is more likely to be misled by manipulation and make wrong passing decisions, which in turn increases the regulator's expected loss from bank's manipulation $ML(s, t_p(s,m))$. Hence, disclosure of s incurs cost for the regulator because it facilitates the bank to manipulate more when the regulator is more susceptible to manipulation. Disclosing s also gives benefit to the regulator. Because the payoff of the asset $x(s,\omega)$ depends both on the regulator's information s and on the fundamental ω, disclosing s reduces the bank's uncertainty about asset payoff. All else equal, the bank manipulates less when the expected gain after passing the test with manipulation is low, i.e., when $q_l(x(s,\omega_l)+B)$ is low. This manipulation choice is beneficial to the regulator. Because when $x(s,\omega_l)$ is low, passing the bank incurs large loss for the regulator. In other words, the regulator demands more informative report when $x(s,\omega_l)$ is low. Disclosing s then makes the regulator's pass/fail decision more accurate. Given the result in Proposition 1, the benefit dominates the cost of disclosing the regulatory signal s when the signal s is small.

In addition, Proposition 2 shows that disclosure of the signal s also changes the expected manipulation level. This additional layer strengthens the existing tradeoff of disclosure. As a result, the optimal disclosure policy follows a simple cutoff rule.

Proposition 3. *The optimal disclosure policy follows a cutoff rule where $D = [\underline{s}, s^*)$ and $N = [s^*, \bar{s}]$. That is, the regulator discloses the signal s when $s < s^*$ and does not disclose the signal s when $s > s^*$, where $s^* \in [\underline{s}, s_D]$ solves*

$$\Big(u(s^*, m_N) - u\big(s^*, m_D(s^*)\big)\Big) f(s^*) = \frac{\partial m_N}{\partial s^*} \int_{s^*}^{\bar{s}} ML_r\big(s, t_p(s, m_N)\big) dF(s). \tag{12}$$

The intuition for a cutoff rule is embedded in the tradeoff of disclosure. It is beneficial for the regulator to disclose the signal s when the manipulation is driven by the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. In this case, the bank's manipulation is increasing in s which implies that the bank's manipulation is less (more) when it causes more (less) losses to the regulator as measured by $ML(s,t_p(s,m))$. In addition, the bank manipulates less in expectation when observing the regulator's signal s. Hence, disclosure improves the regulator's ex ante payoff. However, as the signal s increases, the bank's manipulation is driven by the increases in passing probability after manipulation $\Delta\big(t_p(s,m)\big)$. If the signal is disclosed to the bank, then the bank would manipulate more when the regulator is more susceptible to manipulation. Hence, no disclosure complements the passing threshold to deter the bank's manipulation. The disclosure cutoff point s^* is characterized by equation (12). This equation captures the regulator's tradeoff between the utility gain (loss) from disclosing more information and the loss (gain) from increased (decreased) manipulation in no-disclosure region. No disclosure at all can be optimal if it sufficiently reduces the expected level of manipulation. In sum, the disclosure of s complements the pass/fail decision to minimize the adverse consequence of the bank's manipulation for the regulator.

5 Comparative statics

In this section, I analyze how the optimal disclosure policy changes with the bank's private benefit B when passing the test and the cost of manipulation k.

All else equal, increasing the private benefit B or decreasing the manipulation cost k incentivizes the bank to manipulate more for any given regulatory signal s. Such increase in manipulation occurs no matter the signal s is disclosed or not disclosed to the bank. As a result, the implications on the disclosure policy is unclear. The following lemma shows the effect of manipulation cost k and the bank's private benefit B on disclosure.

Proposition 4. *The optimal disclosure policy is characterized in Proposition 3. If $m_D(s^*) \leq m_N$, then the disclosure cutoff point s^* is increasing in k and decreasing in B. Otherwise, the disclosure cutoff point s^* is decreasing in k and increasing in B.*

The intuition of this result follows the tradeoff underpinning the disclosure cutoff point s^*. Recall equation (12), the regulator determines the disclosure cutoff point by weighing the utility gain from disclosure against the loss resulting from manipulation in the no-disclosure region. When the optimal disclosure policy satisfies $m_D(s^*) \leq m_N$, it implies that the regulator can attain a non-negative utility through disclosure. However, increasing disclosure also increases manipulation in the no-disclosure region, exacerbating the regulator's loss from manipulation.

Consequently, concerns regarding manipulation in the no-disclosure region discourage the regulator from realizing the benefits of disclosure. The cost of manipulation k plays a role in addressing the regulator's concerns about manipulation in the no-disclosure region. As k rises, the regulator can disclose more information to recapture previously forgone gains. Therefore, an increase in the cost of manipulation incentivizes a higher level of disclosure of the regulatory signal. Conversely, if the bank's gain from manipulation B increases, the opposite effect on disclosure would occur.

In the case where $m_D(s^*) > m_N$, the optimal disclosure policy exhibits excessive disclosure. This excessive disclosure is utilized to mitigate the manipulation m_N and the consequent losses arising from it in the no-disclosure region. In other words, the regulator leverages disclosure as a mechanism to reduce manipulation. When the cost of manipulation k rises, the regulator can rely less on disclosure as a means of mitigating manipulation, thereby incurring less losses associated with disclosure. Conversely, as the bank's private benefit B increases, the regulator is forced to use a higher degree of disclosure to counteract the heightened incentive for manipulation by the bank. In sum, the disclosure of the regulatory signal s acts as both a substitute for the cost of manipulation k and a counterforce for the bank's private benefit B in curbing manipulation in no-disclosure region.

6 Discussions

6.1 Cost of inefficient liquidation and inefficient continuation

Assumption 1 assumes that the bank's asset is worth continuing ex ante. This assumption affects how the regulator chooses passing threshold $t_p(s, m)$ in response to the bank's manipulation m (Lemma 2) and how the bank's manipulation changes with the regulatory signal s when s is disclosed (Lemma 3 and Proposition 1). Nevertheless, the main insight for the disclosure of the regulator's signal s does not depend on this assumption. In Appendix B, I derive the results formally.

The main finding of the paper is that the regulator's ex post pass/fail decision alone is insufficient to fully mitigate the adverse consequence of the bank's manipulation. Hence, disclosure of the regulatory signal s is useful. When the signal s is disclosed, the bank's manipulation is determined by two factors: the expected gain after passing the test with manipulation $q_l\big(x(s, \omega_l) + B\big)$ and the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. And the regulator's losses from manipulation $ML_r(s, t_p(s, m))$ depends on the expected losses of inefficient continuation $q_l x(s, \omega_l)$ and the increases in passing probability after manipulation

$\Delta\big(t_p(s,m)\big)$. Disclosure is always beneficial to the regulator when both the bank's manipulation and the regulator's loss from manipulation are driven by the changes in the relative gain from continuing the asset $x(s,\omega_l)$. Conversely, no disclosure is preferred when the changes in manipulation is driven by the increases in passing probability after manipulation $\Delta\big(t_p(s,m)\big)$. The former force is more likely to dominate when the tradeoff of the cost of inefficient liquidation and the cost of inefficient continuation is moderate. Because in such case, the regulator's choice of passing threshold leads to large increases in passing probability if the bank manipulates, i.e., $\Delta\big(t_p(s,m)\big)$ is large. This then incentivizes the bank to care about the gain after passing the test when choosing the manipulation. Hence, the bank's manipulation is more likely to be driven by the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. As discussed above, such manipulation choice benefits the regulator. In any case, the disclosure of s still complements the pass/fail decision.

6.2 No commitment to disclosure policy

Suppose that the regulator cannot commit to any disclosure policy about the signal s. Instead, the regulator decides to disclose or not to disclose the signal s after observing the realization of it. In the following, I show that the only equilibrium in this case is full disclosure.

The intuition is as follows. Consider the no-disclosure set $N=[s_1,s_2]$ with $s_1<s_2$. Denote the bank's manipulation response as m_N. Upon observing the signal s, the regulator would disclose s for which the bank's manipulation is $m_D(s)$ with $m_D(s)<m_N$. This implies that the no-disclosure set must consist of signals s such that $m_N\le m_D(s)$, implying that $\mathbb{E}\left[MB(s,t_p(s,m_N))|s\in[s_1,s_2]\right]\le MB(s,t_p(s,m_D(s))$ for $s\in[s_1,s_2]$. Since $MB(s,t_p(s,m))$ is a continuous function of s, the regulator must be indifferent between disclosing and not disclosing the signals at the boundary of no-disclosure set, i.e., $m_N=m_D(s_1)=m_D(s_2)$. Hence, the following condition must hold

$$\mathbb{E}_s\left[MB(s,t_p(s,m_N)|s\in[s_1,s_2]\right]=MB(s_1,t_p(s_1,m_N))=MB(s_2,t_p(s_2,m_N)).$$

However, given that $MB(s,t_p(s,m))$ is first increasing and then decreasing in s for any given manipulation m, this condition cannot hold if $s_1<s_2$. Hence, $s_1=s_2$ and full disclosure is the equilibrium.

6.3 Real activity

In the baseline model, the bank engages in costly manipulation to affect the report generating process. The manipulation improves the bank's report in the sense of first-order stochastic dominance, but it does not affect the bank's asset payoff. Hence, the disclosure of the regulator's private information only has informational consequence on the bank. It informs the bank about the gain from manipulation and the probability of obtaining the gain.

In Appendix C, I extend the analysis to consider a case where the disclosure of the regulator's private information not only affects the bank's reporting choice but also affects the bank's investment decision. More specifically, the bank exerts costly effort to improve the asset's fundamental and such effort manifests itself in the report. This effort still increases the similarity between the report of low and high fundamental asset, but this similarity arises from actual improvement in the asset's fundamental. As a result, the disclosure of the regulator's private information affects the bank's real activity, i.e., effort choice.

7 Conclusion

This paper presents a tractable model to analyze the optimal disclosure policy about the regulatory assessment models in the presence of the bank's manipulation concern. Disclosing the regulatory models helps the bank understand how its asset performs under different economic environment, which deters the bank's manipulation incentive. However, disclosing the model also makes it easier for the bank to game the assessment. The main message of the paper is that the disclosure policy about regulatory models complements the assessment rule. Additionally, the paper highlights that the accounting regulation and bank internal governance complements the design and improves the effectiveness of regulatory assessment. The implications of this paper extend to regulatory practices such as supervisory stress test and climate risk stress test. By understanding the interactions between performing stress tests and reporting incentives of banks, regulators can improve the design of stress tests and enhance the effectiveness of regulatory assessments.

A Proofs

For ease of exposition, I define the following ratio

$$r(t) \equiv \frac{g^l(t)}{g^h(t)}. \tag{13}$$

Due to the assumption that the density function of report t satisfies MLRP, the ratio $r(t)$ is decreasing in t.

Proof. Lemma 1

All the necessary steps for the cutoff rule are explained in the text. □

Proof. Lemma 2

The regulator chooses the passing threshold based on the signal s and the conjecture about the bank's manipulation $\hat{m}$. I drop the $\hat{}$ for simplicity.

The passing threshold is determined by

$$\mathbb{E}_\omega[x(s,\omega)|t_p,m] = 0.$$

This condition is equivalent to

$$x(s,\omega_h)\frac{q_h g^h(t_p)}{q_h g^h(t_p) + q_l g^l_m(t_p)} + x(s,\omega_l)\frac{q_l g^l_m(t_p)}{q_h g^h(t_p) + q_l g^l_m(t_p)} = 0,$$

Since the density function $g^l(t)$ and $g^h(t)$ have full support, the condition reduces to

$$x(s,\omega_h)q_h g^h(t_p) + x(s,\omega_l)q_l g^l_m(t_p) = 0.$$

This is equivalent to

$$x(s,\omega_h)q_h + x(s,\omega_l)q_l - x(s,\omega_l)q_l(1-m)\big(1-r(t_p)\big) = 0. \tag{14}$$

Apply implicit function theorem, I derive the following two partial derivatives.

$$\frac{\partial t_p}{\partial s} = -\frac{q_h\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)}{(1-m)q_l\big(x(s,\omega_l)\big)^2 r'(t_p)}. \tag{15}$$

Given that the relative gain from continuing the asset $x(s,\omega_l)$ and $x(s,\omega_h)$ are increasing in s and the ratio $r(t)$ is decreasing in t, this derivative is negative.

And the following is the partial derivative of t_p with respect to m,

$$\frac{\partial t_p}{\partial m} = -\frac{q_h x(s, \omega_h) + q_l x(s, \omega_l)}{(1-m)^2 q_l x(s, \omega_l) r'(t_p)}. \tag{16}$$

where $r'(t_p)$ is the derivative of $r(t_p)$ with respect to t_p. Given Assumption 1, the unconditional expected relative gain from continuing the asset is non-negative. Hence, this derivative is non-positive and it equals to zero only when $s = \underline{s}$. □

Proof. Lemma 3

For given passing threshold $t_p(s, m)$, the difference in passing probability between g^l and g^h is $\Delta\big(t_p(s, m)\big)$. I repeat the definition of $\Delta\big(t_p(s, m)\big)$ here

$$\Delta\big(t_p(s, m)\big) \equiv \int_{t \geq t_p(s,m)} \big(g^h(t) - g^l(t)\big) dt.$$

Taking derivative with respect to s, I obtain the following

$$\begin{aligned}
\frac{\partial \Delta\big(t_p(s, m)\big)}{\partial s} &= \frac{d\Delta\big(t_p(s, m)\big)}{dt_p(s, m)} \frac{\partial t_p(s, m)}{\partial s} \\
&= \Big(g^l\big(t_p(s, m)\big) - g^h\big(t_p(s, m)\big)\Big) \frac{\partial t_p(s, m)}{\partial s} \\
&\propto \Big(r\big(t_p(s, m)\big) - 1\Big) \frac{\partial t_p(s, m)}{\partial s}.
\end{aligned}$$

Recall that equation (14) pins down the passing threshold $t_p(s, m)$, and the ratio $r\big(t_p(s, m)\big)$ solves

$$r\big(t_p(s, m)\big) = \frac{m q_l x(s, \omega_l) + q_h x(s, \omega_h)}{m q_l x(s, \omega_l) - q_l x(s, \omega_l)} \geq \frac{m q_l x(s, \omega_l) - q_l x(s, \omega_l)}{m q_l x(s, \omega_l) - q_l x(s, \omega_l)} = 1. \tag{17}$$

The inequality holds because Assumption 1 implies that $q_h x(s, \omega_h) \geq -q_l x(s, \omega_l)$ for all s and equality holds only when $s = \underline{s}$. As a result, the derivative $\frac{d\Delta(t_p(s,m))}{dt_p(s,m)}$ is non-negative. Given the result of Lemma 2 that $\frac{\partial t_p(s,m)}{\partial s} < 0$, the derivative $\frac{\partial \Delta(t_p(s,m))}{\partial s} \leq 0$ and equality holds only when $s = \underline{s}$. □

Proof. Proposition 1

When s is disclosed, the manipulation level is determined by the first-order condition in equation (7). I repeat the first-order condition here,

$$q_l\big(x(s, \omega_l) + B\big)\Delta\big(t_p(s, m_D)\big) - kc'(m_D) = 0.$$

The first term of the left-hand side is $MB_b\big(s, t_p(s, m_D)\big)$. Apply implicit function theorem to the first-order condition, I derive the derivative of m_D with respect to s,

$$\frac{\partial m_D}{\partial s} = \frac{\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}}{kc''(m_D) - \frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial m_D}} = \frac{q_l\left(\Delta\big(t_p(s,m_D)\big)\frac{dx(s,\omega_l)}{ds} + \big(x(s,\omega_l)+B\big)\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial s}\right)}{kc''(m_D) - q_l\big(x(s,\omega_l)+B\big)\frac{\partial\Delta\left(t_p(s,m_D)\right)}{\partial m_D}}. \tag{18}$$

The derivative $\frac{\partial\Delta\left(t_p(s,m)\right)}{\partial m}$ is given by

$$\frac{\partial\Delta\big(t_p(s,m)\big)}{\partial m} = \frac{d\Delta\big(t_p(s,m)\big)}{dt_p(s,m)}\frac{\partial t_p(s,m)}{\partial m} \propto \Big(r\big(t_p(s,m)\big)-1\Big)\frac{\partial t_p(s,m)}{\partial m} \leq 0. \tag{19}$$

I omit the proof, since it is similar to the proof of Lemma 3. Consequently, the following holds

$$\frac{\partial m_D}{\partial s} \propto \frac{\partial MB_b\big(s,t_p(s,m_D)\big)}{\partial s} \propto \Delta\big(t_p(s,m_D)\big)\frac{dx(s,\omega_l)}{ds} + \big(x(s,\omega_l)+B\big)\frac{\partial\Delta\big(t_p(s,m_D)\big)}{\partial s}.$$

For ease of exposition, I introduce the following notation

$$F \equiv \Delta\big(t_p(s,m_D)\big)\frac{dx(s,\omega_l)}{ds} + \big(x(s,\omega_l)+B\big)\frac{\partial\Delta\big(t_p(s,m_D)\big)}{\partial s}$$

In the following, I first show that $F=0$ holds at some $s \in (\underline{s},\bar{s})$ and then I prove that $F=0$ is unique at $s=s_D$.

When $s=\underline{s}$, Assumption 1 assumes that $x(\underline{s},\omega_h)q_h + x(\underline{s},\omega_l)q_l = 0$. According to equation (14), the passing threshold satisfies $r\big(t_p(\underline{s},m)\big)=1$ which implies that $\frac{\partial\Delta\left(t_p(s,m)\right)}{\partial s}=0$, hence, the function F is

$$F|_{s=\underline{s}} = \Delta\big(t_p(\underline{s},m_D)\big)\left.\frac{dx(s,\omega_l)}{ds}\right|_{s=\underline{s}} > 0.$$

When $s=\bar{s}$, Assumption 1 implies that $x(\bar{s},\omega_l)=0$. Hence, the passing threshold is $t_p(\bar{s},m_D)=\underline{t}$ and $\Delta(\underline{t})=0$. Hence, the function F is

$$F|_{s=\bar{s}} = B\left.\frac{\partial\Delta\big(t_p(s,m_D)\big)}{\partial s}\right|_{s=\bar{s}} < 0.$$

By the intermediate value theorem, $F=0$ must hold at some value of $s \in (\underline{s},\bar{s})$.

Next, I show that $F=0$ is unique at $s=s_D$. When $F=0$, the following equation holds,

$$\Delta\big(t_p(s,m_D)\big)\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B} = -\frac{\partial\Delta\big(t_p(s,m_D)\big)}{\partial s}.$$

I drop the indicator D for the manipulation m_D. Then $F = 0$ is equivalent to

$$\frac{G^l\big(t_p(s,m)\big) - G^h\big(t_p(s,m)\big)}{g^h\big(t_p(s,m)\big) - g^l\big(t_p(s,m)\big)} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B} = \frac{\partial t_p(s,m)}{\partial s}. \tag{20}$$

I first show that the left-hand side is increasing in s. I drop the arguments for t_p when no confusion caused. The left-hand side is equivalent to

$$LHS \equiv -\frac{\Delta(t_p)}{\Delta'(t_p)} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}.$$

Where $\Delta'(t_p) \equiv \frac{d\Delta(t_p)}{dt_p}$. The derivative of LHS with respect to s is

$$\frac{\partial LHS}{\partial s} = -\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} \frac{\partial t_p}{\partial s} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B} - \frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds} \frac{\Delta(t_p)}{\Delta'(t_p)}.$$

The derivative $\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p}$ is

$$\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} = \frac{\Delta'(t_p)^2 - \Delta(t_p)\Delta''(t_p)}{\Delta'(t_p)^2}.$$

By assumption, the decreasing hazard rate $\frac{g^i(t)}{1-G^i(t)}$ implies that $g^i(t)$ is decreasing in t. Moreover, the MLRP assumption implies that $r(t)$ is decreasing in t, that is

$$\frac{dr(t)}{dt} = \frac{\frac{dg^l(t)}{dt}g^h(t) - \frac{dg^h(t)}{dt}g^l(t)}{g^h(t)^2} < 0.$$

Recall that at the passing threshold t_p, it holds that $r(t_p) > 1$ which is equivalent to $g^l(t_p) > g^h(t_p)$. Hence, it also holds that $\frac{dg^l(t_p)}{dt_p} < \frac{dg^h(t_p)}{dt_p}$. That is, $\Delta''(t_p) < 0$, which in turn implies that $\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} > 0$. The derivative $\frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds}$ is

$$\frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds} = \frac{-\left(\frac{dx(s,\omega_l)}{ds}\right)^2 + \big(x(s,\omega_l)+B\big)\frac{d^2x(s,\omega_l)}{ds^2}}{\big(x(s,\omega_l)+B\big)^2}.$$

Since $x(s,\omega_l)$ is increasing and concave in s, this derivative is negative. As a result, $\frac{\partial LHS}{\partial s} > 0$.

Now consider the right-hand side of equation (20),

$$RHS = \frac{\partial t_p(s,m)}{\partial s}.$$

And the derivative of the right-hand side is

$$\frac{\partial RHS}{\partial s} = \frac{\partial^2 t_p(s,m)}{\partial s^2}.$$

Recall the derivative $\frac{\partial t_p(s,m)}{\partial s}$ in equation (15). By the chain rule, the second derivative $\frac{\partial^2 t_p}{\partial s^2}$ is,

$$\begin{aligned}
\frac{\partial^2 t_p}{\partial s^2} =& \frac{\partial \frac{\partial t_p}{\partial s}}{\partial s} + \frac{\partial \frac{\partial t_p}{\partial s}}{\partial t_p}\frac{\partial t_p}{\partial s} \\
=& q_h \frac{\mathbb{E}_\omega\left[x(s,\omega)\right] r'(t_p)^2 \frac{dx(s,\omega_l)}{ds}\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)}{(1-m)^2 q_l^2 x(s,\omega_l)^4\big(1-r(t_p)\big)r'(t_p)^3} \\
&+ q_h \frac{\mathbb{E}_\omega\left[x(s,\omega)\right] r'(t_p)^2 x(s,\omega_h)\left(x(s,\omega_l)\frac{d^2x(s,\omega_l)}{ds^2} - \left(\frac{dx(s,\omega_l)}{ds}\right)^2\right)}{(1-m)^2 q_l^2 x(s,\omega_l)^4\big(1-r(t_p)\big)r'(t_p)^3} \\
&+ q_h \frac{\mathbb{E}_\omega\left[x(s,\omega)\right] r'(t_p)^2 x(s,\omega_l)\left(-x(s,\omega_l)\frac{d^2x(s,\omega_h)}{ds^2} + \left(\frac{dx(s,\omega_l)}{ds}\right)\left(\frac{dx(s,\omega_h)}{ds}\right)\right)}{(1-m)^2 q_l^2 x(s,\omega_l)^4\big(1-r(t_p)\big)r'(t_p)^3} \\
&+ q_h \frac{q_h\big(r(t_p)-1\big)r''(t_p)\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)^2}{(1-m)^2 q_l^2 x(s,\omega_l)^4\big(1-r(t_p)\big)r'(t_p)^3}.
\end{aligned} \tag{21}$$

This derivative is negative. Hence, $\frac{\partial RHS}{\partial s} < 0$.

I have shown that when $F = 0$, the left-hand side of equation (20) is increasing in s whereas the right-hand side of equation (20) is decreasing in s, which implies that $F = 0$ has a unique solution s_D. And $F > 0$ for $s < s_D$ and $F < 0$ for $s > s_D$. Recall that $\frac{\partial m_D(s)}{\partial s}$ is proportionate to F, hence, $m_D(s)$ is increasing in s for $s < s_D$ and is decreasing in s for $s > s_D$. Since $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}$ is proportionate to F, s_D also solves $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s} = 0$. □

Proof. Proposition 2

I prove this proposition by contradiction.

Suppose that $\mathbb{E}_s\left[m_D(s)|s \in [\underline{s}, s_D]\right] > m_N$ for $N = [\underline{s}, s_D]$. Denote $\mathbb{E}_s\left[m_D(s)|s \in [\underline{s}, s_D]\right]$ by $\overline{m_D}$,

$$\begin{aligned}
\overline{m_D} - m_N &\propto kc'(\overline{m_D}) - kc'(m_N) \\
&\leq \mathbb{E}_s\left[\, kc'\big(m_D(s)\big)\,\middle|\, s \in [\underline{s}, s_D]\right] - kc'(m_N) \\
&= \frac{\int_{\underline{s}}^{s_D} kc'\big(m_D(s)\big)dF(s)}{\int_{\underline{s}}^{s_D} dF(s)} - kc'(m_N) \\
&\propto \int_{\underline{s}}^{s_D}\Big(kc'\big(m_D(s)\big) - kc'(m_N)\Big)dF(s).
\end{aligned}$$

The inequality is due to the assumption that $kc'(m)$ is weakly convex in m.

The first-order condition for $m_D(s)$ is

$$MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) = kc'\big(m_D(s)\big). \tag{22}$$

And the first-order condition for m_N when $N = [\underline{s}, s_D]$ is

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [\underline{s}, s_D]\right] = kc'(m_N).$$

I can simplify the difference between $\overline{m_D}$ and m_N further.

$$\begin{aligned}
\overline{m_D} - m_N \leq & \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [\underline{s}, s_D]\right]\Big)\, dF(s) \\
\leq & \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [\underline{s}, s_D]\right]\Big)\, dF(s) \\
= & \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big)\Big)\, dF(s) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [\underline{s}, s_D]\right] \int_{\underline{s}}^{s_D} dF(s) \\
= & \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - MB_b\big(s, t_p(s, \overline{m_D})\big)\Big)\, dF(s) \\
= & \int_{\underline{s}}^{s_D} \Big(q_l\big(x(s, \omega_l) + B\big)\Big(\Delta\big(t_p(s, m_D(s))\big) - \Delta\big(t_p(s, \overline{m_D})\big)\Big)\Big)\, dF(s) \\
\leq & 0.
\end{aligned}$$

The first line is obtained by using the first-order condition of m_N and $m_D(s)$. The second line is due to the fact that $MB_b\big(s, t_p(s, m)\big)$ is decreasing in m. This is verified by the following derivative

$$\frac{\partial MB_b\big(s, t_p(s, m)\big)}{\partial m} = q_l\big(x(s, \omega_l) + B\big)\frac{\partial \Delta\big(t_p(s, m)\big)}{\partial m} \leq 0. \tag{23}$$

The derivative $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m}$ is non-positive as shown in equation (19). Then the assumption that $m_N < \overline{m_D}$ implies the second line. The third and fourth line follow from the definition of conditional expectation. The last inequality is obtained by applying FKG inequality, which I now explain in details. The manipulation level $m_D(s)$ is increasing in s when $s < s_D$. And equation (19) shows that $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m} \leq 0$. This means that the term $\Delta\big(t_p(s, m_D(s))\big)$ is decreasing in s through $m_D(s)$. The term $q_l\big(x(s, \omega_l) + B\big)$ is increasing in s. By FKG inequality, the following holds

$$\mathbb{E}_{s \leq s_D}\left[q_l\big(x(s, \omega_l) + B\big)\Delta\Big(t_p\big(s, m_D(s)\big)\Big)\right] \leq \mathbb{E}_{s \leq s_D}\left[q_l\big(x(s, \omega_l) + B\big)\Delta\Big(t_p(s, \mathbb{E}_{s \leq s_D}[m_D(s)])\Big)\right].$$

Where $\mathbb{E}_{s \leq s_D}$ denotes expectation over s conditional on $s \leq s_D$. This implies that the last inequality holds and it contradicts to $m_N < \overline{m_D}$.

Next I prove by contradiction that $\overline{m_D} \geq m_N$ for $N = [s_D, \bar{s}]$. Suppose that the opposite holds, that is, $\overline{m_D} < m_N$ for $N = [s_D, \bar{s}]$. Then the following holds,

$$
\begin{aligned}
&\overline{m_D} - m_N \propto \log\big(kc'(\overline{m_D})\big) - \log\big(kc'(m_N)\big) \\
&\geq \mathbb{E}_s\Big[\log\Big(kc'\big(m_D(s)\big)\Big)\Big| s \in [s_D, \bar{s}]\Big] - \log\big(kc'(m_N)\big) \\
&= \frac{\int_{s_D}^{\bar{s}} \log\Big(kc'\big(m_D(s)\big)\Big) dF(s)}{\int_{\underline{s}}^{s_D} dF(s)} - \log\big(kc'(m_N)\big) \\
&\propto \int_{s_D}^{\bar{s}} \Bigg(\log\Big(kc'\big(m_D(s)\big)\Big) - \log\big(kc'(m_N)\big)\Bigg) dF(s) \\
&= \int_{s_D}^{\bar{s}} \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} dF(s) \\
&\geq \int_{s_D}^{\bar{s}} \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} dF(s) \\
&\propto \int_{s_D}^{\bar{s}} \Big(kc'\big(m_D(s)\big) - kc'(m_N)\Big) dF(s)
\end{aligned}
$$

The first inequality holds because $c'(m)$ is weakly log-concave. By the definition of conditional expectation, I obtain the first equality. I derive the second equality by using mean value theorem, where $kc'(m_s) \in \Big(kc'\big(m_D(s)\big), kc'(m_N)\Big)$ or $kc'(m_s) \in \Big(kc'(m_N), kc'\big(m_D(s)\big)\Big)$ depending on the relation between $kc'(m_N)$ and $kc'\big(m_D(s)\big)$. I now explain the second inequality.

- If $kc'\big(m_D(s)\big) < kc'(m_N)$, then $kc'(m_s) \in \Big(kc'\big(m_D(s)\big), kc'(m_N)\Big)$. Hence, the following holds

  $$\frac{1}{\frac{1}{kc'(m_N)}} \geq \frac{1}{\frac{1}{kc'(m_s)}} \geq \frac{1}{\frac{1}{kc'(m_D(s))}}.$$

  Which implies that

  $$\frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_D(s))}}.$$

- If $kc'\big(m_D(s)\big) > kc'(m_N)$, then $kc'(m_s) \in \Big(kc'(m_N), kc'\big(m_D(s)\big)\Big)$. Hence, the following holds

  $$\frac{1}{\frac{1}{kc'(m_N)}} \leq \frac{1}{\frac{1}{kc'(m_s)}} \leq \frac{1}{\frac{1}{kc'(m_D(s))}}.$$

  Which implies that

  $$\frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_D(s))}}.$$

Hence, regardless of the difference between $kc'\big(m_D(s)\big)$ and $kc'(m_N)$, the second inequality holds.

The first order condition for $m_D(s)$ is the same as in equation (22). And the first-order condition for m_N when $N = [s_D, \bar{s}]$ is

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [s_D, \bar{s}]\right] = kc'(m_N).$$

I further simplify the difference between $\overline{m_D}$ and m_N,

$$\begin{aligned}
\overline{m_D} - m_N \geq & \int_{s_D}^{\bar{s}} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [s_D, \bar{s}]\right]\Big)\, dF(s) \\
\geq & \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [s_D, \bar{s}]\right]\Big)\, dF(s) \\
= & \int_{s_D}^{\bar{s}} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big)\Big)\, dF(s) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [s_D, \bar{s}]\right] \int_{\underline{s}}^{s_D} dF(s) \\
= & \int_{s_D}^{\bar{s}} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - MB_b\big(s, t_p(s, \overline{m_D})\big)\Big)\, dF(s) \\
= & \int_{s_D}^{\bar{s}} \Big(q_l\big(x(s, \omega_l) + B\big)\Big(\Delta\big(t_p(s, m_D(s))\big) - \Delta\big(t_p(s, \overline{m_D})\big)\Big)\Big)\, dF(s) \\
\geq & 0.
\end{aligned}$$

The second inequality uses the assumption that $m_N > \overline{m_D}$. The last inequality is derived by using FKG inequality. The manipulation level $m_D(s)$ is decreasing in s when $s > s_D$. Hence $\Delta\big(t_p(s, m_D(s))\big)$ is increasing in s through $m_D(s)$. Given that the term $q_l\big(x(s, \omega_l) + B\big)$ is increasing in s, FKG inequality implies the last inequality. $\square$

Proof. Lemma 4

I first show that $ML_r\big(s, t_p(s, m)\big)$ is increasing in s for any given m.

$$\frac{dML_r\big(s, t_p(s, m)\big)}{ds} = q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m)\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m)\big)}{ds}.$$

Lemma 3 shows that for any given m, $\Delta\big(t_p(s, m)\big)$ is decreasing in s. Hence, $\frac{d\Delta\left(t_p(s,m)\right)}{ds} < 0$. Since the low fundamental asset has negative value, i.e., $x(s, \omega_l) < 0$, the derivative $\frac{dML_r\left(s, t_p(s,m)\right)}{ds} > 0$. Hence, the derivative $\frac{dML_r\left(s, t_p(s, m_{N_n})\right)}{ds} > 0$ for any no-disclosure set N_n.

Next, consider $ML_r\big(s, t_p(s, m_D(s))\big)$.

$$\begin{aligned}
&\frac{dML_r\big(s, t_p(s, m_D(s))\big)}{ds} = q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m_D(s))\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m_D(s))\big)}{ds} \\
&= q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m_D(s))\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m_D(s))\big)}{dt_p} \left(\frac{\partial t_p(s, m_D(s))}{\partial s} + \frac{\partial t_p(s, m_D(s))}{\partial m} \frac{\partial m_D(s)}{\partial s}\right).
\end{aligned}$$

Lemma 2 shows that $\frac{\partial t_p}{\partial m}$ and $\frac{\partial t_p}{\partial s}$ are non-positive. Therefore, when $\frac{\partial m_D(s)}{\partial s}$ is non-negative, the derivative $\frac{dML_r\big(s,t_p(s,m_D(s))\big)}{ds}$ is positive. In the following, I show that the derivative $\frac{dML_r\big(s,t_p(s,m_D(s))\big)}{ds}$ is non-negative even when $\frac{\partial m_D(s)}{\partial s}$ is negative. When $m_D(s)$ is decreasing in s, the marginal benefit $MB_b\big(s,t_p(s,m_D)\big)$ is decreasing in s for given m_D. Taking into account of the changes in $m_D(s)$, the following shows that the total derivative of $\frac{dMB_b\big(s,t_p(s,m_D(s))\big)}{ds}$ is proportionate to $\frac{\partial MB_b\big(s,t_p(s,m_D)\big)}{\partial s}$,

$$
\begin{aligned}
&\frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} = \frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\left(\frac{\partial t_p(s,m_D)}{\partial s} + \frac{\partial t_p(s,m_D)}{\partial m_D}\frac{\partial m_D(s)}{\partial s}\right)\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right) + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}\frac{\partial m_D(s)}{\partial s}\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right) + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}\left(\frac{\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\right)\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right)\left(1 + \frac{\frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\right)\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right)\frac{kc''(m_D(s))}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\\
&\propto \frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}.
\end{aligned}
$$

Since $\frac{kc''(m_D(s))}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}$ is positive, the total derivative of $MB\Big(s,t_p\big(s,m_D(s)\big)\Big)$ with respect to s is proportionate to the partial derivative of $MB\Big(s,t_p\big(s,m_D(s)\big)\Big)$ with respect to s taking $m_D(s)$ as given.

When $\frac{\partial m_D(s)}{\partial s}$ is negative, the following holds

$$
\frac{\partial m_D(s)}{\partial s} \propto \frac{\partial MB\Big(s,t_p\big(s,m_D\big)\Big)}{\partial s} \propto \frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} < 0.
$$

The total derivative $\frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds}$ equals to

$$
\frac{dMB\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} = q_l\frac{dx(s,\omega_l)}{ds}\Delta\big(t_p(s,m_D(s))\big) + q_l\big(x(s,\omega_l)+B\big)\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds}.
$$

Hence, $\frac{\partial m_D(s)}{\partial s} < 0$ implies

$$
\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds} < -\frac{\Delta\big(t_p(s,m_D(s))\big)}{x(s,\omega_l)+B}\frac{dx(s,\omega_l)}{ds}. \tag{24}
$$

Then the total derivative $\frac{dML_r\big(s,t_p(s,m_D(s))\big)}{ds}$ is

$$\begin{aligned}
\frac{dML_r\big(s,t_p(s,m_D(s))\big)}{ds} &= q_l\frac{dx(s,\omega_l)}{ds}\Delta\big(t_p(s,m_D(s))\big)+q_l x(s,\omega_l)\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds}\\
&\propto \frac{dx(s,\omega_l)}{ds}\Delta\big(t_p(s,m_D(s))\big)+x(s,\omega_l)\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds}\\
&\geq \frac{dx(s,\omega_l)}{ds}\Delta\big(t_p(s,m_D(s))\big)+x(s,\omega_l)\left(-\frac{\Delta\big(t_p(s,m_D(s))\big)}{x(s,\omega_l)+B}\frac{dx(s,\omega_l)}{ds}\right)\\
&= \frac{B}{x(s,\omega_l)+B}\Delta\big(t_p(s,m_D(s))\big)\frac{dx(s,\omega_l)}{ds}\geq 0.
\end{aligned}$$

The first inequality uses the results in equation (24) and the assumption that $x(s,\omega_l)\leq 0$. Hence, the derivative $\frac{dML_r\big(s,t_p(s,m_D(s))\big)}{ds}\geq 0$ always hold. □

Proof. Proposition 3

I complete the proof in two steps. I first show that a cutoff disclosure dominates all other form of disclosures. Next, I solve for the optimal cutoff point s^* and show that $s^* < s_D$.

Suppose that $D=[\underline{s},s_D)$ and $N=[s_D,\bar{s}]$. The regulator's ex ante expected utility with this disclosure policy is denoted as U

$$U=\int_{\underline{s}}^{s_D}u\big(s,m_D(s)\big)dF(s)+\int_{s_D}^{\bar{s}}u(s,m_N)dF(s).$$

In the following, I show that adding more cutoff points to partition the signal space does not improve the regulator's ex ante expected utility. First, I show that adding cutoff point in D does not improve the regulator's ex ante utility. Without loss of generality, consider a disclosure policy which partitions the signal space into $N_2=[\underline{s},s_1]$, $D=(s_1,s_D)$ and $N_1\equiv N=[s_D,\bar{s}]$. The regulator's ex ante expected payoff with such disclosure policy is

$$U'=\int_{\underline{s}}^{s_1}u\big(s,m_{N_2}\big)dF(s)+\int_{s_1}^{s_D}u\big(s,m_D(s)\big)dF(s)+\int_{s_D}^{\bar{s}}u(s,m_N)dF(s).$$

The difference in the regulator's expected utility is

$$
\begin{aligned}
U - U' &= \int_{\underline{s}}^{s_D} u\big(s, m_D(s)\big) dF(s) - \int_{\underline{s}}^{s_1} u\big(s, m_{N_2}\big) dF(s) - \int_{s_1}^{s_D} u\big(s, m_D(s)\big) dF(s) \\
&= \int_{\underline{s}}^{s_1} u\big(s, m_D(s)\big) dF(s) - \int_{\underline{s}}^{s_1} u\big(s, m_{N_2}\big) dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left.\frac{du(s,m)}{dm}\right|_{m=m(s)} dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left(\left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m(s)} + \frac{\partial u(s,m)}{\partial t_p(s,m)} \left.\frac{\partial t_p(s,m)}{\partial m}\right|_{m=m(s)} \right) dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m(s)} dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) ML_r\big(s, t_p(s, m(s))\big) dF(s) \\
&\geq \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) ML_r\big(s, t_p(s, m_{N_2})\big) dF(s) \\
&\propto \mathbb{E}_{s \leq s_1} \Big[m_D(s) ML_r\big(s, t_p(s, m_{N_2})\big) \Big] - m_{N_2} \mathbb{E}_{s \leq s_1} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] \\
&\geq \mathbb{E}_{s \leq s_1} \Big[m_D(s) ML_r\big(s, t_p(s, m_{N_2})\big) \Big] - \mathbb{E}_{s \leq s_1} \big[m_D(s) \big] \mathbb{E}_{s \leq s_1} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] \\
&\geq 0.
\end{aligned}
$$

The first two lines are derived from simplifications of the differences in expected utility. Apply mean-value theorem to the second line gives the third line, where $m(s) \in (m_D(s), m_{N_2})$ if $m_D(s) < m_{N_2}$ or $m(s) \in (m_{N_2}, m_D(s))$ if $m_{N_2} < m_D(s)$. The fourth line shows the total derivative of $u(s,m)$ with respect to m, and it reduces to the fifth line because the passing threshold $t_p(s,m)$ maximizes the regulator's utility $u(s,m)$ for given signal s and given manipulation m, hence, $\frac{\partial u(s,m)}{\partial t_p(s,m)} = 0$. Equation (11) defines $ML_r\big(s, t_p(s,m)\big)$. I now explain the first inequality in details. The following derivative shows that $ML_r\big(s, t_p(s,m)\big)$ is increasing in m,

$$
\frac{\partial ML_r\big(s, t_p(s,m)\big)}{\partial m} = q_l x(s, \omega_l) \frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m}.
$$

Equation (19) implies that this derivative is non-negative. The following proves the first inequality

- If $m_D(s) < m_{N_2}$, then $m(s) \in (m_D(s), m_{N_2})$. Hence, the following holds

$$
ML_r\big(s, t_p(s, m_D(s))\big) \leq ML_r\big(s, t_p(s, m(s))\big) \leq ML_r\big(s, t_p(s, m_{N_2})\big),
$$

which implies that

$$\big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_D(s))\big) \geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m(s))\big)$$
$$\geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_{N_2})\big).$$

- If $m_D(s) > m_{N_2}$, then $m(s) \in (m_{N_2}, m_D(s))$. Hence, the following holds

$$ML_r\big(s, t_p(s, m_D(s))\big) \geq ML_r\big(s, t_p(s, m(s))\big) \geq ML_r\big(s, t_p(s, m_{N_2})\big),$$

which implies that

$$\big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_D(s))\big) \geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m(s))\big)$$
$$\geq \big(m_D(s) - m_{N_2}\big)ML_r\big(s, t_p(s, m_{N_2})\big).$$

Hence, regardless of the difference between $m_D(s)$ and m_{N_2}, the first inequality holds. The second inequality generalize the result from Proposition 2. Since $m_D(s)$ is increasing in s for $s \leq s_1$, and Lemma 4 shows that $ML_r\big(s, t_p(s, m_D(s))\big)$ is increasing in s, hence, the last inequality is obtained by FKG inequality. This proof can be generalized to cases where more than one cutoff points are added on D.

Apply the same approach, I show that adding cutoff point in N does not improve the regulator's ex ante utility. Consider a disclosure policy which partitions the signal space into $D = [\underline{s}, s_D)$, $N_2 = [s_D, s_2)$ and $N_3 = [s_2, \bar{s}]$. The regulator's ex ante expected payoff with such disclosure policy is

$$U' = \int_{\underline{s}}^{s_D} u\big(s, m_D(s)\big)dF(s) + \int_{s_D}^{s_2} u(s, m_{N_2})dF(s) + \int_{s_2}^{\bar{s}} u\big(s, m_{N_3}\big)dF(s).$$

For ease of exposition, I make the following definition in this proof.

$$p_1 \equiv \Pr\big(s \in [s_D, s_2) | s \geq s_D\big) = \frac{\int_{s_D}^{s_2} dF(s)}{\int_{s_D}^{\bar{s}} dF(s)}.$$

And

$$p_2 \equiv \Pr\big(s \geq s_2 | s \geq s_D\big) = \frac{\int_{s_2}^{\bar{s}} dF(s)}{\int_{s_D}^{\bar{s}} dF(s)}.$$

The difference in the regulator's expected utility is

$$
\begin{aligned}
U - U' &= \int_{s_D}^{\bar{s}} u(s, m_N) dF(s) - \int_{s_D}^{s_2} u(s, m_{N_2}) dF(s) - \int_{s_2}^{\bar{s}} u(s, m_{N_3}) dF(s) \\
&= \int_{s_D}^{s_2} \Big(u(s, m_N) - u(s, m_{N_2})\Big) dF(s) + \int_{s_2}^{\bar{s}} \Big(u(s, m_N) - u(s, m_{N_3})\Big) dF(s) \\
&= \int_{s_D}^{s_2} (m_N - m_{N_2}) \left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m_2} dF(s) + \int_{s_2}^{\bar{s}} (m_N - m_{N_3}) \left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m_3} dF(s) \\
&= \int_{s_D}^{s_2} (m_N - m_{N_2}) ML_r\big(s, t_p(s, m_2)\big) dF(s) + \int_{s_2}^{\bar{s}} (m_N - m_{N_3}) ML_r\big(s, t_p(s, m_3)\big) dF(s) \\
&\geq \int_{s_D}^{s_2} (m_N - m_{N_2}) ML_r\big(s, t_p(s, m_{N_2})\big) dF(s) + \int_{s_2}^{\bar{s}} (m_N - m_{N_3}) ML_r\big(s, t_p(s, m_{N_3})\big) dF(s) \\
&\propto m_N \left(p_1 \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] + p_2 \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big) \Big] \right) \\
&\quad - \left(p_1 m_{N_2} \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] + p_2 m_{N_3} \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big) \Big] \right) \\
&\geq (p_1 m_{N_2} + p_2 m_{N_3}) \left(p_1 \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] + p_2 \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big) \Big] \right) \\
&\quad - \left(p_1 m_{N_2} \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big) \Big] + p_2 m_{N_3} \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big) \Big] \right) \\
&\geq 0.
\end{aligned}
$$

The derivations follow the previous proof. Notice that the second inequality is obtained by generalizing the result from Proposition 2. And the last inequality is derived by applying FKG inequality. This proof can be applied to cases where more than one more cutoff points are added on N.

I have shown that the disclosure policy with $D = [\underline{s}, s_D]$ and $N = [s_D, \bar{s}]$ dominates all other forms of disclosure. Next, I solve for the optimal cutoff point. Denote the regulator's ex ante utility with the optimal disclosure policy by U^*,

$$U^* = \int_{\underline{s}}^{s^*} u\big(s, m_D(s)\big) dF(s) + \int_{s^*}^{\bar{s}} u(s, m_N) dF(s).$$

First, the optimal cutoff point $s^* \leq s_D$ must hold. Otherwise, by the previous proof, the regulator can gain by not disclosing the signals $s \in [s_D, s^*]$. But such disclosure policy features two no-disclosure sets which is dominated by the disclosure policy with single no-disclosure set $[s_D, \bar{s}]$. Hence, $s^* \leq s_D$ must hold.

Take derivative of U^* with respect to the cutoff point s^*, the first-order condition determines the optimal cutoff point,

$$\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big) f(s^*) + \frac{\partial m_N}{\partial s^*} \int_{s^*}^{\bar{s}} ML_r\big(s, t_p(s, m_N)\big) dF(s) = 0. \tag{25}$$

The first-order condition for m_N when $N = [s^*, \bar{s}]$ is

$$\mathbb{E}_s \left[MB_b\big(s, t_p(s, m_N)\big) | s \in [s^*, \bar{s}] \right] = kc'(m_N).$$

By implicit function theorem, I derive the derivative $\frac{\partial m_N}{\partial s^*}$,

$$\frac{\partial m_N}{\partial s^*} = \frac{f(s^*)}{\int_{s^*}^{\bar{s}} dF(s)} \frac{\mathbb{E}_{s\geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big)}{kc''(m_N) - \mathbb{E}_{s\geq s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]}.$$

With this derivative, I further reduce the first-order condition in equation (25) to the following

$$\begin{aligned}&\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big)\\ &+ \Big(\mathbb{E}_{s\geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big)\Big) \frac{\mathbb{E}_{s\geq s^*}\left[ML_r\big(s, t_p(s, m_N)\big)\right]}{kc''(m_N) - \mathbb{E}_{s\geq s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]} = 0.\end{aligned} \tag{26}$$

□

Proof. Proposition 4

The no-disclosure set N is $[s^*, \bar{s}]$. The bank's manipulation m_N for $s \in N$ is then determined by the following first-order condition

$$F_N \equiv \mathbb{E}_s \left[MB_b\big(s, t_p(s, m_N)\big) | s \in [s^*, \bar{s}] \right] - kc'(m_N) = 0.$$

By implicit function theorem,

$$\frac{\partial s^*}{\partial k} = -\frac{\frac{\partial F_N}{\partial k}}{\frac{\partial F_N}{\partial s^*}} = \frac{c'(m_N) \int_{s^*}^{\bar{s}} dF(s)}{f(s^*)\Big(\mathbb{E}_{s\geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big)\Big)}.$$

From the first-order condition of s^* in equation (26), the following holds,

$$\begin{aligned}&\mathbb{E}_{s\geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big)\\ &= \Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big) \frac{kc''(m_N) - \mathbb{E}_{s\geq s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]}{-\mathbb{E}_{s\geq s^*}\left[ML_r\big(s, t_p(s, m_N)\big)\right]}.\end{aligned} \tag{27}$$

With this equation, I reduce the derivative $\frac{\partial s^*}{\partial k}$ to the following,

$$\begin{aligned}\frac{\partial s^*}{\partial k} &= \frac{-\mathbb{E}_{s\geq s^*}\left[ML_r\big(s,t_p(s,m_N)\big)\right]c'(m_N)\int_{s^*}^{\bar{s}}dF(s)}{f(s^*)\Big(kc''(m_N)-\mathbb{E}_{s\geq s^*}\Big[\frac{\partial MB_b\left(s,t_p(s,m_N)\right)}{\partial m_N}\Big]\Big)}\frac{1}{\Big(u\big(s^*,m_D(s^*)\big)-u(s^*,m_N)\Big)}\\ &\propto \frac{1}{\Big(u\big(s^*,m_D(s^*)\big)-u(s^*,m_N)\Big)}.\end{aligned}$$

The derivative is positive if $m_D(s^*) \leq m_N$ and it is negative if $m_D(s^*) > m_N$.

Similarly, I derive the following derivative by implicit function theorem,

$$\frac{\partial s^*}{\partial B} = -\frac{\frac{\partial F_N}{\partial B}}{\frac{\partial F_N}{\partial s^*}} = \frac{-\mathbb{E}_{s\geq s^*}\left[q_l\Delta\big(t_p(s,m_N)\big)\right]\int_{s^*}^{\bar{s}}dF(s)}{f(s^*)\Big(\mathbb{E}_{s\geq s^*}\left[MB_b\big(s,t_p(s,m_N)\big)\right]-MB_b\big(s^*,t_p(s^*,m_N)\big)\Big)}.$$

Apply the result in equation (27), the above partial derivative becomes

$$\begin{aligned}\frac{\partial s^*}{\partial B} &= \frac{-\mathbb{E}_{s\geq s^*}\left[ML_r\big(s,t_p(s,m_N)\big)\right]\mathbb{E}_{s\geq s^*}\left[q_l\Delta\big(t_p(s,m_N)\big)\right]\int_{s^*}^{\bar{s}}dF(s)}{f(s^*)\Big(kc''(m_N)-\mathbb{E}_{s\geq s^*}\Big[\frac{\partial MB_b\left(s,t_p(s,m_N)\right)}{\partial m_N}\Big]\Big)}\frac{-1}{\Big(u\big(s^*,m_D(s^*)\big)-u(s^*,m_N)\Big)}\\ &\propto \frac{1}{\Big(u(s^*,m_N)-u\big(s^*,m_D(s^*)\big)\Big)}.\end{aligned}$$

The derivative is negative if $m_D(s^*) \leq m_N$ and it is positive if $m_D(s^*) > m_N$. □

B Cost of inefficiencies

In this appendix, I derive the regulator's optimal disclosure policy assuming that the bank's asset should be liquidated ex ante. That is, I replace Assumption 1 by the following

Assumption B.1. *The bank's asset is ex ante worth liquidating:* $\mathbb{E}_\omega\left[x(s,\omega)\right] \in [q_l x(\underline{s},\omega_l), 0]$ *for* $s \in [\underline{s}, \bar{s}]$.

For given bank's report t with conjectured manipulation level $\hat{m}$ and the signal s, the regulator's pass/fail decision still follows equation (4). As in Lemma 1, the regulator's pass/fail decision is characterized by a cutoff rule on the bank's report t. That is, the regulator passes the bank if and only if the bank's report t is higher than the threshold $t_p(s, \hat{m})$.

Lemma B.1. *For given level of manipulation* m*, the passing threshold* $t_p(s, m)$ *is decreasing in* s*. For given signal* s*, the passing threshold is increasing in* m*.*

Proof. The proof follows the proof of Lemma 2. The only difference is that Assumption B.1 assumes that $q_h x(s, \omega_h) + q_l x(s, \omega_l) < 0$. This assumption implies that $\frac{\partial t_p}{\partial m}$ in equation (16) is positive. □

Compared to Lemma 2, the effect of the signal s on the regulator's choice of the passing threshold remains unchanged. However, the effect of manipulation is the opposite due to Assumption B.1, which assumes that inefficient continuation is more costly than inefficient liquidation. Despite manipulation still increasing the report similarity between low and high fundamental asset, the regulator is more concerned about the inefficient continuation. Hence, when the regulator is facing a report that is less informative, the regulator would raise the passing threshold in order to avoid inefficient continuation.

The regulator's choice of passing threshold determines the difference in passing probability between low and high fundamental asset. The following lemma shows that this difference in passing probability becomes larger as the signal increases.

Lemma B.2. *For given manipulation level* m*,* $\Delta\big(t_p(s, m)\big)$ *is increasing in* s*.*

Proof. The proof is similar to the proof of Lemma 3. The only difference is that Assumption B.1 implies that $q_h x(s, \omega_h) \leq -q_l x(s, \omega_l)$ for all s. As a result, the ratio $r\big(t_p(s, m)\big)$ is less than 1, which then implies that $\Delta\big(t_p(s, m)\big)$ is increasing in s. □

Anticipating the regulator's pass/fail decision, the bank chooses the manipulation level. The bank's manipulation choice depends on whether regulator discloses the regulatory signal s.

Proposition B.1. *When s is disclosed, the level of manipulation $m_D(s)$ is unique and it is increasing in s for all s. When s is not disclosed, the level of manipulation m_{N_n} is unique and it is a constant over the no-disclosure set N_n for $n \in [1, +\infty)$.*

Recall that the bank's manipulation incentive is determined by the increases in the passing probability after manipulation $\Delta\big(t_p(s,m)\big)$ and the expected gain after passing the test with manipulation $q_l\big(x(s,\omega_l)+B\big)$. As the signal s increases, both forces become stronger, leading the bank to manipulation more.

Similar to Proposition 2, the following proposition compares the manipulation level under different disclosure policies.

Proposition B.2. $\mathbb{E}_s\big[m_D(s)|s \in N\big] \leq m_N$ *for any* $N \subseteq S$.

Proof. The proof follows the proof of Proposition 2 when $m_D(s)$ is increasing in s. □

This result states that the disclosure of regulatory signal reduces the expected manipulation level. The rationale for this result is rooted in the interactions between the bank's manipulation choice and the regulator's passing threshold choice when the signal s is disclosed. The same interactions also form the basis for Proposition 2. As manipulation increases with the signal s, the regulator increases the passing threshold $t_p(s,m)$ according to Lemma B.1. This response of the passing threshold reduces the bank's passing probability, and, more importantly, narrows the difference in passing probability between high and low fundamental asset, decreasing the bank's manipulation incentive. In the absence of the disclosure of the signal s, such interactions between the bank's manipulation choice and the regulator's passing threshold choice is muted. Hence, the expected manipulation level is lower when s is disclosed.

I now analyze the regulator's disclosure policy about the signal s. Following Lemma 4, I derive how the regulator's loss caused by the bank's manipulation $ML_r\big(s, t_p(s,m^*)\big)$ changes with the signal s.

Lemma B.3. *If the following condition holds,*

$$\frac{d}{ds}\left(\frac{\frac{d\Delta\left(s,t_p(s,m^*)\right)}{ds}}{\Delta\big(s,t_p(s,m^*)\big)}\right) \leq 0, \tag{28}$$

then $ML_r\big(s,t_p(s,m^*)\big)$ is decreasing in s for $s < s_r$ and increasing in s for $s > s_r$, where $m^* = \{m_D(s), m_{N_n}\}$ and $s_r \in (\underline{s}, \bar{s})$ is the unique solution for $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds} = 0$.

Proof. The derivative of $ML_r\big(s, t_p(s, m^*)\big)$ with respect to s is given by the following,

$$\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds} = q_l \left(x(s, \omega_l) \frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds} + \Delta\big(s, t_p(s, m^*)\big) \frac{dx(s, \omega_l)}{ds} \right).$$

When $s = \underline{s}$, Assumption B.1 assumes that $x(\underline{s}, \omega_h) = 0$. According to equation (14), the passing threshold is $t_p(\underline{s}, m^*)) = \bar{t}$ which implies that $\Delta\big(t_p(\underline{s}, m^*)\big) = 0$, hence, the derivative $\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds}$ is

$$\left.\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds}\right|_{s=\underline{s}} = q_l x(\underline{s}, \omega_l) \left.\frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds}\right|_{s=\underline{s}} < 0.$$

When $s = \bar{s}$, Assumption B.1 implies that $x(\bar{s}, \omega_h) q_h + x(\bar{s}, \omega_l) q_l = 0$. According to equation (14), the passing threshold satisfies $r\big(t_p(\bar{s}, m^*\big) = 1$, which implies that $\left.\frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds}\right|_{s=\bar{s}} = 0$. Hence, the derivative $\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds}$ is

$$\left.\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds}\right|_{s=\bar{s}} = q_l \Delta\big(\bar{s}, t_p(\bar{s}, m^*)\big) \left.\frac{dx(s, \omega_l)}{ds}\right|_{s=\bar{s}} > 0.$$

By the intermediate value theorem, $\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds} = 0$ must hold at some value of $s \in (\underline{s}, \bar{s})$.

Next, I show that $\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds} = 0$ is unique at $s = s_r$. When $\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds} = 0$, the following equation holds,

$$x(s, \omega_l) \frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds} + \Delta\big(s, t_p(s, m^*)\big) \frac{dx(s, \omega_l)}{ds} = 0.$$

This is equivalent to

$$\frac{\frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds}}{\Delta\big(s, t_p(s, m^*)\big)} = -\frac{\frac{dx(s, \omega_l)}{ds}}{x(s, \omega_l)}.$$

The left-hand side is positive for $s \in (\underline{s}, \bar{s})$. In addition, condition in equation (28) ensures that it is weakly decreasing in s. The right-hand side is also positive for $s \in (\underline{s}, \bar{s})$. And the function $x(s, \omega_l)$ is log-concave in s which implies that the right-hand side is weakly increasing in s. Hence, $\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds} = 0$ is unique and the solution is denoted as s_r. □

This result shows that under certain condition, the regulator's marginal loss from manipulation has U-shape. That is, the marginal loss $ML_r\big(s, t_p(s, m^*)\big)$ first decreases in s and then increases in s. The condition in equation (28) means that $\Delta\big(s, t_p(s, m)\big)$ is log-concave in s which ensures that $\Delta\big(s, t_p(s, m)\big)$ is not too convex in s.

Following the intuition of Proposition 3, the optimal disclosure policy should minimize the part of the regulator's loss that cannot be controlled by the optimal pass/fail decision. For given level of manipulation, the disclosure policy should allocate less manipulation to cases where the regulator is more susceptible to it. In addition, the optimal disclosure policy should minimize the expected level of manipulation. To fix the idea, I decompose the regulator's ex ante utility difference between disclosure U_D and no disclosure U_N for a given set of signals S'

$$
\begin{aligned}
U_D - U_N &= \int_{S'} \Big(u\big(s, m_D(s)\big) - u(s, m_N)\Big) dF(s) \\
&= \int_{S'} \big(m_D(s) - m_N\big) ML_r\big(s, t_p\big(s, m(s)\big)\big) dF(s) \\
&= \mathbb{E}_{s\in S'}\Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - m_N\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
&= \mathbb{E}_{s\in S'}\Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - \mathbb{E}_{s\in S'}\big[m_D(s)\big]\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
&\quad + \mathbb{E}_{s\in S'}\big[m_D(s)\big]\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - m_N\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
&= \underbrace{\mathbb{E}_{s\in S'}\Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - \mathbb{E}_{s\in S'}\big[m_D(s)\big]\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big]}_{\text{Distribution effect}} \\
&\quad + \underbrace{\Big(\mathbb{E}_{s\in S'}\big[m_D(s)\big] - m_N\Big)\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big]}_{\text{Expected level effect}}
\end{aligned}
$$

Where m_N is the bank's manipulation response when $N = S'$ and $m(s)$ is the manipulation level that satisfies the mean value theorem. This decomposition shows the two effects of the disclosure of regulatory signal s on the regulator's ex ante payoff. The first two terms capture the "distribution effect", which measures the extent to which disclosure can allocate more manipulation to cases where the regulator suffers less from it. The last two terms are the "expected level effect", which captures the impact of disclosure on the expected level of manipulation. The following proposition characterizes the optimal disclosure policy. It shows that the optimal disclosure policy still follows a single cutoff rule.

Proposition B.3. *Suppose that condition (28) holds. The optimal disclosure policy follows a cutoff rule where $D = (s^*, \bar{s}]$ and $N = [\underline{s}, s^*]$. That is, the regulator discloses the signal s when $s > s^*$ and does not disclose the signal s when $s < s^*$, where $s^* \in [s_r, \bar{s}]$ solves*

$$
\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big) f(s^*) = \frac{\partial m_N}{\partial s^*} \int_{\underline{s}}^{s^*} ML_r\big(s, t_p(s, m_N)\big) dF(s). \tag{29}
$$

Proof. A single cutoff disclosure policy is optimal because the regulator's marginal loss caused by the bank's manipulation $ML_r\big(s, t_p(s, m)\big)$ has U-shape across s. The proof of the optimality of a cutoff disclosure policy is similar to the proof of Proposition 3, hence, omitted. In what follows, I solve the optimal cutoff point s^*. The regulator's ex ante utility with the optimal disclosure policy is U^*,

$$U^* = \int_{\underline{s}}^{s^*} u(s, m_N) dF(s) + \int_{s^*}^{\bar{s}} u\big(s, m_D(s)\big) dF(s).$$

Where the optimal cutoff point s^* solves the following,

$$\Big(u(s^*, m_N) - u\big(s^*, m_D(s^*)\big)\Big) f(s^*) + \frac{\partial m_N}{\partial s^*} \int_{\underline{s}}^{s^*} ML_r\big(s, t_p(s, m_N)\big) dF(s) = 0.$$

The first-order condition for m_N when $N = [\underline{s}, s^*]$ is

$$\mathbb{E}_s \left[MB_b\big(s, t_p(s, m_N)\big) | s \in [\underline{s}, s^*] \right] = kc'(m_N).$$

By implicit function theorem, I derive the derivative $\frac{\partial m_N}{\partial s^*}$.

$$\frac{\partial m_N}{\partial s^*} = \frac{f(s^*)}{\int_{\underline{s}}^{s^*} dF(s)} \frac{MB_b\big(s^*, t_p(s^*, m_N)\big) - \mathbb{E}_{s \leq s^*} \left[MB_b\big(s, t_p(s, m_N)\big) \right]}{kc''(m_N) - \mathbb{E}_{s \leq s^*} \left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N} \right]}.$$

With this derivative, I further reduce the first-order condition to the following

$$\begin{aligned} &\Big(u(s^*, m_N) - u\big(s^*, m_D(s^*)\big)\Big) \\ &+ \Big(MB_b\big(s^*, t_p(s^*, m_N)\big) - \mathbb{E}_{s \leq s^*} \left[MB_b\big(s, t_p(s, m_N)\big) \right] \Big) \frac{\mathbb{E}_{s \leq s^*} \left[ML_r\big(s, t_p(s, m_N)\big) \right]}{kc''(m_N) - \mathbb{E}_{s \leq s^*} \left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N} \right]} = 0. \end{aligned} \tag{30}$$

□

The rationale for implementing a cutoff disclosure policy is analogous to that of Proposition 3. Disclosing the signal s is beneficial for the regulator when the bank's manipulation strategy features reducing manipulation in cases where the regulator is more vulnerable to it. Proposition B.1 shows that as s increases, the bank manipulates more. Additionally, Lemma B.3 indicates that for relatively large values of s, the regulator is less susceptible to manipulation as s increases. Consequently, disclosing s may benefit the regulator when s exceeds a certain threshold, i.e, $s \geq s_r$. No disclosure at all may be optimal, provided that the expected level of manipulation is sufficiently low.

C Real activity

In this appendix, I discuss an extension in which the bank exerts costly effort to improve the fundamental of the asset, and such effort is reflected in the bank's report. Although, similar to the baseline model, the bank is able to improve the report in the sense of first-order stochastic dominance, such improvement in report arises endogenously from the improvement in the asset's fundamental. As a result, the disclosure of the regulatory information now affects the real activity of the bank, i.e., effort choice.

The model is as follows. Suppose that the bank can exert effort m to improve the fundamental of the asset. To keep the notation consistent, I continue to use m to denote the bank's effort. The effort m determines the probability that the asset's fundamental value is ω_h. That is, when the bank exerts effort m and the regulator's signal is s, the relative gain from continuing the asset is $x(s, \omega_h)$ with probability m and $x(s, \omega_l)$ with probability $1 - m$.

The fundamental of the asset determines the report distribution in the same way as in the baseline model. That is, the report t is drawn from a distribution with density $g^i(t)$ when the fundamental is ω_i, where $i = \{h, l\}$. Since the bank's effort determines the asset fundamental, it also determines the report distribution. With effort m, the bank's report generating process is $g^h(t)$ with probability m and $g^l(t)$ with probability $1 - m$. Notice that Assumption 1 no longer holds, since the expected relative gain from continuing the asset for given s is now endogenously determined by the bank's effort. I assume that $x(s, \omega_l) \leq x(\bar{s}, \omega_l) \equiv 0$ and $x(s, \omega_h) \geq x(\underline{s}, \omega_h) \equiv 0$. All other elements of the model remain the same as in Section 2. The following analysis focuses on how the regulator should disclose the regulatory signal to affect the bank's effort choice. I solve the model backwards.

After observing the signal s and the bank's report t, the regulator's pass/fail decision remains the same as in the baseline model. The regulator passes the bank if and only if the expected gain from passing the bank exceeds that from failing the bank. That is,

$$\mathbb{E}_\omega[x(s, \omega)|t, \hat{m}] \geq 0.$$

Where $\hat{m}$ is the regulator's conjecture about the bank's effort. The conditional expectation is

$$\begin{aligned}
\mathbb{E}_\omega[x(s, \omega)|t, \hat{m}] &= x(s, \omega_h) \Pr(\omega = \omega_h|t, \hat{m}) + x_{\hat{m}}(s, \omega_l) \Pr(\omega = \omega_l|t, \hat{m}) \\
&= x(s, \omega_h) \frac{\hat{m} g^h(t)}{\hat{m} g^h(t) + (1 - \hat{m}) g^l(t)} + x(s, \omega_l) \frac{(1 - \hat{m}) g^l(t)}{\hat{m} g^h(t) + (1 - \hat{m}) g^l(t)}.
\end{aligned}$$

The pass/fail decision still features a threshold $t_p(s, \hat{m})$ on the bank's report. The bank passes the test if and only if the report t satisfies $t \geq t_p(s, \hat{m})$. The passing threshold $t_p(s, \hat{m})$ solves $\mathbb{E}_\omega[x(s, \omega)|t_p, \hat{m}] = 0$, which indicates that the regulator is indifferent between passing and failing the bank when the bank's report is $t_p(s, \hat{m})$.

Lemma C.1. *For given level of effort m, the passing threshold $t_p(s, m)$ is decreasing in s. For given signal s, the passing threshold $t_p(s, m)$ is decreasing in m.*

This result echoes to Lemma 2. The explanation for the first result is the same as in Lemma 2. However, the effect of effort is different from that of manipulation. Effort improves the relative gain from continuing the asset $x(s, \omega)$, which endogenously increases relative cost of inefficient liquidation. Consequently, the regulator decreases the passing threshold to pass the bank more often.

Anticipating the regulator's pass/fail decision, the bank chooses the effort level. Suppose that the bank observes the regulator's private signal s. The bank's payoff is

$$V(s, \hat{m}, m) = m\big(x(s, \omega_h)+B\big) \int_{t\geq t_p(s,\hat{m})} g^h(t)dt + (1-m)\big(x(s, \omega_l)+B\big) \int_{t\geq t_p(s,\hat{m})} g^l(t)dt - kc(m).$$

The first-order condition with respect to m determines the bank's effort choice. In equilibrium, the regulator's conjecture about the effort is consistent with the bank's choice. Hence, the equilibrium effort $m_D(s)$ is determined by

$$\big(x(s, \omega_h)+B\big) \int_{t\geq t_p\left(s,m_D(s)\right)} g^h(t)dt - \big(x(s, \omega_l)+B\big) \int_{t\geq t_p\left(s,m_D(s)\right)} g^l(t)dt - kc'\big(m_D(s)\big) = 0. \quad (31)$$

The first two terms are the bank's marginal benefit of exerting effort. I modify the definition of MB_b in equation (8) to the following

$$\begin{aligned} MB_b\big(s, t_p(s, m)\big) &\equiv \big(x(s, \omega_h) + B\big) \int_{t\geq t_p(s,m)} g^h(t)dt - \big(x(s, \omega_l) + B\big) \int_{t\geq t_p(s,m)} g^l(t)dt \\ &= \big(x(s, \omega_h) - x(s, \omega_l)\big) \int_{t\geq t_p(s,m)} g^h(t)dt + \big(x(s, \omega_l) + B\big)\Delta\big(t_p(s, m)\big). \end{aligned} \quad (32)$$

Where $\Delta\big(t_p(s, m)\big)$ is defined in equation (5) and it captures the difference in passing probability between low and high fundamental asset. The first term of MB_b is the bank's gain from improving the asset's fundamental from low to high, provided that the bank passes the test. This term captures *the effect of effort on the relative gain from holding the asset.* The second term is identical to equation (8) and it captures the bank's expected gain from having low fundamental asset pass the test. This term captures *the effect of effort on the bank's report.* This effect is identical to the effect of manipulation in the baseline model.

Lemma C.2. *For given signal s, if*

$$\big(x(s,\omega_h) - x(s,\omega_l)\big)g_h(\underline{t}) < \big(x(s,\omega_l) + B\big)\big(g_l(\underline{t}) - g_h(\underline{t})\big),$$

then $MB_b\big(s, t_p(s,m)\big)$ is increasing in m for $m < m_r(s)$ and it is decreasing in m for $m > m_r(s)$, where $m_r(s)$ solves $\frac{\partial MB_b\left(s,t_p(s,m)\right)}{\partial m} = 0$. Otherwise, $MB_b\big(s, t_p(s,m)\big)$ is increasing in m.

This result shows that the bank's marginal benefit of exerting effort is nonmonotonic in the level of effort m. The intuition is as follows. The first component of $MB_b\big(s, t_p(s,m)\big)$ is $\big(x(s,\omega_h) - x(s,\omega_l)\big) \int_{t \geq t_p(s,m)} g^h(t)dt$, and it represents the bank's gain from improving the asset fundamentals. As the effort m increases, the bank is more likely to have high fundamental asset. In response, the regulator passes the bank more often by lowering the passing threshold. Consequently, the first term $\big(x(s,\omega_h) - x(s,\omega_l)\big) \int_{t \geq t_p(s,m)} g^h(t)dt$ is increasing in the amount of effort, incentivizing the bank to exert more effort. This term captures the bank's incentive to exert effort to improve the fundamental of the asset.

However, the second component $\big(x(s,\omega_l) + B\big)\Delta\big(t_p(s,m)\big)$ may decrease with the effort m, depending on how $\Delta\big(t_p(s,m)\big)$ changes with m. Lemma C.1 shows that the passing threshold $t_p(s,m)$ is decreasing in effort m, meaning that the bank is more likely to pass the test when exerting more effort. When the level of effort is very low (high), the regulator will set the passing threshold very high (low) which makes the bank less (more) likely to pass the test regardless of the fundamental of the asset. In such cases, the difference in passing probability between low and high fundamental asset is small. When the level of effort is intermediate, the regulator will set the passing threshold at an intermediate level which makes the passing probability depend crucially on the fundamental of the asset. Hence, the *difference* in passing probability between low and high fundamental asset $\Delta\big(t_p(s,m)\big)$ is first increasing and then decreasing in the effort level m, which then makes the term $\big(x(s,\omega_l) + B\big)\Delta\big(t_p(s,m)\big)$ follow the same pattern. This term captures the bank's incentive to exert effort only when it leads to higher probability of passing the test.

The condition in Lemma C.2 guarantees that the bank's effort choice is not solely driven by the first component. This condition depends on the bank's private benefit of passing the test. When the private benefit B is relatively small, the first effect ("improving the asset fundamental") always dominates the second effect ("improving the report") and determines the bank's effort choice.

One implications of Lemma C.2 is that the first-order condition in equation (31) may be nonmonotonic in m, hence, the interior solution of $m_D(s)$ may not exist and the solution of

$m_D(s)$ may not be unique. One sufficient condition for the interior solution of $m_D(s)$ to exist is

$$x(s, \omega_h) - x(s, \omega_l) < kc'(1), \ \forall s. \tag{33}$$

This condition means that the effort is costly such that the bank does not have incentive to improve the fundamental to $x(s, \omega_h)$. I assume this condition holds. To avoid having multiple equilibria for the effort $m_D(s)$, I also assume that the bank chooses the highest effort level when it is indifferent. The solution $m_D(s)$ must satisfy the second order condition $\frac{\partial FOC}{\partial m} < 0$.

Proposition C.1. *When s is disclosed, the effort level $m_D(s)$ is increasing in s for $s < s_D$ and it is decreasing in s for $s > s_D$, where s_D is the unique solution for* $\frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial s} = 0$.

Now consider the effort choice when the bank does not observe the regulator's signal s. The equilibrium effort m_N solves,

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_{N_n})\big)\big|\, s \in N_n\right] - kc'(m_{N_n}) = 0.$$

The effort m_N is unique and it is a constant over the regulator's signal s.

The following proposition compares the expected effort level when the bank observes the signal with the expected effort level when the bank does not observes the signal.

Proposition C.2. *If the following holds for all s*

$$\left.\frac{\partial MB_b\big(s, t_p(s, m)\big)}{\partial m}\right|_{m=m_D(s)} >= 0, \tag{34}$$

then $\mathbb{E}_s\left[m_D(s)|s \in N\right] \geq m_N$ *if* $N \subseteq [\underline{s}, s_D]$ *and* $\mathbb{E}_s\left[m_D(s)|s \in N\right] \leq m_N$ *if* $N \subseteq [s_D, \bar{s}]$.

The intuition is as follows. When $s \in [\underline{s}, s_D]$, the effort $m_D(s)$ is increasing in s if s is disclosed. In response, the regulator lowers the passing threshold t_p, which makes the test more lenient regardless of the asset fundamental. Such endogenous response of the passing threshold has two opposite effects on the bank's incentive to exert effort. One the one hand, an easier test allows the bank to pass even without exerting effort, which then decreases the bank's incentive to exert effort. On the other hand, an easier test increases the likelihood that the bank's effort is realized, i.e., the bank passes the test after increase the asset fundamental. This effect increases the bank's incentive to exert effort. (Notice that this second effect is missing in the baseline model.) Depending on the magnitude of the two forces, the bank may increase or decrease effort. When condition (34) holds, the second effect dominates. As a result, the interactions between the regulator's pass/fail decision and the bank's effort choice increases the expected level of effort, comparing to the case when such interactions are absent, i.e. when s is not disclosed.

When $s \in [s_D, \bar{s}]$, the effort $m_D(s)$ is decreasing in s if s is disclosed. In response, the regulator raises the passing threshold t_p to make the test more difficult. Such endogenous response of the pass/fail decision then increases the magnitude of passing probability that can be increased by exerting effort, incentivizing the bank to exert more effort. However, as the test gets more difficult, it also increases the likelihood that the bank may not be paid off by exerting effort. That is, the bank may still fail the test even after exerting effort. (Again, this second effect is missing in the baseline model.) The second effect dominates the bank's effort choice if condition (34) holds. As a result, the interactions between the regulator's pass/fail decision and the bank's effort choice decreases the expected level of effort when s is disclosed compare to the case when s is not disclosed.

Consider the regulator's disclosure policy. For given signal s and equilibrium effort m^*, the regulator's payoff is

$$\begin{aligned} u(s, m^*) &= \int_{t \geq t_p(s,m^*)} \mathbb{E}_\omega[x(s,\omega)|t, m^*] g_{m^*}(t) dt \\ &= m^* x(s, \omega_h) \int_{t \geq t_p(s,m^*)} g^h(t) dt + (1 - m^*) x(s, \omega_l) \int_{t \geq t_p(s,m^*)} g^l(t) dt. \end{aligned}$$

Where $g_{m^*}(t)$ is the unconditional distribution of report t when the bank's effort is m^*. That is,

$$g_{m^*}(t) = m^* g^h(t) + (1 - m^*) g^l(t).$$

Taking derivative of $u(s, m)$ with respect to m, I obtain the marginal effect of bank's effort on the regulator. I modify the definition of ML_r in equation (11) to the following,

$$\begin{aligned} ML_r\big(s, t_p(s,m)\big) &\equiv x(s, \omega_h) \int_{t \geq t_p(s,m)} g^h(t) dt - x(s, \omega_l) \int_{t \geq t_p(s,m)} g^l(t) dt \\ &= \big(x(s,\omega_h) - x(s,\omega_l)\big) \int_{t \geq t_p(s,m)} g^h(t) dt + x(s,\omega_l) \Delta\big(t_p(s,m)\big). \end{aligned}$$

Lemma C.3. *For any disclosure set D or no-disclosure set N_n, $ML_r\big(s, t_p(s, m^*)\big)$ is increasing in s for $m^* = \{m_D(s), m_{N_n}\}$.*

As argued in the baseline model, disclosure is less likely to be optimal when the changes in effort m and changes in the regulator's marginal utility change $ML_r\big(s, t_p(s, m^*)\big)$ are driven by the the difference in passing probability between low and high fundamental asset $\Delta(t_p(s, m))$. This continues to be the case in this extension with effort choice. However, the presence of effort exertion increases the likelihood of disclosure. This is because the regulator's disclosure about the signal s not only informs the bank about the expected gain after passing the test but also provides information about the extent to which exerting effort can improve such gain. Hence, disclosure is more useful to align the interests of the regulator and the bank regarding when effort is more desirable.

Proposition C.3. *When condition (34) holds, the optimal disclosure policy follows a cutoff rule where $D = [\underline{s}, s^*)$ and $N = [s^*, \bar{s}]$. That is, the regulator discloses the signal s when $s < s^*$ and does not disclose the signal s when $s > s^*$, where $s^* \in [\underline{s}, s_D)$.*

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