Belief Polarization, Unconscious Bias, and Financial Markets

Abstract

This paper studies how the social transmission of information with echo chambers affects financial markets. In an equilibrium model, investors trade competitively in the market based on public information revealed by asset prices and private information accumulated through word-of-mouth communication in echo chambers. I show that unconscious biases are endogenously generated in investors' private signals when information percolates with echo chambers. The unconscious biases drive investors' polarized views, lead to belief polarization, generate excess trading volume, and impact assets' expected returns. The information-sharing process amplifies these effects. The public asset prices help dampen belief polarization but do not fully eliminate investors' unconscious biases.

JEL classification: G11; G12; G14; G41.

Keywords: Belief polarization; Echo chambers; Unconscious bias; Information percolation; Word-of-mouth communication.

1 Introduction

There is mounting concern about growing polarization in beliefs (Gentzkow, 2016; Baldassarri and Park, 2020; Boxell et al., 2020), especially among social network communities. People are deeply divided on controversial issues, and such divisions have become increasingly aligned with race, gender, and partisan identities. The exacerbation of social polarization raises a question: How is the financial market affected by this social phenomenon?[1] The paper aims to understand the impacts of polarized beliefs among investors on asset price, expected returns, and trading volume, and how the social transmission of information influences such a belief polarization in the economy.

To account for the polarization in beliefs, my paper begins with a model of two groups of investors endowed with noisy private information about the fundamental value, inclusive of either positive or negative bias.[2] I assume investors have heterogeneous biases and are unconscious of them. The unconscious biases are endogenous results of investors communicating in "*echo chambers*" – an environment where people reject disliked opinions and only receive information that reinforces their existing views (Wojcieszak et al., 2021; Akçay and Hirshleifer, 2021; Cookson et al., 2022). When investors acquire private information through word-of-mouth communications (Duffie et al., 2009; Andrei and Cujean, 2017) in echo chambers, unconscious biases are generated and amplify belief polarization, defined as the difference in average beliefs of the two groups.

These assumptions are closely related to reality. There is extensive evidence about *homophily* in social interaction (e.g., McPherson et al., 2001; Kossinets and Watts, 2009; Golub and Jackson, 2012) that people tend to associate with individuals with similar characteristics. Being surrounded by homogeneous ideas from like-minded people, individuals' preexisting beliefs are substantially reinforced (Bakshy et al., 2015; Jackson, 2019). Also, social media's success, such as Facebook and Twitter, contributes to social polarization by creating *echo chambers* that sheltered people from opposing perspectives (Sunstein, 2001,

[1] There is evidence for the impact of social polarization on the financial market. For example, Goldman et al. (2020) finds political polarization in corporate financial news. Meeuwis et al. (2022) shows there is belief disagreement based on investors' party affiliations.

[2] Investors in the model have unidimensional opinions which can be simply partitioned by the bias value. This is similar to DeMarzo et al. (2003) that people's opinions converge to a left-right spectrum under persuasion bias.

2007). As Jamieson and Cappella (2010) have pointed out, members of an echo chamber actively discredit external sources of information. By making such untrust between insiders and outsiders, the echo chamber isolates members from outside voices and amplifies biased beliefs. Moreover, social media's news feed fundamentally alters the way people encounter ideas and information by leaving them in a *filter bubble*. First identified by Pariser (2012), the filter bubble refers to the result of preference-fitted algorithms developed by social media. The algorithms filter out information people do not want to see and display favorable opinions based on user's personal information, such as search history, past click behavior, and social network connection. As social media has increasingly become the primary source of news for many people,[3] exposure to selective information confines users to polarized views.

The model framework is based on noisy rational expectation equilibrium model (e.g., Admati, 1985), and also captures differences of opinions between investors (e.g., Banerjee, 2011). One key element of my model is that investors have heterogeneous biases which are unobservable and unverified. The bias unconsciously circulates across the population with investors' communications. As shown by simulations, the bias can be endogenously generated from the information percolation process when investors are restricted in echo chambers and selectively accept information. In practice, unconscious biases can come from selective exposure to confirmational information Cookson et al. (2022). Thus, I assume investors prefer to reject disliked information and accept opinions that confirm what they already believe in (Lord et al., 1979; Nickerson, 1998; Rabin and Schrag, 1999; Schulz-Hardt et al., 2000; Roland G. Fryer et al., 2019). The model does not distinguish between the two scenarios because my goal is to explore how biased information affects investors and financial markets instead of figuring out where the bias comes from. The results are not affected by any particular explanation of the bias.

The unconscious bias has two effects. First, it prevents the aggregate private information from perfectly revealing the fundamental value. This imperfection of aggregate wisdom is different from the standard case in rational expectation literature(e.g., Hellwig, 1980; Admati, 1985; He and Wang, 1995). Second, it leads to investors' misperception about their information and thus distorts investors' beliefs. Since investors are unaware of the bias, they

[3]According to Gottfried and Shearer (2018), 68 percent of US adults get news on social media.

misinterpret the composition of private signals and equilibrium asset prices. Such distorted beliefs diffuse in the market with the information-sharing process.

The paper's main results show that the bias-driven polarization generates belief divergence in the economy and results in excess trading volume and biases expected returns. Furthermore, the social transmission of information through investors' communication reinforces belief polarization. It significantly amplifies the belief polarization's impact on financial markets when investors exchange information at a higher frequency.[4] The results imply that when investors cannot correct the bias in their signals and are unwilling to accept opposing ideas, information sharing can enhance the economy's belief divergence instead of speeding up social learning to reach a consensus.[5] I also show that belief polarization can be dampened by public market prices, emphasizing the importance of reliable information disclosure in reducing social divergence.

My work first contributes to the theory of belief polarization in the financial market. It provides a novel explanation for the scenario where investors' beliefs are polarized on some assets: unconscious biases from echo chamber effects lead to belief polarization. This paper also contributes to the literature on social learning. In contrast to the traditional theory of rational social learning that studies how individual decisions are influenced by others' actions(Banerjee, 1992; Hirshleifer and Welch, 1992), I concentrate on the aggregation effect of social interactions in shaping social beliefs, and I show a result of social belief divergence from information transmission. In the AFA presidential address, Hirshleifer (2020) proposes that a positive bias unconsciously and repeatedly added by investors in exchange of information explains the action boom, which is a systematic directional shift in ideas induced by social transactions. My result is consistent with it, but I do not assume investors are more likely to exchange positive information. I attempt to offer another explanation for biased social learning: it results from the information percolation process with echo chambers.

My work adds to the literature on belief dispersion. I focus on belief polarization generated from unconscious information bias and do not assume investors have different priors and agree to disagree. Investors in my model can have heterogeneous beliefs because of

[4] This is consistent with Andrei (2015) that finds higher meeting intensity stimulates trading volume.
[5] See Golub and Jackson (2012) for social learning being slowed down by homophily.

their private noises, while belief polarization only exists when they have unconscious biases. Furthermore, I show that belief polarization amplifies belief dispersion in the economy. In a concurrent working paper, Meeuwis et al. (2022) show evidence that investors' portfolio choice and trade volume are affected by their party affiliations. The paper may provide a theoretical basis for their results. Heterogeneous economic expectations between Republicans and Democrats may arise from slanted news they receive from CNN vs. FOX or their homophilous communities. Communications with people with the same political stance reinforce their belief disagreements and cause polarization.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. I first introduce the setup and information structure of the model in section 2.1 and 2.2. Then I show how unconscious biases can be endogenously generated in communications by simulated results in section 2.3, which provide a theoretical motivation for the unconscious bias. Section 2.4 describes the equilibrium. Section 3 discusses the belief polarization caused by unconscious biases. My main results for asset pricing implications are presented in section 4. Finally, section 6 concludes. All proofs are in Appendix A.

2 Model

2.1 A dynamic setup

I consider a dynamic model with two trading dates, indexed by $t = 0, 1$, and a final liquidation date $t = 2$. The economy is populated by a continuum of investors indexed by $i \in [0, 1]$. With initial wealth W_0^i, each investor trade a risk-free bond and K risky assets indexed by $k \in \{1, \ldots, K\}$ to consume the final wealth W_2^i. The gross return of the risk-free bond is exogenously set as R_f. The K risky assets have equilibrium prices $\mathbf{P}_t = (P_{1,t}, \ldots, P_{K,t})'$ at each trading date $t = 0, 1$, and pay a vector of liquidating dividends $\mathbf{D} = (D_1, \ldots, D_K)'$ at final date $t = 2$:

$$\mathbf{D} = \boldsymbol{\eta} U + \mathbf{e}, \quad U \sim \mathcal{N}(0, \tau_u^{-1}), \quad \mathbf{e} \sim \mathcal{N}_K(\mathbf{0}, \tau_e^{-1}\mathbf{I}) \tag{1}$$

The vector $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_K)'$ denotes risky assets' exposures to a common factor U and is known to investors. The vector $\mathbf{e} = (e_1, \ldots, e_K)'$ represents each risky asset's idiosyncratic risk. U is independent of e_k and e_k are independent across risky assets, $\forall k \in \{1, \dots, K\}$. Thus $D_k \sim \mathcal{N}(\mathbf{0}, H_k^{-1})$ where $H_k^{-1} \equiv (\tau_u^{-1}\eta_k^2 + \tau_e^{-1})^{-1}$.

Investors have CARA utility with risk tolerance parameter γ. Define investor i's information set and portfolio choice at her trading date t by $\mathcal{F}_t^i$ and $\mathbf{x}_t^i = (x_{1,t}^i, \dots, x_{K,t}^i)'$. Then, each investor solves the following optimization problem when she trade in the market:

$$\max_{\mathbf{x}_t^i} \mathbb{E}\left[-e^{-\frac{1}{\gamma}W_2^i}|\mathcal{F}_t^i\right] \tag{2}$$

subject to

$$W_2^i = W_0^i R_f^2 + R_f \mathbf{x}_0^{i\prime}(\mathbf{P}_1 - R_f \mathbf{P}_0) + \mathbf{x}_1^{i\prime}(\mathbf{D} - R_f \mathbf{P}_1) \tag{3}$$

The vector of incremental net supply of the K risky assets in each period is $\mathbf{X}_t = (X_{1,t}, \dots, X_{K,t})'$ and $\mathbf{X}_t \sim \mathcal{N}_K(\mathbf{0}, \tau_x^{-1}\mathbf{I})$. The equilibrium prices of risky assets are determined by the market-clearing condition:

$$\mathbf{X}_t = \int_0^1 \mathbf{x}_t^i di \tag{4}$$

2.2 Information structure and distorted beliefs

Prior to the initial trading date, each investor is endowed with a vector of noisy private signals $\mathbf{S}_0^i = (S_{1,0}^i, \dots, S_{K,0}^i)'$ about the final dividend $\mathbf{D}$.

$$\mathbf{S}_0^i = \boldsymbol{\eta}(U + \epsilon_0^i) + \mathbf{e} = \mathbf{D} + \boldsymbol{\eta}\epsilon_0^i, \quad \epsilon_0^i \sim \mathcal{N}(0, \tau_{s_0}^{-1}) \tag{5}$$

ϵ_0^i denotes investor i's private noise about the economy. The private precision τ_{s_0} is known by investors and investors correctly perceive the components of endowed signals.

Between the two trading dates, investors communicate and accumulate information in the market. At date $t = 1$, each investor i has possessed a new vector of noisy private signals $\mathbf{S}_1^i$ about $\mathbf{D}$. The newly accumulated private signals $\mathbf{S}_1^i$ of investor i is contaminated by an *unconscious bias* δ^i towards the future economy:

$$\mathbf{S}_1^i = \boldsymbol{\eta}(U + \delta^i + \epsilon_1^i) + \mathbf{e} = \mathbf{D} + \boldsymbol{\eta}\delta^i + \boldsymbol{\eta}\epsilon_1^i, \quad \delta^i \sim \mathcal{N}(\bar{\delta}, \nu^2), \quad \epsilon_1^i \sim \mathcal{N}(0, (\tau_{s_1} - \tau_{s_0})^{-1}) \tag{6}$$

Investors have heterogeneous unconscious biases. δ^i is assumed to have a normal distribution with mean $\bar{\delta}$ and variance ν^2. When $\bar{\delta} > (<)0$, the population is on average positively(negatively) biased. In other words, investors with positive(negative) biases dominates the economy. ν represents the heterogeneity of unconscious biases across investors. When ν is large, investors are more dispersed over their unconscious biases. This is different from the heterogeneity of private noises. So, in my model, investors are heterogeneous in terms of private noise as well as unconscious bias. Unconscious bias means investors are unconscious of the existence of δ^i. Here, the unconscious bias δ^i is exogenously specified in the theoretical model. In the next subsection, I will provide an endogenous interpretation for the unconscious bias δ^i and investors' private precision τ_{s1} after one period of communication. In the model, I assume investors perceive the same private precision τ_{s_0} and τ_{s_1}.

Since investors are unconscious of the biases, they misperceive private signals $\mathbf{S}_1^i$ as unbiased. Investor i's belief about her possessed signals at date $t = 1$ is given by $\mathbf{S}_1^{ii} = (S_{1,1}^{ii}, \ldots, S_{K,1}^{ii})'$:

$$\mathbf{S}_1^{ii} = \mathbf{D} + \boldsymbol{\eta}\epsilon_1^i \tag{7}$$

Investor i also misinterprets other investors' private signals. Her belief about $\mathbf{S}_1^j$, the private signals of investor j, is given by $\mathbf{S}_1^{ij} = (S_{1,1}^{ij}, \ldots, S_{K,1}^{ij})'$:

$$\mathbf{S}_1^{ij} = \mathbf{D} + \boldsymbol{\eta}\epsilon_1^j \tag{8}$$

In this paper, I focus on the case where investors are totally unconscious of the biases. I consider an extension that allows investors to partially know the existence of biases for future research.[6]

At each trading date, investor i observes the public asset prices $\mathbf{P}_t$ and uses it as another information source. She conjectures a linear form of equilibrium prices $\mathbf{P}_t$ which depends

[6] One way that generalizes the case where investors are partially conscious of the biases into the current model is to add a parameter $\rho \in [0, 1]$, which controls how much investors believe signals are biased. For instance, equation (8) can be rewritten as $\mathbf{Z}^{ij} = \boldsymbol{\eta}(\rho U + \sqrt{1-\rho^2}\delta^j + \epsilon^j) + \mathbf{e}$. If $\rho = 0$, investors believe other investors' signals have no informativeness about $\mathbf{D}$, but they correctly perceive the bias inclusive in the signals. If $0 < \rho < 1$, investors misestimate how informative and biased other investors' signals are. When ρ is lower, each investor perceives others' signals as less informative and more biased. Throughout the paper, I focus on the case where $\rho = 1$ and investors do not know the existence of biases at all.

on the aggregate private information in the market and the noisy supply $\mathbf{X}_t$. At date $t = 1$, however, investors are unconscious of the biases and misperceive that the aggregate private information reveals the dividend $\mathbf{D}$. Thus, the conjecture $\mathbf{P}_1^i$ is inconsistent with the *true* equilibrium pricing function $\mathbf{P}_1$, which depends on the average bias $\bar{\delta}$ even if investors are unconscious of the bias. I will discuss the aggregate equilibrium effect of the unconscious biases in detail in section 2.4.

2.3 Information percolation and unconscious bias

In the previous subsection, the unconscious bias of investors is specified exogenously. In this subsection, I will show the unconscious bias δ^i can be viewed as being the endogenous result of information percolation with echo chambers from date 0 to date 1 in my model.

I start by respecifying the information structure of investors. Suppose each investor i is born with one noisy private signal Z_0^i about the common factor U:

$$Z_0^i = U + \epsilon_0^i, \quad \epsilon_0^i \sim \mathcal{N}(0, \tau_{s_0}^{-1}) \tag{9}$$

I assume the population consists of two groups of investors, $\mathcal{A}$ and $\mathcal{B}$, with equal mass. Investors with a negative noise belong to group $\mathcal{A}$ and investors with a positive noise belong to group $\mathcal{B}$:

$$i \in \mathcal{A} \quad \text{if} \quad \epsilon_0^i < 0, \quad i \in \mathcal{B} \quad \text{if} \quad \epsilon_0^i > 0 \tag{10}$$

From date 0 to 1, investors randomly meet and sequentially exchange their accumulated private signals in the economy based on information percolation theory (Duffie et al., 2009). Meetings take place continuously at Poisson arrival times with intensity λ. So, information diffuses across the population though communications.

When meeting with someone at time $s \in (0, 1]$, investor i decides whether she accepts the received signals. I denote the average of investor i's accumulated private signals at time s by $\bar{Z}_s^i$. The rules are that each investor i in group $\mathcal{A}$ or group $\mathcal{B}$ only accepts received signals in the interval $(-\infty, \bar{Z}_s^i + \beta_{\mathcal{A}}]$ or $[\bar{Z}_s^i - \beta_{\mathcal{B}}, +\infty)$, respectively. Hence, each investor lives in her own *echo chamber* governed by, $\beta_g > 0, g \in \{\mathcal{A}, \mathcal{B}\}$, a "*tolerance-to-listen*" parameter. Take investor i from group $\mathcal{A}$ as an example. She likes extreme opinions in the interval $(-\infty, \bar{Z}_s^i]$ but are also tolerant to less extreme information in the interval $(\bar{Z}_s^i, \bar{Z}_s^i + \beta_{\mathcal{A}}]$. The larger the parameter β_g, the more tolerant of group g for "disliked" (less extreme) views and are more willing to accept received signals.

When $\beta_g \to +\infty$, investors from group $g \in \{\mathcal{A}, \mathcal{B}\}$ have open minds and are not restricted in echo chambers. They are willing to listen to anyone they meet and accept their opinions. In contrast, when $\beta_g \to 0$, investors from group g have a "*silo*" mentality and do not listen to people with less extreme views. Their echo chambers completely isolate them from disliked voices. In aggregation, group g is polarized, and the polarization of group g becomes less severe when β_g increases from 0. I will show that unconscious biases are endogenously generated by the echo chamber effect in communications. The impact of unconscious biases is controlled by β_g and λ.

I use $\bar{Z}_1^{ip}$ and $\bar{Z}_1^{ib}$ to denote the average of investor i's accumulated signals after one period of communication with and without echo chambers, respectively. The superscript p represents that communicating with echo chambers is the *polarization* case where β_g is finite and investor $i \in g$ selectively accept received information. In contrast, the superscript b implies the *benchmark* case where β_g is infinite, and investor $i \in g$ faces no echo chambers and accept all signals she has received. Clearly, $\bar{Z}_t^{ip}$ is distorted from $\bar{Z}_t^{ib}$ because of the echo chamber effect. Since this distortion is unconsciously generated by investor i in communications, I define it as investor i's unconscious bias δ^i arising from the information percolation process with echo chambers:

$$\delta^i \equiv \bar{Z}_1^{ip} - \bar{Z}_1^{ib} \tag{11}$$

Suppose in the benchmark case, $\bar{Z}_1^{ib} = U + \epsilon_1^i$, where ϵ_1^i is the average of private noises ϵ_0^j investor i has accumulated from communications. Then $\bar{Z}_1^{ip}$ can be decomposed into

$$\bar{Z}_1^{ip} = U + \delta^i + \epsilon_1^i \tag{12}$$

which corresponds to the part in the bracket of equation (6). This provides theoretical foundation for the unconscious bias δ^i towards the future economy in equation (6).

In equilibrium, I am interested in the aggregate effect of unconscious biases:

$$\bar{\delta} = \int_0^1 \bar{Z}_1^{ip} di - \int_0^1 \bar{Z}_1^{ib} di \tag{13}$$

which can be further decomposed into group bias $\bar{\delta}_{\mathcal{A}}$ and $\bar{\delta}_{\mathcal{B}}$:

$$\bar{\delta} = \underbrace{\left(\int_{j\in\mathcal{A}} \bar{Z}_1^{jp} - \int_{j\in\mathcal{A}} \bar{Z}_1^{jb}\right)}_{\equiv\bar{\delta}_{\mathcal{A}}} + \underbrace{\left(\int_{l\in\mathcal{B}} \bar{Z}_1^{lp} - \int_{l\in\mathcal{B}} \bar{Z}_1^{lb}\right)}_{\equiv\bar{\delta}_{\mathcal{B}}} \tag{14}$$

Since investors do not intend to generate such a bias, $\bar{\delta}_{\mathcal{A}}$ and $\bar{\delta}_{\mathcal{B}}$ can be viewed as the average unconscious bias of group $\mathcal{A}$ and group $\mathcal{B}$ after one period of communication.

The echo chamber channel is shut down in the benchmark case by letting $\beta_{\mathcal{A}}, \beta_{\mathcal{B}} \to \infty$. In this case, both groups are tolerant to different views and decide to accept all signals they have received. As the two groups have identical investors and symmetric signals, the population average of information $\int_0^1 \bar{Z}_1^{ib} di$ after communications is equal to U. Also, the group average of information $\int_{l\in\mathcal{A}} \bar{Z}_1^{lb} = -\int_{l\in\mathcal{B}} \bar{Z}_1^{lb}$. Since when there are echo chambers, the average of information does not have a closed-form solution, I will conduct simulations to study the pattern of unconscious biases. The simulation details can be found in appendix A.1.

Figure 1 presents the simulated results of $\bar{\delta}_{\mathcal{A}}$ and $\bar{\delta}_{\mathcal{B}}$ after one period of information percolation when both groups are restricted in echo chambers[7]. In the example, $\beta_{\mathcal{B}}$ is fixed to be 0, which means investors from group $\mathcal{B}$ do not accept any outside information. In contrast, group $\mathcal{A}$'s tolerance to disliked information is adjusted through $\beta_{\mathcal{A}}$. As shown in figure 1, group bias $\bar{\delta}_{\mathcal{A}} = \bar{\delta}_{\mathcal{B}} = 0$ without the information percolation process ($\lambda = 0$). Without communications, investors only hold their initial signals, and thus no distortion is generated in their information after one period. As implied by equation (14), the aggregate unconscious bias $\bar{\delta} = 0$ in this case.

With communications, group $\mathcal{A}$ has negative unconscious bias while group $\mathcal{B}$ possesses

[7]One special case is that echo chambers only exist in one of the groups. Then $\bar{\delta}$ is governed by the "tolerant-to-listen" parameter of that group. $\bar{\delta} < (>) 0$ if $\mathcal{A}(\mathcal{B})$ is the group that is restricted in echo chambers. The current simulation results can be easily extended to include such cases where only $\beta_{\mathcal{A}}$ or $\beta_{\mathcal{B}}$ has an impact.

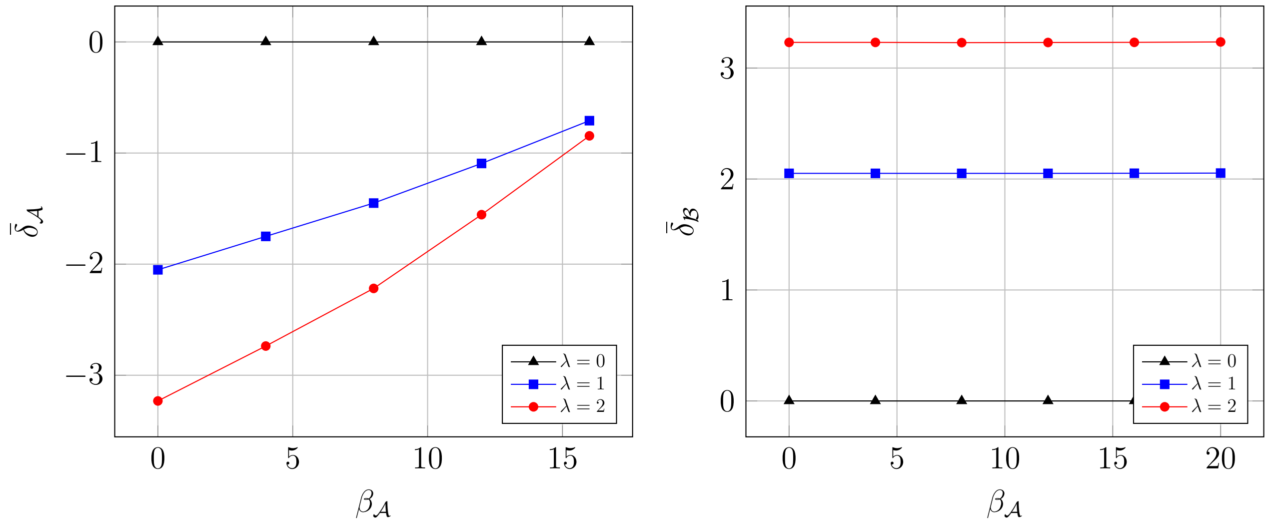

Figure 1: **The unconscious bias of each group**
This figure shows the impact of information percolation with echo chambers on each group's aggregate information. The panel on the left(right) plots the distortion in group $\mathcal{A}(\mathcal{B})$'s average of signals after one-period of information percolation for different values of "tolerance-to-listen" parameter $\beta_{\mathcal{A}}$. $\beta_{\mathcal{B}}$ is fixed as 0. The meeting intensity of the information percolation process is set to be 0, 1 and 2. The number of investors N is $10,000$.

positive unconscious bias. The magnitude of unconscious biases becomes larger when people talk more frequently with each other ($\lambda \uparrow$). The impact of echo chambers is amplified with the information percolation process speeding up. When both groups are restricted in their echo chambers, investors from group $\mathcal{A}(\mathcal{B})$ reject positive(negative) signals from group $\mathcal{B}(\mathcal{A})$ and are negatively(positively) biased. As group $\mathcal{A}$ become more tolerant to outside information ($\beta_{\mathcal{A}} \uparrow$), the absolute value of $\bar{\delta}_{\mathcal{A}}$ decreases. Hence group $\mathcal{A}$ becomes less biased when its investors are more willing to accept outside information. In contrast, $\bar{\delta}_{\mathcal{B}}$ does not vary with $\beta_{\mathcal{A}}$ as investors from group $\mathcal{B}$ reject all the information from group $\mathcal{A}$ and thus are not influenced when the other group becomes less biased. This also implies the absolute value of aggregate bias $\bar{\delta}$ increases when one group becomes less biased $|\bar{\delta}_{\mathcal{A}}| \downarrow$ but the other group does not. As the unconscious bias of each group competes with each other, the group g that has a larger unconscious bias $\bar{\delta}_g$ with a smaller tolerance-to-listen parameter β_g will dominate in aggregation. Therefore, the population is (negatively)positively biased when $\bar{\delta}_{\mathcal{A}}(\bar{\delta}_{\mathcal{B}})$ dominates. The simulation results of aggregate unconscious bias are presented in figure 2.

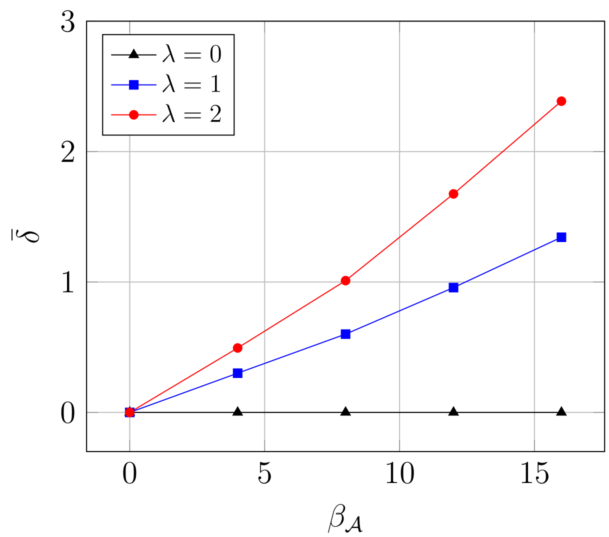

Figure 2: **The population bias**
This figure shows the impact of information percolation with echo chambers on the population. It plots the distortion of population average of signals for different values of "tolerance-to-listen" parameter $\beta_{\mathcal{A}}$. This distortion represents the unconscious bias in aggregation generated from the echo chamber effect. Parameters are consistent with figure 1.

As shown in figure 2, $\bar{\delta} = 0$ in two scenarios: $\lambda = 0$ and $\beta_{\mathcal{A}} = 0$. When there is no information percolation ($\lambda = 0$), echo chambers have no way to affect individual beliefs. The population is unbiased as the information that investors possess does not change. So, communication and exchange of information are needed to generate an aggregate effect of unconscious biases. When $\beta_{\mathcal{A}} = 0$, both group $\mathcal{A}$ and group $\mathcal{B}$ reject all outside information. Since the two groups' populations are symmetric, their average biases become opposite and cancel out each other ($\bar{\delta}_{\mathcal{A}} = -\bar{\delta}_{\mathcal{B}}$). This knife-edged case can be generalized to $\beta_{\mathcal{A}} = \beta_{\mathcal{B}}$. When the two groups have the same level of tolerance with disliked information, group $\mathcal{A}$'s negative bias offsets group $\mathcal{B}$'s positive bias, which leads to zero population bias. So, the difference in the tolerance to disliked information between the two groups is critical to creating non-trivial population bias.

When $\beta_{\mathcal{A}} > 0$ and $\lambda > 0$, $\bar{\delta} > 0$ in figure 2. So, the population average of private information is positively distorted from the benchmark case, and the aggregate unconscious bias is positive. The distortion is sensitive to the "tolerance-to-listen" parameter $\beta_{\mathcal{A}}$, which governs the echo chamber channel of group $\mathcal{A}$. $\bar{\delta}$ increases with $\beta_{\mathcal{A}}$ when $\beta_{\mathcal{B}}$ is fixed, indicating there is more distortion in population when one group becomes more tolerant while the other one is not. Such result arises from the competing effect between $\bar{\delta}_{\mathcal{A}}$ and $\bar{\delta}_{\mathcal{B}}$ which has been discussed in figure 1. When group $\mathcal{B}$ is extremely polarized ($\beta_{\mathcal{B}} = 0$) but group $\mathcal{A}$ is less polarized ($\beta_{\mathcal{A}} > 0$), the echo chamber effect of group $\mathcal{B}$ dominates. The more open group $\mathcal{A}(\beta_{\mathcal{A}} \uparrow)$, the smaller magnitude of $\bar{\delta}_{\mathcal{A}}$, and thus the larger impact of group $\mathcal{B}$'s unconscious bias on the population, which causes larger aggregate distortion. Also, $\bar{\delta}$ is amplified when information percolates at a higher speed which enhances the echo chamber effect in aggregation.

The simulation results in figure 1 and figure 2 have validated the existence of unconscious bias δ^i and its aggregate effect $\bar{\delta}$ across investors. This provides a theoretical basis for each investor's unconscious bias δ^i in equation (6). It is an endogenous result of the information percolation process with echo chambers from time 0 to 1. Furthermore, investors' unconscious biases in aggregation have an impact, which depends on each group's average unconscious bias.

Besides the aggregate effect, the volatility of unconscious biases is also relevant. Figure 3 presented the simulation results for bias volatility ν. Without information percolation, unconscious bias δ^i does not exist, and thus $\nu = 0$. With information percolation and echo chambers, the higher the transmission speed ($\lambda \uparrow$), the larger volatility of investors' unconscious biases. So, given the echo chamber effect $\beta_{\mathcal{A}}$ and $\beta_{\mathcal{B}}$, the unconscious bias becomes more divergent when investors communicate more intensively. On the other hand, when $\beta_{\mathcal{A}}$ increases and group $\mathcal{A}$ becomes more open to disliked information, ν decreases. As investors accept others' signals and average information repeatedly, their biases tend to concentrate. In summary, unconscious bias δ^i becomes more volatile when investors are more restricted by the echo chamber effects, and the information percolation process amplifies its volatility. With the existence of unconscious bias, investors could become more heterogeneous with communications. As I will show in the next sections, ν does not affect the market price in equilibrium and thus can be seen as a secondary parameter in my model. But since ν affects the heterogeneity of the economy, it adds to belief dispersion, and it matters for asset pricing implications like trading volume.

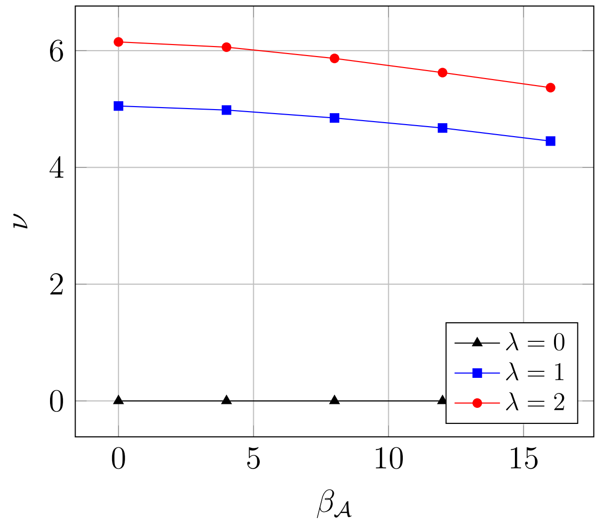

Figure 3: **The bias volatility**
This figure shows the impact of information percolation with echo chambers on the bias volatility. It plots the volatility of δ^i for different values of "tolerance-to-listen" parameter $\beta_{\mathcal{A}}$. Parameters are consistent with figure 1.

Besides the unconscious bias, information percolation also generates heterogeneity in precision across investors, as investors who start with one private signal at date 0 end up with a different number of signals at date 1. Investors who accumulate more signals should have higher precision. However, what really matters is how investors perceive their precision when they trade. To focus on the unconscious bias in a simple setup, I do not consider the heterogeneity in private precision among investors. I assume all investors perceive the same precision in equation (6).

The assumption about the unconscious bias implies investors are unconscious of not only being biased due to the echo chamber effect but also being less precise by selectively accepting information. As implied by the simulation results in figure 4, the average precision increases with the meeting intensity. Thus I assume when meeting intensity λ is large and information percolates at high speed, investors in my model perceive a higher private precision τ_{s_1}.

In summary, this section provides an endogenous interpretation for the unconscious bias δ^i and private precision τ_{s_1} in equation (6). The hypothesis that an unconscious bias arises from the information percolation process with echo chambers has been validated through simulations. It motivates me to specify the unconscious bias in a reduced form in my theoretical model, as in equation (6).

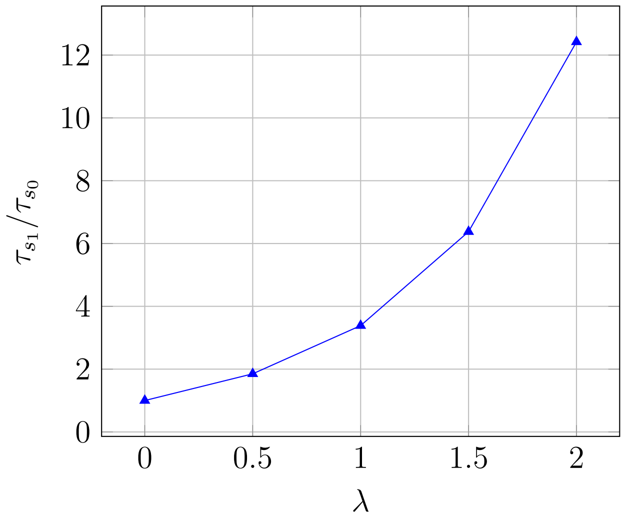

Figure 4: **The average precision**
This figure shows the impact of information percolation on the average precision in the benchmark case. It plots the ratio of average precision τ_{s_1} after the communication over initial precision τ_{s_0} for different values of meeting intensity λ of the information percolation process.

2.4 Equilibrium

At each trading date t, investor i learns about the dividends $\mathbf{D}$ under Gaussian updating through perceived asset prices $\mathbf{P}_t^i$ and the private signals $\mathbf{S}_t^{ii}$. In the minds of investors, $\mathbf{P}_t^i$ and $\mathbf{S}_t^{ii}$ have the same value as $\mathbf{P}_t$ and $\mathbf{S}_t^i$ but misinterpreted components.

Theorem 1. *In equilibrium, the market price of kth risky asset is given by*

$$P_{0,k} = R_f^{-2} L_{0,k}^{-1}(L_{0k} - H_k)D_k - R_f^{-2}\gamma^{-1}L_{0,k}^{-1}(1+\gamma^2\tau_{s_0}\tau_x\eta_k^{-2})X_{0,k} \tag{15}$$

and

$$\begin{aligned} P_{1,k} = {} & R_f^{-1}L_{1,k}^{-1}(L_{1,k}-H_k)D_k + R_f^{-1}L_{1,k}^{-1}[(\tau_{s_1}-\tau_{s_0})\eta_k^{-2} + \gamma^2(\tau_{s_1}-\tau_{s_0})^2\tau_x\eta_k^{-4}]\eta_k\bar{\delta} \\ & - R_f^{-1}\gamma^{-1}L_{1,k}^{-1}(1+\gamma^2\tau_{s_0}\tau_x\eta_k^{-2})X_{0,k} - R_f^{-1}\gamma^{-1}L_{1,k}^{-1}[1+\gamma^2(\tau_{s_1}-\tau_{s_0})\tau_x\eta_k^{-2}]X_{1,k} \end{aligned} \tag{16}$$

where $L_{t,k} \equiv Var^{-1}[D_k|\mathcal{F}_t^i]$ is the conditional precision given by

$$L_{0,k} = H_k + \tau_{s_0}\eta_k^{-2} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x \tag{17}$$

and

$$L_{1,k} = H_k + \tau_{s_1}\eta_k^{-2} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-4}\tau_x \tag{18}$$

Though investors are unconscious of any bias, asset prices are indeed affected by the unconscious bias. The second term on the RHS of (16) shows that unconscious biases have an aggregate effect in equilibrium. The misinterpretation of information distorts investor i's learning process at date 1 as she misunderstands the composition of price and private signals.

Investor i's *perceived* conditional precision L_{1k} depends on the precision of private signals and her perceived asset prices. However, those precisions are not the actual precisions as investor i is unconscious of biases in her perceived signals. She is less precise about $\mathbf{D}$ than she believes when updating her beliefs. At date 1, investor i's conditional expectation places weights on her information in proportion to those signals' relative precisions. However, as I have mentioned, investor i's perceived precisions are not the true precisions. The weight investor i assigns to her signals are biased due to the unconsciousness of biases. This effect is amplified along with the information percolation process and distorts investors' portfolio choices and asset prices in equilibrium.

Asset prices are positively related to the dividends and biases in the economy and negatively associated with the aggregate supply. I am interested in how unconscious bias affects equilibrium prices. Replacing (18) into (16) and rearranging, I have

$$R_f L_{1,k}(L_{1,k} - H_k)^{-1}P_{1,k} = D_k + \frac{(\tau_{s_1} - \tau_{s_0})\eta_k^2 + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\tau_x}{\tau_{s_1}\eta_k^2 + \gamma^2\tau_{s_0}^2\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\tau_x}\eta_k\bar{\delta} - \Delta \tag{19}$$

where Δ includes the terms about X_{0k} and X_{1k}.

$P_{1,k}$ relies more on the average unconscious bias $\bar{\delta}$ when τ_{s_1} the average precision at date 1 increases. Thus the equilibrium prices become more sensitive to the unconscious bias when investors communicate at a higher frequency ($\lambda \uparrow$) with each other. Although at the same time, prices reveal more about dividend D_k, they are more contaminated by the biases in the economy. The price informativeness may decrease, and investors cannot learn more from the price.

At date 1, investor i's conditional expectation about the dividend D_k is given by

$$\begin{aligned} L_{1,k}\mathbb{E}[D|\mathcal{F}_1^i] = (L_{1,k} - H_k)D_k - \gamma\tau_{s_0}\tau_x\eta^{-2}X_{0,k} - \gamma(\tau_{s_1} - \tau_{s_0})\tau_x\eta_k^{-2}X_{1,k} \\ + (\tau_{s_1} - \tau_{s_0})\eta_k^{-1}\delta^i + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-3}\tau_x\bar{\delta} + \tau_{s_0}\eta_k^{-1}\epsilon_0^i + (\tau_{s_1} - \tau_{s_0})\eta_k^{-1}\epsilon_1^i \end{aligned} \tag{20}$$

which is affected by not only her unconscious bias δ^i but also the average bias in the economy. As I have discussed in section 2.3, the magnitude of the average unconscious bias is endogenously determined by the information percolation process with echo chambers.

The more restricted by the echo chamber, the more biased investor i is about her information ($\delta^i \uparrow$), which results in a more positive expectation about the asset dividends when $\delta^i > 0$ and vice versus. Investor i's expectation is also affected by the average bias of the population as she learns from equilibrium market prices. Therefore, even if investor i has no individual bias δ^i because either she rejects echo chambers and listens to all information or she does not communicate with others at all, her expectation is still distorted by the unconscious biases of other investors when there exist echo chambers in the market. When investors meet and talk at a higher frequency, $\tau_{s_1} \uparrow$, the average bias has a larger impact on investor i's conditional expectation. This is because investors believe the market price incorporates more private information when the information-sharing process speeds up. Hence they rely more on the price to update their beliefs, which strengthens the impact of $\bar{\delta}$.

3 Belief Polarization

This section aims to understand how the unconscious bias generated from echo chamber effects in the information percolation process drives belief polarization in financial markets. I first define a measure of belief polarization to discuss the bias's impact.

Definition 1. ***Belief polarization*** $\mathcal{P}_t = (\mathcal{P}_{t,1}, \ldots, \mathcal{P}_{t,K})'$ *of the economy is measured as the distance between the average beliefs about future dividends* $\mathbf{D}$ *of the two groups that trade at date* t*:*

$$\mathcal{P}_t \equiv \int_{i\in\mathcal{A}} \mathbb{E}\left[\mathbf{D}|\mathcal{F}_t^i\right] di - \int_{i\in\mathcal{B}} \mathbb{E}\left[\mathbf{D}|\mathcal{F}_t^i\right] di \tag{21}$$

Substituting the conditional expectations into the definition, I have the measurement of belief polarization as stated in the following proposition.

Proposition 1. *Belief polarization exists only when there are echo chambers in the economy and $\bar{\delta}_{\mathcal{A}} \neq \bar{\delta}_{\mathcal{B}}$.*

$$\mathcal{P}_{0,k} = 0, \quad \mathcal{P}_{1,k} = (\bar{\delta}_{\mathcal{B}} - \bar{\delta}_{\mathcal{A}})\phi_k \tag{22}$$

where ϕ_k is the polarization coefficient for the kth asset, which captures the unconscious biases' impacts on generating belief polarization:

$$\phi_k = \frac{(\tau_{s_1} - \tau_{s_0})(\eta_k^2\tau_e + \tau_u)\eta_k^3}{\tau_u\tau_e\eta_k^4 + (\eta_k^2\tau_e + \tau_u)[\tau_{s_1}\eta_k^2 + \gamma^2\tau_{s_0}^2\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\tau_x]} \tag{23}$$

As implied by equation (22), belief polarization is driven by unconscious biases. It exists only in the second trading session when echo chambers in the information percolation process generate unconscious biases in the economy. The degree of polarization is controlled by the polarization coefficient ϕ_k, which depends on the private precision τ_{s_1} at date 1. As I have discussed in section 2.3 about simulations, τ_{s_1} is monotonically increasing in the information percolation speed λ. So, the belief polarization coefficient ϕ_k is driven by investors' meeting intensity λ. One knife-edged case is when $\bar{\delta}_{\mathcal{A}} = \bar{\delta}_{\mathcal{B}}$ and the two groups have the same average bias. In this case, the population is biased, but there is no belief polarization in the economy as the two groups are actually one.

Figure 5 shows how the impact of unconscious biases on belief polarization of the kth asset, ϕ_k, is affected by its risk exposure η_k and the information percolation speed λ. First, given λ, the magnitude of ϕ_k increases with η_k. It indicates that the unconscious bias's impact on polarization is amplified for assets more exposed to the fundamental risk. So, investors are more polarized on assets with larger risk exposures to the economy.

Second, the polarization coefficient has a hump-shaped pattern, which is mathematically proved in appendix A.3. The shape implies that the unconscious bias's impact is amplified but then attenuated by the information percolation process. This is because private information and market prices are two forces competing with each other when updating beliefs. Individuals' unconscious biases in private signals contribute to the divergence of average beliefs between the two groups. The more intensely investors communicate before they trade, the more unconscious biases have spread within the echo chambers, reinforcing the

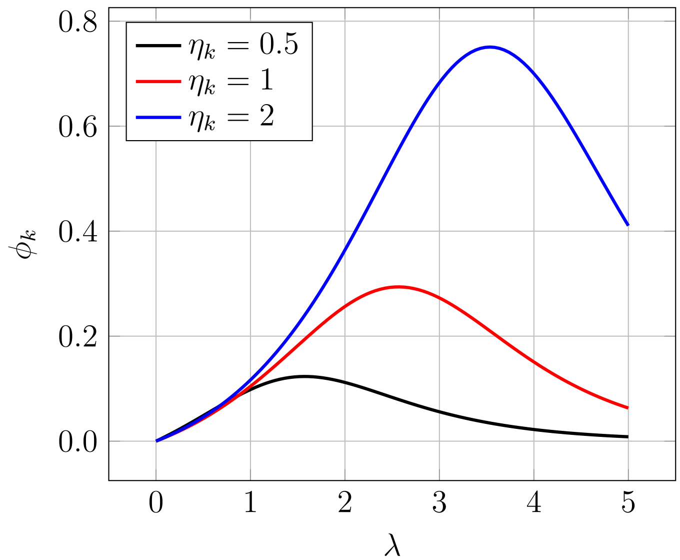

Figure 5: **Polarization coefficient with information percolation speed**
This figure plots the polarization coefficient ϕ_k, the unconscious bias's impact on polarization of the kth asset at date 1, as a function of λ, the information percolation speed. The parameters are set at the following values: $\gamma = 1$, $\tau_u = \tau_x = 2, \tau_e = 1, \tau_{s_0} = 0.05$.

polarization of beliefs. Thus the impact of unconscious biases increases with the information percolation process, especially when the meeting intensity is low and investors rely more on their private information.

On the other hand, the market prices become more informative to investors when the meeting intensity increases. In the eyes of investors who are unconscious of their biases, the market prices are more precise about the payoff when noticing everyone is talking about the news. Investors thus put a larger weight on market prices when updating beliefs. Since all investors condition their expectations about the payoff on the market price in the same way, their beliefs gradually converge, and divergence starts to vanish when the information percolation speed is very high. However, this does not mean unconscious biases disappear. The market prices will not entirely eliminate the belief polarization driven by unconscious biases.

Proposition 2. *Market prices help dampen, through cannot fully eliminate, belief polarization.*

Figure 6 further verifies the effect of market price on dampening polarization by plotting the polarization coefficient when investors use or do not use market prices to update beliefs at

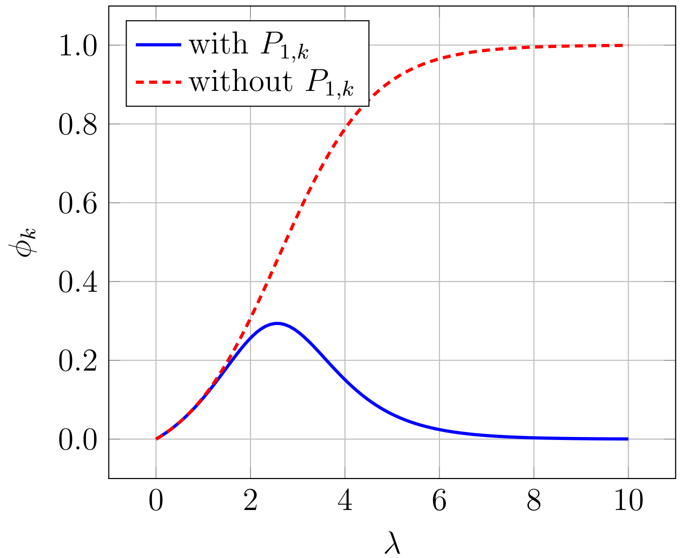

Figure 6: **Polarization coefficient with/without public market prices**
This figure plots the polarization coefficient ϕ_k at date 1, as a function of λ, for two cases. The blud solid line represents the case where investors use market prices at date 1 to update beliefs. The red dashed line represents the case where investors ignores the public market information. The parameters are set at the following values: $\gamma = 1$, $\tau_u = \tau_x = 2, \tau_{s_0} = 0.05, \tau_e = 1, \eta_k = 1$.

date 1. Comparing the two cases, it is clear that unconscious biases' impacts on polarization are smaller with $P_{1,k}$ as an information source. Hence, public market prices help reduce belief polarization. When information percolation intensity is extremely high, market prices can almost eliminate polarization. Overall, the results in figure 6 emphasize the dampening effect of market prices on polarization.

3.1 Polarization amplifies dispersion

In this section, I will discuss the relation between belief polarization and well-documented belief dispersion. I use $\mathcal{D}_{t,k}$ to denote the belief dispersion on the kth asset. Following the common literature (Shalen, 1993; Banerjee, 2011; Atmaz and Basak, 2018), $\mathcal{D}_{t,k}$ is calculated as the cross-sectional variance in investors' posterior expectations:

$$\mathcal{D}_{t,k} \equiv \text{Var}\left[\mathbb{E}[D_k|\mathcal{F}_t^i] - \int_0^1 \mathbb{E}[D_k|\mathcal{F}_t^i]di\right] \tag{24}$$

In literature, belief dispersion is often determined by the noise in private signals. But in my model, belief dispersion is also affected by unconscious biases, through which polarization enters belief dispersion. I have the following proposition about belief dispersion and polarization:

Proposition 3. *Belief dispersion after one period of communication can be decomposed as*

$$\mathcal{D}_{1,k} = \frac{1}{2}\mathcal{D}_{1,k,\mathcal{A}} + \frac{1}{2}\mathcal{D}_{1,k,\mathcal{B}} + \frac{1}{4}\mathcal{P}_{1,k}^2 \tag{25}$$

where

$$\mathcal{D}_{1,k,\mathcal{A}} = Var\left[\mathbb{E}[D_k|\mathcal{F}_1^i, i \in \mathcal{A}] - \int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right] \tag{26}$$

$$\mathcal{D}_{1,k,\mathcal{B}} = Var\left[\mathbb{E}[D_k|\mathcal{F}_1^i, i \in \mathcal{B}] - \int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right] \tag{27}$$

The decomposition shows that polarization is one component of belief dispersion. But since belief polarization $\mathcal{P}_1$ only exists when information percolates with echo chambers, belief dispersion is affected by polarization only when there are echo chambers. When information diffuses with echo chambers, I can write the belief dispersion in equation (24) at date 1 as

$$\mathcal{D}_{1,k} = L_{1,k}^{-2}[\tau_{s_1} + (\tau_{s_1} - \tau_{s_0})^2\nu^2]\eta_k^{-2} \tag{28}$$

So, $\mathcal{D}_{1,k}$ is enhanced by ν^2, the variance of unconscious biases. Thus belief dispersion increases when unconscious bias becomes more volatile across investors. When information percolates without echo chambers, the economy has no polarization, $\mathcal{P}_1 = 0$ as there is no bias, $\nu = 0$. In this benchmark case, the belief dispersion is equal to:

$$\mathcal{D}_{1,k}^* = L_{1,k}^{-2}\eta_k^{-2}\tau_{s_1} \tag{29}$$

The amplification effect of polarization on belief dispersion is presented in figure 7. With polarization, belief dispersion is always more significant than the case without polarization. Also, the relation between belief polarization and information percolation speed is hump-shaped. At the initial, investors rely more on private information to form expectations, which leads to belief divergence because of noisy private signals. However, when information percolation speeds up, and investors talk more frequently, investors' private information tends

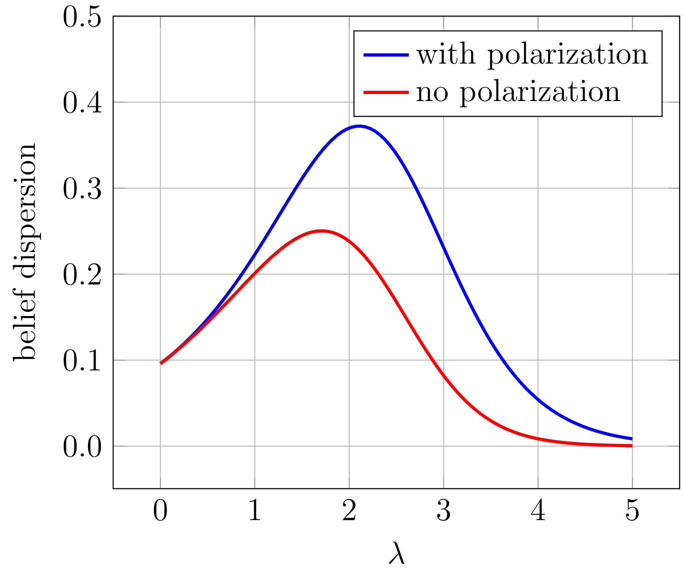

Figure 7: **Belief dispersion**
This figure plots $\mathcal{D}_{1,k}$, belief dispersion after one period of communication, as a function of λ, the meeting intensity. The parameters are set at the following values: $\gamma = 1$, $\tau_u = \tau_x = 2, \eta_k = 1$.

to converge, and thus dispersion starts to decline. When information percolation speed is exceptionally high, the economy gets rid of belief dispersion in the case without polarization, as shown in figure 7. But with polarization, belief dispersion will not be entirely eliminated by information percolation. Polarization generated from echo chamber effects always amplifies belief dispersion in the economy.

4 Asset Pricing Implications

4.1 Trading volume

The first asset pricing implication of my model is about trading volume. I will examine how volumes of trade in the market are influenced by the transmission of unconscious biases across the population. In equilibrium, the traded volume is the cross-sectional average, across investors, of the absolute change in their positions over time. Therefore, the trading volume $\mathcal{V}_1 = (\mathcal{V}_{1,1}, \ldots, \mathcal{V}_{1,K})'$ at date 1 is given by

$$\mathcal{V}_1 \equiv \int_0^1 |\mathbf{x}_1^i - \mathbf{x}_0^i| di \tag{30}$$

Substituting the equilibrium results of $\mathbf{x}_1^i$ and $\mathbf{x}_0^i$ into (30), I have for the kth asset:

$$\mathcal{V}_{1,k} = \int_0^1 |\gamma(\tau_{s_1} - \tau_{s_0})\eta_k^{-1}\epsilon_1^i + \gamma(\tau_{s_1} - \tau_{s_0})\eta_k^{-1}\delta^i - \gamma[(\tau_{s_1} - \tau_{s_0})\eta_k^{-2} + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\tau_x\eta_k^{-4}]\eta_k\bar{\delta} + X_{1,k}| di \tag{31}$$

In the benchmark case, information transmits without echo chambers and there is no unconscious bias in the economy. The corresponding trading volume is denoted by $\mathcal{V}_1^* = (\mathcal{V}_{1,1}^*, \ldots, \mathcal{V}_{1,K}^*)'$:

$$\mathcal{V}_{1,k}^* = \int_0^1 |\gamma(\tau_{s_1} - \tau_{s_0})\eta_k^{-1}\epsilon_1^i + X_{1,k}| di \tag{32}$$

Proposition 4. *The relationship among the trading volume of the kth asset, $\mathcal{V}_{k,1}, \mathcal{V}_{k,1}^*$, is given by*

$$\mathcal{V}_{1,k} > \mathcal{V}_{1,k}^* \tag{33}$$

The unconscious bias generates excess trading volume.

As shown in figure 8, when there is no unconscious bias ($\bar{\delta} = 0$)[8], the market has no excess trading volume[9]. With unconscious biases, the excess trading volume is generated and further enhanced with larger population bias $\bar{\delta}$. The unconscious bias results in excess trading volume in the market as it (i) distorts investors asset allocations; (ii) leads investors trade more aggressively. So, unconscious biases help explain the excess trading volume in the market, especially the spike of trading volume during the period when the market is polarized. In addition, from figure 8, the excess trading volume is decreasing in η_k, the magnitude of risk exposure. Investors trade more intensively on assets less exposed to the

[8]Here by saying no unconscious bias, I mean no echo chamber effects and thus no δ^i is generated. This excludes the knife-edged case where the two groups' biases cancel out

[9]To focus on the excess part generated from my model setup, I assume the benchmark I compare with has included all situations that cause additional trades in the literature. For example, He and Wang (1995) point out information flow is accompanied with large price changes and generate trading volume; Banerjee and Kremer (2010) argue that trade volume spikes up when disagreement is large; Andrei (2015) shows information-sharing among investors cause large trading volume

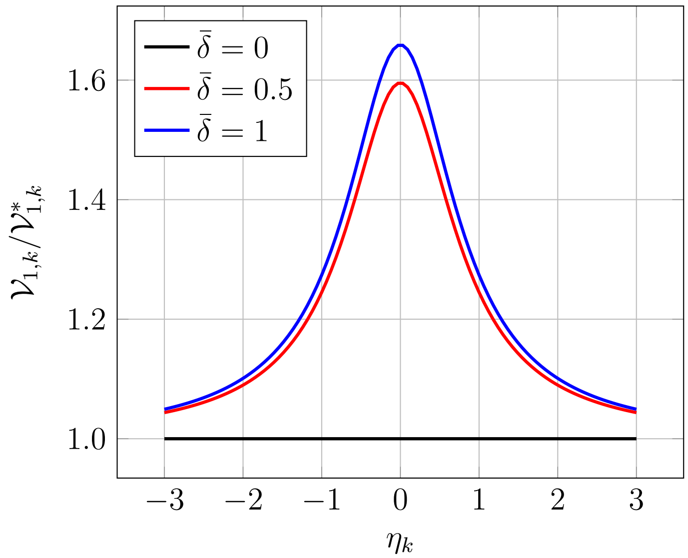

Figure 8: **Excess trading volume with risk exposure**
This figure plots the kth asset's excess trading volume with risk exposure η_k for different values of population bias $\bar{\delta}$. The parameters are set at the following values: $\gamma = 1$, $\tau_u = \tau_x = 2, \tau_e = 1, \lambda = 2, \nu = 2$.

fundamental risk as they have higher precision about those assets.

Figure 9 shows that the excess trading volume generated by the unconscious bias increases with the information percolation speed. Since private precision τ_{s_1} is increasing in the meeting intensity λ, the result corresponds to the intuition that investors trade more aggressively when they communicate more and thus feel they are better informed. Furthermore, the excess trading volume is amplified by higher bias volatility. When δ^i is more volatile, investors are more heterogeneous regarding to their beliefs, which gives rise to excess trading.

Overall, this section shows that unconscious biases can generate "additional" excess trading volume in the economy and the only driven factor is information percolation with echo chambers. This provide a new explanation for the massive volume in the market, especially when beliefs are polarized.

4.2 Expected returns

The expected dollar returns of risky assets at date 1 are given by

$$\mathbb{E}[\mathbf{R}_1] \equiv \int_i \mathbb{E}[\mathbf{D}|\mathcal{F}_1^i]di - R_f\mathbf{P}_1 \tag{34}$$

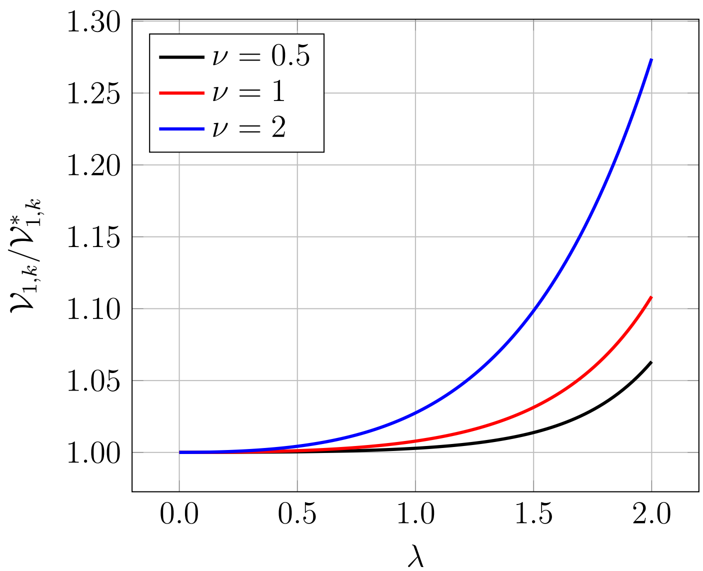

Figure 9: **Excess trading volume with meeting intensity**
This figure plots the kth asset's excess trading volume with meeting intensity λ for different values of bias volatility ν. The parameters are set at the following values: $\gamma = 1$, $\tau_u = \tau_x = 2, \tau_e = 1, \eta_k = 1$.

Denote the expected return in the benchmark case where information percolates without echo chambers by $\mathbb{E}^*[\mathbf{R}_1]$, then I have the following proposition.

Proposition 5. *The relationship between the expected return of the kth asset,* $\mathbb{E}[R_{1,k}]$ *and* $\mathbb{E}^*[R_{1,k}]$, *is given by*

$$\mathbb{E}[R_{1,k}] = \mathbb{E}^*[R_{1,k}] - f_k \eta_k \bar{\delta} \tag{35}$$

where

$$f_k = \frac{\gamma^2(\tau_{s_1} - \tau_{s_0})^2 \tau_x}{\eta_k^2(\tau_u^{-1} + \eta_k^{-2}\tau_e^{-1})^{-1} + \eta_k^2 \tau_{s_1} + \gamma^2 \tau_{s_0}^2 \tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2 \tau_x} \tag{36}$$

From equation (35), when the economy is unconsciously negatively biased ($\bar{\delta} < 0$), I have $\mathbb{E}[R_{1,k}] > (<)\mathbb{E}^*[R_{1,k}]$ for assets with $\eta_k > (<)0$. So, with negative bias towards the economy, investors require higher expected return to hold assets with positive risk exposures. This pattern is verified in figure 10. When the unconscious bias has no aggregate effect ($\bar{\delta} = 0$), expected returns are not affected by information percolation and echo chambers ($\Delta\mathbb{E}[R_{1,k}] = 0$). When $\bar{\delta} > 0$, it has a positive(negative) impact on the price of assets cyclical(countercyclical) with the economy, which leads to a lower(higher) expected return. Moreover, the impact is amplified with the magnitude of population bias $\bar{\delta}$. Hardouvelis

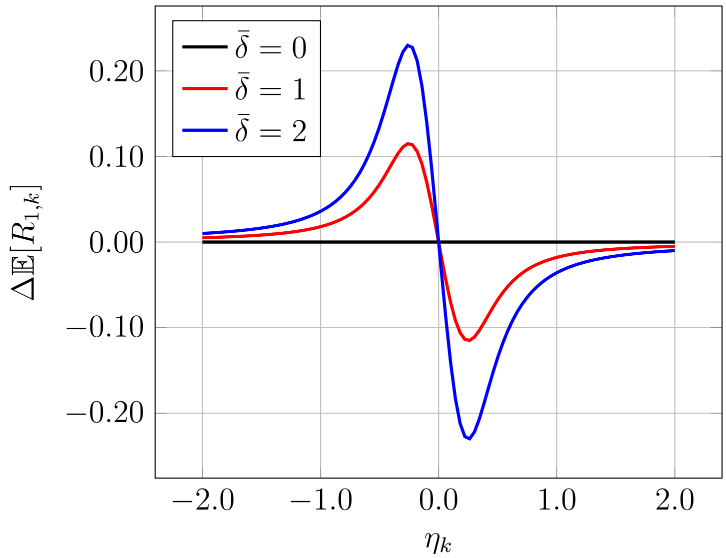

Figure 10: **Expected return with risk exposures**
This figure plots the kth asset's expected return change with its risk exposure η_k for different values of bias volatility ν. The parameters are set at the following values: $\gamma = 1$, $\tau_u = \tau_x = 2, \tau_e = 1, \eta_k = 1$.

et al. (2022) argue that assets with larger polarization earn lower expected returns because of short-sale constraint. In my model, assets that are more exposed to the economy faces larger polarization as shown in section 3. But how expected returns are affected depends on the sign of population bias $\bar{\delta}$ and asset risk exposure η_k.

Figure 11 plots f_k in equation (36) with λ for different values of risk exposure. Clearly, the impact of aggregate unconscious bias increases with the information percolation speed. So, when investors meet and talk at a higher frequency, $\bar{\delta}$ affects assets' expected returns more. This is because market prices relies more on the unconscious bias when information percolation speeds up and investors perceive higher precision. The price movements affect expected returns. Also, figure 11 shows the impact is larger for assets less exposed to the fundamental risk. The expected returns of assets with smaller risk exposure are affected more when there are unconscious biases in the economy.

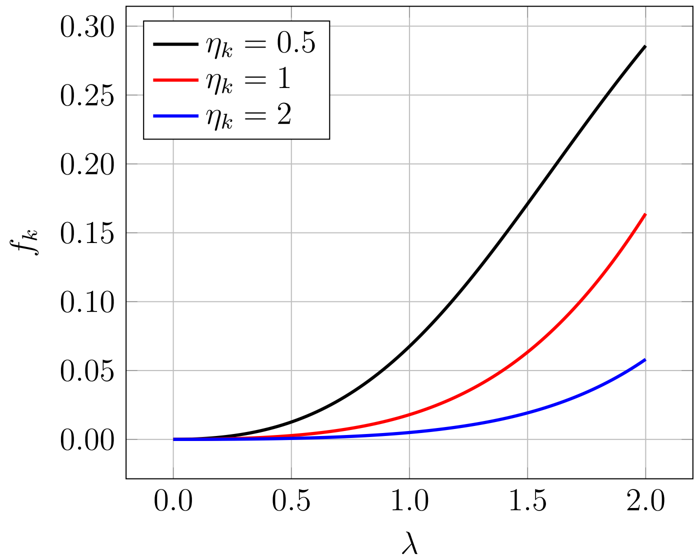

Figure 11: **Expected return with meeting intensity**
This figure plots f_k with meeting intensity λ for different values of risk exposure η_k. The parameters are set at the following values: $\gamma = 1$, $\tau_u = \tau_x = 2, \tau_e = 1$.

5 An implication to political economy: The echo chamber effect on investors' rebalancing behaviors

This section applies to political economy my work on the social transmission of unconscious bias. In this setup, unconscious bias is identified as partisanship bias. Specifically, I consider an extension of my previous model to provide a theoretical basis for politically affiliated investors' rebalancing behaviors after a surprising election result (Meeuwis et al., 2022; Cassidy and Vorsatz, 2021).

WORK IN PROGRESS (COMING SOON)

6 Conclusion

The paper develops an equilibrium model that endogenously generates unconscious biases through information percolation process with echo chambers. The framework nests both rational expectation and differences of opinions. Investors update their beliefs conditional on private signals accumulated through word-of-mouth communication and public information revealed by the market price. The paper's results show that the information bias has an aggregate equilibrium impact on the asset price even if investors are unconscious of it. The unconscious bias is not like a behavioral issue that disappears in aggregation. It indeed affects the equilibrium, and it gives rise to belief polarization in the population. When investors are unconscious of the biases in their signals, more communication is not always good. Instead, information sharing could worsen the situation because more exchange of information amplifies belief polarization instead of speeding up social learning to reach a consensus. The paper presents and discusses some asset pricing implications. The unconscious bias from echo chamber effect leads to excess trading volume. It also affects the expected returns in the financial market.

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A Appendix

A.1 Simulation Steps in Section 2.3:

My simulation involves two parts: the data generating process and information percolation process. The data generating process simulates two groups of investors with normally distributed private noise. The information percolation process determines the communication rules and compute the unconscious bias.

- Step 1: The private noise ϵ_0^i of $N = 100,000$ agents are randomly drawn from a normal distribution with mean $\mu = 0$ and precision $\tau_{s_0} = 0.01$. Agents with a negative bias are marked as group $\mathcal{A}$. a positive noise are marked as group $\mathcal{B}$
- Step 2: For each agent, simulate her meeting time from a Poisson distribution with intensity λ and store as (meeting time, agent). One agent may correspond to several meetings.
- Step 3: Determine the meeting sequences by sorting the meeting time from 0 to 1 and match the adjacent agents to meet and communicate with each other. For example, suppose the simulated meeting sequence is $[(t_0, agent_1), (t_1, agent_2), (t_2, agent_3), (t_3, agent_1), \dots]$, then meeting happens between agent 1 and agent 2, and between agent 3 and agent 4. So, agent 1 first communicate with agent 2 and then communicate with agent 3. (One may notice that when simulated meeting time is odd, there is one agent who cannot be matched with another agent. In this case, I ignore the last meeting which is not a problem considering the large number of agents simulated.)
- Step 4: Determine the exchange of information according to the meeting sequences. Agents from group $\mathcal{A}$ only accept private signals with $\epsilon_0^i \leq \beta_{\mathcal{A}}$ while agents from group $\mathcal{B}$ only accept private signals with $\epsilon_0^i \geq \beta_{\mathcal{B}}$. Otherwise, agents stay with the signals they possess before the meeting. After each meeting, I update agents' private noise as the average noises they have accumulated.

To calculate unconscious bias of each agent and the average bias of each group, we need a benchmark case. Set $\beta_{\mathcal{A}}$ and $\beta_{\mathcal{B}}$ as 10,000 to have agents accepting any information they receive during the meeting. Store the agents' private noise in the benchmark case. Then we can adjust parameters $(\lambda, \beta_{\mathcal{A}}, \beta_{\mathcal{B}})$ in the polarization case. The difference between the updated private noise after all meetings for each agent in the benchmark case and the polarization case is her unconscious bias. Taking the average over all agents I have the population bias and similarly I have the average bias over each group.

A.2 Proof of Theorem 1:

Investor i conjectures the market price of the kth asset as a linear function of the fundamental value and its aggregate supply:

$$P_{0,k}^i = a_{0k} D_k + b_{00k} X_{0,k} = P_{0,k} \tag{A.1}$$

and

$$P_{1,k}^i = a_{1k} D_k + b_{10k} X_{0,k} + b_{11k} X_{1,k} \tag{A.2}$$

while the true market prices at $t = 1$ are given by

$$P_{1,k} = a_{1k} D_k + a_{2k} \eta_k \bar{\delta} + b_{10k} X_{0,k} + b_{11k} X_{1,k} \tag{A.3}$$

Define the normalized price at date 0 as

$$Q_0 \equiv a_{0k}^{-1} P_{0,k}^i = D_k + a_{0k}^{-1} b_{00k} X_{0,k} \tag{A.4}$$

Then

$$\begin{aligned} P_{1,k}^i &= a_{1k} D_k + b_{10k} b_{00k}^{-1} a_{0k} (Q_{0,k} - D_k) + b_{11k} X_{1,k} \\ &= (a_{1k} - b_{10k} b_{00k}^{-1} a_{0k}) D_k + b_{10k} b_{00k}^{-1} a_{0k} Q_{0,k} + b_{11k} X_{1,k} \end{aligned} \tag{A.5}$$

while

$$P_{1,k} = (a_{1k} - b_{10k} b_{00k}^{-1} a_{0k}) D_k + a_{2k} \eta_k \bar{\delta} + b_{10k} b_{00k}^{-1} a_{0k} Q_{0,k} + b_{11k} X_{1,k} \tag{A.6}$$

Therefore, the normalized price perceived by investors at date 1 is given by

$$\begin{aligned} Q_{1,k}^i &\equiv (a_1 - b_{10k} b_{00k}^{-1} a_{0k})^{-1} (P_{1k}^i - b_{10k} b_{00k}^{-1} a_{0k} Q_{0,k}) \\ &= D_k + (a_{1k} - b_{10k} b_{00k}^{-1} a_{0k})^{-1} b_{11k} X_{1,k} \end{aligned} \tag{A.7}$$

while the true normalized price is

$$Q_{1,k} = D_k + (a_{1k} - b_{10k} b_{00k}^{-1} a_{0k})^{-1} a_{2k} \eta_k \bar{\delta} + (a_{1k} - b_{10k} b_{00k}^{-1} a_{0k})^{-1} b_{11k} X_{1,k} \tag{A.8}$$

Each investor i conjectures that

$$Q_{0,k}^i = D_k - \gamma^{-1} \tau_{s_0}^{-1} \eta_k^2 X_{0,k} \tag{A.9}$$

and

$$Q_{1,k}^i = D_k - \gamma^{-1} (\tau_{s_1} - \tau_{s_0})^{-1} \eta_k^2 X_{1,k} \tag{A.10}$$

Date 1: Investor i's optimal position x_{1k}^i is given by the standard result from the CARA utility:

$$x_{1,k}^i = \gamma \mathrm{Var}^{-1}[D_k | \mathcal{F}_1^i] (\mathbb{E}[D_k | \mathcal{F}_1^i] - R_f P_{1,k}) \tag{A.11}$$

Each investor i learns about D_k from Q_{0k}, Q_{1k}, S_{0k}^i and newly accumulated average signals S_{1k}^i following the projection theorem, which states if

$$\begin{pmatrix} \theta \\ s \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mu_\theta \\ \mu_s \end{pmatrix}, \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta s} \\ \Sigma_{s\theta} & \Sigma_{ss} \end{pmatrix} \right] \tag{A.12}$$

then

$$\mathbb{E}[\theta | s] = \mu_\theta + \Sigma_{\theta s} \Sigma_{ss}^{-1} (s - \mu_s) \tag{A.13}$$

$$\mathrm{Var}[\theta | s] = \Sigma_{\theta\theta} - \Sigma_{\theta s} \Sigma_{ss}^{-1} \Sigma_{s\theta} \tag{A.14}$$

Thus the learning results are given by

$$L_{1,k} \equiv \mathrm{Var}^{-1}[D_k | \mathcal{F}_1^i] = H_k + \tau_{s_1} \eta_k^{-2} + \gamma^2 \tau_{s_0}^2 \eta_k^{-4} \tau_x + \gamma^2 (\tau_{s_1} - \tau_{s_0})^2 \eta_k^{-4} \tau_x \tag{A.15}$$

and

$$u_{1,k}^i \equiv \mathbb{E}[D_k | \mathcal{F}_1^i] = L_{1,k}^{-1} [\tau_{s_0} \eta_k^{-2} S_{0,k}^i + (\tau_{s_1} - \tau_{s_0}) \eta_k^{-2} S_{1,k}^i + \gamma^2 \tau_{s_0}^2 \eta_k^{-4} \tau_x Q_{0,k}^i + \gamma^2 (\tau_{s_1} - \tau_{s_0})^2 \eta_k^{-4} \tau_x Q_{1,k}^i] \tag{A.16}$$

The market-clearing condition is given by

$$\begin{aligned}\int_0^1 x_{1,k}^i di &= \gamma \int_0^1 \text{Var}^{-1}[D_k|\mathcal{F}_1^i](\mathbb{E}[D_k|\mathcal{F}_1^i] - R_f P_{1,k})di \\ &= \gamma \int_0^1 L_{1,k}(u_{1,k}^i - R_f P_{1,k})di\end{aligned} \tag{A.17}$$

That is

$$X_{0,k} + X_{1,k} = \gamma L_{1,k} \int_0^1 u_{1,k}^i di - R_f \gamma L_{1,k} P_{1,k} \tag{A.18}$$

Thus

$$P_{1,k} = R_f^{-1} \int_0^1 u_{1,k}^i di - R_f^{-1}\gamma^{-1} L_{1,k}^{-1} X_{0k} - R_f^{-1}\gamma^{-1} L_{1,k}^{-1} X_{1,k} \tag{A.19}$$

Plugging L_{1k} and u_{1k}^i, we have

$$\begin{aligned}P_{1,k} = &R_f^{-1} L_{1,k}^{-1}[\tau_{s_1}\eta_k^{-2} D_k + (\tau_{s_1} - \tau_{s_0})\eta_k^{-2}\bar{\delta} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x Q_{0,k}^i + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-4}\tau_x Q_{1,k}^i] \\ &- R_f^{-1}\gamma^{-1} L_{1,k}^{-1} X_{0,k} - R_f^{-1}\gamma^{-1} L_{1,k}^{-1} X_{1,k}\end{aligned} \tag{A.20}$$

Substituting the conjecture about Q_{0k} and Q_{1k}, we have

$$\begin{aligned}P_{1,k} = &R_f^{-1} L_{1,k}^{-1}[\tau_{s_1}\eta_k^{2} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-4}\tau_x] D_k \\ &+ R_f^{-1} L_{1,k}^{-1}[(\tau_{s_1} - \tau_{s_1})\eta_k^{-2} + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-4}\tau_x(a_{1k} - b_{10k} b_{00k}^{-1} a_{0k})^{-1} a_{2k}]\eta_k\bar{\delta} \\ &- R_f^{-1}\gamma^{-1} L_{1,k}^{-1}(1 + \gamma^2\tau_{s_0}\eta_k^{-2}\tau_x) X_{0,k} - R_f^{-1}\gamma^{-1} L_{1,k}^{-1}[1 + \gamma^2(\tau_{s_1} - \tau_{s_0})\eta_k^{-2}\tau_x] X_{1,k}\end{aligned} \tag{A.21}$$

Therefore,

$$a_{1k} = R_f^{-1} L_{1,k}^{-1}(L_{1,k} - H_k) \tag{A.22}$$

$$b_{10k} = -R_f^{-1}\gamma^{-1} L_{1,k}^{-1}(1 + \gamma^2\tau_{s_0}\tau_x\eta_k^{-2}) \tag{A.23}$$

$$b_{11k} = -R_f^{-1}\gamma^{-1} L_{1k}^{-1}[1 + \gamma^2(\tau_{s_1} - \tau_{s_0})\tau_x\eta_k^{-2}] \tag{A.24}$$

Let's verify the coefficient of $X_{1,k}$ in $Q_{1,k}$, given $a_{0k}^{-1} b_{00k} = -\gamma^{-1}\tau_{s_0}^{-1}\eta_k^{-2}$,

$$\begin{aligned}&(a_{1k} - b_{10k} b_{00k}^{-1} a_{0k})^{-1} b_{11k} \\ &= (a_{1k} + \gamma\tau_{s_0}\eta_k^{-2} b_{10k})^{-1} b_{11k} \\ &= [R_f^{-1} L_{1,k}^{-1}(L_{1,k} - H_k) + \gamma\tau_{s_0}\eta_k^{-2} b_{10k}]^{-1} b_{11k} \\ &= -\gamma^{-1}(\tau_{s_1} - \tau_{s_0})^{-1}\eta_k^2\end{aligned} \tag{A.25}$$

which verifies the conjecture in period 1. Also,

$$\begin{aligned}&R_f^{-1} L_{1,k}^{-1}[(\tau_{s_1} - \tau_{s_0})\eta_k^{-2} + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\tau_x\eta_k^{-4}(a_{1k} - b_{10k} b_{00k}^{-1} a_{0k})^{-1} a_{2k}] \\ =&R_f^{-1} L_{1,k}^{-1}(\tau_{s_1} - \tau_{s_0})\eta_k^{-2} + R_f^{-1} L_{1,k}^{-1}\gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-4}\tau_x(a_{1k} - b_{10k} b_{00k}^{-1} a_{0k})^{-1} a_{2k} \\ =& a_{2k}\end{aligned} \tag{A.26}$$

Then,

$$a_{2k} = R_f^{-1} L_{1,k}^{-1}(\tau_{s_1} - \tau_{s_0})\eta_k^{-2}[1 + \gamma^2(\tau_{s_1} - \tau_{s_0})\eta_k^{-2}\tau_x] \tag{A.27}$$

Therefore,

$$\begin{aligned}P_{1,k} &= R_f^{-1}L_{1,k}^{-1}(L_{1,k}-H_k)D_k + R_f^{-1}L_{1,k}^{-1}[(\tau_{s_1}-\tau_{s_0})\eta_k^{-2}+\gamma^2(\tau_{s_1}-\tau_{s_0})^2\eta_k^{-4}\tau_x]\eta_k\bar{\delta} \\ &\quad - R_f^{-1}\gamma^{-1}L_{1k}^{-1}(1+\gamma^2\tau_{s_0}\eta_k^{-2}\tau_x)X_{0,k} - R_f^{-1}\gamma^{-1}L_{1,k}^{-1}[1+\gamma^2(\tau_{s_1}-\tau_{s_0})\eta_k^{-2}\tau_x]X_{1,k}\end{aligned} \tag{A.28}$$

Date 0: investor i's maximization problem is given by

$$\max_{\mathbf{x}_0^i}\mathbb{E}\left[-e^{-\frac{1}{\gamma}W_2^i}\right] \tag{A.29}$$

subject to

$$W_2^i = W_0^iR_f^2 + R_f\mathbf{x}_0^{i\prime}(\mathbf{P}_1 - R_f\mathbf{P}_0) + \mathbf{x}_1^{i\prime}(\mathbf{D} - R_f\mathbf{P}_1) \tag{A.30}$$

The maximization problem is equivalent to

$$\max_{\mathbf{x}_0^i}\mathbb{E}\left[-e^{-\frac{1}{\gamma}W_0^iR_f^2+\frac{1}{\gamma}R_f^2\mathbf{x}_0^{i\prime}\mathbf{P}_0-\frac{1}{\gamma}R_f\mathbf{x}_0^{i\prime}\mathbf{P}_1-\frac{1}{\gamma}\mathbf{x}_1^{i\prime}\mathbf{D}+\frac{1}{\gamma}R_f\mathbf{x}_1^{i\prime}\mathbf{P}_1}\right] \tag{A.31}$$

Investor i learns about $(\mathbf{D}',\mathbf{P}_1',\mathbf{x}_1^{i\prime})'$ with Gaussian updating under $\mathcal{F}_0^i=\{\mathbf{Q}_0^i,\mathbf{S}_0^i\}$ and her conjecture. Since $D_k, P_{1,k}$ and $x_{1,k}^i)$ only exist in $Q_{0,k}^i$ and $S_{0,k}^i$, investors learn $(D_k,P_{1,k},x_{1,k}^i)'$ with $\{Q_{0,k}^i,S_{0,k}^i\}$. Based on the equilibrium in period 1, I can write

$$\begin{bmatrix} D_k \\ P_{1,k} \\ x_{1,k}^i \\ Q_{0,k}^i \\ S_{0,k}^i\end{bmatrix} = \begin{bmatrix} 1 \\ \frac{L_{1,k}-H_k}{R_fL_{1,k}} \\ 0 \\ 1 \\ 1\end{bmatrix} D_k + \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{1+\gamma^2\tau_{s_0}\eta_k^{-2}\tau_x}{R_f\gamma L_{1,k}} & -\frac{1+\gamma^2(\tau_{s_1}-\tau_{s_0})\eta_k^{-2}\tau_x}{R_f\gamma L_{1,k}} & 0 & 0 \\ 1 & 1 & \gamma\tau_{s_0}\eta_k^{-2} & \gamma(\tau_{s_1}-\tau_{s_0})\eta_k^{-2} \\ -\gamma^{-1}\tau_{s_0}^{-1}\eta_k^2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix} X_{0,k} \\ X_{1,k} \\ \eta_k\epsilon_0^i \\ \eta_k\epsilon_1^i\end{bmatrix} \tag{A.32}$$

I will use projection theorem with $\theta=(D_k,P_{1,k},x_{1,k}^i)'$ and $s=\{Q_{0,k},S_{0,k}^i\}=\mathcal{F}_{0,k}^i$. Here I have,

$$\Sigma_{\theta\theta} = \begin{bmatrix} H_k^{-1} & R_f^{-1}(H_k^{-1}-L_{1,k}^{-1}) & 0 \\ R_f^{-1}(H_k^{-1}-L_{1,k}^{-1}) & Y & -\frac{[2+\gamma^2\tau_{s_1}\eta_k^{-2}\tau_x]}{R_f\gamma\tau_xL_{1,k}} \\ 0 & -\frac{[2+\gamma^2\tau_{s_1}\eta_k^{-2}\tau_x]}{R_f\gamma\tau_xL_{1,k}} & 2\tau_x^{-1}+\gamma^2\tau_{s_1}\eta_k^{-2}\end{bmatrix} \tag{A.33}$$

where

$$\begin{aligned}Y &= R_f^{-2}L_{1,k}^{-1}\{(L_{1,k}-H_k)H_k^{-1}(L_{1,k}-H_k)+\gamma^{-2}(1+\gamma^2\tau_{s_0}\eta_k^{-2}\tau_x)^2\tau_x^{-1}+\gamma^{-2}[1+\gamma^2(\tau_{s_1}-\tau_{s_0})\eta_k^{-2}\tau_x]^2\tau_x^{-1}\}L_{1,k}^{-1} \\ &= \frac{(L_{1,k}-H_k)^2H_k^{-1}}{R_f^2L_{1,k}^2} + \frac{(1+\gamma^2\tau_{s_0}\eta_k^{-2}\tau_x)^2+[1+\gamma^2(\tau_{s_1}-\tau_{s_0})\eta_k^{-2}\tau_x]^2}{R_f^2\gamma^2\tau_xL_{1,k}^2}\end{aligned} \tag{A.34}$$

and

$$\begin{aligned}\Sigma_{ss}^{-1} &= \begin{bmatrix} H_k^{-1}+\gamma^{-2}\tau_{s_0}^{-2}\eta_k^4\tau_x^{-1} & H_k^{-1} \\ H_k^{-1} & H_k^{-1}+\tau_{s_0}^{-1}\eta_k^2\end{bmatrix}^{-1} \\ &= \begin{bmatrix}\gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_xL_{0,k}^{-1}(H_k+\tau_{s_0}\eta_k^{-2}) & -\gamma^2\tau_{s_0}^3\eta_k^{-6}\tau_xL_{0,k}^{-1} \\ -\gamma^2\tau_{s_0}^3\eta_k^{-6}\tau_xL_{0,k}^{-1} & \tau_{s_0}\eta_k^{-2}L_{0,k}^{-1}(H_k+\gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x)\end{bmatrix}\end{aligned} \tag{A.35}$$

and

$$\Sigma_{\theta s} = \begin{bmatrix} H_k^{-1} & H_k^{-1} \\ R_f^{-1} H_k^{-1} + R_f^{-1} L_{1,k}^{-1} \gamma^{-2} \tau_{s_0}^{-1} \eta_k^2 \tau_x^{-1} & R_f^{-1}(H_k^{-1} - L_{1,k}^{-1}) \\ -\gamma^{-1} \tau_{s_0}^{-1} \tau_x^{-1} \eta_k^2 & \gamma \end{bmatrix} \tag{A.36}$$

where I define $L_{0,k} = \mathrm{Var}^{-1}[D_k|\mathcal{F}_0^i] = H_k + \tau_{s_0}\eta_k^{-2} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x$. Applying projection theorem, after tedious computation, I have

$$\mathbb{E}[D_k|\mathcal{F}_0^i] = R_f^{-1}\gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x L_{0,k}^{-1} Q_{0,k} + R_f^{-1}\tau_{s_0}\eta_k^{-2} L_{0,k}^{-1} S_{0,k}^i \equiv m_{D_k} \tag{A.37}$$

and

$$\mathbb{E}[P_{1,k}|\mathcal{F}_0^i] = R_f^{-1}\left(\gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x L_{0,k}^{-1} + \tau_{s_0}\eta_k^{-2} L_{1,k}^{-1}\right) Q_{0,k} + R_f^{-1}\tau_{s_0}\eta_k^{-2}\left(L_{0,k}^{-1} - L_{1,k}^{-1}\right) S_{0,k}^i \equiv m_{P_{1,k}} \tag{A.38}$$

and

$$\mathbb{E}[x_{1,k}^i|\mathcal{F}_0^i] = -R_f^{-1}\gamma\tau_{s_0}\eta_k^{-2} Q_{0,k} + R_f^{-1}\gamma\tau_{s_0}\eta_k^{-2} S_{0,k}^i \equiv m_{x_{1,k}^i} \tag{A.39}$$

Lemma 1. *For a random vector $z \sim \mathcal{N}(0, \Sigma)$,*

$$\mathbb{E}\left[e^{z^\top F z + G^\top z + J}\right] = |I - 2\Sigma F|^{-\frac{1}{2}} e^{\frac{1}{2} G^\top (I - 2\Sigma F)^{-1}\Sigma G + J} \tag{A.40}$$

With (A.37-A.39), I have the demeaned vector of $(z_{D_k}, z_{P_{1,k}}, z_{x_{1,k}^i})'$:

$$z_{D_k} = D_k - m_{D_k}, \quad z_{P_{1,k}} = P_{1,k} - m_{P_{1,k}}, \quad z_{x_{1,k}^i} = x_{1,k}^i - m_{x_{1,k}^i} \tag{A.41}$$

The demeaned vector is normally distributed with mean $\mathbf{0}_3$ and covariance matrix Σ_k given by

$$\Sigma_k = \mathrm{Var}[\theta|s] =$$
$$\begin{bmatrix} L_{0,k}^{-1} & R_f^{-1}(L_{0,k}^{-1} - L_{1,k}^{-1}) & 0 \\ R_f^{-1}(L_{0,k}^{-1} - L_{1,k}^{-1}) & R_f^{-2}(L_{0,k}^{-1} - L_{1,k}^{-1}) + R_f^{-2} L_{1,k}^{-2}(\tau_{s_0}\eta_k^{-2} + \gamma^{-2}\tau_x^{-1}) & -R_f^{-1} L_{1,k}^{-1}(\gamma\tau_{s_0}\eta_k^{-2} + \gamma^{-1}\tau_x^{-1}) \\ 0 & -R_f^{-1} L_{1,k}^{-1}(\gamma\tau_{s_0}\eta_k^{-2} + \gamma^{-1}\tau_x^{-1}) & \tau_x^{-1} + \gamma^2\tau_{s_0}\eta_k^{-2} \end{bmatrix} \tag{A.42}$$

Then the demeaned vector of $(z_{\mathbf{D}}', z_{\mathbf{P}}', z_{\mathbf{x}_1^i}')$ is normally distributed with mean $\mathbf{0}_{3K}$ and covariance matrix $\boldsymbol{\Sigma}$ where $z_{\mathbf{D}} = (z_{D_1}, \ldots, z_{D_K})$, $z_{\mathbf{P}} = (z_{P_{1,1}}, \ldots, z_{P_{1,K}})$ and $z_{\mathbf{x}_1^i} = (z_{x_{1,1}^i}, \ldots, z_{x_{1,K}^i})$.

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_a & \Sigma_b & \mathbf{0}_K \\ \Sigma_b' & \Sigma_c & \Sigma_d \\ \mathbf{0}_K & \Sigma_d' & \Sigma_e \end{bmatrix} \tag{A.43}$$

$\Sigma_a = diag(\Sigma_{111}, \ldots, \Sigma_{K11})$, where Σ_{kij} represents the element at the ith row and jth column in Σ_k, (A.42). Similarly, I have $\Sigma_b = diag(\Sigma_{112}, \ldots, \Sigma_{K12}), \Sigma_c = diag(\Sigma_{122}, \ldots, \Sigma_{K22}), \Sigma_d = diag(\Sigma_{123}, \ldots, \Sigma_{K23})$ and $\Sigma_e = diag(\Sigma_{133}, \ldots, \Sigma_{K33})$.

The quadratic function in the exponent in (A.31) can be written as

$$
\begin{aligned}
& f(z_{\mathbf{D}}, z_{\mathbf{P}_1}, z_{\mathbf{x}_1^i}) \\
= & -\gamma^{-1}(z_{\mathbf{x}_1^i} + m_{\mathbf{x}_1^i})'(z_{\mathbf{D}} + m_{\mathbf{D}}) + R_f\gamma^{-1}(z_{\mathbf{x}_1^i} - m_{\mathbf{x}_1^i})'(z_{\mathbf{P}_1} + m_{\mathbf{P}_1}) \\
& - R_f\gamma^{-1}\mathbf{x}_0^{i\prime}(z_{\mathbf{P}_1} + m_{\mathbf{P}_1}) + R_f^2\gamma^{-1}\mathbf{x}_0^{i\top}\mathbf{P}_0 - R_f^2\gamma^{-1}W_0^i \\
= & \left[z'_{\mathbf{D}}, z'_{\mathbf{P}_1}, z'_{\mathbf{x}_1^i}\right] \underbrace{\begin{bmatrix} \mathbf{0}_N & \mathbf{0}_N & -\frac{1}{2\gamma}\mathbf{I}_N \\ \mathbf{0}_N & \mathbf{0}_N & \frac{R_f}{2\gamma}\mathbf{I}_N \\ -\frac{1}{2\gamma}\mathbf{I}_N & \frac{R_f}{2\gamma}\mathbf{I}_N & \mathbf{0}_N \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} z_{\mathbf{D}} \\ z_{\mathbf{P}_1} \\ z_{\mathbf{x}_1^i} \end{bmatrix} \\
& + \underbrace{\left[-\gamma^{-1}m'_{\mathbf{x}_1^i}, R_f\gamma^{-1}(m'_{\mathbf{x}_1^i} - \mathbf{x}_0^{i\prime}), \gamma^{-1}(R_f m'_{\mathbf{P}_1} - m'_{\mathbf{D}})\right]}_{\mathbf{G}^\top} \begin{bmatrix} z_{\mathbf{D}} \\ z_{\mathbf{P}_1} \\ z_{\mathbf{x}_1^i} \end{bmatrix} \\
& \underbrace{-\gamma^{-1}m'_{\mathbf{x}_1^i}m_{\mathbf{D}} + R_f\gamma^{-1}m'_{\mathbf{x}_1^i}m_{\mathbf{P}_1} - R_f\gamma^{-1}\mathbf{x}_0^{i\prime}m_{\mathbf{P}_1} + R_f^2\gamma^{-1}\mathbf{x}_0^{i\prime}\mathbf{P}_0 - R_f^2\gamma^{-1}W_0^i}_{\mathbf{J}}
\end{aligned}
\tag{A.44}
$$

where $m_{\mathbf{D}} = (m_{D_1}, \ldots, m_{D_K})', m_{\mathbf{P}_1} = (m_{P_1}, \ldots, m_{P_K})'$ and $m_{\mathbf{x}_1^i} = (m_{x_1^i}, \ldots, m_{x_K^i})'$. By applying the Lemma 1 to the expected utility (A.29), I have the optimal demand $\mathbf{x}_0^i$ given by the first order condition:

$$
\frac{\partial(\frac{1}{2}\mathbf{G}'(\mathbf{I}_{3N} - 2\mathbf{\Sigma}\mathbf{F})^{-1}\mathbf{\Sigma}\mathbf{G} + \mathbf{J})}{\partial \mathbf{x}_0^i} = \mathbf{0}_{N\times 1} \tag{A.45}
$$

Substituting $\mathbf{F}, \mathbf{G}$ and $\mathbf{J}$ into the first order condition, after some computations, I have

$$
\frac{x_{0,k}^i}{\gamma^2 L_{0,k}} - \frac{\tau_x x_{0,k}^i}{1+\gamma^2\tau_{s_0}\eta_k^{-2}\tau_x + \gamma^2\tau_x L_{1,k}} + \frac{\frac{1}{\gamma} + \gamma\tau_{s_0}\eta_k^{-2}\tau_x}{1+\gamma^2\tau_{s_0}\eta_k^{-2}\tau_x + \gamma^2\tau_x L_{1,k}}(R_f m_{P_{1,k}} - m_{D_k}) - \frac{R_f}{\gamma}m_{P_{1,k}} + \frac{R_f^2}{\gamma}P_{0,k} = 0 \tag{A.46}
$$

Taking aggregations on both side and applying the market-clearing condition, I have

$$
\begin{aligned}
& \left(\frac{1}{\gamma L_{0,k}} - \frac{\gamma\tau_x}{1+\gamma^2\tau_s\eta_k^{-2}\tau_x + \gamma^2\tau_x L_{1,k}}\right)X_0 \\
= & R_f\int m_{P_{1,k}}di - R_f^2 P_0 - \frac{1+\gamma^2\tau_s\eta_k^{-2}\tau_x}{1+\gamma^2\tau_s\eta_k^{-2}\tau_x + \gamma^2\tau_x L_{1,k}}\int(R_f m_{P_{1,k}} - m_{D_k})di
\end{aligned}
\tag{A.47}
$$

Plugging into $m_{P_{1,k}}$ and m_{D_k}, I have

$$
P_{0,k} = R_f^{-2}L_{0,k}^{-1}(L_{0,k} - H_k)D_k - R_f^{-2}\gamma^{-1}L_{0,k}^{-1}(1+\gamma^2\tau_{s_0}\tau_x\eta_k^2)X_{0,k} \tag{A.48}
$$

Thus the normalize price at date 0 is given by

$$
Q_{0,k} = R_f^2 L_{0,k}(L_{0,k} - H_k)^{-1}P_{0,k} \tag{A.49}
$$

Therefore

$$
P_{0,k} = R_f^{-2}L_{0,k}^{-1}(L_{0,k} - H_k)Q_{0,k} = R_f^{-2}L_{0,k}^{-1}[\tau_{s_0}\eta_k^{-2} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x]Q_{0,k} \tag{A.50}
$$

Substituting $P_{0,k}$ into the FOC for $x^i_{0,k}$, we have the optimal demand vector in period 0 as

$$
\begin{aligned}
x^i_{0,k} &= \gamma\tau_{s_0}\eta_k^{-2}(S^i_{0,k} - R_f^2 Q_{0,k}) \\
&= \gamma\tau_{s_0}\eta_k^{-2}(S^i_{0,k} + \gamma^2\tau_{s_0}\eta_k^{-2}\tau_x R_f^2 Q_{0,k} - \gamma^2\tau_{s_0}\eta_k^{-2}\tau_x R_f^2 Q_{0,k} - R_f^2 Q_{0,k}) \\
&= \gamma(\tau_{s_0}\eta_k^{-2}S^i_{0,k} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x R_f^2 Q_{0,k}) - \gamma(\tau_{s_0}\eta_k^{-2} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x)R_f^2 Q_{0,k} \\
&= \gamma(\tau_{s_0}\eta_k^{-2}S^i_0 + R_f^2\gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x Q_{0,k}) - R_f^2\gamma(H_k + \tau_{s_0}\eta_k^{-2} + \gamma^2\tau_{s_0}^2\eta_k^{-2}\tau_x)P_{0,k} \\
&= \gamma\text{Var}^{-1}[D_k|\mathcal{F}^i_0]\mathbb{E}[D_k|\mathcal{F}^i_0] - \gamma\text{Var}^{-1}[D_k|\mathcal{F}^i_0]R_f^2 P_{0,k} \\
&= \gamma\text{Var}^{-1}[D_k|\mathcal{F}^i_0](\mathbb{E}[D_k|\mathcal{F}^i_0] - R_f^2 P_{0,k})
\end{aligned}
\tag{A.51}
$$

This verifies the optimal demand vector $\mathbf{x}^i_0$ in period 0 has the same form as $\mathbf{x}^i_1$ in period 1 except for the time subscript. The proof can be extended to general case of T period. Therefore, the optimal demand vector of investor i in period t is not affected by future trading opportunities and information.

A.3 Proof of Proposition 1:

Plugging $\mathbb{E}[D_k|\mathcal{F}^i_1]$ into the definition of belief polarization, I have

$$
\begin{aligned}
\mathcal{P}_1 &= \int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}^i_1] - \int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}^i_1] \\
&=(\delta_{\mathcal{B}} - \delta_{\mathcal{A}})(\tau_{s_1} - \tau_{s_0})\eta_k^{-1}L_{1,k}^{-1} \\
&=(\delta_{\mathcal{B}} - \delta_{\mathcal{A}})(\tau_{s_1} - \tau_{s_0})\eta_k^{-1}[H_k + \eta_k^{-2}\tau_{s_1} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-4}\tau_x]^{-1} \\
&=(\delta_{\mathcal{B}} - \delta_{\mathcal{A}})\underbrace{\frac{(\tau_{s_1} - \tau_{s_0})(\eta_k^2\tau_e + \tau_u)\eta_k^3}{\tau_u\tau_e\eta_k^4 + (\eta_k^2\tau_e + \tau_u)[\tau_{s_1}\eta_k^2 + \gamma^2\tau_{s_0}^2\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\tau_x]}}_{\equiv\phi_k(\tau_{s_1})}
\end{aligned}
\tag{A.52}
$$

Define $\phi_k(\tau_{s_1})$ as the polarization coefficient for the kth asset. I am interested in how it varies with information precision τ_{s_1}.

Taking the first derivative with respect to τ_{s_1}:

$$
\frac{d\phi_k(\tau_{s_1})}{d\tau_{s_1}} = \frac{(\eta_k^2\tau_e + \tau_u)\eta_k^3\{\tau_u\tau_e\eta_k^4 + \tau_{s_0}(\eta_k^2\tau_e + \tau_u)\eta_k^2 + (\eta_k^2\tau_e + \tau_u)\gamma^2\tau_{s_0}^2\tau_x - (\tau_{s_1} - \tau_{s_0})^2(\eta_k^2\tau_e + \tau_u)\gamma^2\tau_x\}}{\{\tau_u\tau_e\eta_k^4 + (\eta_k^2\tau_e + \tau_u)[\tau_{s_1}\eta_k^2 + \gamma^2\tau_{s_0}^2\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\tau_x]\}^2}
\tag{A.53}
$$

Therefore, for assets with $\eta_k > 0$, $\frac{d\phi_k(\tau_{s_1})}{d\tau_{s_1}} \geq (<)0$ if

$$
\tau_{s_1} \leq (>)\tau_{s_0} + \sqrt{\frac{\tau_u\tau_e\eta_k^4 + (\eta_k^2\tau_e + \tau_u)(\tau_{s_0}\eta_k^2 + \gamma^2\tau_{s_0}^2\tau_x)}{\gamma^2\tau_x(\eta_k^2\tau_e + \tau_u)}}
\tag{A.54}
$$

So, $|\phi_k(\tau_{s_1})|$ is hump-shaped with τ_{s_1}, which validates the pattern in figure 5. It increases with τ_{s_1} when $\tau_{s_1} \in \left(\tau_{s_0}, \tau_{s_0} + \sqrt{\frac{\tau_u\tau_e\eta_k^4+(\eta_k^2\tau_e+\tau_u)(\tau_{s_0}\eta_k^2+\gamma^2\tau_{s_0}^2\tau_x)}{\gamma^2\tau_x(\eta_k^2\tau_e+\tau_u)}}\right)$ and decreases with τ_{s_1} when $\tau_{s_1} \in \left(\tau_{s_0} + \sqrt{\frac{\tau_u\tau_e\eta_k^4+(\eta_k^2\tau_e+\tau_u)(\tau_{s_0}\eta_k^2+\gamma^2\tau_{s_0}^2\tau_x)}{\gamma^2\tau_x(\eta_k^2\tau_e+\tau_u)}}, +\infty\right)$.

A.4 Proof of Proposition 2:

Suppose now investors do not use market price $P_{1,k}$ to update beliefs about the kth asset. Denote investor i's posterior precision and expectation at date 1 by $L^o_{1,k}$ and $u^{o,i}_{1,k}$. Then,

$$L^o_{1,k} = H_k + \eta_k^{-2}\tau_{s_1} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x \tag{A.55}$$

and

$$u^{o,i}_{1,k} = L^{-1}_{1,k}[\tau_{s_0}\eta_k^{-2}S^i_{0,k} + (\tau_{s_1} - \tau_{s_0})\eta_k^{-2}S^i_{1,k} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x Q_{0,k}] \tag{A.56}$$

Thus the belief polarization $\mathcal{P}^o_1$ without market price information at date 1 is given by

$$\begin{aligned}
\mathcal{P}^o_1 &= \int_{i\in\mathcal{B}} u^{o,i}_{1,k} di - \int_{i\in\mathcal{A}} u^{o,i}_{1,k} di \\
&= (\bar{\delta}_{\mathcal{B}} - \bar{\delta}_{\mathcal{A}})(\tau_{s_1} - \tau_{s_0})\eta_k^{-1}[H_k + \eta_k^{-2}\tau_{s_1} + \gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x + \gamma^2(\tau_{s_1} - \tau_{s_0})^2\eta_k^{-4}\tau_x]^{-1} \\
&= (\bar{\delta}_{\mathcal{B}} - \bar{\delta}_{\mathcal{A}}) \underbrace{\frac{(\tau_{s_1} - \tau_{s_0})(\eta_k^2\tau_e + \tau_u)\eta_k^3}{\tau_u\tau_e\eta_k^4 + (\eta_k^2\tau_e + \tau_u)(\tau_{s_1}\eta_k^2 + \gamma^2\tau_{s_0}^2\tau_x)}}_{\equiv \phi^o_k(\tau_{s_1})}
\end{aligned} \tag{A.57}$$

Comparing ϕ^o with ϕ we can see, given η_k, $|\phi_k| < |\phi^o_k|$. The belief polarization coefficient has larger absolute value when investors do not use public market prices. That is, the unconscious biases have larger impacts on belief polarization when investors ignore the market information $P_{1,k}$. So, market prices help dampen the polarization in the market. But market prices do not fully eliminate belief polarization as $\mathcal{P}^o_1$ exists as long as there are unconscious biases and $\bar{\delta}_{\mathcal{B}} \neq \bar{\delta}_{\mathcal{A}}$.

A.5 Proof of Proposition 3:

The belief dispersion on the kth asset across investors is given by

$$\begin{aligned}
\mathcal{D}_{1,k} &= \text{Var}\left[\mathbb{E}[D_k|\mathcal{F}^i_1] - \int_0^1 \mathbb{E}[D_k|\mathcal{F}^j_1]dj\right] \\
&= \mathbb{E}\left\{\left[\mathbb{E}[D_k|\mathcal{F}^i_1] - \int_0^1 \mathbb{E}[D_k|\mathcal{F}^j_1]dj\right]^2\right\} - \left\{\mathbb{E}\left[\mathbb{E}[D_k|\mathcal{F}^i_1] - \int_0^1 \mathbb{E}\left[D_k|\mathcal{F}^j_1\right] dj\right]\right\}^2
\end{aligned} \tag{A.58}$$

Since

$$\mathbb{E}\left[\mathbb{E}[D_k|\mathcal{F}^i_1] - \int_0^1 \mathbb{E}\left[D_k|\mathcal{F}^j_1\right] dj\right] = \int_0^1 \mathbb{E}[D_k|\mathcal{F}^i_1]di - \int_0^1 \mathbb{E}\left[D_k|\mathcal{F}^j_1\right] dj = 0 \tag{A.59}$$

I have

$$
\begin{aligned}
\mathcal{D}_{1,k} &= \mathbb{E}\left\{\left[\mathbb{E}[D_k|\mathcal{F}_1^i] - \int_0^1 \mathbb{E}[D_k|\mathcal{F}_1^j]dj\right]^2\right\} \\
&= \mathbb{E}\left\{\mathbb{E}^2[D_k|\mathcal{F}_1^i] - 2\mathbb{E}[D_k|\mathcal{F}_1^i]\int_0^1 \mathbb{E}[D_k|\mathcal{F}_1^j]dj + \left(\int_0^1 \mathbb{E}[D_k|\mathcal{F}_1^j]dj\right)^2\right\} \\
&= \int_0^1 \mathbb{E}^2[D_k|\mathcal{F}_1^i]di - 2\left(\int_0^1 \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2 + \left(\int_0^1 \mathbb{E}[D_k|\mathcal{F}_1^j]dj\right)^2 \\
&= \int_0^1 \mathbb{E}^2[D_k|\mathcal{F}_1^i]di - \left(\int_0^1 \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2
\end{aligned} \tag{A.60}
$$

Similarly, the belief dispersion across group $\mathcal{A}$ and group $\mathcal{B}$ is given by

$$
\begin{aligned}
\mathcal{D}_{1,\mathcal{A},k} &= \mathrm{Var}\left[\mathbb{E}[D_k|\mathcal{F}_1^i, i \in \mathcal{A}] - \int_{j\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^j]dj\right] \\
&= \int_{i\in\mathcal{A}} \mathbb{E}^2[D_k|\mathcal{F}_1^j]di - \left(\int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2
\end{aligned} \tag{A.61}
$$

and

$$
\begin{aligned}
\mathcal{D}_{1,\mathcal{B},k} &= \mathrm{Var}\left[\mathbb{E}[D_k|\mathcal{F}_1^i, i \in \mathcal{B}] - \int_{j\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^j]dj\right] \\
&= \int_{i\in\mathcal{B}} \mathbb{E}^2[D_k|\mathcal{F}_1^i]di - \left(\int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2
\end{aligned} \tag{A.62}
$$

Note that $\mathcal{D}_{1,\mathcal{A},k}$ is measured over group $\mathcal{A}$, so the probability measure when doing the integration is different from the probability measure of $\mathcal{D}_{1,k}$.

Belief dispersion $\mathcal{D}_{1,k}$ can be written as

$$
\begin{aligned}
\mathcal{D}_{1,k} &= \frac{1}{2}\int_{i\in\mathcal{A}} \mathbb{E}^2[D_k|\mathcal{F}_1^i]di + \frac{1}{2}\int_{j\in\mathcal{B}} \mathbb{E}^2[D_k|\mathcal{F}_1^i]dj - \left(\frac{1}{2}\int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di + \frac{1}{2}\int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2 \\
&= \frac{1}{2}\int_{i\in\mathcal{A}} \mathbb{E}^2[D_k|\mathcal{F}_1^i]di - \frac{1}{4}\left(\int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2 + \frac{1}{2}\int_{i\in\mathcal{B}} \mathbb{E}^2[D_k|\mathcal{F}_1^i]di - \frac{1}{4}\left(\int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2 \\
&\quad - \frac{1}{2}\int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di \times \int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di \\
&= \frac{1}{2}\mathcal{D}_{1,\mathcal{A},k} + \frac{1}{2}\mathcal{D}_{1,\mathcal{B},k} + \frac{1}{4}\left[\left(\int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2 + \left(\int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right)^2 - 2\int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di \int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right] \\
&= \frac{1}{2}\mathcal{D}_{1,\mathcal{A},k} + \frac{1}{2}\mathcal{D}_{1,\mathcal{B},k} + \frac{1}{4}\left[\int_{i\in\mathcal{A}} \mathbb{E}[D_k|\mathcal{F}_1^i]di - \int_{i\in\mathcal{B}} \mathbb{E}[D_k|\mathcal{F}_1^i]di\right]^2 \\
&= \frac{1}{2}\mathcal{D}_{1,\mathcal{A},k} + \frac{1}{2}\mathcal{D}_{1,\mathcal{B},k} + \frac{1}{4}\mathcal{P}_{1,k}^2
\end{aligned} \tag{A.63}
$$

A.6 Proof of Proposition 4:

The trading volume for the kth asset in period 1 is given by

$$\mathcal{V}_{1,k} \equiv \int_0^1 |x^i_{1,k} - x^i_{0,k}| di \tag{A.64}$$

Since I have proved for $t = 0, 1$

$$\begin{aligned} x^i_{t,k} &= \gamma \mathrm{Var}^{-1}\left[D_k|\mathcal{F}^i_t\right]\left(\mathbb{E}[D_k|\mathcal{F}^i_t] - R_f^{2-t} P_{0,k}\right) \\ &= \gamma L_{t,k} u^i_{t,k} - \gamma R_f^{2-t} L_{t,k} P_{t,k} \end{aligned} \tag{A.65}$$

Substituting $L_{t,k}$, $u^i_{t,k}$ and $P_{t,k}$, I have

$$x^i_{0,k} = \gamma \tau_{s_0} \eta_k^{-1} \epsilon^i_0 + X_{0,k} \tag{A.66}$$

and

$$\begin{aligned} x^i_{1,k} = {} & \gamma \tau_{s_0} \eta_k^{-1} \epsilon^i_0 + \gamma(\tau_{s_1} - \tau_{s_0}) \eta_k^{-1} \epsilon^i_1 + X_{0,k} + X_{1,k} \\ & + \gamma(\tau_{s_1} - \tau_{s_0}) \eta_k^{-1} \delta^i - \gamma[(\tau_{s_1} - \tau_{s_0}) \eta_k^{-2} + \gamma^2 (\tau_{s_1} - \tau_{s_0})^2 \tau_x \eta_k^{-4}] \eta_k \bar{\delta} \end{aligned} \tag{A.67}$$

Define $\Delta x^i_{1,k} \equiv x^i_{1,k} - x^i_{0,k}$, then

$$\Delta x^i_{1,k} = \gamma(\tau_{s_1} - \tau_{s_0}) \eta_k^{-1} \epsilon^i_1 + \gamma(\tau_{s_1} - \tau_{s_0}) \eta_k^{-1} \delta^i - \gamma[(\tau_{s_1} - \tau_{s_0}) \eta_k^{-2} + \gamma^2 (\tau_{s_1} - \tau_{s_0})^2 \tau_x \eta_k^{-4}] \eta_k \bar{\delta} + X_{1,k} \tag{A.68}$$

$\Delta x^i_{1,k}$ is normally distributed with

$$\mathbb{E}[\Delta x^i_{1,k}] \equiv \mu = -\gamma^3 (\tau_{s_1} - \tau_{s_0})^2 \tau_x \eta_k^{-3} \bar{\delta} \tag{A.69}$$

and

$$\mathrm{Var}[\Delta x^i_{1,k}] \equiv \sigma^2 = \tau_x^{-1} + \gamma^2 (\tau_{s_1} - \tau_{s_0})^2 \eta_k^{-2} \tau_{s_1}^{-1} + \gamma^2 (\tau_{s_1} - \tau_{s_0})^2 \eta_k^{-2} \nu^2 \tag{A.70}$$

Thus, $|\Delta x^i_{1,k}|$ follows folded normal distribution and its mean $\int_0^1 |\Delta x^i_{1,k}|$ is given by

$$\sqrt{\frac{2}{\pi}} \sigma e^{-\frac{\mu^2}{2\sigma^2}} + \mu \left[1 - 2\Phi\left(-\frac{\mu}{\sigma}\right)\right] \tag{A.71}$$

where $\Phi(\cdot)$ is the normal cumulative distribution function.

In the benchmark case, there is no echo chamber, $\bar{\delta} = 0$ and $\nu = 0$. The trading volume of kth asset in period 1 in the benchmark case is:

$$\mathcal{V}^*_{1,k} = \sqrt{\frac{2}{\pi} h}, \quad \text{with} \quad h \equiv \tau_x^{-1} + \gamma^2 (\tau_{s_1} - \tau_{s_0})^2 \eta_k^{-2} \tau_{s_1}^{-1} \tag{A.72}$$

In the polarization case, the trading volume is affected the unconscious bias generated from the information percolation process with echo chambers. I have

$$\begin{aligned}\mathcal{V}_{1,k} = &\sqrt{\frac{2}{\pi}[h+\gamma^2(\tau_{s_1}-\tau_{s_0})^2\eta_k^{-2}\nu^2]}\exp\left\{-\frac{\gamma^6(\tau_{s_1}-\tau_{s_0})^4\tau_x^2\eta_k^{-6}\bar{\delta}^2}{2[h+\gamma^2(\tau_{s_1}-\tau_{s_0})^2\eta_k^{-2}\nu^2]}\right\} \\ &-\gamma^3(\tau_{s_1}-\tau_{s_0})^2\tau_x\eta_k^{-3}\bar{\delta}\left[1-2\Phi\left(\frac{\gamma^3(\tau_{s_1}-\tau_{s_0})^2\tau_x\eta_k^{-3}\bar{\delta}}{\sqrt{h+\gamma^2(\tau_{s_1}-\tau_{s_0})^2\eta_k^{-2}\nu^2}}\right)\right]\end{aligned} \tag{A.73}$$

which is increasing in ν^2. Since when $\nu^2=0$, $\mathcal{V}_{k,1}=\sqrt{\frac{2}{\pi}h}$, I have,

$$\mathcal{V}_{1,k} \geq \sqrt{\frac{2}{\pi}h} = \mathcal{V}^*_{1,k} \tag{A.74}$$

Therefore, unconscious biases generate excess trading volume in the economy.

A.7 Proof of Proposition 5:

Substituting $\mathbb{E}[D_k|\mathcal{F}_1^i]$ and $P_{1,k}$ into $\mathbb{E}[R_{1,k}]$, I have

$$\begin{aligned}\mathbb{E}[R_{1,k}] &= \int_0^1 \mathbb{E}[D_k|\mathcal{F}_1^i]di - R_f P_{1k} \\ &= \gamma^{-1}L_{1,k}^{-1}(X_{0,k}+X_{1,k}) - L_{1,k}^{-1}\gamma^2(\tau_{s_1}-\tau_{s_0})^2\tau_x\eta_k^{-3}\bar{\delta} \\ &= \mathbb{E}^*[R_{1,k}] - \frac{\gamma^2(\tau_{s_1}-\tau_{s_0})^2\tau_x\eta_k^{-3}}{(\eta_k^2\tau_u^{-1}+\tau_e^{-1})^{-1}+\eta_k^{-2}\tau_{s_1}+\gamma^2\tau_{s_0}^2\eta_k^{-4}\tau_x+\gamma^2(\tau_{s_1}-\tau_{s_0})^2\eta_k^{-4}\tau_x}\bar{\delta} \\ &= \mathbb{E}^*[R_{1,k}] - \frac{\gamma^2(\tau_{s_1}-\tau_{s_0})^2\tau_x}{\eta_k^2(\tau_u^{-1}+\eta_k^{-2}\tau_e^{-1})^{-1}+\eta_k^2\tau_{s_1}+\gamma^2\tau_{s_0}^2\tau_x+\gamma^2(\tau_{s_1}-\tau_{s_0})^2\tau_x}\eta_k\bar{\delta} \\ &= \mathbb{E}^*[R_{1,k}] - f_k\eta_k\bar{\delta}\end{aligned} \tag{A.75}$$

where $\mathbb{E}^*[R_{1,k}]$ is the expected return of the kth asset when there is no echo chamber and unconscious bias in the benchmark case. f_k is positive.

(1) Given f_k, when $\eta_k\bar{\delta}<0$, $\mathbb{E}[R_{1,k}]>\mathbb{E}[R_{1,k}]^*$, investors require high expected returns for the kth asset. $\eta_k\bar{\delta}<0$ include two cases: (i) the asset is positively related to the fundamental risk ($\eta_k>0$) when the population is negatively biased ($\bar{\delta}<0$); (ii) the asset is negatively related to the fundamental risk ($\eta_k<0$) when the population is positively biased ($\bar{\delta}>0$). We have opposite reasoning when $\eta_k\bar{\delta}>0$.

(2) f_k is influenced by the information percolation speed. When information percolates at a higher speed, $\tau_{s_1}\uparrow$, $f_k>0\downarrow$. But to analyze how communication intensity affects expected returns, I need to fix the sign of $\eta_k\bar{\delta}$. Given $\eta_k\bar{\delta}<0$ and $\mathbb{E}[R_{1,k}]>\mathbb{E}[R_{1,k}]^*$, with information percolation speeding up, the discrepancy between $\mathbb{E}[R_{1,k}]$ and $\mathbb{E}[R_{1,k}]^*$ widens, investors ask for even higher expected returns in the polarization case. In contrast, if $\eta_k\bar{\delta}>0$ and $\mathbb{E}[R_{1,k}]<\mathbb{E}[R_{1,k}]^*$, then when communicating at a higher intensity, the discrepancy between $\mathbb{E}[R_{1,k}]$ and $\mathbb{E}[R_{1,k}]^*$ narrows, investors increase the expected returns though the expected returns are lower than in the benchmark case without unconscious biases.