Pricing Disaster Risk in Corporate Bonds

Cynthia Yin*
The Ohio State University

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Abstract

To explain the "credit spread puzzle" and study the implications of crisis risk on corporate credit spreads, I propose a dynamic capital structural model with long-term bond and disaster risk. The model reproduces the high corporate credit spread and low default rate as observed in the data. Disaster risk affects corporate credit spreads through default risk, risk premium, and corporate capital structure. Default risk dominates other channels in disaster states. With disaster risk in normal times, lower optimal capital levels and firm value lead to higher leverage and credit spread. With more real and financial frictions, firms are more conservative and reduce their leverage, giving rise to lower credit spreads. Following a realized disaster, financially constrained firms lose more equity value, and their credit spreads sharply increase.

*Fisher College of Business, The Ohio State University, 2100 Neil Avenue, Columbus OH 43210. E-mail: yin.708@osu.edu.

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1 Introduction

How does disaster risk affect corporate credit spreads? Standard structural models fail to reconcile the low default probability and high credit spread. This failure, also known as the "credit spread puzzle," is shown to be largely explained by macroeconomic fluctuations. As a type of macroeconomic risk, disaster risk should have direct impacts on credit spreads. As shown in Figure 1, the Baa-Aaa credit spread reached as high as approximately 6%, and the annual default rate peaked at 8% during the Great Depression. Disasters directly drive default probability and loss, so disaster risk has a first-order impact on corporate bond prices. Because disasters are rare events and endogenously change both investment and financing decisions, it is challenging to study empirically. This highlights the necessity of approaching the problem with a dynamic structure.

In this paper, I introduce disaster risk into a capital structure model with long-term defaultable bonds. The stochastic discount factor of the Epstein-Zin utility incorporates the disaster risk, with a small chance of consumption growth crash. Firms are obligated to repay a predetermined amount of coupon and principal each period and can choose to default when the continuation value is negative. The defaultable bonds are priced based on future expected default probability and recovery rate. Firms make endogenous forward-looking investment decisions, which are financed by both equity and bonds. Firms face investment adjustment costs, and fixed and proportional costs when they issue new equity and bonds.

There are two main drivers in the model that affect credit spreads. First, as in other bond pricing models, countercyclical variations in the risk premium, default risk, and default loss drive up credit spreads. In bad times, when investors' marginal utility is high and require a higher rate of return, firms face higher default risk and lower continuation value, leading to increased credit spreads. Disaster risk exaggerates these effects by adding significantly higher default losses and probabilities in the disaster state, and introduces a highly nonlinear pricing kernel. The risk premium channel is critical in normal times, while the default risk channel plays a bigger role in realized disaster states.

On top of the risk premium and default risk channels, firms change their optimal capital structures when they face rare but severe disaster risks. During normal times with the threat of disaster risk, firms choose lower optimal capital levels. Lower capital levels lead to lower equity value and more volatile investments because firms have to sell capital in bad times. Lower capital levels also lead to higher market leverage, generating high credit spreads. [1]

A model with long-term defaultable bonds is computationally challenging to solve, because it does not satisfy the contraction mapping theorem, and the algorithm convergence is not guaranteed. Therefore, I combine the strategies in Chatterjee and Eyigungor (2012) and Kuehn and Schmid (2014). One problem is that there are multiple local optimums, generating oscillations in the algorithm iterations. Following Chatterjee and Eyigungor (2012), I introduce a small transitory income shock to solve it. Another issue is the impact of the initial value. I follow Kuehn and Schmid (2014) to use the model solution for shorter-term bonds as the initial value for longer-term bonds. I start with one-period bonds and gradually achieve five-year maturity. The third issue is the kink of the value function and the numerical errors generated in interpolations. I interpolate on the relatively smoother continuation value function to reduce the numerical errors.

To use the model to study the counterfactual effects in disaster states, careful calibration is critical. I simulate the economy and match the key moments in samples without realized disasters to the empirical observations in the US data. On the representative consumer side, parameter values in the Epstein-Zin utility function reproduce moments that match aggregate consumption growth data and the equity risk premium. On the firm side, parameter values generate moments that match firm-level default rates, credit spreads, and investment and financing policies.

Using the calibrated model, I focus on three main questions: How does disaster risk affect corpo-

[1] Why firms do not lower bond levels to decrease leverage and avoid default in disaster states? On the one hand, long-term bonds are attractive to firms, primarily due to the rapid recovery period following the disaster state. For each period, firms only have to repay a portion of the principal outstanding, providing less liquidity pressure when a disaster hits and more time for the firm to recover in the subsequent recovery state. On the other hand, with less capital, firms are forced to issue new bonds to obtain enough liquidity and repay the promised amount of principal. Firms are also more likely to roll over the bonds or issue new equity when the real and financial frictions are not too high. The incentive to issue bonds outweighs the fear of immediate disaster realization and leads to higher average credit spreads in equilibrium.

rate credit spreads? What are the implications at the firm level, in normal times or disaster states? What are the counterfactual results when firms face higher or lower real and financial frictions?

Disaster risk has a first-order impact on credit spreads. Consistent with empirical observations in normal times, my model generates an average credit spread of 92.79 basis points (bps), close to the $70-100$ bps in the data. Meanwhile, my model reproduces an annual default rate of 0.66%, an annual equity premium of 8.46%, and an average market leverage of 31% [2]. As a counterfactual, a model with a less severe consumption crash or lower disaster probability generates an average credit spread of 33.96 bps and 30.36 bps, respectively. When there is a less severe consumption crash or a lower disaster probability, there is a lower expected default probability and risk premium. Firms are able to build up capital and increase equity value, leading to lower leverage. Less default risk, risk premium, and leverage lead to a decrease in credit spreads. Conversely, a model with a more severe consumption crash or higher disaster probability generates higher leverage and an average credit spread of 254.03 bps and 159.68 bps, respectively.

I then analyze the quantitative results when disasters are realized. Intuitively, when a disaster hits, both the default rate and credit spread increase. My model predicts an annual default rate of 10.45% and an average credit spread of 315.15 bps in samples with realized disasters. Compared to normal times, the average credit spread is more than tripled. The credit spread can vary significantly, with a 90% confidence interval between 111.25 bps and 652.73 bps The variation depends on the severity of the disaster and the economic health before entering the disaster state.

During disaster periods, there is a large drop in investment and new financing, along with an increase in market leverage. These results are driven by a significant negative shock to firms' productivity and a sharp decline in the continuation value. The expected future default rate and losses are high, making it very difficult for firms to issue new bonds. As a result, firms are forced to either default or divest in order to cope with the liquidity shock and repay current liabilities. The

[2] The empirical benchmark of the annual default rate is 0.7%, from Bai (2021). The annual equity premium is 8.577% from CRSP. The empirical market leverage is 29% from Bai et al. (2019).

expectation of high default rates and leverage leads to high credit spreads during disaster periods.

After exploring how disaster risk affects average credit spreads, I study more effects at the cross-section. I first focus on the firm-specific determinants of credit spreads and then explore firms' heterogeneous responses to realized disasters. Consistent with the prediction from the one-period bond model and previous literature, contemporaneous credit spreads are largely explained by credit risk premiums, which have the highest R^2, 0.55, in explaining the variations of credit spreads at the cross-section. In contrast, next-period credit spreads are mostly explained by the market leverage ($R^2 = 0.27$). The results show that the contemporaneous changes in credit spreads respond more to exogenous productivity shocks. The shock is embedded into the endogenous corporate policies, which will be reflected in the credit spreads next period. In contrast, with realized disasters, default risk explains the most variations of both contemporaneous and next-period credit spreads. The default risk channel dominates other economic channels when disasters are materialized.

Following a realized disaster, firms lose equity value and increase credit spreads, but the effects are heterogeneous. Consistent with findings in Fahlenbrach, Rageth, and Stulz (2021) [3], firms with more financial flexibility lose less equity value, and their credit spreads jump less. With a recovery state, the level of credit spreads will converge back to pre-disaster period. Similarly, more profitable firms suffer less in equity value losses and their credit spread stay relatively stable.

How do real and financial frictions play a role? Different from the findings in Kuehn and Schmid (2014), in normal times, more real and financial frictions decrease credit spreads by reducing firms' leverage. When it is more costly to invest or divest, firms choose more conservative corporate policies. Similarly, when the cost of issuing new equity or bonds is high, firms have less flexibility in financing when there is a negative shock. Firms issue fewer bonds and have lower leverage, leading to both lower default rates and credit spreads. Conversely, firms facing fewer real and financial frictions choose more aggressive corporate policies to finance investments. The reason is that it

[3] Fahlenbrach, Rageth, and Stulz (2021) uses cash and book leverage as proxies for financial flexibility. I instead use market leverage because I do not model cash, and it is better calibrated to the data and reflects the flexibility in equity financing.

becomes easier to roll over bonds or use new bonds to solve liquidity pressure, overall generating more defaults and higher credit spreads. I also briefly explore other economic mechanisms. Lower recovery rates and higher fixed operational costs also contribute to higher credit spreads because both forces directly reduce repayments when bonds default.

My paper contributes to three strands of literature. The first is the literature on structural models to quantitatively explain the credit spread puzzle. Huang and Huang (2012) finds that standard structural models fail to generate realistically high credit spreads after being calibrated to observed default rates and losses. Credit spreads were shown to be largely explained by macroeconomic fluctuations (Hackbarth, Miao, and Morellec, 2006; Chen, 2010; Gourio, 2013; Bai, 2021). At the firm level, Kuehn and Schmid (2014) showed that real and financial frictions make corporate bonds risky and generate realistic credit spreads.

Traditional structural credit risk models assume an exogenous evolution of a firm's asset value (Merton, 1974; Leland, 1994) without considring endogenous investment decisions. Such models only align with Modigliani and Miller's (1958) world of perfect separation between financing and investment, contradicting reality. However, most structural models that consider endogenous investments use one-period bonds due to computational challenges. In reality, firms issue bonds with much longer maturity (Eom, Helwege, and Huang, 2004; Driessen, 2005; Dickerson, Mueller, and Robotti, 2023). Compared to short-term bonds, long-term bonds have different default rates and credit spreads, and they have different implications for firms' capital structure (Gourio and Michaux, 2012; Kuehn and Schmid, 2014). I contribute by proposing a new firm-level capital structure model that incorporates disaster risk and long-term bonds, providing another solution to the credit spread puzzle. Also, I show that disaster risk has direct impacts on corporate policies, leading to results on real and financial frictions that differ from previous findings.

Second, my paper contributes to the literature on the effects of disaster risk on asset pricing. The existing quantitative literature has primarily focused on using disaster risk to explain the equity premium (Gourio, 2012; Wachter, 2013; Bai et al., 2019), while bond pricing has received relatively less exploration. On the empirical side, Baron, Xiong, and Ye (2022) uses a sample of 20 advanced economies from 1870 to 2021 to construct a direct measure of disaster risk. The closest two papers are Gourio (2013) and Bai (2021). Both papers study disaster risk and credit risk at the aggregate level with one-period bonds. Gourio (2013) studies credit risk implications of exogenous disaster risk, without considering real and financial frictions. Bai (2021) examines how endogenous disaster risk arises due to labor market frictions and its impact on aggregate credit risk. In contrast, my work differs by focusing on firm-level credit spreads. I also study more realistic long-term bonds, considering both real and financial frictions. Doing so allows for a more detailed understanding of how disaster risk affects corporate policies and credit spreads at the firm level.

Third, and more broadly, my paper contributes to the crisis risk literature. At the aggregate level, crises, mostly financial crises, bring fundamentally important economic questions because it has large negative output shocks that cause real damage compared to normal recessions (Claessens and Kose, 2013; Sufi and Taylor, 2022; Stulz, 2023) [4]. At the cross-section level, firms with more financial flexibility handle the shocks better, reflected in less loss in stock value (Fahlenbrach, Rageth, and Stulz, 2021). Following models in the disaster risk literature, which directly model potential large output crises, my paper sheds light on how firm-level credit spreads respond to underlying and realized crisis risk through endogenous capital structure choices. Therefore, my paper also contributes on how corporate policies respond to crisis risk.

The rest of this paper is organized as follows. Section 2 uses a simplified model to illustrate the key economic mechanism. Section 3 describes the full model. Section 4 discusses the calibration strategy. Section 5 reports the quantitative results. Section 6 concludes.

[4] The severe productivity loss during and after crises are non-negligible (Bordo et al., 2001; Jordà, 2005; Cerra and Saxena, 2008; Reinhart and Rogoff, 2009; Jordà, Schularick, and Taylor, 2013). Crises were triggered by various factors, rational or irrational (see Sufi and Taylor, 2022 for a comprehensive review). The most important ones are credit expansion, asset price growth, and their interaction (Schularick and Taylor, 2012; Jordà, Schularick, and Taylor, 2015; Mian, Sufi, and Verner, 2017; Baron, Verner, and Xiong, 2021; Adrian et al., 2022; Greenwood et al., 2022). In this paper, I do not focus on the causes of crises. I use a partial equilibrium model of heterogeneous firms that takes the feature as given to focus on the implications on firms.

2 An Example of One-Period Bond

Before delving into the full version of the dynamic model with a long-term bond, I provide an example using a one-period bond to explain the economic mechanisms of risk premium. Similar to Kuehn and Schmid (2014), the defaultable one-period bond is priced as follows:

$$Q_{it} = \mathrm{E}_t\left[M_{t+1}\left((1-\mathbb{1}^D_{it+1})(1+c)+\mathbb{1}^D_{it+1}R_{it+1}\right)\right] \tag{1}$$

in which c is coupon payment, $\mathbb{1}^D_{it+1}$ is the default indicator, and R_{it+1} is the recovery rate if default happens. Similarly, the risk-free one-period bond is priced as:

$$Q^f_{it} = (1+c)\mathrm{E}_t[M_{t+1}] \tag{2}$$

The credit spread is the difference between the defaultable and risk-free bond:

$$s_{it} = \frac{1+c}{Q_{it}} - \frac{1+c}{Q^f_{it}} \tag{3}$$

Given equations (1), (2), and (3), the credit spread can be written as:

$$s_{it} = \frac{q_{it}-\chi_{it}}{\mathrm{E}_t[M_{t+1}](1-q_{it})} \approx q_{it}-\chi_{it} \tag{4}$$

$$\text{where} \quad q_{it} = \mathrm{E}_t\left[\frac{M_{t+1}}{\mathrm{E}_t[M_{t+1}]}\mathbb{1}^D_{it+1}\right] \tag{5}$$

$$\chi_{it} = \mathrm{E}_t[M_{t+1}\mathbb{1}^D_{it+1}R^*_{it+1}] \tag{6}$$

The credit spread s_{it} increases with the risk-neutral default probability q_{it} and decreases in the value of the recovery rate χ_{it}. The approximation in equation (4) holds because the risk-free rate $\frac{1}{\mathrm{E}_t[M_{t+1}]}$ is close to one and q_{it} is close to zero. $R^*_{it+1} = \frac{R_{it+1}}{Q_{it}}$ is the recovery rate adjusted by bond price. When there is no default, the wedge between q_{it} and χ_{it} becomes zero, meaning the credit spread s_{it} is zero.

To understand the economic intuitions more explicitly, q_{it} can be decomposed into the actual default probability and the covariance between marginal utility and default risk, while χ_{it} can be decomposed into the expected recovery rate and the covariance between marginal utility and recovery risk. Formally:

$$q_{it} = \mathrm{E}_t[\mathbb{1}^D_{it+1}] + \mathrm{cov}_t\left(\frac{M_{t+1}}{\mathrm{E}_t[M_{t+1}]}, \mathbb{1}^D_{it+1}\right) \tag{7}$$

$$\chi_{it} = \frac{\mathrm{E}_t[R_{it+1}\mathbb{1}^D_{it+1}]}{R_f} + \mathrm{cov}_t\Big(M_{t+1}, R_{it+1}\mathbb{1}^D_{it+1}\Big) \tag{8}$$

To see the economic mechanisms, consider two scenarios: normal times and disaster states. In normal times, compared to a model with no disaster risk, the expected default probability and loss are higher due to the potential rare but large economic crash. Suppose a disaster hits; firms will experience a large negative shock on productivity and a sudden drop in continuation value, which could trigger more defaults than in normal times. The introduction of disaster risk leads to a highly nonlinear pricing kernel, M_{t+1}, with a stronger covariance with default, $\mathbb{1}^D_{it+1}$, and recovery rates, R_{it+1}, generating a larger risk premium. The combined effect of expected default risk and risk premium results in a larger credit spread. However, the real default rate in normal times might be low due to the absence of realized disaster states. Therefore, the inclusion of underlying disaster risk helps explain the credit spread puzzle in normal times, where realized default probability and loss are too low to account for the observed high credit spreads in the data.

In disaster states, the default rates and credit spreads are substantially higher than in normal times. The plummet in firm value is both immediate and persistent. If the economy enters a disaster state, it is expected to stay in that state for several years before entering the recovery state. The persistent low productivity and continuation value lead to more defaults and, as a result, higher credit spreads than in normal times.

However, there is a critical difference between a one-period bond and a long-term bond. When a disaster is realized, a one-period bond defaults immediately because the firm cannot repay the full principal next period, causing its continuation value to drop to negative. On the other hand, a long-term bond only requires a fraction of the principal repayment and has a better chance of taking advantage of the recovery state[5]. This difference significantly impacts a firm's choice of optimal capital structure and, consequently, default risk and credit spreads. The implications will be explored further in the following sections.

3 The Model

I propose a partial equilibrium model with a combination of the dynamic corporate finance model (Hennessy and Whited, 2005; Hennessy and Whited, 2007; Kuehn and Schmid, 2014) and the disaster model (Rietz, 1988; Barro, 2009; Bai et al., 2019). Time is discrete and infinite. Heterogeneous firms face financial frictions when making default, investment, and financing decisions. They encounter both aggregate and idiosyncratic productivity shocks. The representative consumer has Epstein-Zin utility, and the stochastic discount factor is taken by firms as given.

3.1 The Representative Consumer

A representative consumer maximizes Epstein-Zin nonseparable expected utility

$$U_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left(E_t[U_{t+1}^{1-\gamma}] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}} \tag{9}$$

where β is the time discount factor, γ is the relative risk aversion, and ψ is the elasticity of intertemporal substitution. The utility function yields the following pricing kernel

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]} \right)^{1-\frac{1-1/\psi}{1-\gamma}} \tag{10}$$

[5] Giglio, Kelly, and Stroebel (2021) discussed the difference between two types of disaster models. The first is that economic and consumption activity causes disaster risk. The second is that disaster risk drives consumption growth. After a disaster hits, the economy adapts and recovers from the disaster. A firm with a long-term bond adapts to the disaster better compared to a firm with a one-period bond because it has a longer time to repay the full principal. Bai et al. (2019) also discussed that long-term bonds have a lower yield due to similar reasons.

Consumption growth is the only aggregate state variable that captures normal business cycles and disasters. The log growth, denoted g_{ct}, follows

$$\log\left(\frac{C_t}{C_{t-1}}\right) \equiv g_{ct} = \bar{g} + g_t \tag{11}$$

where $\bar{g}$ is the average consumption growth rate, and g_t is the stochastic shock. Consumption growth has three states: disaster state, recovery state, and normal times:

$$g_{t+1} = \begin{cases} (1-\eta)g_t^N + \eta\lambda_D & \text{normal state at time } t \\ \theta\lambda_D + (1-\theta)\lambda_R & \text{disaster state at time } t \\ \nu\lambda_R + (1-\nu)g_t^N & \text{recovery state at time } t \end{cases} \tag{12}$$

In normal times, there is a probability of η that the economy will enter the disaster state, where consumption growth is $\lambda_D < 0$, representing a large negative shock. If the economy enters the disaster state, the probability that it persists is θ. The probability that it exits and enters the recovery state is $1-\theta$, where the consumption growth is $\lambda_R > 0$. If the economy is in the recovery state, it will persist with a probability of ν, or it will return to a normal state. In normal times, the consumption growth follows a standard AR(1) process.

$$g_{t+1}^N = \rho_g g_t^N + \sigma_g \epsilon_{t+1} \tag{13}$$

where ϵ_{t+1} follows a standard normal distribution.

Similar to Bai et al. (2019), I first discretize g_t^N into grids that capture normal economic cycles, and then add a disaster and a recovery state. I use the Rowenhorst (1995) method to discretize g_t^N into five values, $\{g_i\}_{i=1}^5$, and obtain the transition matrix P_*. Specifically,

$$P_* = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{15} \\ p_{21} & p_{22} & \cdots & p_{25} \\ \vdots & \vdots & \ddots & \vdots \\ p_{51} & p_{52} & \cdots & p_{55} \end{bmatrix} \tag{14}$$

where $p_{ij} = P_*(g_{t+1} = g_j | g_t = g_i)$, is the transition probability conditional on current consumption growth.

To add the disaster and recovery states, I insert $g_0 = \lambda_D$ and $g_6 = \lambda_R$ into the g_t grid. Here, $\lambda_D < 0$ and $\lambda_R > 0$ capture the size of consumption growth in the disaster and recovery states, respectively. The transition matrix is modified as follows:

$$P = \begin{bmatrix} \theta & 0 & 0 & \dots & 0 & 1-\theta \\ \eta & p_{11}(1-\eta) & p_{12}(1-\eta) & \dots & p_{15}(1-\eta) & 0 \\ \eta & p_{21}(1-\eta) & p_{22}(1-\eta) & \dots & p_{25}(1-\eta) & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta & p_{51}(1-\eta) & p_{52}(1-\eta) & \dots & p_{55}(1-\eta) & 0 \\ 0 & (1-\nu)/5 & (1-\nu)/5 & \dots & (1-\nu)/5 & \nu \end{bmatrix} \tag{15}$$

From any normal state, there is a probability of η that the economy enters the disaster state. The persistence of the disaster state is measured by θ, which is the probability of staying in the disaster state after entering. The economy only enters the recovery state following a disaster state, with probability $1-\theta$. The persistence of staying in the recovery state is ν. The probability of entering any of the normal states from the recovery state is $(1-\nu)/5$.

3.2 Heterogeneous Firms

Firms are indexed by i and have a production function

$$\Pi_{it} = (1-\tau)(Z_{it}X_t^{1-\alpha}K_{it}^{\alpha} - fK_{it}) \tag{16}$$

where $0 < \alpha < 1$ captures the concavity of production function. X_t is the aggregate productivity, Z_{it} is firm's idiosyncratic productivity, and K_{it} is capital. τ is the tax rate. f is the proportional fixed operating cost.

The growth of aggregate productivity, X_t, is linked to consumption growth. Specifically,

$$\log\left(\frac{X_t}{X_{t-1}}\right) \equiv g_{xt} = \bar{g} + \phi g_t \tag{17}$$

where $\phi > 0$, captures the relative volatility of the aggregate productivity growth, compared to the consumption growth.

The firm's idiosyncratic productivity follows an AR(1) process

$$z_{it+1} = (1 - \rho_z)\bar{z} + \rho_z z_{it} + \sigma_z \epsilon^z_{it+1} \tag{18}$$

where $z_{it} = \log(Z_{it})$. ρ_z captures the persistence of the shock, and $\bar{z}$ is the unconditional mean. ϵ^z_{it+1} is an independently and identically distributed random variable that follows a standard normal distribution. Specifically, ϵ^z_{it+1} and ϵ^z_{jt+1} are uncorrelated, if $i \neq j$. ϵ^z_{it+1} is uncorrelated with aggregate shock ϵ_{t+1}, for $\forall i$.

Investment and Adjustment Cost

Firm i chooses investment I_{it} and next period capital K_{it+1}. Capital accumulates as

$$K_{it+1} = (1 - \delta)K_{it} + I_{it} \tag{19}$$

where δ is the depreciation rate. Following Zhang (2005), capital investment incurs asymmetric adjustment costs:

$$\psi(I_{it}, K_{it}) = \begin{cases} \frac{c^+}{2}(\frac{I_{it}}{K_{it}})^2 K_{it}, & \text{for } I_{it} > 0 \\ 0, & \text{for } I_{it} = 0 \\ \frac{c^-}{2}(\frac{I_{it}}{K_{it}})^2 K_{it}, & \text{for } I_{it} < 0 \end{cases} \tag{20}$$

where $c^- > c^+ > 0$, meaning that it is more costly to divest. The costly divestment is critical in bad times when firms try to sell capital because of liquidity shortage.

Costly Financing

Following Kuehn and Schmid (2014), firm i issues long-term bonds to finance investments and dividend distribution. Each period, firms repay κ of the principal outstanding and a fixed coupon c. Thus, the bond has an average maturity of $\frac{1}{\kappa}$ periods. Net bond issuance or repayment is denoted by J_{it}, and the amount of outstanding bonds evolves as:

$$B_{it+1} = (1-\kappa)B_{it} + J_{it} \tag{21}$$

Bond issuance is not frictionless. I assume a positive fixed and proportional cost if the bond issuance is positive. When firms repay bonds (negative net bond issuance), there is no cost. Formally, the asymmetric bond adjustment cost follows:

$$\Phi(J_{it}) = (\phi_0 + \phi_1 J_{it})\mathbb{1}_{J_{it}>0} \tag{22}$$

After making investment and bond issuance decisions, firm i generates net cash flow

$$e_{it} = \Pi_{it} + m_{it} - I_{it} - \psi(I_{it}, K_{it}) + Q_{it}J_{it} - \Phi(J_{it}) - ((1-\tau)c + \kappa)B_{it} \tag{23}$$

which includes three parts: net output, investment with adjustment cost, and bond issuance and principal repayment. In the net output, $m_{it} \overset{i.i.d.}{\sim} \mathcal{N}(0, 0.001)$ represents the transitory income shock. As shown in Chatterjee and Eyigungor (2012), the i.i.d. transitory income shock helps the model with a long-term bond to converge. To mitigate the impact of the transitory income shock, I choose a relatively narrow standard deviation. The firm's coupon and principal payments depend on the amount of bond outstanding, B_{it}. The tax shield on bond financing is captured by tax-deductible coupon payment. As bonds are risky and defaultable, the price Q_{it} is an endogenous function and will be explained in more detail in the next section.

Positive net cash flow means dividend distribution to shareholders, while negative net cash flow means equity issuance. External equity is costly and includes both proportional cost, λ_1, and fixed cost, λ_0. The total equity issuance cost, denoted as $\Lambda(e_{it})$, is calculated as follows:

$$\Lambda(e_{it}) = (\lambda_0 + \lambda_1|e_{it}|)\mathbb{1}_{e_{it}<0} \tag{24}$$

Default and Bond Valuation

At time t, firm i issues the new bond and repays a portion of the outstanding bonds. The price of the new bond, Q_{it}, depends on the expectation of next-period repayment and recovery rate if the firm defaults. The bond valuation follows:

$$Q_{it} = E_t\left[M_{t+1}\left((1-\mathbb{1}^D_{it+1})(c+\kappa+(1-\kappa)Q_{it+1}) + \mathbb{1}^D_{it+1}\xi R_{it+1}\right)\right] \tag{25}$$

where the recovery rate depends on the next period's productivity and capital level and is bounded by 100%

$$R_{it+1} = \min\left\{1, \frac{\Pi_{it+1}+(1-\delta)K_{it+1}}{B_{it+1}}\right\} \tag{26}$$

The proportional loss in default is captured by ξ.

Value Maximization

Above all, there are four state variables in this model: (1) aggregate productivity X_t, (2) idiosyncratic productivity Z_{it}, (3) capital K_{it}, and (4) bond B_{it}. Let $S(Z_{it}, K_{it}, B_{it}, X_t)$ denote the equity value before the default decision, or the continuation value. When the equity value drops below zero, the firm defaults. The post-default equity value is denoted by

$$V(Z_{it}, K_{it}, B_{it}, X_t, m_{it}) = \max\{0, S(Z_{it}, K_{it}, B_{it}, X_t, m_{it})\} \tag{27}$$

Given the pricing kernel and realized productivity levels X_t and Z_{it}, firm i chooses the optimal investment, K_{it+1}, and bond, B_{it+1}, to maximize its pre-default equity value:

$$S(Z_{it}, K_{it}, B_{it}, X_t, m_{it}) = \max_{K_{it+1}, B_{it+1}} \left\{e_{it} - \Lambda(e_{it}) + E_t\left[M_{t+1}V(Z_{it+1}, K_{it+1}, B_{it+1}, X_{t+1}, m_{it+1})\right]\right\} \tag{28}$$

subject to capital and bond accumulation, and bond pricing equations, the pre-default equity value consists of the current net cash flow and the discounted future post-default equity value.

4 Quantitative Results

In this section, I discuss the quantitative implications of my model, with a focus on credit spreads and default rates. First, I explain the data and how the target moments are measured. Then, I explain my calibration and simulation strategies.

4.1 Data

The firm-level fundamental and financial data are from Compustat's North America Fundamental Quarterly, spanning from 1984 to 2022. Equity market data are sourced from CRSP. The average annual default rate is obtained from Bai (2021). The average five-year default rates and credit spreads are obtained from Kuehn and Schmid (2014). The measurement of cumulative default rate in the simulation follows Moody's (2006).

Investment is measured as the difference between capital expenditure (CAPXY) and the sale of property (SPPEY). Total gross property, plant, and equipment (PPEGTQ) is used to measure capital stock and calculate investment to capital ratio. Equity issuance is the difference between the sale of common and preferred stock (SSTKY) and the purchase of common and preferred stock (PRSTKCY), scaled by lagged total assets (ATQ). The total amount of bond outstanding includes debt in current liabilities (DLCQ) and long-term debt (DLTTQ). Bond issuance is measured by the change of total debt outstanding scaled by lagged total assets. All variables that are year-to-date are transformed into quarterly flow variables based on the fiscal year-end.

The market value of equity is calculated as the product of the price per share (PRCCQ) and the number of shares outstanding (CSHOQ). Market leverage is computed by dividing total debt by the sum of total debt and the market value of equity. Consistent with the standard practice in the literature, firms with SIC codes between 4900 and 4999, 6000 and 6999, or greater than 9000 are excluded from the analysis. Additionally, only firms traded on major exchanges, with exchange codes (EXCHG) between 11 and 14, are included in the study.

4.2 Calibration

I calibrate the model at a quarterly frequency. Solving a model with a long-term bond can be numerically challenging and may not guarantee convergence [6]. To address this, I employ three strategies to improve convergence. First, I use the solution of the model with a one-period bond as the initial value for the model with a longer-term bond, following the approach in Kuehn and Schmid (2014). During calibration, I solve the model with one-quarter, 2-year, and 4-year bonds. Second, to address oscillation issues arising from multiple local optimal solutions, I incorporate a transitory income shock m_{it}, as suggested by Chatterjee and Eyigungor (2012). Third, to reduce numerical errors, I employ interpolation on the pre-default equity value function S_{it}, avoiding interpolation near the kink in the post-default equity value function V_{it}, which is bounded by zero. Although these strategies are generally helpful, there are instances where they may fail. To ensure credible model solutions, I experiment with a wide range of parameter values.

The benchmark parameter values are presented in Table 1. The time discount factor β is set to be 0.955 to match the average annual risk-free rate in normal times. This value is lower than those reported in previous literature due to the introduction of disaster risk. The high marginal utility in the disaster state reduces the risk-free rate in normal times, necessitating a lower time discount factor to get the average risk-free rate consistent with the data. The relative risk aversion, $\gamma = 7.5$, and the elasticity of intertemporal substitution (EIS), $\psi = 2$, are chosen to match the annual equity premium in normal times.

For the consumption dynamics, I match the key parameters to the consumption growth data from the National Income and Product Accounts Table 7.1 Chained (2012) dollars. I use data from 1947 to 2019 to eliminate the special effects of COVID-19[7]. Based on the empirical observations of consumption growth, I set the unconditional mean $\bar{g}$ to be 0.0036, the persistence or autocorre-

[6] The distance between the value function for this and the previous iteration tends to oscillate between some repeated values after reaching some level, for example, 10^{-2}.

[7] If including the COVID-19 period, the autocorrelation of consumption growth becomes -0.1, which is significantly different from the value of 0.3 in normal times as documented in the literature.

lation ρ_g to be 0.3, and the standard deviation σ_g to be 0.005. To ensure that the persistence and conditional volatility of consumption growth are close to those of productivity growth, following Bai et al. (2019), I set the relative volatility of aggregate productivity growth, $\phi = 1$.

For the parameters that govern the disaster dynamics, I set the persistence of the disaster state θ to be 0.914, which is the same as the value in Gourio (2012). The probability of entering the disaster state from normal states η is 0.028/4, implying an annual disaster probability of 2.8% and a quarterly disaster probability of 0.7%. The value is close to Gourio (2012) and Nakamura et al. (2013). To find the persistence of the recovery state ν, the size of consumption crash in the disaster state λ_D, and the size of the recovery state λ_R, I experimented with different values to match the cumulative consumption growth pattern reported in Bai et al. (2019), Gourio (2012), and Nakamura et al. (2013). The procedure yielded $\nu = 0.88$, $\lambda_D = -0.07$, and $\lambda_R = 0.037$. Figure 2 reports the average and median of simulated cumulative consumption growth and its comparison with real data (bottom figure from Gourio (2012)). The maximum short-term negative shock is around 20%, and the long-term fall is around 15%.

To calibrate the firm-side parameters, I set most of the values following what is reported in the literature. The curvature parameter, $\alpha = 0.65$, follows Hennessy and Whited (2007). The quarterly depreciation rate, δ, is 0.03 as in Kuehn and Schmid (2014) and Cooley, Hansen, and Prescott (1995). The tax rate on output, $\tau = 0.3$, is close to the values reported in Hennessy and Whited (2007). The coupon rate is set to 2%. Following Kuehn and Schmid (2014), the parameter that governs bond maturity, κ, is set to be 0.05, to reflect the five-year maturity. Similar to the values reported in Kuehn and Schmid (2014), I set the persistence of the firm's idiosyncratic volatility, ρ_z, to be 0.85, its volatility, σ_z, to be 0.12, and the recovery rate, $\xi = 0.8$, when default happens.

The remaining five parameters are calibrated to match a list of various moments of corporate policies, along with the average historical default rate and credit spreads. The asymmetric investment adjustment costs, $c^+ = 0.4$ and $c^- = 2.8$, are set to match the average and volatility of the investment-to-asset ratio. A relatively high divestment adjustment cost reduces both the level and volatility of the investment-to-asset ratio. As it is more costly to divest in bad times, it discourages firms invest in normal times. The average long-run idiosyncratic productivity, $\bar{z} = -1.8$, is set to roughly match the average credit spreads and default rates. The fixed cost of operation, $f = 0.003$, is further calibrated to match the moments more accurately. I start calibrating the parameters that govern financing frictions following Kuehn and Schmid (2014). The bond adjustment fixed cost ($\phi_0 = 0.03$) and proportional cost ($\phi_1 = 0.078$) are set to match the bond issue moments, both the frequency and the average amount scaled by the capital level. The equity issuance costs, $\lambda_0 = 0.06$ and $\lambda_1 = 0.03$, are calibrated accordingly to match the equity issuance moments.

5 Quantitative Results

In this section, I first discuss the tables that match the aggregate and firm-level moments and explore how disaster risk affects credit spreads in normal times. Then, I show the counterfactual results regarding how the key variables react when a disaster hits. Lastly, I study how real and financial frictions affect credit spreads with the threat of disaster risk.

5.1 Aggregate and Firm-Level Moments

The benchmark parameter values of the representative consumer and firms are reported in Table 1. I independently simulate 5000 samples at a quarterly frequency, each with 5000 firms and 156 quarters (39 years) to match the length of the data. Among the 5000 samples, those without disasters are defined as "normal times." The parameters are calibrated to match key moments in normal times with the data. The chosen moments are sensitive to parameter values to discipline the model. The key variables include the cross-sectional average and dispersion of the investment-to-asset ratio, the cross-sectional frequency, and the average amount of equity and bond financing, the cross-sectional average and dispersion of market leverage, and the cross-sectional average of default rates and credit spreads.

Table 2 reports firm-level moments. The most important moments are the default rates and credit spreads because they are the focus of this study. Bai (2021) reports an average annual default rate of 0.7%, and Kuehn and Schmid (2014) reports a cumulative five-year default rate of 2%. My model generates an annual default rate of 0.66% and a five-year default rate of 3.34%, which is overall consistent with the data.

The average credit spread between Baa-rated and Aaa-rated corporate bonds is reported to be 70-100 bps in Bai (2021) and 100 bps in Kuehn and Schmid (2014). My model generates an average credit spread of 92.79 bps and a 90% confidence interval range between 19.52 and 161.90, which covers the empirical observations.

The market leverage in the data is around 24.3%, while Bai et al. (2019) reports an average market leverage of 30% using international data. My model produces an average market leverage of 31.1%. The result is higher than the empirical observation using Compustat data, but it is close to the number using international data. The 90% confidence interval is between 6.3% and 54.6%, covering both numbers.

My simulation results overall match the investment and financing moments. The model-simulated average investment-to-asset ratio is 0.029, and its volatility is 0.095, which are close to the empirical moments of 0.033 and 0.111, respectively. The equity issuance and bond issuance moments are also overall close to the data, while the average amounts are slightly off track. The average amount of new equity is lower in the model simulation compared to the data, while the new bond issuance amount is higher. This discrepancy is due to the attractiveness of long-term bonds under the threat of disaster risk. The average frequencies of new equity and bond issuance are 0.069 and 0.274 in the data, and the simulation results are 0.181 and 0.268, respectively. Both results fall within the 90% confidence interval.

The real and financial friction parameters jointly affect investment and financing strategies.

Lower investment adjustment costs lead to a lower average investment ratio because firms can easily divest the capital, which also generates higher volatility of the investment ratio. It also leads to more aggressive financing strategies because it is easier for firms to liquidate capital. Less financial frictions have similar effects, as it becomes easier to issue new equity and bonds.

Table 3 reports the aggregate moments. My model generates a 2.75% risk-free rate and 8.46% aggregate equity premium in normal times, which are close to the empirical observations in the data. However, the model generates lower average volatilities of the risk-free rate and aggregate equity return compared to the data. The cyclicality moments are from Bai (2021), including the correlation between consumption growth and average default rate, credit spread, and market leverage. My model generates qualitatively consistent results compared to the data.

To see the economic mechanisms more explicitly, I plot the policy functions of the two endogenous state variables, capital (K) and bond outstanding (B) in Figure 3. When plotting the policy function with respect to one state variable, I fix the other state variables at the average level, except for the consumption growth. The policy functions are plotted at both average consumption growth in normal time and disaster states, to contrast their differences.

Figure 3 Panel (a) plots the investment to capital decision with respect to K and B. The blue line is when the consumption growth shock is zero, and the red line is when the economy is in a disaster state. Firms invest more when the capital level is low, and divest to get liquidity when a disaster hits. Similarly, they divest more when the bond outstanding is higher, to repay the predetermined principal and coupon.

Figure 3 Panel (b) plots the bond issue to the total bond outstanding decision. Firms issue more bonds when the capital level is low because they cannot rely on the output to maintain operations and repay liabilities. In a disaster state, if the firm does not default, the firm issues more bonds to mitigate the large negative productivity and output shock. Firms issue bonds when they have more bonds outstanding, showing a tendency to use new bonds to repay old bonds.

There is heterogeneity across firms' policy functions when they have different amounts of capital and bond outstanding. Figure 4 plots the investment decisions for small (lowest K) and big (highest K) firms, and firms with more (highest B) or less (lowest B) bonds outstanding. Small firms default when the bond outstanding is too high. If they do not default, they invest a lot to accumulate more capital. In comparison, in normal times, big firms invest more when the bond outstanding is not too high, and invest less if they have a large liability outstanding. They divest when a disaster hits regardless of their bond level.

Firms with different levels of bond outstanding have different investment decisions. The investment decision with respect to the capital level generally shows a convex pattern. Firms with less capital invest to accumulate more capital, and firms with a lot of capital invest less because of the decreasing return to scale. Firms with moderate amounts of capital divest to mitigate the negative productivity shock because they lack enough capital to produce and meet operation and liability requirements. Firms with fewer bonds outstanding will most time invest and divest when a disaster hits. However, firms with more bonds outstanding but not high enough capital will divest a lot to repay the liabilities because of the sudden drop of output.

Figure 5 plots the bond issue decisions. Small firms, if not default, are bond issuers and they rely on new bonds to repay old bonds. They are more likely to default and issue less bonds in a disaster state. Big firms, on the other hand, are savers. They rely more on capital and output to operate. Firms with fewer bonds outstanding do not default. In the disaster state, they issue more bonds when the capital level is low, but repay the bonds when the capital level is high. In comparison, firms with more bonds outstanding default when the capital level is low. They, if not default, always issue bonds especially when the capital level is low.

In this section, I show that the model does a reasonable job of matching the key aggregate and firm-level moments. I also show the underlying policy functions that are generating the results. With a calibrated model, I use it to explore other implications in the following sections.

5.2 Disaster Risk Implications

As a solution to the "credit spread puzzle," the benchmark results successfully match the default rate, credit spread, and equity premium in the data. To further explore the underlying economic mechanisms, in this section, I discuss the effects of disaster risk on credit spreads in normal times. I focus on three dimensions: disaster probability (η), the severity of disaster (λ_D), and the persistence of disaster (θ). I separately explore the model under higher and lower η, λ_D, and θ, while keeping all other factors fixed.

As a counterfactual analysis, I explore parameter values that differ but are close to the benchmark values. I do not completely shut down the disaster risk because the model results are very sensitive to parameter values. If I completely remove disaster risk from the model, the results would be fundamentally different. The aggregate and corporate policy moments would deviate significantly from the benchmark moments, making it difficult to compare the default rate and credit spread results. Therefore, I focus on exploring the changes when the disaster risk becomes more or less severe while keeping it present in the model.

Table 4 reports the model simulation results with different scenarios of the disaster risk. Overall, increased disaster risk (higher probability, more severity, or higher persistence) leads to higher credit spreads in normal times. There are two main economic mechanisms driving these results. First, more disaster risk is related to a higher risk premium and expected default rate. Second, firms adopt different bond financing strategies in response to different levels of disaster risk. When the disaster risk is more severe, firms face difficulty in accumulating capital as investors become more hesitant to invest in risky assets. Firms have to issue new bonds to maintain liquidity and roll over existing bonds. Meanwhile, long-term bonds serve as a hedging instrument for firms due to lower promised principal repayment and potential recovery states. With the same amount of bonds, a lower capital level leads to lower equity value and, therefore, higher leverage. This combination of factors causes higher credit spreads and higher default rates when the disaster risk is more severe.

Using the benchmark parameter values, the model generates an average annual default rate of 0.66%, a five-year default rate of 3.34%, an average credit spread of 92.79 bps, and an average market leverage of 31.1%. First, I study two scenarios of disaster probability. With lower disaster probability, the average annual default rate decreases to 0.24%, and the average credit spread decreases to 30.36 bps, along with an average leverage of 12.3%. In contrast, with a higher disaster probability, the model generates an average annual default rate of 2.41%, an average credit spread of 159.68 bps, and an average market leverage of 42.8%.

Then, I study the two scenarios of the consumption growth crash. If I decrease λ_D, which means a model with a more severe consumption growth crash when a disaster is realized, the model generates an annual default rate of 1.36%, an average credit spread of 254.03 bps, and an average market leverage of 0.86%. In contrast, higher λ_D, or less severe consumption growth crash, leads to an annual default rate of 0.55%, an average credit spread of 33.96 bps, and an average market leverage of 9.8%.

Lastly, I study the two scenarios of the persistence of a disaster. If the disaster risk is less persistent, with lower θ, the annual default rate decreases to 0.45%, the average credit spread decreases to 21.18 bps, and the market leverage decreases to 19.5%. In contrast, with more persistent disaster risk, or higher θ, the model generates an annual default rate of 2.12%, an average credit spread of 138.09 bps, and an average market leverage of 37.2%.

5.3 Disaster States

In this section, I report the model predictions during disaster states. Tables 5 and 6 present the key moments in the simulation samples where disasters occur. Compared to normal times, more firms default, and credit spreads increase significantly. The annual default rate is more than ten times higher than in normal times, with a 90% confidence interval from 1.13% to 27.10%. The average credit spread more than triples the normal time average, with a 90% confidence interval from 111.25 bps to 652.73 bps This is consistent with the observations around the Great Depression as plotted in Figure 1. During the Great Depression, the annual default rate peaked at 8% and the credit spread was as high as 6%.

Realized disasters vastly affect corporate policies. Surviving firms reduce investments, or even divest, to compensate for the substantial negative cash flow shock. The average investment ratio drops to slightly negative, -0.005, and the cross-sectional volatility of investment increases to 0.14. Bond financing becomes more challenging than in normal times due to financing frictions, low expected recovery rates, and high expected default rates. The average frequency and amount of new bond issuance both drop to 0.22. Equity financing is almost frozen, with the frequency of new equity issuance dropping to 0.021, and the average amount dropping to 0.008.

To provide a more explicit demonstration of the economic mechanisms, I simulate a long sample of 100,000 quarters after dropping the burning period of 1000 quarters and plot key variables with respect to consumption growth. The key variables are depicted in heatmaps to show the density of observations. As both disaster and recovery states are discrete in my model, the variables are plotted vertically. In normal times, I simulate the economic states in a more continuous way. In each figure, I report the correlation between the variable and the consumption growth to illustrate how they vary with different macroeconomic conditions.

Figure 6 displays the dynamics of default rates and credit spreads. Both have a strong negative correlation with consumption growth. In the disaster state, the average default rate and credit spread are significantly higher. However, the range is wide due to different states before the economy enters the disaster state. If the economy was performing well and firms are financially stronger, there will be less default, and the credit spread will be lower when a disaster is realized.

Figure 7 shows the dynamics of investment. The investment-to-capital ratio is positively correlated with consumption growth, while its dispersion has a strong negative correlation. Firms liquidate capital when there is a large negative productivity and cash flow shock to maintain operation and repay bond principals. The divestment incentives vary largely across firms, depending on their financial conditions, which leads to a large cross-sectional dispersion.

Figure 8 displays the dynamics of bond and equity financing. Equity financing is positively correlated with consumption growth. In the disaster state, the new equity issuance frequency and amount are very close to zero. The new bond issuance frequency is also close to zero in the disaster state, but the amount has a negative correlation with consumption growth. However, most observations are clustered around zero. The negative relationship is driven by the disaster following a good economic condition. Survival firms are financially strong and can take advantage of long-term bonds when a disaster hits. Therefore, the amount of new bond issuance in the disaster state has a wide range, which generates a negative correlation.

Figure 9 demonstrates the dynamics of market leverage and recovery rate. Market leverage is negatively correlated with consumption growth, while the recovery rate has a positive correlation. The high market leverage is driven by the plummet of firm values and persists in the recovery state. The recovery rate has a similar but opposite pattern, driven by the large negative productivity and cash flow shock.

5.4 Decompose the Credit Spread

The results of previous sessions show that in both normal times and realized disaster states, credit spreads change along with endogenous leverage choices. Meanwhile, as shown in the literature, credit spreads are determined by the expected default probability and loss, and the risk premium. In this section, I decompose the credit spreads into three channels (default risk, credit risk premium, and leverage), and study their relative importance. Specifically, I restructure equation (7) and (8) and define the "Expected Loss in Default" as

$$\mathrm{E}_t[\mathbb{1}^D_{it+1}] - \frac{\mathrm{E}_t[R_{it+1}\mathbb{1}^D_{it+1}]}{R_f} \tag{29}$$

which is the difference between the expected default probability and the expected recovery rate if a default happens. The higher (lower) the default rate (recovery rate) is, the higher the "Expected

Loss in Default" (hereafter expected loss) is. I use this term to represent the degree of default risk.

The "Credit Risk Premium" is defined as

$$\mathrm{cov}_t\left(\frac{M_{t+1}}{\mathrm{E}_t[M_{t+1}]}, \mathbb{1}^D_{it+1}\right) - \mathrm{cov}_t\left(M_{t+1}, R_{it+1}\mathbb{1}^D_{it+1}\right) \tag{30}$$

which captures the covariance between the marginal utility and the default. In bad times, marginal utility and default probability are high, while the recovery rate is low. Therefore, the first term is positive and the second term is negative, leading to a positive "Credit Risk Premium".

The definitions above are based on a model with one-period bonds because there is no closed-form solution for long-term bonds. Although not perfect, they are proxies for the default risk and risk premium, which can be used to shed light on how much the credit spreads are explained by the three channels. Furthermore, the default risk and risk premium are firm-specific, giving explanations on firm-level credit spreads.

To decompose the credit spreads, I simulate a panel of firms, and regress credit spreads on market leverage, the expected loss, and the credit risk premium. I use different combinations of the explanatory variable and explore their incremental contribution to R^2. The coefficients and R^2 are averaged across 5000 independent simulations. Following each coefficient estimate, I also report the 5% and 95% percentiles of the estimate across all simulations to show the confidence interval. I report results from pooled panel regressions with and without time-fixed effects.

Table 7 reports the results in normal times and times with realized disasters. Intuitively, all the coefficients on explanatory variables are positive. In normal times, the credit risk premium has the strongest explanatory power, with R^2 of 0.55. In comparison, market leverage and the expected loss alone have R^2 of 0.29 and 0.39, respectively. The combination of the credit risk premium and the expected loss explains around 90% variation of the credit spread. Adding time-fixed effects only marginally increases the R^2. The results are almost unchanged.

In times of realized disasters, the expected loss has the strongest explanatory power. It alone has a R^2 of 0.86, showing that the default risk plays the biggest role in explaining the credit spreads. Adding time-fixed effects increases the explanatory power of all factors, especially market leverage and credit risk premium. However, the results are qualitatively unchanged: default risk still plays the biggest role. Intuitively, in realized disaster periods, the default probability is expected to be much higher because of the persistent disaster state. Meanwhile, the recovery rate is much lower if a default happens because of the plummet in productivity. In contrast, market leverage has limited explanatory power. When a disaster is realized, all firms suffer from the productivity shock. The large negative shock drives firm values directly to negative, regardless of their leverage level.

The comparative statics in Table 4 show that different degrees of disaster risk have direct effects on both the market leverage and the credit spread. The first impression of the result is that market leverage drives the change in credit spreads. However, it is only part of the story. Table 7 shows that the credit risk premium and the expected loss together explain a large portion of the credit spreads.

Then, is the market leverage redundant? The answer is no, because it is still strongly significant after controlling the credit risk premium and the expected loss. To make it more explicit, I run two tests. First, I explore the effects on next-period credit spreads. Then I run more cross-section regressions on other relevant variables to explore what is driving the results. The regressions on next-period credit spreads show that market leverage plays the biggest role in normal times. In comparison, when disasters are realized, default risk still plays the biggest role, and the second explanatory one now becomes market leverage. The striking difference between contemporaneous and lagged regressions is due to the exogenous productivity shocks on firms, which directly impact current period credit risk premium, default risk, and credit spreads. In contrast, next-period credit spread can only be predicted by market leverage, which is endogenously chosen by firms.

To further disseminate the effects, I regress market leverage on the credit risk premium and the expected loss and test if it is fully explained. The first three columns in Table 9 report the regression results. Intuitively, the coefficients are both positive and significant. Higher risk premiums and expected losses are transformed into higher market leverages. The regression results show that the credit risk premium has more explanatory power on market leverage, both in normal times and disaster states. However, the combination of both only explains less than 20% variations in market leverage, showing that the market leverage cannot be subsumed.

To see how the market leverage is affected more clearly, I report regressions using other related variables. Table 9 shows that higher risk premiums and expected losses are transformed into lower capital levels and firm equity values, increasing market leverages. The credit risk premium has stronger effects than the expected loss. They also lower firm bond issuance, leading to a more conservative financing strategy.

In summary, the credit spread can be decomposed into three channels: credit risk premium, default risk, and market leverage. Consistent with previous bond pricing models, the first two channels have strong explanatory power on contemporaneous credit spreads. They also increase market leverage by lowering firm capital level and equity value. The heightened market leverage will be transmitted into next-period credit spreads. Endogenous dynamics of market leverage contribute to the credit spreads in normal times but have limited impacts in disaster states.

5.5 Cross-Section Analysis

The decomposition of credit spread highlights the interesting results at the firm level. Firms' characteristics are important in explaining credit spreads in both normal times and disaster states. In this section, I study more cross-section implications. First, I verify if my model generates cross-section implications that are generally consistent with previous literature (Gomes and Schmid, 2021). Second, I explore how different firms react to the realization of disasters.

Table 10 reports the regression results between credit spread and market leverage, and other key firm characteristics. The data column is from Gomes and Schmid (2021). Overall, the credit spread column is consistent with the data, except for the size. Higher market leverage increases credit spreads, while higher profitability decreases credit spreads. In my model, bigger firms have more capital and output more. All else equal, bigger firms have lower credit spreads. However, the market leverage column is counter-intuitive. The size and profitability coefficients are both opposite to empirical observations. The reason is that bigger firms are more productive and have higher equity value, which decreases the market leverage. When firms are profitable, they have more capacity to hold more bonds which increases the market leverage. This is opposite to the empirical findings that firms, in good time, repay bonds to increase borrowing capacity.

Other than the relationship between contemporaneous characteristics, firms respond differently when a disaster hits. For example, right after the COVID-19 hit, financially more flexible firms lost less value in their stock prices and suffered less during the pandemic period (Fahlenbrach, Rageth, and Stulz, 2021; Stulz, 2023). Figure 10 plots the impulse response functions of average firm equity value and credit spread. I separately plot firms with more or less financial flexibility, which is defined as market leverage [8]. The results are qualitatively the same as the empirical findings. More financially flexible firms have less drop in equity value and a jump in credit spreads. With a recovery state, the lost equity value is partially recovered, and credit spreads decrease to the pre-disaster period. Without a recovery state, the lost equity value and increased credit spreads are not restored.

Table 11 reports the collapse period equity return and credit spread on other firm characteristics. The collapse period is defined as the first quarter when a disaster is realized. Intuitively, more profitable firms lose less equity value, and the credit spreads increase less. Conditional on market leverage, bigger firms lose less equity value. However, they face high divestment costs to liquidate capital, which is transmitted into higher next-period credit spreads.

5.6 Effects of Other Economic Forces

In this section, I study how various economic factors affect credit spreads. I first explore firm-side parameters, and then analyze consumer-side parameters. On the firm side, I focus on real and fi-

[8] Fahlenbrach, Rageth, and Stulz (2021) uses cash and book leverage to define financial flexibility. I use market leverage here for two reasons. First, I do not have cash in my model, and market leverage is well-calibrated to the data moments. Second, market leverage sheds light on the channel of equity issues as a financing instrument.

nancial frictions during normal times. I analyze the model by resolving it with different parameter values that govern investment adjustment costs and financing costs while keeping all other factors constant. Contrary to the findings in Kuehn and Schmid (2014), I find an opposite effect of real and financial frictions. In their paper, they find that real and financial frictions increase credit spread because bonds become riskier. However, in my model, with more friction, both the default rate and credit spread decrease. When it is more costly to roll over bonds or issue equity, the hedging motive becomes less attractive. As a result, firms choose more conservative bond financing strategies, and market leverage drops. Conversely, when there is less friction, firms choose more aggressive financing strategies because it becomes easier to roll over bonds.

Additionally, the effects of the recovery rate (ξ) and fixed operation cost (f) are more intuitive. A lower recovery rate or a higher fixed operation cost leads to an increase in the default rate and credit spread. These findings shed light on how real and financial frictions, and other economic forces, play a crucial role in determining credit spreads in the model, providing a better understanding of the economic mechanisms during normal times.

Table 12 reports the detailed comparative statics during normal times. In Panel A, we observe the effects of more friction, including higher investment adjustment costs and financing costs, lower recovery rates, and higher fixed operation costs. Higher investment adjustment costs cause lower leverage, leading to less default and a lower credit spread. The credit spread decreases from 92.79 bps to less than 40 bps. Similar effects are observed with higher financing costs. Even small changes in fixed equity and bond financing costs have strong effects, reducing the credit spread to less than 20 bps. Lower proportional financing costs have similar but more moderate effects. Lower proportional equity issuance costs lower the credit spread to 70.11 bps, and lower proportional bond issuance costs lower it to 35.54 bps. A lower recovery rate increases the credit spread to 348.15 bps, while a higher fixed operation cost increases it to 150.82 bps.

Panel B of Table 12 presents the effects of less friction, which are generally opposite to Panel

A. Lower investment adjustment costs encourage more investments during normal times. However, firms with more capital become riskier due to the asymmetric investment adjustment costs. It becomes costly to divest when there is a negative shock and firms need to liquidate capital. Higher investment costs increase the credit spread to 93.51 bps, and higher divestment costs increase it to 164.80 bps. Lower costs of equity and bond financing increase default rates and credit spread. As it becomes cheaper to finance, firms opt for more aggressive corporate policies. Higher fixed (proportional) equity issuance costs increase the credit spread to 256.32 bps (161.60 bps). Lower fixed (proportional) bond issuance costs increase the credit spread to 222.92 bps (225.47 bps). Conversely, a higher recovery rate lowers the credit spread to 23.19 bps, and a lower fixed operation cost lowers it to 74.64 bps.

The effects of real and financial frictions in the disaster state are more limited because default risk dominates other economic forces. Although limited, the pattern of default rate, credit spread, and market leverage is similar to the pattern in normal times. With more friction, the average credit spread is around 200 bps, slightly lower than the benchmark, and the annual default rate is around 10%, which is close to the benchmark result. Table 13 reports the detailed results and shows that the negative aggregate productivity shock has a more direct and critical impact, while frictions only have marginal effects when a disaster hits.

Now I explore the effects of the stochastic discount factor. Table 14 reports the comparative statics of key parameters that govern the dynamic of the stochastic discount factor. Panel A reports the results with higher parameter values, with everything else held fixed. Panel B reports results with lower parameter values. I focus on the impacts on market leverage, default rate, and credit spreads.

Overall, more patient (higher β), more risk averse (higher γ), and more volatile aggregate productivity (higher ϕ) lead to higher credit spreads. However, the economic mechanisms are slightly different. More patience leads to lower default rates because it directly increases firm value in all states and decreases the probability of default. Firms choose more aggressive financing decisions, which generates higher market leverage, and then causes credit spreads to increase. In contrast, more risk aversion and more volatile aggregate productivity shock decrease firm value and increase default productivity, which also generates higher market leverage and credit spreads.

Higher elasticity of intertemporal substitution (ψ), persistence (ρ_g), and volatility (σ_g) of consumption growth generate slightly higher default rates, lower market leverages, and lower credit spreads. When ψ is high, the representative consumer is more willing to accept fluctuations, which leads to more diverse firm values and generates higher default rates. In response, firms choose more conservative financing policies. The capital structure decision dominates the default risk in normal times and generates lower credit spreads. When consumption growth (and aggregate productivity growth) is more persistent, firms' output is also more persistent. Firms have lower market leverage, which generates lower credit spreads. When consumption growth (and aggregate productivity growth) volatility is high, firms are more likely to get negative continuation values, which generates high default rates. In response, firms adopt more conservative financing policies, which leads to lower credit spreads.

In summary, both firm-side frictions and consumer-side preferences affect credit spreads. More real and financial friction leads to lower credit spreads. Overall, the effects of market leverages dominate the effects of default rates. The changes in credit spreads align with the direction of the changes in market leverages, which highlights the impacts of endogenous corporate policy choices on credit spreads.

6 Conclusion

Macroeconomic conditions have important implications for corporate credit risk. Disaster risk, as a type of rare but severe macroeconomic downturn, directly impacts corporate credit spreads by affecting default risk and risk premium. Additionally, it critically influences a firm's capital structure due to its effects on corporate policies. To explore how disaster risk affects credit spreads, I develop a dynamic structural model that rationalizes corporate investment, financing, and default decisions. The model simulations are divided into economies with and without realized disasters.

When disaster risk is realized, the model predicts a significantly higher credit spread, default rate, and market leverage. In normal times when disaster risk is not realized, disaster risk influences credit spreads through its impact on default risk, risk premium, and corporate decisions. Higher disaster risk leads to a higher default probability and a more nonlinear pricing kernel. Meanwhile, firms have higher leverage as they face difficulty in accumulating capital and experience lower equity value. Without enough capital and output, firms are forced to issue new bonds to roll over existing ones and maintain bond levels. At the same time, long-term bonds offer the opportunity to take advantage of subsequent recovery states and have less pressure on principal repayment. These factors collectively result in higher market leverage and credit spreads on average.

I also explore the effects of other economic forces, with a focus on real and financial frictions. The results differ from previous findings when there is disaster risk, mainly due to their varying impacts on firm leverage.

Solving a dynamic model with a long-term bond is computationally challenging. To keep the setting tractable, I abstract from many interesting aspects, such as corporate cash holding and Bayesian updates of disaster risk. It would be useful to study more interactions between disaster risk and other aspects of corporate finance in future research.

References

Adrian, Tobias, Grinberg, Federico, Liang, Nellie, Malik, Sheheryar, and Yu, Jie. (2022) The term structure of growth-at-risk. *American Economic Journal: Macroeconomics*, *14*(3), 283–323.

Bai, Hang. (2021) Unemployment and credit risk. *Journal of Financial Economics*, *142*(1), 127–145.

Bai, Hang, Hou, Kewei, Kung, Howard, Li, Erica XN, and Zhang, Lu. (2019) The capm strikes back? an equilibrium model with disasters. *Journal of Financial Economics*, *131*(2), 269–298.

Baron, Matthew, Verner, Emil, and Xiong, Wei. (2021) Banking crises without panics. *The Quarterly Journal of Economics*, *136*(1), 51–113.

Baron, Matthew, Xiong, Wei, and Ye, Zhijiang. (2022) *Measuring time-varying disaster risk: An empirical analysis of dark matter in asset prices* (tech. rep.). Mimeo, Princeton University.

Barro, Robert J. (2009) Rare disasters, asset prices, and welfare costs. *American Economic Review*, *99*(1), 243–64.

Bordo, Michael, Eichengreen, Barry, Klingebiel, Daniela, and Martinez-Peria, Maria Soledad. (2001) Is the crisis problem growing more severe? *Economic policy*, *16*(32), 52–82.

Cerra, Valerie, and Saxena, Sweta Chaman. (2008) Growth dynamics: The myth of economic recovery. *American Economic Review*, *98*(1), 439–457.

Chatterjee, Satyajit, and Eyigungor, Burcu. (2012) Maturity, indebtedness, and default risk. *American Economic Review*, *102*(6), 2674–2699.

Chen, Hui. (2010) Macroeconomic conditions and the puzzles of credit spreads and capital structure. *The Journal of Finance*, *65*(6), 2171–2212.

Claessens, Mr Stijn, and Kose, Mr Ayhan. (2013) Financial crises explanations, types, and implications.

Cooley, Thomas F, Hansen, Gary D, and Prescott, Edward C. (1995) Equilibrium business cycles with idle resources and variable capacity utilization. *Economic Theory*, *6*, 35–49.

Dickerson, Alexander, Mueller, Philippe, and Robotti, Cesare. (2023) Priced risk in corporate bonds. *Available at SSRN*.

Driessen, Joost. (2005) Is default event risk priced in corporate bonds? *The Review of Financial Studies*, *18*(1), 165–195.

Eom, Young Ho, Helwege, Jean, and Huang, Jing-zhi. (2004) Structural models of corporate bond pricing: An empirical analysis. *The Review of Financial Studies*, *17*(2), 499–544.

Fahlenbrach, Rüdiger, Rageth, Kevin, and Stulz, René M. (2021) How valuable is financial flexibility when revenue stops? evidence from the covid-19 crisis. *The Review of Financial Studies*, *34*(11), 5474–5521.

Giglio, Stefano, Kelly, Bryan, and Stroebel, Johannes. (2021) Climate finance. *Annual Review of Financial Economics*, *13*, 15–36.

Gomes, Joao F, and Schmid, Lukas. (2021) Equilibrium asset pricing with leverage and default. *The Journal of Finance*, *76*(2), 977–1018.

Gourio, Francois. (2012) Disaster risk and business cycles. *American Economic Review*, *102*(6), 2734–66.

Gourio, Francois. (2013) Credit risk and disaster risk. *American Economic Journal: Macroeconomics*, *5*(3), 1–34.

Gourio, Francois, and Michaux, Michael. (2012) Financing investment with long-term debt and uncertainty shocks. *Unpublished working paper. University of Southern California*.

Greenwood, Robin, Hanson, Samuel G, Shleifer, Andrei, and Sørensen, Jakob Ahm. (2022) Predictable financial crises. *The Journal of Finance*, *77*(2), 863–921.

Hackbarth, Dirk, Miao, Jianjun, and Morellec, Erwan. (2006) Capital structure, credit risk, and macroeconomic conditions. *Journal of Financial Economics*, *82*(3), 519–550.

Hennessy, Christopher A, and Whited, Toni M. (2005) Debt dynamics. *The Journal of Finance*, *60*(3), 1129–1165.

Hennessy, Christopher A, and Whited, Toni M. (2007) How costly is external financing? evidence from a structural estimation. *The Journal of Finance*, *62*(4), 1705–1745.

Huang, Jingzhi, and Huang, Ming. (2012) How much of the corporate-treasury yield spread is due to credit risk? *The Review of Asset Pricing Studies*.

Jordà, Òscar. (2005) Estimation and inference of impulse responses by local projections. *American economic review*, *95*(1), 161–182.

Jordà, Òscar, Schularick, Moritz, and Taylor, Alan M. (2013) When credit bites back. *Journal of money, credit and banking*, *45*(s2), 3–28.

Jordà, Òscar, Schularick, Moritz, and Taylor, Alan M. (2015) Leveraged bubbles. *Journal of Monetary Economics*, *76*, S1–S20.

Kuehn, Lars-Alexander, and Schmid, Lukas. (2014) Investment-based corporate bond pricing. *The Journal of Finance*, *69*(6), 2741–2776.

Leland, Hayne E. (1994) Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, *49*(4), 1213–1252.

Merton, Robert C. (1974) On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, *29*(2), 449–470.

Mian, Atif, Sufi, Amir, and Verner, Emil. (2017) Household debt and business cycles worldwide. *The Quarterly Journal of Economics*, *132*(4), 1755–1817.

Moody's. (2006) Measuring corporate default rates.

Nakamura, Emi, Steinsson, Jón, Barro, Robert, and Ursúa, José. (2013) Crises and recoveries in an empirical model of consumption disasters. *American Economic Journal: Macroeconomics*, *5*(3), 35–74.

Reinhart, Carmen M, and Rogoff, Kenneth S. (2009) The aftermath of financial crises. *American Economic Review*, *99*(2), 466–472.

Rietz, Thomas A. (1988) The equity risk premium a solution. *Journal of Monetary Economics*, *22*(1), 117–131.

Schularick, Moritz, and Taylor, Alan M. (2012) Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870–2008. *American Economic Review*, *102*(2), 1029–1061.

Stulz, René M. (2023) *Crisis risk and risk management* (tech. rep.). National Bureau of Economic Research.

Sufi, Amir, and Taylor, Alan M. (2022) Financial crises: A survey. *Handbook of International Economics*, *6*, 291–340.

Wachter, Jessica A. (2013) Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, *68*(3), 987–1035.

Zhang, Lu. (2005) The value premium. *The Journal of Finance*, *60*(1), 67–103.

Table 1: Benchmark Parameter Values

This table lists the benchmark parameter values used to solve and simulate the model at a quarterly frequency. I break all the parameters into two groups. The top group includes parameters from the utility function of the representative consumer. The bottom group includes parameters from the firm side. For each parameter, I list its notation, value, and explanation.

Parameter	Value	Meaning
	Consumer	
β	0.955	Subjective discount factor
γ	7.5	Relative risk aversion
ψ	2	Elasticity of intertemporal substitution
$\bar{g}$	0.0036	Average consumption growth
ρ_g	0.3	Persistence of consumption growth
σ_g	0.005	Volatility of consumption growth
η	0.028/4	Probability of entering a disaster state
θ	0.914	Persistence of disaster state
ν	0.88	Persistence of recovery state
λ_D	-0.07	Severity of disaster state
λ_R	0.037	Size of recovery state
	Firm	
α	0.65	Concavity of production function
δ	0.03	Depreciation rate
ϕ	1	Relative volatility of aggregate productivity
τ	0.3	Tax rate
c	0.02/4	Coupon rate
κ	0.05	Bond maturity
ρ_z	0.85	Persistence of idiosyncratic productivity
σ_z	0.12	Volatility of idiosyncratic productivity
ξ	0.8	Recovery rate of bond
c^+	2.6	Investment adjustment cost
c^-	0.4	Divestment adjustment cost
$\bar{z}$	-1.8	Average idiosyncratic productivity
λ_0	0.06	Fixed cost of equity issuance
λ_1	0.03	Proportional cost of equity issuance
ϕ_0	0.03	Fixed cost of bond issuance
ϕ_1	0.078	Proportional cost of bond issuance
f	0.003	Fixed cost of operation

Table 2: Corporate Policies and Credit Spread in Normal Times

This table reports a set of key firm-level moments generated under the benchmark parameters reported in Table 1 in normal times. The data moments for investment, equity issue, bond issue, and market leverage moments are calculated from CRSP-Compustat Merged. The 1-year default rate data moment is from Bai (2021). The 5-year default rate and credit spread data moments are from Kuehn and Schmid (2014). The simulation moments are calculated across 5000 independent simulations, which do not have realized disasters. Each simulation has 156 quarters (39 years).

	Data	Mean	5%	25%	50%	75%	95%
Average Investment-to-Asset	0.033	0.029	0.026	0.028	0.029	0.031	0.033
Volatility Investment-to-Asset	0.111	0.095	0.051	0.078	0.099	0.115	0.128
Frequency of New Equity Issue	0.069	0.181	0.037	0.109	0.185	0.257	0.316
Average New Equity to Asset	0.176	0.098	0.096	0.097	0.098	0.099	0.1
Frequency of New Bond Issue	0.274	0.268	0.054	0.159	0.274	0.379	0.471
Average New Bond to Asset	0.071	0.368	0.363	0.366	0.368	0.369	0.373
Average Market Leverage	0.243	0.311	0.063	0.186	0.317	0.439	0.546
1 Year Default Rate	0.7 (%)	0.656	0.131	0.394	0.663	0.92	1.166
5 Years Default Rate	2 (%)	3.341	0.683	2.023	3.383	4.675	5.918
Credit Spread	100 (b.p.)	92.792	19.524	55.809	94.273	130.941	161.897

Table 3: Aggregate Moments and Cyclicality in Normal Times

This table reports a set of key aggregate and cyclical moments generated under the benchmark parameters reported in Table 1 in normal times. The cyclicality is the correlation between the variable and the consumption growth. All numbers are annualized. The data moments for the average and volatility of the market excess return are from the CRSP value-weighted market return. The data moments for the average and volatility of the risk-free rate are from Kenneth French's website. The cyclicality moments are from Kuehn and Schmid (2014). he simulation moments are calculated across 5000 independent simulations, which do not have realized disasters. Each simulation has 156 quarters (39 years).

	Data	Mean	5%	25%	50%	75%	95%
Average Excess Equity Return	8.577	8.457	7.66	8.2	8.52	8.779	9.035
Volatility of Excess Return	16.967	1.303	0.842	0.987	1.204	1.516	2.087
Risk-Free Rate	3.503	2.752	2.751	2.752	2.752	2.753	2.753
Volatility of Risk-free Rate	2.79	0.003	0.002	0.003	0.003	0.003	0.003
Cyclicality							
Default Rate	-0.33	-0.183	-0.314	-0.236	-0.186	-0.131	-0.049
Credit Spread	-0.36	-0.417	-0.536	-0.47	-0.422	-0.367	-0.285
Market Leverage	-0.11	-0.014	-0.186	-0.085	-0.018	0.055	0.163

Table 4: Disaster Risk Implications in Normal Times

This table reports the impacts of disaster risk in normal times, focusing on three parameters that govern the disaster risk: disaster probability η, size of consumption crash in disaster states λ_D, and disaster persistence θ. For each parameter, I study both more and less disaster risk. Panel A reports the four key moments under the benchmark parameters: 1-year cumulative default rate, 5-year cumulative default rate, average credit spread, and average market leverage. Panel B and Panel C report the comparative statics with lower and higher disaster probability. Panel D and Panel E report comparative statics with more and less severe consumption crashes in disaster states. Panel F and Panel G report comparative statics with more and less persistent disasters. The default rates are reported in percentages. The credit spread is reported in basis points. All the model moments are calculated across 5000 independent simulations, which do not have realized disasters.

	Data	Mean	5%	25%	50%	75%	95%
Panel A: Benchmark ($\eta = 0.007$, $\lambda = -0.07$,$\theta = 0.914$)							
1 Year Default Rate	0.7 (%)	0.656	0.131	0.394	0.663	0.92	1.166
5 Years Default Rate	2 (%)	3.341	0.683	2.023	3.383	4.675	5.918
Credit Spread	100 (b.p.)	92.792	19.52	55.81	94.273	130.94	161.897
Market Leverage	0.243	0.311	0.063	0.186	0.317	0.439	0.546
Panel B: Low $\eta = 0.003$ (Lower Disaster Probability)							
1 Year Default Rate	0.7 (%)	0.239	0.013	0.017	0.167	0.408	0.728
5 Years Default Rate	2 (%)	1.231	0.066	0.091	0.865	2.092	3.721
Credit Spread	100 (b.p.)	30.36	2.651	3.089	21.419	51.006	90.46
Market Leverage	0.243	0.123	0.007	0.009	0.086	0.208	0.375
Panel C: High $\eta = 0.009$ (Higher Disaster Probability)							
1 Year Default Rate	0.7 (%)	2.406	1.482	2.022	2.449	2.821	3.201
5 Years Default Rate	2 (%)	11.639	7.215	9.843	11.887	13.543	15.315
Credit Spread	100 (b.p.)	159.68	82.06	126.8	162.5	195.14	223.54
Market Leverage	0.243	0.428	0.214	0.336	0.436	0.525	0.615
Panel D: Low $\lambda_D = -0.071$ (More Severe Disaster)							
1 Year Default Rate	0.7 (%)	1.363	1.109	1.286	1.38	1.463	1.564
5 Years Default Rate	2 (%)	6.627	5.37	6.222	6.7	7.109	7.673
Credit Spread	100 (b.p.)	254.03	205.5	252.7	260.99	266.88	273.06
Market Leverage	0.243	0.855	0.69	0.856	0.881	0.899	0.912
Panel E: High $\lambda_D = -0.069$ (Less Severe Disaster)							
1 Year Default Rate	0.7 (%)	0.552	0.012	0.106	0.399	0.909	1.541
5 Years Default Rate	2 (%)	2.892	0.065	0.567	2.13	4.774	7.971
Credit Spread	100 (b.p.)	33.96	1.681	7.313	24.854	55.259	92.033
Market Leverage	0.243	0.098	0.002	0.019	0.071	0.161	0.27
Panel F: Low $\theta = 0.85$ (Less Persist Disaster)							
1 Year Default Rate	0.7 (%)	0.445	0.13	0.37	0.633	0.872	1.095
5 Years Default Rate	2 (%)	2.297	0.672	1.888	3.222	4.46	5.555
Credit Spread	100 (b.p.)	21.183	20.37	55.08	93.311	128.82	159.436
Market Leverage	0.243	0.195	0.068	0.189	0.322	0.446	0.555
Panel G: High $\theta = 0.924$ (More Persist Disaster)							
1 Year Default Rate	0.7 (%)	2.121	1.246	1.681	2.119	2.57	2.978
5 Years Default Rate	2 (%)	10.301	6.088	8.232	10.293	12.446	14.365
Credit Spread	100 (b.p.)	138.09	64.18	101.7	138.85	175.92	208.795
Market Leverage	0.243	0.372	0.164	0.268	0.373	0.48	0.573

Table 5: Credit Spread and Default Rate in Disaster States

This table reports the default rates and credit spread in simulations with realized disasters. The first column lists the simulation moments in normal times for comparison. The default rates are reported in percentages. The credit spread is reported in basis points. The moments are calculated across 5000 independent simulations which include realized disasters.

	Normal Times	Disaster States					
	Mean	Mean	5%	25%	50%	75%	95%
1 Year Default Rate	0.656	10.445	1.131	4.125	8.117	14.602	27.104
5 Years Default Rate	3.341	25.207	3.9	15.548	20.655	32.977	55.2
Credit Spread	92.792	315.148	111.25	189.846	272.377	402.774	652.731

Table 6: Corporate Policies in Disaster States

This table reports the corporate policies in simulations with realized disasters. The corporate policies are the same as in Table 2. The first column lists the simulation moments in normal times for comparison. The moments are calculated across 5000 independent simulations which include realized disasters.

	Normal Times	Disaster States					
	Mean	Mean	5%	25%	50%	75%	95%
Average Investment-to-Asset	0.029	-0.005	-0.066	-0.02	0.003	0.018	0.027
Volatility Investment-to-Asset	0.095	0.14	0.092	0.124	0.14	0.157	0.187
Frequency of New Equity Issue	0.181	0.021	0	0	0	0	0.251
Average New Equity to Asset	0.098	0.008	0	0	0	0	0.098
Frequency of New Bond Issue	0.268	0.224	0	0	0.193	0.456	0.559
Average New Bond to Asset	0.368	0.218	0	0	0.375	0.408	0.447
Average Market-to-Book	2.473	1.629	0.919	1.267	1.556	1.932	2.593
Average Market Leverage	0.311	0.563	0.281	0.476	0.586	0.67	0.769

Table 7: Decompose the Credit Spread

This table reports the decomposition of the cross-sectional credit spread, by regressing credit spreads on market leverage (ML), the credit risk premium (CRP), and the expected loss in default (Exp. Loss) using simulated data. Standard errors are in the parenthesis. Each column of the table reports the coefficients and R^2 from regressions with different combinations of dependent variables. The coefficients and R^2 are averaged across 5000 simulations. The standard errors are calculated across simulations. The credit risk premium and the expected loss in default are computed using equations (29) and (30) using a model with one-period bonds. Panel A and Panel B respectively report the results in normal times and times with realized disasters.

Panel A: Normal Times

	(1)	(1)	(2)	(2)	(3)	(3)	(4)	(4)	(5)	(5)	(6)	(6)	(7)	(7)
Constant	0	0	0.004	0.004	0.008	0.009	0	0	0	0	0.003	0.004	0	0
5%	-0.001	-0.001	0.001	0.001	0.002	0.003	0	0	-0.001	-0.001	0.001	0.001	0	0
95%	0	0	0.007	0.007	0.015	0.015	0	0	0	0	0.006	0.006	0	0
ML	0.031	0.031					0.015	0.016	0.028	0.028			0.013	0.013
5%	0.03	0.03					0.015	0.015	0.027	0.027			0.012	0.012
95%	0.031	0.032					0.016	0.016	0.029	0.029			0.014	0.014
CRP			0.597	0.594			0.504	0.504			0.578	0.575	0.5	0.5
5%			0.569	0.564			0.499	0.499			0.554	0.55	0.497	0.496
95%			0.625	0.622			0.509	0.509			0.601	0.598	0.504	0.504
Exp. Loss					0.463	0.462			0.435	0.434	0.442	0.441	0.431	0.431
5%					0.454	0.451			0.432	0.431	0.438	0.436	0.428	0.428
95%					0.474	0.473			0.437	0.436	0.446	0.446	0.434	0.433
Time FE	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
R2	0.285	0.285	0.545	0.548	0.388	0.397	0.605	0.606	0.624	0.625	0.897	0.899	0.939	0.94

Panel B: Realized Disasters

	(1)	(1)	(2)	(2)	(3)	(3)	(4)	(4)	(5)	(5)	(6)	(6)	(7)	(7)
Constant	-0.006	0.008	0.011	0.014	0.012	0.012	-0.003	0.01	-0.001	-0.003	0.005	0.005	0	-0.001
5%	-0.017	-0.002	0.004	0.005	0.005	0.005	-0.008	0	-0.003	-0.007	0.002	0.002	-0.001	-0.002
95%	-0.001	0.025	0.021	0.027	0.017	0.017	0	0.029	0	-0.001	0.007	0.007	0	0
ML	0.064	0.041					0.027	0.01	0.024	0.027			0.011	0.012
5%	0.031	0.032					0.016	-0.006	0.018	0.023			0.008	0.01
95%	0.12	0.06					0.041	0.017	0.028	0.029			0.013	0.013
CRP			1.272	1.058			1.158	1.021			0.565	0.557	0.523	0.512
5%			0.606	0.594			0.537	0.531			0.546	0.541	0.506	0.503
95%			2.54	1.936			2.367	1.932			0.589	0.584	0.546	0.524
Exp. Loss					0.493	0.5			0.479	0.491	0.453	0.457	0.45	0.456
5%					0.466	0.469			0.446	0.452	0.437	0.44	0.431	0.436
95%					0.507	0.513			0.496	0.509	0.46	0.463	0.458	0.464
Time FE	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
R2	0.119	0.392	0.338	0.572	0.863	0.871	0.358	0.58	0.897	0.904	0.972	0.974	0.978	0.98

Table 8: Decompose Next-Period Credit Spread

This table reports the decomposition of the cross-sectional credit spread next period, by regressing credit spreads on market leverage (ML), the credit risk premium (CRP), and the expected loss in default (Exp. Loss) using simulated data. Standard errors are in the parenthesis. Each column of the table reports the coefficients and R^2 from regressions with different combinations of dependent variables. The coefficients and R^2 are averaged across 5000 simulations. The standard errors are calculated across simulations. The credit risk premium and the expected loss in default are computed using equations (29) and (30) using a model with one-period bonds. Panel A and Panel B respectively report the results in normal times and times with realized disasters.

Panel A: Normal Times

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0	0.008	0.01	0	0	0.008	0
5%	0	0.002	0.003	0	0	0.002	0
95%	0	0.015	0.016	0	0	0.015	0
ML	0.03			0.03	0.03		0.03
5%	0.029			0.029	0.029		0.029
95%	0.031			0.031	0.031		0.031
CRP		0.172		0		0.173	0
5%		0.113		-0.006		0.114	-0.006
95%		0.225		0.005		0.225	0.006
Exp. Loss			0.003		-0.027	-0.004	-0.027
5%			-0.009		-0.029	-0.012	-0.029
95%			0.014		-0.025	0.003	-0.025
Time FE	Y	Y	Y	Y	Y	Y	Y
R2	0.272	0.057	0.011	0.272	0.274	0.058	0.274

Panel B: Realized Disasters

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.008	0.037	0.044	0.006	0.015	0.043	0.015
5%	-0.001	0.011	0.013	-0.002	-0.001	0.011	-0.001
95%	0.023	0.085	0.104	0.02	0.049	0.104	0.047
ML	0.042			0.057	0.051		0.055
5%	0.031			0.032	0.031		0.032
95%	0.063			0.108	0.09		0.098
CRP		-0.292		-0.5		0.081	-0.125
5%		-1.094		-1.455		0.002	-0.317
95%		0.127		-0.01		0.169	-0.009
Exp. Loss			-0.263		-0.28	-0.27	-0.27
5%			-0.412		-0.421	-0.414	-0.405
95%			0.003		-0.018	-0.001	-0.018
Time FE	Y	Y	Y	Y	Y	Y	Y
R2	0.39	0.358	0.518	0.43	0.589	0.522	0.591

Table 9: Decompose Other Variables

This table reports the decomposition of the cross-sectional firm variables on the credit risk premium (CRP) and the expected loss in default (Exp. Loss) using simulated data. Standard errors are in the parenthesis. Each column of the table reports the coefficients and R^2 from regressions with different combinations of dependent variables. The coefficients and R^2 are averaged across 5000 simulations. The standard errors are calculated across simulations. The credit risk premium and the expected loss in default are computed using equations (29) and (30) using a model with one-period bonds. Panel A and Panel B respectively report the results in normal times and times with realized disasters.

Panel A: Normal Times						
Dependent	Market Leverage			Equity Value	Capital	Bond Issue
Constant	0.262	0.308	0.261	2.479	0.969	0.059
	(0.004)	(0.004)	(0.004)	(0.013)	(0.003)	(0.001)
CRP	6.001		5.965	-20.174	-4.128	-0.811
	(0.030)		(0.030)	(0.110)	(0.026)	(0.007)
Exp. Loss		1.041	0.809	-2.715	-0.618	-0.008
		(0.006)	(0.004)	(0.014)	(0.003)	(0.001)
R^2	0.179	0.006	0.183	0.140	0.033	0.038

Panel B: Realized Disasters						
Dependent	Market Leverage			Equity Value	Capital	Bond Issue
Constant	0.504	0.545	0.500	1.681	0.844	0.113
	(0.003)	(0.003)	(0.002)	(0.009)	(0.002)	(0.001)
CRP	4.301		3.907	-13.308	-3.062	-1.351
	(0.024)		(0.021)	(0.074)	(0.017)	(0.005)
Exp. Loss		0.572	0.296	-0.988	-0.313	-0.059
		(0.003)	(0.002)	(0.007)	(0.001)	(0.000)
R^2	0.132	0.043	0.139	0.121	0.059	0.058

Table 10: Cross-Section Regressions

This table reports the cross-section regression of credit spread and market leverage on firm characteristics. The numbers in the data column are from Gomez and Schmid (2020). The coefficients are averaged across 5000 independent simulations. Under each coefficient, I report the 5% and 95% quantiles of the coefficient, which indicates the confidence interval.

	Data	Credit Spread	Data	Market Leverage
Market Leverage	0.090	0.034		
5%		0.031		
95%		0.038		
Size	-0.002	-0.004	0.009	-0.408
5%		-0.006		-0.665
95%		-0.002		-0.113
Profitability	-0.071	-0.218	-0.286	1.8
5%		-0.349		0.896
95%		-0.063		2.161
R^2		0.354		0.271

Table 11: Collapse Period Regressions

This table reports the cross-section regression of equity return and credit spread on firm characteristics during the collapse period. The collapse period indicates the first period of a realized disaster. he coefficients are averaged across 5000 independent simulations. Under each coefficient, I report the 5% and 95% quantiles of the coefficient, which indicates the confidence interval.

Collapse Period	Equity Return		Credit Spread	
Market Leverage	-0.78	-0.788	0.354	0.395
5%	-0.83	-0.852	0.337	0.347
95%	-0.749	-0.755	0.376	0.478
Profitability		1.046		-0.278
5%		0.147		-0.671
95%		1.814		0.032
Size		0.033		0.058
5%		-0.013		0.003
95%		0.096		0.182
R^2	0.889	0.904	0.568	0.582

Table 12: Comparative Statics in Normal Times (Firm)

This table reports comparative statics of parameters that govern real and financial friction in normal times, holding all other things fixed. The default rates are in percent. The credit spread is in basis points. The first column reports the simulation results with benchmark parameters. Panel A reports the scenarios with more friction: higher investment and divestment adjustment costs (higher c^+ and c^-), higher fixed and proportional equity issue costs (higher λ_0 and λ_1), higher fixed and proportional bond issue costs (higher ϕ_0 and ϕ_1), less recovery rate (lower ξ), and higher fixed operation cost (higher f). Panel B reports the reverse scenarios with less friction. All the model moments are calculated across 5000 independent simulations, which do not have realized disasters

Panel A: More Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.6	3	0.07	0.06	0.04	0.09	0.6	0.005
1 Year Default Rate	0.656	0.604	0.505	0.44	0.649	0.422	0.517	0.68	1.975
5 Years Default Rate	3.341	3.146	2.585	2.326	3.345	2.231	2.708	3.351	9.631
Credit Spread	92.792	38.019	32.732	18.385	70.114	19.737	35.535	348.147	150.818
Market Leverage	0.311	0.122	0.229	0.051	0.189	0.048	0.098	0.881	0.408
Panel B: Less Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.2	2.55	0.05	0	0.02	0.07	1	0.001
1 Year Default Rate	0.656	0.983	1.248	1.203	1.203	0.683	1.62	0.553	0.411
5 Years Default Rate	3.341	4.983	6.166	5.866	5.963	3.372	7.855	2.889	2.095
Credit Spread	92.792	93.512	164.798	256.315	161.603	222.924	225.47	23.19	74.641
Market Leverage	0.311	0.212	0.524	0.864	0.519	0.929	0.759	0.071	0.233

Table 13: Comparative Statics with Realized Disasters (Firm)

This table reports comparative statics of parameters that govern real and financial friction in samples with realized disasters, holding all other things fixed. The default rates are in percent. The credit spread is in basis points. The first column reports the simulation results with benchmark parameters. Panel A reports the scenarios with more friction: higher investment and divestment adjustment costs (higher c^+ and c^-), higher fixed and proportional equity issue costs (higher λ_0 and λ_1), higher fixed and proportional bond issue costs (higher ϕ_0 and ϕ_1), less recovery rate (lower ξ), and higher fixed operation cost (higher f). Panel B reports the reverse scenarios with less friction. The moments are calculated across 5000 independent simulations which include realized disasters.

Panel A: More Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.6	3	0.07	0.06	0.04	0.09	0.6	0.005
1 Year Default Rate	10.445	11.108	9.140	10.66	10.714	10.838	9.773	11.138	12.175
5 Years Default Rate	25.207	27.411	23.084	27.033	26.293	27.498	25.147	25.531	31.209
Credit Spread	315.148	272.856	175.064	232.268	309.954	243.235	234.899	587.257	388.691
Market Leverage	0.563	0.412	0.468	0.309	0.475	0.309	0.342	0.878	0.644
Panel B: Less Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.2	2.55	0.05	0	0.02	0.07	1	0.001
1 Year Default Rate	10.445	9.809	11.317	11.294	11.206	11.410	11.371	10.827	8.582
5 Years Default Rate	25.207	26.030	27.585	26.923	27.450	26.002	28.253	27.470	22.054
Credit Spread	315.148	284.048	383.353	425.090	370.267	400.790	402.667	198.972	244.477
Market Leverage	0.563	0.423	0.701	0.871	0.687	0.910	0.814	0.337	0.422

Table 14: Comparative Statics in Normal Times (Consumer)

This table reports comparative statics of parameters that govern the stochastic discount factors in normal times, holding all other things fixed. The default rates are in percent. The credit spread is in basis points. The first column reports the simulation results with benchmark parameters. Panel A reports results with higher parameter values of time preference factor (β), risk aversion (γ), relative volatility of aggregate productivity (ϕ), elasticity of intertemporal substitution (ψ), persistence of consumption growth (ρ_g), and consumption growth (σ_g). Panel B reports the results with lower parameter values. All the model moments are calculated across 5000 independent simulations, which do not have realized disasters

Panel A: Higher Values							
	Benchmark	β	γ	ϕ	ψ	ρ_g	σ_g
		0.96	7.7	1.05	2.2	0.4	0.01
1 Year Default Rate	0.656	0	2.541	2.604	1.187	0.703	1.154
5 Years Default Rate	3.341	0	12.092	12.367	5.946	3.602	5.783
Credit Spread	92.792	111.142	295.675	257.199	41.725	62.43	61.929
Market Leverage	0.311	0.706	0.823	0.786	0.104	0.203	0.188
Panel B: Lower Values							
	Benchmark	β	γ	ϕ	ψ	ρ_g	σ_g
		0.95	7	0.95	1.8	0.2	0.002
1 Year Default Rate	0.656	7.349	0.557	0.351	0.484	0.761	0.564
5 Years Default Rate	3.341	31.735	2.897	1.823	2.436	3.852	2.852
Credit Spread	92.792	63.886	37.185	30.673	128.719	107.31	130.235
Market Leverage	0.311	0.053	0.126	0.105	0.494	0.358	0.43

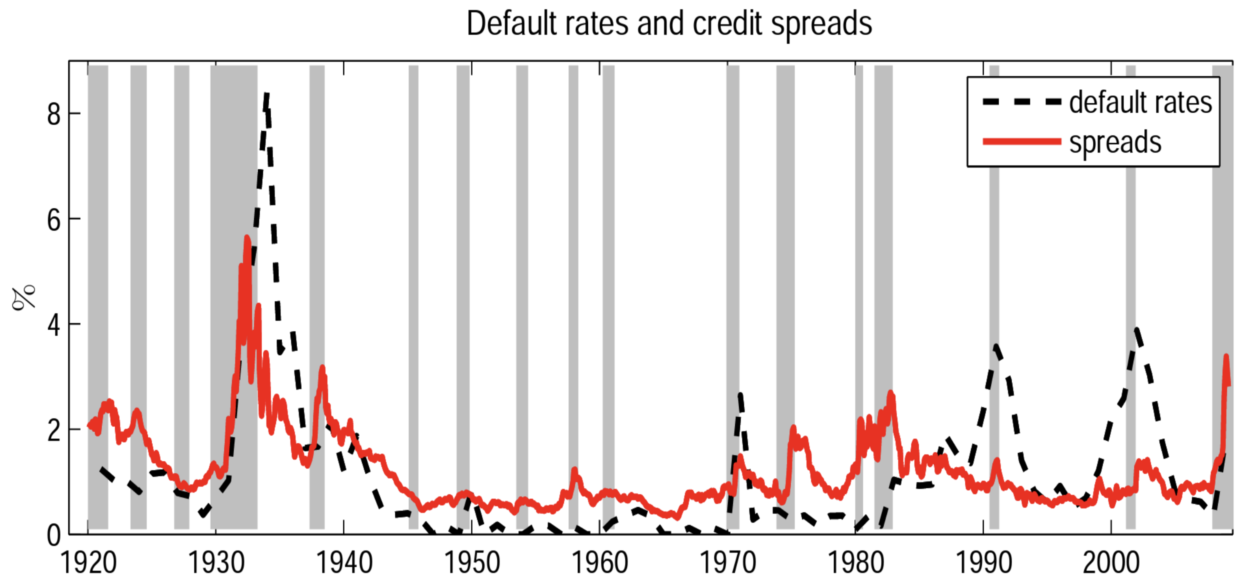


Figure 1: Credit spread and default rate over the business cycle. This figure is from Chen (2010). It plots Moody's annual corporate default rates and the monthly Baa-Aaa credit spreads from 1920 to 2008.

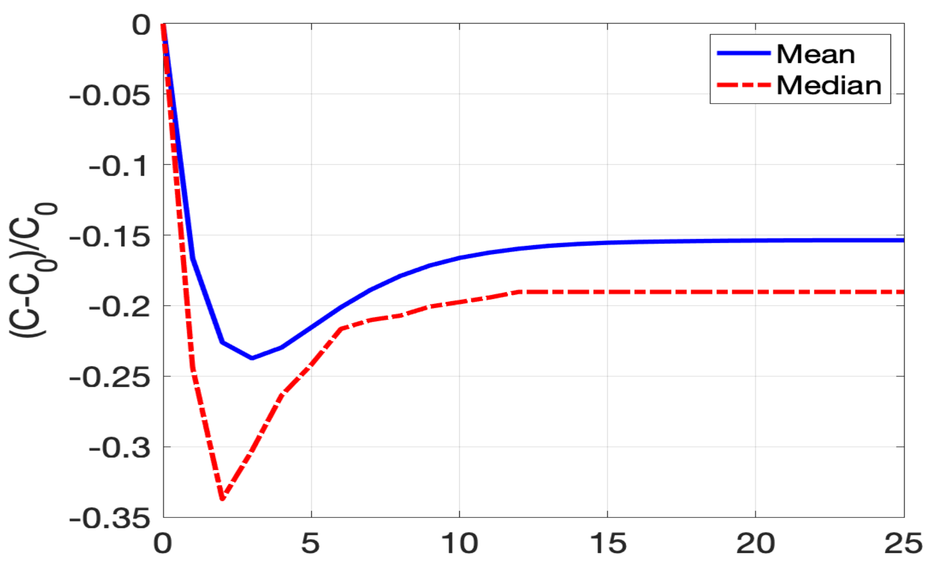
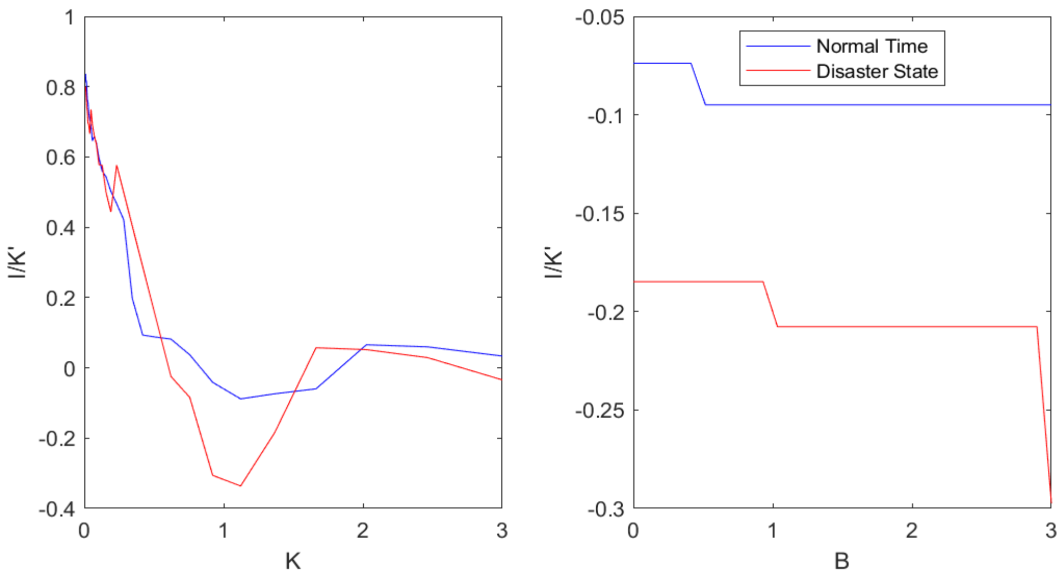
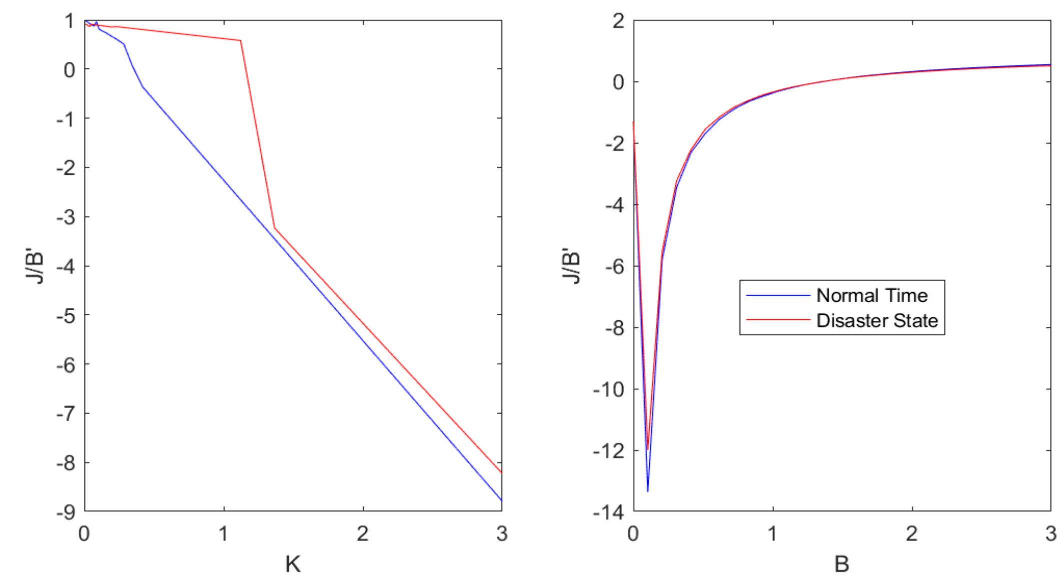


Figure 2: The impulse response of consumption to a disaster shock in the model. In simulations, I calculate the cumulative fractional drop in consumption for 25 years after the economy enters a disaster state. The impulse response is aggregated from the quarterly to annual frequency, and calculated across 10000 simulations. The blue solid line is the mean impulse response, and the red dotted line is the median.

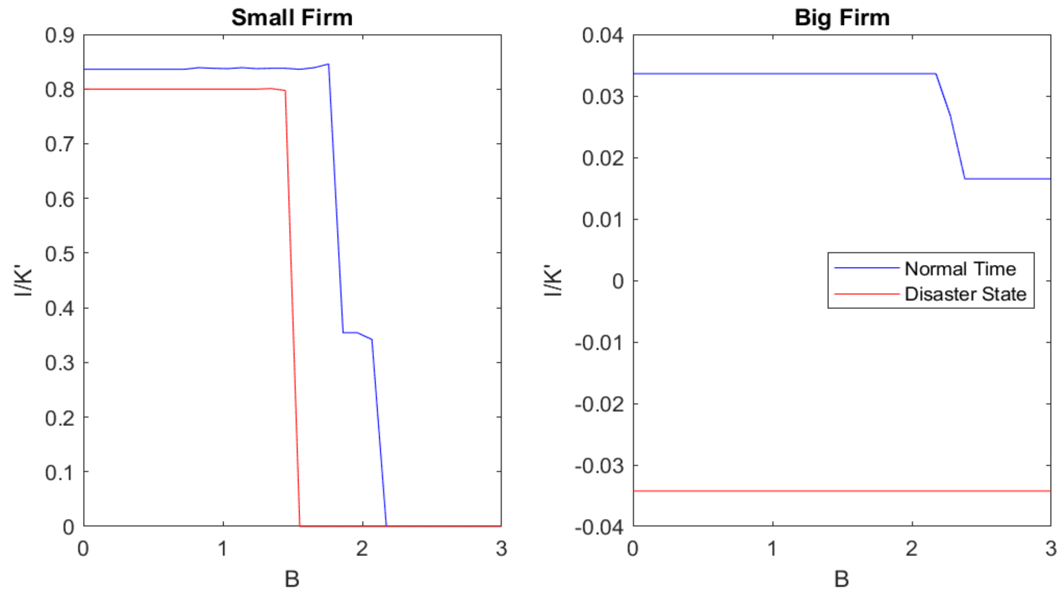


(a) Investment to Capital

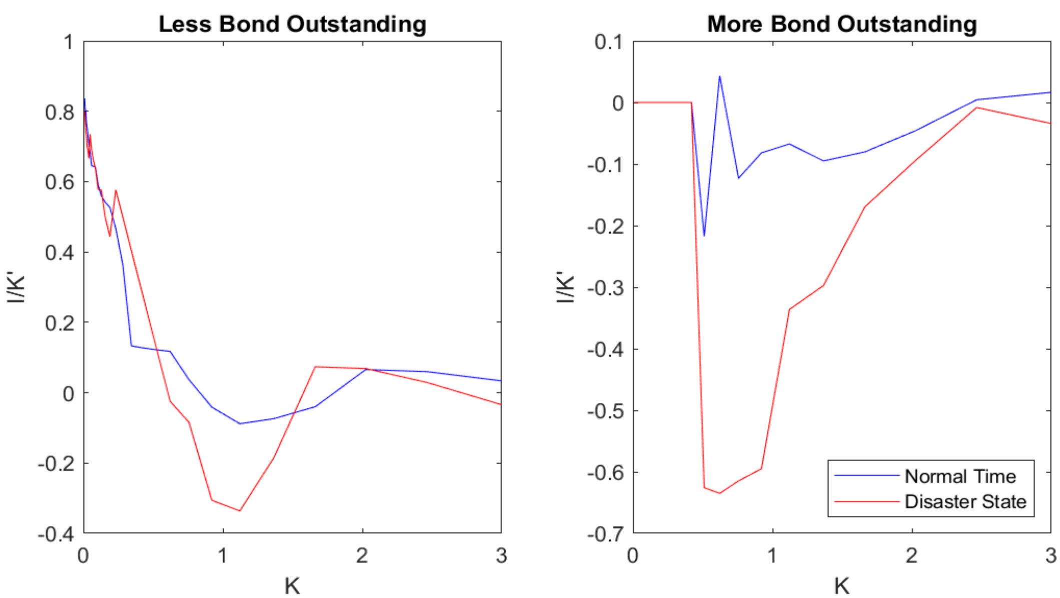


(b) Bond issue to bond outstanding

Figure 3: Policy function at average capital (K) or bond outstanding (B). The upper left figure plots investment-to-capital against K, holding B fixed at the average in the simulation. The upper left figure plots investment-to-capital against B, holding K fixed at the average in the simulation. The lower left figure plots bond issue to bond outstanding against K. The lower right figure plots bond issue to bond outstanding against B. The red line indicates the disaster state. The blue line indicates the average state in normal times.

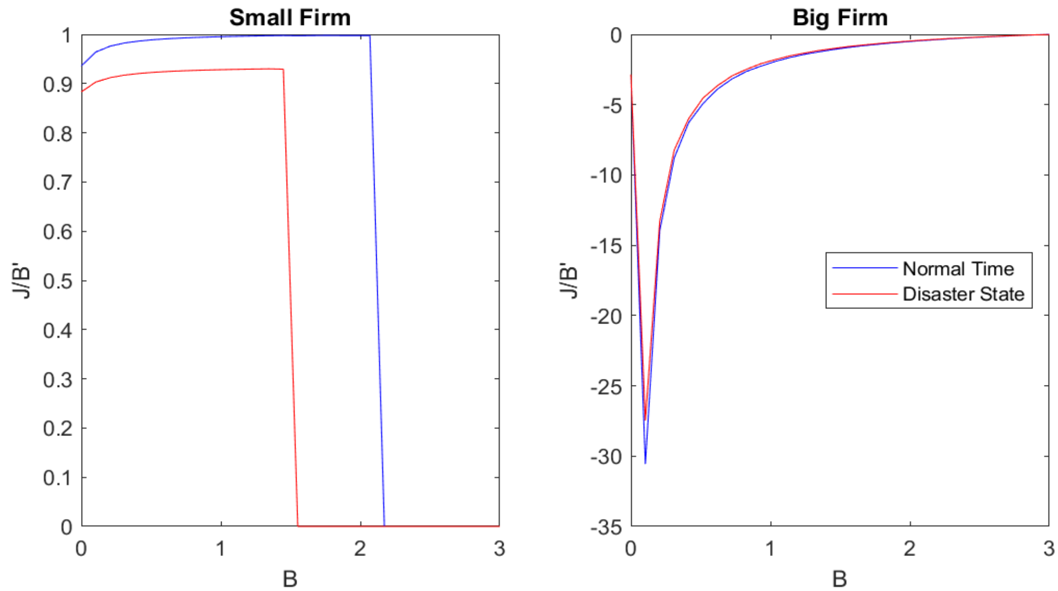


(a) Capital Size

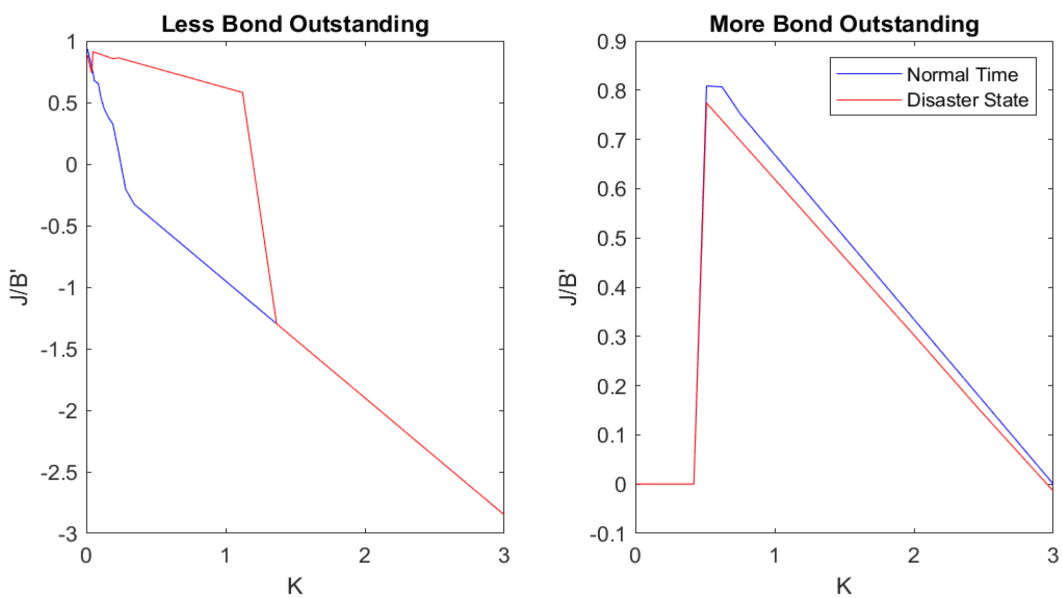


(b) Bond Outstanding

Figure 4: Investment policy function against capital (K) or bond outstanding (B). The upper figures plot investment-to-capital against B, holding K at minimum (left) or maximum value (right). The lower figures plot investment-to-capital against K, holding B at minimum (left) or maximum value (right). The red line indicates the disaster state. The blue line indicates the average state in normal times.

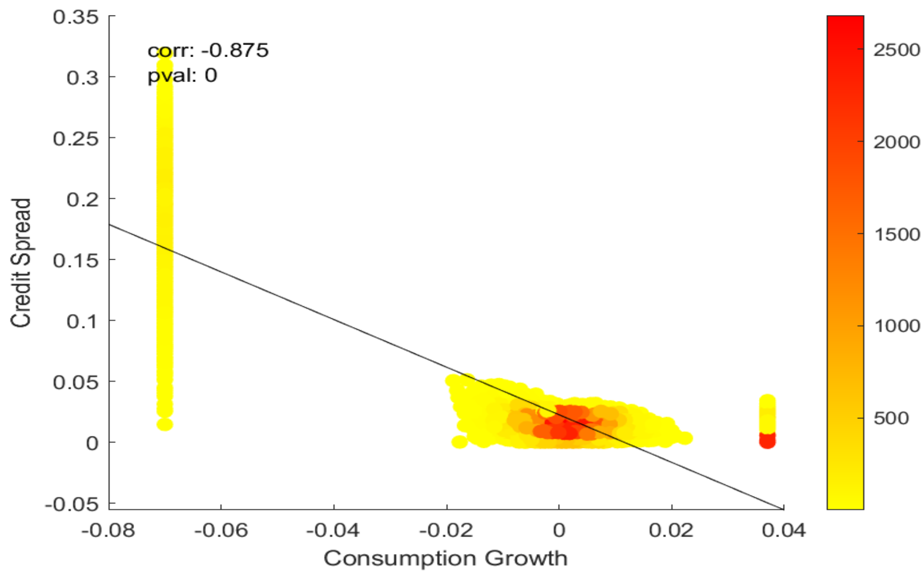


(a) Capital Size

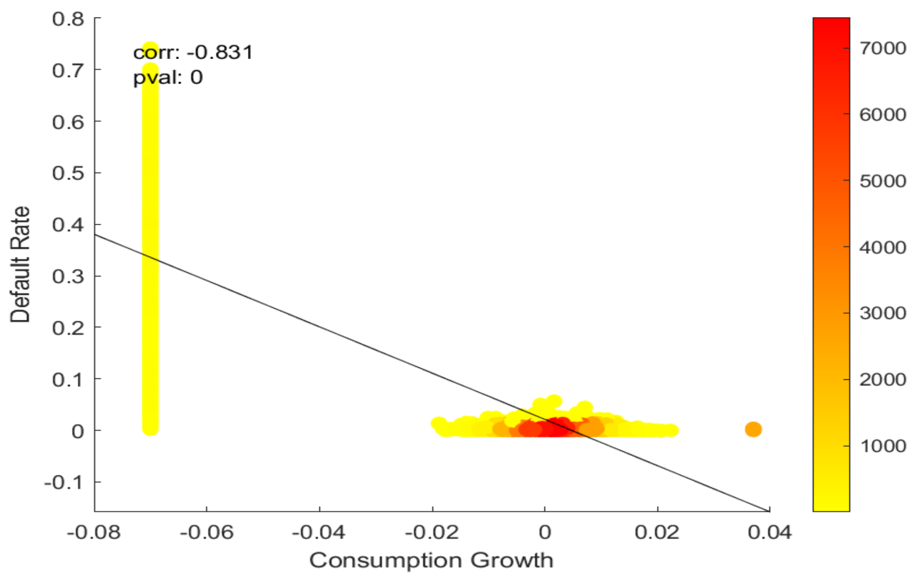


(b) Bond Outstanding

Figure 5: Bond issue policy function against capital (K) or bond outstanding (B). The upper figures plot bond issue to outstanding against B, holding K at minimum (left) or maximum value (right). The lower figures plot bond issue to outstanding against K, holding B at minimum (left) or maximum value (right). The red line indicates the disaster state. The blue line indicates the average state in normal times.

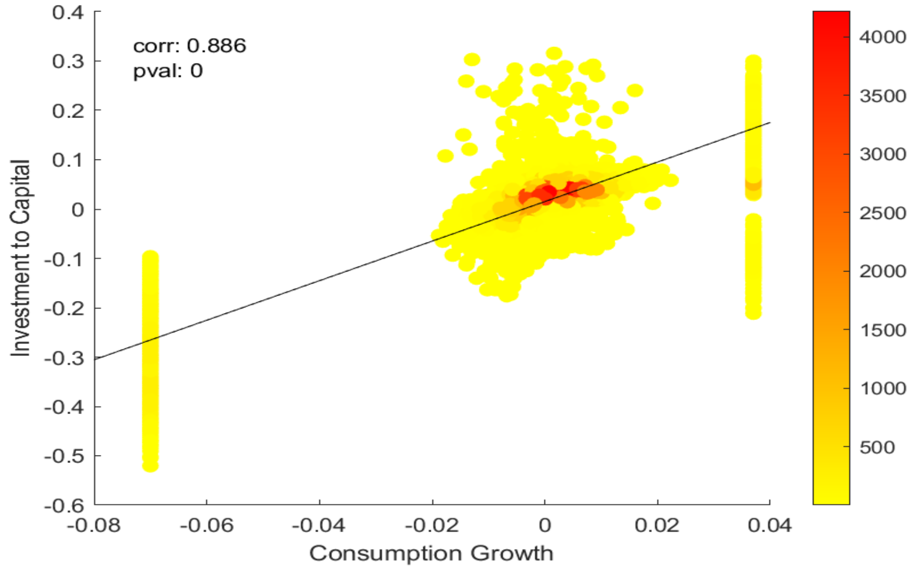


(a) Credit Spread

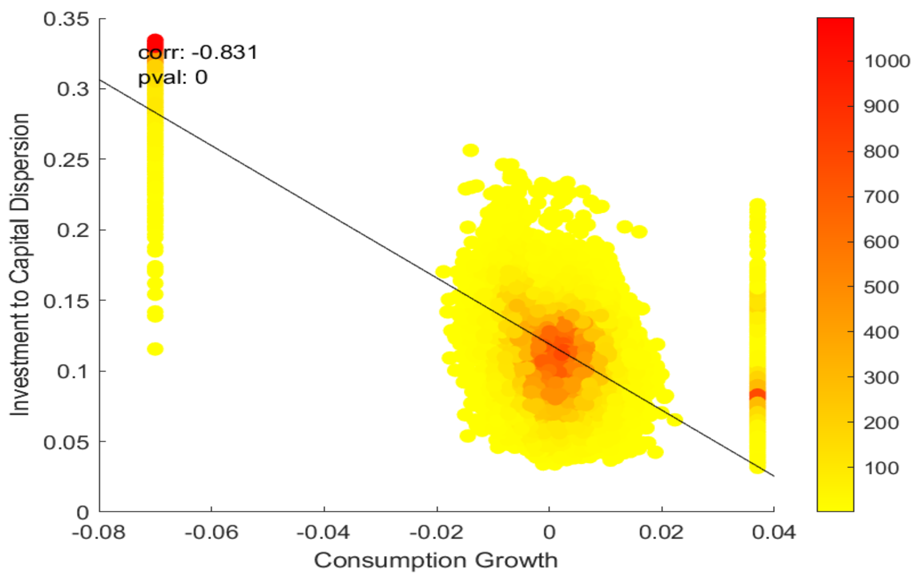


(b) Default Rate

Figure 6: Heatmaps of average cross-section credit spread and default rate against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the average credit spread. Panel B shows the average default rate. The correlation and its p-value are reported on the left top corner. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. In each heatmap, red indicates high density, whereas yellow indicates low density.

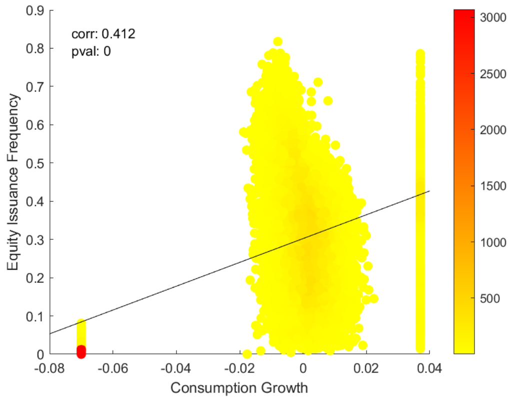


(a) Investment to Capital Rate

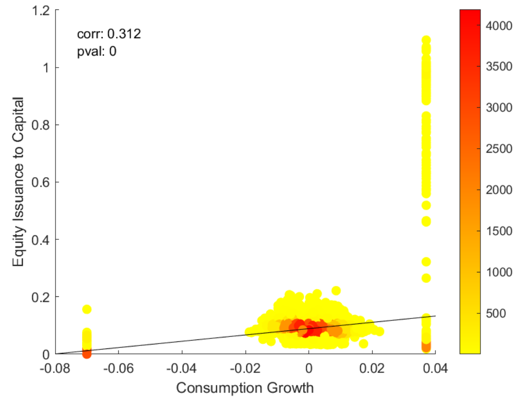


(b) Volatility of Investment to Capital Rate

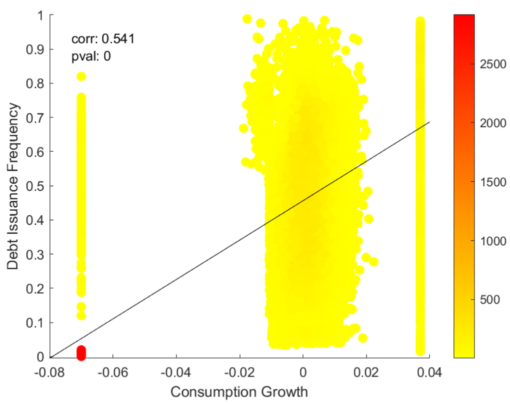
Figure 7: Heatmaps of average and volatility of cross-section investment-to-capital ratio against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the average investment-to-capital ratio. Panel B shows the volatility of the investment-to-capital ratio. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. The correlation and its p-value are reported on the left top corner. In each heatmap, red indicates high density, whereas yellow indicates low density.



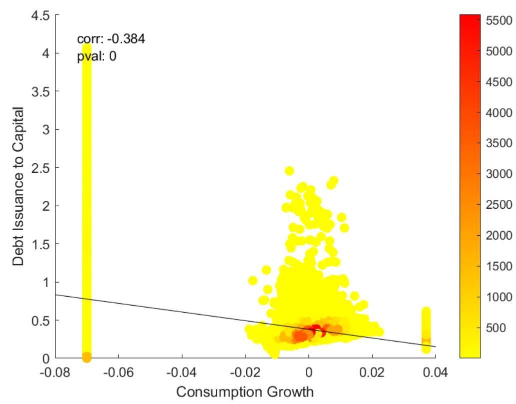
(a) Frequency of New Equity Issue



(b) New Equity Issue to Capital

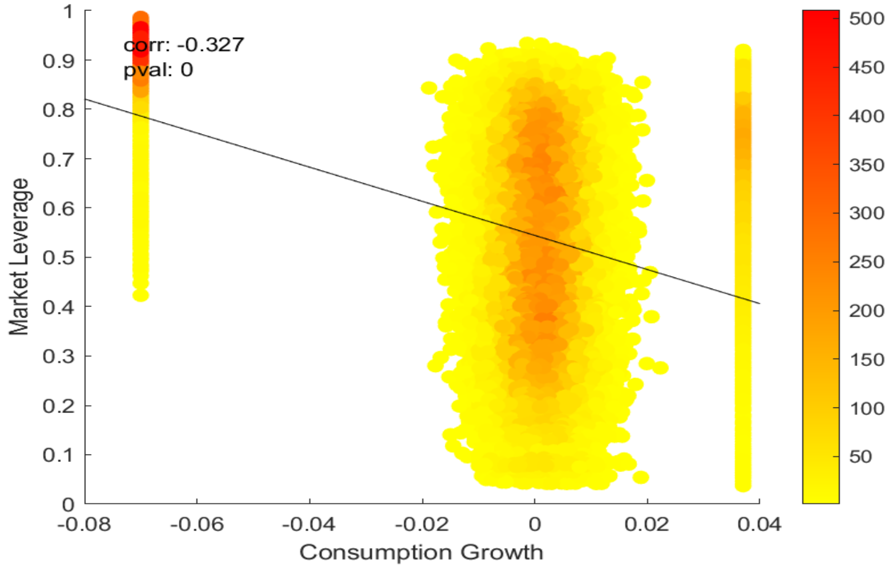


(c) Frequency of New Bond Issue

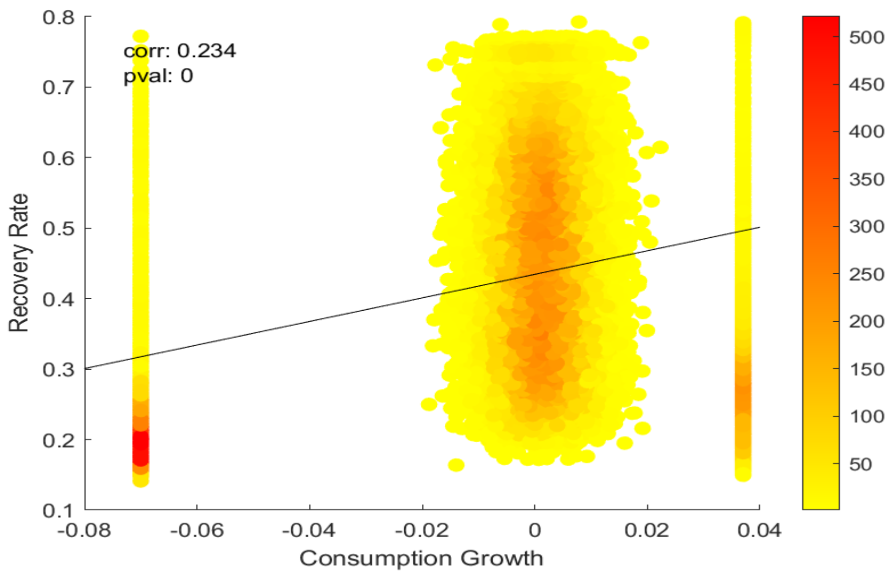


(d) New Bond Issue to Capital

Figure 8: Heatmaps of bond and equity financing against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the frequency of new equity issues. Panel B shows the amount of new equity issues as a portion of capital. Panel C shows the frequency of new bond issues. Panel B shows the amount of new bond issues as a portion of capital. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. The correlation and its p-value are reported on the left top corner. In each heatmap, red indicates high density, whereas yellow indicates low density.

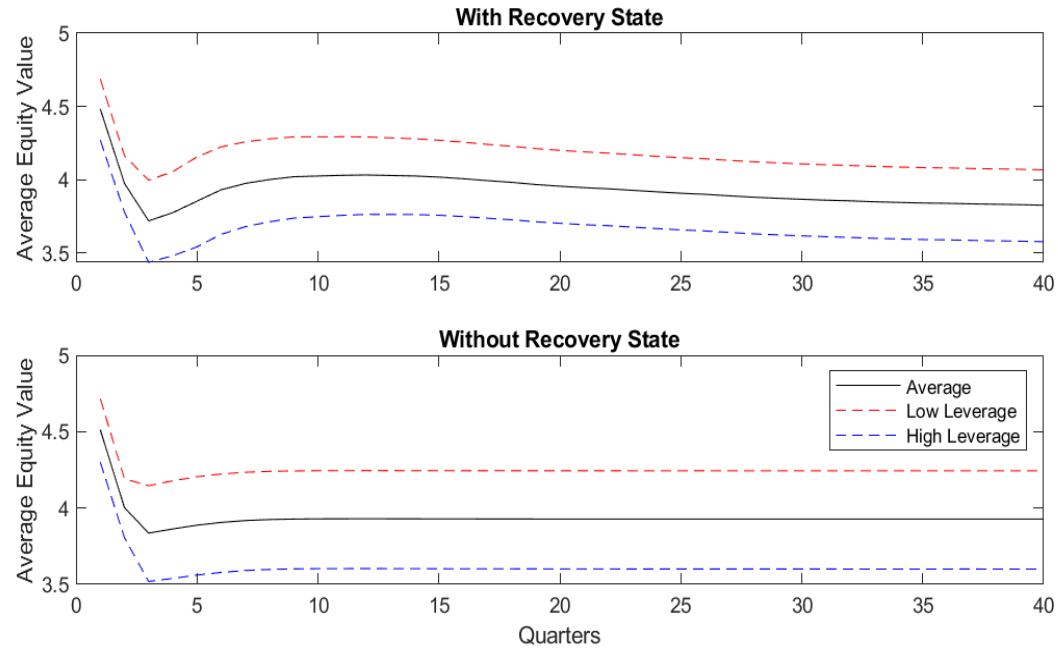


(a) Market Leverage

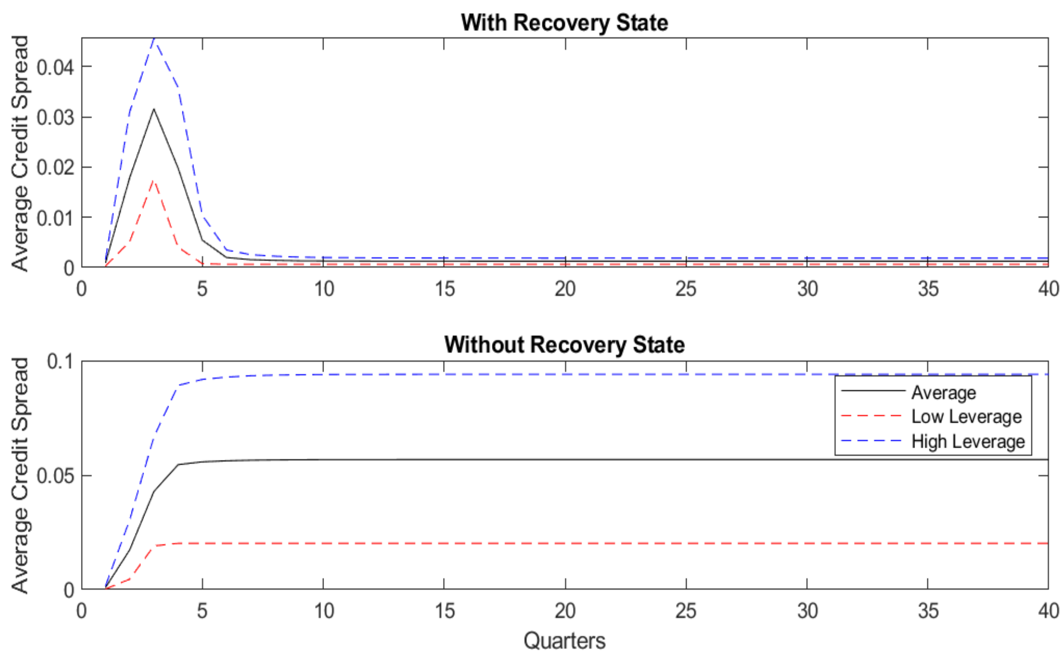


(b) Recovery Rate

Figure 9: Heatmaps of average market leverage and recovery rate against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the average market leverage. Panel B shows the recovery rate. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. The correlation and its p-value are reported on the left top corner. In each heatmap, red indicates high density, whereas yellow indicates low density.



(a) Equity Value



(b) Credit Spread

Figure 10: Impulse response functions of equity value and credit spread after a disaster hits. The black line indicates the average across all survival firms. The red dashed line indicates the average across survival firms with lower market leverage. The blue dashed line indicates the average across survival firms with higher market leverage. The upper figures in Panel A and Panel B include recovery states following the disaster states. The lower figures include only disaster states.

A Numerical Solution

To use the value function iteration method to solve the model, I first detrend the original model to a stationary model without growth. I follow the standard method in macroeconomics literature to use last period aggregate productivity, X_{t-1}, to deflate capital and bond. Deflated capital and bond are respectively denoted as $\widehat{K}_{it} = \frac{K_{it}}{X_{t-1}}$ and $\widehat{B}_{it} = \frac{B_{it}}{X_{t-1}}$. Consumption growth, g_t, replaces X_t as another state variable because productivity growth g_{xt} is a linear function of g_t. The deflated post-default value function is denoted as $\widehat{V}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t) = \frac{V(Z_{it}, K_{it}, B_{it}, X_t)}{X_{t-1}}$.

Other model elements are deflated and denoted in a similar way. Below summarizes the stationary model that will be solved by grid search

$$\widehat{V}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t) = \max\{0, \widehat{S}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t)\} \tag{A.1}$$

$$\widehat{S}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t) = \max_{\widehat{I}_{it}, \widehat{B}_{it+1}} \Big\{\widehat{e}_{it} + \Lambda(\widehat{e}_{it}) + E_t\big[M_{t+1}\widehat{V}(Z_{it+1}, \widehat{K}_{it+1}, \widehat{B}_{it+1}, g_{t+1})\big] exp(\bar{g} + \phi g_t)\Big\} \tag{A.2}$$

$$\textbf{subject to} \tag{A.3}$$

$$\widehat{K}_{it+1} exp(\bar{g} + \phi g_t) = (1 - \delta)\widehat{K}_{it} + \widehat{I}_{it} \tag{A.4}$$

$$\widehat{B}_{it+1} exp(\bar{g} + \phi g_t) = (1 - \kappa)\widehat{B}_{it} + \widehat{J}_{it} \tag{A.5}$$

The bond pricing equation has the following form

$$Q_{it} = E_t\big[M_{t+1}\big((1 - \mathbb{1}_{\widehat{S}_{it+1}<0})(c + \kappa + (1 - \kappa)Q_{it+1}) + \mathbb{1}_{\widehat{S}_{it+1}<0}\xi R_{it+1}\big)\big] \tag{A.6}$$

where the recovery rate is not changed

$$R_{it+1} = \min\{1, \frac{\Pi_{it+1} + (1 - \delta)K_{it+1}}{B_{it+1}}\} = \min\{1, \frac{\widehat{\Pi}_{it+1} + (1 - \delta)\widehat{K}_{it+1}}{\widehat{B}_{it+1}}\} \tag{A.7}$$

and default decision depends on the detrended continuation value

$$\mathbb{1}^{D}_{it+1} = \mathbb{1}_{\widehat{S}_{it+1}<0} \tag{A.8}$$

The model is solved using value function iteration. Both the current states of capital and bonds, K_{it} and B_{it}, are discretized into 30 grid points. I use the Rowenhorst method to discretize aggregate consumption growth, g_t, into 7 grid points, including 5 normal states, 1 disaster state, and 1 recovery state. Similarly, I discretize idiosyncratic productivity, z_{it}, into 9 states, and the transitory income shock, m_{it}, into 5 states using a truncated Gaussian distribution. To obtain more accurate model solutions, I use finer grids for the decision variables, K_{it+1} and B_{it+1}, which are discretized into 300 grids.

Due to the finer grid for decision variables, the bond pricing function is also defined on finer grids. Consequently, I need to interpolate the value function on these finer grids. However, the post-default value function, V_{it}, has a kink at zero, leading to numerical errors during interpolations. To address this issue and prevent convergence problems, I instead interpolate on the pre-default value function, S_{it}, which is a smoother surface. I then obtain the post-default value function based on the interpolated S_{it}, reducing numerical errors and helping the model converge. Additionally, I include a transitory income shock following the approach in Chatterjee and Eyigungor (2012), which helps in solving multiple local optimum problems and prevents oscillations during the solution process.

To solve the model efficiently, I follow the procedure in Kuehn and Schmid (2014). I start by solving the model with a one-period bond and use its solution as the initial point for solving the model with a two-year bond, and so on. For tractability, I solve the model with two and four-year bonds. The convergence threshold for both the value function and bond pricing function is set at 10^{-4}, and in most cases, the value function converges to a level of 10^{-7}. However, it is important to note that these tips do not guarantee convergence, as it also depends on the parameter values. To ensure convergence, I experiment with a wide range of parameter values.