Stop Believing in Reserves*†

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Abstract

The deposit channel of monetary policy is well understood when the Federal Reserve (Fed) tightens monetary policy by raising interest rates. Shadow banks increase their investment yields in response to rising interest rates more than banks, so deposits flow from banks to shadow banks. However, the deposit effects of tightening through the Fed's balance sheet are not as clear. Using a structural model of bank reserve demand and lending, the deposit market, and the repurchase (repo) market, we show that deposits also flow to shadow banks when the Fed tightens monetary policy by reducing the size of its balance sheet. Then, shadow banks lend out these deposits in the repo market to meet increased funding demand. In contrast, when the Fed's balance sheet is large and the Fed tightens through interest rates, shadow banks invest their new deposits at the Fed rather than in the repo market. We show that the size of the Fed's balance sheet is constrained not only by banks' demand for reserves, as is typically considered, but also by the capacity of the repo market, and therefore, an ample reserves monetary policy framework must also consider the demand for money by shadow banks.

Keywords: monetary policy transmission, quantitative tightening, reserves, ON RRP, shadow banks
JEL Classifications: G2, E4, E5

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1 Introduction

Understanding how monetary policy is transmitted to the real economy is a fundamentally important question for policymakers and academics alike. The first step of this process is transmission across money markets. In this paper, we address two aspects of this initial step to understand the implications of tightening policy via an unconventional policy tool—the central bank's balance sheet—as opposed to via the more conventional and well understood policy tool—interest rates.

First, we aim to understand how monetary policy affects the allocation of money between banks and shadow banks when policy is implemented via the central bank's balance sheet rather than through interest rates. The literature establishes two deposit channels of monetary policy transmission: the "bank deposit channel" and the "shadow bank deposit channel." When the Federal Reserve (Fed) tightens monetary policy by raising the federal funds rate, Drechsler et al. (2017) show that this monetary tightening leads to deposits flowing out of the banking system, i.e. the bank deposit channel. At the same time, Xiao (2020) shows that these deposits then flow to shadow banks, or money market mutual funds (MMFs), i.e. the shadow bank deposit channel. These important channels are well understood when the Fed uses interest rates to change monetary policy.

In this paper, we ask how these channels of monetary policy transmission operate when the Fed uses its balance sheet as an unconventional monetary policy tool. The Fed has used its balance sheet as an additional monetary policy tool since 2008 when the federal funds rate was first lowered to its effective lower bound of zero, and has signaled that it will continue to use its balance sheet in this way for the foreseeable future.[1] Because the Fed decided to operate in an "ample reserves framework," meaning that the Fed will continue to use its balance sheet as a monetary policy tool, it is important to understand how these deposit channels of monetary policy transmission operate with the balance sheet compared to with interest rate policy alone.

Second, we discuss two additional channels through which policy tightening via the Fed's balance sheet affects transmission through money markets. These are channels through which balance sheet policy directly affects money markets, rather than the indirect effects of investors choosing to allocate between banks and shadow banks. First is the reserves channel. As the Fed's balance sheet declines, reserves typically decline, reducing bank liquidity and encouraging banks to pay higher rates for the funding they need.[2]

[1] See the January 2019 Statement Regarding Monetary Policy Implementation and Balance Sheet Normalization, which states that the Committee intends to continue to operate in an ample reserves monetary policy framework (`https://www.federalreserve.gov/newsevents/pressreleases/monetary20190130c.htm`).

[2] As the Fed's securities holdings shrink, the Fed's liabilities correspondingly shrink. But, as we will

With this channel, bank demand for reserves determines how much the Fed can shrink its balance sheet and still remain in an ample reserves framework. Second is the securities channel. The decline in the Fed's balance sheet increases the amount of securities that need to be held by private investors. These securities are often funded in the repurchase (repo) market and, as demand to borrow increases, repo rates increase. In this case, the capacity of the repo market determines how much the Fed can shrink its balance sheet. In this paper, we present evidence that the securities channel rather than the more traditional reserves channel may actually be more binding in the current environment.

We begin by providing empirical evidence that while banks, on average, do shed deposits in response to tightening via the balance sheet like they do for increasing interest rates, shadow banks attract substantially more deposits. The magnitude is more than 3 times larger than when monetary tightening occurred through the federal funds rate alone. Our results indicate that balance sheet policy is primarily transmitted through the shadow bank deposit channel, and not the bank deposit channel, and with far larger magnitudes. These results have important implications for how the Fed conducts monetary policy, and understanding how balance sheet runoff and interest rate increases interact.

We develop a tractable theoretical model that captures the flow of deposits between banks and shadow banks, along with the instruments that the Fed uses for monetary policy implementation—the interest rate on reserve balances (IORB) and the size of the Fed's balance sheet.[3] The model highlights the mechanisms that are driving the bank deposit and shadow bank deposit channels when monetary tightening occurs through a decline in the size of the Fed's balance sheet. Then, we calibrate the model using the available moments for the period after the last expansion cycle due to the pandemic, so we can make predictions about how shadow banks will use their deposit funding as the Fed's balance sheet declines.

We are particularly interested in the role of the Overnight Reverse Repo (ON RRP) facility, which the Fed uses to maintain a floor on short-term interest rates. At this facility, shadow banks are eligible to lend deposits to the Fed and receive interest. Therefore, this facility can be thought of as reserves for non-banks. We show that when the Fed tightens monetary policy through interest rates, participation by shadow banks at the ON RRP increases. Shadow banks receive inflows but do not have sufficient investment opportunities for this new money, so turn to the ON RRP. However, when the Fed tightens monetary policy by reducing the size of its balance sheet, shadow banks reduce their participation

discuss and show in the model, reserves may not be the first liability to decline; rather, the Overnight Reverse Repurchase (ON RRP) facility may move first. However, consistent with the Committee's guidance, policy is expected to tighten through the balance sheet until reserves are sufficiently low.

[3] In the model, we think of the securities holdings of the Fed in the System Open Market Account (SOMA) and the total size of the Fed's balance sheet as synonymous, although in reality the Fed holds some assets in addition to outright holdings of securities.

at the ON RRP and lend more in private funding markets. Although they receive even more inflows, shadow banks are able to invest this money in the market since private institutions need to hold more Treasury securities and agency MBS, which are funded in the repo market. Our results have important implications for how shadow banks transmit monetary policy. When tightening occurs through interest rates, much of the shadow banks' increase in deposits is invested at the ON RRP. However, when tightening occurs through the balance sheet, shadow banks lend their deposits in the repo market.

We build upon the models of Armenter and Lester (2017) and Xiao (2020). Households have a deposit endowment and can choose whether to invest their deposits into banks (depository institutions) or shadow banks (money market mutual funds or MMFs) based on the deposit rates available to them. MMFs have two choices of where they can invest these deposits. They can lend to dealers in the private funding (repo) market and receive the repo interest rate, or they can lend to the central bank via the ON RRP and receive the ON RRP rate. Banks also have two investment choices for the deposits they receive from households. They can invest either at the central bank and receive IORB, or in loans and receive an interest rate that reflects the return on loans. Dealers hold a portion of the outstanding securities, which they finance by borrowing from MMFs in the repo market. Figure 1 illustrates the flow of deposits in the model and the relevant interest rates.

In the model, we show that when the Fed raises interest rates by raising IORB, households transfer deposits from banks to MMFs since MMFs raise their rates more with IORB, while bank rates lag. This is consistent with both the bank deposit and shadow bank deposit channels. MMFs then invest most of those additional deposits at the ON RRP, rather than in the repo market, because rising interest rates do not change dealers' demand for repo funding, and the ON RRP rate is very close to the repo rate. Repo volumes remain unchanged, but participation at the ON RRP increases. This result is in line with Afonso et al. (2022). However, when the Fed reduces the size of its balance sheet, which we model by dealers purchasing an exogenous amount of securities that the Fed no longer intends to purchase (and thereby shrinking its balance sheet), dealers' demand for repo borrowing increases to finance these additional security purchases. Therefore, the repo interest rate increases above the ON RRP rate, and MMFs prefer to lend to dealers in the repo market rather than to the ON RRP. MMFs also experience inflows of deposits from households because their yields are higher than banks. Our results show that monetary policy transmission via the Fed's balance sheet occurs through primarily shadow banks, and not banks. Further, shadow banks increase their lending in private funding markets only when monetary tightening occurs through the balance sheet, and not through interest rates. Our results imply that monetary tightening through interest rates can attract deposits back to the Fed via the ON RRP rather than to private markets.

Our model also sheds light on the interaction between interest rate and balance sheet policy. While the two tools are generally thought of as substitutes, we demonstrate a novel complementarity between them. In particular, our results suggest that a central bank can shrink its balance sheet more if it raises interest rates first. This is because, as the Fed raises rates, households shift deposits from banks to MMFs. When MMFs have more deposits, they have more to lend in the repo market as needed when the Fed shrinks its balance sheet. By expanding the relative size of shadow banks versus banks, higher interest rates enable a larger repo market to accommodate a large decline in the Fed's balance sheet.

Next, we calibrate the structural parameters of the model to match moments in the data between March 12 and October 14, 2022.[4] We use this calibration to demonstrate that the model can match the data quite well and to forecast both how much the Fed can shrink its balance sheet and the corresponding level of reserves that are consistent with an ample reserves framework. Rather than solely being constrained by bank demand for reserves (the reserves channel), the size of the Fed's balance sheet is also constrained by the intermediation capacity of the repo market (the securities channel). That is, because dealers have to hold additional securities when the Fed no longer holds them, which they need to finance in the repo market, the amount the Fed can shrink its balance sheet while maintaining rate control also depends on the holdings of MMFs that can be lent in the repo market.[5] Further, our calibration shows that this repo market constraint binds before bank reserve demand binds in the current environment, suggesting that the Fed can shrink its balance sheet by less than would be implied by banks' demand for reserves alone. Given the importance of shadow banks in the US financial system, an ample reserves framework must also consider the demand for money by shadow banks. In particular, our estimates imply that, when IORB is equal to 4.4%, the Fed can shrink its balance sheet by roughly $2.1 trillion and still maintain its ample reserve framework. Our estimates are in line with results in Lopez-Salido and Vissing-Jorgensen (2022), who estimate that the Fed can reduce its balance sheet by roughly $2.2 trillion.

We contribute to three strands of the literature. First, we provide new insights on the implications of the Fed's new monetary policy implementation framework. In this environment, there are still many outstanding questions, such as the minimum size of the Fed's balance sheet consistent with the ample reserves framework, the determinants of reserve demand, and the implications of the Fed's liability composition, in particular the allocation of reserves versus the ON RRP. Lopez-Salido and Vissing-Jorgensen (2022) find that reserve demand is a function of banks' deposits. In a related paper, Acharya

[4]March 11, 2022 was the last day the Fed purchased securities as part of its pandemic asset purchase program.

[5]Dealers can attract more lending to the repo market from MMFs by raising rates, but that would uncouple repo rates from the Fed's administered rates, which is not in line with the FOMC's intention under its ample reserves framework.

et al. (2022) show how increases in the Fed's balance sheet, and therefore reserves, lead to increases in deposits and other bank liabilities. Our results complement these papers by showing that bank deposits decline with interest rate tightening, but not with balance sheet tightening. Therefore, raising rates before or in conjunction with shrinking the balance sheet could reduce reserve demand, thus allowing for a smaller overall size of the Fed's balance sheet, if desired. Further, we provide evidence that bank demand for reserves is not the only constraint on how much the Fed can shrink its balance sheet. Finally, our results also speak to the composition of reserves and ON RRP, which not only affects the ultimate size of the Fed's balance sheet, but also has implications for the supply of safe assets.[6]

Second, we contribute to the literature on the banking channels of monetary policy transmission. There is extensive work done on the banking channels of interest rate policy.[7] However, the work on balance sheet policy is more limited. In a paper similar to ours, Diamond et al. (2022) study the expansionary effects of balance sheet policy and show that reserves crowd out bank lending and, to a lesser extent, crowd in deposits. To the best of our knowledge, we are the first to consider a central bank's balance sheet policy in the context of deposit flows between banks and shadow banks. While increasing interest rates and shrinking the size of the balance sheet are both means of tightening monetary policy, we show that they have very different implications for both commercial and shadow banks, as well as money markets more broadly. Further, the existing work in this area mainly focuses on balance sheet accommodation, while we focus on balance sheet tightening. Current evidence, such as D'Amico and King (2013) and Smith and Valcarel (2022), suggests that balance sheet policy is not symmetric so expansions and contractions may have different effects.

Third, our results also shed light on the substitutability of interest rate and balance sheet policy. Interest rate and balance sheet policy both affect the economy through longer-term interest rates. Various studies have quantified the degree of substitution between the two tools using term premium models.[8] However, these tools affect rates differently.[9] In particular, Kiley (2014) suggests that long-term rates are more influenced by expectations of the short-term rate rather than the term premium, and therefore interest rate policy is more effective.[10] We add to this literature by demonstrating an interdependence of the

[6]The Fed's current guidance, as laid out in the Plans for Reducing the Size of the Federal Reserve's Balance Sheet (`https://www.federalreserve.gov/newsevents/pressreleases/monetary20220504b.htm`), states that the end of balance sheet runoff is tied to the level of reserves. Therefore, the composition of reserves versus ON RRP may affect the stopping point of runoff and the size of the Fed's balance sheet.

[7]See, among others, Kashyap and Stein (2000), Drechsler et al. (2017), and Xiao (2020).

[8]See, among others, Sims and Wu (2020) and Crawley et al. (2022).

[9]Interest rate policy affects longer-term rates through the expected path of short-term rates, while balance sheet policy affects the term premium embedded in longer-term rates.

[10]Existing studies have also discussed the implications of policy tool uncertainty. Brainard (1967) suggests that diversifying across policy tools is optimal when the effectiveness of different tools is uncertain,

effectiveness of the two tools; that is, the amount of tightening that can occur through balance sheet policy is actually dependent on the stance of interest rate policy. In particular, our results suggest that a central bank can shrink its balance sheet more if it raises interest rates first. Further, we highlight the differing effects of the two tools on the relative size of the shadow bank sector, which has important implications for financial stability.

The paper proceeds as follows. Section 2 provides background information on the Fed's current ample reserve framework. Section 3 presents motivating empirical evidence on the bank and shadow bank deposit channels with both interest rate and balance sheet policies. Section 4 describes the model, while Section 5 presents the calibration. Section 6 discusses the key results and Section 7 concludes.

2 The Fed's Monetary Policy Framework

2.1 The Ample Reserves Framework

The Fed has traditionally used adjusting the policy rate to conduct monetary policy. Before the 2008 Global Financial Crisis (GFC), the Fed would adjust the policy rate by conducting open market operations to adjust the supply of reserves and therefore the federal funds rate. The federal funds rate is the interest rate paid by banks to borrow $1 of reserves, which is money held at the Fed, in the federal funds market. During this time, the Fed operated in what was referred as a "scarce reserves regime".

During and after the GFC, the Fed considerably expanded its balance sheet, which meant increasing the supply of reserves in the financial system. Since the supply of reserves was so large, the Fed was unable to use open market operations to adjust the federal funds rate because small changes in the level of reserves did not affect rates. At this time, the Fed began using administered policy rates to control the federal funds rate. The administered rates are the interest on reserve balances rate (IORB) and the ON RRP offering rate. To keep a floor on the federal funds rate, the Fed first introduced the interest on reserves in 2008.[11] Banks are able to place money at the Fed, i.e. reserves, and receive IORB. As a result, banks are not incentivized to lend at lower than IORB because they could always place that money at the Fed instead.

However, IORB was not a sufficiently effective floor because, although banks did not lend below IORB, many non-banks would since they are ineligible to hold reserves and earn

while Williams (2013) suggests that the more certain tool should be primary.

[11] Initially, the Fed had interest rates on required and excess reserves (IORR and IOER, respectively). They have since been replaced by IORB because the Fed no longer has reserve requirements for banks and hence there is no distinction between required and excess reserves.

IORB. This, in turn, put downward pressure on the federal funds rate to trade sometimes below IORB. As a result, the Fed introduced the ON RRP facility in September 2013 to provide a firmer floor to the federal funds rate. At the ON RRP, non-banks, namely MMFs, can lend money to the Fed and receive the ON RRP rate. The ON RRP rate is lower than IORB, providing an effective floor to the federal funds rate.

In January 2019, the Fed officially adopted this new framework, which is referred to as an "ample reserves regime".[12] Figure 2 illustrates this framework. As of June 2022, the amount of reserves in the financial system amounted to $3.3 trillion, denoted by the blue vertical line.

In the ample reserves framework, the Fed can tighten monetary policy in two ways. It can raise IORB and the ON RRP rate, thereby raising the federal funds rate, or it can remove reserves from the financial system. The Fed removes reserves by reducing the size of its balance sheet. The Fed bought Treasury securities and agency mortgage-backed securities (MBS) to create the reserves on its balance sheet. As these securities roll off as they mature, the Fed removes the reserves it initially created. As the Fed lets more securities roll of its balance sheet, the amount of reserves should decline until it reaches a level where the federal funds rate starts responding to changes in reserves, i.e. the amount of reserves is no longer on the flat portion of the reserve demand curve, which is the red line in Figure 2. That level, denoted as $X by the vertical dashed line in the figure, represents the amount of reserves required for the Fed to maintain its ample reserves framework.

3 Empirical Evidence to Motivate the Model

Before we introduce our model, which illustrates the effects of the Fed reducing the size of its balance sheet, we first provide empirical evidence that motivates the set up of our model. Our model starts with households deciding where to place their money, either with banks or non-banks, as the Fed tightens monetary policy. Banks are the only financial intermediary that can hold reserves at the Fed and receive IORB on those balances. Non-banks, namely MMFs, are the main users of the ON RRP facility. Indeed, the vast majority of ON RRP participation amounting to over $2 trillion a day on average in 2022 is from MMFs. Banks and MMFs are the two types of financial intermediaries that directly experience the effects of the Fed reducing the size of its balance sheet, so it is important to understand how these two types of entities respond to tightening monetary policy.

[12] See Ihrig et al. (2020) for a detailed explanation of the ample reserve regime.

3.1 Empirical Specification

Using simple reduced form regressions, we show how bank and MMF deposits change in response to the Fed's two monetary policy tools: increasing its policy rate or reducing the size of its balance sheet. We estimate the following time-series specification quarterly between 1992:Q1 and 2021:Q4, following closely the estimation strategy in Xiao (2020):

$$\begin{aligned} \textit{Growth Rate}_t = \alpha &+ \beta \cdot \sum_{t=-12}^{0} \Delta \text{EFFR}_t + \eta \cdot \left[-1 \cdot \sum_{t=-12}^{0} \Delta \log\left(\text{SOMA}\right)_t \right] \\ &+ \theta \cdot \left\{ \sum_{t=-12}^{0} \Delta \text{EFFR}_t \times \left[-1 \cdot \sum_{t=-12}^{0} \Delta \log\left(\text{SOMA}\right)_t \right] \right\} \\ &+ \sum_{c=1}^{4} \gamma_c X_{c,t} + \lambda t + \varepsilon_t \end{aligned} \tag{1}$$

where we regress either the quarterly year-over-year commercial bank deposit growth rate or the quarterly year-over-year MMF deposit growth rate (also known as assets under management or AUM) on the three-year cumulative change in the effective federal funds rate ($\Delta \textit{EFFR}$), the three-year cumulative change in the logged portfolio value of the System Open Market Account ($-\Delta \log(\textit{SOMA})$), and their interaction ($\Delta \textit{EFFR} \times -\Delta \log(\textit{SOMA})$).[13] We also include four control variables: GDP growth, CPI, the TED spread, and the personal savings rate, in addition to a linear time trend. We have transformed the cumulative change in (log) SOMA by a factor of -1 so that the coefficient estimates on EFFR, SOMA, and their interaction can be read directly as the effects of monetary tightening.

3.2 Data Sources

We make use of the following data sources accordingly.

Deposits. We use quarterly aggregate data on commercial bank deposits and MMF deposits, or assets under management, from 1992:Q1 to 2021:Q4. For commercial banks, we use the sum of interest-bearing savings and transaction deposits booked in domestic banks from the FFIEC Call Reports. For MMFs, we use the total assets under management series from FRED.[14] Growth rates are year-over-year, computed as the percent change from

[13] EFFR is the volume-weighted median rate of overnight federal funds transactions. The System Open Market Account (SOMA) is where the Fed holds its securities. Its value is equivalent to the amount of securities held outright on the Fed's balance sheet.

[14] For the exact series name and description of all data series obtained from FRED that we use, please see the FRED data dictionary in Appendix B.

the previous year.

Monetary Policy. For the federal funds rate, we use the publicly available EFFR data from FRED, which is the daily volume-weighted median of overnight federal funds transactions. For the balance sheet, we retrieve daily data on the total size of the SOMA portfolio of the Federal Reserve.[15] For our deposit growth results, we take quarterly averages of the data for the years 1992 to 2021.

Control Variables. We include contemporaneous quarterly measures of GDP growth, CPI inflation, the TED spread, and the personal savings rate, as controls in our estimation. Using data from FRED, we calculate quarterly GDP growth as the percent change in real GDP from one year ago. Similarly, for CPI inflation, we use the standard index measure from FRED where quarterly inflation is the percent change from one year ago. Finally, quarterly data for both the TED spread and the personal savings rate are again from FRED.

3.3 Deposit Growth Results

Table 1 presents the results from estimating Equation (1) at a quarterly frequency from 1992:Q1 to 2021:Q4. Column 1 shows the results for the bank deposit growth rate while Column 2 shows the results for the MMF deposit growth rate. The direction of the coefficient on $\Delta EFFR$ in both columns confirm the results in Drechsler et al. (2017) and Xiao (2020). That is, when EFFR increased, commercial banks experienced an outflow of deposits while MMFs experienced an inflow of deposits.

Our contribution is the magnitudes and directions of the coefficients on $-\Delta \log(SOMA)$ and $\Delta EFFR \times -\Delta \log(SOMA)$. We observe that a one unit decrease in the SOMA portfolio induces both an outflow of banks deposits from commercial banks, and a significantly larger inflow in MMF deposits. Indeed, the coefficient on $-\Delta \log(SOMA)$ is much higher than $\Delta EFFR$ for both commercial banks and shadow banks, suggesting that monetary tightening through the balance sheet has larger effects on commercial bank deposit outflows and MMF deposit inflows than increasing the policy rate. The interaction term, however, shows that when the Fed tightens monetary policy through its balance sheet and interest rates at the same time, the effects on commercial bank deposit outflows and MMF deposit inflows are only marginally magnified.[16]

[15] While we use internal Federal Reserve System data that gives us SOMA sizes at a daily frequency, a weekly value is made publicly available through the H.4.1 data release. For the public series information, see the FRED data dictionary in Appendix B.

[16] These results are robust to a host of alternative specifications. In particular, our results hold (a) when considering alternative horizon changers in monetary policy (i.e. at the 1-year, 2-year, and 3-year horizons), (b) when using exogenous monetary policy shocks à la Romer and Romer (2004) instead of the effective federal funds rate, (c) when considering only pre-2008 values, and (d) when using the $EFFR - IOR$ spread

Given the fact that Table 1 shows that, while bank deposit growth does respond to monetary tightening through the Fed's balance sheet, the effect on MMF deposits inflows is significantly more pronounced, this suggests that banks are not the main financial intermediary through which transmission of this monetary policy tool occurs. Indeed, Table 1 suggests that the marginal financial intermediary is MMFs. There has been a lot of research on the effect of expanding the Fed's balance sheet, i.e. quantitative easing, on bank lending. However, there has been no research on the effect of the Fed's balance sheet on non-bank investment activity. We fill that gap with this paper, and this is motivated by Table 1 that shows that MMFs receive significant inflows when the Fed shrinks its balance sheet.

3.4 Where do MMFs invest their inflows?

We have shown preliminary empirical evidence that MMFs receive inflows (deposits) when the Fed tightens monetary policy by shrinking its balance sheet. What do they do with that new money? To motivate a main mechanism of our model, we next provide some empirical evidence that when MMFs receive that extra money, they place that money in short-term funding markets, specifically the repurchase (repo) market. A repo is a short-term, often overnight, collateralized loan between a borrower and lender. Alternative investments could include placing the money at the ON RRP or investing the money in Treasury bills. In this section, we provide some empirical evidence of this.

Our data are from the monthly snapshots of the composition of MMF portfolio holdings from the Securities and Exchange Commission filings (Form N-MFP) from 2010 to 2021. These mandated monthly reports provide fund-level data on total assets and holdings broken down into detailed asset categories.[17] We keep only those funds which participate in Treasury repo markets and the ON RRP. Because multiple funds comprise a MMF complex, we sum up to the MMF complex level by summing total assets and each of the holdings categories by month to get monthly complex-level aggregate measures of total assets and total holdings by type.

We estimate the following panel regression monthly between 2010 and 2021 for a given MMF complex c who invests in product p at time t. MMF complex c is comprised of a universe of individual funds f that comprise universe F, a subset of which $S \subseteq F$ are

as our right-hand-side variable. For further notes, see Appendix A.

[17] In particular, these are: Treasuries, Treasury repo, asset-backed commercial paper, commercial paper, certificates of deposit, government agency debt, and government agency repo.

eligible to use the ON RPP facility.

$$\begin{aligned}\Delta Share_{c,p,t} = \alpha + \beta \cdot \Delta \text{EFFR}_{t-1} + \eta \cdot \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big] \\ + \theta \cdot \Big\{ \Delta \text{EFFR}_{t-1} \times \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big] \Big\} \\ + \log\left(\text{AUM}_{c,t}\right) + \log\left(\text{Bills Outsanding}_{t}\right) + \lambda t + \mu_c + \delta_t + \varepsilon_{c,p,t}\end{aligned} \tag{2}$$

Share is defined as the total investment in product p at time t for MMF complex c divided by the sum of total assets under management for MMF complex c at time t.

Formally, *Share* is defined as follows:

$$Share_{c,p,t} = \frac{\sum_{f \in S} \text{Investment}_{f,c,p,t}}{\sum_{f \in S} \text{Assets}_{f,c,t}}$$

$$\text{where } S = \Big\{ f \in F : \ \exists\, t \in T \text{ where } \left(\text{ON RRP}_{f,c,t} > 0\right) \Big\} \subseteq F$$

where p is either the dollar amount of MMF complex c lending in the repo market backed by Treasury collateral, or the dollar amount of take-up at the ON RRP facility on day t.

We regress the monthly change in *Share* on the lagged one month change in EFFR, the lagged one month change in (log) SOMA, and their interaction. We also include several control variables: the total AUM of MMF complex c, the total supply of Treasury bills outstanding, a linear time trend, MMF complex fixed effects, and year $\times$ quarter time fixed effects. As above, we similarly transform the one month change in (log) SOMA by a factor of -1 so that the coefficient estimates on EFFR, SOMA, and their interaction can be read directly as the effects of monetary tightening.

Given, as just shown above, that when monetary tightening is achieved through either rate policy, balance sheet policy, or both, MMFs experience an inflow of deposits, an important following question is naturally what do MMFs do with their increased assets under management? One might expect that, when the policy rate increases, all else equal, because MMFs are endowed with more deposits but the demand in the repo market has remained unchanged, MMFs substitute proportionally away from private repo and into the ON RRP. Conversely, one might expect that, when the balance sheet is reduced, all else equal, the increase in MMF deposits is met with a commensurate increase in demand in the repo market, MMFs substitute proportionally away from the ON RRP and towards the private repo market.

Table 2 presents the results from estimating Equation (2). Column 1 shows the results for when p is MMF lending in Treasury repo and Column 2 shows the results for when p is the amount of MMF take-up at the ON RRP facility. We observe that ON RRP take-up increased when the Fed tightened monetary policy by raising policy rates. This result makes sense because from Table 1 we observed that MMFs receive inflows when the Fed raises interest rates. However, when the Fed tightens monetary policy by reducing the size of its balance sheet, we observe from the coefficients on $-\Delta \log(SOMA)$ and $\Delta EFFR \times -\Delta \log(SOMA)$ that MMFs shift away from the ON RRP and lend more in the private Treasury repo market. The mechanism that we show in the model is that when the Fed rolls off Treasury securities from its balance sheet, the U.S. Treasury must continue to finance that debt and sell more Treasuries to the private market, namely primary dealers. Those dealers then need to finance that extra inventory in the Treasury repo market, thereby putting upward pressure on repo rates. MMFs then shift to lend more in the Treasury repo market instead of placing their money at the ON RRP to meet that heightened demand because the rates are more attractive. The results of Table 2 provide evidence of this mechanism in our model.

4 The Model

In this section, we describe the theoretical model used to analyze the effects of monetary policy tightening through policy rate increases and balance sheet reductions on banks and shadow banks.

4.1 Environment

The theoretical model is an extension of Armenter and Lester (2017) and builds on Xiao (2020). Consider a two-period economy. The economy is populated by five types of agents: banks, broker-dealers (dealers), money market mutual funds (MMFs), households, and firms. Each type has unit measure. Furthermore, there exists a central bank and a government. Agents do not discount between the two periods.

At the beginning of the period 1, households receive an endowment of m_e units of commodity money from the central bank and an endowment of B units of government bonds. Commodity money is backed by a general good that can be consumed in period 2 and can be produced by the central bank at no cost. Each unit of government bond matures in period 2 and yields one unit of commodity money. At the beginning of the first period, the central bank decides how many units of government bonds b^{CB} to buy from households at price p^g. Dealers purchase the remaining government bonds, denoted b^d. We assume that households cannot hold government bonds across periods.[18] The

[18] Alternatively, we could assume that it is costly for households to hold government bonds across periods and that the cost is large enough such that households have an incentive to sell all government bonds at price p^g.

total endowment in units of commodity money that households hold after the sale of their endowment of government bonds is $m = m_e + p^g B$.

After the central bank has made its asset purchases, a deposit market and a repo market open. In the deposit market, households allocate their endowment m to bank deposits in the amount d^b and MMF deposits in the amount d^m. Bank deposits yield an interest rate i_{d^b} and MMF deposits yield an interest rate i_{d^m} in period 2. In order to deposit some of the endowment at a bank account, households and banks are randomly matched and then bargain over the deposit quantity and the deposit rate according to the proportional bargaining solution. After the household and the bank agree on the deposit amount and rate, the remaining funds of households are deposited at MMFs. The market for MMF deposits is assumed to be perfectly competitive and MMFs pay the market clearing interest rate on their deposits. Both banks and MMFs are subject to linear balance sheet costs, k^b and k^m, respectively. The assumption of banks' balance sheet costs is motivated by existing regulation that may limit the size of banks' balance sheets.[19] Balance sheet costs for MMFs are introduced to match the data on MMF deposit rates more accurately.[20]

MMFs can use their deposits obtained from households to lend to dealers in the repo market or invest them at the overnight reverse repo (ON RRP) facility at the central bank. The ON RRP pays an interest rate r. The repo market is assumed to be perfectly competitive and the market clearing interest rate is ρ. We assume that there exists a record-keeping technology in the repo market, such that repayment is perfectly enforceable. Dealers are the borrowers in the repo market as they need to finance their purchase of government bonds from households.

After the deposit and the repo markets have convened, banks decide how to invest the funds received from households. Banks can hold either reserves at the central bank, which yield the interest rate on reserve balances (IORB), R, where $R > r$, or they can make loans ℓ and receive an interest rate i_ℓ, where $i_\ell - R$ is assumed to be positive and constant in R. Following Ennis (2018), banks have some costs associated with investing in loans $\chi(\ell)$ that can be motivated by monitoring costs (see for example, Holmstrom and Tirole, 1997). We implicitly assume that the effort of monitoring is sufficient to guarantee repayment of the loan and therefore abstract from default. $\chi(\ell)$ is assumed to be strictly convex. Banks are furthermore subject to regulation that limits their ability to lend out all their deposits. We assume banks have to hold at least a fraction δ of their deposits as reserves. This assumption is motivated by existing bank regulation such as the liquidity coverage ratio as well as bank liquidity preferences.

[19]For instance, the supplementary leverage ratio (SLR) constrains the size of a bank's balance sheet given its capital.

[20]That is, the MMF balance sheet costs create a wedge between the market repo rate and the MMF yield, consistent with fees that MMFs implement, which keep MMF yields somewhat below repo rates.

After the lending market convenes, a goods market opens, where firms produce the special good and households purchase and consume the special good. We assume that households and firms are anonymous and cannot commit to honor intertemporal promises. Thus, households need a medium of exchange to acquire the consumption good from firms. We assume furthermore that only bank deposits are accepted as a means of payment by firms, whereas MMF deposits are an investment instrument to save for second period consumption. Thus, households pay firms by transferring some of their bank deposits to the firms' bank deposit account. We assume that firms are identical and are uniformly distributed across banks such that the inflow of firm deposits for each bank is identical. Firms receive the average deposit rate on their deposit balances. Note, since all households and banks are identical, the interest rate on deposits will be identical across household and bank matches and across firms. Households receive utility $u(q)$ from consuming q units of the consumption good. Firms can produce the consumption good at linear cost.

In period 2, the central bank pays interest on reserves and ON RRP holdings, dealers repay repo loans, MMFs and banks repay deposits, and banks earn their return on loans. The government redeems bonds for commodity money and dealers repay their debt to households. Lastly, the central bank produces the general consumption good x at no cost that can be consumed by all agents in exchange for commodity money.

4.2 Equilibrium

In the following section, we derive the optimal decisions made by banks, MMF, dealers, households, and firms. We solve the model backwards.

The general goods market. First, the central bank produces the general good x in exchange for commodity money. Denote P the price of the general good. Thus,

$$Px = \Pi^{HH} + \Pi^{F} + \Pi^{B} + \Pi^{D} + \Pi^{MMF}. \tag{3}$$

Denote Π^{HH} and Π^{F} the commodity money holdings of households and firms in period 2, respectively. Further, Π^{B}, Π^{D}, and Π^{MMF} denote the profits of banks, dealers, and MMFs in period 2, respectively.

The special goods market. At the end of period 1, households purchase and consume the special good from firms. The maximization problem for a firm satisfies

$$\max_{q_s} \; -q_s + \phi p q_s (1 + i_{d^b}).$$

Denote ϕ the price of money at the end of period 2 in terms of the general good, such that $P = 1/\phi$, and p the price of the special good. The first-order condition satisfies

$$\phi p(1 + i_{d^b}) = 1. \tag{4}$$

Equation (4) implies that in equilibrium firms are indifferent as to how much to produce if the price of the special good compensates them for the cost of holding bank deposits across periods.

Households can only use bank deposits as a means of payment. Utility maximization implies that households allocate all of their endowment to either bank deposits or MMF deposits. Thus, $m = d^b + d^m$ must hold with equality. The maximization problem of households therefore satisfies

$$\max_q \; u(q) + \phi(d^b - pq)(1 + i_{d^b}) + \phi(m - d^b)(1 + i_{d^m})$$

$$\text{s.t. } d^b - pq \geq 0.$$

The constraint implies that households cannot spend more bank deposits than they have. It has the Lagrange multiplier λ. The first order condition satisfies

$$u'(q) = \phi p(1 + i_{d^b}) + p\lambda.$$

Thus, the optimal quantity consumed satisfies

$$q = \begin{cases} u'^{-1}(1) & \text{if } d^b - pu'^{-1}(1) > 0, \\ d^b/p, & \text{otherwise.} \end{cases} \tag{5}$$

The bank lending market. After banks obtain deposits from households, they can either allocate these funds in a bank lending market or hold reserves at the central bank. We think of the bank lending market as the sum of all investment options that banks might have. Banks receive a return of $(1 + i_\ell)$ on each unit invested in loans. We assume that $i_\ell - R$ is positive and constant such that banks have an incentive to invest in loans and the marginal return on loans does not depend on the level of the interest rate on reserves. Banks face some costs associated with issuing a loan, denoted $\chi(\ell)$, which is strictly convex. Lastly, we assume that banks have to hold at least a fraction δ of deposits as reserves, which are denoted m_r, such that $m_r \geq \delta d^b$ has to hold. The balance sheet identity of banks implies $d^b = \ell + \chi(\ell) + m_r$. Using this identity, the maximization problem of banks can be written as:

$$\max_{\ell} \phi\ell(i_\ell - R) - \phi\chi(\ell) + \phi d^b(R - k^b - i_{d^b})$$

$$\text{s.t. } (1-\delta)\phi d^b - \phi\ell - \phi\chi(\ell) \geq 0.$$

The constraint has the Lagrange multiplier λ_r. The first-order condition satisfies:

$$\phi(i_\ell - R - x'(\ell) - \lambda_r(1 + \chi'(\ell))) = 0. \tag{6}$$

Thus, the optimal quantity of loans satisfies:

$$\ell = \begin{cases} \ell^* & \text{if } (1-\delta)d^b - \chi'^{-1}(i_\ell - R) - \chi(\chi'^{-1}(i_\ell - R)) \geq 0, \\ (1-\delta)d^b & \text{otherwise,} \end{cases} \tag{7}$$

where $\ell^* = \chi'^{-1}(i_\ell - R)$. If the constraint on reserve holdings does not bind, banks will choose to lend until the marginal return of lending one more unit of money equals the marginal cost of issuing a loan. If the constraint is binding, banks hold the required quantity of reserves and invest the rest of their funds in bank loans.

The deposit and repo markets. First, we consider the optimal decisions by dealers and MMFs in the repo market. Denote z^d as the quantity borrowed by dealers in the repo market. Dealers have to finance all of their bond holdings in the repo market and thus $z^d = p^g b^d$. Their bond holdings furthermore yield one unit of commodity money in period $t = 2$. The maximization problem of dealers satisfies

$$\max_{z^d} \phi z^d \left(\frac{1}{p^g} - (1+\rho)\right).$$

The first-order conditions is

$$\frac{1}{p^g} - (1+\rho) = 0. \tag{8}$$

Equation (8) implies that if $1/p^g > 1 + \rho$, dealers want to borrow an infinite amount in the repo market. If $1/p^g < 1 + \rho$, dealers do not want to borrow in the repo market and would therefore not participate in the economy. If $1/p^g = 1 + \rho$, dealers are indifferent as to how much they borrow.

MMFs can use the deposits they receive from households to lend in the repo market or to participate in the ON RRP. Their balance sheet constraint implies $d^{ONRRP} + z^m = d^m$, where d^{ONRRP} denotes balances at the ON RRP facility. ON RRP balances yield the return r, whereas lending in the repo market yields a return ρ for each unit lent. Lastly, MMFs have to pay the deposit rate i_{d^m} on their deposits and have linear balance sheet costs

k^m. Using their balance sheet constraint, the maximization problem of MMFs satisfies

$$\max_{d^m, z^m} z^m(\rho - r) + d^m(r - k^m - i_{d^m})$$

$$\text{s.t. } d^m - z^m \geq 0.$$

The constraint implies that MMFs cannot lend more in the repo market than the amount of deposits they hold and has the Lagrange multiplier λ_M. The first-order conditions satisfy:

$$z^m : \qquad \rho - r - \lambda_M = 0, \tag{9}$$

$$d^m : \qquad r - k^m - i_{d^m} + \lambda_M = 0. \tag{10}$$

If $r - k^m - i_{d^m} > 0$, MMFs are willing to hold an infinite amount of deposits. If $r - k^m - i_{d^m} < 0$, MMFs are not willing to hold any deposits and if $r - k^m - i_{d^m} = 0$, MMFs are indifferent as to how many units of deposits they hold.

Similarly, if the repo rate exceeds the ON RRP rate, $\rho > r$, MMF are willing to lend in the repo market. If, however, the ON RRP rate exceeds the repo rate, $\rho < r$, MMFs prefer to deposit their money at the ON RRP rather than lend in the repo market and if $\rho = r$, MMFs are indifferent between the repo market and the ON RRP facility. Thus, the optimal quantity lent in the repo market satisfies:

$$z^m = \begin{cases} d^m & \text{if } \rho - r > 0, \\ \in [0, d^m] & \text{if } \rho - r = 0, \\ 0, & \text{if } \rho - r < 0. \end{cases} \tag{11}$$

It is straightforward to see that $r > \rho$ cannot be an equilibrium. If the repo rate is below the ON RRP rate, no MMF is willing to lend in the repo market. Thus, the repo rate increases until $\rho = r$. When the repo rate increases to equal the ON RRP rate, MMFs become indifferent between lending in the repo market or depositing at the ON RRP. If $\rho > r$, MMFs have an incentive to lend all of their funds in the repo market. Note, it is possible to have equilibria where $\rho > R$; that is, the repo rate exceeds IORB.

Lastly, we consider the decision of households regarding how many units of bank deposits and MMF deposits to hold. Here, we assume that banks and MMFs have different degrees of market power. In particular, we assume that MMFs compete with each other for deposits in a perfectly competitive market. However, in the bank deposit market, we assume that households and banks are randomly matched and then bargain over the deposit quantity and deposit rate according to the proportional bargaining solution. First, we determine the match surplus between a household and a bank. The match surplus for a household satisfies

$$S_H = u(q) + \phi(d^b - pq)(1 + i_{d^b}) + \phi(m - d^b)(1 + i_{d^m}) - \phi m(1 + i_{d^m}). \tag{12}$$

If the household is not matched with a bank, it can only hold MMF deposits and consequently also not consume the special good.

The match surplus of the bank satisfies

$$S_B = \phi\ell(i_\ell - R) + \phi d^b(R - k^b - i_{d^b}) - \phi\chi(\ell). \tag{13}$$

If a bank is not matched, it does not receive any funds to invest into loans or hold as reserves and thus the value of not being matched is zero. Thus, the match surplus satisfies:

$$S = u(q) + \phi(d^b - pq)(1 + i_{d^b}) - \phi d^b(1 + i_{d^m}) + \phi\ell(i_\ell - R) + \phi d^b(R - k^b - i_{d^b}) - \phi\chi(\ell). \tag{14}$$

The bargaining power of the bank is denoted θ and, consequently, the bargaining power of the household is denoted $1 - \theta$. In reality, households could also use cash as a means of payment. Thus, cash would be an outside option that gives households more bargaining power. Here, we abstract from cash because empirically we do not observe this pattern in the US.[21] Instead, we interpret $(1 - \theta)$ as a reduced form of the outside options that households have and their resulting bargaining power due to these outside options.

The maximization problem of the bank therefore satisfies:

$$\max_{d^b, i_{d^b}} \phi\ell(i_\ell - R) + \phi d^b(R - k^b - i_{d^b}) - \phi\chi(\ell)$$

$$\text{s.t. } \phi\ell(i_\ell - R) + \phi d^b(R - k^b - i_{d^b}) - \phi\chi(\ell) \geq \frac{\theta}{1 - \theta}\left[u(q) + \phi(d^b - pq)(1 + i_{d^b}) - \phi d^b(1 + i_{d^m})\right]$$

The constraint states that the overall match surplus is split proportionally between the household and the bank and has the Lagrange multiplier λ_B. The first-order condition satisfies

$$\begin{aligned} d^b :\ & \phi(R - k^b - i_{d^b}) + \lambda_B\phi(R - k^b - i_{d^b}) - \\ & \lambda_B\frac{\theta}{1 - \theta}\left[u'(q)\frac{\partial q}{\partial d^b} + \phi(1 + i_{d^b}) - \phi p\frac{\partial q}{\partial d^b}(1 + i_{d^b}) - \phi(1 + i_{d^m})\right] = 0, \end{aligned} \tag{15}$$

$$i_{d^b} :\ -\phi d^b - \lambda_B\phi d^b - \lambda_B\frac{\theta}{1 - \theta}\phi(d^b - pq) = 0. \tag{16}$$

[21] In 2021, 81.5% of US households were fully banked and only 4% of households used cash for all transactions (see FDIC, 2021).

Since we observe in the data that the bank deposit rate is less than the yield on MMFs (i.e., $i_{d^b} < i_{d^m}$), we restrict our analysis to this case. Thus, utility maximization implies that households only hold bank deposits to finance their desired quantity of the special good and invest the remaining funds in MMF deposits. Therefore, $d^b = pq$. From this, Equation (16) implies $\lambda = -1$ and therefore:

$$u'(d^b/p) = \frac{1 + i_{d^m}}{1 + i_{d^b}}. \tag{17}$$

From the constraint, the interest rate for bank deposits satisfies:

$$i_{d^b} = (R - k^b) + \frac{\ell\,(i_\ell - R)}{d^b} - \frac{\chi(\ell)}{d^b} - \frac{\theta}{1-\theta}\left[\frac{u(q)}{\phi d^b} - (1 + i_{d^m})\right]. \tag{18}$$

Lastly, the optimal quantity deposited at MMFs satisfies:

$$d^m = m - pu'^{-1}\left(\frac{1 + i_{d^m}}{1 + i_{d^b}}\right). \tag{19}$$

The zero-profit condition for MMFs in the deposit market yields the interest rate paid on MMF deposits:

$$i_{d^m} = r - k^m + \frac{z^m}{d^m}(\rho - r). \tag{20}$$

4.3 Characterization of Equilibrium

By assumption, the demand for repo borrowing is determined by the central bank, since $z^d = p^g b^d = p^g(B - b^{CB})$. Furthermore, in equilibrium, it must be that the price of government bonds, p^g, satisfies $1/p^g - (1 + \rho) = 0$. Market clearing in the repo market requires

$$z^m = p^g b^d. \tag{21}$$

Proposition 1. *There exist two possible equilibrium regimes in the repo market: An excess liquidity regime where $\rho = r$ and $d^{ONRRP} \geq 0$ and a scarce liquidity regime where $\rho \geq r$ and $d^{ONRRP} = 0$.*

The proof of Proposition 1 can be found in Appendix C

Since the demand for liquidity in the repo market is fixed, there are two possible regimes in equilibrium. First, there is an excess liquidity regime in which the aggregate supply of liquidity held by MMFs is larger than the demand for liquidity by dealers. In that case, the repo rate is equal to the ON RRP rate and MMFs lend the quantity demanded by dealers in the repo market and deposit the remaining funds at the ON RRP facility. Second, there is a scarce liquidity regime in which the repo rate is above the ON RRP rate. In that case, MMFs lend all of their deposits in the repo market and ON RRP takeup is zero. Consequently, there exits a quantity of bonds held by dealers, $\tilde{b}^d$, such that demand for liquidity by dealers equals the total available supply of liquidity from MMFs and the repo rate is still at the floor. This critical threshold $\tilde{b}^d$ satisfies $m - d^b = p^g b^d$ at $\rho = r$.

Using Equation (19), the critical threshold $\tilde{b}^d$ at which demand for liquidity in the repo market is equal to the aggregate supply of available liquidity from MMFs (i.e., $z^d = d^m$) satisfies

$$m - pu'^{-1}\left(\frac{1 + i_{d^m}}{1 + i_{d^b}}\right) = p^g \tilde{b}^d$$

when $\rho = r$.

Consequently, the repo rate satisfies

$$\rho = r \tag{22}$$

if $b^d \leq \tilde{b}^d$ and

$$m - pu'^{-1}\left(\frac{1 + i^m_d}{1 + i_{d^b}}\right) = p^g b^d \tag{23}$$

if $b^d > \tilde{b}^d$.

Next, using Equation (20) implies that the interest rate on MMF deposits satisfies

$$i_{d^m} = \rho - k^m. \tag{24}$$

Consequently, the critical threshold $\tilde{b}$ satisfies:

$$\frac{1}{p^g}\left[m - pu'^{-1}\left(\frac{1 + r - k^m}{1 + i_{d^b}}\right)\right] = \tilde{b}^d. \tag{25}$$

Proposition 2 (Definition of Equilibrium). *An equilibrium is a policy* (R, r, ϕ, b^{CB}) *and endogenous variables* $(p^g, \rho, i_{d^b}, i_{d^m}, d^b, d^m, z^m, \tilde{b}^d, \ell, x)$ *satisfying Equations (7), (8), (17), (18), (19), (21), (24), (25),*

$$\frac{x}{\phi} = \ell(1 + i_\ell) + m_r(1 + R) + d^{ONRRP}(1 + r) - m(k^b + k^m) + d^b k^m + d^m k^b + b^d, \tag{26}$$

and Equation (22) if $b^d \leq \tilde{b}^d$ *or Equation (23) if* $b^d > \tilde{b}^d$.

The proof of Proposition 2 can be found in Appendix C.

4.4 Properties of Equilibrium

Define the spread $s = R - r$, which is the difference between the central bank's administered rates. We can therefore redefine the policy rate $r = R - s$. In what follows, we discuss the effects of interest rate hikes and balance sheet reductions under the assumption that the spread s remains constant when the central bank increases the policy rate.

Additionally, denote the minimal level of reserves that banks have to hold $\tilde{m}_r$. $\tilde{m}_r$ satisfies

$$\tilde{m}_r = \delta d^b. \tag{27}$$

$\tilde{m}_r$ is the level of reserves in equilibrium at which the reserve constraint starts binding. If the optimal quantity of loans is such that $m_r < \tilde{m}_r$, banks have to either lend less or raise more funding in the deposit market.

Complementarity between policy rate hikes and balance sheet runoff.

Proposition 3. *The critical value $\tilde{b}^d$ is increasing in the policy rate R.*

The proof of Proposition 3 is in Appendix C.

Proposition 3 implies that the maximum level of borrowing that the repo market can absorb without repo rates rising above the ON RRP offering rate is increasing with the level of the policy rate. The critical threshold $\tilde{b}^d$ can be translated to the minimum size of the central bank's balance sheet with the repo rate equal to the ON RRP rate. Thus, the total amount that the central bank can shrink its balance sheet while maintaining rate control is larger when the policy rate is higher, suggesting a complementarity between the two policy tools.

Transmission of the policy rate to deposit rates. Since $i_{d^m} = \rho - k^m = r - k^m$ in an excess liquidity regime, the interest rate on MMF deposits increases one-for-one with the ON RRP rate due to perfect competition in the market for MMF deposits. Thus, $\mathrm{d}i_{d^m}/\mathrm{d}r = 1$.

Since the market for bank deposits is not perfectly competitive, the bank deposit rate may not adjust one-for-one with the policy rate. Following the proof of Proposition 3, the effect of an increase in the policy rate on the bank deposit rate satisfies:

$$\frac{\mathrm{d}i_{d^b}}{\mathrm{d}r} = \frac{\frac{1}{1-\theta} - (\omega_1 - \omega_2 + \omega_3 - \omega_4)}{1 - \left(\frac{1+i_{d^m}}{1+i_{d^b}}\right)(\omega_1 - \omega_2 + \omega_3 - \omega_4) + \omega_5 - \omega_6 + \omega_7}, \tag{28}$$

where $\omega_1 = \frac{\theta}{1-\theta}\frac{u'(q)}{u''(q)}\frac{1}{q}$, $\omega_2 = \frac{\theta}{1-\theta}\frac{u(q)}{u''(q)}\frac{1}{q^2}$ and $\omega_3 = \frac{\phi(\ell i_{\ell-R})}{u''(q)q^2}$, $\omega_4 = \frac{\phi\chi(\ell)}{u''(q)q^2}$, $\omega_5 = \frac{\theta}{1-\theta}\frac{u(q)}{q}$, $\omega_6 = \frac{\phi(\ell i_{\ell-R})}{q}$, and $\omega_7 = \frac{\phi\chi(\ell)}{q}$. Rearranging, the transmission of increases in the policy rate to the bank deposit rate is less than one-for-one if

$$\theta > \underline{\theta} = \frac{\left(\frac{\phi\ell(i_\ell - R)}{q} - \frac{\phi\chi(\ell)}{q}\right)\left(1 - \frac{i_{d^b}-i_{d^m}}{1+i_{d^b}}\frac{1}{u''(q)q}\right)}{\left(\frac{i_{d^b}-i_{d^m}}{1+i_{d^b}}\right)\left(\frac{u'(q)}{u''(q)q} - \frac{u(q)}{u''(q)q^2} - \frac{\phi\ell(i_\ell-R)}{u''(q)q^2} + \frac{\phi\chi(\ell)}{u''(q)q^2}\right) - 1 + \frac{u(q)}{q} + \frac{\phi\ell(i_\ell-R)}{q} - \frac{\phi\chi(\ell)}{q}} \tag{29}$$

Thus, if the bargaining power of the bank is sufficiently large, the deposit rate will increase by less than one-for-one with the policy rate.

Assuming that this condition holds, increasing the policy rate leads to a one-for-one increase in MMF deposit rates but a less than one-for-one increase in bank deposit rates. This implies that MMF deposits become relatively more attractive and thus more money flows to MMFs as the policy rate increases. This result is consistent with the empirical evidence presented in Section 3. When MMFs hold more deposits, they have more to lend in the repo market. Consequently, the critical threshold $\tilde{b}^d$ is higher because MMFs can lend more in the repo market without the repo rate increasing beyond the ON RRP rate. Thus, as stated in Proposition 3, the higher the policy rate is, the lower the central bank's holdings of government bonds can be.

Excess liquidity in the repo market and excess reserves. We next discuss how excess and scarce liquidity regimes in the repo market relate to excess and scarce reserve holdings by banks. To do so, we will define two threshold values for the quantity of bonds held by the central bank and a corresponding value for the quantity of bank deposits. Values denoted with a tilde represent the thresholds between the excess and scarce liquidity or reserves. In the case of the repo market, this is where $\rho = r$. Values denoted with an upper bar represent the thresholds within the scarce liquidity regime in the repo market between firm and weak rate control; that is, where $\rho = R$.

Define $\bar{b}^d$ as the level of government bonds held by dealers at which the repo rate equals IORB, $\rho = R$. $\bar{b}^d$ satisfies

$$p^g \bar{b}_d = m - d^b, \tag{30}$$

at $\rho = R$. $\bar{b}^d$ can be thought of as the upper bound on dealer bond holdings such that the repo rate does not exceed IORB R and thus the central bank maintains firm rate control.

Rearranging, bank deposits, denoted $\bar{d}^b_d$ when $b^d = \bar{b}^d$ satisfy

$$\bar{d}^b_d = m - p^g \bar{b}^d.$$

Thus, $\bar{d}^b_d$ are bank deposits associated with bond holdings of dealers that are equal to the upper bound. Consequently, $\bar{d}^b_d$ are bank deposits when $\rho = R$.

Denote further bank deposits when $b^d = \tilde{b}^d$, $\rho = r$, and $d^{ONRRP} = 0$ as $\tilde{d}^b_d$. Thus, $\tilde{d}^b_d$ satisfies

$$\tilde{d}^b_d = m - p^g \tilde{b}^d.$$

Thus, $\tilde{d}^b_d$ are bank deposits when bond holdings of dealers are equal to the critical threshold $\tilde{b}^d$ and consequently $\rho = r$.

Lastly, recall that the minimum level of reserves $\tilde{m}_r$ is a fraction δ of bank deposits, thus $\tilde{m}_r = \delta d^b$. Denote the level of bank deposits $\tilde{d}^b_m$, at which the constraint is binding and the quantity invested into loans satisfies $\ell = \ell^*$, where $\ell^* = \chi'^{-1}(i_\ell - R)$. Thus, $\tilde{d}^b_m$ satisfies

$$\tilde{d}^b_m = \frac{\ell + \chi(\ell)}{1 - \delta}.$$

From Equation (17), it is straightforward to see that bank deposits d^b at $b_d = \tilde{b}_d$ are larger than bank deposits at $b_d = \bar{b}_d$ since $R > r$ and therefore $\tilde{d}^b_d > \bar{d}^b_d$ for any R. From Proposition 3, we further know that bank deposits are decreasing in the policy rate R, assuming a constant spread between IORB and the ON RRP rate. Note further that for a constant spread between the lending rate and IORB, $i_\ell - R$, $\tilde{d}^b_m$ is constant in R.

Given these observations, there may exist an intersection at which the level of deposits at $b^d = \tilde{b}^d$ is equal to the level of deposits when the constraint on reserves is binding and banks still invest the optimal quantity $\ell = \ell^*$ in loans. Such an intersection occurs at the point where

$$\frac{\ell + \chi(\ell)}{1 - \delta} = m - p^g \tilde{b}^d.$$

Similarly, there may exist an intersection where the quantity of bank deposits when $b^d = \bar{b}^d$ is equal to the quantity of bank deposits at which the constraint on reserves is binding and banks still invest the optimal quantity $\ell = \ell^*$ in loans. This second intersection occurs at the point where

$$\frac{\ell + \chi(\ell)}{1 - \delta} = m - p^g \bar{b}^d.$$

Assuming that both these intersections exist for positive values of R, the level of bank deposits for each of these three critical thresholds—$\tilde{d}^b_m$, $\tilde{d}^b_d$, and $\bar{d}^b_d$—is depicted in Figure 3. First, for any quantity of deposits that is larger than $\tilde{d}^b_m$ (red line), the constraint on reserves is not binding and consequently, banks hold excess reserves, since the optimal quantity of loans ℓ^* is assumed to be constant in R. At any quantity of bank deposits below this threshold, banks are no longer able to invest the optimal quantity of loans while still holding reserves at or above the constraint. Thus, in this area (below the red line), the constraint on reserve holdings is binding. We refer to this area as the area of scarce reserves. Further, by definition, if the quantity of bonds held by dealers is larger than $\tilde{b}^d$, the repo rate will increase and ON RRP take-up is zero. Recall $p^g \tilde{b}^d = m - d^b$, thus any level of $b^d > \tilde{b}^d$ implies that bank deposits are lower. Therefore, in the area below $\tilde{d}^b_d$ (blue line), $\rho > r$ must hold. Since at this stage, the threshold $\bar{d}^b_d$ has not been reached yet, $R > \rho > r$ must hold. Lastly, by definition if the quantity of bonds held by dealers $b^d > \bar{b}^d$, the repo rate increases above IORB. Therefore, in the area below $\bar{d}^b_d$ (green line), $\rho > R$ must hold.

This yields six possible regimes in equilibrium with either excess or scarce reserves, excess or scarce liquidity in the repo market, and firm or weak rate control.

First, there is a regime in which there is excess liquidity in the repo market, such that the repo rate is equal to the ON RRP rate, there is positive takeup at the ON RRP facility and, simultaneously, banks have enough funding such that they hold excess reserves (above the red and the blue lines). This would be a case when the central bank holds a relatively large quantity of government bonds, such that demand for repo is low relative to the available liquidity by MMFs. Furthermore, banks have a relatively large quantity of bank deposits, such that they can invest the optimal quantity in loans and still hold excess reserves. Such a scenario could for example exist when the marginal return on loans relative to IORB is small and there are substantial costs to issuing a loan. This is likely the starting point before the central bank begins a tightening cycle when none of the constraints on the central bank's balance sheet bind. If the quantity of deposits held by banks is lower, the reserve constraint will start binding, while there still exist excess liquidity in the repo market, such that $\rho = r$ and $d^{ONRRP} \geq 0$ (above the blue line and below the red line). This could be a scenario in which the bargaining power of banks is relatively high such that there is a substantial difference between MMF and bank deposit rates and banks have an incentive to invest all available funds into loans.

Next, as the central bank reduces its quantity of bond holdings, dealers must finance a larger quantity of bonds in the repo market, thereby increasing demand for repo. MMFs first respond by depositing less at the ON RRP facility. Once takeup at the ON RRP facility has reached zero (the blue line), the repo rate must increase in order for MMFs to attract more funding. That is, liquidity becomes scarce in the repo market. However, rate control is still firm since $R \geq \rho \geq r$. At this stage, it can still be possible for banks to hold excess reserves, if the quantity of bank deposits is large and the incentive to lend relatively low. In this regime, $R \geq \rho \geq r$ and $d^{ONRRP} = 0$, while the reserves constraint is not binding (between the green and blue lines and above the red line). If, however, bank deposits are low and the incentive to lend is relatively high, banks may choose to only hold the minimum level of reserves (between the green and blue lines and below the red line).

Finally, if the central bank reduces its quantity of bond holdings further, the increase in demand for repo funding by dealers leads the repo rate to increase beyond IORB, resulting in weak rate control in addition to scarce liquidity in the repo market. In this setting, it may be again the case that banks choose to hold excess reserves as the incentive to lend may be low (below the green and above the red lines). Alternatively, if bank deposits are relatively low and the incentive to lend is relatively high, banks may only choose to hold the minimum level of reserves (below the green and red lines).

Note, these results depend crucially on a few assumptions. First, we assume in the model that banks are not able to participate in the repo market. If banks could lend in the repo market, banks may have an incentive to lend in repo instead of issuing loans or holding reserves. This increase in the supply of liquidity would put downward pressure on the repo rate. Similarly, if banks could borrow in the repo market, an area where the repo rate is equal to the ON RRP rate with scarce reserves may not exist as banks could increase their funding by borrowing in the repo market, which could then be either held as reserves or invested into loans. Moreover, to be consistent with the situation in the US, we assume that dealers cannot hold reserves and therefore cannot earn IORB at the central bank, which implies that R is not a ceiling for the repo rate. Lastly, given the setup of the model, it is possible that only one or even none of the intersections depicted in Figure 3 exist. In such cases, not all of the six regions described here may exist.

We revisit these possible equilibria in our calibrated environment in Section 6 to undersatnd their implications for the size of the central bank's balance sheet in an ample reserve regime.

5 Calibration

In this section, we calibrate the structural parameters of the model to match moments in the data during the Fed's current policy tightening cycle between March 12, 2022 and October 14, 2022. We use this calibration to demonstrate that the model can match the data fairly well and then use the calibrated model to forecast the level of reserves at the critical thresholds $b^d = \tilde{b}^d$ and $b^d = \bar{b}^d$ as well as the minimum level of reserves demand.

5.1 Mapping the model to the data

Identifying agents and trades. Before we calibrate the model, we need to map the model to the data. The decisions and investment options of MMFs in the model best describe the investment decisions of government MMFs. Banks and dealers in the data are mapped to depository institutions and primary dealers, respectively. We map MMF deposits to government MMFs' assets that are invested in repo and at the ON RRP facility and bank deposits to aggregate bank deposits held by depository institutions. Government bond holdings of the central bank are mapped to the Treasury security holdings of the Federal Reserve. Government bonds held by dealers are mapped to the Treasury security holdings of primary dealers. The bank lending rate is mapped it to the weighted average return of commercial bank loans and securities.

Data. To calibrate the model, we use data from March 12, 2022 to October 14, 2022. The Fed's pandemic-era net asset purchases ended on March 11, 2022, which is why we start our calibration period on March 12. On March 16, the Fed began increasing its policy rate. On June 1, the Fed began "balance sheet runoff" and allowed maturing Treasury securities and agency MBS to run off the balance sheet up to monthly caps of $30 and $17.5 billion for Treasury securities and agency MBS, respectively. These caps were increased to $60 and $35 billion for Treasury securities and agency MBS, respectively, beginning on September 1.

For the calibration, we use of the following data sources. The Fed's administered rates—IORB and ON RRP offering rates—are publicly available from the Federal Reserve Bank of New York (FRBNY) website. We use confidential daily data on the Fed's balance sheet to retrieve the Fed's holdings of Treasury securities and reserve balances held at Federal Reserve Banks.[22]

We use data on the amount of government MMF assets invested in Treasury repo from SEC N-MFP filings, which captures both private market repo and the ON RRP, at a monthly frequency. Next, we use confidential ON RRP take-up data to calculate aggregate daily government MMF take-up at the ON RRP.[23] We calculate government MMF lending in the repo market as the difference between total government MMF Treasury

[22]Weekly values of Treasury securities held by the Fed and total reserve balances are public and can be found in the H.4.1 statistical release available on the Board of Governor's website.

[23]Aggregate ON RRP take-up is publicly available on the FRBNY website. ON RRP take-up by counterparty type can also be found on the Office of Financial Research website, which is publicly available with a lag.

repo investments from the N-MFP filings and the total government MMF take-up at the ON RRP. The interest rate on MMF deposits is the net seven day yield for government MMFs from iMoneyNet at a weekly frequency. Our repo rate is the Tri-party General Collateral Rate (TGCR), publicly available daily from the FRBNY website.

Bank deposits are the sum of all interest-bearing deposits (other than large time deposits) for all commercial banks, publicly available at a monthly frequency on the Fed's website from the H.8 data release of the assets and liabilities of commercial banks in the United States. The interest rate for bank deposits is the average rate on interest-bearing checking accounts from RateWatch, available weekly. The average interest rate on loans is the aggregate weighted average return on loans and securities held, using total amounts outstanding and total interest income values taken from the quarterly FFIEC Call Report.

Targets. IORB, the ON RRP rate, the quantity of bonds held by the central bank as well as the aggregate quantity of bonds held by the central bank and dealers, the interest rate on loans, and the minimum reserve-to-deposit ratio are taken directly from the data. R is set equal to the average IORB and r is set equal to the average ON RRP offering rate. The nominal amount of bonds held by the central bank $p^g b^{CB}$ is set equal to the average nominal quantity of Treasuries held by the Federal Reserve and the nominal quantity of government bonds in the environment $p^g B$ is set to equal the sum of Treasury holdings of the Federal Reserve and dealers. The interest rate on loans is set equal to the weighted average return on loans and security holdings during the calibration period.[24] For the parameter δ that governs the minimal acceptable ratio of reserves-to-deposits for banks, we take the average reserves-to-deposits ratio of banks over the first two weeks of September 2019.[25] A summary of these parameters can be found in Table 3.

For the utility function, we assume $u(q) = 1/(1-\alpha)q^{1-\alpha}$, which implies $q = \left(\frac{1+i_{d^b}}{1+i_{d^m}}\right)^{1/\alpha}$ from Equation (17). For the cost of loan origination, we assume $\chi(\ell) = 0.5\beta\ell^2$, which implies $\ell = \frac{i_\ell - R}{\beta}$ from Equation (7), if the constraint is not binding.

With these assumptions, the parameters left to determine are k^b, k^m, θ, α, ϕ, β, and m_e. We calibrate these parameters to match the average TGCR (the repo rate), the average bank deposit rate, the average interest rate on MMF deposits, the average correlation between bank deposit rates and the policy rate, the average markdown of bank deposit rates relative to the weighted average return of bank loans and reserves, average aggregate

[24] The data does not allow us to distinguish between the interest rates on newly issued loans and the interest rate on existing loans. For that reason, we take the weighted average loan rate of newly issued and existing loans during the calibration period.

[25] In mid-September 2019, the repo rate spiked as demand for liquidity increased, suggesting that in that period, the economy was no longer in an ample reserve regime. With this approach, we can be ambiguous about the exact interpretation of the constraint on reserve holdings in the model and approach it as a summary of both regulatory constraints and preferences of banks.

bank deposits, average aggregate reserves held by banks, average aggregate ON RRP take-up, and aggregate bank and MMF deposits less government bond holdings by the central bank. The latter target allows us to back out the level of endowment m_e that households receive in order to match aggregate bank deposits.

Since, in our model, the number of banks, dealers, and MMFs are normalized to one, we divide aggregate bank deposits and loan issuance by the number of banks, denoted n^b, aggregate MMF assets that are invested in the ON RRP or the repo market by the number of MMFs, denoted n^m, and dealer bond positions by the number of dealers, denoted n^d. Recall, in the model, we assume that the spread between the lending rate and IORB, $i_\ell - R$, is constant.

Since our calibration period corresponds to a period of more than ample reserves, the repo rate satisfies

$$\rho = r.$$

Further, in an ample reserves framework, the constraint on reserve demand should not be binding and thus $\ell = (i_\ell - R)/\beta$. Under these assumptions, the interest rate on bank deposits satisfies

$$i_{d^b} = (R - k^b) + \frac{(i_\ell - R)^2}{\beta} \frac{1}{2} \left[\frac{\phi(1 + i_{d^m})^{1/\alpha}}{(1 + i_{d^b})^{(1/\alpha)-1}} \right] - \frac{\theta}{1-\theta} \frac{\alpha}{1-\alpha} (1 + i_{d^m}).$$

The MMF deposit rate satisfies

$$i_d^m = r + k^m.$$

The correlation of bank deposit rates and the policy rate is given by totally differentiating Equations (18), (4), and (17) and satisfies

$$\frac{di_{d^b}}{dr} = \frac{\frac{1}{1-\theta} + \frac{\theta}{1-\theta}\frac{1}{\alpha} - \frac{\theta}{1-\theta}\frac{1}{\alpha(1-\alpha)} + \frac{1}{2}\frac{\phi(i_\ell - R)^2}{\alpha\beta}\left(\frac{1+i_{d^b}}{1+i_{d^m}}\right)^{1-(1/\alpha)}}{1 + \frac{1+i_d^m}{1+i_{d^b}}\left(\frac{\theta}{1-\theta}\frac{1}{\alpha} - \frac{\theta}{1-\theta}\frac{1}{\alpha(1-\alpha)} + \frac{\theta}{1-\theta}\frac{1}{1-\alpha} + \frac{1}{2}\frac{\phi(i_\ell - R)^2}{\alpha\beta}\left(\frac{1+i_{d^b}}{1+i_{d^m}}\right)^{1-(1/\alpha)}\right) - \frac{1}{2}\frac{\phi(i_\ell - R)^2}{\beta}\left(\frac{1+i_{d^b}}{1+i_{d^m}}\right)^{-1/\alpha}}.$$

Following Aruoba et al. (2011), we define the markup μ as price over marginal costs. In a perfectly competitive market, $\mu = 0$, such that $1 + \mu = p/MC = 1$. In our deposit market, perfect competition would imply that the bank deposit rate is set such that banks do not make any profits. We proxy this hypothetical deposit rate by the weighted average return on bank loans and reserves. In our model with the assumptions for the utility function, the markup therefore satisfies

$$1+\mu = \frac{(R-k^b) + \frac{(i_\ell - R)^2}{\beta}\frac{1}{2}\left[\frac{\phi(1+i_{d^m})^{1/\alpha}}{(1+i_{d^b})^{(1/\alpha)-1}}\right] - \frac{\theta}{1-\theta}\frac{\alpha}{1-\alpha}(1+i_{d^m})}{(R-k^b) + \frac{(i_\ell - R)^2}{\beta}\frac{1}{2}\left[\frac{\phi(1+i_{d^m})^{1/\alpha}}{(1+i_{d^b})^{(1/\alpha)-1}}\right]}.$$

Note that the markup in this setup is technically a markdown, as deposit rates tend to be lower relative to a perfectly competitive market. We think of the markup in the deposit market as a negative markup. We argue that the relevant variable in the data for the markup of bank deposit rates is the weighted average return on banks' assets. As banks would not make any profits in a perfectly competitive market, the deposit rate would be equal to the weighted average return on assets.

Further, aggregate average bank deposits satisfy

$$n^b d^b = n^b \left(\frac{1}{\phi}\frac{(1+i_{d^b})^{1/\alpha-1}}{(1+i_{d^m})^{1/\alpha}}\right),$$

Denote reserve holdings m_r. Aggregate average reserve holdings satisfy

$$n^b m_r = n^b \left(d^b - \frac{i_\ell - R}{\beta}\left[1 + \frac{i_{\ell - R}}{2}\right]\right)$$

Aggregate average ON RRP take-up is defined as the difference between MMF deposits and repo lending by MMFs and can be rearranged such that

$$n^m d^{ONRRP} = n^m d^m - \left(\frac{1}{1+\rho}(B - b^{CB})\right).$$

Finally, the initial endowment of commodity money m_e is the difference between the aggregate endowment of households, which can be allocated to either bank or MMF deposits, $d^m + d^b$, and the endowment that stems from the sale of government bonds, $p^g B$. Thus, the initial endowment of commodity money satisfies

$$m_e = n^m d^m + n^b d^b - \frac{1}{1+\rho}B.$$

We calibrate the model by solving for the parameters $\mathcal{P} = \{\alpha, \beta, \theta, k^b, k^m, \phi, m_e\}$ and the equilibrium variables $\mathcal{X} = (\rho, i_{d^b}, i_{d^m}, d^b, m_r, d^{ONRRP})$, such that the squared distance between the parameters that solve the model and the moments in the data are minimized.

$$\min_{X;\mathcal{P}} (\mathcal{S}_{model}(\mathcal{X};\mathcal{P}) - \mathcal{S}_{data})^2$$

$$\text{s.t. } EC(\mathcal{X};\mathcal{P}) = 0.$$

5.2 Results of the Calibration

In this section, we present the results of our calibration. Table 4 shows our calibrated parameters and Table 5 presents how well the calibrated model matches the moments of our data.

From Table 5, we observe that the model matches our moments very well. The repo rate, the interest rate on bank deposits, and the interest rate on MMF deposits are close to the values in the data, with differences of only 3, 1, and 1 basis points, respectively. Regarding quantities, we match aggregate reserves exactly to the data and the model's predictions of bank deposits and ON RRP take up are quite close to the data as well, with a difference of \$60 and \$2 billion, respectively.

Comparing additional implied values from the model to the data within the calibration period, we find that the estimated quantity of issued loans of \$13.12 trillion is relatively close to total commercial bank loans at \$11.45 trillion. The calibrated model furthermore predicts the quantity of \$111 billion in repo lending by MMFs, which corresponds roughly to total government MMF investment in Treasury repo markets of \$149 billion. Lastly, total assets of MMFs in the model are estimated to be \$1.81 trillion, which is very close to the total quantity invested in the ON RRP and repo market by MMFs of \$1.82 trillion.

To assess the fit of the model further, we test how well the model can predict certain variables after the November 2022 FOMC meeting, which is just after the end of our calibration period. At this meeting, the FOMC decided to increase the policy rate by 75 basis points, such that IORB was 3.9% a day after the meeting. For this test, we adjust the values of IORB R, the ON RRP rate r, and the nominal quantity of Treasury securities held by the Fed, as noted in Table 6. The results are summarized in Table 7. The model roughly matched TGCR and the interest rate on MMF deposits, with a difference of 3 and and 7 basis points, respectively. The ratio of bank deposits relative to aggregate deposits is slightly higher in the model by about 0.7 percentage points compared to the data. The share of reserves relative to the sum of bank deposits, MMF deposits, and government bond holdings by the Fed and dealers (M) is also slightly higher in the data, by about 0.6 percentage points. The predicted level of ON RRP take-up is fairly close to the data, with a difference of about \$100 billion. Lastly, the model significantly overshoots the predicted value of the bank deposit rate relative to the data, at 2.38% relative to 12 basis points in the data. This large miss is likely why the model overshoots bank deposits and reserves.

While the model does very well overall, a primary issue is that it predicts a pass-through of the policy rate to the bank deposit rate that seems too high compared to recent experiences. It's possible that the relatively short calibration period leads to an overestimation of this pass-through. A longer time horizon in which the policy rate is higher and bank deposit rates remain low would allow us to calibrate the bargaining power of banks (θ) more accurately to reflect this low pass-through.

Finally, we test the fit of the model for the mid-September 2019 period. At that time, the repo rate had an unprecedented spike from 2.42% to 5.25%, indicating that at this point in time, the economy was no longer in an ample liquidity regime. The model should be able to replicate the switch from an excess to a scarce liquidity regime when using data on the policy rate and Fed holdings of Treasury securities relative to the sum of Fed and dealer Treasury holdings, shown in Table 8. Table 9 depicts the results of this test. First, the model can indeed predict the switch to a scarce liquidity regime with the repo rate far exceeding the ON RRP rate. The model however predicts a TGCR that is about 5.73 percentage points too high relative to the TGCR observed in mid-September 2019. Since we assume that the repo rate is transferred one-to-one to the MMF deposit rate, the model also significantly overshoots the MMF deposit rate. The bank deposit rate interest is roughly in line with the data with a difference of about 34 basis points. The model can furthermore match the ratio of bank deposits to aggregate deposits and the ratio of reserves to endowment (which consists of bank deposits, MMF deposits, and aggregate bond holdings by the central bank and dealers) fairly well. ON RRP take-up was zero in September 2019, which is also roughly what the model predicts.

The very high repo rate that the model predicts in September 2019 suggests that the puzzle of mid-September is not why repo rates spiked so much at that time, but rather why they didn't spike sooner. Given the large portion of government bonds held by dealers that needed to be financed in the repo market as a result of the Fed's balance sheet runoff combined with the size of MMF assets at the time, there was a significant liquidity mismatch in the repo market that should have led to high repo rates. While repo rates were relatively stable and low prior to September 2019, there was some evidence of these emerging imbalances with higher repo rates on quarter-ends throughout 2019. However, the model's significant overestimation of the repo rate in September 2019 may also arise from some features of the model, including the assumption that all dealer bond holdings need to be financed in the repo market, the absence of frictions in the repo market, the assumption that banks cannot participate in the repo market, and the absence of central bank intervention.[26]

[26]Furthermore, we assume in the model that the repo market and the ON RRP are the only investment options for MMFs. Since ON RRP was zero at this time, MMFs in the model needed to attract new funds to invest more in repo. In reality, MMFs also invest in government bonds. Increases in the repo rate would make lending in repo more attractive than investing in government bonds and therefore with the spike in the repo rate, MMFs could also move funds out of government bonds and into the repo market, which would reduce the spike in the repo rate. However, given their constraints, there is little evidence in the data that MMFs did much reallocation like this to take advantage of higher repo rates in mid-September.

Overall, despite some limitations, these tests show that the model does quite well in predicting excess and scarce liquidity regimes in the repo market and the level of ON RRP take-up, reserves, and deposits.

6 Discussion

Using our calibrated model, we now discuss the comparative statics of tightening monetary policy through (i) increasing the policy rate or (ii) reducing the size of the central bank's balance sheet in equilibrium.

6.1 Interest Rate Policy in the Excess Liquidity Regime

In equilibrium, the repo market can either be in a excess liquidity regime or in a scarce liquidity regime. In the excess liquidity regime, due to the competitive deposit market for MMFs and the competitive repo market, an increase in the policy rate is reflected one-for-one in the MMF deposit rate. However, because our model imposes bargaining power over deposits at banks, an increase in the policy rate is not fully passed onto household bank deposits. As a result, households are incentivized to deposit a larger share of their endowment with MMFs rather than banks, leading to an inflow of liquidity to MMFs, as policy rates increase.

Figure 4 shows the results of our model for the allocation of MMF deposits d^m and bank deposits d^b as R, the policy rate, increases. We observe that d^m increases while d^b decreases with R because MMFs increase their interest rate i_{d^m} more than banks increase their rate i_{d^b} as the Fed increases R (shown in Figure 5). In other words, bank deposit rates are stickier than MMF deposit rates as the Fed increases its policy rate, so households shift money from banks to MMFs. The larger pass-through of an increase in the policy rate R to MMF deposit rates relative to the pass-through to bank deposits is as shown in Figure 6 by the increasing spread between MMF deposit rates and bank deposit rates.

6.2 Transitioning from Excess Liquidity to Scarce Liquidity

Proposition 3 implies that there exists a critical value of government bonds held by dealers, $\tilde{b}^d$, where the repo rate equals the offering rate on the ON RRP (i.e., $\rho = r$) and there is no ON RRP take-up (i.e., $d^{ONRRP} = 0$). When b^d, the amount of government bonds held by dealers, is less than $\tilde{b}^d$, the demand by dealers in the repo market is low relative to the the supply of liquidity held by MMFs, $\rho = r$, and ON RRP take-up is above zero. As b^d increases, MMF lend more in repo to meet the increasing demand and ON RRP take-up declines. At $b^d = \tilde{b}^d$, ON RRP take-up has reached zero. At this point, a higher demand for repo borrowing by dealers (i.e. $b^d > \tilde{b}^d$) can only be met by the suppliers of liquidity, the MMFs, if MMFs can attract more funding. As a result, the repo rate increases beyond the ON RRP rate ($\rho > r$) and the MMF deposit rate increases as well. This critical threshold can also be expressed in the quantity of government bonds held by the central bank, denoted $\tilde{b}^{CB}$, where

$$\tilde{b}^{CB} = B - \tilde{b}^d. \tag{31}$$

Figure 7 presents the repo rate ρ as a function of the nominal size of the Fed's balance sheet ($p^g \times b^{cb}$). The vertical dashed line denotes $\tilde{b}^{CB}$, where above this dashed line the supply of liquidity by MMFs in the repo market is larger than b^d and the repo rate ρ equals r, the ON RRP offering rate. However, when b^{cb} is below $\tilde{b}^{CB}$, there is sufficient demand by dealers in the repo market to push ρ above the ON RRP rate and $\rho > r$. Correspondingly, Figure 8 shows that ON RRP take-up equals 0 when $b^{cb} \leq \tilde{b}^{CB}$. When b^{cb} is less than $\tilde{b}^{CB}$, there is scarce liquidity in the repo market in equilibrium.

Figures 9 plots MMF deposits and bank deposits, respectively, as a function of government bond holdings by the Fed ($p^g \times b^{cb}$). Looking at the right side of the graph first, as the Fed reduces the size of its balance sheet ($p^g \times b^{cb}$ declines), MMF and bank deposits are not affected at first. As long as $b^{CB} > \tilde{b}^{CB}$, MMFs lend more in repo by depositing less at the ON RRP and, consequently, there is no effect on the repo rate. Once b^{CB} crosses the threshold, MMF have to attract more funding to meet the increasing demand for repo by dealers. Thus, the repo rate and consequently the MMF deposit rate increase and households shift their deposits from banks to MMFs. Correspondingly, Figure 9 shows a decline in bank deposits as $p^g \times b^{cb}$ declines beyond the critical threshold. Our results are also consistent with the regression results in Table 1 that show that MMFs receive more deposits from households as the Fed decreases the size of its balance sheet.

6.3 Interest Rate and Balance Sheet Policy as Complements

Proposition 3 shows further that the critical threshold $\tilde{b}^d$ is increasing in the policy rate R. From Equation (31), it is straightforward that this implies that $\tilde{b}^{CB}$ is decreasing in the policy rate R, as Figure 10 shows. This result implies that as R increases, the Fed can unwind more of its balance sheet. Further, if the Fed increases R first, it has more room to use its balance sheet as a monetary policy tool. This suggests that, in addition to the standard substitutibility of the two tools, there is also a complementarity.

These results imply that, for example at $R = 4.40\%$, the equilibrium nominal quantity of Treasury securities held by the central bank consistent with firm rate control ($p^g \times \tilde{b}^{cb}$) is approximately \$3.57 trillion. With roughly \$5.75 trillion of Treasury securities on the

Fed's balance sheet during our calibration period (see Table 3), this suggests that, if the Fed ceased hiking rates in January 2023, the Fed's balance sheet could be reduced by approximately \$2.1 trillion and still remain in the excess liquidity regime. However, if the policy rate R were lower, for example at 1.5%, the Fed's balance sheet could only be reduced by about \$1.65 trillion and remain in the excess liquidity regime. On the other hand, if the Fed increased rates to the median projection for the 2023 federal funds rate in the December 2022 Summary of Economic Projections of 5.1%, the Fed's balance sheet could be reduced further, by about \$2.3 trillion cumulatively, and remain in the excess liquidity regime.

6.4 Stop believing in reserves

Lastly, we discuss how the minimum level of reserves demanded by banks relates to the critical threshold at which the economy moves from excess liquidity to scarce liquidity in the repo market. Importantly, as defined by the Federal Reserve, an ample reserves regime does not just require reserves to be above bank reserve demand, but also requires that short-term rate control is achieved via the setting of the Fed's administered rates (IORB and the ON RRP rate).[27] When thinking about the extent to which the Fed's balance sheet can shrink, much attention is on bank reserve demand. However, we show that ample liquidity also needs to be maintained in the repo market in order to maintain rate control. Moreover, in our calibrated model, this latter constraint actually binds first, leading to a larger ultimate size of the Fed's balance sheet than implied by reserve demand alone.

Recall that, as shown in Figure 3, there may exist six different equilibria, depending on the level of the repo rate, whether the constraint on the minimum level of reserves is binding or not, and whether the repo market is in an excess or scarce liquidity regime. Using the calibrated model, we find that three of these areas exist for policy rates between 1.5% and 8.5% (see Figure 11). The blue line represents the level of reserves when the level of government bond holdings b^{CB} equals the critical threshold $\tilde{b}^{CB}$. Consequently, the area above the blue line represents the space where there exists excess liquidity in the repo market, thus $\rho = r$ and $d^{ONRRP} \geq 0$. The area below the blue line represents the space where there exists scarce liquidity in the repo market and therefore $\rho > r$ and $d^{ONRRP} = 0$. The red line represents reserves at the critical threshold $\bar{b}^{CB}$, which satisfies

$$\bar{b}^{CB} = B - \bar{b}^{d}, \tag{32}$$

with $\bar{b}^{d}$ defined in Equation (30). This second critical threshold is the level of government

[27] See the January 2019 Statement Regarding Monetary Policy Implementation and Balance Sheet Normalization (https://www.federalreserve.gov/newsevents/pressreleases/monetary20190130c.htm).

bonds held by the Fed, at which the corresponding level government bonds held by dealers generates a demand for repo borrowing such that the repo rate equals IORB, $\rho = R$. The corresponding level of reserves is represented by the red line. Thus, the area between the blue line and the red line is where there is scarce liquidity in the repo market but the central bank still maintains firm rate control since $R \geq \rho \geq r$. The area below the red line is where the repo rate exceeds IORB, $\rho > R$, and thus the central bank only has weak rate control. Lastly, the orange line represents the level of reserves that are consistent with the minimum level of reserve demand. Consequently, the area above the orange line represents the space where banks hold excess reserves and the space below the orange line represents the space where banks hold the minimum level of reserves.

This implies that the minimum level of reserves demanded by banks is not a sufficient indicator to assess whether the economy is in an ample liquidity regime. Our results show that market rates start increasing at a level of reserves that is much higher than the indicated level of minimum reserves. Moreover, the level of reserves at which the repo rate reaches IORB, which recently has been set 10 basis points below the upper bound of the target range of the fed funds rate, is very close to the level of reserves at which repo rates begin to increase and therefore also much higher than the implied minimum level of reserves. Thus, if the central bank wants to maintain interest rate control and remain in an ample liquidity regime, the level of reserves has to be much higher. In particular, our estimates from the calibration imply that, at IORB of 4.4%, reserves would have to be between \$2.86 and \$2.9 trillion to be consistent with an ample liquidity regime relative to the level of minimum reserve demand of about \$2 trillion.[28]

Thus, for a central bank that wants to maintain an ample reserves framework, the critical threshold $\tilde{b}^{CB}$ is the effective constraint on the size of the central bank's balance sheet, and not the level of minimum reserve demand.

Our numerical estimates for the level of reserves at $b^{CB} = \tilde{b}^{CB}$ and $b^{CB} = \bar{b}^{CB}$ depend on a series of assumptions. First, we assume that all government bonds held by dealers are financed in repo. Historical FR 2004 data shows that in reality dealers only finance about half of their government bond holdings in the repo market. Less reliance by dealers on the repo market would imply that the central bank can shrink its balance sheet further before rate control is threatened, leading to a lower level of reserves. This suggests that our estimates of the level of reserves consistent with an ample liquidity regime are an upper bound.[29] Second, we assume that there are no frictions in the repo market. Frictions

[28]As noted in Section 6.3, maintaining excess liquidity in the repo market and rate control implies that, if the Fed ceased hiking rates in January 2023 at IORB of 4.4%, the Fed's balance sheet could be reduced by approximately \$2.1 trillion and still remain in the excess liquidity regime. However, there is not a direct mapping from the minimum reserve demand to the total size of the Fed's balance sheet since a variety of levels of the ON RRP can be consistent with minimum reserve demand.

[29]A related issue is that we calibrate our model only to the Fed's Treasury securities holdings. In reality,

such as persistent lending relationships could lead to a slower increase in repo rates when $b^{CB} < \tilde{b}^{CB}$. Furthermore, as mentioned above, the repo rate may also increase slower as the level of government bond holdings of the central bank declines beyond $\tilde{b}^{CB}$ if MMFs can move funds out of alternative investments such as government bills to the repo market, rather than needing to attract new funds from households. This would make the repo rate less sensitive to further declines in the central bank's balance sheet, leading to a lower level of reserves when the repo rate reaches IORB. Lastly, our estimate of the minimum level of reserve demand is based on the ratio of bank deposits and reserves at the end of August 2019. We interpret this constraint as a combination of both regulations that require banks to hold a certain amount of reserves and internal preferences of banks for holding reserves. Any changes in either regulations or preferences of banks to hold reserves may move the estimate for the minimum level of reserve demand in either direction.

7 Conclusion

In this paper, we aim to understand the initial transmission of monetary policy when policy is tightened via the Fed's balance sheet. In particular, we address two questions. First, how does balance sheet tightening affect the allocation of money between banks and shadow banks? We assess the deposit channels of monetary policy when policy is tightened via the Fed's balance sheet, rather than via raising interest rates. Empirically, we find evidence that banks do not lose deposits but shadow banks gain significant deposits during balance sheet tightening, and we model this accordingly.

Second, how do the reduction in both reserves and securities holdings of the Fed during balance sheet tightening affect money markets and the size of the Fed's balance sheet? We show that, contrary to the traditional focus on bank reserve demand alone, the repo market's capacity to bear the additional securities that the Fed is running off is actually more likely to constrain the size of the Fed's balance sheet in the current environment. This implies that the Fed's balance sheet will need to be larger than what bank reserve demand alone might suggest and that, within an ample reserves framework, the demand for money by shadow banks also needs to be considered. These findings have significant implications for the Fed's current tightening cycle and the eventual end point of its balance sheet runoff.

the Fed also holds a substantial amount of agency MBS. If dealers also absorb the agency MBS that the Fed runs off its balance sheet and finances all these securities in the repo market, the implications for total amount that the Fed can shrink its balance sheet are the same. However, agency MBS are also held by other types of institutions and, for the portion held by dealers, are not financed completely in repo to an even greater extent than for Treasuries. Less reliance on the repo market suggests that the Fed could shrink its balance sheet further than the estimates shown here.

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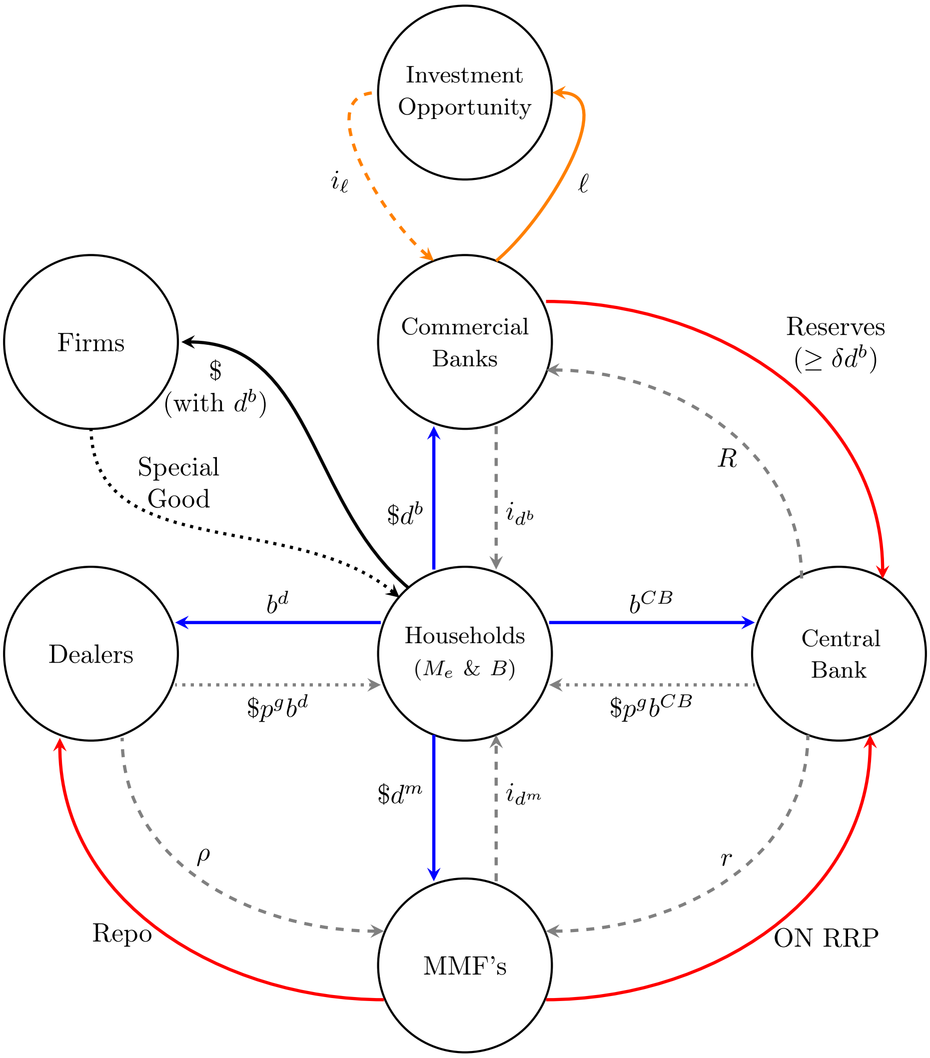
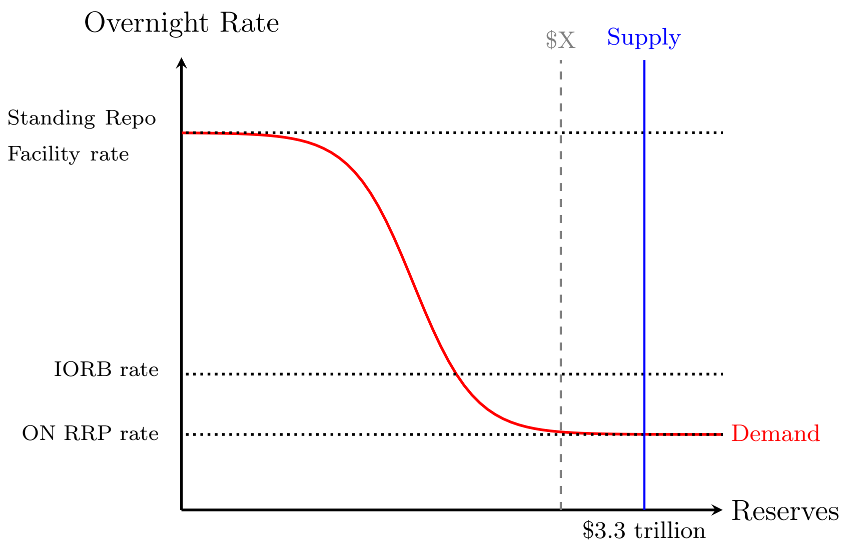
Figures and Tables

Figure 1: Schematic of Model



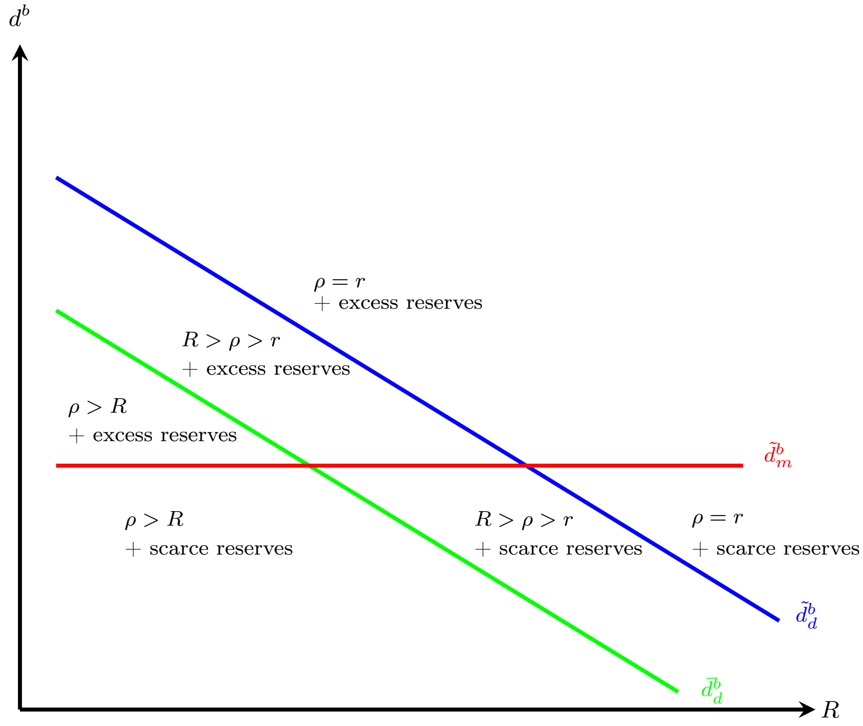
Note. This figure displays a flow chart of our model. Households have an initial bond endowment B. The central bank buys b^{CB} from the household at price p^g, and dealers buy b^d from the household at price p^g such that $B = b^{CB} + b^d$. In addition to their bond earnings $p^g B$, households have an initial money endowment M_e such that their total initial monetary wealth is $M = M_e + p^g B$. Households choose whether to invest their money into banks and earn i_{d^b}, or into MMFs and earn i_{d^m}. Households hold a nonzero amount of commercial bank deposits d^b to use as payment for the special good produced by firms. Dealers must finance their security holdings b^d by borrowing from MMFs in the repo market. MMFs have two choices of where they can invest the deposits they receive from households—they can lend to dealers in the repo market at ρ, or they can lend to the central bank via the ON RRP at r. Similarly, banks have two choices of where they can invest the deposits they receive from households—they can invest either at the central bank and earn IORB R, or they can make loans ℓ to an exogenous outside investment opportunity and earn i_ℓ.

Figure 2: Ample Reserves Framework



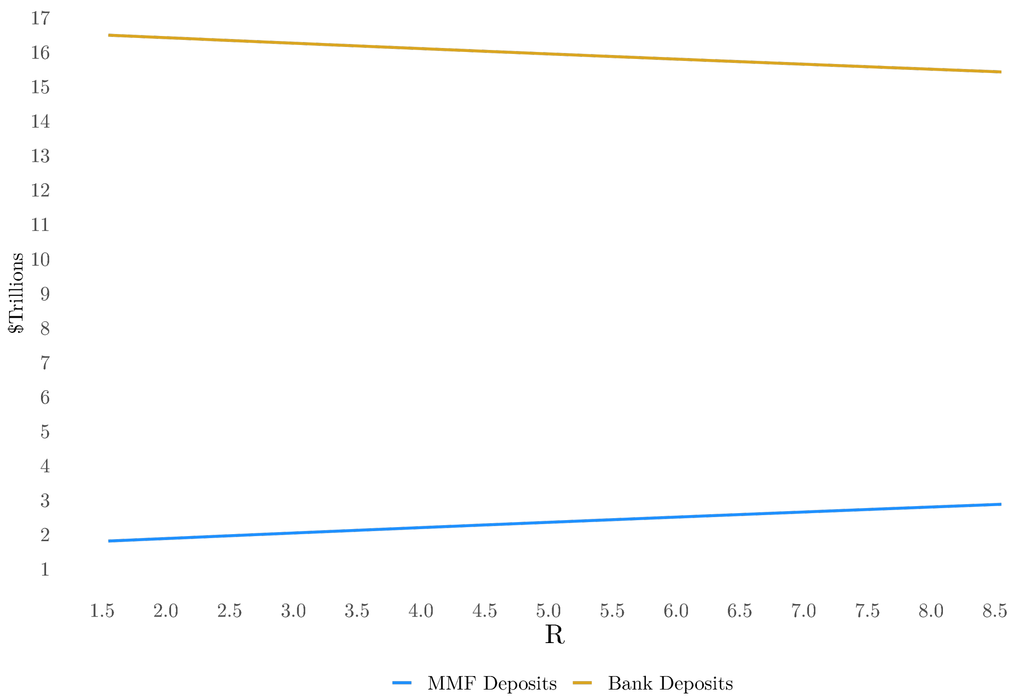
This figure shows the relationship between overnight rates and reserves. The red line denotes reserve demand and the blue line denotes reserve supply. Reserves were $3.3 trillion as of June 2022. $X denotes the minimum level of reserves consistent with an ample reserves environment. IORB rate is the interest rate on reserve balances at which banks can lend to the Fed. ON RRP rate is the offering rate at the Overnight Reverse Repo Facility at which banks and other institutions can lend to the Fed. Standing Repo Facility rate is the rate at which institutions can borrow from the Fed at the Standing Repo Facility.

Figure 3: Liquidity in the repo market and reserves



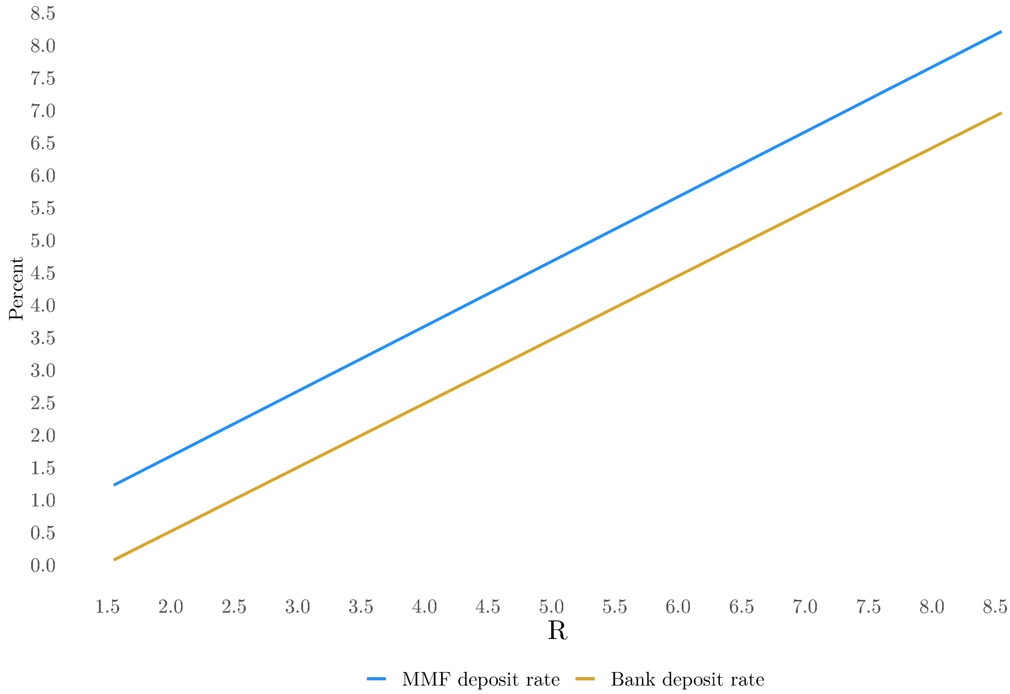
This figure shows the different possible regimes for an equilibrium that may exist. If bank deposits are such that banks hold the minimum amount of reserves, denoted $\tilde{d}^b_m$ (the red line), the economy has scarce reserves. If bank deposits are however large enough so that banks are not constrained in their reserve holdings, the economy has excess reserves. If furthermore bank deposits are larger than bank deposits at the critical threshold $\tilde{b}^d$, denoted $\tilde{d}^b_d$ (the blue line), then there exists excess liquidity in the repo market such that the repo rate is equal to the ON RRP rate and ON RRP take up is positive. If bank deposits are in between the level of bank deposits at $b^d = \tilde{b}^d$ and at $b_d = \bar{b}^d$ (between the blue and the green lines), then the repo market no longer has excess liquidity. The repo rate increases such that $R \geq \rho \geq r$ and ON RRP take-up is zero. Thus, in this area, the central bank still has firm interest rate control. If, however, bank deposits are below the level of bank deposits at $b^d = \bar{b}^d$, denoted $\bar{d}^b_d$ (the green line), then government bond holdings by dealers are large enough such that the repo rate increases beyond IORB, $\rho \geq R$, and the central bank only has weak interest rate control. Thus, there may exist six different regimes: (i) Excess liquidity in the repo market and excess reserves (above the red and the blue lines); (ii) Scarce liquidity in the repo market, firm interest rate control, and excess reserves (above the red line and in between the blue and the green lines); (iii) Scarce liquidity in the repo market such that $\rho \geq R$, that is weak interest rate control, and excess reserves (above the red line and below the green line); (iv) Scarce liquidity in the repo market with weak interest rate control and scarce reserves (below the red and the green lines); (v) Scarce liquidity in the repo market with firm interest rate control and scarce reserves (below the red line and in between the green and the blue lines); and (vi) Excess liquidity in the repo market and scarce reserves (below the red line and above the blue line).

Figure 4: Deposit Allocation and the Policy Rate



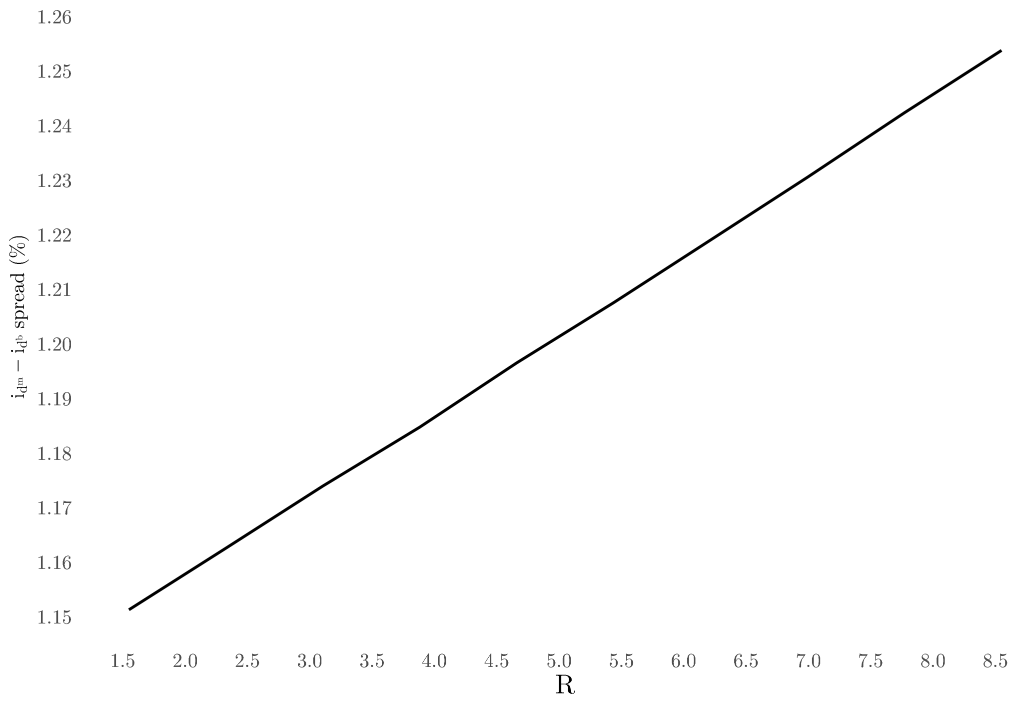
This figure shows how deposits at banks d^b (the orange line) and deposits at MMFs d^m (the blue line) change with the policy rate R. As the policy rate increases, households shift deposits from banks to MMFs.

Figure 5: Deposit Rates and the Policy Rate



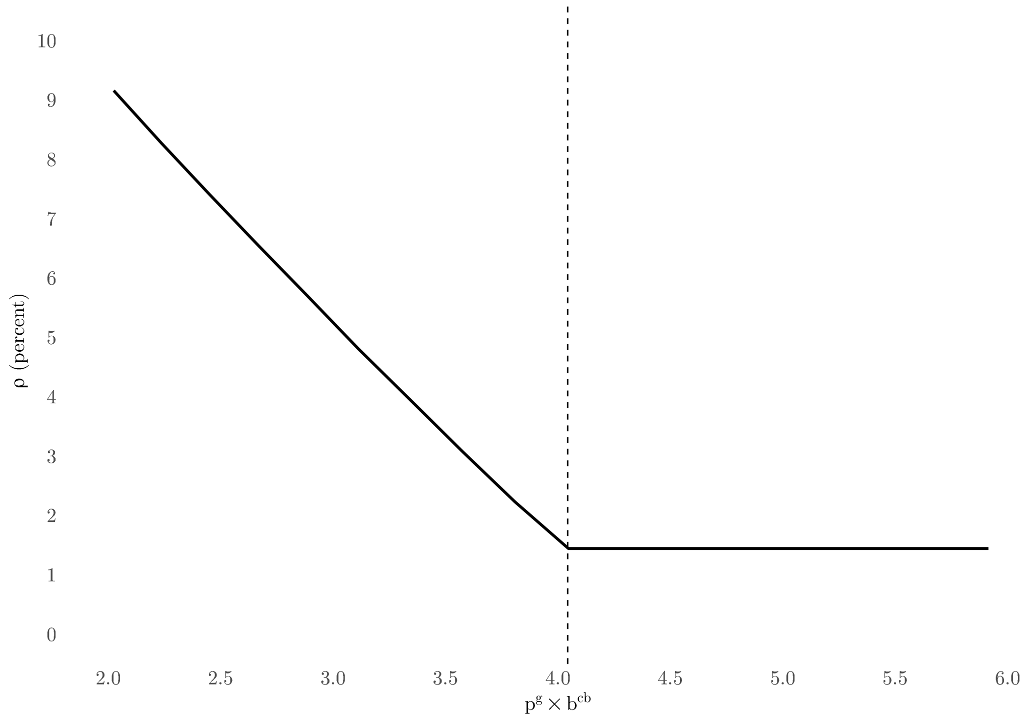
This figure shows how the deposit rate at banks i_{d^b} (the orange line) and the deposit rate at MMFs i_{d^m} (the blue line) change with the policy rate R. As the policy rate increases, both banks and MMFs increase their deposit rate, but MMFs do so at a somewhat faster pace.

Figure 6: The Spread between Deposit Rates



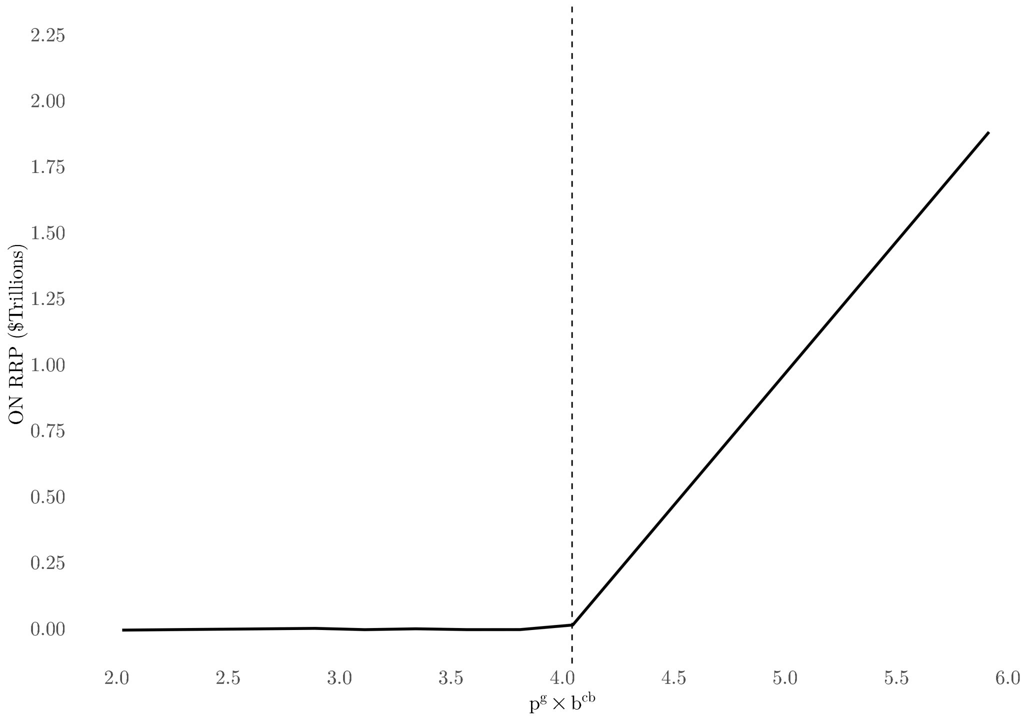
This figure shows the spread between the MMF deposit rate and the bank deposit rate. The positive slope indicates that the pass-through of an increase in the policy rate R is somewhat larger for MMF deposit rates than it is for bank deposit rates.

Figure 7: The Repo Rate and the Fed's Balance Sheet



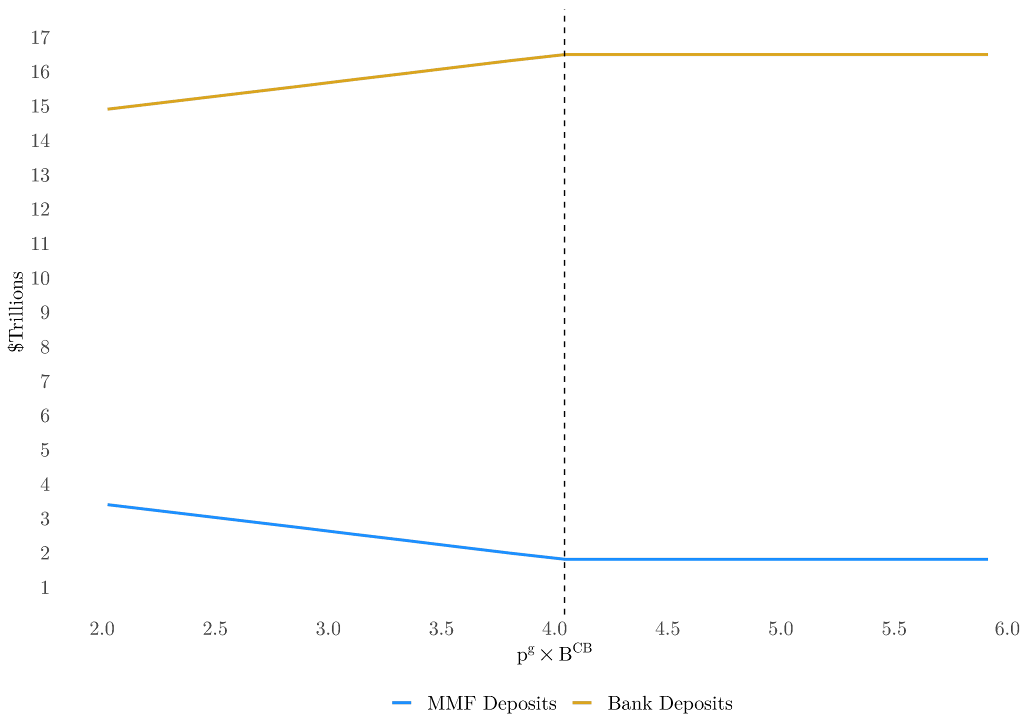
This figure shows how the repo rate ρ changes with the size of the Fed's balance sheet b^{cb}. The dashed line represents $\tilde{b}^{cb}$, the point at which there is a shift from ample liquidity in the repo market (right of the line) to scarce liquidity in the repo market (left of the line). In the ample regime, the repo rate is at the ON RRP rate, whereas in the scarce regime, the repo rate increases as the size of the Fed's balance sheet decreases.

Figure 8: ON RRP Take-up and the Fed's Balance Sheet



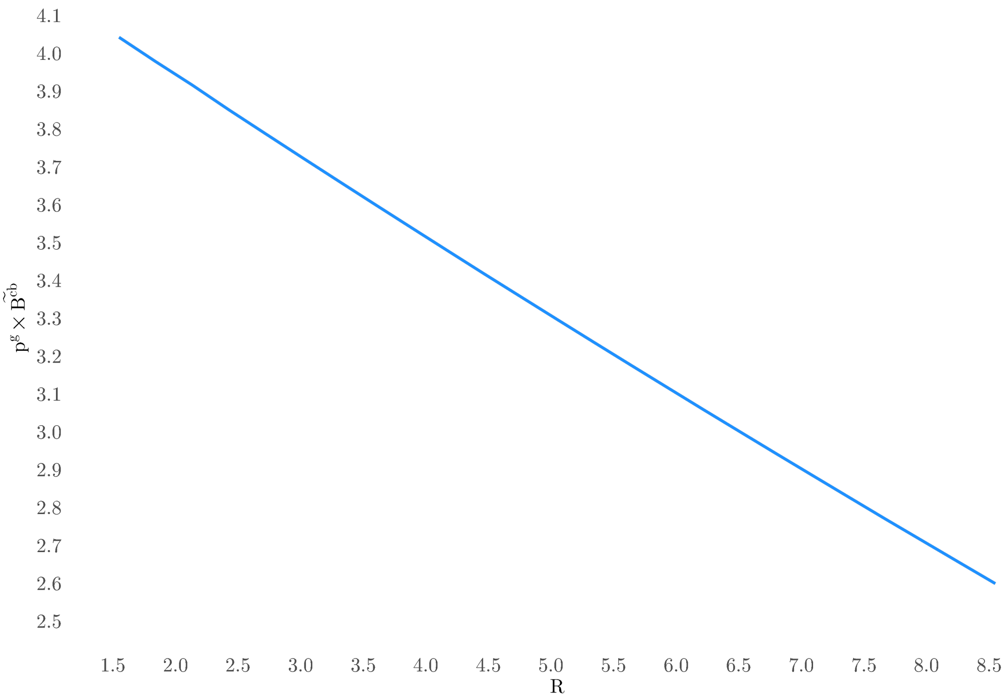
This figure shows how ON RRP take-up changes with the size of the Fed's balance sheet, b^{cb}. The dashed line represents $\tilde{b}^{cb}$, the point at which there is a shift from ample liquidity in the repo market (right of the line) to scarce liquidity in the repo market (left of the line). In the ample regime, ON RRP take-up falls with the size of the balance sheet until it reaches zero, whereas in the scarce regime, ON RRP take-up is constant at zero.

Figure 9: Deposits and the Fed's Balance Sheet



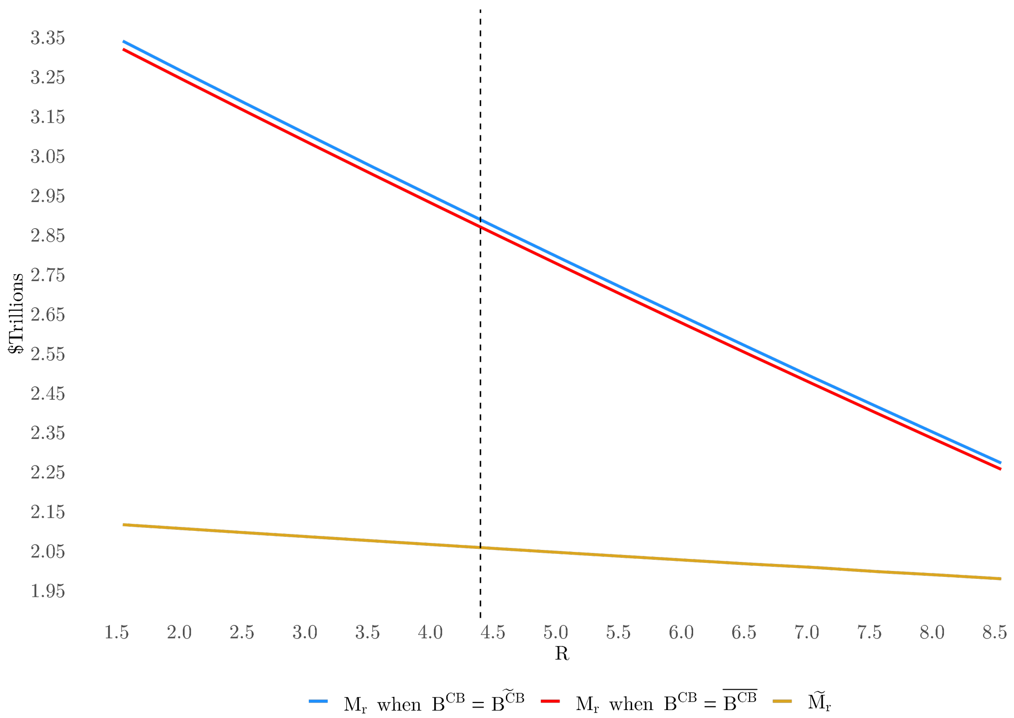
This figure shows how deposit allocation changes with the size of the Fed's balance sheet b^{cb}. The dashed line represents $\tilde{b}^{cb}$, the point at which there is a shift from ample liquidity in the repo market (right of the line) to scarce liquidity in the repo market (left of the line). In the ample regime, deposits are unchanged with the size of the Fed's balance sheet, whereas in the scarce regime, bank deposits decrease and MMF deposits increase as the Fed's balance sheet decreases given the higher rates that MMFs offer relative to bank deposit rates.

Figure 10: The Threshold on Repo Market Liquidity and the Policy Rate



This figure shows how the critical threshold of government bond holdings of the Fed $\tilde{b}^{cb}$ changes with the policy rate R. As the policy rate increases, the critical threshold of the Fed's government bond holdings decreases. If the Fed's government bond holdings are above the blue line, there is ample liquidity in the repo market and if the Fed's government bond holdings are below the line, there is scarce liquidity in the repo market.

Figure 11: Reserves



This figure shows the level of reserves when reserves are equal to the minimum level of reserve demand (orange line), reserves when the Fed's government bond holdings are equal to $\tilde{b}^{CB}$ (blue line), and reserves when the Fed's government bond holdings are equal to $\bar{b}^{CB}$ (red line). This figure shows that there exist four different regimes in equilibrium when the policy rate $R \in [1.5\%, 8.5\%]$. First, if the Fed's government bond holdings are larger than the critical threshold $\tilde{b}^{CB}$, there exists excess liquidity in the repo market and excess reserves (above the blue line). As the Fed reduces its balance sheet beyond $\tilde{b}^{CB}$, the repo rate starts to increase, but banks still hold excess reserves (between the blue line and the red line). If the Fed reduces its balance sheet beyond $\bar{b}^{CB}$, the repo rate increases beyond IORB, $\rho > R$, but banks continue to hold excess reserves (between the red line and the orange line). Lastly, if the Fed reduces its balance sheet further, such that the reserve constraint for banks becomes binding (below the orange line), there exists both scarce liquidity in the repo market and scarce reserves as banks only hold the minimum level of reserve demand.

Table 1: Quarterly Deposit Growth, 1992 to 2021

	(1) CB(YoY)	(2) MMF(YoY)
ΔEFFR	-1.559***	2.520***
	(0.400)	(0.400)
$\big[-1 \cdot \Delta\log(\text{SOMA})\big]$	-2.878*	8.848***
	(1.550)	(1.969)
ΔEFFR $\times \big[-1 \cdot \Delta\log(\text{SOMA})\big]$	-0.954	0.885
	(0.711)	(0.988)
GDP growth	-0.556	-1.400**
	(0.414)	(0.662)
CPI	-0.291	1.488
	(0.626)	(1.120)
TED Spread	-1.095	12.01***
	(2.640)	(4.272)
Personal Savings Rate	0.181	1.429***
	(0.351)	(0.392)
Observations	119	119
Adjusted R-squared	0.508	0.643
Linear Time Trend	Yes	Yes

This table shows the time series regressions of commercial bank and MMF deposit growth rates on conventional (EFFR) and unconventional (SOMA) monetary policy. Following Xiao (2020), changes in the federal funds rate and changes in SOMA are measured as three-year cumulative changes. The data frequency is quarterly from 1992 to 2021. Standard errors in parentheses are computed with Newey-West standard errors with 12 lags. Significance representations are $*p < 0.10$,$**p < 0.05$, $***p < 0.01$.

Table 2: Eligible MMF Complex Holdings, Jan. 2014 to Dec. 2019

	(1) Δ (Private Repo Share)	(2) Δ (ON RRP Share)
ΔEFFR_{t-1}	-0.0669	0.305***
	(0.0398)	(0.0559)
$\big[-1 \cdot \Delta\log(\text{SOMA})_{t-1}\big]$	1.866	-6.501***
	(1.349)	(1.308)
$\Delta\text{EFFR}_{t-1} \times \big[-1 \cdot \Delta\log(\text{SOMA})_{t-1}\big]$	15.55***	-22.56***
	(4.358)	(4.736)
log(AUM)	0.000376	-0.00205
	(0.000993)	(0.00148)
log(Bills Outstanding)	0.0103	-0.0833*
	(0.0296)	(0.0491)
Observations	2167	2167
Number of Clusters	33	33
Adjusted R-squared	0.0860	0.272
Adjusted Within R-Squared	0.0998	0.277
Year-Quarter Fixed Effects	Yes	Yes
MMF Complex Fixed Effects	Yes	Yes
Time Trend	Yes	Yes

This table shows the panel regressions of MMF portfolio allocation on conventional (EFFR) and unconventional (SOMA) monetary policy. The data frequency is monthly from 2014 to 2019. In particular, using the sub-sample of ON RRP eligible MMFs, we regress a MMF's investment share in either private repo or at the ON RRP facility on the one quarter lag in the change in EFFR, the one quarter lag in the change in SOMA, and their interaction. Standard errors in parentheses are clustered at the MMF Complex level. Significance representations are $*p < 0.10$,$**p < 0.05$, $***p < 0.01$.

Table 3: Independent Parameters

Parameter	Description	Value	Source
R	IORB	1.54%	Data
r	ON RRP offering rate	1.44%	Data
i_ℓ	Average interest rate on banks' outside investments	2.03%	Data
$p^g b^{CB}$	Nominal quantity of Treasury securities held by the Fed	\$5.73T	Data
$p^g B$	Nominal quantity bonds in the economy	\$5.81T	Data
δ	Minimal reserve-to-deposit ratio	0.128	Data

This table shows the independent parameters of the model that are used to calibrate the model.

Table 4: Jointly Calibrated Parameters

Parameter	Description	Data	Model
θ	Bargaining power of banks	-	0.006
k^b	Balance sheet costs of banks	-	0.000082%
k^m	Balance sheet costs of MMF	-	0.227%
α	Relative risk aversion	-	0.726
β	Loan cost function	-	1.56
ϕ	Price level in $t = 2$	-	248.97
M_e	Money endowment to households	-	\$12.45T

This table shows the calibrated parameters.

Table 5: Targeted Moments

Parameter	Description	Data	Model	Error
ρ	TGCR	1.41%	1.44%	$+(0.03)$
i_{d^m}	Interest rate on MMF deposits	1.21%	1.22%	$+(0.01)$
i_{d^b}	Interest rate on bank deposits	0.07%	0.06%	$-(0.01)$
D^b	Bank Deposits	\$16.55T	\$16.49T	$-(0.06)$
D^{ONRRP}	Aggregate ON RRP take-up	\$1.68T	\$1.70T	$+(0.02)$
M_r	Aggregate reserves	\$3.33T	\$3.33T	(0)

This table shows the moments that were targeted in the calibration for both the results from the model and the value in the data. The last column displays the difference between the moment from the model and the moment in the data.

Table 6: Parameters for November 2022

Parameter	Description	Value
R	IORB	3.9%
r	ON RRP offering rate	3.8%
$p^g b^{CB}$	Nominal quantity of Treasury securities held by the Fed	\$5.746T

This table shows the values of IORB, the ON RRP offering rate, and the quantity of Treasury securities held by the Fed in November 2022.

Table 7: Post November 2022 FOMC Test

Parameter	Description	Data	Model	Error
ρ	TGCR	3.76%	3.79%	+(0.03)
$D^b/(D^b + D^m)$	Bank Deposit share of Deposits	0.81	0.88	+(0.07)
i_{d^m}	Interest rate on MMF deposits	3.50%	3.57%	+(0.07)
i_{d^b}	Interest rate on bank deposits	0.12%	2.38%	+(2.26)
D^{ONRRP}	Aggregate ON RRP take-up	\$1.78T	\$1.88T	+(0.1)
M_r/M	Aggregate reserve share of M	0.156	0.162	+(0.006)

This table shows the results of running the calibrated model with values for IORB, the ON RRP offering rate, and the quantity of Treasury securities held by the Fed in November 2022.

Table 8: Parameters for September 2019

Parameter	Description	Value
R	IORB	2.1%
r	ON RRP offering rate	2.0%
$p^g b^{CB}$	Treasury securities holdings of the Fed	\$2.098T
$p^g B$	Aggregate Treasury securities holdings of the Fed and dealers	\$2.1886T

This table shows the values of IORB, the ON RRP offering rate, the quantity of Treasury securities held by the Fed, and the aggregate quantity of Treasury securities held by the Fed and dealers at the end of August 2019.

Table 9: September 2019 Test

Parameter	Description	Data	Model	Error
ρ	TGCR	5.25%	10.98%	+(5.73)
$D^b/(D^b + D^m)$	Bank Deposit share of Deposits	0.976	0.995	+(0.019)
i_{d^m}	Interest rate on MMF deposits	1.92%	10.87%	+(8.95)
i_{d^b}	Interest rate on bank deposits	0.14%	0.48%	+(0.34)
D^{ONRRP}	Aggregate ON RRP take-up	\$0.0023T	\$0T	−(0.0023)
M_r/M	Aggregate reserve share of M	0.1263	0.1276	+(0.0013)

This table shows the results of the calibrated model using values of IORB, the ON RRP offering rate, the quantity of Treasury securities held by the Fed, and the aggregate quantity of Treasury securities held by the Fed and dealers at the end of August 2019.

Appendix

A Robustness Checks

A.1 Longer Horizon Policy Changes on Deposit Growth

As a robustness check, we estimate specifications at a variety of horizons on the quarter-to-quarter deposit growth rates - in particular, the one, two, and three year changes in EFFR, SOMA, and their interaction. That is to say concretely, for the time horizons of one, two, and three years at quarterly observation intervals we estimate $\forall h \in \{4, 8, 12\}$

$$\begin{aligned} \textit{Growth Rate}_t = \alpha &+ \beta \cdot \Delta_{t-h}\text{EFFR}_{t-1} + \eta \cdot \big[-1 \cdot \Delta_{t-h}\log(\text{SOMA})_{t-1}\big] \\ &+ \theta \cdot \Big\{\Delta_{t-h}\text{EFFR}_{t-1} \times \big[-1 \cdot \Delta_{t-h}\log(\text{SOMA})_{t-1}\big]\Big\} \\ &+ \sum_{c=1}^{3} \gamma_c X_{c,t} + \lambda t + \varepsilon_t \end{aligned} \tag{A.1}$$

The results from estimating Equation (A.1) are presented in Table A.1, where panels one, two, and three, display the results from the one, two, and three year time horizons respectively. As can be well seen, the results in table A.1 accord well with our main results in table 1. In particular, at all longer time horizons, it is always the case that conventional monetary tightening achieved by increasing the federal funds rate is associated with a decline in commercial bank deposit growth and with an increase in MMF deposit growth. Moreover, at all longer time horizons, it is always the case that unconventional monetary tightening (i.e. reducing the balance sheet) induces MMF deposit inflows larger in magnitude than the deposit growth effect of conventional monetary tightening.

Table A.1: Quarterly Deposit Growth, Long Horizon Policy Changes, 1990-2021

	(1) CB(QtQ)	(2) MMF(QtQ)
Panel 1 - One Year Horizon		
$\Delta_{t-4}\text{EFFR}_{t-1}$	-0.552***	0.924***
	(0.165)	(0.226)
$\left[-1 \cdot \Delta_{t-4}\log(\text{SOMA})_{t-1}\right]$	-0.438	6.924***
	(0.492)	(1.411)
$\Delta_{t-4}\text{EFFR}_{t-1} \times \left[-1 \cdot \Delta_{t-4}\log(\text{SOMA})_{t-1}\right]$	0.0104	3.072***
	(0.546)	(0.868)
Observations	127	127
Adjusted R-squared	0.324	0.286
Linear Time Trend	Yes	Yes
Panel 2 - Two Year Horizon		
$\Delta_{t-8}\text{EFFR}_{t-1}$	-0.265***	0.956***
	(0.0898)	(0.122)
$\left[-1 \cdot \Delta_{t-8}\log(\text{SOMA})_{t-1}\right]$	-0.102	1.996
	(0.583)	(1.800)
$\Delta_{t-8}\text{EFFR}_{t-1} \times \left[-1 \cdot \Delta_{t-8}\log(\text{SOMA})_{t-1}\right]$	0.219	1.070
	(0.269)	(0.995)
Observations	123	123
Adjusted R-squared	0.307	0.318
Linear Time Trend	Yes	Yes
Panel 3 - Three Year Horizon		
$\Delta_{t-12}\text{EFFR}_{t-1}$	-0.211***	1.027***
	(0.0708)	(0.122)
$\left[-1 \cdot \Delta_{t-12}\log(\text{SOMA})_{t-1}\right]$	-0.149	2.912**
	(0.473)	(1.127)
$\Delta_{t-12}\text{EFFR}_{t-1} \times \left[-1 \cdot \Delta_{t-12}\log(\text{SOMA})_{t-1}\right]$	0.0186	1.108***
	(0.133)	(0.385)
Observations	119	119
Adjusted R-squared	0.267	0.372
Linear Time Trend	Yes	Yes

Standard errors in parentheses are Newey-West with 12 lags.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

A.2 Romer-Romer Policy Shocks

As a robustness check we re-estimate our quarterly deposit growth rate results using the exogenous monetary shock measure presented by Romer and Romer (2004), instead of using raw changes in the federal funds rate. Following Romer and Romer (2004) and the methodology outlined in Breitenlechner (2018), we extend the Romer-Romer exogenous monetary policy shock series through to Q4 2016 - that is, using the latest publicly available Tealbook (formerly Greenbook) data. Figure A.1 plots the original and extended Romer-Romer policy shock series.

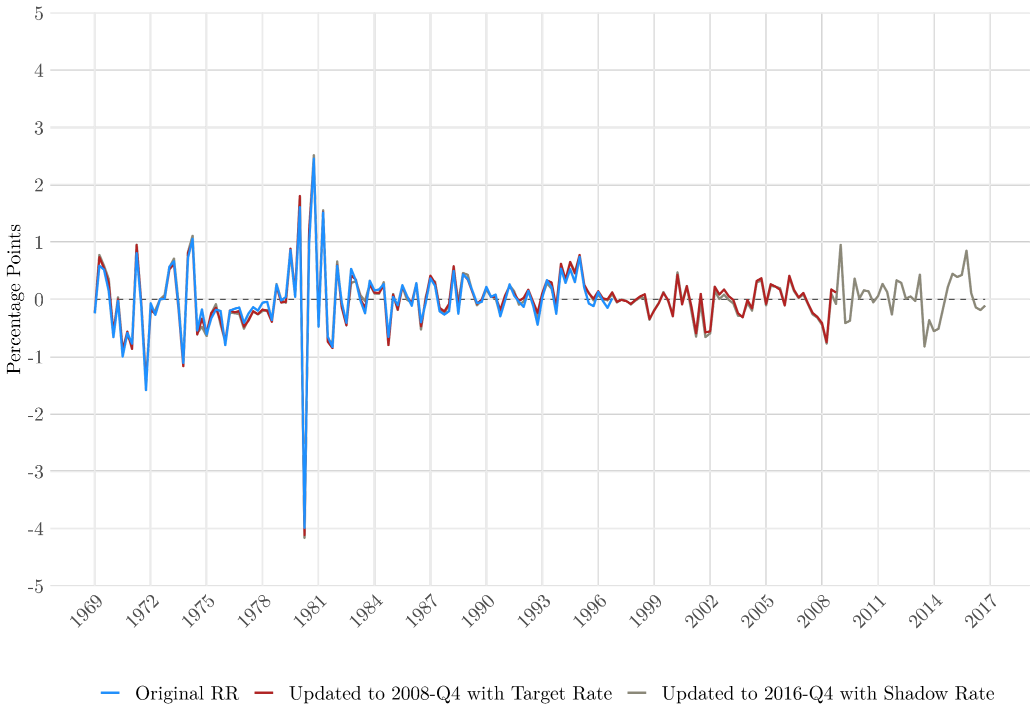


Figure A.1: Original and Extended Romer and Romer (2004) Monetary Policy Shock Series

Specifically, we regress the quarter-to-quarter growth rate of deposits on the one period lag of the monetary policy shock, the one period lag of the change in (log) SOMA, and their interaction, as well as our usual control variables:

$$\begin{aligned} \textit{Growth Rate}_t = \alpha &+ \beta \cdot (\text{RR MP Shock})_{t-1} + \eta \cdot \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big] \\ &+ \theta \cdot \Big\{(\text{RR MP Shock})_{t-1} \times \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big]\Big\} \\ &+ \sum_{c=1}^{3} \gamma_c X_{c,t} + \lambda t + \varepsilon_t \end{aligned} \tag{A.2}$$

The results from Equation (A.2) are displayed in table A.2.

Table A.2: Quarterly Deposit Growth, Romer-Romer Shocks, 1990Q1-2016Q4

	(1) CB(QtQ)	(2) MMF(QtQ)
RR MP Shock$_{t-1}$	-1.678*	2.067*
	(0.901)	(1.061)
$\left[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\right]$	1.905	20.94***
	(2.536)	(6.995)
RR MP Shock$_{t-1}$ $\times \left[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\right]$	2.693	39.34***
	(5.743)	(8.715)
GDP growth	-0.372***	0.127
	(0.0871)	(0.258)
CPI	-0.319*	0.0766
	(0.189)	(0.209)
TED Spread	-0.660	5.713***
	(0.527)	(1.052)
Observations	108	108
Adjusted R-squared	0.223	0.332
Linear Time Trend	Yes	Yes

Standard errors in parentheses are Newey-West with 12 lags.
$^{*}p < 0.10$,$^{**}p < 0.05$, $^{***}p < 0.01$

A.3 EFFR spread to IOR

Here, we consider the case when using the EFFR spread to IOR (the interest on reserve balances) as our right-hand-side variable. In particular, we use the following specification regressing the quarter-to-quarter growth rate of deposits on the one period lag of the EFFR to IOR spread, the one period lag in the (log) SOMA, and their interaction, as well as our usual control variables:

$$\begin{aligned} \textit{Growth Rate}_t = \alpha &+ \beta \cdot (\text{EFFR} - \text{IOR})_{t-1} + \eta \cdot \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big] \\ &+ \theta \cdot \Big\{ (\text{EFFR} - \text{IOR})_{t-1} \times \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big] \Big\} \\ &+ \sum_{c=1}^{3} \gamma_c X_{c,t} + \lambda t + \varepsilon_t \end{aligned} \tag{A.3}$$

The results from Equation (A.3) are displayed in table A.3.

Table A.3: Quarterly Deposit Growth with EFFR-IOR Spread, 2008 Q4 - 2021 Q4

	(1) CB(QtQ)	(2) MMF(QtQ)
$(\text{EFFR} - \text{IOR})_{t-1}$	-0.430	27.88***
	(2.182)	(8.917)
$\big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big]$	0.719	29.22***
	(4.003)	(10.03)
$(\text{EFFR} - \text{IOR})_{t-1} \times \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big]$	15.14	137.8
	(32.44)	(154.8)
GDP growth	-0.597***	-0.290**
	(0.204)	(0.111)
CPI	0.586***	-0.0368
	(0.189)	(0.231)
TED Spread	0.0681	0.822
	(2.103)	(4.580)
Observations	52	52
Adjusted R-squared	0.336	0.360
Linear Time Trend	Yes	Yes

Standard errors in parentheses are Newey-West with 12 lags.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

A.4 Pre-2008

Here, we consider only pre-2008 data, and regress the quarter-to-quarter growth rate of deposits on the one period lag in the change in EFFR, the one period lag in the change in (log) SOMA, and their interaction, as well as our usual control variables:

$$\begin{aligned} \textit{Growth Rate}_t = \alpha &+ \beta \cdot (\Delta \text{EFFR})_{t-1} + \eta \cdot \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big] \\ &+ \theta \cdot \Big\{ (\Delta \text{EFFR})_{t-1} \times \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big] \Big\} \\ &+ \sum_{c=1}^{3} \gamma_c X_{c,t} + \lambda t + \varepsilon_t \end{aligned} \tag{A.4}$$

The results from Equation (A.4) are displayed in table A.4.

Table A.4: Pre-2008 Quarterly Deposit Growth, 1990-2007

	(1) CB(QtQ)	(2) MMF(QtQ)
ΔEFFR_{t-1}	-2.126**	6.014***
	(0.843)	(1.160)
$\big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big]$	-12.63	87.59**
	(22.55)	(34.25)
$\Delta \text{EFFR}_{t-1} \times \big[-1 \cdot \Delta \log(\text{SOMA})_{t-1}\big]$	48.03*	220.9***
	(27.57)	(46.93)
GDP growth	0.0985	-0.974***
	(0.160)	(0.359)
CPI	0.0190	-0.971***
	(0.281)	(0.306)
TED Spread	-0.526	8.735***
	(0.837)	(1.214)
Observations	72	72
Adjusted R-squared	0.486	0.338
Linear Time Trend	Yes	Yes

Standard errors in parentheses are Newey-West with 12 lags.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

B FRED Data Dictionary

Variable	Frequency	Series Name	Series Description
MMF AUM	Quarterly	MMMFFAQ027S	Total Financial Assets Under management for Money Market Mutual Funds in millions, not seasonally adjusted
EFFR	Monthly & Quarterly	EFFR	Effective federal funds rate in percent, not seasonally adjusted
SOMA	Monthly & Quarterly	WALCL	Total assets of the Federal Reserve System Open Market Account, Wednesday level.
GDP Growth	Quarterly	GDPC1_PC1	Real Gross Domestic Product, percent change from one year ago
TED Spread	Quarterly	TEDRATE	Spread between 3-month LIBOR and 3-month Treasury Bill, percent
Personal Savings Rate	Quarterly	PSAVERT	The ratio of personal savings to disposable personal income.

C Proofs

Proof of Proposition 1. Suppose there exists an equilibrium where $\rho > r$ and $d^{ONRRP} > 0$. In that case, MMF have an incentive to move funds out of the ON RRP and lend them in the repo market. The inflow of funds will lead to a decrease in the repo rate ρ such that either $\rho = r$ and MMF are indifferent between the ON RRP facility and the repo market and in that case $d^{ONRRP} \geq 0$ or the repo rate remains above the ON RRP rate $\rho > r$ and MMF move all available funds out of the ON RRP facility to the repo market, such that $d^{ONRRP} = 0$. ■

Proof of Proposition 2. Equations (18), (24), (17), (19), (21), (25), (8) as well as Equations (22) and (23) follow from the derivations given in the main text. It remains to derive Equation (26). First, if households are constrained, they spend all their bank deposit holdings in the goods market. Thus, the money holdings in the beginning of period $t = 2$ satisfy $d^m(1 + i_{d^m})$. Conversely, the money holdings of firms satisfy $d^b(1 + i_{d^b})$, since in equilibrium $pq_s = d^b$.

In period $t = 2$, MMF hold

$$\Pi^{MMF} = z^m(1 + \rho) + (d^m - z^m)(1 + r) - d^m(1 + i_{d^m} - k^m).$$

The liquidity holdings of dealers in period $t = 2$ satisfy

$$\Pi^D = b^d - z^d(1 + \rho)$$

Dealers borrow z^d and use it to purchase bonds. In period $t = 2$, dealers repay their loans and receive the return on government bonds.

Lastly, banks earn a return $(1 + i_\ell)$ on their loans and a return $(1 + R)$ on their reserve holdings. They pay the interest rate i_{d^b} on bank deposits. Their profits therefore satisfy

$$\Pi^B = \ell(1 + i_\ell) + m_r(1 + R) - d^b(R - i_{d^b} - k^b).$$

Adding the money holdings of all agents up and rearranging yields the market clearing condition in $t = 2$,

$$\frac{x}{\phi} = \ell(1 + i_\ell) + m_r(1 + R) + (d^m - z^m)(1 + r) - m(k^b + k^m) + d^b k^m + d^m k^b, \quad \text{(A.5)}$$

which is Equation (26). ■

Proof of Proposition 3. Recall, $\tilde{b}$ is defined as

$$p^g \tilde{b} = m - d^b.$$

Totally differentiating this equation and rearranging yields

$$\mathrm{d}\tilde{b} = \frac{1}{p^g}\left[\mathrm{d}m - \mathrm{d}d^b - \mathrm{d}p^g \tilde{b}\right] \tag{A.6}$$

Recall that $1/p^g = 1 + \rho$. Thus, at $b^d = \tilde{b}$, $1/p^g = 1 + r$. Using this and rearranging, we obtain

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}r} = (1+r)\left[\frac{\mathrm{d}m}{\mathrm{d}r} - \frac{\mathrm{d}d^b}{\mathrm{d}r} + \frac{1}{(1+r)^2}\tilde{b}\right]. \tag{A.7}$$

If the central bank only changes its policy rates, then $\mathrm{d}m/\mathrm{d}r = 0$. Thus, to show that $\tilde{b}$ is increasing in the policy rates, it suffices to show that $\mathrm{d}d^b/\mathrm{d}r$ is decreasing in the policy rates.

Totally differentiating $d^b = pq$ yields

$$\mathrm{d}d^b = \mathrm{d}pq + p\mathrm{d}q. \tag{A.8}$$

Totally differentiating Equation (4) and rearranging yields

$$\mathrm{d}p = -\mathrm{d}i_{d^b}\frac{p}{1+i_{d^b}}. \tag{A.9}$$

Next, totally differentiating Equation (17) and rearranging yields

$$\mathrm{d}q = \frac{1}{u''(q)}\left[\frac{1}{1+i_{d^b}}\mathrm{d}r - \frac{1+r}{(1+i_{d^b})^2}\mathrm{d}\mathrm{d}i_{d^b}\right]. \tag{A.10}$$

Plugging Equations (A.9) and (A.10) into Equation (A.8) and rearranging yields

$$\frac{\mathrm{d}d^b}{\mathrm{d}r} = \frac{1}{u''(q)}\frac{p}{(1+i_{d^b})} - \frac{\mathrm{d}i_{d^b}}{\mathrm{d}r}\left[\frac{pq}{1+i_{d^b}} + \frac{p}{u''(q)}\frac{1+r}{(1+i_{d^b})^2}\right]. \tag{A.11}$$

Totally differentiating Equation (18) and rearranging using Equation 4 and $d^b = pq$ yields

$$\frac{\mathrm{d}i_{d^b}}{\mathrm{d}r} = \frac{\frac{1}{1-\theta} - (\omega_1 - \omega_2 + \omega_3 - \omega_4)}{1 - \left(\frac{1+i_{d^m}}{1+i_{d^b}}\right)(\omega_1 - \omega_2 + \omega_3 - \omega_4) + \omega_5 - \omega_6 + \omega_7}, \tag{A.12}$$

where $\omega_1 = \frac{\theta}{1-\theta}\frac{u'(q)}{u''(q)}\frac{1}{q}$, $\omega_2 = \frac{\theta}{1-\theta}\frac{u(q)}{u''(q)}\frac{1}{q^2}$ and $\omega_3 = \frac{\phi(\ell i_{\ell-R})}{u''(q)q^2}$, $\omega_4 = \frac{\phi\chi(\ell)}{u''(q)q^2}$, $\omega_5 = \frac{\theta}{1-\theta}\frac{u(q)}{q}$ $\omega_6 = \frac{\phi(\ell i_{\ell-R})}{q}$ and $\omega_7 = \frac{\phi\chi(\ell)}{q}$. Combining Equations (A.12) and (A.11) and rearranging yields yields

$$\frac{\mathrm{d}d^b}{\mathrm{d}r} = \frac{1}{u''(q)}\frac{p}{1+i_{d^b}}\left[1 - \frac{\frac{qu''(q)}{1-\theta} - qu''(q)(\omega_1-\omega_2+\omega_3-\omega_4)}{1-\frac{1+i_{d^m}}{1+i_{d^b}}(\omega_1-\omega_2+\omega_3-\omega_4)+\omega_5-\omega_6+\omega_7}\right]$$
$$-\frac{1}{u''(q)}\frac{p}{1+i_{d^b}}\left[\frac{\frac{1}{1-\theta}\frac{1+i_{d^m}}{1+i_{d^b}} - \frac{1+r}{1+i_{d^b}}(\omega_1-\omega_2+\omega_3-\omega_4)}{1-\frac{1+i_{d^m}}{1+i_{d^b}}(\omega_1-\omega_2+\omega_3-\omega_4)+\omega_5-\omega_6+\omega_7}\right] \quad \text{(A.13)}$$

Note, the term $(1/u''(q))(p/(1+i_{d^b}))$ is negative, since $u''(q)<0$. Thus, $\mathrm{d}d^b/\mathrm{d}r<0$, if

$$1-\frac{\frac{qu''(q)}{1-\theta} - qu''(q)(\omega_1-\omega_2+\omega_3-\omega_4)}{1-\frac{1+i_{d^m}}{1+i_{d^b}}(\omega_1-\omega_2+\omega_3-\omega_4)+\omega_5-\omega_6+\omega_7} - \frac{\frac{1}{1-\theta}\frac{1+i_{d^m}}{1+i_{d^b}} - \frac{1+r}{1+i_{d^b}}(\omega_1-\omega_2+\omega_3-\omega_4)}{1-\frac{1+i_{d^m}}{1+i_{d^b}}(\omega_1-\omega_2+\omega_3-\omega_4)+\omega_5-\omega_6+\omega_7} > 0. \quad \text{(A.14)}$$

Using the expressions for ω_j for $j = 1,2,3,4,5,6,7$ in Equation (A.14), rearranging and solving for θ yields

$$\frac{1-u''(q)q-\frac{1+i_{d^m}}{1+i_{d^b}}}{1-u'(q)} > \theta, \quad \text{(A.15)}$$

Note, from Equation (17), $u'(q) = (1+i_{d^m})/(1+i_{d^b})$, thus Equation (A.15) can be simplified to

$$u''(q) - q < 0, \quad \text{(A.16)}$$

which is always true since $u''(q)<0$. Thus, $\tilde{b}^d$ is increasing in r. ■