Fractional homeownership and its impact on life cycle portfolio choice*

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Abstract

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Using a quantitative life cycle model, we study the impact of access to fractional homeownership on individuals' optimal consumption, savings, and housing decisions. Fractional homeownership means that two parties—an individual and an institutional investor—share full ownership of a property. The individual lives in the property full–time and makes periodic rent payments to the institutional investor who sees the property solely as an investment vehicle. The amount of the rent payments depends on the value of the home and the fractional ownership structure that can be time–varying. We find that access to fractional homeownership is most attractive to particularly young and old individuals. Further, it leads to earlier housing market entry, later housing market exit, decreases individuals' loan–to–value ratios and reduces their moving activity at old age; all in comparison to a setting in which the individuals' rent–versus–own decisions are binary.

Key Words: fractional homeownership, housing decisions, loan–to–value ratio, life cycle model, portfolio choice

JEL Classification Codes: D15, E21, G11, G21

1 Introduction

Acquiring real estate is among the biggest financial challenges individuals face throughout their lives. In recent years, the housing affordability crisis rendered purchasing properties even more difficult (Anacker, 2019)—in 2022, housing affordability was at its lowest level since the 2008 financial crisis due to changes in home prices and interest rates applicable to mortgage debt. Nonetheless, the acquisition of real estate was among the primary economic objectives of 75% of the U.S. population.[1]

Due to the unabated desire for homeownership and the housing affordability crisis, alternative financial products for financing real estate are developed. In our manuscript, we study such an alternative by investigating the implications of access to *fractional homeownership* on individuals' lifetime consumption, savings, and housing decisions. Fractional homeownership means that two parties—an individual and an institutional investor—share full ownership of a property. The individual lives in the property full–time and makes periodic rent payments to the institutional investor who sees the property solely as an investment vehicle. The amount of the rent payments depends on the value of the home and the fractional ownership structure that can be time–varying as the individuals have the right to adjust the fraction owned in every period.

Using a rich and realistically calibrated life cycle model of optimal consumption, savings, and housing decisions, we shed, among others, light on the impact of access to fractional homeownership on individuals' housing market entry and exit as well as their loan–to–value ratios. Our results suggest that primarily young, liquidity constrained individuals and older individuals who are in the phase of decumulating their savings make use of fractional homeownership to finance their property. Further, this novel financing option leads to earlier housing market entry, later housing market exit, lower loan–to–value ratios and to reduced moving activity of elderly individuals; all in comparison to the setting in which the individuals' rent–versus–own decisions are binary, i.e., the setting in which individuals can either be renters or owners.[2]

[1] An evolution of housing affordability can be downloaded from https://www.atlantafed.org/. https://www.cnbc.com/ summarizes surveys on individuals' economic objectives. Both sources last accessed on October 10, 2022.

[2] To be clear, we define the individuals' loan–to–value ratios (LTVs) as fractions of their outstanding mortgage debt and the full value of their property, but we do not allow them to borrow more than 80% of the value of the property they effectively own (see Section 3.1). We use this approach for two reasons: (i) The borrowing restriction ensures that individuals do not end up having unsecured loans. (ii) Our calculation of the LTVs highlights that, when living in equally valuable homes, individuals with access to fractional homeownership have lower mortgage debt than those who do not. Yet, we report the fractions of individuals' outstanding mortgage debt and the value of the home they effectively own in Section 4; these ratios are also lower than

On the real estate market, the concept of fractional homeownership is already well–established for second residences. Several individuals own fractions of a property and the time they are allowed to spend in the property depends on the fraction owned, i.e., individuals owning a larger share of the property are allowed to use it more days per year.

In our manuscript, we study a slight alteration of the concept described above as we study fractional homeownership contracts between individuals who want to live in the property permanently and institutional investors for whom the property is purely an investment vehicle. As compensation for not living in the property the institutional investor receives periodical rent payments from the individual; the exact amount of the rent payments depends on the fraction owned by the institutional investor and the value of the property. The market for this variant of fractional homeownership is recently emerging with the first contract being signed in the U.S. in 2023.[3] Further, this concept is related to cooperative housing which is a common form of housing in, for example, Denmark, Norway, and Sweden, but also in larger cities in the U.S. (see, e.g., Balmer and Gerber, 2018; Sørvoll and Bengtsson, 2018; Sazama, 2000).

Fractional homeownership contracts have advantages for both parties—for the individual as well as for the institutional investor. It is well–known that young individuals often face harsh liquidity constraints (Gourinchas and Parker, 2002). Consequently, they struggle, for example, with accumulating sufficient liquid savings for the down payment needed to acquire real estate. We find that access to fractional homeownership alleviates these liquidity issues and thereby enables individuals to live in valuable properties without having to take out very high mortgages, i.e., without facing very high loan–to–value ratios. Another rather recently developed product that is worth mentioning in this context as it also aims at increasing housing affordability and alleviating young individuals' liquidity constraints, are so–called interest–only (IO) loans. This type of loans was introduced in several countries such as Denmark, Sweden, and the Netherlands in the 2000's and is studied, for example, by Amromin et al. (2018), Bian et al. (2018), and Bäckman and Lutz (2020). The development of concepts such as fractional homeownership and IO loans, that facilitate the acquisition of real estate matter as, for example, Goodman and Mayer (2018) document that even after the financial crisis real estate is a valuable asset for individuals and that home acquisitions should be stimulated.

For institutional investors, such as pension funds, real estate is an important asset class.

the LTVs of individuals without access to fractional homeownership.

[3] A press release reporting the first settlement of such a contract can be found on https://www.globenewswire.com/, last accessed April 5, 2023.

Engaging in fractional homeownership contracts has substantial diversification benefits in comparison to owning, for example, one large apartment building. Additionally, fractional homeownership alleviates the principal–agent problem present in many real estate transactions (see, e.g., Arnold, 1992; Daneshvary and Clauretie, 2013; Furst and Evans, 2017). When residents own at least a certain share of their accommodation, they are likely to have stronger incentives to maintain the property's value.

With our manuscript, we contribute to two different strands of literature. First, we contribute to the literature on life cycle portfolio choice. A lot of previous work on life cycle portfolio choice focuses on optimal asset allocation decisions (see, e.g., Cocco et al., 2005; Benzoni et al., 2007) and optimal asset location decisions (see, e.g., Dammon et al., 2004; Fischer and Gallmeyer, 2017). Life cycle models studying optimal consumption, savings, and housing decisions generally assume that the individuals' rent–versus–own decisions are binary, implying that individuals have to fully fund their owner–occupied homes by taking out a mortgage or by accumulating sufficient savings (see, e.g., Cocco, 2005; Kraft and Munk, 2011; Fischer and Stamos, 2013; Schlafmann, 2021). We are, to the best of our knowledge, the first to soften the assumption of the rent–versus–own decision being binary by studying the impact of access to fractional homeownership on life cycle portfolio choice.

Second, we contribute to the literature on fractional ownership of different asset classes. Whitaker and Kräussl (2020), for example, study fractional ownership of art and introduce a novel fractional equity structure for artists' oeuvres. Bogyrbayeva et al. (2021) focus on fractional ownership of autonomous vehicles and provide a solution for the winners determination problem in autonomous vehicles' auctions. There are several papers studying co–ownership in the energy sector of different countries: Johansen and Emborg (2018) focus, for example, on Denmark and Roth et al. (2018) on Germany.

Three papers studying fractional ownership of real estate are Cauley et al. (2007), who study optimal asset allocation when individuals have/do not have the opportunity to sell a fractional interest in their home, Lowies et al. (2018), who identify the demographic structure of fractional homeowners in Australia, and Rong (2023), who discusses how fractional homeownership could be implemented in practice. More specifically, Cauley et al. (2007) document that homeownership constrains individuals' asset allocation decisions. Giving individuals the opportunity to sell fractions of their homes alleviates the constraint homeownership imposes on asset allocation and thus leads to significant welfare gains. Lowies et al. (2018) conduct an experiment and find that younger individuals are rather willing to become fractional homeowners than older individuals and Rong (2023) highlights that tokenization of properties might facilitate entering the housing market, alleviate liquidity problems, and avoid foreclosures of distressed properties.[4] We contribute to this strand of literature by theoretically studying the impact of access to fractional ownership of real estate on optimal consumption, savings, and housing decisions. We extend the model of Cauley et al. (2007) by, among others, explicitly modeling rent–versus–own decisions and mortgages. Further, our theoretical results support the findings of Lowies et al. (2018) as in our manuscript young individuals often finance their homes via fractional ownership. While Rong (2023) focuses on the implementation of fractional homeownership from a legislative perspective, we focus on the impact of access to fractional homeownership on individuals' decision making over the life cycle.

The remainder of this manuscript is structured as follows: Section 2 provides empirical evidence for the housing affordability crisis and discusses the individuals' desire to acquire real estate as well as the details of fractional homeownership contracts. Section 3 outlines our model of optimal consumption, savings, and housing decisions as well as its calibration. The results are presented in Section 4. Finally, Section 5 concludes.

2 Housing affordability/desire and fractional homeownership contracts

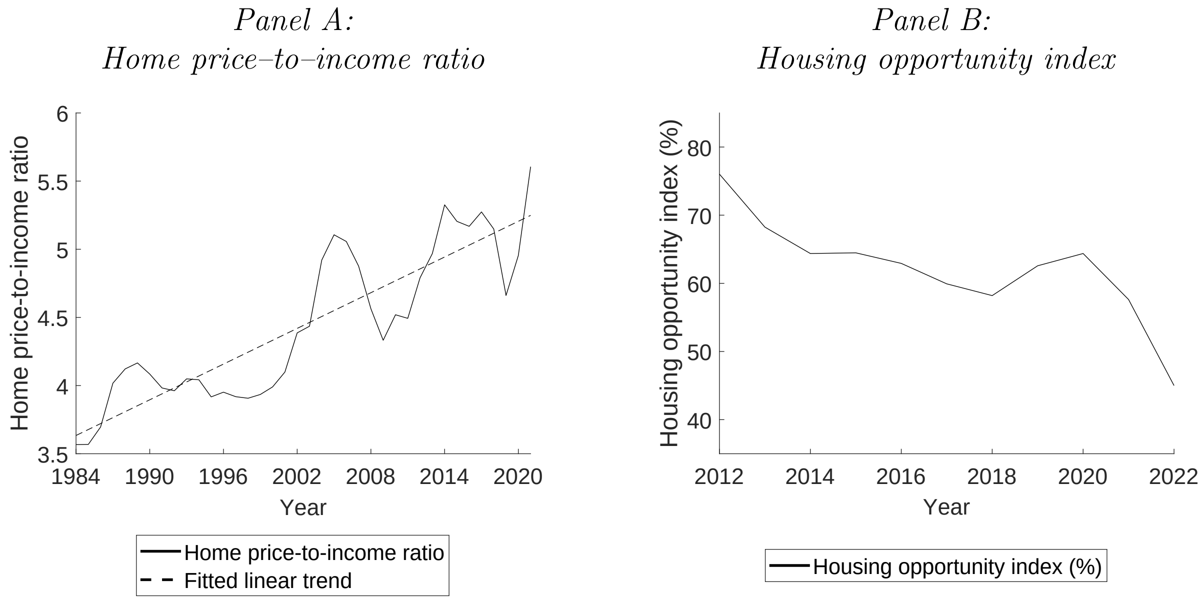
In this section, we (i) motivate the necessity for alternative real estate financing options by analyzing housing affordability and desire in the U.S. and (ii) discuss the design of fractional homeownership contracts.

2.1 Housing affordability/desire

In 2021, 49% of the U.S. population saw the availability of affordable housing as a major problem in their community; a 10 percentage point increase in comparison to 2018, when only 39% reported affordable housing to be a major problem (Pew Research Center, 2022). The housing affordability crisis also received substantial media attention with, for example, CNBC writing "Millennials and Gen Zers do want to buy homes—they just can't afford it, even as adults" and the New York Times trying to identify the drivers of the housing

[4] Tokenization of real estate means that a property is split into several parts (the so–called tokens). These tokens are stored on a blockchain and owning a token of a property results in the same rights and obligations as directly owning the property. Gupta et al. (2020) and Konashevych (2020) provide a thorough overview of real estate tokenization.

Figure 1: **Housing affordability in the U.S. over time**



This figure provides empirical evidence of the current U.S. housing affordability crisis. Panel A depicts the evolution of the annual median U.S. home price–to–income ratio (solid line) and a fitted linear trend (dashed line) from 1984 to 2021. The ratio is constructed as the fraction of annual median sales prices of houses sold in the U.S. and the corresponding annual U.S. median household income. Panel B depicts the U.S. NAHB/Wells Fargo housing opportunity index from 2012 to 2022. It indicates how many percent of homes sold in a given year could have been afforded by a household earning median income in the same year assuming that standard mortgage underwriting criteria apply. As there have been structural changes in the construction of the housing opportunity index, no longer time series than the one from 2012 to 2022 is available. Data for the annual median sales prices of houses sold in the U.S. and for the annual U.S. median household incomes can be downloaded from https://fred.stlouisfed.org/series/MSPUS, last retrieved May 17, 2023, and https://fred.stlouisfed.org/, last retrieved November 2, 2022, respectively. Data for the housing opportunity index can be downloaded from https://www.nahb.org/, last retrieved March 29, 2023.

affordability crisis in an article titled "Is Wall Street Really to Blame for the Affordable Housing Crisis?".[5] The latter article concludes that there are many drivers of the housing affordability crisis, among others, the high inflation rate and the housing shortage, and that institutional investors cannot be held fully responsible for the crisis.

Figure 1 provides further empirical evidence of the housing affordability crisis by showing the evolution of the median home price–to–income ratio and the evolution of the housing opportunity index, respectively. Panel A depicts the evolution of the annual median U.S. home price–to–income ratio from 1984 to 2021 by the solid line and a fitted linear trend by the dashed line. The ratio is constructed as the fraction of annual median sales prices of houses

[5]The articles can be downloaded from https://www.cnbc.com/ and https://www.nytimes.com/, respectively; last accessed November 3, 2022.

sold in the U.S. (accessible via https://fred.stlouisfed.org/, last retrieved May 17, 2023) and the annual U.S. median household income (accessible via https://fred.stlouisfed.org/, last retrieved November 2, 2022). Economically speaking, a home price–to–income ratio of x indicates that a single family home in the U.S. costs x–times the annual U.S. median household income.

The home price–to–income ratio was particularly high for the first time before the beginning of the financial crisis in 2007 and reached its maximum in the last 35 years at the end of 2021 when an average single family home cost more than 5–times the annual median U.S. household income. From the fitted linear trend, we see that the home price–to–income ratio is increasing over time, i.e., the faster increase in home prices than in income levels renders it financially more challenging for individuals to acquire real estate.

Panel B of Figure 1 depicts the U.S. housing opportunity index from 2012 to 2022 (accessible via https://www.nahb.org/, last retrieved March 29, 2023). It indicates how many percent of homes sold in a given year could have been afforded by a household earning median income in the same year assuming that standard mortgage underwriting criteria apply. We observe a sharp decrease in the index' value in the last three years; in 2022 only 45% of the homes sold in the U.S. were affordable for households with median income.

Despite these affordability issues, both, individuals and policymakers, see homeownership as a desirable goal. On the individuals' side, there are various surveys providing evidence of individuals still aiming at becoming homeowners. A survey from Ostrowski (2022), for example, finds that for 74% of the U.S. population owning a home has the highest economic priority. On the policymakers' side, the U.S. government, for example, has special programs that support individuals who aim at acquiring real estate such as government–backed home loans and homeownership vouchers.[6] Further, several countries, such as the Netherlands, Sweden, and the United Kingdom, allow for a deduction of mortgage interest rates (OECD, 2021) and Layton (2022) proposes initiatives that could be implemented by policymakers to increase homeownership rates, such as reducing the labor income tax burden.

Taken together, the housing affordability crisis and the desire for homeownership increase the demand for novel financial products that facilitate the acquisition of real estate. In this manuscript we therefore study such a product, namely fractional homeownership, and outline the details of such a contract in the next section.

[6]Source: https://www.usa.gov/, last accessed January 15, 2023.

2.2 Contract details

The concept of fractional homeownership is not entirely new. There is already a well–developed fractional homeownership market for second residences. Several real estate firms, such as "*Pacaso*" and "*Lazazu*", have specialized on setting up co–ownership contracts between different individuals who are interested in acquiring a second residence.[7] While the exact contract details may vary, most contracts share the feature that the time individuals are allowed to spend in their second residence is proportional to the share of the property they own.

In this manuscript we study a slightly different market, that is currently emerging—the fractional homeownership market for owner–occupied residences. In this variant of fractional homeownership, two parties share full ownership of a property: (i) an individual, i.e., a private investor who is the owner–occupier of the property, and (ii) an institutional investor who sees the real estate unit solely as an investment opportunity. More specifically, we study the impact of access to fractional homeownership of owner–occupied residences on individuals' life cycle portfolio choice.

The fractional homeownership contract is structured as follows: The individual owns only $h\%$ of the property but lives in it permanently. The institutional investor owns $(100 - h)\%$ and sees the property solely as an investment vehicle. As the institutional investor does not derive any utility from living in the property, the individual pays (as compensation) a rent rate to the institutional investor. The exact amount of the rent depends on the value of the property and the fractional homeownership structure. Costs associated with the property, such as maintenance costs, are split proportionally to the ownership structure, i.e., the party who owns $h\%$ of the property pays $h\%$ of the costs, whereas the party who owns $(100-h)\%$ pays $(100 - h)\%$ of the costs.

We assume that individuals can adjust the fraction of the property owned in every period by selling fractions to/buying fractions from the institutional investor. As compensation for these strong rights the individuals gain when entering the contract, we require them to pay a "fractional homeownership premium" that depends on the value of the home to the institutional investor whenever acquiring a new home (not when solely adjusting the fraction owned).

There are several advantages for both parties when entering such a fractional home-

[7]A detailed description of the fractional homeownership products these companies offer can be found on https://www.pacaso.com/ and https://www.lazazu.com/, respectively; all online sources last accessed November 16, 2022.

ownership contract. On the individuals' side, we know that young individuals are often liquidity constrained (Gourinchas and Parker, 2002). Hence they struggle, among others, with accumulating savings needed for the down payment of a mortgage. Access to fractional homeownership decreases the level of savings needed for the down payment and also reduces individuals' total mortgage debt. Other advantages for the individuals are, among others, (i) the reduced exposure to house price volatility as the share of total wealth invested in the housing market is smaller and (ii) the possibility of acquiring a larger home without going on compromise with the consumption levels that can be attained.

On the institutional side, we observe that real estate is becoming an increasingly important asset class. Acquiring fractions of homes in different geographic areas leads to an exceedingly well–diversified portfolio than when acquiring, for example, one large apartment building. Additionally, the principle–agent problem inherent in many rental transactions is alleviated (see, e.g., Arnold, 1992 and Furst and Evans, 2017 for an overview of the principle–agent problem). Individuals who own at least a certain share of the property they live in might have stronger incentives to maintain the property's value than those who are only renting the property. One type of institutional investor, who could be interested in entering such a contract, is pension funds as Preqin (2016) documents that pension funds are by far the largest institutional investors in the real estate industry. About 80% of all private and public pension funds have some real estate investments in their portfolio and the capital invested by pension funds is about 35% of the total capital invested into real estate by institutional investors.

Honestly speaking, there are not only advantages but also downsides that must not be neglected. A disadvantage for the institutional investor is, for example, our assumption, that the institutional investor is required to sell/buy shares from the individual whenever the individual considers it to be optimal to adjust the fraction owned. However, we believe (i) that the "fractional homeownership premium", which is introduced above, creates incentives for the institutional investor to nonetheless enter this type of contract as this premium provides the institutional investor with additional liquidity at the time an individual is acquiring a home and leads to the institutional investor selling the property above market value and (ii) that once the institutional investor owns a sufficiently large number of properties and the individuals who own fractions of these properties are heterogeneous enough, the market might converge to an equilibrium in which individuals who want to buy and those who want to sell fractions of their homes balance each other—given no herd behavior of market participants.[8]

[8] A very interesting direction of future research would be to explicitly model the preferences of the institutional

Moreover, there are high information costs for the institutional investor and it might also prefer to be the majority owner to make important property–related decisions on its own. The latter might also be a preference of the private investor. Given the advantages outlined above, we nonetheless consider fractional homeownership to be an interesting new financial product that deserves to receive attention from research and therefore study it in this manuscript.

Now, assuming that both parties (the individual and the institutional investor) are interested in entering a fractional homeownership contract, the precise contract details such as how to deal with different preferences for maintenance work, have to be discussed.

The prevailing solution to protect the parties engaging in fractional homeownership contracts in case of such disagreements is that they found an LLC ("Limited Liability Company"). The LLC then owns the property whereas the parties of the fractional homeownership contract own parts of the LLC but not of the property itself. As already established in fractional homeownership contracts of second homes, independent real estate agents could mediate between the two parties, set up the LLC, and negotiate the contract details (e.g., the splitting of costs, the rights to sell parts of the property) with both parties in advance. The independent real estate agents should then also have the final say in case of disagreements.[9]

The costs for setting up such an LLC could then either be covered by a part of the "fractional homeownership premium" or be splitted between both parties and be incorporated in the transaction costs when acquiring a new home. Founding an LLC for fractional ownership is not only common in the real estate sector but also, for example, in the energy sector (Lowitzsch, 2019). This procedure is also similar to the management procedures of housing cooperatives in, for example, Sweden.

3 The model

In this chapter, we introduce our life cycle model of optimal consumption, savings, and housing decisions. With this model, we aim at comparing optimal decision making for individuals who have access to fractional homeownership and for individuals who do not have access to fractional homeownership, i.e., for individuals whose rent–versus–own decision is

investor and then maximize the aggregate utility of the private and institutional investor. In case of aggregate utility maximization, it might even be possible to derive a Pareto–optimal cost structure for both parties in fractional homeownership contracts.

[9]A lot of real estate companies who focus on co–ownership of second homes already offer these services to its customers, see, e.g., https://www.pacaso.com/, last accessed November 7, 2022.

binary with each other. Individuals can invest into two financial markets, the housing market as well as the capital market, which are introduced in Sections 3.1 and 3.2, respectively, and make six interrelated decisions at every point in time simultaneously to achieve their goal of maximizing their present discounted lifetime utility as outlined in Section 3.3. The model's calibration is given in Section 3.4.

3.1 Housing market

In life cycle literature, it is common to model rent–versus–own decisions as binary decisions, i.e., individuals can either live in a rented place or in an owner–occupied home (see, e.g., Cocco, 2005; Fischer and Stamos, 2013; Schlafmann, 2021).

In this manuscript, we extend existing life cycle models of optimal consumption, savings, and housing decisions by allowing individuals to partly own the property they live in and to pay a rent rate for the other part, i.e., we allow individuals to be fractional homeowners.

Let Δ_t denote the individual's homeownership status from time t to $t+1$. $\Delta_t \in \{0, 1\}$ takes value zero if the individual is a renter and value one if the individual owns (at least partly) the residence she lives in. In case of fractional homeownership, individuals additionally have to decide upon the fraction of the property owned from time t to $t+1$, $h_t \in (0, 1]$ in case $\Delta_t = 1$. Consequently, they then rent the remaining share of the property, $(1 - h_t)$, from the institutional investor. In the case of a binary rent–versus–own decision, h_t is not needed as decision variable as it follows directly from Δ_t, i.e., in the case of binary homeownership it holds that $\Delta_t = h_t \in \{0, 1\}$.

Owner–occupied homes serve a dual role as (i) a durable consumption good and (ii) an asset. That means, that (i) individuals derive utility from living in the owner–occupied home but (ii) that they also expose themselves to changes in the home's value. In line with, for example, Cocco (2005) and Kraft et al. (2018), we employ a Cobb–Douglas utility function, u, over consumption and housing. As Kiyotaki et al. (2011), we account for individual's preferences of living in an owner–occupied home instead of a rented place by assigning a $\zeta\%$ higher utility per home unit when the individual owns at least a part of the property they live in, i.e., when $\Delta = 1$. When C denotes the amount consumed, Q the home size, and θ

the relative preference over housing consumption, the utility function can be written as[10]

$$u(C, Q) = C^{1-\theta} \left(Q \left(1 + \zeta \mathbb{1}_{\{\Delta=1\}} \right) \right)^{\theta}. \tag{1}$$

We use a log–normal distribution to model returns on investments into the housing market and denote the distribution's mean by μ_H, its standard deviation by σ_H, and its gross return from time t to $t+1$ by G_t^H.

In our model, there are recurring and non–recurring housing market costs. Recurring costs cover the rent rate and/or maintenance costs and have to be paid in every period, whereas non–recurring costs only occur when the individual acquires a new owner–occupied home or adjusts the fraction of the home already owned. These non–recurring costs cover, for examples, fees that have to be paid to a real estate agent and taxes.

As discussed in Section 2.2, we further assume that individuals who have access to fractional homeownership have to pay a "fractional homeownership premium" when they are buying a new property. This premium depends on the value of the home acquired and has to be paid to the institutional investor as compensation for the right to adjust the fraction of the property effectively owned, h_t, at any later point in time.

Let H_t denote the price per home unit at date t. If m_r denotes the rent rate applicable to the home units rented and m_o the maintenance costs applicable to the home units owned, then the recurring costs, $m(\Delta_t, Q_t, H_t, h_t)$, depend on the individual's homeownership status, Δ_t, the value of the home, $Q_t H_t$, and in case of access to fractional homeownership the fraction of the home owned, h_t. Let $\chi \in \{0, 1\}$ be an indicator function taking value zero if individuals do not have access to fractional homeownership and value one if they do. Then the recurring housing costs are given by

$$m(\Delta_t, Q_t, H_t, h_t) = \begin{cases} \left(m_r \mathbb{1}_{\{\Delta_t=0\}} + m_o \mathbb{1}_{\{\Delta_t=1\}} \right) Q_t H_t & \text{if} \quad \chi = 0, \\ \mathbb{1}_{\{\Delta_t=0\}} m_r Q_t H_t + \mathbb{1}_{\{\Delta_t=1\}} \left(m_r (1 - h_t) + m_o h_t \right) Q_t H_t & \text{if} \quad \chi = 1. \end{cases} \tag{2}$$

Let f_p denote the transaction costs, that occur when individuals change their homeownership

[10] It would also be valid to assume that the utility gains are proportional to the fraction effectively owned, i.e., to assume that $u(C, Q) = C^{1-\theta} \left(Q \left(1 + \zeta h_t \right) \right)^{\theta}$. Our assumption that individuals derive a higher utility per home unit if they own at least a part of the property is based on the endowment effect studied, for example, by Reb and Connolly (2007). They argue that the individuals' feelings of ownership towards a property are mainly mediated by subjective feelings of possession rather than by factual ownership. We believe that owning at least a part of a property already leads to such a strong feeling of possession.

status from renter to owner or when moving from one owner–occupied home to another one, and f_f the transaction costs that occur when individuals change the fraction of the owner–occupied home owned. We assume $f_f < f_p$, reflecting that acquiring a new home is more expensive than only adjusting the fraction owned. If f_a denotes the "fractional homeownership premium", that individuals, who have access to fractional homeownership, have to pay whenever acquiring a new property, the non–recurring costs applicable to the investment in the real estate market, n, can be summarized as follows:

$$n(h_t, h_{t-1}, Q_t, Q_{t-1}, \Delta_t, \Delta_{t-1}) = \begin{cases} f_p Q_t H_t \left(\Delta_t (1 - \Delta_{t-1}) + \Delta_t \Delta_{t-1} \mathbb{1}_{\{Q_{t-1} \neq Q_t\}}\right) & \text{if} \quad \chi = 0, \\ f_p h_t Q_t H_t \left(\Delta_t (1 - \Delta_{t-1}) + \Delta_t \Delta_{t-1} \mathbb{1}_{\{Q_{t-1} \neq Q_t\}}\right) & \\ + f_f h_t Q_t H_t \left(\Delta_t \Delta_{t-1} \mathbb{1}_{\{Q_{t-1} = Q_t\}} \mathbb{1}_{\{h_{t-1} \neq h_t\}}\right) & \\ + f_a Q_t H_t \left(\Delta_t (1 - \Delta_{t-1}) + \Delta_t \Delta_{t-1} \mathbb{1}_{\{Q_{t-1} \neq Q_t\}}\right) & \text{if} \quad \chi = 1. \end{cases} \tag{3}$$

In case of $\chi = 1$ in equation (3), the transaction costs (line 1 and line 2 in the second case of the respective equation) are split proportionally between the individual and the institutional investor, i.e., if individuals own a larger fraction, h_t, of the property, they also have to cover a larger share of the transaction costs.

3.2 Capital market

On the capital market, individuals can either invest in a risky asset representing a stock market index or in a risk–free asset paying a constant return r. The risky asset's return from time t to $t+1$ is log–normally distributed with mean μ_S and standard deviation σ_S, and its random gross return from time t to $t+1$ is denoted by G_t^S. We impose a short–selling constraint, i.e., for the individual's investment in the risky asset, S_t, it has to hold that $S_t \geq 0$.

In case of $\Delta_t = 1$, i.e., when individuals are (at least fractional) owners, they can use their home as a collateral and borrow up to a certain share, $\delta \in [0, 1]$, of the fraction of the home's value they own. If B_t denotes the amount invested risk–free (in case of $B_t \geq 0$) or the size of the mortgage (in case of $B_t < 0$), it has to hold that

$$-B_t \leq \begin{cases} \delta Q_t H_t & \text{if} \quad \chi = 0, \\ \delta h_t Q_t H_t & \text{if} \quad \chi = 1. \end{cases} \tag{4}$$

We assume that (as in reality) the interest rate applicable to mortgage debt exceeds the risk–free interest rate. Let $\lambda \in [0,1]$ denote the interest rate margin on mortgage debt and $R_f = 1 + r$ the gross return on investments in the risk–free asset, then $R_D = R_f + \lambda$ denotes the gross interest rate applicable to mortgage debt.

3.3 Optimization problem

In our model, individuals have to make six interrelated decisions at every point in time, $t = \{1, ..., T-1\}$, simultaneously in case they have access to fractional homeownership and five interrelated decisions if their rent–versus–own decision is restricted to be binary. They have to decide upon (i) their consumption level, C_t; (ii) their investment into stocks, S_t; (iii) their homeownership status, Δ_t; (iv) the size of their home, Q_t; (v) the investment in the risk–free asset or the size of their mortgage, B_t, and (vi) upon the fraction of the property owned in case of fractional homeownership, h_t.

The total wealth, W_t, that belongs to the individual at time t (before decisions are made) can be expressed as

$$W_t = \begin{cases} X_t + \mathbb{1}_{\{\Delta_{t-1}=1\}} Q_{t-1} H_t & \text{if} \quad \chi = 0, \\ X_t + \mathbb{1}_{\{\Delta_{t-1}=1\}} h_{t-1} Q_{t-1} H_t & \text{if} \quad \chi = 1, \end{cases} \tag{5}$$

where X_t denotes cash–on–hand, i.e., the sum of (i) labor income received in period t, L_t, (ii) the bond investment from time $t-1$ to t times the gross risk–free rate or the mortgage times the interest rate applicable on mortgage debt, i.e., $B_{t-1}\left(\mathbb{1}_{\{B_{t-1}\geq 0\}} R_f + \mathbb{1}_{\{B_{t-1}<0\}} R_D\right)$, and (iii) the stock investment from time $t-1$ to t times the return earned, i.e., $S_t G^S_{t-1}$.

We use Epstein–Zin preferences (see Epstein and Zin, 1989) to allow for a separation between elasticity of intertemporal substitution, ψ, and risk aversion, γ. When β denotes the individuals time preference, $f(t)$ its probability of survival from period t to $t+1$, and i the inflation rate, the individual's optimization problem is given by[11]

[11] Note that for individuals whose homeownership decision is binary, h_t is not needed as decision variable as it directly follows from Δ_t. To avoid having to state two optimization problems—one for the case with binary homeownership decisions and one for the case with fractional homeownership decisions—we state the optimization problem using both variables. The reader can just apply $\Delta_t = h_t \in \{0,1\}$ when thinking about the case with a binary rent–versus–own decision.

$$\max_{\{C_t,S_t,\Delta_t,Q_t,B_t,h_t\}} V_t(\mathfrak{X}_t) =$$

$$\left((1-\beta)\left(\left(\frac{C_t}{(1+i)^t}\right)^{1-\theta}\left(Q_t\left(1+\zeta\mathbb{1}_{\{\Delta_t=1\}}\right)\right)^{\theta}\right)^{1-\frac{1}{\psi}} + \beta\left(f(t)\,\mathbb{E}_t\left[V_{t+1}(\mathfrak{X}_{t+1})^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{6}$$

subject to

$$\begin{aligned} X_t &= L_t + B_{t-1}\left(\mathbb{1}_{\{B_{t-1}\geq 0\}}R_f + \mathbb{1}_{\{B_{t-1}<0\}}R_D\right) + S_{t-1}G^S_{t-1} \\ &= B_t + S_t + C_t + h_tQ_tH_t - h_{t-1}Q_{t-1}H_t + m(\Delta_t, Q_t, H_t, h_t) + n(h_t, h_{t-1}, Q_t, Q_{t-1}, \Delta_t, \Delta_{t-1}), \end{aligned} \tag{7}$$

$$C_t, Q_t > 0,\ S_t \geq 0, \tag{8}$$

and subject to the borrowing constraint introduced in equation (4) and the budget constraint in equation (5). $\mathfrak{X}_t = [t, \Delta_t, L_t, W_t, h_{t-1}, Q_{t-1}]$ is the vector of state variables required to solve this optimization problem.[12] We explain in greater detail how we solve the optimization problem in Appendix A.

3.4 Calibration

This section outlines the model's base case calibration which is also summarized in Table 1. We assume that individuals enter the life cycle at age $t_0 = 20$, are retired at age $t_{ret} = 66$ and die at the latest at age $t_\Omega = 100$. Consequently, their maximum investment horizon is $T = 81$ years. Prior to t_Ω, we use the empirical estimates of Arias and Xu (2018) to determine the individuals' probabilities of surviving from period t to $t+1$.

We use the estimates of Fischer and Koch (2022), who update the estimates for the labor income process by Cocco et al. (2005) using more recent data, to calibrate the individuals' labor income process. That means that individuals' labor income prior to retirement age is stochastic whereas it is a constant fraction of the last working period's labor income after attaining retirement age. To parameterize the labor income process, we assume—following Fischer and Koch (2022)—that the individuals are married and have attained high school degree.[13]

[12] As h_t is not a decision variable in the case of binary homeownership decisions, h_{t-1} is not needed as a state variable in this case either.

[13] Our results are robust to different parameterizations of the labor income process, i.e., our results are robust

We set the financial market parameters to historical estimates. The annual risk–free rate of return on investments into the risk–free asset is set to $r = 3.9\%$, corresponding to the average U.S. 3–month Treasury Bill rate between 1946 and 2021 and the inflation rate is set to $i = 3.7\%$, the average U.S. inflation rate from 1946 to 2021. We set the interest rate applicable to mortgage debt to $r_D = 7.9\%$, the annual 30–year fixed rate mortgage average from 1971 to 2021.[14] That implies an interest rate margin on mortgage debt of $\lambda = 3.9\%$.

We estimate the average annual return on the S&P500 from 1946 to 2021 to be 12.6% and its standard deviation to be $\sigma_S = 16.6\%$. Following, among others, Claus and Thomas (2001), who doubt that such high equity premia will occur in future periods, we set the equity premium to 3%, implying an expected annual return on the investment into the risky asset of $\mu_S = 6.9\%$.

Using the U.S. S&P's CoreLogic Case–Shiller Home Price Index, we estimate the average annual return on housing from 1946 to 2021 to be $\mu_H = 5\%$ and its standard deviation to be 5.8%. Case and Shiller (1989) and Chinloy et al. (2022) argue that the volatility of individual's home's prices is substantially higher. We follow their consensus and set the standard deviation of the annual expected return on housing to $\sigma_H = 8.7\%$, the empirical estimate of Chinloy et al. (2022) for the U.S. housing market for 1990 to 2020. this is to the best of our knowledge the most recent estimate available.

We mostly use empirical estimates from the literature for parameterizing the housing market. Following Yao and Zhang (2005), we set the individual's relative preference over housing consumption to $\theta = 0.2$, the annual maintenance costs, m_o, to 1.5%, the rent rate to $m_r = 6\%$, and the transaction cost when acquiring a new home, f_p, to 6%. We set the transaction costs that occur when solely adjusting the fraction owned to $f_f = 1\%$ and the "fractional homeownership premium" to $f_a = 2.5\%$.[15] Individuals are allowed to borrow up to 80% of the value of the home they effectively own, i.e., $\delta = 80\%$ (see, e.g., Yao and Zhang, 2005; Bacher, 2021; Paz-Pardo, 2021). Following Kiyotaki et al. (2011) and Fischer and Khorunzhina (2019), we set the individual's preference for living in an owner–occupied home instead of a rented place to $\zeta = 10\%$, i.e., we assign a 10% higher utility per home unit if individuals own at least a fraction of the home they live in.

to individuals' different educational and family statuses.

[14] We use a shorter time horizon for estimating the historical mortgage rate than for estimating the other financial market parameters as the Federal Reserve Bank only provides data from 1971 onwards (source: https://fred.stlouisfed.org/series/MORTGAGE30US; last accessed April 27, 2023).

[15] As we are, to the best of our knowledge, the first to study fractional homeownership using a quantitative life cycle model, there are no empirical estimates for f_f and f_a available yet. Our results are qualitatively similar for different values of $f_f \in [0.5\%, 5.0\%]$ and $f_a \in [0\%, 5.0\%]$ and are available upon request.

Table 1: **Base case calibration**

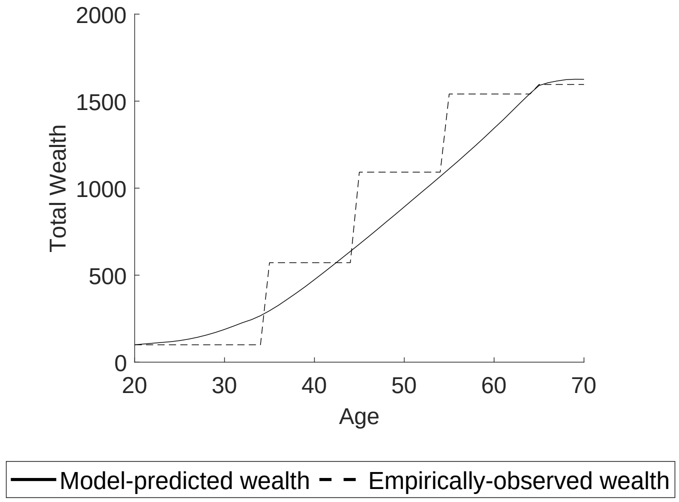
Description	**Symbol**	**Value**	**Source**
Preference parameters			
Degree of risk aversion	γ	5.00	Own choice
Elasticity of intertemporal substitution	ψ	0.10	Own choice
Time preference factor	β	0.96	Own choice
Financial markets			
Expected nominal return on equity	μ_S	6.9%	Claus and Thomas (2001)
Standard deviation of return on equity	σ_S	16.6%	Historical estimate
Nominal risk–free rate	r	3.9%	Historical estimate
Inflation	i	3.7%	Historical estimate
Housing market			
Housing preference parameter	θ	0.2	Yao and Zhang (2005)
Maximum borrowing rate	δ	80.0%	Yao and Zhang (2005)
Risk–free borrowing rate	r_D	7.9%	Historical estimate
Annual rent rate	m_r	6.0%	Yao and Zhang (2005)
Annual maintenance rate	m_o	1.5%	Yao and Zhang (2005)
Transaction costs: home purchase	f_p	6.0%	Yao and Zhang (2005)
Transaction costs: adjustment of fraction	f_f	1.5%	Own choice
Fractional homeownership premium	f_a	2.5%	Own choice
Expected nominal return on housing	μ_H	5.0%	Historical estimate
Standard deviation of return on housing	σ_H	8.7%	Chinloy et al. (2022)
Preference for owning over renting	ζ	10.0%	Kiyotaki et al. (2011)
Correlations			
Stock return and labor income	$\rho_{S,L}$	0.047	Cocco (2005)
Housing return and labor income	$\rho_{H,L}$	0.550	Cocco (2005)
Housing return and stock return	$\rho_{H,S}$	0.000	Cocco (2005)

This table summarizes the base case calibration used in our model.

Following Cocco (2005), we set the correlation between stock return and labor income to $\rho_{S,L} = 0.047$, the correlation between housing return and labor income to $\rho_{H,L} = 0.550$, and we assume that housing and stock returns are uncorrelated, i.e., that $\rho_{H,S} = 0$.

We aim at choosing preference parameters such that we match empirically–observed growth of wealth for individuals not having access to fractional homeownership. Therefore, we set the risk aversion to $\gamma = 5$, the elasticity of intertemporal substitution to $\psi = 0.1$, and the time preference to $\beta = 0.96$. All these values are common in life cycle literature.

Figure 2: **Evolution of model–predicted and empirically–observed growth of wealth**



This figure compares the evolution of model–predicted wealth for individuals not having access to fractional homeownership (solid line) with the evolution of empirically–observed growth of wealth (dashed line) for the accumulation phase of savings from age 20 to age 70. The solid line is the average of 10,000 simulated paths on the policy functions obtained when solving the model outlined in Section 3 but when restricting the homeownership decision to be binary. We use the mean net worth of individuals as reported in the 2019 Survey of Consumer Finances (the most recent survey published) as a proxy for the empirically–observed growth of wealth. The data can be downloaded from https://www.federalreserve.gov/, last retrieved April 8, 2023.

4 Results

In this section, we present the results obtained when solving the quantitative life cycle model outlined in Section 3. First, it is important to show that key model predictions match key features observed in the data.

Figure 2 compares the model–predicted evolution of total savings for individuals whose rent–versus–own decision is restricted to be binary (solid line) with the empirically–observed evolution of total savings (dashed line). The model–predicted evolution of wealth is the average of 10,000 simulated paths on the policy functions obtained when solving the model outlined in Section 3 with the restriction that the rent–versus–own decision is binary. We use the mean net worth of individuals as reported in the 2019 Survey of Consumer Finances (the most recent survey published) as a proxy for empirically–observed growth of wealth.

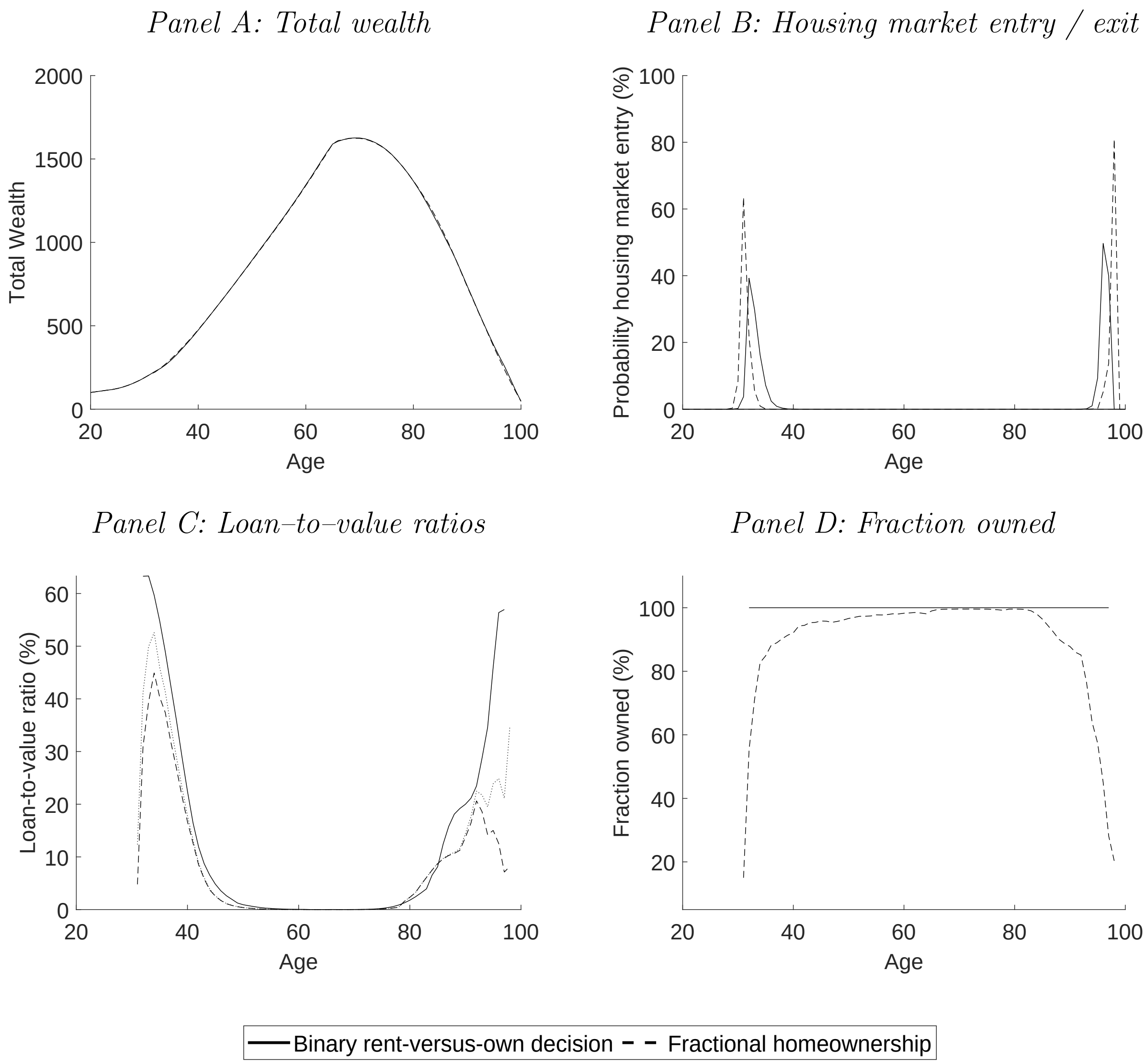
The data can be downloaded from https://www.federalreserve.gov/ (last retrieved April 8, 2023). Our model's predictions match the empirically–observed evolution of total wealth well as the solid line is intersecting with the dashed line for all age intervals the Survey of Consumer Finances provides data for.

Figure 3 compares the model's predictions for the case of binary rent–versus–own decisions (solid line) with the case of fractional homeownership (dashed line). All results are averages of 10,000 simulated paths on the optimal policy functions obtained when solving the optimization problem outlined in Section 3. Panel A compares the evolution of total wealth. Panel B compares the distribution of first–time housing market entry (depicted by the two spikes on the left around age 30) and the distribution of last–time housing market exit (depicted by the two spikes on the right around age 95). The evolution of the loan–to–value ratios is depicted in Panel C. The solid line depicts the loan–to–value ratio for individuals whose rent–versus–own decision is binary. Following up on Footnote 2, the dashed line depicts the loan–to–value ratio that is calculated as the fraction of the outstanding mortgage debt and the value of the home, i.e., the loan–to–value ratio that is calculated as $\frac{-B_t}{Q_t H_t}$, whereas the dotted line depicts the loan–to–value ratio that is calculated as the fraction of the outstanding mortgage debt and the value of the home individuals effectively own, i.e., the loan–to–value ratio that is calculated as $\frac{-B_t}{h_t Q_t H_t}$. Panel D depicts the fraction owned conditional on the individuals being owners, i.e., conditional on $\Delta_t = 1$. Note that the fraction owned is by definition 100% for individuals whose rent–versus–own decision is binary as they are not allowed to own shares of the property they live in.

Panel A of Figure 3 indicates that individuals who have/do not have access to fractional homeownership accumulate roughly the same level of total savings. This is an important finding from the policymakers' perspective as it implies that individuals do not procrastinate on their savings even when they are provided with easier access to the housing market, i.e., this finding provides evidence that fractional homeownership could also be stimulated by policymakers. Instead of leading to procrastination on savings, it is rather the case that individuals with access to fractional homeownership have lower loan–to–value ratios (see Panel C of Figure 3) implying that (i) they can avoid paying the interest rate margin on mortgage debt and (ii) have more liquid wealth available to invest and to consequently earn financial income on.

From Panel B of Figure 3, we see that individuals with access to fractional homeownership enter the housing market earlier and exit it later. The average first–time housing market entrant with access to fractional homeownership is almost two years younger than the average

Figure 3: **Results for binary and fractional homeownership decisions**



This figure compares the model's predictions for individuals whose rent–versus–own decision is binary (solid line) with those for individuals who have access to fractional homeownership (dashed line). Panel A compares the evolution of total wealth and Panel B the distribution of the first–time housing market entry (depicted by the two spikes on the left around age 30) and of the last–time housing market exit (depicted by the two spikes on the right around age 95). Panel C compares the individuals' loan–to–value ratios conditional on the individuals being owners, i.e., conditional on $\Delta_t = 1$. Following up on Footnote 2, the dashed line depicts the loan–to–value ratio that is calculated as the fraction of the outstanding mortgage debt and the value of the home, i.e., as $\frac{-B_t}{Q_t H_t}$, whereas the dotted line depicts the loan–to–value ratio that is calculated as the fraction of the outstanding mortgage debt and the value of the home individuals effectively own, i.e., as $\frac{-B_t}{h_t Q_t H_t}$. Panel D compares the evolution of the fraction owned conditional on the individuals being owners. Note that the fraction owned is not a decision variable when the individuals' homeownership decision is binary but is by definition 100% for all individuals living in owner–occupied homes. All lines are averages of 10,000 simulated paths on the optimal policy functions obtained when solving the optimization problem outlined in Section 3.

first–time housing market entrant who does not have access to fractional homeownership. Further, individuals with access to fractional homeownership exit the housing market for the last time when they are almost one and a half years older than individuals who do not have access to fractional homeownership. Taken these two results together, fractional homeownership extends individuals' time of housing market participation on average by more than three years.[16]

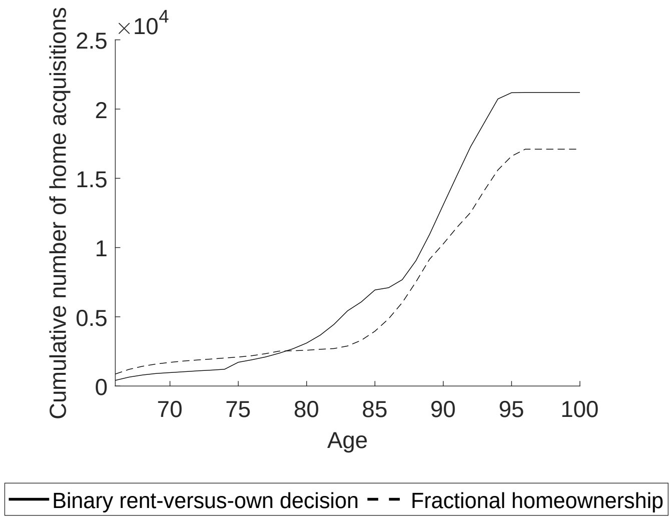
Panel D of Figure 3 provides evidence that the extended housing market participation is really driven by the access to fractional homeownership as particularly young and very old housing market participants make use of the fractional homeownership concept. Younger individuals use the fractional homeownership structure to finance valuable properties while simultaneously keeping their mortgage debt low by just buying small shares of the property they wish to live in. In contrast, old housing market participants reduce the fraction of the home owned instead of borrowing against their housing wealth (see Panel C of Figure 3) or selling the home completely. To put it differently, the elderly with access to fractional homeownership sell fractions of their home to have more liquid wealth available to finance, for example, consumption in their last years instead of fully selling the property or moving to a smaller property.

Panel C of Figure 3 indicates that the loan–to–value ratios are significantly lower for individuals having access to fractional homeownership, irrespective of whether the ratio is calculated as the fraction of outstanding mortgage debt and the full value of the home or as the fraction of outstanding mortgage debt and the value of the home that effectively belongs to the individual. This finding can be explained by the individuals' preferences to only own fractions of a home and borrow less instead of owning the full property, having more outstanding mortgage debt, and facing the interest rate margin on mortgage debt. We find that young individuals have very low loan–to–value ratios when they first enter the housing market but then take out larger mortgages to finance an increase in the fraction of the property effectively owned (see Panel D of Figure 3). However, these mortgages are then repaid at a very fast rate.

In practice, the elderly frequently borrow against the value of their home by entering reverse mortgages contracts (see, e.g., Van Ooijen et al., 2015). In contrast to reverse mortgages/the case with binary rent–versus–own decisions, the loan–to–value ratios of individuals

[16] In our manuscript, we assume that individuals are homogeneous, i.e., that all individuals have the same set of preferences. Therefore, the individuals' housing market entry and exit is concentrated around the ages 30 and 95, respectively. Vestman (2019) shows that the assuming that individuals are heterogeneous, i.e., that individuals have different sets of preferences, would lead to more variation in housing choices.

Figure 4: **Cumulative number of home acquisitions during retirement period**



This figure depicts the cumulative number of home acquisitions during retirement age, i.e., from age 66 to age 100. The results for the case in which the individuals' rent–versus–own decisions are binary are depicted by the solid line and the results for the case in which individuals have access to fractional homeownership are depicted by the dashed line. The lines are constructed using 10,000 simulated paths on the optimal policy functions obtained when solving the optimization problem outlined in Section 3.

with access to fractional homeownership are still relatively low and it is not the case that the house belongs to the bank after the death of the resident (which would be the case if the individual took out a reverse mortgage).

Moving can be stressful, particularly for older individuals, and can even cause adverse health effects such as relocation stress syndrome. Various papers studying moving of the elderly include (i) Yawney and Slover (1973), who document that it is generally more difficult for older individuals to adapt to new living situations, (ii) Venti and Wise (2004), who find—using U.S. panel data—that the majority of individuals does not move even though it would help them to smooth consumption over their life cycle, and (iii) Weeks et al. (2012), who identify drivers (among others gender, age, and household income) of individuals' moving decisions. In this manuscript, we solely study home–to–home relocations for financial reasons but we consider incorporating non–voluntary moving to, for example, nursing homes into life–cycle models of optimal consumption, savings, and housing decisions to be an interesting avenue for further research.

Given that individuals tend to refrain from moving even though it would be optimal for them from a financial perspective, we believe that it would be advantageous if a novel financing option for real estate would require individuals to move less frequently. To shed light on this issue, Figure 4 depicts the cumulative number of home acquisitions during the retirement period, i.e., from age 66 to age 100 by the solid line for the case of binary rent–versus–own decisions and by the dashed line for the case of fractional homeownership. Until the age of 80, individuals acquire homes to a roughly equal extent. After the age of 80, the two lines start to diverge indicating that individuals whose rent–versus–own decision is binary have to move more frequently to make optimal decisions. In contrast, individuals with access to fractional homeownership can just sell fractions of homes they currently live in as indicated by Panel D of Figure 3. Fractional homeownership is thus not only beneficial for younger individuals as it facilitates their housing market entry but also for older individuals as they are not required to move as frequently as individuals whose rent–versus–own decision is binary to still being able to make optimal financial decisions during their retirement phase.

5 Conclusion

The market of fractional homeownership of owner–occupied homes is an emerging market in the U.S., that has up to now received very little attention from researchers. We contribute to closing this gap in the literature by using a realistically calibrated quantitative life cycle model to study the impact of fractional homeownership on optimal consumption, savings, and housing decisions. We define a fractional homeownership contract to be a contract between two parties who share full ownership of a property: (i) an individual who lives in the property permanently and (ii) an institutional investor who sees the property solely as an investment vehicle. As compensation for the right to live in the property full–time, the individual makes periodic rent payments to the institutional investor. The exact amount of these rent payments depends on the value of the home and the fractional ownership structure that can change over time as individuals have the right to adjust the fraction owned in every period.

Our results suggest that, in particular, young individuals facing liquidity constraints and older individuals who are in the phase of decumulating their savings make use of fractional homeownership to finance real estate. Further, we find that individuals who have access to fractional homeownership enter the housing market earlier, exit it later, have lower loan–to–value ratios and have to move less frequently at old age; all in comparison to a setting in which the individuals' rent–versus–own decisions are binary. Nonetheless, individuals in both settings accumulate roughly the same level of total savings over the life cycle.

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A Solution of the optimization problem

To simplify the optimization problem, we normalize with total wealth, W_t, that belongs to the individual at time t. Consequently, $c_t = \frac{C_t}{W_t}$ and $q_t = \frac{Q_t H_t}{W_t}$ are the two decision variables that directly affect the individual's utility. If $V_t(\mathfrak{X}_t) = v_t(\mathfrak{x}_t) \frac{W_t/(1+i)^t}{\left(H_t/(1+i)^t\right)^\theta}$, the simplified optimization problem is:

$$\max_{\{c_t,s_t,\Delta_t,q_t,b_t,h_t\}} v_t(\mathfrak{x}_t) =$$

$$\left((1-\beta)\left(c_t^{1-\theta}\left(q_t\left(1+\zeta \mathbb{1}_{\{\Delta_t=1\}}\right)\right)^{\theta}\right)^{1-\frac{1}{\psi}} + \beta\left(f(t)\,\mathbb{E}_t\left[\left(v_{t+1}(\mathfrak{x}_{t+1})\frac{\frac{W_{t+1}}{W_t(1+i)}}{\left(\frac{H_{t+1}}{H_t(1+i)}\right)^{\theta}}\right)^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}} \tag{9}$$

subject to

$$\begin{aligned} x_t &= \frac{L_t}{W_t} + b_{t-1}\left(\mathbb{1}_{\{b_{t-1}\geq 0\}}R_f + \mathbb{1}_{\{b_{t-1}<0\}}R_D\right) + s_{t-1}G_{t-1}^S \\ &= b_t + s_t + c_t + h_t q_t - h_{t-1}\frac{Q_{t-1}H_t}{W_t} + m(.) + n(.), \end{aligned} \tag{10}$$

$$-b_t \leq \mathbb{1}_{\{\chi=0\}}\delta q_t + \mathbb{1}_{\{\chi=1\}}\delta h_t q_t, \tag{11}$$

$$x_t + \mathbb{1}_{\{\Delta_{t-1}=1\}}\left(\mathbb{1}_{\{\chi=0\}}\frac{Q_{t-1}H_t}{W_t} + \mathbb{1}_{\{\chi=1\}}h_{t-1}\frac{Q_{t-1}H_t}{W_t}\right) = 1, \tag{12}$$

$$c_t, q_t > 0,\ \ s_t \geq 0, \tag{13}$$

in which $s_t = \frac{S_t}{W_t}$, $b_t = \frac{B_t}{W_t}$, $x_t = \frac{X_t}{W_t}$, and $m(.)$ and $n(.)$ are the recurring and non–recurring costs as described in equations (2) and (3), respectively. $\mathfrak{x}_t = \left[t, \Delta_t, \frac{L_t}{W_t}, h_{t-1}, \frac{Q_{t-1}H_t}{W_t}\right]$ is the new vector of state variables needed to solve this optimization problem. For solving the model numerically using backward induction, the continuous state variables h_{t-1}, $\frac{L_t}{W_t}$, and $\frac{Q_{t-1}H_t}{W_t}$ are discretized and the expectation in equation (9) is calculated using Gaussian quadrature. We use parallel computing to expedite our calculations.