How Effective are Portfolio Mandates?[1]

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Abstract

We evaluate the effectiveness of portfolio mandates on equilibrium capital allocation. We show that the impact of mandates crucially depends on firms' demand elasticity of capital. In a production economy with constant returns to scale, firms' demand for capital is infinitely elastic, and mandates can significantly impact the allocation of capital across sectors despite having a negligible impact on the cost of capital. This is in sharp contrast to an endowment economy where inelastic demand for capital implies equilibrium price reactions to mandates, which significantly reduce their effectiveness. Within a canonical real-business-cycle model calibrated to match key asset-pricing and macroeconomic moments, we estimate that a significant portion of the mandate remains effective in shaping equilibrium capital allocation, even when there is little disparity in the cost of capital across sectors. Our analysis challenges the common practice of judging the effectiveness of portfolio mandates by their impact on firms' cost of capital.

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JEL Classification: D53, G11, G12.

1 Introduction

Responsible investing, a strategy aimed at generating social and environmental impact alongside financial returns, has grown tremendously over the last decade. Portfolio "screens" or "mandates" are common implementations of socially responsible investing strategies. Such policies aim to restrict capital allocation to specific firms to increase target firms' cost of capital and make it more costly for them to fund their operations. PricewaterhouseCoopers (2022) forecasts that assets under management that are screened by Environmental, Social, and Governance (ESG) criteria are expected to increase from $18.4tn in 2021 to $33.9tn by 2026, with ESG assets on pace to constitute 21.5% of total global assets under management. Bloomberg Intelligence (2021) expects global ESG assets to exceed $53 trillion by 2025, representing more than a third of total assets under management. On the other hand, partly on the grounds that it reduces investment returns, several states in the US have introduced proposals *against* responsible investing (Donefer, 2023), and twenty-five US states have sued the Biden Administration to halt a Department of Labor rule that prioritizes ESG concepts into retirement-fund regulations (Mayer, 2023).

Despite the large sums of assets allocated to responsible investing and the controversy about its costs and benefits, the academic literature to date provides a skeptical view of its effectiveness. In their pioneering work, Heinkel, Kraus, and Zechner (2001) and, more recently, Berk and van Binsbergen (2021) argue that responsible-investing policies have a negligible impact on targeted firms' cost of capital and are, therefore, ineffective in influencing capital allocation. There is also a large literature that uses the change in the cost of capital as a gauge for the effectiveness of various socially responsible policies.[1]

In this paper, we show that differences in the cost of capital across sectors are generally *not* informative about differences in sectoral capital allocations. Employing both a simplified theoretical model and a quantitative framework, we illustrate that portfolio mandates will likely lead to substantial disparities in sectoral capital allocation, even when the cost-of-capital differences across sectors are minimal. Our analysis applies more broadly beyond socially responsible investing

[1] See, for instance, the article from McKinsey "Why ESG is Here to Stay," which discusses how ESG scores are related to the cost of capital. The article states "... there have been more than 2,000 academic studies, and around 70 percent of them find a positive relationship between ESG scores on the one hand and financial returns on the other, whether measured by equity returns or profitability or valuation multiples. Increasingly, another element is the cost of capital. Evidence is emerging that a better ESG score translates to about a 10 percent lower cost of capital." For a further discussion of the effect of ESG on the cost of capital, see Edmans (2023).

and extends to situations where portfolio constraints are imposed to influence investor behavior for various reasons, including, for example, regulatory compliance and economic sanctions.[2]

Heinkel et al. (2001), Berk and van Binsbergen (2021), and the literature using the cost of capital to measure the effectiveness of ESG-related policies reach their conclusions based on the analysis of an endowment economy. In such an economy, a firm's dividends are *exogenous*, and only its asset returns depend on market-clearing prices. In this paper, we revisit the conclusion that responsible investment policies have a negligible impact on targeted firms' cost of capital and are, therefore, ineffective in influencing capital allocations by studying the equilibrium effect of portfolio mandates in a model with production. In contrast to an endowment economy, in a production economy *both* dividends (payoffs or output) and asset returns are determined endogenously in equilibrium. We show that this has important implications for understanding the real effects of portfolio mandates: in particular, portfolio mandates can lead to large differences in the equilibrium sectoral allocation of physical capital despite negligible differences in the cost of capital.

To understand the intuition driving our result that portfolio mandates can lead to significant changes in capital allocation despite a negligible effect on the cost of capital, we study two versions of a production economy with two sectors, consisting of "green" and "brown" firms, and two groups of investors, one constrained by a portfolio mandate while the other unconstrained. The first version of the model is a stylized two-period frictionless production economy that allows us to develop the key intuition for our findings and *qualitatively* assess the effectiveness of portfolio mandates in equilibrium. The second version of the model extends the stylized model to a multiperiod setting with realistic frictions that allow us to match macroeconomic and asset-pricing moments in the data and hence *quantitatively* assess mandate effectiveness in equilibrium. To highlight the equilibrium effect of mandates, we assume that the green and brown assets are identical, other than the fact that one of the two, e.g., the green asset, is favored by the mandate. Although, in reality, mandates may be imposed in response to externalities, e.g., pollution, we abstract away from modeling the rationale for their existence in the economy.

In the first version of the model, we consider an economy in which firms use capital K supplied by investors to produce output Y according to the production function $Y = A\,K^{\alpha}$, with A

[2]For example, Article 5 of Regulation (EU) No 833/2014, enacted after the onset of the war between Russia and Ukraine, states that "It shall be prohibited to directly or indirectly purchase, sell, provide investment services for or assistance in the issuance of, or otherwise deal with transferable securities" https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32022R0328.

denoting an exogenous productivity shock, and $\alpha \in [0,1]$ the returns-to-scale parameter. The case of $\alpha = 1$, where output is given by $Y = A\,K$, represents the typical "AK" model of a production economy with constant returns to scale. The case of $\alpha = 0$ represents an *endowment* economy, in which output is exogenously given $Y = A\,K^{\alpha} = A$, and hence capital is not used in production. In this economy, the marginal productivity of capital, that is, $R \equiv \alpha A K^{\alpha-1}$, represents the "cost of capital". The relationship between the cost of capital R and capital K represents firms' *demand of capital*. Suppose the economy has two sectors, green (G) and brown (B), and denote by R_G and R_B the cost of capital in each sector, that is, the marginal productivity of sectoral capital. The ratio of the cost of capital is then $R_G/R_B = (A_G/A_B) \times (K_G/K_B)^{\alpha-1}$.

In an endowment economy, $\alpha = 0$ and $R_G/R_B = (A_G/A_B) \times (K_G/K_B)^{-1}$. In this economy, a portfolio mandate designed to increase K_G relative to K_B leads to a corresponding decrease in R_G relative to R_B. In such an economy, the difference in the cost of capital is informative about the difference in capital allocation and, hence, about the effectiveness of a mandate. When the difference in returns is large, unconstrained investors have an incentive to shift their portfolio toward the sector B sector. In equilibrium, the response of the unconstrained investors to the change in relative returns across sectors undoes a part of the intended effect of the portfolio mandate, thus limiting the effectiveness of the mandate. In contrast, in the case of a production economy with constant returns to scale, $\alpha = 1$ and $R_G/R_B = A_G/A_B$. In this economy, a portfolio mandate designed to increase K_G relative to K_B has *no effect* on the cost of capital R_G and R_B. As a result, unconstrained investors have no incentive to shift their portfolio toward the B sector. In equilibrium, the unconstrained investors' response does not offset the portfolio mandate's effect. Thus, mandates can be fully effective in shaping capital allocation even though they have no impact on the sectoral cost of capital.

This raises the question about the appropriate value for the returns-to-scale parameter, α. Empirical estimates from the macroeconomic literature indicate that returns to scale are nearly constant in the US economy, i.e., $\alpha \approx 1$. In a series of influential papers, Hall (1988, 1990) argues that market power and increasing return to scale can explain procyclical productivity in the US. In more recent work, Ahmad, Fernald, and Khan (2019) argue that returns to scale are constant or slightly decreasing; however, they also suggest that there may be increasing returns to scale in specific industries or regions or the presence of factors such as technological progress and network effects. Therefore, increasing returns to scale might be particularly relevant for "green technologies" where learning-by-doing and increased scale have dramatically decreased costs over the past twenty-five years.[3]

To quantify the effectiveness of a portfolio mandate on the sectoral allocation of physical capital in equilibrium, we introduce the concept of "*mandate pass-through*." To illustrate the main idea, consider an economy where both investors have equal wealth and both sectors have identical risk-return tradeoffs so that the optimal unconstrained allocation for both investors is to hold 50% of their portfolio in each sector. Suppose a mandate requires constrained investors to have 75% of their portfolio in the green sector. If we were to ignore the mandate's effect on equilibrium asset prices, the capital allocated to the green sector is $(50\%+75\%)/2 = 62.5\%$, instead of the 50% in the absence of a mandate. We refer to this difference, 12.5%, as the *maximum* mandate pass-through.

In equilibrium, the imposition of a mandate in favor of the green sector may raise the price of green assets and lower that of brown, making the return on brown assets more attractive. As a result of the higher return on brown assets, the unconstrained investor would then invest more than 50% in the brown sector, undoing part of the effect of the portfolio mandate. If, after accounting for general equilibrium effects, the overall allocation of capital to green assets is, say, only 56.25%, then the equilibrium mandate pass-through is only 6.25%. Thus, the *effective* mandate pass-through ratio, defined as the ratio of the equilibrium to maximum mandate pass-through, is $6.25\%/12.5\% = 50\%$; that is, 50% of the mandate survives the equilibrium effects. The mandate pass-through generally depends on the unconstrained household's capital supply elasticity. If unconstrained households are willing to make large shifts to their portfolio in response to even small changes in returns–as would happen, for example, if they are very risk tolerant–then the mandate pass-through will be small. However, when α is close to one, firms' demand elasticity is close to perfectly elastic, making the households' demand elasticity irrelevant and leading to a 100% pass-through.

To assess the *quantitative* effects of portfolio mandates on the financial and real sectors, in the second version of the model, we study a dynamic general-equilibrium production economy that we calibrate to match asset-pricing and macroeconomic moments in the US. For the case of constant returns to scale, $\alpha = 1$, and no portfolio constraints, our model is a canonical real-business-cycle model, similar to that in King, Plosser, and Rebelo (1988) and Jermann (1998), among many others.

[3]For example, Way, Ives, Mealy, and Farmer (2022) argue that, unlike traditional technologies such as oil and gas, clean-energy technologies are on learning curves, where costs drop as a power law of cumulative production. In Section 3.3, we explain in greater detail that estimates from the macroeconomic literature suggest that $\alpha \approx 1$ is the more empirically relevant case.

Just as in the simple single-period model, we consider an economy characterized by two sectors with different technologies, "green" and "brown," and two types of investors, "constrained" and "unconstrained." However, we relax many of the simplifying assumptions made in the single-period model. In particular, we consider an infinite-horizon economy in discrete time where investors have Epstein-Zin recursive preferences, consume in each period, and are endowed with one unit of labor that they supply to firms inelastically. Firms are all-equity financed, incur convex capital-adjustment costs (e.g., Hayashi, 1982), and choose labor and investment to maximize shareholder value subject to a capital-accumulation constraint. We solve for the equilibrium in this economy and then study the effect of a portfolio mandate on the equilibrium stock returns (cost of capital) and capital allocations in the two sectors.

The quantitative multiperiod model confirms the intuition of the simple one-period model. In equilibrium, the optimal portfolio decisions of the unconstrained investor "undo" some of the effects of the portfolio mandate. This occurs because unconstrained investors face a trade-off. On the one hand, the desire to diversify pushes the portfolio towards a 50/50 allocation. On the other hand, by making the brown sector more attractive from a risk-reward perspective, the mandate induces unconstrained investors to tilt their portfolios toward it. We find, however, that portfolio mandates retain a quantitatively significant impact in equilibrium under a realistic calibration that matches asset-pricing and macroeconomic moments of the US economy. For example, under our baseline calibration with a mandate forcing the constrained investor to hold 75% of the portfolio in the green sector, the *effective* mandate pass-through ratio is about 22%.[4] Higher levels of risk aversion, leading to higher and more realistic risk premia, increase the unconstrained investor's desire to hold a diversified portfolio and strengthen the equilibrium real effect of portfolio mandates. Higher values of return to scale also strengthen the real impact of portfolio mandates. In contrast, the effect on the equilibrium cost of capital or Sharpe ratio of the two types of firms remains negligible, consistent with existing evidence, which we discuss below.

In summary, our analysis suggests that in a dynamic general equilibrium production economy designed to match the macroeconomic and asset-pricing moments of the US economy, portfolio mandates can have a quantitatively significant impact on aggregate capital allocation, even if their effect on the cost of capital is negligible. This result sharply contrasts the conclusion drawn from

[4]As we explained in Section 3, the reason the pass-through is not 100% in the dynamic model, as opposed to the 2-period model, is the presence of labor income, its dependence on the share of green versus brown capital, and the households' desire to hedge labor income risk.

studying endowment economies, where, because dividends are exogenous, there is a direct relation between firms' cost of capital and equilibrium capital allocations.

The main contribution of our paper is to study how much of the intended effect of portfolio mandates is undone in equilibrium. Our paper makes two key points. First, we highlight that studying the effects of portfolio mandates in an endowment economy, as most of the finance literature on portfolio mandates has done, is likely to lead to misleading conclusions. In particular, to measure the effectiveness of portfolio mandates, it is essential to focus on the *quantity* of capital flowing to the mandated sectors instead of the effect on the *cost* of capital. This insight is similar to that of Berk and Van Binsbergen (2015), who, in the context of the mutual-fund-performance literature, have emphasized the importance of measuring fund flows instead of risk-adjusted returns. Second, we quantify the impact of portfolio mandates on capital allocation. Specifically, in a general-equilibrium production-economy model calibrated to match key macroeconomic and asset-pricing moments, we show that the real effect of portfolio mandates can be substantial, even if their impact on the cost of capital is negligible.

Our paper relates to the growing literature on socially responsible investing. This literature consists of two main strands: exclusion (exit) and engagement (voice). The first strand of this literature focuses on a "discount-rate channel" in that it studies the effects of limiting (or excluding entirely) investment in certain firms from an investor's portfolio on the cost of capital of targeted firms. The key mechanism in this literature is reduced risk-sharing, which affects the cost of capital in an endowment economy.[5] Notably, Heinkel et al. (2001) and Berk and van Binsbergen (2021) focus on the result that the effect on risk premia is small if profit-seeking investors can substitute for the capital they are restricted from holding. This intuition implies that mandates are effective only if they lead to significantly higher cost of capital for brown firms. A large number of empirical studies find a higher cost of debt and equity financing for "brown" (or "sin") firms although the magnitudes are not substantial, especially for debt financing.[6] Our paper revisits this evidence by

[5] See, e.g., Heinkel et al. (2001); Zerbib (2019, 2022); Berk and van Binsbergen (2021); Pastor, Stambaugh, and Taylor (2021); Pástor, Stambaugh, and Taylor (2022); Pedersen, Fitzgibbons, and Pomorski (2021); Broccardo, Hart, and Zingales (2022); De Angelis, Tankov, and Zerbib (2022); Sauzet and Zerbib (2022); and Cheng, Jondeau, Mojon, and Vayanos (2023).

[6] See, e.g., Goss and Roberts (2011); Chava (2014); Zerbib (2019); Baker, Bergstresser, Serafeim, and Wurgler (2022); Fatica, Panzica, and Rancan (2021); Huynh and Xia (2021); Seltzer, Starks, and Zhu (2022); Pástor et al. (2022); El Ghoul, Guedhami, Kwok, and Mishra (2011); Bolton and Kacperczyk (2021a,b); Aswani and Rajgopal (2022); and Hong and Kacperczyk (2009). A smaller number of studies find insignificant or even lower returns for brown firms, e.g., Larcker and Watts (2020); Flammer (2021); Tang and Zhang (2020); Berk and van Binsbergen (2021); andKontz (2023).

considering a production economy and studies the quantitative effects of portfolio mandates in a calibrated model designed to match key asset-pricing and macroeconomic moments.

In a recent paper, Dangl, Halling, Yu, and Zechner (2023a) study how different types of investor *preferences* affect equilibrium capital allocation. They find that if investments are endogenous, the effect of social preferences on corporate decisions may be sizable even if the difference in the cost of capital between the green and brown sectors is negligible. Dangl, Halling, Yu, and Zechner (2023b) extend this analysis to the case of time-varying social preferences. Unlike them, we show that portfolio mandates can affect capital allocations across sectors—despite small differences in the cost of capital across these sectors—in a standard macroeconomic framework with the portfolio mandate imposed on only a fraction of investors. We also illustrate that the degree of the returns to scale has a crucial impact on the ability of portfolio mandates to influence equilibrium capital allocation.

Finally, Hong, Wang, and Yang (2023) introduce decarbonization capital in a representative-agent dynamic stochastic general-equilibrium model and investigate the effectiveness of sustainable finance mandates in mitigating externalities within the economy. In their economy, the mandate affects all investors and is, therefore, by definition, effective. In contrast, we study an economy where only a fraction of investors is constrained. Because unconstrained investors can trade against constrained investors, in equilibrium, they can potentially undo the effect of mandates. Our finding that mandates can substantially impact equilibrium capital allocation aligns with their conclusion that mandates can effectively address externalities.

The second strand of literature focuses instead on the "cash-flow channel." Broccardo et al. (2022), following Hart and Zingales (2017), conclude that "voice" is more effective than "exit." Oehmke and Opp (2022) focus on activist investors who care about the social cost of investing in brown firms and provide a corporate perspective on the economics of motivated investors: socially responsible activists subsidize firms to adopt clean technologies. Chowdhry, Davies, and Waters (2019) show that if a firm cannot credibly commit to social goals, such subsidies take the form of investment by socially-minded activists. Our paper does not contribute directly to this strand of literature; however, our focus on production economies allows us to consider jointly the cash-flow and discount-rate channels emphasized separately by the engagement and exclusion literature, respectively.

The rest of the paper proceeds as follows. In Section 2, we develop intuition in a simple one-period (two-date) general equilibrium model that we can solve analytically. In Section 3, we assess the real impact of portfolio mandates in a multiperiod general-equilibrium model with heterogeneous investors that is calibrated to match asset-pricing and macroeconomic moments in the US economy. Section 4 concludes.

2 A single-period equilibrium model with portfolio mandates

To understand the economic intuition driving our key results, in this section, we consider a single-period general-equilibrium economy with several simplifying assumptions that make transparent the economic forces at work. Then, to establish the quantitative implications of portfolio mandates, in the next section, we consider a multiperiod model without these simplifying assumptions.

2.1 Setup

The economy consists of a continuum of firms and investors. Investors supply capital to firms. There is one consumption good, which is used as a numéraire. Consumption can be costlessly converted to capital.

Firms. We assume that there are two sectors in the economy, green and brown, and we refer to them using the subscripts G and B, respectively. Each of these sectors consists of a large number of atomistic, identical, all-equity-financed firms. There are no externalities and neither sector is inherently good or bad. The key difference is that some investors have portfolio mandates to hold sector G's equity. Output Y_j in each sector $j = G, B$ is given by the production function[7]

$$Y_j = A_j K_j^{\alpha}, \quad j = G, B, \tag{1}$$

where $\alpha \geq 0$ is the returns-to-scale parameter, A_j denotes a random productivity shock, and K_j is the aggregate capital invested in sector j. We assume that the productivity shocks A_j are uncorrelated normally distributed random variables, that is, $A_j \sim \mathcal{N}(\mu_{A_j}, \sigma_{A_j})$, $j = G, B$. Atomistic firms choose investment K_j in order to maximize the NPV, given by

$$NPV_j = \max_{K_j} \mathbb{E}[\mathbb{M}\tilde{A}_j K_j^{\alpha}] - K_j, \tag{2}$$

[7] Here we assume that capital is the only input of production. The model of Section 3 consider a more general production function with capital and labor as inputs.

with $\mathbb{M}$ denoting the stochastic discount factor that firms take as given. Using the definition of output in equation (1) we obtain that firm j's optimal choice of capital K_j must satisfy

$$\mathbb{E}[\mathbb{M}\alpha\tilde{A}_j K_j^{\alpha-1}] = 1. \tag{3}$$

The Euler equation (3) implicitly defines the aggregate *demand* of capital from firms in sector j and the return on invested capital

$$\tilde{R}_j \equiv \alpha\tilde{A}_j K_j^{\alpha-1} = \alpha\frac{\tilde{Y}_j}{K_j}, \quad j = G, B. \tag{4}$$

The return $\tilde{R}_j$ represents the *cost of capital* for firm j. Because capital is the only input of production and can be costlessly adjusted, the realized profit is[8]

$$\tilde{\Pi}_j = \tilde{A}_j K_j^{\alpha} - \tilde{R}_j K_j = (1-\alpha)\tilde{A}_j K_j^{\alpha}. \tag{5}$$

Constant returns to scale, $\alpha = 1$, imply zero profits. Profits are positive (negative) when return to scale are decreasing (increasing), $\alpha < 1$ ($\alpha > 1$). The NPV in sector j is given by

$$NPV_j = \mathbb{E}[\mathbb{M}\tilde{A}K_j^{\alpha}] - K_j = K_j\left(\frac{1}{\alpha} - 1\right). \tag{6}$$

The NPV is zero for constant returns to scale, positive, for decreasing return to scale, and negative for increasing returns to scale. As $\alpha \to 0$, corresponding the case of an *endowment economy* in which the output $Y_j = \tilde{A}_j$ is entirely exogenous, the optimal capital $K_j \to 0$ and the return to capital and NPV are well defined in the limit. From equation (4) we can infer that firms' *demand of capital* in sector j as a function of the expected cost of capital $\mathbb{E}[R_j]$ is

$$K_j^{\text{demand}} = \left(\frac{\alpha\mu_{A_j}}{\mathbb{E}[\tilde{R}_j]}\right)^{\frac{1}{1-\alpha}}, \tag{7}$$

For $\alpha \in (0,1)$, firm j's demand of capital is inversely related to the expected cost of capital $\mathbb{E}[\tilde{R}_j]$. From equation (4), when $\alpha \to 1$, $\mathbb{E}[R_j] = \mu_{A_j}$ for all K_j, therefore the demand of capital is *infinitely elastic*. When $\alpha \to 0$, $K_j \to 0$, for all $\mathbb{E}[R_j]$, and the demand of capital is *infinitely rigid* at $K_j = 0$.

[8] Because capital can be costlessly adjusted, in this model the marginal price of capital, or Tobin's Q, is always equal to 1. Section 3 generalizes the model to account for the case of convex adjustment costs.

Investors. There is a continuum of identical investors who live for one period (two dates). Each investor is endowed with $e_{0,i}$ units of the consumption good. Consumption can be costlessly converted into capital for sector $j = G, B$. A fraction x of investors faces a constraint that either madate the holdings of green firms ("mandate") or restricts the holdings of brown firms ("screen"). We refer to the constrained invesors using the subscript c. The remaining fraction $1 - x$ is unconstrained, and we refer to them using the subscript u. For tractability, we assume that both types of investors have constant absolute risk aversion (CARA) preferences with an identical coefficient of risk aversion γ.[9]

At $t = 0$, each investor $i = u, c$ choose consumption $c_{0,i}$. Unconstrained investors can chose how to optimally allocate their savings in the G and B sectors and in the risk-free asset, yielding a gross return R_f, to be determined as part of the equilibrium. Constrained investors are restricted to hold a pre-specified fraction of savings in each sector, with the residual invested in the risk-free asset. We denote by $w_{j,i}$ the portfolio weights, as a fraction of savings, that agent $i = u, c$ allocate to sector $j = G, B$. For constrained agents, $w_{j,c}$ are set to

$$w_{G,c} = \overline{w}_G, \quad w_{B,c} = \overline{w}_B, \quad \text{with} \quad \overline{w}_G + \overline{w}_B \leq 1. \tag{8}$$

At time 1, investors terminal consumption $\tilde{c}_{1,u}$ consists of (i) the return on capital invested, determined by the firm's optimiality condition (4), and (ii) a fraction of the total profit $\tilde{\Pi}_j$ from each sector, defined in equation (5). Specifically, each investor i faces the following intertemporal budget constraint

$$\tilde{c}_{1,u} = (e_{0,i} - c_{0,i}) \Big(R_f + w_{B,i}(\tilde{R}_G - R_f) + w_{G,i}(\tilde{R}_B - R_f) \Big) + \tilde{\pi}_{G,i} + \tilde{\pi}_{B,i}, \tag{9}$$

where $\tilde{\pi}_{j,i}$ is investor's i claim to the total profit $\tilde{\Pi}_j$. Because investors are atomistic, when choosing their optimal portfolios $w_{j,i}$, they take the profit share $\tilde{\pi}_{j,i}$ and the return on capital $\tilde{R}_j$ as given. These are quantities that are decided by the firm's optimization problem and are beyond the control of atomistic investors.

The unconstrained agent solves the following problem

$$\max_{\{c_{0,u}, w_{G,u}, w_{B,u}\}} -\frac{e^{-\gamma c_{0,u}}}{\gamma} - \beta \mathbb{E}\left[\frac{e^{-\gamma \tilde{c}_{1,u}}}{\gamma} \right], \tag{10}$$

where β is a time-preference parameter and where $\tilde{c}_{1,u}$ satisfies the intertemporal budget constraint in equation (9). Constrained agents only choose their consumption at time 0 because their portfolio

[9] In the multiperiod model of Section 3, we will allow for the more general case of recursive preferences.

weights are determined exogenous by the mandate. Specifically, the constrained agent solves the following problem

$$\max_{c_{0,c}} -\frac{e^{-\gamma c_{0,c}}}{\gamma} - \beta \mathbb{E}\left[\frac{e^{-\gamma \tilde{c}_{1,u}}}{\gamma}\right], \tag{11}$$

where $\tilde{c}_{1,c}$ is given by the intertemporal budget constraint (9) in which $w_{G,c} = \overline{w}_G$ and $w_{B,c} = \overline{w}_G$. The following proposition characterizes the optimal consumption of both agents, and the optimal portfolio of the unconstrained agent.

Proposition 1. *Given the return on invested capital $\tilde{R}_j = \alpha \tilde{A}_j K_j^{\alpha-1} = \alpha \tilde{Y}_j / K_j$, $j = G, B$ from equation (4) and risk-free rate R_f, the unconstrained and constrained investor capital supply in sector $j = G, B$ is*

$$k_{j,u} \equiv w_{j,u}(e_{0,u} - c_{0,u}) = \frac{\alpha(\mathbb{E}[\tilde{R}_j] - R_f)}{\gamma Var[\tilde{R}_j]} \tag{12}$$

$$k_{j,c} \equiv \overline{w}_j(e_{0,c} - c_{0,c}), \tag{13}$$

where the consumption $c_{0,i}$, $i = u, c$, satisfies equations (A6) and (A19) in Appendix A.

Proposition 1 can be used to infer households' *supply* of capital to sector j, taking as the return $\tilde{R}_j$ and the risk-free rate R_f. Specifically, the total supply of capital to sector j is given by

$$K_j^{\text{supply}} = \underbrace{x k_{j,c}}_{\text{supply of constrained investor}} + \underbrace{(1-x)\frac{\alpha(\mathbb{E}[\tilde{R}_j] - R_f)}{\gamma Var[\tilde{R}_j]}}_{\text{supply of unconstrained investors}}, \quad j = G, B. \tag{14}$$

Equation (14) shows that, for $\alpha \in (0, 1]$ the supply of capital K_j^{supply} increases with the expected return $\mathbb{E}[\tilde{R}_j]$. Furthermore, capital supply becomes more inelastic as risk aversion increases, as the return to scale parameter α decreases, as the fraction of constrained agents x increases, and as the volatility of returns $Var[\tilde{R}_j]$ increases.

Equilibrium. Proposition 1 derives the optimal portfolio consumption choices of atomistic investors *given* the return on capital, $\tilde{R}_j = \alpha \tilde{A}_j K_j^{\alpha-1}$, and the risk-free rate R_f. In equilibrium, these quantities are determined endogenously by imposing that the aggregate supply of capital from households equals the aggregate demand of capital from firms and that the aggregate quantity of risk-free borrowing/lending is zero. The total amount of borrowing/lending in the economy is given by

$$\text{Risk-free borrowing/lending} = \underbrace{x(e_{0,c} - c_{0,c} - k_{G,c} - k_{B,c})}_{\text{constrained risk-free borrowing/lending}} + \underbrace{(1-x)(e_{0,u} - c_{0,u} - k_{G,u} - k_{B,u})}_{\text{unconstrained risk-free borrowing/lending}} \tag{15}$$

In equilibrium, the aggregate capital K_j (and hence the return $\tilde{R}_j$) in sector j and the risk-free rate are determined by the following three market-clearing conditions

$$K_G = xk_{G,c} + (1-x)k_{G,u}, \tag{16}$$

$$K_B = xk_{B,c} + (1-x)(1-x)k_{B,u}, \tag{17}$$

$$0 = x(e_{0,c} - c_{0,c} - k_{G,c} - k_{B,c}) + (1-x)(e_{0,u} - c_{0,u} - k_{G,u} - k_{B,u}), \tag{18}$$

where the portfolio weights $w_{j,u}$ and the consumption $c_{0,u}$ $c_{0,c}$ are given in Proposition 1. The above system of equations does not admit a closed-form solution. In what follows we analyze the equilibrium numerically focusing on the effect of the return-to-scale parameter α.

Assessing the effect of mandates. To quantify the equilibrium effect of portfolio mandates, we introduce the concept of *effective mandate pass-through*, a measure designed to capture the equilibrium impact of a portfolio mandate. We define the effective pass-through as the fraction of the *intended* impact of a mandate that survives in general equilibrium.

Suppose we start from an economy with no mandate, that is, $x = 0$, and denote by $K_j^{x=0}$ the equilibrium capital allocation in such an economy. The introduction of a mandate implies that a mass $x > 0$ of investors are constrained to invest fixed fractions of their savings, $\overline{w}_G$ and $\overline{w}_B$, in the G and B sectors, respectively. By Proposition 1 and the equilibrium conditions (16)–(18) the mandate results in new equilibrium capital allocations, which we denote by K_j^x. Therefore, if the mandate intends to favor sector G, the equilibrium change in capital allocated to this sector is

$$\Delta K_G^{\text{GE}} = K_G^x - K_G^{x=0} = (xk_{G,c} + (1-x)k_{G,u}) - k_{G,u}^{x=0}, \tag{19}$$

where $k_{G,u}^{x=0} = K_G^{x=0}$, because the mass of unconstrained agents is one when $x = 0$. The quantity ΔK_G^{GE}, where the label 'GE' stands for 'General Equilibrium', accounts for the fact that the mandate, by increasing demand of G capital and depressing that of B capital, lowers the return on G and increases that of B. A higher return on B makes it more attractive to the unconstrained investor who will accordingly under-invest in G to take advantage of the higher return in the B

sector. The quantity ΔK_G^{GE} therefore accounts for the fact that the intended effect of the mandate is partially *undone*, in equilibrium, by the portfolio and consumption decisions of the unconstrained investor. The equilibrium change in the portfolio weight in the G sector, implied by the mandate, is then given by the ratio of capital in the G sector to total capital, that is,

$$\Delta w_G^{\text{GE}} = \underbrace{\frac{x k_{G,c} + (1-x) k_{G,u}}{x(e_{0,c} - c_{0,c}) + (1-x)(e_{0,u} - c_{0,u})}}_{\equiv K_G^x/(K_G^x + K_B^x)} - \underbrace{\frac{k_{G,u}^{x=0}}{e_{0,u} - c_{0,u}^{(x=0)}}}_{= K_G^{x=0}/(K_G^{x=0} + K_B^{x=0})} . \tag{20}$$

To assess the equilibrium mandate pass-through, we need a measure of the *intended* effect of the mandate. This is the change in the G capital that would result by ignoring general equilibrium forces on consumption and portfolios of agents. Specifically, if a mandate to hold a proportion $\overline{w}_G$ of savings in the G sector is imposed on a mass x of agents, the intended effect of the mandate is simply

$$\Delta w_G^{PE} = x \left(\overline{w}_G - w_{G,u}^{x=0} \right), \tag{21}$$

where the label 'PE' stands for 'Partial Equilibrium'. We define the mandate pass-though as the equilibrium effect of the mandate from relative to its intended effect, that is

$$\text{Effective mandate pass-through} = \frac{\Delta w_G^{\text{GE}}}{\Delta w_G^{\text{PE}}}. \tag{22}$$

Intuitively, the effective mandate pass-through ratio measures the percentage of the maximum effect of the mandate that is actually achieved in equilibrium.

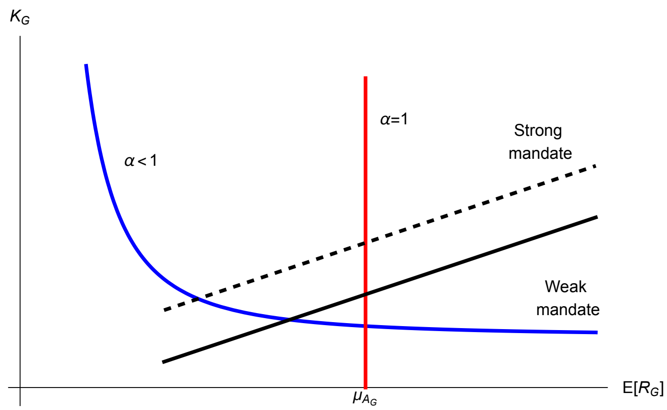
2.2 Results

In this section we illustrate properties of the equilibrium with portfolio mandates. Figure 1 provides an intuitive way to understand how the return to scale α affects the effectiveness of a portfolio mandate in favor of G capital. The upward sloping lines represent capital supply from equation (14) under two different mandates: the solid line refers to a weak mandate, that is low $k_{G,c}$ while the dashed line refers to a stronger mandate. Both supply functions are drawn for a given value of α, taking as given the return investors face.

When $\alpha < 1$ (blue line), both the supply and demand functions for capital matter. Mandating that some investors overweight asset G leads to higher equilibrium capital in asset G, with a

Figure 1: Mandate effectiveness

The figure shows the firms' demand of capital from equation (7) and the investors' supply, from equation (14). The solid black line shows a supply function under a weak mandate, that is, low $k_{G,c}$, while the dashed line shows supply under a strong mandate. Both supply functions are drawn for a given value of α, taking as given the return investors face.



stronger mandate leading to more capital in G. However, because the demand for capital is downward sloping, the effectiveness of the constraint depends on the elasticity of supply. For a very elastic agent, e.g., one with low risk aversion, the supply function is relatively vertical, implying that the unconstrained agents invest a large fraction of their portfolio in brown assets, undoing the intent of the mandate. Graphically, the effectiveness of the mandate can be assessed by the vertical distance between the intersection of the strong and weak mandate lines with the demand for capital function (blue line). As the supply function gets flatter (more inelastic), the vertical distance between the weak and strong mandate crossings shrinks.

When $\alpha = 1$, the demand for capital is perfectly elastic (red line), implying that K_G increases one-for-one with the constrained agent's investment, regardless of the unconstrained capital supply elasticity. Graphically, the vertical distance between the intersection of the strong and weak mandate lines with the demand for capital function is independent of their slope (elasticity of supply) and is always bigger than the $\alpha < 1$ case.

To showcase the properties of the equilibrium with portfolio mandates, we consider a model economy in which $x = 50\%$ of the investors face a mandate to invest $w_{G,c} = \overline{w}_G = 75\%$ of their savings in sector G and $w_{B,c} = \overline{w}_B = 25\%$ in sector B. The quantities of interest obtained from the numerical solution are illustrated in Figures 2 and 3. We consider two identical and independent technologies G and B where the productivity shocks $\tilde{A}_j \sim \mathcal{N}(\mu_j, \sigma_{A_j})$, with $\mu_{A_G} = \mu_{A_B} = 1.05$ and $\sigma_{A_G} = \sigma_{A_B} = 0.2$. A fraction $x = 0.5$ of investors is subject to a portfolio mandate. Investors have CARA preferences with absolute risk aversion γ and are identically endowed with wealth $e_{0,u} = e_{0,c} = 1$. We set the time-preference parameter to $\beta = 0.99$, implying a per-period risk-free rate of $1/\beta = 1.01\%$, in a deterministic, representative agent economy. The left panel in Figures 2 and 3 refer to the case of low risk aversion, $\gamma = 2$, while the right panel refer to the case of high risk aversion, $\gamma = 10$.

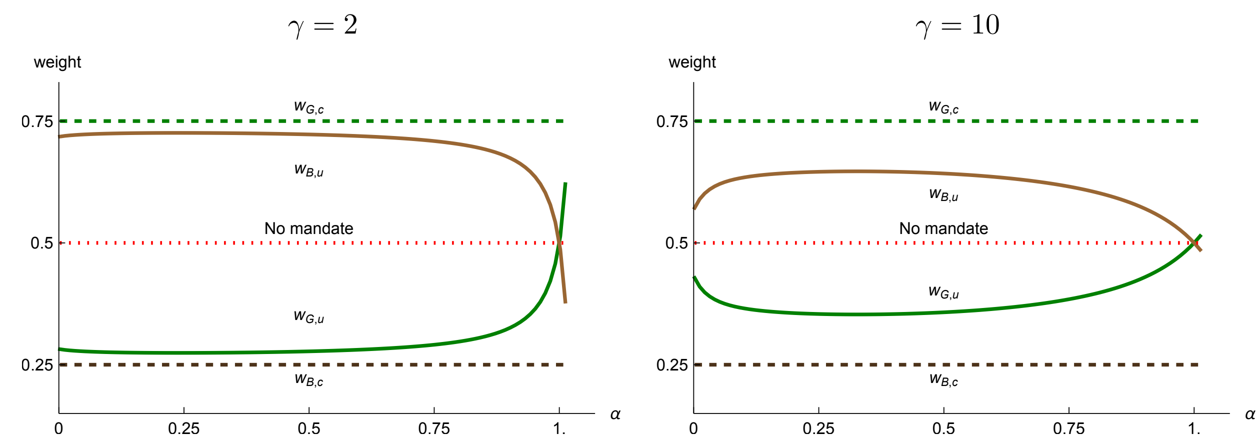
Panel A of Figure 2 shows the portfolio weights of investors who are constrained by the portfolio mandate and who are unconstrained. The dotted red line represents the portfolio weights of all investors if there were no mandates: in this case, each investor would have 50% invested in the green asset and 50% in the brown asset. If a fraction of investors are constrained by the portfolio mandate, their portfolio weights are displayed by the flat dashed lines—$w_{G,c} = \overline{w}_G = 75\%$ for the green asset and $w_{B,c} = \overline{w}_G = 25\%$ for the brown asset. The plot also shows the portfolio weights of the fraction of unconstrained investors. The solid green and brown lines represent $w_{G,u}$ and $w_{B,u}$, respectively. Panel B shows the presence of a mandate increases the expected return of the brown asset relative to that of the green. Because the brown asset becomes more attractive, the portfolio of the unconstrained investor overweights the brown asset and underweight the green. In equilibrium the portfolio chosen by the unconstrained agent undoes part of the effect of the mandate. Comparing the left and right plots of Panel A we see that when risk aversion is low (left panel) the unconstrained agent are more willing to hold undiversified portfolios than when risk aversion is high (left panel). Hence, all else being equal, when risk aversion is low, the choice of unconstrained tend to "undo" the intended effect of the mandate. This effect, however, crucially depend on the level of the return-to-scale parameter α. When α is small, the portfolio weights chosen by unconstrained investors undo most of the benefits of the portfolio mandate; that is, the substantial *decrease* in $w_{G,u}$ undoes the effect of the mandated *increase* in $\overline{w}_{G,c}$, relative to the no-mandated 0.5. However, if $\alpha \approx 1$, the unconstrained agent holds the same portfolio as in the case of no mandate, regardless of the level of risk aversion. In this case, the mandate is most effective.

Panel B of Figure 2 shows that the mandate, by creating excess demand for G capital, increases its price and lowers its required return (cost of capital) relative to the B sector. This effect is amplified by high risk aversion as can be seen by comparing the left to the right plot. However, the panel also shows that, in equilibrium, the spread $\mathbb{E}[R_B] - \mathbb{E}[R_G]$ decreases with α. In fact, for the case where the returns-to-scale parameter is $\alpha = 1$, the difference in the cost of capital

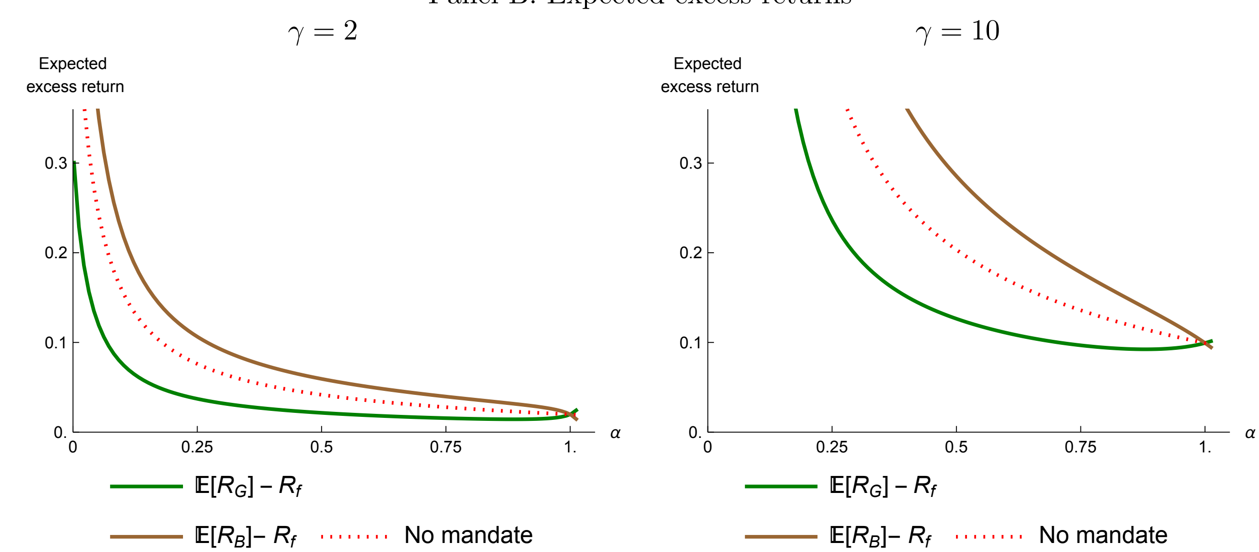
Figure 2: Equilibrium portfolio weights and expected returns

Panel A shows the equilibrium portfolio weights of the unconstrained and constrained investors that are allocated to the G and B sectors. The dashed red line represents these weights in the absence of a portfolio mandate. Panel B shows the equilibrium expected return in excess of the risk-free rate. In the left panels, agents' risk aversion is $\gamma = 2$ and in the right panels $\gamma = 10$. The other parameter values are: $e_{0,u} = e_{0,c} = 1$, $\mu_{A_G} = \mu_{A_B} = 1.05$, $\sigma_{A_G} = \sigma_{A_B} = 0.2$, $x = 0.5$, $\overline{w}_G = 0.75$, $\overline{w}_B = 0.25$. The parameter values used are: $e_{0,u} = e_{0,c} = 1$, $\mu_{A_G} = \mu_{A_B} = 1.05$, $\sigma_{A_G} = \sigma_{A_B} = 0.2$, $\gamma = 2$, $x = 0.5$, $\overline{w}_G = 0.75$, $\overline{w}_B = 0.25$.

Panel A: Portfolio weights



Panel B: Expected excess returns

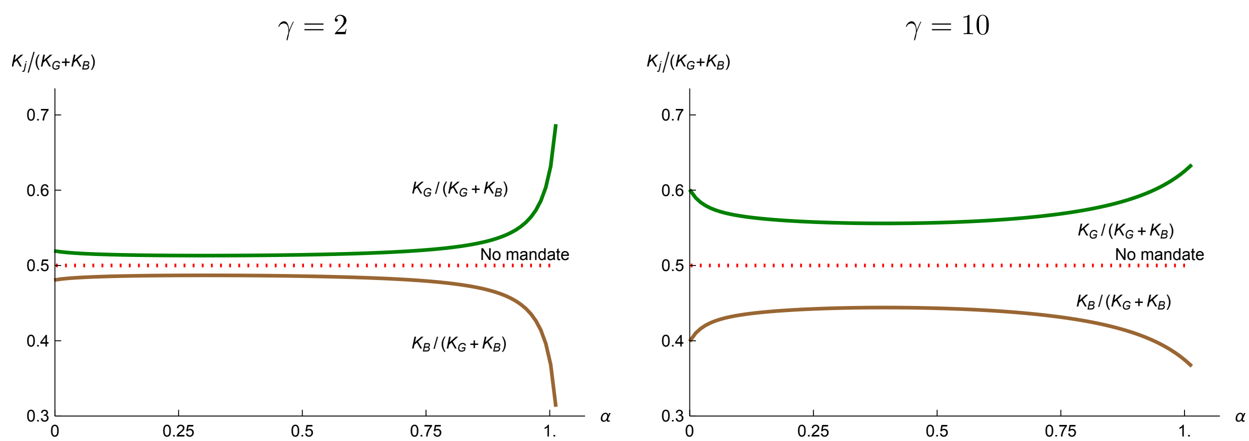


between the two sectors is zero (Panel C), while the difference in the capital allocation is extremely large (Panel A).

Figure 3: Equilibrium capital allocation and mandate pass-through

Panel A shows the equilibrium capital allocation across the G sector (green line) and B sector (brown line) as a function of the returns-to-scale parameter, α. The dashed red line is the capital allocation without a portfolio mandate. Panel B shows the equilibrium mandate pass-through, defined in equation (22). In the left panels, agents' risk aversion is $\gamma = 2$ and in the right panels $\gamma = 10$. The other parameter values are: $e_{0,u} = e_{0,c} = 1$, $\mu_{A_G} = \mu_{A_B} = 1.05$, $\sigma_{A_G} = \sigma_{A_B} = 0.2$, $x = 0.5$, $\overline{w}_G = 0.75$, $\overline{w}_B = 0.25$.

Panel A: Capital allocation



Panel B: Mandate Pass-through

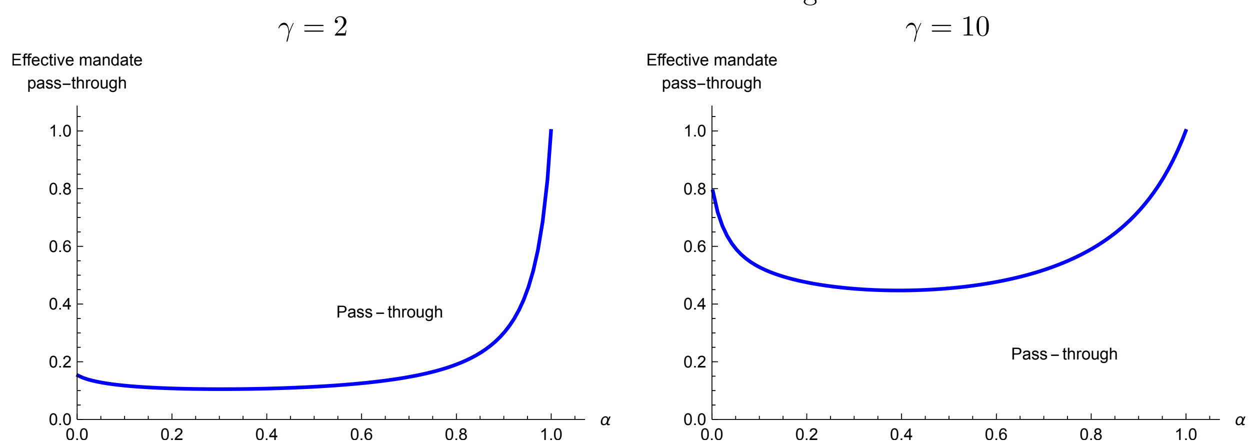


Figure 3 shows the equilibrium sectoral capital allocation, Panel A, and the effective mandate pass-through, Panel B, derived in equation (22). The equilibrium capital allocation of physical capital varies with the returns-to-scale parameter, α. In particular, the mandate increases the allocation of capital in the G sector relative to the benchmark no-mandate case in which the allocation is 50 percent. The equilibrium green capital allocation is closer to the no-mandate case for low values of risk aversion (left plot) and for low values of α. This is a direct implication of the portfolio decisions of unconstrained agents discussed in Figure 2. As α increases, firms' demand of capital is infinitely elastic, the cost of capital of G and B firms is the same and mandates are effective in increasing capital allocation to the G sector.

The effectiveness of mandates is summarized in the passthrough measure shown in Panel B of Figure 3. The pass-through is defined as the capital allocated to the G sector in equilibrium as a fraction of the maximum allocation that would result if we ignored the equilibrium effects on asset prices, as defined in equation (22). Consistent with the patterns of portfolio weights and capital allocation, Panel B shows that the mandate effectiveness is small for low values of the returns-to-scale parameter α but can be substantial as α approaches 1, reaching a value of 100 percent when $\alpha = 1$. Risk aversion significantly impacts mandate effectiveness. All else being equal, a higher risk aversion (right panels) increases the pass-through because unconstrained investors are less willing to hold undiversified portfolios, thus increasing the effectiveness of mandates. However, risk aversion and, more generally, the elasticity of capital supply are irrelevant for the mandate's effectiveness when $\alpha = 1$.

In summary, Figures 2 and 3 show that to fully understand the effectiveness of portfolio mandates, it is essential to consider production models. Models without production, such as the endowment models of Heinkel et al. (2001) and Berk and van Binsbergen (2021), where output is exogenous, can lead to the inference that a low cost-of-capital spread also implies a negligible effect on the allocation of real capital across the B and G sectors, which is not true in general. As the case of constant returns to scale shows, the difference in returns can be zero, yet the mandate's real effect can be substantial. Thus, studying the difference in cost of capital for firms in the B and G sectors is generally not the best way to evaluate whether portfolio mandates are effective; instead, one should directly measure the physical capital in each sector.

So far, we have focused our analysis on the cost of capital of *firms*. But, one could also discuss the implications of portfolio mandates for *investors*. Obviously, the expected return of unconstrained investors' portfolio is higher than that of constrained agent. The reason for this is that when some investors are constrained by the portfolio mandate to invest in the green asset, demand for the brown asset relative to that for the green asset decreases, so the relative price of the brown asset decreases, leading to an increase in its expected return, and unconstrained investors take advantage of this by tilting their portfolio toward brown assets. However, as $\alpha \approx 1$, the difference in expected returns small compared to the case without a mandate. In this case, although mandates do affect equilibrium capital allocation, their effect does not have any impact in the moments of asset returns or the portfolio weights of unconstrained investors.

The results of this section, obtained from an analytically tractable model, are meant to illustrate the *qualitative* impact of portfolio mandates in a general-equilibrium production economy. To assess these claims *quantitatively*, we now turn to a state-of-the-art dynamic general-equilibrium production economy model.

3 A multiperiod equilibrium model with portfolio mandates

In this section, we embed portfolio mandates in a canonical neoclassical general equilibrium model with production that is then calibrated to match empirical macroeconomic and asset-pricing moments. Our model, when returns to scale are constant ($\alpha = 1$) and there are no portfolio constraints, is a canonical real-business-cycle model, similar to King et al. (1988) and Jermann (1998), among many others.[10] We use this model to assess quantitatively the impact of portfolio mandates in equilibrium.

In the baseline version of the model, we assume that the technologies for the firms in the green and brown sectors are identical. In the absence of mandates, the equilibrium in this economy implies that each investor allocates an equal fraction of its risky portfolios to the two sectors. As a result, in equilibrium, capital is equally distributed between the green and brown sectors. Portfolio mandates distort this allocation directly, through the portfolio constraint, and indirectly through the equilibrium effect on prices. Solving for the equilibrium in this economy allows us to assess the magnitudes of these distortions quantitatively. In particular, the analysis in this section highlights that the qualitative effects identified in the simple model of Section 2 are also quantitatively substantial. In particular, portfolio mandates can significantly impact the allocation of real capital even when the difference in the cost of capital in the two sectors is negligible.

3.1 The multiperiod model with frictions

Below, we describe how we model investors, firms, and labor and conclude by specifying the conditions for equilibrium.

[10] For example, our model is identical to King et al. (1988) if we shut down capital adjustment costs and set the utility of leisure to zero, and is similar to Jermann (1998) with the only difference being the adjustment cost specification—we use quadratic adjustment costs as in Hayashi (1982).

3.1.1 Investors

We consider an infinite-horizon economy in discrete time $t = \{0, 1, \ldots\}$. Just as in the previous section, the economy is populated by a continuum of measure-one investors who are infinitely lived and supply labor and invest in firms with one of two production technologies: G and B. A fraction x of the investors is constrained (c) in that it is subject to a portfolio mandate to hold the risky assets in a given fixed proportion. The remaining fraction $1 - x$ of investors is unconstrained (u).

Let $W_{i,t}$, $C_{i,t}$, and $L_{i,t}$ represent, respectively, the net worth, consumption, and labor supply of investor $i = \{u, c\}$. Investors are endowed with one unit of labor that they supply inelastically, that is, $L_{i,t} = 1$ for all i and t for the wage ω_t. Let $B_{u,t+1}$ denote the face value at time $t+1$ of the one-period risk-free bond held by the unconstrained investors and by $R_{f,t}$ the risk-free rate; hence, $B_{u,t+1}/R_{f,t}$ represents the time t value of the holdings of the risk-free bond. We denote by $w_{G,i,t}$ and $w_{B,i,t}$ the share of the investible wealth of investor i that is invested in the G and B sectors, respectively. The cum-dividend time-t values of the green and brown firms are, respectively, $V_{G,t}$ and $V_{B,t}$, with dividends $D_{G,t}$ and $D_{B,t}$.

We assume investors have Epstein-Zin recursive preferences with risk aversion γ, elasticity of intertemporal substitution ψ and time-discount parameter β. The unconstrained investor solves

$$U_u(W_{u,t}) = \max_{\{C_{u,t}, w_{G,u,t}, w_{B,u,t}\}} \left\{ (1-\beta) C_{u,t}^{1-1/\psi} + \beta \left(\mathbb{E}_t[U_u(W_{u,t+1})^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \tag{23}$$

subject to the intertemporal budget constraint

$$W_{u,t+1} = (W_{u,t} + \omega_t L_{u,t} - C_{u,t}) \left(R_{f,t} + w_{G,u,t}(R_{G,t+1} - R_{f,t}) + w_{B,u,t}(R_{B,t+1} - R_{f,t}) \right) + \Upsilon_{u,t+1}, \tag{24}$$

where the return $R_{j,t+1} = V_{j,t+1}/V_{j,t}$, $j = \{G, B\}$, with $V_{j,t}$ denoting firm j's value, defined later in equation (26). The term $\Upsilon_{u,t+1}$ in equation (24), represents a small net lump-sum transfers received by unconstrained investors. Allowing for such transfers helps with the stability of the numerical solution. Without transfers, the constrained investors' wealth share can drift toward zero or one for long periods.[11] In aggregate, the lump-sum transfer is zero-sum, that is, $x\Upsilon_{c,t} + (1-x)\Upsilon_{u,t} = 0$.

[11] We assume that the net transfer to unconstrained investors is a small fraction ξ of the difference in the wealth of the constrained and unconstrained investors. Hence the transfer $\Upsilon_{u,t+1}$ to an unconstrained investor is positive if the constrained investor's wealth is larger than that of the unconstrained and negative otherwise. Since the wealth of each type of investor is close to 50% in equilibrium, the size of the net transfer is very small.

The optimality conditions for the problem (23)–(24) results in three standard Euler equations, one for each of the three financial assets, that is, the bond and the stocks for G and B firms.

The constrained investors' problem is identical to that of the unconstrained investor, with the only difference being that constrained investors cannot choose their equity shares; instead, they face a mandate to invest in the G and B sectors in given proportions, $\overline{w}_{j,c} \in (0,1)$, $j = \{G, B\}$. As a result, the optimality conditions for constrained investors consist of a single Euler equation, characterizing the optimal consumption decision.

3.1.2 Firms

There are two types of firms, G and B, which make optimal hiring and investment decisions to maximize shareholders' value. As in a standard neoclassical model, we assume that firms incur convex capital-adjustment costs when making investment decisions (e.g., Hayashi, 1982). We assume that firms are all-equity financed, with investors being the shareholders. Investors' consumption and portfolio decisions result in a flow of capital $K_{j,t}$, $j = \{G, B\}$ into the two sectors of the economy. Firms operate in a perfectly competitive market and produce identical goods but are subject to different productivity shocks.

Firms produce output $Y_{j,t}$ according to a Cobb-Douglas production function

$$Y_{j,t} = (K_{j,t})^{\alpha\theta}(A_{j,t}L_{j,t})^{(1-\theta)}, \tag{25}$$

where $\theta \in [0,1]$ controls the relative importance of capital in the production and $\alpha \in [0,1]$ is a returns-to-scale parameter. The production function exhibits constant returns to scale if $\alpha = 1$ and declining returns to scale if $\alpha < 1$. The quantity $A_{j,t}$ in equation (25) denotes a stochastic process representing neutral (TFP) productivity shocks. This shock may contain aggregate or firm-specific components; the aggregate component may have stationary and non-stationary components.

Firms choose labor $L_{j,t}$ and investment $I_{j,}$ to maximize shareholder value. Formally, firm j's value $V_{j,t}$ results from the solution of the following problem

$$V_{j,t}(K_{j,t}) = \max_{L_{j,t}, I_{j,t}} D_{j,t}(K_{j,t}) + \mathbb{E}_t\left[\mathbb{M}_{u,t+1}V_{j,t+1}(K_{j,t+1})\right], \tag{26}$$

where $\mathbb{M}_{u,t+1}$ is the stochastic discount factor (SDF) of the unconstrained investors, the marginal investors in this economy. When maximizing shareholder value, firms take $\mathbb{M}_{u,t+1}$ as given. The optimization in (26) is subject to the capital accumulation equation, which, using $\delta > 0$ to denote capital depreciation, is

$$K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t}. \tag{27}$$

As is well known, the firm value $V_{j,t}(K_{j,t})$ can be written as $V_{j,t}(K_{j,t}) = K_{j,t} \times Q_{j,t}$, with Qj,t denoting Tobin's Q, or the market-to-book ratio. In the absence of adjustment costs, Tobin's Q is equal to 1, in which case, $V_{j,t}(K_{j,t}) = K_{j,t}$ because capital can be instantaneously transferred to and from consumption. In the presence of adjustment costs, there is a wedge between the price of installed capital (firm value) and uninstalled capital (consumption), and therefore, Tobin's Q will, in general, be different from one.

3.1.3 Labor

In equation (26), $D_{j,t}(K_{j,t})$ represents the dividends firm j distributes to its shareholders. To define this quantity, we need to describe how wages are set in the model. If labor markets were perfectly flexible, the aggregate wage would be far too volatile, having the same properties as output; this would also counterfactually imply that profits and dividends are counter-cyclical and that equity volatility is too low. As shown by Favilukis and Lin (2016), introducing wage rigidity into a production-economy model makes wages, profits, and dividends behave more like in the data and improves the model's asset-pricing performance. Because asset prices are crucial for our mechanism, we introduce wage rigidity in a reduced-form manner.

Specifically, we assume that firms must hire at least labor $\overline{L} < L_{j,t}$ at wage $\overline{\omega}_t$, but are free to choose how much remaining labor, $L_{j,t} - \overline{L}$, to hire, and that labor is paid a competitive wage $\tilde{\omega}_t$ that clears labor markets. Because labor supply is inelastic and set to $L_{j,t} = 1$, the average wage paid is therefore $\omega_t L_{j,t} = \overline{\omega}_t \overline{L} + \tilde{\omega}_t (L_{j,t} - \overline{L})$, which, in equilibrium, is smoother than $\tilde{\omega}_t$. Note that the firm's first-order condition is independent of $\overline{L}$; therefore, this reduced-form way of modeling wage rigidity does not affect the firm's investment choice. However, it does affect dividends, wage paid, firm value, and equity return. Firm j's dividends are therefore given by

$$D_{j,t}(K_{j,t}) = Y_{j,t} - \omega_t L_{j,t} - I_{j,t} - \eta \left(\frac{I_{j,t}}{K_{j,t}} - \hat{\delta} \right)^2 K_{j,t}, \quad \eta > 0,\ \hat{\delta} > 0, \tag{28}$$

where $Y_{j,t}$ is output, defined in equation (25), $\hat{\delta} = \delta + g$ is capital depreciation δ gross of the growth rate g, and the term $\eta \left(\frac{I_{j,t}}{K_{j,t}} - \hat{\delta} \right)^2 K_{j,t}$ represents a quadratic adjustment-cost function.[12]

3.1.4 Equilibrium

An equilibrium of this economy consists of the following: (i) investors' consumption and portfolio policies, $\{C_{i,t}, w_{G,i,t}, w_{B,i,t}\}$; (ii) firms' investment and hiring policies, $\{I_{j,t}, L_{j,t}\}$; (iii) wages $\tilde{\omega}_t$; (iv) prices of the two risky assets, $\{V_{G,t}, V_{B,t}\}$, and the risk-free rate, $R_{f,t}$, such that: investors maximize their lifetime utility in equation (23), firms maximize shareholder value in equation (26), and the markets for labor, the two risky assets, and the risk-free asset clear. By Walras' law, the goods market automatically clears; that is, the aggregate budget constraint holds.

3.2 Calibration

We solve numerically for an equilibrium in the economy described above using dynamic programming. We calibrate the model's parameters at an annual frequency to match key macroeconomic and asset-pricing moments. Table 1 shows the parameter values used in our baseline calibration. In our benchmark case, we consider a coefficient of relative risk aversion $\gamma = 5$. However, we also solve the model with higher risk aversion, up to $\gamma = 50$, to explore the model's implications with a more realistic value of the equity risk premium. We set the elasticity of intertemporal substitution (EIS) to $\psi = 0.2$ so that for the benchmark case of $\gamma = 5$, the investors' preferences are time-separable CRRA. We set $\beta = 0.9422$ to target a ratio of capital to output K/Y of around 2.9 in the steady state and an aggregate growth rate of $g = 1.5\%$. We assume that 50% of investors are subject to a portfolio mandate ($x = 0.5$) requiring them to hold their wealth in the ratio 75% to 25% between the G and B sectors. We set the wealth transfer parameter at the end of each period to $\xi = 0.01$.

We choose parameters for the Markov chain describing the TFP process to match the volatility and autocorrelation of Hodrick-Prescott (H-P) filtered output.[13] Specifically, we assume that the firm's productivity is separated into aggregate and industry-level components: $A_t^j = A_t Z_t^j$. The aggregate component $A_t = (1 + g)^t$ captures the growth trend. The industry component

[12] Because we set gross depreciation to $\hat{\delta} = \delta + g$, adjustment costs are zero in the steady state.

[13] We use a filtering parameter of 100, as proposed by Backus and Kehoe (1992).

Table 1: Parameter values

The table reports the values for the parameters used in the benchmark calibration of the multiperiod model described in this section.

Parameter	Symbol	Benchmark value
Investors		
Relative risk aversion	γ	5.0
Elasticity of intertemporal substitution	ψ	0.2
Time discount rate	β	0.9422
Fraction of constrained investors	x	0.50
Portfolio mandate in G capital	$\overline{w}_G$	0.75
Portfolio mandate in B capital	$\overline{w}_B$	0.25
Faction of labor receiving government fixed	$\overline{L}$	0.5
Investors' wealth transfer fraction	ξ	0.01
Firms		
Aggregate growth rate	g	0.015
Low TFP shock realization	L	0.912
High TFP shock realization	H	1.088
Probability of remaining in current state	p	0.82
Depreciation rate	δ	0.06
Capital adjustment cost	η	5.0
Parameter controlling the capital share	θ	0.35
Return to scale	α	1.0

$Z_t^j = 1 + z_t^j$ drives the business cycle and follows a 2-state Markov chain, with $L = 0.912$ and $H = 1.088$, and with probability $p = 0.82$ of staying in the current state.

We set the capital adjustment cost $\eta = 5$ to match investment volatility. We set the fraction of labor receiving a fixed wage $\overline{L} = 0.50$ so that the volatility of wages is about half that of output, which also implies reasonable values for the volatility and procyclicality of dividends and profits. We set depreciation $\delta = 0.06$, a standard value in the literature. We set capital share $\theta = 0.35$ so that 65% of output is paid to labor. In our baseline model, we set returns to scale to be constant, that is, $\alpha = 1.0$. We also solve models with decreasing returns to scale: $\alpha = 0.90$ and $\alpha = 0.80$. For these models, for the model to match the target moments (specifically, labor share and the capital-to-output ratio) β rises to 0.952 and 0.963, respectively, and $\alpha\theta$ falls to 0.32 and 0.28. Finally, to explore the implications of increasing returns to scale, we also solve the model for $\alpha = 1.02$.

In our calibration, we allow for the existence of a government sector which enables us to distinguish between total and private-sector GDP. It is well known that the latter is much more

Table 2: Macroeconomics moments

The table shows macroeconomics moments from the model and compares them to corresponding quantities in the data. All variables, other than the Share of GDP, are H-P filtered. Volatility is in annual percentage units. GDP-P refers to private sector GDP. The values in the "Model" columns are obtained by solving a version of the model with a portfolio mandate that constrains 50% of investors to invest 75% of their wealth in firms in the G sector and 25% in firms in the B sector. The model is calibrated at an annual frequency. Parameter values are reported in Table 1.

	Share of GDP		Volatility (%)		Corr with GDP		Autocorr	
	Data	Model	Data	Model	Data	Model	Data	Model
GDP	1.00	1.00	2.33	2.32	1.00	1.00	0.54	0.35
GDP-P	0.80	0.81	2.74	2.85	0.91	1.00	0.48	0.35
Consumption	0.63	0.64	1.72	1.60	0.91	0.99	0.53	0.34
Investment	0.17	0.18	7.60	7.42	0.78	0.99	0.45	0.34
Wages	—	—	1.17	1.42	0.49	1.00	0.58	0.35

volatile than the former. To model the government in a simple way, we assume that the actual amount of labor supplied by investors is 1.35 instead of 1.0, as described in the model section above, with 1.0 working in the private sector and 0.35 in government. Unlike private-sector employees, government employees are paid a constant wage adjusted for growth. That is, the government wage rate is set to $\overline{\omega}(1+g)^t$ where $\overline{\omega}$ is the unconditional average of the detrended market-clearing wage. Hence, total government expenses are equal to $0.35 \times \overline{\omega}$ and total labor income is then $(\omega_t \times 1) + (\overline{\omega} \times 0.35)$. We assume that government expenditure equals a lump-sum tax levied on total labor income. With this assumption, the problem's solution is independent of government size. The only quantity affected by government expenditure is total GDP, which is equal to the sum of private-sector GDP and government expenditure. The choice of 1.35 for total labor implies that private sector GDP is 80% of total GDP, as in the data.[14]

Table 2 compares macroeconomic moments in the data to corresponding quantities in the multiperiod model, under the assumption that 50% of investors face a mandate to invest 75% of their wealth in firms in the G sector and 25% in the B sector. The values reported in the table are obtained by simulating the model for 10,000 years and using a 100-year burn-in period. The table reports five quantities: total GDP, private-sector GDP (GDP-P), Consumption, Investment, and Wages. For each quantity, we compute the share of GDP, the volatility, the correlation with GDP, and the autocorrelation and compare them to the corresponding values in the data. The table shows

[14] Note that labor is approximately 65% of output, so if private labor is 1.0, then private output is 1.0/0.65=1.54. Government labor, which equals government output, is 35%. Therefore private output as a share of total output is 1.54/(1.54+0.35)=81%.

Table 3: Asset-pricing moments

The table shows the annual mean and volatility of the risk-free rate, $\mathbb{E}[R_f]$ and $\sigma(R_f)$, and of the market risk premium, $\mathbb{E}[R^M - R_f]$ and $\sigma(R^M - R_f)$ obtained from a model with a portfolio mandate that constrains 50% of investors to invest 75% of their wealth in firms in the G sector and 25% in firms in the B sector. The model is calibrated at an annual frequency. The equity return is levered using a leverage ratio of 2. Parameter values are reported in Table 1. Values in the Data column are based on the sample period 1950–2021 and are from Ken French's website.

		Model				
	Data	$\gamma = 5$	$\gamma = 10$	$\gamma = 25$	$\gamma = 40$	$\gamma = 50$
$\mathbb{E}[R_f]$	0.91	5.78	5.62	5.06	4.53	4.17
$\sigma(R_f)$	2.27	3.27	3.24	3.21	3.23	3.29
$\mathbb{E}[R^M - R_f]$	8.99	1.42	1.76	2.84	3.92	4.36
$\sigma(R^M - R_f)$	17.89	16.32	16.28	16.16	16.00	16.28

that the model matches key macroeconomic moments reasonably well under the baseline parameters of Table 1. These moments remain largely unaffected by different values of risk aversion; therefore, in the table, we report results only for the benchmark case of $\gamma = 5$. The only moments significantly different from the data are the correlations of investment and wages with GDP, which, in the data, are much less than in the model. This is not surprising because, with only one aggregate shock, model correlation with GDP tends to be close to 1.

Table 3 reports the asset-pricing moments: the annual mean and volatility of the risk-free rate and of the equity-market risk premium. The equity return used to compute the market risk premium is levered using a factor of two, equivalent to an economy-wide 50/50 debt/equity ratio. The table shows that the model does a good job of matching the volatility of the risk-free rate and of the equity risk premium in the data. However, not surprisingly, for the case of low risk aversion, $\gamma = 5$, the risk-free rate is too high, and the equity risk premium is too low. This is just a manifestation of the equity-premium puzzle. Higher values of risk aversion in the table result in values of the equity premium and risk-free rate that are closer to the data.[15]

3.3 Equilibrium effects of portfolio mandates

Below, we describe the equilibrium effects of portfolio mandates in the multiperiod model of a production economy. Table 4, which is the counterpart to Figures 2 and 3 for the simple model of Section 2, contains our main quantitative results about the equilibrium effects of portfolio mandates

[15] Note that as we change risk aversion, EIS stays constant, which explains why the macroeconomic moments don't change much.

in the multiperiod model with frictions. The fundamental goal of the analysis is to contrast the capital allocation (Panels A and B) and the cost of capital (Panels C and D) for different values of the returns-to-scale parameter, α.

Table 4 considers values of the returns to scale parameter ranging from $\alpha = 0.80$ (decreasing returns to scale) to $\alpha = 1.02$ (increasing returns to scale). Empirical estimates from the macroeconomic literature indicate that returns to scale are nearly constant in the US economy. A series of influential papers Hall (1988, 1990) argues that market power and increasing return to scale can explain procyclical productivity in the US. In subsequent work, Basu and Fernald (1997) estimate constant or slightly decreasing returns to scale in the US economy. They note, however, that estimates of returns to scale vary at different levels of industry aggregation. While a typical industry exhibits decreasing returns, the total manufacturing and private economy show increasing returns. They suggest that there may be economies of scale at the aggregate level in that as the scale of production increases, the average cost of production decreases. Ahmad et al. (2019) present new estimates of returns to scale for the US economy based on two separate industry datasets and compare them to previous estimates in the literature. They find evidence of constant or slightly decreasing returns to scale at the aggregate level in the US economy over 1989–2014, consistent with a relatively small aggregate markup in the post-1990 period. While their evidence points to constant or declining returns to scale, Ahmad et al. (2019) do not rule out the possibility of increasing returns to scale in specific industries or regions or the presence of certain factors, such as technological progress or network effects. Increasing returns to scale might be particularly relevant for "green technologies" where learning by doing and increased scale have led to a dramatic decline in costs over the past 25 years; for example, Way et al. (2022) argue that, unlike traditional technologies such as oil and gas, clean-energy technologies are on learning curves, where costs drop as a power law of cumulative production. This might be particularly relevant for "green technologies" in a model with different types of capital. In light of this evidence, in our analysis in Table 4, we allow for both decreasing, constant, and (slightly) increasing returns to scale.

Panel A of Table 4 reports the equilibrium fraction of capital flowing to firms in the G sector, $\frac{K^G}{K^G+K^B}$. Because we assume that the technologies of firms in the two sectors are identical, the optimal unconstrained investor's portfolio is equally weighted between the G and B sectors. Hence, all entries in Panel A would equal 0.50 in a world without mandates. The values in the table refer, however, to the case in which constrained investors are mandated to hold 75% of wealth in firms in the G sector and 25% in the B sector, implying that the *maximum* proportion of capital allocated to the G sector as a result of the mandate is $(50\% + 75\%)/2 = 62.5\%$. Panel A shows that the *equilibrium* allocation of capital to the G sector varies between 50% to 55.5% depending on the returns-to-scale parameter α and risk aversion γ. The deviation from the unconstrained 50/50 allocation is particularly strong for the case of constant returns to scale ($\alpha = 1$) and high risk aversion. For example, the equilibrium capital allocation to the G sector is 55.5% for $\alpha = 1$ and $\gamma = 50$. Although levels of risk aversion of $\gamma = 25$ or 50 are clearly unreasonable, they are considered here as a reduced-form way of capturing high risk premia in the economy arising from, e.g., limited participation, taxes, and intermediary frictions.[16]

To evaluate the magnitude of the equilibrium effect of mandates on capital allocation, we compute the "effective mandate pass-through," defined in (22), which we report in Panel B of Table 4. To construct the pass-through, we first compute the maximum effectiveness of a mandate, ignoring any equilibrium consideration. In our setting, because the constrained investor represents 50% of the entire mass of investors, a portfolio mandate of 75% in G and 25% in B implies that 62.5% ($= 0.5 \times 75\% + 0.5 \times 50\%$) of the entire capital should be allocated to the G sector. Under this "partial equilibrium" intuition, the *maximum* deviation from the unconstrained 50/50 allocation is, therefore, $12.5\% = 62.5\% - 50\%$. Using the equilibrium allocation to G in Panel A, denoted by $K_G\big|_{\alpha}^{x=50\%}$, we can then construct the effective mandate pass-through ratio of the mandate as the equilibrium pass-through expressed as a percentage of the maximum pass-through:

$$\text{Effective mandate pass-through ratio} = \frac{K_G\big|_{\alpha}^{x=50\%} - 0.50}{0.125}. \tag{29}$$

The values of the effective mandate pass-through ratio in Panel B of Table 4 show that, although general-equilibrium effects undo part of the mandate, a significant part remains effective. For example, with a risk aversion of 5 and constant returns to scale ($\alpha = 1$), about 21.60% of the mandate remains effective. Intuitively, by increasing the cost of capital of firms in the B sector, the mandate makes them more attractive to unconstrained investors who trade off higher returns for worse diversification. As risk aversion increases and risk premia increase, the equilibrium allocation further deviates from the unconstrained 50%. For relative risk aversion of $\gamma = 50$ and constant returns to scale, the pass-through is 44%. Thus, the results in Panel B show that the effect of

[16] For example, in standard habit models, (e.g., Campbell and Cochrane, 1999), while the curvature parameter in the utility function is 2, the average effective risk aversion is around 80.

Table 4: Equilibrium effects of portfolio mandates

The table shows the equilibrium effect of portfolio mandates on capital allocation and cost of capital for different values of the risk aversion and returns-to-scale parameters. These values are obtained from a model with a portfolio mandate that constrains 50% of investors to invest 75% of their wealth in firms in the G sector and 25% in the B sector. Panel A reports the share of capital allocated to the green sector as a fraction of total capital, $K_G/(K_G+K_B)$; Panel B computes the effective mandate pass-through ratio defined in equation (29); that is, the fraction of the intended pass-through effect that survives general equilibrium effects; Panel C reports the difference in the equilibrium cost of capital between G and B sectors; and Panel D reports the difference in Sharpe ratios between firms in the G and B sectors. Parameter values are reported in Table 1.

Return to scale	Risk aversion $\gamma = 5$	$\gamma = 10$	$\gamma = 25$	$\gamma = 40$	$\gamma = 50$
	Panel A: Capital allocation, $K_G/(K_G + K_B)$				
$\alpha = 1.02$	0.549	0.558	0.564	0.564	0.565
$\alpha = 1.00$	0.527	0.539	0.546	0.552	0.555
$\alpha = 0.90$	0.503	0.505	0.511	0.515	0.517
$\alpha = 0.80$	0.501	0.501	0.502	0.503	0.504
	Panel B: Effective mandate pass-through ratio (%)				
$\alpha = 1.02$	39.20	46.56	50.96	51.52	51.60
$\alpha = 1.00$	21.60	31.20	36.80	41.60	44.00
$\alpha = 0.90$	2.40	4.00	8.80	12.00	13.60
$\alpha = 0.80$	0.80	0.80	1.60	2.40	3.20
	Panel C: Difference in cost of capital, $R_B - R_G$ (%)				
$\alpha = 1.02$	-0.02	-0.02	0.00	0.00	0.00
$\alpha = 1.00$	0.04	0.08	0.12	0.14	0.14
$\alpha = 0.90$	0.06	0.10	0.18	0.19	0.17
$\alpha = 0.80$	0.05	0.07	0.10	0.12	0.13
	Panel D: Difference in Sharpe ratio, $SR_B - SR_G$				
$\alpha = 1.02$	-0.0004	0.0012	0.0063	0.0106	0.0120
$\alpha = 1.00$	0.0040	0.0070	0.0115	0.0148	0.0131
$\alpha = 0.90$	0.0033	0.0057	0.0104	0.0124	0.0114
$\alpha = 0.80$	0.0030	0.0037	0.0054	0.0066	0.0070

mandates on the allocation of real capital is particularly strong when risk premia are close to their values in the data.

Unlike the single-period model of Section 2, the mandate pass-through for $\alpha = 1$ is less than 100%. The reason why it is lower here is because of households' desire to hedge labor income risk. If the constrained agent overweights G and the unconstrained does not overweight B in response, overall output in the economy is more correlated with shocks to G than shocks to B. Since labor income is perfectly correlated with output, labor income is also more correlated with G shocks.

Thus, the unconstrained agent can hedge labor income risk by overweighting asset B, thus reducing the effectiveness of the mandate. This mechanism may be less relevant to the real world than to our model because (i) labor income tends to be imperfectly correlated with output and (ii) the marginal investor tends to be different from the average labor income earner. Therefore, our model may be underestimating the mandate pass-through ratio.

Panels C and D report the effect of mandates on the firms' cost of capital and Sharpe ratios, respectively. Unlike the significant impact documented in Panels A and B, the effect on the cost of capital (Panel C) and Sharpe ratios (Panel D) are minimal. For example, in our baseline calibration ($\gamma = 5, \alpha = 1.0$), the cost of capital of firms in the B sector is only four basis points higher than that of firms in the G sector (Panel C), and the Sharpe ratio is only 0.004 units higher. This negligible difference in the cost of capital contrasts with the significant mandate pass-through effect of 21.6% reported in Panel B. The contrast between the mandate's "real" and "financial" effects is even stronger when risk premia are closer to their value in the data ($\gamma = 50$). In this case, the difference in the cost of capital under constant returns-to-scale is 14 basis points, while the mandate pass-through is 44%.

In sum, the results from our quantitative model support the central intuition developed in the simple model of Section 2. Specifically, in an economy with production, the difference in the cost of capital is a poor metric to assess the real impact of portfolio mandates in equilibrium. Mandates can significantly impact capital allocation while having a negligible effect on firms' cost of capital. These findings caution against using the cost of capital to measure the effectiveness of portfolio mandates in equilibrium; instead, one should measure the flow of capital.

4 Conclusion

In this paper, we examine the impact of portfolio mandates on the allocation of physical capital in a general-equilibrium economy with production and heterogenous investors. In contrast to the existing literature that has studied responsible investing in models of an endowment economy, we consider a production economy that nests the endowment economy as a special case.

To assess the quantitative importance of the effect of portfolio mandates, we study a dynamic general equilibrium production economy. Under a realistic calibration of the multiperiod model that matches asset-pricing and macroeconomic moments of the US economy, we find that the effect of portfolio mandates on the allocation of physical capital across sectors can be substantial. In contrast, the impact on the equilibrium cost of capital and Sharpe ratio of firms in the two sectors remains negligible, consistent with existing evidence.

Thus, a key takeaway of our analysis is that judging the effectiveness of portfolio mandates by studying their effect on the cost of capital of affected firms can be misleading: small differences in the cost of capital across sectors can be associated with significant differences in the allocation of physical capital across these sectors.

A Proofs

Proof of Proposition 1

Unconstrained agents. Because productivity shocks are normally distributed, we can explicitly write the expectation in equation (10) as follows

$$\max_{\{c_{0,u},w_{G,u},w_{B,u}\}} -\frac{e^{-\gamma c_{0,u}}}{\gamma} - \beta\frac{e^{-\gamma\mathbb{E}[\tilde{c}_{1,u}]+\frac{\gamma^2}{2}Var[\tilde{c}_{1,u}]}}{\gamma}, \tag{A1}$$

where, from the intertemporal budget constraint in equation (9) and using the fact that productivity shocks $\tilde{A}_j$ are independent,

$$\mathbb{E}[\tilde{c}_{1,u}] = (e_{0,u} - c_{0,u})(R_f + w_{G,u}(\mathbb{E}[\tilde{R}_G] - R_f) + w_{B,u}(\mathbb{E}[\tilde{R}_B] - R_f)) + \mathbb{E}[\tilde{\pi}_{G,u} + \tilde{\pi}_{B,u}] \tag{A2}$$

$$\begin{aligned} Var[\tilde{c}_{1,u}] &= (e_{0,u} - c_{0,u})^2(w_{G,u}^2 Var[\tilde{R}_G] + w_{B,u}^2 Var[\tilde{R}_B]) + Var[\tilde{\pi}_{G,u}] + Var[\tilde{\pi}_{B,u}] \\ &\quad 2(e_{0,u} - c_{0,u})(w_{G,u}Cov(\tilde{R}_G, \tilde{\pi}_{G,u}) + w_{B,u}Cov(\tilde{R}_B, \tilde{\pi}_{B,u}). \end{aligned} \tag{A3}$$

In choosing their optimal portfolio and consumption, agents take the return $\tilde{R}_j$ and the profit $\tilde{\pi}_{j,u}$ as given.

We assume that investor i is entitled to a fraction of the total profit $\tilde{\Pi}_j$, proportional to the amount invested. Specifically, denoting by $k_{j,i} \equiv e_{0,i}(1 - c_{0,i})w_{j,i}$ the amount agent i invest in sector j, we obtain that

$$\tilde{\pi}_{j,i} = \frac{k_{j,i}}{K_j}\tilde{\Pi}_j = (e_{0,i} - c_{0,i})w_{j,i}(1-\alpha)\tilde{A}_j K_j^{\alpha-1}, \tag{A4}$$

where the last equality follows from the definition of firm j profit in equation (5). Note that, although the individual profit $\tilde{\pi}_{j,i}$ depend on the portfolio weights $w_{j,i}$, because investors are atomistic, when choosing their optimal portfolios $w_{j,i}$, they take the profit share $\tilde{\pi}_{j,i}$ and the return on capital $\tilde{R}_j$ as given.

The first-order condition with respect to $w_{j,u}$, $j = G, B$, yields

$$w_{j,u} = \frac{\mathbb{E}[\tilde{R}_j] - R_f}{\gamma(e_{0,u} - c_{0,u})\alpha\sigma_{A_j}^2 K_j^{2(\alpha-1)}}, \quad j = G, B. \tag{A5}$$

The first-order condition with respect to $c_{0,u}$ yields

$$c_{0,u} = \mathbb{E}[\tilde{c}_{1,u}] - \frac{\gamma}{2}Var[\tilde{c}_{1,u}] - \frac{1}{\gamma}\log\beta\Gamma_u, \tag{A6}$$

with

$$\Gamma_u \equiv R_f + w_{G,u}(\mathbb{E}[\tilde{R}_G] - R_f) + w_{B,u}(\mathbb{E}[\tilde{R}_B] - R_f) -$$
$$\gamma\alpha(e_{0,u} - c_{0,u})\left(w_{G,u}^2\sigma_{A_G}^2 K_G^{2(\alpha-1)} + w_{B,u}^2\sigma_{A_B}^2 K_B^{2(\alpha-1)}\right). \tag{A7}$$

Using the portfolio weight from equation (A5) it is immediate to see that $\Gamma_u = R_f$, therefore the optimality condition for consumption is

$$c_{0,u} = \mathbb{E}[\tilde{c}_{1,u}] - \frac{\gamma}{2}Var[\tilde{c}_{1,u}] - \frac{1}{\gamma}\log\beta R_f. \tag{A8}$$

In order to derive an expression for $c_{0,u}$ we need to explicitly compute $\mathbb{E}[\tilde{c}_{1,u}]$ and $Var[\tilde{c}_{1,u}]$, defined in equations (A2) and (A3). Using the definition of returns $\tilde{R}_j$ from equation (4) and the definition of individual profit $\tilde{\pi}_{j,u}$ from equation (A4) we have that, for $j = G, B$,

$$\mathbb{E}[\tilde{R}_j] = \alpha\mu_{A_j}K_j^{\alpha-1} \tag{A9}$$

$$Var[\tilde{R}_j] = \alpha^2\sigma_{A_j}^2 K_j^{2(\alpha-1)} \tag{A10}$$

$$\mathbb{E}[\tilde{\pi}_{j,u}] = (e_{0,u} - c_{0,u})w_{j,u}(1-\alpha)\mu_{A_j}K_j^{\alpha-1} \tag{A11}$$

$$Var[\tilde{\pi}_{j,u}] = (e_{0,u} - c_{0,u})^2 w_{j,u}^2(1-\alpha)^2\sigma_{A_j}^2 K_j^{2(\alpha-1)} \tag{A12}$$

$$Cov[\tilde{R}_j, \tilde{\pi}_{j,u}] = (e_{0,u} - c_{0,u})w_{j,u}\alpha(1-\alpha)K_j^{2(\alpha-1)}. \tag{A13}$$

Substituting these expressions in equation (A8) and simplifying we arrive at the following equation that implicitly characterize optimal consumption $c_{0,u}$

$$c_{0,u} = (e_{0,u} - c_{0,u})\mathbb{E}[\tilde{y}_u] - \frac{\gamma}{2}(e_{0,u} - c_{0,u})^2 Var[\tilde{y}_u] - \frac{1}{\gamma}\log\beta R_f, \tag{A14}$$

where

$$\tilde{y}_u = R_f + w_{G,u}\left(\frac{\tilde{Y}_G}{K_G} - R_f\right) + w_{B,u}\left(\frac{\tilde{Y}_B}{K_B} - R_f\right). \tag{A15}$$

Constrained agents. Constrained agent only solve an intertemporal consumption/saving problem, as their portfolio weights are fixed by the mandate at $w_{G,c} = \overline{w}_G$ and $w_{B,c} = \overline{w}_B$. Specifically, they solve

$$\max_{c_{0,c}} -\frac{e^{-\gamma c_{0,c}}}{\gamma} - \beta\frac{e^{-\gamma\mathbb{E}[\tilde{c}_{1,c}] + \frac{\gamma^2}{2}Var[\tilde{c}_{1,c}]}}{\gamma}, \tag{A16}$$

with

$$\mathbb{E}[\tilde{c}_{1,c}] = (e_{0,c} - c_{0,c})(R_f + \overline{w}_G(\mathbb{E}[\tilde{R}_G] - R_f) + \overline{w}_B(\mathbb{E}[\tilde{R}_B] - R_f)) + \mathbb{E}[\tilde{\pi}_{G,u} + \tilde{\pi}_{B,c}] \tag{A17}$$

$$Var[\tilde{c}_{1,c}] = (e_{0,c} - c_{0,c})^2(\overline{w}_G^2 Var[\tilde{R}_G] + \overline{w}_B^2 Var[\tilde{R}_B]) + Var[\tilde{\pi}_{G,c}] + Var[\tilde{\pi}_{B,c}]$$
$$2(e_{0,c} - c_{0,c})(\overline{w}_G Cov(\tilde{R}_G, \tilde{\pi}_{G,c}) + \overline{w}_B Cov(\tilde{R}_B, \tilde{\pi}_{B,c}). \quad \text{(A18)}$$

The first-order condition with respect to $c_{0,c}$ yields

$$c_{0,c} = \mathbb{E}[\tilde{c}_{1,c}] - \frac{\gamma}{2} Var[\tilde{c}_{1,c}] - \log \beta \Gamma_c, \quad \text{(A19)}$$

with

$$\Gamma_c \equiv R_f + \overline{w}_G(\mathbb{E}[\tilde{R}_G] - R_f) + \overline{w}_B(\mathbb{E}[\tilde{R}_B] - R_f) -$$
$$\gamma\alpha(e_{0,u} - c_{0,u}) \left(\overline{w}_G^2 \sigma_{A_G}^2 K_G^{2(\alpha-1)} + \overline{w}_B^2 \sigma_{A_B}^2 K_B^{2(\alpha-1)} \right) \quad \text{(A20)}$$
$$= \mathbb{E}[\tilde{R}_{p,c}] - \gamma\alpha(e_{0,c} - c_{0,c}) Var[\tilde{y}_c]. \quad \text{(A21)}$$

where

$$\tilde{y}_c \equiv R_f + \overline{w}_G \left(\frac{\tilde{Y}_G}{K_G} - R_f \right) + \overline{w}_B \left(\frac{\tilde{Y}_B}{K_B} - R_f \right), \quad \text{(A22)}$$

$$\tilde{R}_{p,c} \equiv R_f + w_{G,c} \left(\tilde{R}_G - R_f \right) + w_{B,c} \left(\tilde{R}_B - R_f \right). \quad \text{(A23)}$$

Using equations (A9)–(A13) to express $\mathbb{E}[\tilde{c}_{1,c}]$ and $Var[\tilde{c}_{1,c}]$ in equation (A19) and simplifying we arrive at the following equation that implicitly characterize optimal consumption $c_{0,c}$

$$c_{0,c} = (e_{0,c} - c_{0,c})\mathbb{E}[\tilde{y}_c] - \frac{\gamma}{2}(e_{0,c} - c_{0,c})^2 Var[\tilde{y}_c] - \frac{1}{\gamma} \log \beta \Gamma_c \quad \text{(A24)}$$

∎

B Endowment economy as a limit of a production economy

In this appendix we show that an endowment economy can be thought of as the limit of a production economy where the return to scale parameter goes to zero. We prove this result in the context of a representative agent economy with no uncertainty and log investors for which we can obtain an equilibrium in closed form. With this analytic solution we can show that the decentralized equilibrium corresponds to the social planner equilibrium and that the equilibrium for the production economy converges to the equilibrium in an endowment economy as the return to scale parameter goes to zero.

We consider an economy in which agents have log preferences and live for two periods. Agents are endowed with wealth W_0 and have access to a deterministic production technology $Y =$

AK^α. Agent choose the amount of investment K in order to maximize their lifetime consumption. We first consider the social planner problem and then verify that the equilibrium in the planned economy corresponds to that in the decentralized economy.

Social planner problem. The social planner chooses capital allocation K solves the following problem

$$\max_K \log C_0 + \beta \log C_1 \tag{B1}$$

where

$$C_0 = W_0 - K - b/R_f \tag{B2}$$

$$C_1 = AK^\alpha + b, \tag{B3}$$

with b denoting the amount of borrowing/lending at time 0 and R_f the risk-free rate, to be determined as part of the equilibrium. The first-order condition with respect to b yields

$$\frac{1}{W_0 - K - b/R_f}\frac{1}{R_f} = \frac{\beta}{AK^\alpha + b} \tag{B4}$$

In equilibrium there is no borrowing or lending, therefore we set $b = 0$ and obtain

$$R_f = \frac{1}{\beta}\frac{AK^\alpha}{W_0 - K}. \tag{B5}$$

The first-order condition with respect to K yields

$$\frac{1}{W_0 - K - b/R_f} = \frac{\alpha\beta AK^{\alpha-1}}{AK^\alpha + b} \tag{B6}$$

Setting $b = 0$ we obtain

$$K(1 + \alpha\beta) = \alpha\beta W_0, \tag{B7}$$

implying that the equilibrium capital is

$$K^* = \frac{\alpha\beta}{1 + \alpha\beta} W_0. \tag{B8}$$

Substituting K^* in the expression of the risk-free rate (B5), we have that the equilibrium risk-free rate is

$$R_f^* = \frac{1}{\beta}\frac{A}{W_0^{1-\alpha}}\frac{(\alpha\beta)^\alpha}{(1 + \alpha\beta)^{\alpha-1}} \tag{B9}$$

Note that

$$\lim_{\alpha\to 0} R_f^* = \frac{1}{\beta}\frac{A}{W_0} \tag{B10}$$

and, because $\lim_{\alpha \to 0} (\alpha\beta)^\alpha = 1$,

$$\lim_{\alpha \to 0} K^* = 0. \tag{B11}$$

Hence as $\alpha \to 0$ the economy becomes an endowment economy in which $C_0 = W_0$ and $C_1 = A$ is the exogenous consumption process. The ratio A/W_0 in the risk-free rate equation (B10) represents therefore consumption growth.

Decentralized economy. The decentralized economy consists of atomistic firms owned by atomistic households.

Firms. Firms take the discount rate R as given and choose capital K to maximize NPV, that is

$$\max_K \frac{AK^\alpha}{R} - K. \tag{B12}$$

The first order condition yields

$$R = \alpha A K^{\alpha - 1} \tag{B13}$$

and the firm profit is

$$\pi = AK^\alpha - RK = (1-\alpha)AK^\alpha. \tag{B14}$$

Hence the firm earn positive profits if returns to scale are declining, $\alpha < 1$ and zero profit if returns to scale are constant, $\alpha = 1$.

Households. Households take as given the risk-free rate R_f from lending/borrowing, the return R from investing in the firms and the profit π originated by firms. They solve the following problem

$$\max_{C_0, w} \log C_0 + \beta \log C_1 \tag{B15}$$

where

$$C_1 = (W_0 - C_0)(R_f + w(R - R_f)) + \pi. \tag{B16}$$

Note that, because the profit π is non-zero when $\alpha \neq 1$, we need to include it in the consumption of the household, as they receive it in the form of dividend by owning the firm. Household, however, take this profit as given. The fist-order conditions with respect to w and C_0 yield,

$$\frac{\beta}{C_1}(R - R_f) = 0 \quad \text{and} \quad \frac{1}{C_0} = \frac{\beta}{C_1}(R_f + w(R - R_f)). \tag{B17}$$

The first condition implies that $R = R_f$, as it should be given that there is no uncertainty in this economy. Because in equilibrium there cannot be any borrowing or lending, it must be that

$C_0 = W_0 - K$, where K is the amount of households capital investment. Hence, from the first order condition (B17) and the budget constraint(B16) we have

$$\frac{1}{W_0 - K} = \frac{\beta R_f}{K R_f + \pi}. \tag{B18}$$

Because $R_f = R$, using the definition of return in equation (B13), we obtain

$$K^* = \frac{\alpha\beta}{1 + \alpha\beta} W_0. \tag{B19}$$

Therefore, the optimal investment K^* in the decentralized economy corresponds to the one in the social planner economy, derived in equation (B8). This is just an implication of the First Fundamental Theorem of welfare economics, see, e.g., Mas-Colell, Whinston, Green, et al. (1995). Substituting K^* from equation (B19) in the expression of the risk-free rate (B13), we have that the equilibrium risk-free rate is

$$R_f^* = \alpha A \left(\frac{\alpha\beta}{1 + \alpha\beta} W_0 \right)^{\alpha - 1} = \frac{1}{\beta} \frac{A}{W_0^{1-\alpha}} \frac{(\alpha\beta)^\alpha}{(1 + \alpha\beta)^{\alpha - 1}}, \tag{B20}$$

which corresponds to the equilibrium risk-free rate derived in equation (B9) for the social planner. As $\alpha \to 0$, $K^* \to 0$ and $R_f^* \to A/W_0$, that is, the production economy converges to an endowment economy where the exogenous consumption is given by $C_0 = W_0$ and $C_1 = A$.

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