The Power of Reserve Tiering:
Financial Institution Heterogeneity and Monetary Policy Pass-through

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Abstract

This paper studies how reserve tiering, which involves remunerating different reserve tiers at different rates, can promote monetary policy pass-through to the loan market. The data shows a sharp increase in low-interest-rate loans and a decline in medium-interest-rate loans in Japan following the reserve tiering policy. Using a heterogeneous agent model to connect interbank and loan markets, I show that financial institutions' heterogeneous interest rate exposures to the tiered system can explain the changes. Higher-risk banks borrow more under higher tiering rates from non-depository institutions (NDIs) and get additional cheaper funding through interbank trading to cut loan rates. At the same time, banks' profit function shifts, and the optimal loan rate for banks decreases. Further, I show that reserve tiering brings risks to the financial system since banks with higher risk exposure cut loan rates more. The paper suggests that to cool down the overheating economy, the central bank can implement reserve tiering with ascending interest rates by remunerating the reserve above a threshold at a higher interest rate instead. Reserve tiering with ascending interest rates is more effective, less costly, and can stabilize the financial system's health compared with alternative monetary policies.

JEL Classification: E52, E58, G21

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Keywords: Reserve tiering, monetary policy pass-through, loan interest rate, financial institution heterogeneity

1 Introduction

Central banks typically have reserve requirements for financial institutions in the system to ensure that financial institutions are able to meet liabilities in case of sudden withdrawals. At the same time, central banks pay a unified interest rate to the reserve balances of financial institutions to get more control over the interbank rates. However, faced with the problem of the lengthy center bank balance sheet, low inflation rate, and failure of the monetary policy pass-through due to the zero lower bound for the deposit rate, as shown in Figure 1, during 2014-2016, the Swiss National Bank (SNB), European Central Bank (ECB), the Denmarks Nationalbank (DN) have implemented the negative interest rate policy with the tiered system on their reserves. Specifically, reserve tiering means that banks' reserve holdings at the central bank below a certain threshold are remunerated at a higher interest rate. At the same time, reserves above the threshold are remunerated at a lower rate, which is typically negative.

According to the ECB, *"the two-tier system aims to support the bank-based transmission of monetary policy, while preserving the positive effect that negative rates can have on the accommodative stance of monetary policy, and towards the sustained convergence of inflation to the ECB's aim."* For central banks, interest rate cut policy in the low-interest-rate environment is not powerful. Banks are reluctant to cut the deposit rate into negative territory and have a shrinking interest margin, thus hindering the monetary policy pass-through. As a result, the reserve tiering is designed to protect the banks' interest margin, reduce the hurt of the negative interest rate policy on the financial institutions' health, and stimulate the monetary policy pass-through. Therefore, it is worth investigating how reserve tiering differs from conventional monetary policy. Specifically, how does reserve tiering bring changes to the monetary policy pass-through? What is the mechanism behind

Country	Central Bank	Rate	Tiers	Instrument	Start Date
Denmark	**Denmarks Nationalbank (March 19, 2020)**	-0.20% (July 6, 2012) -0.10% (January 25, 2013) +0.05% (April 25, 2014) -0.05% (September 5, 2014) -0.20% (January 20, 2015) -0.35% (January 23, 2015) -0.50% (January 30, 2015) -0.75 % (February 6, 2015) -0.65 % (January 8, 2016) -0.75% (September 13, 2019) -0.60% (March 20, 2020) -0.50% (March 19, 2021) -0.60% (October 1, 2021)	Two Tiers	Certificates of deposit rate. Current-account rate (-0.50% on March 19, 2021; and -0.60% on October 1, 2021) and lending rate (-0.35% on March 19, 2021; and -0.45% on October 1, 2021) went negative later.	Jul. 6, 2012- Apr. 24, 2014, Sept. 5, 2014
Hungary	**Hungarian National Bank (Magyar Nemzeti Bank, MNB)**	-0.05% (March 23, 2016) -0.15% (September 20, 2017) -0.05% (March 27, 2019) +0.25% (September 28, 2021)	No Tiers	Deposit rate	Mar. 23, 2014 - Sep. 27 2019
Japan	**Bank of Japan (BoJ)**	-0.10% (February 16, 2016)	Three Tiers	Deposit rate	Feb. 16, 2016
Switzerland	**Swiss National Bank (SNB)**	-0.25% (December 18, 2014) -0.75% (January 15, 2015)	Two Tiers	Policy Rate: Sight deposits at SNB (SNB moved the target range downwards to between -1.25% and -0.25%.)	Jan. 15, 2015
Sweden	**Swedish Riksbank(SR)**	-0.10% (February 18, 2015) -0.25% (March 25, 2015) -0.35% (July 8, 2015) -0.50% (February 17, 2016) -0.25% (September 1, 2019) 0.00% (August 1, 2020)	No Tiers	Policy Rate: Repo rate. Deposit rate (-0.50% on September 7, 2014; -0.75% on October 29, 2014; -0.85% on February 18, 2015; -1.00% on March 25, 2015; -1.10% on August 7, 2015; -1.25% on February 17, 2016; -1.00% on September 1, 2019; -0.35% on October 30, 2019; -0.10% on August 1, 2020) and lending rate also went negative	Feb. 18, 2015 - July 31, 2020
	European Central Bank (ECB)	-0.10% (June 11, 2014) -0.20% (September 10, 2014) -0.30% (December 9, 2015) -0.40% (March 16, 2016) -0.50% (Sepember 18, 2019)	Two Tiers	Deposit rate	Jun. 11, 2014, with two tiers started in Oct. 30, 2019.

Source: Jobst et al. (2016). Also, the table is updated based on the information on the websites of Central Banks.

Figure 1: Overviews of Countries with Negative Interest Rate Policy

that? Furthermore, is reserve tiering good or bad? Does reserve tiering bring changes to economic welfare?

The paper first uses the Bank of Japan as a case study and finds that reserve tiering stimulates the monetary policy pass-through to increase the low-interest-rate loans and decrease the medium-interest rate loans. From the financial institutions' reserve balance and the interbank market trading data from the BOJ, the paper finds that the critical feature of the monetary policy pass-through under reserve tiering is the financial intuition's heterogeneous exposure to the central bank interest rates. Bigger banks and non-depository institutions (NDIs) typically receive higher interest rates than smaller banks. Moreover, higher-risk smaller banks have less significant negative interest rate balances at the central bank and have incentives to borrow more. The heterogeneous interest rate exposures stimulate interbank trading because NDIs and bigger banks are highly incentivized to lend out their negative interest rate balances to avoid the negative interest rate at the central bank. Consequentially, higher-risk smaller banks borrow more in the interbank market from bigger banks and NDIs and get extra funding to offer more low-interest-rate loans. Since banks are net borrowers while NDIs are net lenders in the interbank market, there are more low-interest-rate loans in the market as a whole.

This paper builds a model with two types of financial institutions of banks and non-depository institutions, and two markets of interbank and loan markets to characterize how reserve tiering changes monetary policy pass-through. In the model, the central bank sets the reserve tiering threshold. The financial institution's reserve below the threshold is remunerated at the regular interest rate, while the reserve above the threshold is remunerated at the negative interest rate. Furthermore, the central bank sets the reserve requirement for financial institutions and offers a lending facility to allow financial institutions to borrow to meet the reserve requirement.

The model shows that reserve tiering promotes monetary policy pass-through to the loan market via four channels. The first channel is **the liquidity channel**. Banks get extra funding to offer loans from the NDIs through interbank trading since NDIs would like to lend out the extra balances to avoid negative reserve balances at the central bank. As a result, the reserve tiering threshold is a new policy tool for the central bank to implement the monetary policy. The second channel is **the interest rate channel**. Cutting interest rates in different tiers brings down the interbank rate due to the NDIs' incentive to lend out the funding. Then, the decline in the interbank rate passes to the loan rate, and the loan rate goes down together. The paper also points out that cutting the lending facility rate and negative interest rate tier rate is more effective when the difference between the required reserve and the reserve tiering threshold is low. The third channel is **the bank**

interest margin channel. Due to the extra funding opportunities, banks' profit function shifts to the left, and the optimal loan rate declines compared with the non-tiered reserve system. The fourth channel is **the loan risk reallocation channel**. Banks with a higher risk exposure cut the loan rates by higher magnitudes. Intuitively, high-risk banks have higher leverage and lower reserve balances. As a result, there is more room for high-risk banks to borrow from the interbank market to offer loan rate cuts.

Consistent with the loan risk reallocation channel, the paper further uses an extended model to capture the interest rate distribution changes as shown in the BOJ data, i.e., the low-interest loan declines while the medium-interest-rate loan increases. The model shows that entrepreneurs with a higher risk exposure are more likely to benefit from the reserve tiering policy by receiving a lower loan interest rate exposure. As a result, when reserve tiering is implemented, banks with medium- and high-risk portfolios will have a high-magnitude loan rate decline until hitting the zero lower bound. This is because risky banks have higher leverage and fewer reserves, thus having more room to borrow from the interbank market. And low-risk portfolio entrepreneurs will get a small magnitude interest rate cut-down. As a result, after the reserve tiering is implemented, there is an increase in the low-interest-rate loans while there is a decrease in the medium-interest-rate loans. And the reserve tiering policy brings risk to the financial system's health as it results in a reduced risk premium for high-risk loans. To deal with the risk, the central bank can increase the lending facility rate since high-risk banks are more likely to suffer bank runs and will have a higher funding cost to meet the reserve requirement.

Finally, faced with the possibility of an overheating economy after COVID-19, the central bank can implement reserve tiering with ascending interest rates to cool down the economy. More specifically, the central bank can set a lower interest rate for reserve balances below a specific threshold, while a higher interest rate is applied to reserves exceeding the established threshold. The reserve tiering with ascending interest rates can work through the four above channels to stimulate the loan rate increase. The paper further emphasizes the advantages of reserve tiering by comparing reserve tiering with different monetary policy tools. On the one hand, reserve tiering with ascending interest rates works through the additional liquidity and loan risk reallocation channels to amplify the policy impact than simply raising interest rates. On the other hand, compared with increasing reserve requirements, reserve tiering can help stabilize the financial system's health using the power of the market by a higher-magnitude loan rate increase in higher-risk banks. Finally, reserve tiering is less costly than open market operations and quantitative easing.

This paper examines the impact of reserve tiering, a concept that has been relatively understudied in previous literature. Firstly, the paper presents a formal model that establishes the interconnections between the interbank and loan markets, thereby providing insights into the transmission mechanism of reserve tiering. Additionally, the paper characterizes how heterogeneous rate exposure to the tiered system can impact monetary policy pass0-through via different channels. By offering a more comprehensive framework, this paper contributes to a broader understanding of the policy's implications under different economic conditions. Lastly, the paper explores changes in loan distribution resulting from monetary policy actions, an aspect that has received limited attention in prior research that predominantly focuses on a uniform loan rate within the loan market.

The following paper is organized as follows. Section 2 reviews the related literature and specifies the paper's contributions. Section 3 offers a case study of BOJ to illustrate the mechanism. Section 4 is the baseline model to characterize the impact of the reserve tiering. Section 5 characterizes how reserve tiering passes through to the loan market via different channels. Section 6 discusses the loan rate distribution changes using an extended model. Section 7 consists of the application of reserve tiering in today's economic conditions. Section 8 concludes.

2 Related Literature and Contributions

Firstly, this paper is related to the literature that analyzes the impact of the Negative Interest Rate Policy (NIRP) on the loan market, which are mainly empirical works. On the one hand, the impact is realized through the portfolio rebalancing channel. Banks have a higher incentive to lend out balances through the loan market to avoid negative balances. And banks tend to lend to riskier firms and shrink balance sheets. Demiralp et al. (2021) identifies that banks that are most reliant on retail deposits and hold excess liquidity significantly increase the loan provision after NIRP using euro data. Basten and Mariathasan (2020) uses the Swiss bank data and finds that going negative can interrupt pass-through to deposit and mortgage rates, and the interruption can be reduced by up to 90% through tiered remuneration. Bittner et al. (2020) further confirms that banks prefer establishing new lending relationships with risky firms than increasing loan volume to the existing borrower using German data. In contrast, some papers find that the NIRP increases the funding cost of high-deposit banks, and thus banks reduce net worth. Heider et al. (2019) finds that NIRP introduced by ECB brings more risk-taking and less lending by euro-area banks with a greater reliance on deposit funding using syndicated loan data. Moreover, Eggertsson et al. (2019) finds that NIRP pass-through to lending rates and credit volumes is substantially lower once the deposit rate declines to a particular level using Sweden bank data. Similarly, Arce et al. (2021) uses the bank survey data in Spain and finds that banks suffering NIRP decrease their loan supply and increase their lending rates, especially when lowly capitalized and lending to risky firms.

On the other hand, the loan increase is also associated with the liquidity channel. Using Italy data, Bottero et al. (2019) find NIRP affects banks with higher ex-ante net short-term interbank positions or more liquid balance sheets. NIRP-affected banks rebalance their portfolios from liquid assets to credit, especially for riskier and smaller firms. Bottero et al. (2019) further explains a bank's rebalancing by the wide spread between the safe asset yield and loan yield. Similarly, Arseneau (2020) uses the supervisory data of the U.S. bank expectation on the NIRP and finds banks with a lot of short-term liquid assets expect to suffer the most significant adverse impact if rates were to become negative.

Another thread of literature this paper is associated with is the monetary policy impact on the interbank market. On the one hand, a limited number of papers discuss the tiered monetary policy's impact on the interbank market. Fuhrer et al. (2020) use a unique dataset from Switzerland and a theoretical model and find that reserve tiering stimulates sufficient activities in the interbank market. Similarly, Altavilla et al. (2022) uses ECB data and finds that reserve tiering reduces money market fragmentation through inter-bank trading. On the other hand, this paper is connected to the papers that formally model the interbank market as either a Walrasian or Over-the-Counter market. Fuhrer et al. (2020) models the central bank market as a centralized Walrasian market and incorporates it into the general equilibrium model similar to past literature such as Ennis (2018) and Bech and Keister (2017). In contrast, Afonso et al. (2019) discusses the interbank market's over-the-counter feature due to different types of financial institution participants. Afonso et al. (2019) uses a random search model and captures the landscape of the federal funds market under both abundant and scarce supply of aggregate reserves.

The main contributions of this paper are as follows. Firstly, the impact of reserve tiering is not well understood by the literature. This paper formally models the connections between the interbank market and loan market to explain why tiered monetary policy can stimulate monetary policy transmission by offering a higher volume of low-interest loans. The channels characterized in this paper are consistent with the empirical findings in some recent papers (Altavilla et al., 2022; Baldo et al., 2022; Basten and Mariathasan, 2020). Also, this paper contributes to characterizing how different exposure to tiered NIRP can impact financial institution trading behavior and profits asymmetrically in the interbank market. In addition, the paper provides a more generalized framework to understand the policy implications in different economic conditions. Finally, the paper also characterizes how the loan rate distribution changes following the monetary policy, which received limited attention in the past literature since a uniform loan rate is characterized.

3 A Case Study of Bank of Japan (BOJ)

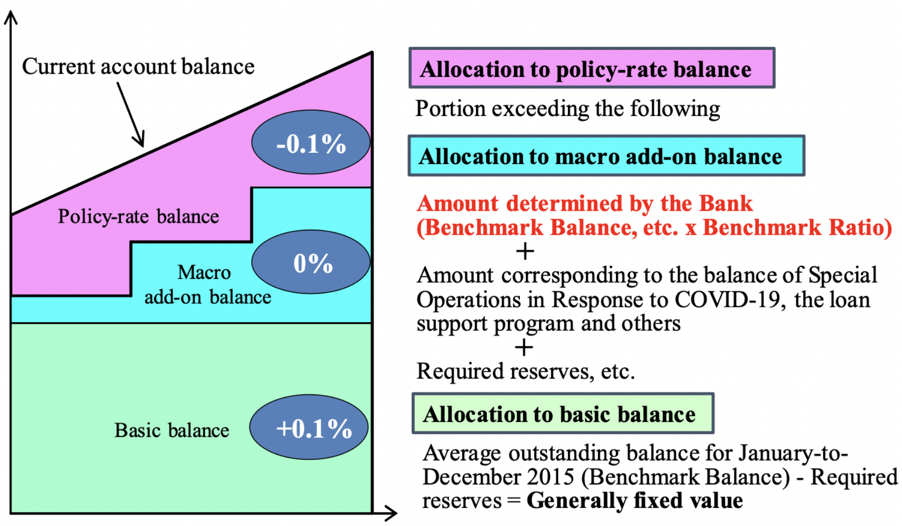
One typical case of reserve tiering is the Bank of Japan (BOJ). Though not the first to go negative or the first to tier rates, the BOJ is the first central bank to use a three-tier structure, applying different interest rates to different portions of each financial institution's outstanding balance. On January 29, 2016, the Bank of Japan (BOJ) surprised the market by introducing "QQE with a negative interest rate" on marginal excess reserves. This policy was effective on February 16, 2016. The three tiers are: (1) Basic balance (rate: +0.1%, amount: approx. ¥210 trillion); (2) Macro add-on balance (rate: 0.0%, amount: ¥40 trillion); (3) Policy rate balance (rate: -0.1%, amount: ¥10-30 trillion). Therefore, only the current balance exceeding ¥250 trillion gets exposed to NIRP. To address the concern that financial institutions may significantly increase the zero-interest-rate cash holdings under NIRP, BOJ states that it will deduct the cash holding increasing amount from the zero interest rate tiers of current account balances.[1]

Each financial institution has different interest rate exposures. As shown in Figure 2, the financial institution's average outstanding balance for January to December 2015 is calculated as the benchmark balance. The benchmark balance minus the required reserves receives the +0.1% interest rate. [2] And the amount determined by the bank, which equals the benchmark balance times the benchmark ratio plus the required reserves, in addition to the balance of loan support program, and the required reserve receive 0% interest rate. And the balance in excess of those two tiers receives -0.1% interest rate. Under the

[1]Source: https://www.boj.or.jp/en/announcements/release_2016/k160129b.pdf

[2]According to the BOJ, the required reserve rate is different for different types of financial intuitions and balances. For banks, the reserve requirement ratio for time deposits is 0.05% for balances from 50 billion - 1.2 trillion yen, 0.9% for balances from 1.2 trillion yen to 2.5 trillion yen, and 1.2% for balances more than 2.5 trillion yen. For other deposits in banks, BOJ has a reserve requirement ratio of 0.1% for balances from 50 billion to 500 billion yen, 0.8% for balances from 500 billion to 1.2 trillion yen, 1.3% for more than 1.2 trillion yen. The Reserve requirement ratio on the outstanding principal balance of money in trust is 0.1%. Source: https://www.boj.or.jp/en/statistics/boj/other/reservereq/junbi.htm/

three-tier system arrangement, bigger banks generally face higher required reserve levels and tend to have a smaller proportion of basic balances, a larger proportion of macro add-on balances, and larger policy-rate balances. As a result, larger banks receive lower overall reserve interest rates compared to smaller banks. The specific details regarding the institutional differences will be presented shortly.



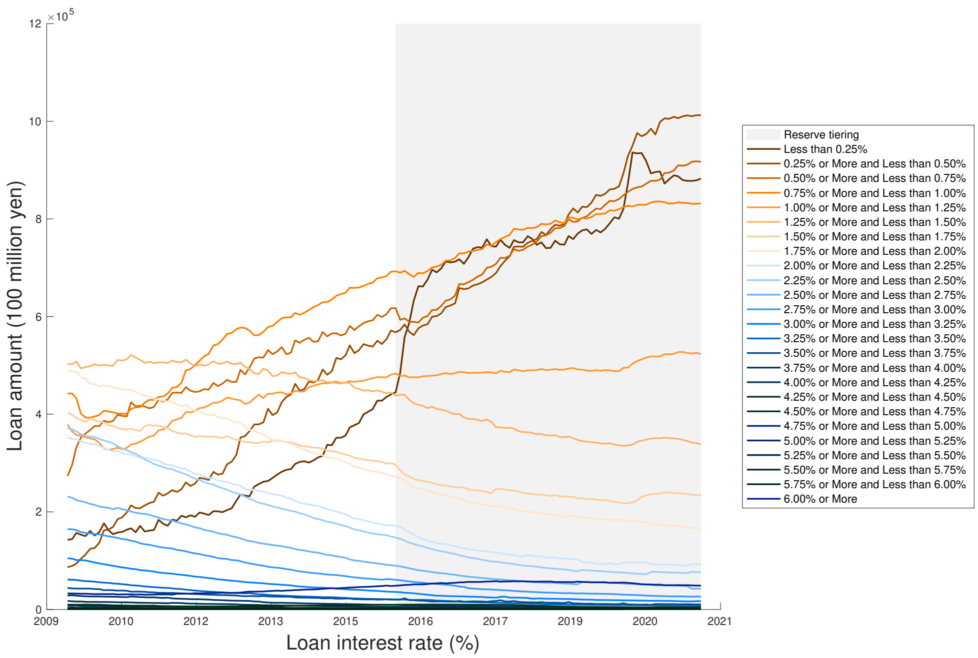
Source: Bank of Japan, Arao et al. (2022)

Figure 2: Framework of the Three-Tier System of NIRP

The loan rate is an important indicator in analyzing the monetary policy pass-through. When observing the outstanding loan breakdown by interest rate, as shown in Figure 3, there is a sharp increase in the low-interest-rate loan outstanding and a decline in the high-interest-rate loans after NIRP. Specifically, there are ¥22 trillion increase in the loans outstanding with the rate of less than 0.25% from January to June 2016.[3] This trend is

[3]This increase in the low-interest rate loan outstanding is abnormal after considering special low-interest loan programs offered around the same time of NIRP. One of them is ¥300 billion 0% loans provided through the Fund-supplying operations to support financial institutions in disaster areas of the 2016 Kumamoto Earthquake (https://www.boj.or.jp/en/mopo/measures/mkt_ope/ope_u/index.htm/). The other is the loan support programs, including the Fund-Provisioning Measure to Stimulate Bank Lending, the Fund-Provisioning Measure to Support Strengthening the Foundations for Economic Growth, and the Funds Supplying Operations Support Financial Institutions in Disaster Areas affected by the Great East Japan Earthquake. From January 2016 to June 2016, the total new loan disbursement is around ¥9.8 trillion

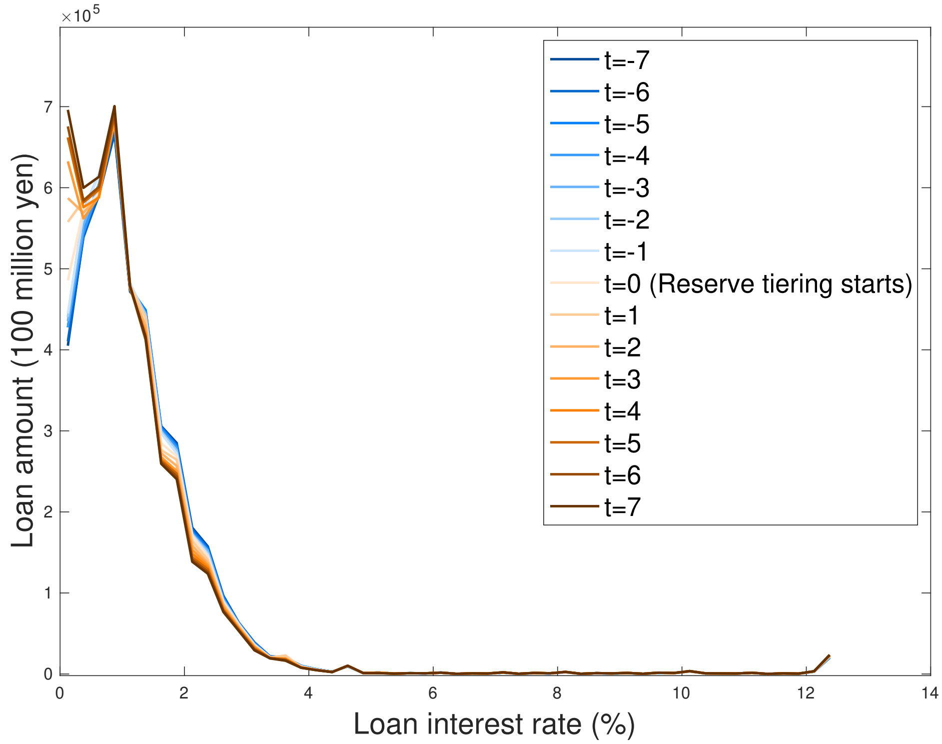
unlikely to be caused by the negative interest rate itself but is associated with the reserve tiering due to the limited pass-through to loan rate following the negative interest rate policy (Heider et al., 2021; Arce et al., 2021; Basten and Mariathasan, 2020; Heider et al., 2019; Eggertsson et al., 2019), which is because the NIRP hurts banks' interest margin due to the zero lower bound in the deposit rate. Also, the increasing low-interest-loan trend is more sound than the interest rate cut within positive territory. Furthermore, if further looking at the loan amount distribution across different loan rates, as shown in Figure 4, there is a sharp increase in the low-interest-rate loans and a decline in the medium-interest-rate loans three months following the reserve tiering. Therefore, it is worth investigating how reserve tiering caused this abnormal increase in bank lending in Japan.



Source: Bank of Japan

Figure 3: Loans and Discount Outstanding by Interest Rate in Japan

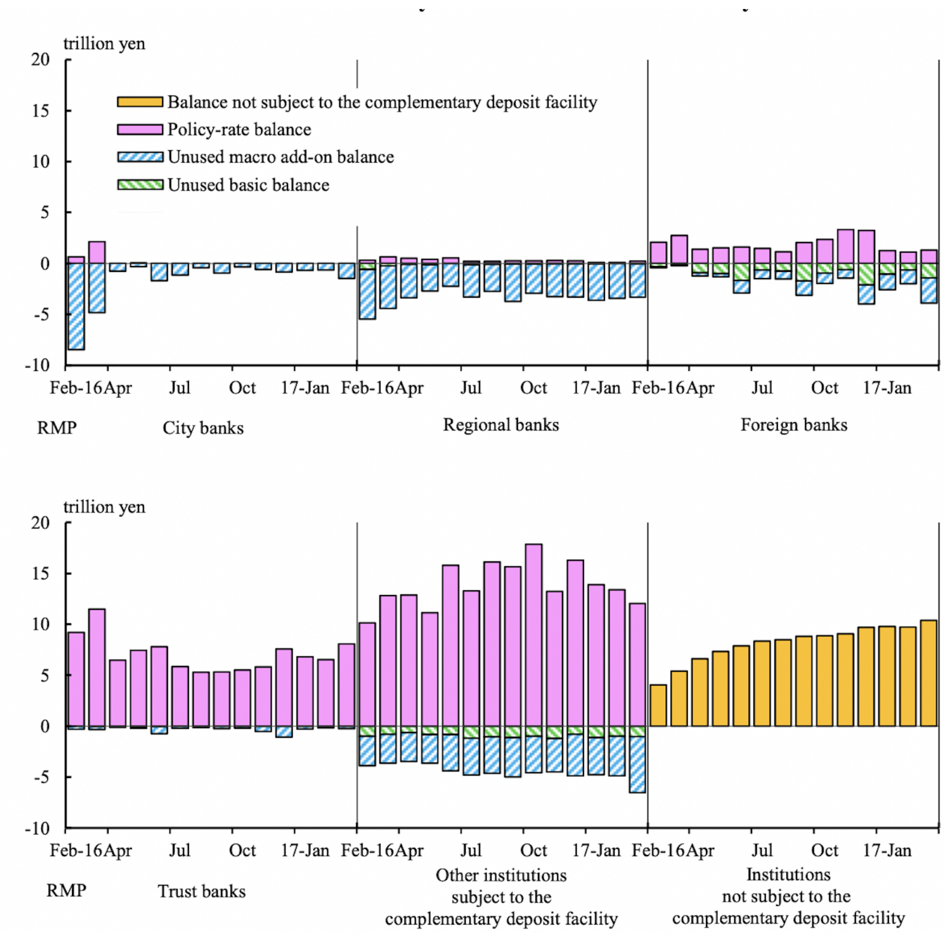
(https://www.boj.or.jp/en/mopo/measures/mkt_ope/len_b/index.htm/). These two, in total, only take around 50% of the total increase in the loan outstanding with interest less than 0.25%, without considering the loan payback impact on the loan outstanding.



Source: Bank of Japan

Figure 4: Loans and Discount Distribution by Interest Rate in Japan (t is monthly)

One conjecture is that the phenomenon is caused by the financial institution heterogeneity exposed to the reserve rates. Not all financial institutions have current account balances at the central bank exposed to the negative policy rate balance. According to Figure 5 in the BOJ report, foreign banks, trust banks, and other institutions subject to the complementary deposit facility, including investment trusts, mainly get exposed to the NIRP. Besides trust banks, those institutions are not the main consumer loan providers. These institutions hold negative interest rate balances for different reasons. According to BOJ, those financial institutions mainly get exposed to NIRP because they want to pile up idle money due to the rising stock price and hold more cash to purchase JGBs in expectation of a lower interest rate. Any increase in cash holding is deducted from the zero interest rate tiers of the current account balance to prevent financial institutions from increasing cash holdings significantly. Therefore, those institutions get a negative interest rate balance accordingly. Moreover, some foreign banks also like to hold a negative rate balance as long as they can secure profits since the yen funding cost in the foreign exchange swap market is lower than the negative rate cost. Finally, trust banks have negative balances since money reserve funds (MRFs) and other investment trusts transfer their funds to bank accounts of trust banks through ”lending to banking accounts.” However, this trend decreased significantly after the April 2016 reserve maintenance period due to some policy constraints in doing these lendings. [4]

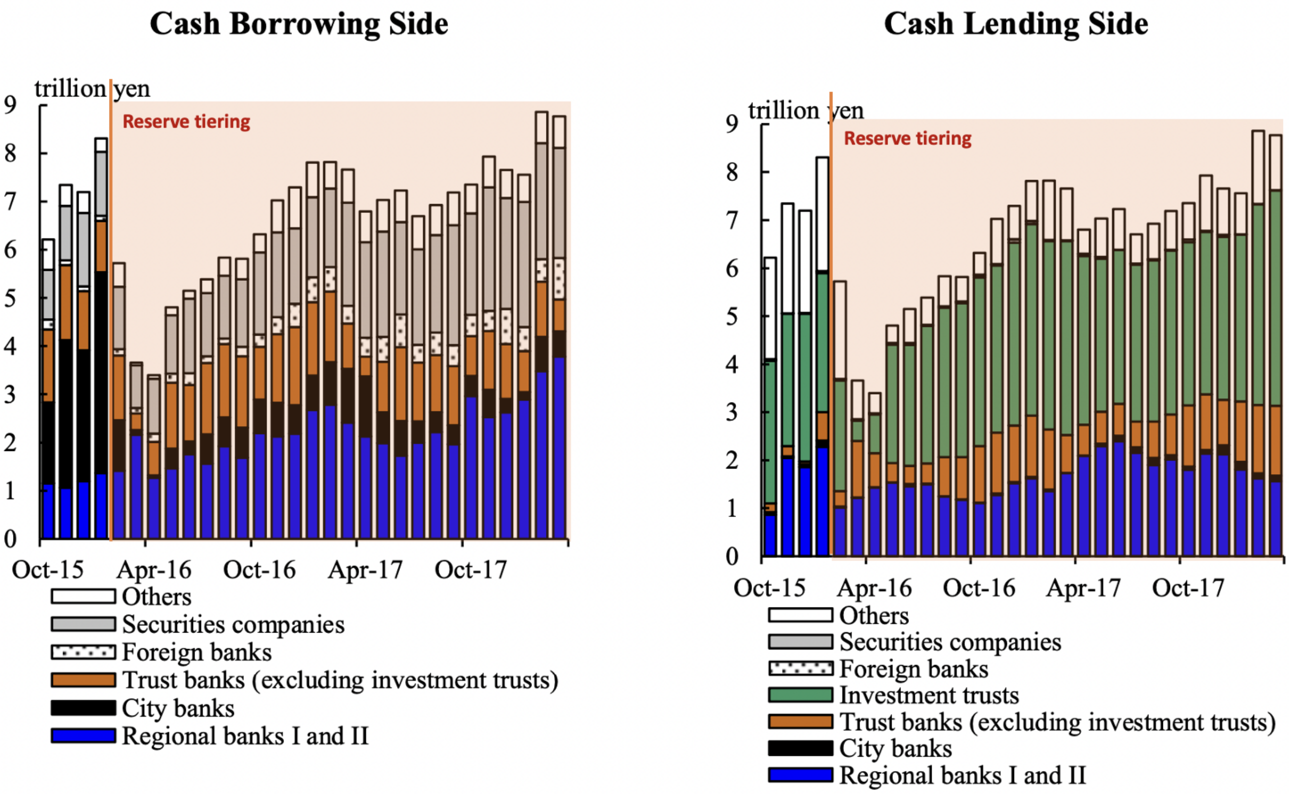


Source: Bank of Japan

Figure 5: BOJ Current Accounts by Sector

[4]Source: BOJ Market Operations in Fiscal 2016 https://www.boj.or.jp/en/research/brp/ron_2017/data/ron170714a.pdf

On the contrary, city banks and regional banks only have a few negative interest rate balances. Together with trust banks, they are the main consumer loan providers. Moreover, city banks, typically big banks, have higher negative interest rate balances than regional banks, usually small banks.



Source: Bank of Japan

Figure 6: Japan's Amounts Outstanding in the Uncollateralized Call Market Breakdown by Sector

In the interbank market, over half of the trade volume in the uncollateralized call market is contributed by non-depository institutions ("NDI") such as investment trusts. If we focus on the financial institutions that are required to have reserves at the central bank, investment trusts and trust banks are mainly lenders in the interbank market. While at the same time, banks, including city banks and regional banks, serve as net borrowers.[5]

[5] The uncollateralized call market and general collateral repo market are the two markets where financial

As shown in Figure 6, after the reserve tiering is implemented. City banks, typically large and less risky banks that suffer higher negative interest rate balances, borrow less. And regional banks, which are risky and have lower negative interest rate balances, borrow more. Trust banks and investment trusts, which have large negative interest rate balances, lend more in the market. As a result, banks get extra funding from NDIs through interbank trading. If we further classify city banks and trust banks as big banks, and regional banks as small banks, small banks borrow more from both large banks and NDIs.

When facing the tiered negative interest rate, small banks, as net borrowers, make more profits in the interbank market since they can borrow at a low and even negative rate from the lenders and gain positive or zero profits when they deposit those balances in the central bank. Therefore, they have more cheaper funding from big banks and NDIs to offer loans through interbank trading. While large banks that typically face a lower interest rate at the central bank, borrow less after the reserve tiering policy. At the same time, trust banks and investment trusts mainly serve as lenders due to their initial high balance exposed to the negative interest rate policy. Although they want to lend out all their excess balances in the interbank market to avoid the negative interest rate balance in the central bank, they cannot find enough borrowers. Thus, they suffer losses due to the negative rate policy.

4 Baseline Model

In this section, I use a simple model to capture the monetary transmission mechanism of the tiered monetary policy. The model has six types of agents. Capital investors offer capital to banks and NDIs. Entrepreneurs take loans from banks and carry out risky projects. Depositors put deposits at banks. The central bank sets the amount of the

institutions can trade with each other. The sector decomposition data is only available for the uncollateralized call market. And the sector decomposition for the general collateral repo market is only available after 2020. But the sector decomposition since 2020 shows that those two markets have very similar trends. Therefore, I focus on the uncollateralized call market here.

reserve requirements and tiering thresholds for each bank and NDIs, sets the liquidity ratio requirement of banks, pays tiered interest on reserves, and offers lending facilities to financial institutions. There are two types of market participants in the interbank market: banks and non-depository institutions (NDIs). Banks offer loans to the market, receive deposits, face liquidity constraints and reserve requirements, and trade in the interbank market. NDIs can trade with banks and face reserve requirements. NDIs do not receive deposits and do not offer loans.[6] This section introduces the timeline of events in the market and then describes the model setup.

4.1 Model Timeline

The model to capture the interbank market activities follows Fuhrer et al. (2020). Moreover, the model incorporates two new features. Firstly, the model includes the NDI agents in the analysis. The model is used to characterize the feature that NDIs mainly serve lenders' role due to the negative policy rate balance. At the same time, banks are net borrowers, although a small fraction of banks also acts as lenders due to the initial balances. Secondly, the model incorporates the loan and deposit markets, making the loan amount and deposit amounts endogenously determined. This can be used to capture the loan market features, as observed above.

Central Bank: As shown in Figure 7, the central bank pays a positive interest rate r_M on required reserves and pays negative rates r_R on the reserves exceeding a level M^i for bank i and $\tilde{M}$ for NDIs. The central bank sets the reserve requirement K^i for bank i and also sets the liquidity ratio q. The central bank also offers lending facilities at the r_X, through which financial institutions can borrow to satisfy reserve requirements. The central bank sets up the rate such that $r_R < r_M < r_X$.

[6]In Afonso et al. (2019), similar institutions are called government-sponsored enterprises (GSEs). The model in Afonso et al. (2019) captures the fact that nowadays, activity in the Federal Funds market has been dominated by GSEs, which look for some yield on their overnight balances rather than the trades occurring between banks.

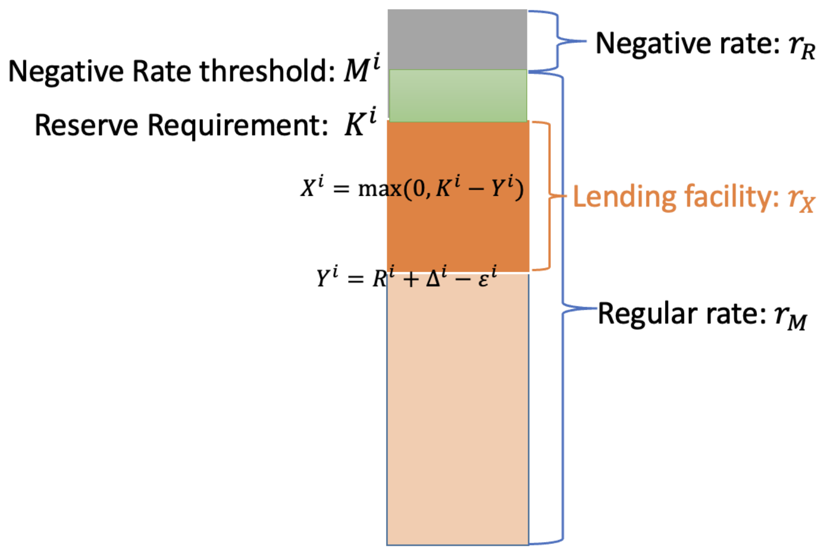


Figure 7: Central Bank Reserve Requirement

Banks: There is a continuum of banks of measure one indexed by i to maximize the expected profits. Bank i has the initial balance sheet as follows: reserves (R^i), deposits (D^i), loans (L^i), and capital (E^i). Bank i faces the reserve requirement (K^i) and the negative interest rate threshold (M^i). The loan rate r_L and the deposit rate r_D are determined by the aggregate loan amount and deposit amount in the market, respectively.

Assets	Liabilities
Reserves (R^i)	Deposits (D^i)
Loans (L^I)	Capital (E^i)

Figure 8: Pre-shock balance sheet

NDI: There is a continuum of non-depository institutions ("NDI") with measure Γ. These institutions already have pre-determined balances at the central bank exceeding the reserve tiering threshold $\tilde{M}$ and thus have the negative policy rate balance and receive r_R from the central bank for sure. They maximize profits or minimize losses in the market.

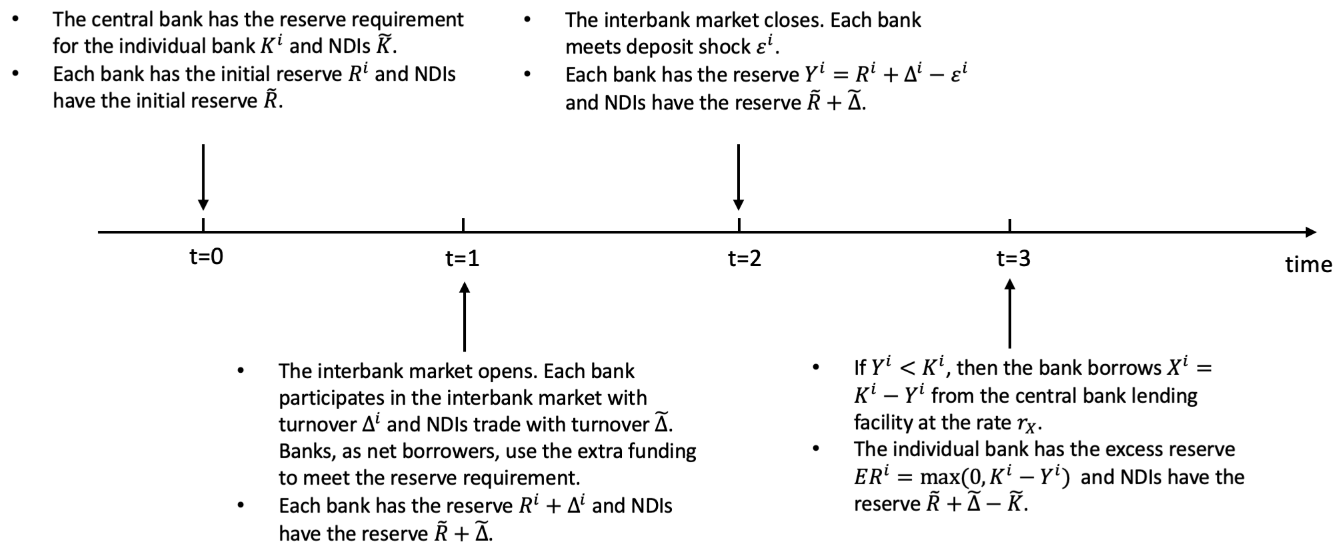


Figure 9: Model Timeline

There are four periods in the model, as shown in Figure 9.

t=0 (Initial conditions): The central bank has reserve requirement for each bank K^i and NDIs $\tilde{K}$. Each bank has the initial reserve balance R^i at the central bank, and NDIs have the initial reserve $\tilde{R}$.

t=1 (The interbank market opens): The interbank market opens. Banks participate in the interbank market with a turnover or trading volume Δ^i, which is positive for borrowers and negative for lenders. Banks either trade with each other or trade with NDIs. NDIs are homogeneous and receive exogenous capital $\tilde{R}$ exceeding the negative interest rate balance $\tilde{M}$. Thus, NDIs participate in the trade as lenders only and want to lend as much as possible. The trading interest rate in the interbank market is r_Δ.

t=2 (Deposit shocks): When the interbank market closes, the banks receive a deposit shock ε^i. These shocks are i.i.d. shocks with the CDF G. Bank i's reserve after the deposit shock is $Y^i = R^i + \Delta^i - \varepsilon^i$.

t=3 (The central bank lending facility): If $Y^i < K^i$, i.e., the reserve is below the requirement, bank i has to borrow from the central bank's lending facility at the rate r_X to

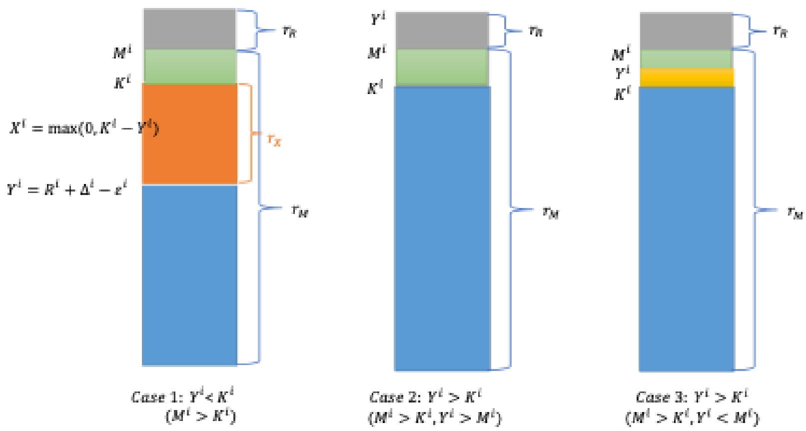


Figure 10: Three bank reserve cases

meet the requirement K^i, and the borrowing is $X^i = max(0, K^i - Y^i)$. On the contrary, if the reserve is above the requirement, i.e, $Y^i > K^i$, then the excess reserves $ER^i = max(0, Y^i - K^i)$ are deposited at the central bank. The central bank offers a negative rate r_R on the reserve exceeding M^i and offers a positive rate r_M on the reserve below M^i.

4.2 Model setup

4.2.1 Banks

In this model, banks face three different reserve cases after shocks, as shown in Figure 10. Bank i's reserve balance is either below the required reserve threshold K^i, in between the required reserve threshold and the negative interest rate threshold M^i, or greater than the negative interest rate threshold M^i. The three cases can be summarized using the following banks' profit functions:

$$
\begin{aligned}
\pi^i = -r_\Delta \Delta^i + r_M K^i - r_X \max(0, K^i - Y^i) + r_M \max(0, Y^i - K^i) + (r_R - r_M) \max(0, Y^i - M^i) \\
+ r_L L^i - r_D (D^i - \varepsilon^i) - \chi(L^i) - r_E E^i
\end{aligned}
$$

where r_L is the loan interest rate offered by the bank and is endogenous in this model.

L^i is the loan supplied by the bank. The loan volume and the loan rate are the key variables of interest in my analysis. The loan market has a negative elasticity of demand. Thus L^i is negatively associated with the interest rate r_L, which will be derived in detail in the following part. When offering loans, banks have a cost $\chi(L^i)$, which is strictly increasing, strictly convex, and twice continuously differentiable. These assumptions imply that bank has some monopoly power in the loan market as the loan volume increases. Also, there is a fixed cost of offering loans, i.e., $\lim_{L^i \to 0} = \underline{\chi} > 0$ and $\underline{\chi}$ is a constant. [7] r_D is the deposit rate with the zero lower bound $r_D \geq 0$.

The first term $-r_\Delta \Delta^i$ in the profit function captures the cost or gain associated with interbank trading. $r_M K^i$ is the interest income on the required reserve. $-r_X \max(0, K^i - Y^i)$ is the lending cost in the central bank discount window. $r_M \max(0, Y^i - K^i)$ is the profits of the balances exceeding the required reserve but lower than the threshold of negative interest rate. $(r_R - r_M)\max(0, Y^i - M^i)$ is the term to capture the cost of the negative interest rate balance and the adjustment of the benefits received on this part of the balance. $r_L L^i$ is the return on loans. And $\chi(L^i)$ is the loan cost. $-r_D(D^i - \varepsilon^i)$ is the cost associated with the deposits after shock. $r_E E^i$ is the cost for the capital.

Assets	Liabilities
Reserves $(R^i + \Delta^i)$	Deposits $(D^i - \varepsilon^i)$
Loans (L^i)	capital (E^i)
	Interbank borrowing (Δ^i)

Figure 11: Balance sheet at stage 2 after borrowing $(\Delta^i > 0)$

Assets	Liabilities
Reserves $(R^i + \Delta^i)$	Deposits $(D^i - \varepsilon^i)$
Loans (L^i)	Capital (E^i)
Interbank lending (-Δ^i)	

Figure 12: Balance sheet at stage 2 after lending $(\Delta^i < 0)$

[7] The assumption of the cost of loans follows Ennis (2018).

Due to the balance sheet accounting features, banks have the same balance sheet constraint before and after interbank trading. So I have

$$R^i + L^i = D^i - \varepsilon_i + E^i$$

At the same time, following Zarruk and Madura (1992) and Ennis (2018), banks face a required capital-to-asset ratio q. Capital E^i of the bank i is tied to be a fixed proportion of the bank's asset, i.e., $E^i \geq q(R^i + \Delta^i + L^i)$ for borrowers and $E^i \geq q(R^i + L^i)$ for lenders. Therefore, banks have the following budget constraint for the after-interbank trading balance sheet. [8]

$$E^i \geq qR^i + qL^i + q\max(0, \Delta^i)$$

Meanwhile, banks face minimum deposit amount constraint $D^i \geq \underline{D}^i$. Intuitively, this is because depositors have cash hoarding costs, which encourage them to deposit the cash at the bank even facing no interest rate. On the other hand, this is consistent with the zero lower bound on the deposit rate, which will be discussed further in the description of the depositors.

Banks will borrow from the central bank if $Y^i < K^i$, i.e. $\varepsilon^i > R^i + \Delta^i - K^i$. When banks face $M^i > Y^i > K^i$, i.e. $\varepsilon^i < R^i + \Delta^i - K^i$, banks face interest rate r_M. Alternatively, when $Y^i > M^i$, i.e. $\varepsilon^i < R^i + \Delta^i - M^i$, banks face the interest rates r_M and r_R. Therefore, the bank's expected payoff can be written as:

$$E[\pi^i] = -r_\Delta \Delta^i + r_M K^i - r_D D^i + r_L L^i - \chi(L^i) - r_E E^i - r_X \int_{R^i+\Delta^i-K^i}^{\infty} (\varepsilon^i - (R^i + \Delta^i - K^i)) dG(\varepsilon^i) +$$
$$r_M \int_{-\infty}^{R^i+\Delta^i-K^i} ((R^i + \Delta^i - K^i) - \varepsilon^i) dG(\varepsilon^i) + (r_R - r_M) \int_{-\infty}^{R^i+\Delta^i-M^i} ((R^i + \Delta^i - M^i) - \varepsilon^i) dG(\varepsilon^i)$$

[8] It should be noticed that in this model, there is no cash on the bank's balance sheet. This is to accommodate the design of the tiered monetary policy that banks cannot hoard cash to avoid a low-interest-rate policy balance. When the central bank observes that banks hoard cash, they will adjust the policy rate balance down to ensure that banks have the policy rate balance. For the same reason, the required reserve level K^i is not calculated as a constant times the deposit of that bank like Ennis (2018).

Therefore, the bank problem can be summarized as follows. Given r_M, r_X, r_R, r_E, K^i, M^i, r_Δ, r_L, r_D, bank i chooses the interbank trading amount Δ^i, loan amount L^i, deposit amount D^i, and capital holding E^i to maximize the expected payoff $E[\pi^i]$ subject to the liquidity constraint, the balance sheet constraint and the deposit constraint.

$$\max_{\Delta^i,L^i,D^i,E^i} E[\pi^i]$$

$$\text{s.t.} R^i + L^i = D^i - \varepsilon_i + E^i$$

$$E^i \geq qR^i + qL^i + q\max(0,\Delta^i)$$

$$D^i \geq \underline{D}^i$$

Substituting the expression of E^i using the first budget constraint back to the profit function and the second budget constraint, the Lagrange multiplier attached to the second constraint and the third constraint is λ_1 and λ_2, with $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. The FOCs for Δ^i, L^i, D^i respectively are as follows:

$$[\Delta^i] \quad -r_\Delta + r_X(1-G(R^i+\Delta^i-K^i)) + r_M G(R^i+\Delta^i-K^i) + (r_R - r_M)G(R^i+\Delta^i-M^i) - \lambda_1 q 1_{(\Delta^i>0)} = 0$$

$$[L^i] \quad r_L - \chi^{'}(L^i) - r_E + \lambda_1(1-q) = 0$$

$$[D^i] \quad -r_D + r_E - \lambda_1 + \lambda_2 = 0$$

4.2.2 NDIs

As discussed above, non-depository institutions are mainly lenders. In this model, suppose those NDIs have an initial balance $\tilde{R}$ and lend $-\tilde{\Delta} > 0$ at the rate r_Δ. Also, NDIs have required reserve balance $\tilde{K}$ and $\tilde{R} + \tilde{\Delta} > \tilde{K}$. Suppose those NDIs always have balances suffering the negative policy rate r_R, i.e., they always have balances exceeding their negative reserve tier $\tilde{M}$ if not trading. NDIs have capital $\tilde{E}$ and the reserve of NDIs is an exogenous portion $\tilde{q}$ of the capital. I assume NDIs are homogeneous. Given $\tilde{R}, \tilde{M}, r_R, r_M$ given, NDIs' problem to maximum payoff can be summarized as follows:

$$\max_{\tilde{\Delta}, \tilde{E}} \tilde{\pi} = r_R(\tilde{R} + \tilde{\Delta} - \tilde{M}) + r_M \tilde{M} - r_\Delta \tilde{\Delta} - r_E \tilde{E}$$

$$\text{s.t. } \tilde{R} = \tilde{q}\tilde{E}$$

$$\tilde{R} + \tilde{\Delta} \geq \tilde{M}$$

Substitute the first constraint into the objective function, and suppose the Lagrange multipliers for the second constraint is μ_1 with $\mu_1 \geq 0$. The FOC of $\tilde{\Delta}$ is as follows:

$$[\tilde{\Delta}] \quad r_R - r_\Delta + \mu_1 = 0$$

In equilibrium, when the budget constraint binds ($\tilde{R} + \tilde{\Delta} = \tilde{M}$), i.e., when the NDIs lend out all their balances exceeding the budget constraint, the interest rates satisfy $r_R < r_\Delta$. Intuitively, it is beneficial for the NDIs to lend out current account balances in the interbank market, and demand in the interbank market is sufficient. On the contrary, when the budget constraint does not bind ($\tilde{R} + \tilde{\Delta} > \tilde{M}$), i.e., there is no sufficient demand in the interbank market for NDIs to lend out as much as they want. In this case, since the demand is less than the supply, the interbank trading interest rate r_Δ will be pushed down until the central bank reserves top tier interest rate r_R. For NDIs, there is no difference between lending out the balance exceeding the low-interest-rate tier or putting the balance at the central bank. To accommodate the reality that the interbank rate is usually located within a corridor system of the reserve rate and the discount window rate, in the following parts, I assume the first case is true and $\mu_1 > 0$. Therefore, in the model, $r_R < r_\Delta$ holds. [9]

[9] NDIs are homogenous in this model, which can be modified to be heterogeneous if using the search model in the interbank market. The heterogeneous setting of the NDIs will help to figure out the order of the bank-to-bank trading or bank-to-NDI trading. A similar result is discussed in Afonso et al. (2019).

4.2.3 Entrepreneurs

Following Ennis (2018), there is one continuum of entrepreneurs in the economy. Entrepreneurs j get a loan L^j with interest rate r_L from a bank at time t, and invest in one project that gives a return r_P at the next period. At the time $t+1$, entrepreneurs must repay the loans. Projects are risky and heterogeneous in returns r_P.The returns have the CDF $H(r_P)$ and $r_P \in [0, \bar{r_P}]$. An entrepreneur will borrow a loan from the bank and carry out the project if $r_P > r_L$. The total loan demand is:

$$L^d(r_L) = \int_{r_L}^{\bar{r_P}} dH(r_P) \equiv 1 - H(r_L)$$

Therefore, the total loan demand is a decreasing function in the loan interest rate since $H(r_L)$ is a CDF, which is consistent with the settings in the bank part.

4.2.4 Depositors

Suppose there is one continuum of depositors in the economy. Depositors put deposits D in the banks and receive the rate r_D. And the total deposit is:

$$D^d(r_D) = \underline{D} + \kappa r_D$$

where $\kappa > 0$ and $\underline{D} > 0$. So the deposit supply is an increasing function of the deposit rate. Moreover, when the deposit rate $r_D = 0$, there is still a minimum amount of deposit $\underline{D}$ in banks. This setting is consistent with the reality that people would like to put money in the bank even if they get no interest rate due to cash storage costs. Moreover, banks are reluctant to cut the deposit rate into the negative territory and always set $r_D \geq 0$. So the model always has $D \geq \underline{D}$.

4.2.5 Capital investors

Suppose there is one continuum of capital investors, which has an inelastic supply of capital to banks and NDIs. Those capital investors receive a return r_E on the capital supplied. To ensure that the capital constraint binds, the model has $r_E > r_D$.

4.2.6 Central bank

As discussed in the previous part, the central bank determines the reserve requirement of each bank K^i and NDIs $\tilde{K}$. The central bank also determines the tiering monetary policy threshold for each individual bank M^i and NDIs $\tilde{M}$. The central bank pays negative rates r_R on the reserves exceeding a level M^i for bank i and $\tilde{M}$ for NDIs. The central bank pays positive rate r_M on the reserve lower than M^i for bank i and $\tilde{M}$ for NDIs. The central bank also offers lending facilities at the r_X, through which financial institutions can borrow to satisfy reserve requirements. The central bank sets up the rate such that $r_R < r_M < r_X$. The central bank also determines the capital-to-asset ratio requirement q. Bank i has reserve R^i and NDIs have reserve $\tilde{R}$. The central bank has the following aggregate conditions for the reserve tiering threshold M, the aggregate reserve R, and the reserve requirement K.

$$\int_i M^i di + \Gamma\bar{M} = M$$

$$\int_i R^i di + \Gamma\bar{R} = R$$

$$\int_i K^i di + \Gamma\bar{K} = K$$

4.3 Equilirium

In equilibrium, the following market clearing conditions hold.

$$\text{Interbank market:} \quad \int_i \Delta^i di + \Gamma\tilde{\Delta} = 0$$

$$\text{Deposit market:} \quad \int_i D^i di = D = D^d(r_D)$$

$$\text{Loan market:} \quad \int_i L^i di = L = L^d(r_L)$$

$$\text{Capital market:} \quad \int_i E^i di + \Gamma\tilde{E} = E$$

Definition 1. *The equilibrium is a set of* $\left\{\Delta^i, D^i, L^i, E^i, \tilde{\Delta}, \tilde{E}\right\}_{\forall i}$ *and rates* $\{r_\Delta, r_L, r_D\}$ *such that, given rates* $\{r_M, r_R, r_X, r_E\}$*, allocations* $\left\{K^i, M^i, R^i,\right\}_{\forall i}$ *and* $\left\{\tilde{K}, \tilde{M}, \tilde{R}\right\}$ *chosen by the central bank, banks, NDIs, depositors, entrepreneurs solve their maximumization problems as described above and interbank market, deposit market, loan market and capital market clear.*

The summary of the equilibrium solutions is shown in Appendix A.

5 Monetary Policy Pass-through

Reserve tiering can stimulate the monetary policy pass-through through four different channels: the liquidity channel, the interest rate channel, the bank interest margin channel, and the bank loan risk reallocation channel. The first three channels will be characterized in this section. And the fourth channel will be introduced in the next section about the interest rate distribution.

5.1 The liquidity channel

The liquidity channel is the key channel for reserve tiering to promote the monetary policy pass-through. Faced with the negative interest rate at the central bank, NDIs have incentives to lend out their balances to avoid the low interest rate. As a result, banks get extra liquidity from NDIs through interbank trading.

In equilibrium, the liquidity channel can be summarized using the following Propositions.

Proposition 5.1. *[The loan rate and the NDIs' tiering threshold] In equilibrium, the loan rate and the NDI tiering threshold satisfy the following equation:*

$$\frac{\partial r_L^*}{\partial \tilde{M}} = \frac{qg\Gamma(r_M - r_R)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big) + q^2\alpha(\frac{1}{1-q} + 2h)} > 0$$

When the central bank moves down the negative interest rate balance threshold of the NDIs, or increases the lower interest rate portion of reserves, the loan rate decreases.

Proof. See Appendix A.4. □

Proposition 5.2. *[The borrowing banks' trading volume and the NDI tiering threshold] In equilibrium, $\frac{\partial \beta}{\partial \tilde{M}} < 0$. The borrowing banks' total trading volume in the interbank market is negatively associated with the NDI reserve tiering threshold. Therefore, when the central bank cuts the negative interest rate balance threshold for the NDIs, borrowing banks have incentives to borrow more in the interbank market.*

Proof. See Appendix A.4. □

Intuitively, Proposition 5.1 and Proposition 5.2 indicate that when the central bank cuts the reserve tiering threshold for NDIs, NDIs have incentives to lend out more balances and banks have incentives to borrow more in the interbank market. As a result, banks have this part of liquidity through interbank trading to finance their loans. Therefore, to stimulate the monetary policy pass-through, it is optimal for the central bank to decrease the reserve tiering threshold for NDIs.

The magnitude of the response of the loan rate in response to the reserve tiering balance change depends on the mass of NDIs Γ. When the mass of NDIs increases, banks have more funding sources through the interbank trade, which offers them more funding to offer loans. Moreover, the magnitude of the r_L in response to the change in $\tilde{M}$ also depends on the risk of the loan projects h. If the projects that the entrepreneurs invest in are highly risky, i.e., h is low, for every unit of decrease in $\tilde{M}$, banks cut the loan rate more. This indicates that the reserve tiering policy brings new risks to the financial system due to decreasing risk premiums for high-risk projects.

Therefore, compared with the traditional interest rate cut, tiered monetary promotes the interest rate pass-through through the liquidity channel. On the one hand, the tiering threshold offers the central bank another channel to stimulate the loan rate cut. On the other hand, the trading between NDIs and banks provides a new source of liquidity to the banks. Thus, banks have extra funding from the NDIs to supply more loans and cut the loan rate.

5.2 The interest rate channel

Reserve tiering's impact on the loan rate also realizes through the interaction between the loan rate and the interbank rate, as shown in the following proposition.

Proposition 5.3. *[The connection between the interbank rate and the loan rate] In equilibrium, the relationship between the loan rate and the interbank rate is as follows:*

$$-r_\Delta + r_X(1-G(\delta_1)) + r_M G(\delta_1) + (r_R - r_M)G(\delta_2) = \alpha q r_E - \alpha\frac{q}{1-q}(r_L - \chi^{'}(L) - q r_E) \qquad (1)$$

where $\delta_1 = R - \Gamma\tilde{M} - K + \Gamma\tilde{K}$ *and* $\delta_2 = R - M$. *Since* $\frac{\partial r_L}{\partial r_\Delta} = \frac{1-q}{\alpha q(1+\chi^{''}(L)H^{'}(r_L))} \approx \frac{1}{\alpha q(\frac{1}{1-q}+2h)} > 0$, *there is a positive relationship between the loan rate and the interbank trading rate.*

Proof. See Appendix A.4. □

Proposition 5.4 shows that the central bank interest rate cut works through the interbank market and transmits to the loan market. Futhermore, proposition 5.4 also implies $\frac{\partial r_\Delta}{\partial r_X} = 1 - G(\delta_1) > 0$, $\frac{\partial r_\Delta}{\partial r_R} = G(\delta_2)$, and $\frac{\partial r_\Delta}{\partial r_M} = G(\delta_1) - G(\delta_2) > 0$. Therefore, the central bank can choose to cut the negative-interest-rate reserve balance interest rate r_R, the regular reserve balance interest rate r_Δ, or the discount window interest rate r_X to push the loan rate down through the interbank market rate. Intuitively, when the banks have a cheaper funding source from the interbank market, they have incentives to cut the loan rate.

With tiering monetary policy, another channel worth noticing is the gap between the reserve requirement K and the negative interest rate balancing threshold M. On the one hand, when the gap between K and M increases, the monetary policy pass-through is realized better by cutting the regular reserve rate r_M since the magnitude of $\frac{\partial r_\Delta}{\partial r_M}$ increases. While cutting discount window rate r_X and negative interest rate balance rate r_R may not work well since the magnitudes of the $\frac{\partial r_\Delta}{\partial r_X}$ and $\frac{\partial r_\Delta}{\partial r_R}$ is not high due to a lower K and higher M. On the contrary, when the gap between K and M declines, cutting the r_R and r_X seem to work better to stimulate the monetary policy pass-through. In conclusion, to get the optimal policy pass-through through the interbank market rate r_Δ, the central bank can choose a high reserve requirement K and low-interest-rate tiering threshold M gap and cut the regular reserve rate r_M. Alternatively, the central bank can set a low K and M gap and cut reserve rate r_R and discount window rate r_X instead. Intuitively, the low-interest-rate tiering rate will work better only when the tiering balance is large enough, i.e., when the M is low enough. Similarly, the discount window rate will work better if the borrowing from the central bank is high, i.e., when the K is high. And the increasing gap between M and K indicates a higher balance of reserve suffering r_M, so cutting r_M works better.

Tiered monetary policy also promotes the monetary policy pass-through through the interest rate channel. Combined with the reserve requirement and tiered monetary policy threshold settings, the central bank can cut interest rates for different tiers to further stimulate the monetary policy pass-through through interbank trading. At the same time, banks get cheaper funding from the interbank market to cut the loan rate.

5.3 Bank interest margin channel

Another channel the tiered monetary policy work is the bank interest margin channel. The reserve tiering shifts the banks' profit function and changes the optimal policy of banks from increasing the loan rate to decreasing the loan rate.

Combining the FOCs of L^i, D^i, and market clearing conditions, I can get the following proposition.

Proposition 5.4 (The connection between the loan rate and the deposit rate)**.** *When the deposit rate is not binding, for a particular bank i, the relationship between loan rate r_L and deposit rate r_D is as follows:*

$$r_L - \chi^{'}(L) - r_E + (r_E - r_D)(1-q) = 0.$$

Consequentially, $\frac{\partial r_D}{\partial r_L} = \frac{1+\chi^{''}(L)H^{'}(r_L)}{1-q} \approx \frac{1}{1-q} + 2h > 1$, *and the deposit rate and the loan rate change in the same direction.*

Proof. See Appendix A.4. □

Proposition 5.4 is consistent with the fact that banks would like to maintain the interest margin by cutting loan and deposit rates together when the expansionary monetary policy is implemented. [10] At the same time, banks will cut the deposit rate more than the loan rate to increase their interest margin. This is caused by the capital requirement, the loan cost, and the loan risk. Intuitively, when the capital requirement increases, banks have to finance more by the bank capital, and cutting the deposit rate is less harmful to banks since they do not need to attract many deposits. Moreover, if we further assume $\chi^{'''}(L) > 0$, it can be found that banks that have higher loan volume cut their deposit rate more for every unit of the loan rate cut. This is consistent with the phenomenon that large banks with large loan volumes typically offer a lower deposit rate. Moreover, if the projects that the entrepreneurs invest in are less risky, i.e., $H^{'}(r_L)$ is higher, when banks cut the loan

[10]Note that in this model, banks choose the loan and deposit amount, and the loan and deposit rate are consequences of the market-clearing conditions. However, when there is a one-to-one relationship between the bank and the borrower or depositor, choosing the interest rate and amount is equivalent. And many past works of literature studying the bank's problem allow banks to choose the loan and deposit rates (Gertler and Karadi, 2011; Ennis, 2018; Ravn and Sterk, 2021).

rates, they cut the deposit rate more. This is because banks prefer to finance those low-risk projects and have a safe return to pay the deposit rate.

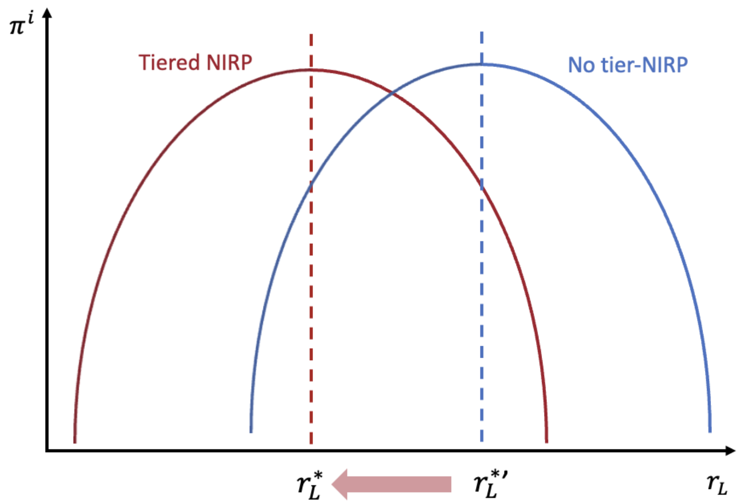


Figure 13: Bank loan rate cut mechanism

One key feature of the low interest rate or negative interest rate is the shrinking interest rate margin due to zero lower bound for the deposit rate. The past literature discusses that banks are reluctant to cut the deposit rate into negative territory even at the risk of hurting the interest margin.

Proposition 5.5. *When the deposit rate is close to zero. The expected profit function of the bank can be approximated by a quadratic function of r_L: $E(\pi^i) \approx -r_\Delta \Delta^i - r_D D^i + r_L L^i - \chi(L^i) - r_E L^i + C_1$, where C_1 is a constant that has minor impact on the loan rate chosen. And then the optimal loam rate $r_L^* \approx -[\frac{\alpha q}{(2+2h)h}(\frac{1}{1-q} + 2h)]\int_i \Delta^i di + \frac{2+r_E+\frac{1}{h}}{2+2h}$.*

Proof. See Appendix B. □

Compared with the non-tiered monetary policy, the optimal loan interest rate r_L^* is lowered by the first term when we consider that banks are net borrowers and the NDIs are lenders. Therefore, the borrowing banks mainly drive the interest rate cut. And the higher the borrowing volume, the more influential the role the banks play in cutting the loan rate.

Intuitively, as shown in Figure 13, the optimal loan rate in the system goes down from $r_L^{*'}$ to the lower one r_L^* under the tiered monetary policy due to the profit curve shifts to the left for borrowing banks.

6 The Loan Rate Distribution Changes

An extended model with heterogeneous banks can capture the interest rate distribution changes as shown in Figure 4 when doing some modifications. When reserve tiering is introduced, the low-interest-rate loan increases while the medium-interest-rate loan declines.

I add the following modifications to the baseline model. Firstly, each bank has a different risk preference and is matched with one entrepreneur with the corresponding risk exposure in the loan market. Secondly, suppose the entrepreneur i gets loans from the bank i, and the portfolio return CDF $H^i(r_P)$ is a uniform distribution CDF with the support $r_p \in [0, \bar{r_P}^i]$. So the CDF is $H^i(r_P) = h^i r_P$ for r_P in the support, where $h^i = \frac{1}{\bar{r_P}^i}$ is the risk aversion. Therefore, each bank has different $\bar{r_P^i}$ and h^i. Thirdly, in equilibrium, each bank has a different loan rate r_L^i. So each bank has an additional constraint $1 - H^i(r_L^i) = L^i$.

6.1 Loan rate reallocation channel: an analysis with approximation

Proposition 6.1. *When the deposit rate is close to zero, the optimal loan rate for bank i is* $r_L^i \approx \frac{(r_E + 2 + r_\Delta \frac{1-q}{q}) + \frac{1}{h^i}}{2 + 2h^i}$. *And* $\frac{\partial r_L^i}{\partial h^i} = \frac{4(r_E + 1 + r_\Delta \frac{1-q}{q})}{(4 + 4h^i)^2} > 0$.

Proof. See Appendix C.1. □

When reserve tiering is introduced, the loan rate offered by the individual bank is impacted mainly by two factors: the individual bank's risk aversion h^i and the interbank trading rate r_Δ. Banks that have a higher risk exposure or lower h^i choose to cut the interest rate by a higher magnitude following the reserve tiering policy than banks with lower risk exposure. This indicates that there will be an increase in systematic risk due to the lower risk premium for high-risk loans. On the contrary, banks are very cautious in adjusting their portfolio to cut loan rates less for entrepreneurs with lower risk exposure.

Intuitively, a risky bank is more likely to have higher leverage and a lower reserve balance. As a result, a risky bank has more room to borrow in the interbank market, resulting in a large cheaper funding inflow to offer further cuts in the loan rate.

However, the central bank can mitigate the systematic risk when a bank run happens. Supposing the bank run risk or deposit shock is proportional to the bank shock, for example,. $h^i = \frac{1}{\varepsilon^i}$. Intuitively, when a bank has a higher risk exposure or a lower h^i, the bank run is higher. So risky banks are more likely to borrow from the central bank lending facility. By increasing the lending facility rate, the central bank can increase the funding cost of risky banks and reduce the system risk in the economy.

6.2 The loan rate distribution changes

When the reserve tiering is implemented, entrepreneurs with low-risk portfolios will have a low magnitude loan rate decline until hitting the zero lower bound. But the medium-interest rate loan interest rate declines at a high magnitude. As a result, there is a sharp increase in low-interest-rate loans and a decline in medium-interest-rate loans due to the higher magnitude loan rate shift for medium-risk entrepreneurs. At the same time, the entrepreneurs with high-risk portfolios will receive a high-magnitude loan interest rate cut-down. However, the high-risk portfolio entrepreneurs have received high loan rates and low loan volume before reserve tiering. So a high magnitude of the loan rate cut will be almost non-observable in the chart.

7 Application Today: Interest Rate Ordering in Tiers

7.1 Reserve tiering with ascending interest rates

An interesting extension of the reserve tiering policy is the ordering of the interest rate on different tiers. As discussed in the monetary policy pass-through section, the central bank can stimulate demand in the market by remunerating the reserve above a certain threshold at a lower interest rate and remunerating the reserve below a certain threshold at a higher interest rate. It is natural to consider the case that when the central bank wants to cool down the overheating economy like today, it can pay a lower interest rate for the reserve balance below a certain threshold and pay a higher interest rate for the reserve balance above the threshold. In the model, this corresponds to the case that $r_R > r_M$. I call this case *reserve tiering with ascending interest rates*.

In this case, banks and NDIs still have heterogeneous interest rate exposures to the central bank reserve tiering system. However, NDIs receive higher interest rates than banks from the central bank in this case. The interbank rate r_Δ is higher than the lower interest rate for the reserves r_M but lower than the central bank lending facility rate r_X. The four monetary policy pass-through channels work together in the opposite direction to promote the loan rate increase.

Considering the liquidity channel and the bank interest margin channel, if $r_\Delta < r_R$, i.e., the interbank trading rate is lower than the upper interest rate, the loan rate will increase. On the one hand, NDIs have incentives to borrow funding from the interbank market and put it in the central bank reserve system to gain the profit $r_R - r_\Delta$. Also, NDIs have fewer incentives to lend out balances in the interbank market to gain profits since they can simply put their balance in the central bank to gain interest profit. At the same time, since $r_\Delta > r_M$, banks have incentives to lend out funding to the interbank market until the reserve requirements since they are less likely to pass the reserve tiering threshold due to the initial low balance at the central bank and they will gain higher profit by lending out in the interbank market. As a result, banks become net lenders, i.e., $\int_i \Delta^i di < 0$. As a result, the loan rate $r_L^* \approx -[\frac{\alpha q}{(2+2h)h}(\frac{1}{1-q} + 2h)]\int_i \Delta^i di + \frac{2+r_E+\frac{1}{h}}{2+2h}$ will be higher and the loan volume will decrease correspondingly.

On the contrary, if $r_\Delta > r_R$, both NDIs and banks have incentives to lend out balances up to the reserve requirement because they can gain higher interest rate payments in the interbank market. However, there are not as many borrowers in the interbank market, and the trading volume in the interbank market declines. As a result, the loan rate declines since banks do not have extra funding from the interbank market.

Also, the positive relationship between the loan rate r_L and r_Δ still holds in the model. An increase in the policy rate by adjusting the upper interest rate still promotes the loan rate increase through the interest channel. Banks' financing is more expensive, and the loan rate goes up.

The loan risk reallocation channel also works to increase the loan rate. When $r_\Delta < r_R$, lower-risk banks are net borrowers, and higher-risk banks are net lenders in the interbank market. This results in a higher mignitude loan rate increase for higher-risk banks. As a result, the financial system health improves due to an increased risk premium for bank loans.

7.2 Policy discussions: advantages of reserve tiering

To deal with the overheating economy after COVID-19 stimulating policies, the reserve tiering with a higher interest rate for the reserve balance above the threshold is an optional policy tool. Reserve tiering can stimulate the policy pass-through to the loan market than simply raising interest rates to reduce the loan demand. In addition to the regular interest rate channel in the monetary policy transmission mechanism, reserve tiering also works through the additional liquidity and the loan risk reallocation channels via interbank market trading. Moreover, compared with increasing the reserve requirements, reserve tiering with ascending interest rates can also help to stabilize the financial system's health using the power of the market. As described above, risky projects will receive a higher magnitude loan interest rate increase with the reserve tiering policy. Otherwise, the central bank has to adjust the policy carefully to achieve similar results. At the same time, reserve tiering can help the central bank absorb the funding in the market rather than more costly measures such as open market operations and quantitative easing.

8 Conclusion

I have built a simple model with both banks and non-depository institutions to connect the interbank and the loan markets to understand how reserve tiering promotes the monetary policy pass-through to the loan market. As observed in Japan's data, there is a sharp increase in low-interest-rate loans and a decline in medium-interest-rate loans following the BOJ's Negative Interest Rate Policy with tiers. The key driver is the financial institution's heterogeneous interest rate exposure to the tiered reserve system. Motivated by the low-interest-rate exposure at the central bank, NDIs are incentivized to lend out funding to the banks through interbank trading. Therefore, banks have cheaper funding sources from the NDIs to offer lower-interest-rate loans. At the same time, the optimal loan rate also declines due to the bank's profit function shift. The paper further shows banks with higher risk exposures cut the loan rates by higher magnitudes due to additional funding from less risky banks. As a result, the reserve tiering policy brings new risks to the financial system due to a decline in the risk premium for high-risk loans.

The findings in this paper have several policy implications. On the one hand, central banks can adjust different rates and monetary policy threshold combinations to achieve the optimal policy impact. Specifically, when there is a large volume of low-interest-rate balances at the central bank, cutting the interest rate for that portion works better to promote the monetary policy pass-through. On the other hand, the central bank can stimulate demand in the market by remunerating the bank's reserve holdings at the central bank above a certain threshold at a lower interest rate, like in the BOJ case. When the central bank wants to cool down the overheating economy, the central bank can remunerate the reserve above a threshold at a higher interest rate instead. The reserve tiering with ascending order can promote the loan rate increase via different policy pass-through channels.

The paper further compares reserve tiering with ascending interest rates with other monetary policy tools. Reserve tiering with ascending interest rates can work through the additional liquidity and loan rate reallocation channels than increasing a unified interest rate. Additionally, reserve tiering can help stabilize the financial system's health than increasing the reserve requirements. Finally, reserve tiering is less costly than open market operations and quantitative easing.

References

Afonso, Gara, Roc Armenter, and Benjamin Lester (2019) "A model of the federal funds market: yesterday, today, and tomorrow," *Review of Economic Dynamics*, 33, 177–204.

Altavilla, Carlo, Miguel Boucinha, Lorenzo Burlon, Mariassunta Giannetti, and Julian Schumacher (2022) "Money markets and bank lending: Evidence from the adoption of tiering."

Arao, Takuto et al. (2022) "Market Operations under the Three-Tier System-Explanation Using the Reserve Demand Curve Model,"Technical report, Bank of Japan.

Arce, Oscar, Miguel Garcia-Posada, Sergio Mayordomo, and Steven Ongena (2021) "Adapting lending policies in a "negative-for-long" scenario," *Available at SSRN 3161924*.

Arseneau, David M (2020) "How Would US Banks Fare in a Negative Interest Rate Environment?" *International Journal of Central Banking*.

Baldo, Luca, Florian Heider, Peter Hoffmann, Sigaux Jean-David, and Olivier Vergote (2022) "How do banks manage liquidity? Evidence from the ECB's tiering experiment," *Working paper*.

Basten, Christoph and Mike Mariathasan (2020) "Interest rate pass-through and bank risk-taking under negative-rate policies with tiered remuneration of Central Bank Reserves," *Swiss Finance Institute Research Paper* (20-98).

Bech, Morten and Todd Keister (2017) "Liquidity regulation and the implementation of monetary policy," *Journal of Monetary Economics*, 92, 64–77.

Bittner, Christian, Diana Bonfim, Florian Heider, Farzad Saidi, Glenn Schepens, and Carla Soares (2020) "Why so negative? The effect of monetary policy on bank credit supply across the euro area," *Unpublished working paper*.

Bottero, Margherita, Ms Camelia Minoiu, José-Luis Peydró, Andrea Polo, Mr Andrea F Presbitero, and Enrico Sette (2019) "Negative monetary policy rates and portfolio rebalancing: Evidence from credit register data," *International Monetary Fund*.

Demiralp, Selva, Jens Eisenschmidt, and Thomas Vlassopoulos (2021) "Negative interest rates, excess liquidity and retail deposits: Banks' reaction to unconventional monetary policy in the euro area," *European Economic Review*, 136, 103745.

Eggertsson, Gauti B, Ragnar E Juelsrud, Lawrence H Summers, and Ella Getz Wold (2019) "Negative nominal interest rates and the bank lending channel,"Technical report, National Bureau of Economic Research.

Ennis, Huberto M (2018) "A simple general equilibrium model of large excess reserves," *Journal of Monetary Economics*, 98, 50–65.

Fuhrer, L, Matthias Jüttner, Jan Wrampelmeyer, Matthias Zwicker et al. (2020) "Reserve tiering and the interbank market,"Technical report.

Gertler, Mark and Peter Karadi (2011) "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58 (1), 17–34.

Heider, Florian, Farzad Saidi, and Glenn Schepens (2019) "Life below zero: Bank lending under negative policy rates," *The Review of Financial Studies*, 32 (10), 3728–3761.

——— (2021) "Banks and negative interest rates," *Annual Review of Financial Economics*.

Jobst, Andreas Andy, Huidan Huidan Lin, and Mahmood Pradhan (2016) "Negative Interest Rate Policy (NIRP): Implications for Monetary Transmission and Bank Profitability in the Euro Area," *IMF Working Papers*, 2016 (172).

Ravn, Morten O and Vincent Sterk (2021) "Macroeconomic fluctuations with HANK & SAM: An analytical approach," *Journal of the European Economic Association*, 19 (2), 1162–1202.

Zarruk, Emilio R and Jeff Madura (1992) "Optimal bank interest margin under capital regulation and deposit insurance," *Journal of Financial and Quantitative analysis*, 27 (1), 143–149.

A Equilibirum Solutions for the Baseline Model

Suppose there is a mass of α banks are borrowers in the interbank market. i.e. $\int_{i\,|\,\Delta^i>0} di = \alpha$. And the total trading volume of the borrowers are $\int_{i\,|\,\Delta^i>0} \Delta^i = \beta \int_i \Delta^i di$, where $\beta > 1$ following the condition that banks are net borrowers in the interbank market. There are eight endogenous variables $\left\{\beta, \int_i \Delta^i di, \tilde{\Delta}, D, L, r_\Delta, r_L, r_D\right\}$ in the model.

A.1 The conditions for banks' lending and borrowing choices

- When a bank is a borrower in the interbank market, i.e., $\Delta^i > 0$, taking the equilibrium interest rates r_Δ, r_L, r_D as given, then its equilibrium conditions are:

  1. Banks' balance sheet conditions:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i + q\Delta^i \tag{2}$$

  2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1-G(R^i+\Delta^i-K^i)) + r_M G(R^i+\Delta^i-K^i) + (r_R-r_M)G(R^i+\Delta^i-M^i) = q(r_E-r_D) \tag{3}$$

  3. The relationship between the loan rate and the deposit rate:

$$r_L - \chi^{'}(L^i) - r_E + (r_E - r_D)(1-q) = 0 \tag{4}$$

- When a bank is a lender in the interbank market, i.e., $\Delta^i < 0$, taking the equilibrium interest rates r_Δ, r_L, r_D as given, then its equilibrium conditions are:

  1. Banks' balance sheet conditions:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i \tag{5}$$

2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1 - G(R^i + \Delta^i - K^i)) + r_M G(R^i + \Delta^i - K^i) + (r_R - r_M)G(R^i + \Delta^i - M^i) = 0 \quad (6)$$

3. The relationship between the loan rate and the deposit rate:

$$r_L - \chi^{'}(L^i) - r_E + (r_E - r_D)(1 - q) = 0 \quad (7)$$

To simplify the expressions, assume the following conditions are true.

Assumption A.1. *The exogenous shock* ε^i *follows a uniform distribution* $G(\varepsilon^i) = \frac{1}{\bar{\varepsilon}}\varepsilon^i = g\varepsilon^i$ *in the interval* $[0, \bar{\varepsilon}]$.

Assumption A.2. *The project return* r_P *follows a uniform distribution* $H(r_L) = \frac{1}{\bar{r_P}} r_L = hr_L$ *in the interval* $[0, \bar{r_P}]$.

Assumption A.3. *The loan cost function* $\chi(L) = L^2 + \underline{\chi}$ *in the interval* $(0, \infty)$ *and is strictly increasing, strictly convex, and twice continuously differentiable function. And* $\underline{\chi} > 0$.

Under those assumptions, from equation 3, the borrowing bank trading volume $\Delta_b^i(R^i, K^i, r_E, r_D, r_D)$ is

$$\Delta_b^i(R^i, K^i, r_E, r_D, r_D) = \frac{r_\Delta - r_X + q(r_E - r_D) + (r_X - r_R)gR^i - (r_X - r_M)gK^i + (r_R - r_M)gM^i}{-(r_X - r_R)g}. \quad (8)$$

And from equation 6, the lending bank trading volume $\Delta_l^i(R^i, K^i, r_\Delta)$ is

$$\Delta_l^i(R^i, K^i, r_\Delta) = \frac{r_\Delta - r_X + (r_X - r_R)gR^i - (r_X - r_M)gK^i + (r_R - r_M)gM^i}{-(r_X - r_R)g} \quad (9)$$

Since $\Delta_b^i > 0$, I can derive that banks will be borrowers in the interbank market if

$$R^i - \frac{r_X - r_M}{r_X - r_R}K^i - \frac{r_M - r_R}{r_X - r_R}M^i < -\frac{r_\Delta - r_X}{(r_X - r_R)g} - \frac{(r_E - r_D)q}{(r_X - r_R)g}. \quad (10)$$

Intuitively, a bank is more likely to borrow in the interbank market if it has a lower initial reserve R^i, a higher reserve requirement K^i, or a higher reserve tiering threshold M^i.

Since $\Delta_l^i < 0$, I can derive that banks will be lenders in the interbank market if

$$R^i - \frac{r_X - r_M}{r_X - r_R} K^i - \frac{r_M - r_R}{r_X - r_R} M^i > -\frac{r_\Delta - r_X}{(r_X - r_R)g}. \tag{11}$$

Intuitively, a bank is more likely to lend in the interbank market if it has a higher initial reserve R^i, a lower reserve requirement K^i, or a lower reserve tiering threshold M^i.

A.2 Aggregate conditions

There are seven aggregate equilibrium equations.

1. Banks' balance sheet condition:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i + q\max(0, \Delta^i)$$

   Aggregating across all banks yields the following equation:

$$\begin{aligned}(1-q)R - (1-q)\Gamma\tilde{R} + (1-q)L &= D - E(\varepsilon) + q\int_{i\,|\,\Delta^i>0} \Delta^i di \\ &= D - E(\varepsilon) + q\beta \int_i \Delta^i di\end{aligned}$$

2. The relationship between the interbank rate and the deposit rate:

$$\begin{aligned}-r_\Delta + r_X(1 - G(R - \Gamma\tilde{R} + \int_i \Delta^i di - K + \Gamma\tilde{K})) + r_M G(R - \Gamma\tilde{R} + \int_i \Delta^i di - K + \Gamma\tilde{K}) \\ +(r_R - r_M)G(R - \Gamma\tilde{R} + \int_i \Delta^i di - M + \Gamma\tilde{M}) = \alpha q(r_E - r_D)\end{aligned}$$

3. The relationship between the loan rate and the deposit rate:

$$r_L - \chi^{'}(L) - r_E + (r_E - r_D)(1-q) = 0$$

4. The trading condition of the NDIs:

$$\tilde{R} + \tilde{\Delta} = \tilde{M}$$

5. The loan market clears:

$$1 - H(r_L) = L$$

6. The deposit market clears:

$$\underline{D} + \kappa r_D = D$$

7. The interbank market clears:

$$\int_i \Delta di + \Gamma \tilde{\Delta} = 0$$

A.3 An additional equation of β

Given $\int_{i\,|\,\Delta^i>0} \Delta^i di = \beta \int_i \Delta^i di$ and $\int_{i\,|\,\Delta^i>0} \Delta^i di + \int_{i\,|\,\Delta^i<0} \Delta^i di = \int_i \Delta^i di$, combining with equation 9, I can derive that

$$(1-\beta) \int_i \Delta^i di = \frac{r_\Delta - r_X + (r_X - r_R) g \int_{i\,|\,\Delta^i<0} R^i di - (r_X - r_M) g \int_{i\,|\,\Delta^i<0} K^i di + (r_R - r_M) g \int_{i\,|\,\Delta^i<0} M^i di}{-(r_X - r_R) g}$$

$$\Rightarrow \beta = 1 - \frac{r_\Delta - r_X + (r_X - r_R) g R^l - (r_X - r_M) g K^l + (r_R - r_M) g M^l}{(r_X - r_R) g \Gamma(\tilde{M} - \tilde{R})} \tag{12}$$

where $R^l = \int_{i\,|\,\Delta^i<0} R^i di$, $K^l = \int_{i\,|\,\Delta^i<0} K^i di$, and $M^l = \int_{i\,|\,\Delta^i<0} M^i di$. In the model calibration part, by setting the distributions of R^i, K^i, and M^i and α, β aims to match the data from Japan.

Let $\mathcal{A} \equiv -\frac{1}{(r_X - r_R) g \Gamma(\tilde{M} - \tilde{R})}$ and $\mathcal{B} \equiv 1 - \frac{-r_X + (r_X - r_R) g R^l - (r_X - r_M) g K^l + (r_R - r_M) g M^l}{(r_X - r_R) g \Gamma(\tilde{M} - \tilde{R})}$, then equation 12 can be rewritten as $\beta^* = \mathcal{A} r_\Delta^* + \mathcal{B}$.

When combining all eight equations, the eight endogenous variables can be solved.

A.4 Proofs of Propositions

A.4.1 Proposition 5.4

Proof. From the relationship between the loan rate and the deposit rate, it can be derived that

$$\begin{aligned} r_D &= \frac{1}{1-q}(r_L - \chi^{'}(L) - qr_E) \\ &= \frac{1}{1-q}(r_L - 2(1 - H(r_L)) - qr_E) \\ &= (\frac{1}{1-q} + 2h)r_L - \frac{(qr_E + 2)}{1-q} \end{aligned} \tag{13}$$

Plugging back the loan market clearing condition and the deposit market clearing condition back to the aggregate banks' balance sheet condition and substituting the equation (13),

$$(1-q)R - (1-q)\Gamma\tilde{R} + (1-q)(1 - H(r_L)) = \underline{D} + \kappa(\frac{1}{1-q}(r_L - \chi^{'}(L) - qr_E)) - E(\varepsilon) - q\beta\Gamma(\tilde{M} - \tilde{R})$$

$$\Rightarrow (1-q)H(r_L^*) + \frac{\kappa}{1-q}(r_L^* - \chi^{'}(L^*) - qr_E) = (1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\beta\Gamma(\tilde{M} - \tilde{R})$$

$$\Rightarrow r_L^* = \frac{\beta^* q\Gamma(\tilde{M} - \tilde{R}) + (1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \tag{14}$$

Taking derivative w.r.t. $\tilde{M}$ on both sides, it can be derived that

$$\frac{\partial r_L}{\partial \tilde{M}} = \frac{\beta^* q\Gamma}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} > 0$$

Therefore, $\frac{\partial r_L}{\partial \tilde{M}} > 0$.

Let $\mathcal{C} \equiv \frac{q\Gamma(\tilde{M} - \tilde{R})}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h}$ and $\mathcal{D} \equiv \frac{(1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h}$, then equation 14 can be rewritten as $r_L^* = \mathcal{C}\beta^* + \mathcal{D}$.

□

A.4.2 Proposition 5.3

Proof. When plugging back equation (13) into the relationship between the interbank rate and the deposit rate, it can be derived that:

$$\begin{aligned}-r_\Delta + r_X(1 - G(R - \Gamma\tilde{R} + \int_i \Delta^i di - K + \Gamma\tilde{K})) + r_M G(R - \Gamma\tilde{R} + \int_i \Delta^i di - K + \Gamma\tilde{K}) \\ +(r_R - r_M)G(R - \Gamma\tilde{R} + \int_i \Delta^i di - M + \Gamma\tilde{M}) = \alpha q(r_E - \frac{1}{1-q}(r_L - \chi^{'}(L) - qr_E))\end{aligned}$$

$$\Rightarrow -r_\Delta + r_X(1 - G(\delta_1)) + r_M G(\delta_1) + (r_R - r_M)G(\delta_2) = \alpha q r_E - \alpha\frac{q}{1-q}(r_L - \chi^{'}(L) - qr_E)$$

$$\begin{aligned}\Rightarrow r_\Delta^* = \alpha q(\frac{1}{1-q} + 2h)r_L^* + r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} \\ +g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q\end{aligned} \tag{15}$$

where $\delta_1 = R - \Gamma\tilde{M} - K + \Gamma\tilde{K}$ and $\delta_2 = R - M$. Taking derivative w.r.t. r_Δ on both sides, it can be derived that

$$\frac{\partial r_L}{\partial r_\Delta} = \frac{1}{\alpha q(\frac{1}{1-q} + 2h)} > 0.$$

Let $\mathcal{S} \equiv \alpha q(\frac{1}{1-q} + 2h)$ and $\mathcal{T} \equiv r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q$ and the equation 15 can be rewritten as $r_\Delta^* = \mathcal{S}r_L^* + \mathcal{T}$. □

A.4.3 Proposition 5.1

Proof. Combining equations 12, 14 and 15, I can find that:

$$\beta^* = \frac{\mathcal{ADS} + \mathcal{AT} + \mathcal{B}}{1 - \mathcal{ACS}} \tag{16}$$

$$r_L^* = \frac{\mathcal{ACT} + \mathcal{BC} + \mathcal{D}}{1 - \mathcal{ACS}} \tag{17}$$

$$r_\Delta = \frac{\mathcal{BCS} + \mathcal{CT} + \mathcal{D}}{1 - \mathcal{ACS}}. \tag{18}$$

And I can derive that

$$
\begin{aligned}
\mathcal{ACS} &= -\frac{1}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})} \frac{q\Gamma(\tilde{M} - \tilde{R})}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \alpha q(\frac{1}{1-q} + 2h) \\
&= -\frac{q^2 \alpha(\frac{1}{1-q} + 2h)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{ACT} = -\frac{1}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})} \frac{q\Gamma(\tilde{M} - \tilde{R})}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \Big(& r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} \\
& + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q\Big)
\end{aligned}
$$

$$
= -\frac{q\Big(r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q\Big)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}
$$

$$
\begin{aligned}
\mathcal{BC} &= \Big(1 - \frac{-r_X + (r_X - r_R)gR^l - (r_X - r_M)gK^l + (r_R - r_M)gM^l}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})}\Big) \frac{q\Gamma(\tilde{M} - \tilde{R})}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \\
&= \frac{q\Big((r_X - r_R)g\Gamma(\tilde{M} - \tilde{R}) + r_X - (r_X - r_R)gR^l + (r_X - r_M)gK^l - (r_R - r_M)gM^l\Big)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}
\end{aligned}
$$

So I can derive that

$$
r_L^* = \frac{\mathcal{ACT} + \mathcal{BC} + \mathcal{D}}{1 - \mathcal{ACS}}
$$

$$
= \frac{qg\Gamma(r_M - r_R)\tilde{M} + g(r_X - r_R)R + qg(r_R - r_M)M - qg(r_X - r_M)K + qg\Gamma(r_X - r_M)\tilde{K} - (r_X - r_R)g\Gamma\tilde{R} + \alpha q^2(1+q)r_E + 2\alpha q^2 - q + (r_X - r_R)g(1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big) + q^2\alpha(\frac{1}{1-q} + 2h)}
$$

Therefore, $\frac{\partial r_L^*}{\partial \tilde{M}} = \frac{qg\Gamma(r_M - r_R)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big) + q^2\alpha(\frac{1}{1-q} + 2h)} > 0.$ □

A.4.4 Proposition 5.2

Proof. Following the proof of Proposition 5.1, I can derive the formula parts for β.

$$
\mathcal{ADS} = -\frac{1}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})} \frac{(1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa}{(1-q)h + \frac{\kappa}{1-q} + 2\kappa h} \alpha q(\frac{1}{1-q} + 2h)
$$

$$= -\frac{\alpha q(\frac{1}{1-q} + 2h)\Big((1-q)R - (1-q)\Gamma\tilde{R} + 1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa\Big)}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big)}$$

$$\mathcal{AT} = -\frac{\Big(r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q\Big)}{(r_X - r_R)g\Gamma(\tilde{M} - \tilde{R})}$$

So I can derive that:

$$\beta = \frac{\mathcal{ADS} + \mathcal{AT} + \mathcal{B}}{1 - \mathcal{ACS}}$$

$$= \frac{-\mathcal{M}g\Gamma(r_X - r_M)\tilde{M} + [\Gamma\alpha q + 2h\alpha q(1-q)\Gamma - \mathcal{M}(r_X - r_R)g\Gamma]\tilde{R} + [-\alpha - 2\alpha qh(1-q) + g(r_R - r_X)\mathcal{M}]R + \mathcal{M}g(r_R - r_M)M - \mathcal{M}g(r_X - r_M)K + \mathcal{M}g\Gamma(r_X - r_M)\tilde{K} + \mathcal{N}}{\Gamma(\tilde{M} - \tilde{R})[(r_X - r_R)g\mathcal{M} - q^2\alpha(\frac{1}{1-q} + 2h)]}$$

where $\mathcal{M} \equiv (1-q)h + \frac{\kappa}{1-q} + 2\kappa h$ and $\mathcal{N} = -\alpha q(\frac{1}{1-q} + 2h)(1 - q + \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa) + \mathcal{M}[\alpha q(1+q)r_E + 2\alpha q + r_X - (r_X - r_R)R^l - (r_X - r_M)gK^l + (r_R - r_M)gM^l]$.

I can further derive that:

$$\frac{\partial\beta}{\partial\tilde{M}} = \frac{[-\mathcal{M}g\Gamma(r_M - r_R) + \Gamma\alpha q + 2h\alpha q(1-q)\Gamma]\tilde{R} + [-\alpha - 2\alpha qh(1-q) + g(r_R - r_X)\mathcal{M}]R + \mathcal{M}g(r_R - r_M)M - \mathcal{M}g(r_X - r_M)K + \mathcal{M}g\Gamma(r_X - r_M)\tilde{K} + \mathcal{N}}{\{\Gamma(\tilde{M} - \tilde{R})[(r_X - r_R)g\mathcal{M} - q^2\alpha(\frac{1}{1-q} + 2h)]\}^2} < 0$$

since the term $\mathcal{M}g(r_R - r_M) + \mathcal{N}$ is the dominant factor and is negative. □

A.5 The interbank trading rate $\mathbf{r}_\Delta^*$

The model solution is based on the assumption that the interbank trading rate $r_\Delta \geq r_R$.

Proof. Following the proof of Proposition 5.1, I can derive the formula parts for r_Δ.

$$r_\Delta^* = \mathcal{S}r_L^* + \mathcal{T}$$

$$= \alpha q(\frac{1}{1-q} + 2h)r_L^* + r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q$$

$$= \alpha q(\frac{1}{1-q} + 2h)\frac{qg\Gamma(r_M - r_R)\tilde{M} + g(r_X - r_R)R + qg(r_R - r_M)M - qg(r_X - r_M)K + qg\Gamma(r_X - r_M)\tilde{K} - (r_X - r_R)g\Gamma\tilde{R} + \alpha q^2(1+q)r_E + 2\alpha q^2 - q + (r_X - r_R)g(1 - q - \underline{D} + E(\varepsilon) + q\kappa r_E + 2\kappa)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big) + q^2\alpha(\frac{1}{1-q} + 2h)}$$

$$+ r_X + g(r_R - r_X)R - g(r_R - r_M)M + g\Gamma(r_X - r_M)\tilde{M} + g(r_x - r_M)K - g\Gamma(r_X - r_M)\tilde{K} - \alpha q(1+q)r_E - 2\alpha q$$

And I can further derive that

$$\frac{\partial r_\Delta^*}{\partial \tilde{M}} = \alpha q(\frac{1}{1-q} + 2h)\frac{\partial r_L^*}{\partial \tilde{M}} + g\Gamma(r_X - r_M)$$

$$= \frac{\alpha(\frac{1}{1-q} + 2h)q^2 g\Gamma(r_M - r_R)}{(r_X - r_R)g\Big((1-q)h + \frac{\kappa}{1-q} + 2\kappa h\Big) + q^2\alpha(\frac{1}{1-q} + 2h)} + g\Gamma(r_X - r_M) > 0$$

Therefore, as the central bank adjusts down the bank reserve tiering threshold for the NDIs, the interbank lending rate r_Δ^* declines accordingly.

Also, it can be derived that

$$\frac{\partial r_\Delta^*}{\partial R} = \frac{\alpha q(\frac{1}{1-q} + 2h)(r_x - r_R)g}{(r_X - r_R)g[(1-q)h + \frac{\kappa}{1-q} + 2\kappa h] + q^2\alpha(\frac{1}{1-q} + 2h)} - g(r_X - r_R).$$

$$= g(r_X - r_R)[\frac{\alpha q(\frac{1}{1-q} + 2h)}{(r_X - r_R)g[(1-q)h + \frac{\kappa}{1-q} + 2\kappa h] + q^2\alpha(\frac{1}{1-q} + 2h)} - 1]$$

□

B Bank Interest Margin Channel

B.1 Proposition 5.5

The proof for Proposition 5.5 is as follows.

Proof. The expected profit of the bank can be approximated by

$$E(\pi^i) \approx -r_\Delta \Delta^i - r_D D^i + r_L L^i - \chi(L^i) - r_E L^i + C_1 \approx -r_\Delta \Delta^i - r_D D^i + (r_L - r_E)L^i - \chi(L^i) + C_1$$

where C_1 is a constant that has minor impact on the loan rate chosen. Aggregating across all banks, the expected profit is

$$E(\pi) \approx -r_\Delta \int_i \Delta^i di - r_D D + r_L L - \chi(L) - r_E L + C_1 \approx -r_\Delta \int_i \Delta^i di - r_D D + (r_L - r_E)L - \chi(L) + C_1$$

Taking the F.O.C. with respect to L when r_D is close to zero.

$$-r_\Delta \frac{\partial \int_i \Delta^i di}{\partial L} - \frac{\partial r_\Delta}{\partial L} \int_i \Delta^i di + (r_L - r_E) + \frac{\partial r_L}{\partial L}(1 - hr_L) - \chi'(L) = 0 \tag{19}$$

And I can derive that

$$\frac{\partial r_\Delta}{\partial r_L} = \frac{\partial r_\Delta}{\partial L} \frac{\partial L}{\partial r_L}$$

$$\Rightarrow \frac{\partial r_\Delta}{\partial L} = \frac{\frac{\partial r_\Delta}{\partial r_L}}{\frac{\partial L}{\partial r_L}} = \frac{\alpha q(\frac{1}{1-q} + 2h)}{-h} = -\frac{\alpha q}{h}(\frac{1}{1-q} + 2h)$$

Also, using the interbank market aggregate condition, I can derive that

$$\frac{\partial \int_i \Delta^i di}{\partial L} = \frac{\partial - \Gamma(\tilde{M} - \tilde{R})}{\partial L} = 0$$

Also using $1 - hr_L = L$, I can derive that

$$\frac{\partial r_L}{\partial L} = -\frac{1}{h}$$

So the equation 19 is

$$\Rightarrow \frac{\alpha q}{h}(\frac{1}{1-q} + 2h) \int_i \Delta^i di + r_L - r_E - \frac{1}{h} + r_L - 2(1 - hr_L) = 0$$

$$\Rightarrow r_L^* \approx -[\frac{\alpha q}{(2+2h)h}(\frac{1}{1-q} + 2h)] \int_i \Delta^i di + \frac{2 + r_E + \frac{1}{h}}{2 + 2h}$$

C Equilibrium Solutions for the Extended Model

Suppose there is a mass of α banks are borrowers in the interbank market and $\int_{i\,|\,\Delta^i>0} = \alpha$. In the extended model, suppose each bank i offers loans only to one entrepreneur at the rate r_L^i. And the endogenous variables are $\left\{\Delta^i, \tilde{\Delta}, D^i, L^i, r_\Delta, r_L^i, r_D\right\}_{\forall i}$. The equilibirum conditions are as follows.

- When a bank is a borrower in the interbank market, i.e., $\Delta^i > 0$, taking the equilibrium interest rates r_Δ, r_L^i, r_D as given, then its equilibrium conditions are:

  1. Banks' balance sheet conditions:

  $$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i + q\Delta^i \tag{20}$$

  2. The relationship between the interbank rate and the deposit rate:

  $$-r_\Delta + r_X(1 - G(R^i + \Delta^i - K^i)) + r_M G(R^i + \Delta^i - K^i) + (r_R - r_M)G(R^i + \Delta^i - M^i) = q(r_E - r_D) \tag{21}$$

  3. The relationship between the loan rate and the deposit rate:

  $$r_L^i - \chi^{'}(L^i) - r_E + (r_E - r_D)(1 - q) = 0 \tag{22}$$

  4. The relationship between the loan volume and loan rate:

  $$1 - H^i(r_L^i) = L^i \tag{23}$$

- When a bank is a lender in the interbank market, i.e., $\Delta^i < 0$, taking the equilibrium interest rates r_Δ, r_L^i, r_D as given, then its equilibrium conditions are:

1. Banks' balance sheet conditions:

$$R^i + L^i = D^i - \varepsilon^i + qR^i + qL^i \tag{24}$$

2. The relationship between the interbank rate and the deposit rate:

$$-r_\Delta + r_X(1 - G(R^i + \Delta^i - K^i)) + r_M G(R^i + \Delta^i - K^i) + (r_R - r_M)G(R^i + \Delta^i - M^i) = 0 \tag{25}$$

3. The relationship between the loan rate and the deposit rate:

$$r_L^i - \chi^{'}(L^i) - r_E + (r_E - r_D)(1 - q) = 0 \tag{26}$$

4. The relationship between the loan volume and loan rate:

$$1 - H^i(r_L^i) = L^i \tag{27}$$

Additionally, I can also get the following conditions.

- The trading condition of the NDIs:

$$\tilde{R} + \tilde{\Delta} = \tilde{M} \tag{28}$$

- The deposit market clears:

$$\underline{D} + \kappa r_D = D \tag{29}$$

- The interbank market clears:

$$\int_i \Delta^i di + \Gamma \tilde{\Delta} = 0 \tag{30}$$

Suppose assumption A.1 and assumption A.3 still hold. And the assumption A.2 can be revised as follows.

Assumption C.1. *Each entrepreneur j has different risk exposure and gets loans from a different bank i, the project return r_P follows a uniform distribution $H(r_L) = \frac{1}{\bar{r_P}^i} r_L = h^i r_L$ in the interval $[0, \bar{r_P}^i]$.*

Assumption C.1 corresponds to the condition that each bank has a different risk tolerance level and is exposed to entrepreneurs with different risks.

Combing equation 20, equation 24 and the aggregate trading conditions of NDIs and the interbank market clearing condition, I can get the following condition between r^*_Δ and r^*_D:

$$r^*_\Delta = r_X(1-G(R-\Gamma\tilde{M}-K+\Gamma\tilde{K}))+r_M G(R-\Gamma\tilde{M}-K+\Gamma\tilde{K})+(r_R-r_M)G(R-M)-\alpha q(r_E-r^*_D) \quad (31)$$

The equilibrium can be solved using the following algorithm:

1. Apply bisection with bounds $\bar{r_D} - \epsilon_D$ and $\bar{r_D} + \epsilon_D$.

2. At iteration n:

   (a) Use $r_D^{(n)}$ and equation 31 to solve for $r_\Delta^{(n)}$

   (b) Given $(r_D^{(n)}, r_\Delta^{(n)})$, given $\{R^i, K^i, M^i, \tilde{R}, \tilde{K}, \tilde{M}, \varepsilon^i\}_{\forall i}$ solve for $\{\Delta^i, D^i, L^i, r_L^i, \tilde{\Delta}\}_{\forall i}$

   (c) Compute the aggregate deposit supply $\int_i D^i di$ and the aggregate demand $\underline{D}+\kappa r_D$ for convergence. Following the Walras law, if the deposit market clears, the interbank market also clears.

C.1 Proposition 6.1

Proof. If focusing on the individual bank i,

$$E(\pi^i) \approx -r_\Delta \Delta^i - r_D(D^i - \varepsilon^i) + r_L^i L^i - \chi(L^i) - r_E L^i + C_1 \approx -r_\Delta \Delta^i - r_D(D^i - \varepsilon^i) + (r_L^i - r_E)L^i - \chi(L^i) + C_1$$

Given r_Δ and r_D, taking F.O.C. with respect to L^i,

$$-r_\Delta \frac{\partial \Delta^i}{\partial L^i} - r_D \frac{\partial D^i}{\partial L^i} + \frac{\partial r_L^i}{\partial L^i} L^i + r_L^i - r_E - \chi^{'}(L^i) = 0 \tag{32}$$

When the bank is a borrower, taking partial derivative with respect to L^i,

$$1 = \frac{\partial D^i}{\partial L^i} + q + q\frac{\partial \Delta^i}{\partial L^i}$$

$$\Rightarrow \frac{\partial \Delta^i}{\partial L^i} = -\frac{1}{q}\frac{\partial D^i}{\partial L^i} + \frac{1-q}{q}$$

Also, using $1 - H^i(r_L^i) = 1 - h^i r_L^i = L^i$, I have

$$\frac{\partial r^i}{\partial L^i} = -\frac{1}{h^i}$$

When the deposit rate is close to zero lower bound, using the loan market clearing condition, the aggregate loan volume is close to $\underline{D}$ and is fixed for each individual bank. Therefore, I have $\frac{\partial D^i}{\partial L^i} \approx 0$.

Plugging back to the equation 32,

$$-r_\Delta(-\frac{1}{q}\frac{\partial D^i}{\partial L^i} + \frac{1-q}{q}) - r_D\frac{\partial D^i}{\partial L^i} - \frac{1}{h^i}(1 - h^i r_L^i) + r_L^i - r_E - 2(1 - h^i r_L^i) = 0$$

$$\Rightarrow -r_\Delta \frac{1-q}{q} - \frac{1}{h^i}(1 - h^i r_L^i) + r_L^i - r_E - 2(1 - h^i r_L^i) = 0$$

$$\Rightarrow r_L^i \approx \frac{(r_E + 2 + r_\Delta \frac{1-q}{q}) + \frac{1}{h^i}}{2 + 2h^i}$$

$$\frac{\partial r_L^i}{\partial h^i} = \frac{4(r_E + 1 + r_\Delta \frac{1-q}{q})}{(4 + 4h^i)^2} > 0$$

□